Asset Prices, Wealth Inequality and Background Risk with Durability and Habits

Georgios Koimisis, M.Phil.[1]; Christos Giannikos, Ph.D.[2]

[1]CUNY Graduate Center, [2]CUNY Baruch College

Abstract

In an exchange economy with identical agents, except for their initial endowment, we examine how wealth inequality affects the equilibrium level of the equity premium and the risk free rate, when there is a single durable good and the agents' preferences are habit forming. We measure inequality by introducing a mean preserving transfer of endowment. This creates the departure from an egalitarian distribution of wealth. Preferences are modeled either as external or internal habits. For our calibrations we introduce two and three classes of wealth, in a simple two period setting. We also explore the effects of the addition of a small uninsurable labor income risk. It seems that wealth inequality is important for all versions of the model.

Introduction

The consumption-based capital asset pricing model (CCAPM) with standard preferences (power utility) fails to explain sufficiently the high observed equity premium, the high volatility of returns and the countercyclical variation in the equity premium (Lucas, 1978; Grossman and Shiller, 1981; Mehra and Prescott, 1985).

The objective of this paper is to study the relationship between (i) asset prices, (ii) the heterogeneity of consumers' wealth, (iii) market incompleteness, (iv) the nature of the good and (v) habit persistence. We implement wealth inequality in a simple Arrow-Debreu1 exchange economy with identical agents.

Methodologically we build on Gollier (2001) and Franke et al. (1998) and we extend their work by introducing a single durable good in the economy, instead of a perishable good. We demonstrate that durability may increase asset risk premia under certain conditions.

Next, we introduce habit persistence. For our analysis, we consider internal and external habit formation. The difference between the two habit types rests on the effect that current consumption has on future habit. Our work suggests that wealth inequality produces also different equity premia and risk free rates for different types and levels of habit formation. Several specifications are considered throughout our study, although we keep the habit strength parameter uniform across agents.

Theoretical Model and Methodology

Simple Arrow-Debreu exchange economy with a single durable good

vNM utility u on consumption at each period, with ($u' > 0$) and ($u'' < 0$).

Two dates, indexed by $t = 0$ and $t = 1$.

Uncertainty at $t = 1 : S$ states of the world where $s = 1,\dots s,\dots S$ with probability p_s of state s.

Large number of n identical agents. Each agent $i = 1,\dots,n$ is endowed with w_{i0} units of the single durable good at $t = 0$, and with a bundle of contingent claims $(w_{i1},\dots,w_{is},\dots,w_{iS})$.

$\sum_{i=1}^{n}\frac{1}{n}w_{is} = z_s$ i.e. the average endowment is random.

Frictionless competitive market for contingent claims that takes place at $t = 0$.

Let π_0 and π_s be the price to be paid at $t = 0$ for the delivery of 1 unit of good respectively at $t = 0$ and $t = 1$, conditional on state s.

The problem becomes:

$$\max_{D_i} \sum_{s=1}^{S} q_s u(D_{is}, x_{is})$$
$$\sum_{s=1}^{S} \pi_s u(D_{is} - w_{is}) = 0$$
$$D_{is} = (1-\delta)D_{i0} + C_{is}$$

where $q_0 = \frac{1}{\beta}$, $q_s = \pi_s$ for $s = 1,\dots s,\dots S$, D_{is} is the stock of durable good at state s, C_{is} are the units of durable good at state s, and x_{is} are habit levels. In each state $s = 1,\dots s,\dots S$ the good has $D_s = (1-\delta)D_0 + C_s$. For $\delta = 1$, the good is perishable.

External Habit Formation: $x_{is} = \varepsilon D_s, 0 < \varepsilon < 1$, where $\mathrm{D}_s = \sum_{i=1}^{n}\frac{1}{n}\mathrm{D}_{is}$

Internal Habit Formation: $x_{is} = \varepsilon D_{i0}$

In both specifications, parameter ε denotes the habit strength.

The competitive equilibrium satisfies the following set of necessary and sufficient conditions.

$q_s u'(D_{is}, x_{is}) = \lambda_i \pi_s$, $i = 1,\dots,n$, $s = 0,\dots s,\dots S$

$\sum_{i=1}^{n}\frac{1}{n}\mathrm{D}_{is} = \sum_{i=1}^{n}\frac{1}{n}\mathrm{w}_{is}$, $s = 0,\dots s,\dots S$

where λ_i is the Lagrange multiplier associated to the budget constraint of agent i.

Background Risk: Agent's wealth at time $t = 1$ is given by $z + y$, where y is the background risk. Utility is given by $v(z,x) = Eu(z + y, x)$.

For no background risk, utility reduces to $v(z,x) = Eu(z,x)$.

Asset pricing: The relative price of equity with respect to bonds is given by the following simple formula:

$$\Pi = \frac{\sum_{s=1}\pi_s z_s}{\sum_{s=1}\pi_s}$$

The risk free asset provides one unit of consumption good at $t = 1$ with probability 1. The price to be paid at $t = 0$ for that asset will be

$$\frac{1}{R} = \frac{\sum_{s=1}\pi_s}{\pi_0}$$

R is the gross risk-free rate $(1 + \mathrm{r}_f)$

Preferences

Complete Markets: For HARA, wealth inequality has no effect on asset pricing, when markets are complete (Gollier 2001). Therefore, we use the following non-linear absolute risk tolerance specification (Guiso and Paiella, 2008):

$u'(z,x) = \exp[-r\frac{(z-x)^{1-b}}{1-b}]$. The absolute risk tolerance is concave when $b \in [0,1)$ and convex for $b > 1$.

For $b = 1$, the utility exhibits CRRA. We assume a coefficient of relative risk aversion $r = 2$.

Incomplete Markets: All agents in the economy have hyperbolic absolute risk aversion (HARA) utility for wealth at the end of a single time period, i.e. exhibit linear absolute risk tolerance. We define the HARA type as

$v(z,x) = -\zeta\left[\eta + \frac{(z-x)+y}{r}\right]^{1-r}$ for some constants ζ and η.

Contact Information

Georgios Koimisis
The City University of New York, Graduate Center
Email: gkoimisis@gradcenter.cuny.edu

Christos Giannikos
The City University of New York, Baruch College
Email: Christos.Giannikos@baruch.cuny.edu

For more information about our paper, visit: http://gkoimisis.org/index.php/research/

Data and Calibrations

The GDP per capita is as follows:

$$z_s = \begin{cases} z_0 = 1, & \text{with certainty} \\ z_1 = 0.99, & \text{future recession with } prob = 61.756\% \\ z_2 = 1.0632, & \text{future growth with } prob = 38.244\% \end{cases}$$

Wealth Partition in the U.S. Economy

Class	Population (%)	GDP % 1997	GDP % 2007	GDP/capita % 1997	GDP/capita % 2007
Poor	40%	1.35%	0.9%	3.375%	2.25%
Middle	55%	45.15%	49.9%	82.09%	90.73%
Rich	5%	53.5%	49.2%	1070%	984%

Two Classes: Two equally weighted social classes, the poor and the rich. The poor are endowed with a share of $(1 - h)$ of the GDP per capita in each state, whereas the rich get a share of $(1 + h)$ of it.

Parameter h is at the same time the standard deviation and the coefficient of variation of the wealth distribution.

Three Classes: Three unequally weighted social classes, 40% of poor, 5% of rich, and 55% of middle class (SCF, Diaz-Gimenez et al.; 1997, 2011).

The background risk is distributed as $(-k, 1/2;\ +k, 1/2)$. Parameter k is the standard deviation of the growth of labor income.

Results and Discussion

By making the good more durable the equity premium increases further. Durability and wealth inequality yield more reasonable results in a three-classes society. Figure 1 plots the equity premium (in %) as the curvature of ART changes from concave to convex. for a perishable and a durable good (δ = 0.97). As long as the agents have concave ART, the equity premium is larger for a durable good. As the ART becomes more convex there is practically no difference between holding a durable good from a perishable one, even in an unequally distributed economy.

As the habit strength increases the asset prices move closer to their observed levels. A striking result arises: In the unequally weighted three classes economy the asset prices decrease and move closer to their observed levels, as we increase the habit strength. At the same time, the equity premium, although marginally, rises.

Figure 2 shows the difference between equity premia for a non-habit forming economy and one with external habits ($\varepsilon = 0.04$). Figure 3 shows the difference between equity premia for a non-habit forming economy and one with internal habits ($\varepsilon = -0.04$). When agents exhibit concave ART, internal habits raise the equity premium in an unequal economy. We observe that internal habits have a higher impact on risk premia. For an $\varepsilon = -0.9$ we get an equity premium of 3.84%.

In the case of incomplete markets, we analyze the effect of a small uninsurable labor income risk ($k = 0.25$) on asset prices, in a society with two equally weighted classes of wealth, poor and rich, endowed with 50% and 150% of the GDP per capita, respectively.

Wealth inequality raises the equity premium, but only marginally (Table 2). Also, we observe that as income uncertainty increases, the premium increases up to a maximum k of around 35% for the non habit economy and up to 40% for the economy with internal habits. From these points and after, wealth inequality decreases the equity premium (Figures 4-6).

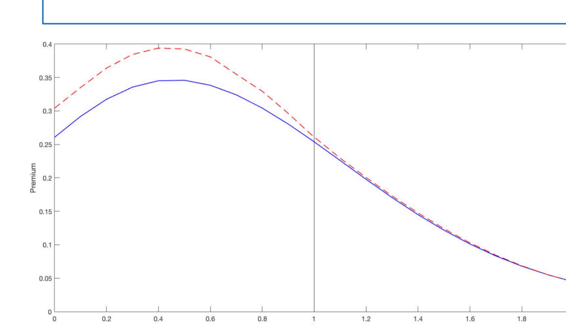

Figure 1. Three Classes, No Habits, Durable Good (red dashed line)

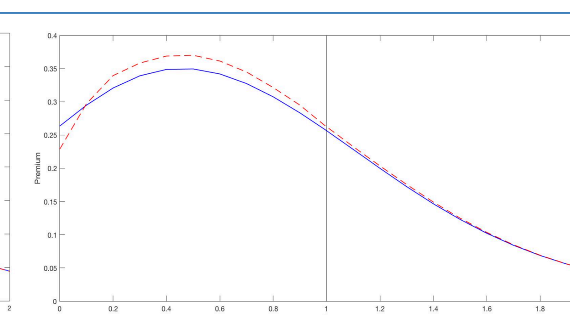

Figure 2. Three Classes, External Habits (red dashed line)

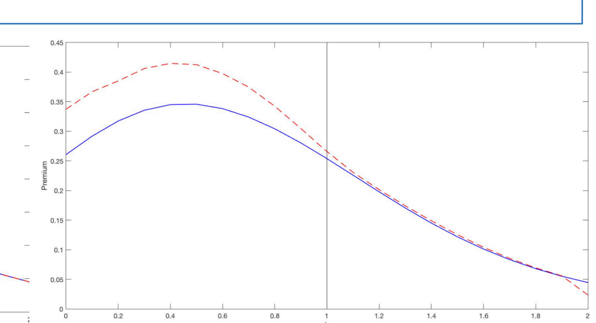

Figure 3. Three Classes, Internal Habits (red dashed line)

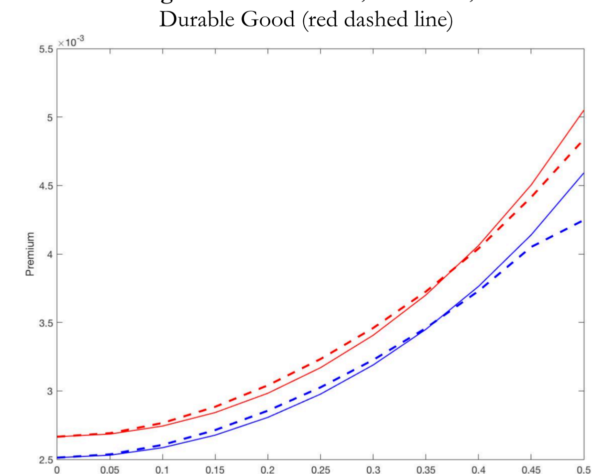

Figure 4. Two Classes, Background Risk, Durable Good (dashed lines)

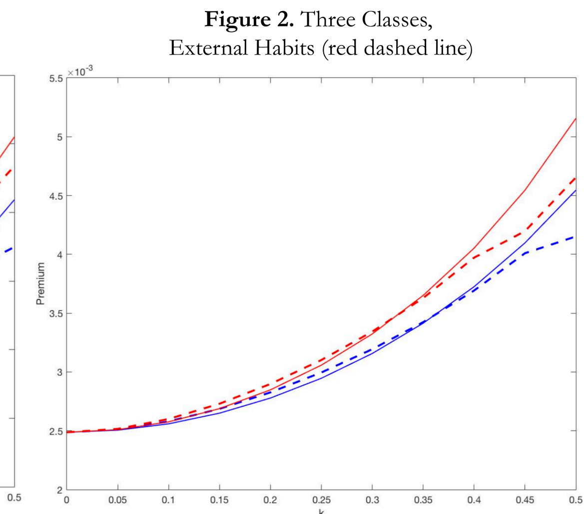

Figure 5. Two Classes, Background Risk, External Habits (dashed lines)

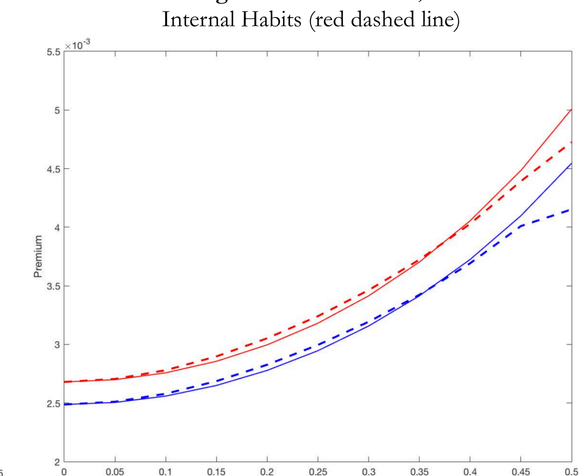

Figure 6. Two Classes, Background Risk, Internal Habits (dashed lines)

Table 1. Three Classes, Complete Markets: Equity Premium, in %

	Durability (δ)	External Habits (ε)	Internal Habits ($-\varepsilon$)
0.00	-	0.35	0.35
0.10	3.80	1.93	0.57
0.30	2.81	0.60	1.31
0.60	1.67	0.50	2.48
0.90	0.56	0.54	3.84
1.00	0.35	-	-

Table 2. Three Classes, Incomplete Markets: Equity Premium, in %

	Durability (δ)	External Habits (ε)	Internal Habits ($-\varepsilon$)
0.00	-	0.2995	0.2995
0.10	0.5960	0.3102	0.3589
0.30	0.5312	0.3459	0.4629
0.60	0.4332	0.7959	0.5978
0.90	0.3335	0.2429	0.7191
1.00	0.2995	-	-

Conclusion

Our conclusion is that wealth inequality, durability and habit formation are serious features that can contribute modestly to the resolution of the equity premium puzzle, in both complete and incomplete markets. In a complete market setting, wealth inequality raises the equity premium when the absolute risk tolerance is concave. The results for the equity premium become higher when the nature of a good or habit formation are considered and can approach the observed historical levels, under certain conditions. Furthermore, our calibrations show an improved equity premium when we consider a more realistic economy.

We also examine the effect of wealth inequality in a model including an idiosyncratic background risk. We show that the presence of a small uninsurable background risk biases asset prices towards a larger equity premium. Furthermore, we document the following interesting result. While Gollier (2001) showed that wealth inequality raises the equity premium when income uncertainty is low but decreases it for large income uncertainty, we show that with the addition of durability or habit formation in our model, the magnitude of the background risk required to decrease the equity premium changes. When the good is durable or agents exhibit internal habit persistence (with substitutability of consumption), a higher background risk is required to reduce the equity premium. The opposite happens in the case of external habits.

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