Fake news

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Abstract

Fake news has been influential and topical recently. The senders intentionally produce fake news to benefit financially or politically from leveraging them to mislead the receivers . We propose and fully solve a game theoretic model which captures the tension between the sender and the receiver of fake news. We have a potentially infinite horizon continuous time model with two agents with asymmetric information where the receiver does not know whether the sender is sending fake news. The receiver receives a stream of news from the sender, which contains both true news and fake news if it is a fake news sender. The fake news differentiates from the true news in that their content follow different distributions. Based on the news the receiver observes, she updates her belief on whether the sender is sending fake news, then she dynamically decides whether to continue getting news from this source. The sender dynamically decides the volume of the fake news facing a trade-off between the immediate gain from making the receiver reads more fake news and the loss in the future due to the loss of trust of the receiver. We prove the existence and uniqueness of Markov equilibrium and show insights from the equilibrium strategies and payoffs. Practically, fake news senders are specialized while the receivers are relative naive, therefore we model and analyze an off-equilibrium case where the receiver is not accurately anticipating the fake news sender's behavior.

1 Introduction

Fake news is not a new issue but it has become topical recently as it is argued to be very effective to society these days (Parkinson 2016). Similar to Allcott and Gentzkow (2017), we define fake news as false news articles that are intentionally designed to mislead readers. These articles could be spread through traditional media like newspaper, TV or social media. The goal of this paper is to provide a model that captures the tension between a fake news sender, which could be a traditional news outlet or an identity on some social media, and a news receiver, who will benefit from true news while harmed by fake news.

To illustrate the strategies and payoffs of the fake news sender (referred to as "sender" hereafter) and the news receiver (referred to as "receiver" hereafter), we propose and fully solve a game theoretic model. We have a potentially infinite horizon continuous time model with two agents with asymmetric information where the receiver does not know whether the sender is sending fake news at the beginning and learns this information as the game proceeds. The receiver observes a stream of news, which is a stream of true news if the sender is not a fake news sender whereas a mixture of true news and fake news if the sender is a fake news sender. To fit with the real-time feature of the news, we model that the news are observed piece by piece discretely but can be at any moment on the continuous time line. Also, in our model, both the sender and the receiver are making decisions dynamically at every moment contingent on what they have observed in the game. Specifically, the receiver decides whether to trust the sender, which from the receiver's point of view, has some possibility of containing fake news. The sender decides the intensity of fake news in the stream while facing the trade-off between the immediate gain from misleading the receiver and the potential loss in the future due to the increasing suspicion of the receiver. To capture the distinction between fake news and ordinary news, we model each piece of news as a random variable and assume that the distribution of a piece of fake news is different from that of true news, which is consistent with the characterization from the machine learning literature that aims at detecting fake news (for example, Conroy et al. (2015) and Rubin et al. (2015)). The distributions can be interpreted is various ways, including the likelihoods of occurrence of events, the linguistic features or the likelihoods of passing some fact checking tools.

We prove the existence and uniqueness of a Markov equilibrium in our model. To provide insights, we then use some numerical examples to illustrate the players' strategies and payoffs. On the sender's side, we show that the sender's optimal strategy is to be careful and not too aggressive to keep the receiver from abandoning this source. Also, we compare the sender's payoffs when the receiver anticipates the distribution of fake news correctly and incorrectly. On the other hand, we show that no matter whether the receiver knows the distributions, the most important thing for her is to have at least some suspicion about the source at the beginning. We also show the receiver's benefit from having better understanding of the distributions, while in practice the receiver needs to balance this benefit with the cost of learning the distributions.

2 Literature Review

Our paper is mainly related to three streams of literature in economics, including communication games, media market analysis and deception games. In this section, we will discuss the relationship between our model and earlier literature and why it is necessary to make the refinements to fit with the fake news settings.

In communication game literature, researchers are interested in the commitment power of the sender. For example, Crawford and Sobel (1982) characterized the case where there is no commitment while Kamenica and Gentzkow (2011) explored the case where there is full commitment. We argue that in the fake news settings, the commitment power is actually somewhere between the two extremes — the receivers are not clear about whether the sender is trustful or not. Especially, the trust of the receiver towards the sender is updated dynamically through observing the sender's behavior. For example, the receiver will trust the sender more when the receiver does some fact checking and the sender's message passes it and vice versa. Therefore, in our model, the receiver is dynamically Bayesian learning the sender's commitment power through time. Such a learning process can also be interpreted as a lying cost in the Kartik (2009) model, because the sender is suffering from a loss of trust in the future if he lies.

In traditional media market research, including Mullainathan and Shleifer (2002) and Gentzkow and Shapiro (2006), the incentives of the media are assumed to be attaining more revenue by selling more products. However, we argue that in the fake news settings, the fake news media are aiming at misleading more readers. The difference is that, in our model, we specify how the sender's payoff differs based on the receivers' trust, even when the receivers consume the news. Also, we use a dynamic continuous time model to specify the continuation payoff of the sender, which was only generally characterized in previous literature. This continuous time model can serve as a basis for more fruitful research around this topic, for example, how the sender would set the specific timing of the fake news to make it most efficient.

Finally, our research is related to the deception games literature because the fake news sender can be seen as a deceiver trying not to be detected as fake. Our model is most related to the dynamic deception game model developed by Anderson and Smith (2013) because in both models, the sender can deceive due to some noise and we are using the same solution concept, namely Markov equilibrium. This is also one of our major differences from Gentzkow and Shapiro (2006): They focus on the aspect that the fake news senders can deceive because the receivers have their own belief about the reality, while we explore the aspect on how random noise help the senders to deceive, which we believe is complementary to their work. Compared to Anderson and Smith (2013), we generalized their model because in the fake news settings, each piece of news is observed discretely and can follow arbitrary distributions.

3 Model development

To model the traffic of news and the interaction between the sender and the receiver, we propose a game-theoretic model based on point processes. In this section, we first provide a brief overview of point processes and then specify the players' information and payoff structures in our model.

3.1 Point processes

A point process is a type of stochastic process that models the occurrence of events as a series of random points in time or geographic space (Xu et al. (2014)). For example, in the context of this study, the receiver's observation of each piece of news can be modeled as a point occurring along the time line. We can describe such a point process by $N(t)$, which is an increasing non-negative integer-valued counting process such that $N(t_2) - N(t_1)$ is the total number of points that occurred within the time interval $(t_1, t_2]$. Most point processes can be fully characterized by the *conditional* *intensity function* defined as follows (Daley and Vere-Jones (2007)):

$$\lambda(t|\mathcal{H}_t) = \lim_{\Delta_t \to 0} \frac{\Pr(N(t+\Delta t) - N(t) > 0|\mathcal{H}_t)}{\Delta t} \tag{1}$$

where $\mathcal{H}_t$ is the *history* up to time instant t, which includes all the information before t. The intensity measures the probability of instantaneous point occurrence given the previous history. To be specific, given the history $\mathcal{H}_t$, the probability of a point occurring within $(t, t+\Delta t]$ is $\lambda(t|\mathcal{H}_t)\Delta t$.

It is worth noting that the commonly used Poisson processes can be seen as a special kind of point processes where the intensity $\lambda(t|\mathcal{H}_t)$ is independent of the history $\mathcal{H}_t$. If the intensity $\lambda(t|\mathcal{H}_t)$ is constant over the whole process, then the point process reduces to a homogeneous Poisson process, and if the intensity $\lambda(t|\mathcal{H}_t)$ is not constant but can be a deterministic function of time t and independent of the history, then the point process reduces to a nonhomogeneous Poisson process.

3.2 Players' information structure

In the game we are modeling, there are two players—the sender and the receiver—and there is information asymmetry in the game: the sender has perfect information about the receiver, whereas the receiver does not know whether the sender is sending fake news or not. We will discuss their information structure in detail in this subsection.

In the game, the receiver observes the incoming traffic, which is a mixture of true news and fake news in the case we are interested in whether the sender is a fake news sender. We assume that the true news is exogenous whereas the fake news is endogenously decided by the sender. We need to model two characteristics of each piece of news: the timing and the content. We use two point processes to model the timing of each piece of fake news and true news. Specifically, we use $N_a(t)$ to denote the stochastic process that counts the number of pieces of fake news up to time t and similarly use $N_0(t)$ to denote the count of true news. Following (1), denote $\Lambda_0(t|\mathcal{H}_t)$ and $\Lambda_a(t|\mathcal{H}_t)$ as the conditional intensity functions of N_0 and N_a, respectively. We assume that Λ_a, the intensity of fake news, is decided by the sender, whereas Λ_0, the intensity of true news, is exogenous and constant for the entire process, which implies that N_0 is a Poisson process. For the content, as mentioned in the introduction, we use two random variables to model the content of each piece of fake news versus true news. Formally, denote $Z_0, Z_a : \Omega \to \mathbb{R}$ to be the random variables that characterizes true news and fake news. Practically, the event space Ω is determined by how the receiver perceives fake news against true news. In the present model, we do not go into the receiver's decision model of Ω. We assume Ω to be exogenous. Denote the distributions of Z_0, Z_a to be P_0, P_a respectively. P_a characterizes the fake news generation technology of the sender. The smaller the difference between P_0 and P_a, the better technology the sender has because the sender can be more deceptive. We assume P_0 and P_a to be exogenous and invariant through the game. To summarize, the receiver observes the incoming news traffic as a stochastic process Y that subjects to

$$dY = Z_0 dN_0 + Z_a dN_a \tag{2}$$

However, with this observation of Y, the receiver does not know whether the sender is sending fake news or not. Formally speaking, there are two states of world: In State 1, the sender is sending fake news, and Y comes from (2). In State 2, the sender is only sending true news, and Y subjects to $dY = Z_0 dN_0'$, where N_0' is a point process whose intensity Λ_0' satisfies $\Lambda_0' = \Lambda_0 + \Lambda_a$. At each time t, the receiver has a belief $q(t) \in [0, 1]$ that the world is in State 1 and a belief $1 - q(t)$ that the world is in State 2. q is Bayesian updated through the observation of Y and follows:

$$dq = \frac{q(1-q)(\frac{\Lambda_a P_a(dY)+\Lambda_0 P_0(dY)}{\Lambda_0+\Lambda_a} - P_0(dY))}{q \cdot \frac{\Lambda_a P_a(dY)+\Lambda_0 P_0(dY)}{\Lambda_0+\Lambda_a} + (1-q)P_0(dY)} \tag{3}$$

When proposing (3), we are making three assumptions. First, the receiver is learning whether it is fake news sender from and only from the content of news, which means that the timing of news is not informative to the receiver. We assumes that $\Lambda_0' = \Lambda_0 + \Lambda_a$ in State 2 to make sure that the timing of news in both states is expected by the receiver to be the same, which makes the timing does not include information of the existence of the sender. Second, we assume that the receiver knows P_0 and P_a beforehand, which could come from some previous experience or research. In the latter part of our paper, we relax this assumption to analyze and compare the case where the receiver does not have accurate information about the distributions. Third, we assume that the receiver has correct anticipation of Λ_0 and Λ_a, conditional on the world in State 1. This assumption is mandatory when one is trying to analyze the Bayesian Nash equilibrium of a game-theoretic model. The solution concept we are using, Markov equilibrium, is a subset of Bayesian Nash equilibrium, which makes this assumption required. We will discuss our solution concept in detail in Section 3.

The information is asymmetric in the game. While the receiver does not know whether the world is in State 1 or State 2, the sender knows that he is sending fake news, meaning that the world is in State 1. Besides the timing and content of the fake news he sends, we assume that the sender also knows the timing and content of occurred true news, which is natural. We also assume that the sender knows the initial belief of the receiver, $q(t_0)$. Then from (3), the sender knows the receiver's belief, $q(t)$, at any moment t.

3.3 Players' payoff structure

With the information structure above, next we model the decision variables and payoff structure for both players.

First, for the receiver, it is worth noting that she is deciding whether to continue using this source of news, instead of picking the pieces of news from this source that look authentic and using them. Thus, the receiver needs to decide whether to abandon this news source, fully depend on it, or partly depend on it. Denote $p(t) \in [0, 1]$ as the receiver's dependence on the focal news source at time t, which is her decision variable. Normalize her payoff from the outside option, like relying on other news sources, as 0. We assume that the receiver will get a unit positive payoff from depending on each piece of true news. On the other hand, we assume that the receiver suffers L loss when she depends on a piece of fake news. With a belief process $q(t)$, through dynamically choosing $p(t)$ based on the history before t, the receiver maximizes her expected payoff:

$$E[\int_0^\infty p((1-q)(\Lambda_0 + \Lambda_a) + q(\Lambda_0 - L\Lambda_a))dt] \tag{4}$$

The sender's goal is to mislead the receiver. Therefore, his payoff depends on how much fake news is taken by the receiver and how much the receiver depends on the fake news. Through dynamically choosing the fake news intensity $\Lambda_a(t)$, the sender maximizes his payoff:

$$E[\int_0^\infty e^{-rt} p\Lambda_a dt] \tag{5}$$

in which r is the time discount factor. We did not assume a time discount factor for the receiver, and we will explain why in the equilibrium analysis. We assume that the capacity of the sender is c, that is, $\Lambda_a(t) \in [0, c]$ for all t.

4 Equilibrium analysis

In the present research, we focus on the Markov equilibrium of the game, where both players' strategies are Markovian and at each time t depends only on the receiver's current belief $q(t)$, which is also known by the sender. In other words, we want to analyze the equilibrium strategies where both decision variables of the players, p and Λ_a, can be written as deterministic functions of q.

To simplify notation, define function $g(\cdot; q, \Lambda_a)$ as

$$g(\cdot) = \frac{q(1-q)(\frac{\Lambda_a P_a(\cdot)+\Lambda_0 P_0(\cdot)}{\Lambda_0+\Lambda_a} - P_0(\cdot))}{q \cdot \frac{\Lambda_a P_a(\cdot)+\Lambda_0 P_0(\cdot)}{\Lambda_0+\Lambda_a} + (1-q)P_0(\cdot)}$$

Then, equation (3) can be rewritten as

$$dq = g(dY)$$

Based on the property of point processes, we know that at any time period dt, the likelihood of $dN_0 + dN_a \geq 2$ is $O(dt^2)$. Therefore, with likelihood $1 - O(dt^2)$, dN_0 and dN_a are either 0 or 1 and are not both 1. Therefore,

$$dq = g(Z_0)dN_0 + g(Z_a)dN_a \tag{6}$$

Now, we start analyzing the equilibrium strategies of the players.

Because both players' strategies are only dependent on q and equation (6) suggests that the receiver's strategy p will not influence the evolution of q, the receiver's current decision will not have any impact in the future. Therefore, the receiver's dynamic optimization problem is equivalent to optimization at each static time point t. This is the reason why we did not assume time discount factor for the receiver, since it does not affect her strategy. With this argument, the receiver's optimization problem (4) can be rewritten as

$$p(q) \in \arg\sup_{p\in[0,1]} p((1-q)(\Lambda_0+\Lambda_a)+q(\Lambda_0-L\Lambda_a)) \quad \forall q\in[0,1] \tag{7}$$

For the sender, denote $V(q)$ as the maximal payoff he can get, given the receiver's strategy p, if the initial belief of the receiver, $q(t_0)$, is q. Formally,

$$V(q) \doteq E(\int_0^\infty e^{-rt} p\Lambda_a dt | q_0 = q)$$

Then, with the Hamilton-Jacobi-Bellman equation, we have

$$rV = \sup_{\Lambda_a\in[0,c]} p\Lambda_a + \frac{1}{dt}E(dV) \quad \forall q\in[0,1]$$

That is,

$$rV = \sup_{\Lambda_a\in[0,c]} p\Lambda_a + V'(E[g(Z_0)|q,\Lambda_a]\Lambda_0 + E[g(Z_a)|q,\Lambda_a]\Lambda_a) \quad \forall q\in[0,1]$$

which can be separated into two conditions:

$$\Lambda_a(q) \in \arg\sup_{\Lambda_a\in[0,c]} p\Lambda_a + V'(E[g(Z_0)|q,\Lambda_a]\Lambda_0 + E[g(Z_a)|q,\Lambda_a]\Lambda_a) \quad \forall q\in[0,1] \tag{8}$$

and

$$rV = p\Lambda_a + V'(E[g(Z_0)|q,\Lambda_a]\Lambda_0 + E[g(Z_a)|q,\Lambda_a]\Lambda_a) \quad \forall q\in[0,1] \tag{9}$$

Thus, in all, a Markov equilibrium is a 3-tuple (p, V, Λ_a), where each entry is a function of q, such that conditions (7),(8),(9) are satisfied.

With the definition above, we have the following result:

Theorem 1: There exists a unique Markov equilibrium.

Proof: We prove this by solving both players' equilibrium strategies from (7), (8), (9). For details of proof and equilibrium strategies, see appendix.

5 An illustrative example

In this section, to illustrate and analyze the equilibrium strategies, we use an example where both distributions of Z_0 and Z_a, P_0 and P_a, are binary distributions. Other than the computational convenience, another advantage of choosing the binary distributions is that it is easier to measure the efficiency of the sender's technology for producing fake news. To be specific, assume that there are two states in the event space of Z_0 and Z_a, Ω, denoted as State M and State N. Z_0 is realized as State M with probability p_0, realized as State N with probability $1 - p_0$; Z_a is realized as State M with probability p_a, realized as State N with probability $1 - p_a$. The efficiency of the sender's technology can be measured by the difference between P_0 and P_a, which in this case reduces to $|p_0 - p_a|$. When $|p_0 - p_a| = 0$, the sender's technology is flawless and the receiver has no way to distinguish between true and fake news, which makes our research trivial. Thus, we study the case where $|p_0 - p_a| \neq 0$.

Our focus in this section is to illustrate both players' strategies and payoffs and understand how the sender's technology will influence them. First, we are interested in how the game proceeds as the time evolves and we have the following result when both players are playing their equilibrium strategies.

Theorem 2: When $0 < q < 1$, $E[dq/dt] > 0$. Therefore, there are only two absorbing states: $q = 0$ and $q = 1$. If the receiver's initial belief $q_0 > 0$, $q \to 1$ when $t \to \infty$.

Proof: See appendix.

From Theorem 2, we observe that if the receiver's initial belief is strictly between 0 and 1, then this game is like a Ponzi Scheme: At the end of the game, the receiver learns that there is a fake news sender; however, she lost some value through the learning process, which can be seen as the cost of learning, and the sender also gains as a result of the game.

To visualize both players' strategies and payoffs, we use a numerical example. Because we are interested in the influence of the sender's technology, we set parameters including L, c, r, Λ_0, p_0 to be fixed and analyze the change of p_a. For example, we set $L = 3, c = 3, r = 0.1, \Lambda_0 = 1, p_0 = 0.1$, and compare strategies and payoffs between cases where $p_a = 0.3$ and $p_a = 0.4$.

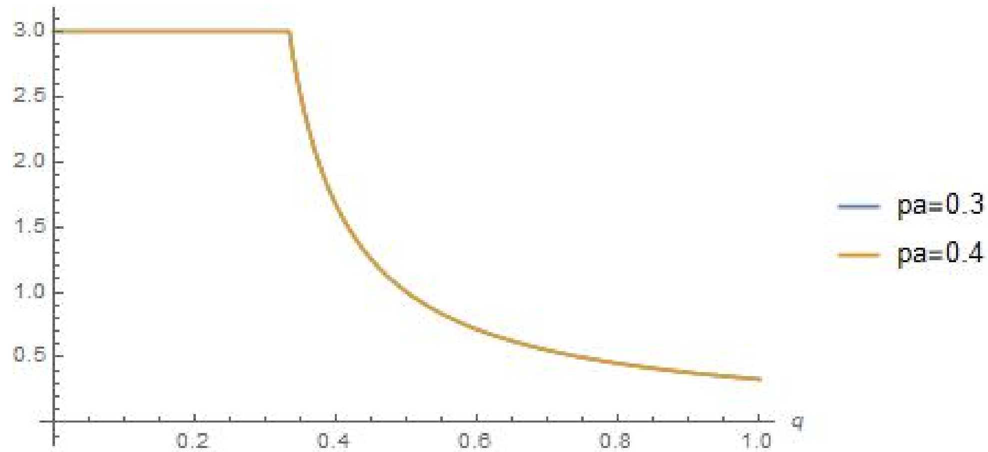

Figure 1: *Comparison of sender's strategies*

Figure 1 depicts the sender's strategies in the two cases. In both cases, the sender fully employs his capability when the receiver's belief is below a threshold, and starts declining after the threshold with the increase of the belief. The intuition is that when the receiver is more suspicious of the news source, the sender needs to be more cautious to keep the receiver from abandoning the source. The sender's strategies fully overlap in the two cases, suggesting that the strategies remain the same with different technologies. This is because given the belief of the receiver, the receiver's optimization problem (7) is independent of the sender's technology, which makes the intensity needed by the sender to keep the receiver using the source is independent of the sender's technology.

Figure 2 depicts the receiver's strategies. With a lower belief, implying a higher trust of the news source, the receiver will fully depend on the source. As the belief increases, the receiver

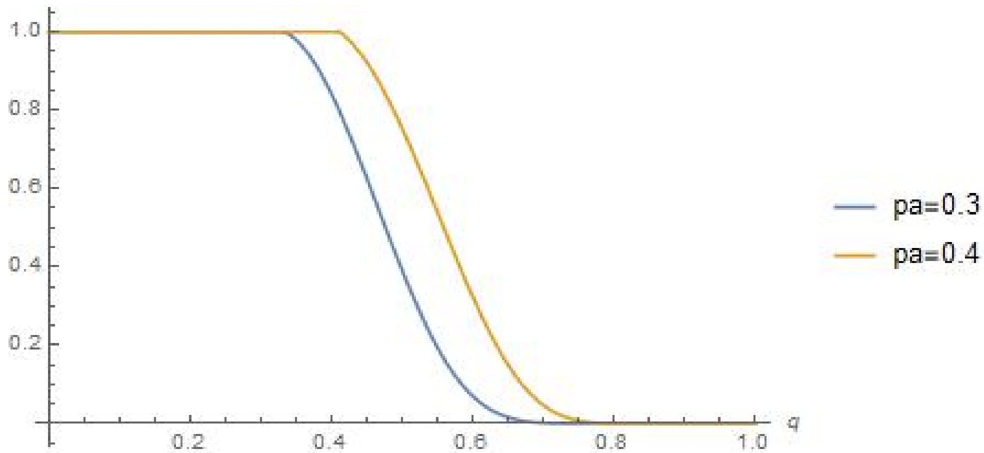

Figure 2: *Comparison of receiver's strategies*

will rely less on the source. The dependence converges to 0 as the belief converges to 1. It is worth noting that the receiver's strategies *converge* to 0 continuously because, in the equilibrium, the sender is also carefully managing his intensity to ensure that he is not too aggressive, which provides the receiver some incentive to keep using the news source even if she has significant belief that it could be a fake news sender. Another observation from Figure 2 is that when the sender has a better technology ($p_a = 0.3$), the receiver will be less dependent on the news source, given the same belief.

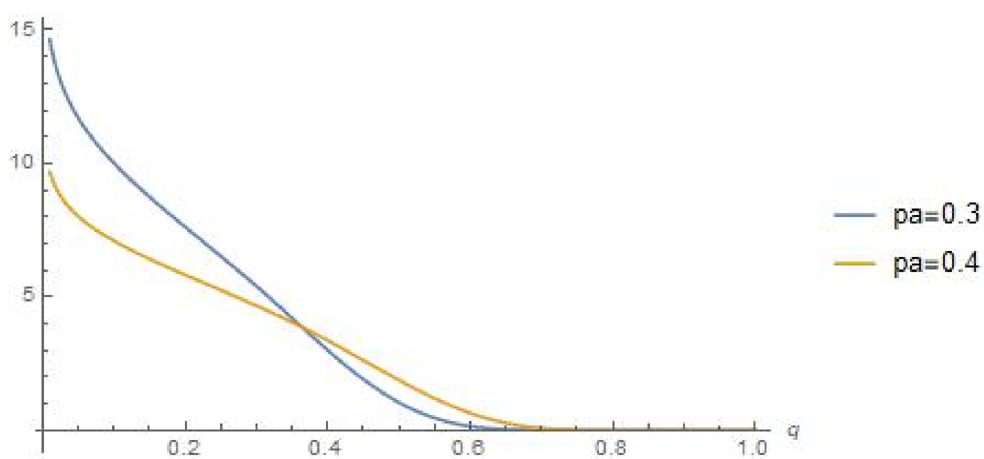

Figure 3: *Comparison of sender's payoffs*

Figure 3 depicts the sender's value function $V(q)$, which can be interpreted as the sender's expected payoff throughout the game when the receiver's initial belief $q(t_0) = q$. It is straightforward to demonstrate that V is monotonously decreasing with q because the more trust the receiver has at the beginning, the more room for the sender to extract profits. Comparing the two cases where the sender's technology differs, we observe that if the initial belief of the receiver is low, the sender is better off with a superior technology; meanwhile, if the initial belief of the receiver is high, the sender is better off with an inferior technology. The intuition here is that having a superior technology has two effects: First, the difference between the distributions of true and fake news is less, therefore the receiver will update her belief slower. Second, as shown in Figure 2, the receiver will be more cautious and depend on the news source less, especially when her belief is high. When the initial belief of the receiver is lower, there is a longer period in terms of belief where the receiver's strategies—given different technologies of the sender—do not diverge, which makes the first effect overwhelms the second effect more. On the other hand, when the initial belief of the receiver is relatively high, the second effect overwhelms the first effect. This contrast between these two effects leads to the comparison of the sender's payoffs, shown in Figure 3.

With theorem 2, the receiver's expected payoff (4) can be calculated (for details, see appendix) and is shown in Figure 4.

When there is a fake news sender, it is no surprise that the receiver needs to incur more cost of learning if she starts with a lower initial belief, and this cost converges to infinity when the initial belief is approaching 0. Therefore, having an initial belief that is strictly larger than 0 can significantly reduce this cost. On the other hand, from Figure 2 we see that the receiver's strategy holds the same when her belief is under a threshold, implying that when the sender is only sending true news, having a reasonably low initial belief that is different from 0 will not hurt the receiver

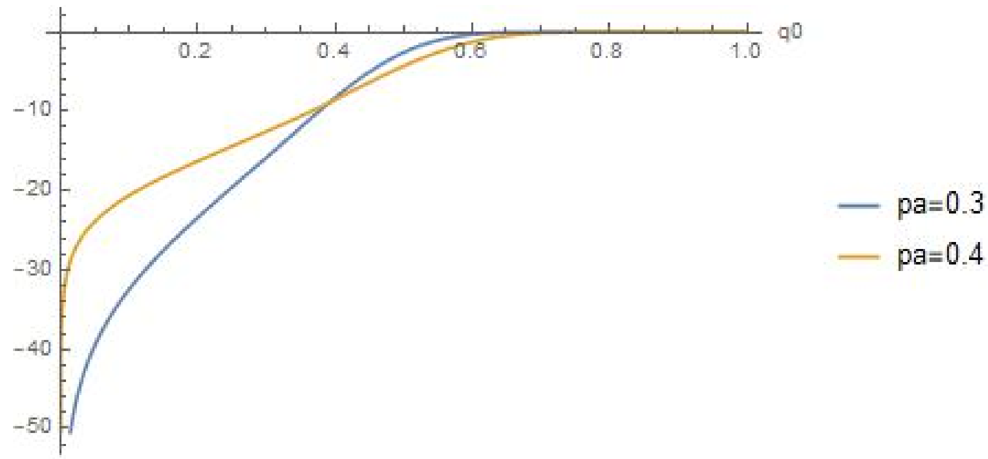

Figure 4: *Receiver's expected payoffs*

at all. Taking both possibilities into account, it is suggested that the receiver should have some suspicion (a reasonably low initial belief) for any news source.

6 Off-equilibrium analysis

In the previous section, we illustrated equilibrium strategies and payoffs, assuming that the receiver knows accurately what technology the fake news sender would be using. However, in practice, this assumption could be too restrictive. In behavioral game theory, it is argued that players may need time and effort to learn the structure of the game (for example, Fudenberg and Levine (2016) and Sargent (1999)), which makes the Nash equilibrium concept not applicable. In our case, the fake news sender is specialized in producing fake news while the receiver can be an ordinary person which is relatively naive. Therefore, assuming that the receiver understands perfectly the structure of the game at the beginning is not realistic. In this section, we relax this assumption and explore a case where the receiver does not perfectly anticipate the sender's technology to get more insights.

In this case, following the same arguments as before, we keep assuming Z_0 and Z_a to be binary distributions, and keep using previous notations p_0 and p_a to model the distributions of Z_0 and Z_a. However, in this case, the receiver is anticipating the sender's technology, p_a, to be p'_a, which is not equal to p_a. Therefore, instead of using the equilibrium strategy where the technology is p_a, the receiver uses the equilibrium strategy where the technology is p'_a, which is suboptimal in this case. Also, the receive updates the belief q based on her wrong anticipation, meaning that in function $g(\cdot)$ the parameter p_a is substituted by p'_a, and Λ_a is substituted by the receiver's anticipation on the sender's attacking intensity, which is the sender's equilibrium strategy when his technology is p_a. Assume that the sender knows how the receiver estimates his technology and therefore updates his strategy to be optimal given the receiver's strategy. In other words, in this case, equation (7) does not hold because the receiver is not optimizing, whereas equations (8) and (9) hold given a certain p, which is the equilibrium strategy of the receiver where the technology is p'_a.

We say that the receiver *underestimates* the sender's technology if $p_0 < p_a < p'_a$ or $p'_a < p_a < p_0$. The larger $(p_0 - p'_a)/(p_0 - p_a)$ is, we say that the sender is *more underestimated*. Practically, this underestimation is likely to come from the sender's improvement of technology, which is not known by the receiver.

Analogous to Theorem 2, we are still interested in how the game proceeds as time evolves. However, the result is different with the following property:

Theorem 3: Assume that $Lc > \Lambda_0$. If the sender is underestimated, then there exists a unique $q_e \in (0, 1)$, such that:

i) when $0 < q < q_e$, $E[dq/dt] > 0$

ii) when $q = q_e$, $E[dq/dt] = 0$

iii) when $q_e < q < 1$, $E[dq/dt] < 0$

Therefore, other than $q = 0$ and $q = 1$, q_e is another absorbing state. If $0 < q(t_0) < 1$, as $t \to \infty$, q

will fluctuates around q_e and the sender's intensity fluctuates around $\Lambda_a(q_e)$. Specifically, $\Lambda_a(q_e)$ satisfies:

i) $\Lambda_a(q_e) = \frac{p_0 - p'_a}{Lp_0 + p'_a - p_a - Lp_a}\Lambda_0$, if $\frac{p_0 - p'_a}{p_0 - p_a} < \frac{c}{c+\Lambda_0}(1 + L)$.

ii) $\Lambda_a(q_e) = c$, if $\frac{p_0 - p'_a}{p_0 - p_a} \geq \frac{c}{c+\Lambda_0}(1 + L)$

Proof: See appendix.

The assumption at the beginning is trivial: When $Lc < \Lambda_0$, there is actually no game because the sender is not capable enough to hurt the receiver as much as her gain, so the receiver will always be fully dependent on the news source and still benefit even when it contains fake news. From Theorem 3, as $t \to \infty$, q does not converge to 1, therefore the receiver's expected payoff at each time does not converge to 0, so we cannot use equation (4) to evaluate her expected payoff throughout the game. Instead, we use the intensity of fake news in the stream in the long term to evaluate the receiver's payoff. If the receiver's initial belief is $q(t_0) = 1$, then she will abandon the source at all time; and if the receiver's initial belief is $q(t_0) = 0$, then she will be receiving fake news at the sender's capacity, c, at all time. Other than those two trivial cases, in the long term, the fake news intensity will fluctuates around $\Lambda_a(q_e)$. As $(p_0 - p'_a)/(p_0 - p_a)$ characterizes how the sender is underestimated, we see that as long as the receiver does not underestimates the sender too much, she will ends up with $\Lambda_a(q_e) = \frac{p_0 - p'_a}{Lp_0 + p'_a - p_a - Lp_a}$, which is easy to show that it is smaller than c. Actually, in practical cases like the numerical example we raise in the following, this difference is significant. Therefore, following the same arguments as in the equilibrium case, the receiver is suggested to have a low but non-zero initial belief because this will not hurt the receiver at all if there is no sender but benefits the receiver significantly if there is a sender. In addition, we get the following result:

Theorem 4: If the receiver has initial belief $0 < q(t_0) < 1$ and underestimates the sender's technology, she will receive more fake news in the long run if the sender is more underestimated. Or formally, $\Lambda_a(q_e)$ is an increasing function with respect to $(p_0 - p'_a)/(p_0 - p_a)$.

Proof: See appendix.

This result shows that it is beneficial for the receiver to have better understanding of the sender's technology, even with some upfront cost, because it will benefit her payoff in the long term.

In this section, we still use a numerical example, to show the effects of underestimating the sender's technology. Specifically, we keep assuming $L = 3, c = 3, r = 0.1, \Lambda_0 = 1, p_0 = 0.1$ while setting $p_a = 0.3$ and $p'_a = 0.4$.

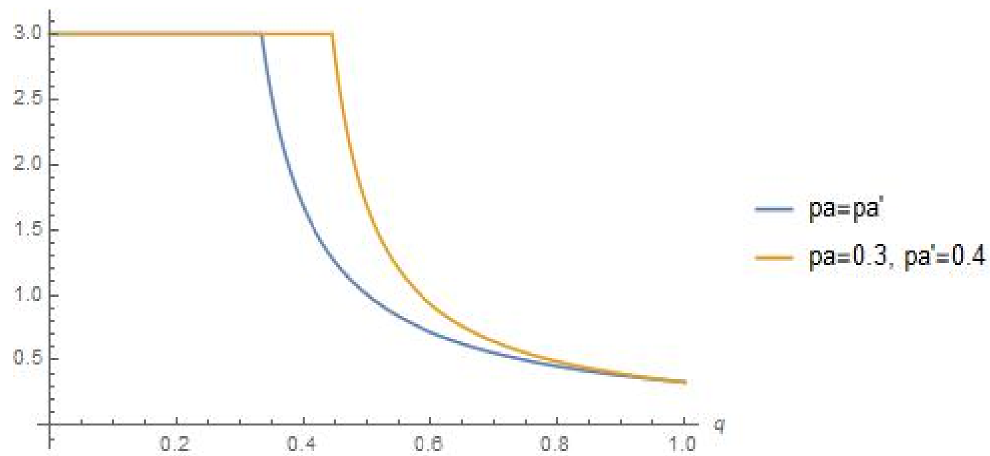

Figure 5: *Comparison of sender's equilibrium strategy and strategy when the receiver is underestimating the sender's technology*

Figure 5 depicts the sender's strategy when the receiver is underestimating her technology. As expected, under the same belief, he is attacking with a higher intensity than in the equilibrium case because he benefits more from the receiver's higher dependence on the data source than in the equilibrium strategy (as shown in Figure 2).

Figure 6 depicts the comparison between the sender's payoff in the off-equilibrium case where $p_a = 0.3$ and $p'_a = 0.4$; and in those two equilibrium cases where $p_a = 0.3, 0.4$. First, the sender

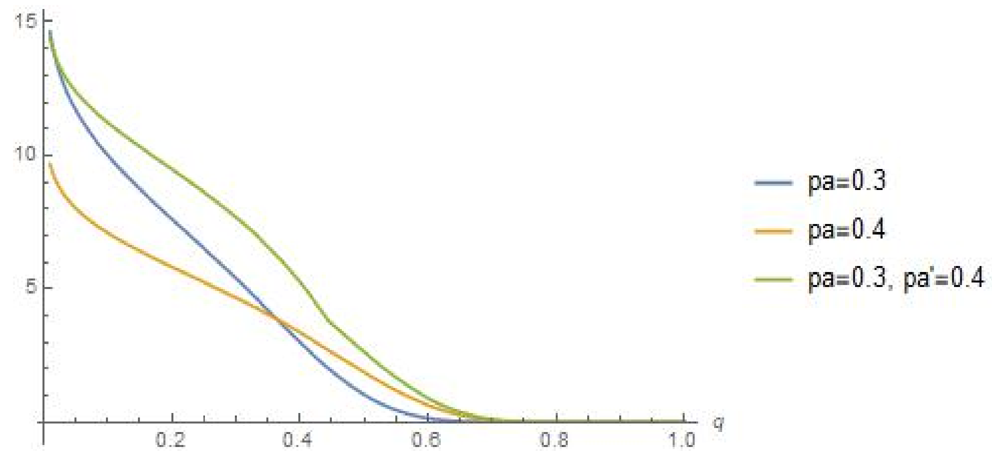

Figure 6: *Comparison of sender's payoffs when the receiver is underestimating the sender's technology*

is strictly better off in the case where equilibrium strategies are played as $p_a = 0.4$. The intuition is straightforward that the receiver is using the same strategy in these two cases while the sender has a superior technology. The comparison between the off-equilibrium case and the $p_a = 0.3$ equilibrium case is more complicated. In this comparison, the sender's technologies are the same, so the difference comes from the receiver's underestimation of the sender's technology, which can also be interpreted as the improvement of the sender's technology.

This underestimation has two effects: First, as shown in Figure 2, the receiver has higher dependence when her belief is high. Second, the Bayes's Rule suggests that, for each piece of suspicious news that is realized as State M, the receiver will updates her belief more toward there being a sender when she underestimates the sender's technology. Therefore, when the receiver's belief is relatively low such that she's fully dependent on the source, this underestimation will make her belief updated faster and, as a result, reduce the sender's payoff. Aggregating these two effects leads to what we observe in Figure 6: If the receiver's initial belief is high, the sender is better off by this underestimation, however, when the initial belief gets lower, this difference shrinks.

Because improving technology to generate fake news is costly for the sender, this result provides insight for the sender to make the decision on how much effort he should exert on sharpening his technology. In either case where the belief is low or the belief is high, improving the technology will not give him a big boost in payoff; however, when the belief is moderate, the sender may want to spend more effort to improve his technology.

7 Conclusions and future directions

In the present research, we proposed a game-theoretic model based on a point processes to model the news traffic that potentially contains fake news and to provide insights for both the sender and the receiver of fake news. Specifically, we show that for the receiver, having some suspicion of the news source initially can be greatly helpful, even when the receiver does not accurately anticipate about the fake news sender's technology.

Discussions remain open in many aspects around this issue. First, it will be interesting to understand more synergies between pieces of news. The synergies can include both timing and content: For example, the sender may need to be consistent on their fake news and gives reinterpretation when true news happens. Second, practically speaking, the receiver can decide the event space of each piece of news, Ω, by choosing different strategies to check them. An Ω with a higher dimension means higher cost for the receiver but also allows the receiver to have better understanding on whether there is fake news. How to balance this trade-off is also an important issue for the receiver. Third, the fake news sender's technology of generating fake news may be endogenous, especially practically, senders may face a trade-off between how deceptive the news is and how much the news can mislead readers.

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