Gravity in FX R-Squared: Understanding the Factor Structure in Exchange Rates

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Abstract

We relate the risk characteristics of currencies to measures of physical, cultural, and institutional distance. Currencies of countries which are more distant from other countries are more exposed to systematic currency risk. This is due to a gravity effect in the factor structure of exchange rates: When a currency appreciates against a basket of other currencies, its bilateral exchange rate appreciates more against currencies of distant countries. As a result, currencies of peripheral countries are more exposed to systematic variation than currencies of central countries. Trade network centrality is the best predictor of a currency's average exposure to systematic risk.

Keywords: Exchange Rates, Factor Models, Gravity Equation, Home Bias, Trade Network, Centrality

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Exchange rates appear to be disconnected from macroeconomic quantities: macro variables cannot reliably forecast changes in spot exchange rates (Meese and Rogoff, 1983; Engel and West, 2005) and exchange rates are only weakly correlated with macro variables (Backus and Smith, 1993; Kollmann, 1995).[1] However, exchange rates do strongly co-vary: common factors explain a large share of the variation in bilateral exchange rates (Verdelhan, 2015). We show that exchange rate co-variation follows very regular patterns which are determined by measures of physical, cultural, and institutional distance between countries. Specifically, when a currency appreciates against a basket of other currencies, it tends to appreciate more against currencies of distant countries than currencies of close countries. Our findings show that a large part of systematic exchange rate movements are explained by fundamental differences between countries, despite being disconnected from macroeconomic quantities.

We measure currency co-variation in a way that is analogous to measuring market betas of stocks. For example, starting from US dollar exchange rates, we construct the US dollar base factor as the average appreciation of the dollar versus a basket of foreign currencies. The US dollar base factor measures systematic variation in the US dollar — similar to the return on the equity market. To measure individual currencies' exposure to this systematic US dollar variation, we regress changes in their US dollar exchanges rates on the US dollar base factor. We refer to these regression coefficients as base factor loadings. For example, the Canadian dollar's loading on the US dollar base factor is 0.5 while the New Zealand dollar's loading is 1.3. This tells us that when the US dollar systematically appreciates by 1% versus a basket of currencies, on average, it appreciates by 0.5% versus the Canadian dollar and 1.3% versus the New Zealand dollar. We show that these base factor loadings are increasing in measures of distance between the base country and the foreign country.

The measures of distance which explain the factor structure in exchange rates include not only physical distance, but also shared language, legal origin, shared border and colonial linkages. By construction, the average loading for a given base currency is one. Doubling the distance between a country and the base country increases the loading by 15% for an average country. A shared language lowers the loading between 11 and 15%. In the case of U.S. based exchange rates, the loading on the dollar factor decreases by 50% when the other country uses English as one of its main languages. Shared border lowers the loading by another 8 to 14%, while colonial linkages lower the loadings by up to 32%. The explanatory power of these gravity variables remains fairly constant over time, except during the global

[1] Froot and Rogoff (1995); Frankel and Rose (1995) survey the empirical literature that tests the empirical predictions of standard theories of exchange rates.

financial crisis.

By understanding differences in base factor loadings, we learn about differences in exposure to common factors which drive exchange rate covariation. Other moments of exchange rates, such as bilateral exchange rate volatility, depend on foreign country idiosyncratic shocks. Base factors diversify away the idiosyncratic components of exchange rates and isolate common variation — simply because they are averages across all exchange rates with respect to a base currency. As a result, the base factor loading of an exchange rate measures exposure to common factors.

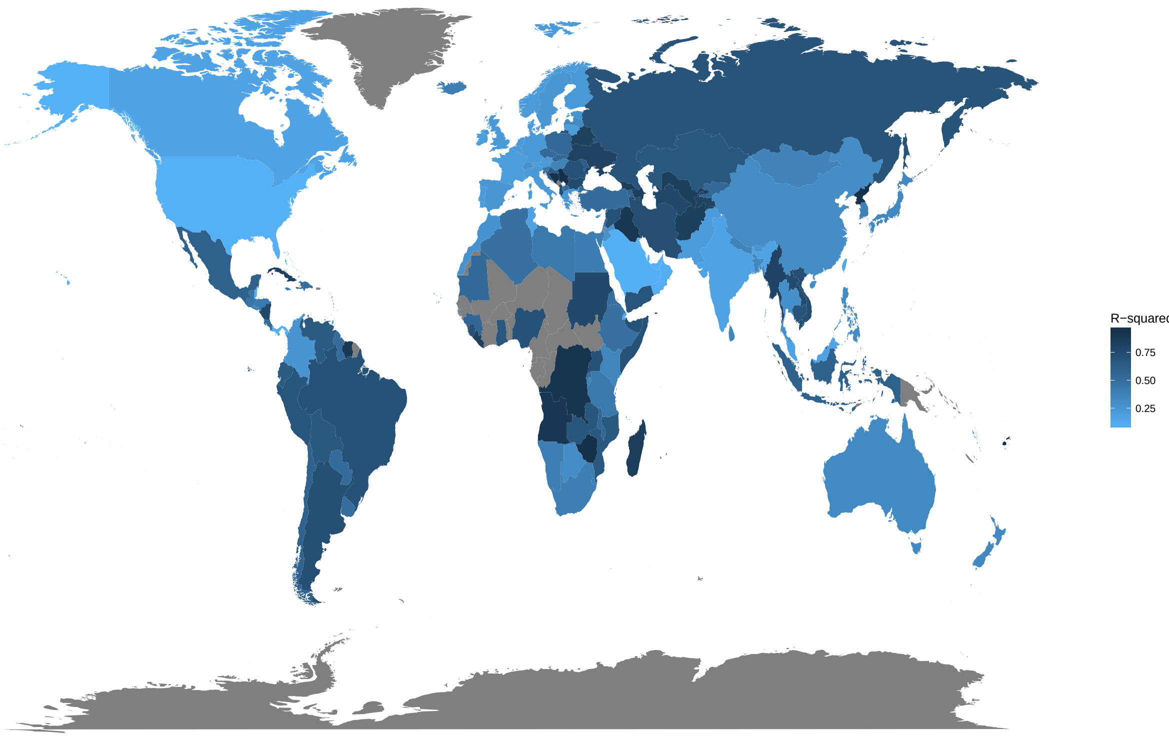
Exposure to the base factors explains a substantial amount of the time series variation in bilateral exchange rates. For the average exchange rate in our sample, the R^2 in the regression of exchange rate changes on the base factor is 47%. From the perspective of a base country investor, the average of this R^2 across foreign currencies represents the average amount of systematic risk which they face in FX markets. Figure (1) plots the average R^2 on a map. Peripheral countries, which are distant from most other countries, have high average R^2. Conversely, central countries have low average R^2 due to being close to most other countries. This is due to the gravity effect in the factor structure of exchange rates: bilateral exchange rates exposure to the common base factor is increasing in distance.

While we largely understand the determinants of stock return loadings (e.g. financial leverage or growth options), much less is known about the determinants of currency loadings with respect to risk factors. A currency's loadings on global risk factors determine its risk characteristics and returns.[2] Our paper identifies the fundamental determinants of an exchange rate's exposure to these risk factors because our base factors are different linear combinations of the underlying global FX factors (e.g., the carry trade factor in Lustig et al. (2011), the USD factor in Lustig et al. (2014); Verdelhan (2015), etc.). Our approach does not require us to commit ex-ante to a specific set of global FX risk factors.

Base factor loadings also teach us about exposures of stochastic discount factors to common global shocks. In models with complete spanning in international financial markets, the change in the spot exchange rate measures changes in the difference between foreign and domestic state prices. Spot exchange rates only need to adjust if foreign and domestic state prices diverge. Therefore, differences in base factor loadings map to differences in exposures of stochastic discount factors to common global factors. To illustrate this, we derive expressions for base factor loadings in a multi-country affine term structure model, based on Lustig

[2]See Lustig and Verdelhan (2007); Lustig et al. (2011); Menkhoff et al. (2012); Lustig et al. (2014); Lettau et al. (2014); David et al. (2014).

Figure 1: Average R^2 by Base Factor



Map of cross-sectional average R-squared from the regression $\Delta s_t = \alpha + \varphi^* \Delta base_t + e_t$ for each possible base currency. $\Delta base_t$ is the average appreciation of the US dollar at time t relative to all available currencies, excluding the bilateral exchange rate on the right hand side from the basket. Spot rates are monthly from January 1973 until December 2014 for 162 countries from Global Financial Data.

et al. (2011), and a multi-country long run risks model with global shocks, based on Colacito and Croce (2011); Bansal and Shaliastovich (2013); Lewis and Liu (2015); Colacito et al. (2015). We show how the base factor loadings of exchange rates can be used to measure the underlying exposure to common sources of risk in these models. The similarity of the loadings on global factors of countries' stochastic discount factors is decreasing in distance.

Measures of distance also explain the intensity of trade and financial holdings between countries. One of the most robust empirical findings in international trade is the gravity equation's success in accounting for trade flows: the size of trade flows between two countries is inversely proportional to the distance between two countries (Tinbergen, 1962). Interestingly, we find that the gravity effect in the factor structure in exchange rates is robust to controlling for trade intensity and financial holdings.[3] Cross-border trade and financial holdings alone explain only 1 to 3% of the variation in base factor loadings relative to up to 33% for the gravity variables. Due to the low explanatory power of trade and financial holdings, our findings impute a quantitatively important role to correlated shocks — in addition to risk sharing — in accounting for the stylized fact that closer countries have more correlated business cycles (see Frankel and Rose, 1998). In support of the correlated shocks hypothesis, we find that resource similarity between two countries significantly lowers the factor loadings[4].

Although bilateral trade intensity does not explain the variation in base factor loadings, trade network centrality (Richmond, 2015) is the best predictor of a currency's exposure to systematic risk, as measured by a currency's average R^2. There are two reasons why trade network centrality explains average R^2, but bilateral trade intensity fails to explain the variation in base factor loadings. First, base factor loadings measure exposure to common global shocks, while bilateral trade may be subject to idiosyncratic shocks. The existence of idiosyncratic shocks to bilateral trade may significantly decrease trade's ability to explain the differential exposure to the common shocks measured by the base factor loadings. In contrast, trade network centrality is specifically constructed to measure a countries overall position in the global trade network, which in turn determines its exposure to global shocks. Similarly, because average R^2 averages over all values for a particular base currency, the idiosyncratic

[3]The same gravity effects that drive variation in base factor loadings may lead countries to peg their nominal exchange rate. In particular, countries may be more likely to peg their exchange rate to countries which are closer (Tenreyro, 2007), which would lower the base factor loading. We confirm this finding and show that the gravity effect persists even after controlling for pegs or removing pegs altogether.

[4]Chen and Rogoff (2003) report evidence that the exchange rates of commodity exporters respond similarly to the dollar price of commodities, while Chen et al. (2010) find that commodity exporters' exchange rates forecast commodity prices.

effects of a given foreign currency are mitigated. This results in a measure that captures exposure to global shocks for the currency, similar to trade network centrality. A second reason that bilateral trade intensity may fail to explain the variation in base factor loadings is that an increase in bilateral trade could be associated with more specialization. This specialization would imply a lower correlation of shocks between the countries, increasing the base factor loadings, and mitigating any negative effect trade has on base factor loadings.

In a seminal paper, Engel and Rogers (1996) find that the distance between cities in the U.S. and Canada is the main determinant of relative price variability across cities, but they document a large U.S.-Canada border effect (see also Parsley and Wei, 2001; Hau, 2002, for more recent evidence of openness and distance on exchange rate volatility). Our findings imply that distance between countries is a significant determinant of differences in exposures to common factors which drive covariation in relative prices. In the context of the border effect documented by Engel and Rogers (1996), this suggests that a subtantial amount of relative price variation is drive by country specific shocks.

Gravity models have a long history in international trade (see Anderson and van Wincoop, 2004; Costinot, 2014; Head and Mayer, 2014, for recent surveys). The elasticity of trade flows with respect to distance is large and remarkably stable over time (Leamer and Levinsohn, 1995). Economists have long understood proximity to be a source of comparative advantage in international trade, even though standard theories of international trade do not create a direct role for distance (see Chaney, 2013, for a recent survey of the limited role of distance in modern trade theory). Obstfeld and Rogoff (2000) argue that costs of international trade can account for most of the outstanding puzzles in international trade. Importantly, distance can also proxy for the costs of acquiring information (see Coval and Moskowitz, 1999, for evidence from mutual fund industry).[5] Portes et al. (2001); Portes and Rey (2005) find that gravity models can also explain cross-border financial flows, while Flavin et al. (2002); Lucey and Zhang (2010) show that asset return correlations depend on distance.

There is a large literature in international finance on common or global risk factors, mostly focused on equities. This literature includes world arbitrage pricing theory, developed by Adler and Dumas (1983); Solnik (1983); a world consumption-capital asset pricing model (CAPM), Wheatley (1988); a world CAPM, Harvey (1991); world latent factor models, Campbell and Hamao (1992); Bekaert and Hodrick (1992); Harvey et al. (2002); world multi-

[5]Recently, Eaton et al. (2016) conduct the following experiment: they remove trade frictions in a calibrated version of the Eaton et al. (2016) model of international trade. Interestingly, this experiment eliminates many of the standard international finance puzzles, including the Backus and Smith (1993) exchange rate disconnect.

loading models, Ferson and Harvey (1993); and more recently work on time-varying capital market integration by Bekaert and Harvey (1995); Bekaert et al. (2009). We contribute by identifying distance as the key determinant of a bilateral exchange rate's loadings on these global risk factors. Recently, Hassan (2013), Ready et al. (2013) and Richmond (2015) develop theories that shed light on the origins of currency loadings. Hassan (2013) points out that larger countries' currencies will tend to appreciate in response to adverse global shocks and hence offer a hedge. In an equilibrium model of international trade, Ready et al. (2013) distinguish between commodity exporters and final goods producers. In their model, the real exchange rate of commodity exporters depreciates in response to an adverse global shock. Richmond (2015) shows how the global trade network generates common global risk, which central countries are more exposed to. This causes central countries' currencies to appreciate in bad global states, which drives down their interest rates and currency risk premia.

In a large class of complete market models, stochastic discount factors have to be highly correlated across countries in order to confront the much lower volatility of exchange rates, as pointed out by Brandt et al. (2006). In a long-run risks model, Colacito and Croce (2011); Lewis and Liu (2015) argue that the persistent component of consumption growth is highly correlated across countries. Using a preference-free approach, Lustig et al. (2016) show that the permanent component of the pricing kernel needs to be highly correlated. Our work on base factor loadings implies that the correlation of stochastic discount factors across countries is subject to gravity effects, consistent with the notion that there is more risk sharing between countries that are closer. In the context of a long run risks model, we show how the base factor loadings can be used to back out the exposures of each country's long-run consumption growth to global factors. More work is needed to infer the size of implied trading costs needed to rationalize these exposures in models with endogenous risk sharing.

Our findings also have important portfolio implications. Equities and other financial securities of distant countries that are most appealing to, say, a U.S. investor from a diversification perspective will tend to impute more non-diversifiable currency risk to her portfolio. Provided that hedging currency risk is costly, our findings may shed additional light on the home bias puzzle in equities (French and Poterba, 1991; Cooper and Kaplanis, 1994).[6]

The rest of this paper is organized as follows. Section (1) describes a complete markets model of exchange rate covariation and motivates the study of base factor loadings. Section (2) documents the factor structure in bilateral exchange rates and its relation to measures

[6] Lewis (1999) surveys the evidence on the home bias in equities.

of distance. Section (3) tests the gravity model of exchange rate co-variation. Section (4) studies the relation between trade and the factor structure in bilateral exchange rates. Section (5) presents a calibrated long-run risks model using our gravity results. Section (6) checks the robustness of our findings. Section (7) concludes.

1 A Simple Theory of Exchange Rate Covariation

We begin by presenting a simple theory of exchange rate covariation. This theory motivates our empirical approach of studying base factor loadings. By studying base factor loadings, we learn about the determinants of exposure of the stochastic discount factor to global shocks. To illustrate this, we derive expressions for these loadings in a multi-country affine term structure model, based on Lustig et al. (2011) and a multi-country long run risks model with global shocks, based on Colacito and Croce (2011); Bansal and Shaliastovich (2013); Lewis and Liu (2015); Colacito et al. (2015). The same arguments apply in the context of multi-country versions of other leading complete market models of exchange rate determination (see, e.g., Verdelhan, 2010; Farhi and Gabaix, 2016, for leading examples).

1.1 Complete Market Models of Exchange Rates

The starting point for our analysis is a class of flexible, affine models of interest rates and exchange rates. This extends earlier work by Backus et al. (2001), Hodrick and Vassalou (2002), Brennan and Xia (2006), Leippold and Wu (2007), Lustig et al. (2011) and Sarno et al. (2012). Specifically, we adopt a version of the model developed by Lustig et al. (2011); Verdelhan (2015). We provide a consumption-based example in the following section.

Single-Factor SDF Model

There are $N+1$ countries, one of which we classify as the home country without loss of generality. All foreign country values are denoted with $*$. There is no time variation in factor loadings in the model. The real log SDF m^*_{t+1} in the foreign countries is given by:

$$-m^*_{t+1} = \alpha + \chi \sigma^{*,2} + \xi^* (\sigma^g)^2 + \tau^* \sigma^* u^*_{t+1} + \kappa^* \sigma^g u^g_{t+1},$$

where u^*_{t+1} are local shocks and u^g_{t+1} is a common shock that originates in the home country, all of which are zero mean and variance 1. To give content to the notion that u^g originates in the home country, we impose that $0 \leq \kappa^* \leq 1$. To keep the analysis simple, we have also abstract from time-variation in the σ's. The assumption that the common shock originates in the base country is only to simplify exposition. This single common shock model is a simplified version of a richer model with K common shocks, which we present in Section (1.1). In that model, we do not constrain where the shock originates and all results carry through.

By no arbitrage, when markets are complete, the change in the log exchange rate in foreign currency per unit of home currency is given by $\Delta s_{t+1} = m_{t+1} - m^*_{t+1}$. This yields the following expression for changes in exchange rates:

$$\Delta s_{t+1} = (\alpha^* - \alpha) + (\xi^* - \xi)(\sigma^g)^2 + (\chi^* \sigma^{*,2} - \chi\sigma^2) + (\tau^*\sigma^* u^*_{t+1} - \tau\sigma u_{t+1}) + (\kappa^* - 1)\sigma^g u^g_{t+1}.$$

The expected excess return on foreign currency is given by: $E_t[rx_{t+1}] + \frac{1}{2}Var_t[\Delta s_{t+1}] = \tau\sigma^2 + (1-\kappa^*)\sigma^{g,2}$. This model produces a factor structure in bilateral exchange rates, driven by the common factor u^g_{t+1}. We define the base factor for the home currency as the equal-weighted average of the log changes in bilateral exchange rates $\Delta base_{t+1} = \frac{1}{N}\sum_{j=1}^{N} \Delta s_{t+1}$, which can be shown to yield the following expression:

$$\Delta base_{t+1} = (\overline{\alpha^*} - \alpha) + (\overline{\xi^*} - \xi)(\sigma^g)^2 + (\overline{\chi^*\sigma^{*,2}} - \chi\sigma^2) + (\overline{\tau\sigma^* u^*_{t+1}} - \tau\sigma u_{t+1}) + (\overline{\kappa^*} - 1)\sigma^g u^g_{t+1}.$$

The base factor measures the systematic variation in the home country's currency versus all foreign currencies. For large N, we have the following simple expression for currency i's base factor, which only depends on the base-country-specific shock and common shock:

$$\lim_{N\to\infty} \Delta base_{t+1} = (\overline{\alpha^*} - \alpha) + (\overline{\xi^*} - \xi)(\sigma^g)^2 + (\overline{\chi^*\sigma^{*,2}} - \chi\sigma^2) - \tau\sigma u_{t+1} + (\overline{\kappa^*} - 1)\sigma^g u_{g,t+1}.$$

In this model, different bilateral exchange rates will have different exposures to the base factor generated by different values of κ^*. The slope coefficient φ^* in a projection of the bilateral exchange rate changes, Δs, on the base factor, $\Delta base$, governs how much systematic risk the bilateral exchange rate is exposed to. This coefficient is determined the SDFs' loadings on the common shocks: all else equal, the lower it is, the higher the slope coefficient φ^*.

Proposition 1. *The variance of the base factor, the covariance of the exchange rate with* *the base factor, the loadings on the base factor and the R^2 are given by, respectively:*

$$Var\left(\lim_{N\to\infty} \Delta base_{t+1}\right) = \tau^2\sigma^2 + (\overline{\kappa^*}-1)^2 \left(\sigma^g\right)^2$$

$$Cov\left(\Delta s_{t+1}, \lim_{N\to\infty} \Delta base_{t+1}\right) = \tau^2\sigma^2 + (\kappa^*-1)(\overline{\kappa^*}-1) \left(\sigma^g\right)^2$$

$$\varphi^* = \frac{Cov\left(\Delta s_{t+1}, \lim_{N\to\infty} \Delta base_{t+1}\right)}{Var\left(\lim_{N\to\infty} \Delta base_{t+1}\right)} = \frac{\tau^2\sigma^2 + (\kappa^*-1)(\overline{\kappa^*}-1) \left(\sigma^g\right)^2}{\tau^2\sigma^2 + (\overline{\kappa^*}-1)^2 \left(\sigma^g\right)^2},$$

$$R^2 = \frac{\varphi^{*,2}\, Var\left(\lim_{N\to\infty} \Delta base_{t+1}\right)}{Var\left(\Delta s_{t+1}\right)} = \frac{\left[\tau^2\sigma^2 + (\kappa^*-1)(\overline{\kappa^*}-1) \left(\sigma^g\right)^2\right]^2}{\left[\tau^2(\sigma^2+\sigma^{*,2}) + (\kappa^*-1)^2 \left(\sigma^g\right)^2\right]\left[\tau^2\sigma^2 + (\overline{\kappa^*}-1)^2 \left(\sigma^g\right)^2\right]}.$$

Unlike bilateral exchange rate volatility, the slope coefficient φ^* does not depend on the idiosyncratic volatility of the foreign SDF. The only source of cross-sectional variation is κ^*. That makes φ^* a natural object to study for international economists. The slope coefficient is monotonically decreasing in κ^*, hence, it is a natural measure of exposure to the common shock. A country with average exposure has a loading of one, an exposure that is less than average translates into a loading larger than one, and vice-versa. In addition, these loadings determine currency risk premia.

Proposition 2. *The expected excess return on foreign currency as given by:*

$$E_t[rx_{t+1}] + \frac{1}{2}Var_t[\Delta s_{t+1}] = \lambda_0 + \varphi^*\lambda_1,$$

where $\lambda_0 = \tau\sigma^2\frac{\overline{\kappa^*}}{(\overline{\kappa^*}-1)}$ *is the expected excess return on the zero-beta currency, and* $\lambda_1 = \frac{Var(\lim_{N\to\infty}\Delta base_{t+1})}{1-\overline{\kappa}}$ *is the price of base factor risk.*

The only foreign country-specific variable that enters the expression for currency risk premia is the foreign loading φ^*, which measures the exposure to base factor risk.

We hypothesize that the common shock exposure, κ^*, decreases monotonically in distance from the foreign country, $*$, to the home country:

Assumption 1. *The common shock exposure is always largest in the base country* $0 < \kappa^* \leq 1$, *and* κ^* *decreases monotonically in distance from* $*$ *to the home country.*

In the context of gravity models of trade and financial flows, this assumption is sensible. When countries trade more or have more bilateral financial flows, countries share more risks and their pricing kernels will be more exposed to the same common shock. Distance governs the correlation of the pricing kernel: as the distance to $*$ declines and κ^* increases, the covariance of the pricing kernels at home and abroad increases. More generally, let $\boldsymbol{G}_{i,j}$ denote the gravity variables for the country pair i, j. Then our assumption is that the partial derivate of $\kappa^*(\boldsymbol{G}_{i,j})$ with respect to each element of the gravity vector is negative if that variable is increasing in distance.

Armed with Assumption 1, we can interpret the model quantities. First, the variance of the base factor, $\text{Var}\left(\lim_{N\to\infty} \Delta base_{t+1}\right)$, is higher in 'peripheral' countries that are more distant from other countries. These are countries with larger $|\overline{\kappa^*} - 1|$. $(|\overline{\kappa^*} - 1|)^{-1}$ is a related to standard measures of network centrality for the home country; 'network closeness' is defined as the inverse of the average distance. While the bilateral R^2 is not in general monotonic in κ^*, it is decreasing in the average loading $\overline{\kappa^*}$. This shows that as the loading on the common factor of the average foreign country decreases from 1, the R^2 for the average foreign country will increase. Therefore, countries which are on average distant from other countries will on average have high R^2 in our base factor regressions.

Next we interpret the base factor loadings, φ^*. The only source of cross-sectional variation is κ^* — the exposure to the common shock. The foreign country-specific shocks are averaged out and do not matter for the loadings on the base factor. Since the home country loads more than average on the common factor, then $\varphi^* \geq 0$ is always positive since we imposed that $\kappa^* < 1$. Given our assumptions, φ^* are bounded by:

$$\left[\frac{\tau^2\sigma^2}{\tau^2\sigma^2 + (\overline{\kappa^*} - 1)^2 \left(\sigma^g\right)^2}, \frac{\tau^2\sigma^2 - \left(\overline{\kappa^*} - 1\right)\left(\sigma^g\right)^2}{\tau^2\sigma^2 + (\overline{\kappa^*} - 1)^2 \left(\sigma^g\right)^2}\right].$$

The lower bound is attained when $\kappa^* = 1$. This is the case of perfect risk sharing when commodity baskets and preferences are identical. The upper bound is attained when $\kappa^* = 0$. This is the case of no exposure to common risks. In addition, φ^* increases as κ^* decreases, or equivalently, as distance increases. As κ^* drops below $\overline{\kappa^*}$, φ increases above one. In a trade context, this implies that lower trade intensity goes together with higher exposures to the base factor. More generally, our assumption implies that the partial derivative of $\varphi^*_{i,j}(\boldsymbol{G}_{i,j})$ with respect to each element of the gravity vector is positive if that variable is increasing in distance.

In Section (3), we test the prediction of the model that distance has a significant effect on the currency factor structure.

Multi-Factor SDF Model

A richer model would allow for multiple common factors. Most of the analysis carries through. The log SDF, m^*_{t+1} in each country is given by:

$$-m^*_{t+1} = \alpha^* + \chi^* \sigma^{*,2} + \sum_{k=1}^{K} \xi^*_k \left(\sigma^g_k\right)^2 + \tau^* \sigma^* u^*_{t+1} + \sum_{k=1}^{K} \kappa^*_k \sigma^g_k u^g_{k,t+1},$$

where u_{t+1} are local shocks and $u^g_{k,t+1}$ are common global shocks, all of which are zero mean and variance 1. Exchange rates changes are

$$\Delta s_{t+1} = m_{t+1} - m^*_{t+1} = (\alpha^* - \alpha) + \sum_{k=1}^{K} (\xi^*_k - \xi_k) \left(\sigma^g_k\right)^2 + \tau^* \sigma^* u^*_{t+1} - \tau \sigma u_{t+1} + \sum_{k=1}^{K} (\kappa^*_k - \kappa_k) \sigma^g_k u^g_{k,t+1}$$

For large N, we have the following simple expression for currency i's base factor:

$$\lim_{N \to \infty} \Delta base_{t+1} = (\overline{\alpha^*} - \alpha) + \sum_{k=1}^{K} (\overline{\xi^*_k} - \xi_k) \left(\sigma^g_k\right)^2 - \tau \sigma u_{t+1} + \sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k) \sigma^g_k u^g_{k,t+1}, \qquad (1)$$

where the last term is a particular linear combination of the global shocks.

Proposition 3. *The variance of the base factor, the covariance of the exchange rate with the base factor and the loadings on the base factor are given by, respectively:*

$$Var\left(\lim_{N \to \infty} \Delta base_{t+1}\right) = \tau^2 \sigma^2 + \sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k)^2 \left(\sigma^g_k\right)^2$$

$$Cov\left(\Delta s_{t+1}, \lim_{N \to \infty} \Delta base_{t+1}\right) = \tau^2 \sigma^2 + \sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k)(\kappa^*_k - \kappa_k) \left(\sigma^g_k\right)^2$$

$$\varphi^* = \frac{Cov\left(\Delta s_{t+1}, \lim_{N \to \infty} \Delta base_{t+1}\right)}{Var\left(\lim_{N \to \infty} \Delta base_{t+1}\right)} = \frac{\tau^2 \sigma^2 + \sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k)(\kappa^*_k - \kappa_k) \left(\sigma^g_k\right)^2}{\tau^2 \sigma^2 + \sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k)^2 \left(\sigma^g_k\right)^2}$$

The base factor loading φ^* varies due to differences in loadings on the K common factors. The term $\sum_{k=1}^{K} (\overline{\kappa^*_k} - \kappa_k)(\kappa^*_k - \kappa_k) \left(\sigma^g_k\right)^2$ measures this difference. For each factor k, we distinguish two cases. First, when $\overline{\kappa^*_k} < \kappa_k$, the factor k is relatively important for home country. We refer to these as the home country's 'own factors'. In this case, the base factor loading φ^* increases if the factor is less important for j ($\kappa^*_k < \kappa_k$). Second, when $\overline{\kappa^*_k} > \kappa_k$ the factor is less important for the home country. In this case, the base factor loading φ^* increases if the factor is more important for the foreign country ($\kappa_k^* > \kappa_k$).

Finally, it is easy to check that the expected excess return on a long position in the basket of foreign currencies is given by: $E_t[rx_{t+1}] + \frac{1}{2}Var_t[\Delta base_{t+1}] = \tau\sigma^2 - \sum_{k=1}^{K} \kappa_k (\overline{\kappa_k^*} - \kappa_k) \left(\sigma_k^g\right)^2$.

For the multi-factor model, we make the following assumption:

Assumption 2. *The weighted difference in factor exposures* $\sum_{k=1}^{K} (\overline{\kappa_k^*} - \kappa_k)(\kappa_k^* - \kappa_k) \left(\sigma_k^g\right)^2$ *increases monotonically in log distance from* $*$ *to home.*

When the foreign and home countries are more distant from each other, it is natural to assume that the foreign country is less exposed to the home country's 'own factors' ($\overline{\kappa_k^*} < \kappa_k$, case 1) and $*$ is more exposed to the other factors ($\kappa_k^* > \kappa_k$, case 2). Assumption 2 implies that $\text{Var}\left(\lim_{N\to\infty} \Delta base_{t+1}\right) = \tau^2\sigma_i^2 + \sum_{k=1}^{K} (\overline{\kappa_k^*} - \kappa_k)^2 \left(\sigma_k^g\right)^2$ is larger for peripheral countries that are farther from the average country. Countries that are distant from each other have different factor loadings — more so for factors that are important for countries' base factor variation ($\overline{\kappa_k^*} \ll \kappa_k$). Given Assumption 2 , the exchange rate loadings on the base factor also increase with distance.

Long-Run Risk Interpretation

Our factor model nests the long run risks model pioneered in FX by Colacito and Croce (2011); Bansal and Shaliastovich (2013); Lewis and Liu (2015); Colacito et al. (2015). To illustrate this, we present a model that has two common factors in consumption growth. For simplicity, we refer to these as the North American (NA) and European (E) factors. The consumption growth process in the home and foreign countries are given by:

$$\begin{aligned}
\Delta c_{t+1} &= \mu + \kappa^{NA} x_t^{NA} + \kappa^E x_t^E + \sigma\eta_{t+1}, &(2)\\
\Delta c_{t+1}^* &= \mu + \kappa^{NA,*} x_t^{NA} + \kappa^{E,*} x_t^E + \sigma\eta_{t+1}^*, &(3)\\
x_{t+1}^{NA} &= \rho_x x_t^{NA} + \varphi_e \sigma e_{t+1}^{NA}, &(4)\\
x_{t+1}^{E} &= \rho_x x_t^{E} + \varphi_e \sigma e_{t+1}^{E}, &(5)
\end{aligned}$$

where $(\eta_t, \eta_t^*, e_t^{NA}, e_t^E)$ are i.i.d. mean-zero, variance-one innovations. As in Colacito and Croce (2011); Colacito et al. (2015), each country's consumption growth contains transient and persistent components. We adopt their approach, but include multiple common components: x_t^g for $g \in \{NA, E\}$. Colacito and Croce (2011); Colacito et al. (2015) impute a high degree of correlation to the SDFs through the persistent component of consumption growth.

The inclusion of a common persistent component confronts the Brandt et al. (2006) puzzle: complete market models can only reconcile the low volatility of exchange rate changes Δs with the high volatility of m if m and m^* are highly correlated. To simplify the analysis, we abstract from a country-specific persistent consumption growth component, because these would not affect the loadings of bilateral exchange rates on the common base factor.

We use the following notation: $\theta = (1-\alpha)/(1-\rho)$ and $\psi = 1/\rho$, where α is the risk aversion and ψ is the intertemporal elasticity of substitution. β is the time discount factor. With Epstein-Zin preferences, the log SDF in the home country is a function of log consumption changes and the log total wealth return:

$$m_{t+1} = \frac{1-\alpha}{1-\rho}\log\beta - \frac{1-\alpha}{1-\rho}\rho\Delta c_{t+1} + \left(\frac{1-\alpha}{1-\rho} - 1\right) r^A_{t+1}.$$

By no arbitrage, when markets are complete, the change in the log exchange rate is given by:

$$\Delta s_{t+1} = m_{t+1} - m^*_{t+1} = -\frac{1-\alpha}{1-\rho}\rho(\Delta c_{t+1} - \Delta c^*_{t+1}) + \left(\frac{\rho-\alpha}{1-\rho}\right)(r^A_{t+1} - r^{*,A}_{t+1}).$$

We define the log price-consumption ratio z_t as $p_t - c_t$. We now have a log-linear approximation of the return on wealth: $r^A_{t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}$, where $\kappa_1 = exp(p-c)/(1+exp(p-c))$ and $\kappa_0 = \log(1+exp(p-c)) - (p-c)\kappa_1$. In the Section (A.1), we show that the log price-consumption ratio z_t is linear in the state variables: $z_t = A^0 + \kappa^{NA}\frac{1-\rho}{1-\kappa_1\rho_x}x^{NA}_t + \kappa^E\frac{1-\rho}{1-\kappa_1\rho_x}x^E_t$. To simplify the notation, we use the following shorthand: $\sigma^g = \frac{(\alpha-\rho)\kappa_1}{1-\kappa_1\rho_x}\varphi_e\sigma$. The innovation to the exchange rate is given by

$$\Delta s_{t+1} - E_t[\Delta s_{t+1}] = \alpha(\sigma\eta^*_{t+1} - \sigma\eta_{t+1}) + \left(\kappa^{NA,*} - \kappa^{NA}\right)\sigma^g e^{NA}_{t+1} + \left(\kappa^{E,*} - \kappa^E\right)\sigma^g e^E_{t+1}$$

Exchange rates respond to the local temporary consumption shocks in the home country, the foreign shocks, as well as the common persistent shocks. The base factor (without dropping the foreign currency) for currency i is simply given by:

$$\Delta base_{t+1} - E_t[\Delta base_{t+1}] = -\alpha\sigma\eta_{t+1} + \left(\kappa^{NA,*} - \kappa^{NA}\right)\sigma^g e^{NA}_{t+1} + \left(\kappa^{E,*} - \kappa^E\right)\sigma^g e^E_{t+1}$$

For large N, we have the following simple expression for currency i's base factor, which only depends on the base-country-specific and the common shock:

$$\lim_{N\to\infty}\Delta base_{t+1} - E_t[\Delta base_{t+1}] = -\alpha\sigma\eta_{t+1} + \left(\overline{\kappa^{NA,*}} - \kappa^{NA}\right)\sigma^g e^{NA}_{t+1} + \left(\overline{\kappa^{E,*}} - \kappa^E\right)\sigma^g e^E_{t+1}.$$

Proposition 4. *The variance of the base factor, the covariance of the exchange rate with* *the base factor and the loadings on the base factor are given by, respectively:*

$$
\begin{aligned}
Var_t\left(\lim_{N\to\infty}\Delta base_{t+1}\right) &= \alpha^2\sigma^2 + (\overline{\kappa^{NA,*}} - \kappa^{NA})^2\,(\sigma^g)^2 + (\overline{\kappa^{E,*}} - \kappa^{E})^2\,(\sigma^g)^2 \\
Cov_t\left(\Delta s_{t+1}, \lim_{N\to\infty}\Delta base_{t+1}\right) &= \alpha^2\sigma^2 + (\sigma^g)^2\left[(\kappa^{NA,*} - \kappa^{NA})(\overline{\kappa^{NA,*}} - \kappa^{NA}) + (\kappa^{E,*} - \kappa^{E})(\overline{\kappa^{E,*}} - \kappa^{E})\right]
\end{aligned}
$$

$$
\varphi_t^* = \frac{Cov_t\left(\Delta s_{t+1}, \lim_{N\to\infty}\Delta base_{t+1}\right)}{Var_t\left(\lim_{N\to\infty}\Delta base_{t+1}\right)} = \frac{\alpha^2\sigma^2 + (\sigma^g)^2\left[(\kappa^{NA,*} - \kappa^{NA})(\overline{\kappa^{NA,*}} - \kappa^{NA}) + (\kappa^{E,*} - \kappa^{E})(\overline{\kappa^{E,*}} - \kappa^{E})\right]}{\alpha^2\sigma^2 + (\overline{\kappa^{NA,*}} - \kappa^{NA})^2\,(\sigma^g)^2 + (\overline{\kappa^{E,*}} - \kappa^{E})^2\,(\sigma^g)^2},
$$

where $(\sigma^g)^2 = \left(\frac{(\alpha-\rho)\kappa_1}{1-\kappa_1\rho_x}\varphi_e\right)^2\sigma^2$.

As an example, consider the U.S. as the home country. The U.S. is more exposed than the average country to the American factor. As a result, a lower exposure of the foreign country to the American factor (e.g., switching from Canada to Norway) results in an increased loading of its bilateral exchange rate on the dollar factor. In Section (5), we calibrate this model to illustrate how the empirical base factor loadings, φ_t^*, can be used to infer exposures to the common factors in consumption growth, $\kappa^{g,*}$ for $g \in \{NA, E\}$.

2 The Factor Structure in Exchange Rates

We now turn to empirically measuring the base factor loadings and their determinants. We start by describing the data. Next, we document the empirical properties of the base factors and their relation to systematic currency risk. In Section (3) we document the gravity effect in the factor structure of exchange rates.

2.1 Data Description

FX Data

We obtain FX data from Global Financial Data (GFD) for 162 countries from January 1973 until December 2014. All FX data is with respect to the US dollar and is end-of-month. CPI data used to calculate real exchange rate changes is monthly from GFD. Currency's are omitted after they secede to the Euro, beginning in 1999. Throughout the paper we include the Euro (beginning in 1999) when constructing base factor loadings, but the Euro is omitted in regressions including gravity variables due to lack of gravity data. Our main results restrict the sample to 24 developed and 23 emerging countries as classified by MSCI in August 2015. In Section (6) we present robustness tests on the full and developed samples. We provide additional details of the sample construction in Appendix B.

Gravity Data

Most gravity data is available from Head et al. (2010) and Mayer and Zignago (2011). Distance is the population weighted average between large cities in each country pair (Mayer and Zignago (2011)). Common language is 1 if a language is spoken by over 9% of the population in both countries (Mayer and Zignago (2011)). Common legal origins is from Porta et al. (2007), linguistic similarity from Desmet et al. (2012), and genetic distance from Spolaore and Wacziarg (2009). The data on pegs is from Shambaugh (2004). The peg classification is based upon bilateral exchange rate volatility being less than 2% in two consecutive years. For full sample tests, the peg dummy is 1 if either currency was pegged to the other or both currencies were pegged to the same currency at any point in the sample. For the 5-year rolling tests, the peg dummy is 1 if either currency was pegged to the other or they were pegged same currency at any point in the prior 6 years.

Trade data is from United Nations COMTRADE and The Center for International data. Bilateral asset holdings are from the IMF Coordinated Portfolio investment survey. GDP data are from the World Bank's World Development Indiactors. Finally, we construct a new variable which measures the natural resource similarity between two countries. To do this, we obtain and clean the list of natural resources by country from the CIA world factbook. Using the list of natural resources, we construct vectors of dummy variables — 1 if a country has the resource, 0 otherwise. Natural resource similarity between two countries is the cosine similarity of the vectors of resource dummy variables.

2.2 Estimating Base Factor Loadings

Base factor loadings are estimated for all base currencies in the sample against all other currencies following the procedure in Verdelhan (2015). Specifically, base factor loadings, $\varphi^*_{i,j}$, are estimated from the regression

$$\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j} \Delta base_{i,t} + u_{i,j,t} \ , \tag{6}$$

where $s_{i,j,t}$ is the time t exchange rate in units of currency j per unit of currency i. An increase in $s_{i,j,t}$ implies an appreciation of currency i relative to currency j, and $\Delta base_{i,t}$ is the average appreciation of the currency i against all other currencies at time t. Starting with US based spot rates, we convert all rates to a specific base currency i. To avoid a mechanical relation between exchange rate changes and base factors, we calculate a separate base factor for each currency j, which omit that currency. For example, we construct the

US dollar factor, $\Delta base_{\$,t} = \frac{1}{N-1}\sum_{k\neq j}\Delta s_{\$j,t}$, by averaging the change in the exchange rate across all bilateral exchange rates against the USD. When we study the relation between the USD/GBP bilateral exchange rate, $\Delta s_{\$,£,t}$, and the USD base factor, we drop the USD/GBP bilateral exchange rate from the construction of the base factor. Conditional base factor loadings, $\varphi^{*}_{i,j,t}$, are estimated using 60 month rolling windows. The regression must have 48 months of available data for the conditional base factor loading to be estimated. Monthly rolling factor loadings are averaged to generate yearly observations.

The base factors are closely related to the first principal component of bilateral exchange rate changes. To show this, we compute the first principal component of the bilateral exchange rates $\Delta s_{i,j,t}$ for each base currency i. For example, instead of the dollar base factor, we could use the first principal component of all bilateral exchange rates against the dollar[7]. Table (B3) in Section (B.4) reports the correlations of the 1st principal component and the base factor by base currency. For most currencies, the first principal component is essentially the base factor: The equal-weighted average of bilateral exchange rate changes also turns out to to the linear combination that explains most of the variation. This alleviates concerns about the equal weights assigned to all bilateral partners. The only exception is Singapore with a correlation of 0.86. As a result, we simply proceed by analyzing the base factors.

We study base factor loadings with respect to each base currency in our data. To understand why studying multiple base factors is reasonable, consider a latent statistical factor model for exchange rate variation — this model nests the models presented in Section (1). Exchange rates changes are given by:

$$\Delta s_{i,j,t} = \alpha_{i,j} + \boldsymbol{\gamma}'_{i,j}\boldsymbol{f}_t + u_{i,j,t} \ , \tag{7}$$

where $s_{i,j,t}$ denotes the time t log exchange rate in units of currency j per unit of currency i and $\boldsymbol{f}_t$ denotes a $K\times 1$ vector of orthogonal, global FX factors. This vector could include the USD factor and the FX carry trade factor studied by Lustig et al. (2011, 2014); Verdelhan (2015), a Chen and Rogoff (2003) commodity factor, regional FX factors, etc. The base factor for country i is simply a linear combination of the underlying factors: $\Delta base_{i,t} = \frac{1}{N-1}\sum_{k\neq j}\Delta s_{i,k,t} = \frac{1}{N-1}\sum_{k\neq j}\gamma'_{i,k}\boldsymbol{f}_t + \frac{1}{N-1}\sum_{k\neq j}u_{i,k,t}$.

In a world with multiple global FX factors, switching base currencies reveals new information to the econometrician, in spite of triangular arbitrage. From triangular arbitrage,

[7] To compare base factors and 1st principal components, it is necessary to construct a different sample because a balanced panel is needed. For this comparison only, all observations from countries which join the euro are dropped, except for Germany. The German exchange rate becomes the Euro starting in 1999. Using this sample, base factors and 1st principal components are calculated for each potential base currency.

we know that $\Delta s_{i,j,t} - \Delta s_{i,k,t} = \Delta s_{k,j,t}$. That implies the following restriction on the loadings: $\gamma'_{i,j} - \gamma'_{i,k} = \gamma'_{k,j}$. The base factor for country k can be constructed from the base factor for country i using the triangular arbitrage relation: $\Delta base_{k,t} = \frac{1}{N-1}\sum_{j\neq k}\Delta s_{k,j,t} = \frac{1}{N-1}\sum_{j\neq k}\left(\boldsymbol{\gamma}'_{i,j} - \boldsymbol{\gamma}'_{i,k}\right)\boldsymbol{f}_t + \frac{1}{N-1}\sum_{j\neq k}(u_{i,j,t} - u_{i,k,t})$. In a world with $K > 1$ common factors, we cannot identify the $N \times K$ coefficients from only $N-1$ different independent loadings with respect to one base factor. Section A.2 of the appendix provides some examples. We assume there are multiple global FX factors, but do not attempt to identify the right factors. Instead, we study base factor loadings with respect to all base currencies and account for potential correlation between the base factor loadings in our tests. Details can be found in Section (3.1) and Section (B.3) of the appendix.

2.3 Variance Decomposition

The base factor loadings impact numerous important quantities in foreign exchange markets. Consider the R^2 of the regression in Equation (6):

$$R^2_{i,j} = \frac{\left(\widehat{\varphi}^*_{i,j}\right)^2 \sum_t \left(\Delta base_{i,t} - \overline{\Delta base_i}\right)^2}{\sum_t \left(\Delta s_{i,j,t} - \overline{\Delta s}_{i,j}\right)^2} \ . \tag{8}$$

This is a measure of the amount of systematic currency risk faced by a domestic investor in the base country who takes long positions in foreign currency. All else equal, countries j with a larger loading on the base factor will tend to have a higher R^2. In addition, base countries i with more volatile base factors tend to have higher average R^2. We use the regression in equation Equation (6) to decompose the variance of changes in log exchange rates. Table (1) presents this decomposition of exchange rate variance for each base country[8]. The first column reports the average variance of the bilateral exchange rates. The second column reports the average, across currencies j, of the variance explained by the base factor (the numerator of Equation (8)). The third column reports the idiosyncratic variance of the bilateral exchange rates. The numbers in the first column are the sum of the numbers in the second and third column. All three columns are multiplied $\times 100$. The fourth column reports average R^2.

There is substantial cross-country variation in the variance attributable to the base factor. The average explained variance is 0.68 for developed countries and 3.37 for emerging market countries. In some countries, a high explained variance reflects the effects of high and volatile

[8] Table (B10) reports the same results for real exchange rates, computed using the ratio of the countries' CPIs.

Table 1: Variance Decomposition of Bilateral Exchange Rates by Base Currency

	FX Var	Base Var	Id. Var	R^2 Mean		FX Var	Base Var	Id. Var	R^2 Mean
Developed Countries					**Emerging Countries**				
Australia	2.81	0.95	1.86	0.50	Brazil	12.57	11.05	1.52	0.89
Austria	3.20	0.64	2.55	0.31	Chile	10.75	9.06	1.69	0.82
Belgium	3.29	0.67	2.63	0.32	China	2.81	1.07	1.74	0.53
Canada	2.31	0.42	1.89	0.33	Colombia	2.48	0.67	1.81	0.45
Denmark	2.31	0.52	1.78	0.29	Czech Republic	6.22	4.40	1.82	0.76
Euro Area	1.11	0.48	0.64	0.41	Egypt	3.71	2.01	1.69	0.66
Finland	3.24	0.51	2.74	0.31	Greece	3.14	0.67	2.47	0.39
France	3.24	0.61	2.63	0.29	Hungary	3.08	1.32	1.76	0.58
Germany	3.32	0.70	2.62	0.33	India	2.36	0.44	1.92	0.34
Hong Kong	2.29	0.41	1.88	0.30	Indonesia	6.03	4.33	1.70	0.78
Ireland	3.23	0.53	2.70	0.29	Korea	3.23	1.32	1.91	0.56
Israel	4.17	2.48	1.70	0.69	Malaysia	2.30	0.40	1.90	0.30
Italy	3.28	0.55	2.73	0.33	Mexico	8.19	6.42	1.77	0.81
Japan	2.94	1.04	1.90	0.51	Peru	16.29	14.71	1.57	0.88
Netherlands	3.30	0.68	2.62	0.32	Philippines	2.94	1.04	1.90	0.52
New Zealand	2.87	0.96	1.91	0.50	Poland	6.36	4.60	1.76	0.77
Norway	2.31	0.46	1.85	0.28	Qatar	2.22	0.46	1.76	0.32
Portugal	3.26	0.55	2.71	0.29	Russian Federation	8.76	7.80	0.96	0.87
Singapore	2.11	0.22	1.89	0.17	South Africa	3.31	1.40	1.91	0.58
Spain	3.40	0.65	2.75	0.37	Taiwan	2.31	0.43	1.87	0.32
Sweden	2.41	0.54	1.86	0.34	Thailand	2.66	0.78	1.89	0.45
Switzerland	2.59	0.77	1.82	0.42	Turkey	4.47	2.78	1.69	0.72
United Kingdom	2.45	0.55	1.90	0.37	United Arab Emirates	2.21	0.44	1.77	0.31
United States	2.25	0.41	1.84	0.30					
All	2.83	0.68	2.15	0.36	All	5.15	3.37	1.77	0.59

Summary statistics of data from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$ for each possible base currency i. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. FX Var, Base Var, and Id Var are cross-sectional means of the time series variances for each base currency. FX Var is the total variance, Base Var is the variance attributed to the base factor, and Id Var is the remaining idiosyncratic variance. The numbers in the first column are the sum of the numbers in the second and third column. All three columns are multiplied $\times 100$. R^2 mean is the cross-sectional mean of the R^2 for each base currency. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

inflation episodes — the explained variances for Brazil, Peru and Israel are respectively 11.05, 14.71 and 2.48.

The composition of the variances are different as well. The average R^2 is 0.36 for developed countries' currencies, compared to an average R^2 of 0.59 for emerging market currencies. This reflects the fact that the ratio of the explained variance to exchange rate variance is higher for emerging market currencies than developed currencies. Figure (1) plots the average R^2 on a map. Due to the gravity effect in the factor structure of bilateral exchange rates, peripheral countries, which are distant from most other countries, have high average R^2. The next section formally documents this gravity effect.

3 The Gravity Effect in the Factor Structure

In the previous section, we established that variation in base factor loadings drives important differences in the properties of exchange rates. In this section, we show that variation in base factor loadings can largely be understood as a function of measures of distance between countries.

We begin by summarizing the key variables in our dataset. Table (2) reports summary statistics for all of the variables in our main sample. There are a total of 2,070 base country/foreign country combinations. There is a lot of variation in the loadings across currencies. The average loadings are close to one. The average standard deviation of the loadings across countries for a given base currency is 0.33. Similarly, there is a lot of variation in the R^2. The average R^2 is 0.47 while the cross-sectional standard deviation is 0.29. The average distance between a base currency and its counterparts is 8.62 (in logs) or 5541 km. On average, 13% (4%) of the countries share a language (border) with the base currency. The average resource similarity with the base currency is 0.24. 2% share the same colonizer with the base currency. 28% of the currencies have been pegged to the base currency or have shared a peg with the base currency to another currency at any point in the sample.

Table (B2) reports summary statistics for the rolling sample. In the rolling sample, only 12% of the currencies are pegged to or share a peg with the base currency. In the 5-year rolling samples, the peg dummy is 1 if either currency was pegged to other or they were pegged same currency at any point in the 6 years prior.

Table 2: Full Sample Summary Statistics

	N	Mean	Median	Sd	Min	Max
Loading	2,070	0.95	1.00	0.33	-0.15	2.95
Loading (Real)	1,640	0.93	0.99	0.33	-0.16	3.25
R-squared	2,070	0.47	0.46	0.29	0.00	0.98
R-squared (Real)	1,640	0.47	0.45	0.29	0.00	0.99
Log Dist	2,070	8.62	9.00	0.93	5.08	9.88
Common Language	2,070	0.13	0.00	0.34	0.00	1.00
Shared Border	2,070	0.04	0.00	0.20	0.00	1.00
Resource Similarity	2,070	0.24	0.23	0.17	0.00	0.82
Linguistic Proximity	957	1.06	0.22	2.23	0.00	15.00
Genetic Distance	1,023	0.72	0.78	0.52	0.00	2.67
Colonial Linkage	2,070	0.02	0.00	0.14	0.00	1.00
Peg Dummy	2,070	0.28	0.00	0.45	0.00	1.00

Summary statistics of the factor loadings and gravity data. Factor loadings, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

3.1 Understanding the variation in the loadings

A large class of complete market models predict that the loadings $\varphi^*_{i,j}\left(\boldsymbol{G}_{i,j}\right)$ are increasing in distance. To explain the variation in base factors loadings, we regress the full sample loadings, $\varphi^*_{i,j}$, on various exogenous measures of the economic distance between i and j. We include physical distance, shared language, shared legal origin, share border, colonial link, resource similarity, genetic distance and linguistic similarity. All of the regressions indicate that an increase in the economic distance between i and j increases $\varphi^*_{i,j}$, the sensitivity of the bilateral exchange rate to the base factor.

We run the following cross-sectional regression of the loadings on the gravity variables:

$$\varphi^*_{i,j} = \delta + \beta \boldsymbol{G}_{i,j} + e_{i,j}.$$

The dependent variable in our model is estimated. This does not bias the estimates, but may introduce heteroskedasticity into the residuals (Lewis and Linzer, 2005). Additional correlation in the residuals arises due to the interdependent nature of exchange rates. Therefore, in all tables we report standard errors correcting for heteroskedasticity (White, 1980), clustering on base factor or foreign country (Cameron et al., 2011), or clustering on country pairs (Aronow et al., 2015) — depending on the specification. Additional details are in Section (B.3).

Table (3) reports the results for MSCI developed and emerging countries. In this sample, physical distance, shared language, colonial linkages and resource similarity all have robust effects on the loading. The average loading for a given base factor is one, while the cross-sectional standard deviation is 0.42 (0.23) for developed (emerging market) countries. A one standard deviation in log distance (the equivalent of approx. 8,500 km) increases the loading by about 0.13. This number is robust across different specifications, except the no peg specification. Shared language lowers the loading by about 0.11. Shared border lowers the loading by 0.13. Colonial linkages lower the loadings by up to 0.23. Legal origin, linguistic proximity, and genetic distances, do not have a statistically significant effect on the currency loadings. This specification accounts for 1/4 of all the variation in the loadings. Given the measurement error in these loadings, this is a remarkably high number.

Finally, resource similarity also lowers the loadings. If two countries were perfectly similar in terms of their resource endowments, that would lower the loadings by up to 0.17. This result echoes the findings of Chen and Rogoff (2003) who report that the exchange rates of commodity exporters move in lockstep with the dollar price of commodities. Strictly speaking, resource similarity is not a gravity variables, because countries that are similar in their resource endowments are less likely to trade. Instead, this variable measures the correlation of shocks.

Specifications (1), (2) and (3) do not control for pegs. For completeness, specification (4) introduces a peg dummy. The peg dummy is one if the currencies were ever pegged to each other or the same currency at any point in the 1973-2014 sample. Controlling explicitly for pegs mitigates most of these 'economic distance' effects. This is not surprising. We will establish in the next section that the decision to peg is driven the same largely determined by the same exogenous 'economic distance' variables. The broader claim that economic distance determines currency covariation (with or without currency pegs) is still valid. Note that resource similarity is no longer significant in specification (4). That is not surprising, given that resource similarity was a major determinant of the decision to peg. If a currency has been pegged to the base currency, or if they both have been pegged to the same currency in our sample, this lowers the loadings by another 0.25. This effect is not entirely mechanical: the peg dummy is one if the currencies were pegged at any point during the sample.

Finally, specification (5) excludes all currencies that were pegged at some point in the

Table 3: Full Sample Regressions with Nominal Loadings

	All (1)	All (2)	All (3)	All (4)	No Pegs
Log Distance	0.156	0.162	0.141	0.116	0.083
	(0.036)	(0.045)	(0.038)	(0.029)	(0.037)
Shared Language			−0.110	−0.088	−0.123
			(0.035)	(0.034)	(0.043)
Shared Legal		−0.039	−0.005	0.013	0.028
		(0.033)	(0.027)	(0.025)	(0.032)
Shared Border		−0.084	−0.130	−0.083	−0.114
		(0.047)	(0.040)	(0.048)	(0.093)
Colonial Link		−0.078	−0.234	−0.210	−0.310
		(0.065)	(0.102)	(0.100)	(0.082)
Resource Similarity		−0.172	−0.146	−0.097	−0.081
		(0.082)	(0.064)	(0.076)	(0.099)
Linguistic Proximity		−0.002			
		(0.006)			
Genetic Distance		−0.053			
		(0.038)			
Peg Dummy				−0.239	
				(0.061)	
R^2	0.189	0.212	0.230	0.322	0.095
Num. obs.	2070	903	2070	2070	1498

Regressions $\varphi^*_{i,j} = \delta + \beta \boldsymbol{G}_{i,j} + e_{i,j}$ of base factor betas on gravity variables. $\boldsymbol{G}_{i,j}$ is a set of gravity variables. Base factor betas, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j} base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Standard errors are clustered on country dyads using Aronow et al. (2015).

1973-2014 sample. This reduces the number of country pairs from 2,070 to 1,498. The R^2 drops from 23.0% to 9.5%. However, distance, language, and colonial link effects are statistically significant at the 5% level. We will use rolling sample regressions in order to have a more targeted control for currency pegs below.

In Table (B4), we compare the nominal loadings to the real loadings. The real loadings are computed by running the same regression of real exchange rate changes on the real base factor. Specifications (1) and (2) report results without a peg dummy for nominal and real loadings respectively. Specifications (3) and (4) report the same regression with a peg dummy. Both pairs of regressions are on matched samples. In both cases, the magnitude and significance of the regression coefficients are similar. This is consistent with Mussa (1986)'s observation that real exchange rates largely track nominal ones.

3.2 Marginal Propensity to Peg and Rolling Sample Estimates

Exchange rate regimes are endogenous. The decision to peg is largely governed by distance between the countries and other measures of economic distance. To show this, Table (4) reports the estimation results for a logit model similar to Tenreyro (2007). The dependent variable is a peg dummy which measures whether two currencies were ever pegged to each other or to the same currency. Because the peg dummy is symmetric and the gravity data is symmetric, the models are only estimated on unique pairs of countries.

Distance, resource similarity, genetic distance, and common legal origins are significant determinants of whether currencies are pegged. Distance reduces the likelihood of a peg. In specification (3), a one unit increase in log distance from its mean (8.73 to 9.73 in logs or 6,186km to 16,815km) decreases the peg probability by approximately 5%. An increase in resource similarly from its mean of .19 to .29 increases the peg probability by 2% in specification (3). Finally, having common legal origins increases the peg probability by 7% in specification (2).

To control for the effect of pegs in a targeted way, we use the rolling estimates of the base loadings. Table (5) reports the results of regressions of base factor loadings computed over 60-month rolling windows on time fixed effects and the gravity variables. The peg dummy is now defined differently; it is one only if the currencies were pegged to each other or to the same currency at any point in the prior 72 months. Overall, the r-squareds in the rolling regressions are substantially lower, presumably because the loading estimates are noisier.

As before, the peg dummy in the fourth specification mitigates some of these economic distance effects, because these same effects ultimately determine the likelihood of a peg.

Table 4: Marginal Propensity to Peg in Full Sample

	(1)	(2)	(3)
Log Distance	−0.061	−0.072	−0.045
	(0.018)	(0.030)	(0.022)
Shared Language			0.059
			(0.040)
Shared Legal		0.068	0.049
		(0.032)	(0.026)
Shared Border		0.070	0.119
		(0.064)	(0.061)
Colonial Link		−0.015	−0.004
		(0.058)	(0.047)
Resource Similarity		0.309	0.226
		(0.132)	(0.099)
Linguistic Proximity		−0.002	
		(0.008)	
Genetic Distance		0.057	
		(0.028)	
Num. obs.	12699	7652	12403

Logit models of peg dummy on gravity data. Peg dummy measures whether countries were ever pegged to each other or to the same currency during the sample. A currency pair is considered pegged if the bilateral exchange rate volatility is less than 2% in 2 consecutive years (Shambaugh (2004)). The table reports marginal effects at the mean. Data is yearly from 1973 until 2014 for the 162 countries in the Global Financial Data dataset. Standard errors are clustered on country dyads using Aronow et al. (2015).

Specification (4) controls for pegs while (1)-(3) do not. Overall, the size of the coefficients in specification (4) are somewhat smaller than those in specification (3). The distance coefficient is still around 0.14 in specification (4). The shared language effect is -0.10. The effects of a shared border is around -0.11. The effect of a colonial linkage has decreased from -0.28 to -0.2, while the effect of resource similarity is roughly constant. A one standard deviation increase in resource similarity reduces the loading by 0.04. Specification (5) excludes the pegs altogether. Reassuringly, the magnitudes of these slope coefficients does not differ significantly between specification (4) and specification (5).

Table 5: Rolling Sample Regressions with Nominal Factor loadings

	All (1)	All (2)	All (3)	All (4)	No Pegs
Log Distance	0.155	0.163	0.138	0.119	0.121
	(0.036)	(0.043)	(0.040)	(0.035)	(0.037)
Shared Language			−0.122	−0.096	−0.107
			(0.040)	(0.031)	(0.035)
Shared Legal		−0.041	−0.019	−0.033	−0.032
		(0.039)	(0.030)	(0.026)	(0.027)
Shared Border		−0.055	−0.126	−0.076	−0.113
		(0.051)	(0.049)	(0.046)	(0.044)
Colonial Link		−0.144	−0.281	−0.200	−0.225
		(0.063)	(0.083)	(0.044)	(0.060)
Resource Similarity		−0.198	−0.166	−0.151	−0.165
		(0.080)	(0.064)	(0.062)	(0.074)
Linguistic Proximity		−0.003			
		(0.005)			
Genetic Distance		−0.037			
		(0.029)			
Peg Dummy				−0.472	
				(0.054)	
Within R^2	0.086	0.114	0.114	0.185	0.086
Num. obs.	61130	27021	61130	58298	53532

Regressions $\varphi^*_{i,j,t} = \delta + \kappa_t + \lambda \boldsymbol{G}_{i,j} + e_{i,j,t}$ of base factor loadings on gravity variables. $G_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t} \Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \dots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Standard errors are clustered on country dyads using Aronow et al. (2015).

Interestingly, when the shared language is English, the effects are much larger. For example, when we only consider the USD factor, the loading decreases by 0.53 when the other country has English as one of its major languages (see Table (B5) in the Section (B.4)).

Finally, Table (B6) checks the results of the nominal against the real base factor loadings in the rolling sample regressions. The samples are matched on the available of CPI data. In the real specifications (1)-(3), some of the coefficients are smaller in absolute value. In particular, colonial linkages are no longer statistically significant. However, the distance is even stronger. The r-squareds in the real specifications are slightly lower than in the nominal specifications.

We obtain similar results for real exchange rates, echoing Mussa (1986); Flood and Rose (1995)'s observation that real exchange rates largely track the nominal ones, even if the nominal exchange rate is fixed. The sensitivity of changes in the real exchange rate to the base factor is governed by the same economic forces, and the coefficients have similar magnitudes. The only exception is the effect of colonial linkages. Engel (1999) attributes most of the variation in U.S. real exchange rates to the relative prices of tradeables. Based on extrapolation of Engel (1999)'s decomposition, our findings imply that the relative prices of tradeables in countries that are economically distant, and hence trade less, will be more sensitive to the common factor. Conversely, the factor structure in relative prices will be weaker in countries that are close and trade more intensely. In product-level data, there is evidence that producer-currency pricing (price stickiness) may account for some of these effects[9] (see, e.g., Nakamura and Steinsson, 2008; Gopinath and Rigobon, 2008). Recent evidence suggests that these effects are not entirely due to price stickiness. Burstein and Jaimovich (2009) find evidence in U.S-Canadian product-level data that active pricing-to-market, i.e. changes in the mark-ups contingent on the location of the sale, accounts for a lot of the variation in the relative prices of tradeables. Interestingly, we even find similar effects of distance on real exchange rate co-variation within the Euro zone in Section (6.5).

4 Trade and Currency Risk

Given that bilateral trade flows and financial flows are log-linear combinations of gravity variables (see Anderson and van Wincoop, 2004; Costinot, 2014; Head and Mayer, 2014,

[9]In these models, flexible exchange rates are a good substitute for flexible prices and facilitate the adjustment to country-specific shocks. (For an equilibrium model, see Obstfeld and Rogoff, 1995)

for recent surveys of the gravity equation in international trade), it is natural to use trade intensity as the right-hand-side variable in the cross-sectional regressions. If the gravity effects that we have reported merely capture the effect of gravity on trade flows, risk sharing and hence exchange rates, then trade intensity would potentially drive out these gravity effects in a horse race. That is not what we find.

4.1 Bilateral Trade and Financial Asset Holdings

Table (6) presents results controlling for bilateral imports plus exports to GDP of the foreign country and for the GDP share of the base country. Column (1) presents results only controlling for bilateral exports and imports normalized by foreign country's GDP. More trade between the foreign country and the base country, relative to the foreign country's GDP, lowers the base factor loading. Surprisingly, the R-squared in this regression is only 2.4%, despite the fact that gravity variables explain trade intensities quite well, much smaller than the R^2 we obtained for the gravity regression. Column (3) includes a control for the size of the base country. Column (4) includes both trade to GDP and GDP shares. Introducing the gravity variables increases the R^2 fivefold.

When we introduce the gravity variables, this lowers the size of the trade intensity coefficient by 66%, but the gravity coefficients are not significantly altered. We cannot rule out that the limited explanatory power of trade intensity is partly due to measurement error in the trade intensity measure. More importantly, bilateral trade intensities may be inadequate predictors of the foreign country's exposure to the base country's pricing kernel shocks: Countries that trade more with each other, may opt to specialize motivated by comparative advantage, thus reducing the correlation of the fundamental shocks to which they are exposed. This in turn may mitigate the risk sharing effect of increased trade.[10] However, given the size of the increase in R^2, we conclude that correlated shocks may play a more important role than previously considered.

[10] Frankel and Rose (1998) do find that on average countries with more significant trade linkages have more correlated business cycles.

Table 6: Rolling Sample Regressions controlling for Trade and GDP (MSCI Developed and Emerging Subset)

	All (1)	All (2)	All (3)	No Pegs
Log Distance		0.140	0.144	0.122
		(0.042)	(0.042)	(0.038)
Shared Language		−0.107	−0.106	−0.100
		(0.037)	(0.037)	(0.034)
Shared Legal		−0.030	−0.031	−0.030
		(0.030)	(0.031)	(0.027)
Shared Border		−0.102	−0.091	−0.105
		(0.056)	(0.060)	(0.052)
Colonial Link		−0.293	−0.275	−0.219
		(0.091)	(0.094)	(0.063)
Resource Similarity		−0.191	−0.204	−0.185
		(0.063)	(0.065)	(0.077)
Trade/GDP (Foreign)	−2.111	−0.721	−0.897	−0.838
	(0.563)	(0.268)	(0.400)	(0.402)
log GDP Share (Base)			0.012	0.018
			(0.009)	(0.014)
Within R^2	0.024	0.124	0.130	0.094
Num. obs.	54884	54884	54338	49946

Regressions $\gamma_{ijt}^{base} = \alpha + \kappa_t + \beta \boldsymbol{G}_{ij} + e_{ij}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{ij}$ is a set of gravity variables. Base factor loadings, γ_{ijt}^{base}, are from 60-month rolling regressions $\Delta s_{ij\tau} = \alpha + \gamma_{ijt}^{base} base_{i\tau} + e_{ij\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{it}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Real exchange rate changes include relative differences in inflation. Standard errors clustered on base country and foreign country using Cameron et al. (2011)).

Finally, we know that bilateral asset holdings also satisfy a gravity equation. Table (7) presents results controlling for various forms of bilateral asset holdings as reported by the IMF Coordinated Portfolio Investment Survey. Portfolio investment is defined as 'cross-border transactions and positions involving debt or equity securities, other than those included in direct investment or reserve assets.' Asset holdings are measured as total assets held by each foreign country in the base country and are normalized by GDP or total assets held. The results echo those of the trade regressions – asset holdings do lower base factor loadings, but the amount of variance that is explained by the asset holdings is small compared to that of the gravity variables.

Table 7: Rolling Sample Regressions with Bilateral Asset Holdings (MSCI Developed and Emerging Subset)

	All (1)	All (2)	All (3)	All (4)	All (5)	No Pegs	No Pegs
Log Distance			0.138	0.127	0.130	0.122	0.121
			(0.040)	(0.033)	(0.032)	(0.032)	(0.033)
Shared Language			−0.122	−0.086	−0.102	−0.068	−0.067
			(0.040)	(0.035)	(0.036)	(0.024)	(0.026)
Shared Legal			−0.019	−0.041	−0.040	−0.053	−0.053
			(0.030)	(0.044)	(0.042)	(0.040)	(0.037)
Shared Border			−0.126	−0.082	−0.078	−0.067	−0.060
			(0.049)	(0.080)	(0.086)	(0.054)	(0.051)
Colonial Link			−0.281	−0.355	−0.320	−0.237	−0.221
			(0.083)	(0.115)	(0.111)	(0.061)	(0.063)
Resource Similarity			−0.166	−0.116	−0.096	−0.091	−0.094
			(0.064)	(0.048)	(0.044)	(0.068)	(0.068)
Assets to GDP (Foreign)	−0.497			−0.091		0.301	
	(0.379)			(0.303)		(0.196)	
Assets to GDP (Base)	−0.902			−0.495		−0.278	
	(0.299)			(0.206)		(0.329)	
Assets to Total (Base)		−0.446			0.757		2.186
		(2.575)			(2.026)		(1.844)
Assets to Total (Foreign)		−3.765			−1.475		−2.402
		(1.144)			(1.901)		(1.898)
Within R^2	0.010	0.001	0.114	0.120	0.117	0.095	0.091
Num. obs.	14522	16446	61130	14522	16316	13892	15016

Regressions $\gamma_{ijt}^{base} = \alpha + \kappa_t + \varphi \boldsymbol{G}_{ij} + e_{ij}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{ij}$ is a set of gravity variables. Base factor loadings, γ_{ijt}^{base}, are from 60-month rolling regressions $\Delta s_{ij\tau} = \alpha + \gamma_{ijt}^{base} base_{i\tau} + e_{ij\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{it}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Bilateral asset holdings are from the IMF Coordinated Portfolio Investment Survey. Standard errors are clustered on country dyads using Aronow et al. (2015).

Bilateral trade and asset holdings explain only a small fraction of the cross-country variation in the base factor loadings. Hence, the risk sharing channel is only part of the explanation. Our results imply that the fundamental shocks are more correlated for countries that are closer together. The large effects of resource similarity offer direct support for this correlated shock hypothesis.

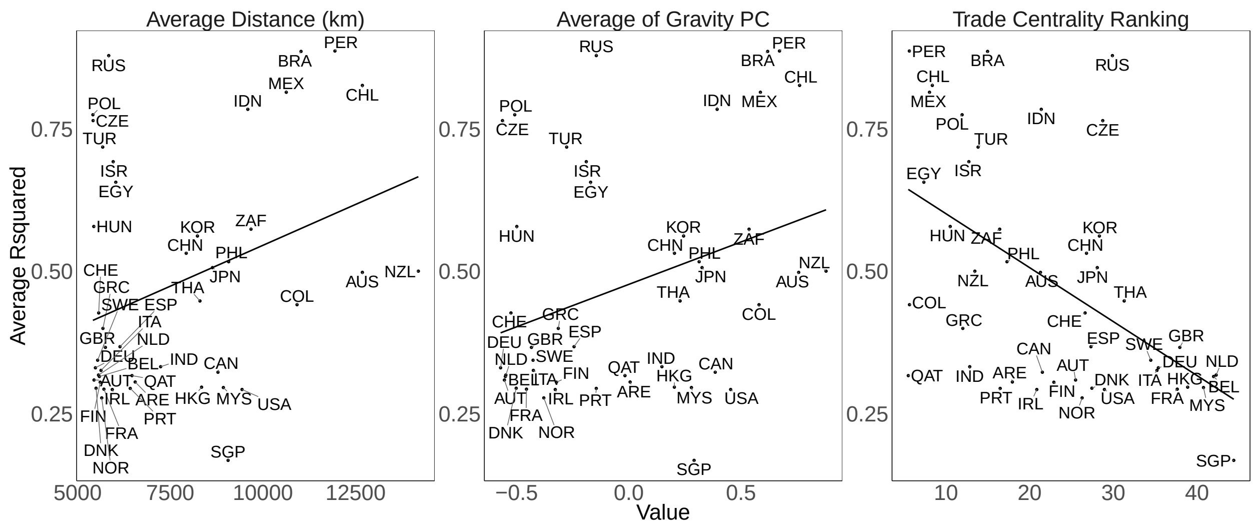
4.2 Trade Centrality

The average R^2 for a base currency measures how much systematic risk a domestic currency investor is exposed to when she takes a long position in a basket of foreign currencies. The theory predicts that these average R^2 should covary positively with the base country's average distance from other countries.

Figure (2) shows this relation. The first panel plots average R-squared versus average distance to all other countries. The second panel uses a measure of average distance which is the average by base country of the first principal component of bilateral gravity variables. The third panel uses a measure of countries' position in the global trade network from (Richmond, 2015), referred to as trade network centrality. Richmond (2015) shows how trade network centrality captures a country's overall exposure to global shocks — central countries being more exposed to these shocks than peripheral countries. The centrality measure is calculated as the lead eigenvector of a matrix of bilateral trade intensities and is motivated by an international model with a global production network. Trade network centrality ranking is the time series average ranking, where the maximum rank is normalized to 1. Trade network centrality turns out to explain a subtantial amount of the variation in average R^2 of currencies. Even within the group of developed currencies, countries that are central in the global trade network tend to have low R^2: the R^2 of Belgium, Singapore, and Hong Kong are 0.32, 0.17, and 0.30, respectively. Countries in the periphery of the global trade network tend to have high R^2s: the R^2 is 0.50 for Australia and New Zealand. This is consistent with the notion that trade network centrality measures a country's exposure to global shocks due to their position in the global trade network.

Table (8) reports the results of a regression of the average R^2 for each country on measures of average distance of that country from others. These regressions correspond to Figure (2). As expected, countries which are on-average more distant from other countries have bilateral exchange rates which are more exposed to systematic risk. While bilateral trade intensity had limited explanatory power for the base factor loadings, trade network centrality is a powerful predictor of how much systematic currency risk currencies are exposed to. Centrality explains

Figure 2: Average R-Squared vs Measures of Average Distance



Plots of average R-squared versus measures of average distance and trade network centrality. R-squared values, $R^2_{i,j}$, are from regressions $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j} \Delta base_{i,t} + e_{i,j,t}$. $E[R^2_{i,j}]$ is the cross-sectional average R-squared for each i. For each currency j, $\Delta base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Average distance is measured in km for each country to all other countries in the sample. Gravity PC is the first principal component of bilateral distance, shared language, shared legal origins, shared colonial origins, resource similarity, and shared border. Trade centrality is alpha centrality of a network with bilateral trade intensity as weights as in Richmond (2015). Trade centrality ranking is the time series average ranking where rankings are normalized to the maximum number of countries in the sample. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

about 25% of the cross-sectional variation. Unlike bilateral trade intensity, trade network centrality has an unambiguously positive effect on exposure: countries that are central in the trade network are more exposed to global risk than peripheral countries (see Richmond, 2015, for empirical evidence).

Table 8: Regressions of Average RSquared on Measures of Average Distance

	(1)	(2)	(3)	(4)	(5)
Intercept	0.476	0.477	0.695	0.670	0.671
	(0.029)	(0.029)	(0.061)	(0.063)	(0.064)
Average log Distance	0.069			0.039	
	(0.029)			(0.028)	
Average of Gravity PC		0.149			0.072
		(0.066)			(0.063)
Centrality Ranking			−0.426	−0.376	−0.378
			(0.107)	(0.111)	(0.114)
Adj. R^2	0.094	0.086	0.253	0.270	0.258
Num. obs.	45	45	45	45	45

Regressions $E[R^2_{i,j}] = \alpha + \kappa H_i + e_i$ of average R-squared on measures of average distance and trade network centrality. R-squared values, $R^2_{i,j}$, are from regressions $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$. $E[R^2_{i,j}]$ is the cross-sectional average R-squared for each i. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Average distance is measured in km for each country to all other countries in the sample. Gravity PC is the first principal component of bilateral distance, shared language, shared legal origins, shared colonial origins, resource similarity, and shared border. Trade centrality is alpha centrality of a network with bilateral trade intensity as weights as in Richmond (2015). Trade centrality ranking is the time series average ranking where rankings are normalized to the maximum number of countries in the sample. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

5 Calibrated LRR Model

As illustrated in Section (1.1), the base factor loadings provide a preference-free measure of exposure to common shocks. To demonstrate how the base factor loadings relate to the structural parameters of a long run risk model, we calibrate the two factor model in Section (1.1). Our calibration follows Colacito and Croce (2011), with the specific parameters given in Table (9).

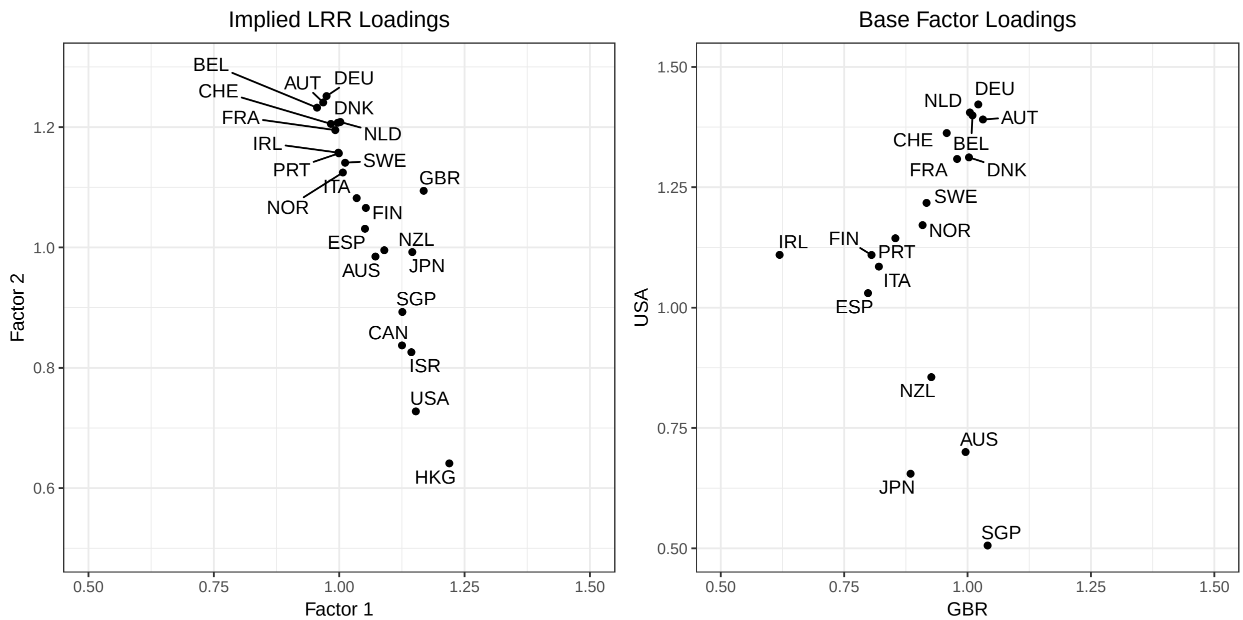
Table 9: Parameters of Calibrated LRR Model

β	.998
α	4.25
ψ	2
ρ	$1/\psi$
μ_c	0.0016
σ_c	0.0068
ϕ_e	.048
σ_x	$\phi_e \times \sigma_c$
ρ_x	.987

For this calibration we focus on the sample of 23 developed countries, which excludes the Euro area. Given that we assume there are two common factors in consumption growth, there are 46 exposures to the common factors, $\kappa^{g,*}$ for $g \in \{NA, E\}$. Using Proposition (4) and a set of the base factor loadings, we can recover the implied exposures to the common factors. For this calibration we target each dollar exchange rates's exposure to the dollar factor, each pound exchange rate's exposure to the pound factor, as well as the USD/JPY and GBP/JPY exposures to the JPY factor. These $46 = 22 + 22 + 2$ base factor loadings allow us to recover the 46 implied exposures to the common factors in consumption growth.

The left panel of Figure (3) displays the implied loadings on the two common factors. A clear pattern emerges that is consistent with our findings of gravity in the factor structure of exchange rates. European countries, other than the UK, have implied loadings that are very similar to each other — high loadings on the second factor and low loadings on the first. On the other hand, countries like the US and Canada have a high implied loading on the first factor and a low loading on the second factor. To illustrate the source of the hetereogenity in LRR factor exposures, the right panel of Figure (3) displays loadings on the USD and GBP factors. The substantial hetereogeneity in base factor loadings across the two base factors maps into the hetereogenity in loadings on the two underlying LRR factors of the model. It is important to note that while we calibrated the model to base factor loadings primarily from the USD and GBP, this does not necessarily imply that the underlying sources LRR shocks originate in the US and the UK. If we had a perfectly specified model, including the correct number of factors, we could calibrate to any set of base factor loadings and obtain the same implied exposures to the underlying sources of risk.

Figure 3: Implied Exposures to Common Global Factors and Base Factor Loadings



Calibrated exposures, $\kappa^{g,*}$ for $g \in \{1,2\}$, to common global factors in a two-factor long-run risk model and base factor loadings for the USD and GBP exchange rates. The $\kappa^{g,*}$ are calibrated to match base factor loadings in a long-run risk model with heterogeneous exposure to two global factors. Base factor loadings, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 23 developed countries, as classified by MSCI.

To better understand how this heterogeneity in exposure to the two factors relates to our gravity findings, we return to Assumption 2. This assumption stated that the weighted difference in factor exposures increases distance, where the weighted difference was given by:

$$\sum_{k=1}^{K} (\overline{\kappa_k^*} - \kappa_k)(\kappa_k^* - \kappa_k)\left(\sigma_k^g\right)^2 .$$

Using the calibrated values for the factor exposures in our two factor model, we can calculate this weighted difference for each of the country pairs and see it relates to our gravity variables. The results are in Table (10). Columns 1 and 3 present regressions of the weighted difference of factor exposures on gravity variables. For comparison, columns 2 and 4 present the same specification, but with base factor loadings on the left hand side. As predicted, the weighted difference in implied factor exposures is increasing in the distance between countries.

Table 10: Regressions of Weighted Difference in Factor Exposures on Gravity Variables

	Weighted κ	Loading	Weighted κ	Loading
Log Distance	0.016	0.052	0.015	0.056
	(2.244)	(2.769)	(2.000)	(3.058)
Shared Language			0.005	−0.074
			(0.567)	(−1.570)
Shared Legal			−0.018	−0.060
			(−2.044)	(−1.132)
Shared Border			0.010	0.021
			(0.709)	(0.488)
Colonial Link			−0.022	−0.282
			(−4.543)	(−6.327)
Resource Similarity			−0.045	−0.029
			(−1.074)	(−0.737)
Adj. R^2	0.096	0.142	0.122	0.299
Num. obs.	506	506	506	506

Regressions of weighted differences in implied factor exposures κ^* from a calibrated two-factor LRR model on gravity variables. κ^* are calibrated to match base factor loadings in a long-run risk model with heterogeneous exposure to global factors. Base factor loadings, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j} \Delta base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 23 developed countries, as classified by MSCI. Standard errors are clustered on country dyads using Aronow et al. (2015).

The fact that the weighted difference in factor exposures have a positive relation with distance is a direct consequence of the negative relation between base factor loadings and κ^* in Proposition (4) and that the base factor loadings are increasing in distance. As explained in Section (1.1), this result is intuitive: If countries are closer to each other it is plausible that they will be more exposed to similar risks than countries which are more distant from each other. This calibration illustrates how the base factor loadings provide a new way to measure exposure to global shocks in international asset pricing models: base factor loadings isolate exposure to common shocks, thus providing additional moments for international models of exchange rate determination to match, and also providing additional insight into the fundamental source of heterogeneity in exposure to global risk. Whether this degree of distance-dependent heterogeneity in exposures of consumption growth can be rationalized in a model with shipping costs is a question that we leave for future research.

6 Robustness

The gravity effects that we have documented are quite robust. The following sections present various robustness checks.

6.1 Developed Currencies

Table (B7) considers only the subset of developed countries, using the MSCI designation of developed countries. In this subsample, the distance effect is even stronger. In specifications (1)-(3), the effect of log distance on the loading is around 0.23, compared to 0.14. Some of the other variables are no longer enter significantly. Shared legal origin lowers the loading by more than 0.3 when pegs are removed. These variables jointly account for about 1/3 of the variation in the loadings.

6.2 All Currencies

Table (B8) presents results using data for all 162 countries in our sample. When we expand beyond the subset of MSCI developed and emerging countries, all gravity effects remain significant, but the coefficients are mitigated. A log point increase in distance increases the base factor loading by 3 bps, compared to 14 bps in the developed and emerging subset.

6.3 Fixed Effects

Table (B9) presents results with different fixed effects. Column (1) is without any fixed effects, column (2) has year fixed effects, column (2) has base-country year fixed effects, and column (4) has base-coutry year and year fixed effects. The key takeaway is that the different fixed effects do not affect the qualitative or quantitve implications of our gravity model.

6.4 Persistence

It could be the case that the increase in global integration has lowered the explanatory power of gravity variables on base factor loadings over time. To examine this, Figure (B1) plots 60-month rolling sample R-squared values of a regression of base factor loadings on log distance, common language, commodity distance, common legal origins, and common colonial origins. To ensure that we are examining the same set of countries over time, the sample is limited to 13 countries for which we have a balanced panel of exchange rates: Australia, Canada, Denmark, Germany, Hong Kong, Japan, New Zealand, Norway, Singapore, Sweden, Switzerland, United Kingdom, and United States. The German exchange rate becomes the Euro in January 1999. Interestingly, the R-squared remained quite stable until the 2008 financial crisis, when the explanatory power drops from above 40% to a low of 10%. The dramatic decline in global trade volumes had a major impact on the factor structure of exchange rates that cannot be explained by gravity variables. Since the crisis, the R-squared has gradually increased back to 23%.

6.5 Currency Unions

This section presents regressions for just the euro subset. Base factors are constructed only using real data on the subset of euro area countries. The results are from 1999-2014. Table (B11) reports the results. Even in this Euro subset, the real exchange rate co-variation is consistent with the gravity effects we have documented. In a univariate regression of the loadings on log distance, the slope coefficient is 0.13, similar to the effects we have documented in the full sample. Similarly, the coefficient on shared language is -0.29.

7 Conclusion

When Fed chairman Bernanke signaled an end to large-scale asset purchases in May 2013, some emerging market currencies subsequently depreciated by more than 25% against the USD, while other currencies did not depreciate at all (Nechio et al., 2014). What governs the differential response of currencies to a monetary policy shock, or any other shocks, in the U.S.? Are these mostly due to differences in policies and economic conditions across countries? Our paper shows that the differential response of currencies to these types of shocks are determined to a large extent by initial conditions that are completely outside of the control of monetary and fiscal policy. Furthermore, our findings suggest that models of exchange rate determination should generate a factor structure in exchange rates that is explained by measures of distance between countries.

References

Adler, M. and B. Dumas (1983). International portfolio choice and corporation finance: A synthesis. *Journal of Finance 38*(3), pp. 925–984.

Anderson, J. E. and E. van Wincoop (2004, September). Trade costs. *Journal of Economic Literature 42*(3), 691–751.

Aronow, P. M., C. Samii, and V. A. Assenova (2015, July). Cluster-robust variance estimation for dyadic data. Working Paper Yale University.

Backus, D., S. Foresi, and C. Telmer (2001). Affine models of currency pricing: Accounting for the forward premium anomaly. *Journal of Finance 56*, 279–304.

Backus, D. and G. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*, 297–316.

Bansal, R. and I. Shaliastovich (2013). A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies 26*(1), 1–33.

Bekaert, G. and C. R. Harvey (1995). Time-varying world market integration. *Journal of Finance 50 (2)*, 403–444.

Bekaert, G. and R. J. Hodrick (1992). Characterizing predictable components in excess returns on equity and foreign exchange markets. *The Journal of Finance 47*(2), 467–509.

Bekaert, G., R. J. Hodrick, and X. Zhang (2009). International stock return comovements. *Journal of Finance 64*, 2591–2626.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006). International risk-sharing is better than you think (or exchange rates are much too smooth). *Journal of Monetary Economics 53*, 671 – 698.

Brennan, M. J. and Y. Xia (2006). International Capital Markets and Foreign Exchange Risk. *Review of Financial Studies 19*(3), 753–795.

Burstein, A. and N. Jaimovich (2009). Understanding movements in aggregate and product-level real exchange rates.

Cameron, A. C., J. B. Gelbach, and D. L. Miller (2011). Robust inference with multiway clustering. *Journal of Business & Economic Statistics 29*(2).

Cameron, A. C. and D. L. Miller (2014). Robust inference for dyadic data.

Campbell, J. Y. and Y. Hamao (1992). Predictable stock returns in the united states and japan: A study of long-term capital market integration. *The Journal of Finance 47(1)*, pp. 43–69.

Chaney, T. (2013, August). The gravity equation in international trade: An explanation. Working Paper 19285, National Bureau of Economic Research.

Chen, Y. and K. Rogoff (2003). Commodity currencies. *Journal of International Economics 60*(1), 133 – 160. Emperical Exchange Rate Models.

Chen, Y.-C., K. S. Rogoff, and B. Rossi (2010). Can exchange rates forecast commodity prices?*. *The Quarterly Journal of Economics 125*(3), 1145–1194.

Colacito, R. and M. M. Croce (2011). Risks for the long run and the real exchange rate. *Journal of Political Economy 119*(1), 153–181.

Colacito, R., M. M. Croce, F. Gavazzoni, and R. C. Ready (2015). Currency risk factors in a recursive multi-country economy.

Cooper, I. and E. Kaplanis (1994). Home bias in equity portfolios, inflation hedging, and international capital market equilibrium. *The Review of Financial Studies 7*(1), 45–60.

Costinot, A. (2014). Trade theory with numbers: Quantifying the consequences of globalization. *Handbook of International Economics 4*, 197.

Coval, J. D. and T. J. Moskowitz (1999). Home bias at home: Local equity preference in domestic portfolios. *The Journal of Finance 54*(6), 2045–2073.

David, J. M., E. Henriksen, and I. Simonovska (2014, December). The risky capital of emerging markets. Working Paper 20769, National Bureau of Economic Research.

Desmet, K., I. Ortuno-Ortin, and R. Wacziarg (2012). The political economy of linguistic cleavages. *Journal of development Economics 97*(2), 322–338.

Eaton, J., S. Kortum, and B. Neiman (2016). Obstfeld and rogoff's international macro puzzles: a quantitative assessment. *Journal of Economic Dynamics and Control 72*, 5 – 23. Trade and macroeconomics: new approaches to enduring puzzles.

Eaton, J., S. Kortum, B. Neiman, and J. Romalis (2016, November). Trade and the global recession. *American Economic Review 106*(11), 3401–38.

Engel, C. (1999). Accounting for u.s. real exchange rate changes. *Journal of Political Economy 107*(3), pp. 507–538.

Engel, C. and J. H. Rogers (1996). How wide is the border? *The American Economic Review*, 1112–1125.

Engel, C. and K. D. West (2005). Exchange rates and fundamentals. *Journal of Political Economy 113*(3), 485–517.

Farhi, E. and X. Gabaix (2016). Rare disasters and exchange rates *. *The Quarterly Journal of Economics 131*(1), 1.

Ferson, W. E. and C. R. Harvey (1993). The risk and predictability of international equity returns. *Review of Financial Studies 6*, 527–566.

Flavin, T. J., M. J. Hurley, and F. Rousseau (2002). Explaining stock market correlation: A gravity model approach. *The Manchester School 70*(S1), 87–106.

Flood, R. P. and A. K. Rose (1995). Fixing exchange rates a virtual quest for fundamentals. *Journal of Monetary Economics 36*(1), 3 – 37.

Frankel, J. A. and A. K. Rose (1995). Chapter 33 empirical research on nominal exchange rates. Volume 3 of *Handbook of International Economics*, pp. 1689 – 1729. Elsevier.

Frankel, J. A. and A. K. Rose (1998). The endogenity of the optimum currency area criteria. *The Economic Journal 108*(449), 1009–1025.

French, K. R. and J. M. Poterba (1991). Investor diversification and international equity markets. *The American Economic Review 81*(2), 222–226.

Froot, K. A. and K. Rogoff (1995). Chapter 32 perspectives on ppp and long-run real exchange rates. Volume 3 of *Handbook of International Economics*, pp. 1647 – 1688. Elsevier.

Gopinath, G. and R. Rigobon (2008). Sticky borders. *The Quarterly Journal of Economics 123*(2), pp. 531–575.

Harvey, C. R. (1991). The world price of covariance risk. *Journal of Finance 46*, 111–117.

Harvey, C. R., B. Solnik, and G. Zhou (2002). What determines expected international asset returns? *Annals of Economics and Finance 3*(2), 249–298.

Hassan, T. (2013). Country size, currency unions, and international asset returns. *Journal of Finance 68*(6), 2269–2308.

Hau, H. (2002). Real exchange rate volatility and economic openness: Theory and evidence. *Journal of Money, Credit and Banking 34*(3), 611–630.

Head, K. and T. Mayer (2014). Chapter 3 - gravity equations: Workhorse,toolkit, and cookbook. In E. H. Gita Gopinath and K. Rogoff (Eds.), *Handbook of International Economics*, Volume 4 of *Handbook of International Economics*, pp. 131 – 195. Elsevier.

Head, K., T. Mayer, and J. Ries (2010). The erosion of colonial trade linkages after independence. *Journal of International Economics 81*(1), 1–14.

Hodrick, R. J. and M. Vassalou (2002). Do we need multi-country models to explain exchange rate and interest rate and bond return dynamics? *Journal of Economic Dynamics and Control 26*, 1275–1299.

Kollmann, R. (1995). Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance 14*(2), 191 – 211.

Leamer, E. E. and J. Levinsohn (1995). Chapter 26 international trade theory: The evidence. Volume 3 of *Handbook of International Economics*, pp. 1339 – 1394. Elsevier.

Leippold, M. and L. Wu (2007). Design and estimation of multi-currency quadratic models. *Review of Finance 11*(2), 167–207.

Lettau, M., M. Maggiori, and M. Weber (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics 114*(2), 197 – 225.

Lewis, J. B. and D. A. Linzer (2005). Estimating regression models in which the dependent variable is based on estimates. *Political Analysis 13*(4), 345–364.

Lewis, K. K. (1999). Trying to explain home bias in equities and consumption. *Journal of Economic Literature 37*(2), 571–608.

Lewis, K. K. and E. X. Liu (2015). Evaluating international consumption risk sharing gains: An asset return view. *Journal of Monetary Economics 71*(Supplement C), 84 – 98.

Lucey, B. M. and Q. Zhang (2010). Does cultural distance matter in international stock market comovement? evidence from emerging economies around the world. *Emerging Markets Review 11*(1), 62 – 78.

Lustig, H., N. Roussanov, and A. Verdelhan (2011). Common risk factors in currency markets. *Review of Financial Studies 24 (11)*, 3731–3777.

Lustig, H., N. Roussanov, and A. Verdelhan (2014). Countercyclical currency risk premia. *Journal of Financial Economics 111*(3), 527–553.

Lustig, H. and A. Verdelhan (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*(1), 89–117.

Lustig, H. N., A. Stathopoulos, and A. Verdelhan (2016). Nominal exchange rate stationarity and long-term bond returns.

Mayer, T. and S. Zignago (2011). Notes on cepii's distances measures: The geodist database.

Meese, R. A. and K. Rogoff (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics 14*, 3–24.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf (2012). Carry trades and global foreign exchange volatility. *The Journal of Finance 67*(2), 681–718.

Mussa, M. (1986). Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications. In *Carnegie-Rochester Conference series on public policy*, Volume 25, pp. 117–214. Elsevier.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics 123*(4), 1415–1464.

Nechio, F. et al. (2014). Fed tapering news and emerging markets. *FRBSF Economic Letter 6*, 014–06.

Obstfeld, M. and K. Rogoff (1995). Exchange rate dynamics redux. *Journal of Political Economy 25*, 338–390.

Obstfeld, M. and K. Rogoff (2000). The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual 15*, 339–390.

Parsley, D. C. and S.-J. Wei (2001). Explaining the border effect: the role of exchange rate variability, shipping costs, and geography. *Journal of International Economics 55*(1), 87 – 105.

Porta, R. L., F. Lopez-de Silanes, and A. Shleifer (2007). The economic consequences of legal origins. Technical report, National Bureau of Economic Research.

Portes, R. and H. Rey (2005). The determinants of cross-border equity flows. *Journal of International Economics 65*(2), 269 – 296.

Portes, R., H. Rey, and Y. Oh (2001). Information and capital flows: The determinants of transactions in financial assets. *European Economic Review 45*(4), 783 – 796. 15th Annual Congress of the European Economic Association.

Ready, R., N. Roussanov, and C. Ward (2013). Commodity trade and the carry trade: A tale of two countries. Technical report, National Bureau of Economic Research.

Richmond, R. (2015, August). Trade network centrality and the currency carry trade. Working Paper UCLA Anderson School of Management.

Sarno, L., P. Schneider, and C. Wagner (2012). Properties of foreign exchange risk premiums. *Journal of Financial Economics 105*(2), 279 – 310.

Shambaugh, J. C. (2004). The effect of fixed exchange rates on monetary policy. *The Quarterly Journal of Economics*, 301–352.

Solnik, B. (1983). International arbitrage pricing theory. *The Journal of Finance 38(2)*, 449–457.

Spolaore, E. and R. Wacziarg (2009). The diffusion of development. *The Quarterly Journal of Economics 124*(2), 469–529.

Tenreyro, S. (2007). On the trade impact of nominal exchange rate volatility. *Journal of Development Economics 82*(2), 485–508.

Tinbergen, J. (1962). Shaping the world economy: suggestions for an international trade policy. *Twentieth Century Fund Press, New York*.

Verdelhan, A. (2010). A habit-based explanation of the exchange rate risk premium. *Journal of Finance 65*(1), 123–145.

Verdelhan, A. (2015). The share of systematic variation in bilateral exchange rates. *Journal of Finance (Forthcoming)*.

Wheatley, S. (1988). Some tests of international equity integration. *Journal of Financial Economics 21(2)*, 177–212.

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica: Journal of the Econometric Society*, 817–838.

A Model Appendix: For Online Publication

A.1 Long Run Risks Model

We first use a log-linear approximation of the wealth return. We then assume that the price-consumption ratio is linear in the state variables. Finally, we check this conjecture and compute the corresponding coefficients by using the log-linear Euler equation.

Let us look again at the Campbell-Shiller decomposition. Start from $1 = (R^A_{t+1})^{-1} R^A_{t+1} = (R^A_{t+1})^{-1}(P_{t+1} + C_{t+1})/P_t$. Multiply both sides by the price-consumption ratio P_t/C_t:

$$\frac{P_t}{C_t} = (R^A_{t+1})^{-1}(1 + \frac{P_{t+1}}{C_{t+1}})\frac{C_{t+1}}{C_t}.$$

Taking logs leads to:

$$p_t - c_t = -r^A_{t+1} + \Delta c_{t+1} + \log(1 + e^{p_{t+1} - c_{t+1}}).$$

A first-order Taylor approximation of the last term around the mean price-consumption ratio P/C gives:

$$\begin{aligned} p_t - c_t &= -r^A_{t+1} + \Delta c_{t+1} + \log(1 + \frac{P}{C}) + \frac{P/C}{1 + P/C}(p_{t+1} - c_{t+1} - (p - c)), \\ &\simeq -r^A_{t+1} + \Delta c_{t+1} + \kappa_0 + \kappa_1(p_{t+1} - c_{t+1}), \end{aligned}$$

where $\kappa_1 = exp(p-c)/(1 + exp(p-c))$ and $\kappa_0 = \log(1 + exp(p-c)) - (p-c)\kappa_1$. Define the log price-consumption ratio z_t as $p_t - c_t$. We now have a log-linear approximation of the return on wealth:

$$r^A_{t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}.$$

Guess and verify that the log price-consumption ratio z_t is linear in the state variables

$$z_t = A^0 + A^{NA} x^{NA}_t + A^E x^E_t$$

Assume joint conditional normality of consumption growth, x, and the variance of consumption growth. Verify the conjecture above from the Euler equation:

$$E_t[e^{m^*_{t+1} + r^A_{t+1}}] = 1 \Leftrightarrow E_t\left[m^*_{t+1}\right] + E_t\left[r^A_{t+1}\right] + \frac{1}{2}Var_t\left[m^*_{t+1}\right] + \frac{1}{2}Var_t\left[r^A_{t+1}\right] + Cov_t\left[m^*_{t+1}, r^A_{t+1}\right] = 0$$

. With Epstein-Zin preferences, we have shown that the log SDF is a function of log consumption changes and the log total wealth return:

$$m_{t+1}^{*}=\frac{1-\alpha}{1-\rho}\log\varphi-\frac{1-\alpha}{1-\rho}\rho\Delta c_{t+1}^{*}+\left(\frac{1-\alpha}{1-\rho}-1\right)r_{t+1}^{*,A}.$$

Substituting in the expression for the log total wealth return r^{A} into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned}
m_{t+1}^{*}-E_{t}\left[m_{t+1}^{*}\right] &= -\lambda_{m,\eta}^{*}\sigma\eta_{t+1}^{*}-\lambda_{m,e}^{NA,*}\sigma e_{t+1}^{NA}-\lambda_{m,e}^{E,*}\sigma e_{t+1}^{E}\\
E_{t}\left[m_{t+1}^{*}\right] &= m_{0}\\
&\quad+[-\frac{1-\alpha}{1-\rho}\rho\kappa^{NA,*}+\frac{\rho-\alpha}{1-\rho}(\kappa^{NA,*}+A^{NA,*}(\kappa_{1}\rho_{x}-1))]x_{t}^{NA}\\
&\quad+[-\frac{1-\alpha}{1-\rho}\rho\kappa^{E,*}+\frac{\rho-\alpha}{1-\rho}(\kappa^{E,*}+A^{E,*}(\kappa_{1}\rho_{x}-1))]x_{t}^{E}\\
Var_{t}\left[m_{t+1}^{*}\right] &= \left(\left(\lambda_{m,\eta}^{*}\right)^{2}\sigma^{2}+\left(\lambda_{m,e}^{NA,*}\right)^{2}\sigma^{2}+\left(\lambda_{m,e}^{E,*}\right)^{2}\sigma^{2}\right)
\end{aligned}$$

where $\lambda_{m,\eta}^{*}=\alpha$, $\lambda_{m,e}^{NA,*}=\frac{\alpha-\rho}{1-\rho}B^{NA,*}$, $\lambda_{m,e}^{E,*}=\frac{\alpha-\rho}{1-\rho}B^{E,*}$, $B^{NA,*}=\kappa_{1}A^{NA,*}\varphi_{e}$, and $B^{E,*}=\kappa_{1}A^{E,*}\varphi_{e}$.

Likewise, using the Campbell-Shiller approximation of r^{A}, we compute innovations in the consumption claim return, and its conditional mean and variance:

$$\begin{aligned}
r_{t+1}^{*,A}-E_{t}\left[r_{t+1}^{*,A}\right] &= \sigma\eta_{t+1}^{*}+B^{NA,*}\sigma e_{t+1}^{NA}+B^{E,*}\sigma e_{t+1}^{E}\\
E_{t}\left[r_{t+1}^{*,A}\right] &= r_{0}+[\kappa^{NA,*}+A^{NA,*}(\kappa_{1}\rho_{x}-1)]x_{t}^{NA}+[\kappa^{E,*}+A^{E,*}(\kappa_{1}\rho_{x}-1)]x_{t}^{E}\\
Var_{t}\left[r_{t+1}^{*,A}\right] &= \left(1+(B^{NA,*})^{2}+(B^{E,*})^{2}\right)\sigma^{2}.
\end{aligned}$$

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations:

$$Cov_{t}\left[m_{t+1}^{*},r_{t+1}^{*,A}\right]=\left(-\sigma^{2}\lambda_{m,\eta}^{*}-\sigma^{2}\lambda_{m,e}^{NA,*}B^{NA,*}-\sigma^{2}\lambda_{m,e}^{E,*}B^{E,*}\right)$$

Using the method of undetermined coefficients , we can solve for the constants:

$$\begin{aligned}
A_{1}^{NA,*} &= \kappa^{NA,*}\frac{1-\rho}{1-\kappa_{1}\rho_{x}},\\
A_{1}^{E,*} &= \kappa^{E,*}\frac{1-\rho}{1-\kappa_{1}\rho_{x}},
\end{aligned}$$

The log price-consumption ratio z_t is linear in the state variable x_t^g: $z_t^*=A_0^*+\kappa^*\frac{1-\rho}{1-\kappa_1\rho_x}x_t^g$.

We use A_1^* to denote the loading on the state variable: $A_1^* = \kappa^* \frac{1-\rho}{1-\kappa_1\rho_x}$. Using the expression for the innovation to the log SDF $m_{t+1}^* - E_t\left[m_{t+1}^*\right]$, we can back out the innovation to the equilibrium exchange rate as:

$$\Delta s_{t+1} - E_t[\Delta s_{t+1}] = \left(-\lambda_{m,\eta}\sigma\eta_{t+1} - \lambda_{m,e}^{NA}\sigma e_{t+1}^{NA} - \lambda_{m,e}^{E}\sigma e_{t+1}^{E}\right) - \left(-\lambda_{m,\eta}^{*}\sigma\eta_{t+1}^{*} - \lambda_{m,e}^{NA,*}\sigma e_{t+1}^{NA} - \lambda_{m,e}^{E,*}\sigma e_{t+1}^{E}\right),$$

This expression in turn can be simplified as

$$\Delta s_{t+1} - E_t[\Delta s_{t+1}] = \alpha(\sigma\eta_{t+1}^* - \sigma\eta_{t+1}) + \frac{\alpha-\rho}{1-\kappa_1\rho_x}\kappa_1\varphi_e\sigma\left[(\kappa^{NA,*} - \kappa^{NA})e_{t+1}^{NA} + (\kappa^{E,*} - \kappa^{E})e_{t+1}^{E}\right].$$

A.2 Statistical Factor Models of Exchange Rates

We consider a simple, statistical (latent) factor model for exchange rate variation. There are multiple latent factors driving exchange rate variation:

$$\Delta s_{i,j,t} = \alpha_{i,j} + \boldsymbol{\gamma}_{i,j}'\boldsymbol{f}_t + u_{i,j,t}\ , \tag{9}$$

where $s_{i,j,t}$ denotes the time t log exchange rate in units of currency j per unit of currency i and $\boldsymbol{f}_t$ denotes a $K \times 1$ vector of orthogonal factors. An increase in $s_{i,j,t}$ implies an appreciation of currency i relative to currency j. Collecting terms, we can write this factor model in vector notation: $\Delta \boldsymbol{s}_{i,t} = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_i\boldsymbol{f}_t + \boldsymbol{u}_{i,t}$, where $\boldsymbol{\Gamma}_i$ is the $N \times K$ matrix of loadings. The variance-covariance matrix of exchange rates is $\boldsymbol{\Gamma}_i\boldsymbol{\Gamma}_i' + \boldsymbol{\Sigma}_{e,i}$. From triangular arbitrage, we know that $\Delta s_{i,j,t} - \Delta s_{i,k,t} = \Delta s_{k,j,t}$. That implies the following restriction on the loadings: $\gamma_{i,j}' - \gamma_{i,k}' = \gamma_{k,j}'$. Triangular arbitrage implies that the matrix of loadings satisfy the following restrictions : $\boldsymbol{e}_j'\boldsymbol{\Gamma}_i - \boldsymbol{e}_k'\boldsymbol{\Gamma}_i = \boldsymbol{e}_j'\boldsymbol{\Gamma}_k$, and the disturbances satisfy the following restrictions: $\boldsymbol{e}_j'\boldsymbol{u}_i - \boldsymbol{e}_k'\boldsymbol{u}_i = \boldsymbol{e}_j'\boldsymbol{u}_k$, where $\boldsymbol{e}_j'$ is an $N \times 1$ vector of zero with a one in the j-th position. Hence, the variance-covariance matrix is singular.

The latent factors can include global FX factors such as the dollar factor and the carry trade factors. Our setup also allows for N local factors in $\boldsymbol{f}_t$, i.e. factors that are specific to country i and only affects bilateral exchange rates between i and some other country j, but have no effect on other bilateral exchange rates that do not involve i. These are fixed effects for home country and time. Factor i is local to country i if and only if $\gamma_{i,j}'(i) = \gamma_{i,k}'(i)$, implying that $\gamma_{k,j}'(i) = 0$ for all k, j. Complete market models give rise to local factors if their SDFs are subject to country-specific shocks.

Each base factor is a different linear combination of the underlying factors $base_{i,t} = \boldsymbol{\delta}_i'\boldsymbol{f}_t$, given by

$$\Delta base_{i,t} = \frac{1}{N-1}\sum_{k\neq j}\Delta s_{i,k,t} = \frac{1}{N-1}\sum_{k\neq j}\gamma'_{i,k}\boldsymbol{f}_t + \frac{1}{N-1}\sum_{k\neq j}u_{i,k,t}.$$

As $N \rightarrow \infty$, the L.L.N. implies that last term converges to zero. The base factor eliminates idiosyncratic noise. We construct the base factor for country k, which is a different linear combination of the underlying factors:

$$\Delta base_{k,t} = \frac{1}{N-1}\sum_{j\neq k}\Delta s_{k,j,t} = \frac{1}{N-1}\sum_{j\neq k}\left(\boldsymbol{\gamma}'_{i,j} - \boldsymbol{\gamma}'_{i,k}\right)\boldsymbol{f}_t + \frac{1}{N-1}\sum_{j\neq k}(u_{i,j,t} - u_{i,k,t}).$$

Note that the local i-factor will drop out from the base factor. That follows immediately from the restriction on the factor loadings. Since this is a different linear combination, each country's loadings on the new base factor will differ as well. Why examine different base currencies? We cannot identify the $N \times K$ coefficients from only $N-1$ different independent loadings with respect to one base factor. Hence, we exploit the entire cross-section. If there are N local factors, then each base currency adds novel information.

Simple Example with Two Factors To build some intuition, we consider a simple example with two exchange rate factors:

$$\Delta s_{i,j,t} = \alpha_{i,j} + \boldsymbol{\gamma}_{i,j}(1)f_{t,1} + \boldsymbol{\gamma}_{i,j}(2)f_{t,2} + u_{i,j,t}\ . \tag{10}$$

We construct the base factor,

$$\Delta base_{i,t} = \frac{1}{N-1}\sum_{k\neq j}\Delta s_{i,k,t} = \frac{1}{N-1}\sum_{k\neq j}\left(\boldsymbol{\gamma}_{i,k}(1)f_{t,1} + \boldsymbol{\gamma}_{i,k}(2)f_{t,2}\right) + \frac{1}{N-1}\sum_{k\neq j}u_{i,k,t}$$

As $N \rightarrow \infty$, the L.L.N. implies that last term converges to zero. The base factor eliminates idiosyncratic noise.

In our paper, the object of interest is the slope coefficient in a projection of the exchange rate changes on the base factor. For large N, this slope coefficient is given by:

$$\varphi^*_{i,j} = \frac{\boldsymbol{\gamma}_{i,j}(1)\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}(1)\sigma^2_f(1) + \boldsymbol{\gamma}_{i,j}(2)\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}(2)\sigma^2_f(2)}{\frac{1}{N-1}\left(\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}(1)\right)^2\sigma^2_f(1) + \frac{1}{N-1}\left(\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}(2)\right)^2\sigma^2_f(2)}, \tag{11}$$

where we have used the orthogonality of the factors. The slope coefficient only depends on the currency factor loadings and the volatility of the factors. When we switch to a new base country, the change in spot exchange rates are given by:

$$\Delta s_{kjt} = (\alpha_{i,j} - \alpha_{i,k}) + \left(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(1)\right) f_{t,1} + (\gamma_{i,j}(2) - \gamma_{i,k}(2))\, f_{t,2} + u_{i,j,t} - u_{i,k,t}. \quad (12)$$

We construct the base factor for country k, which is a different linear combination of the underlying factors:

$$\Delta base_{k,t} = \frac{1}{N-1}\sum_{j\neq k}\left((\gamma_{i,j}(1) - \gamma_{i,k}(1))\, f_{t,1} + (\gamma_{i,j}(2) - \gamma_{i,k}(2))\, f_{t,2}\right) + \frac{1}{N-1}\sum_{j\neq k}(u_{i,j,t} - u_{i,k,t})$$

As $N \to \infty$, the L.L.N. implies that last term converges to zero. The new slope factor in a projection of the exchange rate changes on the base factor is given by:

$$\begin{aligned}
\varphi^*_{k,j} &= \frac{\left(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(1)\right)\sum_{l\neq j}\left(\boldsymbol{\gamma}_{i,l}(1) - \boldsymbol{\gamma}_{i,k}(1)\right)\sigma_f^2(1)}{\frac{1}{N-1}\left(\sum_{k\neq j}(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(1))\right)^2\sigma_f^2(1) + \frac{1}{N-1}\left(\sum_{k\neq j}(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(2))\right)^2\sigma_f^2(2)} \\
&+ \frac{\left(\boldsymbol{\gamma}_{i,j}(2) - \boldsymbol{\gamma}_{i,k}(2)\right)\sum_{l\neq j}\left(\boldsymbol{\gamma}_{i,l}(2) - \boldsymbol{\gamma}_{i,k}(2)\right)\sigma_f^2(2)}{\frac{1}{N-1}\left(\sum_{k\neq j}(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(1))\right)^2\sigma_f^2(1) + \frac{1}{N-1}\left(\sum_{k\neq j}(\boldsymbol{\gamma}_{i,j}(1) - \boldsymbol{\gamma}_{i,k}(2))\right)^2\sigma_f^2(2)}
\end{aligned}$$

In general, there is no simple mapping from one set of base factor loadings to another, because the new base factor is a different linear combination of the fundamental exchange rate factors. However, suppose that the first factor is country i's local factor. Then k's base factor is not exposed to i's local factor:

$$\Delta base_{k,t} = \frac{1}{N-1}\sum_{j\neq k}\Delta s_{k,j,t} = \frac{1}{N-1}\sum_{j\neq k}(\gamma_{i,j}(2) - \gamma_{i,k}(2))\, f_{t,2} + \frac{1}{N-1}\sum_{j\neq k}u_{i,k,t}$$

and the loadings only measure covariance with the second factor:

$$\varphi^*_{k,j} = \left(\boldsymbol{\gamma}_{i,j}(2) - \boldsymbol{\gamma}_{i,k}(2)\right)\frac{\sum_{l\neq j}\left(\boldsymbol{\gamma}_{i,l}(2) - \boldsymbol{\gamma}_{i,k}(2)\right)}{\frac{1}{N-1}\left(\sum_{l\neq j}\left(\boldsymbol{\gamma}_{i,l}(2) - \boldsymbol{\gamma}_{i,k}(2)\right)\right)^2}$$

Hence, local factors to country i are eliminated when we switch base factor.

Simple Example with Single Factor Suppose that there is a single latent factor (e.g. the dollar factor) driving all of the currency variation; we can easily derive the loadings for any bilateral exchange rate. Suppose we switch to a new base currency k. Theloadings on base factor i are given by:

$$\varphi^*_{i,j} = \gamma_{i,j}\frac{\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}}{\frac{1}{N-1}\left(\sum_{k\neq j}\boldsymbol{\gamma}_{i,k}\right)^2}$$

The loadings on base factor k are given by:

$$\varphi^*_{k,j} = \gamma_{i,j}\frac{\sum_{l\neq j}(\gamma_{i,l}-\gamma_{i,k})}{\frac{1}{N-1}\left(\sum_{l\neq j}(\gamma_{i,l}-\gamma_{i,k})\right)^2} - \gamma_{i,k}\frac{\sum_{l\neq j}(\gamma_{i,l}-\gamma_{i,k})}{\frac{1}{N-1}\left(\sum_{l\neq j}(\gamma_{i,l}-\gamma_{i,k})\right)^2},$$

which is an affine transformation of the $\varphi^*_{i,j}$. There is no additional information from switching to a different base currency in a single factor world. Essentially, the same single factor model applies for base currency k:

$$\Delta s_{kjt} = (\alpha_{i,j}-\alpha_{i,k}) + (\gamma_{i,j}-\gamma_{i,k}) f_t + u_{i,j,t} - u_{i,k,t}. \tag{13}$$

In this single factor world, we only really need to analyze one base currency. The new slope coefficients $\gamma_{k,j} = (\gamma_{i,j}-\gamma_{i,k})$ can be backed out from the other ones. A single factor specification counterfactually implies that the bilateral exchange rate for equidistant countries from the base country (e.g. the U.S.) does not load on the (dollar) factor.

B Data Appendix: For Online Publication

B.1 FX and CPI Data

Spot rates in foreign currency per US dollar are from Global Financial Data (GFD). The sample is monthly from January, 1973 to December 2014 for 162 countries: Afghanistan, Albania, Algeria, Angola, Argentina, Armenia, Aruba, Australia, Austria, Azerbaijan, Bahamas, Bahrain, Bangladesh, Barbados, Belarus, Belgium, Belize, Bermuda, Bhutan, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Brunei Darussalam, Bulgaria, Burundi, Cabo Verde, Cambodia, Canada, Cayman Islands, Chile, China, Colombia, Comoros, Congo, Costa Rica, Croatia, Cuba, Cyprus, Czech Republic, Denmark, Djibouti, Dominican Republic, Ecuador, Egypt, El Salvador, Eritrea, Estonia, Ethiopia, Europe, Fiji, Finland, France, Gambia, Georgia, Germany, Ghana, Greece, Guatemala, Guinea, Guyana, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Korea, Kuwait, Kyrgyzstan, Lao People's Democratic Republic, Latvia, Lebanon, Lesotho, Liberia, Libya, Lithuania, Luxembourg, Macao, Macedonia, Madagascar, Malawi, Malaysia, Maldives, Malta, Mauritania, Mauritius, Mexico, Moldova, Mongolia, Morocco, Mozambique, Myanmar, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Nigeria, Norway, Oman, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Russian Federation, Rwanda, Samoa, Sao Tome and Principe, Saudi Arabia, Serbia, Seychelles, Sierra Leone, Singapore, Slovakia, Slovenia, Somalia, South Africa, Spain, Sri Lanka, Sudan, Suriname, Swaziland, Sweden, Switzerland, Syrian Arab Republic, Taiwan, Tajikistan, Tanzania, Thailand, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Arab Emirates, United Kingdom, Uruguay, Uzbekistan, Vanuatu, Venezuela, Viet Nam, Yemen, Zambia, and Zimbabwe.

Spot rates for countries which adopt the euro are omitted after the adoption date. The euro series starts on January 1, 1999.

CPI data is from GFD and is used to calculate real exchange rate changes. For countries which only provide quarterly CPI data, we interpolate a monthly series. CPI observations where month-over-month continuously compounded inflation is greater than 50% are omitted. We also omit Armenia, Ukraine, Herzegovina, Serbia, Nicaragua, Peru, and Brazil from the CPI data due to hyperinflation episodes.

Country classifications (developed, emerging, and frontier) are from MSCI [11] as of August 2015.

B.2 Gravity Data

Below is a description and source for each of the gravity variables in our dataset.

Distance — Population weighted average distance in kilometers between large cities' of each country pair (Mayer and Zignago (2011)).

Shared Language — Common language is 1 if a language is spoken by over 9% of the population in both countries (Mayer and Zignago (2011)).

Shared Legal — Dummy variable from a classification of countries' legal origins. See Porta et al. (2007) for a description and discussion.

Colonial Link — A dummy variable which is 1 if countries have shared a common colonizer after 1945. See Mayer and Zignago (2011).

Resource similarity — We obtain a list of natural resources by country from the CIA world factbok[12]. Using this list, we construct vectors of dummy variables — 1 if a country has the resource, 0 otherwise. Natural resource similarity between two countries is the cosine similarity of the vectors of resource dummy variables.

Linguistic similarity — Population weighted measure of linguistic proximity based upon language trees. A higher value implies that the average language spoken within the two countries diverged more recently. Data is from Desmet et al. (2012).

[11] Available at https://www.msci.com/market-classification

[12] Available at https://www.cia.gov/library/publications/the-world-factbook/fields/2111.html

Genetic distance — Weighted genetic distance between population subgroups within country pairs. Genetic distance is calculated off of differences in allele frequency. A higher value implies that the population within the two countries diverged genetically at a more recent date. The data is from Spolaore and Wacziarg (2009).
Peg Dummy — A currency is considered pegged if the bilateral exchange rate volatility is less than 2% in two consecutive years. The peg dummy is 1 if either currency was pegged to the other or both currencies were pegged to the same currency at any point in the sample. For the 5-year rolling samples, the peg dummy is 1 if either currency was pegged to other or they were pegged same currency at any point in the prior 6 years. The data on pegs is from Shambaugh (2004).

B.3 Calculation of Standard Errors

The triangular arbitrage condition for exchange rates requires careful calculation of standard errors in our regressions. Consider the general factor model in Equation (9):

$$\Delta s_{i,j,t} = \alpha_{i,j} + \boldsymbol{\gamma}'_{i,j}\boldsymbol{f}_t + e_{i,j,t}. \tag{14}$$

From triangular arbitrage, $\Delta s_{i,k} = \Delta s_{i,j} - \Delta s_{k,j}$, which implies $\boldsymbol{\gamma}_{i,k} = \boldsymbol{\gamma}_{i,j} - \boldsymbol{\gamma}_{k,j}$. This relation is true for any factors $\boldsymbol{f}$, including base factors, which are a linear combination of the underlying factors. This implies that *base* factor loadings may be correlated if they contain the same base or foreign country. As a result, there may be correlation in the errors in our primary regression specifications:

$$\varphi^*_{i,j} = \delta + \lambda G_{i,j} + e_{i,j}.$$

We accommodate for this by using dyadic clustering as in Cameron and Miller (2014) and Aronow et al. (2015). The latter paper uses the multi-way clustering algorithm of Cameron et al. (2011), which we apply in this paper. These standard errors allow for arbitrary correlation when an observation contains the same country — whether base or foreign. Specifically, we assume that

$$E\left[e_{i,j}e_{i',j'}|G_{i,j}, G_{i',j'}\right] = 0 \text{ unless } i = i' \text{ or } j = j' \text{ or } i = j' \text{ or } j = i'.$$

Table (B1) illustrates the importance of correctly estimating the standard errors. Columns 1 and 2 only cluster on base country or foreign country respectively. Column 3 clusters on both base country and foreign country. All three of these columns have smaller standard error estimates than column 4 which uses dyadic clustering. Clustering on base country and foreign country (column 3) produces standard errors that are closest to the dyadic clustering, consistent with the findings of Cameron and Miller (2014).

Table B1: Rolling Sample Regressions with Nominal Factor loadings (MSCI Developed and Emerging Subset) Comparing Different Variance Estimates

	Base Cluster	Foreign Cluster	Both Cluster	Dyad Cluster
Log Distance	0.139	0.139	0.139	0.139
	(0.023)	(0.021)	(0.030)	(0.040)
Shared Language	−0.120	−0.120	−0.120	−0.120
	(0.031)	(0.026)	(0.035)	(0.040)
Shared Legal	−0.020	−0.020	−0.020	−0.020
	(0.020)	(0.022)	(0.024)	(0.030)
Shared Border	−0.126	−0.126	−0.126	−0.126
	(0.041)	(0.037)	(0.041)	(0.049)
Colonial Link	−0.278	−0.278	−0.278	−0.278
	(0.057)	(0.053)	(0.065)	(0.084)
Resource Similarity	−0.165	−0.165	−0.165	−0.165
	(0.049)	(0.042)	(0.051)	(0.063)
Within R^2	0.114	0.114	0.114	0.114
Num. obs.	61130	61130	61130	61130

Regressions $\varphi^*_{i,j,t} = \delta + \kappa_t + \lambda \boldsymbol{G}_{i,j} + e_{i,j}$ of base factor loadings on gravity variables. $G_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t} \Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Standard errors clustered on base country, foreign country, or both using Cameron et al. (2011)). Standard errors are clustered on country dyads using Aronow et al. (2015).

Table B2: Rolling Sample Summary Statistics

	N	Mean	Median	Sd	Min	Max
Loading	61,260	0.92	0.98	0.48	-4.17	4.84
Loading (Real)	47,613	0.92	0.97	0.44	-3.13	5.78
R-squared	61,236	0.48	0.49	0.30	0.00	1.00
R-squared (Real)	47,613	0.47	0.49	0.28	0.00	1.00
Log Dist	86,715	8.62	9.00	0.93	5.08	9.88
Common Language	86,715	0.13	0.00	0.34	0.00	1.00
Shared Border	86,715	0.04	0.00	0.20	0.00	1.00
Resource Similarity	86,715	0.24	0.23	0.17	0.00	0.82
Linguistic Proximity	40,184	1.06	0.22	2.23	0.00	15.00
Genetic Distance	42,956	0.72	0.78	0.52	0.00	2.67
Colonial Linkage	86,715	0.02	0.00	0.14	0.00	1.00
Peg Dummy	83,160	0.12	0.00	0.33	0.00	1.00

Summary statistics of the factor loadings and gravity data. Factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t}\Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

Table B3: Correlation of 1st Principal Components and Base Factors by Country

Base	Correlation
Australia	-1.00
Brazil	1.00
Canada	0.99
Chile	1.00
China	0.99
Colombia	1.00
Czech Republic	1.00
Denmark	0.99
Egypt	-1.00
Germany	0.99
Hong Kong	0.99
Hungary	1.00
India	0.99
Indonesia	-1.00
Israel	1.00
Japan	1.00
Korea	1.00
Malaysia	-0.99
Mexico	-1.00
New Zealand	1.00
Norway	0.99
Peru	-0.98
Philippines	-0.99
Poland	1.00
Qatar	-0.99
Russian Federation	-1.00
Singapore	0.86
South Africa	1.00
Sweden	0.99
Switzerland	1.00
Taiwan	0.95
Thailand	1.00
Turkey	1.00
United Arab Emirates	0.99
United Kingdom	0.99
United States	-0.99

For each base currency i, the 1st p.c. of all bilateral exchange rate changes $\Delta s_{i,j,t}$ is computed. The base factor $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial for 24 developed and 23 emerging countries, as classified by MSCI.

Table B4: Full Sample Regressions with Nominal and Real Base Factor loadings

	Nominal (1)	Real (1)	Nominal (2)	Real (2)
Log Distance	0.159	0.139	0.130	0.112
	(0.036)	(0.033)	(0.028)	(0.024)
Shared Language	−0.130	−0.143	−0.111	−0.126
	(0.043)	(0.036)	(0.039)	(0.032)
Shared Legal	−0.025	−0.041	−0.007	−0.025
	(0.027)	(0.029)	(0.028)	(0.031)
Shared Border	−0.092	−0.121	−0.032	−0.065
	(0.038)	(0.047)	(0.041)	(0.052)
Colonial Link	−0.038	0.017	−0.048	0.008
	(0.045)	(0.062)	(0.057)	(0.080)
Resource Similarity	−0.084	−0.070	−0.046	−0.035
	(0.057)	(0.049)	(0.080)	(0.067)
Peg Dummy			−0.239	−0.222
			(0.061)	(0.054)
R^2	0.286	0.243	0.376	0.320
Num. obs.	1640	1640	1640	1640

Regressions $\varphi^*_{i,j} = \delta + \lambda G_{i,j} + e_{i,j}$ of base factor loadings on gravity variables. $G_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Real exchange rate changes include relative differences in inflation. Standard errors are clustered on country dyads using Aronow et al. (2015).

Table B5: Rolling Sample Regressions with Nominal Factor loadings (US Base Factor Only)

	All (1)	All (2)	All (3)	No Pegs
Log Distance	−0.095	−0.188	0.030	−0.008
	(0.228)	(0.188)	(0.092)	(0.129)
Shared Language		−0.523	−0.516	−0.737
		(0.176)	(0.120)	(0.181)
Shared Legal		0.020	0.027	0.191
		(0.206)	(0.122)	(0.201)
Resource Similarity		−0.136	−0.025	−0.015
		(0.433)	(0.289)	(0.345)
Peg Dummy			−0.866	
			(0.110)	
Within R^2	0.003	0.120	0.345	0.204
Num. obs.	1500	1500	1462	1146

Regressions $\varphi^{base}_{\$,j,t} = \alpha_{\$,j} + \kappa_t + \varphi \boldsymbol{G}_{\$,j} + e_{\$,j}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{\$,j}$ is a set of gravity variables. Base factor loadings, $\varphi^{base}_{\$,j,t}$, are from 60-month rolling regressions $\Delta s_{\$,j,\tau} = \alpha_{\$,j} + \varphi^{base}_{\$,j,t} base_{\$,\tau} + e_{\$,j,\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{\$,t}$ is the average appreciation of of the US dollar at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Standard errors are clustered on foreign country using Cameron et al. (2011)).

Table B6: Rolling Sample Regressions with Nominal and Real Base Factor loadings

	Nominal (1)	Real (1)	Nominal (2)	Real (2)
Log Distance	0.171	0.154	0.151	0.136
	(0.039)	(0.035)	(0.033)	(0.029)
Shared Language	−0.157	−0.163	−0.126	−0.134
	(0.047)	(0.045)	(0.038)	(0.036)
Shared Legal	−0.023	−0.035	−0.037	−0.046
	(0.033)	(0.033)	(0.030)	(0.032)
Shared Border	−0.080	−0.084	−0.023	−0.032
	(0.052)	(0.048)	(0.042)	(0.037)
Colonial Link	−0.110	−0.094	−0.084	−0.077
	(0.050)	(0.045)	(0.035)	(0.031)
Resource Similarity	−0.100	−0.100	−0.090	−0.093
	(0.048)	(0.054)	(0.063)	(0.063)
Peg Dummy			−0.445	−0.412
			(0.047)	(0.045)
Within Adj. R^2	0.160	0.156	0.226	0.217
Num. obs.	47493	47493	45002	45002

Regressions $\varphi^*_{i,j,t} = \delta + \kappa_t + \lambda \boldsymbol{G}_{i,j} + e_{i,j}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t} \Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Real exchange rate changes include relative differences in inflation. Standard errors are clustered on country dyads using Aronow et al. (2015).

Table B7: Rolling Sample Regressions with Nominal Factor loadings (MSCI Developed Countries)

	All (1)	All (2)	All (3)	All (4)	No Pegs
Log Distance	0.222	0.233	0.222	0.172	0.176
	(0.043)	(0.049)	(0.040)	(0.030)	(0.031)
Shared Language			−0.115	−0.101	−0.047
			(0.110)	(0.084)	(0.084)
Shared Legal		−0.231	−0.231	−0.233	−0.292
		(0.123)	(0.118)	(0.096)	(0.103)
Shared Border		0.074	−0.007	0.022	0.014
		(0.041)	(0.108)	(0.074)	(0.064)
Colonial Link		−0.076	−0.267	−0.316	−0.316
		(0.097)	(0.086)	(0.072)	(0.080)
Resource Similarity		−0.126	−0.048	−0.083	−0.090
		(0.133)	(0.141)	(0.131)	(0.136)
Linguistic Proximity		−0.010			
		(0.008)			
Genetic Distance		−0.109			
		(0.097)			
Peg Dummy				−0.418	
				(0.078)	
Within R^2	0.239	0.309	0.327	0.382	0.260
Num. obs.	13840	5757	13840	13840	12160

Regressions $\varphi^*_{i,j,t} = \alpha_{i,j} + \kappa_t + \lambda \boldsymbol{G}_{i,j} + e_{i,j}$ of base factor loadings on gravity variables. $G_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t} \Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \dots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed countries, as classified by MSCI. Standard errors are clustered on country dyads using Aronow et al. (2015).

Table B8: Rolling Sample Regressions with Nominal Factor loadings

	All (1)	All (2)	All (3)	All (4)	No Pegs
Log Distance	0.047	0.028	0.033	0.030	0.033
	(0.012)	(0.018)	(0.013)	(0.015)	(0.015)
Shared Language			−0.074	−0.063	−0.061
			(0.022)	(0.017)	(0.017)
Shared Legal		−0.038	−0.025	−0.018	−0.023
		(0.017)	(0.012)	(0.011)	(0.013)
Shared Border		−0.082	−0.093	−0.067	−0.073
		(0.033)	(0.030)	(0.025)	(0.029)
Resource Similarity		−0.094	−0.042	−0.051	−0.075
		(0.051)	(0.042)	(0.035)	(0.038)
Linguistic Proximity		−0.009			
		(0.004)			
Genetic Distance		−0.004			
		(0.025)			
Peg Dummy				−0.343	
				(0.041)	
Within R^2	0.002	0.002	0.004	0.026	0.003
Num. obs.	664507	239311	645845	565960	481296

Regressions $\varphi^*_{i,j,t} = \alpha_{i,j} + \kappa_t + \lambda \boldsymbol{G}_{i,j} + e_{i,j}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j,t}$, are from 60-month rolling regressions $\Delta s_{i,j,\tau} = \alpha_{i,j} + \varphi^*_{i,j,t} \Delta base_{i,\tau} + e_{i,j,\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 162 countries. Standard errors are clustered on country dyads using Aronow et al. (2015).

Table B10: Variance Decomposition of Real Bilateral Exchange Rates by Base Currency

	Base Var	FX Var	Id Var	R^2 Mean	Load Sd		Base Var	FX Var	Id Var	R^2 Mean	Load Sd
Developed Countries						**Emerging Countries**					
Australia	1.02	3.29	2.26	0.49	0.15	Chile	18.07	19.99	1.92	0.87	0.04
Austria	0.61	3.86	3.25	0.27	0.63	China	1.54	3.46	1.92	0.58	0.19
Belgium	0.62	3.98	3.36	0.27	0.60	Colombia	0.79	3.02	2.23	0.46	0.34
Canada	0.46	2.78	2.32	0.32	0.31	Czech Republic	5.02	7.24	2.22	0.77	0.07
Denmark	0.51	2.74	2.23	0.26	0.51	Egypt	2.63	4.87	2.24	0.68	0.07
Finland	0.48	3.96	3.48	0.28	0.33	Greece	0.86	4.01	3.15	0.45	0.16
France	0.55	3.93	3.38	0.25	0.60	Hungary	1.62	3.58	1.96	0.60	0.18
Germany	0.66	4.02	3.36	0.29	0.59	India	0.72	3.07	2.35	0.41	0.14
Hong Kong	0.54	2.85	2.31	0.35	0.30	Indonesia	5.03	6.95	1.92	0.79	0.05
Ireland	0.49	3.94	3.45	0.26	0.52	Korea	1.50	3.80	2.30	0.56	0.12
Israel	6.43	8.59	2.15	0.81	0.13	Malaysia	0.50	2.82	2.32	0.31	0.32
Italy	0.51	3.98	3.47	0.30	0.34	Mexico	8.26	10.35	2.09	0.83	0.12
Japan	1.08	3.41	2.33	0.49	0.15	Philippines	1.27	3.59	2.32	0.53	0.15
Netherlands	0.61	3.98	3.37	0.27	0.61	Poland	11.33	13.33	2.00	0.86	0.19
New Zealand	1.01	3.36	2.34	0.49	0.12	Russian Federation	17.77	18.71	0.94	0.94	0.05
Norway	0.43	2.72	2.30	0.24	0.39	Taiwan	0.64	2.93	2.29	0.38	0.34
Portugal	0.68	4.17	3.49	0.36	0.32	Thailand	0.91	3.23	2.31	0.46	0.22
Singapore	0.31	2.62	2.31	0.21	0.49	Turkey	4.30	6.11	1.81	0.77	0.09
Spain	0.61	4.12	3.50	0.35	0.23						
Sweden	0.51	2.82	2.31	0.32	0.31						
Switzerland	0.74	3.01	2.27	0.39	0.35						
United Kingdom	0.54	2.88	2.34	0.35	0.20						
United States	0.46	2.74	2.28	0.31	0.45						
All	0.86	3.64	2.78	0.34	0.41	All	4.60	6.73	2.13	0.62	0.18

Summary statistics of data from the regression $\Delta s_{i,j,t} = \alpha_{i,j} + \varphi^*_{i,j}\Delta base_{i,t} + e_{i,j,t}$ for each possible base currency i. $\Delta s_{i,j,t}$ is the log real change in the bilateral exchange rate calculated by substracting differences in log inflation. For each currency j, $base_{i,t}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Base Var, FX Var, and Id Var are cross-sectional means for each base currency. Base Var is the variance attributed to the base factor, FX Var is the total variance, and Id Var is the remaining idiosyncratic variance. R^2 mean is the cross-sectional mean of the R^2 for each base currency. Load Sd is the standard deviation of the loadings $\varphi^*_{i,j}$ for each base currency i. Spot rates are monthly from January 1973 until December 2014 from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI.

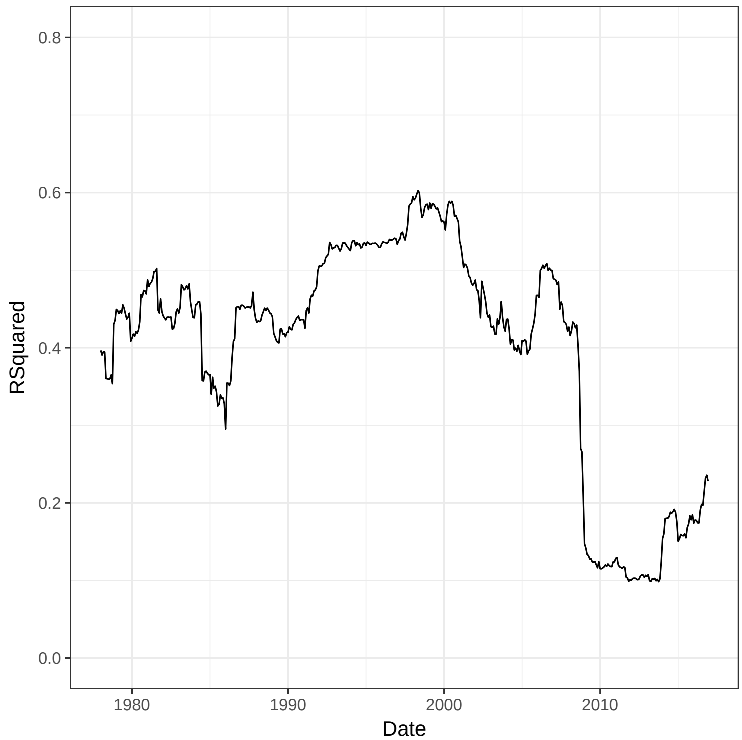
B.4 Additional Tables and Figures

Table B9: Rolling Sample Regressions with Nominal Factor Loadings GFD Data (MSCI Developed and Emerging Subset) Comparing FEs

	None	Year	Base-Year	Base-Year/Year
Log Distance	0.119	0.147	0.119	0.146
	(0.035)	(0.033)	(0.035)	(0.033)
Shared Language	−0.095	−0.085	−0.096	−0.084
	(0.031)	(0.035)	(0.031)	(0.035)
Shared Legal	−0.033	−0.030	−0.033	−0.031
	(0.026)	(0.025)	(0.026)	(0.025)
Shared Border	−0.076	−0.044	−0.076	−0.039
	(0.046)	(0.046)	(0.046)	(0.046)
Colonial Link	−0.202	−0.174	−0.200	−0.173
	(0.044)	(0.058)	(0.044)	(0.058)
Resource Similarity	−0.152	−0.164	−0.151	−0.162
	(0.062)	(0.068)	(0.062)	(0.068)
Peg Dummy	−0.471	−0.494	−0.472	−0.534
	(0.054)	(0.060)	(0.054)	(0.063)
Within R^2	0.186	0.198	0.185	0.206
Num. obs.	58298	58298	58298	58298

Regressions $\gamma_{ijt}^{base} = \alpha + \kappa_t + \beta \boldsymbol{G}_{ij} + e_{ij}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{ij}$ is a set of gravity variables. Base factor loadings, γ_{ijt}^{base}, are from 60-month rolling regressions $\Delta s_{ij\tau} = \alpha + \gamma_{ijt}^{base} base_{i\tau} + e_{ij\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{it}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Spot rates are from Global Financial Data for 24 developed and 23 emerging countries, as classified by MSCI. Real exchange rate changes include relative differences in inflation. Standard errors clustered on base country and foreign country using Cameron et al. (2011)).

Figure B1: 60-Month rolling sample R-squareds of base factor loadings on gravity variables



Plots of 60-month rolling sample R-squared from regressions $\gamma_{ijt}^{base} = \alpha + \kappa_t + \varphi \boldsymbol{G}_{ij} + e_{ij}$ of base factor loadings on gravity variables. $\boldsymbol{G}_{ij}$ is a set of gravity variables. Gravity variables are log distance, common language, commodity distance, common legal origins, and common colonial origins. Base factor loadings, γ_{ijt}^{base}, are from 60-month rolling regressions $\Delta s_{ij\tau} = \alpha + \gamma_{ijt}^{base} base_{i\tau} + e_{ij\tau}$ with $\tau = t - 59 \ldots t$. For each currency j, $base_{it}$ is the average appreciation of currency i at time t relative to all available currencies, excluding currency j. Monthly spot rates are from Global Financial Data for a balanced panel of 13 countries where the German rate becomes the euro rate in 1999.

Table B11: Euro Subsample Real Base Factor loadings vs Gravity

	Model 1	Model 2
Log Distance	0.130	0.100
	(0.038)	(0.047)
Shared Legal		0.025
		(0.046)
Shared Border		−0.022
		(0.101)
Shared Language		−0.294
		(0.073)
Adj. R^2	0.036	0.050
Num. obs.	306	306

Regressions $\varphi^*_{i,j} = \delta + \lambda \boldsymbol{G}_{i,j} + e_{i,j}$ of real base factor loadings on gravity variables. $\boldsymbol{G}_{i,j}$ is a set of gravity variables. Base factor loadings, $\varphi^*_{i,j}$, are from the regression $\Delta s_{i,j,t} = \alpha + \varphi^*_{i,j} \Delta base_{i,t} + e_{i,j,t}$. For each currency j, $base_{i,t}$ is the average real appreciation of currency i at time t relative to all available currencies, excluding currency j. Real spot rate changes are from Barclays and Reuters for 18 Euro area countries from 1999 through 2013.