# Information-driven Business Cycles: A Primal Approach*

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**Abstract**

We develop a methodology to characterize equilibrium in DSGE models, free of parametric restrictions on information. First, we define a "primal" economy in which deviations from full information are captured by wedges in agents' expectations. Then, we provide conditions ensuring some information-structure can implement these wedges. We apply the approach to estimate a business cycle model where firms and households have dispersed information. The estimated model fits the data, attributing the majority of fluctuations to a single shock to households' expectations. The responses are consistent with an implementation in which households become optimistic about local productivities and gradually learn about others' optimism.

**Keywords:** Business cycle, dispersed information, DSGE models, information-robust characterization, primal approach, sentiments.

**JEL Classification:** E32, D84.

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# 1 Introduction

Many prominent theories in macroeconomics are based on incomplete information. Among their applications, such theories offer a structural interpretation of cyclical fluctuations, formalizing the widespread idea that business cycles are driven by waves of optimism and pessimism among consumers and firms (Lorenzoni, 2009; Angeletos and La'O, 2013; Benhabib, Wang and Wen, 2015). Yet, few of these models have been investigated quantitatively, mainly because of technical difficulties arising from the introduction of dispersed information in general equilibrium frameworks and the challenge of specifying ex-ante plausible information structures. This paper develops a new approach that avoids these difficulties, and uses it to explore the quantitative potential of dynamic stochastic general equilibrium (DSGE) models with incomplete information.

Our approach defines a "primal" economy in which deviations from full information are summarized by wedges in agents' equilibrium expectations. We then provide necessary and sufficient conditions for the existence of an information structure that is consistent with the expectation errors captured by these wedges. Subject to these implementability conditions, the set of dynamics that are feasible in the primal economy is the same as the set of dynamics that are feasible in the incomplete-information economy. Exploiting this equivalence, we derive a simple *information-robust* characterization of all equilibria in DSGE models with incomplete information.

We apply our approach to a dispersed-information version of an otherwise frictionless business cycle model. Shocks to productivity are the only fundamental source of aggregate volatility. The model allows households and firms to be imperfectly informed about both local and aggregate economic conditions. While the incomplete-information version of our economy is hard to solve, the corresponding primal economy permits a simple aggregate representation. Specifically, in the aggregate representation, expectation errors by households and firms are summarized by a "demand wedge" in the aggregate Euler equation and a "labor wedge" that governs the output gap of the economy. The behavior of these wedges is constrained by the implementability conditions characterized by our approach.

As a first main result, we show that if one does not impose any restriction on learning beyond Bayesian rationality, the model can fully account for any joint process in output, employment and inflation. This, firstly, reinforces the insight of Maćkowiak and Wiederholt (2015) that a single informational friction can replace the multitude of frictions typically used in DSGE models to generate empirically plausible responses to productivity and monetary shocks. Second, going beyond the results established by Maćkowiak and Wiederholt, our result also demonstrates that one does not even need any fundamental source of business cycle fluctuations. Instead, our benchmark result establishes that purely expectational shocks in the tradition of Lorenzoni (2009), Angeletos and La'O (2013) and Benhabib, Wang and Wen (2015) can, in principle, perfectly account for business cycle data.

Motivated by our benchmark result, we estimate our model using U.S. business cycle data on output, employment, inflation, and interest rates to assess the nature of information-frictions that is required to explain the data. Because our empirical specification includes more data targets than expectation wedges, the model is no longer able to generate a perfect fit. Nevertheless, we find that the estimated model does a remarkably good job at matching business cycle comovements, essentially replicating the vast majority of (auto) covariances within the confidence region of the data.

Our key findings are as follows. First, essential for the empirical performance of the model is its ability to generate wedges that are correlated across equations. In particular, we estimate a strong positive correlation between the demand wedge and the labor wedge and a negative correlation between the two wedges and productivity shocks. While it is typically difficult for full-information models with structurally uncorrelated shocks to generate perturbations that are correlated across equations, the expectation wedges in our model are naturally correlated. This is both because information can be correlated across households and firms and because expectation errors by households are likely to affect both their consumption and labor supply.

Second, decomposing the estimated labor wedge into a firm- and household-side component, we show that virtually all of the fluctuations in the labor wedge can be attributed to household errors. More generally, we cannot reject the null hypothesis that firms make no expectational errors. This is in line with the theoretical insights of Hellwig and Venkateswaran (2014), who demonstrate that firms need to observe only a small set of sufficient statistics in order to make optimal production plans. While our results do not require that firms have superior information relative to households, they do indicate that firms behave *as if* they had full information. In contrast, we can generally reject the hypothesis that households are aware of aggregate economic conditions for at least one year after their realization. For inflation and interest rates we do not reject household knowledge after a lag of one year; for output and productivity growth we do not reject household knowledge after a lag of roughly two years.

Third, the primary driver of business cycles is a purely expectational shock unrelated to productivity growth. The effects of this shock are driven jointly by the demand and labor wedge, inducing procyclical fluctuations in output, employment and inflation akin to a "demand shock". We demonstrate that the response is consistent with a simple parametric information structure, in which households receive news about productivity, generating a demand wedge as in Lorenzoni (2009). While Lorenzoni (2009) relies on nominal rigidity to generate real effects from household optimism, our implementation generates a comovement in the labor wedge by introducing dispersed information across households as in Lucas (1972, 1973) and Woodford (2003). As a result, households only gradually learn about the fluctuations in aggregate demand that are driven by the errors of other households. Accordingly, they supply labor *as if* wages were sticky.

Finally, in contrast to the full-information version of our economy in which hours and inflation do not respond to productivity, the estimated model gives technology a modest but positive role in driving these variables. This is because households learn about aggregate productivity innovations only gradually, implying a slow adjustment of output in response to TFP shocks with a negative output gap and deflation in the interim period. Productivity shocks account for roughly 30 percent of the business-cycle fluctuations in hours and roughly 40 percent of inflation.[1]

The methodology developed in this paper is related to the literature on information-robust predictions by Bergemann and Morris (2013, 2016) and Bergemann, Heumann and Morris (2014). These papers demonstrate the equivalence between Bayes equilibria in games with incomplete information and Bayes correlated equilibria. The primal approach developed in this paper is similar in that it also demonstrates the equivalence between a class of incomplete-information economies with another class of full-information models. It is more general, however, as it is not limited to static game environments, but equally applies to dynamic market economies. Moreover, the primal approach developed in this paper gives straightforward implementability conditions, and it extends to arbitrary "minimal information requirements" that can be imposed by the researcher.

On the applied side, our analysis relates to a recent literature exploring business cycle models with incomplete information. While the literature is mostly theoretical, there are now a few studies with a quantitative focus. In particular, Angeletos, Collard and Dellas (2015) and Huo and Takayama (2015*a*) explore a version of Angeletos and La'O (2013); Blanchard, L'Huillier and Lorenzoni (2013) estimate a simplified version of Lorenzoni (2009); Melosi (2014, 2016) estimates a variant of Woodford (2003); and Maćkowiak and Wiederholt (2015) calibrate a particular DSGE model with rational inattention. A notable difference with

[1]The estimated contribution to output fluctuations at business cycle frequencies is 10 percent. The small contribution of productivity to output is consistent with both recent DSGE estimations and the structural VAR literature, which rarely finds that productivity shocks explain more than one quarter of output cyclicality (Shapiro and Watson, 1988; King et al., 1991; Cochrane, 1994; Gali, 1999; Christiano, Eichenbaum and Vigfusson, 2003; Smets and Wouters, 2007).

respect to these papers is the flexibility of expectation dynamics considered in this paper. In particular, our approach does not require us to take an ex-ante stand on which agents are affected by information-frictions, how information is shared in the cross-section of agents, or any other parametric properties of the information structure. Instead it allows us to evaluate the empirical performance across *all* information structures and let the data decide which provides the best fit.

At a methodological level, the closest to our approach are Jurado (2016)—who estimates a model with near-rational belief distortions—and Angeletos, Collard and Dellas (2015)—who bypass the computational difficulties of incomplete information by relaxing the common prior assumption. Our approach allows for a similar generality of application, while ensuring that expectation errors are consistent with rational expectations of all agents. In addition, our approach is also particularly tractable, since our characterization of incomplete-information equilibria can be obtained using standard tools developed for full-information economies.

In its ability to reduce the computational burden of solving (and estimating) incomplete information models, our approach paper also relates to Rondina and Walker (2014), Acharya (2013) and Huo and Takayama (2015*b*), who use frequency-domain techniques to obtain analytical solutions in certain models, and Nimark (2009) who explores the asymptotic accuracy of a finite-state approximation approach to a class of dispersed information models.

This paper is also related to the business cycle accounting literature in the tradition of Chari, Kehoe and McGrattan (2007). These papers consider simple economies augmented by a number of reduced-form wedges to equilibrium conditions. The approach developed in this paper connects the incomplete-information literature to these frameworks by mapping a generic DSGE model with incomplete information into a full-information wedge-economy. In contrast to business cycle accounting exercises, we approach wedges with a single structural interpretation in mind, reflected by implementability conditions on these wedges that ensure consistency with an incomplete-information economy. As we argue above, the incomplete-information interpretation is attractive as it can jointly generate the behavior of all wedges (including their co-movement patterns) with a simple structural narrative.

The paper is structured as follows. Section 2 sets up the model economy. Section 3 describes the primal approach. Section 4 describes implications of the primal approach for the aggregate economy. It establishes our benchmark "business cycle accounting" result and further derives restrictions on aggregate comovement from alternative specifications of information. Section 5 details our empirical strategy and presents the baseline empirical results. Section 6 explores a simple information structure that is consistent with our estimated expectation wedges. Section 7 concludes.

# 2 The Model Economy

## 2.1 Setup

The model is a standard RBC economy without capital, augmented with imperfect information. Households and firms are located on a continuum of islands, indexed by $i \in [0,1]$. On each island, a representative household interacts with a representative firm in a local labor market. Firms use the labor provided by households to produce differentiated intermediate goods, which are aggregated by a competitive final goods sector operating on the mainland.

**Households** Preferences on island $i$ are given by

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}U(C_{i,t+\tau},N_{i,t+\tau})\,|\,\mathcal{I}_{i,t}\right\},$$

where $\beta \in (0,1)$ is the discount factor, $N_{i,t}$ is hours worked, $C_{i,t}$ is final good consumption, and $\mathcal{I}_{i,t}$ is the set of information available in island $i$ at time $t$. The utility flow $U$ is given by

$$U(C,N) = \log C - \frac{1}{1+\zeta}N^{1+\zeta},$$

where $\zeta \geq 0$ is the inverse of the Frisch elasticity of labor supply. The household's budget constraint is

$$P_t C_{i,t} + Q_t B_{i,t} \leq W_{i,t}N_{i,t} + B_{i,t-1} + D_{i,t},$$

where $P_t$ is the price of the final good, $Q_t$ is the nominal price of a riskless one-period bond, $B_{i,t}$ are local bond holdings, $W_{i,t}$ are local wage rates, and $D_{i,t}$ are profits of the local firm.[2] Bonds are in zero net supply, so market clearing requires $\int_0^1 B_{i,t}\,\mathrm{d}i = 0$. No other financial assets can be traded across islands, which implies that households are exposed to idiosyncratic income risks.

**Intermediate-goods producers** Each good $i$ is produced by a monopolistically competitive firm with access to a linear production technology,

$$Y_{i,t} = A_{i,t}N_{i,t}. \tag{1}$$

[2]Following Maćkowiak and Wiederholt (2015), we assume that bond positions adjust to clear the budget constraint independently of the information available to households.

Firms choose $N_{i,t}$ to maximize expected profits, $\mathbb{E}[P_{i,t}Y_{i,t} - W_{i,t}N_{i,t}|\,\mathcal{I}_{i,t}]$, subject to an inverse demand curve specified below. The wage rate $W_{i,t}$ is determined competitively.[3] The productivity $A_{i,t}$ consists of an aggregate and an island-specific component,

$$\log A_{i,t} = \log A_t + \Delta a_{i,t},$$

where the aggregate component follows a random walk process

$$\log A_t = \log A_{t-1} + \epsilon_t.$$

The innovation $\epsilon_t$ is i.i.d. across time with zero mean and constant variance. The island-specific component $\Delta a_{i,t}$ follows a time-invariant, stationary random process that is i.i.d. across islands and is normalized so that $\int_0^1 \Delta a_{i,t}\,\mathrm{d}i = 0$.

**Final-good sector** A competitive final-goods sector aggregates intermediate input goods $i \in [0,1]$, using the technology

$$Y_t = \left(\int_0^1 Z_{i,t} Y_{i,t}^{\frac{\theta-1}{\theta}}\,\mathrm{d}i\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution among goods, $Y_{i,t}$ denotes the input of intermediate good $i$ at time $t$, and $Z_{i,t}$ is an island-specific demand shifter following a time-invariant, stationary process that is i.i.d. across islands and satisfies $\int_0^1 \log(Z_{i,t})\,\mathrm{d}i = 0$. Profit maximization yields the inverse input demands, given by

$$P_{i,t} = \left(\frac{Y_{i,t}}{Y_t}\right)^{-1/\theta} Z_{i,t} P_t, \tag{2}$$

where the aggregate price index $P_t$ is defined by

$$P_t = \left(\int_0^1 Z_{i,t}^{\theta} P_{i,t}^{1-\theta}\,\mathrm{d}i\right)^{\frac{1}{1-\theta}}.$$

**Monetary policy** We close the model by specifying a simple interest rate rule, pinning down the equilibrium rate of inflation, $\pi_t \equiv \log(P_t/P_{t-1})$. Specifically, we assume that the

[3] Formally, firm $i$ is representative of a continuum of firms, $j \in [0,1]$, competing in the local labor market. Each of these firms produces a separate variety $(i,j)$ that are aggregated to $Y_{i,t}$ by the final goods sector using the technology $Y_{i,t} = (\int_0^1 Y_{ij,t}^{1-1/\theta}\,\mathrm{d}j)^{\theta/(\theta-1)}$ where the elasticity of substitution across varieties $\theta$ is the same as in the final good technology specified below.

central bank sets nominal bond prices such that

$$i_t = \phi \pi_t, \tag{3}$$

where $\phi > 1$ and $i_t = -\log(Q_t)$.[4]

**Information** Our approach is aimed at providing a general characterization of equilibria that are consistent with a theory of incomplete information where agents use Bayes law to form expectations. To this aim, we do not take a parametric stand on the signals available to agents. Instead we allow for all information structures subject to the following three restrictions.

**Assumption 1** (Information bounds)**.** $\Theta_{i,t} \subseteq \mathcal{I}_{i,t} \subseteq \mathcal{I}_t^*$.

Assumption 1 defines a lower and an upper bound on information available in island $i$ at date $t$. The upper bound, $\mathcal{I}_t^*$, defines the history of all variables that are realized at date $t$, so that agents cannot learn more than what is potentially knowable under full information.[5] The lower bound, $\Theta_{i,t}$, includes at least the actions of the agents living in island $i$. Apart from this basic requirement of rationality, our approach allows $\Theta_{i,t}$ to be specified arbitrarily, allowing the researcher to explore a variety of informational assumptions. In our baseline specification, we pick a conservative lower bound $\Theta_{i,t}$, requiring only that firms and households are aware of the full history of their own actions as well as local productivities:

$$\Theta_{i,t} = \{A_{i,t}, C_{i,t}, N_{i,t}, Y_{i,t}\} \cup \Theta_{i,t-1}. \tag{4}$$

Some alternative specifications for $\Theta_{i,t}$ are discussed in Section 4.2.

Next, we make the usual assumption that all agents perfectly recall past information.

**Assumption 2** (Recursiveness)**.** $\mathcal{I}_{i,t-1} \subseteq \mathcal{I}_{i,t}$.

Finally, we impose ex-ante symmetry across islands and time to streamline the exposition. This does not restrict behavior of the aggregate economy.

**Assumption 3** (Ex-ante symmetry)**.** *The unconditional distribution over $(\mathcal{I}_{i,t}, A_{i,t}, Z_{i,t})$ is identical across all $i$ and $t$.*

[4] The rule also contains a constant intercept ensuring consistency with the natural rate at the zero-inflation steady state. The term is omitted since it drops out after we log-linearize the model below.

[5] Notice that which variables are realized at date $t$ is to some extent definitional. In particular, $\mathcal{I}_t^*$ could contain future innovations if they are realized at date $t$ as in the news literature. In our application, we abstract from this form of news, assuming all innovations to $(a_{i,t}, z_{i,t})$ realize at date $t$.

**Comment on prices and market clearing** It is worth noting that our minimal assumptions do not impose that agents perfectly learn from prices. Assuming so would be unnecessary restrictive, and our approach allows us to avoid specifying the details of how agents learn from prices without compromising market clearing.[6] We can do this because imposing market clearing in the parallel primal economy, which we describe in the next section, ensures that agents are sufficiently well informed about prices for them to perform their market clearing role (see Appendix C for details).

## 2.2 Equilibrium conditions

We work with a log-linear approximation to the model around the balanced growth path of the economy with no heterogeneity and full information. Lower-case letters denote log-deviations of a variable from the stochastic steady state where $y_{i,t} = a_t$ for all $i$ and $\pi_t = 0$.

The households' Euler equation is given by

$$c_{i,t} = \mathbb{E}[c_{i,t+1} - \phi\pi_t + \pi_{t+1} \,|\, \mathcal{I}_{i,t}]. \tag{5}$$

Combining firms' demand for labor with households' supply, local labor market clearing requires

$$y_{i,t} = \xi \, \mathbb{E}[x_{i,t} \,|\, \mathcal{I}_{i,t}] + a_{i,t}, \tag{6}$$

where $x_{i,t} \equiv y_{i,t} - c_{i,t} + p_{i,t} - p_t$ is the nominal trade-balance on island $i$, the linearized price index $p_t$ is given by $p_t = \int_0^1 p_{i,t}\, \mathrm{d}i$, and $\xi \equiv 1/(\zeta + 1)$. The linearized demand relation and budget constraint take the form

$$p_{i,t} = \theta^{-1}(y_t - y_{i,t}) + z_{i,t} + p_t \tag{7}$$

and

$$\beta b_{i,t} = b_{i,t-1} + x_{i,t}, \tag{8}$$

where $b_{i,t} \equiv B_{i,t}/\mathbb{E}_{i,t}[P_t Y_t]$ is kept in levels rather than logs, since $B_{i,t}$ can take negative values. Given a process for fundamentals and information $\{a_{i,t}, z_{i,t}, \mathcal{I}_{i,t}\}$, an equilibrium of

[6] Limiting the ability of agents to learn from prices most importantly limits their ability to learn about the aggregate state. Lorenzoni (2009) argues that, in practice, learning about aggregates is likely to be impaired by a large number of shocks, model misspecification, and the presence of structural breaks. One specific approach to capture these effects within a simple model like ours would be to decentralize markets so that local prices no longer reflect aggregate states (e.g., Lorenzoni, 2009; Angeletos and La'O, 2013; Chahrour and Ulbricht, 2017). An alternative interpretation is offered by the rational inattention literature (e.g., Maćkowiak and Wiederholt, 2015; Vives and Yang, 2017) where information sets do not reflect all the information that is in principle attainable from prices due to finite processing constraints.

the model is a set of processes $\{c_{i,t}, y_{i,t}, b_{i,t}, p_{i,t}\}$ and $\{y_t, \pi_t\}$ that are consistent with (5)–(8), with Bayesian updating, and with market clearing for goods,

$$y_t = \int_0^1 y_{i,t}\,\mathrm{d}i = \int_0^1 c_{i,t}\,\mathrm{d}i,$$

and bonds.

# 3 Primal Approach

In this section, we present the methodological innovation in this paper. We begin by defining a fictitious full-information version of our model in which all expectation errors are treated as exogenous wedges. This parallel "wedge economy" can be solved using standard full-information tools. We then derive necessary and sufficient conditions on these wedges such that they can be supported as expectation errors in an equilibrium of the incomplete information economy. The set of equilibria in the fictitious wedge-economy satisfying these conditions is equivalent to the full set of possible equilibria in the incomplete information economy. Our equivalence result hence provides a tractable method of characterizing the full set of incomplete-information equilibria, without making any parametric assumptions about information.

## 3.1 Primal Representation

Let $\mathbb{E}_t[\cdot] \equiv \mathbb{E}_t[\cdot|\mathcal{I}_t^*]$ denote the full-information expectations operator. To arrive at the "primal" analogue of the economy characterized in Section 2.2, we replace all expectation operators $\mathbb{E}[\cdot|\mathcal{I}_{i,t}]$ with $\mathbb{E}_t[\cdot]+\tau_{i,t}$, where $\{\tau_{i,t}\}$ are treated as exogenous "expectation" wedges. Specifically, in our case, we replace equations (5) and (6) with the corresponding primal equations:[7]

$$c_{i,t} = \mathbb{E}_t[\left(c_{i,t+1} - \tau^c_{i,t+1}\right) - \phi\pi_t + \pi_{t+1}] + \tau^c_{i,t} \tag{9}$$
$$y_{i,t} = \xi(x_{i,t} + \tau^x_{i,t}) + a_{i,t}. \tag{10}$$

Here $\tau^c_{i,t}$ and $\tau^x_{i,t}$ have the interpretation of prediction errors, relative to full-information, regarding household $i$'s consumption target and island $i$'s terms-of-trade, $p_{i,t} - p_t$, respectively.[8]

[7] Here $\tau^c_{i,t}$ is specified after rewriting (5) in its non-recursive form. With this normalization, $\tau^c_{i,t}$ defines the gap relative to the optimal level of consumption that household $i$ would choose if it had full information at $t$ and all future dates.

[8] Notice that as $y_{i,t}, c_{i,t} \in \Omega_{i,t}$, all uncertainty about $x_{i,t}$ can be attributed to the terms-of-trade, $p_{i,t} - p_t$.

Note that both wedges are defined relative to the full-information target that obtains given the behavior of the rest of the economy (i.e., given the expectation errors made on other islands).

## 3.2 Implementation Theorem

We now characterize implementability of the expectation wedges. Let $\mathcal{T}$ denote a stochastic process for $\mathcal{T}_t \equiv \{\tau^c_{i,t}, \tau^x_{i,t}\}_{i\in[0,1]}$ and let $\mathcal{E}(\mathcal{T})$ denote an equilibrium in the primal economy induced by $\mathcal{T}$.[9] We assume $\mathcal{E}(\mathcal{T})$ to have a stationary Gaussian distribution that is ex-ante symmetric across islands (see below for a discussion of how our results extend to non-stationary, non-symmetric and non-Gaussian cases). The following theorem states the implementation result.

**Theorem 1.** *Fix an $\mathcal{E}(\mathcal{T})$. Then there exists an information structure consistent with Assumptions (1)–(3) that implements $\mathcal{T}$, and hence $\mathcal{E}(\mathcal{T})$, in the incomplete-information economy if and only if for all $i$ and $t$ it holds that (i) $\mathbb{E}[\mathcal{T}_t] = 0$ and (ii)*

$$\mathbb{E}[\tau^c_{i,t}\theta] = 0 \ \textit{for all} \ \theta \in \Theta_{i,t}, \tag{11a}$$

$$\mathbb{E}[\tau^x_{i,t}\theta] = 0 \ \textit{for all} \ \theta \in \Theta_{i,t}. \tag{11b}$$

The theorem gives two conditions that are jointly necessary and sufficient for $\mathcal{T}$ to be implemented by some information structure. Condition (i) is a simple rationality requirement that agent's beliefs cannot be perpetually biased. Condition (ii) is an orthogonality requirement between the expectation wedges and the corresponding lower bounds on information $\Theta_{i,t}$. The necessity of this restriction is the familiar principle that expectation errors must be orthogonal to all available information. The novel part of our result is the sufficiency of this condition. For any $\mathcal{E}(\mathcal{T})$ with $\mathbb{E}[\mathcal{T}_t] = 0$, we can always construct an information structure that implements the joint process $\mathcal{E}(\mathcal{T})$ as long as it satisfies (11).

The following example illustrates this in a simple case. The general proof is given in Appendix A.1.

[9] There is no need for the equilibrium in the primal economy to be unique. If there are multiple $\mathcal{E}(\mathcal{T})$ for a given $\mathcal{T}$, our results hold with respect to each of them. Even if the equilibrium $\mathcal{E}(\mathcal{T})$ in the primal economy is unique given $\mathcal{T}$, the incomplete-information economy may still feature multiplicity as $\mathcal{T}$ itself may be driven by sunspot-realizations as in, e.g., Benhabib, Wang and Wen (2015).

**Example** Consider an economy defined by a single equilibrium condition, $y_t = \mathbb{E}[a_t|\mathcal{I}_t]$, where $\mathbb{E}[a_t] = 0$, and let $\Theta_t = \{y_{t-s}\}_{s\geq 0}$. The primal economy is given by

$$y_t = a_t + \tau_t. \tag{12}$$

Let $\mathcal{E}_t = (y_t, a_t, \tau_t)$ be a stationary Gaussian process satisfying (12). Theorem 1 states that $\mathcal{E}_t$ is implementable by some $\{\mathcal{I}_t\}$, satisfying $y_t \in \mathcal{I}_t$ for all $t$, if and only if (i) $\mathbb{E}[\tau_t] = 0$ and (ii) $\mathbb{E}[\tau_t y_{t-s}] = 0$ for all $s \geq 0$. The necessity of conditions (i) and (ii) is immediate, since optimal inference requires that expectation errors are orthogonal to variables in the information set and are unpredictable.

To see why the conditions are also sufficient, suppose that $\mathcal{I}_t = \{\omega_{t-s}\}_{s\geq 0}$ where $\omega_t = a_t + \tau_t$. That is, each period, the agent receives a new signal $\omega_t$ that has the same joint distribution over $(\omega_t, \mathcal{E}_t)$ as the "equilibrium" belief $y_t$ that we wish to implement. Projecting $a_t$ onto $y^t \equiv \{y_{t-s}\}_{s\geq 0}$, we have

$$\mathbb{E}[a_t|\mathcal{I}_t] = \mathrm{Cov}(a_t, y^t)[\mathrm{Var}(y^t)]^{-1} y^t. \tag{13}$$

Notice that

$$\mathrm{Cov}(y_t, y^t) = \begin{bmatrix} 1 & 0 & 0 & \cdots \end{bmatrix} \mathrm{Var}(y^t). \tag{14}$$

Further notice that (12) in combination with condition (ii) gives $\mathrm{Cov}(a_t, y^t) = \mathrm{Cov}(y_t - \tau_t, y^t) = \mathrm{Cov}(y_t, y^t)$. We can thus use (14) to substitute out $\mathrm{Cov}(a_t, y^t)$ in (13) to get

$$\mathbb{E}[a_t|\mathcal{I}_t] = y_t.$$

We conclude that as long as conditions (i) and (ii) hold, there exists a simple information-structure $\{\mathcal{I}_t\}$ that implements $\mathcal{T}_t$. Intuitively, observing the equilibrium expectation $y_t$ is a sufficient statistic for forming $\mathbb{E}[a_t|\mathcal{I}_t]$, giving us a simple means of implementing $\mathcal{T}_t$.[10]

The full proof in the appendix generalizes to dynamic economies involving an arbitrary number of equations, variables, and information sets. As $\mathcal{E}(\mathcal{T})$ is an equilibrium in the primal economy, we can use the logic above to implement the beliefs implied by the primal economy for each information set, being assured that the remaining equilibrium conditions of the incomplete-information economy continue to hold by construction. The full proof further

[10]The argument is related to the one given in Bergemann and Morris (2016) who show the equivalence between Bayes correlated equilibria and static Bayesian games with incomplete information. Our approach of formulating a primal economy and characterizing implementability in terms of a simple orthogonality condition is more general, however, as it straightforwardly applies to dynamic market economies and allows for arbitrary minimal information requirements.

extends the argument to arbitrary minimal information requirements $\{\Theta_{i,t}\}$.

**Remarks** Although our notation in presenting Theorem 1 is motivated by our model economy, the proof of the theorem is generic and can be applied to virtually any rational expectations DSGE model. Nevertheless we make a few assumptions that are worth discussing.

First, we require stationarity of $\mathcal{E}$. On the one hand, this rules out non-stationary processes of $\mathcal{T}_t$. On the other hand, this requires the primal economy to be stationary. In many cases, an appropriate transformation can be used to induce stationarity in the primal economy, even when the economy is fundamentally non-stationary. E.g., in our case, it suffices to define the primal economy in terms of the output gap $\hat{y}_t$ as in (15)–(16) below, ensuring stationarity of $\mathcal{E}$ as long as $\mathcal{T}$ is stationary.[11]

Second, we require $\mathcal{E}$ to be ex-ante symmetric across islands to be consistent with Assumption 3. Dropping Assumption 3, the Theorem applies to non-symmetric $\mathcal{E}$ without modification.

Third, while we assume $\mathcal{E}$ to be Gaussian, the assumption is not needed when one is only interested in implementing the auto-covariance structure of $\mathcal{E}_t$. In our empirical application, we make sure that our estimator indeed only uses information regarding the covariance structure, so that we do not need to make any distributional assumptions regarding $\mathcal{T}$ to invoke our theoretical results.

Fourth, in our case firms and households operate under the same information sets and share the same lower bounds $\Theta_{i,t}$, but this does not need to be the case. An immediate corollary to Theorem 1 is that whenever two distinct information sets $\mathcal{I}_1$ and $\mathcal{I}_2$ share the same lower bound $\Theta$, the set of expectation wedges supported by $(\mathcal{I}_1, \mathcal{I}_2)$ is identical to the one supported under the additional requirement that $\mathcal{I}_1 = \mathcal{I}_2$. In our case this implies that imposing common knowledge within islands places no additional restrictions on $\mathcal{T}_t$ compared to the case where firms and households operate under possibly distinct information sets but share the same lower bounds $\Theta_{i,t}$.

# 4 Characterizing the Aggregate Economy

If the researcher is interested in the ability of the incomplete information model to match aggregate data, as we are in this paper, then the crucial question is to what degree the orthogonality conditions in Theorem 1 restrict the feasible dynamics of the aggregate econ-

[11]Our model also features a non-stationary process for island-specific consumption and output, but can be stationarized in terms of growth rates. See the proof of Theorem 2 for details.

omy. In this section, we show that in many cases the answer to this question is surprisingly straightforward to derive using the equivalence from Section 3.

## 4.1 Aggregation and Equilibrium in the Primal Economy

We begin by providing an explicit characterization of aggregate equilibrium in the primal economy. Unlike the incomplete-information economy, in which aggregation involves average expectation operators and hence depends on the cross-sectional distribution of beliefs, the primal economy permits a simple aggregate representation. Letting $\tau_t^c = \int_0^1 \tau_{i,t}^c \, \mathrm{d}i$ and $\tau_t^x = \int_0^1 \tau_{i,t}^x \, \mathrm{d}i$ and integrating over (9) and (10), we get

$$\hat{y}_t = \xi \tau_t^x \tag{15}$$

$$\hat{y}_t = \mathbb{E}_t[\hat{y}_{t+1} - \tau_{t+1}^c - \phi\pi_t + \pi_{t+1}] + \tau_t^c \tag{16}$$

where $\hat{y}_t \equiv y_t - a_t$ is the output gap relative to potential output under full information.

Equations (15) and (16) define the aggregate dynamics in the primal economy. Common prediction errors in the Euler equation, captured by $\tau_t^c$, show up as a standard Euler wedge (aka "demand shock"). Similarly, it can be shown that common prediction errors regarding the terms-of-trade, captured by $\tau_t^x$, correspond to the usual definition of the labor wedge.

Note that $\tau_t^c$ and $\tau_t^x$ are the sole drivers of the aggregate output gap and inflation. Accordingly, if all agents had full information ($\tau_t^c = \tau_t^x = 0$), the aggregate economy would be in its stochastic steady state where output reaches its potential in every period ($y_t = a_t$) and inflation is zero.

In general, for any joint process $(\tau_t^c, \tau_t^x)$ a solution can be obtained using standard numerical tools. In our case, a closed form solution is also available. Substituting out $\hat{y}_t$ in (16), $\pi_t$ is characterized by the prediction formula

$$\pi_t = \phi^{-1}\mathbb{E}_t[\mathrm{d}\tau_{t+1}^x - \mathrm{d}\tau_{t+1}^c + \pi_{t+1}]. \tag{17}$$

Following Hansen and Sargent (1980, 1981), one can obtain an explicit solution for inflation, given by

$$\pi_t = \begin{bmatrix} 1 & -1 \end{bmatrix} \frac{(1-L)A(L) - (1-\phi^{-1})A(\phi^{-1})}{\phi L - 1} u_t \tag{18}$$

where $A(L)u_t \equiv (\tau_t^x, \tau_t^c)'$ is the Wold representation of $(\tau_t^x, \tau_t^c)$, and $u_t$ are the corresponding innovations.

## 4.2 Implementability of Aggregate Wedges

To characterize feasible dynamics of the aggregate economy, we now explore how the restrictions on island-specific expectation errors given by Theorem 1 constrain the fluctuations of the aggregate wedges $\tau_t^c$ and $\tau_t^x$. The answer depends on what is contained in the lower bound on information, $\Theta_{i,t}$. In the following, we explore several possible specifications for $\Theta_{i,t}$, demonstrating how one can use specific assumptions about minimal information to derive restrictions on the aggregate economy.

**Baseline Specification:** $\quad \Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}\} \cup \Theta_{i,t-1}$.

With $\Theta_{i,t}$ given by (4), there are essentially no restrictions on the aggregate wedge process $(\tau_t^c, \tau_t^x)$—and hence on $(\hat{y}, \pi_t)$—if the local shocks $(\Delta a_{i,t}, z_{i,t})$ are sufficiently volatile.

**Theorem 2.** *Fix a process $(y_t, \pi_t, a_t)$ where $(\hat{y}_t, \pi_t)$ has a finite Wold representation with $\mathbb{E}[(\hat{y}_t, \pi_t)] = 0$. Then there exists a process for the idiosyncratic fundamentals $(\Delta a_{i,t}, z_{i,t})$ such that $(y_t, \pi_t, a_t)$ is an equilibrium of the incomplete information economy.*

Theorem 2 provides a strong benchmark result emphasizing the potential of information-driven theories of the business cycle. Of course, not every combination of information structures and local shocks consistent with a particular process for $(y_t, \pi_t, a_t)$—or, equivalently, $(y_t, \pi_t, n_t)$—will be economically plausible. We return to the question of economic plausibility in our quantitative exploration in Sections 5 and 6.

To build an intuition for the "everything goes" result established in Theorem 2, notice that $(\tau_t^c, \tau_t^x)$ must be supported by expectation errors that are *correlated* across islands. The main insight is that by using correlated errors about *local* shocks, we can implement arbitrary processes for the average errors across islands as long as the local shocks are sufficiently volatile. For instance, consider a variant of the example given in Section 3.2, where $y_i = a_i + \tau_i$ and $\Theta_i = \{y_i\}$ for $i \in [0, 1]$. The orthogonality condition in Theorem 1 imposes two restrictions on the distribution over $(y_i, a_i, \tau_i)$. First, it implies that $\mathrm{Var}[a_i] = \mathrm{Var}[y_i] + \mathrm{Var}[\tau_i]$, constraining the variance of $\tau_i$ to satisfy $\mathrm{Var}[\tau_i] \leq \mathrm{Var}[a_i]$. Second, it pins down the covariation of $\tau_i$ with $a_i$, $\mathrm{Cov}[a_i, \tau_i] = -\mathrm{Var}[\tau_i]$. Now consider $(\bar{y}, \bar{a}, \bar{\tau}) = \int_0^1 (y_i, a_i, \tau_i) \mathrm{d}i$. By varying the correlation of $(\tau_i, a_i)$ in the cross-section, one can implement any distribution over $(\bar{y}, \bar{a}, \bar{\tau})$ that satisfies

$$\mathrm{Var}[\tau_i] \geq \frac{(\mathrm{Cov}[\bar{\tau}, \bar{a}] + \mathrm{Var}[\tau_i])^2}{\mathrm{Var}[a_i] - \mathrm{Var}[\bar{a}]} + \mathrm{Var}[\bar{\tau}]$$

and $\bar{y} = \bar{a} + \bar{\tau}$. Clearly for any $\mathrm{Var}[\tau_i] > \mathrm{Var}[\bar{\tau}]$, the condition is non-binding for some sufficiently volatile $a_i$.[12] Intuitively, when the idiosyncratic shocks $(a_i - \bar{a})$ are sufficiently volatile, we can support aggregate expectation wedges using correlated errors about the idiosyncratic variations in the agents' objectives, regardless of the degree of aggregate uncertainty about $\bar{a}$.

We now consider several alternative specifications in which the result in Theorem 1 places tighter bounds on the behavior of the aggregate economy.

**Alternative 1:** $\Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}, a_t\} \cup \Theta_{i,t-1}$.

The information bound in Alternative 1 contains all the same variables as in our baseline specification, but adds aggregate productivity as an observable. Following the proof of Theorem 2, it is straightforward to see that any stationary joint dynamics for the output gap and inflation, $(\hat{y}_t, \pi_t)$, is feasible. The additional orthogonality restriction imposed by adding aggregate productivity, however, implies that neither expectation error can be correlated with current or past productivity. Thus, while the model can support large independent fluctuations in beliefs, the aggregate response to productivity shocks must be exactly that of the full information economy.

**Alternative 2:** $\Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}, \hat{y}_t\} \cup \Theta_{i,t-1}$.

In Alternative 2, agents have access to the baseline minimal information plus a signal revealing current and past realizations of the output gap, $\hat{y}_t$. E.g., agents may observe aggregate output $y_t$ and either further observe aggregate productivity $a_t$ or live in an economy where there are no aggregate productivity shocks. In this case, output always follows its natural rate, $y_t = a_t$, regardless of any confusion that agents may have regarding their local conditions. To see this, note that the orthogonality condition of Theorem 1 implies $\text{Cov}[\tau^x_{i,t}, \hat{y}_t] = \text{Var}[\hat{y}_t] = 0$. Hence, $y_t = a_t$.

**Alternative 3:** $\Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}, s_{t-\bar{h}}\} \cup \Theta_{i,t-1}$ for $s_t \equiv (y_t, \pi_t, i_t, n_t)$ and $\bar{h} \geq 1$.

In Alternative 3, agents are assumed to observe all aggregate variables with some lag $\bar{h}$. While the contemporaneous size of aggregate expectation errors is *not* bounded, their persistence is, since the additional orthogonality conditions now imply that the autocorrelation of expectation errors must die out at lags $\bar{h}$ and greater. This result generalizes a result in Acharya, Benhabib and Huo (2017), which bounds the persistence of a class of sentiment

[12]E.g., fix $\text{Var}[\tau_i] = 2\text{Var}[\bar{\tau}] + \text{Cov}[\bar{\tau}, \bar{a}]$. Then the condition holds for any $\text{Var}[a_i] \geq \text{Var}[\bar{a}] + 4(\text{Var}[\bar{\tau}] + \text{Cov}[\bar{\tau}, \bar{a}])$, which also suffices to satisfy the idiosyncratic variance bound $\text{Var}[\tau_i] \leq \text{Var}[a_i]$ (the covariance condition between $\tau_i$ and $a_i$ holds by construction).

shocks. Together, Alternatives 2 and 3 demonstrate that aggregate information is a powerful constraint on the scope for agents to make correlated errors.

**Alternative 4:** $\Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}, s_t\} \cup \Theta_{i,t-1}$ for some public signal $s_t = \hat{y}_{t-1} + u_t$.

Alternative 4 demonstrates that, with noisy observations of past aggregate outcomes, the effects of beliefs again can be bounded in interesting ways. In this case, we assume that agents also observe a noisy indicator regarding the previous period's output gap. In this case, we have that $\text{Cov}[\hat{y}_{t-1} + u_t, \hat{y}_t] = 0$ implies $\text{Cov}[\hat{y}_t, \hat{y}_{t-1}] = -\text{Cov}[\hat{y}_t, u_t]$. Standard estimates of the output gap are strongly positively autocorrelated, implying that positive noise in aggregate statistics, if they are internalized by agents, should be contractionary. Without making any assumption on the process for the signal noise, we can immediately conclude that it is impossible for the common error in a signal regarding recently past aggregate conditions to drive a positive output gap today.

**Alternative 5:** $\Theta_{i,t} = \{a_{i,t}, c_{i,t}, n_{i,t}, y_{i,t}, y_t\} \cup \Theta_{i,t-1}$.

Here $y_t$ is known, but not $a_t$. A few steps of algebra show that

$$\text{Var}[\hat{y}_t] - \text{Cov}[\hat{y}_t, \hat{y}_{t-1}] = -\text{Cov}[\hat{y}_t, \epsilon_t].$$

Since the left hand side of the above equation is always weakly positive, this implies that $\text{Cov}[\hat{y}_t, \epsilon_t] < 0$. In other words, shocks to aggregate technology should have contractionary effects on the output gap, and therefore on labor. High autocorrelation of the output gap implies that in practice, $\text{Cov}[\hat{y}_t, \epsilon_t]$ should be modest. Both of these implications are consistent with aggregate US data.

**Discussion** While the extra restrictions arising from aggregate observations are quite interesting, previous literature has argued that they may be too strong. The rational inattention literature, in particular, has argued that many sources of information that are in principle publicly available may not be much used by private agents for the purpose of information extraction (see, e.g., the discussion in Woodford, 2003). Maćkowiak and Wiederholt (2009) show that this argument is further strengthened when applied to aggregate information, since aggregate conditions typically play a relatively small role for individuals in their decision making. In that spirit, our quantitative approach below imposes only minimal restrictions on the information used by agents from the outset and explores *ex post* what type of information-based stories are consistent with the expectation wedges that we estimate.

# 5 Quantitative Exploration

In this section, we use our theoretical results to quantify how well incomplete information can account for U.S. business cycle data on output, inflation, hours, and the nominal interest rate. Because we target more data series than we have expectations wedges, the wedges that we estimate in this section are over-identified.[13] Nevertheless, we find they do an excellent job matching the comovement in these four series. After assessing its empirical performance, we explore the properties of the estimated expectation wedges and study their implications for agents' learning. Finally, we use wage data to decompose the labor wedge into a portion attributable to the errors of households and a portion attributable to the errors of firms. We find that nearly all of the deviations from full information can be attributed to household expectation errors.

## 5.1 Econometric Methodology

Building on the equivalence result in Theorem 1, our empirical approach formulates the information structure directly in terms of a stochastic process for the average expectation wedges $\bar{\mathcal{T}}_t \equiv (\tau_t^c, \tau_t^x)$. Practical concerns lead us to adopt a specific parametric specification for $\bar{\mathcal{T}}_t$, but the approach is non-parametric regarding the underlying information structure. Since we do not have strong priors regarding how agents collect their information (or on the distributional properties of the noise terms associated with these channels) this seems a natural starting point for an empirical investigation.

We estimate the model based on the comovements of the data at business cycle frequencies. Accordingly, we augment our baseline specification of $\Theta_{i,t}$ with the aggregate orthogonality conditions resulting from lagged observation of aggregates $s_t = (y_t, \pi_t, i_t, n_t)$, as in Alternative 3:

$$\mathbb{E}[\tau_t^c s_{t-\bar{h}-j}] = 0,\ \forall j \geq 0 \tag{19}$$

$$\mathbb{E}[\tau_t^x s_{t-\bar{h}-j}] = 0,\ \forall j \geq 0. \tag{20}$$

We set $\bar{h}$ to 32 quarters. Conditions (19) and (20) ensure that the fit of our economy will not be driven by longer-run confusion about things that lie outside our estimation horizon and are therefore not disciplined by the data.

The relevant structural parameters for the aggregate model are $\zeta$ and $\phi$. We fix these

[13] We also work with a lower bound on information that reveals aggregate statistics at long lags, providing further overidentifying restrictions on the model economy.

parameters in our estimation procedure as they are only weakly identified given the specification of $\bar{\mathcal{T}}$ adopted below. Specifically, we set the inverse Frisch elasticity of labor supply $\zeta$ to 0.25 and the Taylor rule coefficient $\phi$ to 2, which is in the range of values typically used in the business cycle literature.

We assume that $\bar{\mathcal{T}}_t$ follows a first-order auto-regressive process,

$$\bar{\mathcal{T}}_t = \Lambda \bar{\mathcal{T}}_{t-1} + \omega_t,$$

where $\omega_t$ is i.i.d. across time with zero mean and covariance matrix $\Psi$. Since agents may not be immediately aware of aggregate productivity shocks, we allow $\omega_t$ to be correlated with the productivity innovations, $\epsilon_t$. The joint covariance matrix is denoted $\tilde{\Psi} = \mathrm{Var}[(\omega_t, \epsilon_t)]$. In total, this gives us 10 parameters that are to be estimated, which are collected in the vector $\gamma \equiv \{vec(\Lambda), vech(\tilde{\Psi})\}$.

Let $\Gamma$ denote the set of parameters consistent with the implementability conditions given in (19)–(20). Following the logic of Theorem 2, any $\gamma \in \Gamma$ is implementable if the island-specific shocks are sufficiently volatile (see Appendix A.2 for details). Accordingly, we ignore for now all orthogonality conditions other than (19)–(20) and provide ex post a process for the local shocks that ensures that the estimated process for $\bar{\mathcal{T}}_t$ is indeed implementable.

We estimate the model parameters $\gamma$ using the generalized method of moments (GMM) to minimize the distance between the model's covariance structure and the data, subject to $\gamma \in \Gamma$.[14] Let

$$\tilde{\Omega}_T = vech\{\mathrm{Var}[(\tilde{s}^d_t, \ldots, \tilde{s}^d_{t-k})]\},$$

denote the empirical auto-covariance matrix of frequency-filtered quarterly US data, $\tilde{s}^d_t$, on real per-capita output, inflation, nominal interest rates, and per-capita hours.[15] We target auto-covariances between zero and $k = 8$ quarters. For the filtering, we use the Baxter and King (1999) approximate high-pass filter with a truncation horizon of 32 quarters; i.e., $\tilde{s}^d_t = HP_{32}(s^d_t)$ where $s^d_t$ are empirical observations of the model variables $s_t = (y_t, \pi_t, i_t, n_t)$.[16]

[14]Given our parametric specification for $\bar{\mathcal{T}}_t$, (19)–(20) can not hold exactly unless $\bar{\mathcal{T}}_t = 0$ for all $t$. We therefore allow for a small numerical deviation, ensuring that the estimated model satisfies all orthogonality conditions within a tolerance of $2 \cdot 10^{-6}$.

[15]Data range from 1960Q1 to 2012Q4. Real output is given by nominal output divided by the GDP deflator. Inflation is defined as the log-difference in the GDP deflator. Interest rates are given by the Federal Funds Effective rate. Hours are given by hours worked in the non-farm sector. Variables are put in per-capita terms using the non-institutional population over age 16.

[16]The Baxter and King (1999) filter requires specification of a lag-length $\bar{\tau}$ for the approximation. We set $\bar{\tau}$ to their recommended value of 12.

Our estimator is thus given by

$$\hat{\gamma} = \underset{\gamma \in \Gamma}{\operatorname{argmin}} (\tilde{\Omega}_T - \tilde{\Omega}(\gamma))' W (\tilde{\Omega}_T - \tilde{\Omega}(\gamma)), \tag{21}$$

where $\tilde{\Omega}(\gamma)$ is the model analogue to $\tilde{\Omega}_T$ and $W$ is a weighting matrix set to an estimate of $\left[\mathrm{Var}\{T^{1/2}\tilde{\Omega}_T\}\right]^{-1}$ (see Appendix B.2 for details). Following the suggestion of Gorodnichenko and Ng (2009), our model analogue $\tilde{\Omega}(\gamma)$ is computed after applying the same filtering procedure to the model that we have applied to the data. In Appendix B.1, we derive a closed-form transformation from $\Omega \equiv vech\{\mathrm{Var}[(\mathrm{d}s_t, \ldots, \mathrm{d}s_{t-K})]\}$ to $\tilde{\Omega} = \Xi\Omega$ for a constant matrix $\Xi$ and $K = k + 2\bar{\tau}$. Using the transformation, we can equivalently express (21) as

$$\hat{\gamma} = \underset{\gamma \in \Gamma}{\operatorname{argmin}} (\Omega_T - \Omega(\gamma))' \tilde{W} (\Omega_T - \Omega(\gamma)), \tag{22}$$

where now the unfiltered model is estimated (in first differences) on unfiltered data and the filtering is achieved by replacing $W$ with $\tilde{W} \equiv \Xi' W \Xi$. Using (22) in place of (21), estimation becomes straightforward as the mapping from $\gamma$ to $\Omega(\gamma)$ is available in closed form.

All confidence intervals and hypothesis tests are based on a bootstrapped distribution, $\{\hat{\gamma}_b\}_{b=1}^{B}$, with $B = 2,500$ replications. As the bootstrap data generating process we use a VAR(10) estimated on $\mathrm{d}s_t^d$. In each sample $b$, we first construct $W_b$ according to the steps described in Appendix B.2, and then use (22) to estimate $\hat{\gamma}_b$ where the target moments $(\Omega_b - \Omega(\gamma))$ are recentered about their population mean to adjust for overidentification.

## 5.2 Empirical Results

In this section we explore the properties of the estimated model and assess its ability to account for business cycle comovements in the data. A full listing of the estimated parameter values is given in Table 4 in the appendix.

**Predicted moments** We begin by assessing the empirical performance of the estimated model. Figure 1 compares the predicted model moments with the targeted data moments. The dashed black lines show the empirical covariance structure $\tilde{\Omega}_T$ along with 90-percent confidence intervals (depicted by the shaded areas). The solid blue lines show the corresponding moments for the estimated model. Each row $i$ and column $j$ in the table of plots shows the covariances between $\tilde{s}_t^i$ and $\tilde{s}_{t-k}^j$ with lags $k \in \{0, 1, \ldots, 8\}$ depicted on the horizontal axis.

The estimated model does a very good job at capturing the auto-covariance structure of the four time series. In particular, the model captures the positive contemporaneous

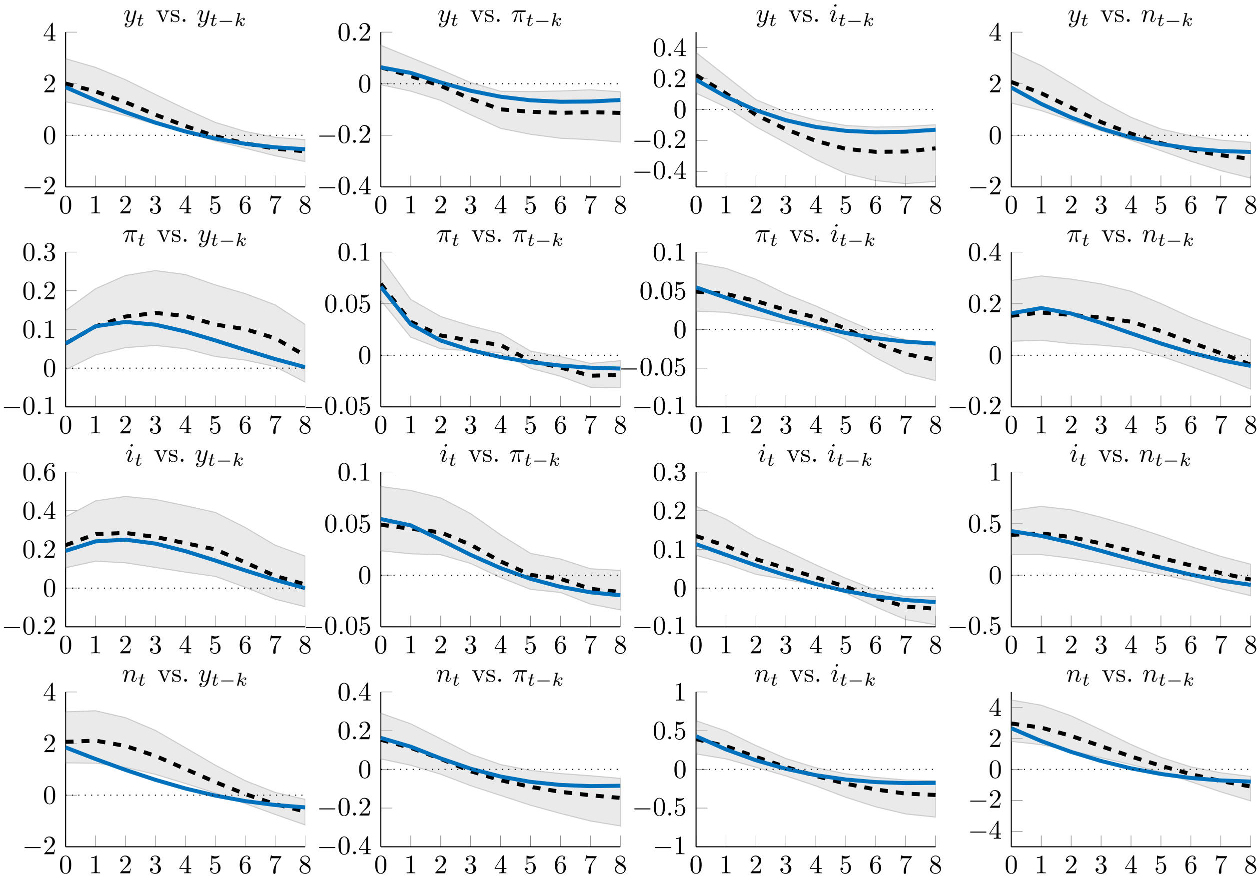


**Figure 1:** Business cycle comovements in the data and predicted by the estimated model. Note.—Dashed black lines show the empirical covariance structure $\tilde{\Omega}_T$ together with 90 percent confidence intervals depicted by the shaded areas. Solid blue lines show the corresponding model moments $\tilde{\Omega}(\hat{\gamma})$. Each row $i$ and column $j$ in the table shows the covariances between $\tilde{s}_t^i$ and $\tilde{s}_{t-k}^j$ with lags $k \in \{0, 1, \ldots, 8\}$ depicted on the x-axis.

comovement of output, hours, and inflation visible in the data, which is typically difficult for productivity-driven models to accommodate. The estimated model also does an excellent job at capturing the autocorrelation structure found in the data, in particular the rising profile of inflation's comovement with lagged GDP (second row, first column), as well as the falling autocorrelations of GDP, inflation, and hours (along the diagonal).

**Correlation of wedges** Essential for the empirical performance of the model is that the Euler wedge and the labor wedge are correlated across time and among each other. As reported in Table 1, the contemporaneous correlation between $\tau_t^c$ and $\tau_t^x$ is above 99 percent. While most business cycle models generate shocks that are correlated across time, it is typically difficult for full-information models with structurally uncorrelated shocks to generate perturbations that are correlated across equations. By contrast, the incomplete-information wedges are naturally correlated both because information can be correlated across house-

**Table 1:** Correlation structure of estimated expectation wedges

| | Standard deviation | First-order autocorr. | Contemporaneous correlation with $\tau_t^c$ | with $\tau_t^x$ | with $\epsilon_t$ |
|---|---|---|---|---|---|
| $\tau_t^c$ | 0.0534 | 0.88 | 1.00 | · | · |
| $\tau_t^x$ | 0.0318 | 0.89 | 0.99 | 1.00 | · |
| $\epsilon_t$ | 0.0074 | – | -0.29 | -0.29 | 1.00 |

holds and firms and because expectation errors by household are likely to affect both their consumption and labor supply. Below we decompose the labor wedge into a household and firm-side component, illustrating that virtually all of the fluctuations in the labor wedge can be attributed to expectation errors by households. In Section 6, we then illustrate in a particular simple example how waves of optimism and pessimism among households regarding the local productivity $a_{i,t}$ coupled with incomplete information regarding other households' optimism and aggregate TFP can account for the correlation patterns in Table 1.

**Identifying shocks** The estimated process for $\bar{\mathcal{T}}_t$ reflects two distinct channels through which incomplete information affects the dynamics of the economy. First, agents may have incomplete information regarding aggregate productivity, potentially modifying the economy's response to productivity shocks. Second, with incomplete information, agents can also make correlated errors that introduce an independent source of business cycle fluctuations. We now decompose the process of the estimated expectation wedges to isolate responses to productivity innovations and different types of expectational shocks.

Since we assume that productivity is exogenous to $\bar{\mathcal{T}}_t$, we can separate out the two roles of incomplete information by projecting $\bar{\mathcal{T}}_t$ on current and past productivity shocks $\epsilon_t$. We get

$$\bar{\mathcal{T}}_t = P(L)\epsilon_t + v_t, \tag{23}$$

where $P(L)\epsilon_t \equiv \mathbb{P}[\bar{\mathcal{T}}_t|\epsilon_t, \epsilon_{t-1}, \dots]$ denotes the projection. The lag-polynomial $P$ identifies the average expectation errors in (16) and (15) that are associated with productivity innovations. The remaining residuals, $v_t$, identify purely expectational business cycle shocks.

Let

$$v_t = B_1(L)\eta_{1,t} + B_2(L)\eta_{2,t}, \tag{24}$$

where $B_1(L)$ and $B_2(L)$ are lag-polynomials in nonnegative powers of $L$, and $\eta_{1,t}$ and $\eta_{2,t}$ are orthogonal white-noise processes. As in the structural VAR literature, $B_1(L)$ and $B_2(L)$ cannot be uniquely identified without additional identifying assumptions. To provide an

**Table 2:** Unconditional variance decomposition

| | GDP | Inflation | Hours |
|---|---|---|---|
| $\eta_{1,t}$ | 0.90 | 0.27 | 0.66 |
| $\eta_{2,t}$ | 0.01 | 0.31 | 0.01 |
| $\epsilon_t$ | 0.09 | 0.43 | 0.33 |

Note.—Contributions are to unconditional variances in the frequency filtered model.

economically interesting interpretation of our shocks, we identify the first shock $\eta_{1,t}$ as the shock that contributes most to the unconditional variance of output which is orthogonal (at all leads and lags) to technology. The second shock $\eta_{2,t}$ then captures the remaining (non-technology-driven) movements in the economy. The identification strategy is closely related to Uhlig's (2003; 2004) approach of identifying shocks based on their contribution to some finite forecast error variance.

Table 2 shows the contributions of the three shocks to the unconditional variances of the high-pass filtered model variables. While productivity contributes substantially to fluctuations in inflation and hours, the majority of the fluctuations in output, inflation and hours are accounted for by the expectational shocks $\eta_{1,t}$ and $\eta_{2,t}$. Specifically, the first expectational shock is the dominant driver of output and employment fluctuations, whereas the second expectational shock plays a moderate role in explaining inflation.

**Impulse responses to productivity shock** The solid lines in Figure 2 represent the responses of the estimated model to a one-standard-deviation innovation in productivity. Shaded bands capture the 90% confidence regions from the bootstrapped distribution of responses.

The random walk assumption we have placed on productivity implies that, under full information, output would immediately jump to the new potential output level (depicted by the black-dotted line). As unanticipated permanent changes in potential output have no effect on the output gap, there would be no responses in inflation, hours, or the interest rate under full information.

With incomplete information, the responses are quite different. Output hardly moves on impact and then only slowly adjusts to its new potential level over roughly four years. The negative output gap over the period of transition is reflective of the negative labor wedge, $\tau_t^x$, over the same period. The negative effect of the shock on inflation follows from equation (17) and the qualitatively similar, but larger, fall in the Euler wedge, $\tau_t^c$. Overall, the picture shows a delayed response to the productivity shock, with a short run fall in labor and a negative output gap. This pattern is consistent with the VAR-based evidence by Basu, Fernald and

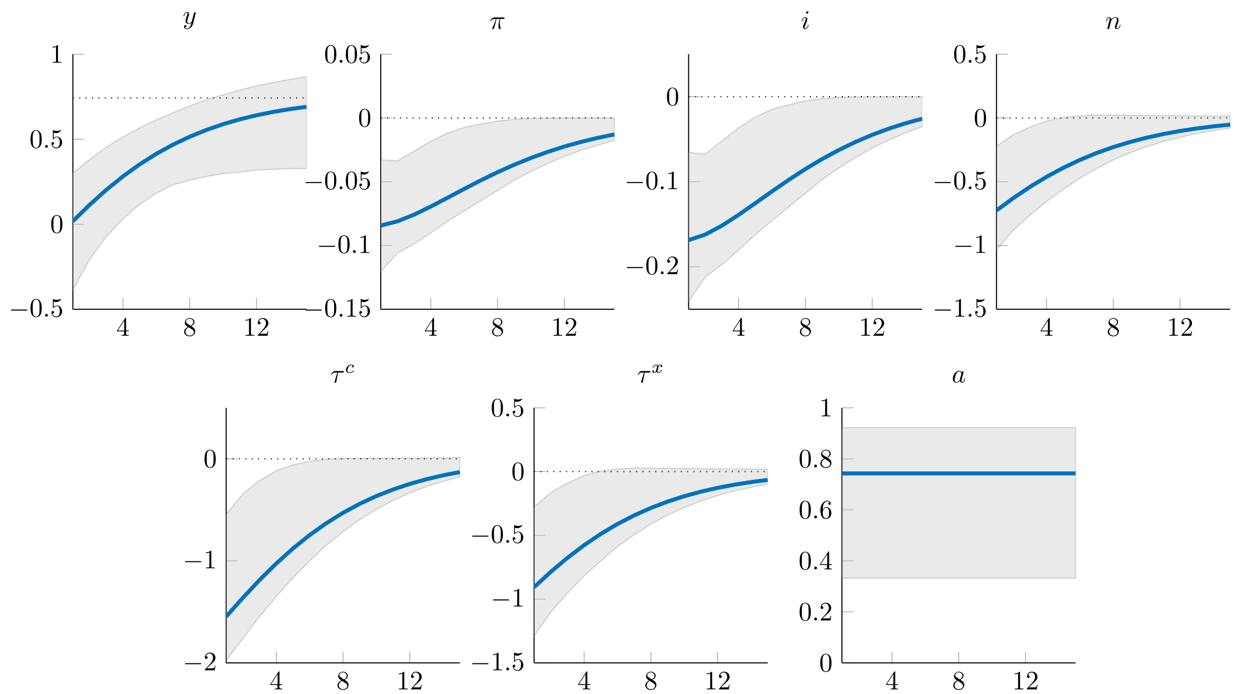


**Figure 2:** Impulse responses to a productivity shock. Note.—Responses are for a one-standard-deviation shock to $\epsilon_t$ and are depicted in percentage deviations from the steady state. Dotted black lines depict the full-information benchmark. Shaded bands capture the 90% confidence regions from the bootstrapped distribution of responses.

Kimball (2006).

**Impulse responses to expectational shocks** Figure 3 shows impulse response functions to a one-standard-deviation shock in $\eta_{1,t}$ (the shock contributing most to observed output fluctuations).

The shock is propagated by a sharp and persistent surge in both expectation wedges, with the fluctuation in the household's intertemporal condition once again exceeding that of the labor wedge. The resulting increase in household demand leads to a joint increase in output, inflation, interest rates, and employment. In terms of output, the shock implies a peak response of approximately 1 percent and has a half-live of roughly two years. Overall, the shock delivers dynamic responses that resemble a prototypical demand shock.

We now turn to the second expectational shock $\eta_{2,t}$. Figure 4 gives the impulse responses. At the point estimate, the shock has a substantial impact effect on inflation. But the estimated responses are transitory and statistically insignificant. Indeed, in our bootstrap simulations, the shock $\eta_{2,t}$ is often estimated to have zero variance, resulting in a statistically insignificant variance across draws.

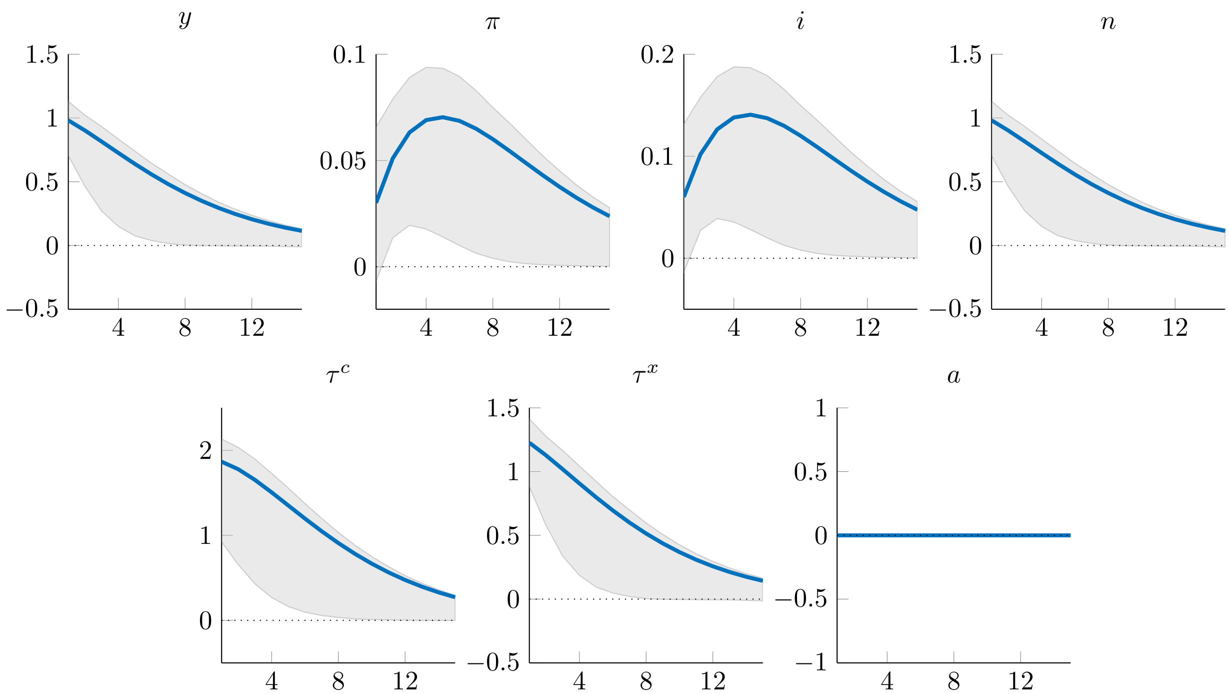


**Figure 3:** Impulse responses to an expectational shock in $\eta_{1,t}$. Note.—Responses are for a one-standard-deviation shock to $\eta_{1,t}$ and are depicted in percentage deviations from the steady state. Dotted black lines depict the full-information benchmark. Shaded bands capture the 90% confidence regions from the bootstrapped distribution of responses.

**Testing for information** Figure 5 reports the cross-correlation coefficients of the expectation wedges vis-à-vis current and past output growth, inflation, the Federal funds rate and productivity growth. The shaded regions correspond to 90 percent confidence intervals. Since expectation errors must be orthogonal to variables in the corresponding information set, significantly non-zero correlations between a wedge and a particular variable indicate that the variable could not have been in the information set of agents at the time they made their choices. Conversely, since the orthogonality requirements of Theorem 1 are also sufficient, a statistically insignificant correlation indicates that our estimates are consistent with agents knowing the corresponding statistic.

Our test suggests that private sector agents are generally unaware of aggregate conditions within a 1-year horizon. In particular, if our estimated wedges are driven by information, then we can clearly reject that agents are aware of contemporaneous inflation, interest rates, or productivity. Regarding past information, we can reject orthogonality for inflation and interest rates up to four quarters, while for productivity we can reject orthogonality for up to five quarters. Meanwhile, the case for output is somewhat less clear, as orthogonality is less clearly rejected over the same period.

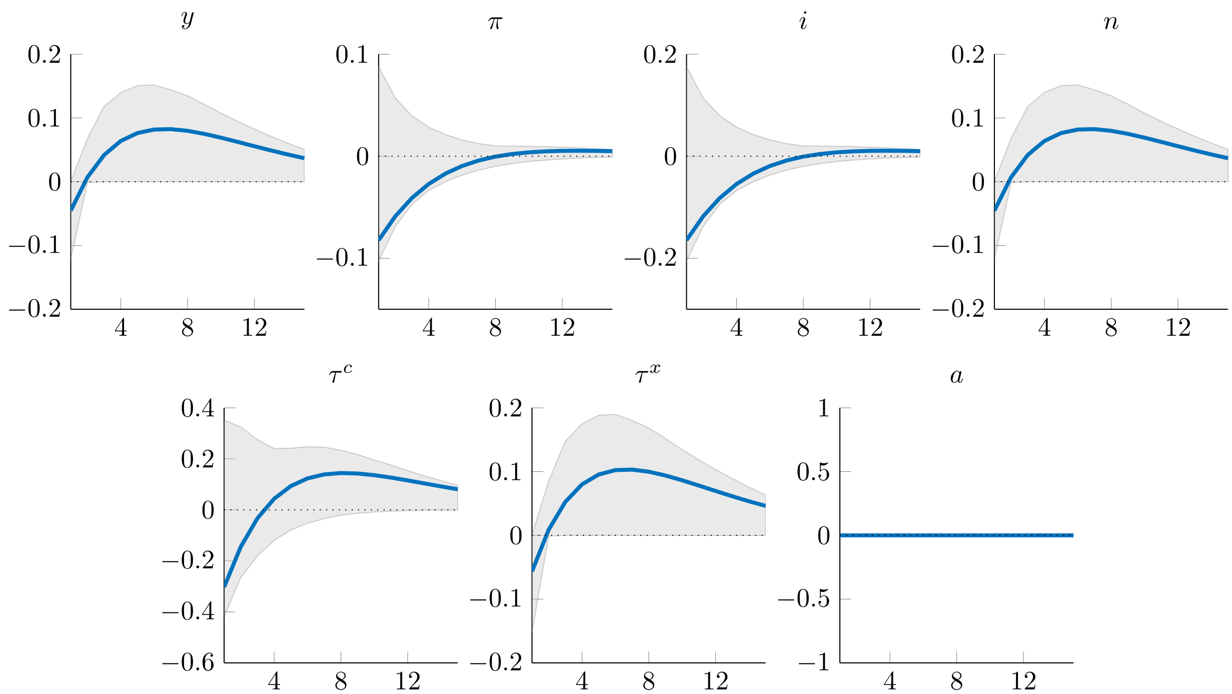


**Figure 4:** Impulse responses to an expectational shock in $\eta_{2,t}$. Note.—Responses are for a one-standard-deviation shock to $\eta_{2,t}$ and are depicted in percentage deviations from the steady state. Dotted black lines depict the full-information benchmark. Shaded bands capture the 90% confidence regions from the bootstrapped distribution of responses.

**Decomposing the labor wedge** While the intertemporal wedge is unambiguously driven by the choices of the household, the labor wedge is potentially influenced by both household and firm choices. We now decompose the labor wedge into a household and firm-side error.

Specifically, from the households' and firms' labor supply and demand choices, the two components of the labor wedge are defined by

$$\tau_t^{x,h} = \int_0^1 \mathbb{E}[p_t | \mathcal{I}_{i,t}] \, \mathrm{d}i - p_t \tag{25}$$

$$\tau_t^{x,f} = \int_0^1 \mathbb{E}[p_{i,t} | \mathcal{I}_{i,t}] \, \mathrm{d}i - p_t, \tag{26}$$

where the combined labor wedge is given by

$$\tau_t^x = \tau_t^{x,f} - \tau_t^{x,h}. \tag{27}$$

That is, the household-side of the labor wedge is driven by expectation errors vis-à-vis the import price, while the firm-side component is driven by errors vis-à-vis the export price. Let $\hat{w}_t = w_t - p_t - a_t$ be the stationarized average of local equilibrium wages, where $w_t =$

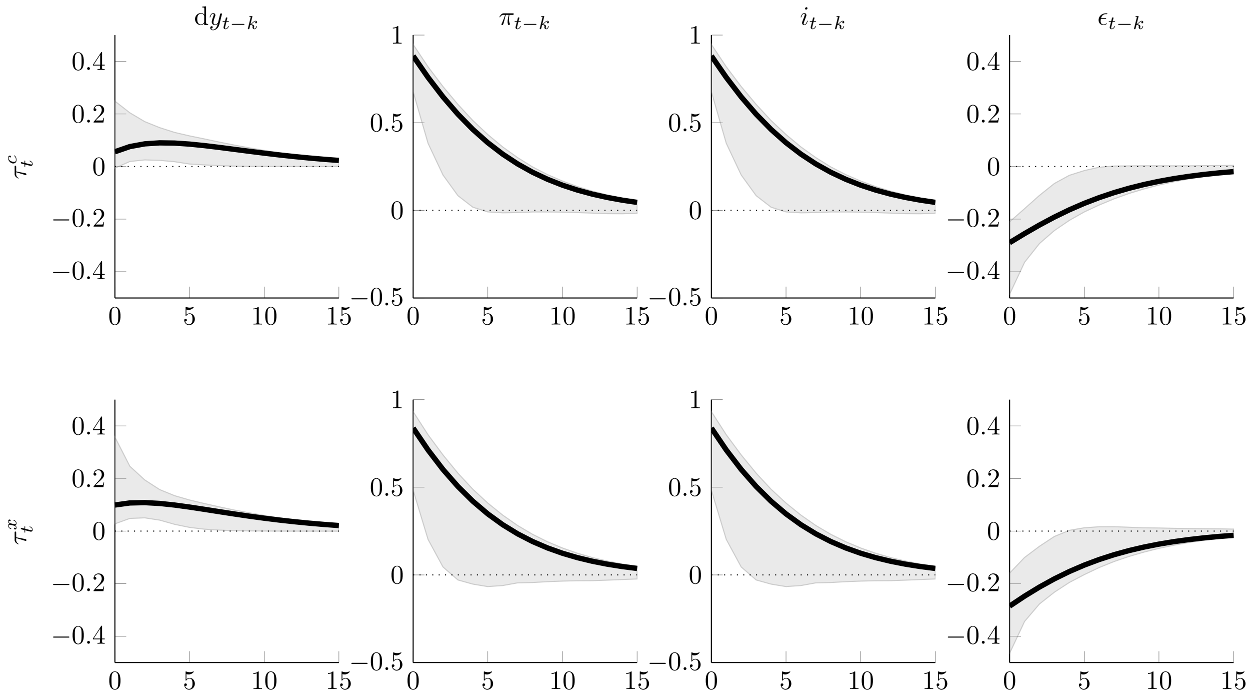


**Figure 5:** Correlation between expectation-wedges and aggregate statistics. Note.—The plot shows the (auto) correlation coefficients of the estimated expectation-wedges vis-à-vis output growth, inflation, the fed funds rate and productivity growth. The order of the autocorrelation is on the x-axis. Shaded areas depict 90 percent confidence intervals.

$\int_0^1 w_{i,t}\,\mathrm{d}i$. Then the individual optimality conditions imply that the average household/firm errors regarding the marginal benefits of supplying/demanding an additional hour of work satisfy

$$\tau_t^{x,h} = \hat{w}_t - \tau_t^x \tag{28}$$

$$\tau_t^{x,f} = \hat{w}_t. \tag{29}$$

With data on the average wage faced by agents in the economy, it is thus possible to decompose the labor wedge into a portion attributable to household errors and portion attributable to firm errors.[17] To do this, we fix our baseline estimates for the combined wedges and then augment our original system with equations (28) and (29). We then estimate a subsidiary process

$$\tau_t^{x,f} = D_1\bar{\mathcal{T}}_{t-1} + D_2(\eta_{1,t}, \eta_{2,t}, \epsilon_t)'$$

[17] Although our interpretation is different, this exercise resembles that of Karabarbounis (2014). He finds qualitatively similar results.

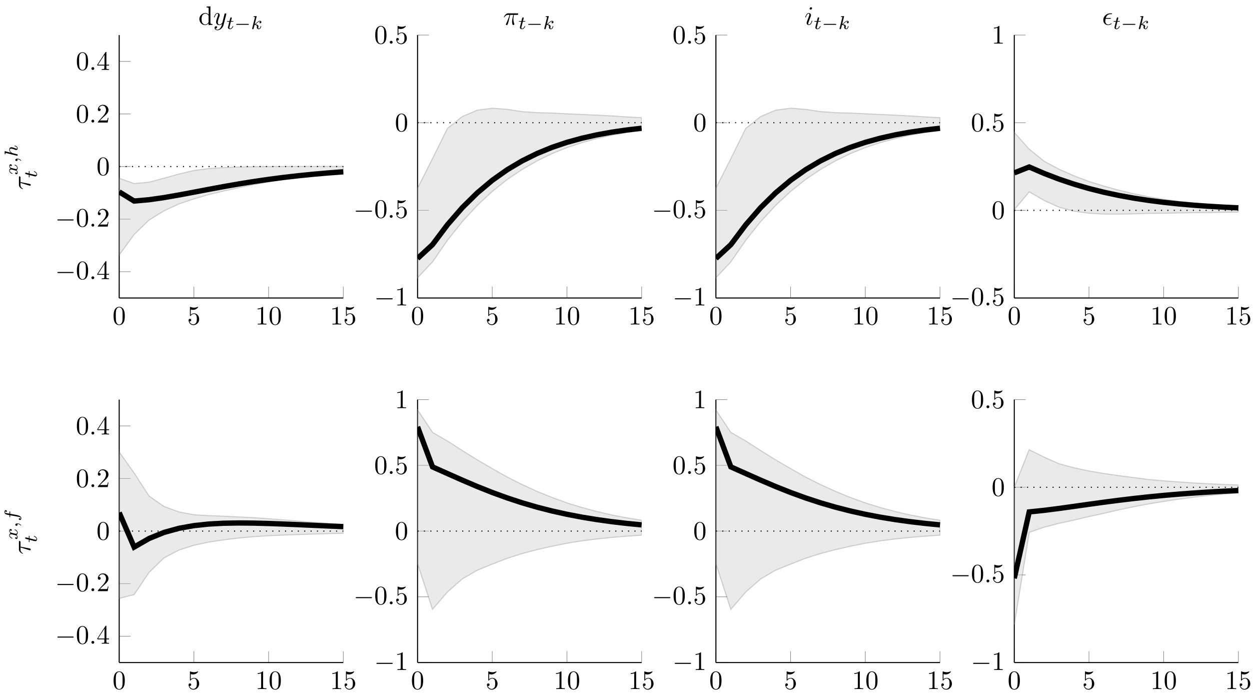


**Figure 6:** Correlation between components of $\tau_t^x$ and aggregate statistics. Note.—The plot shows the (auto) correlation coefficients of the estimated expectation-wedges vis-à-vis output growth, inflation, the fed funds rate and productivity growth. The order of the autocorrelation is on the x-axis. Shaded areas depict 90 percent confidence intervals.

where $\tau_t^{x,f}$ is governed by a $1 \times 2$ vector $D_1$ of weights on the aggregate expectation wedges and a $1 \times 3$ vector $D_2$ capturing the impact of the three innovations in the economy on the firm-side labor wedge, which from (27) in turn pins down the household-side labor wedge. These five additional parameters are then estimated to match the empirical covariances, $\mathrm{Cov}[\tilde{w}_t^d, (\tilde{w}_{t-s}^d, \tilde{s}_{t-s}^d)]$, where $\tilde{w}_t^d$ denotes Baxter and King (1999)-filtered real compensation-per-hour in the non-farm business sector.[18]

The results of this exercise are described in Figure 6, which now plots the correlation of the household- and firm-side components of $\tau_t^x$ with various aggregate statistics. Only in the case of the household contribution to the wedge, $\tau_t^{x,h}$, can we ever reject the null of orthogonality with respect to any of the aggregate statistics shown. In other words, for the purpose of accounting for aggregate data we cannot reject that firms have full information: An information-based theory of the business cycle must be driven by a lack of information on the part of households rather than of firms.[19] These results are in line with forecast-based

[18]The estimation follows the same GMM-procedure as our baseline estimation outlined in Section 5.1.

[19]Further supporting this conclusion, we cannot reject that the estimated coefficients $D_1$ and $D_2$ defining the process for $\tau_t^{x,f}$ are zero. Similarly, the ratio of $\mathrm{Var}[\tau_t^{x,h}]/\mathrm{Var}[\tau_t^{x,f}]$ is roughly 25:1.

**Table 3:** Standard deviations of island-specific shocks

| Noise process | $\Delta\tau^c_{i,t}$ | $\Delta\tau^x_{i,t}$ | $\Delta a_{i,t}$ | $z_{i,t}$ |
|---|---|---|---|---|
| Standard deviation | .0267 | .0190 | .0350 | .0551 |

evidence which finds that households appear to be less informed than other agents in the economy (e.g., Carroll, 2003). The fit of the economy with and without firm-side wedge compared to the data is documented by Figures 9 and 10 in Appendix F.

Notice, however, that our results neither imply that firms have full information, nor that firms' information sets are necessarily distinct from the ones of households. For instance, consider the case where $p_{i,t} \in \mathcal{I}_{i,t}$. Then from (26) we clearly have that $\tau^{x,f}_t = 0$ for all $t$, approximately consistent with the evidence presented above. Yet, firms neither have superior knowledge compared to household nor do they need to know the aggregate state of the economy. Intuitively, knowledge of $(a_{i,t}, w_{i,t}, p_{i,t})$ suffices for firms to behave optimally, inducing them to behave *as if* they would have perfect information (see also Hellwig and Venkateswaran, 2014).

**Minimal volatilities** Finally, we return to the question of what volatility for the idiosyncratic noise terms, $\{\Delta a_{i,t}, z_{i,t}\}$, is required to implement the processes for aggregate expectation errors described above. In practice, many different processes for these shocks are able to support the estimated belief processes. For our implementation, we treat the local shocks as MA(32) processes that can be arbitrarily correlated with both each other and the island-specific component of the expectation wedges (see Appendix D for details). We then search numerically for a feasible implementation of estimated belief processes that minimizes the sum of unconditional variances of these shocks. Table 3 reports the corresponding standard deviations for these shocks. While local conditions must exhibit substantial volatility, the order of magnitude of these volatilities is on par with aggregate conditions in the economy. These values fall well within the plausible range for the volatility of local conditions, which are often calibrated or estimated to be much larger.

# 6 A Simple Model of Information

In this section, we explore whether our estimated impulse responses can be implemented by a fully parametric—and simple—information structure. From Theorem 2, an exact implementation exists, given by the signals constructed in the proof to Theorem 1. In general, however, there are many other possible implementation strategies. Here we explore a simple story of consumer confidence coupled with imperfect awareness regarding the errors or others that gives rise to dynamics similar to the ones estimated in Section 5. Specifically, our model builds on Lorenzoni (2009) in order to generate temporary fluctuations in aggregate demand from consumer confidence. Rather than using exogenous price rigidity to translate those fluctuations into real terms, we depart from Lorenzoni (2009) and combine the model with elements from Lucas (1972, 1973) and Woodford (2003) to generate real output fluctuations by making agents imperfectly aware of the aggregate component of consumer sentiment. This gives rise to a "real Keynesian" theory of the business cycle. We show how the model is able to generate the correlation structure of the expectation wedges discussed above.

A full implementation of the model would require a complicated numerical solution. In the interest of transparency, we limit ourselves here to a stylized illustration of our ideas that can be characterized in closed-form, and focus on implementing a single impulse response at a time. Because the response to the second expectational shock has been found to be insignificant in Section 5.2, we focus on implementing the economy's response to productivity and expectational shocks of the first type.

## 6.1 Setup

The model is given by the one in Section 2. Local productivities are given by

$$a_{i,t} = \bar{a}_0 + \mu_{i,0} + \epsilon_{i,t},$$

where $\bar{a}_0$ is aggregate productivity, $\mu_{i,0}$ is a fully-persistent island-specific factor, and $\epsilon_{i,t}$ is a transient island-specific factor. The productivity components $\bar{a}_0$, $\mu_{i,0}$ and $\epsilon_{i,t}$ as well as the local demand shocks, $z_{i,t}$, are independently normally distributed white noise processes with variances $\kappa_{\bar{a}}^{-1}$, $\kappa_{\mu}^{-1}$, $\kappa_{\epsilon}^{-1}$ and $\kappa_z^{-1}$. Information is given by

$$\mathcal{I}_{i,t} = \{\Theta_{i,t}, p_{i,t}, s^{\mu}_{i,0}, s^{\bar{a}}_{i,t}, \mathcal{I}^*_{t-\bar{h}}\} \cup \mathcal{I}_{i,t-1}$$

where

$$s^{\mu}_{i,0} = \mu_{i,0} + \eta_0$$
$$s^{\bar{a}}_{i,t} = \bar{a}_0 + \psi_{i,t},$$

and $\eta_0$ and $\psi_{i,t}$ are normally distributed white noise processes with variances $\kappa_\eta^{-1}$ and $\kappa_\psi^{-1}$. Full-information is revealed with a lag of $\bar{h} = 32$ quarters. Notice that $p_{i,t} \in \mathcal{I}_{i,t}$. So in line with our discussion in Section 5.2, both expectation wedges are driven exclusively by household errors. In addition to $p_{i,t}$, agents receive two exogenous signals. First, $s^{\mu}_{i,0}$ reveals the local productivity trend $\mu_{i,0}$ with some noise $\eta_0$. As $\eta_0$ is correlated across islands, it induces aggregate optimism regarding the local productivity, generating aggregate fluctuations similar to the estimated response to $\eta_{1,t}$. Second, together with $a_{i,t}$ (contained in $\Theta_{i,t}$), $s^{\bar{a}}_{i,t}$ governs the speed at which agents learn about the aggregate productivity component $\bar{a}_0$. Due to the one-shot nature of our model, uncertainty about $\mu_{i,0}$ and $\bar{a}_0$ will be largest at date 0 and will die out as $t \to \infty$, generating single impulse-response paths to the realizations of $\eta_0$ and $\bar{a}_0$ at date 0.

For our numerical exercise, we parametrize the model as follows. From Section 5.1, we have $\zeta = 0.25$ and $\phi = 2$. Next we set $\beta = 0.99$ and $\theta = 21$ (implying markups of 5 percent as in Huang, Zheng and Phaneuf, 2004). In line with the estimated standard deviation for $a_t$, we set $\kappa_{\bar{a}} = 0.0074^{-2}$. Similarly, we ensure that the response to $\eta_0$ has the right scale by setting $\kappa_\eta = 0.01^{-2}$. The idiosyncratic productivity processes is parametrized by $\kappa_\mu = 0.02^{-2}$ and $\kappa_\epsilon = 0.05^{-2}$. Idiosyncratic demand shocks are given by $\kappa_z = 0.0025^{-2}$. Finally, the signal about aggregate productivity has a precision of $\kappa_\psi^{-2} = 0.0060$.

## 6.2 Response to consumer sentiment shocks

**Characterization** For simplicity, suppose $\kappa_{\bar{a}}^{-1} = 0$, so that the only source of aggregate fluctuations is the noise term $\eta_0$ that materializes at date 0. A positive realization of $\eta_0$ implies a positive signal about the persistent component of the local productivities, which translates into increased demand by the local consumer. Because $\eta_0$ is (perfectly) correlated across all islands, this translates into an increase in aggregate demand. In particular, solving the island-economy, it can be shown that the consumers' Euler equation aggregates to

$$\hat{y}_t = \phi_\mu \bar{\mathbb{E}}_t[\mu_{i,0}] + \bar{\mathbb{E}}_t[\hat{y}_{t+1} - \phi_r(\phi\pi_t - \pi_{t+1})] + \phi_r(\bar{\mathbb{E}}_t[\bar{a}_0] - \bar{a}_0) \tag{30}$$

where $\phi_\mu = \beta(1-\theta^{-1})$ and $\phi_r = 1$. Whether an increase in aggregate demand due to the first term on the right-hand side translates into real output fluctuations depends on the response of labor markets. Specifically, integrating over (6) with $p_{i,t} \in \mathcal{I}_{i,t}$, we have

$$\hat{y}_t = \xi(p_t - \bar{\mathbb{E}}_t[p_t]). \tag{31}$$

Note that if households and firms were to be perfectly aware of the consumer confidence shock $\eta_0$, then $\bar{\mathbb{E}}_t[p_t] = p_t$ and $\hat{y}_t = 0$. As in a standard real business cycle model, the economy would never deviate from the natural level of output, and fluctuations in demand would be offset by price changes.

To solve for the equilibrium of this economy, conjecture a law of motion

$$\begin{pmatrix} \hat{y}_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} \Lambda_t^y \\ \Lambda_t^\pi \end{pmatrix} \eta_0.$$

Integrating over $s_{i,0}^\mu$, we have that

$$\bar{\mathbb{E}}_t[\mu_{i,0}] = \eta_0 - \bar{\mathbb{E}}_t[\eta_0].$$

It hence follows that all expectations in (30) and (31) can be cast in terms of $\bar{\mathbb{E}}_t[\eta_0]$. Using the Kalman filter to process the information in $\mathcal{I}_{i,t}$ and integrating over islands, we have

$$\bar{\mathbb{E}}_t[\eta_0] = \frac{\bar{\kappa}_{t-1}\bar{\mathbb{E}}_{t-1}[\eta_0] + \lambda_t^2\kappa_z + \kappa_\epsilon}{\bar{\kappa}_t}$$
$$\bar{\kappa}_t = \bar{\kappa}_{t-1} + \lambda^2\kappa_z + \kappa_\epsilon,$$

where the filter is initialized at $\bar{\mathbb{E}}_{-1}[\eta_0] = \kappa_\mu/\bar{\kappa}_{-1}$ and $\bar{\kappa}_{-1} = \kappa_\eta + \kappa_\mu$, and where $\lambda_t = \theta^{-1}\Lambda_t^y + \sum_{s=0}^t \Lambda_s^\pi$. Noting that $\hat{y}_s = \pi_s = 0$ for all $s \geq \bar{h}$, the model can be solved by guessing and verifying a process $(\Lambda_t^y, \Lambda_t^\pi)$.

**Baseline response** The impulse response to a one-standard deviation shock to $\eta_0$ is shown in Figure 7. An increase in consumer confidence driven by optimism regarding the local productivity process (depicted in the third panel) raises aggregate demand by consumers. Initially firms and households are unable to distinguish the increase in aggregate demand from island-specific demand shocks. Accordingly, they underpredict the presence of the aggregate demand shock (depicted in the fourth panel) so that wages and prices only adjust slowly and the demand shock translates into real output fluctuations.

Qualitatively, the model is able to generate the positive comovement in $\hat{y}_t$ and $\pi_t$ estimated in Section 5.2, but the size of output fluctuations is much too small relative to inflation. To see why this is the case, note that from (31), we have that

$$\Lambda_0^y = \xi\Lambda_0^\pi \left(1 - \frac{\bar{\mathbb{E}}_0[\eta_0]}{\eta_0}\right).$$

Given our choice of $\mathcal{I}_{i,t}$, the degree of awareness regarding the confidence shock, $\bar{\mathbb{E}}_t[\eta_0]/\eta_0$, is

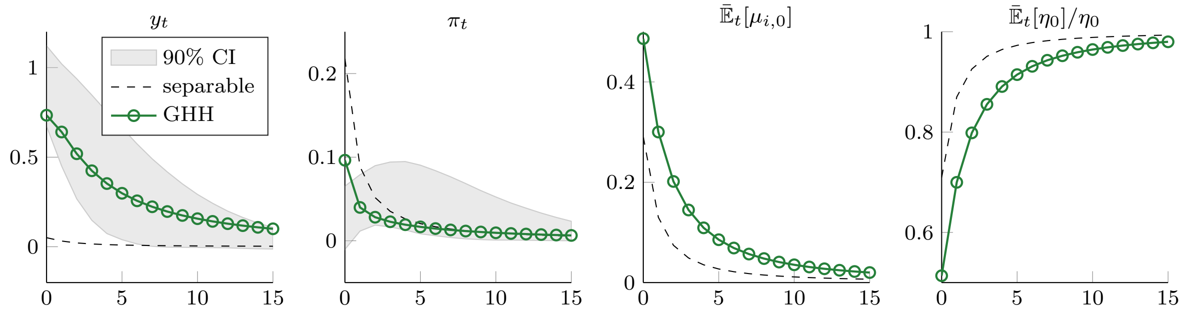


**Figure 7:** Impulse responses to an expectational shock in $\eta_0$ in the parametric model. Dashed lines correspond to the baseline setting with additively separable preferences. Solid green lines correspond to the case with GHH preferences and roundabout production. Shaded areas depict 90 percent confidence intervals to the response to $\eta_{1,t}$ of the estimated model.

bounded between 0 and 1. Hence, it follows that

$$\Lambda_0^y \leq \xi \Lambda_0^\pi,$$

bounding the impact response of output relative to inflation by a factor of $\xi \leq 1$.

Before exploring how the bound on the impact response of output can be relaxed within our setup, we make two observations about the nature of the bound. First, the bound emerges because we assume $p_{i,t} \in \mathcal{I}_{i,t}$, which, from equation (26), shuts down any firm-side fluctuations in the labor wedge. If firms were unaware of the (inverse) demand at the time of production, then systematic confusion about $z_{i,t}$ could support fluctuations in $\bar{\mathbb{E}}_{i,t}[p_{i,t}] - p_t$, uncoupling the labor wedge from $p_t$. Here we do not go this route since the results in Section 5.2 suggest that firm-side fluctuations in the labor wedge do not play a major role in accounting for the estimated impulse responses.

Second, an alternative way to increase the response of the output gap relative to inflation would be to introduce exogenous price rigidity. Propagation via sticky prices is essential for the expectation shocks of Lorenzoni (2009) and Blanchard, L'Huillier and Lorenzoni (2013) to have real effects. In a previous draft of this paper (Chahrour and Ulbricht, 2017), we show how including nominal frictions helps generating sizeable output fluctuations in a narrative similar to the one explored here.

**GHH and roundabout production** Rather than assume sticky prices, we explore two minor changes to preferences and production technology that taken together relax the bound significantly. First, we tame the wealth effect on labor supply by considering a version of

Greenwood, Hercowitz and Huffman (1988) (GHH) preferences, given by

$$U(C_{i,t}, N_{i,t}, A_{i,t}) = \log\left(C_{i,t} - \frac{1}{1+\zeta} A_{i,t} N_{i,t}^{1+\zeta}\right),$$

where the dependency on $A_{i,t}$ ensures consistency with a balanced-growth path. Second, we introduce a round-about production structure as in Basu (1995) and Huang, Zheng and Phaneuf (2004) where

$$Y_{i,t} = M_{i,t}^{\alpha}(A_{i,t}N_{i,t})^{1-\alpha}.$$

Material inputs $M_{i,t}$ are in terms of the final consumption good, so that market clearing requires $\int_0^1 (M_{i,t} + C_{i,t})\mathrm{d}i = Y_t$. We set the share of material inputs to $\alpha = 0.7$, which is in the range of values considered plausible by Huang, Zheng and Phaneuf (2004).

Given the modifications, equations (30) and (31) continue to hold subject to adjustments in the definitions of $\phi_\mu$, $\phi_r$ and $\xi$. Most relevantly, we now have $\xi = (\zeta(1-\alpha))^{-1}(1 + \alpha\zeta)$, evaluating to approximately 15.7 under our parametrization.[20] (See Appendix E for expressions of the remaining parameters and a full derivation).

The responses to the modified economy are depicted by the solid lines in Figure 7. All the responses closely fall into the confidence region of our estimate. The path is driven by substantial optimism regarding average local productivities (depicted in the third panel). As time passes, islands gradually learn about the origin of their confidence—i.e., the average expectation regarding $\eta_0$ converges toward $\eta_0$ as seen in the last panel—and, correspondingly, average beliefs about local conditions converges back to zero.

## 6.3 Response to aggregate productivity shocks

We now turn to the implementation of the response to an aggregate productivity shock. For simplicity, we again focus on a single impulse response in the case where productivity is the only source of aggregate fluctuations ($\kappa_\eta^{-1} = 0$). To implement the estimated response, it suffices to slow down learning with respect to the realization of $\bar{a}_0$ sufficiently. The environment adopted in this section naturally achieves this goal. Using similar steps as for the response to $\eta_0$, we can solve the model again by guessing and verifying a response path that is consistent with (30) and (31). Figure 8 shows the solution under our standard preferences and the GHH/roundabout structure. Again, the roundabout structure falls into confidence

[20] Similarly, the primal economy of the modified model spans exactly the same dynamics as the original economy for $\bar{\mathcal{T}}_t^{\text{GHH}} = M\bar{\mathcal{T}}_t$ given a $2 \times 2$ matrix $M$. Clearly the transformation to $\bar{\mathcal{T}}_t$ preserves the parametric shape, so there is no need to re-estimate the model. In particular, all estimated impulse response functions and confidence bands will be exactly the same as in our baseline estimation in Section 5.2.

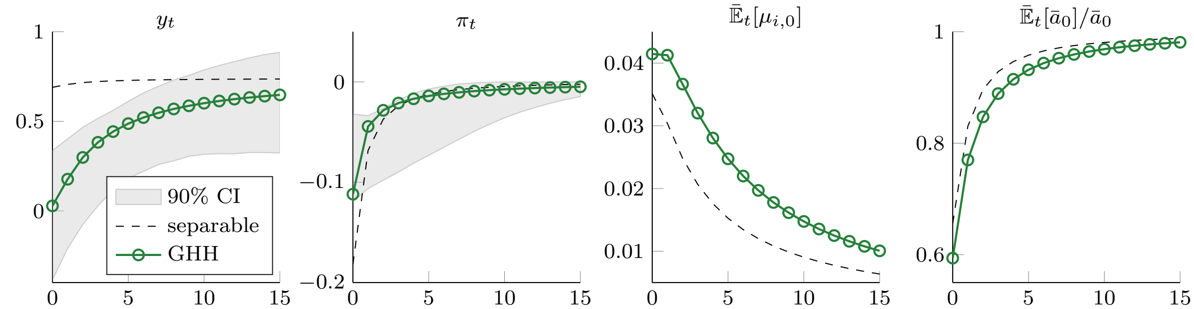


**Figure 8:** Impulse responses to a productivity shock in $\bar{a}_0$ in the parametric model. Dashed lines correspond to the baseline setting with additively separable preferences. Solid green lines correspond to the case with GHH preferences and roundabout production. Shaded areas depict 90 percent confidence intervals to the response to $a_t$ of the estimated model.

region of our estimate.

On impact, agents are on average aware of about 60% of the aggregate productivity realization, leading to less adjustment in output than optimal under full information. As agents gradually learn about the realized productivity, output converges to its new potential and deflation ceases.

# 7 Concluding Remarks

We have established the equivalence between a primal economy characterized by a set of reduced-form wedges and the class of economies driven by incomplete information. Applying our result, we show how to estimate a macroeconomic model with incomplete information without parametric assumptions on information structures. Our approach is, at once, straightforward to use and can be easily adapted to myriad contexts. We use the approach to explore a dispersed-information version of a simple RBC economy. Our exploration suggests that standard macroeconomic time series could in principle be fully explained by information. The most important ingredient in accounting for the data is incomplete information on the part of households. By contrast the data does not call for significant departures from full information behavior on the side of firms.

# A Mathematical Appendix

## A.1 Proof of Theorem 1

Consider any expectation wedge $\tau \in \mathcal{T}$. Let $(\hat{a}, \mathcal{I}^*)$ denote the corresponding expectation target and full-information set contained in $\mathcal{E}$, so that

$$y \equiv \mathbb{E}[\hat{a}|\mathcal{I}^*] + \tau \tag{32}$$

defines the equilibrium "belief" implied by the primal economy. Also let $\Theta$ be the corresponding lower bound on $\mathcal{I}$ in the incomplete information economy. We want to show that conditions (i) and (ii) are jointly necessary and sufficient for the construction of some $\mathcal{I} \in \Theta$ such that

$$\mathbb{E}[\hat{a}|\mathcal{I}] = y. \tag{33}$$

**Necessity** The necessity is immediate, since optimal inference requires that expectation errors are orthogonal to variables in the information set and are unpredictable. E.g., combining (32) and (33), implementation requires

$$\tau = \mathbb{E}[\hat{a}|\mathcal{I}] - \mathbb{E}[\hat{a}|\mathcal{I}^*]. \tag{34}$$

Computing the unconditional expectation over (34) yields $\mathbb{E}[\tau] = 0$. Similarly, postmultiplying (34) by $\Theta'$ gives $\mathbb{E}[\tau\Theta'] = \mathbb{E}[\hat{a}\Theta'|\mathcal{I}] - \mathbb{E}[\hat{a}\Theta'|\mathcal{I}^*]$ as $\Theta' \subseteq \mathcal{I} \subseteq \mathcal{I}^*$. Again computing the unconditional expectation, we have $\mathbb{E}[\tau\Theta'] = 0$.

**Sufficiency** We demonstrate sufficiency by construction. Let $\mathcal{I} = \Theta$. Notice that in dynamic settings, Assumption 2 holds as long as $\Theta$ is recursive, ensuring consistency with the dynamic nature of agents' beliefs.

Let $\tilde{\Theta} \equiv \Theta \setminus \{y\}$ and let $a \equiv \mathbb{E}[\hat{a}|\mathcal{I}^*]$. From the law of iterated expectations, we have $\mathbb{E}[\hat{a}|\mathcal{I}] = \mathbb{E}[a|\mathcal{I}]$ as $\mathcal{I} \subseteq \mathcal{I}^*$. Projecting $a$ onto $(y, \tilde{\Theta})$ we thus have[21]

$$\mathbb{E}[\hat{a}|\mathcal{I}] = \begin{bmatrix} \Sigma_{ay} & \Sigma_{a\tilde{\Theta}} \end{bmatrix} \begin{bmatrix} \Sigma_{yy} & \Sigma_{y\tilde{\Theta}} \\ \Sigma'_{y\tilde{\Theta}} & \Sigma_{\tilde{\Theta}\tilde{\Theta}} \end{bmatrix}^{-1} \begin{bmatrix} y \\ \tilde{\Theta} \end{bmatrix},$$

where we use $\Sigma_{ab}$ as shorthand for $\mathrm{Cov}[a, b']$. Combining (32) with condition (ii) of the

[21]When the vector $\Theta$ contains co-linear variables, the proof follows after replacing $\Sigma^{-1}_{\Theta\Theta}$ with the generalized inverse $\Sigma^{\dagger}_{\Theta\Theta}$ and using the standard properties of the projection matrix, $\Sigma_{\Theta\Theta}\Sigma^{\dagger}_{\Theta\Theta}$.

Theorem yields

$$\mathrm{Cov}(a, y') = \mathrm{Cov}(y - \tau, y') = \mathrm{Cov}(y, y')$$
$$\mathrm{Cov}(a, \tilde{\Theta}') = \mathrm{Cov}(y - \tau, \tilde{\Theta}') = \mathrm{Cov}(y, \tilde{\Theta}').$$

Noting that

$$\begin{bmatrix} \Sigma_{yy} & \Sigma_{y\tilde{\Theta}} \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} \Sigma_{yy} & \Sigma_{y\tilde{\Theta}} \\ \Sigma'_{y\tilde{\Theta}} & \Sigma_{\tilde{\Theta}\tilde{\Theta}} \end{bmatrix},$$

we therefore get

$$\mathbb{E}[\hat{a}|\mathcal{I}] = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} y \\ \tilde{\Theta} \end{bmatrix} = y.$$

As the proof applies to any $\tau \in \mathcal{T}$, we conclude that as long as conditions (i) and (ii) holds, we can replicate $\mathcal{T}$ by including an exogenous signal $a + \tau$ into each information set that has the same distributional properties as the primal "belief" $y$. Moreover, because $\mathcal{E}(\mathcal{T})$ is an equilibrium in the primal economy, all equilibrium conditions in the incomplete-information economy hold by construction, concluding the proof of the theorem.

## A.2 Proof of Theorem 2

Fix a lower bound $\Theta_{i,t} = \{c_{i,t}, y_{i,t}, a_{i,t}\} \cup \Theta_{i,t-1}$.[22] As $\Theta_{i,t}$ is non-stationary, we work with the informationally equivalent signals $\mathcal{S}_{i,t} = (\mathrm{d}c_{i,t}, \mathrm{d}y_{i,t}, \mathrm{d}a_{i,t})'$. By Theorem 1, $\mathcal{T}_t$ can be implemented if $\mathbb{E}[(\tau^x_{i,t}, \tau^c_{i,t})] = 0$ and

$$\mathrm{Cov}[(\tau^x_{i,t}, \tau^c_{i,t}), \mathcal{S}_{i,t-s}] = 0 \quad \forall s \geq 0. \tag{35}$$

Let $\mathcal{S}_t$ and $\Delta\mathcal{S}_{i,t}$ define the aggregate and idiosyncratic components of $\mathcal{S}_{i,t}$ so that

$$\mathcal{S}_{i,t} = \underbrace{\begin{pmatrix} \mathrm{d}y_t \\ \mathrm{d}y_t \\ \epsilon_t \end{pmatrix}}_{\mathcal{S}_t} + \underbrace{\begin{pmatrix} \Delta\mathrm{d}c_{i,t} \\ \Delta\mathrm{d}y_{i,t} \\ \Delta\mathrm{d}a_{i,t} \end{pmatrix}}_{\Delta\mathcal{S}_{i,t}},$$

[22]Here we can drop $n_{i,t}$ as it contains no additional information beyond observing $a_{i,t}$ and $y_{i,t}$. Orthogonality with respect to $(a_{i,t}, y_{i,t})$ automatically implies orthogonality with respect to $n_{i,t}$.

and define $\Gamma_s \equiv -\text{Cov}[(\tau_t^c, \tau_t^x), \mathcal{S}_{t-s}]$. We can then rewrite (35) as

$$\text{Cov}[(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x), \Delta\mathcal{S}_{i,t-s}] = \Gamma_s \quad \forall s \geq 0. \tag{36}$$

Note that the aggregate wedges $(\tau_t^c, \tau_t^x)$ can be obtained from $(\hat{y}_t, \pi_t)$ and are zero-mean under the requirements of the theorem. To prove the theorem, it thus suffices to show that for any aggregate process characterized by $\{\Gamma_s\}$, we can find a process $(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x, \Delta a_{i,t}, z_{i,t})$ such that (36) holds. Note that by construction $(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x, \Delta a_{i,t}, z_{i,t})$ does not affect the aggregate dynamics of the economy, so that it is indeed consistent with any aggregate process that one wishes to implement.

We begin by solving for the equilibrium dynamics of the "Δ-economy", giving us $(\text{d}\Delta c_{i,t}, \text{d}\Delta y_{i,t})$ as a function of $(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x, \Delta a_{i,t}, z_{i,t})$. With the solution at hand, we then derive an explicit expression for (36) and show how for any $\{\Gamma_s\}$ we can construct a valid process $(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x, \Delta a_{i,t}, z_{i,t})$ that satisfies the orthogonality requirements.

**Characterizing the Δ-economy** Suppose that the aggregate economy is in equilibrium.[23] The equilibrium dynamics of the (fictitious) "Δ-economy" are characterized by the following equations:

$$\begin{aligned}
\Delta c_{i,t} &= \mathbb{E}_t[\Delta c_{i,t+1} - \Delta\tau_{i,t+1}^c] + \Delta\tau_{i,t}^c \\
\Delta y_{i,t} &= \xi(x_{i,t} + \Delta\tau_{i,t}^x) + \Delta a_{i,t} \\
\beta b_{i,t} &= b_{i,t-1} + x_{i,t} \\
x_{i,t} &= \Delta y_{i,t} - \Delta c_{i,t} + \Delta p_{i,t} \\
\Delta p_{i,t} &= -\theta^{-1}\Delta y_{i,t} + z_{i,t}.
\end{aligned}$$

The system can be rewritten as

$$\mathbb{E}_t[\text{d}\Delta y_{i,t+1}] = \delta\mathbb{E}_t[\xi^{-1}\text{d}\Delta a_{i,t+1} + \text{d}z_{i,t+1} + \text{d}\Delta\text{d}\tau_{i,t+1}^x - \text{d}\Delta\tau_{i,t+1}^c] \tag{37}$$

$$\beta b_{i,t} = b_{i,t-1} + \xi^{-1}(\Delta y_{i,t} - \Delta a_{i,t}) - \Delta\tau_{i,t}^x \tag{38}$$

where $\delta \equiv (\theta^{-1} + \xi^{-1} - 1)^{-1}$, and consumption is determined by

$$\Delta c_{i,t} = -\delta^{-1}\Delta y_{i,t} + z_{i,t} + \Delta\tau_{i,t}^x + \xi^{-1}\Delta a_{i,t}. \tag{39}$$

Fix some process $(\Delta\tau_{i,t}^c, \Delta\tau_{i,t}^x, \Delta a_{i,t}, z_{i,t})' = A(L)\upsilon_{i,t}$, where $A(L)$ is a square-summable

[23] I.e., $\hat{y}_t = n_t = \xi\tau_t^x$ and $\pi_t$ is given by (18).

matrix-polynomial in non-negative powers of the lag operator $L$ and the vector $\upsilon_{i,t}$ are white noise shocks. Conjecture

$$\Delta y_{i,t} = \xi(\beta - 1)b_{i,t-1} + \Phi(L)\upsilon_{i,t}. \tag{40}$$

Substituting (40) in (38), it must be that

$$\Phi(L)\upsilon_{i,t} = \xi\beta \mathrm{d}b_{i,t} + \xi\Delta\tau^x_{i,t} + \Delta a_{i,t}. \tag{41}$$

Using (40) to eliminate $\Delta \mathrm{d}y_{i,t+1}$ in (37), we have

$$(\beta - 1)\xi \mathrm{d}b_{i,t} + \left[(L^{-1} - 1)\Phi(L)\right]_+ \upsilon_{i,t} = \begin{bmatrix}-\delta & \delta & \delta\xi^{-1} & \delta\end{bmatrix} \left[(L^{-1} - 1)A(L)\right]_+ \upsilon_{i,t} \tag{42}$$

where $[\cdot]_+$ sends the negative powers of $L$ to zero. Further using (42) to eliminate $\mathrm{d}b_{i,t}$ in (41) and applying the $z$-transform, we obtain the following functional equation

$$\begin{aligned}(1 - \beta^{-1}z)\Phi(z) = &\\ &\begin{bmatrix}-\delta & \delta & \delta\xi^{-1} & \delta\end{bmatrix} [(1 - z)A(z) - A_0] + \Phi_0 + (1 - \beta^{-1}) \begin{bmatrix}0 & \xi & 1 & 0\end{bmatrix} A(z)z.\end{aligned} \tag{43}$$

Evaluating (43) at $z = \beta$, pins down $\Phi_0$ and $\Phi(z)$, from which we obtain the following equilibrium process for $\mathrm{d}\Delta y_{i,t} \equiv \mathrm{d}y(L)\upsilon_{i,t}$ and $\mathrm{d}\Delta c_{i,t} \equiv \mathrm{d}c(L)\upsilon_{i,t}$:

$$\mathrm{d}y(z) = \begin{bmatrix}-\delta & \delta & \delta\xi^{-1} & \delta\end{bmatrix} (1 - z)A(z) + \begin{bmatrix}\delta & \xi - \delta & 1 - \delta\xi^{-1} & -\delta\end{bmatrix} [(1 - \beta)A(\beta)] \tag{44}$$

and

$$\mathrm{d}c_{i,t} = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} (1 - z)A(z) + \begin{bmatrix}-1 & 1 - \delta^{-1}\xi & \xi^{-1} - \delta^{-1} & 1\end{bmatrix} [(1 - \beta)A(\beta)]. \tag{45}$$

**Implementation step** Using (44) and (45), (36) can be rewritten as

$$\begin{aligned}\Gamma_s = \mathrm{Cov}&\left[\begin{pmatrix}\Delta\tau^c_{i,t}\\ \Delta\tau^x_{i,t}\end{pmatrix}, \begin{bmatrix}1 & 0 & 0 & 0\\ -\delta & \delta & \delta\xi^{-1} & \delta\\ 0 & 0 & 1 & 0\end{bmatrix}(1 - L)A(L)\xi_{i,t-s}\right] + \\ &+ \mathrm{Cov}\left[\begin{pmatrix}\Delta\tau^c_{i,t}\\ \Delta\tau^x_{i,t}\end{pmatrix}, \begin{bmatrix}-1 & 1 - \delta^{-1}\xi & \xi^{-1} - \delta^{-1} & 1\\ \delta & \xi - \delta & 1 - \delta\xi^{-1} & -\delta\\ 0 & 0 & 0 & 0\end{bmatrix}(1 - \beta)A(\beta)\xi_{i,t-s}\right] \quad \forall s \geq 0.\end{aligned} \tag{46}$$

Post-multiplying both sides by

$$M \equiv \begin{bmatrix} 1 & 1 & 0 \\ 0 & \delta^{-1} & 0 \\ 0 & -\xi^{-1} & 1 \end{bmatrix}$$

and applying the $z$-transform, (46) is equivalent to the following functional equation

$$\tilde{\Gamma}(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \left\{ \left[A(z)(1-z^{-1})A(z^{-1})'\right]_+ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}' + \right.$$
$$\left. + A(z)(1-\beta)A(\beta)' \begin{bmatrix} -1 & 1-\delta^{-1}\xi & \xi^{-1}-\delta^{-1} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}' \right\} \quad (47)$$

where $\tilde{\Gamma}(z) \equiv \mathcal{Z}\{\Gamma_s M\}_{s\geq 0}$ is the (one-sided) $z$-transform of $\{\Gamma_s M\}$ . Write

$$A(L) = \begin{bmatrix} A_\tau(L) \\ A_a(L) \\ A_z(L) \end{bmatrix}$$

where $A_\tau(z)$ is a lag-polynomial of size $2 \times n$, $A_a(z)$ and $A_z(z)$ are each lag-polynomials of size $1 \times n$, and $n$ is an arbitrary number of innovations. Then (47) can be further rewritten as

$$\tilde{\Gamma}_1(z) + \Omega(z) = \left\{(1-z^{-1})A_\tau(z)A_\tau(z^{-1})'\right\}_+ + \Psi(z) + A_\tau(z)A_\tau(\beta)'\Lambda \quad (48)$$

and

$$\tilde{\Gamma}_2(z) = \left\{(1-z^{-1})A_\tau(z)A_a(z^{-1})'\right\}_+, \quad (49)$$

where $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ correspond to the first two and third column of $\tilde{\Gamma}$, respectively, and where

$$\Psi(z) \equiv \left\{A_\tau(z)\left[(1-\beta)A_z(\beta)' \quad (1-z^{-1})A_z(z^{-1})'\right]\right\}_+$$

and

$$\Omega(z) \equiv -(1-\beta)(\xi^{-1}-\delta^{-1})\left[A_\tau(z)A_a(\beta)' \quad 0\right]$$

and

$$\Lambda \equiv \begin{bmatrix} -(1-\beta) & 0 \\ (1-\beta)(1-\delta^{-1}\xi) & 0 \end{bmatrix}.$$

Fix $N$ as the largest non-zero power of $z$ in $\tilde{\Gamma}$. Consider the following parametric structure for $A_\tau$, $A_a$, and $A_z$:

$$\begin{bmatrix} A_\tau(z) \\ A_a(z) \\ A_z(z) \end{bmatrix} = \begin{bmatrix} \lambda_\tau(z) & I \\ \lambda_a(z) & (1-z)^{-1}\lambda_{a,0} \\ 0 & \lambda_{z,0} + \lambda_{z,1} z \end{bmatrix}$$

with

$$\lambda_\tau(z) = \begin{bmatrix} \lambda_{\tau,1} + \rho z & \cdots & \lambda_{\tau,N} + \rho^N z^N \end{bmatrix}$$

and

$$\lambda_a(z) = \begin{bmatrix} (1-z)^{-1}\lambda_{a,1} & \cdots & (1-z)^{-1}\lambda_{a,N} \end{bmatrix},$$

and where $\{\lambda_{a,j}, \lambda_{z,j}\}$ are of size $1 \times 2$ and $\{\lambda_{\tau,j}\}$ are of size $2 \times 2$.

Condition (49) then simplifies to

$$\tilde{\Gamma}_2(z) = \lambda_\tau(z)\lambda_a' + \lambda_{a,0}'.$$

So for any $\lambda_\tau$, it suffices to set

$$\lambda_{a,s} = \rho^{-s}\tilde{\Gamma}_{2,s}' \quad \forall s \geq 1, \quad \text{and}$$
$$\lambda_{a,0} = \tilde{\Gamma}_{2,0}' - \sum_{j=1}^{N} \lambda_{\tau,j}'\lambda_{a,j}$$

in order to satisfy orthogonality with respect to $a_{i,t}$.

Regarding condition (48), we have that

$$\Pi(z) \equiv \tilde{\Gamma}_1(z) + \Omega(z) - \Lambda - I = \left\{(1-z^{-1})\tau_\tau(z)\tau_\tau(z^{-1})'\right\}_+ + \Psi_0 + \lambda_\tau(z)\lambda_\tau(\beta)'\Lambda$$

where

$$\Omega(z) = -\tilde{\Gamma}_2(z)\begin{bmatrix} (\xi^{-1} - \delta^{-1}) & 0 \end{bmatrix}$$

and

$$\Psi_0 \equiv \Psi(z) = \begin{bmatrix} (1-\beta)\left(\lambda_{z,0} + \beta\lambda_{z,1}\right) & \lambda_{z,0} \end{bmatrix}.$$

Notice that (i) the left-hand side, $\Pi(z)$, is exogenously determined by the aggregate economy that we are trying to implement, and (ii) we have $\Psi_0$ as a degree of freedom to induce an arbitrary unconditional covariance on the right-hand side. Writing out the right-hand side in the time-domain, we have

$$\Pi_0 = \Psi_0 - \rho\lambda'_{\tau,1} + \frac{\rho^2}{1-\rho^2} + \sum_{j=1}^{N} \lambda_{\tau,j}\lambda'_{\tau,j}(I+\Lambda) + \sum_{j=1}^{N} \rho^j \beta^j \lambda_{\tau,j}\Lambda \tag{50}$$

$$\Pi_s = \rho^s \lambda'_{\tau,s}(I+\Lambda) - \rho^{s+1}\lambda'_{\tau,s+1} + \rho^{2s}\beta^s\Lambda. \tag{51}$$

Initialized at $\lambda_{N+1} = 0$, (51) can be solved recursively backwards for a sequence $\{\lambda_{\tau,s}\}$ that ensures orthogonality with respect to $(c_{i,t-s}, y_{i,t-s})_{s\geq 1}$. Finally, orthogonality with respect to $(c_{i,t}, y_{i,t})$ is achieved by setting $\Psi_0$ to satisfy (50), completing the proof.

# B Details of the Econometric Methodology

## B.1 Applying the Frequency-filter

Let

$$J = \left(\tilde{\Omega}_T - \tilde{\Omega}(\gamma)\right)' W \left(\tilde{\Omega}_T - \tilde{\Omega}(\gamma)\right) \tag{52}$$

denote the penalty function in terms of BK-filtered moments, where the filter is applied to both the data and the model. In this appendix, we demonstrate how the penalty can be expressed in terms of the variance over unfiltered first-differenced moments, $\Omega \equiv vech\left\{\mathrm{Var}\left(\mathrm{d}s^t_{t-K}\right)\right\}$, where d is the first-difference operator, and $K \equiv k + 2\bar{\tau}$ with $\bar{\tau}$ denoting the approximation horizon of the BK-filter.[24] Specifically, for any positive-semidefinite $W$ we show that $J$ in (52) is equivalent to

$$J = \left(\Omega_T - \Omega(\gamma)\right)' \tilde{W} \left(\Omega_T - \Omega(\gamma)\right), \tag{53}$$

with $\tilde{W} = \Xi' W \Xi$ replacing $W$ (a closed-form expression for $\Xi$ is given below).

The Baxter and King (1999) filtered version of $s_t$ takes the form

$$\tilde{s}_t = \sum_{j=-\bar{\tau}}^{\bar{\tau}} a_j s_{t-j}$$

where $\tilde{s}_t$ is stationary by construction. For the high-pass filter used in this paper, the weights

[24] The first-difference filter is applied to the unfiltered variables to ensure stationarity for variables that have a unit root. Our transformation includes an adjustment term that corrects for the fact that the filtered moments in $\tilde{\Omega}$ are about levels rather than first-differences.

$\{a_j\}$ are given by

$$a_j = \tilde{a}_j - \theta, \quad \theta = \frac{1}{2\bar{\tau}+1} \sum_{j=-\bar{\tau}}^{\bar{\tau}} \tilde{a}_j$$

with

$$\tilde{a}_0 = 1 - \bar{\omega}/\pi, \quad \tilde{\alpha}_{j\neq 0} = -\sin(j\bar{\omega})/(j\pi), \quad \bar{\omega} = 2\pi/32.$$

To construct the filter-matrix $\Xi$, rewrite $\tilde{s}_t$ in terms of growth rates to get

$$\tilde{s}_t = \sum_{j=-\bar{\tau}}^{\bar{\tau}} \sum_{l=0}^{\infty} a_j \mathrm{d}s_{t-j-l}.$$

Noting that $\sum_{j=-\bar{\tau}}^{\bar{\tau}} a_j = 0$, we can simplify to get

$$\tilde{s}_t = B \mathrm{d}s_{t-\bar{\tau}-j}^{t+\bar{\tau}}$$

where

$$B = [b_{-\bar{\tau}}, \ldots, b_{\bar{\tau}}] \otimes I_n, \tag{54}$$

$n = 4$ is the number of variables in $\tilde{s}_t$, and $b_s = \sum_{j=-\bar{\tau}}^{s} \alpha_j$.

Letting $L_j$ define the backshift matrix

$$L_j = \left[ 0_{n(2\bar{\tau}+1),nj}, \quad I_{n(2\bar{\tau}+1)}, \quad 0_{n(2\bar{\tau}+1),n(k-j)} \right], \tag{55}$$

we then have that

$$\tilde{\Sigma}_j \equiv \mathrm{Cov}(\tilde{s}, \tilde{s}_{t-j}) = B L_0 \Sigma^K L_j' B',$$

or, equivalently,

$$vec(\tilde{\Sigma}_j) = (B L_j \otimes B L_0)\, vec(\Sigma^K).$$

To complete the construction of $\Xi$, define selector-matrices $P_0$ and $P_1$ such that

$$vech(\tilde{\Sigma}^k) = P_0 \begin{bmatrix} vec(\tilde{\Sigma}_0) \\ \vdots \\ vec(\tilde{\Sigma}_k) \end{bmatrix}$$

and

$$vec(\Sigma^K) = P_1 vech(\Sigma^K).$$

Stacking up $vec(\tilde{\Sigma}_j)$, we then get

$$\tilde{\Omega} = \Xi\Omega$$

where

$$\Xi = P_0 \begin{bmatrix} BL_0 \otimes BL_0 \\ \vdots \\ BL_k \otimes BL_0 \end{bmatrix} P_1 \tag{56}$$

with $B$ and $L_j$ as in (54) and (55). Substitution in (52) yields (53).

## B.2 Estimation of the optimal weighting matrix

Our estimation of

$$S \equiv \text{Var}\left\{T^{1/2}\left(\tilde{\Omega}_T - \tilde{\Omega}(\gamma_0)\right)\right\} = \text{Var}\left\{T^{1/2}\tilde{\Omega}_T\right\}$$

is based on a bootstrap identical to the one described in the main text (with 5000 replications). Let $\tilde{S} = \text{Var}\{T^{1/2}\tilde{\Omega}_b\}$ where the variance is across bootstrap samples with $\tilde{\Omega}_b = \Xi\Omega_b$ being the target moments in a given sample $b \in \{1, \dots, 5000\}$. It is well-known that estimations of covariances of covariance structures are prone to small-sample bias due to the estimation of fourth moments, which tend to correlate with the targeted covariance structure (e.g., Abowd and Card 1989 and Altonji and Segal, 1996). In addition, we find that $\tilde{S}$ is near singular. We follow Christiano, Trabandt and Walentin (2010) and dampen the off-diagonal elements of $\tilde{S}$ relative to the diagonal to improve the small-sample efficiency of $\tilde{S}$. Specifically, let $\tilde{S}_{i,j}$ denote the $(i,j)$-th block of $\tilde{S}$ corresponding to the cross-sample covariance between $\text{Cov}\{\text{d}s_t, \text{d}s_{t-i}\}$ and $\text{Cov}\{\text{d}s_t, \text{d}s_{t-j}\}$. We replace $\tilde{S}$ by $\tilde{S}^{(\nu_1,\nu_2)}$ where each block $\tilde{S}_{i,j}^{(\nu_1,\nu_2)}$ in $\tilde{S}^{(\nu_1,\nu_2)}$ is given by $\varsigma_{i,j}(\nu_1)(M(\nu_2) \circ \tilde{S}_{i,j})$ with

$$\varsigma_{i,j} = \left(1 - \frac{|i-j|}{k+1}\right)^{\nu_1}, \quad \nu_1 \geq 0$$

and

$$M(\nu_2) = 1 - \nu_2 + \nu_2 I_{n^2}, \quad 0 \leq \nu_2 \leq 1,$$

where $\circ$ is the element-wise (Hadamard) product and $n = 4$ is the number of elements in $s_t$. Intuitively, $\nu_2$ is a dampening factor applied to the off-diagonal elements within each $\tilde{S}_{i,j}$ block and $\nu_1$ is a dampening applied to the covariance between different auto-covariance-blocks that is increasing in $|i-j|$. For $\nu_1 = \nu_2 = 0$, the resulting matrix $\tilde{S}^{(\nu_1,\nu_2)}$ equals $\tilde{S}$. For $\nu_1 \to \infty$, $\tilde{S}^{(\nu_1,\nu_2)}$ becomes a block-diagonal version of $\tilde{S}$, so that the GMM criterion does not depend on the cross-block co-variation $\text{Cov}\left\{\text{Cov}\left[\tilde{s}_t, \tilde{s}_{t-i}\right], \text{Cov}\left[\tilde{s}_t, \tilde{s}_{t-j}\right]\right\}$ for $i \neq j$. For $\nu_2 = 1$, each block $\tilde{S}_{i,j}^{(\nu_1,\nu_2)}$ becomes diagonal, so that the GMM criterion does not depend on the cross-variable co-variation $\text{Var}\left\{\text{Cov}\left[\tilde{s}_t^{(m)}, \tilde{s}_{t-i}^{(n)}\right]\right\}$ for any $m \neq n$.[25] In either case, the criterion continues to make full use of all targeted moments $\tilde{\Omega}_T$. To have a consistent estimator of $S$, we need that $\nu_1 \to 0$ and $\nu_2 \to 0$ as $T \to \infty$, but do not restrict the small sample behavior of $\nu_1$ and $\nu_2$. Our choice of $\nu_1$ and $\nu_2$ is aimed at maximizing the small sample efficiency of $\tilde{S}^{(\nu_1,\nu_2)}$. Specifically, we set $\nu_1$ and $\nu_2$ to minimize the RMSE in a simulation experiment where we generate time series of the length of our original data sample, treating the bootstrap DGP described in the main text as the truth. The efficient estimator is given by $\nu_1 = 5$ and $\nu_2 = 0.5$, which also suffices to make $\tilde{S}^{(\nu_1,\nu_2)}$ well-conditioned. Collecting, we have $W = [\tilde{S}^{(\nu_1,\nu_2)}]^{-1}$ and $\tilde{W} = \Xi'[\tilde{S}^{(\nu_1,\nu_2)}]^{-1}\Xi$.

# C Market Clearing in the Primal Economy

Imposing market clearing in the primal economy, the consumer Euler equation reads

$$\mathbb{E}_t[r_t + \mathrm{d}\tau_{t+1}^c] = \mathbb{E}_t[\mathrm{d}\hat{y}_{t+1}] = \xi\mathbb{E}_t[\mathrm{d}\tau_{t+1}^x]. \tag{57}$$

For markets to clear, the real interest rate $r_t = \phi\pi_t - \mathbb{E}_t\pi_{t+1}$ has to adjust so that consumers' demand—taking into account households' errors in their consumption decisions $\tau_t^c$—matches the output gap as determined by $\tau_t^x$. Clearly, for any stationary process for $\tau_t^c$ and $\tau_t^x$, there exists a process for $r_t$ so that (57) holds. Intuitively, by fixing $\tau_t^c$ first, the real interest rate endogenously adjusts so that the real interest rate *as perceived* by consumers, $r_t + \mathrm{d}\tau_{t+1}^c$, clears the markets. This is fundamentally different to a parametric structure in which the expectation error is determined endogenously and there may not be any solution to (57). For instance, suppose consumers are perfectly informed about local conditions and the only source of uncertainty is the real interest rate. In that case, we can interpret $\bar{\mathbb{E}}_t[r_t] = r_t + \mathbb{E}_t[\mathrm{d}\tau_{t+1}^c]$. With a parametric information structure, we would need to ensure that $\bar{\mathbb{E}}_t[r_t]$ is sufficiently responsive to $\tau_t^x$, which may, e.g., fail if consumers have virtually no information regarding $r_t$. By contrast, the primal approach endogenously pins down $\bar{\mathbb{E}}_t[r_t]$ as the market-clearing object. Fixing $\tau_t^c$ merely determines how $r_t$ has to adjust so that $r_t + \mathbb{E}_t[\mathrm{d}\tau_{t+1}]$ clears the market: If consumers make larger errors, then interest rates can adjust by more to ensure an effective degree of awareness that suffices to clear the market.

[25] Here $\tilde{s}_t^{(m)}$ denotes the $m$-th element of $\tilde{s}_t$.

# D Computing the Volatility-Minimizing Local Shocks

From the proof of Theorem 2, we have that the aggregate process for $\bar{\mathcal{T}}_t$ is implementable if (47) holds. For any parametric process for the island-specific shocks, $(\Delta\tau^c_{i,t}, \Delta\tau^x_{i,t}, \Delta a_{i,t}, z_{i,t})$, we can thus simply search numerically over the process that minimizes $\mathrm{Var}[a_{i,t}] + \mathrm{Var}[z_{i,t}]$ subject to (47).

Specifically, with our MA(32) parametrization we have

$$\begin{bmatrix} \Delta\tau^c_{i,t} \\ \Delta\tau^x_{i,t} \\ \Delta a_{i,t} \\ z_{i,t} \end{bmatrix} = \sum_{j=0}^{32} \underbrace{A_j}_{4\times 4} \upsilon_{i,t-s}. \tag{58}$$

Substituting (58) into (47), the implementability constraint becomes

$$\tilde{\Gamma}_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \left( G_s \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}' + H_s \begin{bmatrix} -1 & 1-\delta^{-1}\xi & \xi^{-1}-\delta^{-1} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}' \right), \qquad s \in \{0, 1, \dots, 32\}$$

where

$$G_s = \sum_{j=0}^{32-s} (A_{j+s} - A_{j+s+1}) A'_j$$

$$H_s = (1-\beta) A_s \times \left( \sum_{j=0}^{32} \beta^j A'_j \right)$$

$$\delta = (\theta^{-1} + \xi^{-1} - 1)^{-1}$$

and

$$\tilde{\Gamma}_s \equiv -\mathrm{Cov}\left[ \begin{pmatrix} \tau^c_t \\ \tau^x_t \end{pmatrix}, \begin{pmatrix} \mathrm{d}y_{t-s} \\ \mathrm{d}y_{t-s} \\ \epsilon_{t-s} \end{pmatrix} \right] \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & \delta^{-1} & 0 \\ 0 & -\xi^{-1} & 1 \end{bmatrix}$$

is given by the aggregate estimate.

# E Characterization of the Economy with GHH-Preferences and Roundabout Production

In this appendix, we derive the equations characterizing the economy with GHH preferences and roundabout production. In analogue to (5) and (6), the local Euler equation and labor market clearing condition are given by

$$y_{i,t} = \xi \mathbb{E}_{i,t}[p_{i,t} - p_t] + a_{i,t} \tag{59}$$

and

$$\begin{aligned} \lambda c_{i,t} + (1-\lambda)\left(\xi^{-1}(y_{i,t} - a_{i,t}) + y_{i,t}\right) = \\ \mathbb{E}_{i,t}[\lambda c_{i,t+1} + (1-\lambda)\left(\xi^{-1}(y_{i,t+1} - a_{i,t+1}) + y_{i,t+1}\right) - (\phi\pi_t - \pi_{t+1})] \end{aligned} \tag{60}$$

where

$$\xi \equiv \frac{1+\zeta\alpha}{(1-\alpha)\zeta} \qquad \lambda \equiv \left(1 - \frac{1}{1+\zeta} \cdot \frac{1-\alpha}{\mu-\alpha}\right)^{-1}.$$

Optimal material demands are given by

$$m_{i,t} = \mathbb{E}_{i,t}[p_{i,t} - p_t] + y_{i,t} = \xi^{-1}(y_{i,t} - a_{i,t}) + y_{i,t}. \tag{61}$$

Following steps similar to the first step in the proof to Theorem 2, we have that—conditional on $a_{i,t}$—the response of $\mathbb{E}[c_{i,t+1}]$ and $\mathbb{E}[y_{i,t+1}]$ to $\mathbb{E}[\mu_{i,t+1}]$ are given by

$$\mathbb{E}[\Delta y_{i,t+1}] = \frac{\theta}{\theta+\xi}\mathbb{E}[\mu_{i,t+1}] \tag{62}$$

$$\mathbb{E}[\Delta c_{i,t+1}] = \frac{\theta-1}{\theta+\xi}\left\{1 - \lambda^{-1}(1-\beta)\right\}\mathbb{E}[\mu_{i,t+1}]. \tag{63}$$

Aggregating over (59) and (60), we have

$$\hat{y}_t = \xi \bar{\mathbb{E}}[p_{i,t} - p_t]$$

and

$$\lambda c_t + (1-\lambda)\left((\xi^{-1}+1)\hat{y}_t + a_t\right) = \\ \bar{\mathbb{E}}_t[\lambda \Delta c_{i,t+1} + (1-\lambda)\left(\xi^{-1}(\Delta y_{i,t+1} - \mu_{i,t+1}) + \Delta y_{i,t+1}\right)] \\ + \bar{\mathbb{E}}_t[\lambda c_{t+1} + (1-\lambda)\left((\xi^{-1}+1)\hat{y}_{t+1} + a_{t+1}\right) - (\phi\pi_t - \pi_{t+1})].$$

The aggregate resource constraint is $y_t Y = c_t C + m_t M$ or, substituting for steady state shares,

$$y_t = (1-\alpha\mu^{-1})c_t + \alpha\mu^{-1}m_t.$$

Combining with optimal material demands (61), we have

$$c_t = y_t - \xi^{-1}\frac{\alpha\mu^{-1}}{1-\alpha\mu^{-1}}\hat{y}_t. \tag{64}$$

Substituting (62)–(64) into (60), we get after some algebra,

$$\hat{y}_t = \phi_\mu \bar{\mathbb{E}}_t[\mu_{i,0}] + \bar{\mathbb{E}}_t[\hat{y}_{t+1} - \phi_r(\phi\pi_t - \pi_{t+1})] + \phi_r(\bar{\mathbb{E}}_t[\bar{a}_0] - \bar{a}_0)$$

where

$$\phi_\mu = \beta\frac{\theta-1}{\theta+\xi}\phi_r$$
$$\phi_r = \left[1 + \xi^{-1}\left(1 - \frac{\lambda}{1-\alpha\mu^{-1}}\right)\right]^{-1}.$$

# F Additional tables and figures

**Table 4:** Parameters of the estimated VAR(1) process for the expectation wedges

| Coefficient matrix $\Lambda$ on lagged states | Coefficient matrix $R$, where $\tilde{\Psi} = RR'$ |
|---|---|
| $\begin{bmatrix} \underset{(-0.914,\,0.918)}{0.289} & \underset{(-0.243,\,2.500)}{1.010} \\ \underset{(-1.222,\,0.263)}{-0.281} & \underset{(0.354,\,2.500)}{1.347} \end{bmatrix}$ | $\begin{bmatrix} \underset{(0.976,\,2.140)}{1.891} & - & \underset{(-1.977,\,-0.545)}{-1.546} \\ \underset{(0.868,\,1.402)}{1.220} & \underset{(0.000,\,0.369)}{0.141} & \underset{(-1.291,\,-0.279)}{-0.907} \\ - & - & \underset{(0.332,\,0.922)}{0.743} \end{bmatrix}$ |

Note.—Numbers in parenthesis are 90% confidence intervals.

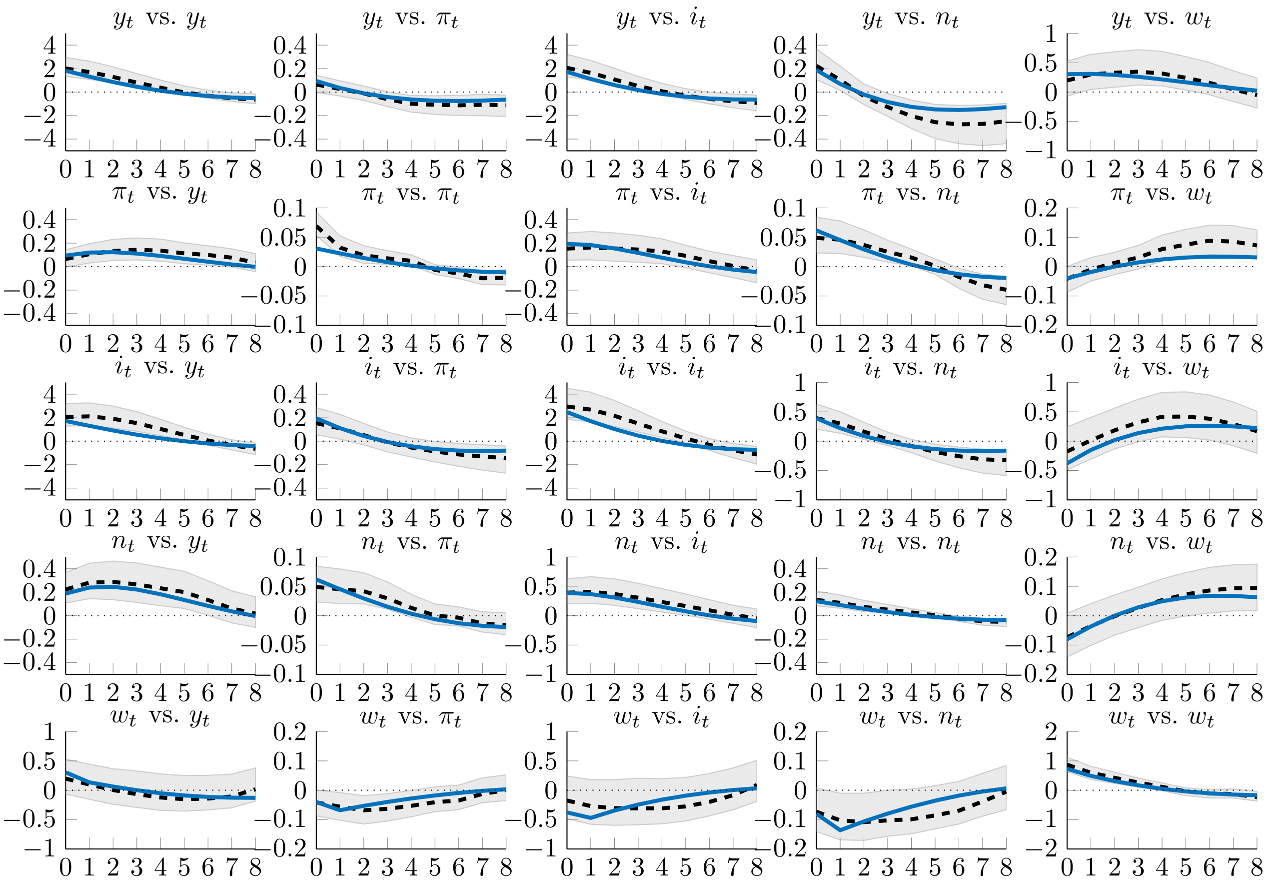

**Figure 9:** Business cycle comovements in the wage-augmented data and predicted by the estimated model with both firm- and household-side errors. Note.—Dashed black lines show the empirical covariance structure $\tilde{\Omega}_T$ together with 90 percent confidence intervals depicted by the shaded areas. Solid blue lines show the corresponding model moments $\tilde{\Omega}(\hat{\gamma})$. Each row $i$ and column $j$ in the table shows the covariances between $\tilde{s}_t^i$ and $\tilde{s}_{t-k}^j$ with lags $k \in \{0, 1, \ldots, 8\}$ depicted on the x-axis.

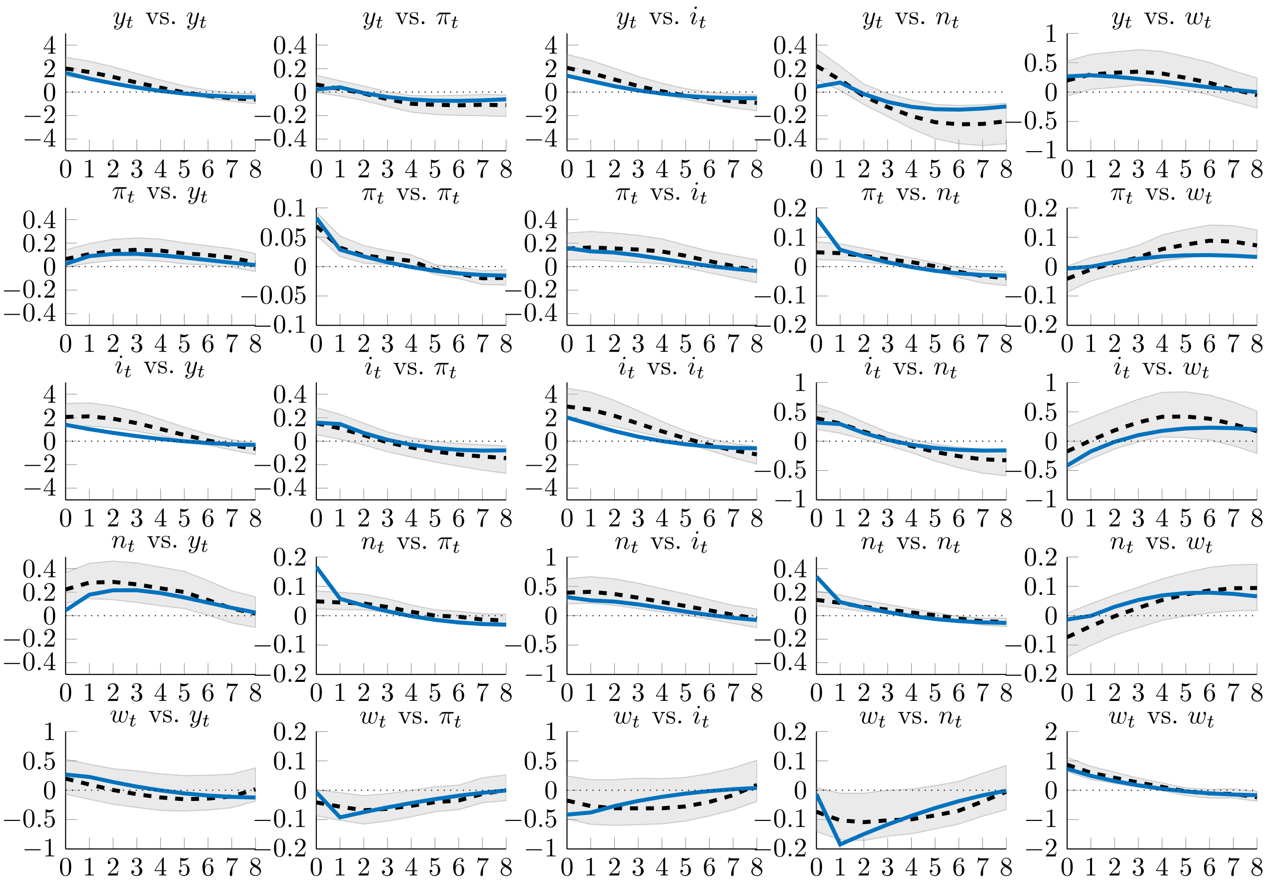


**Figure 10:** Business cycle comovements in the wage-augmented data and predicted by the estimated model with only household-side errors. Note.—Dashed black lines show the empirical covariance structure $\tilde{\Omega}_T$ together with 90 percent confidence intervals depicted by the shaded areas. Solid blue lines show the corresponding model moments $\tilde{\Omega}(\hat{\gamma})$. Each row $i$ and column $j$ in the table shows the covariances between $\tilde{s}_t^i$ and $\tilde{s}_{t-k}^j$ with lags $k \in \{0, 1, \ldots, 8\}$ depicted on the x-axis.

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