Optimal Defaults with Normative Ambiguity

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Abstract

A large and growing literature suggests that decision-makers are more likely to select options presented to them as the default. Numerous decision-making models can potentially explain the presence of default effects. However, such models differ in their implications for the link between choices and welfare, and are difficult to distinguish empirically. In this paper, we study how alternative explanations for default effects shape conclusions about optimal policy. We utilize a simple and general framework that nests most models of default effects that have been described in the literature. The model parameterizes the degree to which default effects arise due to a welfare-relevant preference versus a mistake on the part of decision-makers. When this parameter is unknown – a situation we refer to as normative ambiguity – determining the optimal default is often impossible. We apply this framework to data on 401(k) plan contributions and find that the optimal policy is to promote active choices when less than 6 to 9 percent of employees' revealed opt-out costs (about $120 to $180) are welfare-relevant, and otherwise to minimize opt-outs.

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A growing body of empirical research finds that decision-makers are more likely to select an option when that option is presented as the default. There are numerous models of decision-making that can potentially explain this behavior; a few examples are status quo bias, limited attention, or a desire by decision-makers to avoid exerting mental effort. In most settings in which default effects are observed, these alternative decision-making models are difficult to distinguish empirically. And in many cases, decision-makers may be hetereogeneous in the reason they exhibit default sensitivity.

In this paper, we study optimal policymaking in settings in which there is uncertainty about the reason decision-makers are sensitve to a default. To do so, we propose a simple framework for modeling default effects that captures this uncertainty, and use it to derive new insights about the relationship between optimal policy and the source of observed default effects.

The starting point for our approach is the fact that for a broad class of decision-making models, the effect of defaults on behavior can be characterized in terms of two ingredients: (1) decision-makers' utility over the menu of available options, and (2) an "as-if" cost to selecting an option that is not the default. This implied cost to opting out of the default is defined so as to rationalize decision-makers' observed behavior; decision-makers behave *as if* they face an opt out cost of this magnitude. Unlike standard models, we do not impose that as-if costs actually reduce the welfare of the decision-makers who opt out of the default. Instead, we parameterize the degree to which as-if costs are normative (that is, the degree to which they enter into decision-makers' welfare). We use the phrase *normative ambiguity* to capture uncertainty in the degree to which as-if costs are normative.

Our motivation for studying this issue is that alternative candidate models of default effects imply different conclusions about the degree to which as-if costs are normative. For example, one possible explanation for default effects is that decision-makers rationally seek to avoid exerting the mental effort required to choose between non-default options. In this model, all as-if costs will be normative. Alternatively, decision-makers might seek to avoid exerting mental effort, but systematically over-estimate the amount of effort that will be required to choose between the non-default options. In this model, some – but not all – as-if costs will be normative. Finally, decision-makers might inadvertantly fail to consider making a decision in the first place, in which case none of the as-if costs will be normative. Although falsifying specific candidate models might be possible with the right data, it is difficult to conceive of a convincing empirical test for determining the share of as-if costs that enter into decision-makers' welfare. This dilemma is worsened by the fact that decision-makers may be heterogeneous with respect to the model of decision-making that explains their behavior, and hence, in the degree to which their as-if costs are normative. Because it is difficult for outside observers to determine the share of as-if costs that are welfare-relevant, normative ambiguity is likely to arise whenever default effects are observed.

We use our framework to characterize the optimal default in terms of three ingredients: the distribution of (1) decision-makers' preferences over the available options; (2) as-if costs, and (3) the share of as-if costs that are normative. Our result has a sufficient statistics flavor: when these ingredients are known, the optimal default can be determined without additional knowledge of the underlying positive model (or models) of behavior. Standard revealed preferences techniques can be used to recover the first two ingredients, but not the third. Hence, our proposed approach is to identify (1) and (2) from observed choice data, and then to determine the optimal default as a function of (3), based on the plausible range of decision-making models in the setting at hand.

We show that when as-if costs are mostly non-normative, the optimal policy induces decision-makers to make an active choice. Depending on the setting, the planner can implement this policy directly, by eliminating the presence of any default option from the decision, or indirectly, by setting as the default an option that most decision-makers will find sufficiently undesirable. In contrast, when as-if costs are mostly normative, forcing active choice is not only undesirable, doing so actually *minimizes* social welfare. Instead, we show that a better approach in such settings is to set a default that leads relatively few individuals to opt out, because then many individuals receive an option that is close to the best option for them and few individuals incur real costsof opting out. The intuition for optimal policy in this case resembles the intuition behind a rule thumb proposed in the literature to minimize opt-outs (Thaler and Sunstein, 2003); we provide conditions under which this rule of thumb is guaranteed to achieve the social optimum.

Following the presentation of our main results, we relax the assumptions of our basic model in two ways. First we relax the assumption that individuals make no mistakes other than potentially inflating opt-out costs. When the choices of active decision-makers are sub-optimal, the effect of defaults on welfare is complicated by the fact that those who do not opt out of the default may end up with a better option than if they were to have opted out. Second, we calculate our main welfare effects with the addition of variable as-if costs rather than fixed as-if costs only, so that choosing an option further away from the default incurs larger as-if costs. Such variable costs allow our welfare framework to nest some additional behavioral models of default effects, such as anchoring and adjustment models. We show that our key findings regarding the desirability of active choices or minimizing opt-outs are largely unchanged in this extension to the model.

We illustrate our approach by applying it to data on employee contribution decisions to a 401(k) retirement plan. We characterize the optimal default as a function of the degree to which the as-if costs implied by employees' observed default sensitivity is normative. For the firm we study, we show that one cannot identify the optimal policy without taking a stance on the fraction of employees' as-if costs that are normative. The critical threshold in our data is whether the normative share of as-if costs is sufficiently low, i.e.less than 6 to 9 percent of total as-if costs, which corresponds to \$120-\$180 for the median employee. When the normative share of as-if costs is below this threshold, the optimal plan design is one that induces employees to make an active contribution decision. In contrast, when the normative share of as-if costs exceeds this threshold, the optimal policy is to set the default at the contribution rate that minimizes employee opt-outs, which, in this context, corresponds to the contribution rate that maximizes the employer match.

Optimal policy therefore turns on the question of whether normative opt-out costs are higher or lower than the estimated threshold. We discuss additional exercises that could shed some light on this question. In the 401(k) setting we study, such exercises suggest that normative opt-out costs are so low that active choice is likely to be optimal. However, we find that uncertainty as to the normative componentof opt-out costs tilts the optimal policy toward minimizing opt-outs, since the desirability of active choice is much more sensitive than minimizing opt-outs to the resolution of normative ambiguity.

Our results contribute to a growing literature on the welfare economics of default options. A recent paper that is closely related to ours is Bernheim, Fradkin and Popov (2015) ("BFP"), which analyzes the welfare economics of 401(k) plan defaults. BFP consider a range of potential positive models for default effects in the 401(k) context and derive tools for analyzing welfare within each model. The authors then apply these results to data from a large employer and evaluate the optimal default under each model. They find that the optimal default contribution rate is quite stable between models for the firms in their data.

Our results complement and build on BFP in several important respects. First, the general framework we develop applies to a broad class of models (both within and outside the 401(k) plan context) that includes the specific positive models they consider. Whereas their approach allows for some degree of normative ambiguity as to the proper welfare criterion within a given positive model, ours accommodates normative ambiguity both within and between alternative positive models, thus imposing significantly less restrictive assumptions about the underlying model of decision-making. In addition, because our framework is not tied to a specific positive model, it can easily incorporate heterogeneity in the model that explains an observed default effect. This innovation allows us to shed light on the generic reason why BFP find such stability in the optimal default across models: in the general class of models we consider, normative ambiguity will tend not to affect the optimal default when the set of feasible policies is sufficiently restricted (i.e., when policies that induce active choices are ruled out) and preferences over contribution rates are sufficiently well-behaved. In addition, the generality of our approach dramatically simplifies some key features of the problem relative to the prior literature, making our framework transparent and easy to apply.

The second way in which we build on BFP is by expanding the policy space under consideration. In many settings (including 401(k) contributions), policymakers have the ability to force decision-makers to make a choice without any default. In other settings, policymakers have the ability to set the default to a sufficiently undesirable option that decision-makers will be motivated to opt-out. We show that when either of these policies is feasible, determining the optimal policy is impossible without resolving normative ambiguity to at least some degree.[1] Our approach allows us to explicitly identify the map from normative judgements to optimal policy in this expanded policy space. This expanded focus yields important policy payoffs: in the 401(k) context we consider, we find that the optimal policy plausibly takes the form of promoting active choice, rather than setting a default to the contribution rate that maximizes the employer match (as BFP concluded).

Two other papers are also closely related to our own. Carroll et al. (2009) was the first to study when policies that force active choice are preferable to setting a default. Within a model of time inconsistency, they show that the desirability of active choice depends on the degree of time inconsistency that decision-makers exhibit. We extend this result beyond the specific decision-making model Carroll et al. consider to models in which default effects arise for reasons unrelated to time inconsistency. More recently, in independent work, Chesterley (2017) also studies the welfare effect of default options. Chesterley's setup is similar to ours in some respects, but complementary in focus. For example, he assumes the social planner has perfect information about the extent to which decision-makers' sensitivity to the default reflects a welfare-relevant cost, and the only positive model he considers is one in which default effects are magnified because of present bias. Here too, our contribution lies in highlighting the role of model uncertainty and normative ambiguity in shaping the conclusions that can be drawn about optimal policy.

Another related strand of the literature attempts to disentanle various mechanisms for default effects. A few recent papers examine the implications of inattention for estimates of switching costs, often in the context of choosing a health insurance plan (Abaluck and Adams, 2017; Heiss et al., 2016). Using different strategies, both of these papers estimate that as-if costs are in the thousands of dollars without accounting for attention, which is consistent with the estimates from pension plans we discuss below, but that removing the effect of inattention the cost of opting out of the default is in the hundreds of dollars. Another recent paper, Blumenstock, Callen and Ghani (2017), conducts an experiment to test the mechanisms for default effects in a savings context, finding that the cognitive costs of choosing a plan appear to be the largest driver of default effects. These results can inform the normative judgements policy-makers must make, but they do not resolve normative ambiguity. For instance, believing that large estimates of as-if costs from default effects are entirely driven by inattention would imply that the planner must then determine whether and to what extent paying attention incurs a normative cost. Understanding the role of normative judgements in policy is therefore complementary to understanding the mechanisms for default effects.

[1] BFP do discuss the possibility of penalizing decision-makers who select the default, which is in effect a method of forcing active choice. However, like Carroll et al. 2009, below, their analysis considers this question only within the context of a single positive model (time inconsistency). Hence, it does not reveal the general role that normative ambiguity plays in shaping the desirability of this approach.

The remainder of the paper is laid out as follows: Section 1 sets out our model. Section 2 develops tools for comparing welfare between two defaults and derives a formula for the optimal policy. Section 3 considers policiespromoting active choice. Section 4 considers the rule of thumb to select the default that minimizes the number of opt-outs. Section 5 illustrates our results using data on 401(k) plan contribution defaults. Section 6 considers extensions to our basic results incorporating mistaken active choices and a more flexible notion of decision costs. Section 7 concludes.

1 Model

1.1 Notation and Assumptions

Consider a population of decision-makers of measure 1. Decision-makers choose from a fixed menu X, where $x_i \in X$ denotes the option chosen by individual i. One option from the available menu is presented to decision-makers as the default, which we label by $d \in X$. Decision-makers have well-behaved preferences over the elements of X, represented by utility function $u_i(\cdot)$.[2] To facilitate welfare analysis, we assume that $u_i(\cdot)$ is cardinal and comparable across individuals. Preferences over X do not depend on the default.

Individual behavior is characterized by the solution to the following optimization problem:

$$x_i(d) = \arg\max_{x \in X} \; u_i(x) - \gamma_i \, 1_{\{x \neq d\}} \tag{1}$$

where $\gamma_i \geq 0$ for all i. We will refer to γ_i as the *as-if cost* to selecting an option that is not the default. Let $x_i^* = \arg\max\limits_{x \in X} u_i(x)$ denote the choice that maximizes (1) when $\gamma_i = 0$. We assume that decision-makers indifferent between selecting the default and a different option will select the default. Under these assumptions, behavior is given by

$$x_i(d) = \begin{cases} x_i^* & u_i(x_i^*) - u_i(d) > \gamma_i \\ d & u_i(x_i^*) - u_i(d) \leq \gamma_i \end{cases} \tag{2}$$

It will be useful to define an index of the degree to which an individual prefers opting out of the default, $a_i(d) = u_i(x_i^*) - u_i(d) - \gamma_i$. We will refer to a decision-maker with $a_i(d) > 0$ as *active* at default d and a decision-maker with $a_i(d) \leq 0$ as *passive* at default d.[3] We denote the cumulative distribution of $a_i(d)$ over

[2] Note that the differentiability of $u_i(x)$ may fail in some relevant applications. For example, in the context of default contribution rates to 401(k) plans with an employer match, $u(x)$ might exhibit an interior kink point at the contribution rate at which the match kicks in or at which the match is maximized. We discuss this further below.

[3] Note that a decision-maker who "actively" considers each option in the choice set before settling on the option that happens to be the default would still be referred to as "passive" in our terminology. In addition, we assume in equation (3), below, that such a decision-maker's welfare would be the same as that of a decision-maker who selected the default option without

the population of decision-makers at a given default by $F_{a;d}$.

Our main results will apply to the class of models generating behavior that can be represented by (2). Masatlioglu and Ok (2005) derives necessary and sufficient restrictions on behavior that a model with this representation must satisfy.[4] For example, (2) requires that if a decision-maker would choose x over y when y is the default, she must also choose x over y when x is the default. Similarly, if a decision-maker is active when x is the default and passive when y is the default, she must select y over x when choosing between the two when no default is available.

Equation (2) describes individual behavior. The following equation characterizes individual welfare in our model:

$$w_i(x, d) = u_i(x) - \pi_i \, \gamma_i \, \mathbf{1}_{\{x \neq d\}} \tag{3}$$

where $\mathbf{1}_{\{x \neq d\}}$ indicates whether the decision-maker selects an option other than the default, and $\pi_i \in [0, 1]$ reflects the degree to which the as-if costs are normative – that is, the extent to which they affect the decision-maker's welfare.[5] Using the language of Kahneman, Wakker and Sarin (1997), one can think of the maximand in (1) as "decision utility" and the utility function in (3) as "experienced utility." When $\pi_i = 1$, a decision-maker's sensitivity to the default is rational. When $\pi_i = 0$, default sensitivity represents a complete mistake; the decision-maker behaves as if selecting a non-default option would reduce his welfare, but if he were to actually select a non-default option, his welfare would not decrease. When $\pi_i \in (0, 1)$, the decision-maker exhibits too much sensitivity to the default; it would be rational for him to exhibit at least some sensitivity, but his behavior implies that the welfare reduction from opting out is greater than it would actually be.[6]

considering any alternatives. This assumption is innocuous for purposes of deriving the optimal default under our model, since a decision-maker who considers each option even when his most-preferred option is the default would also consider each option under alternative defaults.

[4] For ease of exposition, we focus on a slightly less general representation than the one implied by the axioms considered by Masatlioglu and Ok (2005). Our results are unchanged when using the more general representation associated with the axioms described in that paper.

[5] One might extend our approach to settings in which $\pi_i > 1$, which may occur, for example, when opt-out costs are not fully salient.

[6] Close readers of BFP may wonder about the difference between the role of π in our model and the role of "frame-dependent weights" in theirs. The idea behind frame-dependent weights is that within certain positive models, the extent to which a decision-maker accounts for the welfare-relevant portion of opt-out costs will vary based on the choice environment (i.e., the "frame"). For example, if we assume that default effects are generated by present-bias, we would observe a different degree of the welfare-relevant opt-out costs reflected in the decision-maker's behavior depending on whether the opt-out decision was made during the same time period in which the opt-out costs were to be incurred (as opposed to during a prior period). If an observer wished to remain agnostic about whether behavior in one frame or another frame better represented decision-makers' preferences, one approach for doing so would be to use the framework developed by Bernheim and Rangel (2009) to construct bounds on welfare that reflect uncertainty about which set of observed choices should be used to infer preferences. The use of frame-dependent weights by BFP reflects this idea – it captures uncertainty about the proper perspective on welfare for a decision-maker when a given positive model implies that the decision-maker's behavior will vary based on some condition in the decision-making environment. Mechanically, frame-dependent weights enter into BFP's analysis in a similar way that π enters into our analysis, but the interpretation and use of the two concepts is quite different. In particular, frame-dependent weights reflect uncertainty in welfare stemming from uncertainty about the proper perspective on welfare within a given positive model. In contrast, we primarily use π to capture different implications for welfare stemming from variation between alternative positive models. More importantly, in their empirical application, BFP assume a particular value for their frame-dependent weights that strikes them as ex ante reasonable; our approach is to remain agnostic about the share of as-if costs that are normative to

We denote a decision-maker's indirect utility by $v_i(d) \equiv w_i(x_i(d), d)$. Aggregate social welfare under default d is given by

$$W(d) \equiv \int_i v_i(d)\, di.$$

An *optimal default* $d^* \in X$ is an option that yields the highest social welfare when presented as the default, $W(d^*) \geq W(d)\ \forall d \in X$.

To summarize, the decision-maker behaves as if selecting the non-default option incurs utility cost γ_i. However, selecting the non-default option in fact reduces the decision-maker's welfare by only $\pi_i\gamma_i$. Because the social welfare function incorporates γ_i only to the extent of π_i, the model generates a wedge between behavior and welfare whenever $\pi_i \neq 1$. For this reason, we label $\pi_i\gamma_i$ the *normative opt-out cost* and $(1-\pi_i)\gamma_i$ the *behavioral opt-out cost.*

1.2 Relationship to Positive Models of Default Effects

In this sub-section we briefly review alternative behavioral models that have been proposed to explain default effects and discuss the extent to which they do or do not map into our framework. The main insight is that although many behavioral models are consistent with our representation, each implies a different conclusion regarding the share of the as-if costs that are normative (π).

1.2.1 Real Opt-Out Costs

The real opt-out costs model is defined by $\pi_i = 1$. Decision-makers select from among the available options according to their preferences over the available items (u_i), while rationally accounting for the welfare-relevant costs associated with selecting an option that is not the default. These costs might include monetary costs, such as administrative fees for selecting a non-default option, or non-monetary costs such as the hassle or mental effort required to determine one's most-preferred option from the available menu. Although the latter category of costs are not present in neoclassical models, to the extent they are welfare-relevant, it is rational for decision-makers to account for them when determining whether to opt out of the default. Because $\pi_i = 1$, this positive model implies that decision utility (2) and experienced utility (3) are identical.

1.2.2 Status Quo Bias

Another proposed explanation for default effects is that decision-makers are biased towards following the status quo, and interpret the default option to be a continuation of the status quo (Masatlioglu and Ok,

highlight how assumptions of this type shape the welfare conclusions that emerge.

2005). Decision-makers in this model follow a psychological heuristic in which they behave as if following the status quo is associated with some additional benefit $b_i \geq 0$:

$$x_i(d) = \arg\max_{x \in X} \; u_i(x) + b_i 1_{\{x=d\}} \tag{4}$$

Calling the status quo effect a "bias" suggests that this propensity to avoid deviating from the status quo option does not actually increase decision-makers' welfare:

$$w(x_i, d) = u_i(x) \tag{5}$$

The fact that b_i affects behavior but not welfare is what differentiates this positive model from the real opt-out costs model described above. It is easy to see that status quo bias maps into our framework with $\gamma_i = b_i$ and $\pi_i = 0$.

1.2.3 Endowment Effect

A related possibility is that default effects may be driven by an endowment effect, in which decision-makers perceive themselves as endowed with the default option and exhibit reluctance to exchange that endowment for other options Tversky and Kahneman (1991). Whether this additional reluctance enters into decision-makers' welfare is controversial (see Zeiler, 2017, for a discussion of this point). Behaviorally, default effects driven by the endowment effect can be modeled in the same way as default effects driven by status quo bias. When the endowment effect is fully normative, $\pi_i = 1$; when it is fully a bias, $\pi_i = 0$. It is also easy to imagine the endowment effect operates partly as a non-standard preference and partly as a bias, in which case $\pi_i \in (0, 1)$.

1.2.4 Quasi-Hyperbolic Discounting

In many cases, the as-if costs implied by observed default effects appear implausibly large. Consequently, a number of papers have considered behavioral models in which decision-makers behave as if the normative opt-out costs associated with a decision were magnified (i.e., $\pi_i < 1$). One way in which researchers have done this is by incorporating present-bias into a model of default effects (Carroll et al., 2009; Bernheim, Fradkin and Popov, 2015).

To illustrate how present bias fits into our framework, suppose that the decision-maker decides whether to opt-out from the default in the first period. In the second and all future periods, the decision-maker receives flow utility from the option she selected in the previous period, and decides again whether to opt out from the default. We assume for simplicity that opt-out costs and flow utility functions are fixed across periods;

allowing individuals to realize a new, potentially lower, opt-out cost in future periods is a straightforward extension (see Carroll et al., 2009).[7]. Because opt-out costs and flow utility are fixed, the individual faces the same decision problem in each period and will make the same choice in each period. In this framework, we can therefore think of $u_i(x)$ utility for some option x received in perpetuity. As in Laibson (1997), $\delta_i \in (0,1]$ denotes the discount rate and $\beta_i \in (0,1)$ denotes the degree of present-bias. The contemporaneous cost of opting out is denoted by c_i.

Suppose first that the agent is *sophisticated,* so that she correctly anticipates her future opt-out decisions. In this case, choices are described by:

$$x_i(d) = \arg\max_{x \in X} \; \delta_i \beta_i u_i(x) - c_i 1_{\{x \neq d\}} \tag{6}$$

and welfare is described by:

$$w(x_i, d) = \delta_i \, u_i(x) - c_i 1_{\{x \neq d\}} \tag{7}$$

Normalizing these preferences shows that (6) and (7) are equivalent to (1) and (3), with $\gamma_i = \frac{c_i}{\delta_i \beta_i}$ and $\pi_i = \beta_i$.

Next suppose that the individual is *naïve*, so that she may choose not to opt out today but expect to opt out at some point in the future. As in BFP, we consider the case of *partial naïveté*, with the degree of naïveté summarized by $\kappa_i \in [0,1]$. To evaluate her utility in the next period (after a negligible delay) if she opts out, the agent places weight κ_i on the case in which she decides whether to opt out in that period according to her long-run preferences ($\beta = 1$), and weight $(1 - \kappa_i)$ on the case in which she continues to be present-biased indefinitely (and thus continues to opt-out). The perceived payoff to selecting the defaultis is derived by BFP and given by:

$$\beta_i \kappa_i \max\{\delta_i u_i(x^*) - c_i, \;\delta_i u_i(d)\} + \beta_i (1 - \kappa_i) \delta_i u_i(d)] \tag{8}$$

As before, the agent believes that if she opts out, she receives $\beta_i \delta_i u_i(x^*) - c_i$. Comparing these two and simplifying, the agent opts out if and only if

$$u_i(x^*) - u_i(d) < \frac{1 - \beta_i \kappa_i}{\beta_i - \beta \kappa_i} \frac{c_i}{\delta_i}. \tag{9}$$

This model therefore simplifies to our costly opt-out model with $\gamma_i = \frac{1-\beta_i\kappa_i}{\beta_i - \beta_i\kappa_i} \frac{c_i}{\delta_i}$ and $\pi_i = \frac{\beta_i - \beta_i \kappa}{1 - \beta_i \kappa}$. Note that

[7] The main difference in this extension is that there is potentially an option value to waiting for a lower cost in order to opt out in a later period.

when the agent is fully naive, i.e. $\kappa_i = 1$, the agent will procrastinate indefinitely and never opt out. In this case one would estimate empirically that the as-if costs were arbitrarily large for such an agent (or whatever fraction of such agents there are in the population), $\gamma_i \to \infty$, and, though they are never incurred, such costs would be totally irrelevant for welfare, $\pi_i = 0$.

1.2.5 Inattention

Another potential explanation for default effects is that some decision-makers neglect to make an active choice, and therefore fail to consider either the utility of the available options or the (real or perceived) opt-out costs associated with selecting the non-default option (Chetty, 2012; Goldin and Lawson, 2016). Following Masatlioglu, Nakajima and Ozbay (2012) we model inattention by supposing that decision-makers maximize utility over some subset of the available options, $\Gamma_i(X, d) \subseteq X$, where Γ_i represents what Masatlioglu, Nakajima and Ozbay refer to as an *attention filter:*

$$x_i(d) = \arg \max_{\Gamma_i(X,d)} \; u_i(x)$$

The following intuitive restriction on the possibilities for Γ_i permits us to import this model into our framework:

$$\forall i, \; \Gamma_i(X, \, d) \in \{\{d\}, \, X\}$$

In words, the individual either pays attention only to the default (passive choice) or she pays attention to the full menu (active choice).

Closing the model requires specifying a process by which $\Gamma_i(X, d)$ is determined. There are two intuitive possibilities. One is a heuristic model of attention, in which Γ_i is exogenous to the utility stakes of the decision being considered. In this model there are simply two (exogenously determined) types of agents: attentive choosers ($i \in \mathbf{A}$) and inattentive choosers ($i \notin \mathbf{A}$):

$$\Gamma_i(X, d) = \begin{cases} X, & i \in \mathbf{A} \\ \{d\}, & i \notin \mathbf{A} \end{cases}$$

This behavior maps into our model with $\gamma_i \in \{0, \infty\}$ and $\pi_i = 0$.

Alternatively, the set of options to which a decision-maker is attentive may depend on the utility gain from choosing actively. Let $\tilde{\gamma}_i$ denote the perceived utility cost to making an active choice (e.g., mental effort) and let $\tilde{\pi}_i \tilde{\gamma}_i$ denote the actual utility costs to doing so. Welfare is given by:

$$w_i(X,d) = \begin{cases} u_i(x_i^*) - \tilde{\pi}_i \tilde{\gamma}_i, & \Gamma_i = X \\ u_i(d), & \Gamma_i = \{d\} \end{cases}$$

Individual i chooses to be active if the perceived utility gains from doing so exceed the associated costs:

$$\Gamma_i(X,d) = \begin{cases} X, & u_i(x^*) - u(d) > \tilde{\gamma}_i \\ \{d\}, & \text{otherwise} \end{cases}$$

It is apparent that this model is equivalent to the general model of default sensitivity laid out above, where the ambiguity over the welfare consequences of following the default is simply pushed back a level to the welfare consequences of choosing actively or passively: $\gamma_i = \tilde{\gamma}_i$ and $\pi_i = \tilde{\pi}_i$.

1.2.6 Combinations of the Above Models

In practice, default effects may be generated by combinations of the above models, in which decision-makers are more likely to select the default option partly because doing so avoids a normatively relevant cost (e.g., mental effort) and partly due to a bias or heuristic. In such cases, the as-if costs are neither fully normative nor fully behavioral, $\pi_i \in (0,1)$. Similarly, decision-makers may be heterogeneous with respect to the decision-making model that explains the source of their default sensitivity.

1.2.7 Anchoring Effects

A possible mechanism by which defaults shape behavior is through a psychological anchoring effect, in which the default induces decision-makers to select an option closer to the default than they would otherwise choose (Tversky and Kahneman, 1974). Models of defaults as anchors cannot be represented using the opt-out cost representation that is our focus because they imply that the default potentially affects the behavior of all decision-makers, not only those who ultimately select it.[8]

Luckily, it is sometimes possible to distinguish anchoring models of defaults from opt-out cost models of default effects by investigating whether defaults induce peaks or troughs in the options near to them (Bernheim, Fradkin and Popov, 2015). Although it is possible that defaults operate through anchoring effects in certain contexts, the empirical evidence reviewed in section 1.3 suggests that there are many contexts in which the opt-out cost models appear to better fit the data. We nevertheless consider an extension to our framework that can incorporate anchoring effects in Section 6.2.

[8] Technically, models of anchoring violate the axiom that Masatlioglu and Ok (2005) label Status Quo Independence (SQI*).

1.2.8 Defaults as Advice

Decision-makers might select the default option if they themselves are uncertain over which option is most consistent with their preferences and they believe that the planner's choice of default provides an informative signal as to which option is best for them. The optimal policy prescriptions we consider are geared towards a world in which the planner lacks ex ante information as to which option is most consistent with decision-makers' preferences, suggesting that rational (well-informed) decision-makers would not treat the default signal as having any informational content. Nonetheless, decision-makers might mistakenly construe the default as a suggestion by the planner and treat it as containing some informational content. One possibility is that decision-makers treat the suggestion as "take it or leave it" advice – i.e., they either follow the suggestion exactly or ignore it altogether, perhaps by gathering so much information on their own that the original suggestion has negligible signal value. Such a model is isomporphic to the status quo bias model when the default has no true signal value. Alternatively, decision-makers may take the suggested option into account, even if they do not accept it, and choose something closer to the default than what they otherwise would have chosen. In this case, the default affects decision-making like an anchor, where the effect of the default on a decision-maker's behavior depends on the strength of the decision-maker's prior and the perceived reliability by the decision-maker of the informational signal embodied in the choice of default.

1.3 Empirical Plausibility

In practice, it is often difficult to directly test the axiomatic foundations of particular behavioral models. With respect to models of default effects, for example, difficulties may arise because individuals have heterogeneous preferences and opt-out costs, or it may be impossible to observe the same individual choosing under alternative defaults, holding everything else fixed.

One prediction of our model that, with modest additional structure, does lend itself to testing is the idea that fewer individuals will select any given option when the default is close to that option than when the default is far from that option. Intuitively, decision-makers that prefer the option in question will be more likely to settle for the default – thus avoiding the opt-out costs – when the utility gains from selecting the non-default are relatively low. More formally, this prediction can be stated as follows:

Suppose that the menu X is ordered, and $u_i(\cdot)$ is single-peaked. Then (2) implies that for any two defaults $d^{'}$ and $d \in X$ such that $d^{'} > d$, it follows that $P(x_i(d) = x) \geq P(x_i(d^{'}) = x)$ for $x > d^{'}$, and $P(x_i(d) = x) \leq P(x_i(d^{'}) = x)$ for $x < d$.

Evidence consistent with this prediction has been documented across a range of settings, including: 401(k) contributions (e.g., Madrian and Shea 2001, Figure IIc; Choi et al., 2006, Figure 2), charitable contributions

(Altmann et al., 2016); taxi ride tips (Haggag and Paci, 2014); and even thermostat temperature settings in office buildings (Brown et al., 2013). These findings support the empirical relevance of the class of behavioral models we study. Notably, the anchoring model of defaults discussed in Section 1.2.7 makes the opposite prediction, suggesting for example that we should observe $P(x_i(d) = x) < P(x_i(d') = x)$ for $x > d' > d$, at least at values of x that are sufficiently close to d'. Note that although research such as Choi et al. (2012) that reports evidence consistent with anchoring effects does not do so in the case of default options.

2 Characterizing the Optimal Default

In this section we characterize the optimal default in terms of the components of the model described in Section 1.1.

Our first result highlights that the welfare achieved under a default can be decomposed between active and passive choosers as follows:

Lemma 1:

$$W(d) = E[u_i(x_i^*) - \pi_i\gamma_i \,|\, a_i(d) > 0]\; (1 - F_{a;d}(0)) + E[u_i(d) \,|\, a_i(d) \leq 0)\, F_{a;d}(0), \tag{10}$$

Lemma 1 simplifies the evaluation of welfare by showing that we can think of the welfare effect of a given default d in terms of two groups: (1) active choosers selecting x_i^* and incurring normative costs $\pi_i\gamma_i$, and (2) passive choosers selecting d.

Consider a change in the default from d_0 to d_1. From Lemma 1, it is apparent that such a change affects welfare directly for passive choosers, for whom it changes the option they select, and may in addition affect the composition of active and passive decision-makers (see Chesterley, 2017, for a discussion of this point). To study the welfare effects of this change, it will be useful to partition the population into four groups of decision-makers based on their behavior under the old default (d_0) and the new default (d_1):

Group Name	**Behavior when default is:**		**Characterization**	**Fraction of Population**
	d_0	d_1		
Always Active	$a_i(d_0) > 0$	$a_i(d_1) > 0$	$u_i(x_i^*) - \max\{u_i(d_0),\ u_i(d_1)\} > \gamma_i$	$p(AA)$
Always Passive	$a_i(d_0) \leq 0$	$a_i(d_1) \leq 0$	$u_i(x_i^*) - \min\{u_i(d_0),\ u_i(d_1)\} \leq \gamma_i$	$p(PP)$
Active-to-Passive	$a_i(d_0) > 0$	$a_i(d_1) \leq 0$	$u_i(x_i^*) - u_i(d_0) > \gamma_i \geq u_i(x_i^*) - u_i(d_1)$	$p(AP)$
Passive-to-Active	$a_i(d_0) \leq 0$	$a_i(d_1) > 0$	$u_i(x_i^*) - u_i(d_1) > \gamma_i \geq u_i(x_i^*) - u_i(d_0)$	$p(PA)$

The table describes how the composition of these four groups isdetermined in terms of the behavioral parameters from equation (2). We denote the fraction of the population in each of these groups by $p(j)$ for $j \in \{AA,\ PP,\ PA,\ AP\}$. Intuitively, the passive-to-active group is composed of decision-makers for whom the original default is close enough to their preferred option to acquiesce to, but the new default is not, $i \in PA \implies u_i(d_0) > u_i(d_1)$. Similarly, decision-makers in the active-to-passive group are sufficiently dissatisfied with the old default to make an active choice, but content to choose passively under the new default, $i \in AP \implies u_i(d_1) > u_i(d_0)$.

The following proposition uses this decomposition to characterize the welfare effect of a change in default policy.

Proposition 1 *For any two defaults* $d_0,\ d_1 \in X$:

$$W(d_1) - W(d_0) = E\left[u_i(x^*) - u_i(d_0) - \pi_i\gamma_i \,|\, PA\right]\, p(PA) - E\left[u_i(x^*) - u_i(d_1) - \pi_i\gamma_i \,|\, AP\right]\, p(AP)$$

$$+E\left[u_i(d_1) - u_i(d_0) \,|\, PP\right]\, p(PP) \tag{11}$$

Several features of (11) are notable. First, the always-active choosers, group AA, do not enter into the welfare effect of the default change. These individuals incur the same normative cost ($\pi_i\gamma_i$) and make the same choice (x_i^*) under both defaults. Second, for those who are passive at d_0 and active at d_1 (group PA), the change induces a utility gain from choosing actively, $u_i(x_i^*) - u_i(d_0)$, but also causes them to incur normative cost $\pi_i\gamma_i$. The first term in equation (11) reflects the change in social welfare from these individuals. The second term is the analogous contribution from individuals who are active at d_0 but not at d_1 (group PA). The third term reflects individuals who are passive under both defaults (group PP). The overall effect on this group's welfare depends on whether they (on average) prefer the new default or the original default.

One instructive special case concerns the situation when all individuals prefer the same option, $x_i^* = x^*$ for all i. Not surprisingly, the optimal policy is such settings is to set the default equal to decision-makers' most-preferred option:

Corollary 1.1 *Suppose* $x_i^* = x^*$ *for all* i. *Then* x^* *is the optimal default.*

When everyone prefers the same option, that option is the optimal default, regardless of the π_i's. Intuitively, complete homogeneity in preferences eliminates normative ambiguity because it eliminates the need to compare the welfare of active choosers with the welfare of passive choosers (see e.g. equation (10)); this is because no one incurs (potentially normatively relevant) opt-out costs.

Note that Proposition 1 holds regardless of the nature of the menu X – it might be discrete, continuous, or of multiple dimensionality. The next result considers situations where X is a real interval, which occurs in many applied contexts.

Proposition 2 *Let X be any interval in $\mathbb{R}$, and suppose $u_i(x)$ is everywhere differentiable for all i. If d^* represents an interior solution to the optimal default problem, the following first-order condition is satisfied:*

$$\begin{aligned} 0 = W^{'}(d^*) \quad &= \quad E[(1-\pi_i)\gamma_i \,|\, a_i(d^*) = 0,\ u_i'(d^*) < 0]\, f_{a|u'<0}(0)\, F_{u'}(0) \\ &- \quad E[(1-\pi_i)\gamma_i \,|\, a_i(d^*) = 0,\ u_i'(d^*) > 0]\, f_{a|u'>0}(0)\ (1 - F_{u'}(0)) \\ &+ \quad E\left[u'(d^*) \,|\, a_i(d^*) < 0\right]\, F_{a;d^*}(0) \end{aligned} \tag{12}$$

where $f_{a|u'>0}$ is the probability density function of $a_i(d^*)$ conditional on $u_i'(d^*) > 0$; $F_{u'}$ is the cumulative density function of $u_i'(d^*)$; and, as above, $F_{a;d^*}$ is the cumulative density function of $a_i(d^*)$.

As in Proposition 1, the three terms represent the welfare effects of the default change on decision-makers in the AP, PA, and PP groups. The first term represents the PA group; a decision-maker for whom $a_i(d) = 0$ and $u_i^{'}(d) < 0$ will be passive at the original default and active following a marginal increase in the default (which they prefer slightly less than the original default). Similarly, the second term represents decision-makers in the AP group, who are slightly better off after the marginal increase in the default, and therefore more willing to acquiesce to it. Decision-makers in the third group, with $a_i(d) < 0$, remain passive even after a small change in the desirability of the default.

How does the normative share of as-if costs affect the optimal default? Proposition 2 highlights that πmatters for weighting the relative welfare effects of a change in the default for decision-makers in the PA and AP groups against the welfare effects for decision-makers in the PP group. When $\pi_i = 1$, the welfare effect depends only on decision-makers in the PP group, who experience a marginal change in welfare from moving to a slightly better or slightly worse default. The reason why is that decision-makers in the PA and AP groups behave as though they are indifferent between following the default and making an active choice ($a_i(d) = 0$). When $\pi_i = 1$ for decision-makers in these groups, that behavior fully reflects their welfare, and the envelope theorem implies that their welfare is not affected by a policy change that makes them active or passive.

In contrast, when $\pi_i < 1$, the welfare of the PA and AP groups will be weighted more heavily in determining the optimal default. The reason why is that decision-makers in the PA group were choosing to remain passive when their welfare would have been higher had they become active, and are better off after being induced to become active by the change in default. Conversely, those in the AP group would have higher welfare from being active, even after the change in the default induces them to become passive. The further π is from 1, the larger are these effects. In addition, although the fraction of the population in the AP and PA groups will generally be smaller than the fraction of the population in the PP group for marginal changes in the default, decision-makers in the former groups experience a *discrete* welfare change from the change in the default, whereas those in the PP group experience only a marginal change in their welfare from ending up with a slightly better or slightly worse default. We explore further how the optimal policy depends on π_i in the next two Sections.

3 Forcing Active Choices

The framework developed thus far can shed light on the policy, often discussed in the literature, of forcing decision-makers to make active choices. In practice, such policies might take the form of (1) setting the default to an option so undesirable that the vast majority of decision-makers are likely to opt out, or (2) simply requiring decision-makers to make an active choice (e.g. Carroll et al., 2009). As an example of the former approach, one could imagine setting intestacy law – law governing inheritances in the absence of a will – so that individuals who die without leaving a will would have all of their assets taxed at a 100% rate. An example of the latter approach would be requiring new employees to make an active decision about how much to contribute to their 401(k) plans as a condition of employment.[9] We will refer to both of these policies as "penalty defaults" in the spirit of Ayres and Gertner (1989).

We define a *penalty default* as some option $d_p \in X$ for which $a_i(d) > 0$ for all i. It is straightforward to show that whenever $u_i(d_p)$ is sufficiently low for all individuals, d_p will be a penalty default. Comparing a change in the default to a penalty default d_p from an arbitrary alternative d using Proposition 1, we have that

$$W(d_p) - W(d) = E[u_i(x^*) - u_i(d) - \pi_i \gamma_i | PA]\, p(PA) \tag{13}$$

Because individuals are never passive at d_p, only the first term of (11) matters for welfare.

The following proposition, which stems from (13), highlights the importance of resolving normative ambiguity when policies that promote active choice are available:

Proposition 3 Suppose that X is any menu and there exists a penalty default $d_p \in X$.

(3.1) *There exists a threshold $\underline{\pi} \in [0, 1)$ such that $\pi_i \leq \underline{\pi}$ for all i implies d_p maximizes social welfare.*

[9] Another possible way to induce active choices is to reduce the costs of opting out of a default, considered by Chesterley (2017), or by taxing decision-makers who select the default option, considered by BFP.

(3.2) There exists a threshold $\overline{\pi} \in (0,1]$ such that $\pi_i \geq \overline{\pi}$ for all i implies d_p minimizes *social welfare.*

Proposition 3 shows that when forcing active choice is a feasible policy, it is never possible to identify the optimal default without taking a stance on whether or to what degree opt-out costs are normative.[10] Moreover, the stakes are high: forcing active choices can be either the best or the worst possible outcome for social welfare, depending on what π turns out to be. Note that Proposition 3 applies to any menu, not only real-valued X.

To interpret (3.1), start from the benchmark case where $\pi_i = 0$ for everyone. In that case, setting a penalty default to force active choices is a first-best default: everyone receives the option they prefer and no one incurs any normative opt-out costs. The result in (3.1) generalizes this idea to the case where π is small but not necessarily zero. In situations where most but not all individuals choose actively under the penalty default, the consideration of welfare becomes somewhat murkier. In such cases, selecting a penalty default to encourage active choice may have strong negative effects on the (relatively small) share of individuals who nevertheless choose passively under the penalty default. This provides a rationale why forcing choices without any default may sometimes be a better means of encouraging active choices than setting a penalty default.

We can see from (13) that when the π_i's are large and as-if costs of opting out are normatively relevant, the active choice policy considered in (3.1) may not be desirable. The implication of (3.2) and is that requiring active choices may be *extremely* undesirable for high values of π. Note that when $\pi_i = 1$ for all i, the right-had side of (13) must be negative; this is because individuals who are passive at default d have $u_i(x^*) - u_i(d) < \gamma_i$. Such individuals reveal a preference for choosing passively. Hence, when $\pi_i = 1$ for all i, forcing active choice is not only dominated by other potential defaults that allow for some passive choice, but in fact forcing active choice is dominated by *every other potential default.* The result in (3.2) generalizes the same reasoning to sufficiently high values of π that may nevertheless be less than 1.

4 Minimizing Opt-Outs

A commonly discussed rule of thumb for setting defaults, first proposed by Thaler and Sunstein (2003), is to select as the default whichever option minimizes the number of decision-makers who opt-out (i.e., who select any non-default option as their choice). Translated into our notation, the opt-out minimizing default, d^m, is defined as the value of d that maximizes: $W^m(d) \equiv F_{a;d}(0)$, where, as above, $F_{a;d}(\cdot)$ is the cumulative

[10] The case in which x_i^* is homogenous is a knife's edge exception to this statement. In that case, setting $d = x^*$ achieves the highest possible social welfare for any value of π. If, however, $\pi_i = 0 \ \forall i$ and there is any heterogeneity in x_i^*, forcing active choice becomes socially preferable.

density function of $a_i(d)$.

Evaluating this expression at two possible defaults, d_0 and d_1, it is straightforward to derive that under W^m, social welfare is improved by changing the default from d_0 to d_1 if and only if $p(PA) < p(AP)$. That is, the default change must cause more decision-makers to become passive than it causes to become passive. To illustrate how this condition relates to welfare in our model, note that we may decompose (11) as:

$$\begin{aligned} W(d_1) - W(d_0) = \quad & \underbrace{(p(AP) - p(PA))\,\overline{\pi\gamma}}_{1} \\ & + \underbrace{p(AP)\,E\left[\pi_i\gamma_i - \overline{\pi\gamma}\,|\,AP\right] - p(PA)\,E\left[\pi_i\gamma_i - \overline{\pi\gamma}\,|\,PA\right]}_{2} \\ & + \underbrace{E[u_i(x^*) - u_i(d_0)|PA]\,p(PA) - E[u_i(x^*) - u_i(d_1)|AP]\,p(AP)}_{3} \\ & + \underbrace{E[u_i(d_1) - u_i(d_0)|PP]\,p(PP)}_{4} \end{aligned} \tag{14}$$

where $\overline{\pi\gamma} = E[\pi_i\gamma_i]$.

As an initial matter, note that term 1 compares $p(AP)$ and $p(PA)$ exactly as in W^m. Individuals who are active at d_1 but not d_0 (group PA) will incur opt-out costs under d_1 valued at $\pi\gamma$, which has a negative effect on their welfare. The opposite is true for the AP group, who incur costs under d_0 but not d_1. Term 1 therefore favors whichever default minimizes opt-outs. Therefore, when all of the other terms in 14 are negligible or have the same sign as the first term, the opt-out minimizing default coincides with the optimal default.

However, the other terms in 14 represent factors that may cause the optimal default to diverge from the default that minimizes opt-outs. Term 2 reflects the fact that even when the size of the AP and PA groups are the same, the magnitude of the normative opt-out costs of each may differ. Similarly, term 3 reflects that, aside from whatever cost they incur from being active, the AP group receives a utility gain from choosing x^* under d_1 instead of the default under d_0, and similarly for the PA with respect to d_0. The magnitude of these utility gains and losses from changes in whether decision-makers opt-out may be different between the AP and PA group. Finally, the fourth term captures how the change in the default affects welfare for the decision-makers who remain passive. Notably, the preferences of this group are completely neglected by the minimizing opt-outs rule, even though the choices of this group are directly affected by what the default is. When the preferences of group PP differ systematically from those of the PA and AP groups, the default selected by W^m may be suboptimal because it fails to reflect the preferences of the decision-makers who remain passive under both defaults.

The relative importance of the terms in equation 14 depend on the magnitude of π and the relative sizes of the various groups. When the PP group is small, the overall welfare effect will be dominated by the other terms. If, in addition, $\pi_i = 1$, we can conclude that the net welfare effect for the PA group is negative, i.e., $E[u_i(x^*) - u_i(d_0) - \gamma_i|PA] < 0$. The opposite is true for the AP group, for whom we will have a net positive welfare effect when $\pi = 1$. Thus, when π is large, the size of the PP group is small, and when the magnitude of the welfare effects on the AP and PA groups is similar, the minimizing opt-out rule will tend to approximate the optimal default. In contrast, when π is small, or when the PP group is large and tends to prefer defaults that induce many decision-makers to opt-out, the minimizing opt-outs rule of thumb may perform poorly.

The following proposition provides sufficient conditions under which minimizing opt-outs yields the optimal default:

Proposition 4 *Suppose that $X = [x_{min}, x_{max}] \subseteq \mathbb{R}$ and that:*

(A4.1) *As-if costs γ_i are distributed independently of x_i^*.*

(A4.2) *Preferences are given by $u_i(x) = u(x - x_i^*)$ for some map $u : \mathbb{R} \to \mathbb{R}$, with $u'(0) = 0$, $u'' < 0$ and $u(c) = u(-c)$ for any c.*

(A4.3) *x_i^* follows a single-peaked and symmetric distribution about some mode x^m.*

Under these conditions, there exists a threshold $\overline{\pi} \in (0, 1]$ such that $\pi_i \geq \overline{\pi}$ for all i implies that the optimal default is the default that minimizes opt-outs.

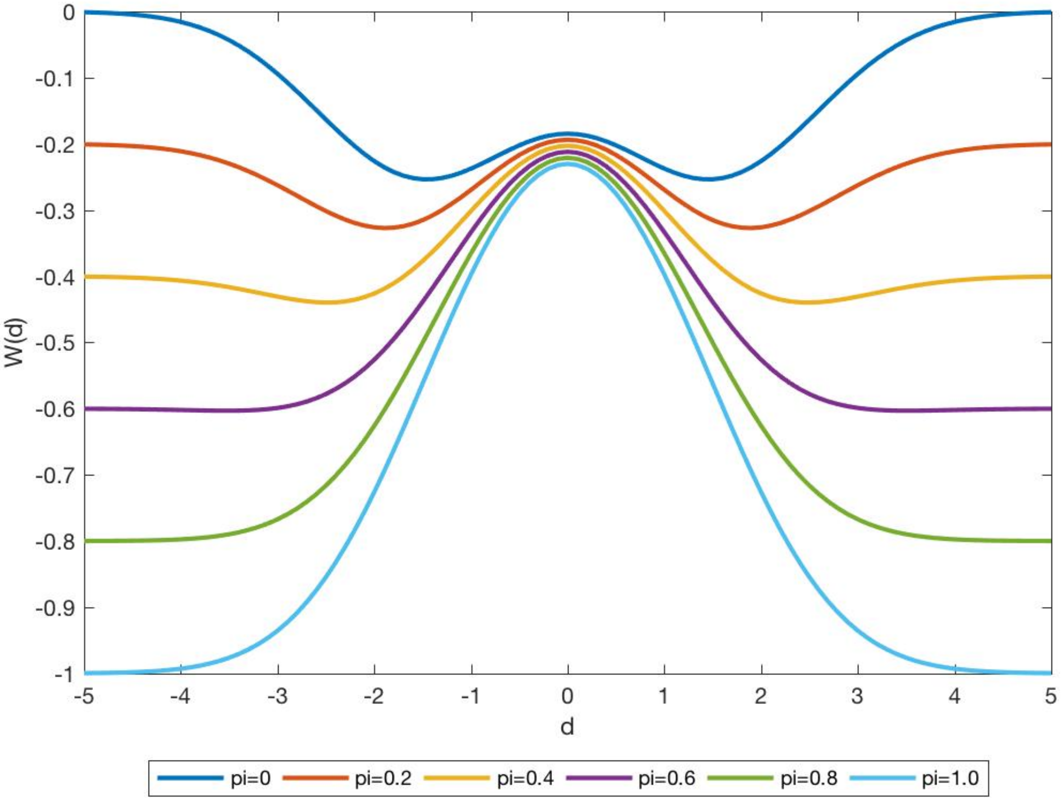
Proposition 4 provides conditions under which minimizing opt-outs yields the optimal policy. Loosely speaking, these conditions occur when as-if costs are sufficiently normative, the distributions of the underlying behavioral parameters are independent, and decision-makers' preferences are well-behaved. We can understand the sufficient conditions in terms of their implications for the various terms in Equation (14). In particular, (A4.1) rules out a relationship between as-if costs γ_i and preferences that could cause the sign of term 2 in Equation (14) to have the opposite sign of term 1. Next, (A4.2) makes the comparison of the utility differences in the last two terms of Equation (14) straightforward, as all heterogeneity in $u_i(\cdot)$ derives from heterogeneity in the distribution of optimal choices x_i^*. Third, (A4.3) rules out features of the distribution of x_i^* that could pull the optimal default away from the opt-out-minimizing default via the third and fourth terms in Equation (14). For example, suppose x_i^* were distributed according to a single peaked distribution around some x^m, but with an extra point mass at $x' < x^m$. In that case, it would be possible that opt-outs were minimized at x_m, but that switching the default from x^m to some $d' < x^m$ would increase welfare by giving the point-mass of individuals (who are assumed to be passive under $d = x^m$ and d') an option closer to their preferred option x'. Together, (A4.2) and (A4.3) guarantee that the effect of a change in the default in opt-outs among the PA and AP groups in (14) is a strong signal about the change in welfare of the AP group. Finally, when all or nearly all of the as-if costs are normative (A4.4), the average utility gain associated with selecting a more-preferred option is guaranteed to outweigh the utility costs associated with making an active choice, as discussed above.

Together, Propositions 3 and 4 imply that, under the regularity conditions given by (A3.1) - (A3.3), the optimal policy rule is relatively simple. When π is sufficiently large, the social planner should minimize opt-outs. When π is sufficiently small and a penalty default is available, the social planner should force active choices. For intermediate values of π, other policies may be optimal. When the regularity conditions in (A3.1)-(A3.3) are not satisfied, minimizing opt-outs may not be the optimal policy for large π, but one can use the expression in Equation 14 to correct for asymmetries in $u(\cdot)$ or the distribution of x^*, or a correlation between as-if costs γ_i and optimal choices x_i^*.

Figure (1) plots social welfare for a stylized model that satisfies (A4.1)-(A4.3). To fill out the model, we assume that π_i is uniform across decision-makers, x_i^* follows a Gaussian distribution in the population, and $u(x - x_i^*)$ is quadratic – i.e., $u(x - x_i^*) = -\alpha(x - x_i^*)^2$ for suitably chosen $\alpha > 0$. To interpret the figure, recall that forcing active choice is equivalent in the model to selecting a default sufficiently extreme that all decision-makers choose to opt out, and, because we plot $W(d)$ assuming that $x^m = 0$, setting a default of zero will minimize opt-outs. The figure show shows that as π varies, the default that minimizes opt-outs remains a local optimum; the feature that varies with π is the relative attractiveness of forcing active choice. As suggested by the figure, the optimal policy in this stylized setting takes the form of a threshold rule around some threshold $\overline{\pi} \approx 0.2$. When $\pi > \overline{\pi}$, setting the default to minimize opt-outs is optimal. Instead, when $\pi < \overline{\pi}$, the optimal policy is to force active choice.

Our results in this section also shed additional light on previous results from the literature. Specifically, Carroll et al. (2009) consider the optimal policy within a model of default effects similar to the one we describe in Section 1.2.4, where present bias magnifies opt-out costs relative to true opt-out costs and the individual is a sophisticated quasi-hyperbolic discounter according to some (homogeneous) factor β. The authors add some additional structure to the model, namely a uniform distribution of costs over some finite interval (c.f. our Assumption A3.1), quadratic loss preferences (c.f. our assumption A3.2), and a uniform density of optimal choices x_i^* over some finite interval (c.f. our assumption A3.3). Within this model, they show that active choices are optimal when (1) β is sufficiently low and (2) optimal choices (x_i^*) are sufficiently heterogeneous. Conversely, when (1) β is sufficiently high and (2) preferences are less heterogeneous, the optimal choice will tend to be a "center default" that minimizes opt-outs (c.f. x^m in Proposition 4). Recall that in the sophisticated present bias model with the long-run view of welfare, $\pi = \beta$. Our Proposition 3 therefore illuminates the generic reason why active choices are optimal when β is low: *these are the*

Figure 1: Social Welfare with Quadratic Preferences and Gaussian x_i^*



Note: This figure plots the function $W(d)$ for a simulated model in which $u(x - x_i^*) = -\alpha(x - x_i)^2$ with $\alpha = 0.25$, x_i^* follows a Gaussian distribution with mean zero and standard deviation one, and $\gamma_i = 1$ for all individuals. We assume π_i is homogeneous across decisionmakers, and we plot $W(d)$ for several values of π. Setting $d = E[x_i^*] = 0$ will minimize opt-outs in this model, and setting an extreme default will force active choices. The simulation thus shows that the optimal policy follows a threshold rule over π, so that minimizing opt-outs is optimal for high values of π, and forcing active choices is optimal for low values of π.

situations when as-if costs are deemed normatively irrelevant. Similarly, our Proposition 4 illuminates the generic reason why minimizing opt-outs is optimal when β is close to 1.

5 Empirical Illustration

This section illustrates our results using data on 401(k) plan contribution decisions. We choose to focus on this setting for two reasons. The first is that it is a setting in which defaults have been shown to affect behavior and in which the choice of default is of significant practical importance. The second is that it has been the focus of a recent and influential literature on optimal default policy; holding the setting constant in our analysis relative to this prior literature helps clarify the value added by our approach.

To preview our results, we draw two substantive contributions from this analysis. First, we generalize the result from BFP that the uncertainty over optimal defaults is small when the range of policies considered does not include policies that promote active choice. Specifically, we estimate the mapping between values of π and the optimal default. This allows us to conclude that the optimal policy BFP identifies applies not only for the illustrative models they consider, but rather for all behavioral models within a more general class (i.e., any model that is consistent with the opt-out cost representation). Our second contribution is to show that normative ambiguity *does* generate meaningful uncertainty as to optimal policy when the policy space is expanded to include policies that promote active choice. When the as-if costs associated with default effects are mostly irrelevant from a welfare perpsective, the optimal policy is to adopt a penalty default (e.g., setting a very high default contribution rate) or to require active choice as a condition of employment. In contrast, these policies are dominated when even a modest fraction of the observed as-if opt-out costs are normatively relevant. Finally, we discuss additional reasoning that may help to resolve the normative ambiguity over π, and we discuss how the planner's uncertainty over π would matter for welfare.

Apart from these substantive conclusions about optimal policy, the illustration highlights two of the chief benefits of our approach, namely the simplicity with which it can be applied and the transparency between our assumptions and the welfare conclusions that emerge.

The data we use consists of 401(k) contribution rates for newly eligible employees of three firms first anlayzed by Choi et al. (2004, 2006) and Beshears et al. (2008). We describe the relevant features of these data below, and refer readers to the earlier studies for additional detail.

The first step in our approach is to estimate the distribution of parameters in the opt-out cost representation of behavior. To do so, we follow BFP and rely on a fitted structural as-if costs model of behavior to contribution rate data for each employer.[11] The model incorporates the employer match (which is either 50

[11] Specifically, we use the estimated structural model from the BFP publicly available replication files, as the underlying

or 100 percent, depending on the firm) for up to 6 percent of employee earnings. The cap on the employer match creates a large kink in the budget constraint at 6 percent of earnings, which induces bunching in the optimal contribution rate at 6 percent. The model also predicts bunching at the corner solutions of 0 and the maximum contribution rate (15 or 25 percent, depending on the firm). As in BFP, we assume the distribution of the parameter governing employees' optimal savings rate is independent of the distribution of as-if costs. As-if costs are assumed to follow a distribution that allows some fraction of decision-makers to have zero costs, and the rest draw a cost from an exponential distribution. Finally, we evaluate $W(d)$ using equivalent variation relative to a benchmark in which all individuals receive their most-preferred option x_i^* without incurring any costs. As the units of x_i are in percentages of annual salaries contributed to a 401(k) plan, the units of welfare thus correspond to the percentage of annual salary that would make individuals receiving x_i^* (without any costs) willing to switch to a default d. As welfare is lower under a given default d than under the benchmark, equivalent variation is typically negative.

Throughout this analysis, we assume a uniform value of π_i for the entire population. The calculation of welfare for any distribution of π_i, including those in which it covaries with other heterogeneous parameters, is straightforward; restricting our anlaysis to a uniform value of π allows us to display important insights in a simple fashion.

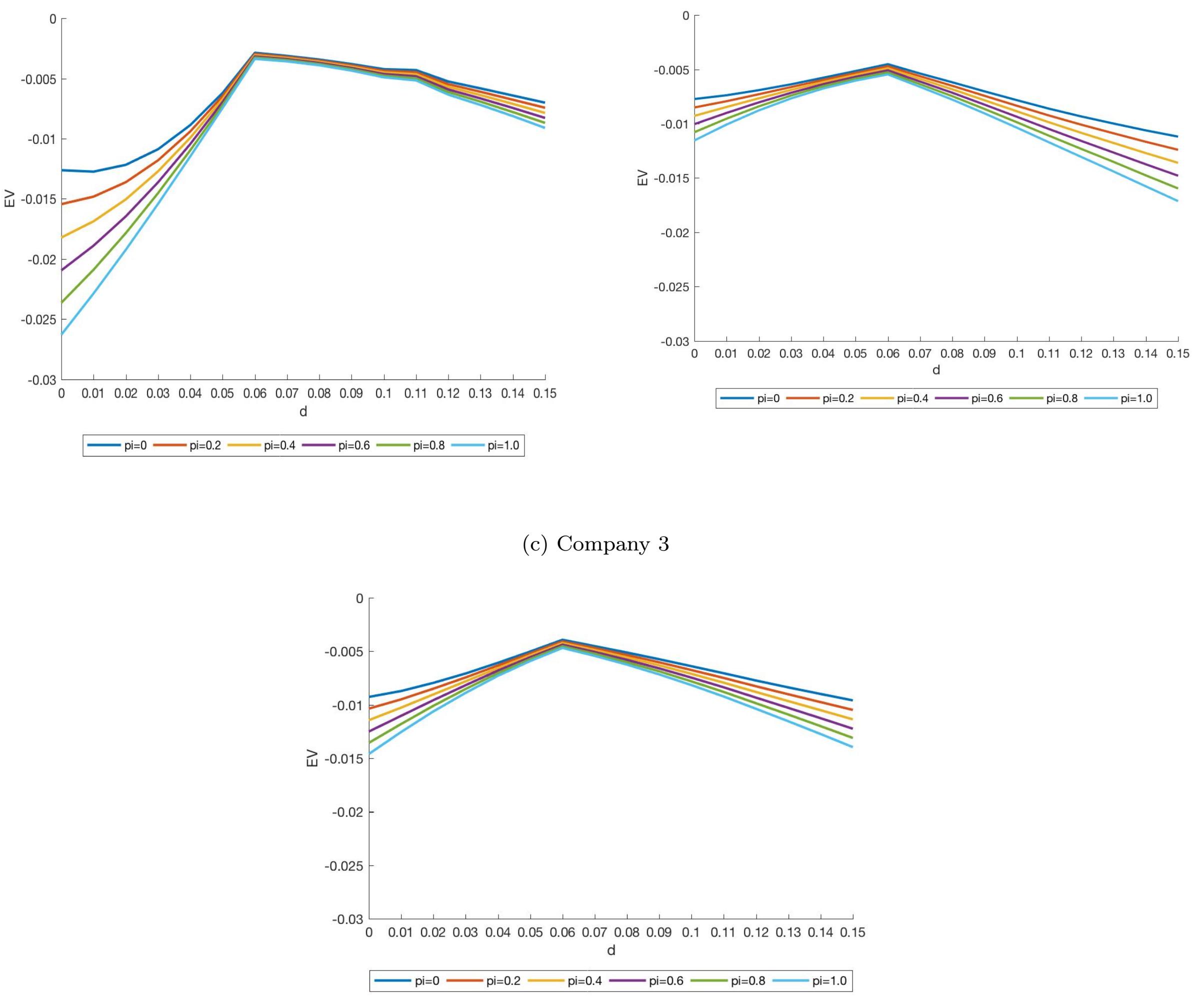
Our first analysis is to solve for the optimal default as a function of the normative relevance of the as-if costs, π. Figure 2 depicts equivalent variation for alternative default contribution rates between 0 and 15 percent of earnings, for values of π ranging from zero to one. We find that regardless of π, the optimal policy is to set a default of 6 percent of earnings, which is where, for all three firms, employer matching contributions are maximized. This analysis generalizes the main finding in BFP to any positive model of default effects consistent with the opt-out cost srepresentation, and to any view of welfare within such a model.

We next extend the analysis to consider policies that promote active choice. We add this policy choice by extending the space of contribution rate defaults we consider. Proposition 3 suggests that extending the set of feasible policies in this way will introduce uncertainty into the optimal policy. Indeed, extending the policy space from that considered in Figure 2 yields exactly this result. As shown in Figure 3, extremely high defaults dominate when π is low, but the 6 percent default dominates when π is moderate or large. As described above, the intuition for this result is that high defaults prompt many decision-makers to make an active choice, and when π is sufficiently low, the welfare cost of making this choice is low as well.[12] There is

contribution rate data are proprietary and not available to us.

[12] A similar phenomenon is evident in Figures A.7 to A.12 in the Online Appendix of BFP, who do not focus on it because the extreme contribution rates exceed statutory limits on 401(k) contributions. However, setting an extremely high default contribution rate is not the only way to force employees to make active choices – the employer could simply require it as a condition of employment, as in Carroll et al. (2009). In addition, .statutory limits in this policy area may be modified (see e.g.

Figure 2: Equivalent variation for different default contributions, by the welfare relevance of as-if costs


(a) Company 1

(b) Company 2

(c) Company 3

Note: This figure depicts welfare in equivalent variation units for five values of π (pi) ranging from zero, in which as-if costs are not at all welfare-relevant, to one, in which costs are fully welfare relevant, for three different firms. We observe that regardless of the value of π, welfare is maximized by setting the default contribution at 6 percent of earnings, where employer matching contributions are maximized. The relatively large increase in EV as the default is increased from zero to 6 percent in Company 1 is attributable to the relatively large 100 percent employer matching contributions in Company 1, compared to 50 percent matching contributions in Companies 2 and 3. The maximum contribution in Company 3 is 25 percent. We bound the domain of d at 15 percent for consistency and clarity; doing so does not change the implications for welfare.

a strong qualitative similarity between the stylized model in Figure 1 and the estimated model in Figure 3; the main difference between these is the non-differentiable spike in the latter at the 6 percent default, which is caused by the kink in the budget constraint from employer matching contributions.

Figure 4 compares welfare under the 6 percent default to an active choice regime, for values of π ranging from zero to one. First, when $\pi = 0$, the active choice regime leads to exactly the same outcome as our benchmark in which all individuals costlessly receive x_i^*, so its equivalent variation is zero. Consistent with Propositions 3 and 4, we find that at higher values of π, the 6 percent, employer-contribution-maximizing default dominates the active choice default. The 6 percent default is also the default that minimizes opt-outs.[13] The optimal policy thus takes the form of a threshold rule: active choices dominate below the threshold, the 6 percent default dominates above the threshold. The threshold below which active choice dominates is about $\pi = 0.06$ for Company 1, $\pi = 0.09$ for Company 2, and $\pi = 0.08$ for Company 3.

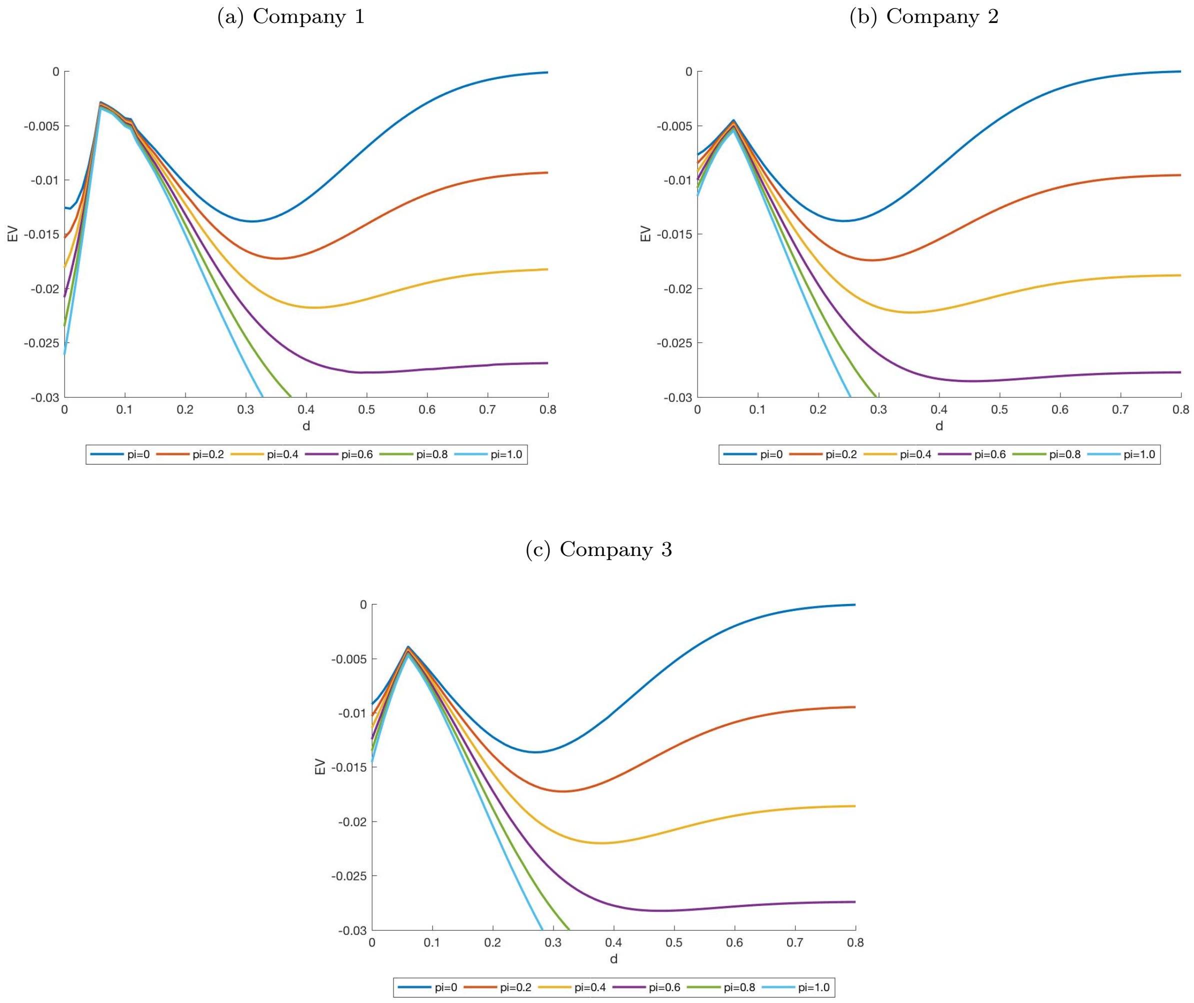
Discussion

What is the optimal policy with respect to default 401(k) contribution rates? As described in the prior section, the answer depends on the degree to which employees' observed default sensitivity reflects costs that are normative. Recall that γ reflects the magnitude of costs that would be required to rationalize employees' observed sensitivity to the default. For an employee with the median salary in the data ($40,000), the estimated distribution of γ_i for the 60 percent of employees estimated to have strictly positive costs has a mean of $3,386. The mean cost for *all* employees is about 5.07 percent of their salary, or $2,028 at the median salary. Ten percent of all employees are estimated to have an as-if cost that exceeds 15 percent of their salary, i.e. $6,000 for an employee with median salary. . For an employee at company 3, we found that the threshold value of π for which active choice is optimal is $\pi = 0.08$. Hence, determining optimal policy in this setting requires determining whether the mean reduction in welfare that an employee would incur by opting out of the default is at least 8% of $2,028, or $162, which corresponds to about 0.4 percent of the median employee salary. When opting out of the default reduces welfare by at least $162 on average, the optimal policy is to set the default to the contribution rate that minimizes opt-outs, which here corresponds to the contribution rate that maximizes the employer match, or 6 percent. In contrast, if the normative component of opt-out costs is below $162 on average, the optimal policy takes the form of setting the default to an extremely high contribution rate (perhaps as high as 80%), or, more realistically, requiring employees to make an active choice about how much they wish to contribute.

the Pension Protection Act of 2006, which allowed employers to automatically enroll employees in 401(k) plans).

[13] The conditions imposed on preferences by the structural model are similar to the regularity conditions laid out in Proposition 4, notably including the independence of the determinant of optimal savings rates and as-if opt-out costs. The main difference derives from the presence of a large kink in the budget constraint due to the employer match, and the other corner solutions at zero and the maximum contribution.

Figure 3: Equivalent variation for different default contributions for an expanded menu of contribution rates



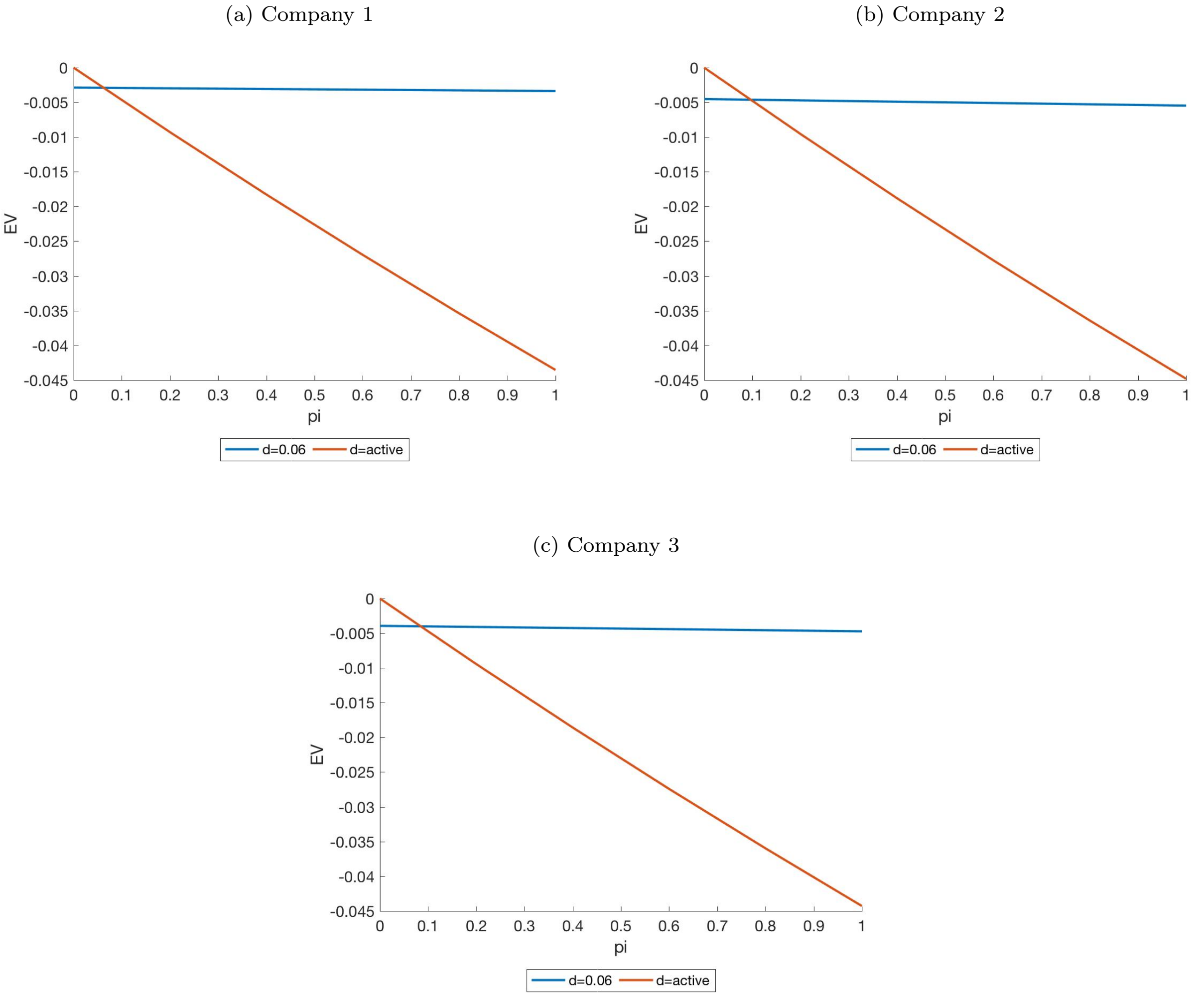
(a) Company 1

(b) Company 2

(c) Company 3

Note: This figure depicts welfare in equivalent variation units for five values of π (pi) ranging from zero, in which as-if costs are not at all welfare-relevant, to one, in which costs are fully welfare relevant, for three different firms. We extend the range of possible options all the way to 80 percent. We observe that, unlike in Figure 1, normative ambiguity occurs for low values of π.

Figure 4: Welfare Under Active Choice versus Minimizing Opt-Outs in 401(k) Plans


(a) Company 1

(b) Company 2

(c) Company 3

Note: This figure compares welfare under the 6 percent contribution default (d=0.06) with welfare under an active choice regime (d=active) for three firms. We observe that at low values of π, the active choice regime leads to higher welfare.

Identifying the normative component of opt-out costs poses a methodological challenge, since, as described above, it cannot be directly inferred from observed choice data. Nonetheless, some conclusions can be drawn in cases in which other relevant information is known. For example, one might imagine surveying employees about the amount they would be willing to pay to to avoid the process of opting out of a 401(k) default and actively choosing their own contribution rate. Although stated preferences may be biased for other reasons, they may shed light on welfare when decision-makers' observed choices do not. Alternatively, an observer might estimate the normative component of opt-out costs from choices made in other contexts, which do not exhibit the same biases from default effects. For example, one might look to the price decision-makers are willing to pay to avoid filling out other forms or making other decisions of similar complexity, using data on the price of paid tax preparers or financial planners. Finally, an observer might estimate the number of non-work hours required to actively choose one's contribution rate and fill out the form, and price that time according to the employee's implied wage rate.

In the 401(k) setting we focus on, this type of analysis tends to suggest that normative componeent of as-if costs is smaller than 5 to 8 percent, which would imply that active choice is optimal. Suppose that employees value their time at \$19 per hour, which is the equivalent hourly wage rate of an employee in our sample with median salary and a 40-hour work week. The threshold value of \$162 would imply that, on average, the process of opting out and making an active choice would need to take more than 8 hours of the employee's time in order for the 6 percent default to dominate an active choice policy. This amount of time is on the high side of what most casual observers would likely consider plausible. The actual process of filling out forms and selecting an option typically takes less than an hour, though of course researching the available options to determine which one will be best takes more time. Relatedly, given the cognitive difficulty of making pension choices, the individual may value the time spent completing it at a higher rate than the \$19/hour benchmark implied by wage rates.[14] Still, if we suppose the process takes 2 hours, employees would need to value the time it takes to opt out at about \$80/hour, which seems high for an individual making \$40,000 per year. These conclusions match the intuition of others as well – BFP calibrate some of their models using parameters that effectively impose a value of $\pi = 0.01$.

Our results also imply, however that while active choice may be optimal under plausible restrictions on normative costs, it is in a sense a riskier policy than minimizing opt-outs. For company 3, Figure 4 shows that minimizing opt-outs leads to equivalent variation ranging from 0.4 to 0.5 percent of annual earnings (about \$160 to \$200 at the median earnings) relative the first-best benchmark, for values of π from zero to one. The active choice policy leads to equivalent variation ranging from 0 to 4.4 percent of annual earnings

[14] In addition, this calculation uses the employee's average wage and is therefore likely to underestimate the employee's reservation wage for marginal hours worked. On the other hand, it uses the pre-tax wage rate, which likely overestimates the true opportunity cost of the employee's time.

($0 to $1,760 at the median earnings). One could directly incorporate this idea into optimal policy analysis by formalizing the planner's uncertinaty over π and calculating expected social welfare.

Suppose the planner has some subjective probabilitiy distribution over the value of π. Given a uniform distribution over some range of π deemed plausible by the planner, we can compare the expected equivalent variation between active choices and minimizing opt-outs by integrating the difference between $W(d)$ for these two policies in Figure 4, over the relevant range of π.[15]With a uniform distribution over $[0,1]$, the expected equivalent variation from active choice is -2.3 percent of annual earnings and the expected equivalent variation of minimizing opt-outs is -0.4 percent of earnings for company 3. The expected equivalent variation of minimizing opt-outs is much higher partly as a consequence of Propositions 3 and 4: when π is small, minimizing opt-outs is sub-optimal, but remains a local optimum; in contrast, when π is large, active choice minimizes social welfare. More fundamentally, when minimizing opt-outs, π only matters for the welfare of active choosers, who tend to have low as-if costs already; when setting an active choice policy, π matters for the welfare of all decision-makers, even those with very high as-if costs. One can of course impose restrictions on the distribution of π deemed subjectively plausible using similar arguments to those above. For example, if an observer were willing to impose that the normative component of opt-out costs was no more than 0.5% of an employee's annual salary, this would correspond to a maximum value for π of approximately 0.1. With π distributed uniformly between [0, 0.1], the expected equivalent variation from active choice is -0.24 percent of earnings, compared to -0.40 percent of earnings from setting a default contribution rate of 6%.

6 Extensions

In this section we consider two extensions of the model used in earlier sections. The first extension considers the case when active choices may not maximize true welfare-relevant preferences over the menu objects, which allows us to consider situations where policymakers wish to correct some internality using a default. The second extension allows for a richer notion of opt-out costs, which allows the framework to nest more complicated positive models of default effects. We focus on re-deriving the the necessary condition for the optimal default for continuous problems, i.e. Proposition 2, which conveys the most important intuitions for these extensions. We also briefly discuss the implications of these extensions for optimal default policies.

[15] With a subjective probability distribution that is not uniform, one naturally takes a weighted intrgral of this difference where the weights are the planners subjective likelihood for each value of π.

6.1 Mistaken Active Choices

The only deviation from rational decision-making considered thus far is the presence of normatively irrelevant opt-out costs. In some settings, policymakers may believe that choices are plagued by biases unrelated to defaults. For example, in the retirement pension participation decision, present bias may cause employees to under-save even when they make active choices. We will refer to such biases as “internalities” (Herrnstein et al., 1993). The presence of internalities is frequently invoked as a justification for using defaults to “nudge” behavior (e.g., Thaler and Sunstein, 2008).[16]

To set up this extension, we first assume that individual behavior is described as above by Equation (1). Hence, whether an indivdiual is active or passive for a given default is the same as before. Now, however, we assume that individual welfare is given by

$$w_i(x) = u_i(x) + m_i(x) + \pi_i \gamma_i \{x \neq d\}, \tag{15}$$

where $m_i(x)$ is the internality imposed on the individual by his or her choice of x, i.e., the component of the welfare effect of x that is not taken into account by the decision-maker. Here, $a_i(d) > 0 \implies x_i(d) = x_i^a$. The active choice x_i^a maximizes $u_i(x)$ but not $u_i(x) + m_i(x)$ due to the internality.

We will focus on the case in which X is a real interval. We assume that both $u_i(x)$ and $m_i(x)$ are differentiable. In the pension contributions example, for an active chooser who under-saves, we would have $u_i'(x_i^a) = 0$ and $m_i'(x_i^a) > 0$. In this case, adopting a default that increases savings for that active chooser would increase his or her welfare on the margin, holding all else constant. Indirect utility and social welfare are defined as above.

We will describe how the optimal policy changes with the addition of internalities, first in general and then under some restrictions that provide additional intuition. Let $W_0^{'}(d)$ be the effect of a marginal change in the default on welfare in our original model, as derived in Proposition 2. With internalities, the analogue to this exprssion is given by:

$$\begin{aligned} W^{'}(d) \quad &= \quad W_0^{'}(d) + E\left[m_i(x_i^a) - m_i(d) \mid a_i(d) = 0,\ u_i'(d) < 0\right]\ f_{a|u'<0}(0)\, F_{u'}(0) \\ &- \quad E\left[m_i(x_i^a) - m_i(d) \mid a_i(d^*) = 0,\ u_i'(d^*) > 0\right]\ f_{a|u'<0}(0)\ (1 - F_{u'}(0)) \\ &+ \qquad\qquad E\left[m_i'(d) \,|\, a_i(d) < 0\right]\ F_{a;d}(0) \end{aligned} \tag{16}$$

[16] Prior literature on optimal defaults does consider the possibility that certain biases may inflate opt-out costs but does not allow such biases to affect decisions by active choosers. For example, both Carroll et al. (2009); Bernheim, Fradkin and Popov (2015) consider models in which present bias causes decision-makers to choose according to inflated opt-out costs, but neither model allows present bias to affect savings decisions by active choosers (whose decisions are assumed to be optimal). Relatedly, many situations in which defaults are employed as policy, such as organ donation, exhibit significant externalities. Adding externalities to our model is straightforward.

Re-writing this expression using the definitions of the active-or-passive groups from before, we have

$$\begin{aligned} W^{'}(d) \quad &= \quad W_0^{'}(d) + E\left[m_i(x_i^a) - m_i(d)\,|\,PA\right]\ P(PA) \\ &- \quad E\left[m_i(x_i^a) - m_i(d)\,|\,AP\right]\ P(AP) \\ &+ \quad E\left[m_i'(d)\,|\,PP\right]\ P(PP). \end{aligned} \tag{17}$$

There are two changes in this expression relative to the one in Proposition 2. First, the PA and AP groups exprience a discrete change in the internality from becoming active or becoming passive as the default changes. Second, the always-passive (PP) group in the third term of Equations (16) and (17) experience an additional marginal welfare effect, m_i', from the change in the default. As before, the welfare of always-active choosers does not enter into the evaluation of the welfare effect of a change in the default.

In order to compare optimal policy with and without internalities, it is instructive to place some intuitive restrictions on $m_i(x)$. The following proposition provides intuition for how the presence of internalities affects the determination of the optimal default derived above:

Proposition 5 *In the model with internalities, suppose that*

(A5.1) *Marginal internalities are constant for each individual,* $m_i(x) = m_i\ x$

(A5.2) *Preferences are given by* $u_i(x) = u(|x_i^a - x|)$ *for some map* $u : \mathbb{R}^+ \to R$, *with* $u'(0) = 0$, $u' \leq 0$, *and* $u'' < 0$.

(A5.4) *The marginal internality* m_i *is independent of* x_i^a *and* γ_i.

Let μ_m *denote the mean of* m_i *across decision-makers, let* $W_0(d)$ *denote social welfare without internalities, and let* $X(d) \equiv E[x_i(d)]$ *denote the total amount of* x *undertaken, aggregating accross all individuals. Under these conditions, if* d^* *represents an interior solution to the optimal default problem, the following first-order condition is satisfied:*

$$0 = W^{'}(d^*) = W_0^{'}(d^*) + \mu_m\, X^{'}(d^*). \tag{18}$$

Proposition 5 highlights that the optimal policy balances the concerns of the previous model, summarized by $W_0^{'}(d^*)$, with a new goal, which is to correct the internality generated by the decisions of the active choosers.[17] For example, if $\mu_m > 0$ represents the average degree of under-saving among a population of decision-makers, the optimal savings contribution default would induce more saving than when the social

[17] The assumptions under which the simple expression in Equation (18) obtains are instructive but not guaranteed. Assumption (A5.1) simply makes the problem more tractable by assuming the internality is approximately linear. Assumptions (A5.2) and (A5.3) are less innocuous, requiring that the marginal internality m_i is independent of other structural parameters governing individual behavior (though not necessarily of the *other* paramter summarizing mistakes, π_i). Relaxing (A5.2) and (A5.3) but

planner assumed no internalities were present. The larger the mean internality, μ_m, the further the deviation from the no-internality optimum.

In addition, it is straightforward to show that

$$X'(d) = E\left[x_i^a - d|PA\right]\ P(PA) + E\left[d - x_i^a|AP\right]\ P(AP) + P(PP). \tag{20}$$

Thus, the effect of a change in the default on total activity (x) has two components. First, both of the marginally active groups reduce their activity discretely – recall that $x_i^a < d$ for the PA group, and $x_i^a > d$ for the AP group. Second, the always-passive group increases their activity by a marginal amount.

To understand how optimal policy differs in this model relative to the model without internalities, suppose we initially have a default $\hat{d}$ that is optimal when no internalities are present, so $W_0^{'}(\hat{d}) = 0$. If, as in the under-saving example, $\mu_m > 0$, then a deviation from this default in whichever direction *increases* $X(d)$ would constitute an improvement in social welfare. Importantly, whether total activity increases with an increase in the default or with a decrease in the default is an empirical question. For example, one might assume that the presence of positive internalities from saving would lead the planner to prefer a higher default savings rate. Our results in this section show, however, that this intuition is only correct when there are relatively few marginally active choosers. In the case where total saving is relatively unaffected by the default, which may in fact be approximately correct in the oft-studied 401(k) setting (see e.g. Choi et al., 2004), the presence of internalities from under-saving is irrelevant for the optimal default.

In addition to affecting the optimal default, the presence of internalities is also relevant for assessing the desirability of forcing active choice. As in Section 3, consider the comparison between a penalty default d^p (under which all individuals chosose actively) and a generic default d (under which at least some individuals choose passively). Adapting Equation (13) to the case of internalities yields:

$$W(d^p) - W(d_0) = E\left[u_i(x^a) - u_i(d) - \pi_i\gamma_i + m_i(x_i^a) - m_i(d)\,|\,PA\right]\ p(PA) \tag{21}$$

Unlike the case in Proposition 3.1, it is no longer the case that a sufficiently small value of π guarnatees that d^p is the optimal default. To see why, suppose that $\pi_i = 0$ for all individuals. As before, we know that

maintaining (A5.1), it is straightforward to derive that:

$$\begin{array}{rcc} W^{'}(d) & = & W_0^{'}(d) + \mu_m X'(d) \\ & + & E[(m_i - \mu_m)(x_i^a - d)|PA]P(PA) \\ & - & E[(m_i - \mu_m)(x_i^a - d)|AP]P(AP) \\ & + & E[m_i - \mu_i|PP]P(PP) \end{array} \tag{19}$$

Equation (19) shows how to modify the expression in Proposition 5 to account for the fact that the mean marginal internality may be different accross the three groups of decision-makers we are interested in. For example, we might suppose that the PA group, who in general have low values of x_i^a, are especially bad under-savers. In this case, the second term in equation (19) would be non-negligible.

$u_i(x_i^a) > u_i(d)$. However, it may be the case that $m_i(x_i^a) < m_i(d)$. When the difference $m_i(d) - m_i(x_i^a)$ is sufficiently large for enough individuals, d will be a better default than d^p. In the under-saving example, it is possible – though not assured – that a default under which some individuals choose passively will increase total saving relative to an active choice regime to such an extent that the active choice regime is not optimal, even when as-if costs are completely irrelevant for welfare.

To summarize, incorporating mistakes by active choosers into the model means that the planner may be able to raise social welfare by choosing a default that makes those mistakes less likely to occur, and reduces the benefits of penalty defaults that cause decision-makers to choose actively. Uncertainty by the planner over the distribution of m thus creates a new difficulty for determining the optimal default that parallels the normative ambiguity caused by uncertainty over π.[18]

6.2 Variable Opt-Out Costs

Thus far we have assumed that as-if opt-out costs are constant (for a given individual) and do not depend on which non-default option the decision-maker selects. An alternative behavioral model is that defaults "pull" decision-makers towards options near the default in addition to making them more likely to select the default itself. For example, defaults may serve as an anchor (Example 1.2.7).

Ultimately, the question of whether defaults effects can be better described by including variable as-if costs in the model is an empirical question. Empirical evidence, reviewed in Section 1.3, regularly finds that increases in the default can affect choices far away from the default, suggesting that fixed costs are likely present. A variable costs model alone, such as a model of anchoring and adjustment where a higher default tends to lead to higher $x_i(d)$, would not predict, for example, that the fraction of individuals who contribute nothing to their pension would increase when the default rate of contribution is increased. Whether adding variable costs gives the model *additional* explanatory power relative to the fixed-cost-only model is more difficult to test. One possibility is to look closely at choices around the default. The fixed costs model with no variable cost predicts a "hole" in the observed distribution of choices around the default, whereas adding variable costs model predicts a "hill" around the defult when fixed costs are sufficiently low. Still, given that both fixed and variable costs are plausibly heterogeneous, separately identifying these two components of decision-makers' revealed preferences without strong assumptions about distributions of the two costs is difficult. Here, we show how the inclusion of a variable costs affects the conclusions of our main analysis, especially the desirability of active choices versus minimizing opt-outs.

[18] In some cases it is possible to identify potential internalities by varying framing, just as one can identify candidate values of π_i by varying framing. For example, with under-saving due to present bias, one can infer the size of the internality by varying the timing of (active) decisions relative to payoffs to identify the present bias parameter (β_i). Even in this example, however, using such a technique to infer the size of the internality requires adopting the long-run view of welfare, which is a normative judgement (Bernheim, 2009). See Lockwood and Taubisnky (2017) for more examples.

We focus on the case where X is a real interval. Suppose that instead of (1), individual behavior is given by

$$x_i(d) = \arg\max_{x \in X}\; u_i(x_i) - c_i(x_i - d) - \gamma 1\{x_i \neq d\}. \tag{22}$$

For simplicity, we will assume that u_i is single-peaked, with $u_i'(x_i^*) = 0$ and $u_i'' < 0$ everywhere. For this extension, we assume that the as-if cost associated with choosing a non-default option increases the further the chosen option is from d, so that $c_i'(x_i - d) \geq 0$ when $x_i - d > 0$, and $c_i'(x_i - d) \leq 0$ when $x_i - d < 0$. The as-if cost function is twice differentiable, with $c'' \geq 0$. We normalize $c(0)$ to zero. In this model individuals choose the default when passive, or $\tilde{x}(d) = \arg\max u_i(x_i) - c_i(x_i - d)$ when active. The individual is active if $\tilde{a}_i(d) \equiv [u_i(\tilde{x}_i(d)) - c_i(\tilde{x}_i(d) - d)] - u_i(d) - \gamma_i > 0$.

Similar to before, welfare is given by

$$w_i(x) = u_i(x) - \rho_i c_i(x - d) - \pi_i \gamma_i 1\{x_i \neq d\}, \tag{23}$$

where ρ_i denotes the normative relevance of variable costs $c_i(\cdot)$ and π_i the normative relevance of fixed costs as before. Indirect utility and social welfare are also defined similarly to before.

Given any change in the default, we can divide individuals into four groups as before, except now these groups are based on $\tilde{a}_i(d)$. Taking a derivative of the welfare function with respect to d, we have that the necessary condition from Proposition 2 becomes, with the addition of variable costs,

$$\begin{array}{rcrcl} 0 & = & W'(d) & = & E\left[\rho_i c_i' + (1-\rho_i) c_i' \frac{c_i''}{c_i'' - u_i''} \middle| AA\right] P(AA) \\ & & & + & E\left[(1-\rho)c + (1-\pi_i)\gamma_i \middle| PA\right] P(PA) \\ & & & - & E\left[(1-\rho)c + (1-\pi_i)\gamma_i \middle| AP\right] P(AP) \\ & & & + & E[u'(d)|PP]P(PP). \end{array} \tag{24}$$

where all components involving $c_i(\cdot)$ are evaluated at $x = \tilde{x}(d)$.

Adding variable costs changes this expression in two ways. First, the always-active choosers (AA) are affected by a change in the default. The sign of the welfare effect on an always-active chooser is positive if and only if $x_i^* < d$. For an individual with $x_i^* < d$, we will have that $x_i^* < x_i(d) < d$, and an increase in the default makes it costlier to choose an option close to x_i^*. The $\rho_i c_i'$ term of the welfare effect for members of the AA group in Equation 23 corresponds to the direct welfare effect of increasing this cost. Such an individual also increases x_i in response to this change in costs: it is straightforward to show that $\tilde{x}_i'(d) = \frac{c_i''}{c_i'' - u_i''} \in [0, 1)$. The second term of the welfare effect for the AA group corresponds to the welfare impact of this change in behavior.[19] As before, when as-if costs are fully normatively relevant for all individuals, $\rho_i = 1$, and the

[19] Note that the behavioral reponse is $\tilde{x}'(d) = 0$ when costs are linear, i.e. $c'' = 0$.

envelope theorem eliminates the indirect welfare effect from the behavioral response. However, when $\rho_i < 1$, the individual over-reacts to the increase in costs, reducing their welfare. The opposite intuition applies when $x_i^* > d$; such individuals in the AA group are made better off by an increase in the default. The second addition to the welfare calculation is the extra variable cost incurred by marginally active decision-makers in the PA and AP groups. As it changes welfare discretely when the individual switches between choosing actively and choosing passively, this component affects welfare in exactly the same fashion as the fixed cost.

Our key result that forcing active choice is optimal when default effects are driven purely by behavioral frictions will still be true in this model, but properly examining an active choice policy requires subtle reasoning here. In this model, setting an extreme default so that everyone opts out will not necessarily be equivalent to forcing active choices directly. One might naturally suppose that when forcing active choices, the planner sets no anchor, which eliminates the variable costs, whereas when a penalty default acts as an anchor, the variable costs will matter for behavior and welfare. Suppose there is a policy that forces decision-makers to make active choices and eliminates variable costs (i.e. it does not set an anchor). It is straightforward to show that such a policy will be globally optimal when π_i is sufficiently small for all individuals (regardless of ρ_i), exactly as in Proposition 3. However, whether such a policy becomes extremely undesirable when π_i and ρ_i are sufficiently high is not clear in this model, because the policy that forces active choices also eliminates the variable costs and this can improve welfare. Conversely, a penalty default will surely minimize welfare when π_i and ρ_i are sufficiently high, but due to the large distortions on active choices it may have through the variable costs, it may not be optimal when π_i and ρ_i are large.

By a very similar procedure to the one we use in Proposition 4, one can show that minimizing opt-outs is optimal when π_i and ρ_i are sufficiently large, under some regularity conditions. Specifically, we could maintain Assumptions (A4.1)-(A4.3), and add the assumption that the variable cost function is the same for all individuals, $c_i(x_i - d) = c(x_i - d)$. Under these assumptions minimizing opt-outs will still be globally optimal when default effects are driven by real components of individual welfare.

7 Conclusion

Uncertainty over the underlying positive model that generates a particular behavior is a pervasive source of difficulty in behavioral economics. Under a range of positive models of default effects, decision-makers' behavior can be described using "as-if" preferences over opt-out costs revealed by their observed choices. Revealed preference analysis can recover information about these as-if preferences, but cannot answer whether these as-if preferences accurately reflect individuals' welfare. Our analysis of the optimal default problem clarifies the conditions in which optimal policy determinations do and do not depend on the degree to which these as-if opt-out costs are normatively relevant.

In general, we find that uncertainty as to the fraction of opt-out costs that are normative – as would be associated with uncertainty over the underlying behavioral model that generates default effects – poses a serious problem for determining the optimal default. In a limited number of cases, robustness criteria like those proposed by Hansen and Sargent (2008) and Bernheim and Rangel (2009) will allow us to examine cases where optimal policy is invariant to the share of opt-out costs that are normatively relevant. However, these special cases are the exception and not the rule. They obtain only when there is little heterogeneity in decision-makers' preferences or when the policy space is restricted to rule out policies that promote active choice. Active choice policies include forcing decision-makers to make a choice without any default at all, or setting the default to an option that most decision-makers will find extremely undesirable. Alternatively, when most opt-out costs are known to be normatively relevant and preferences are sufficiently well-behaved, we show that minimizing the fraction of decision-makers' opting out of the default provides a good rule of thumb for achieving the optimal policy.

When policies that promote active choice are feasible, determining the optimal default requires making a normative judgement. Two kinds of empirical evidence can help with these judgements. The first is to use various interventions attempting to reduce or enhance default effects to doing so shed light on the positive mechanisms driving default effects, as in Blumenstock, Callen and Ghani (2017). Even with evidence regarding the mechanism driving default effects, however, one must make a normative judgement determining whether that mechanism acts by imposing normative costs or by driving a wedge between choices and welfare. The second potential strategy is to gather external evidence to make an informed judgement. This approach requires assumptions about what choices in other settings, such as those that reveal the monetary value of workers' time, or those in which workers state hypothetically how much they would accept to make an active choice, tell us about the normative relevance of as-if costs. We give examples of this type of reasoning in our examination of pension plan defaults. While both of these strategies can help the planner make an informed choice, neither can resolve the problem entirely objectively.

Several empirical phenomena in behavioral economics, such as reference dependence, can be rationalized by adding components to as-if preferences. These problems introduce similar hard questions about welfare. Is reference dependence purely a behavioral friction resulting from consumers' employing some heuristic, or does it reflect a real discontinuity in marginal welfare around the reference point? Studying the role normative judgements play in the policy determination of reference points, such as those created by statutory retirement ages in many countries (Seibold, 2017), is an important question for future research.

Finally, the particular difficulty in determining optimal policy we identify is new but not unprecedented. The modern theory of optimal redistributive taxation also incorporates normative judgements, in this case judgements about the social value of equity (Mirrlees, 1971; Saez, 2001). Barring clever attempts to ascertain the value of equity itself by revealed preference (Kuziemko et al., 2015) – an analogue of which might be attempted in behavioral problems – determining the optimal redistributive tax policy requires normative judgements that cannot be resolved via revealed preference alone. Often these judgements are parameterized, using welfare weights or the curvature of a social welfare function. Indeed the whole of classic optimal policy problems may be divided into those where revealed preferences alone yield the optimal policy via Pareto comparisons (Ramsey, 1927; Kaldor, 1939; Hicks, 1939), and those such as the optimal redistributive tax problem where normative judgements are necessary. Our results here suggest that a similar division, between welfare comparisons that require normative judgements and those that do not, will be fruitful for "behavioral" optimal policy problems.

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A Appendix: Proofs

Lemma 1:

$$W(d) = E[u_i(x_i^*) - \pi_i\gamma_i | a_i(d) > 0]\ (1 - F_{a;d}(0)) + E[u_i(d) | a_i(d) \leq 0)\, F_{a;d}(0),$$

where $F_{a;d}(\cdot)$ denotes the cumulative density function of $a_i(d)$.

Proof: From the definition of the social welfare function we know that $W(d) = E[v_i(d)]$. By the law of iterated expectations,

$$W(d) = E[v_i(d) | a_i(d) > 0] P(a_i(d) > 0) + E[v_i(d) | a_i(d) \leq 0] P(a_i(d) \leq 0)$$

We know from the consumer's problem and the definition of $a_i(d)$ that 1) $a_i(d) \leq 0 \implies x_i(d) = d$ and 2) $a_i(d) > 0 \implies x_i(d) = x_i^* = \arg\max u_i(x)$. Substituting these into $v_i(d) = w_i(x_i(d), d) = u_i(x_i(d)) - \pi_i\gamma_i 1\{x_i(d) \neq d\}$ gives the result. ■

Proposition 1: *For any two defaults $d_0, d_1 \in X$:*

$$W(d_1) - W(d_0) = E[u_i(x^*) - u_i(d_0) - \pi_i\gamma_i | PA]\, p(PA) - E[u_i(x^*) - u_i(d_1) - \pi_i\gamma_i | AP]\, p(AP) + E[u_i(d_1) - u_i(d_0) | PP]\, p(PP).$$

Proof: We know that $W(d_1) - W(d_0) = E[v_i(d_1) - v_i(d_0)]$. We partition individuals into the four groups (PA, AP, PP and AA) and apply the law of iterated expectations to express the change in welfare as a probability-weighted sum over these four groups. As before, $a_i(d) \leq 0 \implies x_i(d) = d$ and 2) $a_i(d) > 0 \implies x_i(d) = x_i^* = \arg\max u_i(x)$. In the PA group, $a_i(d_1) > 0$ so $v_i(d_1) = u_i(x_i^*) - \pi_i\gamma_i$,and $a_i(d_0) \leq 0$, so

$v_i(d_0) = u_i(d_0)$. Thus $E[v_i(d_1) - v_i(d_0)|PA] = E[u_i(x^*) - u_i(d_0) - \pi_i\gamma_i|PA]$. Proceeding similarly for the other four groups and substituting in the resulting expressions yields the desired result. ■

Proposition 2: *Let X be any interval in $\mathbb{R}$. If d^* represents an interior solution to the optimal default problem, the following first-order condition is satisfied:*

$$\begin{aligned} 0 = W^{'}(d^*) \quad &= \quad E[(1-\pi_i)\gamma_i|a_i(d^*) = 0,\ u_i'(d^*) < 0]\, f_{a|u'<0}(0)\, F_{u'}(0) \\ &- \quad E[(1-\pi_i)\gamma_i|a_i(d^*) = 0,\ u_i'(d^*) > 0]\, f_{a|u'<0}(0)\ (1 - F_{u'}(0)) \\ &+ \quad E\left[u'(d^*)\,|\,a_i(d^*) < 0\right]\ F_{a;d^*}(0) \end{aligned}$$

where $f_{a|u'>0}$ is the probability density function of $a_i(d^*)$ conditional on $u_i'(d^*) > 0$; $F_{u'}$ is the cumulative density function of $u_i'(d^*)$; and, as above, $F_{a;d^*}$ is the cumulative density function of $a_i(d^*)$.

Proof: One can obtain this result by direct calculation of the derivative of the welfare function, as divided into active and passive choosers in Lemma 1 (i.e. expressing the expectations as integrals and applying Leibniz rule). One can also obtain the result by plugging in $d_1 = d_0 + \Delta d$ in Proposition 1, taking the limit as Δd approaches zero, plugging in the definitions of the primitives, and noting that the PA and AP groups now both have $a_i(d) = 0$, which implies that $u_i(x^*) - u_i(d) = \gamma_i$ by construction. ■

Proposition 3 Suppose that there exists a penalty default $d_p \in X$.

(3.1) *There exists a threshold $\underline{\pi} \in [0, 1)$ such that $\pi_i \leq \underline{\pi}$ for all i implies d_p maximizes social welfare.*

Proof: We will prove the existence of a threshold $\underline{\pi} \in [0, 1)$ such that when $\pi_i \leq \underline{\pi}$, $W(d^p) \geq W(d)$ for any d.

Let $X^A \subset X$ be the subset of X such that for any $d \in X^A$, $P(a_i(d) \leq 0) > 0$.

Let $d \in X$ be an arbitrary default. We know $W(d^p) \geq W(d)$ is trivially true when d is also a penalty default, i.e. $d \notin X^A$ as then $W(d) = W(d_p)$ for any π. Next suppose $d \in X^A$, so $p(PA) > 0$. Let $\tilde{\pi}(d) = \inf_{i \in PA(d)} \pi_i$ be the smallest possible value of π_i for the PA group for default d. We know from Equation (13) that

$$W(d_p) - W(d) \geq p(PA)\{E[u_i(x^*) - u_i(d)|PA] - \tilde{\pi}(d)E[\gamma_i|PA]\} \tag{25}$$

The RHS of this expression is a continuous and strictly monotonically decreasing function of $\tilde{\pi}(d)$ (so long as $E[\gamma_i|PA] > 0$, which must be true because PA individuals choose passively). When $\tilde{\pi}(d) = 0$, the RHS of this expression is strictly positive because $u_i(x^*) \geq u_i(d)$ for all i. When $\tilde{\pi}(d) = 1$, the RHS is strictly negative because $u_i(x^*) - u_i(d) < \gamma_i$ for all individuals that are passive at d, which is the PA group in this situation. The Intermediate Value Theorem then implies there is a value of $\tilde{\pi}(d)$, such that we know that the expression on the RHS of (25) is 0. Denoting this threshold by $\underline{\pi}(d)$, we have that $W(d_p) - W(d) \geq 0$ when $\pi_i \leq \underline{\pi}(d)$ for all i. The result then follows from letting $\underline{\pi} = \inf_{d \in X^A} \underline{\pi}(d)$, so that $\pi_i < \underline{\pi}$ implies $W(d_p) - W(d) \geq 0$ for any d. ■

(3.2) *There exists a threshold* $\overline{\pi} \in (0, 1]$ *such that* $\pi_i \geq \overline{\pi}$ *for all* i *implies* d_p minimizes *social welfare.*

Proof: The proof is analogous to the proof of (3.1). For any default d, let $\hat{\pi}(d) = \inf_{i \in PA(d)} \pi_i$. Using equation (13) and a similar Intermediate Value Theorem argument to the above we derive that there is a threshold $\overline{\pi}(d)$, such that $\pi_i \geq \overline{\pi}(d)$ implies $W(d_p) - W(d) \leq 0$. The result then follows from letting $\overline{\pi} = \sup_{d \in X^A} \overline{\pi}(d)$. ■.

Proposition 4 *Suppose that* $X = [x_{min}, x_{max}] \subseteq \mathbb{R}$ *and that:*

(A4.1) *As-if costs* γ_i *are distributed independently of* x_i^**.*

(A4.2) *Preferences are given by* $u_i(x) = u(x - x_i^*)$ *for some map* $u : \mathbb{R} \to \mathbb{R}$*, with* $u'(0) = 0$*,* $u'' < 0$ *and* $u(c) = u(-c)$ *for any* c*.*

(A4.3) x_i^* *follows a single-peaked and symmetric distribution about some mode* x^m*.*

Under these conditions, there exists a threshold $\overline{\pi} \in (0, 1]$ *such that* $\pi_i \geq \overline{\pi}$ *for all* i *implies that the optimal default is the default that minimizes opt-outs.*

Proof: We provide the proof the theorem for the case when $\pi_i = 1$ for all i. It is straightforward to show that if the theorem holds when $\pi_i = 1$ for all i, it must hold for sufficiently high π_i.

Starting from the case where $\pi_i = 1$ for all i, we first prove that $W'(x^m) = 0$, $W'(d) > 0$ for $d < x^m$, and $W'(d) < 0$ for $d > x^m$., which implies that W has a unique global maximum at x^m. We then prove that opt-outs are minimized under x^m. We start by letting $d \in X$ be some default.

Step 1: Characterizing the first and second derivative of $W(d)$.

By (A4.1) we know that $W_i(d) = \int_\gamma E[v_i(d)|\gamma_i = \gamma] f(\gamma) d\gamma$. Let $W_\gamma(d) = E[v_i(d)|\gamma_i = \gamma]$. Note that to prove our result, it suffices to prove that for any fixed γ, $W_\gamma^{'}(d) = 0$ if $d = x^m$, and $W_\gamma''(d) < 0$ always.

We first introduce some notation involving the function $u()$. Without loss of generality $u(0) = 0$. Taking γ as given, by (A4.2) there is some unique value ξ such that $u(\xi) = u(-\xi) = \gamma$. Note that when $x^* = d - \xi$, utility at the default is given by $u(d - x^*) = u(d - (d - \xi)) = u(\xi) = \gamma$, and similarly when $x^* = d + \xi$, $u(d - (d + \xi)) = \gamma$. By (4.2), an individual is active when $x_i^* \leq d - \xi$ or $x_i^* \geq d + \xi$.

We next characterize $W'_\gamma(d)$. Assuming $\pi_i = \pi$ is homogeneous for all i (which is true when $\pi_i = 1$ for all i), welfare at d is given by

$$W_\gamma(d) = \int\limits_{x^*=-\infty}^{d-\xi} \gamma f(x^*)dx^* + \int\limits_{x^*=d-\xi}^{d+\xi} u(d-x^*)f(x^*)dx^* + \int\limits_{x^*=d+\xi}^{\infty} \gamma f(x^*)dx^*,$$

Where $f(x^*)$ is the pdf of x_i^*. Note that $f(x^*)$ does not depend on γ by (A4.1). Differentiating the above with respect to d and applying $u(d - x_i^*) = \gamma$ at $x_i^* = d - \xi$ or $d + \xi$, we obtain

$$W'_\gamma(d) = \gamma(1-\pi)[f(d-\xi) - f(d+\xi)] + \int\limits_{x^*=d-\xi}^{d+\xi} u'(d-x^*)f(x^*)dx^* \tag{26}$$

This is an analogue of Proposition 2 for some fixed γ, with the added structure of (A4.2). When $\pi = 1$ (A4.4), the first term of this expression, which corresponds to the PA and AP groups, vanishes, leaving only the PP group, which we now splitinto those with $x^* < d$ and those with $x^* > d$:

$$W'_\gamma(d) = \int\limits_{x^*=d-\xi}^{d} u'(d-x^*)f(x^*)dx^* + \int\limits_{x^*=d}^{d+\xi} u'(d-x^*)f(x^*)dx^*. \tag{27}$$

Step 2: For any constant ζ, $f(d+\zeta) \geq f(d-\zeta) \iff x^m \geq d$.

Suppose $x^m \geq d$ and take a constant ζ. If $x^m > d + \zeta > d - \zeta$, the result immediately follows from the assumption in (A4.3) that $f()$ is single-peaked. If $d + \zeta \geq x^m \geq d > d - \zeta$ take a constant c such that $d + \xi - x_m = x_m - c$. By symmetry about x^m, $f(c) = f(d + \zeta)$. We know that $c < x_m$, because $x_m - (d+\zeta) \leq 0$. We also know that $c \geq d - \zeta$, because we presumed $x_m \geq d$. We then have $x^m \geq c \geq d - \zeta$. The single-peaked assumption then implies $f(d + \zeta) = f(c) \geq f(d - \zeta)$.

Supposing $x^m < d$ and proceeding analogously proves the converse.

Step 3: $x^m \geq d \iff W'_\gamma(d) \geq 0$.

Starting from equation (27), note that by (A4.2) the first term is positive ($u' > 0$ when $x^* < d$) and the second term is negative ($u' < 0$ when $x^* < d$). We can compare the signs of the two terms in the previous expression by re-writing this equation, using the symmetry of the utility function, as:

$$W'_\gamma(d) = \int\limits_{x^*=d-\xi}^{d} u'(d-x^*)[f(x^*) - f(\tilde{x})]dx^*$$

where $\tilde{x} = 2d - x^*$, so that $d - x^* = -(d - \tilde{x})$. We know from symmetry that when $d = x^m$, $f(x^*) = f(\tilde{x})$, so $W'(x^m) = 0$.

As $u'(d-x^*) > 0$ in the range of integration we use above. When $x^m > d$, the result in Step 3 implies that $f(x^*) \geq f(\tilde{x})$ for $x^* \in [d-\zeta, d]$, so we know that $W'_\gamma(d) \geq 0$. When $x^m < d$, the result in step 2 implies that $f(x^*) \leq f(\tilde{x})$ for $x^* \in [d-\zeta, d]$, and we know that $W'_\gamma(d) \leq 0$.

Step 3 proves that there is a unique global maximum of W at x^m.

Step 4: Setting $d = x^m$ minimizes opt-outs.

Let the frequency of opt-outs be given by $A(d) = P(a_i(d) > 0)$. Using $\xi = u^{-1}(\gamma)$ from before and letting F be the cdf of x_i^*, we know that

$$A(d) = F(d-\xi) + 1 - F(d+\xi)$$

Taking a derivative with respect to d, we have that

$$A'(d) = f(d-\xi) - f(d+\xi).$$

Setting $d = x^m$, it is straightforward to verify using (A4.3) that $A'(d) = 0$ if $d = x^m$, $A'(d) < 0$ if $d < x^m$, and $A'(d) > 0$ if $d > x^m$, which is sufficient to prove that x^m minimizes $A(d)$. ■

B Relationship to the Axiomatization of Masatlioglu and Ok (2005)

Masatlioglu and Ok (2005) provides an axiomatic characterization of a model very similar to the fixed as-if cost model we use. Their paper seeeks to rationalize status quo bias; recall that we showed in Section 1.2.2 that giving extra utility to the status quo is the same as having a fixed cost of not choosing the status quo (see equation (4)). The representation of choices used by Masatlioglu and Ok (see their equations (3) and (4)) is isomorphic to our own (see our equation (2), and also equation (4)), with one exception: the fixed as-if cost could depend on the default in their model. Whether and to what extent γ depends on d is difficult to test empirically, but we know of no evidence suggesting that it does. Nevertheless, here we discuss further the implications of our restriction that γ does not depend on d by relaxing it and examining welfare.

Consider a model that is identical to our beseline model except that the fixed cost is a function of d for each individual, denoted $\gamma_i(d)$. It is straightforward to show that the derivative from Proposition 2 becomes

$$\begin{aligned} 0 \;=\; W'(d) \;=\; & \quad E\left[\pi_i \gamma_i'(d) \middle| AA\right] P(AA) \\ & + \; E\left[(1-\pi_i)\gamma_i(d) \middle| PA\right] P(PA) \\ & - \; E\left[(1-\pi_i)\gamma_i(d) \middle| AP\right] P(AP) \\ & + \quad E[u'(d)|PP]P(PP). \end{aligned} \tag{28}$$

This expression is identical to the expression in Proposition 2 except for the first term. In our basic model, individuals that are always active for a change in the default do not experience any change in their welfare. When the fixed costs depend on d and $\pi_i > 0$, changing the default can affect the welfare of these decision-makers because . The analogue of equation (11) is also straightforward to derive for this model.

First, we note that the argument in Proposition 3 (see the proof above) for active choices being optimal for sufficiently low π is unaffected by this addition. When as-if costs are not normative, forcing active choices still leads all individuals to receive x_i^* without incurring any costs. Whether forcing active choices *minimizes* welfare for sufficiently high π is unclear. The difficulty is that the penalty default d^p could in principle have a lower fixed cost ($\gamma(d^m)$) than other defaults, which can make the penalty default relatively more attractive than some other defaults.

We know by the same logic as Proposition 4 (proof above) that the last three terms of (28) will all be zero under (A3.1)-(A3.3) when we minimize opt-outs, and that ignoring the changes in $\gamma(d)$ for active choosers we would get to a global optimum by minimizng opt-outs when π_i is sufficiently high for all individuals. The additional term in Equation (28) therefore implies that minimizing opt-outs will not be optimal in general when the change in $\gamma(d)$ for a marginal change in the default is zero. Intuitively, if increasing the default from the opt-out minimizing default would reduce the cost incurred by active decision-makers, we know the aggregate effect on all other decision-makers is zero (by Proposition 3), so such an increase in the default would be an improvement on minimizing opt-outs. For a more extreme example, suppose there is a default d^* such that $\gamma_i(d^*) = 0$ for all i. Such a default is obviously the optimal default regardless of the π_i's.[20]

To summarize, our result that active choices are desirable when default effects are purely driven by behavioral frictions survives the extension implied by the model of Masatlioglu and Ok (2005). Minimizing opt-outs will still be a good rule of thumb when default effects are real costs and the dependence between the costs and the default is not too strong, but if the costs vary strongly with the default it may be possible to improve on the opt-out minimization rule of thumb.

[20] When $\pi = 0$, both the active choice policy and the default d^* are optimal defaults.