The Relation Between Idiosyncratic Volatility and Expected Returns: A Statistical Artifact of Temporary Changes in Idiosyncratic Volatility

Hongyan Li and Raman Kumar *

December 31, 2017

Abstract

We document a systematic pattern of temporary increases in the estimated idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility in a given month. A large portion of this temporary increase in the estimated idiosyncratic volatility is reversed in the subsequent month, which suggests the possibility of relatively large positive estimation errors. This temporary increase in the idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility is associated with relatively large positive returns (positive abnormal returns) in the estimation month and relatively low returns (negative abnormal returns) in the subsequent month. Our evidence shows that these temporary changes in the estimated idiosyncratic volatility and the related positive and negative abnormal returns in the estimation and subsequent months, respectively, create a negative relation between the estimated idiosyncratic volatility and subsequent month returns documented in the prior literature (Ang et al. 2006). After controlling for the (negative) relation with the past month's return, there is no significant relation between idiosyncratic volatility and subsequent month's returns as predicted by traditional asset pricing models. Moreover, we find no significant relation between idiosyncratic volatility and subsequent returns for subsets of stocks that do not exhibit any significant changes in idiosyncratic volatility despite large differences in the levels of their idiosyncratic volatility. Finally, there is no relation between the estimated idiosyncratic volatility and subsequent returns after a lag of 3 months when the abnormal returns associated with temporary changes are no longer present. Overall, our results are consistent with the notion that there is no relation between the true underlying idiosyncratic volatility and expected returns, and that the previously documented negative relation between estimated idiosyncratic volatility and subsequent month's returns is being driven by estimation errors (temporary one-month changes) in the estimated idiosyncratic volatility and the associated abnormal returns.

* Hongyan Li (email: hongyan@vt.edu, Tel: 540-231-4144) and Raman Kumar (email: rkumar@vt.edu, Tel: 703-538-8372) are both from the Department of Finance, Pamplin College of Business, Virginia Tech. Blacksburg, VA, 24060. Hongyan Li acknowledges the support of 2015 Pamplin College of Business Summer Research Grant.

I. Introduction

Rational asset pricing models in which investors hold well-diversified portfolios, and are not exposed to unsystematic risk, imply that there should be no relation between the idiosyncratic volatility (IVOL) and the expected returns. Merton (1987) suggests that idiosyncratic risk and expected return should be positively related as investors with incomplete information will hold under-diversified portfolios. However, researchers such as Ang, Hodrick, Xing and Zhang (hereafter AHXZ 2006) have documented a negative relation between estimated idiosyncratic volatility and subsequent realized returns. The direction of the relationship and what may explain this relationship are disputed. Fu (2009) using an EGARCH model documents a significant positive relationship between idiosyncratic volatility and returns. Cao et al. (2013) finds a positive (negative) relation between idiosyncratic risk and returns among relatively undervalued (overvalued) stocks. Bali et al. (2008) and Han et al. (2011) show that there is no relation between the two. The observed relation between the idiosyncratic (firm-specific) risk of a firm and the stock returns goes against the predictions of the traditional asset pricing models and remains an unresolved puzzle.

We document a systematic pattern of temporary increases in the estimated idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility in a given month. A large portion of this temporary increase in the estimated idiosyncratic volatility is reversed in the subsequent month, which suggests the possibility of relatively large positive estimation errors. This temporary increase in the idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility is associated with positive abnormal returns in the estimation month and negative abnormal returns in the subsequent month. Our evidence shows that these estimation errors or temporary changes in the estimated idiosyncratic volatility and the related positive and negative abnormal returns in the estimation and subsequent months, respectively, create a negative relation between the estimated idiosyncratic volatility and subsequent month returns documented in the prior literature (Ang et al. 2006). After controlling for the (negative) relation with the past month's return, there is no significant relation between idiosyncratic volatility and subsequent month's returns as predicted by traditional asset pricing models. Moreover, we find no significant relation between idiosyncratic volatility and subsequent returns for subsets of stocks that do not exhibit any significant changes in idiosyncratic volatility despite large differences in the levels of their idiosyncratic volatility. `Moreover, we find no significant relation between idiosyncratic volatility and subsequent returns for subsets of stocks that do not exhibit any significant changes in idiosyncratic volatility despite large differences in the levels of their idiosyncratic volatility. Finally, there is no relation between the estimated idiosyncratic volatility and subsequent returns after a lag of 2 months when the abnormal returns associated with temporary changes are no longer present. Overall, our results are consistent with the notion that there is no relation between the true underlying idiosyncratic volatility and expected returns, and that the previously documented negative relation between estimated idiosyncratic volatility and subsequent month's returns is being driven by estimation errors (temporary one-month changes) in the estimated idiosyncratic volatility and the associated abnormal returns.

Based on our results we conjecture that there are two possible scenarios, which may explain why the level of idiosyncratic risk does not matter, but the changes in IVOL matter.

First, an inefficient and/or incomplete market reaction to unexpected (negative) events could induce both higher temporary idiosyncratic risk and return predictability in observed returns and this in turn induces a relation between estimated idiosyncratic volatility and subsequent returns. Second, the changes in firm characteristics and idiosyncratic risk may result in trades among investors to adjust their portfolios. When this process is completed and new equilibrium is reached, the level of idiosyncratic risk does not matter.

This paper adds to the current understanding of the IVOL puzzle. We contribute by documenting the importance of separating IVOL levels and temporary IVOL changes and their separate effects on future return, respectively. Overall, we conclude that the IVOL puzzle is mostly driven by firms with negative past performance which also experience temporary increases in their idiosyncratic volatility and continue to have abnormal negative returns in the subsequent month. The level of idiosyncratic risk does not matter for firms that do not undergo changes in their idiosyncratic volatility.

The rest of this paper proceeds as follows: section II briefly reviews the related literature and presents our motivation; section III documents the data and the methodology; section IV examines the relation between the IVOL level and returns; section V investigates the relation between IVOL changes and subsequent returns; and section VI concludes the paper.

II. Related Literature and Motivation

Traditional asset pricing models in which investors hold well-diversified portfolios imply that there should be no relation between the idiosyncratic volatility (IVOL) and the expected returns. However, Ang, Hordrick, Xing and Zhang (hereafter AHXZ 2006) document that stocks with high idiosyncratic volatility earn low subsequent returns. The presence of significant relation between idiosyncratic risk and return both in US market and in international markets (AHXZ 2009) has puzzled many researchers.

Several papers have attempted to empirically resolve this puzzle and some theories have been proposed to explain this puzzle. Merton (1987) suggests that idiosyncratic risk and expected return should be positively related when investors with incomplete information hold under-diversified portfolios. Consistent with Merton (1987), Fu (2009) uses an EGARCH model and finds a significant positive relation between expected idiosyncratic risk and returns. However, Fink, Fink and He (2012) suggest that the Fu (2009) EGARCH model introduces a look-ahead bias. They find that there is no relation between expected returns and expected idiosyncratic volatility when only information up to time t-1 is used to estimate idiosyncratic volatility. Pontiff (2006) shows that idiosyncratic risk is the single most holding cost faced by arbitrageurs, and therefore, suggests that the negative relation between idiosyncratic volatility and returns may be a result of subsequent price correction of overpriced high IVOL stocks which may be too costly to arbitrage. Shleifer and Vishney (1997) suggest that arbitrage has limits, risk and costs. Consistent with the limits of arbitrage argument, Cao et al (2013) and Stambaugh et al (2013) show a positive relation between idiosyncratic risk and return among relative undervalued stocks, and a negative relation between IVOL and return among relative overvalued stocks. Stambaugh et al (2013) further suggest that the IVOL effect is related to investor sentiment. Yet the debate continues: Bali et al (2008) show that the relation between IVOL and return is not robust when using different weighting schemes and different data frequencies. Han and

Lesmond (2011) suggest that the bid-ask bounce biases the estimation of idiosyncratic volatility, and show that the relation between IVOL and return diminishes when CRSP mid-quote based price is used in estimation during sample period 1984 to 2008. Despite these attempts, the overall significantly negative relation between the estimated idiosyncratic volatility and the subsequent month's return continues to be a puzzle. This study will provide a possible resolution for the puzzle by documenting a relatively large temporary increase in the estimated volatility and the associated abnormal return behavior for the sub-group of stocks with the highest estimated idiosyncratic volatility, and by documenting that there is no relation between idiosyncratic volatility and returns for stocks which do not have such temporary increases in idiosyncratic volatility.

We first replicate the AHXZ (2006) results while extending the sample period from July 1963 to December 2013. We find a systematic pattern of relatively large and temporary increases in the estimated idiosyncratic volatility in the estimation month for the quintile of stocks with the highest estimated idiosyncratic volatility and associated positive abnormal returns in the estimation month followed by negative abnormal returns for these stocks in the subsequent month. The large and temporary increases in the estimated idiosyncratic volatility for a sub-group of stocks and the associated abnormal return behavior for these stocks suggests the need to isolate the effect of the IVOL level from the temporary IVOL changes. Otherwise, we may draw a false or spurious conclusion about the relation between IVOL and return. A systematic relation between temporary changes in the estimated idiosyncratic volatility for a sub-group of stocks in a given month and the abnormal return behavior in the subsequent month can create an empirical relation between the estimated idiosyncratic volatility and subsequent month's return even when there is no relation between the true underlying idiosyncratic volatility and expected returns.

III. Data and Methodology

The data include all NYSE, AMEX, and NASDAQ stocks with share code 10 or 11 from CRSP for the period from July 1963 to December 2013. Following AHXZ (2006), we require a minimum of 17 trading days in a month.

Following AHXZ (2006) and the literature, we define idiosyncratic volatility relative to the Fama-French 3 factor model. For each stock *i*, we run the following regression using daily returns within the month:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i$$

and $\sqrt{Var(\varepsilon_t^i)}$ is defined as the idiosyncratic risk for stock *i* in that month.

IV. Idiosyncratic Volatility and Returns

For every month in our sample period we form five quintile portfolios based on the estimated idiosyncratic volatility in that month, and calculate the average IVOL for each portfolio in the estimation month and in the preceding and subsequent months. Panel A of Table 1 presents the average IVOL levels for the five portfolios from month *t-3* to month *t+3*. *IVOL1* is the portfolio of firms with the lowest estimated IVOL in month *t* and *IVOL5* is the portfolio with the highest estimated IVOL in month *t*. For the portfolio *IVOL5*, there is a sharp increase in the average IVOL from 0.0505 to 0.0611 from month *t-1* to month *t* and then a drop back to 0.0510 in month *t+1*. Similarly, portfolio *IVOL1* displays a temporary drop in the average IVOL in month *t*. These results are not surprising because forming portfolios on a variable not only groups them by the true levels of the variable but also on the temporary changes. Moreover, as we will show later, these temporary changes in IVOL are associated with abnormal returns in the current and subsequent months. Figure 1a shows the patterns of the average levels of idiosyncratic volatility from month t-12 to month t+12 for the five portfolios. A similar pattern of temporary changes is observed.

Panel B of Table 1 presents the average returns of the five quintile portfolios from month t-3 to month t+3. Portfolio *IVOL5* has relatively large negative average monthly returns (less than -1%) in the preceding months, a relatively large positive return of 1.95% in month *t*, and then a drop back to 0.09% in month *t+1*. Thus, the firms in *IVOL5* which exhibit a large temporary one-month increase in IVOL in month *t* also exhibit a large increase in average returns in month *t* from negative returns in month t-1 to 1.95% in month *t* and then a large drop to 0.09% in month *t+1*. It is also noteworthy that *IVOL5* has almost twice as high returns than the other portfolios in month *t*, while exhibiting lower returns in the preceding and subsequent months with large differences. It appears that portfolio *IVOL5* includes firms which have large temporary increases in IVOL and abnormal positive returns in month *t*, and abnormal negative returns in month *t+1*. Figures 1b and 1c show the patterns of these returns for these portfolios from month *t-12* to month *t+12*. The sharp and temporary changes in the estimated idiosyncratic volatility for the extreme portfolios and the changes in their observed returns reveal the need to isolate the effect of IVOL level from the temporary IVOL changes. Before examining the relation between the level of IVOL and subsequent returns, it is critical to control for the effect of temporary changes in IVOL on subsequent returns.

In order to isolate the effects of IVOL level from the temporary IVOL changes, we create a subsample of stable IVOL firms for every month *t* in our sample period. We do this by first calculating the average IVOL for each stock over the preceding 12 months (*t-12* to *t-1*) and then calculating the change in IVOL in month *t* from the past 12-month average. Finally, we select 80% of the stocks for our portfolio of stable IVOL firms for each month by excluding 20% of the firms with the highest absolute value of the change in IVOL in month *t*. We further subdivide the 20% of the firms with the largest absolute value of the changes into the IVOL increase group (*IVOLINC*) and the IVOL decrease group (*IVOLDEC*) based on the sign of the change. We also form five revised quintile portfolios (*IVOLS1* to *IVOLS5*) from the 80% subset of stable IVOL firms every month.

Table 2 presents the summary statistics for the IVOL in month *t* (Panel A), the past 12-month average IVOL (Panel B), and the IVOL change (IVOL – past 12-month average IVOL; Panel C) for the original five quintile portfolios which include all firms. The results show the summary statistics for the distribution of IVOL in month t for the five portfolios, the distribution of the past 12-month average IVOL, and the distribution of the changes in IVOL in month *t*. The mean IVOL in month *t* ranges from a low of 0.0087 for portfolio *IVOL1* to 0.0611 for portfolio *IVOL5*, as compared to the past 12-month average IVOL of 0.0141for portfolio *IVOL1* and 0.0463 for portfolio *IVOL5*. This suggests that monthly estimates of IVOL may include a large temporary change component. The mean IVOL change is -0.0054 for portfolio *IVOL1* and 0.0149 for portfolio *IVOL5*.

Table 3 presents the summary statistics for the IVOL in month *t* (Panel A), the past 12-month average IVOL (Panel B), and the IVOL change (IVOL – past 12-month average IVOL; Panel C) for the revised five quintile portfolios (*IVOLS1 – IVOLS5*) which include the 80% stable IVOL firms, and for the two portfolios with IVOL increases (*IVOLINC*) and the IVOL decreases (*IVOLDEC*). The mean IVOL in month t ranges from a low of 0.0085 for portfolio *IVOLS1* to 0.0433 for portfolio *IVOLS5*, as compared to the past 12-month average IVOL of 0.0122 for portfolio *IVOLS1* and 0.0408 for portfolio *IVOLS5*. As expected, the monthly estimates of IVOL are much closer to their 12-month averages for the revised quintiles of stable firms. The mean IVOL change is -0.0037 for portfolio *IVOLS1* and 0.0026 for portfolio *IVOLS5*, as compared to -0.0054 and 0.0149 for *IVOL1* and *IVOL5*, respectively, demonstrating that we have reduced the magnitude of the change. As expected, portfolio *IVOLDEC* exhibits a mean change of -0.0330, and portfolio *IVOLINC* exhibits a mean change of 0.0235. The distribution of the IVOL levels in Panel A of the table show that the subset of the stable firms which include 80% of the firms continues to exhibit a large variation in their IVOL levels. The 75th percentile (Q3) for portfolio *IVOLS5* is 0.0541, which is 7.84 times larger than the 25% percentile (Q1) of 0.0069 for portfolio *IVOLS1*. The mean and the median IVOL for portfolio *IVOLS5* are about 5 times larger than the corresponding values for portfolio *IVOLS1*.

Panel A of Table 4 presents the average IVOL levels for the seven revised portfolios from month t-3 to month t+3. As expected, portfolios *IVOLS1* and *IVOLS5* exhibit smaller temporary changes in IVOL in month *t*, as compared to *IVOL1* and *IVOL5*, respectively, in Table 1, and *IVOLDEC* and *IVOLINC* picking up the bigger changes. Panel B of Table 4 exhibits a similar attenuation of the changes in the returns for *IVOLS1* and *IVOLS5*, with bigger changes picked up by *IVOLDEC* and *IVOLINC* portfolios. To further reduce the magnitude of the changes in IVOL in month *t* for our subset of stable firms, we repeated all of our analysis by successively excluding 30% or 40% of the largest absolute of the IVOL changes, instead of excluding only 20%. The results are consistent with the conclusions of the paper of no relation between the IVOL level and subsequent returns for the subset of stable firms. However, while the magnitude of the IVOL changes becomes smaller, the range of the IVOL level for the subset of stable firms also become smaller. We chose to present the results excluding only 20% of the largest absolute IVOL changes to retain most of the variation in the IVOL levels, while excluding only the most extreme temporary changes in IVOL.

Figure 2a presents the IVOL levels for the seven portfolios. The relatively flat lines for *IVOLS1* through *IVOLS5* suggest that these firms do not experience large changes in IVOL in month t. Moreover, they exhibit stable levels of IVOL not only in the period prior to formation (by design), but also in the subsequent 12 month period while maintaining the relatively large differences in the IVOL levels.

Figure 2b presents the plots of the average value-weighted monthly returns for the seven revised portfolios. The plots suggest that firms with high stable IVOL levels are firms with volatile and often negative past performance, while firms with low to medium stable IVOL levels earn relatively stable and similar past and future returns of about 1% per month. Moreover, when the returns of the stable high IVOL portfolios stabilize in month +5, they stabilize at about the same levels as those of the low to medium stable IVOL levels. These results suggest that the return differences across portfolios with different IVOL levels are being driven by short term and temporary changes in IVOL and the related return volatility, and not because of any relationship between the IVOL levels and expected return. As stated earlier, excluding 30% or 40% of the firms (instead of 20%) with the largest absolute value of the changes in IVOL from the subset of the stable IVOL firms makes the IVOL levels and the returns more stable for the stable IVOL portfolios, and the results of this paper continue to be consistent with the conclusions of the paper that there is no relation between idiosyncratic volatility and subsequent returns.

V. Cross-Sectional Regression results

We further examine the cross-sectional relation between the IVOL level and subsequent return at the firm level using Fama-Macbeth regressions. Specifically, each month from July 1963 to December 2013 we run a firm-level cross-sectional regression as the following:

$$R_{i,t+1} = \alpha_{0t} + \gamma_{1,t} IVOL_{i,t} + \gamma_{2,t} beta_{i,t} + \gamma_{3,t} size_{i,t} + \gamma_{4,t} BM_{i,t} + \gamma_{5,t} R_{i,t} + \varepsilon_{it}$$

(1)

The Dependent Variable ($R_{i,t+1}$) is the realized stock return of firm *i* in month *t+1*. Beta is estimated by CAPM model using previous 36 monthly returns. Following Bali et al (2011) and existing literature, firm size is measured by the natural logarithm of the market value of equity (a stock's price times shares outstanding in millions of dollars) at the month of *t* for each stock; following Fama and French (1992) and Bali et al (2011), we compute a firm's book-to-market ratio using the market value of its equity at the end of December of the previous year and the book value of common equity plus balance-sheet deferred taxes for the firm's latest fiscal year ending in the prior calendar year. [1]

Table 5 presents the time-series averages of the slopes of cross sectional regressions of all firms using the standard Fama and MacBeth(1973) methodology. In column (1) we show the results of the simple regression of the subsequent month return on IVOL. The average slope co-efficient of -0.118 is significant. In column (2), wherein we control for beta, firm size and the book-to-market ratio, the average slope coefficient on IVOL of regression on all firms is negative, -0.140, and significant at 1% level. These significant negative coefficients on IVOL mirror the negative relation between idiosyncratic risk and future returns documented in prior research. However, when we include the return in month *t* as a control variable in columns (3) and (4), the average slope coefficient on IVOL drops in magnitude by about 50% and becomes insignificant. This suggests that the observed negative relation between the estimated idiosyncratic volatility for a given month and the subsequent month's return is a manifestation of the abnormal return behavior for a subset of the firms. The significant negative average slope coefficient on the return for month *t,* and the implied return reversal from month *t* to *t+1* is consistent with the pattern of returns for the highest IVOL quintile of firms as documented in Panel B of Table 1 and Figures 1b and 1c.

Panel A of Table 6 presents the regression results for the subsample of stable firms which exclude the 20% of the firms each month with the largest absolute value of IVOL changes in the estimation month. The magnitude of the average estimated slope coefficient

[1] Flowing literature, the book-to-market ratio and size are winsorized at the 1% and 99% level to avoid issues of extreme observations.

on IVOL for the simple regression drops from -0.118 to -0.042 and becomes insignificant. Moreover, it remains insignificant when other control variables are included. These results provide additional evidence that the previously documented negative relation between the estimated monthly idiosyncratic volatility and subsequent month's return is being a driven by a subsample of firms with large temporary changes in idiosyncratic volatility and the associated abnormal return behavior in the estimation and subsequent months. Panel B of Table 6 presents the regression results for a smaller (70%) subsample of stable IVOL stocks by excluding the 30% of the stocks with the highest absolute value of the change in volatility in the estimation month over its 12-month average. The results are consistent with the results in Panel A of the Table.

Panel A (Panel B) of Table 7 presents the regression results for the firms in the *IVOLINC* (*IVOLDEC*) portfolio, which have a positive (negative) change in IVOL from the 20% subset of the firms which were excluded from the subsample of stable firms because of large absolute value of IVOL changes. For the *IVOLINC* group of firms, the average slope coefficient on IVOL is negative and significant in the simple regression, and when beta, size and book-to-market are included as control variables, and becomes insignificant only when the return in month *t* is included. For the *IVOLDEC* group of firms, there is no significant relation between the estimated IVOL and the subsequent month return. These results suggest, along with the results for the stable subsample, suggest that the observed negative relation between the estimated idiosyncratic volatility in a given month and the subsequent month's return is being primarily driven by the subsample of firms which experience a large temporary increase in the estimated idiosyncratic volatility and relatively large positive returns in the estimation month which are reversed in the subsequent month. Excluding such firms or controlling for return reversals takes away the significance of the relation between idiosyncratic volatility and returns.

We suggest two possible explanations for why there may be no relation between IVOL level and subsequent returns when there is no change in IVOL, but a significant negative relation between IVOL level and returns when firm experience IVOL increases. First, an inefficient and/or incomplete market reaction to unexpected (negative) events could induce both higher temporary idiosyncratic risk and return predictability in observed returns and this in turn induces a relation between estimated idiosyncratic volatility and subsequent returns. Second, the changes in firm characteristics and idiosyncratic risk may result in trades among investors to adjust their portfolios. When this process is completed and new equilibrium is reached, the level of idiosyncratic risk does not matter.

Lastly, we examine the persistence of the effect of IVOL on future returns. Specifically, we run Fama-Macbeth regressions for all firms with the returns for subsequent months as the dependent variables, as follows

$$R_{i,t+n} = \alpha_{0t} + \gamma_{1,t} IVOL_{i,t} + \gamma_{2,t} beta_{i,t} + \gamma_{3,t} size_{i,t} + \gamma_{4,t} BM_{i,t} + \varepsilon_{it} \qquad (2)$$

where n = 1, 2, 3, 4 and 6.

Other variables are as specified as in equation (1). Results are reported in Table 8.

Results show that the average slope coefficients on IVOL continues to be significant for the return in month *t+2* but with a smaller absolute magnitude. The relation continues to weaken, and for months *t+3*, *t+4* and *t+6*, there is no significant relation between the estimated idiosyncratic volatility and returns. In unreported results, we find no relation between the idiosyncratic volatility and returns for month 7 and beyond also. These results mirror the graph in Figures 1b, 1c and 2b, and are noteworthy because despite the large differences in IVOL in month t, the returns start to converge and are not different from each other among different IVOL portfolios after 3 to 6 months, although their IVOL levels still stay different. The IVOL level of high IVOL portfolios remains high, and the IVOL level of low IVOL portfolio remains low, but the returns converge.

VI. Conclusions

We document a systematic pattern of temporary increases in the estimated idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility in a given month. A large portion of this temporary increase in the estimated idiosyncratic volatility is reversed in the subsequent month, which suggests the possibility of relatively large positive estimation errors. This temporary increase in the idiosyncratic volatility for the quintile of stocks with the highest estimated idiosyncratic volatility is associated with relatively large positive returns (positive abnormal returns) in the estimation month and relatively low returns (negative abnormal returns) in the subsequent month. Our evidence shows that these temporary changes (specifically temporary increases) in the estimated idiosyncratic volatility and the related positive and negative abnormal returns in the estimation and subsequent months, respectively, create a negative relation between the estimated idiosyncratic volatility and subsequent month returns documented in the prior literature (Ang et al. 2006). After controlling for the (negative) relation with the past month's return, there is no significant relation between idiosyncratic volatility and subsequent month's returns as predicted by traditional asset pricing models. Moreover, we find no significant relation between idiosyncratic volatility and subsequent returns for subsets of stocks that do not exhibit any significant changes in idiosyncratic volatility despite large differences in the levels of their idiosyncratic volatility. Finally, the negative relation between idiosyncratic volatility and subsequent returns starts to weaken as lags between the estimated volatility and returns are introduced to control for the problem created by the relation between the temporary changes and associated returns in the estimation and subsequent months. By month *t+3,* the negative relation is no longer present, despite large continued differences in their estimated volatilities. Overall, our results are consistent with the notion that there is no relation between the true underlying idiosyncratic volatility and expected returns, and that the previously documented negative relation between estimated idiosyncratic volatility and subsequent month's returns is being driven by temporary changes in the estimated idiosyncratic volatility and the associated abnormal returns.

References:

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006. The cross-section of volatility and expected returns, *Journal of Finance* 61, 259-299.

Ang, Andrew, Robert J. Hodrick, Yuang Xing, and Xiaoyan Zhang, 2009, High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence, *Journal of Financial Economics*, 91, 1-23.

Bali, Turan G., and Nusret Cakici, 2008. Idiosyncratic volatility and the cross-section of expected returns, *Journal of Financial and Quantitative Analysis* 43, 29-58.

Bali, Turan G., Nusret Cakici and Robert F. Whitelaw, 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns, *Journal of Financial Economics* 99, 427-446.

Cao, Jie, and Bing Han, 2013. Idiosyncratic risk, costly arbitrage, and the cross-section of stock returns, working paper, http://ssrn.com/abstract=1291626.

Fama, E.F., French, K.R., 1992. Cross-section of expected stock returns. *Journal of Finance* 47, 427-465.

Fama, E.F., Macbeth, J.D., 1973. Risk and return: some empirical tests. *Journal of Political Economy* 81, 607-636.

Fink, J., Fink., and He, H. 2012. Expected idiosyncratic volatility measures and expected returns. *Financial Management* 41, 519-553.

Fu, Fangjian, 2009. Idiosyncratic risk and the cross-section of expected returns, *Journal of Financial Economics* 91, 24-37.

Han, Yufeng, and David Lesmond, 2011. Liquidity biases and the pricing of cross-sectional idiosyncratic volatility, *The Review of Financial Studies* 24, 1590-1629.

Merton, Robert C., 1987. A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483-510.

Miller, Merton, and Myron Scholes, 1972. Rates of return in relation to risk: A re-examination of some recent findings, in “Studies in the Theory of Capital Markets”.

Miller, Edward M., 1977. Risk, uncertainty, and divergence of opinion, *Journal of Finance*, 1151-1168.

Pontiff, J., 2006. Costly arbitrage and the myth of idiosyncratic risk. *Journal of Accounting & Economics* 42, 35-52.

Roll, Richard, and Stephen Ross, 1980. An empirical investigation of the arbitrage pricing theory, *Journal of Finance* 35, 1073-1103.

Shleifer, Andrei, and Robert W. Vishny, 1997. The limits of arbitrage, *Journal of Finance* 52, 35-55.

Stambaugh, Robert F., Jianfeng Yu and Yu Yuan, 2013. Arbitrage asymmetry and the idiosyncratic volatility puzzle, working paper.

Walkshausl, Christian, 2013, The high returns to low volatility stocks are actually a premium on high quality firms. *Review of Financial Economics* 22, 180-186.

Figure 1. a

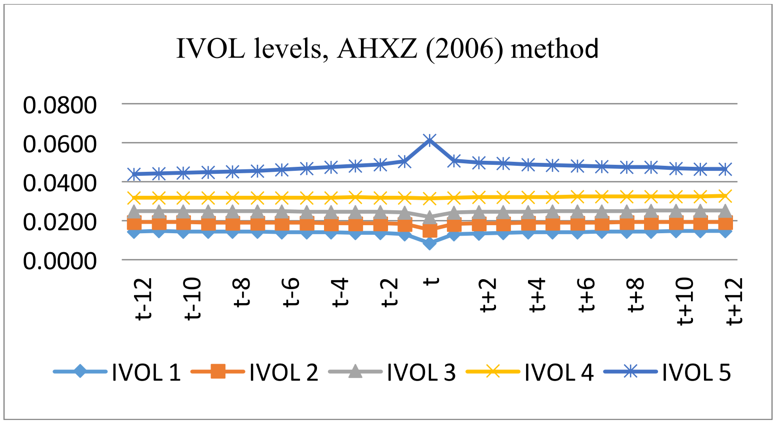

Figure 1.b

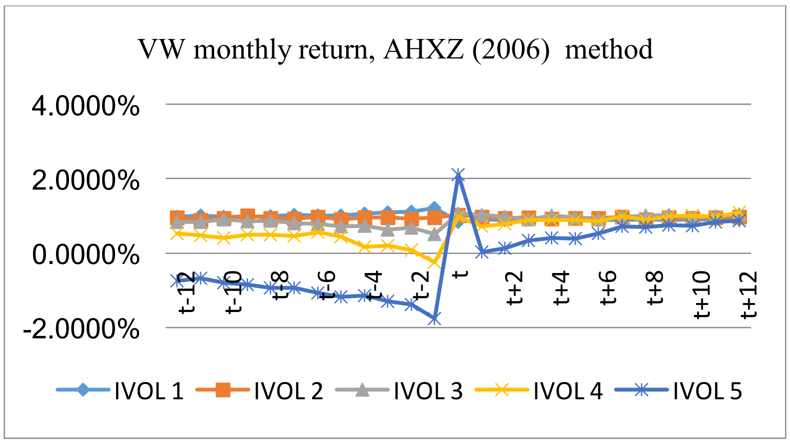

Figure1. c

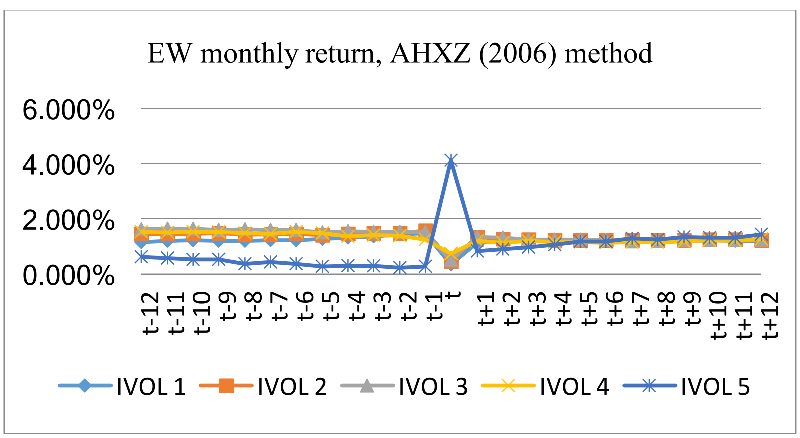

Figure 1 shows the IVOL level (a), value weighted average monthly return (b), equal weighted average monthly return (c) each month for 12 months before and after the portfolio formation. AHXZ (2006) method is used to estimate idiosyncratic volatility and to rank portfolios, i.e., we estimated idiosyncratic risk relative to ff-3 model using daily returns within that month. Then stocks are ranked according to their idiosyncratic risk level each month from July 1963 to December 2013. Portfolio formation month is the month *t* in the graph.

Figure 2. a

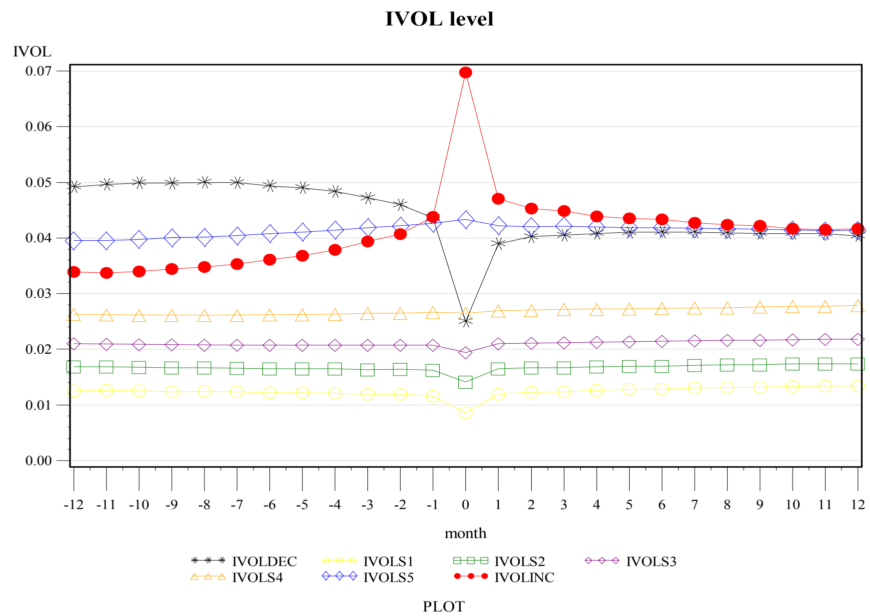

Figure 2. b

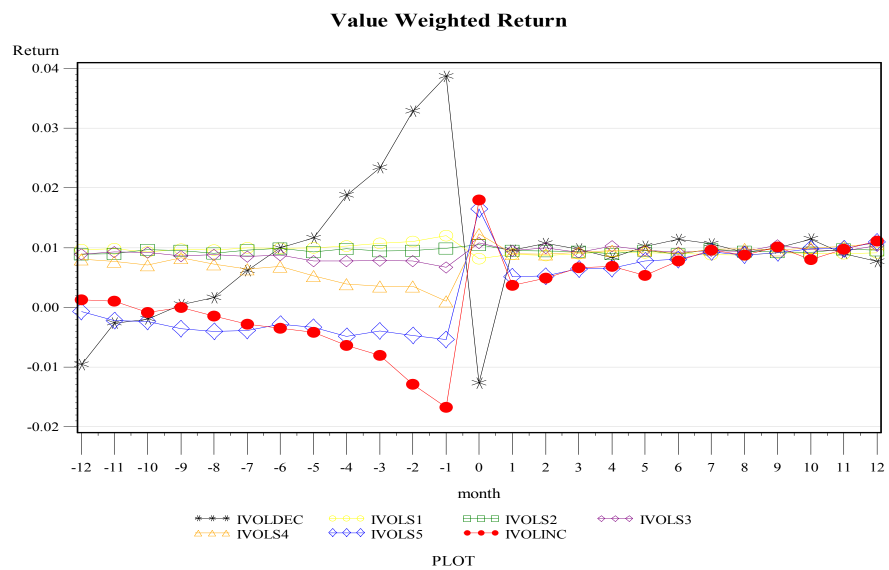

Figure 2a presents the IVOL levels for the seven revised portfolios. **Figure 2b** presents the plots of the average value-weighted monthly returns for the seven revised portfolios. *IVOLS1-IVOLS5:* We first calculate the average IVOL for each stock over the preceding 12 months (t-12 to t-1) and then calculate the change in IVOL in month t from the past 12-month average. We select 80% of the stocks for our portfolio of stable IVOL firms for each month by excluding 20% of the firms with the highest absolute value of the change in IVOL in month t. We form five revised quintile portfolios (IVOLS1 to IVOLS5) from the 80% subset of stable IVOL firms every month. *IVOLINC and IVOLDEC:* We further subdivide the 20% of the firms with the largest absolute value of the changes into the IVOL increase group *(IVOLINC)* and the IVOL decrease group (*IVOLDEC*) based on the sign of the change.

Table 1 Idiosyncratic Volatility and Returns for All Firms

For every month in our sample period we form five quintile portfolios based on the estimated idiosyncratic volatility in that month, and calculate the average IVOL for each portfolio in the estimation month and in the preceding and subsequent months. Panel A of Table 1 presents the average IVOL levels for the five portfolios from month t-3 to month t+3. IVOL1 is the portfolio of firms with the lowest estimated IVOL in month t and IVOL5 is the portfolio with the highest estimated IVOL in month t. Panel B of Table 1 presents the average returns of the five quintile portfolios from month t-3 to month t+3.

Panel A **IVOL**	t-3	t-2	t-1	***t***	t+1	t+2	t+3
IOVL1	0.0137	0.0136	0.0131	0.0087	0.0130	0.0135	0.0136
IOVL2	0.0184	0.0184	0.0181	0.0152	0.0182	0.0184	0.0185
IOVL3	0.0242	0.0241	0.0239	0.0217	0.0240	0.0242	0.0243
IOVL4	0.0317	0.0317	0.0316	0.0311	0.0316	0.0318	0.0320
IOVL5	0.0483	0.0490	0.0505	0.0611	0.0510	0.0501	0.0496

Panel B **VW Returns**	t-3	t-2	t-1	***t***	t+1	t+2	t+3
IOVL1	0.0108	0.0111	0.0121	0.0082	0.0089	0.0088	0.0090
IOVL2	0.0093	0.0089	0.0094	0.0103	0.0097	0.0095	0.0095
IOVL3	0.0064	0.0068	0.0049	0.0103	0.0099	0.0099	0.0090
IOVL4	0.0014	0.0007	-0.0028	0.0096	0.0077	0.0080	0.0093
IOVL5	-0.0127	-0.0140	-0.0180	0.0195	0.0009	0.0032	0.0047

Table 2 Summary Statistics for the IVOL, Average IVOL and the IVOL change for All Firms

For every month in our sample period we form five quintile portfolios based on the estimated idiosyncratic volatility in that month. Table 2 presents the summary statistics for the IVOL in month t (Panel A), the past 12-month average IVOL (Panel B), and the IVOL change (IVOL – past 12-month average IVOL; Panel C) for the original five quintile portfolios which include all firms.

Panel A IVOL	Mean	Std	P10	Q1	Median	Q3	P90
IOVL1	0.0087	0.0025	0.0059	0.0068	0.0083	0.0101	0.0113
IOVL2	0.0152	0.0039	0.0112	0.0124	0.0143	0.0174	0.0201
IOVL3	0.0217	0.0059	0.0157	0.0172	0.0201	0.0256	0.0297
IOVL4	0.0311	0.0090	0.0221	0.0240	0.0282	0.0375	0.0437
IOVL5	0.0611	0.0209	0.0406	0.0444	0.0537	0.0769	0.0900

Panel B Past 12 month Average IVOL	Mean	Std	P10	Q1	Median	Q3	P90
IOVL1	0.0141	0.0027	0.0113	0.0122	0.0137	0.0153	0.0176
IOVL2	0.0188	0.0042	0.0146	0.0156	0.0175	0.0216	0.0239
IOVL3	0.0244	0.0062	0.0185	0.0197	0.0224	0.0291	0.0332
IOVL4	0.0316	0.0086	0.0231	0.0246	0.0283	0.0393	0.0447
IOVL5	0.0463	0.0142	0.0325	0.0344	0.0397	0.0610	0.0686

Panel C IVOL Change	Mean	Std	P10	Q1	Median	Q3	P90
IOVL1	-0.0054	0.0023	-0.0084	-0.0071	-0.0050	-0.0039	-0.0027
IOVL2	-0.0035	0.0027	-0.0063	-0.0047	-0.0036	-0.0021	-0.0009
IOVL3	-0.0028	0.0036	-0.0065	-0.0044	-0.0030	-0.0011	0.0012
IOVL4	-0.0005	0.0050	-0.0053	-0.0028	-0.0009	0.0016	0.0047
IOVL5	0.0149	0.0114	0.0048	0.0084	0.0124	0.0182	0.0280

Table 3 Summary Statistics for the IVOL, Average IVOL and the IVOL change for the revised seven Portfolios

IVOLS1-IVOLS5: We first calculate the average IVOL for each stock over the preceding 12 months (t-12 to t-1) and then calculate the change in IVOL in month t from the past 12-month average. We select 80% of the stocks for our portfolio of stable IVOL firms for each month by excluding 20% of the firms with the highest absolute value of the change in IVOL in month t. We form five revised quintile portfolios (IVOLS1 to IVOLS5) from the 80% subset of stable IVOL firms every month. *IVOLINC and IVOLDEC:* We further subdivide the 20% of the firms with the largest absolute value of the changes into the IVOL increase group *(IVOLINC)* and the IVOL decrease group (*IVOLDEC*) based on the sign of the change. Table 3 presents the summary statistics for the IVOL in month t (Panel A), the past 12-month average IVOL (Panel B), and the IVOL change (IVOL – past 12-month average IVOL; Panel C) for the revised five quintile portfolios (*IVOLS1 – IVOLS5*) and for the two portfolios with IVOL increases *(IVOLINC)* and the IVOL decreases (*IVOLDEC*).

Panel A **IVOL**	Mean	Std	P10	Q1	Median	Q3	P90
IVOLS1	0.0085	0.0022	0.0062	0.0069	0.0083	0.0096	0.0108
IVOLS2	0.0141	0.0034	0.0106	0.0117	0.0134	0.0159	0.0183
IVOLS3	0.0194	0.0048	0.0144	0.0157	0.0183	0.0225	0.0259
IVOLS4	0.0265	0.0068	0.0196	0.0213	0.0246	0.0315	0.0360
IVOLS5	0.0433	0.0117	0.0316	0.0337	0.0388	0.0541	0.0614
IVOLINC	0.0697	0.0243	0.0435	0.0494	0.0613	0.0908	0.1066
IVOLDEC	0.0250	0.0089	0.0157	0.0182	0.0216	0.0327	0.0398

Panel B **Past 12 month** **Average IVOL**	Mean	Std	P10	Q1	Median	Q3	P90
IVOLS1	0.0122	0.0025	0.0096	0.0104	0.0118	0.0133	0.0155
IVOLS2	0.0166	0.0035	0.0131	0.0140	0.0155	0.0189	0.0207
IVOLS3	0.0208	0.0048	0.0161	0.0171	0.0193	0.0243	0.0275
IVOLS4	0.0263	0.0065	0.0197	0.0214	0.0241	0.0315	0.0356
IVOLS5	0.0408	0.0108	0.0302	0.0317	0.0361	0.0515	0.0579
IVOLINC	0.0367	0.0122	0.0242	0.0267	0.0316	0.0485	0.0556
IVOLDEC	0.0485	0.0161	0.0329	0.0352	0.0422	0.0626	0.0725

Panel C **IVOL Change**	Mean	Std	P10	Q1	Median	Q3	P90
IVOLS1	-0.0037	0.0016	-0.0055	-0.0042	-0.0035	-0.0029	-0.0023
IVOLS2	-0.0024	0.0020	-0.0045	-0.0033	-0.0024	-0.0015	-0.0003
IVOLS3	-0.0014	0.0024	-0.0037	-0.0024	-0.0015	-0.0003	0.0009
IVOLS4	0.0002	0.0028	-0.0022	-0.0009	-0.0001	0.0011	0.0027
IVOLS5	0.0026	0.0029	0.0005	0.0012	0.0020	0.0031	0.0049
IVOLINC	0.0330	0.0131	0.0173	0.0238	0.0306	0.0411	0.0512
IVOLDEC	-0.0235	0.0084	-0.0352	-0.0286	-0.0222	-0.0174	-0.0133

Table 4 Idiosyncratic Valotility and Returns for the revised seven Portfolios

IVOLS1-IVOLS5: We first calculate the average IVOL for each stock over the preceding 12 months (t-12 to t-1) and then calculate the change in IVOL in month t from the past 12-month average. We select 80% of the stocks for our portfolio of stable IVOL firms for each month by excluding 20% of the firms with the highest absolute value of the change in IVOL in month t. We form five revised quintile portfolios (*IVOLS1 to IVOLS5*) from the 80% subset of stable IVOL firms every month. *IVOLINC and IVOLDEC:* We further subdivide the 20% of the firms with the largest absolute value of the changes into the IVOL increase group *(IVOLINC*) and the IVOL decrease group (*IVOLDEC*) based on the sign of the change. Panel A of Table 4 presents the average IVOL levels for the seven revised portfolios from month t-3 to month t+3. Panel B of Table 4 presents the average returns for the seven revised portfolios from month t-3 to month t+3.

Panel A **IVOL Level**	t-3	t-2	t-1	***t***	t+1	t+2	t+3
IVOLS1	0.0119	0.0119	0.0116	0.0085	0.0119	0.0123	0.0124
IVOLS2	0.0164	0.0164	0.0163	0.0141	0.0165	0.0167	0.0167
IVOLS3	0.0207	0.0207	0.0207	0.0194	0.0210	0.0211	0.0212
IVOLS4	0.0265	0.0265	0.0266	0.0265	0.0270	0.0270	0.0272
IVOLS5	0.0419	0.0422	0.0426	0.0433	0.0422	0.0420	0.0421
IVOLINC	0.0394	0.0407	0.0438	0.0697	0.0471	0.0453	0.0448
IVOLDEC	0.0472	0.0460	0.0436	0.0250	0.0389	0.0402	0.0405

Panel B **VW Returns**	t-3	t-2	t-1	***t***	t+1	t+2	t+3
IVOLS1	0.0107	0.0110	0.0120	0.0081	0.0089	0.0087	0.0090
IVOLS2	0.0094	0.0095	0.0099	0.0105	0.0095	0.0094	0.0094
IVOLS3	0.0079	0.0078	0.0067	0.0108	0.0095	0.0100	0.0092
IVOLS4	0.0035	0.0036	0.0010	0.0123	0.0090	0.0089	0.0092
IVOLS5	-0.0039	-0.0047	-0.0054	0.0165	0.0051	0.0053	0.0065
IVOLINC	-0.0081	-0.0129	-0.0168	0.0180	0.0037	0.0049	0.0067
IVOLDEC	0.0234	0.0328	0.0387	-0.0127	0.0096	0.0107	0.0097

Table 5 Fama-MacBeth Regression of Returns on Idiosyncratic Volatility and Firm Characteristics (All Firms)

The table presents the time-series averages of the slopes in cross sectional regressions using the standard Fama and MacBeth(1973) methodology. The Dependent Variable $R_{i,t+1}$ is the realized stock return of firm i in month t+1. Beta is estimated by CAPM model using previous 36 monthly return. Size and book-to-market ratio are defined as Fu(2009). Standard errors are Newey-West method corrected. P-value are in parenthesis. *, ** and *** indicates 10%, 5% and 1% significance level respectively.

$$R_{i,t+1} = \alpha_{0t} + \gamma_{1,t} IVOL_{i,t} + \gamma_{2,t} beta_{i,t} + \gamma_{3,t} size_{i,t} + \gamma_{4,t} BM_{i,t} + \gamma_{5,t} R_{i,t} + \varepsilon_{it}$$

	All firms			
	(1)	(2)	(3)	(4)
$Ivol_t$	-0.118**	-0.140***	-0.066	-0.056
	(0.011)	(0.000)	(0.203)	(0.102)
beta		0.001		0.001
		(0.210)		(0.295)
Size		-0.002***		-0.001***
		(0.000)		(0.005)
BtoM		0.002***		0.003***
		(0.003)		(0.000)
Rt			-0.053***	-0.062***
			(0.000)	(0.000)
Average Adjusted Rsq	0.017	0.042	0.025	0.050

Table 6 Fama-MacBeth Regression of Returns on Idiosyncratic Volatility and Firm Characteristics (*Stable IVOL Firms*)

We first calculate the average IVOL for each stock over the preceding 12 months (t-12 to t-1) and then calculate the change in IVOL in month t from the past 12-month average. We select 80% of the stocks for our portfolio of stable IVOL firms for each month by excluding 20% of the firms with the highest absolute value of the change in IVOL in month t.

The table presents the time-series averages of the slopes in cross sectional regressions using the standard Fama and MacBeth(1973) methodology. The Dependent Variable $R_{i,t+1}$ is the realized stock return of firm i in month t+1. Beta is estimated by CAPM model using previous 36 monthly return. Size and book-to-market ratio are defined as Fu(2009). tandard errors are Newey-West method corrected. P-value are in parenthesis. *, ** and *** indicates 10%, 5% and 1% significance level respectively.

$$R_{i,t+1} = \alpha_{0t} + \gamma_{1,t} IVOL_{i,t} + \gamma_{2,t} beta_{i,t} + \gamma_{3,t} size_{i,t} + \gamma_{4,t} BM_{i,t} + \gamma_{5,t} R_{i,t} + \varepsilon_{it}$$

Panel A	**Stable IVOL FIRMS (80% Firms)**			
	(1)	(2)	(3)	(4)
$Ivol_t$	-0.042	-0.089	-0.038	-0.050
	(0.591)	(0.103)	(0.639)	(0.379)
beta		0.001		0.001
		(0.156)		(0.186)
Size		-0.001***		-0.001***
		(0.000)		(0.005)
BtoM		0.002***		0.002***
		(0.007)		(0.001)
Rt			-0.053***	-0.061***
			(0.000)	(0.000)
Average Adjusted Rsq	0.022	0.048	0.030	0.055

Panel B	**Stable IVOL FIRMS (70% Firms)**			
	(1)	(2)	(3)	(4)
$Ivol_t$	-0.034	-0.085	-0.036	-0.052
	(0.677)	(0.142)	(0.674)	(0.391)
beta		0.001		0.002
		(0.143)		(0.153)
Size		-0.001***		-0.001***
		(0.000)		(0.009)
BtoM		0.002***		0.002***
		(0.008)		(0.002)
Rt			-0.053***	-0.062***
			(0.000)	(0.000)
Average Adjusted Rsq	0.024	0.050	0.032	0.057

Table 7 Fama-MacBeth Regression of Returns on Idiosyncratic Volatility and Firm Characteristics (*IVOLINC* and *IVOLDEC*)

Firms belong to the *IVOLINC* portfolio if they are among the 20% of the firms with the highest absolute value of the change in IVOL in month t from the past $12-$month average ($|IVOL_t - \overline{IVOL_{t-1},\ IVOL_{t-12}}|$) and if the change is positive. Firms belong to the *IVOLDEC* portfolio if they are among the 20% of the firms with the highest absolute value of the change in IVOL in month t from the past $12-$month average ($|IVOL_t - \overline{IVOL_{t-1},\ IVOL_{t-12}}|$) and if the change is Negative.

The table presents the time-series averages of the slopes in cross sectional regressions using the standard Fama and MacBeth(1973) methodology. The Dependent Variable Ri,t+1 is the realized stock return of firm i in month t+1. Beta is estimated by CAPM model using previous 36 monthly return. Size and book-to-market ratio are defined as Fu(2009). tandard errors are Newey-West method corrected. P-value are in parenthesis. *, ** and *** indicates 10%, 5% and 1% significance level respectively.

$$R_{i,t+1} = \alpha_{0t} + \gamma_{1,t}IVOL_{i,t} + \gamma_{2,t}beta_{i,t} + \gamma_{3,t}size_{i,t} + \gamma_{4,t}BM_{i,t} + \gamma_{5,t}R_{i,t} + \varepsilon_{it}$$

	Panel A IVOLINC				***Panel B IVOLDEC***			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Ivol_t$	-0.108***	-0.136***	-0.061	-0.056	-0.029	-0.012	-0.076	0.076
	(0.008)	(0.002)	(0.147)	(0.222)	(0.737)	(0.905)	(0.407)	(0.592)
beta		-0.001		-0.000		0.003**		0.002
		(0.487)		(0.889)		(0.025)		(0.112)
Size		-0.006***		-0.004***		-0.004***		-0.002
		(0.000)		(0.000)		(0.000)		(0.223)
BtoM		0.002*		0.004***		0.003**		-0.000
		(0.057)		(0.001)		(0.021)		(0.911)
Rt			-0.047***	-0.053***			-0.074***	-0.057***
			(0.000)	(0.000)			(0.000)	(0.002)
Average Adj Rsq	0.015	0.034	0.027	0.046	0.017	0.030	0.024	0.039

Table 8 Fama-MacBeth Regression of Returns on Idiosyncratic Volatility and Firm Characteristics

The table presents the time-series averages of the slopes in cross sectional regressions using the standard Fama and MacBeth(1973) methodology. The Dependent Variables $R_{i,t+1}$, $R_{i,t+2}$, $R_{i,t+3}$, $R_{i,t+4}$, and $R_{i,t+6}$ are the realized stock return of firm *i* at month t+1 to t+4, and t+6, respectively. Beta is estimated by CAPM model using previous 36 monthly return. Size and book-to-market ratio are defined as Fu(2009). Newey-West P-value are in parenthesis. *, ** and *** indicates 10%, 5% and 1% significance level respectively.

$$R_{i,t+n} = \alpha_{0t} + \gamma_{1,t} IVOL_{i,t} + \gamma_{2,t} beta_{i,t} + \gamma_{3,t} size_{i,t} + \gamma_{4,t} BM_{i,t} + \varepsilon_{it}$$

$Where\ n = 1, 2, 3, 4, and\ 6$, respectively

All Firms	Dependent Variable				
	Ret_{t+1} (1)	Ret_{t+2} (2)	Ret_{t+3} (3)	Ret_{t+4} (4)	Ret_{t+6} (5)
$Ivol_t$	-0.140***	-0.083**	-0.029	-0.021	-0.003
	(0.000)	(0.011)	(0.376)	(0.535)	(0.942)
beta	0.001	0.001	0.001	0.001	0.001
	(0.210)	(0.238)	(0.465)	(0.346)	(0.516)
Size	-0.002***	-0.001***	-0.001***	-0.001***	-0.001**
	(0.000)	(0.001)	(0.007)	(0.009)	(0.021)
BtoM	0.002***	0.002***	0.003***	0.003***	0.003***
	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)
Average Adjusted Rsq	0.042	0.041	0.040	0.040	0.039