Formalization and Welfare in a Dualistic Economy: A General Equilibrium Analysis with Application to Indonesia

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Abstract:

This paper presents a novel general equilibrium model of formal and informal sectors of a dualistic developing economy, to enable both supply-side and demand-side analyses of formalization and welfare. In a multi-region setting with perfect labor mobility, a consumer taste for diversity, skill heterogeneity, and regional disparity in regulatory and trade costs, high-skill entrepreneurs self-select to pay the regulatory costs in the formal sector to trade globally and lower-skill entrepreneurs choose the informal sector to trade locally to avoid the regulatory costs. In equilibrium, formalization reflects the balance between consumption diversity, which expands with local informal entrepreneurship, and productivity, which increases with formal-sector employment. The equilibrium accounts for the sectoral disparity in entrepreneur skill, labor productivity, and firm size documented in La Porta and Shelifer (2014). It further accounts for the concurrent rises in education, export, and formalization and for regional sorting of formal entrepreneur skill found in Indonesia. An extended model is calibrated to Indonesia data to show the impact of an income-tax-rate reduction on formalization, welfare, and regional tax revenue.

Key Words: *Informality; Human Capital; Trade; Welfare; Development; Indonesia.*

JEL Classification Codes: *O17; R13*

1, Introduction

Public infrastructure is significant for economic development on lowering trade cost and improving efficiency, hence the investment in skills and capital can be more profitable. Without enough infrastructure investment leads to a large informal sector (Acoca, Shahana and Susan, 2014), which does not contribute to public finance (Johnson, Kauffman and Shleifer,1997; 2010 UN-HABITAT) and intensifies the shortage of public service. According to census data from IPUMS International shown in Figure 1, nearly all developing economies, especially Indonesia, the largest economy among ASEAN countries, are trapped in high informality, though they have experienced impressive growth and rapid urbanization process in the last few decades (McKinsey Global Institute, November 2014). During the first decade of 21st Century, Indonesian formal employment share in total employment is observed to be less than 50 percent in Figure 1. It is only half of the average level, far lagged behind by other developing economies all over the world.

[Figure 1 here]

The pervasive dualistic economy is contradictory to the expectation that the informal sector will disappear as the economy develops in literature. Part of the researchers emphasizes the positive impact of the informal sector and argues that the informal sector is an integral part of the urban economy (Lewis, 1954; J. Ihrig and K.S. Moe, 2004; Banerjee and Duflo, 2011) for reducing the unemployment ratio (Todaro, 1969; Harris and Todaro, 1970; Glomm, 1992) and lowering the living cost (Porta and Shleifer, 2014; 2010 UN-HABITAT) in developing countries. However, much more literature focuses on the inefficiency of the informal sector and the distortion caused by it. The productivity of the informal sector is extremely low, compared to the formal sector. The inefficiency of informal firms can be partially explained by their low-skill entrepreneurs and the smaller capital-labor ratio (Bernard and Jensen, 1995; Stokey, 1996; Paula and Scheinkman, 2011; Gennaioli, Porta, Silanes and Shleifer, 2012; N.A. Loayza, 2016). On the contrary, the human capital gap between formal and informal sectors is not obvious on the level of workers (Porta and Shleifer, 2014). Moreover, although the scale of informal economy is huge, informal firms are typically small (Porta and Shleifer, 2014). As a result of not paying taxes, informal firms can not participate in trade (Marx, Stoker, and Suri 2013), but only sell their goods in the local market, making them unproductive and small. In addition, large informal sector lowers the quality of public service (N.A. Loayza, 1996), competes with formal firms unfairly (Farrell, 2004) and distorts the policy to a greater extent (R. Arnott, 2008). Finally, informality becomes less important as the economy develops. Under the trade liberalization, the economic growth and productivity innovation mainly come from formal sector (Lucas, 2009; Melitz and Redding, 2012), while its low efficiency makes informal firms stagnant and disappear over time (Ardagna and Lusardi, 2008; De Mel, McKencie and Woodruff, 2008; Porta and Shleifer, 2008; De Andrade, Henrique, Bruhn and McKencie, 2014; Perla, Tonetti and Waugh, 2015).

In order to provide new insight on the interaction between formal sector and informal sector, this research put forward a coherent micro-foundation of the literature to achieve three objectives. Firstly, developing the model in Behrens, Duranton and Robert-Nicoud (2014) and Dixit and Stiglizs (1978), this paper formulates a general equilibrium model, which includes location sorting and occupation selection, to account for both demand-side and supply-side determinants of formality. Compared to the closed economy model in Behrens, Duranton and Robert-Nicoud (2014), the domestic commodity market is connected to the international market, but not separated into a number of local markets. Trade liberalization stimulates the interaction of the formal and informal sector because it has been considered as an important driving force of promoting resource reallocation from non-exporters to exporters (Lucas, 2009; Melitz and Redding, 2012; Perla, Tonetti and Waugh, 2015). In addition, since the scale of international trade is much larger than domestic trade across regions, the model does not analyze inter-city trade separately. By doing so, the computation becomes much easier and the implication of local specific trade cost turns to be more obvious. Unlike the standard iceberg cost in classical NEG model in Fujita, Krugman and Venables' book (2001), trade cost in our model depends on local trade infrastructure and efficiency (Cosar and Demir, 2016). Local specific trade cost affects formalization significantly, especially when discussing the international trade. For example, according to the experience from Brazil, the informal employment is found to be increasing sharply in the locations which are more difficult and less exposed to foreign trade (Dix-Carneiro and Kovak, 2017). Moreover, there is formalization cost for the allowance to join the international trade, which is paid and only paid by the formal firms, including national income tax and local specific fixed cost. The fixed cost refers to anything limiting formal firms, such as business licenses, regulatory compliances, and financial, legal services and the premium of land rent like regulation. The reason for having formalization cost is the shortage of public infrastructure in the developing countries like Indonesia, compared to the developed countries, hence the formal firms have to pay for using the infrastructure. Formalization cost distinguishes entrepreneurs into formal and informal because the informal firms with lower marginal production would like to stay in the informal sector to control the fixed cost (Melitz and Redding, 2012).

Motivated by the important feature from employment data of Indonesia, this paper studies the incentive of formalization and geographic pattern of formal and informal employment. With continuous skill distribution of labor endowment, constant elasticity of substitution (CES) preference and monopolistic competition market as those in Behrens, Duranton and Robert-Nicoud (2014) and Melitz (2003), the difference in income motivates various occupation selection. The selection outcome is similar to the finding in Lucas's (1978) classical theory of entrepreneurial span of control for firms, which predicts that the talent of small-firm managers lies in the middle of the skill distribution, such that the people in the left tail become employees and the people in the right tail run bigger firms. However, the skill variation affects not only the firms' size but also the location choice across multiple regions with heterogeneous access to the global market. Subjected to free mobility setting based on the fact of huge internal labor flow in Indonesia, it is found that cities with better public infrastructure for trade are more preferred by the more productive formal entrepreneurs, implying that there is potential regional competition in public good provision. The third objective is applying our model to the context of Indonesia. Using the Indonesian data, it is the first time that the determinants of formalization are calibrated. The numerical comparative statics sheds light on the impact of various exogenous factors and policy interventions on economic development, employment formalization and welfare. The calibrated structure variables and exogenous parameters allow counterfactual analysis evaluating the influence of reducing income tax rate on public finance, formal employment share and personal welfare across locations and labor force vary in skill. The model and its calibration result, which concerns the geographic pattern of both the infrastructure and heterogeneous workers and entrepreneurs' sorting, make progress in the study of Quantitative Spatial Economy (QSE) (Redding and Rossi-Hansberg, 2016).

Our study expands the research of informality. Part of the previous paper focuses on the informal sector on the policy-level, discussing what is the optimal size of informality or which policy is the best one to control its size (N.A. Loayza, 1996; A. De. Paula and J.A. Scheinkman, 2011; J. Ihrig and K.S. Moe, 2004). However, this group of papers ignores the fact that being informal is the result of occupation selection in the market. Therefore, they do not pay much attention to the interaction between informality and other factors, such as trade, labor migration, and development of the formal economy, which decides the supply and demand of formalization. Departing from the literature which analyzes the supply and demand determinants of formalization separately, this paper combines these two factors and studies formalization from urban economics perspective by examining the formal employment share across cities (or regions) in an open and emerging economy. On the perspective of demand for formality, Murphy, Shleifer and Vishny (1989) points out that both the local and external market size are important for the expansion of the formal economy. Loayza (2016) investigates the demand for the formal sector in labor migration and economic growth due to its higher capital-labor ratio, where formality and informality are exogenously given. Rauch (1991) is a representative supply-side analysis of formalization, in which the model is similar to those in Behrens, Duranton and Robert-Nicoud (2014). Unlike Loayza

(2016), Rauch (1991) endogenizes the occupation selection of entrepreneurs between formal and informal sector, taking managerial skill and labor cost as dominant factors of formalization selection. The shortage of Rauch's work is that it distinguishes formal firms and informal firms only with their size. Compared to the literature, this paper makes new achievement. The spatial sorting under free mobility assumption enables us to account for the geographic pattern of the formal and informal sector across regions due to the difference of local trade infrastructure and regulatory cost. Furthermore, with monopolistic competition and heterogeneous skill setting and highlighting the competitive advantage of formal sector in trade, our model compensate the gap of Rauch (1991) and identify the factors motivating the labor force to enter the formal sector, such as trade cost, business fee, and tax. Besides latent variables which have been partially studied in Azuma and Grossman (2002) and De Mel, McKencie and Woodruff (2013), this paper also develops the supply-side analysis of formalization in the influence of human capital endowment.

The model in this paper also contributes to the theory of the relationships among trade, development, labor migration and heterogeneous firms. On the growth dimension of the New Economic Geography (NEG) model in previous literature, the typical case is product innovation or capital accumulation (Ethier,1982; Desmet and Rossi-Hansberg, 2010). To be different, our theory emphasizes the role of public infrastructure, which motivates investment in the formal sector by lowering their trade cost. Melitz and Redding (2012) studies trade and behavior of heterogeneous firms in the background of U.S. Under the monopolistic competitive market structure, they come to a conclusion that the firms serving as exporters are more competitive. Firms decide whether to join export sector based on the tradeoff between benefit from external market and transportation cost in ice-berg form. However, their model can not apply in developing countries without enough infrastructure, because it fails to capture the reaction of firms to local public goods and tax. Also, there is still scope for further study of the heterogeneous firms' spatial sorting in Melitz and Redding's (2012) work, as well as other recent literature on quantitative spatial models (Redding and Rossi-Hansberg, 2016). This paper fulfills the literature gap by focusing on local formalization cost and trade efficiency's impact on the spatial variation of the formal and informal sector. Ma and Tang (2016) analyzes the welfare effect of the internal migration in China. There are tradable sector and non-tradable sector in each city, and the productivity of each firm is unknown before entering anyone sector. Only after paying operation cost to enter one sector, then the productivity is randomly driven from one Pareto distribution across all the locations. The uncertainty in productivity can not explain the preference of people with various human capital for occupation, and the same skill distribution all over the country fails to describe the location preference of different sectors. Our work makes improvement through studying the spatial skill sorting and how the reallocation of human capital affects development.

Furthermore, our research extends the usage of NEG and QSE by applying the model to the context of Indonesia to estimate the effect of the tax-reducing policy. The tax rate is closely related to formalization process because it is the formal sector that bearing the corporate tax. The high tax rate motivates a large number of informal firms, which do not contribute to public finance (Johnson, Kauffman and Shleifer,1997; Porta and Shleifer, 2014; 2010 UN-HABITAT). Less tax burden and regulatory barrier for firms are significant for economic development through making the investment for skills and capital more profitable (Glaeser and Xiong, 2017). In the previous literature of Indonesia's tax reform, it is broadly found that the reduction of personal and corporate income tax promote the economic growth (Easterly and Rebelo, 1993; Engen, 1996; Marx, 2003; Ikhsan, Trialdi and Syahrial, 2005) and lower the incidence of poverty (Timmer, 2006; Yusuf, 2013). On the other hand, the tax reform leads to an increase in income inequality because the tax cut is more beneficial to households in the highest income categories (Samuelson, 1986; Amir, Asafu-Adjaye and Ducpham, 2013). Borgne, Silvani, Brondolo & Bosch (2008) and Putra (2014) focuses on the long-term effect of Indonesia's tax reform. The nominal tax revenue is found to rise significantly in this period, but the tax reform is not the only driven force. The high inflation rate, rapid economic growth and tax authorities' professionalism are also claimed to contribute to the steep increase in public income. Applying our employment formalization framework enriches the literature by explaining whether and how the tax base is broadened. The model also sheds light on the connection between the impact of cutting tax and other parameters in development, such as human capital, external demand, consumer preference, local transportation cost and regulatory barrier.

Combining the theory and its numerical simulation result obtains three key findings. Firstly, the coexistence of the formal sector and the informal sector in equilibrium reflects the balance between consumption diversity, which expands with local informal entrepreneurship, and productivity, which increases with formal-sector employment. When the preference for diversity decreases, productivity sorting of formal entrepreneurs become more significant, making the formal employment share increases in the location with lower trade cost. Secondly, larger scale of external trade encourages formalization by supporting the import of consumption diversity and making formal firms more profitable and bigger. Thirdly, from the perspective of supply-based analysis, improvement in skill distribution raises formalization by supplying a greater number of highly-productive formal firms and increasing export. Moreover, although both better trade-support infrastructure and lower business tax promote formalization, there is productivity sorting for the formal entrepreneurs, meaning that more productive formal entrepreneurs sort for the location with more efficient trade service, while less productive formal entrepreneurs select the location with smaller fixed cost. Our model implies that low education level, high trade cost and regulatory cost due to the shortage of public infrastructure and service in emerging economy weaken the supply determinants of formalization, limiting the size of the formal sector and its ability to provide diversity. To fulfill this gap, the demand for the informal sector is much stronger thus formal employment share is much lower than the developed countries.

Several propositions are driven from the calibration and counterfactual analysis results. Firstly, thanks to the broadened tax base, reducing income tax rate from 25% to 20% is estimated to have significant welfare and productivity effect. The calibrated benefit from more formal entrepreneurship dominates the cost of lower tax rate and raises local public income by over 20%, and therefore, the tax reform is predicted to be sustainable. In addition, the policy reducing tax improves average welfare by 6.76% and promotes formal-share by about 6.1%. Secondly, lower income tax rate contributes to the growth of public income and formal sector more in the provinces with lower trade cost. Thirdly, entrepreneurs whose skill are slightly smaller than the formal cutoff benefits the most from the tax reform because they can become formal and export to the external market. Higher real wage rate raises employee's utility and reduces the motivation to be informal entrepreneurs. Through encouraging more formal entrepreneurship, all the impact above are strengthened when trade cost becomes smaller, the regulatory barrier is relaxed and the human capital endowment is improved.

Section 2 presents some stylized facts about formal and informal employment in Indonesia to motivate the model described and solved in section 3. Section 4 introduces the comparative static based on the numerical simulation of the model. Section 5 expands the theoretical framework and analyzing the policy impact of tax-reduction. Section 6 concludes.

2, Stylized Facts of Formality and Informality in Indonesia

The empirical evidence presented here are based on the National Labor Force Survey of Indonesia (NLFS) in the last two decades (1995, 2000, 2005, 2010 and 2015) from Badan Pusat Statistik (BPS), Indonesia Census Data in the first decade of 21st Century (2000,2005 and 2010) from IPUMS International and CEIC dataset from the library of National University of Singapore.[1] For each observation, NLFS provides personal information, such as location, education level, age, employment status, wage, and so on. Census data complements NLFS with personal migration data in the last five years and the information on the county level, including urban status, quality of public service, employment size and high school

[1] The Census data of Indonesia from IPUMS international is cited from: Minnesota Population Center. *Integrated Public Use Microdata Series, International: Version* 6.5 [dataset]. Minneapolis: University of Minnesota, 2017. http://doi.org/10.18128/D020.V6.5.

graduate share. Both the original census data and NLFS data are on the individual level, but interviewees of the various year are different. As a result, in the dynamic analysis, the smallest research unit is the county, but not the individual. In addition, CEIC statistics fulfills the gap on the macro data of Indonesia, for example, the provincial scale of trade, road statistics, tax revenue and direct expenditure on public goods.

In this research**, *formality*** is measured with NLFS data. ***Formal workers*** are defined as wage or salary workers. ***Formal entrepreneurs*** are those with high education (at least graduated from secondary school) or who employ wage or salary workers. All the other labor force not in the formal sector are considered as working in the informal sector. Since this research focuses on employment activities, the individuals not employed or with unknown "class of work" are excluded from analysis.[2] After the adjustment, there are about 9.38 million, 0.45 million and 15.32 million observations in 2000, 2005 and 2010 Census datasets, respectively; the sample sizes of NLFS from 1995 to 2015 are 367728, 53590, 103414, 513553 and 320344. In the census data, one observation represents 10 individuals, but it is more than 10 in NLFS of each year and census data of 2005 for their much smaller sample size. Based on the definition of formal workers and entrepreneurs, ***formal employment share*** refers to the share of formal employment in total employment.

Based on the literature, two significant difference between the formal and informal sectors are the advantage of the formal sector in trade and entrepreneurs' education level (Melitz and Redding, 2012; Porta and Shleifer, 2014). During 1995 to 2015, the national and urban formal employment share, high school graduate share of the labor force and growth of goods export in Indonesia are shown in Figure 2. To prevent bias for the relatively small sample size of NLFS 2000-2010, the sample of Figure 2 are Census data 2000-2010 and NLFS 1995, 2015.

[Figure 2 here]

[2] The observations whose "Class of work" are "Not in the universe" and "Unknown" are dropped. "Not in the universe" means that the individuals are under 10 years-old or not employed, and "Unknown" makes it impossible to identify whether they are formal or informal.

According to the Figure 2, except for several years, the growth of goods export is always bigger than 0 before 2011. In 2011, the scale of goods export in Indonesia is four times as big as those in 1995. Supported by rapidly growing international demand, the formal employment share increases by about 10 percents during 1995 to 2015, while declines in years when the growth of export fluctuates, meaning that the development of the formal economy is much relying on export. However, there is usually significant external shocks in years with the negative growth of export. Noted that there is Asian Financial Crisis in 1997-1998, Crash of Dot Com Bubble around 2001 and Global Financial Crisis in 2008, hence the fluctuation is temporary (Nasution, 2002). Besides the influence of expanding export, Figure 2 shows the formal-share increases with education level in Indonesia from 1995 to 2015. Driven from NLFS, the formal-share of the educated individuals (at least graduated from secondary school) is around 90%, while it is no more than 30% of the less educated people, implying the positive correlation between education and formalization.

The variance of education and productivity between the formal and informal sector are mainly reflected on the entrepreneurs level. Figure 3 exhibits the high school graduate share of large firms (with at least 20 employees) and small firms' (with less than 20 employees) entrepreneurs. It is found that the share of educated entrepreneurs is much bigger among large firms which are all formal and nearly twice of the level among small firms. In addition, compared to the average level of all labor force shown in Figure 2, the high school graduate share of the entrepreneurs, especially the large firms', is higher and growing faster.

[Figure 3 here]

The difference in productivity between formal firms and informal firms in Indonesia is further proved by the mean of entrepreneurs' school years in Figure 4. The education experience of the formal entrepreneurs is longer than the informal entrepreneurs by around seven school years. Jointly analyzing Figure 3 and Figure 4, we come to a conclusion that the more educated entrepreneurs are more likely to be formal, and the scale of their firms are larger than those managed by less skillful entrepreneurs.

[Figure 4 here]

Previous research argues that the informal sector is an integral part of the urban economy (J. Ihrig and K.S. Moe, 2004; Lucas, 2004; Porta and Shleifer, 2014; 2010 UN-HABITAT). Thus the reaction of the informal sector to the expanding formal sector is to be further studied. Figure 5 shows the urbanization rate of employment across sectors between 1995 and 2015. The rapidly increasing urbanization rate from 33.18% to 52.07% in the last two decades, and the accompanying huge internal labor flow among regions, which is more than ten million from 2000 to 2010, making Indonesia be a good example to study the spatial variation of formality. Two patterns can be observed from Figure 5. Firstly, the national formal employment share and urbanization rate are nearly growing together. Secondly, the urbanization rate of formal employment is much higher than the informal employment, being consistent with the higher urban formal employment share in Figure 2.

[Figure 5 here]

[Table 1 here]

Table 1 shows the estimated Zipf's coefficients of the county employment size distribution from 1995 to 2015.[3] There are two groups of estimates, one for the whole county, another for the urban area. The estimates are derived from the regression of log county employment size on log rank. A smaller absolute value of Zipf's coefficient means more uneven employment size distribution across counties. Three patterns can be observed from Table 1. Firstly, comparing to the whole county, concentration trend is more significant in the urban area. Secondly, the concentration degree is higher in the formal sector than the informal sector. This characteristic is also supported by the spatial distribution of employment shown in Figure 6 because the formal sector consistently has a flatter upper tail than the informal sector. Thirdly, the spatial dispersion increases during 2005 and 2015. The first pattern implies that urban-rural difference in local conditions, such as transportation networks and trade infrastructure, significantly affecting location choice of the labor force, especially for the

[3] Table 1 is estimated based on the largest 100 counties each year.

formal employment. The second pattern shows that the formal firms are more willing to concentrate, possibly for the benefit from agglomeration economies, while the last pattern represents that the motivation of concentration rises since 2005.

[Figure 6 here]

The heterogeneous spatial distribution pattern between the formal sector and the informal sector, driven from Figure 5 and Table 1 raises a question: what kind of location is more preferred by the growing formal sector? One potential explanation is that urban-rural difference in local conditions, such as transportation networks and trade infrastructure, significantly affecting location choice of the labor force, especially for the formal employment. Formal employment share is more likely to be higher in the relatively developed area, such as Java region where the capital Jakarta locates, because of its better infrastructure and more efficient trade service.

Figure 7 supports this hypothesis through the much higher average school years of formal entrepreneurs in Java than other regions in Indonesia. It proves that more skillful formal entrepreneurs sort for locations with more and better road and ports. On the contrary, informal entrepreneur's education doesn't vary significantly across regions. Differing with the entrepreneurs, there is no obvious difference on school experience between formal workers and informal workers, based on Figure 8. Moreover, similar to the informal entrepreneurs, workers' education is nearly the same across regions.

[Figure 7 here]

[Figure 8 here]

Location sorting of formal entrepreneurs contributes to the bigger formal employment share in Java region than other regions in Figure 9. Although lower trade cost makes Java region more attractive to the productive formal firms, the formal employment share exhibits convergence across regions, under rapid growth of goods export and education level. The faster-rising formal-share in other regions of Indonesia implies that there are some other factors affecting the spatial distribution of the formal and informal employment, for example, the difference in fixed cost across regions and change of consumer preference as time goes by. We leave this hypothesis to the model section, in which a coherent micro-foundation of the literature and the stylized facts in Indonesia are provided, so as to shed light on the motivation of formalization and spatial dispersion of formal employment and informal employment.

[Figure 9 here]

Besides formal employment share, the share of local tax revenue in aggregate public income is observed to be much bigger in Java region than all other regions of Indonesia in Figure 9, meaning that the public finance of locations with higher formal employment share relies more on tax revenue, which is consistent with Figure 10. The increasing fixed line between provincial tax income share and formal employment share not only further shows the formal economy is the main source of tax income, but also implies that the influence of tax-cutting policy varies across provinces, possibly due to different local latent variables, which is waiting to be checked. In addition, the convergence of formal employment share means that the spatial distribution of formal sector become more even in 2015 than 2005.

[Figure 10 here]

Besides the infrastructure and trade cost, human capital is supposed to affect the structure of local public finance significantly. Based on Figure 11, local tax income share is positively related to the percentage of labor force with at least high school education. It implies that the impact of reducing tax may be different on individual vary in skill. For example, the middle-skill people hesitating between the formal and informal sector are more likely to run formal firms once the corporate tax decreases. We left this hypothesis in the counterfactual analysis.

[Figure 11 here]

3, The Model

3.1 Model Setting

We present a stylized open-city model to enable both supply-side and demand-side analyses of formalization and welfare. The model shows the influence of two key policy variables, namely the trade cost and fixed cost, on the general equilibrium formal employment share and the spatial distribution of formal and informal employment across cities.

Endowment and Sequences of Location and Occupation Choices

We consider two cities with the inelastic supply of housing in an open economy, varying on exogenous and independent trade cost $\boldsymbol{T_c}$ and fixed cost $\boldsymbol{\theta_c}$. Each trading with the world market subject to a city-specific trade cost $\boldsymbol{T_c}$, which depends on city location as well as the city's investment in infrastructure, such as ports and trade-support institutions. The economy is endowed with a continuum of $\boldsymbol{N}$ units of utility-maximizing worker-consumers with perfect mobility and heterogeneous skills $\boldsymbol{\varphi}$, such that $\boldsymbol{\varphi} \in [\boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{max}}]$. The distribution of skill is summarized by the continuously differentiable cumulative truncated Pareto distribution function $\boldsymbol{G}(\boldsymbol{\varphi})$ over $[\boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{max}}]$. [4] Every agent sorts for a place to locate and selects an occupation: employee, the informal or formal entrepreneur, based on full knowledge of his skill $\boldsymbol{\varphi}$ to maximize personal utility. When working as entrepreneurs, people receive firms' profit as income and their personal productivity is the marginal product of labor. Being different with informal firms which can not participate in trade, formal firms join trade with a fixed cost $\boldsymbol{\theta_c}$ and national income tax $\boldsymbol{\tau}$, where the latter is also applicable to the formal employees. With a job of employees, everyone supplies one unit of labor for an endogenous local wage rate $\boldsymbol{w_c}$, no matter how high his skill level is. Although the nominal wage rate in the formal sector is higher, the income tax for the formal workers makes the local real wage rate $\boldsymbol{w_c}$ equalize across sectors.

Population Structure

The size of each city is endogenous. In the equilibrium, each person should live in one of these two cities. Total population $\boldsymbol{N}$, local population $\boldsymbol{N_c}$ and distribution of skill $\boldsymbol{g}(\boldsymbol{\varphi})$ in the economy satisfy equation (1) and (2) :

[4] The probability density function is $\boldsymbol{g}(\boldsymbol{\varphi}) = \boldsymbol{k\varphi_m}/\boldsymbol{\varphi^{k+1}}$, where $\boldsymbol{\varphi_m} = \frac{\varphi_{min}{}^k \varphi_{max}{}^k}{\varphi_{max}{}^k - \varphi_{min}{}^k}$, such that $\int_{\varphi_{min}}^{\varphi_{max}} \boldsymbol{g}(\boldsymbol{\varphi})\, \mathrm{d}\boldsymbol{\varphi} = 1$. $\boldsymbol{\varphi_{min}}$ and $\boldsymbol{\varphi_{max}}$ are the lower bound and upper bound of the skill's interval, respectively.

$$\mathrm{N_c} = \int_{\varphi_{\min}}^{\varphi_{\max}} \mathrm{N_c}(\varphi)\, d\varphi \tag{1}$$

$$\mathrm{Ng}(\varphi) = \sum_{c=1,2} \mathrm{N_c}(\varphi) \tag{2}$$

Equation (1) means that local population can be decomposed into groups differ in productivity. Equation (2) states that the total amount of people with skill $\boldsymbol{\varphi}$ in the economy is equal to the mass of individual with $\boldsymbol{\varphi}$ between these two cities. Adding up equation (2) across the set of skill leads to $\boldsymbol{N} = \boldsymbol{N_1} + \boldsymbol{N_2}$, which satisfies the full population condition that all agents live in these two cities.

Personal utility is proportional to local amenity $\boldsymbol{A_c}$:

$$\mathrm{A_c} = \mathrm{N_c}^{-a_c} \tag{3}$$

Where $\boldsymbol{a_c}$ is an exogenous parameter relates to the congestion within a city. Higher $\boldsymbol{a_c}$ is, faster the amenity $\boldsymbol{A_c}$ decreases with local labor size. For simplicity, $\boldsymbol{a_c}$ is assumed to be homogeneous across cities.

Consumers

In each city, agents consume all the varieties of goods in the market. Consumers can purchase not only goods produced by local firms, formal or informal, but also goods shipped from the rest of world. Individuals are risk-neutral so that personal utility is proportional to local amenity and consumption bundle, which is equal to income divided by the local price index.[5] The personal preference follows Dixit-Stiglitz type with a constant elasticity of substitution for any two variety $\boldsymbol{\sigma}$, equaling to $\boldsymbol{1} + \frac{\boldsymbol{1}}{\boldsymbol{\varepsilon}}$, where $\boldsymbol{\varepsilon} > 0$. Let subscript $\boldsymbol{F}$ and $\boldsymbol{I}$ indicate the formal and informal sectors, respectively, $\boldsymbol{u_c}(\boldsymbol{\varphi})$ and $\boldsymbol{y_c}(\boldsymbol{\varphi})$ indicate the utility and the income of a worker with skill $\boldsymbol{\varphi}$ in city $\boldsymbol{c}$, $\boldsymbol{u_c}(\boldsymbol{\varphi})$. The utility and the budget constraint are given by:

[5] Without losing generality, our model abstracts away land by assuming that personal utility or output are independent with consuming more land, which implies land rent is standardized to be zero. The zero land rent is not only the necessary condition for free mobility of labor and inelastic supply of housing but also capturing the fact of extremely low land rent in developing countries. This fact is reflected by the extremely high ownership ratio of housing all over Indonesia, which is above 82% during 2005 to 2010, according to the census data. As mentioned in Lucas (2002), Desmet and Rossi-Hansberg (2013) and Behrens, Duranton and Robert-Nicoud (2014), production takes place in the center of cities, so that there is a potential premium on land rent for the formal sector. Without losing generality, this model includes the premium in $\boldsymbol{\theta}$, the local fixed cost for the formal sector.

$$\max_{x(j)} u_c(\varphi) = A_c \left(\int_{(\Omega_{Ic}+\Omega_{Fc}+\Omega_O)} x(j)^{\frac{1}{1+\varepsilon}} dj \right)^{1+\varepsilon} \tag{4}$$

$$\int_{(\Omega_{Ic}+\Omega_{Fc}+\Omega_O)} p_c(j)x(j)\, dj = y_c(\varphi)$$

where $\boldsymbol{\Omega_{Ic}}, \boldsymbol{\Omega_{Fc}}$ are endogenous sets of the local informal sector and local formal sector, while $\boldsymbol{\Omega_O}$ is the exogenous set of the imported goods from the world which is used to keep the trade account balance. For simplicity, assuming that the import good with price $\mathbb{P}_o$ in the international market, which is exogenous given global price index. $y_c(\varphi)$ is income of the agents with skill φ in city c and spent only in the consumption goods. $x(j)$ stands for the personal demand of goods j, and $p_c(j)$ is the price of goods j in city c.

To maximize personal utility, based on the utility function equation (4) and the budget constraint, demand for goods j of an agent with skill φ in city c is

$$x(j) = \left[\frac{p_c(j)}{\mathbb{P}_c}\right]^{-\sigma} \frac{y_c(\varphi)}{\mathbb{P}_c} \tag{5}$$

Hence the indirect utility is equal to

$$u_c(y_c(\varphi)) = \frac{A_c y_c(\varphi)}{\mathbb{P}_c} \tag{6}$$

Where $\mathbb{P}_c$ is the price index of all goods can be access to in city c, including import goods, local formal and informal goods, such that

$$\mathbb{P}_c = \left[\int_{(\Omega_{Ic}+\Omega_{Fc}+\Omega_O)} p_c(j)^{-\frac{1}{\varepsilon}} dj\right]^{-\varepsilon} \tag{7}$$

Producers

All firms are monopolistically competitive, having only one entrepreneur and producing only one variety goods. The only input for each firm, formal or informal, is labor, and the marginal product is equal to the entrepreneur's skill. Output of the j'th firm in city c, $X_c(j)$, is equal to[6]

$$X_c(j) = \varphi_c(j) l_c(j) \tag{8}$$

[6] With a production function in the form of equation (8) like those in Behrens, Duranton and Robert-Nicoud (2014), the setting of heterogeneous firms in a constant-elasticity-of-substitution demand system is the sufficient condition for monopolistic competition market structure, because the difference in marginal product of labor prevents the case of perfect competition in the market.

$\boldsymbol{l_c(j)}$ is the amount of labor employed by firm $\boldsymbol{j}$ in city $\boldsymbol{c}$, and $\boldsymbol{\varphi_c(j)}$ is personal productivity of its entrepreneur, taking a form similar as those in Behrens, Duranton and Robert-Nicoud (2014) and Lucas (1978). The marginal cost $\boldsymbol{d(w_c l_c(j))/dX_c(j) = w_c/\varphi}$ decreases with $\boldsymbol{\varphi}$, implying that the firms with higher skill management are more productive. Although the after-tax income for formal employees is the same as those in the informal sector, which equals to an endogenous local wage rate $\boldsymbol{w_c}$, formal firms suffer a higher labor cost $\boldsymbol{w_c/(1-\tau)}$. Besides the labor cost and income tax $\boldsymbol{\tau}$, formal firms incur a fixed cost $\boldsymbol{\theta_c}$ for intermediate services, such as business licenses, regulatory compliances, and financial, legal services and the premium of land rent:

$$\max_{p_c(j)} \pi_c(j) = (1-\tau)\left[p_c(j)X_c(j) - \frac{w_c}{(1-\tau)} l_c(j) - \theta_c\right], j \in \Omega_{Fc} \tag{9}$$

$$\max_{p_c(j)} \pi_c(i) = p_c(j)X_c(j) - w_c l_c(j), j \in \Omega_{Ic} \tag{10}$$

Compared to the informal firms producing and selling locally, formal firms sell their goods in both local and international market with different prices, due to the loss of transportation, which is determined by $\boldsymbol{T_c}$. $\boldsymbol{T_c}$ is the trade cost in city $\boldsymbol{c}$, where $\boldsymbol{T_c} \in (1, +\infty)$. The bigger $\boldsymbol{T_c}$ is, the higher trade cost for local export and import paid by the buyers will be. For every unit of goods sent from city $\boldsymbol{c}$, only $1/\boldsymbol{T_c}$ unit arrives its destination. Similarly, for every one unit of goods shipped from the world, only $1/\boldsymbol{T_c}$ unit arrives city $\boldsymbol{c}$. In order to obtain the same revenue, $\boldsymbol{p_{Oc}(j) = T_c p_c(j)}$, where $\boldsymbol{p_{Oc}(j)}$ is the international price of goods produced by the $\boldsymbol{j}$'th formal firm in city $\boldsymbol{c}$.

Total spending on intermediate services, which are procured from local suppliers, is

$$R_c = \theta_c \int_{\Omega_{Fc}} dj + \tau\left[\int_{\Omega_{Fc}} (p_c(j)X_c(j) - \theta_c)\, dj\right] \tag{11}$$

Definition 1: The equilibrium is the allocation $\{\boldsymbol{N_c^*}, \boldsymbol{G_c^*(\varphi)} \mid \boldsymbol{c} \in \{1, 2\}\}$ and market-clearing price indexes $\{\mathbb{P}_c^*, \boldsymbol{w_c^*} \mid \boldsymbol{c} \in \{1, 2\}\}$, given the parameters set $\{\boldsymbol{\varepsilon}, \boldsymbol{k}, \boldsymbol{a_c}, \boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{max}}\} \in \boldsymbol{R_+^*}$, endowment $\{\boldsymbol{N}, \boldsymbol{G(\varphi)}\}$, policy variables $\{\boldsymbol{\tau}, \boldsymbol{T_c}, \boldsymbol{\theta_c}\}$ and world market condition $\mathbb{P}_{\boldsymbol{o}}$, such that the endogenous variables $\{\boldsymbol{A_c}, \boldsymbol{X_c}, \boldsymbol{L_c}, \boldsymbol{Y_c}\}$ meet the conditions below:

(1) Each household chooses a location, occupation and consumption bundle to maximize individual utility;

(2) Each formal and Informal differentiated good producer chooses price and labor employment to maximize profit;

(3) Labor markets and product markets all clear.

Based on the algorithm in Appendix A, there is and only is one solution to this equilibrium. ■

3.2 Local Production and Employment Mix

We first solve the workers' choice between an occupation as an employee and that as an entrepreneur given the labor size $\boldsymbol{N_c}$ and skill distribution $\boldsymbol{G_c(\varphi)}$ in the city. Since the informal sector only sells its products locally, while formal firms participate in trade, the aggregate demand of the $\boldsymbol{j}$'th formal firm and informal firm depend on their price $\boldsymbol{p_c(j)}$

$$X_{Fc}(j) = p_c(j)^{-\sigma}\left(\mathbb{P}_c^{\frac{1}{\varepsilon}}Y_c + T_c^{-\sigma}\mathbb{P}_O^{\frac{1}{\varepsilon}}Y_O\right),\ j \in \Omega_{Fc} \tag{12}$$

$$X_{Ic}(j) = p_c(j)^{-\sigma}\mathbb{P}_c^{\frac{1}{\varepsilon}}Y_c,\ j \in \Omega_{Ic} \tag{13}$$

Where

$$Y_c = \int_{\varphi_{min}}^{\varphi_{max}} y_c(\varphi)\, d\varphi + R_c$$

is aggregate income in city $\boldsymbol{c}$. $\mathbb{P}_{\boldsymbol{O}}$ and $\boldsymbol{Y_O}$ are exogenous external price index and demand respectively. Both $\boldsymbol{X_{Fc}(j)}$ and $\boldsymbol{X_{Ic}(j)}$ increase with local price index $\mathbb{P}_{\boldsymbol{c}}$, because bigger $\mathbb{P}_{\boldsymbol{c}}$ means the advantage of price for all firms in the local market. From equation (12) as well as (13), the own-price elasticity of demand in both sectors are the same, $-\boldsymbol{\sigma}$. However, compared to the informal firms, formal firms' aggregate demand is negatively related to local trade cost $\boldsymbol{T_c}$, global price index $\mathbb{P}_{\boldsymbol{O}}$ and external demand $\boldsymbol{Y_O}$. Substituting equation (12) and (13) into equation (9) and (10), yields the profit-maximizing price

$$p_{cF}(\varphi) = \frac{(1+\varepsilon)w_c}{\varphi(1-\tau)} \tag{14A}$$

$$p_{cI}(\varphi) = \frac{(1+\varepsilon)w_c}{\varphi} \tag{14B}$$

As a result, the profit-maximizing price is equal to the marginal cost plus markup.

Adding up equation (14) over the set of variety of consumption goods in each city, obtaining the local price index:

$$\mathbb{P}_c = \frac{(1+\varepsilon)w_c}{\Phi_c{}^{\varepsilon}} \tag{15}$$

Where

$$\Phi_c = \Phi_{Ic} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{Fc} + \left(\frac{w_c}{T_c w_O}\right)^{\frac{1}{\varepsilon}}\Phi_O \tag{16}$$

$$\Phi_{Ic} = \int_{\Omega_{Ic}} \varphi(j)^{\frac{1}{\varepsilon}}\,dj \tag{17}$$

$$\Phi_{Fc} = \int_{\Omega_{Fc}} \varphi(j)^{\frac{1}{\varepsilon}}\,dj \tag{18}$$

Equation (17) and (18) are the definition of sectors' capacity, hence $\boldsymbol{\Phi_c}$ is local aggregate capacity for all kind of goods supplied in city $\boldsymbol{c}$, including local and overseas variety. Rewriting equation (12) and (13) with equation (14), which holds for demand of formal firms $\boldsymbol{X_{Fc}(j)}$ and informal firms $\boldsymbol{X_{Ic}(j)}$

$$X_{cF}(\varphi) = (1-\tau)^{\sigma}\left[\left(\frac{\varphi}{\Phi_c{}^{\varepsilon}}\right)^{\sigma}\frac{Y_c}{\mathbb{P}_c} + \left(\frac{w_O\varphi}{T_c w_c \Phi_O{}^{\varepsilon}}\right)^{\sigma}\frac{Y_O}{\mathbb{P}_O}\right] \tag{12'}$$

$$X_{cI}(\varphi) = \left(\frac{\varphi}{\Phi_c{}^{\varepsilon}}\right)^{\sigma}\frac{Y_c}{\mathbb{P}_c} \tag{13'}$$

Noticed that endogenous $\boldsymbol{\Phi_c}$ and exogenous $\boldsymbol{\Phi_O}$ are the aggregate capacity of the firms in the local and global commodity market, hence $\boldsymbol{\varphi/\Phi_c{}^{\varepsilon}}$ and $\boldsymbol{w_O\varphi/T_c w_c \Phi_O{}^{\varepsilon}}$ represents the capacity share of a firm. Since bigger $\boldsymbol{\Phi}$ means tougher competition, equation (12') and (13') show that a firm with the higher capacity share is more competitive and able to occupy the bigger proportion of the market. Larger market share for the most efficient firm improves national productivity but losing diversity in the market. Holding $\boldsymbol{\Phi_O}, \mathbb{P}_{\boldsymbol{O}}$ and $\boldsymbol{Y_O}$ constant, the wage rate of global labor market $\boldsymbol{w_O}$ is also exogenously given, satisfying $\mathbb{P}_{\boldsymbol{O}} = \frac{(1+\varepsilon)w_O}{\Phi_O{}^{\varepsilon}}$.[7] Bigger global-local wage-rate ratio $\boldsymbol{w_O/w_c}$ means the local advantage in labor cost thus export price, which contributes to the increase in external demand and decrease of import. Item $\boldsymbol{Y_c}/\mathbb{P}_{\boldsymbol{c}}$ and $\boldsymbol{Y_O}/\mathbb{P}_{\boldsymbol{O}}$

[7] The structure of the global market is set to be the same as the local market so that there is no difference on the relation between parameters. However, due to the much larger size, the parameters of the global market, like $\boldsymbol{\Phi_O}, \mathbb{P}_{\boldsymbol{O}}, \boldsymbol{Y_O}$ and $\boldsymbol{w_O}$ are all exogenous.

are the total demand of city $\boldsymbol{c}$ and the international market respectively, and producers benefit from markets with a larger scale of demand.

Combining (12'), (13'), (14) and (15), producers' profit becomes

$$\pi_{Fc}(\varphi) = \frac{(1-\tau)^{\sigma}}{\sigma}\left[Z_c + \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}} \frac{Z_O}{{T_c}^{\sigma}}\right] \varphi^{\frac{1}{\varepsilon}} - (1-\tau)\theta_c \tag{19}$$

$$\pi_{Ic}(\varphi) = \frac{Z_c}{\sigma}\varphi^{\frac{1}{\varepsilon}} \tag{20}$$

Defining $\boldsymbol{Z_c = Y_c/\Phi_c}$, since $\boldsymbol{\Phi_c}$ is the aggregate capacity of the variety supplied in the local market, while $\boldsymbol{Y_c}$ is the total consumption expenditure in city $\boldsymbol{c}$, thus $\boldsymbol{Z_c}$ represents the unit return of capacity in the local market, which can be viewed as productivity. According to the profit function, both $\boldsymbol{\pi_{Fc}(\varphi)}$ and $\boldsymbol{\pi_{Ic}(\varphi)}$ rises with $\boldsymbol{Z_c}$. Similar to the implication of the demand functions, profit function of formal firms decreases with local wage rate but increases with the total demand. More importantly, lower local trade cost $\boldsymbol{T_c}$ not only contributes to larger revenue for the formal producers but also implies a bigger elasticity of income for the global demand $\boldsymbol{Y_O}$.

The individual selects occupation to maximize personal utility. When working as an employee, the after-tax labor income is equal to $\boldsymbol{w_c}$, being independent with personal skill. To be an informal entrepreneur, on the other hand, personal income is the profit of the firm as shown in equation (20). The utility tradeoff between workers and informal entrepreneurs is exhibited in Figure 12.

[Figure 12 here]

Denoting "**entrepreneurship threshold**" $\underline{\boldsymbol{\varphi_c}}$ as the local skill threshold between informal entrepreneurs and workers, and with equation (20), yields

$$u_{Ic}\left(\underline{\varphi}_c\right) - u_{Wc}\left(\underline{\varphi}_c\right) = \frac{A_c Z_c}{\sigma \mathbb{P}_c}\underline{\varphi}_c^{\frac{1}{\varepsilon}} - \frac{A_c w_c}{\mathbb{P}_c} = 0 \Rightarrow \underline{\varphi}_c = \left(\frac{\sigma w_c}{Z_c}\right)^{\varepsilon} \tag{21}$$

The entrepreneurship requirement becomes lower as local total income $\boldsymbol{Y_c}$ increases ($\partial\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}/\partial\boldsymbol{Y_c} < 0$), while higher wage rate ($\partial\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}/\partial\boldsymbol{w_c} > 0$), while tougher local competition ($\partial\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}/\partial\boldsymbol{\Phi_c} > 0$) makes it more difficult to be an entrepreneur.

Defining "**formalization threshold**" $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ as the skill cutoff between formal and informal entrepreneurs and identifying it by using the personal indirect utility (6) with (19) and (20):

$$u_{Fc}(\overline{\varphi}_c) - u_{Ic}(\overline{\varphi}_c) = 0 \Rightarrow \overline{\varphi}_c = \left[\frac{\sigma(1-\tau)\theta_c}{(1-\tau)^{\sigma}\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{Z_O}{T_c{}^{\sigma}} - [1-(1-\tau)^{\sigma}]Z_c}\right]^{\varepsilon} \tag{22}$$

Comparing the indirect utility function between the formal and informal firms, both of them benefit from the local return for productivity increases, while only formal firms paying the tax can be more profitable when the global return for productivity rises. Hence sector selection is a tradeoff between extra revenue from the external market and local additional cost for a formal firm, shown in Figure 13. For a sufficiently small $\boldsymbol{\tau}$ or $\boldsymbol{\theta}$, we may have $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}} \leq \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$, in which case the informal sector disappears. We will focus on the case where $\boldsymbol{\tau}$ and $\boldsymbol{\theta}$ are sufficiently high so that $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}} > \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ and the formal and informal sectors coexist.

[Figure 13 here]

Equation (22) implies that higher $\boldsymbol{w_c}$ means labor cost rises, and therefore, running a formal firm becomes less profitable and more difficult. In addition, $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ is positively related to formalization cost $\boldsymbol{\theta_c}$ and income tax rate $\boldsymbol{\tau}$, while negatively depending on the scale of external market $\boldsymbol{Y_O}$. Formalization is easier to take place in a city with more efficient trade service ($\boldsymbol{\partial\overline{\varphi}_c/\partial T_c} > 0$).

Substituting $\boldsymbol{\Omega_{Ic}}$ and $\boldsymbol{\Omega_{Fc}}$ with $\boldsymbol{N_c}$, $\boldsymbol{G_c(\varphi)}$, $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ and $\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ in (17) and (18) [8]

$$\Phi_{Ic} = N_c \int_{\underline{\varphi}_c}^{\overline{\varphi}_c} \varphi^{1/\varepsilon}\, dG_c(\varphi) \tag{17'}$$

$$\Phi_{Fc} = N_c \int_{\overline{\varphi}_c}^{\varphi_{max}} \varphi^{1/\varepsilon}\, dG_c(\varphi) \tag{18'}$$

[8] To guarantee the aggregate productivity of the formal sector $\boldsymbol{\Phi_{Fc}}$ converges when $\boldsymbol{\varphi_{max}} \to +\infty$, $\boldsymbol{k} > 1/\varepsilon$.

In the equilibrium, local labor market clears by equalizing labor supply L_c^S and labor demand L_c^D. Labor is supplied by low skill agents with skill smaller than $\underline{\varphi}_c$: $L_c^S = N_c \int_{\varphi_{min}}^{\varphi_c} d\,G_c(\varphi)$. Labor demand L_c^D can be decomposed with sectors, L_{Ic}^D and L_{Fc}^D. The amount of labor employed by each firm is $X(\varphi)/\varphi$, which is increasing with managerial skill $\varphi^{\frac{1}{\varepsilon}}$, as the pattern in Figure 3. With equation (12'), (13'), (19) and $L_c^S = L_{Ic}^D + L_{Fc}^D$, the clear condition of the local labor market is equation (23):

$$L_{Ic}^D = N_c \int_{\underline{\varphi}_c}^{\overline{\varphi}_c} \left(\frac{\varphi}{{\Phi_c}^{\varepsilon}}\right)^{\sigma} \frac{Y_c}{\varphi \mathbb{P}_c} dG_c(\varphi) = \frac{\Phi_{Ic} Z_c}{(1+\varepsilon) w_c}$$

$$L_{Fc}^D = N_c \int_{\overline{\varphi}_c}^{\varphi_{\max}} (1-\tau)^{\sigma} [\left(\frac{\varphi}{{\Phi_c}^{\varepsilon}}\right)^{\sigma} \frac{Y_c}{\varphi \mathbb{P}_c} + \left[\frac{\varphi}{(1+\varepsilon) w_c T_c}\right]^{\sigma} \frac{Y_O}{\varphi}]\, dG_c(\varphi) = \frac{(1-\tau)^{\sigma} \Phi_{Fc}}{(1+\varepsilon) w_c} [Z_c + \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}} \frac{Z_O}{{T_c}^{\sigma}}]$$

$$(1+\varepsilon) w_c N_c \int_{\varphi_{\min}}^{\varphi_c} d\,G_c(\varphi) = [\Phi_{Ic} + (1-\tau)^{\sigma} \Phi_{Fc}] Z_c + (1-\tau)^{\sigma} \Phi_{Fc} \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}} \frac{Z_O}{{T_c}^{\sigma}} \quad (23)$$

The LHS of equation (23) is equal to total local labor income plus markup and therefore is the aggregate value of output in city c. It depends on the two parts in RHS, which is positively related to Z_c and Z_O, the producer productivity in the local and global market, respectively. We can prove that Z_c rises with local aggregate formal capacity Φ_{Fc}. Therefore, the total value of local output is an increasing function of Φ_{Fc}, implying that more developed formal economy contributes to income growth. The formal employment share S_c can be expressed by L_{Ic}^D and L_{Fc}^D:

$$S_c = \frac{L_{Fc}^D}{L_{Fc}^D + L_{Ic}^D} = \frac{1}{1 + L_{Ic}^D / L_{Fc}^D} = \frac{1}{1 + 1 \Big/ \left[\frac{(1-\tau)^{\sigma} \Phi_{Fc}}{\Phi_{Ic}} \left(1 + \frac{(w_O / T_c w_c)^{\frac{1}{\varepsilon}} Z_O}{Z_c}\right)\right]} \quad (24)$$

Equation (24) shows that formal employment share positively depends on the ratio of aggregate capacity between formal sector and informal sector, and $\frac{(w_O/T_c w_c)^{\frac{1}{\varepsilon}} Z_O}{Z_c}$, which represents the relative ability to satisfy diversity demand by international trade. Therefore, formal employment share is higher in a city which has lower wage rate w_c and trade cost T_c.

Decomposing Y_c by occupation as $Y_c = Y_{Wc} + Y_{Ic} + Y_{Fc} + R_c$, where Y_{Wc}, Y_{Ic}, Y_{Fc} and R_c is the total income of workers, informal entrepreneurs, formal entrepreneurs and suppliers of intermediate services in city c, respectively, such that

$$Y_{Wc} = w_c N_c \int_{\varphi_{min}}^{\underline{\varphi}_c} d\, G_c(\varphi)$$

$$Y_{Ic} = N_c \int_{\underline{\varphi}_c}^{\overline{\varphi}_c} \pi_{Ic}(\varphi)\, dG_c(\varphi) = \frac{\Phi_{Ic} Z_c}{\sigma}$$

$$Y_{Fc} = N_c \int_{\overline{\varphi}_c}^{\varphi_{max}} \pi_{Fc}(\varphi)\, dG_c(\varphi) = \frac{(1-\tau)^{\sigma}}{\sigma}\left[Z_c + \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}} \frac{Z_O}{{T_c}^{\sigma}}\right]\Phi_{Fc} - (1-\tau)\theta_c N_c \int_{\overline{\varphi}_c}^{\varphi_{max}} d\, G_c(\varphi)$$

$$R_c = \frac{\tau w_c L_{Fc}^{D}}{(1-\tau)} + \frac{\tau(1-\tau)^{\frac{1}{\varepsilon}}}{\sigma}\left[Z_c + \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}} \frac{Z_O}{{T_c}^{\sigma}}\right]\Phi_{Fc} + (1-\tau)\theta_c N_c \int_{\overline{\varphi}_c}^{\varphi_{max}} d\, G_c(\varphi)$$

Hence $\boldsymbol{Y_c}$ is expressed by

$$Y_c = w_c\left[L_{Ic}^{S} + \frac{L_{Fc}^{S}}{(1-\tau)}\right] + \frac{1}{\sigma}\left[\left(\Phi_{Ic} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{Fc}\right)Z_c + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{Fc}\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{Z_O}{{T_c}^{\sigma}}\right] \quad (25)$$

The export income $(\boldsymbol{1-\tau})^{\frac{1}{\varepsilon}}\boldsymbol{\Phi_{Fc}}\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{Z_O}{{T_c}^{\sigma}}$ satisfies the trade balance condition

$$\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{(1-\tau)^{\frac{1}{\varepsilon}}\Phi_{Fc}Z_O}{{T_c}^{\sigma}} = \left(\frac{w_c}{T_c w_O}\right)^{\frac{1}{\varepsilon}}\Phi_O Z_c \Rightarrow Z_c = \left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\left(\frac{\mathbb{P}_O}{\mathbb{P}_c}\right)^{\frac{1}{\varepsilon}}\frac{(1-\tau)^{\frac{1}{\varepsilon}}\Phi_{Fc}Z_O}{T_c\Phi_c} \quad (26)$$

Implying that the export income is equal to expenditure on import goods. The aggregate productivity $\boldsymbol{Z_c}$ in city $\boldsymbol{c}$ increases with world productivity, local price competitiveness $\left(\frac{\mathbb{P}_O}{\mathbb{P}_c}\right)^{\frac{1}{\varepsilon}}$, wage competitiveness $\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}$ and formal capacity $\boldsymbol{\Phi_{Fc}}$, but decreases with trade cost $\boldsymbol{T_c}$.

3.3 Inter-City Equilibrium of Skill Sorting

We turn to the spatial sorting issue in this section. Individuals choose a city to maximize their personal utility which equals real income. Free mobility ensures that utility offered by each city is equalized for workers and the informal entrepreneurs with any skill level, hence no one has the incentive to deviate[9]:

$$u_{W1}(\varphi) = u_{W2}(\varphi) \Rightarrow \frac{A_1 w_1}{\mathbb{P}_1} = \frac{A_2 w_2}{\mathbb{P}_2}, \varphi \in [\varphi_{min}, \underline{\varphi}_c] \quad (27)$$

$$u_{I1}(\varphi) = u_{I2}(\varphi) \Rightarrow \frac{A_1 Z_1}{\sigma\mathbb{P}_1}\varphi^{\frac{1}{\varepsilon}} = \frac{A_2 Z_2}{\sigma\mathbb{P}_2}\varphi^{\frac{1}{\varepsilon}}, \varphi \in [\underline{\varphi}_c, \overline{\varphi}_c] \quad (28)$$

The indifferent conditions for workers and the informal entrepreneurs (27) and (28) are robust. Regardless of personal skill, income for workers is local wage rate $\boldsymbol{w_c}$, and therefore, if

[9] Corner solution in which all workers and firms concentrate in one city is both empirically counterfactual and theoretically meaningless. Therefore, we assume that the total population size $\boldsymbol{N}$ and the congestion coefficient $\boldsymbol{a_c}$ in the economy is sufficiently large such that corner solution is irrational.

sorting equilibrium exists for any worker, all workers do not satisfy the indifferent condition, driving them to move to the city with higher real income. However, bigger size of labor supply raises the urban dis-amenity $\boldsymbol{A_c}$, which incurs welfare loss. More importantly, if all workers are in one city, that means all firms have to locate in the same city, too, but it is impossible because the total population size $\boldsymbol{N}$ in the economy is sufficiently large that corner solution in which all workers and firms concentrate in one city is irrational. As a result, sorting equilibrium is infeasible for workers, and equation (27) is robust. Combining equation (15) and (27), indifference condition on workers' utility across locations leads to

$$\frac{\mathbb{P}_2}{\mathbb{P}_1} = \frac{\mathrm{A}_2 \mathrm{w}_2}{\mathrm{A}_1 \mathrm{w}_1} \Rightarrow \frac{\mathbb{P}_2}{\mathbb{P}_1} = \frac{\mathrm{A}_2 \mathrm{w}_2}{\mathrm{A}_1 \mathrm{w}_1} \tag{29}$$

$$\left(\frac{\Phi_2}{\Phi_1}\right)^{\varepsilon} = \frac{\mathrm{A}_2}{\mathrm{A}_1} \tag{30}$$

When employees' utility is indifferent across cities, equation (29) claims that local price index is positively related to wage rate, discounting the benefit from higher wage rate and therefore preventing workers concentrating in the city with a bigger return of labor. Equation (30) tells fact that local aggregate capacity, which includes not only local firms but also the producers of import goods, is inversely proportional to urban congestion. Since the congestion is positively depended on population size, the inter-city ratio of aggregate capacity increases with the local city size. It means that although larger labor size raises urban friction and incurs welfare loss, it can be compensated by a lower local price index, because of higher aggregate capacity. This implication is supported by equation (29), which states that the price index decreases with labor size.

Noticed that the right part of (28) is independent with $\boldsymbol{\varphi}$, hence if there is sorting equilibrium for the informal entrepreneurs, all informal firms will concentrate in one city, while the formal firms are in both cities. However, the concentration of informal employment reduces local producer productivity but raises local wage rate and congestion, driving part of the informal firms to another city.[10] According to (29) and (30), rewrites (28) with the definition of informal firms' profit in (20), yields

$$\frac{Y_2}{Y_1} = \frac{w_2}{w_1}\left(\frac{A_1}{A_2}\right)^{\frac{1}{\varepsilon}} \Rightarrow \frac{Z_2}{Z_1} = \frac{w_2}{w_1} \tag{31}$$

Jointly analyzing equation (29) and (31), the equal cross-city ratio of real price index $\mathbb{P}_c/\boldsymbol{A}_c$, the productivity $\boldsymbol{Z}_c$ and wage rate $\boldsymbol{w_c}$ mean that stable spatial pattern requires same relative return for variety, human capital, and labor in city 1 and city 2.

Lemma 1 ***The skill threshold for the informal-sector entrepreneur are invariant across cities.***

Proof. Using equation (21), (29) and (31), the ratio between $\underline{\boldsymbol{\varphi}}_{\mathbf{1}}$ and $\underline{\boldsymbol{\varphi}}_{\mathbf{2}}$ is

$$\underline{\varphi}_1/\underline{\varphi}_2 = \left(\frac{w_1 Z_2}{w_2 Z_1}\right)^{\varepsilon} = 1 \tag{32}$$

Lemma 1 states that the skill threshold for the informal-sector entrepreneur is invariant across cities, given utility indifference condition for workers and the informal entrepreneurs. Any variation on the skill thresholds for the informal entrepreneurs across locations is contradictory to equation (27) and (28), which imply $\boldsymbol{u_{I1}}(\boldsymbol{\varphi}) = \boldsymbol{u_{I2}}(\boldsymbol{\varphi})$ and $\boldsymbol{u_{W1}}(\boldsymbol{\varphi}) = \boldsymbol{u_{W2}}(\boldsymbol{\varphi})$. Suppose $\underline{\boldsymbol{\varphi}}_{\mathbf{1}} < \underline{\boldsymbol{\varphi}}_{\mathbf{2}}$, then for any $\boldsymbol{\varphi} \in [\underline{\boldsymbol{\varphi}}_{\mathbf{1}}, \underline{\boldsymbol{\varphi}}_{\mathbf{2}}]$, such that $\boldsymbol{u_{I1}}(\boldsymbol{\varphi}) > \boldsymbol{u_{W1}}(\boldsymbol{\varphi}) = \boldsymbol{u_{W2}}(\boldsymbol{\varphi}) > \boldsymbol{u_{I2}}(\boldsymbol{\varphi})$, leading to a paradox. As a result of Lemma 1, in the equilibrium, $\underline{\boldsymbol{\varphi}}_{\mathbf{1}} = \underline{\boldsymbol{\varphi}}_{\mathbf{2}} = \underline{\boldsymbol{\varphi}}$. ■

Combining the indifferent condition (28) and the same education level of informal entrepreneurs exhibited in Figure 7 implies that the average size of informal firms is the same in city 1 and city 2:

$$\frac{L_{I1}^{D}}{N_1 \int_{\underline{\varphi}}^{\overline{\varphi}_1} d\, G_1(\varphi)} = \frac{L_{I2}^{D}}{N_2 \int_{\underline{\varphi}}^{\overline{\varphi}_2} d\, G_2(\varphi)} \Rightarrow \frac{\Phi_{I1} Z_1}{(1+\varepsilon) w_1 N_1 \int_{\underline{\varphi}}^{\overline{\varphi}_1} d\, G_1(\varphi)} = \frac{\Phi_{I2} Z_2}{(1+\varepsilon) w_2 N_2 \int_{\underline{\varphi}}^{\overline{\varphi}_2} d\, G_2(\varphi)} \tag{33}$$

Lemma 2 ***Sorting of formal entrepreneurs and trade cost.***

More skillful formal entrepreneurs benefit more from being located in a city with lower trade cost, hence the individuals in the upper tail of skill distribution sorts for the location with

[10] Since the informal producers totally rely on the local market and earn their living through "free-ride" behavior, they raise urban dis-amenity and labor without obtaining benefit from external market, lowering the skill premium for local producers.

lower trade cost $\boldsymbol{T_c}$.

Proof. Since the indirect utility of formal entrepreneurs is such that

$$\frac{\partial^2 u_{Fc}(\varphi)}{\partial\varphi\,\partial T_c} = \frac{\partial^2\left[\frac{(1-\tau)^{\sigma}A_c}{\sigma\mathbb{P}_c}\varphi^{\frac{1}{\varepsilon}}\left[Z_c+\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{Z_O}{{T_c}^{\sigma}}\right]-\frac{(1-\tau)A_c\theta_c}{\mathbb{P}_c}\right]}{\partial\varphi\,\partial T_c} = \frac{-(1-\tau)^{\sigma}A_c(w_O/w_c)^{\frac{1}{\varepsilon}}Z_O}{\varepsilon\mathbb{P}_c(T_c)^{\sigma+1}}\varphi^{\frac{1}{\varepsilon}-1} < 0 \quad (34)$$

■

Equation (34) implies that there is complementarity among entrepreneurship skill, lower trade cost, and external demand. According to the labor demand function, more skillful entrepreneur represents larger employment size. Lemma 2 states that cities with lower trade cost attract the large formal firms to run their business because their products can be more competitive in export. Furthermore, since the demand from global market is expected to rise in the long term, more efficient trade service not only means more profitable currently but also brings the expectation of faster growth in revenue in the future, because of the bigger elasticity to the scale of external demand. Assuming that $\boldsymbol{T_1} < \boldsymbol{T_2}$, the more skillful formal entrepreneurs locate in city 1 due to the lower trade cost, based on Lemma 2. Therefore, compared to the perfectly-mixed-skill for workers and informal entrepreneurs, the spatial pattern of location sorting for the formal entrepreneurs is imperfectly-mixed-skill, and the utility function of the formal entrepreneurs are shown in Figure 14.

[Figure 14 here]

The utility of formal entrepreneurs $\boldsymbol{u_{F1}(\varphi)}$ and $\boldsymbol{u_{F2}(\varphi)}$ are equal at $\boldsymbol{\varphi_0}$:

$$u_{F1}(\varphi_0) = u_{F2}(\varphi_0) \Rightarrow \frac{(1-\tau)^{\frac{1}{\varepsilon}}{w_O}^{\frac{1}{\varepsilon}}Z_O[(w_1T_1)^{-\sigma}-(w_2T_2)^{-\sigma}]{\varphi_0}^{\frac{1}{\varepsilon}}}{\sigma} = \frac{\theta_1}{w_1} - \frac{\theta_2}{w_2} \Rightarrow \varphi_0 = \left[\frac{\sigma(\frac{\theta_1}{w_1}-\frac{\theta_2}{w_2})}{(1-\tau)^{\frac{1}{\varepsilon}}{w_O}^{\frac{1}{\varepsilon}}Z_O[(w_1T_1)^{-\sigma}-(w_2T_2)^{-\sigma}]}\right]^{\varepsilon} \quad (35)$$

The formal entrepreneurs with skill $\boldsymbol{\varphi} \subset [\boldsymbol{\overline{\varphi}_2}, \boldsymbol{\varphi_0})$ choose city 2, while individuals with $\boldsymbol{\varphi} \subset [\boldsymbol{\varphi_0}, \boldsymbol{\varphi_{max}})$ select city 1, implying that lower trade cost $\boldsymbol{T_1}$ and formalization cost $\boldsymbol{\theta_1}$ attract more formal entrepreneurs to city 1. The skill sorting of labor between city 1 and city 2 is summarized in Figure 15.

[Figure 15 here]

The sufficient and necessary condition of this pattern is

$$\begin{cases} (w_1T_1)^{-\sigma} - (w_2T_2)^{-\sigma} > 0 \Leftrightarrow \frac{T_2}{T_1} > \frac{w_1}{w_2} \\ \frac{\theta_1}{w_1} - \frac{\theta_2}{w_2} > 0 \Leftrightarrow \frac{\theta_1}{\theta_2} > \frac{w_1}{w_2} \\ \varphi_0 > \overline{\varphi}_1 > \overline{\varphi}_2 \end{cases} \tag{36}$$

The first condition represents the bigger slope of formal entrepreneurs' utility in city 1 requires lower real trade cost $\boldsymbol{w_c T_c}$. In order not to be dominated by city 1 in the formal entrepreneurs' sorting, the real formalization cost $\boldsymbol{\theta_c / w_c}$ is supposed to be smaller in city 2. However, if the third condition does not hold, then the order in the LHS will be opposite, $\boldsymbol{\varphi_0} \leq \boldsymbol{\overline{\varphi}_1} \leq \boldsymbol{\overline{\varphi}_2}$, so that all formal firms still concentrate in city 1 even though the second condition hold. Combining these three conditions in (36) obtains $\boldsymbol{\theta_1 T_1} > \boldsymbol{\theta_2 T_2}$, since $\boldsymbol{T_1} < \boldsymbol{T_2}$, $\boldsymbol{\theta_1} > \boldsymbol{\theta_2}$, meaning that the equilibrium requires the fixed cost is lower in the location with worse trade service.

4, Numerical Simulation

In this section, the computational experiments focus on the interaction between supply determination of formalization, with respect to local fundamentals and skill endowment, and demand factors of the formal sector, such as the elasticity of substitution and external market condition. The results of numerical simulation not only illustrate the implications of the model for formal employment share as well as spatial distribution of the formal employment but also demonstrate how the employment and skill shifts across sectors and regions in response to the change of endowment, local fundamentals, and external market condition through numerical comparative statics.

4.1 Baseline Case

The numerical simulation for the sorting equilibrium is based on the algorithm in Appendix A. It relies on the parameters set $\{\boldsymbol{\varepsilon}, \boldsymbol{k}, \boldsymbol{\tau}, \boldsymbol{a_c}, \boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{max}} \mid \boldsymbol{k} > 1/\boldsymbol{\varepsilon}\} \in \boldsymbol{R}_+^*$, endowment $\{\boldsymbol{N}, \boldsymbol{G(\varphi)}\}$, policy variables $\{\boldsymbol{T_c}, \boldsymbol{\theta_c}\}$ and world market condition $\{\mathbb{P}_{\boldsymbol{O}}, \boldsymbol{w_O}, \boldsymbol{Y_O}\}$. We set the total population size $\boldsymbol{N} = 100$, the dis-amenity coefficient $\boldsymbol{a_1} = \boldsymbol{a_2} = 0.065$, the minimum skill level $\boldsymbol{\varphi_{min}} = \mathbf{1}$, representing the people with no education, and the maximum skill level $\boldsymbol{\varphi_{max}} = \mathbf{30}$, which is bigger than the upper bound of school-year to include the working experience. Based on the current income tax rate in Indonesia (Corporate tax rates table, KPMG), $\boldsymbol{\tau}$ is set to be 0.25. According to the classical NEG model in Fujita, Krugman and Venables' book (2001), the smaller the elasticity of substitution $\boldsymbol{\sigma}$ is, the more likely that the symmetric equilibrium is unstable. In order to avoid the noise from symmetric solution, the elasticity is assumed to be equal to 3. To capture the low education level in developing countries, the parameter $\boldsymbol{k}$ in truncated Pareto distribution $\boldsymbol{G}(\boldsymbol{\varphi})$ is set to be 4.5. Normalizing world price $\mathbb{P}_{\boldsymbol{O}}$ to be 1, the global market condition $\{\mathbb{P}_{\boldsymbol{O}}, \boldsymbol{\Phi}_{\boldsymbol{O}}, \boldsymbol{Y}_{\boldsymbol{O}}\}$ is {1, 5000, 10000}, such that the equilibrium share of export in total income is between 20% to 30%, being close to the real level in Indonesia. Due to the requirement for the equilibrium's existence in equation (36), assuming $\boldsymbol{T}_{\mathbf{1}} < \boldsymbol{T}_{\mathbf{2}}$, while $\boldsymbol{\theta}_{\mathbf{1}} > \boldsymbol{\theta}_{\mathbf{2}}$. That means city 1 has lower trade cost, while city 2 has lower formalization cost. In the baseline model, we set $\boldsymbol{T}_{\mathbf{1}} = 1.5$, $\boldsymbol{T}_{\mathbf{2}} = 2$, $\boldsymbol{\theta}_{\mathbf{1}} = 11$ and $\boldsymbol{\theta}_{\mathbf{2}} = 8$. The numerical result of the model in the baseline case with parameters $\{\boldsymbol{N} = \mathbf{100}, \boldsymbol{\sigma} = \mathbf{3}, \boldsymbol{k} = \mathbf{4.5}, \boldsymbol{\tau} = \mathbf{0.25}, \boldsymbol{a}_{\mathbf{1}} = \boldsymbol{a}_{\mathbf{2}} = \mathbf{0.065}, \boldsymbol{\varphi}_{\boldsymbol{min}} = \mathbf{1}, \boldsymbol{\varphi}_{\boldsymbol{max}} = \mathbf{30}, \boldsymbol{Y}_{\boldsymbol{O}} = \mathbf{10000}, \boldsymbol{\Phi}_{\boldsymbol{O}} = \mathbf{5000}, \mathbb{P}_{\boldsymbol{O}} = \mathbf{1}, \boldsymbol{T}_{\mathbf{1}} = \mathbf{1.5}, \boldsymbol{T}_{\mathbf{2}} = \mathbf{2}, \boldsymbol{\theta}_{\mathbf{1}} = \mathbf{200}, \boldsymbol{\theta}_{\mathbf{2}} = \mathbf{100}\}$ equal to is listed in the first column of Table 2.

[Table 2 here]

From the numerical simulation result of the model in the baseline case, it is found that city 1 has higher population size, total income, and aggregate capacity. Most importantly, city 1's wage rate and formal employment share are higher, because the most productive formal firms sort for the city with more efficient trade service. In addition, since workers and informal entrepreneurs are indifferent across locations, the average utility is greater in city 1, in which the most productive formal entrepreneurs locate.

Substituting equation (23) into (24), rewrites (25') as

$$\mathrm{Y_c} = (1 + \varepsilon)\mathrm{w_c N_c G_c}(\underline{\varphi}) \tag{25''}$$

With equation (25'') and (31), yields

$$\frac{\mathrm{Y_2}}{\mathrm{Y_1}} = \frac{\mathrm{w_2}}{\mathrm{w_1}}\left(\frac{\mathrm{A_1}}{\mathrm{A_2}}\right)^{\frac{1}{\varepsilon}} = \frac{(1+\varepsilon)\mathrm{w_2 N_2 G_2}(\underline{\varphi})}{(1+\varepsilon)\mathrm{w_1 N_1 G_1}(\underline{\varphi})} \Rightarrow \left(\frac{\mathrm{A_1}}{\mathrm{A_2}}\right)^{\frac{1}{\varepsilon}} = \frac{\mathrm{N_2 G_2}(\underline{\varphi})}{\mathrm{N_1 G_1}(\underline{\varphi})} \tag{37}$$

Using equation (29)-(31) and (37), yields

$$\frac{U_1}{U_2} = \frac{A_1 Y_1/\mathbb{P}_1}{A_2 Y_2/\mathbb{P}_2} = \left(\frac{A_2}{A_1}\right)^{\frac{1}{\varepsilon}} = \frac{\Phi_1}{\Phi_2} = \frac{N_1 G_1(\underline{\varphi})}{N_2 G_2(\underline{\varphi})} \tag{38}$$

The item $\boldsymbol{\Phi}_1/\boldsymbol{\Phi}_2$ represents not only the relative size of aggregate capacity but also the ratio of the diversity of goods. Driven from equation (16), $\boldsymbol{\Phi_c}$ can be decomposed into two parts, local varieties and import varieties. Bigger $\boldsymbol{\Phi_{Fc}}$ leads to more export thus more import. Due to its large scale, though the informal sector does not participate trade, it can still compensate the gap of diversity significantly. The diversity of goods improves social welfare through pulling down the price index, allowing larger population and employment size. This is the reason why the ratio of local population scale, labor supply, and total utility are positively depending on $\boldsymbol{\Phi}_1/\boldsymbol{\Phi}_2$ in equation (38), which is supported by the simulation. Furthermore, a smaller elasticity of substitution $\boldsymbol{\sigma}$ implies stronger diversity preference and strengthens the positive correlation among diversity, employment size, and welfare. Rewriting the right part of (38), obtain

$$\frac{U_1/N_1}{U_2/N_2} = \frac{G_1(\underline{\varphi})}{G_2(\underline{\varphi})} \tag{39}$$

Equation (39) argues that average utility $\boldsymbol{U_c/N_c}$ is proportion to $\boldsymbol{G_c(\underline{\varphi})}$. Noticed that $\boldsymbol{G_c(\underline{\varphi})}$ is the share of labor supply to the local population size, which will be maximized when all workers are employed by the most productive firm. It implies that the average welfare can be improved if the production concentrates on the most efficient enterprise. However, by doing so, the diversity suffers loss thus brings a negative effect on total utility and economy scale. The implication from equation (38) and (39) claims that the coexistence of formality and informality is essentially the result of a tradeoff between diversity and efficiency. Productivity sorting of formal entrepreneurs promotes efficiency, while the informal sector provides extra diversity. In developing countries, lack of skillful labor force and relatively high cost for the formal sector incurred by the shortage of public infrastructure limit the scale of the formal sector thus its ability to provide diversity. Therefore, the demand for the informal sector is much stronger, leading to a much lower formal share, compared to the developed countries.

4.2 Comparative Statics

Impact of Adjustment in Supply Factors and Demand Factors of Formalization

Now that the diversity preference is an important factor of local formal employment share and skill sorting across locations, in order to further investigate its impact on the equilibrium, we do comparative static through controlling all other parameters but adjusting the elasticity of substitution $\boldsymbol{\sigma}$ to 2.8, from 3 in the baseline model. The second column of Table 2 shows the result. When the preference for diversity rises, the number of firms increases, while the total labor supply reduces. Hence the wage rate rises but average size of firms decreases. When $\boldsymbol{\sigma} = \mathbf{1} + 1/\varepsilon$ become smaller, the reduce of firms' size can also be explained by the model, because it depends on $\boldsymbol{\varphi}^{1/\varepsilon}$. In addition, a smaller elasticity of substitution brings positive effect on the welfare through improving both aggregate utility and average utility, due to more output, higher wage rate, greater utility slope of the entrepreneurs, increasing export but lower price index. Most importantly, stronger diversity preference makes $\boldsymbol{\Phi_{F1}/\Phi_{F2}}$ smaller, meaning the motivation of sorting reduces, which is proven by Lemma 3.

Lemma 3 ***Sorting of formal entrepreneurs and elasticity of substitution.***
Taking $\boldsymbol{\Phi_{F1}/\Phi_{F2}}$ to measure the dispersion of formal employment across locations, then it increases with the elasticity of substitution $\boldsymbol{\sigma}$.

Proof. Seen in Appendix B. ■

Lemma 3 predicts that when the elasticity of substitution $\boldsymbol{\sigma}$ reduces, which implies the preference for diversity increases, the motivation of the formal entrepreneurs' sorting behavior is weakened. Therefore, the formal employment share intends to be convergent across locations. Being consistent with Lemma 3, as the demand for diversity and labor cost goes up, the relative size of formal capacity $\boldsymbol{\Phi_{F1}/\Phi_{F2}}$ become smaller, while the case in the informal sector is opposite, which implies the incentive of formal firms' sorting is weakened. The convergence of formal employment share is caused by faster expansion of the formal sector in city 2 and higher growth of the informal sector in city 1.

In the context of Indonesia, both the determinants of formalization on the demand-side, with respect to the scale of export, and supply-side, with respect to the share of educated labor, are found to have grown rapidly during 1995-2015 in Figure 2. Controlling all other parameters, the influence of increasing export (changing $\boldsymbol{Y_O}$ from 10000 to 12000,) and higher education level of the adult labor force (Reducing $\boldsymbol{k}$ from 4.5 to 4) are estimated, listed in the third and fourth column of Table 2.[11] Firstly, as global demand expands and skill endowment is improved, the gap on utility slope of entrepreneurs become bigger, more formal firms are created, while part of the informality is driven out and transferred to be workers thus the labor supply is raised. Therefore, part of the labor force is reallocated from the informal sector to the formal sector, making formal employment share and export income rise in the economy. Secondly, the level of utility becomes higher, average or aggregate, local or national, because of more output, higher wage rate, and increasing export. Furthermore, due to the complementarity between entrepreneurship skill, lower trade cost and external demand proved by Lemma 2, the larger proportion of productive entrepreneurs and bigger $\boldsymbol{Y_O}$ imply the formal sector become more efficient, motivating sorting behavior of formal entrepreneurs. Hence the formal employment and export are more concentrating to city 1 for its lower trade cost, and the dispersion of formal capacity, formal employment share and wage increases.

Based on the comparative static analysis of demand-based and supply-based determinants, combining the effect of improvement on both sides of formalization give us deeper insight. The fifth column of Table 2 shows the interaction of lower $\boldsymbol{\sigma}$ thus stronger preference for diversity, higher $\boldsymbol{Y_O}$ thus greater external demand and smaller $\boldsymbol{k}$ thus more supply of human capital. The joint change influences the equilibrium in three aspects. Firstly, strengthening the impact of each other, the adjustment of conditions improves income and welfare further, locally and nationally. Secondly, rising utility slope for entrepreneurs causes the number of firms increases, meaning that the preference for diversity is the dominants in deciding labor

[11] When the parameter k decreases, the truncated Pareto distribution has a flatter right tail, implying that the proportion of high-skill labor force is bigger.

supply. On the other hand, larger external market and bigger human capital endowment offset the negative effect on aggregate and formal capacity when lower $\boldsymbol{\sigma}$ only. Thirdly, less employment and higher income not only implies workers become more productive and wage dispersion increases but also means formal share rises but converges due to the informal sector is more concentrated to city 1.

Except for those affecting formalization directly, change of other parameters also influence the occupation selection and skill sorting. The sixth column of Table 2 exhibits the simulated effect of smaller $\boldsymbol{a}$ than the baseline case, which contributes to mitigating congestion. Less friction raises the utility slope of entrepreneurs, improves the personal, local and national utility and encourages concentration of firms and employment. However, lower congestion does not change the export income, skill sorting of formal entrepreneurs and wage, proved by the algorithm in Appendix A. Therefore, it only makes the informal firms and employment more concentrated in city 1 thus formal employment share converge somewhat, instead of promoting productivity.

Impact of Local Latent Variable's Change on Formalization

The local specific trade cost $\boldsymbol{T_c}$ and fixed cost $\boldsymbol{\theta_c}$ are important latent variables on the supply side of formalization. In developing countries, shortage of trade-supporting infrastructure and low quality of public service prevent $\boldsymbol{T_c}$ and $\boldsymbol{\theta_c}$ being reduced. As a result, the motivation to be formal is weakened, limiting the size of the formal sector and its ability to provide diversity. In order to fulfill this gap, the demand for the informal sector is much stronger thus formal employment share is much lower, compared to the developed countries. Table 3 and Table 4 reports the simulated result of the cost-reducing policy on $\boldsymbol{T_c}$ and $\boldsymbol{\theta_c}$, under different supply and demand conditions of formalization, respectively.

[Table 3 here]

[Table 4 here]

The impact of lowering the transport cost is more significant than the fixed cost, though the directions of their trend are nearly the same, which intensifies the advantage of formalization in city 1. Therefore, the formal capacity, total income and aggregate utility increase in city 1, partially offset by the reduction in city 2, where the effect is stronger as the formal entrepreneur demand and supply become greater. Moreover, the wage rate rises in city 1 but decreases in city 2, with a bigger effect under a stronger preference for diversity. Formal share rises in both cities, which is little affected by economic conditions. The increase in formal employment share and wage rate in city 1 are attributed to its more productive and competitive formal sector under lower $\boldsymbol{T_1}$ or $\boldsymbol{\theta_1}$, however, the formal employment share is pushed up due to the lower labor cost in city 2. The various reason for formalization across cities is reflected in the average utility, which is improved much faster in city 1 than city 2. Furthermore, the decrease of city 2's average utility in Table 4 implies that, unlike lower trade cost which contributes to productivity and export income, adjustment of the fixed cost only affects the allocation of the formal sector without promoting efficiency.

In order to provide more insight on the policy to reduce trade cost and fixed formalization cost, Table 5 shows the decomposition of the impact of lower trade cost. Firstly, we keep occupation and population constant, only allow export income to change with trade cost and observe its effect on aggregate income, local capacity, and welfare. Secondly, we introduce the change of local occupation selection without migration and figure out the influence. Finally, we compute the total effect of reducing $\boldsymbol{T_1}$ from 1.5 to 1.4 through the adjustment of all endogenous variables.

[Table 5 here]

On the perspective of export effect, it is found that lower local trade cost contributes to the growth of income, improvement of welfare and greater capacity from import. As formal economy become more profitable while the slope of informal entrepreneurs' utility become smaller, the employment size increases and part of the labor force are reallocated from the informal sector to the formal sector. Higher formal employment share promotes efficiency and raises aggregate income further. However, the bigger wage rate due to greater labor demand brings negative impact on export income, offsetting part of the benefit of the export effect on welfare. When there is migration across locations, workers and formal entrepreneurs are further concentrated in city 1, because of higher wage rate and lower trade cost, respectively, while part of the informal sector leaves for city 2. Therefore, wage rate, formal capacity, export income and aggregate welfare continue to grow. Differing with the influence of reducing trade cost, improvement in the fixed cost for the formal firms has no export effect and occupation selection effect, because of the skill sorting of the formal entrepreneurs over the whole economy. Comparing the counterfactual results of decreasing $\boldsymbol{T_c}$, $\boldsymbol{\theta_c}$, and $\boldsymbol{a_c}$, adjustment of trade cost affects formalization most through changing the productivity of formal sector, occupation selection and skill threshold of sorting, while diminishing fixed cost is also useful to create more formal firms, though it does not contribute to economic efficiency. Smaller amenity coefficient $\boldsymbol{a}$ influences neither the supply side nor the demand side of formalization, but only reallocates some informal firms to the larger city.

Furthermore, Figure 16 shows how the equilibrium aggregate income changes with local trade cost. It supports that the second order derivative of $\boldsymbol{w_1}$ with respect to $\boldsymbol{T_1}$ is positive, which is an indication of agglomeration economies in public good provision: a greater concentration of formal employment in city 1 increases the return to $\boldsymbol{T_1}$ improvement for city 1. This pattern sheds light on the future research about the influence of reducing local trade cost on development in the long-term.

[Figure 16 here]

5, Impact of the tax-reducing policy

Recently, the government of Indonesia is discussing a policy to reduce the corporate tax rate from 25% to 20%, so as to attract investment to spur the economy (The Strait Times, 11th April 2016). Through this tax reform, the Finance Minister said that Indonesia seeks to be in line with other ASEAN countries which have already cut their tax rates for the enterprises to

20%, such as Vietnam since 2015 and Thailand since 2013 (Corporate tax rates table, KPMG). However, the impact of the policy is still waiting to be estimated. According to the numerical simulation in section 4, high tax rate limits the supply of formal employment. In order to prove the validity of the theoretical framework, we develop the model to $\boldsymbol{M}$ ($\boldsymbol{M} > 2$) cities case and applies it to the context of Indonesia.[12] The endowment $\{\boldsymbol{N}, \boldsymbol{G}(\boldsymbol{\varphi})\}$, national income tax rate $\boldsymbol{\tau}$, consumer preference $\boldsymbol{\sigma}$, urban amenity coefficient $\boldsymbol{a_c}$ and local latent variables $\{\boldsymbol{T_c}, \boldsymbol{\theta_c}\}$ are estimated. Using the estimated results, we run a number of counterfactual exercises to simulate the impact of tax reform on formalization, tax revenue, and welfare across provinces and individuals with various skill in Indonesia.

5.1 Method of Estimation and Calibration

For simplicity, we normalize the world price $\mathbb{P}_{\boldsymbol{O}}$ to be 1. The parameters set $\{\boldsymbol{k}, \boldsymbol{\varepsilon}, \boldsymbol{\tau}, \underline{\boldsymbol{\varphi}}, \overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}, \boldsymbol{a_c}, \boldsymbol{Y_O}\}$ are estimated by the simulated method of moments (SMM).[13] The SMM estimator minimizes the moment condition, which is a weighted distance measure between statistics and their predicted value in the model. The weights are given by the inverse of estimated variances of the sample. Most of the observation in moment condition base on National Labor Force Survey of Indonesia (NLFS) 2005, 2010 and 2015. Done by Badan Pusat Statistik (BPS), NLFS provides information of personal employment status across provinces of Indonesia, making it possible to identify the occupation. Therefore, the size of labor supply and the number of firms, formal or informal, can be calculated. Moreover, the wage rate of workers and personal school years can be observed directly in the dataset, where personal

[12] Without studying inter-city trade separately, the outcome of local employment mix does not change as the number of cities increase. The spatial employment distribution and sorting behavior are still consistent with Lemma 1 and Lemma 2, shown in Figure 15. Similar to the 2-city equilibrium in definition 1, there is and only is one solution for the multiple-city case, which is proved in Appendix A.

[13] The parameters are identified by the structure model and skill distribution. The set of local amenity coefficient $\boldsymbol{a_c}$ is defined by minimizing the observed-predicted gap on local adult labor force size. Identification of $\boldsymbol{\varepsilon}$ follows labor market clearing condition and local aggregate income constraint. Estimated $\boldsymbol{Y_O}$ guarantees that local trade balance condition hold, while simulated $\boldsymbol{\tau}$ equalizes the real wage rate of the formal and informal employees. Optimal distributional parameter $\boldsymbol{k}$ minimizes the variance between observed and predicted proportion of different skill. Being simulated by the labor supply and the number of formal firms, $\underline{\boldsymbol{\varphi}}$ and $\overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}$ are used for calibrating trade cost and fixed cost in section 5.2. In the counterfactual analysis, they are endogenous rather than estimated.

skill $\boldsymbol{\varphi}$ is set to be his school years.[14] As a complement, the macro statistics of Indonesia, including tax revenue, provincial export income, and domestic household expenditure are obtained from CEIC dataset through the library of National University of Singapore. The variables used in the moment condition (2005, 2010, 2015) are listed as below: [15]

1, NLFS data

(a) The proportion of labor force with skill $\boldsymbol{\varphi}$, $\forall\boldsymbol{\varphi}$.

(b) The provincial aggregate wage income of employees, including formal and informal.

(c) The provincial ratio between average wage rate of formal and informal employees.

(d) The national scale of employees, including formal and informal.

(e) The national number of formal firms.

(f) The provincial size of employment adult labor force.

2, CEIC data

(a) The provincial aggregate household expenditure.

(b) The provincial scale of export.

(c) The provincial government revenue.

Since the local productivity $\boldsymbol{Z_c}$ is calibrated by equation (21)

$$\mathrm{Z_c} = \frac{\sigma \mathrm{w_c}}{\underline{\varphi}^{1/\varepsilon}} \tag{40}$$

Therefore, local trade cost $\boldsymbol{T_c}$ satisfies

$$\mathrm{T_c} = (1+\varepsilon)\mathrm{w_c}\left[\frac{\mathrm{Z_c}}{(\text{Export income})_c}\right]^{\varepsilon} \tag{41}$$

Because of the rewriting trade balance condition

[14] To be robust, personal skill is supposed to be assumed as $\boldsymbol{\varphi} = \boldsymbol{School\,Years} + \boldsymbol{\mu}$, where $\boldsymbol{\mu} \sim \boldsymbol{N}(\mathbf{0}, \boldsymbol{\psi}^{\mathbf{2}})$. However, since there isn't applicable observation for estimating $\boldsymbol{\psi}^{\mathbf{2}}$, $\boldsymbol{\psi}^{\mathbf{2}}$ is fixed at 0 so far. In order to guarantee positive $\boldsymbol{\varphi}$, $\boldsymbol{\varphi_{min}}$ is fixed at 3, representing the labor force do not complete primary school, and $\boldsymbol{\varphi_{max}} = \mathbf{16}$, indicating individuals graduating from college.

[15] In the empirical investigation, all the statistics of trade account, household expenditure, public income, and wage rate are annual and counted by the price index of 2000.

$$(1-\tau)^{\frac{1}{\varepsilon}}[(1+\varepsilon)w_c]^{-\frac{1}{\varepsilon}}T_c{}^{-\sigma}Y_O\Phi_{Fc}=\left[\frac{(1+\varepsilon)w_c}{T_c}\right]^{\frac{1}{\varepsilon}}Z_c=(\text{Export income})_c \tag{26'}$$

Local fixed cost $\boldsymbol{\theta_c}$ for the formal firms is calibrated by

$$\begin{cases}(\text{Number of Formal Firms})_1=N\varphi_m\left(\varphi_{1,2}{}^{-k}-\varphi_{max}{}^{-k}\right)\\(\text{Number of Formal Firms})_c=N\varphi_m\left(\varphi_{c,c+1}{}^{-k}-\varphi_{c-1,c}{}^{-k}\right),c\in\{2,3,\dots\dots,M-1\}\\\theta_M=\frac{\overline{\varphi}_M{}^{1/\varepsilon}\left[(1-\tau)^{\sigma}[(1+\varepsilon)w_M]^{-1/\varepsilon}T_M{}^{-\sigma}Y_O-[1-(1-\tau)^{\sigma}]Z_M\right]}{\sigma(1-\tau)}\end{cases} \tag{42}$$

The number of formal firms can be observed in NLFS directly, while the conditions of province $\boldsymbol{M}$ with highest real trade cost, such as $\overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}$, $\boldsymbol{Z_M}$, and $\boldsymbol{T_M}$, have been simulated in section 5.1 or calibrated in equation (40) and (41).

[Table 6 here]

[Table 7 here]

[Table 8 here]

The set of endowment $\{\boldsymbol{N},\boldsymbol{k}\}$, estimated parameters $\{\boldsymbol{\varepsilon},\boldsymbol{Y_O},\boldsymbol{\tau},\boldsymbol{a_c}\}$, calibrated variables $\{\boldsymbol{T_c},\boldsymbol{\theta_c}\}$ and occupation cutoff $\{\underline{\boldsymbol{\varphi}},\overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}\}$ are listed in Table 6 to Table 8, where the aggregate population $\boldsymbol{N}$ is equal to the employed adult labor force size of Indonesia in 2005, 2010 and 2015.[16] The simulated income tax $\boldsymbol{\tau}$ for the formal sector is about 25% and close to the real tax rate, showing the validity of the theoretical model. According to the result of estimation in the context of Indonesia, the decreasing amenity coefficient $\boldsymbol{a_c}$, which reflects the improvement of domestic transportation efficiency, allows labor force size to grow by 20% without too much amenity loss during 2005 to 2015. In the same period, the elasticity of substitute $\boldsymbol{\sigma}$ fluctuates around 2.3, which is quite low and implies the strong preference for diversity. On the demand side of formalization, the real external demand $\boldsymbol{Y_O}$ doubles in the last decade, as exhibited in Figure 2. The supply determinants are also found to be better off. Regardless of the fluctuation of fixed formalization cost $\boldsymbol{\theta_c}$, the average trade cost $\boldsymbol{T_c}$ across provinces decreases significantly in this decades. More importantly, the lower $\boldsymbol{k}$ means the proportion of

[16] The asymptotic standard errors are reported in brackets. Let $\boldsymbol{G}$ be the matrix of derivative of the moments with respect to the model parameters, and $\boldsymbol{S}$ be the variance-covariance matrix of the moments. The variance-covariance matrix of the parameter estimates is given by $(\boldsymbol{G'WG})^{-1}\boldsymbol{G'WSWG}(\boldsymbol{G'WG})^{-1}$ where weighting matrix $\boldsymbol{W}$ is given by the inverse of a diagonal matrix that contains variances of the moments.

skillful labor force grows rapidly. Jointly analyzing the skill distribution and the two cutoffs for occupation selection, the formal economy develops while the scale of the informal sector becomes smaller from 2005 to 2015.

The employment spatial distribution across provinces in Indonesia is coherent with the numerical simulation in section 4. It is found that provinces with lower trade cost have lager labor force size and formal sector in Indonesia. Moreover, they have higher total income, export income, aggregate utility and price index. Most importantly, as Figure 15 shows, more productive formal firms select provinces which provide more efficient trade service thus the wage rate is also higher. In addition, since workers and informal entrepreneurs are indifferent across locations, the variance of average utility is decided by the utility of formal entrepreneurs. Therefore, provinces in which the most productive formal entrepreneurs locate has higher average utility.

5.2 Outcome of Counterfactual Analysis

Based on the estimated parameters and the algorithm in Appendix A, we are able to evaluate the influence of the policy that reducing income tax for the formal sector from 25% to 20% by doing counterfactual exercise in the context of Indonesia, 2005, 2010 and 2015, and the result is listed in Table 9.

[Table 9 here]

It is predicted that the tax revenue of local government will increase by over 20% due to this tax reform, implying the benefit from broadening tax bases dominates the cost from smaller $\boldsymbol{\tau}$ thus the policy reducing tax rates is sustainable. On average, lower income tax promotes formal employment share by 6.1%. As the formal economy develops, the efficiency of the economy become better off, and therefore, the estimated utility rises by about 6.5% to 7%. In addition, thanks for less tax burden, more competitive formal sector is expected to raise average wage, export scale, and aggregate income, while running the informal business become less profitable due to the smaller utility slope.

When we compare the predicted influence of the tax reform among different years, it is observed that growth of the formal economy and tax income is faster in 2015, compared to the case in 2005. This pattern can be partially explained by the calibrated determinants of formalization in Table 8. During 2005 and 2015, both the supply factors, with respect to human capital, trade cost and regulatory barrier, and the demand factors, such as external demand, are improved significantly. The better endowment, market size, and policy condition strengthen the motivation of the formal sector's expansion, which is the main channel that lower the tax rate contributes to economic development.

Figure 17 shows how the impact of tax-cutting policy on the formal sector interacts with other determinants of formalization. It is observed that the formal employment share grows faster in locations with lower local trade cost $\boldsymbol{T_c}$, which can be proved by Lemma 4 below.

[Figure 17 here]

Lemma 4 ***Impact of reducing income tax and trade cost.***
Formal entrepreneurs in locations with lower trade cost $\boldsymbol{T_c}$ benefit more from reducing income tax $\boldsymbol{\tau}$.
Proof. Since the indirect utility of formal entrepreneurs is such that

$$\frac{\partial^2 u_{Fc}(\varphi)}{\partial\tau\,\partial T_c} = \frac{\partial^2\left[\frac{(1-\tau)^{\sigma}A_c}{\sigma\mathbb{P}_c}\varphi^{\frac{1}{\varepsilon}}\left[Z_c+\left(\frac{w_O}{w_c}\right)^{\frac{1}{\varepsilon}}\frac{Z_O}{{T_c}^{\sigma}}\right]-\frac{(1-\tau)A_c\theta_c}{\mathbb{P}_c}\right]}{\partial\tau\,\partial T_c} = \frac{\sigma(1-\tau)^{\sigma-1}A_c(w_O/w_c)^{\frac{1}{\varepsilon}}Z_O}{\mathbb{P}_c{T_c}^{\sigma+1}}\varphi^{\frac{1}{\varepsilon}} > 0 \quad (43)$$

■

Equation (43) implies that there is complementarity between lower trade cost and tax rate. Lemma 4 claims that less tax burden strengthens the advantage of smaller local trade cost $\boldsymbol{T_c}$ in attracting formal firms because running formal firms and export become more profitable. Moreover, improvement in trade cost also raises the benefit of reducing the tax rate. Therefore, in Figure 17, the calibrated change of formal employment share in 2010 and 2015 are much higher than the situation in 2005, due to the stronger driven force from the supply side of formalization.

Similar to the situation of formal employment share's change, the growth of provincial tax revenue is predicted to be decreasing with trade cost, though there is greater fluctuation in Figure 18. Because of the complementarity between $\boldsymbol{\tau}$ and $\boldsymbol{T_c}$, provinces with lower trade cost attract more and larger formal firms, stimulating tax base to be broadened faster.

[Figure 18 here]

Besides the effect across provinces, the welfare improvement among people vary in skill shown in Figure 19 is also an important concern of the policy makers. In the left tail of skill distribution, since the real wage rate increases in the tax reform, employee's utility become better off. On the other hand, individuals with middle-skill who work as informal entrepreneurs suffer welfare loss due to higher labor cost and more competitive formal firms, hence part of the labor force are reallocated to the formal sector. For the formal entrepreneurs in the right tail, they benefit from lower tax rates, but their welfare improvement is much less than the people with skill slightly smaller than formal cutoff before the tax reform. When the tax burden is relaxed, the formal threshold in skill turns to be smaller. Therefore, it allows more firms to be formal and participate in the external market, leading to significant growth in profit thus income. Again, since the determinants of formalization are improved as time goes by, the expected effect of the tax reform is more obvious in 2010 and 2015 than 2005.

[Figure 19 here]

6, Conclusion

Inadequate public infrastructure raises the cost to run formal business and incurs high informality. Large informal sector constraints public finance and intensifies the infrastructure shortage in developing countries. In Indonesia, the elementary empirical work implies that formal employment share increases in the whole economy but still varies considerably across regions, and development of the formal economy is accompanying with a large informal sector. To provide new insight, this paper formulates a coherent micro-foundation to study the determinants of formalization and geographic pattern of the formal sector, developed from the theoretical framework in Behrens, Duranton and Robert-Nicoud (2014) and Dixit and Stiglizs

(1978). Combining the theory and its numerical simulation result obtains three key findings. Firstly, the coexistence of the formal sector and the informal sector in equilibrium reflects the balance between consumption diversity, which expands with local informal entrepreneurship, and productivity, which increases with formal-sector employment. When the preference for diversity decreases, productivity sorting of formal entrepreneurs become more significant, making the formal employment share increases in the location with lower trade cost. Secondly, larger scale of external trade encourages formalization by supporting the import of consumption diversity and making formal firms more profitable and bigger. Thirdly, from the perspective of supply-based analysis, improvement in skill distribution raises formalization by supplying a greater number of highly-productive formal firms and increasing export. Moreover, although both better trade-support infrastructure and lower business tax promote formalization, there is productivity sorting for the formal entrepreneurs, meaning that more productive formal entrepreneurs sort for the location with more efficient trade service, while less productive formal entrepreneurs select the location with smaller fixed cost. Our model implies that low education level, high trade cost and regulatory cost due to the shortage of public infrastructure and service in emerging economy weaken the supply determinants of formalization, limiting the size of the formal sector and its ability to provide diversity. To fulfill this gap, the demand for the informal sector is much stronger thus formal employment share is much lower than the developed countries.

The theoretical framework is applicable to examine the formal employment share across provinces of Indonesia, hence it enables us to understand the influence of reducing tax in an open and emerging economy further. We formulate moment conditions to estimate the model parameters and exogenous location fundamentals based on Indonesia regional and national data. In addition, the calibrated structure model can be used for counterfactual analysis, which simulates the regional employment and welfare impact of tax reform in the Indonesia context. Several propositions are driven from the calibration and counterfactual analysis results. Firstly, thanks to the broadened tax base, reducing income tax rate from 25% to 20% is estimated to have significant welfare and productivity effect. The calibrated benefit from more formal entrepreneurship dominates the cost of lower tax rate and raises local public income by over 20%, and therefore, the tax reform is predicted to be sustainable. In addition, the policy reducing tax improves average welfare by 6.76% and promotes formal-share by about 6.1%. Secondly, lower income tax rate contributes to the growth of public income and formal sector more in the provinces with lower trade cost. Thirdly, entrepreneurs whose skill are slightly smaller than the formal cutoff benefits the most from the tax reform because they can become formal and export to the external market. Higher real wage rate raises employee's utility and reduces the motivation to be informal entrepreneurs. Through encouraging more formal entrepreneurship, all the impact above are strengthened when trade cost becomes smaller, the regulatory barrier is relaxed and the human capital endowment is improved.

The structure model sheds light on the directions for further research. In the next stage, we are going to study the regional dynamics in capacity growth in dualistic sectors and infrastructure investment by local governments. We intend to estimate the positive influence from formalization on the structure variables and policy parameter, investigating whether the formalization process is self-reinforced thus sustainable. Moreover, the general equilibrium framework can account for economic incentives for internal labor migration and human capital accumulation by workers. We are working to develop the model for more reliable identification of such incentives and help us formulate testable hypotheses about human capital accumulation and employment spatial distribution, which will be brought to Indonesia data. In addition, we will examine the productivity growth in formal and informal sectors in relation to export growth, regional infrastructure improvement, and internal labor migration.

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Appendix A Algorithm of the Equilibrium in Definition 1

In this section, we prove that there is and only is one equilibrium for Definition 1. According to the analysis in section 3.2 and 3.3, the equilibrium defined in Definition 1can be rewritten as Definition 1':

Definition 1': Sorting equilibrium is the allocation $\{\boldsymbol{N_c^*},\ \boldsymbol{G_c^*}(\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}),\ \boldsymbol{G_c^*}(\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}) |\ \boldsymbol{c} \in \{1, 2\}\}$ and market-clearing price indexes $\{\mathbb{P}_c^*,\ \boldsymbol{w_c^*},\ \boldsymbol{Z_c^*} \mid \boldsymbol{c} \in \{1, 2\}\}$, given the parameters set $\{\boldsymbol{\varepsilon}, \boldsymbol{k}, \boldsymbol{\tau}, \boldsymbol{a_c}, \boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{max}} \mid \boldsymbol{k} > 1/\boldsymbol{\varepsilon}\} \in \boldsymbol{R}_+^*$, endowment $\{\boldsymbol{N}, \boldsymbol{G}(\boldsymbol{\varphi})\}$, policy variables $\{\boldsymbol{T_c}, \boldsymbol{\theta_c}\}$ and world market condition $\{\mathbb{P}_{\boldsymbol{O}}, \boldsymbol{\Phi_O}, \boldsymbol{Y_O}\}$, such that the endogenous variables $\{\boldsymbol{\varphi_0}, \boldsymbol{A_c}, \boldsymbol{\Phi_{Ic}}, \boldsymbol{\Phi_{Fc}}, \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \boldsymbol{Y_c}\}$ follow the conditions below

(1) $\boldsymbol{A_c}$ satisfies the full population condition $\boldsymbol{N} = \boldsymbol{N_1} + \boldsymbol{N_2}$;

(2) $\boldsymbol{Z_c}, \boldsymbol{Y_c}, \boldsymbol{w_c}$ and $\mathbb{P}_c$ satisfy equation (15);

(3) $\boldsymbol{Z_c}, \boldsymbol{Y_c}, \boldsymbol{\Phi_{Ic}}, \boldsymbol{\Phi_{Fc}}$ and $\boldsymbol{w_c}$ satisfy equation (16);

(4) $\boldsymbol{\Phi_{Ic}}, \boldsymbol{N_c},\ \overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ and $\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ satisfy equation (17');

(5) $\boldsymbol{\Phi_{Fc}}, \boldsymbol{N_c}$ and $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ satisfy equation (18');

(6) $\boldsymbol{Z_c}, \boldsymbol{w_c}$ and $\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ satisfy equation (21);

(7) $\boldsymbol{w_c}$ and $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ satisfy equation (22);

(8) $\boldsymbol{Z_c}, \boldsymbol{\Phi_{Ic}}, \boldsymbol{\Phi_{Fc}}, \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ and $\boldsymbol{w_c}$ satisfy equation (23) (labor market clearing condition);

(9) $\boldsymbol{Z_c}, \boldsymbol{\Phi_{Ic}}, \boldsymbol{\Phi_{Fc}}, \boldsymbol{w_c}$ and $\boldsymbol{Y_c}$ satisfy equation (25');

(10) $\boldsymbol{Z_c}, \boldsymbol{\Phi_{Fc}}$ and $\boldsymbol{w_c}$ satisfy equation (26) (trade balance condition);

(11) $\boldsymbol{A_c}, \boldsymbol{w_c}$ and $\mathbb{P}_c$ satisfy equation (27) for $\boldsymbol{c} \in \{1, 2\}$ (utility indifference condition for workers);

(12) $\boldsymbol{A_c}, \boldsymbol{Z_c}$ and $\mathbb{P}_c$ satisfy equation (28) for $\boldsymbol{c} \in \{1, 2\}$ (utility indifference condition for the informal entrepreneurs);

(13) $\boldsymbol{\Phi_{Ic}}, \boldsymbol{w_c}, \boldsymbol{N_c},\ \overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ and $\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$ satisfy equation (33) for $\boldsymbol{c} \in \{1, 2\}$; (equal average size of informal firms condition);

(14) $\boldsymbol{\varphi_0}$ and $\boldsymbol{w_c}$ satisfy equation (35) for $\boldsymbol{c} \in \{1, 2\}$;

(15) $\boldsymbol{T_c}, \boldsymbol{\theta_c}$ and $\boldsymbol{w_c}$ satisfy condition (36) for $\boldsymbol{c} \in \{1, 2\}$.

■

Equilibrium Wage Rate and Labor Supply

Using equation (35'), yields

$$\frac{\partial \varphi_0^{\frac{1}{\varepsilon}}}{\partial w_1} = \frac{\sigma\left[(\sigma-1)(w_1T_1)^{-\sigma}\left(\frac{\theta_1}{w_1}-\frac{\theta_2}{w_2}\right)+\left[\frac{\theta_1}{w_1}(w_2T_2)^{-\sigma}-\frac{\theta_2}{w_2}(w_1T_1)^{-\sigma}\right]\right]}{(1-\tau)^{\frac{1}{\varepsilon}}w_O^{\frac{1}{\varepsilon}}Z_O w_1[(w_1T_1)^{-\sigma}-(w_2T_2)^{-\sigma}]^2} > 0 \tag{44A}$$

$$\frac{\partial \varphi_0^{\frac{1}{\varepsilon}}}{\partial w_2} = \frac{-\sigma\left[(\sigma-1)(w_2T_2)^{-\sigma}\left(\frac{\theta_1}{w_1}-\frac{\theta_2}{w_2}\right)+\left[\frac{\theta_1}{w_1}(w_2T_2)^{-\sigma}-\frac{\theta_2}{w_2}(w_1T_1)^{-\sigma}\right]\right]}{(1-\tau)^{\frac{1}{\varepsilon}}w_O^{\frac{1}{\varepsilon}}Z_O w_2[(w_1T_1)^{-\sigma}-(w_2T_2)^{-\sigma}]^2} < 0 \tag{44B}$$

Because of condition (36). Equation (44) proves the partial derivative of the sorting cutoff $\boldsymbol{\varphi_0}^{\frac{1}{\varepsilon}}$ with respect to $\boldsymbol{w_1}$ is positive, while the partial derivative with respect to $\boldsymbol{w_2}$ is negative, implying that the incentive of sorting for city 1 decreases with $\boldsymbol{w_1}$ but rises with $\boldsymbol{w_2}$.

According to the Pareto distribution of skill $\boldsymbol{g(\varphi)} = \boldsymbol{k\varphi_m/\varphi^{k+1}}$, where $\boldsymbol{\varphi_m} = \frac{\varphi_{min}{}^k\varphi_{\max}{}^k}{\varphi_{\max}{}^k-\varphi_{min}{}^k} > 0$ for all $\boldsymbol{\varphi} \in [\boldsymbol{\varphi_{min}}, \boldsymbol{\varphi_{\max}}]$ in the economy, combining equation (18') and Figure 15, yields

$$G_1(\overline{\varphi}_1) = G_1(\varphi_0) = 1 - \frac{N\varphi_m(\varphi_0{}^{-k}-\varphi_{\max}{}^{-k})}{N_1} \tag{45}$$

$$G_2(\overline{\varphi}_2) = 1 - \frac{N\varphi_m\left(\overline{\varphi}_2{}^{-k}-\varphi_0{}^{-k}\right)}{N_2} \tag{46}$$

$$\Phi_{F1} = \frac{Nk\varphi_m}{(k-\frac{1}{\varepsilon})}\left(\varphi_0{}^{\frac{1}{\varepsilon}-k} - \varphi_{\max}{}^{\frac{1}{\varepsilon}-k}\right) \tag{47A}$$

$$\Phi_{F2} = \frac{Nk\varphi_m}{\left(k-\frac{1}{\varepsilon}\right)}\left(\overline{\varphi}_2{}^{\frac{1}{\varepsilon}-k} - \varphi_0{}^{\frac{1}{\varepsilon}-k}\right) \tag{47B}$$

Based on equation (22), (35') and (47), $\boldsymbol{\varphi_0}^{\frac{1}{\varepsilon}}$ thus $\boldsymbol{\Phi_{F2}}$ increases with $\boldsymbol{T_1}$ and $\boldsymbol{\theta_1}$, but decreases with $\boldsymbol{T_2}$ and $\boldsymbol{\theta_2}$, which is opposite to the case of $\boldsymbol{\Phi_{F1}}$. It implies that lower trade cost and formalization cost encourages the growth of the formal sector.

With equation (21) and (47), trade balance condition (26) can be rewritten as

$$w_1{}^{1+\frac{2}{\varepsilon}} = \underline{\varphi}^{\frac{1}{\varepsilon}}\frac{(1-\tau)^{\frac{1}{\varepsilon}}Nk\varphi_m w_O{}^{\frac{2}{\varepsilon}}Z_O}{\sigma\left(k-\frac{1}{\varepsilon}\right)\Phi_O T_1}\left(\varphi_0{}^{\frac{1}{\varepsilon}-k} - \varphi_{\max}{}^{\frac{1}{\varepsilon}-k}\right) \tag{26'A}$$

$$w_2{}^{1+\frac{2}{\varepsilon}} = \underline{\varphi}^{\frac{1}{\varepsilon}}\frac{(1-\tau)^{\frac{1}{\varepsilon}}Nk\varphi_m w_O{}^{\frac{2}{\varepsilon}}Z_O}{\sigma\left(k-\frac{1}{\varepsilon}\right)\Phi_O T_2}\left(\overline{\varphi}_2{}^{\frac{1}{\varepsilon}-k} - \varphi_0{}^{\frac{1}{\varepsilon}-k}\right) \tag{26'B}$$

Combining the rewritten trade balance condition in city 1 and city 2, obtains

$$\frac{T_1}{T_2}\left(\frac{w_1}{w_2}\right)^{1+\frac{2}{\varepsilon}} = \frac{\Phi_{F1}}{\Phi_{F2}} = \frac{\varphi_0{}^{\frac{1}{\varepsilon}-k}-\varphi_{\max}{}^{\frac{1}{\varepsilon}-k}}{\overline{\varphi}_2{}^{\frac{1}{\varepsilon}-k}-\varphi_0{}^{\frac{1}{\varepsilon}-k}} \tag{48}$$

According to equation (44), for any $\boldsymbol{w_2}$, $\boldsymbol{\varphi_0}$ is a monotonically increasing function of $\boldsymbol{w_1}$, hence $\boldsymbol{\Phi_{F1}}/\boldsymbol{\Phi_{F2}}$ is monotonically decreasing with $\boldsymbol{w_1}$. When $\frac{w_1}{w_2} \to \frac{T_2{}^{1+\varepsilon}\theta_2{}^{\varepsilon}}{T_1{}^{1+\varepsilon}\theta_1{}^{\varepsilon}}$, $\boldsymbol{\varphi_0} \to \overline{\varphi}_2$ thus $\frac{\Phi_{F1}}{\Phi_{F2}} \to +\infty$; when $\frac{w_1}{w_2}$ is sufficiently big that $\boldsymbol{\varphi_0} \to \boldsymbol{\varphi_{max}}$, $\frac{\Phi_{F1}}{\Phi_{F2}} \to \mathbf{0}$. As a result, for any $\boldsymbol{w_2}$, there is and only is one $\boldsymbol{w_1^*}(\boldsymbol{w_2})$ and $\boldsymbol{\varphi_0^*}(\boldsymbol{w_2})$ such that equation (48) holds. Since the total derivative of equilibrium $\boldsymbol{\varphi_0^{*\frac{1}{\varepsilon}}}$ for endogenous variables set $(\boldsymbol{w_1}, \boldsymbol{w_2})$ equal to zero, using equation (44), obtain

$$d\varphi_0^{*\frac{1}{\varepsilon}} = \frac{\partial\varphi_0^{*\frac{1}{\varepsilon}}}{\partial w_1} dw_1 + \frac{\partial\varphi_0^{*\frac{1}{\varepsilon}}}{\partial w_2} dw_2 = 0 \Rightarrow \frac{dw_2}{dw_1} = \frac{w_2}{w_1} > 0 \tag{49}$$

Adding up labor market clear condition (23) in city 1 and city 2, using (21), yields

$$N\int_{\varphi_{min}}^{\underline{\varphi}} d\,G(\varphi) = \frac{1}{\varepsilon\underline{\varphi}^{\frac{1}{\varepsilon}}} N\int_{\underline{\varphi}}^{\varphi_{max}} \varphi^{1/\varepsilon}\, dG(\varphi) + \frac{(1-\tau)^{\frac{1}{\varepsilon}} w_O{}^{\frac{1}{\varepsilon}} Z_O}{1+\varepsilon}\left[\frac{\Phi_{F2}}{(w_2T_2)^{\sigma}} + \frac{\Phi_{F1}}{(w_1T_1)^{\sigma}}\right] \tag{50}$$

Equations (26'A), (26'B) and (50) and variables $(\underline{\boldsymbol{\varphi}}, \boldsymbol{w_1}, \boldsymbol{w_2})$ comprise a three equations and three variables system. When the entrepreneur threshold $\underline{\boldsymbol{\varphi}}$ rises from $\boldsymbol{\varphi_{min}}$ to $\boldsymbol{\varphi_{max}}$, it raises the LHS monotonically from 0 to a positive number, while depresses the RHS monotonically from a positive number to 0. Therefore, there is and only is a $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ clearing labor market. For any w_2, the RHS of (50) is negatively related to $\boldsymbol{w_1}$, hence the function $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ driven from equation (50) is negatively related to $\boldsymbol{w_1}$. It implies when $\boldsymbol{w_1}(\boldsymbol{w_2})$ rises, the demand for labor reduces thus the entrepreneur threshold $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ become smaller.

Combining the trade balance condition and labor market clear condition obtains

$$N\int_{\varphi_{min}}^{\underline{\varphi}} d\,G(\varphi) = \frac{1}{\varepsilon\underline{\varphi}^{\frac{1}{\varepsilon}}} N\int_{\underline{\varphi}}^{\varphi_{max}} \varphi^{1/\varepsilon}\, dG(\varphi) + \frac{\Phi_O}{\varepsilon w_O{}^{\frac{1}{\varepsilon}}}\left[\left(\frac{w_1}{T_1\underline{\varphi}}\right)^{\frac{1}{\varepsilon}} + \left(\frac{w_2}{T_2\underline{\varphi}}\right)^{\frac{1}{\varepsilon}}\right] \tag{51}$$

Similar to equation (50), the LHS of equation (50) is monotonically increasing with $\underline{\boldsymbol{\varphi}}$, while the RHS is monotonically decreasing with $\underline{\boldsymbol{\varphi}}$, hence there is and only is a $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ satisfying trade balance condition. Since the RHS of (51) increases with $\boldsymbol{w_1}$, so that for any $\boldsymbol{w_2}$, the function $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ driven from equation (51) is positively related to $\boldsymbol{w_1}$. It means when $\boldsymbol{w_1}(\boldsymbol{w_2})$ rises, the supply of labor rises thus the entrepreneur threshold $\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2})$ become bigger.

[Figure 20 here]

Combining equation (50), (51), both of which are functions of $(\underline{\boldsymbol{\varphi}}(\boldsymbol{w_1}, \boldsymbol{w_2}), \boldsymbol{w_1}(\boldsymbol{w_2}))$ obtains Figure 20. When $\boldsymbol{w_1} > \boldsymbol{w}_1^*$ and $\underline{\boldsymbol{\varphi}} > \underline{\boldsymbol{\varphi}}^*$, the labor market in city 1 is not clear because of too little demand, so that $\boldsymbol{w_1}$ decreases, making part of the workers run their own business and then $\underline{\boldsymbol{\varphi}}$ becomes smaller. When $\boldsymbol{w_1} > \boldsymbol{w}_1^*$ and $\underline{\boldsymbol{\varphi}} < \underline{\boldsymbol{\varphi}}^*$, more local labor force serve as workers, push up $\underline{\boldsymbol{\varphi}}$ and pull down $\boldsymbol{w_1}$. Finally, the labor market clearing condition guarantee there is a unique solution $(\underline{\boldsymbol{\varphi}}^*(\boldsymbol{w}_1^*, \boldsymbol{w_2}), \boldsymbol{w}_1^*(\boldsymbol{w_2}))$ of the equilibrium for any $\boldsymbol{w_2}$. Calculating $\frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{w_1}}$ and $\frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{w_2}}$ with labor market clearing condition (50)

$$\frac{\partial\underline{\varphi}}{\partial w_1} = -\frac{\frac{\left(w_1^{\frac{1}{\varepsilon}}T_1^{-\frac{1}{\varepsilon}}+w_2^{\frac{1}{\varepsilon}}T_2^{-\frac{1}{\varepsilon}}\right)w_O^{\frac{1}{\varepsilon}}Z_O\Phi_{F1}}{\varepsilon T_1 w_1^{2\sigma}}+\frac{(w_O w_2/T_2)^{\frac{1}{\varepsilon}}Z_O\Phi_{F1}}{\varepsilon(1+\varepsilon)T_1 w_1^{2\sigma}}+\frac{k w_O^{\frac{1}{\varepsilon}}Z_O\varphi_0^{-k}}{\varepsilon(1+\varepsilon)N\varphi_m}\left(\frac{w_1^{\frac{1}{\varepsilon}}T_1^{-\frac{1}{\varepsilon}}+w_2^{\frac{1}{\varepsilon}}T_2^{-\frac{1}{\varepsilon}}}{T_1 w_1^{1+\frac{2}{\varepsilon}}}\right)\frac{\partial\varphi_0^{\frac{1}{\varepsilon}}}{\partial w_1}}{N\varphi_m\left[\frac{(\varepsilon k+k-1)k}{(\varepsilon k-1)}\underline{\varphi}^{-(k+1)}-\frac{k\varphi_{max}^{\frac{1}{\varepsilon}-k}}{\varepsilon(\varepsilon k-1)}\underline{\varphi}^{-(\frac{1}{\varepsilon}+1)}\right]} < 0 \qquad (52A)$$

$$\frac{\partial\underline{\varphi}}{\partial w_2} = \frac{\frac{(w_O/T_2)^{\frac{1}{\varepsilon}}Z_O w_2^{\frac{1}{\varepsilon}-1}\Phi_{F1}}{\varepsilon(1+\varepsilon)T_1 w_1^{1+\frac{2}{\varepsilon}}}-\frac{k w_O^{\frac{1}{\varepsilon}}Z_O\varphi_0^{-k}}{\varepsilon(1+\varepsilon)N\varphi_m}\left(\frac{w_1^{\frac{1}{\varepsilon}}T_1^{-\frac{1}{\varepsilon}}+w_2^{\frac{1}{\varepsilon}}T_2^{-\frac{1}{\varepsilon}}}{T_1 w_1^{1+\frac{2}{\varepsilon}}}\right)\frac{\partial\varphi_0^{\frac{1}{\varepsilon}}}{\partial w_2}}{N\varphi_m\left[\frac{(\varepsilon k+k-1)k}{(\varepsilon k-1)}\underline{\varphi}^{-(k+1)}-\frac{k\varphi_{max}^{\frac{1}{\varepsilon}-k}}{\varepsilon(\varepsilon k-1)}\underline{\varphi}^{-(\frac{1}{\varepsilon}+1)}\right]} > 0 \qquad (52B)$$

In the equilibrium, $\underline{\boldsymbol{\varphi}}^*$ has zero total derivative with respect to $(\boldsymbol{w_1}, \boldsymbol{w_2})$:

$$d\underline{\varphi}^* = \frac{\partial\underline{\varphi}^*}{\partial w_1}dw_1 + \frac{\partial\underline{\varphi}^*}{\partial w_2}dw_2 = 0 \Rightarrow \frac{dw_2}{dw_1} > 0 \qquad (49')$$

Based on (52A) and (52B). The derivative of $\boldsymbol{w_1}$ for $\boldsymbol{w_2}$ such that

$$\frac{\partial\underline{\varphi}^*}{\partial w_1}w_1 + \frac{\partial\underline{\varphi}^*}{\partial w_2}w_2 = -\frac{\frac{\left(w_1^{\frac{1}{\varepsilon}}T_1^{-\frac{1}{\varepsilon}}+w_2^{\frac{1}{\varepsilon}}T_2^{-\frac{1}{\varepsilon}}\right)w_O^{\frac{1}{\varepsilon}}Z_O\Phi_{F1}}{\varepsilon T_1 w_1^{1+\frac{2}{\varepsilon}}}+\frac{k w_O^{\frac{1}{\varepsilon}}Z_O\varphi_0^{-k}}{\varepsilon^2 N\varphi_m}\left(\frac{w_1^{\frac{1}{\varepsilon}}T_1^{-\frac{1}{\varepsilon}}+w_2^{\frac{1}{\varepsilon}}T_2^{-\frac{1}{\varepsilon}}}{T_1 w_1^{\sigma}}\right)\left[T_1^{-\sigma}-(\frac{w_1}{w_2})^{\sigma}T_2^{-\sigma}\right]\left(\theta_1-\frac{w_1\theta_2}{w_2}\right)}{N\varphi_m\left[\frac{(\varepsilon k+k-1)k}{(\varepsilon k-1)}\underline{\varphi}^{-(k+1)}-\frac{k\varphi_{max}^{\frac{1}{\varepsilon}-k}}{\varepsilon(\varepsilon k-1)}\underline{\varphi}^{-(\frac{1}{\varepsilon}+1)}\right]} < 0 \Rightarrow \frac{w_1}{w_2} < \frac{dw_1}{dw_2} \qquad (53)$$

[Figure 21 here]

Combining the two different curves of $\boldsymbol{w}_1^*(\boldsymbol{w_2})$ driven from trade balance condition (48) and labor market clearing condition (50), we have the fixed-point mapping of the equilibrium Figure 21. Area I~IV of the mapping represents ("Excess Labor Supply, Excess Export"), ("Excess Labor Demand, Excess Export"), ("Excess Labor Demand, Excess Import") and ("Excess Labor Supply, Excess Import"), respectively. Figure 21 shows that not only there is and only is one solution of $(\boldsymbol{w}_1^*, \boldsymbol{w}_2^*)$ thus the whole equilibrium, but also any point $(\boldsymbol{w_1}, \boldsymbol{w_2})$ in the four areas of the mapping converges to these two curves and finally the equilibrium point $\boldsymbol{O}(\boldsymbol{w}_1^*, \boldsymbol{w}_2^*)$.

Equilibrium Population Size and Informal Capacity

The local amenity $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ satisfy the full population condition $\boldsymbol{N = N_1 + N_2}$:

$$A_1^{-\frac{1}{a_1}} + A_2^{-\frac{1}{a_2}} = N \tag{54}$$

Adding up local informal capacity $\boldsymbol{\Phi_{I1}}$ and $\boldsymbol{\Phi_{I2}}$, yields:

$$\Phi_{I1} + \Phi_{I2} = N \int_{\underline{\varphi}^*}^{\overline{\varphi}_2^*} \varphi^{\frac{1}{\varepsilon}}\, dG(\varphi) \tag{55}$$

Where $\underline{\boldsymbol{\varphi}}^*$ and $\overline{\boldsymbol{\varphi}}_{\boldsymbol{2}}^*$ have been solved in the last section. Based on equation (16) and (30), we have

$$\left(\frac{A_1}{A_2}\right)^{\frac{1}{\varepsilon}} = \frac{\Phi_{I2} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{F2} + \left(\frac{w_2}{T_2 w_O}\right)^{\frac{1}{\varepsilon}}\Phi_O}{\Phi_{I1} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{F1} + \left(\frac{w_1}{T_1 w_O}\right)^{\frac{1}{\varepsilon}}\Phi_O} \tag{56}$$

Noticed that $\boldsymbol{\Phi_{Fc}^*}$ and $\boldsymbol{w_c^*}$ have also been calculated. Finally, rewriting the equal average size of informal firms condition (33) as

$$\frac{\Phi_{I1}}{A_1^{-\frac{1}{a_1}} - N\varphi_m\left(\varphi_0^{-k} - \varphi_{max}^{-k}\right) - \frac{A_2^{\frac{1}{\varepsilon}} N\varphi_m(\varphi_{min}^{-k} - \underline{\varphi}^{-k})}{A_1^{\frac{1}{\varepsilon}} + A_2^{\frac{1}{\varepsilon}}}} = \frac{\Phi_{I2}}{A_2^{-\frac{1}{a_2}} - N\varphi_m\left(\overline{\varphi}_2^{-k} - \varphi_0^{-k}\right) - \frac{A_1^{\frac{1}{\varepsilon}} N\varphi_m(\varphi_{min}^{-k} - \underline{\varphi}^{-k})}{A_1^{\frac{1}{\varepsilon}} + A_2^{\frac{1}{\varepsilon}}}} \tag{57}$$

Because of equation (31), (37), (45) and (46). Combining equation (54) to (57) obtains equilibrium $\{\boldsymbol{A_1^*}, \boldsymbol{A_2^*}, \boldsymbol{\Phi_{I1}^*}, \boldsymbol{\Phi_{I2}^*}\}$. Therefore, the whole equilibrium in definition 1 is solved.

Algorithm of the Equilibrium with M cities

In any location $\boldsymbol{c}$, where $\boldsymbol{c} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots\dots, \boldsymbol{M}\}$, with equation (40) and the estimated skill distribution $\boldsymbol{g(\varphi)}$, trade balance condition (26) can be rewritten as

$$\begin{cases} w_1^{1+\frac{2}{\varepsilon}} = \underline{\varphi}^{\frac{1}{\varepsilon}} \dfrac{(1-\tau)^{\frac{1}{\varepsilon}} N k \varphi_m Y_O}{\sigma(1+\varepsilon)^{\frac{2}{\varepsilon}}\left(k-\frac{1}{\varepsilon}\right)T_1} \left(\varphi_{1,2}^{\frac{1}{\varepsilon}-k} - \varphi_{max}^{\frac{1}{\varepsilon}-k}\right) \\ w_c^{1+\frac{2}{\varepsilon}} = \underline{\varphi}^{\frac{1}{\varepsilon}} \dfrac{(1-\tau)^{\frac{1}{\varepsilon}} N k \varphi_m Y_O}{\sigma(1+\varepsilon)^{\frac{2}{\varepsilon}}\left(k-\frac{1}{\varepsilon}\right)T_c} \left(\varphi_{c,c+1}^{\frac{1}{\varepsilon}-k} - \varphi_{c-1,c}^{\frac{1}{\varepsilon}-k}\right), c \in \{2,3, \dots\dots, M-1\} \\ w_M^{1+\frac{2}{\varepsilon}} = \underline{\varphi}^{\frac{1}{\varepsilon}} \dfrac{(1-\tau)^{\frac{1}{\varepsilon}} N k \varphi_m Y_O}{\sigma(1+\varepsilon)^{\frac{2}{\varepsilon}}\left(k-\frac{1}{\varepsilon}\right)T_M} \left(\overline{\varphi}_M^{\frac{1}{\varepsilon}-k} - \varphi_{M-1,M}^{\frac{1}{\varepsilon}-k}\right) \end{cases} \tag{58}$$

Adding up labor market clear condition (23) across locations, using (21), yields

$$N \int_{\varphi_{min}}^{\underline{\varphi}} dG(\varphi) = \frac{N}{\varepsilon \underline{\varphi}^{\frac{1}{\varepsilon}}}\left[\int_{\underline{\varphi}}^{\overline{\varphi}_M} \varphi^{1/\varepsilon}\, dG(\varphi) + (1-\tau)^{\sigma} \int_{\overline{\varphi}_M}^{\varphi_{max}} \varphi^{1/\varepsilon}\, dG(\varphi)\right] + (1-\tau)^{\sigma} Y_O \sum_{c=1}^{M} \frac{\Phi_{Fc}}{[(1+\varepsilon) w_c T_c]^{\sigma}} \tag{59}$$

Jointly analyzing equation (58) and (59), there are $(\boldsymbol{M}+\mathbf{1})$ conditions for $(\boldsymbol{M}+\mathbf{1})$ variables $\{\underline{\boldsymbol{\varphi}}^{*}, \boldsymbol{w}_{\boldsymbol{c}}^{*} \mid \boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}\}$. With the equilibrium $\underline{\boldsymbol{\varphi}}^{*}$ and $\boldsymbol{w}_{\boldsymbol{c}}^{*}$, the set of sorting cutoff of formal entrepreneurs $\{\boldsymbol{\varphi}_{\boldsymbol{c},\boldsymbol{c}+\mathbf{1}}^{*}, \overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}^{*} \mid \boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}-\mathbf{1}\}\}$ and local formal capacity $\{\boldsymbol{\Phi}_{\boldsymbol{Fc}}^{*} \mid \boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}\}$ are figured out, such that $\boldsymbol{T}_{\boldsymbol{\alpha}} < \boldsymbol{T}_{\boldsymbol{\beta}}$ if and only if $\boldsymbol{\alpha} < \beta$, where $\boldsymbol{\alpha},\boldsymbol{\beta} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}$.

The local amenity $\boldsymbol{A}_{\boldsymbol{c}}$, $\boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}$, satisfy the full population condition $\boldsymbol{N} = \sum_{\boldsymbol{c}=\mathbf{1}}^{\boldsymbol{M}} \boldsymbol{N}_{\boldsymbol{c}}$

$$\sum_{c=1}^{M} A_c^{-\frac{1}{a_c}} = N \tag{60}$$

Adding up local informal capacity $\{\boldsymbol{\Phi}_{\boldsymbol{Ic}} \mid \boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}\}$ across locations, obtain

$$\sum_{c=1}^{M} \Phi_{Ic} = N \int_{\underline{\varphi}^{*}}^{\overline{\varphi}_M^{*}} \varphi^{\frac{1}{\varepsilon}}\, dG(\varphi) \tag{61}$$

Where $\underline{\boldsymbol{\varphi}}^{*}$ and $\overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}^{*}$ have been solved. Based on equation (16) and (30), for any $\boldsymbol{\alpha},\boldsymbol{\beta} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}$

$$\left(\frac{A_\alpha}{A_\beta}\right)^{\frac{1}{\varepsilon}} = \frac{\Phi_{I\beta} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{F\beta} + \left[\frac{(1+\varepsilon)w_\beta}{T_\beta}\right]^{\frac{1}{\varepsilon}}}{\Phi_{I\alpha} + (1-\tau)^{\frac{1}{\varepsilon}}\Phi_{F\alpha} + \left[\frac{(1+\varepsilon)w_\alpha}{T_\alpha}\right]^{\frac{1}{\varepsilon}}} \tag{62}$$

Finally, rewriting the equal average size of informal firms condition (33) as

$$\frac{\Phi_{I\alpha}}{A_\alpha^{-\frac{1}{a_\alpha}} - N\varphi_m\left(\varphi_{\alpha,\alpha+1}^{-k} - \varphi_{\alpha-1,\alpha}^{-k}\right) - \frac{L_{I\alpha}^{D}+L_{F\alpha}^{D}}{(1+\varepsilon)w_\alpha}} = \frac{\Phi_{I\beta}}{A_\beta^{-\frac{1}{a_\beta}} - N\varphi_m\left(\varphi_{\beta,\beta+1}^{-k} - \varphi_{\beta-1,\beta}^{-k}\right) - \frac{L_{I\beta}^{D}+L_{F\beta}^{D}}{(1+\varepsilon)w_\beta}} \tag{63}$$

For any $\boldsymbol{\alpha},\boldsymbol{\beta} \in \{\mathbf{2},\mathbf{3},\ldots\ldots,\boldsymbol{M}-\mathbf{1}\}$. When $\boldsymbol{\alpha} = \mathbf{1}$, $\boldsymbol{\varphi}_{\boldsymbol{\alpha}-\mathbf{1},\boldsymbol{\alpha}} = \boldsymbol{\varphi}_{\boldsymbol{max}}$; when $\boldsymbol{\alpha} = \boldsymbol{M}$, $\boldsymbol{\varphi}_{\boldsymbol{\alpha},\boldsymbol{\alpha}+\mathbf{1}} = \overline{\boldsymbol{\varphi}}_{\boldsymbol{M}}^{*}$. The item $(\boldsymbol{L}_{\boldsymbol{I\beta}}^{\boldsymbol{D}} + \boldsymbol{L}_{\boldsymbol{F\beta}}^{\boldsymbol{D}})$ is a function of $\boldsymbol{\Phi}_{\boldsymbol{Ic}}$, seen in the RHS of equation (23). Combining equation (60) to (63), there are $\mathbf{2}\boldsymbol{M}$ conditions, from which the equilibrium variables set $\{\boldsymbol{A}_{\boldsymbol{c}}^{*}, \boldsymbol{\Phi}_{\boldsymbol{Ic}}^{*} \mid \boldsymbol{c} \in \{\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{M}\}\}$ with a size of $\mathbf{2}\boldsymbol{M}$ can be solved. Therefore, the equilibrium with $\boldsymbol{M}$ cities is solved.

Appendix B Proof of Lemma 3

Lemma 3 ***Sorting of formal entrepreneurs and elasticity of substitution.***

Taking $\boldsymbol{\Phi_{F1}/\Phi_{F2}}$ to measure the dispersion of formal employment across locations, then it increases with the elasticity of substitution $\boldsymbol{\sigma} = (1+\frac{1}{\boldsymbol{\varepsilon}})$.

Proof. Using equation (48), obtain

$$\frac{\Phi_{F1}}{\Phi_{F2}} = \frac{\varphi_0^{\frac{1}{\varepsilon}-k} - \varphi_{max}^{\frac{1}{\varepsilon}-k}}{\overline{\varphi}_2^{\frac{1}{\varepsilon}-k} - \varphi_0^{\frac{1}{\varepsilon}-k}} \tag{64}$$

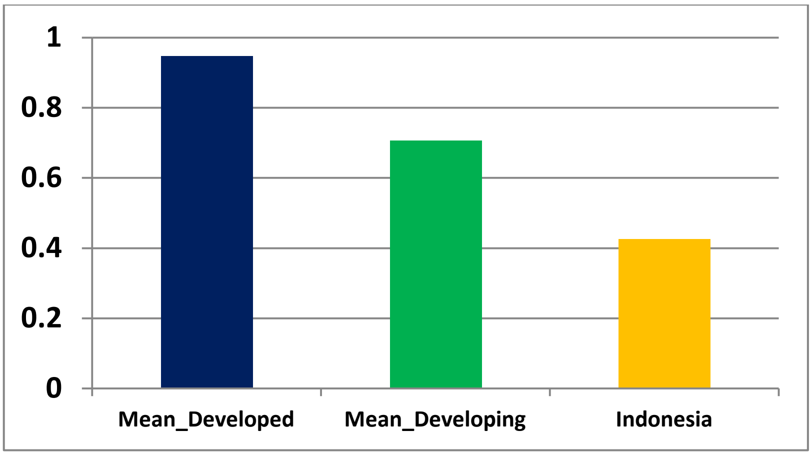
When the elasticity of substitution $\boldsymbol{\sigma} = (1+\frac{1}{\boldsymbol{\varepsilon}})$ reduces thus $\boldsymbol{\varepsilon}$ rises, $\boldsymbol{\sigma^{\varepsilon}}$ increases, while $\boldsymbol{w_O}^{\frac{1}{\varepsilon}}$ and $\frac{1}{T_1{}^{\sigma}w_1{}^{\sigma}} - \frac{1}{T_2{}^{\sigma}w_2{}^{\sigma}}$ decreases because $\frac{T_2}{T_1} > \frac{w_1}{w_2}$. As a result, $\frac{\partial\varphi_0}{\partial\varepsilon} > 0$, similarly, we have $\frac{\partial\overline{\varphi}_2}{\partial\varepsilon} > 0$. Combine equation (22) and (35'), yields

$$\overline{\varphi}_2^{\frac{1}{\varepsilon}} - \varphi_0^{\frac{1}{\varepsilon}} = \frac{\sigma T_2{}^{\sigma} w_2{}^{\frac{1}{\varepsilon}}\left(\frac{\theta_2}{T_1{}^{\sigma}w_1{}^{\frac{1}{\varepsilon}}} - \frac{\theta_1}{T_2{}^{\sigma}w_2{}^{\frac{1}{\varepsilon}}}\right)}{(1-\tau)^{\frac{1}{\varepsilon}} w_O{}^{\frac{1}{\varepsilon}} Z_O w_1\left(\frac{1}{T_1{}^{\sigma}w_1{}^{\sigma}} - \frac{1}{T_2{}^{\sigma}w_2{}^{\sigma}}\right)} \tag{65}$$

When elasticity of substitution $(1+\frac{1}{\varepsilon})$ decreases, $\left(\frac{w_O}{w_2}\right)^{\frac{1}{\varepsilon}}\boldsymbol{T_2}^{-\sigma}$ reduces, while $\frac{\theta_2}{T_1{}^{\sigma}w_1{}^{\frac{1}{\varepsilon}}} - \frac{\theta_1}{T_2{}^{\sigma}w_2{}^{\frac{1}{\varepsilon}}}$ rises, due to $\frac{w_1}{w_2} > \frac{T_2{}^{1+\varepsilon}\theta_2{}^{\varepsilon}}{T_1{}^{1+\varepsilon}\theta_1{}^{\varepsilon}}$. Therefore, $\boldsymbol{\overline{\varphi}_2}^{\frac{1}{\varepsilon}} - \boldsymbol{\varphi_0}^{\frac{1}{\varepsilon}}$ is positively related to $\boldsymbol{\varepsilon}$. According to this relation and $\frac{\partial\varphi_0}{\partial\varepsilon} > 0$, we come to a conclusion that smaller elasticity of substitution diminishes $\boldsymbol{\Phi_{F1}/\Phi_{F2}}$, which implies that higher diversity preference weakens the motivation of the formal entrepreneurs' sorting behavior. ■

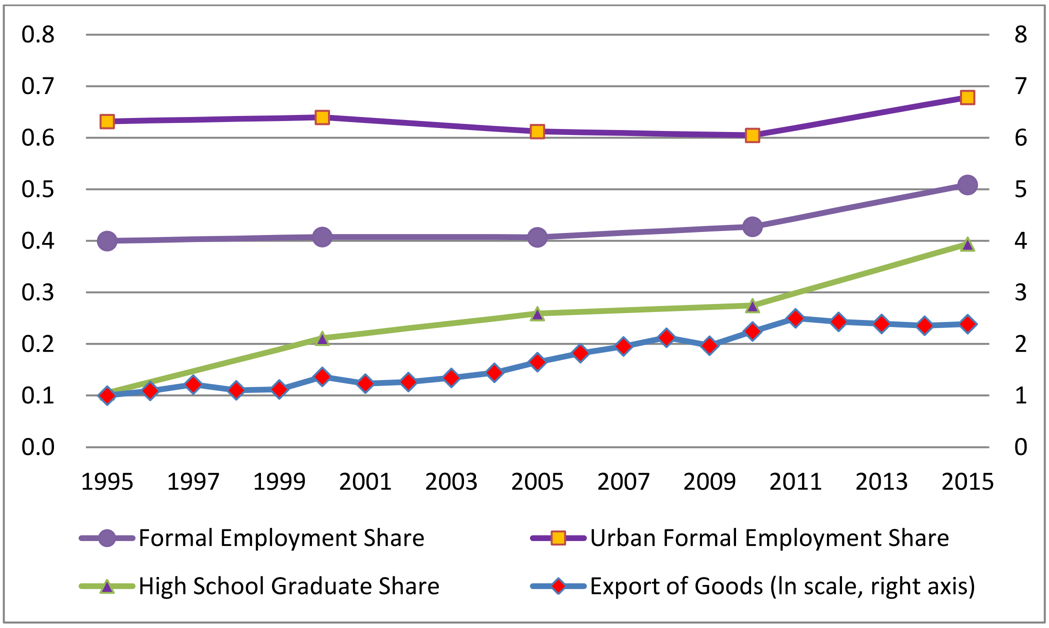
Appendix C: Figures

Figure 1. **Formal Employment Share of Different Countries around 2000**



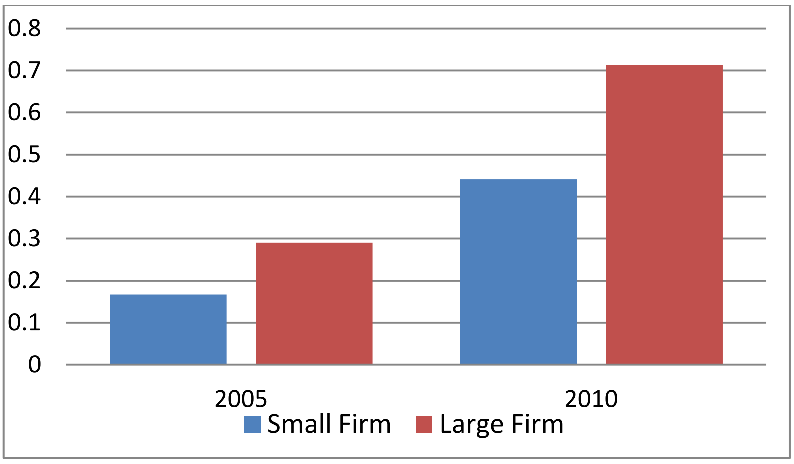
Source: IPUMS International

Figure 2. **Export, Education and Formal Employment Share**



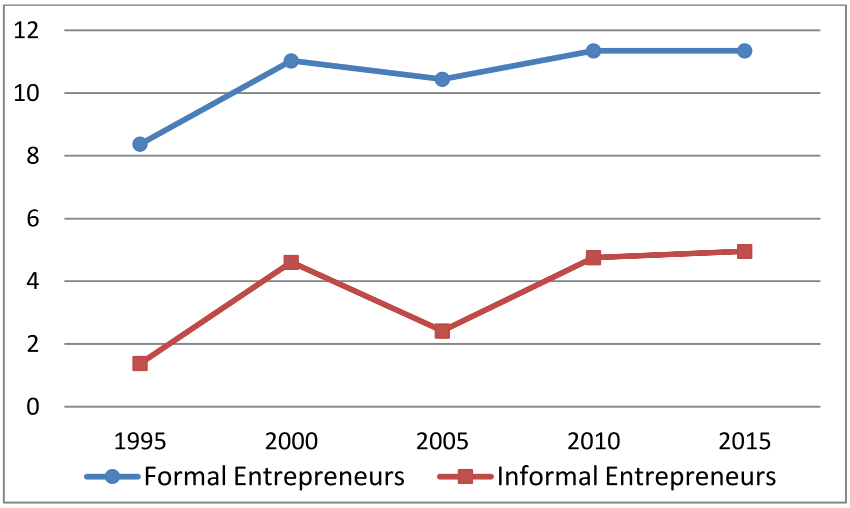
Source: CEIC, Census data 2000-2010 and NLFS 1995, 2015

Figure 3. **High School Graduate Share of Entrepreneurs by Firm Size**



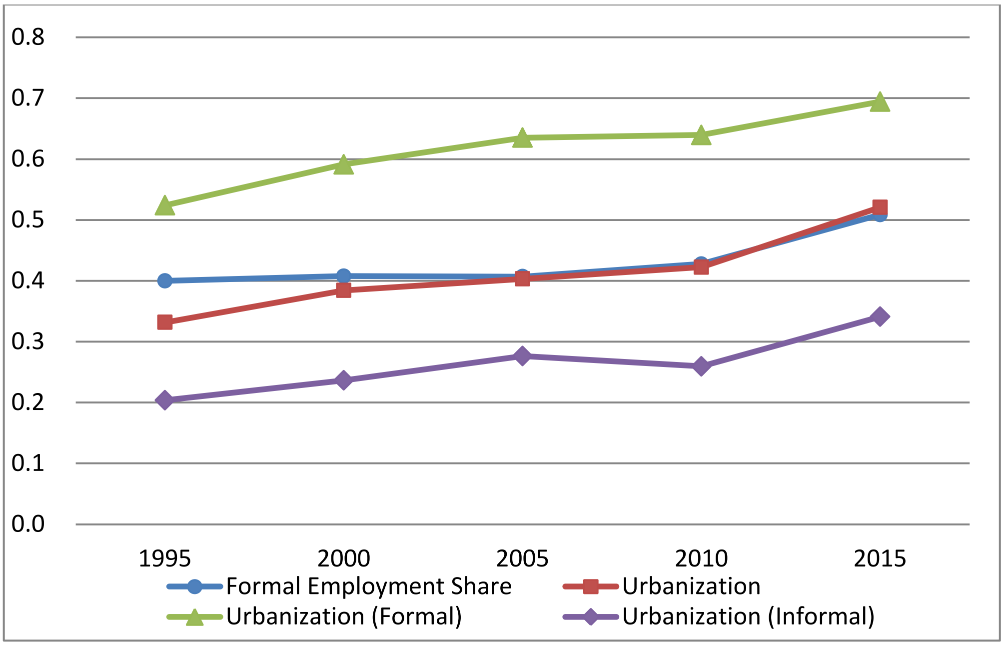
Source: NLFS 2005 and 2010

Figure 4. **Average School Years of Entrepreneurs, Formal V.S. Informal**



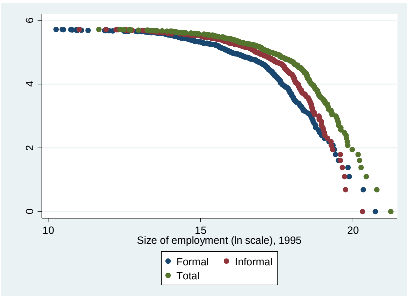
Source: NLFS 1995, 2000, 2005, 2010 and 2015

***Figure 5.* Urbanization Rate and Formal Employment Share, Java V.S. Outside Java**



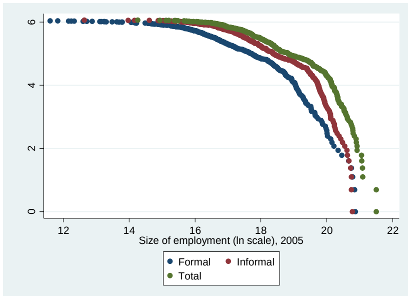
Source: NLFS 2005, 2010 and 2015

***Figure 6-1.* The spatial Distribution of Employment across counties, 1995**



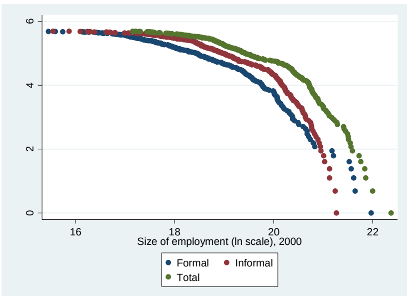
Source: NLFS

***Figure 6-3.* The spatial Distribution of Employment across counties, 2005**



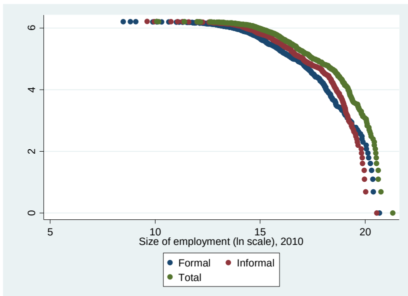
Source: NLFS

***Figure 6-2.* The spatial Distribution of Employment across counties, 2000**



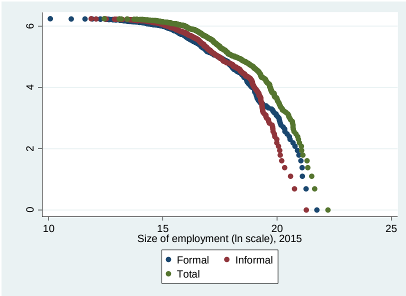
Source: NLFS

***Figure 6-4.* The spatial Distribution of Employment across counties, 2010**



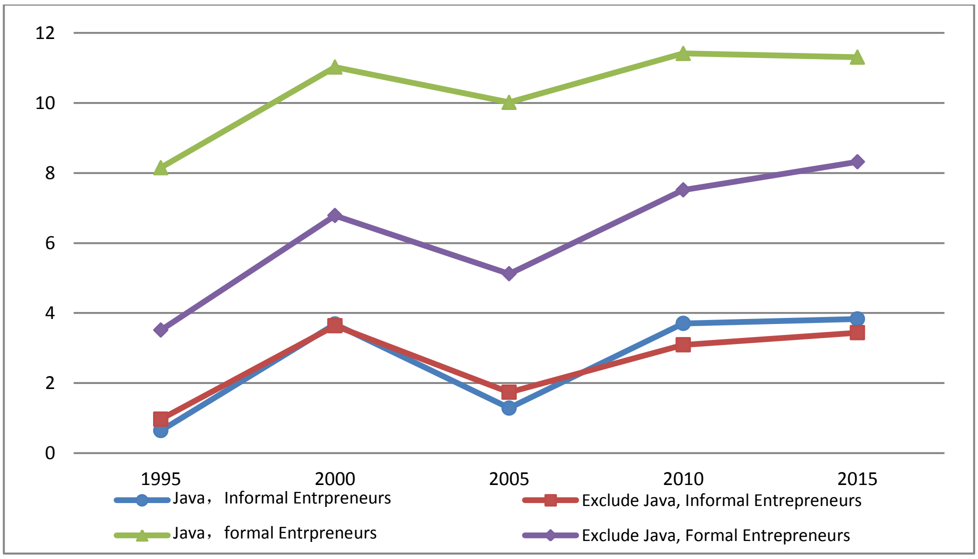
Source: NLFS

Figure 6-5. **The spatial Distribution of Employment across counties, 2015**



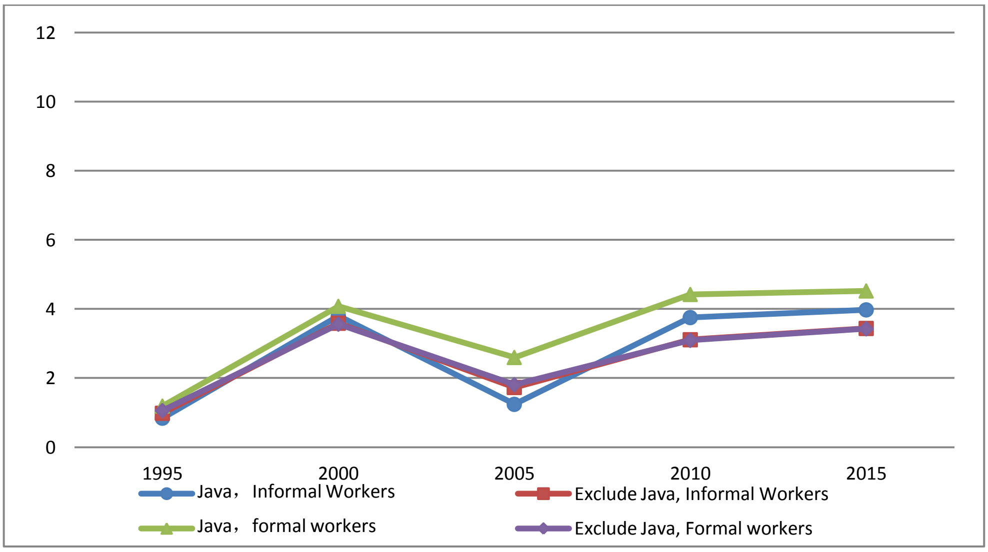
Source: NLFS

Figure 7. **Average School Years of Entrepreneurs, Java V.S. Outside Java**



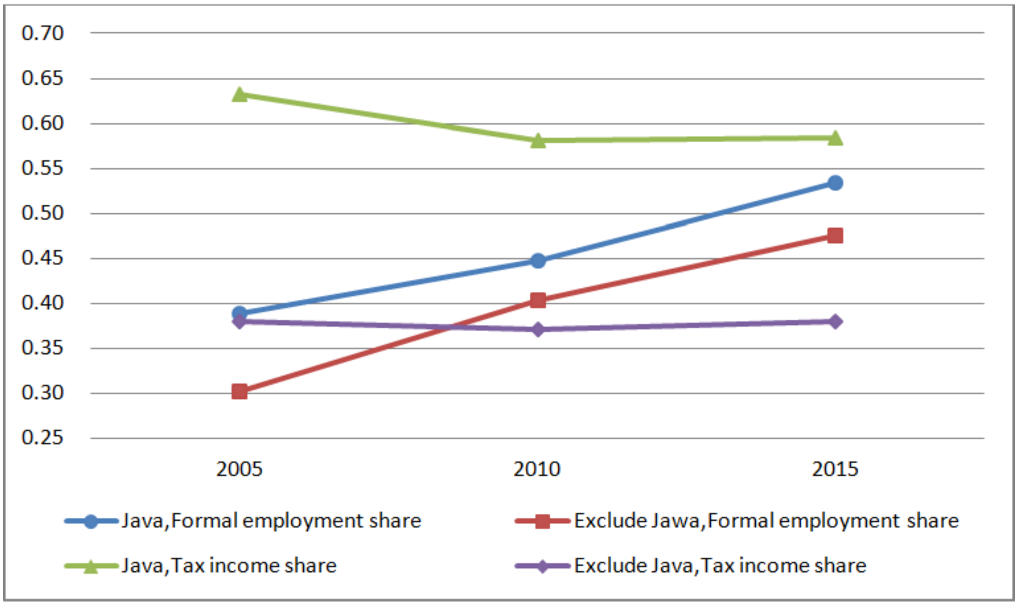
Source: NLFS 1995, 2000, 2005, 2010 and 2015

Figure 8. **Average School Years of Employees, Java V.S. Outside Java**



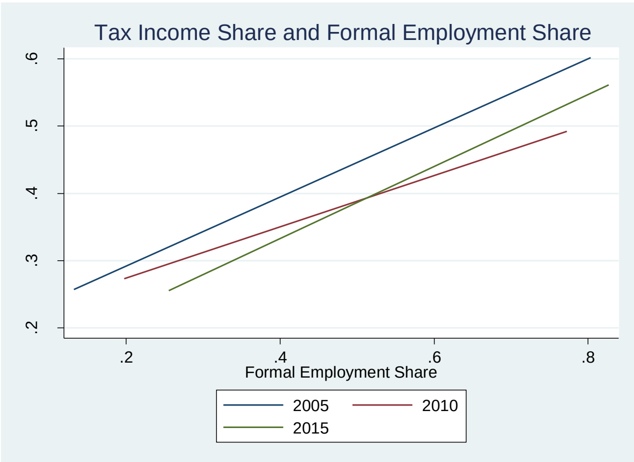
Source: NLFS 1995, 2000, 2005, 2010 and 2015

***Figure 9.* Formal Employment Share and Tax Income Share, Java V.S. Outside Java**



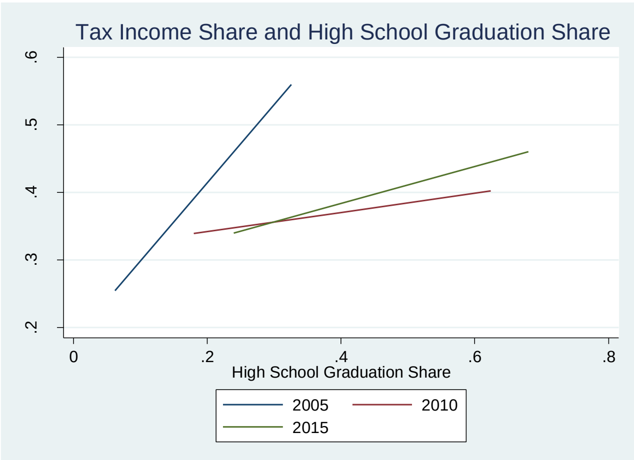
Source: CEIC and NLFS 2005, 2010 and 2015

***Figure 10.* Tax Income Share and Formal Employment Share**



Source: CEIC and NLFS 2005, 2010 and 2015

***Figure 11.* Tax Income Share and High School Graduation Share**



Source: CEIC and NLFS 2005, 2010 and 2015

Figure 12. **Utility Tradeoff between Workers and Informal Entrepreneurs**

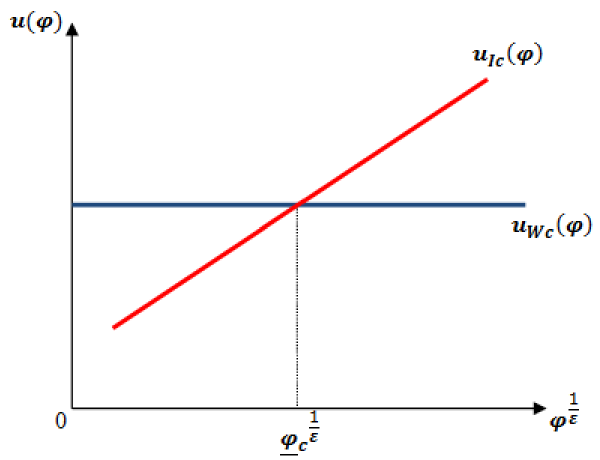


Figure 13. **Utility Tradeoff between Informal and Formal Entrepreneurs**

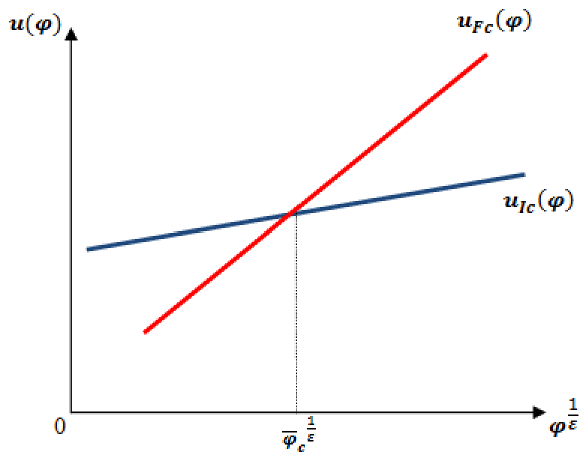


Figure 14. **$u_{F1}(\varphi)$ and $u_{F2}(\varphi)$ (Sorting Equilibrium)**

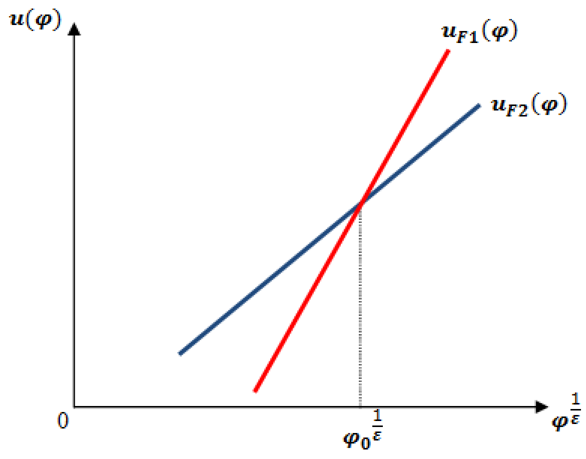


Figure 15. **Skill Sorting between City 1 and City 2**

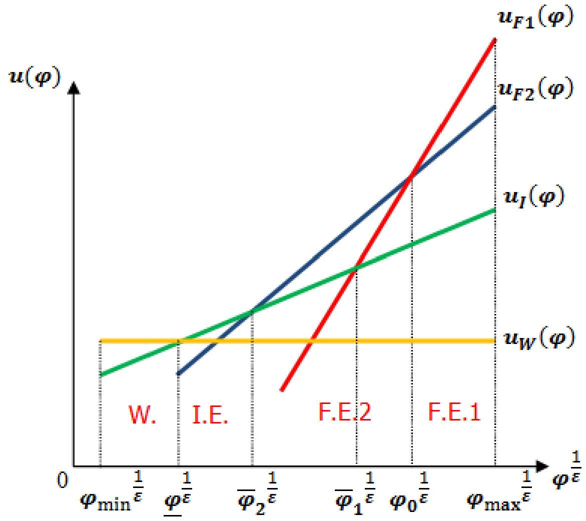


Figure 16. Trade Cost and Aggregate Income

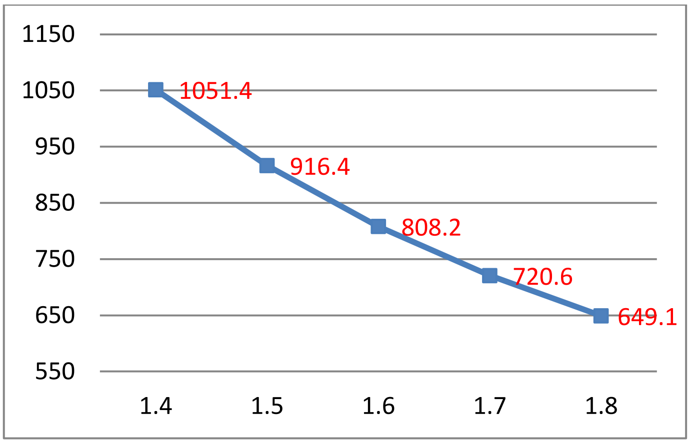
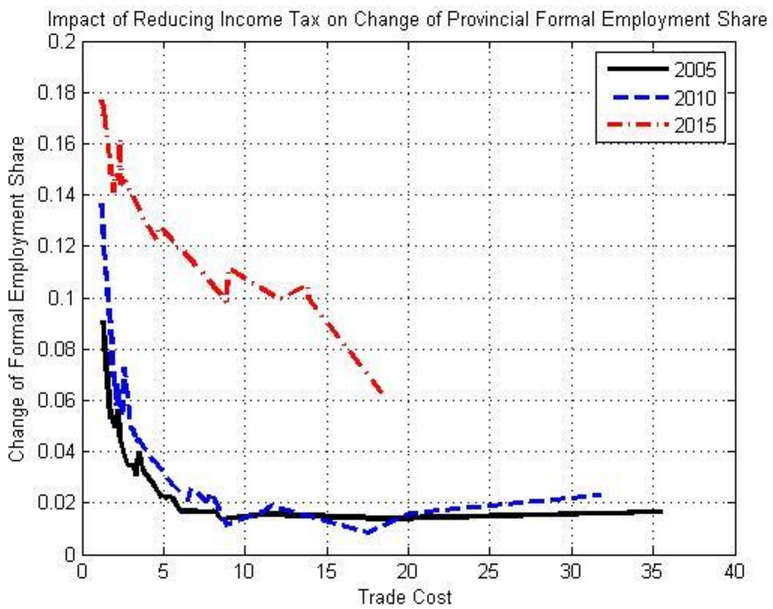
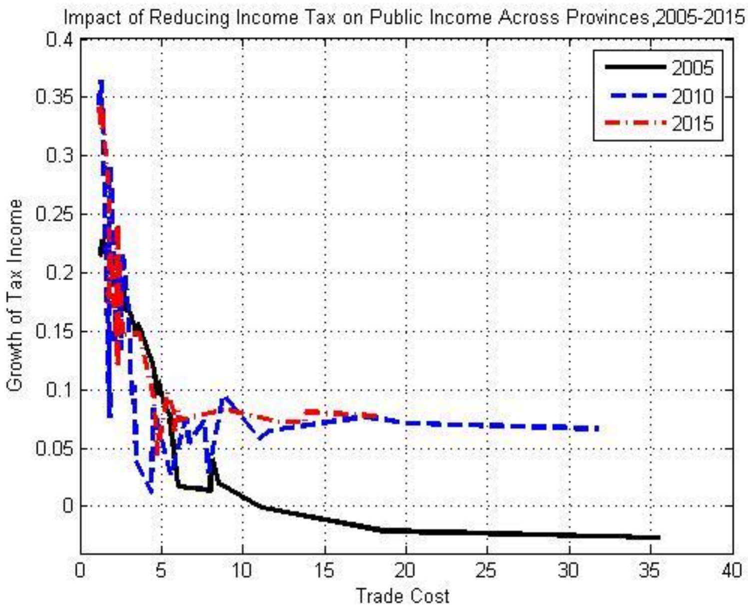


Figure 17. **Trade Cost and Change of Formal Employment Share**



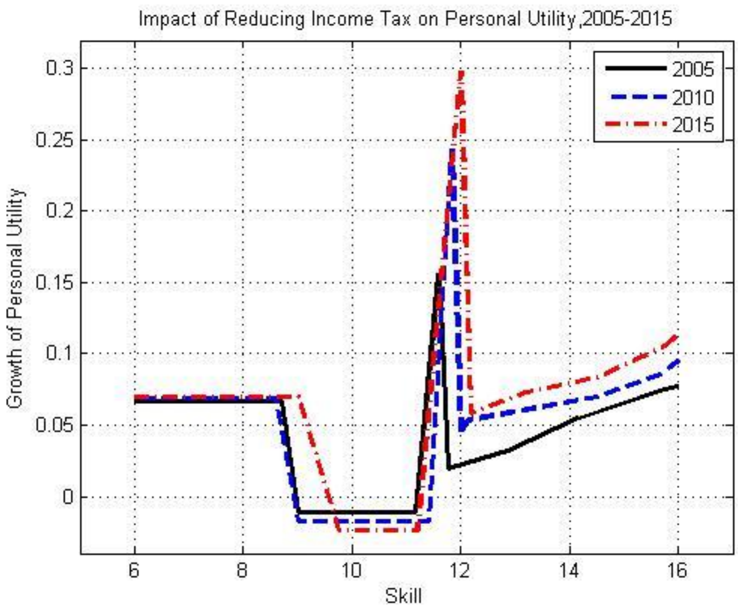
Source: CEIC and NLFS 2005, 2010 and 2015

Figure 18. **Trade Cost and Growth of Tax Revenue**



Source: CEIC and NLFS 2005, 2010 and 2015

Figure 19. **Personal Skill and Growth of Personal Utility**



Source: CEIC and NLFS 2005, 2010 and 2015

Figure 20. $\underline{\varphi}^*(w_1, w_2)$ **and** $w_1^*(w_2)$

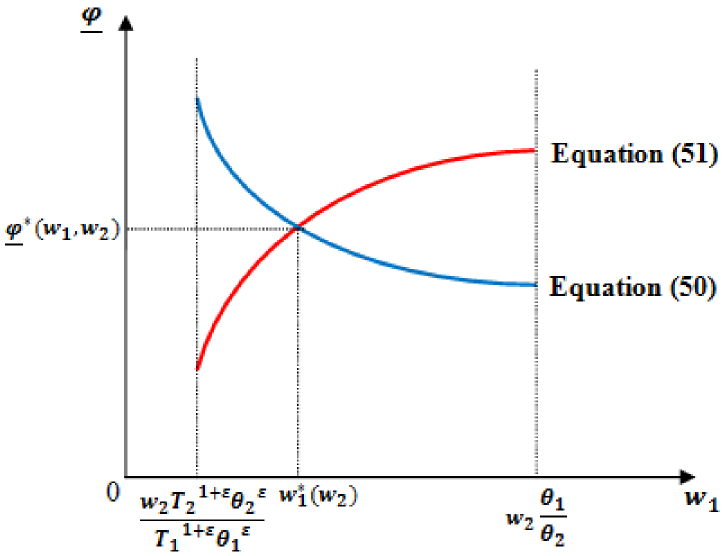
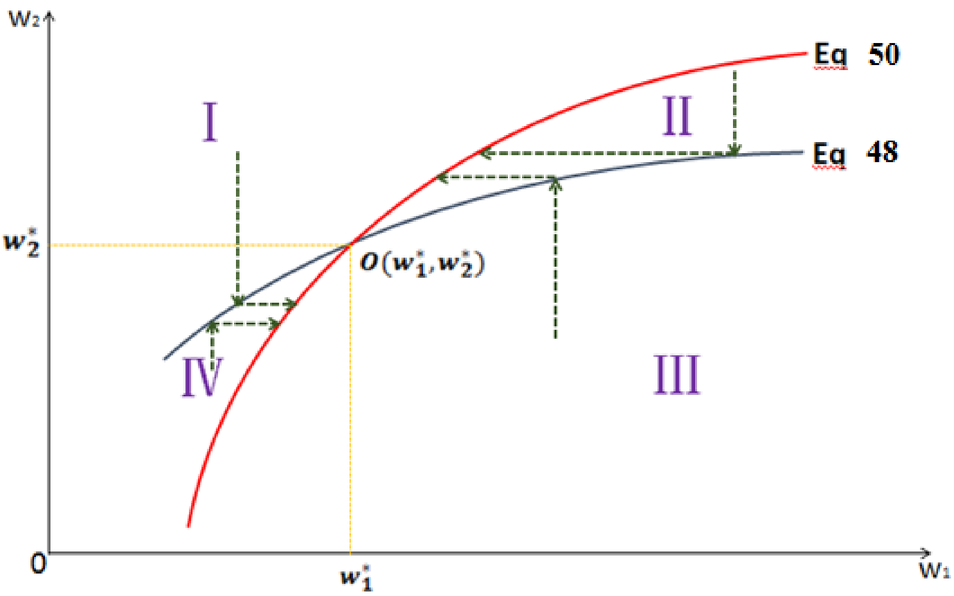


Figure 21. **Fixed-point Mapping of Equilibrium in Definition 1**



Appendix D: Tables

***Table 1.* Zipf's Coefficient of Employment Size Distribution across Top 100 Counties**

	Labor	Formal	Informal	Labor (urban)	Formal (urban)	Informal (urban)
Log-Rank 1995	-1.491	-1.135	-1.349	-0.990	-0.921	-1.040
Log-Rank 2000	-2.073	-1.071	-1.688	-1.202	-1.095	-1.138
Log-Rank 2005	-2.240	-1.293	-1.668	-1.267	-0.996	-1.384
Log-Rank 2010	-1.423	-0.947	-1.301	-0.920	-0.807	-1.017
Log-Rank 2015	-1.356	-0.977	-1.197	-0.874	-0.785	-0.941

Source: NLFS 1995, 2000, 2005, 2010 and 2015

Table 2. Result of Numerical Simulation

Parameter set	Baseline	σ =2.8	Y_O=12000	k =4	σ =2.8, Y_O=12000, k =4	$a_1 = a_2$ =0.06
Population 1	65.4	68.9	66.1	67	68	69
Population 2	34.6	31.1	33.9	33	32	31
Employee's Size 1	48.3	48.2	49.3	51.2	48.2	50.5
Employee's Size 2	22.4	18.6	23.1	23.6	19.2	20.2
Wage Rate 1	10.61	12.2	11.2	12.43	15.12	10.61
Wage Rate 2	6.58	7.63	6.95	7.63	9.38	6.58
Aggregate Income 1	916	1069.1	998.7	1171.7	1473.4	951.3
Aggregate Income 2	248	263.7	260.3	286	329.3	226.2
Average Utility 1	0.44	0.49	0.46	0.53	0.61	0.53
Average Utility 2	0.37	0.43	0.38	0.43	0.52	0.45
Aggregate Capacity 1	58.1	56.1	60.8	74.9	73	60.4
Aggregate Capacity 2	25.4	22.1	25.5	29.8	26.3	23.1
Formal Capacity 1	6.85	5.52	7.47	10.58	8.69	6.85
Formal Capacity 2	3.80	3.33	4.14	5.8	5.19	3.80
Φ_{F1}/Φ_{F2}	1.802	1.657	1.803	1.824	1.675	1.802
Formal Employment Share 1	0.577	0.508	0.617	0.681	0.631	0.552
Formal Employment Share 2	0.36	0.392	0.4	0.463	0.532	0.399

Table 3. Impact of Reducing T_1

Parameter set	Baseline	σ =2.8	Y_O=12000	k =4	σ =2.8, Y_O=12000, k =4
Policy variable	T_1=1.5/ 1.4				
Wage Rate 1	10.61/ 11.15	12.2/ 12.85	11.2/ 11.76	12.43/ 13.06	15.12/ 15.92
Wage Rate 2	6.58/ 6.39	7.63/ 7.4	6.95/ 6.74	7.63/ 7.44	9.38/ 9.12
Aggregate Income 1	916/ 1051.4	1069.1/ 1226.9	998.7/ 1148.2	1171.7/ 1345.7	1473.4/ 1689.2
Aggregate Income 2	248/ 221.9	263.7/ 231.9	260.3/ 232.7	286/ 253.8	329.3/ 288.7
Average Utility 1	0.44/ 0.48	0.49/ 0.52	0.46/ 0.5	0.53/ 0.57	0.61/ 0.65
Average Utility 2	0.37/ 0.38	0.43/ 0.45	0.38/ 0.39	0.43/ 0.44	0.52/ 0.55
Formal Capacity 1	6.85/ 8.13	5.52/ 6.62	7.47/ 8.85	10.58/ 12.32	8.69/ 10.23
Formal Capacity 2	3.80/ 3.53	3.33/ 3.13	4.14/ 3.84	5.8/ 5.33	5.19/ 4.82
Formal Employment Share 1	0.577/ 0.654	0.508/ 0.572	0.617/ 0.697	0.681/ 0.764	0.631/ 0.699
Formal Employment Share 2	0.36/ 0.397	0.392/ 0.442	0.4/ 0.441	0.463/ 0.517	0.532/ 0.597

Table 4. Impact of Reducing θ_1

Parameter set	Baseline	σ =2.8	Y_O=12000	k =4	σ =2.8, Y_O=12000, k =4
Policy variable	θ_1=200/ 180				
Wage Rate 1	10.61/ 10.71	12.2/ 12.35	11.2/ 11.3	12.43/ 12.51	15.12/ 15.27
Wage Rate 2	6.58/ 6.52	7.63/ 7.54	6.95/ 6.88	7.63/ 7.58	9.38/ 9.29
Aggregate Income 1	916/ 934	1069.1/ 1096.5	998.7/ 1019.1	1171.7/ 1189.9	1473.4/ 1504.1
Aggregate Income 2	248/ 241.8	263.7/ 255.1	260.3/ 253.7	286/ 280.1	329.3/ 319.8
Average Utility 1	0.44/ 0.45	0.489/ 0.492	0.46/ 0.463	0.527/ 0.529	0.606/ 0.608
Average Utility 2	0.366/ 0.365	0.427/ 0.427	0.377/ 0.376	0.426/ 0.426	0.522/ 0.522
Formal Capacity 1	6.85/ 7.22	5.52/ 5.89	7.47/ 7.87	10.58/ 11.06	8.69/ 9.2
Formal Capacity 2	3.80/ 3.71	3.33/ 3.25	4.14/ 4.05	5.8/ 5.66	5.19/ 5.06
Formal Employment Share 1	0.577/ 0.585	0.508/ 0.517	0.617/ 0.628	0.681/ 0.691	0.631/ 0.639
Formal Employment Share 2	0.36/ 0.366	0.392/ 0.402	0.4/ 0.409	0.463/ 0.472	0.532/ 0.544

Table 5. Decomposition of the Impact of Reducing T_1 in City 1

Parameter set	Baseline	Export Effect	Occupation Selection Effect	Total Effect
Policy variable	T_1=1.5/ 1.4			
Population Size	65.4	65.4	65.4	68.3
Wage Rate	10.6	10.6	10.8	11.2
Informal E's Utility Slope	0.166	0.165	0.163	0.164
Aggregate Income	916.4	955.7	975.3	1051.4
Export Income	350.4	426.7	408.9	454
Aggregate Utility	29	33	32.2	32.6
Average Utility	0.44	0.5	0.49	0.48
Aggregate Capacity	58.1	65.4	65	65.2
Formal Capacity	6.85	6.85	6.85	8.13
Informal Capacity	21.6	21.6	20.2	17.3
Total Employment	48.3	48.3	50	51.6
Formal Employment	27.9	27.9	30.7	33.8
Formal Employment Share	0.577	0.577	0.614	0.654

Table 6. Endowment

	2005		2010		2015	
$\boldsymbol{N}$	9.09E+07		1.03E+08		1.09E+08	
$\boldsymbol{k}$	2.37	(0.046)	2.27	(0.039)	1.93	(0.028)

Source: CEIC and NLFS 2005, 2010 and 2015

Table 7. Estimation Results of Parameters

	2005		2010		2015	
$\boldsymbol{\sigma}$	2.3	(0.032)	2.3	(0.035)	2.3	(0.03)
$\boldsymbol{Y_0}$	5.18E+15	(3.43E+13)	6.57E+15	(4.76E+13)	9.05E+15	(6.27E+13)
$\boldsymbol{\tau}$	0.266	(0.0011)	0.248	(0.0008)	0.254	(0.001)
Mean ($\boldsymbol{a_c}$)	0.149		0.142		0.137	
SD ($\boldsymbol{a_c}$)	0.059		0.057		0.052	

Source: CEIC and NLFS 2005, 2010 and 2015

Table 8. Calibration Results of Variables

	2005	2010	2015
Mean (T_c)	6.619	6.15	5.305
SD (T_c)	8.151	6.743	4.676
Mean (θ_c)	2.36e+6	2.16e+6	1.15e+6
SD (θ_c)	6.13e+6	5.88e+6	3.37e+6
$\overline{\varphi}_M$	13.021	11.85	12.472
$\underline{\varphi}$	8.158	8.32	9.902

Source: CEIC and NLFS 2005, 2010 and 2015

Table 9. Impact of Reducing τ from 25% to 20%

	2005	2010	2015	Average
Average Wage Rate	5.72%	6.20%	6.78%	6.23%
Aggregate Income	9.72%	10.43%	12.59%	10.91%
Export Income	6.68%	7.99%	9.60%	8.09%
Tax Income	21.96%	25.53%	29.27%	25.58%
Aggregate/Average Utility	6.50%	6.79%	6.98%	6.76%
Worker Utility	6.71%	6.89%	7.04%	6.88%
Utility Slope of I.E.	-1.14%	-1.71%	-2.37%	-1.74%
Formal Employment Share	3.74%	5.26%	9.30%	6.10%

Source: CEIC and NLFS 2005, 2010 and 2015