A Portfolio Perspective on the Multitude of Firm Characteristics*

Victor DeMiguel Alberto Martín-Utrera Francisco J. Nogales Raman Uppal

December 15, 2017

*DeMiguel, London Business School, e-mail: avmiguel@london.edu; Martín-Utrera, Lancaster University Management School, e-mail: a.martinutrera@lancaster.ac.uk; Nogales, Universidad Carlos III de Madrid, e-mail: FcoJavier.Nogales@uc3m.es; Uppal, Edhec Business School and CEPR, e-mail: raman.uppal@edhec.edu. We gratefully acknowledge comments from Torben Andersen, Turan Bali, Pedro Barroso, Kerry Back, Malcolm Baker, Alexandre Belloni, Jonathan Berk, Harjoat Bhamra, Svetlana Bryzgalova, Andrea Buraschi, Veronika Czellar, Serge Darolles, Alex Edmans, Wayne Ferson, Lorenzo Garlappi, René Garcia, Francisco Gomes, Amit Goyal, Nick Hirschey, Christian Julliard, Petri Jylha, Bige Kahraman, Nishad Kapadia, Ralph Koijen, Robert Kosowski, Apostolos Kourtis, Anton Lines, Abraham Lioui, Raphael Markellos, Lionel Martellini, Spyros Mesomeris, Maurizio Montone, Narayan Naik, Ľuboš Pástor, Barbara Ostdiek, Anna Pavlova, Julien Penasse, Ludovic Phalippou, Ilaria Piatti, Jeffrey Pontiff, Riccardo Rebonato, Scott Richardson, Thierry Roncalli, James Sefton, Georgios Skoulakis, Stephen Taylor, Nikolaos Tessaromatis, Grigory Vilkov, Christian Wagner, Michael Weber, Dacheng Xiu, Paolo Zaffaroni, Frank Zhang, Lu Zhang, and seminar participants at Cass Business School, Citi Global Quant Research Conference, Conference on Computational and Financial Econometrics, Deutsche Bank Global Quantitative Conference, Edhec Business School, European Finance Association, French Finance Association, HEC Paris, Imperial College Business School, INET Econometric Seminar Series (Oxford), INFORMS Annual Meeting, INQUIRE Europe, INQUIRE UK, Invesco, KU Leuven, Lancaster University Management School, London Business School, London School of Economics, New Methods for the Empirical Analysis of Financial Markets Conference, Northern Finance Association, Norwich Business School, Rice University, Saïd Business School, Universitat Pompeu Fabra, Université Catolique de Louvain, Université de Nantes, Vienna University of Economics and Business, 5th Luxembourg Asset Management Summit, 10th International Conference on Computational and Financial Econometrics (Seville), 30th Annual Seminar of the London Quant Group (Oxford), and XXIV Finance Forum (Madrid). Nogales acknowledges support by the Spanish government from project MTM2013-44902-P.

Abstract

We investigate how many characteristics matter jointly for an investor who cares not only about average returns but also about portfolio risk, transaction costs, and out-of-sample performance. We find only a small number of characteristics—six—are significant without transaction costs. With transaction costs, the number of significant characteristics *increases* to 15 because the trades in the underlying stocks required to rebalance different characteristics often net out. Thus, transaction costs increase the dimension of the cross section of stock returns because combining characteristics helps to reduce transaction costs. We also show that investors can improve out-of-sample performance net of transaction costs by exploiting a large set of characteristics instead of the small number considered in prominent asset-pricing models.

Keywords: anomalies, risk, transaction costs, cross section of stock returns.
JEL Classification: G11

1 Introduction

Hundreds of variables have been proposed to predict the cross-section of stock returns; see, for instance, Harvey, Liu, and Zhu (2015), McLean and Pontiff (2016), and Hou, Xue, and Zhang (2017).[1] This abundance of cross-sectional predictors leads Cochrane (2011) to ask, "Which characteristics really provide independent information about average returns?" Likewise, Goyal (2012) states that "these days one has a multitude of variables that seem to explain the cross-sectional pattern of returns. The amount of independent information in these variables is unclear as no study to date [...] has conducted a comprehensive study to analyze the joint impact of these variables."

Our goal is to investigate the dimension of the cross section of stock returns from a *portfolio perspective*. In other words, how many firm-specific characteristics matter *jointly* from the perspective of an investor who cares not only about average returns but also about portfolio risk, transaction costs, and out-of-sample performance. As highlighted in Pastor and Stambaugh (2000), a portfolio perspective is important because it provides an economic metric for judging differences across models. It also allows one to assess how many characteristics matter jointly because for portfolio allocation it is optimal to trade *combinations* of characteristics to reduce both portfolio risk and transaction costs.

To achieve our goal, we consider a dataset with more than 50 firm-specific characteristics and focus on three research questions. First, how many characteristics are jointly significant from a portfolio perspective and why? Second, how does the answer to this question change with transaction costs? Third, can an investor improve out-of-sample performance net of transaction costs by exploiting a large set of characteristics instead of the small number considered in prominent asset pricing models?

To address our research questions from a portfolio perspective, we extend the parametric portfolio framework in Brandt, Santa-Clara, and Valkanov (2009). Parametric portfolios are obtained by adding to a benchmark portfolio a linear combination of the long-short portfolios associated with each of the firm-specific characteristics considered. To determine which characteristics are jointly significant, we use a screen-and-

[1]See also the survey papers Subrahmanyam (2010), Richardson, Tuna, and Wysocki (2010), and Nagel (2013), and the book Bali, Engle, and Murray (2016).

clean method to test which characteristics have parametric portfolio weights that are significantly different from zero. Finally, we demonstrate analytically and empirically in Appendix A how our methodological approach based on the parametric portfolios relates to the time-series and cross-sectional regression approaches.

Our answers to the three research questions are as follows. First, in the absence of transaction costs, only a small number of characteristics—about six—are significant. Five characteristics—unexpected quarterly earnings, return volatility, asset growth, 1-month momentum, and gross profitability—are significant because they increase the mean returns and also help to reduce the risk of the portfolio of characteristics. A sixth characteristic, beta, is significant *only* because of its ability to reduce the risk of the other characteristics, in particular, the return-volatility characteristic.[2] We also find that traditional characteristics such as 12-month momentum and book to market are not significant because, although they have high mean returns, they do not offer a sufficiently attractive tradeoff between portfolio mean return and risk.

Second, in contrast to what one would find if evaluating characteristics in isolation, we find that the presence of transaction costs *increases* the number of jointly significant characteristics from six to 15. This is because the trades in the underlying stocks required to rebalance different characteristics often cancel each other out and thus, combining characteristics allows one to substantially reduce transaction costs. We show analytically that if one assumes that the trades in a particular stock required to rebalance K different characteristics are independently and identically distributed with zero mean, then the transaction cost required to rebalance an equally weighted portfolio of the K characteristics in combination is $1/\sqrt{K}$ of that required to rebalance them separately. Essentially, combining characteristics allows one to diversify *trading*, just as combining them allows one to diversify risk. Empirically, we find that the marginal transaction cost associated with trading the stocks underlying a characteristic is reduced by around 65% on average when they are combined. As a result, certain characteristics that would

[2]The returns of the beta and return-volatility characteristics are highly correlated over time, but while return volatility has a large (negative) mean return, beta has a negligible mean return. Thus, the investor optimally goes long the beta characteristic to hedge the risk of her short position in the return-volatility characteristic, without compromising its mean return.

require a large amount of trading in the underlying stocks if exploited in isolation, such as the short-term-reversal characteristic (1-month momentum), continue to be significant in the presence of transaction costs because of the trading diversification possible from combining characteristics. The main takeaway from this finding is that transaction costs *increase* the dimension of the cross section of stock returns.

Third, we show that an investor can exploit a large set of characteristics in the presence of transaction costs to achieve an out-of-sample Sharpe ratio that is larger than that obtained by exploiting a small set of characteristics. For instance, we find the investor achieves an out-of-sample Sharpe ratio of returns net of transaction costs around 100% larger than that from exploiting the three traditional characteristics considered in Brandt et al. (2009) and 25% higher than that from exploiting a set of four characteristics that include investment and profitability characteristics such as those highlighted in Fama and French (2015) and Hou, Xue, and Zhang (2014).[3] These out-of-sample results confirm that in the presence of transaction costs the cross section of stock returns is not fully explained by a small set of characteristics.[4]

1.1 Characteristics versus factors

We now discuss the relation between firm-specific characteristics and risk factors in the context of our work. Firm-specific characteristics are variables that can be computed using individual-firm data, e.g., the historical stock-return volatility of a firm. Factors, on the other hand, are variables that proxy for a common source of risk, e.g., the market return. Firm-specific characteristics are related to factors because the return of a long-short portfolio based on a characteristic can be used as a proxy for an underly-

[3]Fama and French (1992) argues that the cross-section of expected stock returns could be explained with only three factors: market, size, and book to market. Following Jegadeesh and Titman (1993) and Fama and French (1996), the academic community largely accepted a so-called Fama and French (1993) and Carhart (1997) four-factor model containing also momentum. More recently, investment and various forms of profitability have emerged as important factors. Novy-Marx (2013) proposes a four-factor model containing market, value, momentum, and *gross* profitability; Hou et al. (2014) proposes a four-factor model containing market, size, investment, and profitability (*return on equity*); and Fama and French (2015) proposes a five-factor model with market, size, book to market, investment, and *operating* profitability.

[4]This out-of-sample analysis also alleviates the data-mining concerns raised in Fama (1991), Kogan and Tian (2013), Harvey et al. (2015), Bryzgalova (2015), McLean and Pontiff (2016), and Linnainmaa and Roberts (2016), and Chordia, Goyal, and Saretto (2017).

ing unknown risk factor. For instance, Fama and French (1993) finds that returns on long-short portfolios based on size and book to market explain the cross-section of stock returns, and thus argues that these characteristics are proxies for common risk factors.

The relation between characteristics and risk factors, however, is not always clear. For instance, Daniel and Titman (1997) challenges the findings in Fama and French (1993) and claims that it is the size and book-to-market characteristics themselves rather than the covariance structure that explains the cross-section of expected stock returns. Pastor and Stambaugh (2000) explains that once model uncertainty and margin constraints are taken into account, the difference between characteristic-based and risk-factor-based models is small from an investment perspective. In addition, Kozak, Nagel, and Santosh (2016) argues that there is no clear distinction between risk-factor pricing and behavioral asset pricing. Therefore, we consider 50 firm-specific characteristics and are agnostic about whether a particular characteristic is a proxy for the loading on a common risk factor or not; instead, we account for risk directly through the mean-variance utility of the investor.[5]

1.2 Relation to literature on asset pricing

The asset pricing literature can be classified by the following three methodologies: cross-sectional regression, time-series regression, and the stochastic discount factor approach. In this section we discuss how our portfolio approach relates to these three approaches.

One of the most popular *cross-sectional approaches* is the Fama and MacBeth (1973) procedure, which runs a cross-sectional regression of stock returns on firm-specific characteristics at each date, and then tests the significance of the risk premia, defined as the average of the regression slopes over time. One advantage of this approach when studying the dimension of the cross section is that it allows one to test which characteristics are *jointly* significant. Indeed, Green, Hand, and Zhang (2017) considers 94 characteristics and finds using Fama-MacBeth regressions that 12 are jointly signifi-

[5]In addition to the 50 firm-specific characteristics, we consider also beta (i.e., the exposure of each stock to the market-return factor) because of its importance for investment management, as shown in Frazzini and Pedersen (2014). Although beta is a risk-factor loading rather than a characteristic, for simplicity of exposition we refer to all 51 variables as characteristics.

cant, Freyberger, Neuhierl, and Weber (2016) considers 36 characteristics and finds using *nonparametric* cross-sectional regressions that 15 provide independent information, and Messmer and Audrino (2017) considers 68 characteristics and finds by applying *adaptive lasso* to cross-sectional regressions that 14 provide independent information.

There are two main differences between these papers and our work. First, while cross-sectional regressions focus on mean returns, our portfolio approach accounts for *both* mean and variance of returns. Second, our portfolio approach accounts for transaction costs. Analytically, we show that even in the absence of transaction costs our approach based on the parametric portfolios produces results that are different from those of Fama-MacBeth regressions *unless* the covariance matrix of asset returns is diagonal. That is, if assets are correlated, a given characteristic could have a zero slope in cross-sectional regressions and yet result in a nonzero parametric portfolio weight. This is the case, for instance, when the correlation of the return of a characteristic with the returns of other characteristics can be exploited by the investor to reduce risk.[6]

Empirically, we find that this is indeed the case. For instance, Fama-MacBeth regressions find that while return volatility is significant, the beta characteristic is not. A closer look reveals that the cross-sectional slopes of return volatility and beta are highly correlated over time. Consequently, our portfolio approach, which takes risk into account, finds that the investor optimally goes short return volatility and goes long beta to reduce risk, and hence, we find that both characteristics are jointly significant. [7]

The *time-series approach* regresses the return of a characteristic-based long-short portfolio on the returns of a few commonly accepted factors, such as the Fama and French (1993) and Carhart (1997) four factors. If the intercept of this time-series regression is statistically significant, then the return on the characteristic is not fully explained by the commonly accepted factors. Gibbons, Ross, and Shanken (1989) shows that testing

[6]The slopes in cross-sectional regressions can be estimated using either ordinary least squares (OLS) or generalized least squares (GLS). Lewellen, Nagel, and Shanken (2010) recommends using GLS because its R^2 captures the mean-variance efficiency of the model's factor-mimicking portfolios. Our analytical results show that both OLS and GLS cross-sectional regressions produce results that are different in general from those of our portfolio approach.

[7]Our out-of-sample analysis is also related to Lewellen (2015), which shows that Fama-MacBeth regressions provide good out-of-sample estimates of stock *expected returns*. Our out-of-sample analysis, however, focuses on estimating directly *portfolio weights*, which incorporate information about expected returns as well as risk and transaction costs.

the significance of the intercept is equivalent to testing whether the characteristic long-short portfolio can improve the Sharpe ratio of a mean-variance investor who already has access to the commonly accepted factors. Consequently, this approach captures the tradeoff between mean return and risk. Recently, Novy-Marx and Velikov (2016) develops a "generalized alpha" that extends the time-series regression to capture the impact of transaction costs.

A disadvantage of the time series approach is that it focuses on the significance of the *intercept*, and therefore, tests the significance of a single characteristic when it is added to a set of commonly accepted factors.[8] This is a limitation because the result of the statistical inference depends on the sequence in which variables are selected. For instance, a time-series regression of the return on the beta characteristic onto the returns of the Fama and French (1993) and Carhart (1997) four factors finds that beta is not significant, but a time-series regression of the beta return onto these four factors *and* the return of the return-volatility characteristic finds that beta is significant. We show analytically that, in the absence of transaction costs, our approach of testing the significance of the characteristics for mean-variance parametric portfolios is *equivalent* to testing the significance of the *slopes* of a particular time-series regression. The advantage of our approach based on slope significance is that it allows one to consider *all* characteristics simultaneously rather than sequentially. This is crucial because both risk and transaction costs depend critically on how characteristics are combined.

There are also papers that combine elements from both cross-sectional and time-series regressions. Back, Kapadia, and Ostdiek (2015) first runs cross-sectional regressions to estimate risk premia and then runs time-series regressions of these risk premia on factors. The advantage of this procedure is that it avoids the errors-in-variables problem. Feng, Giglio, and Xiu (2017) combine the double-selection lasso in Belloni, Chernozhukov, and Hansen (2014) with two-pass regressions to estimate risk prices and evaluate the marginal contribution of a new factor with respect to an existing high-dimensional set

[8]Note that one can also regress the returns of *multiple* assets with respect to the commonly accepted factors. Gibbons, Ross, and Shanken (1989) shows that in this case, testing whether the intercepts of these regressions are jointly equal to zero is equivalent to testing whether the *multiple* assets can improve the Sharpe ratio of an investor who already has access to the commonly accepted factors. The Gibbons, Ross, and Shanken test, however, does not identify *which* of the multiple assets are significant.

of factors. The advantage of this approach is that it explicitly accounts for potential model-selection errors, and thus, avoids the biases associated with omitted variables. Nevertheless, the inference in the two aforementioned approaches depends on the *sequence* in which characteristics are tested, just like in time-series regressions.

Baker, Luo, and Taliaferro (2017) studies the relevance of cross-sectional and time-series regressions for a mean-variance investor. The paper shows that a risk-neutral investor facing quadratic transaction costs cares only about characteristics that are significant in cross-sectional regressions, a mean-variance investor facing no transaction costs cares only about time-series regressions, and a mean-variance investor facing quadratic transaction costs cares about both types of regressions. We sidestep the choice between cross-sectional and time-series regressions by focusing directly on the parametric portfolio problem of a mean-variance investor facing transaction costs.

Finally, the *stochastic discount factor* (SDF) approach is the most closely related to our portfolio approach because one can show that for every mean-variance efficient portfolio there is an SDF that is an affine function of the portfolio return. Ghosh, Julliard, and Taylor (2016a,b) uses a model-free robust approach to estimate the SDF that fits a cross section of asset returns by minimizing its entropy relative to the physical probability measure. Using this approach, Ghosh, Julliard, and Taylor (2016b) identifies a novel source of risk not captured by the Fama and French (1993) and Carhart (1997) factors.

Kozak, Nagel, and Santosh (2017) proposes a robust SDF by imposing an economically-motivated prior on SDF coefficients that can shrink the contributions of both low-variance principal components of characteristics as well as individual characteristics with low risk prices. They find that principal-component-sparse SDFs explain the cross section better than characteristic-sparse SDFs. A distinguishing feature of our work is that we study the impact of *transaction costs* on the dimensionality of the cross section of stock returns. Our main finding is that transaction costs *increase* the number of characteristics that are significant for portfolio construction, and thus, transaction costs provide another economic rationale for non-sparse characteristic-based asset-pricing models.

1.3 Relation to literature on transaction costs

Several papers study the transaction costs associated with trading *particular* characteristics: Korajczyk and Sadka (2004) studies the market-impact costs associated with exploiting momentum and find that this characteristic can be exploited on only a relatively modest scale. Novy-Marx and Velikov (2016) considers 23 different anomalies and finds that strategies to minimize transaction costs significantly reduce the impact of transaction costs on the profitability of anomaly-based trading strategies. Chen and Velikov (2017) considers 135 anomalies and shows that if, in addition to transaction costs, one takes into account the post-publication decay, the profitability of anomaly-based trading strategies is substantially diminished. The aforementioned papers use publicly available datasets to estimate the costs of an average investor. Frazzini, Israel, and Moskowitz (2015), using proprietary data from an institutional money manager, finds that the trading costs associated with exploiting size, momentum, and book to market can be quite small for large institutional investors, and that these managers can exploit these characteristics to a much larger extent than previously thought.

Very few papers consider the transaction costs associated with trading multiple characteristics *jointly*. Hanna and Ready (2005) shows that the long-short stock-selection strategy considered in Haugen and Baker (1996), which is based on a combination of more than 50 characteristics, does not outperform the portfolios based solely on book to market and momentum once transaction costs are taken into account. Hand and Green (2011) considers parametric portfolios with three accounting-based characteristics in addition to size, book to market, and momentum and finds that accounting-based characteristics can improve performance substantially, but transaction costs reduce the benefits from exploiting accounting-based characteristics. We show that by combining characteristics the investor can alleviate the impact of transaction costs significantly because of trading diversification.

Other papers have also found that combining characteristics helps to reduce transaction costs. For instance, Frazzini, Israel, and Moskowitz (2015) considers size, value, and momentum and explains that "value and momentum trades tend to offset each other, resulting in lower turnover which has real transaction costs benefits." Barroso and Santa-

Clara (2015) considers currency portfolios based on six characteristics and explains that "transaction costs depend crucially on the time-varying interaction between characteristics." Novy-Marx and Velikov (2016) studies "filtering", a cost mitigation technique that allows investors trading one strategy to opportunistically take small positions in another at effectively negative trading costs. We build on these three papers and show how to quantify precisely the reduction in transaction costs when an investor *optimally* rebalances a portfolio based on several characteristics.

The rest of this paper is organized as follows. Section 2 describes the data. Section 3 explains how we apply and extend the methodology of parametric portfolios. Our three research questions are addressed in three distinct sections: Section 4 studies how many characteristics matter in the absence of transaction costs, Section 5 examines how transaction costs affect the dimension of the cross section, and Section 6 investigates whether investors can exploit a large set of characteristics to achieve better out-of-sample performance relative to exploiting a small set of characteristics. Section 7 concludes. Appendix A studies analytically and empirically how our portfolio approach relates to the cross-sectional and time-series regression approaches. Appendix B contains proofs for all analytical results in the manuscript and the Internet Appendix contains robustness checks studying how our results depend on: the presence of *quadratic* transaction costs as opposed to *proportional* transaction costs that we consider in the main body of the manuscript, exploiting characteristics only after their publication as in McLean and Pontiff (2016), excluding microcaps, firm size, shortsale constraints, applying the reality check in White (2000), expanding our dataset to also consider characteristics with a large number of missing observations, different subperiods, the constraint on maximum turnover, risk-aversion, and using different methods to standardize firm characteristics.

2 Data

We combine U.S. stock-market information from three databases, CRSP, Compustat, and I/B/E/S, covering the period from January 1980 to December 2014. We start by compiling data on the 100 firm-specific characteristics considered in Green, Hand, and Zhang

(2014),[9] but we drop characteristics with a large proportion of missing observations.[10] Specifically, we first drop characteristics with more than 5% of missing observations for more than 5% of those firms with CRSP returns available for the entire sample from 1980 to 2014. In addition, we drop characteristics without any observations for more than 1% of these firms. The resulting dataset contains 51 characteristics, which include the 24 variables that Green et al. (2014) finds significant in Fama-MacBeth regressions, except *fgr5yr* (forecasted growth in five-year-earnings per share) and *sfe* (scaled analyst forecast of one-year-ahead earnings). Table 1 lists the 51 characteristics together with their definitions, the name of the author(s) who identified it, and the date and journal of publication.

Our initial database contains every firm traded on the NYSE, AMEX, and NASDAQ exchanges. We then remove firms with negative book-to-market ratios. Like Brandt et al. (2009), we also remove firms below the 20th percentile of market capitalization because these are very small firms that are difficult to trade. Our final dataset considers 51 firm-specific characteristics for a total of 17,930 firms of which an average of 3,071 firms have return data every month.

As in Green et al. (2014), we cross-sectionally winsorize each characteristic; that is, we replace extreme observations that are beyond a certain threshold with the value of the threshold. Specifically, we set equal to the third (first) quartile plus (minus) three times the interquartile range any observations that are above (below) this threshold.

Finally, as in Brandt et al. (2009), we standardize each characteristic so that it has a cross-sectional mean of zero and a cross-sectional standard deviation of one. The resulting standardized characteristic is a long-short portfolio that goes long on stocks whose characteristic is above the cross-sectional average, and short on stocks whose characteristic is below the cross-sectional average.

[9] As in Green et al. (2014), when constructing monthly characteristics at time t, we assume that annual (quarterly) accounting data is available at the end of month $t-1$ if the firm's fiscal year ended at least six (four) months earlier.

[10] To ensure that our results are reliable, in our main analysis we consider only characteristics with a small proportion of missing observations. However, in Section IA.7 of the Internet Appendix, we run our experiments using all 100 characteristics and find that our main results are robust.

3 Methodology

To study how many characteristics matter jointly from a portfolio perspective, we adopt and extend the parametric portfolio methodology in Brandt et al. (2009). This section explains parametric portfolios and our extensions. Section 3.1 applies the parametric portfolio framework to the case with mean-variance utility and Section 3.2 shows how to include transaction costs. Section 3.3 shows how the portfolio optimality conditions can be used to identify the marginal contribution of each characteristic to the investor's mean-variance utility. Section 3.4 introduces the *regularized* parametric portfolios, which are designed to deal with a large number of characteristics, and Section 3.5 describes a *screen and clean* method to test whether the parametric portfolio weights corresponding to the different characteristics are significantly different from zero.

3.1 Mean-variance parametric portfolios

Parametric portfolios use a set of firm-specific characteristics to *tilt* the benchmark portfolio toward stocks that help to increase the investor's utility. The portfolios are obtained by adding to the benchmark portfolio a linear combination of long-short portfolios obtained by standardizing K firm-specific characteristics so that they have zero mean and unit standard deviation. The resulting *parametric portfolio* at time t, $w_t(\theta) \in \mathbb{R}^{N_t}$, can be written as

$$w_t(\theta) = w_{b,t} + (x_{1,t}\theta_1 + x_{2,t}\theta_2 + \ldots + x_{K,t}\theta_K)/N_t, \tag{1}$$

where $w_{b,t} \in \mathbb{R}^{N_t}$ is the *benchmark portfolio* at time t, $x_{k,t} \in \mathbb{R}^{N_t}$ is the long-short portfolio obtained by standardizing the kth firm-specific characteristic at time t, θ_k is the weight of the kth characteristic in the parametric portfolio, and N_t is the number of firms at time t.[11] As in Brandt et al. (2009), we consider a portfolio that is fully invested in risky assets.[12] The parametric portfolio can also be written in compact matrix notation by

[11]The weights of the characteristics in the parametric portfolio are scaled by the number of stocks N_t so that they are meaningful for the case with a varying number of stocks. Without this scaling parameter, increasing the number of stocks while keeping the weights fixed would result in more aggressive portfolio allocations.

[12]Consequently, the parametric portfolio weights on the stocks need to sum to one. Because the weights on the stocks in the long-short portfolios sum to zero, this implies that the parametric weight on the benchmark portfolio must equal one.

defining $X_t \in \mathbb{R}^{N_t \times K}$ to be the matrix whose kth column is $x_{k,t}$:

$$w_t(\theta) = w_{b,t} + X_t\theta/N_t, \tag{2}$$

where $\theta \in \mathbb{R}^K$ is the *parameter vector*, whose kth component is the weight of the kth characteristic θ_k, and $X_t\theta/N_t$ is the characteristic portfolio at time t.

The parametric portfolio return at time $t+1$, which we denote as $r_{p,t+1}(\theta)$, can thus be rewritten as

$$\begin{aligned} r_{p,t+1}(\theta) &= w_{b,t}^\top r_{t+1} + \theta^\top X_t^\top r_{t+1}/N_t \\ &= r_{b,t+1} + \theta^\top r_{c,t+1}, \end{aligned} \tag{3}$$

where $r_{t+1} \in \mathbb{R}^{N_t}$ is the return vector at time $t+1$, $r_{b,t+1} = w_{b,t}^\top r_{t+1}$ is the benchmark portfolio return at time $t+1$, and $r_{c,t+1} = X_t^\top r_{t+1}/N_t$ is the *characteristic return vector* at time $t+1$, which contains the returns of the long-short portfolios corresponding to the K characteristics scaled by the number of firms N_t.[13] Equation (3) shows that the parametric-portfolio return is the benchmark-portfolio return plus the return of the characteristic portfolio.

We assume the investor optimizes a mean-variance utility. The advantages of mean-variance utility, as we will show below, are that it allows us to identify the marginal contribution of each characteristic to the investor's utility and to compare analytically the parametric portfolio weights to the results from time-series and cross-sectional regressions.[14] In particular, we assume the investor solves the following problem:

$$\min_{\theta} \quad \frac{\gamma}{2}\mathrm{var}_t[r_{p,t+1}(\theta)] - E_t[r_{p,t+1}(\theta)], \tag{4}$$

where γ is the risk-aversion parameter and $\mathrm{var}_t[r_{p,t+1}(\theta)]$ and $E_t[r_{p,t+1}(\theta)]$ are the variance and mean of the parametric portfolio return, respectively.

Given T historical observations of returns and characteristics, the following proposition shows that the parametric portfolio problem can be formulated as a tractable quadratic optimization problem.

[13] Note that we use only lagged values of characteristics to build portfolios; thus, the returns of the characteristic portfolio formed at time t, $X_t\theta/N_t$ are evaluated using the return at time $t+1$; that is, $\theta^\top X_t^\top r_{t+1}/N_t$.

[14] We have run our empirical analysis also for power utility, as in Brandt et al. (2009), and the main insights are unchanged.

Proposition 3.1 *The mean-variance parametric portfolio problem in* (4) *can be rewritten as*

$$\min_{\theta} \quad \underbrace{(\gamma/2)\theta^\top \widehat{\Sigma}_c \theta}_{var(char)} + \underbrace{\gamma \theta^\top \widehat{\sigma}_{bc}}_{cov(bench)} - \underbrace{\theta^\top \widehat{\mu}_c}_{mean}, \tag{5}$$

where $\widehat{\Sigma}_c$ *and* $\widehat{\mu}_c$ *are the sample covariance matrix and mean of the characteristic-return vector* r_c*, and* $\widehat{\sigma}_{bc}$ *is the sample vector of covariances between the benchmark portfolio return* r_b *and the characteristic-return vector* r_c*.*

Proposition 3.1 shows that the mean-variance parametric portfolio problem is to find the parameter vector θ that offers the optimal tradeoff between the variance of the characteristic portfolio return, $(\gamma/2)\theta^\top \widehat{\Sigma}_c \theta$; the covariance of the characteristic portfolio return with the benchmark portfolio return, $\gamma\theta^\top \widehat{\sigma}_{bc}$; and the mean characteristic portfolio return, $\theta^\top \widehat{\mu}_c$.

3.2 Transaction costs

As in Brandt et al. (2009) and Hand and Green (2011), we consider an investor who faces proportional transaction costs that decrease with firm size and over time. Proportional transaction costs are a reasonable assumption for the average investor; see Novy-Marx and Velikov (2016) and Chen and Velikov (2017). Nevertheless, Section IA.1 of the Internet Appendix shows that our main findings are robust to using quadratic transaction costs that are often used to model the price impact costs of large investors. We define the proportional transaction cost parameter for the ith stock at time t as

$$\kappa_{i,t} = y_t z_{i,t}, \tag{6}$$

where y_t and $z_{i,t}$ capture the variation of the transaction cost parameter with time and firm size, respectively. Following Brandt et al. (2009) and Hand and Green (2011), we assume y_t decreases linearly from 3.3 in January 1980 to 1.0 in January 2002, and after that it remains at 1.0. We set $z_{i,t} = 0.006 - 0.0025 \times me_{i,t}$, where $me_{i,t}$ is the market capitalization of firm i at time t after being normalized cross-sectionally so that it takes values between zero and one.[15]

[15]Brandt et al. (2009) defines y_t so that transaction costs in 1974 are four times larger than in 2002. Therefore, if we decrease y_t uniformly until 1980, we would have a starting value for y_t approximately

Given T historical observations of returns and characteristics, the transaction cost associated with implementing the parametric portfolios can be estimated as

$$\mathrm{TC}(\theta) = \frac{1}{T-1} \sum_{t=1}^{T-1} \|\Lambda_t (w_{t+1}(\theta) - w_t^+(\theta))\|_1, \tag{7}$$

where the transaction cost matrix at time t, Λ_t, is the diagonal matrix whose ith diagonal element contains $\kappa_{i,t}$, $\|a\|_1 = \sum_{i=1}^N |a_i|$ is the 1-norm of the N-dimensional vector a, and w_t^+ is the portfolio before rebalancing at time $t+1$, that is,

$$w_t^+ = (w_{b,t} + X_t \times \theta / N_t) \circ (e_t + r_{t+1}), \tag{8}$$

where e_t is the N_t-dimensional vector of ones and $x \circ y$ is the Hadamard or componentwise product of vectors x and y.

Combining (5) and (7), the mean-variance parametric portfolio problem with transaction costs is

$$\min_\theta \quad \underbrace{(\gamma/2)\theta^\top \widehat{\Sigma}_c \theta}_{var(char)} + \underbrace{\theta^\top \gamma \widehat{\sigma}_{bc}}_{cov(bench)} - \underbrace{\theta^\top \widehat{\mu}_c}_{mean} + \underbrace{\mathrm{TC}(\theta).}_{transaction\ costs} \tag{9}$$

3.3 Understanding why a characteristic matters

To understand why particular characteristics are significant from a portfolio perspective, it is useful to consider the first-order optimality conditions for the mean-variance parametric portfolio problem with transaction costs, that is, the problem in (9).

By decomposing the variance of the characteristic portfolio return, $\theta^\top \widehat{\Sigma}_c \theta$, into a term associated with the characteristic *own-variances*, $\theta^\top \mathrm{diag}(\widehat{\Sigma}_c)\theta$, and a term associated with the characteristic covariances, $\theta^\top (\widehat{\Sigma}_c - \mathrm{diag}(\widehat{\Sigma}_c))\theta$, where $\mathrm{diag}(\widehat{\Sigma}_c)$ is the diagonal matrix whose kth diagonal element contains the variance of the kth characteristic return, the mean-variance parametric portfolio problem with transaction costs can

equal to 3.3. This functional form results in proportional transaction costs of 180 basis points for the smallest firms and 100 basis points for the largest firms in the 1980s, and about 60 basis points for the smallest firms and 35 basis points for the largest firms after 2002. See also French (2008, p. 1553) for a discussion of the time evolution of transaction costs.

be rewritten as

$$\min_{\theta} \quad \underbrace{(\gamma/2)\theta^\top \mathrm{diag}(\widehat{\Sigma}_c)\theta}_{own-var(char)} + \underbrace{(\gamma/2)\theta^\top(\widehat{\Sigma}_c - \mathrm{diag}(\widehat{\Sigma}_c))\theta}_{cov(char)} + \underbrace{\theta^\top \gamma \widehat{\sigma}_{bc}}_{cov(bench)} - \underbrace{\theta^\top \widehat{\mu}_c}_{mean} + \underbrace{\mathrm{TC}(\theta).}_{transaction\ costs} \tag{10}$$

Note that the transaction cost term $\mathrm{TC}(\theta)$ is a convex function of the parameter θ, but it is not differentiable at values of θ for which there exist i and t such that $w_{i,t+1}(\theta) = w^+_{i,t}(\theta)$. Therefore, the optimality conditions must be formally defined in terms of the subdifferential $\partial\mathrm{TC}(\theta)$.

Proposition 3.2 *The first-order optimality conditions for problem* (10) *are*

$$0 \in \underbrace{\gamma diag(\widehat{\Sigma}_c)\theta}_{own-var(char)} + \underbrace{\gamma(\widehat{\Sigma}_c - diag(\widehat{\Sigma}_c))\theta}_{cov(char.)} + \underbrace{\gamma\widehat{\sigma}_{bc}}_{cov(bench.)} - \underbrace{\widehat{\mu}_c}_{mean} + \underbrace{\partial TC(\theta)}_{\text{costs}}, \tag{11}$$

where the ith component of the subdifferential of the transaction cost term is

$$\partial_{\theta_i} TC(\theta) = \frac{1}{T-1}\sum_{t=1}^{T-1} sign(w_{t+1}(\theta) - w^+_t(\theta))^\top(\Lambda_t[(X_{t+1})_{\bullet,i} - (X_t)_{\bullet,i} \circ (e_t + r_{t+1})]), \tag{12}$$

where $A_{\bullet,i}$ is the ith column of matrix A, and

$$sign(w_{j,t+1}(\theta) - w^+_{j,t}(\theta)) = \begin{cases} +1 & if \quad w_{j,t+1}(\theta) > w^+_{j,t}(\theta), \\ -1 & if \quad w_{j,t+1}(\theta) < w^+_{j,t}(\theta), \\ [-1,1] & if \quad w_{j,t+1}(\theta) = w^+_{j,t}(\theta). \end{cases} \tag{13}$$

The first-order optimality conditions in (11) allow us to identify the *marginal* contribution of each characteristic to the investor's mean-variance utility. Specifically, the kth component of the right-hand side in (11) is the marginal contribution of the kth characteristic to the parametric portfolio mean-variance utility; that is, the marginal change to mean-variance utility associated with a unit increase in the weight that the parametric portfolio assigns to the kth characteristic. Moreover, the five terms on the right-hand-side of (11) are: the marginal contributions of the kth characteristic to the characteristic own-variance, $\gamma\mathrm{diag}(\widehat{\Sigma}_c)\theta$; the characteristic covariance with other characteristics, $\gamma(\widehat{\Sigma}_c - \mathrm{diag}(\widehat{\Sigma}_c))\theta$; the covariance between the characteristic and benchmark portfolios, $\gamma\widehat{\sigma}_{bc}$; the characteristic portfolio mean, $-\widehat{\mu}_c$; and the transaction cost, $\partial\,\mathrm{TC}(\theta)$. By evaluating each of these five terms for each of the characteristics, we can identify its contribution to the investor's mean-variance utility.

Finally, to gauge the size of the trading diversification benefit associated with combining characteristics, it will be useful to compute the marginal contribution to transaction costs of trading the ith characteristic in isolation (that is, without the benchmark or any other characteristics), which is

$$\partial_{\theta_i}\mathrm{TC}(\theta) = \frac{1}{T-1}\sum_{t=1}^{T-1} \|\Lambda_t[(X_{t+1})_{\bullet,i} - (X_t)_{\bullet,i} \circ (e_t + r_{t+1})]\|_1, \tag{14}$$

where $(X_t)_{\bullet,i}$ is a vector with the standardized values of characteristic i at time t across all the firms.

3.4 The regularized parametric portfolios

Although the parametric portfolios only require estimating one parameter per characteristic, we consider a large number of characteristics. To deal with such high-dimensional setting, we propose a new class of parametric portfolios, which we term the *regularized parametric portfolios*. These portfolios are obtained by imposing a lasso[16] constraint on the parametric portfolio framework to achieve two goals. First, the lasso constraint helps to avoid overfitting, reducing the impact of estimation error. Second, the lasso constraint is a variable selection method that results in parametric portfolios where only the relevant characteristics receive a nonzero parameter. This allows us to characterize the dimension of the cross section and study how it changes with transaction costs.

In contrast, the minimum-entropy SDF approach used in Ghosh et al. (2016b) results in SDFs that assign a nonzero weight to every characteristic, and thus, it is not suitable to study the dimension of the cross section. The elastic-net SDF approach in Kozak et al. (2017) shrinks the contributions to the SDF of *both* low-variance principal

[16]The term lasso originated as the acronym for *least absolute shrinkage and selection operator*. The lasso was originally proposed in Tibshirani (1996) in the context of statistical learning and has become a prominent tool in the age of machine learning. See Hastie, Tibshirani, and Wainwright (2015) for an in-depth treatment of the lasso, and for a Bayesian interpretation of the lasso constraint in the context of portfolio choice, see DeMiguel, Garlappi, Nogales, and Uppal (2009a).

components of characteristics as well as individual characteristics with low risk prices. Shrinking *only* the contributions of low-risk-price characteristics allows us to characterize how transaction costs affect the number of characteristics that are significant for portfolio construction.

The regularized parametric portfolios are obtained by solving problem (9) subject to the lasso constraint, that is, by solving

$$\min_{\theta} \quad \frac{\gamma}{2}\theta^{\top}\widehat{\Sigma}_c\theta + \theta^{\top}\gamma\widehat{\sigma}_{bc} - \theta^{\top}\widehat{\mu}_c + \mathrm{TC}(\theta), \tag{15}$$

$$\text{s.t.} \quad \|\theta\|_1 \leq \delta, \tag{16}$$

where $\|\theta\|_1 = \sum_{k=1}^{K} |\theta_k|$ is the 1-norm of the parameter vector, and δ is the threshold parameter. To gain intuition about the meaning of the threshold parameter δ, note that for the case with threshold parameter $\delta = \infty$, we recover the standard parametric portfolios, and for the case with $\delta = 0$, we recover the benchmark portfolio; that is, we get $\theta = 0$. Thus as one increases the threshold parameter δ, the regularized parametric portfolios move from the benchmark (value-weighted) portfolio toward the standard parametric portfolio.

3.5 Testing the significance of characteristics considered jointly

We now explain how we test whether the parametric portfolio weights corresponding to the different characteristics are significantly different from zero. Chatterjee and Lahiri (2011) shows that it is challenging to carry out statistical inference in the presence of a lasso constraint, such as the one imposed on the regularized parametric portfolios. To address this issue, we use a two-stage *screen-and-clean* method similar to the methods proposed in Wasserman and Roeder (2009), Meinshausen and Yu (2009), and Meinshausen, Meier, and Buhlmann (2009). In the first stage, we *screen* the characteristics by using the regularized parametric portfolios. Specifically, we use five-fold cross-validation, as explained in Hastie et al. (2015, Section 2.3), to select the lasso threshold δ that optimizes the mean-variance criterion.[17] For the resulting optimal lasso threshold, we compute the

[17] In particular, we divide the sample of monthly observations into five intervals of equal length. For j from 1 to 5, we remove the jth-interval from the sample and use the remaining sample returns to

regularized parametric portfolios and "screen" or remove any characteristics with a zero parameter.

In the second stage, we *clean* the characteristics that were not removed in the first stage. That is, we compute the parametric portfolios using the characteristics that were not removed in the first stage, but now without a lasso constraint, thus circumventing the concerns highlighted in Chatterjee and Lahiri (2011), and apply a bootstrap method to establish which of these characteristics have parametric portfolio weights that are significantly different from zero.[18] Specifically, we apply the percentile-interval method described in Efron and Tibshirani (1993, Section 13.3) and Hastie et al. (2015, Section 6.2) to establish the significance of the selected characteristics.[19]

Other approaches have been considered in the literature to identify characteristics that are jointly relevant. For instance, Freyberger et al. (2016) and Messmer and Audrino (2017) use a refinement of the lasso approach known as *adaptive lasso* to select characteristics in the context of cross-sectional regressions. The adaptive lasso is complementary to our approach as it could be used as the variable selection method for the screen stage of our screen-and-clean approach. Like the approaches above, our screen-and-clean method considers all characteristics simultaneously. Alternatively, one might think of using a *sequential* bootstrap method to test the significance of adding one more characteristic to an existing parametric portfolio. This approach would be similar

compute the regularized parametric portfolio for several values of δ. We then evaluate the return of the resulting portfolios on the jth-interval. After completing this process for each of the five intervals, we have out-of-sample portfolio returns for the entire sample for each value of δ. Finally, we compute the mean-variance utility of these out-of-sample returns and select the value of δ that corresponds to the portfolio with the largest mean-variance utility.

[18]Barroso and Santa-Clara (2015) uses a one-stage bootstrap method essentially equivalent to our "clean" stage to test the statistical significance of the different characteristics in a *currency* parametric portfolio. This method is appropriate in the context of that paper because it considers only *five* characteristics and thus does not require a variable selection methodology like lasso.

[19]In detail, we first generate $1,000$ bootstrap samples from the original dataset using sampling with replacement. Second, we estimate the optimal parametric portfolio for the remaining characteristics and for each bootstrap sample. Finally, we declare as significant at the 5% level those characteristics whose estimated parameter is strictly positive (strictly negative) for at least 95% of the bootstrap samples, and compute the p-value as the proportion of bootstrap samples for which the parameter is less than or equal to zero (greater than or equal to zero). Note that the parametric portfolio approach relies on the assumption that, conditional on firm-specific characteristics, stock returns are independently and identically distributed (iid). Therefore, we employ an *iid* bootstrap method. Nevertheless, to gauge the importance of the iid assumption, we have repeated the tests using the stationary bootstrap in Politis and Romano (1994), which takes serial dependence into account, and we have found that the results are robust. In particular, we have run the (nonstudentized) stationary bootstrap with expected block sizes of two and six months, and we have found that this does not affect the significance results.

in spirit to the methodology proposed in Harvey and Liu (2015) in the context of sequential factor selection. From a portfolio perspective, however, a sequential significance test would not capture the risk and trading-diversification benefits from adding *several* characteristics simultaneously. This is crucial because both risk and transaction costs depend critically on how characteristics are combined.

Finally, in results not reported to conserve space, we find that our main finding that transaction costs increase the number of significant characteristics is robust to the choice of significance test. The reason for this is that our main insight is obtained by *comparing* the number of significant characteristics for the cases with and without transaction costs. We find that, independently of the test or data sample used, trading diversification results in an increased number of significant characteristics for the case with transaction costs.

4 How many characteristics matter?

We now study how many characteristics matter jointly from a portfolio perspective. This section considers the case without transaction costs, and Section 5 studies the effect of transaction costs.

4.1 How many characteristics are jointly significant and why?

We apply the screen-and-clean method described in Section 3.5 to test the significance of the characteristics when they are considered *jointly* in the absence of transaction costs. We consider a risk-aversion parameter $\gamma = 5$, we use the value-weighted portfolio as the benchmark, and we run the bootstrap test on the 319 monthly observations from May 1988 to December 2014.[20]

The results from the "screen" stage, not reported to conserve space, establish that the optimal lasso threshold for the case without transaction costs is $\delta = 25$, and only 10 characteristics survive the screening. We then run the "clean" stage test for these

[20]Although our dataset covers the period from January 1980 to December 2014, we drop the first 100 months so that the significance test is run on the exact same sample as the out-of-sample analysis in Section 6. Also, in Section IA.10 of the Internet Appendix, we consider other values of risk-aversion: $\gamma = 2$ and 10.

10 characteristics plus the three characteristics considered in Brandt et al. (2009): size, book to market, and momentum. Table 2 reports the significance and marginal contributions of each characteristic in the parametric portfolios. For each characteristic, the first column gives the acronym, the second the optimal value of the parameter together with its significance level, and the last four columns give the marginal contribution of the characteristic to: (i) the characteristic own-variance, (ii) the covariance of the characteristic with the rest of the characteristics in the portfolio, (iii) the covariance of the characteristic with the benchmark portfolio, and (iv) the characteristic mean. Marginal contributions that drive the characteristic to be nonzero are in blue sans serif font, and marginal contributions that drive the characteristic toward zero are in red *italic* font.[21]

We observe from Table 2 that, in the absence of transaction costs, six characteristics are significant. Two characteristics are significant at the 1% confidence level: unexpected quarterly earnings (*sue*) and return volatility (*retvol*); three characteristics at the 5% level: asset growth (*agr*), 1-month momentum (*mom1m*), and gross profitability (*gma*); and one characteristic, beta, is significant at the 10% level. From a return-prediction perspective, Hou et al. (2014) and Fama and French (2015) show that four and five variables, respectively, are enough to predict expected returns. Our result confirms that, in the absence of transaction costs, a small number of characteristics are sufficient also from a portfolio perspective.

Moreover, in line with Hou et al. (2014) and Fama and French (2015), we also find that an investment characteristic (asset growth) and a profitability characteristic (the gross profitability in Novy-Marx (2013)) are significant at the expense of the value characteristics book to market (*bm*) and industry-adjusted book to market (*bm_ia*), which are not significant.[22] In addition, consistent with recent findings in Ang, Hodrick, Xing, and Zhang (2006, 2009) on the low-volatility characteristics, we find that return volatility

[21] Note that for characteristics with a positive parametric portfolio weight, negative (positive) marginal contributions help to decrease (increase) the objective function in the minimization problem (9) and thus increase (decrease) the investor's mean-variance utility. Therefore for characteristics with positive parametric portfolio weights, negative (positive) marginal contributions are in blue sans serif font (red *italic* font). The opposite color and font convention applies to characteristics with negative parametric portfolio weights.

[22] See Novy-Marx (2013) for a comprehensive analysis of the relation between gross profitability and value.

is significant. Consistent with the findings in Novy-Marx (2015), we find that unexpected quarterly earnings is significant at the expense of 12-month momentum (*mom12m*), which is not significant. Finally, we find that a short-term reversal characteristic, 1-month momentum, is significant in the absence of transaction costs, which is consistent with the results in Lo and MacKinlay (1990) in the context of contrarian strategies.

The marginal contributions in Table 2 show that the three most significant characteristics—unexpected quarterly earnings (*sue*), return volatility (*retvol*), and asset growth (*agr*)—matter from a portfolio perspective because they increase mean returns and reduce the risk of both the benchmark portfolio and the portfolio of characteristics. For instance, return volatility has the largest mean return (marginal contribution 0.00323), negative return covariance with the other characteristics (marginal contribution 0.02914), and negative return covariance with the benchmark (marginal contribution 0.00292).[23] The next two most significant characteristics (1-month momentum (*mom1m*) and gross profitability (*gma*)) are significant because they increase mean return and reduce the risk of the portfolio of characteristics, although they increase the risk of the benchmark portfolio because their returns covary positively with the benchmark portfolio return.

The aforementioned five characteristics are significant because they help to reduce the risk of the portfolio of characteristics and increase its mean return. The beta characteristic is significant at the 10% level *only* because of its ability to reduce the risk of the portfolio of characteristics. To see this, note that Table 2 shows that, consistent with the findings in the existing literature (see Black (1993) and the references therein), the marginal contribution of *beta* to mean return is very small. However, the beta return has a large negative covariance with the returns of the other characteristics (marginal contribution −0.01381), and this is what makes it relevant from a portfolio perspective. This is illustrated in Figure 1, which depicts the marginal contributions of the six significant

[23]The marginal contributions to covariance with the benchmark and other characteristics and to mean return are counteracted at the optimal parameter $\theta_{retvol} = -10.85$ by its own-variance (marginal contribution -0.03529). Note that the marginal contribution to own-variance grows linearly with the characteristic parameter, and thus, it tends to dominate for characteristics with a large optimal parameter θ_k.

characteristics, and shows that *beta* has a large marginal contribution to the covariance with the other characteristics that helps to reduce the overall portfolio risk.[24]

Table 2 also explains why size, book to market, and momentum are *not* significant when evaluated from a portfolio perspective. For instance, 12-month momentum (*mom12m*) is not significant, even though its expected return is large (marginal contribution -0.00275), because its return has a very large positive covariance with the returns of the other characteristics in the portfolio. That is, 12-month momentum does not offer a good tradeoff between mean return and portfolio risk diversification. Likewise, book to market (*bm*) is not significant, even though it offers a substantial mean return (marginal contribution -0.00205), because its return covaries positively with the returns of the other characteristics (marginal contribution 0.00023).[25] Unlike *mom12m* and *bm*, market capitalization (*mve*) offers an insignificant mean return, and although it helps to diversify the characteristic portfolio, the magnitude of this diversification benefit is not sufficiently large to make it significant, consistent with the findings in the existing literature; see, for example, the discussion in Asness, Frazzini, Israel, Moskowitz, and Pedersen (2015).

4.2 How are the characteristics correlated?

As discussed above, the contribution of characteristics to portfolio risk plays an important role. Thus, the correlations between the characteristic returns matter from a portfolio perspective. To further understand the correlation structure of the most significant characteristics, Table 3 reports the correlation matrix for the returns of the six significant characteristics and the three characteristics considered in Brandt et al. (2009): size, book to market, and momentum. We first observe that the returns of the size, book to market, and momentum characteristics are not very highly correlated, with correlation coefficients smaller than 20%. Intuitively, this is what one may expect from the returns of a small set of factors that explain the cross section of expected stock returns. On the other hand,

[24]Note that the marginal contribution of beta to the portfolio mean is difficult to see in the figure because it is close to zero.

[25]Industry adjusted book to market covaries negatively with the other characteristics, but its mean return is substantially smaller than that of book to market and it covaries positively with the benchmark, and as a result, it is not significant either.

the returns of the six significant characteristics we identify are more highly correlated. In particular, we observe from Table 3 that the beta return is highly correlated with the return of both return volatility (correlation of 93%) and gross profitability (54%). In addition, the returns of asset growth and gross profitability are also highly correlated (56%).

To understand why these characteristics with highly correlated returns are jointly significant for portfolio choice, consider the case of return volatility and beta. The returns of these two characteristics are highly positively correlated, but the mean return of beta is very small. As a consequence, the investor optimally goes long the beta characteristic to hedge the risk of her short position in the return-volatility characteristic, while preserving most of its mean return. The benefit of this strategy is illustrated in Panel (a) of Figure 2, which shows the cumulative returns from going long the beta characteristic and shorting the return-volatility characteristic. The strong correlation between the monthly returns of the beta and return-volatility characteristics is evident from the figure. Moreover, the cumulative return of shorting return volatility increases over time, while the cumulative return of being long beta is flat. Panel (a) also shows the cumulative return of a blended strategy that assigns a -50% weight to return volatility and a 50% weight to beta. This blended strategy has increasing cumulative returns and very low volatility.

Asness, Moskowitz, and Pedersen (2013) finds that the returns of value and momentum are negatively correlated and a blended strategy of these two characteristics performs well. We compare the return volatility and beta blended strategy with the value and momentum blended strategy. Panel (b) in Figure 2 shows the cumulative return of these two blended strategies, where we have scaled them so that they have the same volatility. We find that the return-volatility and beta blend attains a cumulative return of 110%, whereas the value and momentum blend attains a cumulative return that is slightly less than 80%.

Our finding that, despite the high correlation between the return volatility and beta characteristics, the return-volatility characteristic commands a much higher average return than beta is consistent with results in the existing literature. As explained in Bali et al. (2016), return volatility and idiosyncratic volatility are very similar in the cross section.[26] Therefore, the high average return of the return-volatility characteristic can be traced back to the high average return of the idiosyncratic-volatility characteristic, which is documented in Ang et al. (2006). Moreover, Bali et al. (2016, Table 15.7) shows that the idiosyncratic risk characteristic commands a high average return mostly when computed from daily data over short horizons, which is how return volatility is computed in our analysis. Beta, on the other hand, is computed from weekly returns over the past three years, and thus delivers much lower average returns; for a detailed analysis of the relation between beta and idiosyncratic volatility, see Liu, Stambaugh, and Yuan (2016).

Finally, Appendix A compares analytically and empirically our methodological approach based on the parametric portfolios with the cross-sectional (Section A.1) and time-series (Section A.2) regression approaches.

5 What is the effect of transaction costs?

In this section, we examine how transaction costs affect the dimension of the cross section of stock returns. As explained in Section 3.2, we consider an investor who faces proportional transaction costs that decrease with firm size and over time, as specified in Brandt et al. (2009) and Hand and Green (2011). Proportional transaction costs are a reasonable assumption for the average investor; see Novy-Marx and Velikov (2016) and Chen and Velikov (2017). However, for large investors a common assumption is that their price impact is linear on the amount traded, and thus, they face quadratic transaction costs; see, for instance, Korajczyk and Sadka (2004). In Section IA.1 of the Internet Appendix, we show that our main findings are robust to the presence of quadratic transaction costs.

Intuitively, one may expect that in the presence of transaction costs *fewer* characteristics would be significant. Indeed, we find that this is the case if one were to consider each characteristic individually: 21 characteristics are *individually* significant in the absence of transaction costs, but only 14 in the presence transaction costs.[27] However, when

[26](Bali et al., 2016, p. 365) states that "idiosyncratic volatility and total volatility are very similar in the cross section. While total volatility is a function of idiosyncratic volatility and systematic risk (captured by beta in the CAPM model), it is important for a researcher to recognize that these variables are highly similar empirically."

[27]In results that are not reported to conserve space, we study the significance of the 51 single-characteristic portfolios that are obtained by solving the problem defined in (9) for the case where only

considered jointly, we find that the number of characteristics that are *jointly* significant at the 5% level *increases* from five in the absence of transaction costs to 15 in the presence of transaction costs. The reason for this is that the additional characteristics help to reduce the amount of trading required to rebalance the portfolio of stocks underlying the characteristics. The main takeaway is that transaction costs *increase* the dimension of the cross-section of stock returns.

5.1 How many characteristics are jointly significant and why?

Table 4 gives the significance and marginal contributions of the characteristics for the parametric portfolios in the presence of transaction costs. For each characteristic, the first column gives the acronym, the second the optimal value of the parameter and its significance level, and the next five columns the marginal contribution of the characteristic to: (i) the characteristic own-variance, (ii) the covariance of the characteristic with the rest of the characteristics in the portfolio, (iii) the covariance of the characteristic with the benchmark portfolio, (iv) the characteristic mean, and (v) the transaction cost. The last column reports the marginal contribution of the characteristic to transaction costs when traded in *isolation*, that is, independently from the benchmark portfolio and the other characteristics. Contributions that drive the characteristic to be nonzero are in blue sans serif font, and contributions that drive the characteristic toward zero are in red *italic* font (cf. Footnote 21).

The explanation for the result that the number of significant characteristics is *larger* in the presence of transaction costs can be found by comparing the marginal contribution to transaction cost of the characteristics when traded jointly (column seven in Table 4) and in isolation (column eight in Table 4). The transaction costs associated with trading combinations of characteristics are *much* smaller than those associated with trading characteristics in isolation. We find that the marginal transaction cost associated

one characteristic is available. Because we are considering a single characteristic at a time, we do not need to use the first step of the screen-and-clean test, and instead we just run the bootstrap significance test on each of the 51 single-characteristic parametric portfolios. Finally, note that here we consider 51 *individual* significance tests and thus, following the suggestion in Harvey et al. (2015), we apply Bonferroni's adjustment. In particular, we require that p-values should be no greater than $\alpha/51$ for individual characteristics to be significant at the α level.

with trading the 15 significant characteristics is reduced by around 65% on average when they are combined.

A stark example of the trading diversification benefits from combining characteristics is the short-term reversal characteristic (*mom1m* in the 14th row of Table 4), which has an enormous marginal contribution to transaction costs if traded in isolation (marginal contribution 0.00857), but a four times smaller marginal contribution to transaction cost when traded in combination (marginal contribution 0.00211). This is illustrated in Figure 3, which graphs the marginal contributions to transaction costs of the 15 significant characteristics for the case when the characteristics are traded jointly and in isolation. The figure highlights the dramatic reduction to the marginal contribution to transaction costs of 1-month momentum when traded in combination with the other characteristics. As a result, the short-term reversal characteristic is significant even in the presence of transaction costs. This result contrasts sharply with DeMiguel, Nogales, and Uppal (2014) and Novy-Marx and Velikov (2016) that find that the short-term reversal characteristic is not profitable after transaction costs *when traded in isolation.*[28]

The following proposition characterizes the reduction in transaction costs obtained by combining characteristics.

Proposition 5.1 *Assume that the trades in the ith stock required to rebalance K different characteristics, that is, the quantities*

$$trade_{i,k} = (X_{t+1})_{i,k} - (X_t)_{i,k}(1 + r_{i,t+1}), \qquad k = 1, 2, \ldots, K \tag{17}$$

are independently and identically distributed as a Normal distribution with zero mean and standard deviation σ. Then, the average transaction cost of the trade in the ith stock required to rebalance an equally weighted portfolio of the K characteristics is $1/\sqrt{K}$ of that required to rebalance the kth characteristic in isolation.

The intuition behind this proposition is that, just as we get diversification of risk when we combine stocks, we get diversification in trading when we combine characteristics. To

[28]DeMiguel et al. (2014) finds that a short-term reversal (contrarian) strategy is not profitable in the presence of even modest proportional transaction costs of 10 basis points. Novy-Marx and Velikov (2016) finds that the short-term reversal strategy does not improve the investment opportunity set of an investor with access to the Fama-French factors, even when a buy-and-hold transaction-cost-mitigation strategy is employed.

see this, note that rebalancing the long-short portfolio associated with *each* characteristic requires trading in the *same* underlying stocks. Thus, exploiting multiple characteristics allows one to cancel out some of the trades in the underlying stocks required to rebalance the characteristic long-short portfolios. For instance, if to rebalance a particular characteristic long-short portfolio one needs to buy a particular stock, whereas to rebalance another characteristic one needs to sell the same stock, then the net amount of trading required to exploit these two characteristics in combination is smaller than that required to exploit them in isolation.

Proposition 5.1 relies on the assumption that the trades required to rebalance different characteristics are independently distributed. This is not a particularly restrictive assumption because the average correlation among the trades required to rebalance the 15 significant characteristics in our dataset is very small at 5.42%. Nevertheless, the following proposition extends the result in Proposition 5.1 to the general case.

Proposition 5.2 *Assume that* $trade_{i,k}$ *for* $k = 1, 2, \ldots, K$ *are jointly distributed as a multivariate Normal distribution with zero mean and covariance matrix* Ω. *Then,*

1. *The average transaction cost of the trade in the* i*th stock required to rebalance an equally weighted portfolio of the* K *characteristics is* $\sqrt{e^\top \Omega e}/(K\sqrt{\Omega_{kk}})$ *of that required to rebalance the* k*th characteristic in isolation, where* $e \in \mathbb{R}^K$ *is the vector of ones and* Ω_{kk} *is the variance of* $trade_{i,k}$.

2. *If, in addition, the covariance matrix* Ω *is symmetric with respect to the* K *characteristics; that is, if the variances of the trades in the* i*th stock required to rebalance the* K *different characteristics are all equal to* σ^2, *and the correlations between the trades in the* i*th stock required to rebalance the* K *different characteristics are all equal to* ρ, *then the average transaction cost of the trade in the* i*th stock required to rebalance an equally weighted portfolio of the* K *characteristics is*

$$\sqrt{1 + \rho(K-1)}/\sqrt{K} \tag{18}$$

of that required to rebalance the k*th characteristic in isolation.*

Expression (18) demonstrates that provided the rebalancing trades of different characteristics are not perfectly correlated, combining characteristics will result in trading diversification and a reduction in transaction costs.

Finally, Section A.3 in Appendix A compares empirically the results from using our parametric portfolio approach from those from using the generalized alpha approach in Novy-Marx and Velikov (2016).

6 Out-of-sample analysis

The previous sections studied the significance of the different characteristics for portfolio choice *in-sample*; that is, for our full sample of observations. In this section, we study whether an investor can improve *out-of-sample* performance net of transaction costs by exploiting a large set of characteristics instead of a small number of characteristics. To answer this question, we use the regularized parametric portfolios described in Section 3.4.

This section is organized as follows. Section 6.1 describes the methodology that we use to evaluate out-of-sample performance, Section 6.2 reports the performance of the different portfolios, and Section 6.3 studies how the out-of-sample returns of the regularized parametric portfolios load on three prominent factor models.

6.1 Methodology for out-of-sample evaluation

We compare the out-of-sample performance of the regularized parametric portfolios to that of two parametric portfolios that exploit sparse sets of characteristics. To evaluate the out-of-sample performance of the different portfolios we use a "rolling-horizon" procedure similar to that used in DeMiguel, Garlappi, and Uppal (2009b). First, we choose a window over which to perform the estimation. The total number of monthly observations in the dataset is $T_{tot} = 419$. We choose an estimation window of $T = 100$ monthly observations. Second, using the return data over the estimation window, we compute the various portfolios we study. Third, we repeat this "rolling-window" procedure for the next month, by including the data for the next month and dropping the data for the earliest month. We continue doing this until the end of the dataset is reached. At the end of this process, we have generated $T_{tot} - T = 319$ portfolio-weight vectors for each strategy; that is, w_t^j for $t = T, \ldots, T_{tot} - 1$ and for each strategy j. Holding the portfolio w_t^j for one month gives the *out-of-sample* return net of transaction costs at time $t+1$:

$$r_{t+1}^j = (w_t^j)^\top r_{t+1} - \|\Lambda_t(w_t^j - (w_{t-1}^j)^+)\|_1,$$

where Λ_t is the transaction cost matrix at time t defined in Section 3.2, and $(w_{t-1}^j)^+$ is the portfolio for the jth strategy before rebalancing at time t; that is

$$(w_{t-1}^j)^+ = w_{t-1}^j \circ (e_{t-1} + r_t),$$

where e_{t-1} is the N_{t-1} dimensional vector of ones, and $x \circ y$ is the Hadamard or componentwise product of vectors x and y. Then, for each portfolio we study, we compute the monthly turnover, and the out-of-sample annualized mean, standard deviation, and Sharpe ratio of returns net of transaction costs:

$$\begin{aligned}
\text{turnover} &= \frac{1}{T_{tot} - T} \sum_{t=T}^{T_{tot}-1} \|w_t^j - (w_{t-1}^j)^+\|_1, \\
\hat{\mu}^j &= \frac{12}{T_{tot} - T} \sum_{t=T}^{T_{tot}-1} (w_t^j)^\top r_{t+1}, \\
(\hat{\sigma}^j)^2 &= \frac{12}{T_{tot} - T} \sum_{t=T}^{T_{tot}-1} \left((w_t^j)^\top r_{t+1} - \hat{\mu}^j\right)^2, \qquad \text{and} \\
\widehat{\text{SR}}^j &= \frac{\hat{\mu}_j}{\hat{\sigma}_j}.
\end{aligned}$$

To test the statistical significance of the difference between the Sharpe ratio of the regularized parametric portfolio and those of the other benchmark and parametric portfolios we consider, we use the iid bootstrap method in Ledoit and Wolf (2008), with 10,000 bootstrap samples to construct a one-sided confidence interval for the difference between Sharpe ratios. We use three/two/one asterisks (*) to indicate that the difference is significant at the 0.01/0.05/0.10 level.[29]

[29]Note that to reduce computation time, we compute the optimal parameter vector θ only in January of each year, and use this parameter vector to compute the parametric portfolios for every month of the

6.2 Out-of-sample performance

Table 5 reports the out-of-sample performance of the different portfolios in the presence of transaction costs and risk-aversion parameter $\gamma = 5$. Panel A reports the performance for the portfolios that do not use any characteristics, which are the benchmark value-weighted portfolio (VW) and the equally weighted portfolio ($1/N$). Panel B reports the performance of two parametric portfolios that exploit a small number of characteristics and two parametric portfolios that exploit large sets of characteristics. The first parametric portfolio exploits the *three* characteristics considered in Brandt et al. (2009): size, book to market, and momentum. The second parametric portfolio exploits *four* characteristics: size, book to market, asset growth, and gross profitability, which include investment and profitability characteristics such as those highlighted in Fama and French (2015) and Hou et al. (2014). The first parametric portfolio that exploits a large set of characteristics is based on the 15 characteristics that are significant in the presence of transaction costs as reported in Table 4; note that this portfolio benefits from look-ahead bias because these 15 characteristics were identified using the entire sample period. Finally, the regularized parametric portfolio that exploits a large set of 51 characteristics, where the lasso threshold is calibrated each year using five-fold cross-validation to maximize mean-variance utility. The regularized parametric portfolios only use past data. Therefore, this approach does not have the advantage of look-ahead bias.

We observe from Table 5 that the parametric portfolios based on size, book to market, and momentum outperform the benchmark value-weighted and equally weighted portfolios. The parametric portfolios based on size, book to market, asset growth, and gross profitability outperform the parametric portfolios based just on size, book to market, and momentum. The parametric portfolios based on the 15 significant characteristics over the entire sample period perform even better than those based on size, book to mar-

year. We use the cross-validation methodology explained in Section 3.5 to calibrate the lasso threshold, but using only the 100 observations in each estimation window. Also, we find that the regularized parametric portfolios that solve problem (15)–(16) result in very large turnovers. Although we find that these portfolios are profitable even after transaction costs (see Section IA.9 of the Internet Appendix), they may not be implementable for institutional investors facing turnover constraints. Therefore, we report the results for the parametric portfolios after scaling them to control for turnover. Specifically, we scale the optimal parameter vector θ so that the portfolio monthly turnover is around 100%. Section IA.9 of the Internet Appendix reports the results in the absence of turnover controls.

ket, asset growth, and gross profitability, but, as mentioned above, these parametric portfolios benefit from look-ahead bias. The regularized parametric portfolios achieve a similar Sharpe ratio, but *without* the benefit of look-ahead bias.[30]

The gains from exploiting a large set of characteristics are significant: the regularized parametric portfolios achieve an out-of-sample Sharpe ratio that is 100% higher than that of the parametric portfolios based on size, book to market, and momentum and 25% higher than that of the parametric portfolios based on size, book to market, asset growth, and gross profitability, with the differences being statistically significant. The magnitude of the economic gains is evident also from Figure 4, which depicts the out-of-sample cumulative returns of the value-weighted portfolio and the four parametric portfolios we consider, after scaling them so that they all have the same volatility. These out-of-sample results confirm that in the presence of transaction costs the cross section of stock returns is not fully explained by a small number of characteristics.

6.3 Can factor models explain regularized portfolio returns?

The previous section demonstrates that the regularized parametric portfolios significantly outperform the parametric portfolios that exploit size, book to market, and momentum, and the parametric portfolios that exploit size, book to market, asset growth, and gross profitability. To check the robustness of this result, we run a time-series regression of the out-of-sample returns of the regularized parametric portfolio onto three sparse factor models from the literature: the Fama and French (1993) and Carhart (1997) four-factor model (FFC), the Fama and French (2015) five-factor model (FF5), and the Hou et al. (2014) four-factor model (HXZ). All factors are obtained from Kenneth French's and Lu Zhang's websites.

Table 6 shows that none of these three sparse factor models fully explains the returns of the regularized parametric portfolios, which achieve an economically and sta-

[30] Note that although the parametric portfolios that exploit the 15 significant characteristics benefit from look-ahead bias because the 15 characteristics are selected using the entire dataset, for the out-of-sample experiment we estimate the optimal *weights* for these 15 characteristics using only past data. Therefore, these portfolios suffer from estimation error, which explains why it is possible for the regularized parametric portfolios to have similar performance despite not benefiting from look-ahead bias.

tistically significant abnormal average monthly return of about $\alpha = 1\%$ for each of the three models.[31]

7 Conclusion

A multitude of variables have been proposed in the literature to predict the cross-section of expected stock returns. The existing literature takes a return-prediction perspective to understand which variables provide independent information about average returns. In contrast, we take a *portfolio perspective* that takes into account not only average returns but also risk, transaction costs, and out-of-sample performance.

In response to the question posed by Cochrane, which we highlighted at the start of the manuscript, we find that in the absence of transaction costs, out of the 51 characteristics we consider, only a small number—about six—are jointly significant. In the presence of transaction costs, the number of significant characteristics *increases* from six to 15 because combining characteristics helps to reduce transaction costs in trading the stocks underlying the characteristics. Kozak et al. (2017) argue that "the empirical asset-pricing literatures multi-decade quest for a sparse characteristics-based factor model [...] is ultimately futile". We find that transaction costs increase the number of characteristics that are significant for portfolio construction. Thus, our results provide another rationale for non-sparse characteristic-based factor models.

[31]The table also shows that the regularized parametric portfolio returns load significantly on the market, value (HML), and momentum (UMD) factors for the FFC model, on the market, value, and investment (CMA) factors for the FF5 model, and on the market, investment (I/A), and profitability (ROE) factors for the HXZ model.

A Relation to regression approaches

A.1 Relation to Fama-MacBeth regressions

In this section, we study analytically and empirically the relation between our approach and the Fama-MacBeth regressions in the absence of transaction costs. The Fama-MacBeth procedure can be described as running cross-sectional regressions of stock returns, r_t, onto firm-specific characteristics at each date t:

$$r_t \; = \; X_{t-1}\lambda_t + \epsilon_t, \tag{A.1}$$

where $X_{t-1} \in \mathbb{R}^{N_{t-1}\times K}$ is the matrix of firm-specific characteristics at time $t-1$,[32] $\lambda_t \in \mathbb{R}^K$ is the vector of slopes at time t, and $\epsilon_t \in \mathbb{R}^{N_{t-1}}$ is the vector of pricing errors at time t. The Fama-MacBeth approach then tests the significance of the average of the slopes over time, $\overline{\lambda}$.

Most of the existing literature estimates the Fama-MacBeth cross-sectional regressions using ordinary least squares (OLS). Lewellen et al. (2010), however, recommends using generalized least squares (GLS) cross-sectional regressions because their goodness-of-fit metric has a clear economic interpretation. In particular, they extend a result in Kandel and Stambaugh (1995) to show that the GLS R^2 measures the mean-variance efficiency of the model's factor-mimicking portfolios.[33] The following proposition clarifies the relation between our portfolio approach and the Fama-MacBeth OLS and GLS regressions.

Proposition A.1 *Assume that the standardized firm characteristics are constant through time so that $X_t = X$. Then, the OLS and GLS Fama-MacBeth average slopes are*

$$\overline{\lambda}_{OLS} = (X^\top X)^{-1}X^\top\widehat{\mu}_r, \qquad \textit{and} \tag{A.2}$$

$$\overline{\lambda}_{GLS} = (X^\top\widehat{\Sigma}_r^{-1}X)^{-1}X^\top\widehat{\Sigma}_r^{-1}\widehat{\mu}_r, \tag{A.3}$$

[32]For the sake of simplicity and without loss of generality, we can assume that X_{t-1} is divided by the number of firms at time $t-1$, as we do for parametric portfolios.

[33]Lewellen et al. (2010) studies two-pass cross-sectional regressions, rather than Fama-MacBeth regressions; see (Cochrane, 2009, Sections 12.2 and 12.3). For our theoretical analysis, we make the simplifying assumption that the characteristics are time invariant, and in this case the cross-sectional regressions coincide with the Fama-MacBeth regressions. In addition, we use firm-specific characteristic data, rather than factor data, and thus all of our analysis is based on a single pass regression of stock returns onto characteristics.

where $\widehat{\mu}_r \in \mathbb{R}^N$ is the sample mean of stock returns and $\widehat{\Sigma}_r \in \mathbb{R}^{N\times N}$ is the sample covariance matrix of stock returns. Assume also that the sample vector of covariances between the benchmark portfolio return and the characteristic portfolio return vector is zero ($\sigma_{bc} = 0$). Then the optimal mean-variance parametric portfolio is

$$\theta^* = \frac{1}{\gamma}(X^\top \widehat{\Sigma}_r X)^{-1} X^\top \widehat{\mu}_r. \tag{A.4}$$

Proposition A.1 shows that the OLS and GLS Fama-MacBeth slopes differ in general from the mean-variance parametric portfolio weights; that is, testing the significance of Fama-MacBeth slopes is different from testing the significance of the weights a mean-variance investor assigns to each characteristic. Note, in particular, that the OLS and GLS Fama-MacBeth slopes are different in general from the mean-variance parametric portfolio weights *unless* the sample covariance matrix of asset returns is equal to the identity matrix ($\Sigma_r = I$).

The following corollary provides further insight into the difference between the parametric portfolio weights and the OLS Fama-MacBeth slopes.

Corollary A.2 *Let the assumptions in Proposition A.1 hold, and assume in addition that the columns of the firm-specific characteristic matrix X are orthonormal; that is, $X^\top X = I$. Then, the optimal mean-variance parametric portfolio is*

$$\theta^* = \frac{1}{\gamma}\widehat{\Sigma}_c^{-1}\overline{\lambda}_{OLS}, \tag{A.5}$$

where $\widehat{\Sigma}_c$ is the sample covariance matrix of characteristic returns and γ is the risk-aversion parameter.

Corollary A.2 shows that, for the particular case in which the columns of the firm-specific characteristic matrix are orthonormal, there is a componentwise one-to-one relationship between mean-variance parametric portfolio weights and OLS Fama-MacBeth slopes *only if* the sample covariance matrix of characteristic returns, $\widehat{\Sigma}_c$, is diagonal.[34] If, on the other hand, characteristic returns are correlated, then a given

[34]To see this, note that if $\widehat{\Sigma}_c$ is diagonal, then $\theta^*_k = (\overline{\lambda}_{OLS})_k/(\gamma(\widehat{\Sigma}_c)_{kk})$, where $(\widehat{\Sigma}_c)_{kk}$ is the kth element of the diagonal of $\widehat{\Sigma}_c$, and thus there is a one-to-one correspondence between the kth component of θ^* and the kth component of $\overline{\lambda}_{OLS}$.

characteristic k could have a zero OLS Fama-MacBeth slope ($\overline{\lambda}_k = 0$), and yet have a nonzero parametric portfolio weight ($\theta^*_k \neq 0$). This is the case, for instance, when the correlation of the kth characteristic return with the returns on the other characteristics can be exploited by the investor to reduce risk and thus improve her overall mean-variance utility.

The above theoretical results demonstrate that testing the significance of Fama-MacBeth slopes will, in general, produce results that are different from those of testing the significance of the weights that a mean-variance investor assigns to each characteristic. We now compare empirically the significance results from OLS Fama-MacBeth regressions with those of our approach.[35] Table 7 reports the significance of the Fama-MacBeth slopes for the six characteristics we found to be significant in Section 4.1 plus size, book to market, and momentum. The first column lists the name of the characteristics, the second column reports the multiple regression slopes and Newey-West t-statistics (in brackets),[36] and the third column reports the individual regression slopes and Newey-West t-statistics.

We see from Table 7 that the five characteristics that are significant at the 5% level in Section 4.1 are also jointly significant for cross-sectional regressions. However, in contrast to the finding in Section 4.1, beta is not significant in the Fama-MacBeth regressions even at the 10% level. This is because, as shown in Proposition A.1, Fama-MacBeth slopes differ in general from parametric portfolio weights when the returns on the characteristics are correlated over time and the investor can exploit this to reduce the risk of the mean-variance portfolio. Regarding the book-to-market and momentum characteristics, we see from Table 7 that both book to market (*bm*) and 12-month momentum (*mom12m*) are significant for multiple cross-sectional regressions, whereas they were not significant from a portfolio perspective. Intuitively, these characteristics are significant in multiple cross-sectional regressions because these regressions ignore the large contribution of these characteristics to the risk of the overall portfolio of characteristics, which reduces their appeal from a portfolio perspective.

[35]We do not run GLS Fama-MacBeth regressions because the sample covariance matrix of stock returns is singular for our case with thousands of stocks and only hundreds of monthly dates.

[36]We compute t-statistics with Newey-West adjustments of 12 lags, as in Green et al. (2014).

A.2 Relation to time-series regressions

In this section, we study analytically and empirically the relation of our portfolio approach to the time-series regression approach in the absence of transaction costs. The time-series approach may be described as regressing the return of a *new* characteristic onto the returns of K_c *commonly* accepted characteristics; that is,

$$r_{n,t} = \alpha_{TS} + \beta_{TS}^\top r_{c,t} + \epsilon_t, \tag{A.6}$$

where $r_{n,t} \in \mathbb{R}$ is the return of the *new* characteristic at time t, $r_{c,t} \in \mathbb{R}^{K_c}$ is the return of the commonly accepted characteristics at time t, the error term $\epsilon_t \in \mathbb{R}$ follows a Normal distribution with zero mean and standard deviation σ_ϵ, $\alpha_{TS} \in \mathbb{R}$ is the intercept of the regression, and $\beta_{TS} \in \mathbb{R}^{K_c}$ is the slope vector. If the intercept in this regression is significant, the return on the new characteristic is not fully explained by the return of the commonly accepted characteristics. Gibbons et al. (1989) shows that a significant intercept implies that the new characteristic-based long-short portfolio improves the investment opportunity set of a mean-variance investor who already has access to the returns on the set of commonly accepted characteristics.

As explained above, the time-series regression approach tests the significance of the intercept. In contrast, the following proposition shows that, in the absence of transaction costs, our approach is equivalent to testing the significance of the *slopes* in a particular constrained time-series multiple regression. Britten-Jones (1999) shows that the tangency mean-variance portfolio can be identified by solving a linear regression. We extend this result to the context of *any* parametric portfolio on the mean-variance efficient frontier by introducing a constraint on the mean return of the portfolio.

Proposition A.3 *For a given risk-aversion parameter γ, the optimal parameter θ^* for the mean-variance parametric portfolio problem without transaction costs* (5) *is equal to the ordinary least square (OLS) estimate of the slope vector in the following time-series regression model:*

$$r_{b,t} = \alpha - \beta^\top r_{c,t} + \epsilon_t, \tag{A.7}$$

where $r_{b,t} \in \mathbb{R}$ is the return of the benchmark portfolio, $r_{c,t} \in \mathbb{R}^K$ is the return on the characteristics, $\alpha \in \mathbb{R}$ is the intercept, and $\beta \in \mathbb{R}^K$ is the slope vector, subject to the *constraint that*

$$\beta^\top \mu_c = (\theta^*)^\top \mu_c, \tag{A.8}$$

where μ_c *is the mean characteristic return vector and* $(\theta^*)^\top \mu_c$ *is the average return of the mean-variance parametric portfolio.*

The advantage of the parametric-portfolio approach is that by focusing on the slopes, it allows one to test the significance of the different characteristics when they are considered *jointly*. The traditional time-series approach, on the other hand, is designed to test only the significance of a single characteristic when it is added to a set of commonly accepted characteristics; see also Footnote 8. This is a limitation of the time-series regression because the result of the statistical inference depends on the sequence in which variables are selected. For instance, when regressing the return of each characteristic onto the returns of the four Fama and French (1993) and Carhart (1997) factors downloaded from Kenneth French's website, we find that eight characteristics are significant in the absence of transaction costs, but beta *is not* significant.[37] Beta, however, *is* significant when its returns are regressed onto the four Fama and French (1993) and Carhart (1997) factors *plus* the return of the return-volatility long-short portfolio, because beta helps to hedge the return-volatility characteristic.[38] Accordingly, beta *matters* if one controls for return volatility.[39] Our portfolio approach considers all characteristics simultaneously and finds that return volatility and beta are jointly significant together with four other characteristics. These empirical results highlight the importance of considering all characteristics simultaneously. Other advantages of our portfolio approach are that it allows one to consider transaction costs in a straightforward manner and identify the marginal contribution of each characteristic to the investor's utility.

[37]We run 48 significance tests corresponding to the 51 characteristics except size, value, and momentum and thus, following Harvey et al. (2015) we apply Bonferroni's adjustment and require that p-values should be no greater than $\alpha/48$ for individual characteristics to be significant at the α level.

[38]We again apply Bonferroni's adjustment.

[39]This result is analogous to that in Asness et al. (2015), which finds that despite the weak performance of the *size* characteristic when evaluated in isolation, it becomes significant once it is considered in combination with a quality characteristic.

A.3 Relation to generalized alpha

In this section, we compare empirically the results from our portfolio approach in the presence of transaction costs with those from using the generalized alpha developed in Novy-Marx and Velikov (2016), which extends the traditional time-series regression framework to take transaction costs into account. Novy-Marx and Velikov (2016) proposes computing the returns of the mean-variance portfolio in the presence of transaction costs for the commonly accepted characteristics, MVE_X, and the returns of the mean-variance portfolio in the presence of transaction costs for the commonly accepted characteristics plus the new characteristic, $\text{MVE}_{X,y}$. Then it runs the following regression:

$$\text{MVE}_{X,y}/w_y = \alpha + \beta \text{MVE}_X + \epsilon, \tag{A.9}$$

where w_y is the weight of the mean-variance portfolio on the new characteristic. Novy-Marx and Velikov (2016) shows that in the absence of transaction costs, the generalized alpha in (A.9) equals the alpha from the traditional time-series approach. In the presence of transaction costs, this approach tests the significance of adding the new characteristic to a set of commonly accepted characteristics taking transaction costs into account.[40]

As discussed in Section A.2, the main advantage of our portfolio approach with respect to the time-series approach is that it considers all characteristics simultaneously and tests their significance when considered jointly, whereas the time-series regressions are designed to consider one characteristic at a time; see Footnote 8. To illustrate this, we compute the generalized alpha for each of our characteristics with respect to the four Fama and French (1993) and Carhart (1997) factors downloaded from Kenneth French's website. We find that, in the presence of transaction costs, *none* of the characteristic portfolios has a significant generalized alpha with respect to the four factors.[41] However, in the absence of transaction costs, Section A.2 showed that eight characteristics were significant with respect to the four factors. That is, the number of characteristics that are significant with respect to the four factors for the time-series approach *decreases* in

[40] Although the implementation in Novy-Marx and Velikov (2016) considers the transaction cost associated with each characteristic independently, here we extend the approach in Novy-Marx and Velikov (2016) to capture trading diversification.

[41] To address the multiple testing problem, we again apply Bonferroni's adjustment because we carry out 48 significance tests corresponding to our 51 characteristics except size, value, and momentum.

the presence of transaction costs when the characteristics are considered in isolation.[42] In contrast, our portfolio approach shows that the number of significant characteristics *increases* in the presence of transaction costs. This is because our approach allows one to consider all characteristics simultaneously and identify the optimal combination of characteristics that results in substantial trading diversification.

[42]This result regarding the significance of characteristics when considered in isolation is consistent with the results in Novy-Marx and Velikov (2016), which finds that fewer characteristics are significant in the presence of transaction costs than in the absence of transaction costs.

B Proofs for all propositions

Proof of Proposition 3.1

Equation (3) shows that the parametric portfolio is a combination of the benchmark portfolio and the K standardized firm-specific characteristics, scaled by the number of firms N_t. Therefore, we can define this combination as $w = [1,\ \theta] \in \mathbb{R}^{K+1}$ and the vector of benchmark and characteristic returns as $R_t = [r_{b,t}\ ,\ r_{c,t+1}/N_t]$. Under this specification, the mean-variance parametric portfolio problem takes the familiar form:

$$\min_{w} \quad \frac{\gamma}{2} w^\top \widehat{\Sigma} w - w^\top \widehat{\mu}, \tag{B.1}$$

$$\text{s.t.} \quad w_1 = 1, \tag{B.2}$$

where $w = [w_1,\ \theta] \in \mathbb{R}^{K+1}$ and $\widehat{\Sigma}$ and $\widehat{\mu}$ are the sample covariance matrix and mean of $R_t = [r_{b,t}\ ,\ r_{c,t+1}]$. The result follows by using straightforward algebra to eliminate the decision variable w_1 and the constraint, and then removing terms in the objective function that do not depend on the parameter vector θ.

Proof of Proposition 3.2

The marginal contributions of the characteristics are given by the subdifferential of the objective function in (10) with respect to θ. Note that the first four terms in (10) are differentiable with respect to θ and thus their subdifferentials coincide with their gradient. It is straightforward to show that the gradients of these four terms are given by the first four terms in the right-hand side of (11).

The only term that is not differentiable is the implied transaction cost from trading asset j at time $t+1$. According with expression (7), we can define the transaction cost term for asset j at time $t+1$ as

$$u_{j,t+1} = |\Lambda_{jj,t} \left(w_{j,t+1}(\theta) - w^+_{j,t}(\theta) \right) |, \tag{B.3}$$

where $\Lambda_{jj,t}$ is the associated transaction cost parameter for asset j at time t. Therefore, it suffices to characterize the subdifferential of expression (B.3).[43] Note that the function inside the absolute value is differentiable with respect to θ. Thus, applying the chain rule for subdifferentials, we have that the subdifferential of $u_{j,t+1}$ with respect to the ith

[43] See Rockafellar (2015) for an extensive treatment of subdifferentials.

parametric portfolio weight θ_i is equal to the subdifferential of the absolute value function times the differential of $\Lambda_{jj,t}\left(w_{j,t+1}(\theta) - w_{j,t}^+(\theta)\right)$.

Note that $\Lambda_{jj,t} > 0$ and thus, the subdifferential of the absolute value function is given by the sign function as precisely defined in (13). Finally, the differential of the term $\Lambda_{jj,t}\left(w_{j,t+1}(\theta) - w_{j,t}^+(\theta)\right)$ is

$$\frac{d[\Lambda_{jj,t}\left(w_{j,t+1}(\theta) - w_{j,t}^+(\theta)\right)]}{d\theta_i} = \Lambda_{jj,t}[(X_{t+1})_{ji} - (X_t)_{ji}(1 + r_{j,t+1})].$$

The result follows by adding the subdifferentials of $u_{j,t+1}$ for $j = 1, 2, \ldots, N_t$, and then combining the subdifferentials with respect to θ_i for $i = 1, 2, \ldots, K$ into a single vector.

Proof of Proposition 5.1

The trade in the ith stock required to rebalance an equally weighted portfolio of K characteristics is:

$$\text{trade}_i^{ew} = \frac{1}{K}\sum_{k=1}^{K}\text{trade}_{i,k} = \frac{1}{K}\sum_{k=1}^{K}[(X_{t+1})_{i,k} - (X_t)_{i,k}(1 + r_{i,t+1})]. \tag{B.4}$$

Because $\text{trade}_{i,k}$ for $k = 1, 2, \ldots, K$ are independently and identically distributed as a Normal distribution with zero mean and standard deviation σ, we have that trade_i^{ew} is distributed as a Normal distribution with zero mean and standard deviation $\sigma/\sqrt{K}$.

Therefore the average cost of the trade in the ith stock required to rebalance an equally weighted portfolio of the K characteristics is κ_i times the mean absolute deviation of trade_i^{ew}, where κ_i is the transaction-cost parameter for the ith stock. Geary (1935) shows that the mean absolute deviation of a Normally distributed random variable is $\sqrt{2/\pi}$ times its standard deviation. Therefore, the average cost of the trade in the ith stock required to rebalance an equally weighted portfolio of K characteristics is

$$\text{TC}(\text{trade}_i^{ew}) = \kappa_i \times \sqrt{2/\pi} \times \sigma/\sqrt{K}. \tag{B.5}$$

Following a similar argument, the average cost of the trade in the ith stock required to rebalance the kth characteristic in isolation is $\text{TC}(\text{trade}_i^k) = \kappa_i \times \sqrt{2/\pi} \times \sigma$, which is $\sqrt{K} \times \text{TC}(\text{trade}_i^{ew})$.

Proof of Proposition 5.2

The trade in the ith stock required to rebalance an equally weighted portfolio of K characteristics is $\text{trade}_i^{ew} = \frac{1}{K}\sum_{k=1}^{K}\text{trade}_{i,k}$. Because $\text{trade}_{i,k}$ for $k = 1, 2, \ldots, K$ are jointly distributed as a multivariate Normal distribution with zero mean and and covariance matrix Ω, we have that trade_i^{ew} is distributed as a Normal distribution with zero mean and standard deviation $\sqrt{e^\top \Omega e}/K$. The results follows from simple algebra and the arguments in the proof of Proposition 5.1.

Proof of Proposition A.1

Let us consider the following cross-sectional regression model:

$$r_t = X\lambda_t + \epsilon_t, \tag{B.6}$$

where $r_t \in \mathbb{R}^N$ is the vector of stock returns at time t, $X \in \mathbb{R}^{N\times K}$ is the matrix of standardized firm characteristics, $\lambda_t \in \mathbb{R}^K$ is the vector of slopes at time t, and $\epsilon_t \in \mathbb{R}^N$ is the vector of pricing errors at time t.[44] The OLS and GLS Fama-MacBeth slopes of model (B.6) are

$$\overline{\lambda}_{OLS} = (X^\top X)^{-1}X^\top \widehat{\mu}_r \tag{B.7}$$

$$\overline{\lambda}_{GLS} = (X^\top \widehat{\Sigma}_r^{-1}X)^{-1}X^\top \widehat{\Sigma}_r^{-1}\widehat{\mu}_r, \tag{B.8}$$

where $\widehat{\mu}_r$ is the vector of sample mean returns. It is straightforward to see that $\overline{\lambda}_{OLS}$ and $\overline{\lambda}_{GLS}$ are identical when $\widehat{\Sigma}_r$ is the identity matrix. On the other hand, we know that the solution of a mean-variance parametric portfolio is

$$\theta^* = \frac{1}{\gamma}\widehat{\Sigma}_c^{-1}\widehat{\mu}_c - \widehat{\Sigma}_c^{-1}\widehat{\sigma}_{bc}. \tag{B.9}$$

Now, given the assumption that firm characteristics are constant, we can define the vector of mean characteristic-portfolio returns and the covariance matrix of characteristic-portfolio returns as $\widehat{\mu}_c = X^\top \widehat{\mu}_r$ and $\widehat{\Sigma}_c = X^\top \widehat{\Sigma}_r X$, respectively. Assuming that the covariance between characteristic portfolio returns and the benchmark portfolio is zero, expression (B.9) can be then redefined as

$$\theta^* = \frac{1}{\gamma}(X^\top \widehat{\Sigma}_r X)^{-1}X^\top \widehat{\mu}_r. \tag{B.10}$$

[44] Note that we now assume that characteristics X_t and the number of firms N_t are constant through time and therefore we can drop subscripts t.

Therefore, one can see that $\overline{\lambda}_{OLS}$, $\overline{\lambda}_{GLS}$ and θ^* will be equivalent when $\widehat{\Sigma}_r$ is the identity matrix of dimension N and the covariance between characteristic portfolio returns and the benchmark portfolio is zero.

Proof of Corollary A.2

The result in Corollary A.2 follows from the assumption that $X^\top X = I$, which implies that $\overline{\lambda}_{OLS} = X^\top \widehat{\mu}_r = \widehat{\mu}_c$. Then, if the covariance between characteristic-portfolio returns and the benchmark portfolio is zero, we can define the solution of a mean-variance parametric portfolio as

$$\theta^* = \frac{1}{\gamma}\widehat{\Sigma}_c^{-1}\overline{\lambda}_{OLS}. \tag{B.11}$$

Proof of Proposition A.3

We can estimate model (A.7) with OLS. The corresponding optimization problem, in matrix form, is

$$\begin{aligned} \min_{\alpha,\beta} \quad & r_b^\top r_b + \alpha^2 T + \beta^\top r_c^\top r_c \beta - 2\alpha r_b^\top e_T + 2 r_b^\top r_c \beta - 2\alpha e_T^\top r_c \beta \\ \text{s.t.} \quad & \widehat{\mu}_c^\top \beta = \mu_0, \end{aligned}$$

where e_T is a T-dimensional vector of ones. Now, given that $\widehat{\Sigma}_c = r_c^\top r_c - \widehat{\mu}_c \widehat{\mu}_c^\top$, $\widehat{\sigma}_{bc} = r_b^\top r_c - \widehat{\mu}_b \widehat{\mu}_c^\top$, and $e_T^\top r_c = T\widehat{\mu}_c$, we can write the above problem as

$$\begin{aligned} \min_{\alpha,\beta} \quad & r_b^\top r_b + \alpha^2 T + \beta^\top \widehat{\Sigma}_c \beta + \beta^\top \widehat{\mu}_c \widehat{\mu}_c^\top \beta - 2\alpha r_b^\top e_T + 2(\widehat{\sigma}_{bc} + \widehat{\mu}_b \widehat{\mu}_c)^\top \beta - 2\alpha T \widehat{\mu}_c^\top \beta \\ \text{s.t.} \quad & \widehat{\mu}_c^\top \beta = \mu_0, \end{aligned}$$

and now, because $\widehat{\mu}_c^\top \beta$ is constant in the feasible region, we can obtain the OLS slopes of (A.7) as the solution to the following problem:

$$\begin{aligned} \min_{\beta} \quad & \beta^\top \widehat{\Sigma}_c \beta + 2\widehat{\sigma}_{bc}\beta \\ \text{s.t.} \quad & \widehat{\mu}_c^\top \beta = \mu_0, \end{aligned}$$

which is a quadratic mean-variance optimization problem. If we set μ_0 equal to the solution of the mean-variance parametric portfolio problem times the vector of mean characteristic portfolio returns, this is $\theta^{*\top}\widehat{\mu}_c$, the OLS slopes of the time-series model in (A.7) coincide with the solution of the mean-variance parametric portfolio problem in (5).

Table 1: List of characteristics considered

This table lists the characteristics we consider ordered alphabetically by acronym. The first column gives the number of the characteristic, the second column gives the characteristic's definition, the third column gives the acronym, and the fourth and fifth columns give the authors who analyzed them, and the date and journal of publication. Our definitions and acronyms match those in Green et al. (2014).

#	Characteristic and definition	Acronym	Author(s)	Date and Journal
1	Abnormal volume in earnings announcement: Average daily trading volume for 3 days around earnings announcement minus average daily volume for 1-month ending 2 weeks before earnings announcement divided by 1-month average daily volume. Earnings announcement day from Compustat quarterly	aeavol	Lerman, Livnat & Mendenhall	2007, WP
2	Asset growth: Annual percent change in total assets	agr	Cooper, Gulen & Schill	2008, JF
3	Bid-ask spread: Monthly average of daily bid-ask spread divided by average of daily spread	baspread	Amihud & Mendelson	1989, JF
4	Beta: Estimated market beta from weekly returns and equal weighted market returns for 3 years ending month $t-1$ with at least 52 weeks of returns	beta	Fama & MacBeth	1973, JPE
5	Book to market: Book value of equity divided by end of fiscal-year market capitalization	bm	Rosenberg, Reid & Lanstein	1985, JPM
6	Industry adjusted book to market: Industry adjusted book-to-market ratio	bm_ia	Asness, Porter & Stevens	2000, WP
7	Cash productivity: Fiscal year-end market capitalization plus long term debt minus total assets divided by cash and equivalents	cashpr	Chandrashekar & Rao	2009 WP
8	Industry adjusted change in asset turnover: 2-digit SIC fiscal-year mean adjusted change in sales divided by average total assets	chatoia	Soliman	2008, TAR
9	Change in shares outstanding: Annual percent change in shares outstanding	chcsho	Pontiff & Woodgate	2008, JF
10	Industry adjusted change in employees: Industry-adjusted change in number of employees	chempia	Asness, Porter & Stevens	1994, WP
11	Change in 6-month momentum: Cumulative returns from months $t-6$ to $t-1$ minus months $t-12$ to $t-7$	chmom	Gettleman & Marks	2006 WP
12	Industry adjusted change in profit margin: 2-digit SIC fiscal-year mean adjusted change in income before extraordinary items divided by sales	chpmia	Soliman	2008, TAR
13	Change in tax expense: Percent change in total taxes from quarter $t-4$ to t	chtx	Thomas & Zhang	2011 JAR
14	Convertible debt indicator: An indicator equal to 1 if company has convertible debt obligations	convind	Valta	2016 JFQA
15	Dollar trading volume in month $t-2$: Natural log of trading volume times price per share from month $t-2$	dolvol	Chordia, Subrahmanyan & Anshuman	2001, JFE
16	Dividends-to-price: Total dividends divided by market capitalization at fiscal year-end	dy	Litzenberger & Ramaswamy	1982, JF
17	3-day return around earnings announcement: Sum of daily returns in three days around earnings announcement. Earnings announcement from Compustat quarterly file	ear	Kishore, Brandt, Santa-Clara & Venkatachalam	2008, WP
18	Change in common shareholder equity: Annual percent change in book value of equity	egr	Richardson, Sloan, Soliman & Tuna	2005, JAE
19	Earnings to price: Annual income before extraordinary items divided by end of fiscal year market cap	ep	Basu	1977, JF
20	Gross profitability: Revenues minus cost of goods sold divided by lagged total assets	gma	Novy-Marx	2013 JFE
21	Industry sales concentration: Sum of squared percent of sales in industry for each company	herf	Hou & Robinson	2006, JF
22	Employee growth rate: Percent change in number of employees	hire	Bazdresch, Belo & Lin	2014 JPE
23	Idiosyncratic return volatility: Standard deviation of residuals of weekly returns on weekly equal weighted market returns for 3 years prior to month-end	idiovol	Ali, Hwang & Trombley	2003, JFE
24	Industry momentum: Equal weighted average industry 12-month returns	indmom	Moskowitz & Grinblatt	1999, JF

Table 1 continued: List of characteristics considered

#	Characteristic and definition	Acronym	Author(s)	Date and Journal
25	Leverage: Total liabilities divided by fiscal year-end market capitalization	lev	Bhandari	1988, JF
26	Change in long-term debt: Annual percent change in total liabilities	lgr	Richardson, Sloan, Soliman & Tuna	2005, JAE
27	12-month momentum: 11-month cumulative returns ending one month before month-end	mom12m	Jegadeesh	1990, JF
28	1-month momentum: 1-month cumulative return	mom1m	Jegadeesh	1990, JF
29	36-month momentum: Cumulative returns from months $t-36$ to $t-13$	mom36m	De Bondt & Thaler	1985, JF
30	6-month momentum: 5-month cumulative returns ending one month before month-end	mom6m	Jegadeesh & Titman	1990, JF
31	Market capitalization: Natural log of market capitalization at end of month $t-1$	mve	Banz	1981, JFE
32	Industry-adjusted firm size: 2-digit SIC industry-adjusted fiscal year-end market capitalization	mve_ia	Asness, Porter & Stevens	2000, WP
33	Δ% CAPEX - industry Δ% AR: 2-digit SIC fiscal-year mean adjusted percent change in capital expenditures	pchcapx_ia	Abarbanell & Bushee	1998, TAR
34	Δ% gross margin - Δ% sales: Percent change in gross margin minus percent change in sales	pchgm_pchsale	Abarbanell & Bushee	1998, TAR
35	Δ% sales - Δ% AR: Annual percent change in sales minus annual percent change in receivables	pchsale_pchrect	Abarbanell & Bushee	1998, TAR
36	Price delay: The proportion of variation in weekly returns for 36 months ending in month t explained by 4 lags of weekly market returns incremental to contemporaneous market return	pricedelay	How & Moskowitz	2005, RFS
37	Financial-statements score: Sum of 9 indicator variables to form fundamental health score	ps	Piotroski	2000, JAR
38	R&D to market cap: R&D expense divided by end-of-fiscal-year market capitalization	rd_mve	Guo, Lev & Shi	2006, JBFA
39	Return volatility: Standard deviation of daily returns from month $t-1$	retvol	Ang, Hodrick, Xing & Zhanf	2006, JF
40	Return on assets: Income before extraordinary items divided by one quarter lagged total assets	roaq	Balakrishnan, Bartov & Faurel	2010, JAE
41	Revenue surprise: Sales from quarter t minus sales from quarter $t-4$ divided by fiscal-quarter-end market capitalization	rsup	Kama	2009, JBFA
42	Sales to cash: Annual sales divided by cash and cash equivalents	salecash	Ou & Penman	1989, JAE
43	Sales to inventory: Annual sales divided by total inventory	saleinv	Ou & Penman	1989, JAE
44	Sales to receivables: Annual sales divided by accounts receivable	salerec	Ou & Penman	1989, JAE
45	Annual sales growth: Annual percent change in sales	sgr	Lakonishok, Shleifer & Vishny	1994, JF
46	Volatility of dollar trading volume: Monthly standard deviation of daily dollar trading volume	std_dolvol	Chordia, Subrahmanyan & Anshuman	2001, JFE
47	Volatility of share turnover: Monthly standard deviation of daily share turnover	std_turn	Chordia, Subrahmanyan & Anshuman	2001, JFE
48	Cashflow volatility: Standard deviation for 16 quarters of cash flows divided by sales	stdcf	Huang	2009, JEF
49	Unexpected quarterly earnings: Unexpected quarterly earnings divided by fiscal-quarter-end market cap. Unexpected earnings is I/B/E/S actual earnings minus median forecasted earnings if available, else it is the seasonally differenced quarterly earnings before extraordinary items from Compustat quarterly file	sue	Rendelman, Jones & Latane	1982, JFE
50	Share turnover: Average monthly trading volume for most recent 3 months scaled by number of shares outstanding in current month	turn	Datar, Naik & Radcliffe	1998, JFM
51	Zero trading days: Turnover weighted number of zero trading days for most recent month	zerotrade	Liu	2006, JFE

Table 2: Significance and marginal contributions without transaction costs

This table reports the significance and marginal contributions for the parametric portfolios without transaction costs, for risk-aversion parameter $\gamma = 5$. We run a screen-and-clean significance test. For the screen step, we calibrate the regularized parametric portfolios with five-fold cross-validation and find that the lasso threshold that maximizes investor's utility is $\delta = 25$. For the clean step, we run the bootstrap experiment for the parametric portfolios using those characteristics with nonzero θ's from the previous step plus the three characteristics considered in Brandt et al. (2009): size, book to market, and momentum. Characteristic p-values are computed using the percentile method discussed in Section 3.5. We assign three/two/one asterisks (*) to those characteristics whose p-values are lower than 0.01/0.05/0.1, respectively. For each characteristic, the first column gives the acronym, the second the optimal value of the parameter and the significance asterisks, and the next four columns give the marginal contribution of the characteristic to: (i) the characteristic own-variance, (ii) the covariance of the characteristic with the other characteristics in the portfolio, (iii) the covariance of the characteristic with the benchmark portfolio, and (iv) the characteristic mean. Contributions that drive the characteristic to be nonzero are in blue sans serif font, and contributions that drive the characteristic toward zero are in red *italic* font (cf. Footnote 21).

		Marginal contributions to			
Characteristic	Param.	variance	cov (char.)	cov (bench.)	mean
sue	20.12^{***}	*0.00341*	−0.00068	−0.00019	−0.00254
retvol	-10.85^{***}	*−0.03529*	0.02914	0.00292	0.00323
agr	-10.37^{**}	*−0.00397*	0.00050	0.00057	0.00290
mom1m	-3.10^{**}	*−0.00509*	0.00454	*−0.00109*	0.00164
gma	5.97^{**}	*0.00252*	−0.00255	*0.00069*	−0.00066
beta	2.36^{*}	*0.00971*	−0.01381	*0.00419*	−0.00008
bm_ia	6.49	*0.00337*	−0.00328	*0.00072*	−0.00081
chcsho	−5.89	*−0.00210*	*−0.00111*	0.00092	0.00228
rd_mve	6.01	*0.00215*	−0.00096	*0.00045*	−0.00164
std_turn	8.53	*0.01442*	−0.01576	*0.00214*	−0.00080
bm	3.10	*0.00264*	*0.00023*	−0.00082	−0.00205
mve	−4.02	*−0.00136*	0.00148	*−0.00034*	0.00022
mom12m	−4.42	*−0.00784*	0.01125	*−0.00066*	*−0.00275*

Table 3: Correlations of significant characteristics

This table reports the correlation matrix for the returns of the six characteristics that are most significant in the absence of transaction costs and the returns of the three characteristics considered in Brandt et al. (2009): size (mve), book to market (bm), and momentum ($mom12m$).

Characteristics	sue	retvol	agr	mom1m	gma	beta	bm	mve	mom12m
Unexpected quarterly earnings (sue)	1.00	−0.43	−0.08	0.18	−0.18	−0.36	−0.05	0.41	0.45
Return volatility (retvol)	−0.43	1.00	0.22	−0.18	0.45	0.93	−0.46	−0.63	−0.17
Asset growth (agr)	−0.08	0.22	1.00	−0.33	0.56	0.33	−0.64	0.03	−0.17
1-month momentum (mom1m)	0.18	−0.18	−0.33	1.00	−0.23	−0.26	0.14	0.19	0.28
Gross profitability (gma)	−0.18	0.45	0.56	−0.23	1.00	0.54	−0.62	−0.24	−0.06
Beta (beta)	−0.36	0.93	0.33	−0.26	0.54	1.00	−0.54	−0.52	−0.21
Book to market (bm)	−0.05	−0.46	−0.64	0.14	−0.62	−0.54	1.00	−0.05	−0.08
Market capitalization (mve)	0.41	−0.63	0.03	0.19	−0.24	−0.52	−0.05	1.00	0.20
12-month momentum (mom12m)	0.45	−0.17	−0.17	0.28	−0.06	−0.21	−0.08	0.20	1.00

Table 4: Significance and marginal contributions with transaction costs

This table reports the significance and marginal contributions for the parametric portfolios in the presence of transaction costs, for risk-aversion parameter $\gamma = 5$. We run a screen-and-clean significance test. For the screen step, we calibrate the regularized parametric portfolios with five-fold cross-validation and find that the lasso threshold that maximizes investor's utility is $\delta = 25$. For the clean step, we run the bootstrap experiment for the parametric portfolios using those characteristics with nonzero θ's from the previous step plus the three characteristics considered in Brandt et al. (2009): size, book to market, and momentum. Characteristic p-values are computed using the percentile method discussed in Section 3.5. We assign three/two/one asterisks (*) to those characteristics whose p-values are lower than 0.01/0.05/0.1, respectively. For each characteristic, the first column gives the acronym, the second the optimal value of the parameter and the significance asterisks, and the next five columns give the marginal contribution of the characteristic to: (i) the characteristic own-variance, (ii) the covariance of the characteristic with the other characteristics in the portfolio, (iii) the covariance of the characteristic with the benchmark portfolio, (iv) the characteristic mean, and (v) the transaction cost. The last column reports the marginal contribution of the characteristic to transaction costs when it is traded in isolation. Contributions that drive the characteristic to be nonzero are in blue sans serif font, and contributions that drive the characteristic toward zero are in red *italic* font (cf. Footnote 21).

		Marginal contributions to					Indiv.
Characteristic	Param.	variance	cov (char.)	cov (bench.)	mean	tran. cost	tran. costs
rd_mve	11.85^{***}	*0.00425*	−0.00333	*0.00045*	−0.00164	*0.00027*	*0.00055*
agr	-7.27^{***}	*−0.00278*	*−0.00012*	0.00057	0.00290	*−0.00057*	*0.00125*
sue	3.00^{***}	*0.00051*	*0.00077*	−0.00019	−0.00254	*0.00146*	*0.00240*
turn	-3.41^{***}	*−0.00806*	0.00502	0.00279	0.00068	*−0.00043*	*0.00177*
retvol	-1.92^{***}	*−0.00623*	0.00148	0.00292	0.00323	*−0.00139*	*0.00468*
std_turn	1.28^{***}	*0.00217*	−0.00433	*0.00214*	−0.00080	*0.00082*	*0.00493*
zerotrade	-1.53^{***}	*−0.00129*	0.00284	*−0.00205*	0.00124	*−0.00075*	*0.00235*
chatoia	4.51^{**}	*0.00029*	*0.00008*	−0.00005	−0.00077	*0.00046*	*0.00116*
chtx	1.36^{**}	*0.00026*	−0.00022	*0.00015*	−0.00123	*0.00104*	*0.00232*
beta	3.39^{**}	*0.01398*	−0.01829	*0.00419*	−0.00008	*0.00021*	*0.00126*
ps	4.94^{**}	*0.00156*	−0.00027	−0.00068	−0.00127	*0.00066*	*0.00140*
gma	6.60^{**}	*0.00278*	−0.00298	*0.00069*	−0.00066	*0.00016*	*0.00090*
herf	-5.78^{**}	*−0.00144*	0.00061	0.00041	0.00061	*−0.00019*	*0.00077*
mom1m	-0.62^{**}	*−0.00102*	0.00258	*−0.00109*	0.00164	*−0.00211*	*0.00857*
bm_ia	2.85^{**}	*0.00148*	−0.00168	*0.00072*	−0.00081	*0.00029*	*0.00128*
stdcf	-5.05^{*}	*−0.00259*	0.00101	0.00068	0.00114	*−0.00024*	*0.00067*
pchgm_pchsale	3.46^{*}	*0.00034*	*0.00006*	−0.00003	−0.00079	*0.00042*	*0.00122*
chcsho	-3.11^{*}	*−0.00111*	*−0.00166*	0.00092	0.00228	*−0.00044*	*0.00123*
bm	1.74^{*}	*0.00148*	*0.00122*	−0.00082	−0.00205	*0.00017*	*0.00121*
chmom	−0.67	*−0.00065*	0.00166	*−0.00073*	0.00044	*−0.00072*	*0.00404*
baspread	0.55	*0.00240*	−0.00795	*0.00329*	*0.00279*	−0.00053	*0.00322*
ep	1.27	*0.00206*	*0.00045*	−0.00166	−0.00104	*0.00018*	*0.00125*
idiovol	−1.80	*−0.00680*	0.00194	0.00308	0.00187	*−0.00008*	*0.00109*
roaq	−0.12	*−0.00014*	0.00292	*−0.00114*	*−0.00215*	0.00051	*0.00186*
mve	−2.28	*−0.00077*	0.00092	*−0.00034*	0.00022	*−0.00003*	*0.00045*
mom12m	−0.61	*−0.00109*	0.00418	*−0.00066*	*−0.00275*	0.00031	*0.00265*

Table 5: Out-of-sample performance

This table reports the out-of-sample performance of the regularized parametric portfolios in the presence of transaction costs, for risk-aversion parameter $\gamma = 5$. Panel A reports the performance for the portfolios that do not use any characteristics, which are the benchmark value-weighted portfolio (VW) and the equally weighted portfolio ($1/N$). Panel B reports the performance of four parametric portfolios: the parametric portfolio that exploits the size, book-to-market, and momentum characteristics (Size/val./mom.), the parametric portfolio that exploits the size, book-to-market, asset growth, and gross profitability characteristics (Size/val./inv./prof.), the parametric portfolio that exploits the 15 most significant characteristics identified using the entire sample (Fifteen significant characteristics), and the regularized parametric portfolio that identifies the characteristics ex ante (Regularized). The lasso threshold is calibrated using cross-validation over the estimation window. For each portfolio, the first column reports the monthly turnover, and the next three columns report the out-of-sample annualized mean, standard deviation, and Sharpe ratio of returns, net of transaction costs. We test the significance of the difference of the Sharpe ratio of each portfolio with that of the regularized parametric portfolio. Three/two/one asterisks (*) indicate that the difference is significant at the 0.01/0.05/0.1 level.

Policy	Turnover	Mean	SD	SR
Panel A: Portfolios with no characteristics				
VW	0.050	0.085	0.150	0.567***
1/N	0.134	0.085	0.177	0.482***
Panel B: Portfolios with characteristics				
Size/val./mom.	0.754	0.145	0.215	0.675***
Size/val./inv./prof.	0.963	0.236	0.220	1.072**
Fifteen significant characteristics	1.065	0.223	0.166	1.343
Regularized	0.979	0.241	0.178	1.356

Table 6: Factor loadings of regularized parametric portfolios

This table reports the intercept, slopes, and t-statistics (in brackets) from regressing the out-of-sample regularized portfolio returns onto three different factor models: (1) the Fama and French (1993) and Carhart (1997) four-factor model (FFC) that includes the market, size (SMB), value (HML), and momentum (UMD) factors; (2) the Fama and French (2015) five-factor model (FF5) that includes the market, size, value, profitability (RMW), and investment (CMA) factors; and, (3) the Hou et al. (2014) four-factor model (HXZ) that includes the market, size, investment (I/A), and profitability (ROE) factors. We report t-statistics with Newey-West adjustments of 12 lags. Factors are obtained from Kenneth French's and Lu Zhang's websites.

FFC	Coefficient	FF5	Coefficient	HXZ	Coefficient
α	0.0115	α	0.0102	α	0.0095
	[4.12]		[3.59]		[2.89]
Market	0.8898	Market	0.9747	Market	0.9147
	[15.29]		[15.35]		[11.90]
SMB	0.0745	SMB	0.1212	SMB	0.2547
	[0.49]		[0.84]		[1.37]
HML	0.3697	HML	−0.2640	I/A	0.7491
	[1.84]		[−1.71]		[2.65]
UMD	0.3249	RMW	0.2554	ROE	0.3316
	[2.46]		[1.31]		[1.69]
		CMA	1.0852		
			[3.64]		

Table 7: Fama-MacBeth regressions for significant characteristics

This table reports the slope coefficients from Fama-MacBeth regressions and the corresponding t-statistics (in brackets) with Newey-West adjustments of 12 lags. We report the results for multiple and individual regressions for the six most significant characteristics in the absence of transaction costs, and the three characteristics considered in Brandt et al. (2009): size (mve), book to market (bm), and momentum ($mom12m$).

Characteristic	Multiple	Individual
Unexpected quarterly earnings (sue)	0.0019	0.0027
	[7.38]	[7.10]
Return volatility (retvol)	−0.0037	−0.0032
	[−4.42]	[−2.22]
Asset growth (agr)	−0.0026	−0.0031
	[−5.39]	[−5.09]
1-month momentum (mom1m)	−0.0033	−0.0017
	[−4.67]	[−2.13]
Gross profitability (gma)	0.0020	0.0007
	[3.80]	[1.34]
Beta (beta)	0.0013	0.0001
	[0.99]	[0.04]
Book to market (bm)	0.0016	0.0021
	[2.11]	[2.17]
Market capitalization (mve)	−0.0007	−0.0002
	[−1.76]	[−0.40]
12-month momentum (mom12m)	0.0026	0.0030
	[2.43]	[2.45]

Figure 1: Marginal contributions of significant characteristics

This figure shows the marginal contributions to the investor's utility of the six significant characteristics in the absence of transaction costs. The horizontal axis gives the labels of the significant characteristics: unexpected quarterly earnings (unexp. earn.), return volatility (ret. vol.), asset growth, 1-month momentum (reversals), grow profitability (profit.), and beta. Contributions that drive the characteristic to be nonzero are depicted with positive bars, and contributions that drive the characteristic toward zero are depicted with negative bars; cf. Footnote 21.

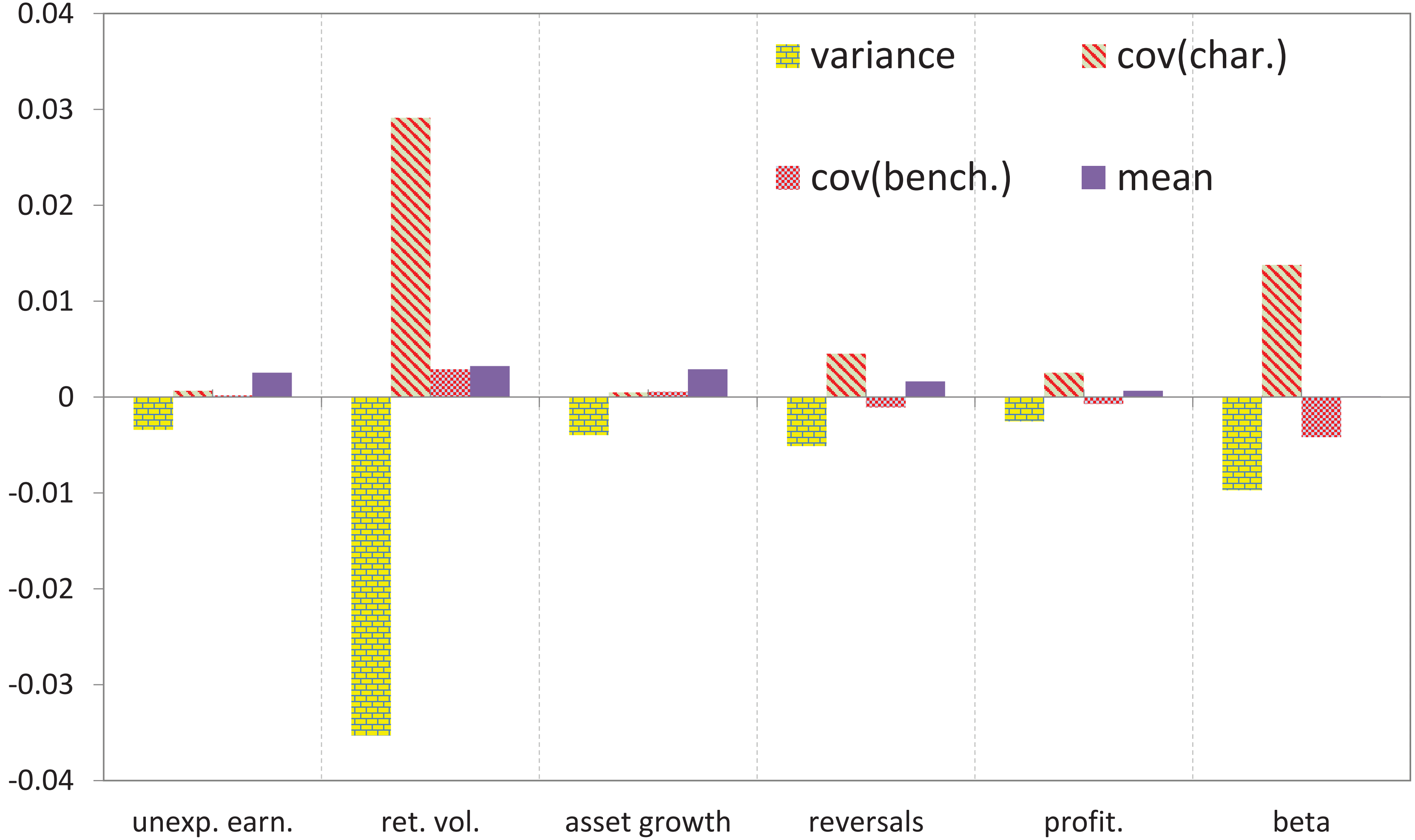
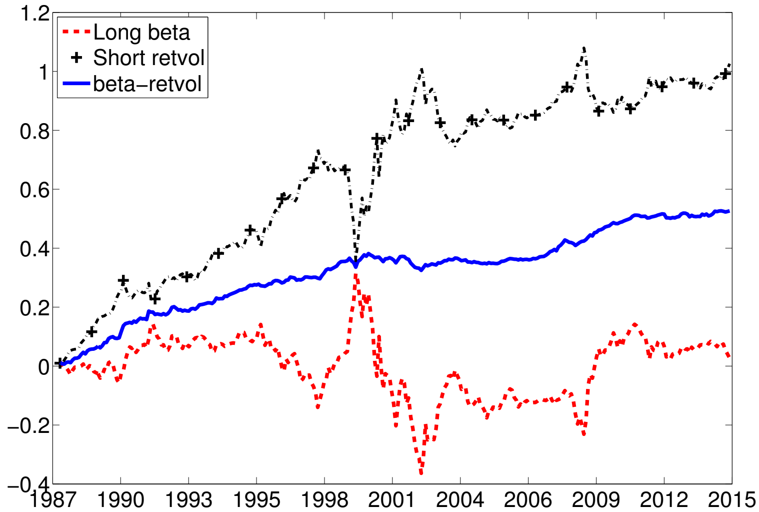


Figure 2: Beta and return-volatility cumulative returns

This figure shows the cumulative returns of a strategy that goes long on beta (long beta) and a strategy that goes short on return volatility (short retvol). Panel (a) depicts the cumulative returns for long *beta* and short *retvol*, together with the cumulative return of a blended strategy formed by assigning 50% weight to *beta* and −50% to *retvol*. Panel (b) depicts the cumulative returns of the blended strategy with *beta* and *retvol* and the cumulative returns of a blended strategy that assigns 50% to book to market (*bm*) and 50% to 12-month momentum (*mom*12*m*). For comparison purposes in Panel (b) we normalize both strategies so that they have the same volatility.

(a) Retvol, beta, and blend



(b) Two blended strategies

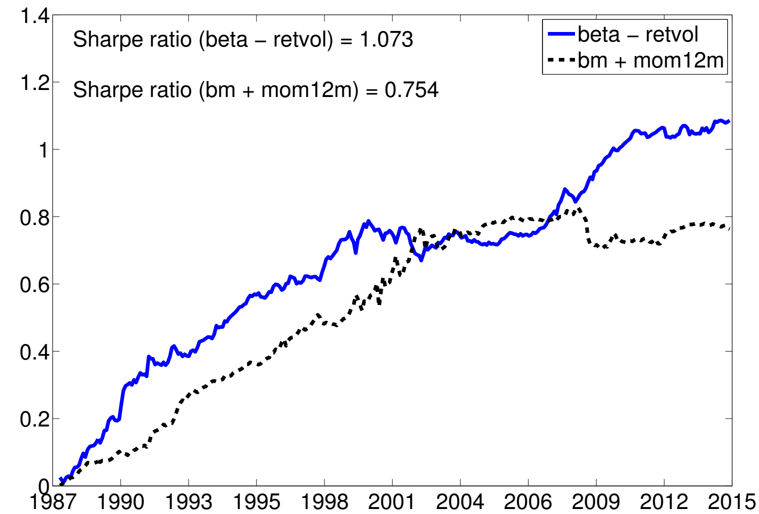


Figure 3: Marginal contribution to transaction costs of characteristics traded in combination and individually

This figure shows the marginal contribution to transaction costs (in absolute value) when characteristics are traded in combination with other characteristics (Combination), and when characteristics are traded in isolation (Individual). We plot the marginal contribution to transaction costs of the 15 most significant characteristics in Table 4.

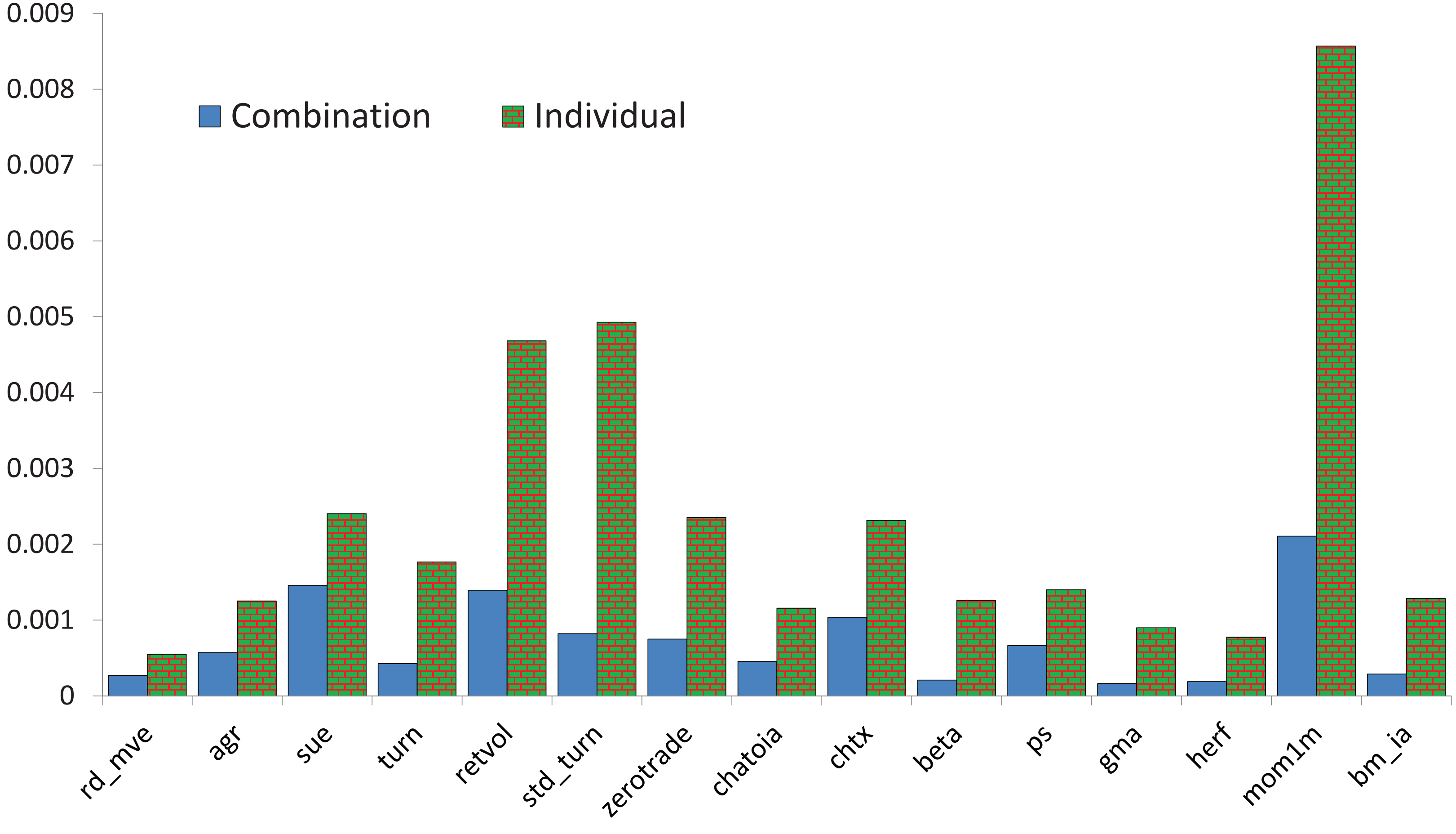
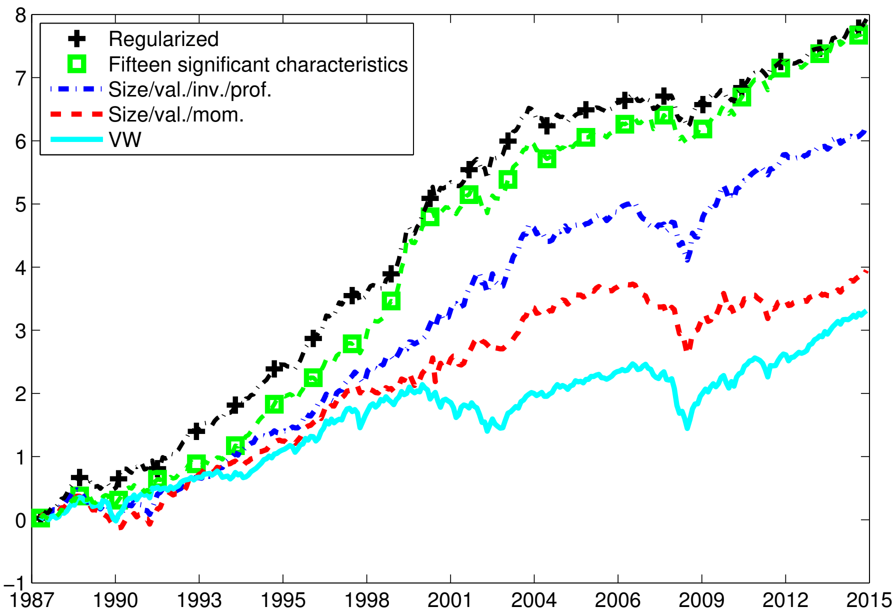


Figure 4: Out-of-sample cumulative returns

This figure shows the out-of-sample cumulative returns of the value-weighted portfolio (VW) and four different parametric portfolios in the presence of transaction costs, for risk-aversion parameter $\gamma = 5$. The four parametric portfolios are: the parametric portfolio that exploits the size, book-to-market, and momentum characteristics (Size/val./mom.), the parametric portfolio that exploits the size, book-to-market, asset growth, and gross profitability characteristics (Size/val./inv./prof.), the parametric portfolio with the 15 most significant characteristics identified using the entire sample (Fifteen significant characteristics), and the regularized parametric portfolio that identifies the characteristics ex ante (Regularized). The lasso threshold is calibrated using cross-validation over the estimation window. For comparison purposes we normalize all portfolio returns so that they have the same volatility.



References

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *The Journal of Finance* 61, 259–299.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2009, High idiosyncratic volatility and low returns: International and further U.S. evidence, *Journal of Financial Economics* 91, 1–23.

Asness, Clifford S., Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse H. Pedersen, 2015, Size matters, if you control your junk, SSRN working paper.

Asness, Clifford S, Tobias J. Moskowitz, and Lasse H. Pedersen, 2013, Value and momentum everywhere, *The Journal of Finance* 68, 929–985.

Back, Kerry, Nishad Kapadia, and Barbara Ostdiek, 2015, Testing factor models on characteristic and covariance pure plays, SSRN working paper.

Baker, Malcolm, Patrick Luo, and Ryan Taliaferro, 2017, Detecting anomalies: The relevance and power of standard asset pricing tests, hBS working paper.

Bali, Turan G., Robert F. Engle, and Scott Murray, 2016, *Empirical Asset Pricing: The Cross Section of Stock Returns* (John Wiley & Sons, Hoboken, New Jersey).

Barroso, Pedro, and Pedro Santa-Clara, 2015, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis* 50, 1037–1056.

Belloni, Alexandre, Victor Chernozhukov, and Christian Hansen, 2014, Inference on treatment effects after selection among high-dimensional controls, *The Review of Economic Studies* 81, 608–650.

Black, Fischer, 1993, Beta and return, *The Journal of Portfolio Management* 20, 8–18.

Brandt, Michael W., Pedro Santa-Clara, and Rossen Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, *Review of Financial Studies* 22, 3411–3447.

Britten-Jones, Mark, 1999, The sampling error in estimates of mean-variance efficient portfolio weights, *The Journal of Finance* 54, 655–671.

Bryzgalova, Svetlana, 2015, Spurious factors in linear asset pricing models, Stanford GSB working paper.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *The Journal of Finance* 52, 57–82.

Chatterjee, Arindam, and Soumendra Nath Lahiri, 2011, Bootstrapping lasso estimators, *Journal of the American Statistical Association* 106, 608–625.

Chen, Andrew, and Mihail Velikov, 2017, Accounting for the anomaly zoo: A trading cost perspective, federal Reserve Board working paper.

Chordia, Tarun, Amit Goyal, and Alessio Saretto, 2017, p-hacking: Evidence from two million trading strategies, HEC Lausanne working paper.

Cochrane, John H., 2009, *Asset Pricing: Revised Edition* (Princeton University Press).

Cochrane, John H., 2011, Presidential address: Discount rates, *The Journal of Finance* 66, 1047–1108.

Daniel, Kent, and Sheridan Titman, 1997, Evidence on the characteristics of cross sectional variation in stock returns, *The Journal of Finance* 52, 1–33.

DeMiguel, Victor, Lorenzo Garlappi, Francisco J. Nogales, and Raman Uppal, 2009a, A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms, *Management Science* 55, 798–812.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal, 2009b, Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?, *Review of Financial Studies* 22, 1915–1953.

DeMiguel, Victor, Francisco J. Nogales, and Raman Uppal, 2014, Stock return serial dependence and out-of-sample portfolio performance, *Review of Financial Studies* 27, 1031–1073.

Efron, Brad, and Rob Tibshirani, 1993, *An Introduction to the Bootstrap* (Chapman & Hall).

Fama, Eugene F., 1991, Efficient capital markets: II, *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *The Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *The Journal of Finance* 51, 55–84.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *The Journal of Political Economy* 81, 607–636.

Feng, Guanhao, Stefano Giglio, and Dacheng Xiu, 2017, Taming the factor zoo, Chicago Booth working paper.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2015, Trading costs of asset pricing anomalies, Fama-Miller working paper.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

French, Kenneth R, 2008, Presidential address: The cost of active investing, *The Journal of Finance* 63, 1537–1573.

Freyberger, Joachim, Andreas Neuhierl, and Michael Weber, 2016, Dissecting characateristics nonparametrically, Chicago Booth School of Business working paper.

Geary, Roy C., 1935, The ratio of the mean deviation to the standard deviation as a test of normality, *Biometrika* 27, 310–332.

Ghosh, Anisha, Christian Julliard, and Alex Taylor, 2016a, What is the consumption-capm missing? an information-theoretic framework for the analysis of asset pricing models, *The Review of Financial Studies* 30, 442—504.

Ghosh, Anisha, Christian Julliard, and Alex P Taylor, 2016b, An information-theoretic asset pricing model, Technical report, London School of Economics.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Goyal, Amit, 2012, Empirical cross-sectional asset pricing: A survey, *Financial Markets and Portfolio Management* 26, 3–38.

Green, Jeremiah, John R. M. Hand, and Frank Zhang, 2014, The remarkable multidimensionality in the cross-section of expected U.S. stock returns, SSRN working paper.

Green, Jeremiah, John R.M. Hand, and X. Frank Zhang, 2017, The characteristics that provide independent information about average us monthly stock returns, *Forthcoming in The Review of Financial Studies* .

Hand, John R.M., and Jeremiah Green, 2011, The importance of accounting information in portfolio optimization, *Journal of Accounting, Auditing & Finance* 26, 1–34.

Hanna, J. Douglas, and Mark J. Ready, 2005, Profitable predictability in the cross section of stock returns, *Journal of Financial Economics* 78, 463–505.

Harvey, Campbell R., and Yan Liu, 2015, Lucky factors, SSRN working paper.

Harvey, Campbell R., Yan Liu, and Heqing Zhu, 2015, ...and the cross-section of expected returns, *The Review of Financial Studies* 29, 5–68.

Hastie, Trevor, Robert Tibshirani, and Martin Wainwright, 2015, *Statistical Learning with Sparsity: The Lasso and Generalizations* (CRC Press).

Haugen, Robert A., and Nardin L. Baker, 1996, Commonality in the determinants of expected stock returns, *Journal of Financial Economics* 41, 401–439.

Hou, Kewei, Chen Xue, and Lu Zhang, 2014, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Hou, Kewei, Chen Xue, and Lu Zhang, 2017, Replicating anomalies, Technical report, National Bureau of Economic Research.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *The Journal of Finance* 48, 65–91.

Kandel, Shmuel, and Robert F. Stambaugh, 1995, Portfolio inefficiency and the cross-section of expected returns, *The Journal of Finance* 50, 157–184.

Kogan, Leonid, and Mary H. Tian, 2013, Firm characteristics and empirical factor models: A data-mining experiment, FRB International Finance discussion paper.

Korajczyk, Robert A., and Ronnie Sadka, 2004, Are momentum profits robust to trading costs?, *The Journal of Finance* 59, 1039–1082.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2016, Interpreting factor models, Ross School of Business working paper.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2017, Shrinking the cross section, available at SSRN: https://ssrn.com/abstract=2945663.

Ledoit, Olivier, and Michael Wolf, 2008, Robust performance hypothesis testing with the Sharpe ratio, *Journal of Empirical Finance* 15, 850–859.

Lewellen, Jonathan, 2015, The cross-section of expected stock returns, *Critical Finance Review* 4, 1–44.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial Economics* 96, 175–194.

Linnainmaa, Juhani T., and Michael R. Roberts, 2016, The history of the cross section of stock returns, Technical report, National Bureau of Economic Research.

Liu, Jianan, Robert F. Stambaugh, and Yu Yuan, 2016, Absolving beta of volatility's effects, The Wharton School working paper.

Lo, Andrew W., and A. Craig MacKinlay, 1990, When are contrarian profits due to stock market overreaction?, *Review of Financial Studies* 3, 175–205.

McLean, R. David, and Jeffrey Pontiff, 2016, Does academic research destroy stock return predictability?, *The Journal of Finance* 71, 5–32.

Meinshausen, Nicolai, Lukas Meier, and Peter Buhlmann, 2009, p-values for high-dimensional regression, *Journal of the American Statistical Association* 104, 1671–1681.

Meinshausen, Nicolai, and Bin Yu, 2009, Lasso-type recovery of sparse representations for high-dimensional data, *Annals of Statistics* 37, 246–270.

Messmer, Marcial, and Francesco Audrino, 2017, The (adaptive) lasso in the zoo-firm characteristic selection in the cross-section of expected returns, university of St.Gallen working paper.

Nagel, Stefan, 2013, Empirical cross-sectional asset pricing, *Annual Review of Financial Economics* 5, 167–199.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Novy-Marx, Robert, 2015, Fundamentally, momentum is fundamental momentum, Simon Graduate School of Business working paper.

Novy-Marx, Robert, and Mihail Velikov, 2016, A taxonomy of anomalies and their trading costs, *Review of Financial Studies* 29, 104–147.

Pastor, Lubos, and Robert F. Stambaugh, 2000, Comparing asset pricing models: An investment perspective, *Journal of Financial Economics* 56, 335–381.

Politis, Dimitris N., and Joseph P. Romano, 1994, The stationary bootstrap, *Journal of the American Statistical Association* 89, 1303–1313.

Richardson, Scott, İrem Tuna, and Peter Wysocki, 2010, Accounting anomalies and fundamental analysis: A review of recent research advances, *Journal of Accounting and Economics* 50, 410–454.

Rockafellar, R. Tyrrell, 2015, *Convex Analysis* (Princeton university press).

Subrahmanyam, Avanidhar, 2010, The cross-section of expected stock returns: What have we learnt from the past twenty-five years of research?, *European Financial Management* 16, 27–42.

Tibshirani, Robert, 1996, Regression shrinkage and selection via the lasso, *Journal of the Royal Statistical Society. Series B (Methodological)* 58, 267–288.

Wasserman, Larry, and Kathryn Roeder, 2009, High-dimensional variable selection, *Annals of Statistics* 37, 2178–2201.

White, Halbert, 2000, A reality check for data snooping, *Econometrica* 68, 1097–1126.