Timing of Auctions of Real Options*

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Abstract

This paper endogenizes auction timing and initiation in auctions of real options. Revenue-maximizing timing deviates from welfare-maximizing or bidders' preferred timing because the information rent the seller pays makes her face a "virtual strike price" higher than the option exercise cost. The irreversible nature of time endows a seller potential control over the winning bidder's eventual option exercise. As long as she does not strongly prefer early exercise, she inefficiently delays the auction; otherwise auction timing is efficient, but option exercises are always inefficiently delayed. When the seller lacks commitment to auction timing and offer finality, bidders always initiate in equilibrium regardless of the divergence in their and the seller's preferred option exercise. The model also predicts that bidder initiation corresponds to faster option exercise, consistent with empirical evidence from the selling and drilling of oil and gas tracts.

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1 Introduction

Auctions of real options are prevalent in licensing and patent acquisitions, leasing of natural resources, real estate development, M&A deals, venture capital and private equity markets, and privatization of large national enterprises.[1] The application of auction theory to corporate finance has also been extensive (Hansen (2001) and Dasgupta and Hansen (2007)). Many decisions in organizations under these settings entail optimal timing of taking certain actions such as shutting down a plant, going public, adopting a new technology, and launching a new product.[2] Yet the original owner of the firm, namely the entrepreneur and initial investors, may have preferences over the optimal exercise of these real options different from the decision makers.

At first glance, this divergence in preference should not matter because, after all, the new owners of the real options make exercise decisions and sellers seem powerless once the assets are sold. This is why many studies treat an auction's taking place as exogenous. However, whereas in classical studies on auctions the assets' values are independent of the agents' post-auction actions, real options derive their values from the holders' timely exercise. Consequently, the irreversible nature of time endows the seller partial control over the option exercise, which implies that auction timing and initiation can affect the competitive bidding and real outcomes. It is thus insufficient to focus on the auction stage alone and ignore endogenous auction timing, especially in corporate finance settings in which assets are typically embedded with real options and timing the sales can be strategic.

To understand the impact of auction timing, I combine the real options framework with that of auctions. Specifically, a seller and multiple potential bidders are risk-neutral and

[1] Bolton, Roland, Vickers, and Burda (1992) describe the privatization policies in Central and Eastern Europe. Pakes (1986) and Schwartz (2004) discuss patents as real options. For many M&A deals, the post-auction exercise problem lies in integrating the two firms or in deciding on new investments viable only when firms are combined (e.g., if they require pooling patents.)

[2] The internet pioneer, Yahoo, which peaked in 2000 with a market cap around $125 billion, was already well past its prime when co-founder Jerry Yang turned down Microsoft's $44.6 billion buyout offer in 2008, dismissing it as inadequate. The company later sold its core internet assets to Verizon for $4.8 billion. Groupon is another example inciting questions about its timing of sale. The daily-deal site rejected a $6 billion buyout from Google in 2010, only to see its shares falling below that shortly after a 2011 IPO.

maximize their expected payoffs. The seller owns a real option that only the bidders have the expertise to exercise. Each bidder has private information about his exercise cost. They interact in continuous time in three sequential stages. In the first stage, the seller (or potentially a bidder once we allow bidder initiation) strategically initiates the auction. In the second stage, participating bidders bid (potentially with securities) and the seller allocates the asset. In the final stage, the winning bidder rationally times the exercise of the investment option (and delivers the contingent payment, if any, to the seller).

In reality, a seller may have intrinsic preferences for the project's future that differ from the bidders. The inventor of a technology or a product might develop emotional attachment (Tjan (2011) and Matyszczyk (2015)), prefer an earlier commercialization, or favor later product shutdown; the manager of a firm may not fully internalize the reputation cost of the entrepreneur who originally founded the company, but instead care more about firm performances in her relatively short tenure. I therefore allow the seller to derive a differential benefit b upon option exercise.

I then use a mechanism design approach to show that the seller times the auction to maximize the option value less the information rent while a bidder times the auction to maximize the information rent, neither of which maximizes social welfare. Because of the information rent the seller has to pay, she effectively holds options with an augmented exercise cost. As long as she is not too biased towards early exercise (b is not too positive), the seller endogenously delays the auction beyond what is socially optimal because the irreversible nature of time allows her to hold up the bidders—the bidder can exercise the option only after getting the ownership right through the auction, but not before—, which forces the bidder to partially incorporate her preference on option exercise. Doing so would push the bidders' options more in the money on average, which compresses the valuation distribution and increases bidder competition. When she strongly prefers early exercise, however, she loses the partial control because of the uni-directional flow of time: she may accelerate the auction, but the winning bidder would still wait to exercise the option.

Surprisingly, even when $b = 0$ and the social cost of holding an auction is infinitesimal, cash auctions traditionally deemed efficient are inefficiently delayed, but never inefficiently accelerated. Abandonment options tend to be sold late if they are associated with the seller's preference for late exercise, for example when the entrepreneur may view the closing of plants or the company as a termination to his legacy. Similar arguments hold for investment options associated with winning bidders who prefer early exercise for empire building. In such cases the seller's endogenous auction timing partially mitigates inefficient option exercises, but introduces inefficiencies in itself. In cases where the bidders prefer a late exercise, for example due to preference for quiet life by managers, the seller efficiently times the auction, but option exercises are always inefficiently late because the winning bidder does not internalize the seller's benefit from an early exercise.

Can a seller always time the auction? She can in oil lease auctions, wireless spectrum auctions, or privatization auctions. But many other sales to competitive buyers feature sellers lacking such commitment; that is, bidders can instead approach the seller to trigger an auction, as seen in corporate takeovers and project finance where bidders decide what to offer and often can initiate the contact or negotiation. One prominent M&A case involves Microsoft's \$8.5 billion cash acquisition of the voice-over-IP service Skype—its largest acquisition to date—for the portolio of real options such as Windows phone integration. Google's largest acquisition with \$12.5 billion for Motorola Mobility Company in 2012 also gave it the option to develop the portfolio of patents Motorola held. Both deals are bidder initiated and against the backdrop of potential rival bids, are effectively auctions. Still others, such as licensing agreements and contracts in the entertainment industry, appear in both categories.

When the seller lacks commitment to auction timing and cannot resist dynamic adjustments of offers, she effectively holds a second-price or English auctions. When allowed to initiate in these auctions, bidders always do in equilibrium, regardless of the divergence in preference for option exercise. The intuition is that both the seller and bidders dynamically update their beliefs about the type of bidders present based on (the absence of) initiation, yet the seller's option value starts to be eroded later than that of the bidders' because the seller receives the second best real option in second price auctions, which becomes "in the money" after the best real option does. Even when the seller prefers early option exercise, bidders internalize the seller's preference because they anticipate the endogenous reserve price the seller charges in order to at least break even in expectation.

While natural settings to test auction timing and initiation are hard to come by, the leasing and drilling of oil and gas tracts in the Gulf of Mexico provide a unique testing ground for some of the model implications. In particular, the model predicts that a bidder initiates only when his real option is in the money and therefore the investment option is exercised more quickly on average when bidders initiate than when a seller initiates strategically or randomly. Utilizing the introduction of Area Wide Leasing in May 1983 as a natural change, I find the data is suggestive of 10%-40% higher likelihood to explore and drill under bidder initiation, lending evidence to the endogeneity of auction timing in practice.

The main results and intuition in the paper are robust to introducing security bids in the auctions.[3] To show that, I introduce auction timing to the framework of security bid auctions in DeMarzo, Kremer, and Skrzypacz (2005) and Cong (2017). Even though security bids alter the bidders' option exercises, the seller can still use auction timing to partially align bidders' option exercise timing with her preference, if the bidders tend to exercise the option later than what she prefers. When the seller lacks commitment to auction timing and security design, the bidders still always initate because in equilibrium their bids are all cash-like, as shown in Cong (2017).

Literature

This paper is foremost related to the literature on auction theory (e.g. Krishna (2009)), especially its applications to model corporate finance transactions, such as mergers and

[3] Prima facie, the type of bids should not matter as a cash equivalent always exists. One advantage to contingent bids is that they enhance the seller's revenue by effectively linking payoff to a variable affiliated with bidders' private information—the "linkage" principle in Milgrom (1985). Contingent bids also mitigate liquidity or legal constraints and reduce valuations gaps among various parties.

acquisitions and sales of scarce resources (e.g., Bulow, Huang, and Klemperer (1999); Boone and Mulherin (2007); Povel and Singh (2010)). Extant papers typically focus on the auction design and outcomes when an asset is already up for sale and leave out the endogenous timing of auctions. Two exceptions are Gorbenko and Malenko (2017) which connects bidders' financial constraints to their incentives to initiate deals, bids, and payment methods, and Gorbenko and Malenko (2016) which endogenizes auction initiation of first-price auctions with changing valuations.

Although my model also applies to the settings they consider, the focus is on selling real options with post-auction exercise. Because of this distinction, the irreversibility of time now endows the seller partial control over the option exercise, making auction timing a strategic decision variable. Moreover, complementary to Gorbenko and Malenko (2017, 2016), bidder initiation in this paper is based on Bayesian learning about the aggregate market absent initiation instead of financial constraints, and focuses on second-price and English auctions. Also closely related is Chen and Wang (2015) that considers two-sided private information and endogenous initiation in M&A deals. This paper differs in the focus on auctions instead of one-on-one negotiations, post-auction optionality, and the irreversible nature of time.

An emerging literature also examines the irreversibility of time in dynamic corporate decisions, and depending on the direction of preference divergence between seller and buyer, draw rather asymmetric conclusions. Grenadier, Malenko, and Malenko (2016) examine the scope for and the structure of delegation and communication in a dynamic environment with an uninformed principal and an informed but biased agent. The direction of the bias crucially affects the agent's ability to credibly communicate information and full revelation only occurs when the agent is biased towards late decision making. In a distinct environment, Guo (2016) studies the same topic by considering the optimal mechanism without transfers in an experimentation setting where the agent prefers to experiment longer than the principal, and shows that the optimal contract is time-consistent if and only if the agent prefers to experiment longer than the principal. Again the direction of the conflict of interest (timing) shapes the results dramatically. While such asymmetry appears in my settings as well, competitive bidding and bidder's information rent interact with the conflict of interests, and the asymmetry no longer manifests itself based on a simple dichotomy of the direction of bias. Orlov, Skrzypacz, and Zryumov (2017) study vertical issues and authority in organizations marrying Bayesian persuasion with the dynamic setups in Grenadier, Malenko, and Malenko (2016) and Guo (2016), and find that there could be initial information pipeting followed by full delegation and agent's impatience can lead to full and immediate information revelation. Overall, instead of communications or contracting between principals and agents, this paper focuses on a somewhat more basic economic activity: auction – the transfer of ownership and control to competitive buyers with endogenous entry and time-varying valuations.

In this regard, closely related is Malenko and Tsoy (2017) which studies auction design when buyers rely on biased experts. The authors show that while revenue equivalence theorem holds in static mechanisms, advisors communicate information gradually, resulting in more efficient allocation and higher or lower revenues depending on the direction of expert bias. These results derive from the irreversibility of running prices in Dutch or English auctions, which is similar to the irreversibility of time. That said, the paper does not consider endogenous auction timing, and focuses on transmission of non-verifiable information, rather than the post-auction option exercise that I examine.

Finally, my paper complements the emerging literature on agency issues in a real-options framework broadly. For example, Grenadier and Wang (2005), Cong (2012), and Gryglewicz and Hartman-Glaser (2015) study distortion of investment incentives due to adverse selection and moral hazard. Grenadier and Malenko (2011) and Morellec and Schürhoff (2011) examine signaling through option exercises. Most closely related is Cong (2017) which analyzes the post-auction moral hazard in auctions of real options with security bids, both when the seller commits to security design and when she does not, but all without allowing auction timing. Also related is Board (2007), which allows seller's state-dependent allocation post-auction, and characterizes the seller's revenue-maximizing mechanism, but only considers the case with seller's commitment power and no private benefits. This paper focuses on the transfer of ownership and control in the absence of incentive contracts, and is the first to explicitly demonstrate how auction timing is used as a tool to influence option exercises.

The remainder of the paper proceeds as follows. Section 2 describes the economic environment and sets up the model. Section 3 derives optimal option exercise, bidding equilibria, and endogenous auction timing. Section 4 allows bidder initiation and characterizes auction timing for second-price and English auctions, before providing empirical evidence corroborating model implications. Section 5 discusses auction timing and bidding with securities. Section 6 concludes. The appendix contains all the proofs.

2 Auction Environment

A risk neutral revenue-maximizing seller with discount rate $r > 0$ owns a project with an embedded option. We can think of the seller as a technology-oriented entrepreneur and the option as the product's commercialization or the startup's sale to other companies, investors in the public market, or business managers.

The state variable that summarizes the project's potential profit is public and evolves stochastically according to a geometric Brownian Motion (GBM):

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \tag{1}$$

where B_t is a standard Brownian motion under the equivalent martingale measure, μ is the instantaneous conditional expected percentage change per unit time in X_t, and σ is the instantaneous conditional standard deviation per unit time. I assume $\mu < r$ to ensure a finite value of the option. Note X_t could represent the present value of a stream of future cash flows.

The seller does not have the expertise to exploit the option but can auction the project to N risk neutral potential bidders with the same discount rate r who have the expertise to exercise the option at a cost θ_i, which are i.i.d. with positive support $[\underline{\theta}, \overline{\theta}]$.[4] Denote the cumulative distribution and density function by $F(\theta)$ and $f(\theta)$, respectively. As is standard in the auction literature (e.g., Myerson (1981)), we assume the distribution is *regular*, i.e., $z(\theta) \equiv \theta + F(\theta)/f(\theta)$ is increasing. The project operated by type θ_i produces cashflows whose present value at the time of option exercise t is $X_t - \theta_i$. For now I consider the case in which the entrepreneur sells the entire company, but discuss the seller's shares retention later. We assume X_0 is low enough so that the divergence in the two parties' preferences for auction timing and option exercise is non-trivial.

When the seller holds the auction, she incurs a cost $Y > 0$, which can be viewed as her continuous private utility from control rY, or a differential utility she gets relative to the buyers.[5] The project is thus worth Y to her if it is never developed. Y could also represent the effort cost of disclosure to potential bidders, pre-contract costs, or costs associated with revealing proprietary information to rivals (French and McCormick (1984); Hansen (2001); Gorbenko and Malenko (2017)). In reality, bidders may also need to expend effort and initial resources to learn about the project or hire illiquid human capital, which does not alter our results. What is important is that the total social cost (represented by Y in the current setup) is positive to rule out trivial, degenerate auction timing (always holding auction right away).[6]

[4]For clarity, I refer to the seller as female and the bidders as male.

[5]The seller may lose the benefits from alternative uses of the asset before the auction, or the option value of more efficient allocation of the asset when technology improves. Bleakley and Ferrie (2014) show that after an initial allocation of the frontier land in Georgia, land use took over a century to converge to post-allocation efficiency and land value was depressed by 20%. Another example is the FCC spectrum auction, where the government selling certain bandwidth to multiple firms has to consider the cost of losing the option to allocate it in the future to firms with better technology, because even for the federal government, repurchasing the bandwidth is hard because of the well-known hold-up problem involved in multilateral bargaining.

[6]In an earlier draft, I show the results are robust to modeling seller and bidders' costs separately, or allowing Y to depend on X in simple forms. Y may be insignificant, especially when a winning bidder can contract with the seller to continue the original use before the option is exercised. But such cases are rare. For example, the federal government typically auctions areas of land or sea involving multiple leases in a shared ecosystem, and cannot contract with individual winners to keep certain areas intact while allowing drilling in a neighboring tract. Due to political and ideological differences, national parks and environmental organizations are unlikely to collaborate with energy firms to maintain their operations before the energy

In the same spirit as Grenadier, Malenko, and Malenko (2016) and Malenko and Tsoy (2017), we assume the seller derives a private utility of b when the option is exercised. As in these papers, b is common konwledge but non-contractible. Positive b indicates her preference for early option exercise; negative b indicates her preference for late option exercise. We note that b is really a measure of divergence in preference rather than a bias only for the seller. For example, the new manager of a firm may enjoy a private benefit from acquiring a target firm that would show up as a negative b in our model.

When the auction is held at time t_a, bidders compete by offering cash bids. We allow combinations of upfront cash payment and contingent payment (security bids) in section 5. The agents interact in continuous time as shown in Figure 1. To analyze the dynamics, I work backward to first solve for the optimal investment strategy for the winning bidder, then derive the bidding equilibrium given the bidders' valuations based on their investment strategies, and then study the impact of strategically timing the auction.

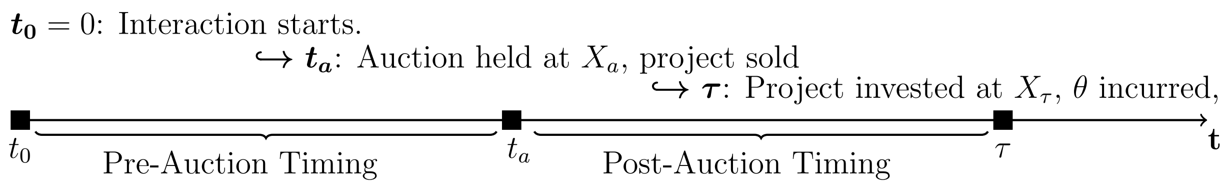

Figure 1: Timeline

We first consider the baseline case in which the seller designs and commits to the auction rule and timing, and discuss bidder initiation absent such commitment. I focus on first-price auctions (FPAs) and second-price auctions (SPAs) in which the bidder with the highest bid wins and pays the highest bid or the second-highest bid, respectively. Other common auction forms such as Dutch auctions or English auctions are equivalent to FPAs and SPAs respectively. I assume the seller commits to no renegotiation post-auction, and to no contracting or resale to losing or non-participating bidders.

Welfare in this paper is defined by the total payoff to the seller and bidders, and efficiency in this paper means constrained efficiency from a global optimizer's perspective; that is,

firms start exploiting.

welfare maximizing under the same informational or institutional constraints as individual agents.

3 Endogenous Auction Timing

3.1 Optimal Stopping and Bidding Strategies

A bidder of type θ owns the project entirely upon winning, and optimally develops the project at time $t \geq t_a$ to maximize $\mathbb{E}[e^{-r(t-t_a)}(X_t - \theta)]$. The optimal strategy for this standard problem involves immediate investment upon reaching an upper threshold $X^*(\theta)$.[7] Let X_a denote the cash-flow level when the auction is held. The value of the investment option W and $X^*(\theta)$ are independent of Y and t, and are given by

$$X^*(\theta) = \max\left\{X_a, \frac{\beta}{\beta - 1}\theta\right\}, \quad \textit{where} \quad \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \quad (2)$$

$$\textit{and} \quad W(X_a; \theta) = D(X_a; X^*(\theta))(X^*(\theta) - \theta), \quad \textit{where} \quad D(X; X') = \left(\frac{\min\{X, X'\}}{X'}\right)^\beta. \quad (3)$$

Note that $D(X_t; X')$ corresponds to the time-t price of an Arrow-Debreu security that pays one dollar when the first moment threshold $X' \geq X_t$ is reached. The option value of the project is simply the total value of Arrow-Debreu securities that replicate the payoff of the investment option at exercise.

Bidder i's private valuation is then $W(X_a; \theta_i)$, which decreases in θ_i, and his bidding strategies are the same as those in standard cash FPAs and SPAs. In particular, in SPAs bidder of type θ bids $W(X_a; \theta)$ under weakly undominated strategy. Because post-auction investments are not distorted, FPAs and SPAs generate equivalent revenues to the seller, and allocate the project to type $\theta_{(1)}$ if $\theta_{(1)} \leq \hat{\theta}$, where $\theta_{(j)}$ is the jth lowest realized θ, and $\hat{\theta}$ is the cutoff-type that participates. I derive $\hat{\theta}$ shortly, but for now note that if it is interior, the entrepreneur optimally sets a reserve price such that the cutoff type contributes zero

[7] See, for example, McDonald and Siegel (1986) and Dixit and Pindyck (1994).

revenue upon winning. The cases with other reserve price or an entry fee are similar.

3.2 Seller's Real Option and Optimal Timing

Given the equilibrium bids and option exercise, a seller chooses auction timing t_a to maximize her expected revenue.

Lemma 1. *The seller's expected utility in FPAs and SPAs held at t_a is given by*

$$\mathbb{E}\left[e^{-rt_a}\mathbb{1}_{\{\theta_{(1)}\leq\hat{\theta}\}}\left[e^{-r(\tau_1^*-t_a)}(X_{\tau_1^*}-z(\theta_{(1)})+b)-Y\right]\right], \tag{4}$$

where $\theta_{(1)}$ is the smallest realized cost, and τ_1^* is the bidder's first hitting time to $X^*(\theta_{(1)})$.

The term e^{-rt_a} simply discounts the auction revenue to its present value; $\mathbb{1}_{\{\theta_{(1)}\leq\hat{\theta}\}}$ indicates that the best type can afford the reserve price and participates; τ_1^* is the first-hitting time to $X^*(\theta_{(1)})$. The surplus upon option exercise is $X_{\tau_1^*}-\theta_{(1)}$, but the seller's payoff depends on the "virtual valuation" of the best type rather than the actual valuation (Bulow and Roberts (1989)). Therefore information asymmetry means that she pays a rent equivalent to an added cost $F(\theta_{(1)})/f(\theta_{(1)})$ at the time of exercise. In other words, the seller essentially owns the best type's real option with a stochastic "virtual strike price" $z(\theta_{(1)})+b$. In general, the winning bidder's optimal investment timing differs from the seller's, depending on b and Y.

Another

Corollary 1. *Denote θ_c as the solution to*

$$D(X_a;X^*(\theta))[X^*(\theta)-z(\theta)+b]=Y, \quad \theta\in[\underline{\theta},\overline{\theta}]. \tag{5}$$

If θ_c exists, the seller optimally sets reserve price $R=W(X_a;\theta_c)$; otherwise, the seller does not set reserve price and bidders either all abstain ($\hat{\theta}=\underline{\theta}$) or all participate ($\hat{\theta}=\overline{\theta}$).

As such, if the cutoff type is interior, then $\hat{\theta} = \theta_c$. Next comes the paper's first key result regarding auction timing.

Proposition 1. *An optimal threshold strategy for timing an auction exists. As long as the seller does not have strong relative preference for early exercise, she inefficiently delays the auction and never sells the project when she expects no chance of immediate investment. Option exercises are almost surely inefficiently late when $b > 0$, but could be early or late when $b < 0$.*

Intuitively, option values erode as X_a increases, thus it would not be optimal to postpone the auction indefinitely. But because the seller effectively bears Y, she can profitably delay the incidence of this cost, especially if she expects no bidder to invest right away and she can wait for greater participation. The bigger Y is, the more the seller endogenously delays the auction. Similarly, the smaller b is, the more the seller delays the auction. That said, our result is not solely driven by Y or b. In truth, the irreducible force behind the delay lies in the fact that the seller faces a lower virtual valuation than a social planner does.

Corollary 2. *Even when $b = 0$ and $Y \to 0$, the seller still inefficiently delays the sale.*

Even though auction timing depends on the exogenous divergence in preferences b and auction cost Y, we obtain inefficient delay in selling the asset even without these considerations ($b = 0$ and $Y \to 0$). The information asymmetry between the seller and bidders alone generates their divergence preferences over option exercise.

This result may appear straightforward ex post, is specific to real options that involve post-auction actions. A priori, it is not obvious that cash auctions in general are endogenously held late, not to mention the subtle asymmetry in the divergence in preferences.

By delaying beyond what is socially optimal, the seller "compresses" the spread in bidders' valuations and reduces the winner's information rent through enhancing the competition. The winning bidder cannot go back in time to exercise the option at a lower threshold, and this irreversibility gives the seller partial control over option exercise even though the sale transfers ownership and control to the winning bidder.

Figure 2 illustrates this effect by plotting time zero present values of the expected revenues and welfare from cash auctions held when X_t first reaches X_a. Hence, for regulators concerned with welfare, auction timing is as important a consideration as market power.

Corollary 3. *When the seller strongly prefers early exercise (b is sufficiently positive), the auction timing is socially efficient, but option exercises are inefficiently late.*

The uni-directional nature of time flow (and Second Law of Thermodynamics) again creates the asymmetry that timing the auction early would not help the seller control the option exercise because the winning bidder can wait longer.

3.3 Robustness and Discussion

Before moving on to bidder initiation, I remark on whether the main tradeoffs are robust to alternative settings.

Information Asymmetry and Contingent Bids

In the proof of Lemma 1 in the appendix, I actually allow for potentially interdependent values and general forms of contingent bids, to show that the results are not driven by my assumptions of private-value or cash bids.

In general, the asymmetric information could be related to the stochastic process X_t, or have some interactions with it. For example, the results all go through when we allow exercise payoffs of the form $\theta_i X_t - \theta_o$, where θ_i is bidder i's private information, and θ_o is equal and commonly known across all agents. What is important is that the the bidder has private information, which translates into an information rent that the seller faces. From the proof of Lemma 1 and Proposition **??**, as long as the bidder's utility as a function of his type is differentiable everywhere, the information asymmetry would lead to the seller's facing real options with increased strike price or reduced cash flow, and the economic intuition goes through. That said, if the private type involves beliefs on the stochastic process such as its drift, we may not obtain closed-form solutions. Moreover, higher dimensions of informational asymmetry would also complicate the analysis. These constitute interesting future work.

Entrepreneur's Share Retention

As often observed in real life, instead of selling the entire project, the entrepreneur may retain certain shares of the company even after ceding control. How does share retention affect the timing of sale?

I first argue that share retention does not qualitatively change the results. Suppose the entrepreneur retains α shares of the project, then her payoff upon option exercise becomes $\alpha X_\tau + b$. Normalizing the payoff by α, we can re-label $b' = \frac{b}{\alpha}$. Because of risk-neutrality and the absence of ex post transfer, the main tradeoffs present in auction timing still remain, though the solution changes quantitatively.

From a social planner's perspective, share retention can potentially mitigate inefficient auction timing. To see this, note that Proposition 1 implies the auction timing is only inefficient for negative or small positive values of b. Requiring the seller to retain a large enough share would make her align more with the social optimal. However, retaining too much reduces the profit of the winning bidder who has to pay the private cost θ, risking inefficiently delaying option exercise further.

Abandonment Option

Is our result driven by the fact that the entrepreneur is selling an investment option? In corporate finance, abandonment options play equally important roles as investment options (e.g., Leland (1994) and Grenadier, Malenko, and Strebulaev (2014)). Suppose the seller owns an abandonment option: because of the information rent she pays, holding the auction late (at a lower cash flow level) still allows her to postpone the option exercise. The only difference now is that when bidders can initiate (discussed in the next section), the monotone initation strategy is increasing in cash flow, i.e., better types initiate earlier at higher cash flow because they recover more upon abandonment.

4 Bidder Initiation

Many economic interactions such as corporate takeovers, competition for supply contracts, and talent recruitment have characteristics of auctions because buyers are competing with one another to make offers. Yet the seller may not have full control over auction timing and a bidder may approach the seller with an offer to trigger an auction.[8] Is there still inefficient delay in auctions when bidders can initiate? How does it alter the seller's revenue and welfare?

These are not only of theoretical interests, but also have empirical implications. In fact, in M&As and patent sales, bidders often can initiate the auction (Fidrmuc, Roosenboom, Paap, and Teunissen (2012) and Gorbenko and Malenko (2017)). Aktas, De Bodt, and Roll (2010) and Masulis and Simsir (2015) document that bidding premium in M&As crucially depends on whether a bidder or a target initiates; Hackbarth and Miao (2012) analyze the timing of mergers in a real options framework. Nevertheless, theoretical research on the initiation of M&A deals are very limited, with Gorbenko and Malenko (2017, 2016) and Chen and Wang (2015) as notable exceptions.

To gain deeper insights into the process, one has to formalize the game. Typically the seller's lack of commitment is not merely restricted to auction timing, but also concerns renegotiation and bidders' offer adjustment. As McAdams and Schwarz (2007) point out, in real life committing to a sealed-bid auction is hard, especially in corporate acquisitions.[9] The board of directors of a target firm has to disclose all bids to shareholders, and considers subsequent offers to avoid shareholder lawsuits. In reality, these sales either entail sellers

[8]When an asset of a Delaware corporation is for sale, the Revlon rule imposes upon directors a duty to solicit competitive bids to maximize shareholders' value. It may seem that many takeovers occur after one-on-one negotiations, but as demonstrated in Aktas, De Bodt, and Roll (2010), even in such cases latent competition such as the threat of sale to a rival buyer is significant.

[9]Even in formal auctions, such a commitment is difficult to maintain. in "Lawsuit Seeks to Block Sale of G.M. Building", New York Times, September 20, 2003, Charles Bagli documents how General Motors entertained a late offer after auctioning its Manhattan building in a first-price auction.

and buyers' engagement in multiple rounds of negotiations and repeated communications, or manifest themselves in two-stage auctions used in privatization, takeover, and merger and acquisitions (e.g. Frankel (2011)). The former resembles an informal English auction in which buyers raise their bids until one winner emerges. Perry, Wolfstetter, and Zamir (2000) show the latter are typically robust mechanisms equivalent to an English auction. As such, I focus on SPAs, which are immune to offer adjustments due to the fact that bidding one's own value is the weakly undominated strategy. English auctions are equivalent to SPAs in terms of revenue, allocation, and welfare. I discuss them in the next section when we introduce security bids.

The fact that all agents dynamically update their beliefs about the distribution of types complicates the game in general. Although a bidding equilibrium with FPA can be derived, it involves asymmetric bidders and does not survive bidders' offer adjustments and renegotiation.[10] Asymmetries among bidders do not affect bidding behavior in SPAs, however; it is still a weakly dominant strategy for each bidder to bid his or her value (Krishna (2009)). Therefore, focusing on SPAs and English auctions also helps us illuminate the key economic mechanism under relatively transparent bidding and option exercise strategies.

The equilibrium concept is Perfect Bayesian and as is standard in the auctions literature, we focus on symmetric equilibria with a weakly monotone threshold for initiation; that is, if $\theta < \theta'$, the initiation threshold for bidder θ is weakly lower than that for θ'. This means that if the auction has not been initiated at X_t, everyone updates their beliefs about types that are present.

4.1 Endogenous Initiation by Sellers or Bidders

The seller times the auction to maximize the second-highest valuation, whereas a bidder times the auction to maximize the present value of informational rent (difference between

[10] See Gorbenko and Malenko (2016) for a detailed discussion of bidder initiation in FPAs absent post-auction option exercises. In FPAs, bidding strategies depend on the dynamically updated beliefs about the types present. In a monotone equilibrium, a better type that initiates reveals his type, and is not guaranteed to win unless he bids his own valuation, because a slightly worse type can outbid him.

his valuation and the second-highest valuation). The latter starts to erode earlier than the former as the initiation threshold X_a increases. Therefore, the seller always waits in such an equilibrium.

Proposition 2. *An SPA or English auction of real option admits an essentially unique auction timing equilibrium*[11]*, whereby bidders always initiate with threshold* $\max\{X_I(\theta), X_0\}$*, where* $X_I(\theta)$ *is weakly increasing in* θ *and uniquely solves*

$$\int_{\theta}^{\hat{\theta}} \frac{d}{dX} \left. \frac{W(X;\theta) - W(X;\theta')}{X^{\beta}} \right|_{X=X_I} f(\theta')[1-F(\theta')]^{N-2} d\theta' = 0, \tag{6}$$

and $\hat{\theta}$ *solves* $W(X_I;\hat{\theta}) = Y$*.*

If by cash flow level X the auction has not been initiated, everyone updates their beliefs about types that are present. The seller times the auction to maximize the second-highest valuation, whereas a bidder of type $X_I^{-1}(X)$ times the auction to maximize the present value of informational rent. The best type's real option becomes in the money before the second best type's, therefore the best type is always more eager to initiate than the seller in such an equilibrium.

The prediction that bidders initiate is broadly consistent with empirical evidence. For example, patent holders rarely organize an auction and instead are often approached by acquirers. Also, acquisitions by strategic bidders in informal negotiations are primarily bidder-initiated.[12]

A bidder gets the difference between his valuation and the second highest valuation, and does not bear the cost of ownership transfer Y unless he is the only participant, because this cost impacts the highest valuation and the second highest valuation in the same way. He also does not internalize the seller's preference b. It in turn means that when Y increases

[11]This is a Markovian Perfect Bayesian Equilibrium if we use both X_t, and $\hat{X} := \sup\{X_{t'}, t' \leq t\}$ as state variables, though on the equilibrium path they coincide

[12]For example, Fidrmuc, Roosenboom, Paap, and Teunissen (2012) document almost 80% are bidder-initiated. Note the current model is more applicable to strategic acquisitions where bidders are more likely to have private information regarding valuation than in financial acquisitions.

or b decreases, both the seller and the bidders would prefer a higher threshold for initiation, but the seller is differentially affected more. As such, conditional on knowing the least-cost type, the auction should be held later to maximize social welfare.

Corollary 4. *Bidder-initiated informal auctions are inefficiently accelerated ex post.*

Finally, since a bidder would not initiate until the investment trigger for his real option is reached, whereas a seller may time an auction when no real option is actually in the money, but only the second best type's is "in the money" in expectation. The following corollary ensues.

Corollary 5. *The real option is exercised more quickly on average in auctions allowing bidder initiation than in those only initiated by the seller.*

This corollary links the auction timing to the exercise timing, and is evidence that endogenous auction initiation and timing have material impact on equilibrium outcomes.

4.2 Empirical Illustration using Oil Lease Auctions

Natural settings to compare auctions of real options where bidders can or cannot initiate are rare to come by, therefore most of our model predictions are hard to test. However, corollary 5 predicts that a bidder initiates only when his real option is in the money, which implies an investment option is, on average, exercised more quickly when bidders initiate than when a seller initiates strategically or randomly. The leasing and exploration of oil and gas tracts in the Gulf of Mexico provide a unique setting for testing this model implication. What follows is an empirical illustration corresponding to testing Corollary 5, which is not meant as proof for causal relationship but provides suggestive evidence.

4.2.1 Institutional Background

Offshore drilling activities in the gulf of Mexico date back to the 1940s. The US Congress passed the Outer Continental Shelf Lands Act (OCSLA) in 1953 to grant the Department of the Interior the authority for conducting lease auctions, collecting royalties, and overseeing all activities associated with the drilling in federal waters. The Minerals Management Service (MMS) traditionally conducted the lease auctions, but due to a reorganization in response to the Deepwater Horizon oil spill in 2010, it was replaced by the Bureau of Ocean Energy Management (BOEM) and the Bureau of Safety and Environmental Enforcement (BSEE). Most leases were sold in "Bonus-bid" auctions, where the royalty rate on future revenue is fixed and the bidders bid upfront cash. The current royalty rate is standardized at 18.75%, but has historically taken on different values at various times and for different leases.

One important policy change utilized here is the introduction of Area Wide Leasing (AWL) in May 1983, which marked an important break in the lease auction and drilling environment for offshore tracts. Prior to AWL, potential bidders could nominate most near-shore tracts (less than 200 meters of water depth) and certain deep-water tracts (exceeding 200 meters of water depth) to be auctioned. Following comments by other interested parties, such as fishery and environmental interests, BOEM carried out lease sales which were typically on the order of a few hundred tracts. AWL eliminated the nomination process and made most of the offshore blocks in a region available in each sale, including thousands of tracts in deep water areas.[13]

Data on the lease auctions, drilling, and mineral production are from the Minerals Management Service of the Department of Interior. I observe detailed lease-level variables, borehole-level variables, lease sale data, ownership data, and production data. For the cost of drilling and equipping a borehole that varies by year, region, well type, and well depth, I use John Beshears's inflation-adjusted estimates based on annual surveys by the American Petroleum Institute (API) and GDP implicit price deflator index from the Bureau of Economic Analysis. The detailed description of various variables are in Beshears (2013). The estimation related to AWL uses leases auctioned in 1978-1989 with a total of 5399 leases.

[13]Moreover, some lease tenures were increased from 5 or 8 years to 10 years, and the royalty rates on tracts with water depth of more than 400 meters were lowered from 1/6 to 1/8. Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010) give more details.

Monthly prices for oil and natural gas are obtained from the World Bank Commodity Price Data (Index Mundi Data Set and the Energy Information Administration (EIA) also contain futures prices, but are not monthly). The prices are inflation-adjusted using monthly CPI data from the Federal Reserve Bank in St. Louis.[14] The discussion in this paper uses average spot prices and the results are robust to using different categories of crude oil and natural gas (such as West Texas Intermediate, Brent Crude Oil, etc.).

The key variables in the empirical exercises are listed below:

- ***Event***: The event is first exploratory drill. For this, I take the spud date for the first exploratory borehole drilled in each lease tract.

- ***AWL***: Dummy for the implementation of AWL policy. It takes the value 0 for leases auctioned before May 1983 and 1 afterward.

- ***RTY***: The royalty rate specified in the lease agreement. It is typically 1/6 or 1/8 in this data set.

- ***DUR***: The number of days from lease auction to expiration. Most lease terms are 5-10 years.

- ***DEPTH***: The water depth of the leased tract. The results reported use minimum water depth, and are robust to alternative specifications using maximum water depth or average water depth.

- ***SIZE***: The area covered under each lease. Most leases had an area of approximately 5,000 acres, though some were smaller.

- ***MKT***: The number of leases sold in the same sale as a proxy for the market demand for oil and gas leases. When the market demand is high, the quality of the marginal lease sold may be low in the sense that the reserve quantity is small or there is huge uncertainty, which in turn affect drilling decisions.

[14] $http://research.stlouisfed.org/fred2/graph/?s[1][id] = CPIAUCSL.$

- ***P(t)***: Average spot price for oil and gas. The results are robust to the inclusion of oil and gas prices separately, or prices lagged by 1-5 months.
- ***VOL(t)***: Average trailing-12-month volatilities for oil and gas spot prices. The results are robust to the inclusion of oil and gas price volatilities separately and to variations within one year of the trailing window.
- ***COST(t)***: The industry average drilling cost for dry, oil, and gas boreholes. The drilling cost is the initial cost plus the equipping cost.
- ***Firm f.e.***: I control for firm-specific characteristics by including the firm ID as a factor, and its interaction with time for firms owning multiple leases. Over the years, it tends to be the same group of firms that bid for leases. For jointly-owned leases, I use the largest shareholder. The results are robust to using the second-largest shareholder.
- ***Info.Ext.***: I control for information externality by using subsamples with lease areas exceeding $4,000$ acres and $5,000$ acres (full control) respectively. Prior studies (Lin (2009, 2012)) have shown that information externality is not significant for exploratory drills, and decreases with the size of the tract. Even for development drills, information externality is negligible for tracts larger than $5,000$ acres.

I employ a Cox proportional hazard model to test whether Auction initiation has any impact on how quickly tracts are explored. They make the assumption that the hazard rate $\kappa(t)$ of exploratory drill at time t conditional on lack of drill until time t is $\kappa(t) = \psi(t)[exp(\vec{V}(t)^T\vec{\gamma})]$, where $\psi(t)$ is the baseline hazard rate that is completely unrestricted, and $V(t)$ is a vector of independent variables listed earlier. This specification allows censoring of observations and allows time-varying covariates. There is no survivorship bias or response bias because the leases are sampled at birth (the auction), and all leases are recorded by the Department of the Interior.

The results are reported in Table 1. The coefficient on the AWL dummy is consistently negative and significant, indicating that ceteris paribus, the rate to exploratory drill decreased from that in the years with bidder initations. I consider the years 1978-1989, though the results are robust to the size of the window, and AWL consistently reduces the likelihood of exploratory drilling by at least 10%. I also estimate the model with year dummies instead of AWL, and Figure 4 clearly shows the break in the estimated coefficients, which corresponds to a reduction of 40% in the hazard ratio to explore and drill for leases auctioned after the implementation of AWL. These findings are consistent with the fact that options are typically exercised faster under bidder initiation because the initiating bidder's real option comes into money earlier than that for the seller.

5 Timing Auctions with Security Bids

Thus far we have focused on cash auctions to illustrate the economic intuitions. In reality, we routinely observe competing bids in combinations of cash and contingent securities. If the contingent securities are entirely written on $X_\tau - \theta$ when the option is exercised at $t = \tau$, they do not distort the winning bidder's incentives for option exercise, then they are equivalent to cash bids. However if the security is written on X, it introduces post-auction moral hazard as in Cong (2017): due to the non-contractibility of θ, the winning bidder bears the private cost, but only claims partial cash flow from the exercise We now extend our analysis to this setting.

5.1 Introducing Security Bids

The inefficient delay depends on the specific security used because the latter affects auction timing through τ_1^*, yet the main tradeoff remains. As long as the seller's virtual valuation differs from true option value, timing auctions leads to substantial variations in revenues, potentially at the expense of welfare.

We follow Cong (2017) to define ordered standard security bids as follows.

DEFINITION. *A **standard security bid** is an upfront cash payment $C \in \mathbb{R}$ and a contingent payment at the time of investment τ given by continuous function $S(X_\tau) \in \mathbb{R}$.*

*An **ordered set of securities** ranked by index s is defined by a left-continuous map $\Pi(s) = \{C(s), S(s, \cdot)\}$ from $[s_L, s_H] \subset \mathbb{R}$ to the set of standard security bids such that for each voluntary participant of type θ, $\boldsymbol{V(s,\theta) \equiv \tilde{V}(C(s), S(s,\cdot), \theta)}$ is non-negative and non-increasing in s on $[s_L, \tilde{s}]$ and negative on $(\tilde{s}, s_H]$ for some $\tilde{s} \in [s_L, s_H)$.*

Standard security bids are simple and intuitive and, as shown in Cong (2017), can implement the optimal auction design in an extended space of securities. In addition to being standard, an ordered set of securities admits one-dimensional ranking with index s for any payoff from the project, and permissible bids cover a wide range such that each participant earns a non-negative profit by bidding low enough but earns no profit by bidding too high. Any such sets can be represented by the mapping defined above up to an order-preserving transformation of the index. s could be the fraction of shares α in a pure equity auction $\{C(\alpha) = 0, S(\alpha, X) = \alpha X\}$, the (negative) strike price k in a call-option auction $\{C(-k) = 0, S(-k, X) = \max\{X - k, 0\}\}$, or the bonus B in a bonus-bid auction with royalty rate ϕ fixed $\{C(B) = B, S(B, X) = \phi X\}$. M&As, VC contracts, and lease auctions routinely use such securities, and indeed the bidder offering the highest s wins.[15]

The numerical illustrations to follow sometimes include another common form of security: a fixed promise of payment B from the project's payoff—essentially debts without interests, $S(B, X) = \min(X, B)$, also known as friendly debt, or in Islamic finance, Qard/Qardul hassan.[16]

[15] In M&As with the acquirer's stocks as bids, C simply corresponds to the value of the acquirer's cash flows that are independent of the acquisition, and X is the payoff from the acquired assets and projects, and the synergy created. Cong (2017) discusses how the seller may want to restrict the range of allowable bids.

[16] Interestless debts are used frequently in contractual agreements in Islamic banking and microfinance, and are equivalent to granting the winning bidder instead of the seller call options - the exact opposite situation to that for call option bids.

Now define

$$\tilde{V}(C, S(\cdot), \theta) = \max_{\tau \geq t_a} \mathbb{E}_X[e^{-r(\tau - t_a)}(X_\tau - S(X_\tau) - \theta)] - C, \tag{7}$$

where τ is any stopping time. $S(X)$, being of general form, distinguishes this problem from traditional real-options models. Cong (2017) shows that under mild regularity conditions, a threshold investment strategy exists that is optimal among all stopping times. Moreover, the valuation $\tilde{V}(C, S(\cdot), \theta)$ is continuously decreasing in θ.

Formal Auctions

We first consider "formal auctions" where the seller can commit to the security design and auction timing. Cong (2017) characterizes the bidding strategies in FPAs and SPAs for a fixed auction time t_a. As shown in the appendix, Lemma 1 and Proposition 1 still hold, and the proofs are only modified with a new stopping time by the winning bidder.

Auction timing continues to matter with security bids, as seen in Figure 3(a). It also affects the ranking of security designs: among several pure contingent securities, equity gives the highest expected revenue and call option the lowest at $X_a = 280$, whereas call option is the highest and debt is the lowest at $X_a = 360$. The worst security design at $X_a = 300$ more than doubles the revenue from the best security design at $P_a = 220$. Welfare is similarly affected (Figure 3(b)). In this regard, strategic timing could be as important as security design.

Optimal Auction

Even with security bids, Lemma 1 still holds. The optimal auction thus involves both security design and auction timing. As shown in Cong (2017), even with interdependent values, the optimal security design can be implemented using an auction with standard security bids. The key insight lies in that the seller can use a bid-specific royalty rate to incentivize the winning bidder to exercise at $X^*(z(\theta_{(1)}))$, thus making the bidder face the same optimization problem. The optimal design complements this royalty rate with a corresponding bonus payment, to ensure truth-telling in the bidding stage. Though the seller's and bidders' preferences for option exercise are now aligned, we note that the "virtual strike price" $z(\theta)$ means the auction still occurs later than a constrained-efficient formal auction.

These results relate to Myerson (1981)'s analysis in a static setting on the wedge between the seller's revenue and welfare. Take the case of $b = 0$: in addition to bidder exclusion, option exercises are inefficiently delayed under the current settings to increase revenue. Moreover, auction timing leads to several distinct features in an optimal auction. Although the seller still excludes bidders, the auction is held under better market conditions (higher X_a), which encourages participation and mitigates the exclusion. This implies that in real life one may not see sellers excluding bidders as much using entry fees or reserve prices, because she has the alternative tool of choosing a more propitious time to hold the auction. For example, an entrepreneur selling a startup seldom excludes potential acquirers, but rather waits for the product to have higher valuations before going onto the market. Inefficiencies in dynamic settings are thus multi-dimensional.

Informal Auctions

In many cases the seller cannot commit to a security design and entertains all forms of offers. I follow DeMarzo, Kremer, and Skrzypacz (2005) to call these transactions informal auctions.[17] Is there still inefficient option exercise in informal auctions and how would auction timing change? Answers to these questions would allow us to apply the insights derived earlier much more broadly.

If the seller commits to neither a pre-specified timing of the auction nor a bidding and allocation rule, she holds the auction at the most opportune time, and then chooses the bid that gives her the highest expected payoff based on her beliefs regarding the type of each

[17] DeMarzo, Kremer, and Skrzypacz (2005) do not consider offer adjustments from winning and losing bidders as I do in ascending informal auctions defined later.

bidder at the time the auction is held. Because auction timing, bidding, and investment involve sequential actions, the equilibrium concept for informal auctions is **Perfect Bayesian Equilibrium**. Informal auctions therefore exhibit signaling through security choices. In particular, it is the most natural to formally define an English auction with security bids:

1. The seller initiates the auction at some time t_a and all agents enter the bidding stage.

2. The seller gradually increases a numerical score R from $R = Y$, and a bidder remains in the auction if he can deliver an informal bid from a "feasible set" $\{\Pi : R(\Pi) \geq R\}$. The auction ends when only one bidder is left, and he chooses an informal bid from the final "feasible set."

3. The winning bidder i pays the upfront cash C^i and the initial cost X at t_a, and then invests rationally at $\tau^i_{\theta_i}$ and makes the contingent payment $S^i(P_{\tau^i_{\theta_i}})$, where C^i and S^i are given by his chosen final bid.

Note this variant of the English auction is equivalent to SPAs, in which bidders bid a score they generate and the winner pays the second-highest score bid.[18] This a priori is different from SPAs in which the winning bidder pays the informal bid corresponding to the second highest score. The distinction is important because the same security bids generally cost the buyers differently.

Cong (2017) shows that in informal auctions, investments are always efficient conditional on auction timing when the seller cannot commit to pre-specified security design, and in equilibrium, every bid is equivalent to cash. The intuition is that the least information-sensitive bids allows a better type to separate from worse types in the cheapest way: to outbid an opponent by one dollar costs the same for all types, but to outbid an opponent by one percent of equity costs a better type strictly more. Moreover cash bids creates the biggest social surplus. Therefore, in equilibrium everyone bids cash (Proposition 5 in Cong

[18] Defining an ascending auction with multiple security bids is challenging. See also Gorbenko and Malenko (2017), one of the first studies to formalize an English auction with security bids (both cash and equity).

(2017)). As such, Lemma 1, and Propositions 1 and 2 apply without modification. The seller times the auctions inefficiently late as long as b is not too big. Moreover, when the bidders can initiate in an ascending informal auction, they always do so in equilibrium.

6 Conclusion

Auctions of real options are ubiquitous, involve tremendous financial resources, and have policy implications. Because the irreversible nature of time endows a seller potential control over the post-auction option exercise, endogenous auction timing and initiation matter. I find that as long as the seller is not too biased towards early option exercise, she inefficiently delays the auction, which mitigates the divergence in her and the bidders' preferences for option exercise; otherwise, auction timing is efficient but option exercise is always inefficiently delayed. When bidders can initiate, they always do so in equilibrium and options are exercised faster on average.

To better understand corporate transactions and business sales in reality, I extend the analysis to security-bid auctions of real options. I find that both endogenous auction initiation and the seller's commitment to the auction design significantly influence equilibrium outcomes. Taken together, the results of the paper complement earlier approaches that treat auction initiation and timing as exogenous, and are especially relevant for sales of assets embedded with real options.

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Appendix: Derivations and Proofs

A.1 Proof of Lemma 1

Proof. Let $Q(\tilde{\theta}_i, \theta_{-i})$ be the probability of allocating the project to bidder i, who has investment cost $K(\theta_i, \theta_{-i})$, where K is symmetric in other bidders' report types, and has positive derivative in θ_i denoted by K_1 that is uniformly bounded by a constant $A > 0$. In the main model of this paper, $K(\theta_i, \theta_{-i}) = \theta_i$, but this specification allows other cases with interdependent values such as common-value auctions.

The expected utility at time zero to type θ_i upon participating and optimally investing is

$$U(\theta_i, \tilde{\theta}_i) = \mathbb{E}_{\theta_{-i}}\left[Q(\tilde{\theta}_i, \theta_{-i}) \max_{\tau \geq t_a} \mathbb{E}_X\left[e^{-r\tau}(X_\tau - K(\theta_i, \theta_{-i})) - \int_{t_a}^{\infty} e^{-rt} S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t) dt - e^{-rt_a} Y\right]\right],$$

where $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ is a general security payment (including cash) that potentially depends on everyone's report and the common information filtration for both the seller and winning bidder. In the baseline model, $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t) = rB(\tilde{\theta}_i)$, which makes the integral term the cash bid $B(\tilde{\theta}_i)$ at the time of auction. As $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ could be artificially constructed that an optimal stopping time for exercising the real option may not exist, it is reasonable to focus attention on the set of $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ such that an optimal stopping time exists for all types under a direct mechanism. With this restriction, let $\tau^*(\theta_i, \tilde{\theta}_i, \theta_{-i})$ denote the optimal stopping time that is almost surely bigger than t_a, and $\tau_i^* = \tau^*(\theta_i, \theta_i, \theta_{-i})$. Incentive compatibility requires $U(\theta_i) \equiv U(\theta_i, \theta_i) \geq U(\theta_i, \tilde{\theta}_i)$ and the individual rationality requires $U(\theta_i) \geq 0$.

The IC constraint can be written as $\theta_i \in \text{argmax}_{\tilde{\theta}_i \in [\underline{\theta}, \overline{\theta}]} U(\theta_i, \tilde{\theta}_i)\ \forall\ i$. Let $a = (\tau, \tilde{\theta})$ denote the action pair of reporting $\tilde{\theta}$ and rationally exercise following the stopping time τ. Let

$$g(a, \theta) = Q(\tilde{\theta}, \theta_{-i}) \mathbb{E}_X\left[e^{-r\tau}(X_\tau - K(\theta_i, \theta_{-i})) - \textstyle\int_{t_a}^{\infty} e^{-rt} S(\tilde{\theta}, \theta_{-i}, \mathcal{I}_t) dt - e^{-rt_a} Y\right]$$

Then following the argument in Milgrom and Segal (2002), for any θ',$\theta'' \in [\underline{\theta}, \overline{\theta}]$ with $\theta' < \theta''$,

$$\begin{aligned}
|U(\theta') - U(\theta'')| =& \mathbb{E}_{\theta_{-i}}\left[\left|\sup_{a'} g(a', \theta') - \sup_{a''} g(a'', \theta'')\right|\right] \\
\leq& \mathbb{E}_{\theta_{-i}}\left[\sup_a |g(a, \theta') - g(a, \theta'')|\right] = \mathbb{E}_{\theta_{-i}}\left[\sup_a \left|\int_{\theta''}^{\theta'} g_\theta(a, \theta) d\theta\right|\right] \\
\leq& \mathbb{E}_{\theta_{-i}}\left[\int_{\theta'}^{\theta''} \sup_a |g_\theta(a, \theta)| \, d\theta\right] \leq A|\theta'' - \theta'|
\end{aligned}$$

This implies $U(\theta)$ is absolutely continuous, and thus differentiable everywhere. $U(\theta) = U(\hat{\theta}) -$

$\int_{\theta}^{\hat{\theta}} U'(\theta')d\theta'$. By Theorem 1 in Milgrom and Segal (2002), $U'(\theta) = g_\theta(a^*, \theta)$. Writing it in the integral form gives that any incentive compatible and individually rational mechanism satisfies

$$U(\theta_i) = \mathbb{E}_{\theta_{-i}}\left[\int_{\theta_i}^{\hat{\theta}} Q(\theta_j, \theta_{-i})\mathbb{E}_X[e^{-r\tau_j^*}]K_1(\theta_j, \theta_{-i})d\theta_j\right] + U(\hat{\theta}) \tag{8}$$

where $U(\hat{\theta}) \geq 0$. Moreover $\tau_i \geq t_a$, $\forall i$ for time consistency.

The ex-ante social welfare is $N\mathbb{E}_{\vec{\theta}}[Q(\theta_i, \theta_{-i})\left(\mathbb{E}_X[e^{-r\tau_i^*}(X_{\tau_i^*} - K(\theta_i, \theta_{-i}) + b)] - e^{-rt_a}Y\right)]$, and the seller's *ex ante* revenue is the social welfare less the agents' ex-ante utilities:

$$N\mathbb{E}_{\vec{\theta}}[Q(\theta_i, \theta_{-i})(\mathbb{E}_X[e^{-r\tau_i^*}(X_{\tau_i^*} - K(\theta_i, \theta_{-i}) + b)] - e^{-rt_a}Y)] - N\mathbb{E}_{\vec{\theta}}[U(\theta_i)].$$

Using (8) and taking expectations over the winning bidder's type, it becomes

$$N\mathbb{E}_{\vec{\theta}}[Q(\theta_i, \theta_{-i})(\mathbb{E}_X[e^{-r\tau_i^*}(X_{\tau_i^*} - K(\theta_i, \theta_{-i}) - F(\theta_i)/f(\theta_i) + b)] - e^{-rt_a}Y)] - NU(\hat{\theta})$$

The seller optimally sets $U(\hat{\theta}) = 0$. When $K(\theta_i, \theta_{-i}) = \theta_i$, this simplifies to

$$N\mathbb{E}_{\vec{\theta}}[Q(\theta_i, \theta_{-i})(\mathbb{E}_X[e^{-r\tau_i^*}(X_{\tau_i^*} - z(\theta_i) + b)] - e^{-rt_a}Y)].$$

To maximize revenue, the seller allocates to the best type, if at all. The optimal cutoff type is set to contribute zero revenue upon winning, therefore θ_c solves

$$D(X_a; X^*(\theta))[X^*(\theta) - z(\theta) + b] - Y = 0 \tag{9}$$

Notice the LHS is monotone in θ, we thus get the results in the corollary. $\square$

A.2 Proof of Proposition 1

Proof. With cash bids, $X_{\tau^*} = X^*(\theta_{(1)})$, but for other security-bid FPAs or SPAs, X_{τ^*} may depend on $\theta_{(2)}$ in SPAs. Denote $X_{\tau^*(\theta_{(1)}, \theta_{(2)})}$ as X_{τ^*}. The seller's expected utility for holding auction when X_a is first reached can be written as

$$D(X_0; X_a)\int_{\underline{\theta}}^{\hat{\theta}} d\theta \int_{\theta}^{\overline{\theta}} d\theta' \frac{N(N-1)}{2} f(\theta)f(\theta')[1-F(\theta')]^{N-2}\left[D(X_a; X_{\tau^*})\left[X_{\tau^*} - \theta - \chi\frac{F(\theta)}{f(\theta)} + b\right] - Y\right]. \tag{10}$$

where $\chi = 1$, $X_{\tau^*}(\theta)$ is the winning bidder's investment threshold, and $\hat{\theta}$ potentially depends on X_a. Here θ' is the best realized type from the remaining $N-1$ bidders. The derivative w.r.t. X_a is

$$\frac{D(X_0; X_a)}{X_a} \int_{\underline{\theta}}^{\hat{\theta}} d\theta \int_{\theta}^{\overline{\theta}} d\theta' \frac{N(N-1)}{2} f(\theta) f(\theta') [1-F(\theta')]^{N-2} \left[\beta Y + \mathbf{I}_{\{X_a > X_{\tau^*}\}} \left[\beta(z(\theta) - b) - (\beta - 1) X_a\right]\right], \tag{11}$$

where I have used the Liebniz formula and the fact that marginal revenue from an interior cutoff type is zero. This expression is continuous in X_a with derivative positive for $X_a \leq \underline{X} \equiv \frac{\beta}{\beta-1}\underline{\theta}$ and negative for $X_a \geq \overline{X} \equiv \frac{\beta}{\beta-1}(Y + z(\overline{\theta}) - b)$. Thus there exists X_a in the compact region $[\underline{X}, \overline{X}]$ that maximizes (10). This proves the existence of optimal threshold strategy for auction timing, and the fact that the seller never holds the auction when no bidder would exercise immediately. When we have security bids and X_{τ^*} is not necessarily $X^*(\theta_{(1)})$, we can redefine $\underline{X} \equiv \min_\theta X_{\tau^*}(\theta)$.

Now apply the above argument to welfare, an efficient threshold strategy exists, and the derivative of social surplus w.r.t. X_a is

$$\frac{D(X_0; X_a)}{X_a} \int_{\underline{\theta}}^{\hat{\theta}} d\theta \int_{\theta}^{\hat{\theta}} d\theta' \frac{N(N-1)}{2} f(\theta) f(\theta') [1-F(\theta)]^{N-2} \left[\beta Y + \mathbf{I}_{\{X_a > X_{\tau^*}\}} \left[\beta(\theta - b) - (\beta - 1) X_a\right]\right], \tag{12}$$

which is smaller than (11) for every X_a. At the optimal threshold X_{opt}, integrating (11) over $[X_{opt}, X_a]$ must be weakly negative for any $X_a > X_{opt}$. Thus integrating (12) over $[X_{opt}, X_a]$ must be also weakly negative for any $X_a > X_{opt}$, implying the efficient threshold $X_{eff} \leq X_{opt}$. Now at X_{eff}, (12) is necessarily zero, otherwise it is not a local maximum. Because $X_{eff} \geq \underline{X}$, integrating (11) over $[X_{eff}, X_{eff} + \epsilon]$, where $\epsilon > 0$ is infinitesimal, must be positive. This implies $X_{eff} < X_{opt}$. Another way to see this is that Equation (10) with $\chi = 0$ corresponds to welfare, and the equation is supermodular in (X_a, χ). Thus a seller optimally delays the auction beyond the socially efficient threshold given the same security design and allocation rule. When $b < 0$, the option exercise by the winning bidder could be inefficiently early ($X_a < X^*(\theta_{(1)} - b)$) or late ($X_a < X^*(\theta_{(1)} - b)$); when $b > 0$, it is always inefficiently late.

Now if b is sufficiently large, i.e. $b > Y + \overline{\theta} - \underline{\theta}$, even though the seller prefers an early option exercise, the socially optimal auction threshold is lower than $X^*(\underline{\theta})$, making (11) and (12) equal. Therefore the maximizer for the seller's payoff must also be the maximizer for the social planner. Auction is thus efficiently timed, and the option exercise by the winning bidder is inefficiently late because he does not internalize the positive b.

Finally, we remark that we obtain inefficient delay even absent divergence in preference for option exercise ($b = 0$) and auction cost ($Y \to 0$). This is stated in the corollary. The information asymmetry alone creates divergence in preference for option exercise. $\square$

A.3 Proof of Proposition 2

Proof. Conjecture that in equilibrium bidder θ initiates the auction with threshold $X_I(\theta)$, which is increasing, and the seller initiates with a threshold X_S. Let $\theta(X_a) = sup\{\theta : X_I(\theta) \le X_a\}$. As discussed before, we denote $\hat{\theta}$ as the worst type that participates when the seller can rationally charge a reserve price $R = W(X_a; \hat{\theta}(X_a))$. The expected payoff to the bidder θ following initiation threshold $X_a \le X_S$ is

$$\int_{\underline{\theta}}^{\theta(X_a)} d\theta' \frac{(N-1)f(\theta')}{[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_I(\theta')}\right)^{\beta} \left[[W(X_I(\theta');\theta) - \max\left\{W(X_I(\theta');\hat{\theta}(X_I(\theta'))), W(X_I(\theta');\theta')]\right\}\right]^{+}$$
$$+ \int_{\theta(X_a)}^{\overline{\theta}} d\theta' \frac{(N-1)f(\theta')}{[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_a}\right)^{\beta} \left[[W(X_a;\theta) - \max\left\{W(X_a;\hat{\theta}(X_a)), W(X_a;\theta')]\right\}\right]^{+} \tag{13}$$

where θ' is basically the first-order statistic of the remaining $N-1$ bidders; similarly the payoff when $X_a > X_S$ is

$$\int_{\underline{\theta}}^{\theta(X_S)} d\theta' \frac{(N-1)f(\theta')}{[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_I(\theta')}\right)^{\beta} \left[[W(X_I(\theta');\theta) - \max\left\{W(X_I(\theta');\hat{\theta}(X_I(\theta'))), W(X_I(\theta');\theta')]\right\}\right]^{+}$$
$$+ \int_{\theta(X_S)}^{\overline{\theta}} d\theta' \frac{(N-1)f(\theta')}{[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_S}\right)^{\beta} \left[[W(X_S;\theta) - \max\left\{W(X_S;\hat{\theta}(X_S)), W(X_S;\theta')]\right\}\right]^{+} \tag{14}$$

When $X_a \le X_S$, $\left(\frac{X_0}{X_I(\theta')}\right)^{\beta} \left[[W(X_I(\theta');\theta) - \max\left\{W(X_I(\theta');\hat{\theta}(X_I(\theta'))), W(X_I(\theta');\theta')]\right\}\right]^{+}$, if positive, decreases w.r.t. X_a when $X_a > X^*(\theta' - b + Y)$. Therefore, differentiating (13) w.r.t. X_a and applying Leibniz's formula gives that in equilibrium $X_I(\theta) \le X^*(\theta - b + Y)$.

Now for the seller, if she uses threshold X_a, the expected payoff is,

$$\int_{\underline{\theta}}^{\theta(X_a)} d\theta' \int_{\underline{\theta}}^{\theta'} d\theta \frac{N(N-1)f(\theta)f(\theta')}{2[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_I(\theta)}\right)^{\beta} \left[W(X_I(\theta);\theta') + D(X_a;X^*(\theta))b - Y\right]^{+}$$
$$+ \int_{\theta(X_a)}^{\overline{\theta}} d\theta' \int_{\underline{\theta}}^{\theta'} d\theta \frac{N(N-1)f(\theta')f(\theta)}{2[1-F(\theta')]^{2-N}} \left(\frac{X_0}{X_a}\right)^{\beta} \left[W(X_a;\theta') + D(X_a;X^*(\theta))b - Y\right]^{+}. \tag{15}$$

Suppose $X_a < X_I(\overline{\theta})$. For any $\theta' > \theta(X_a)$, the earlier argument leads to $X_a < X^*(\theta - b + Y)$, for otherwise θ would initiate earlier than X_a - a contradiction. This implies $X_a < X^*(\theta' - b + Y)$. Applying the Leibniz formula again, the derivative of (15) is then weakly positive path-by-path because $\left(\frac{X_0}{X_a}\right)^{\beta} [W(X_a;\theta') + D(X_a;X^*(\theta))b - Y]$ is increasing in X_a. This implies the integrand is weakly positive path-by-path for any $X_S = X_a$ unless $X_S = X_I(\overline{\theta})$. Thus almost surely the seller never initiates.

Now if the bidders initiate, their problem is reduced to expression (13). The derivative at X_a has the same sign as

$$\int_{\theta(X_a)}^{\overline{\theta}} d\theta' f(\theta')[1-F(\theta')]^{N-2} \frac{d}{dX} \left. \frac{\left[[W(X;\theta) - \max\left\{ W(X;\hat{\theta}(X)), W(X;\theta') \right] \right\} \right]^+}{X^\beta} \right|_{X=X_a}, \tag{16}$$

which is positive at $\hat{X}(\theta)$ and non-positive at $X^*(\theta+Y)$. The integrand is weakly monotone in X_a path-by-path, thus (16) changes sign at a unique $X_a = X_I(\theta)$.

Given (13) is concave in X_a with non-negative cross-partial in X_a and θ, and there exists unique maximizer $X_I(\theta)$, Implicit Function Theorem gives that $X_I(\theta)$ is indeed non-decreasing. A similar argument would rule out a decreasing equilibrium in which the initiator always loses. This ensures (16) is continuous, establishing the optimality of X_I and the FOC in the proposition. There could be multiple equilibria with different initiation thresholds below X_0, but in terms of initiation outcome and payoffs, they are all equivalent, making the proposed equilibrium essentially unique.

Given that a bidder's threshold for holding the auction is lower than his threshold if he were maximizing social welfare, the initiation is accelerated in the ex post sense. Moreover, he would invest in the project right away, making the exercise of the real option faster than in auctions where only the seller can initiate and the realized winning type might still wait after the auction. $\square$

Tables and Figures

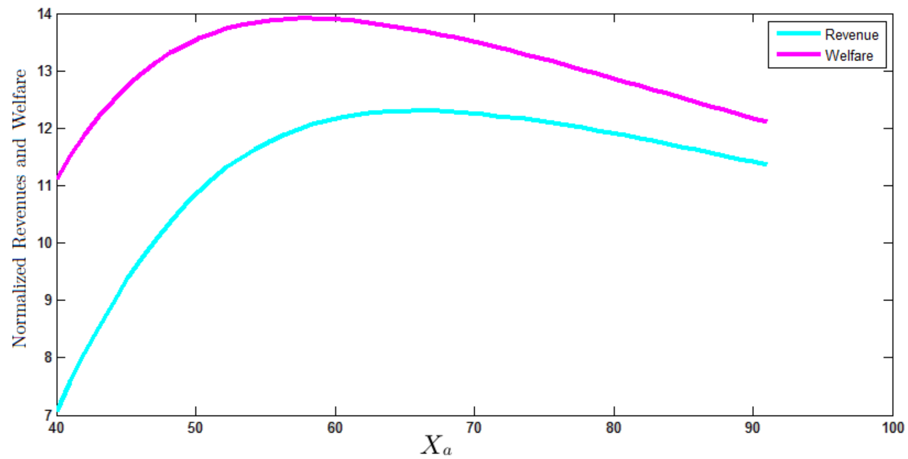
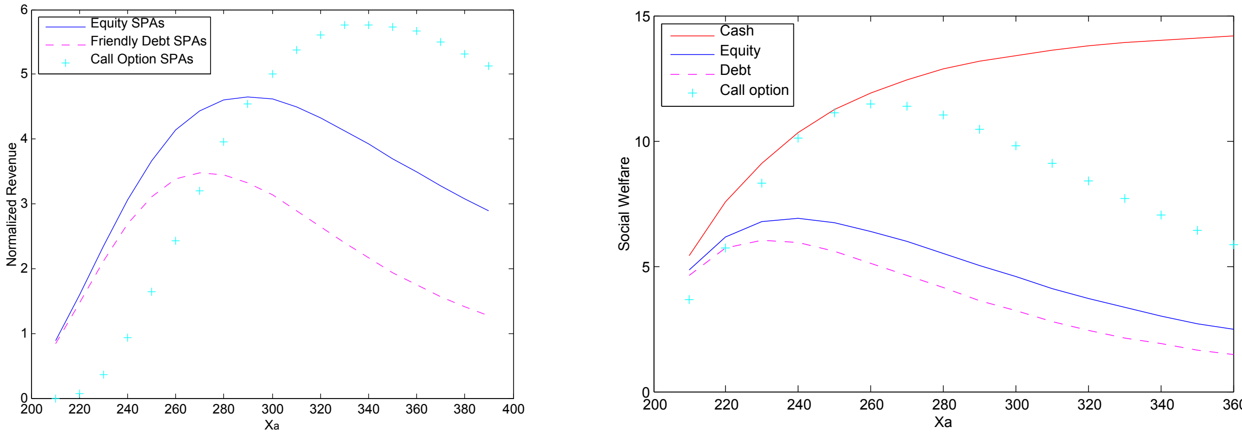


Figure 2: Revenues and Welfare for cash auctions following threshold timing X_a. 200,000 simulations for $\theta \sim Unif[10, 40]$, $r = 0.06$, $\mu = 0.01$, $\sigma = 0.2$, $Y = 15$, $N = 7$, $X_a = 40$. Welfare-maximizing auction timing threshold is lower than revenue-maximizing timing.



(a) $\beta = 5$, $Y = 10$, $N = 5$ (b) $\beta = 6$, $Y = 8$, $N = 30$

Figure 3: Plots of expected seller's revenue and social welfare against the auction threshold for SPAs with equities, friendly debts as defined in Section 5.1, and call options. One million simulations for θ uniformly distributed in $[200, 500]$, $P_0 = 210$. For exposition, β is specified instead of the primitives r, μ, and σ.

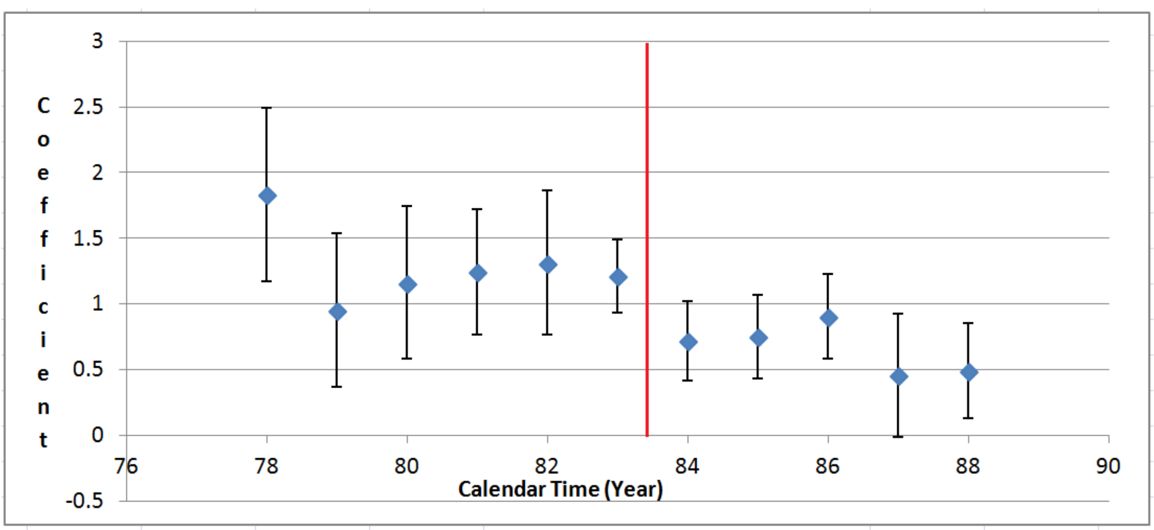


Figure 4: Coefficients for year dummies in the Cox estimation after controlling for all observable covariates, firm fixed effect, and information externality. The estimates are reported with 95% confidence interval. The red line marks the commencement of Area Wide Leasing (AWL).

Table 1: Exploration of Oil and Gas Tracts

This table presents estimates from a Cox regression with time-varying covariates. The dependent variable is time-to-exploratory-drill, which measures the number of days from the lease auction to the first exploration. The independent variables are the variable of interest *AWL* indicating the absence of bidder initiation, oil and gas price measure $P(t)$, price volatility $VOL(t)$, drilling cost $COST(t)$, royalty rate *RTY*, water depth *DEPTH,* lease length *DUR*, tract size *SIZE*, market demand *MKT*, firm fixed effects *FIRM f.e.*, and control for information externality *INFO.* If the dependent variable is observed without any realization, it is treated as a censored event. Model χ^2 reports the joint significance of the estimates. Hazard ratio indicates the impact of AWL on likelihood to drill.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
AWL	-0.5674***	-0.3875***	-0.1856*	-0.3310***	-0.2329*	-0.3558*	-0.1623*
	(0.0698)	(0.0820)	(0.1123)	(0.1008)	(0.1316)	(0.1839)	(0.0889)
MKT		0.0118**		0.0089	0.0144***	0.9153***	0.0071
		(0.0054)		(0.0061)	(0.0055)	(0.3169)	(0.0061)
RTY		-0.0200		-0.0591*		-0.0341	-0.0611*
		(0.0227)		(0.0318)		(0.0588)	(0.0315)
DUR		-0.0073*		-0.0106**		-0.2728	-0.0108**
		(0.0043)		(0.0046)		(82.07)	(0.0046)
DEPTH		-0.0005***		0.0004***		-0.0002	-0.0004***
		(0.0001)		(0.0001)		(0.0004)	(0.0001)
SIZE		-0.0003		-0.0022***			-0.0022***
		(0.0002)		(0.0005)			(0.0005)
P(t)			0.0214***		0.0212***	-0.0495	0.0144**
			(0.0058)		(0.0065)	(0.1019)	(0.0068)
VOL(t)			0.0037		-0.0076	-0.3401*	-0.0277
			(0.0159)		(0.0170)	(0.2053)	(0.0187)
COST(t)			0.0004		-0.0003	-0.0112	0.0002
			(0.0004)		(0.0004)	(0.0073)	0.0004
Info.Ext.	No	No	No	Yes	Yes	Yes (Full)	Yes
Firm f.e.	No	No	No	Yes	Yes	No	Yes
Hazard Ratio	0.5670	0.6787	0.8306	0.7182	0.7922	0.7006	0.8502
No. of Leases	5399	5399	5399	4983	4983	3266	4983
Model χ^2	66.05	210.7	90.43	445.1	437.7	23.92	488.1

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$