Should Corporate Pension Funds Invest in Risky Assets?

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Abstract

Whether defined-benefit corporate pension plans should invest in risky assets has always been subject to debate, and the risky pension asset allocation frequently causes concerns. In this study, we model corporate pension decisions in a setting where a firm balances its risk management concern with employees' preference for systematic risk exposure. For a reasonable set of parameter values, the optimal pension investment risk-taking and its relations with a firm's bankruptcy probability and pension funding ratio predicted by the model are consistent with empirical observations. We show that the inefficient systematic risk sharing by defined benefit pension plans may cause pensions to take even more investment risk than what employees desire if they were to manage their own retirement wealth. Further, firms may substantially reduce their overall pension funding costs under an alternative arrangement with employees bearing all systematic investment risk. This is consistent with the observed shifting from defined benefit plans to defined contribution plans.

I. Introduction

Defined benefit corporate pension plans (herafter DB plans or simply pensions) are legal entities set up by companies to provide a stable stream of incomes to retired employees. Despite a long history of pension evolution, how pension plans should invest remains an unsettled issue. At one extreme, many corporate pensions consider themselves as patient, long-term investors, and are major holders of illiquid, risky assets such as real estate, hedge funds, and private equities. At the other extreme, some pensions managers have subscribed to a complete "de-risking" strategy, holding only safe fixed-income securities or annuities. Collectively, U.S. corporate pensions' allocation to stocks and alternative risky assets currently hovers above 50%.[1] Media and policymakers have frequently raised the concern that aggressive pension investments, coupled with prevalent pension underfunding, endanger the retirement incomes of millions of workers.

Academic studies have questioned the rationale for risky pension investments. Early studies such as Black (1980) and Tepper (1981) argue that corporate pensions should hold only fixed-income securities, due to the favorable corporate tax treatment on such investments by pensions. The theory of corporate risk management provides an even stronger reason against pension investment in risky assets. As Froot and Stein (1998) point out, any risky investment without a positive alpha – including pension investment – does not create value for corporate shareholders; meanwhile, risky investment may reduces firm value by forcing firms to raise costly financing. Therefore, to maximize shareholder value, pensions should only invest in riskfree assets that match the horizon of pension obligations. This risk management based argument effectively serves as a foundation for the pension de-risking approach proposed by practitioners (e.g., Cooper and Bianco, 2003). From this perspective,

[1] See Section II for more detailed discussions.

the observed risky asset allocation by corporate pensions poses a puzzle.[2]

Some researchers (e.g., Sharpe 1976; Treynor 1977) link the risky investments by corporate pensions to a moral hazard problem, in a way similar to the risk-shifting problem for firms with high financial leverage. In addition, Lucas and Zeldes (2006) and Sundaresan and Zapatero (1997) point out that pensions may invest in stocks to hedge against the growth of pension obligations. These explanations however do not completely rhyme with empirical evidence. Studies such as Bodie, Light, Morck, Taggart (1985), Ruah (2009), and An, Huang, and Zhang (2013) find that firms with higher bankruptcy risks – and thus stronger risk-shifting incentives –have lower pension asset risk. Lucas and Zeldes (2006) note that equity allocation by pension plans with low future wage growth remains quite high and cannot be explained by their calibrated hedging demand. It suffices to say that the search is still on for a better understanding of investment policies followed by most corporate pensions.[3]

In this study, we take a stakeholder approach to model key pension decisions by a firm, including the investment decision as well as benefit level choice and pension funding. Different from the assumption in the typical corporate risk management literature, firms in our model balances the risk management concern of shareholders with employees' preference for systematic risk exposure.[4] The model predicts substantial risky assets in optimal pension portfolios. For a reasonable set of parameters, the model-predicted optimal level of pension investment risk and its relations with the bankruptcy risk and pension funding ratio are

[2]Possibly, some firms invest pension money in risky assets with a hope for positive alphas. But evidence provided by a long stream of academic studies, from Lakonishok, Shleifer, and Vishny (1992) to Busse, Goyal, and Wahal (2010), suggests that actively equity portfolios managed on behalf of pension funds on average fail to generate positive alphas. Further, the alpha-seeking motivation does not explain why pensions invest substantially in index portfolios. According to French (2008), during the period of 2000-2006, about 30% of DB plans' equity investments are passively managed.

[3]Existing studies have identified additional factors affecting pension investment decisions. These factors include pension assets as corporate financial slack (Bodie et al. 1985), accounting manipulation (Bergstresser, Desai, and Ruah 2006), the correlation between pension investment risk and corporate operating risk (Broeders 2010), and labor unions (Ippolito 1985). However, these effects are proposed to explain the different investment decisions across pensions, instead of explaining the average pattern of large pension allocation to risky assets.

[4]Specifically, in our model, a firm makes pension decisions to maximize shareholder value subject to an employee participation constraint in the form of a reservation utility. The objective function of this constrained optimization problem takes the form of a weighted average of shareholder value and employees' utility.

consistent with those reported by empirical studies. We also show that the typical defined benefit pensions offer inefficient risk sharing between shareholders and employees, which exacerbates pension risk taking.

Risk sharing within a firm is a time-honored topic that has produced many insightful observations. For example, risky profits to shareholders and fixed wages (and pension benefits) to employees can be viewed as the natural outcome of risk sharing between risk-neutral shareholders and risk-averse employees. It is important to note that the shared risk in such compensation settings is firm specific and diversifiable. But the risk ensuing from diversified pension investments is systematic, which neither shareholders nor employees can diversify away.

As it turns out, employees may have a stronger capacity or appetite for systematic risk than shareholders, which a critical element of our model. To understand why this is the case, think about a standard assumption in existing studies (e.g., Froot and Stein 1998) that shareholders hold an optimally diversified portfolio that maximizes their utility. This assumption means that shareholders are indifferent to a small change in the level of systematic risk brought about by an incremental investment, as long as the investment is fairly valued (i.e., having a zero alpha). Employees, on the other hand, have a substantial part of their wealth tied up in safe wages and do not have substantial outside wealth. Because of this wealth constraint, employees may be under-exposed to systematic risk. As a consequence, they may desire systematic risk exposure in their pension payoffs, even when such exposure does not deliver positive alpha. In sum, the different capacities for systematic risk are not because shareholders and employees have different utility functions, but rather because of the different levels of systematic risk they are already exposed to.

Another important element of our model is that despite the fixed pension benefits promised by firms, employees are to an extent exposed to pension investment risk. The division of pension cash flows between shareholders and employees are shaped by various explicit and implicit features of pension contracts – pension benefits promised by the firm, pension assets as collateral to pension benefits, pension funding provided by firm, firm bankruptcy, and pension surplus sharing. Sifting through these complexities, employees' share of pension asset risk intuitively comes from two channels. First, bankruptcy exposes employees to the downside of pension investment risk. When a firm goes bankrupt, employees' pension payoff depends on pension asset value relative to promised pension benefits. Second, employees are exposed to the upside of pension investment risk via pension surplus sharing. When the value of pension assets exceeds the value of promised benefits, employees are entitled to at least a fraction of the excess.[5]

The difference in risk capacity and pension risk sharing described above affect pension investment decisions in the following way. Suppose a pension chooses between a fairly-valued stock index fund and the riskfree asset. Although shareholders are indifferent to a small systematic risk increase in terms of their utility, an increase in risk reduces their value by increasing the firm's expected financing cost. Therefore, if shareholders are to make pension decisions solely on their own, as predicted in the standard corporate risk management setting, they would strictly prefer riskfree investment. However, pension investment risk is attractive to employees. Under reasonable assumptions for the financing cost function and employees' utility function, there is an optimal level of pension investment risk, born by both shareholders and employees. In addition, we show that decisions on the level of promised pension benefit and pension funding level can be understood in association with pension risk sharing and investment decisions. To compensate shareholders for their share of pension risk, employees may have to accept a reduced level of pension benefits. Further, pension funding level affects the magnitude of upside risk and downside risk born employees and shareholders. Thus for firms with different bankruptcy probabilities, optimal investment

[5]The Employee Retirement Income Security Act (ERISA) of 1974 requires that pensions assets are managed to the exclusive benefit of beneficiaries, which can be construed as that beneficiaries have claims on pension surplus. However, the laws in this aspect are incomplete, and firms' effective control rights on pensions matter. Firms have means to grab at least part of the surplus, from reducing pension contribution, to pension termination, to merging overfunded pensions with underfunded ones, to diverting pension assets to for operating costs and restructuring costs. However, various restrictions prevent firms from recapturing all the pension surplus and encourage firms to share pension surplus with employees. For example, firms face a punitive 50% exercise tax on surplus reversion in outright pension terminations, but the exercise tax rate is reduced to 20% if employees receive at least 20% of such surplus reversion. Section II provides more institutional details on pension surplus sharing. The fact that employees may enjoy a fraction of pension surplus has been noted by early studies such as Miller and Scholes (1981) and Bulow and Scholes (1983).

risk and optimal funding level will all be different.

We show that with reasonable parameter values, the model outcomes match several important empirical observations about pension investments. For example, consider a 30-year retirement horizon and an annual bankruptcy probability of 0.5% for the firm (typical for BBB-rated firms), with employees' share of pension surplus at a modest 20% and with other parameters such as market risk premium, market volatility, and financing costs are either calibrated to historical data or taken from estimates of existing studies. The optimal equity weight in this case is around 55%, matching well with the observed level of risky allocation by corporate pensions. By varying model parameters, we also find that the optimal portfolio weight on equity decreases with the bankruptcy probability, and increases with the pension funding ratio in most cases. These two patterns are consistent with empirical findings reported in existing studies (e.g., Bodie, Light, Morck, Taggart 1985; Ruah 2009; An, Huang, and Zhang 2013).

Our analysis further shows the relation between pension investment risk and employees' share of pension risk tends to be substitutive. When employees' share of pension risk is lower (but above a minimum threshold), the pension invests more aggressively. This relation, although surprising at first, can be intuitively understood as an effect to maintain a desirable level of systematic risk exposure in employees' pension payoffs. To further understand this relation, we explore a hypothetical pension arrangement where employees bear all the pension investment risk – somewhat similar to a defined-contribution plan (plus the longevity risk-sharing feature of a defined benefit plan). We find that under the alternative arrangement, the optimal level of investment risk is lower than under the typical defined-benefit arrangement. Further, the alternative arrangement could substantially reduce a firm's total pension funding cost. This is consistent with the observation that driven by the desire to reduce pension funding cost, many firms have shifted away from defined benefit plans to defined contribution plans.

Using various extensions of the model, we further study the effect of pension insurance, pension surplus reversion tax, and dynamic decisions of pension contribution and pension investments.

Overall, our paper provides a new perspective to understand the risky asset allocation policies pursued by corporate pensions. The insight of our model is that pension risk taking could be driven by employees' preference for systematic risk exposure. In this regard, our paper is related to, but quite different from Love, Smith, and Wilcox (2011). The main purpose of their paper is to study the effect of PBGC insurance on pension risk taking. A crucial difference is their assumption that employees can hold optimal portfolios and therefore price systematic risk the same way as shareholders. As a result, their model does not give rise to any systematic risk sharing, and has a strong prediction that pensions should invest riskfree as long as PBGC insurance is fairly priced.

The rest of the paper is organized as follows. Section II provides background information on defined benefit pension plans. In Section III, we describe a one-period model of pension investments and the analytical results. Section IV reports numeric analysis of the one-period model. Section V provides a dynamic model of pension investments. Finally, Section VI has our concluding remarks.

II. Background on Corporate Pensions and Pension Investments

II.1 Pension and Pension Investments: A Brief History

The origin of U.S. corporate-sponsored pensions could be traced to the 1880s, when companies in the booming railroad industry used pensions benefits to recruit workers. In 1875 American Express – a railway company at the time – established the first pension plan with a defined benefit feature. By 1929, there were about four hundred corporate pensions in operation, sponsored by many large corporations of the time (Munnell, 1982). Corporate pensions took a hit during the Great Depression, but recovered afterwards and grew rapidly post World War II, covering 25%, 41%, 45%, 46%, and 43% of all private-sector workers in 1950, 1960, 1970, 1980, and 1990 respectively (McDonnell, 1998). In the recent decades, however, with the rise of defined contribution (DB) plans and individual retirement accounts (IRAs) since late 1970s and 1980s, the importance of defined benefit pensions in overall retirement savings has declined. According to recent statistics, in 2014 DB plan assets stand at \$3.96 trillion, compared with \$5.32 trillion for DC plans and \$6.23 trillion for IRAs.[6] The decreasing popularity of corporate DB plans relative to DC plans and other alternatives has been well noted and discussed in existing studies; see, e.g., Munell and Soto (2008), and Rauh and Stefanescu (2009).

Prior to the stock market boom of the 1950s, corporate pensions invested only in safe assets such as bank deposits, government bonds and corporate bonds. In 1950, a DB plan of General Motors became the first to invest in the stock market (McDonnell, 1998). Over time, pensions have shifted toward substantial allocation to risky assets. By mid-1960s, aggregate corporate pension allocation to stocks exceeded allocation to fixed income assets.[7] According to various recent statistics and surveys, corporate pension allocation to risky assets (e.g., stocks, hedge funds, private equities, real estates) ranges between 50%s to 60%s (e.g., Stockton 2012; Andonov, Bauer, and Cremers 2012; Panis and Brien 2015; Willis Towers Watson 2016). For example, based on corporate filings (Department of Labor Form 5500) in 2013, Panis and Brien (2015) find that on average 50.6% of pension assets are invested in stocks, with another 12% in real estate and alternative assets. They also note that pension allocation to risky assets is trending down in recent years.

Corporate pensions' venture into the stock market in the 1950s was related to the market boom during that period. In the 1980s, corporate pensions were among the early investors (together with public pensions and insurers) in junk bonds. Due to the long-term liability structure, DP pension plans are considered long-term investors who can afford to invest in illiquid and risky assets (e.g., Campbell and Viceira 2005). Starting from the 1990s, corporate

[6] Statistics for DB and DC plans are from Private Pension Plan Bulletin, Abstract of 2014 Form 5500 Annual Reports, by Employee Benefits Security Administration, Depart of Labor. Available at https://www.dol.gov/agencies/ebsa/researchers/statistics/retirement-bulletins/private-pension-plan. Statistics for IRAs are from The IRA Investor Profile: Traditional IRA Investor's Activity 2007-2015. Available at https://www.ici.org/pdf/rpt_17_ira_traditional.pdf.

[7] Based on statistics provided by Federal Reserve Board Flow of Funds data (Z.1 Statistical Release).

pensions also become pioneering investors, and remain a major force today, in alternative assets such as hedge funds and private equities. Meanwhile, their relative importance in the public equity market sees a peak in mid 1990s and has since been on a decline (French 2008).

The notion of liability-driven investing (LDI) for pensions was developed in the 1980s (Liebowitz, 1986; Ang 2014). LDI takes into account pension liabilities when making investment decisions. An extreme version of this approach is to complete de-risk pension assets, i.e., investing in annuities and avoiding any form of market risk. A more flexible version of LDI is to set the pension investment objective to be a concave function of pension surplus ratio (i.e., the ratio of pension assets to pension liabilities), thus introducing a hedging component in the optimal portfolio against the interest rate risk of pension liabilities. LDI strategies have gained traction among pension managers in recent years (e.g., Cooper and Bianco, 2003; Leibowitz and Ilmanen, 2016).

II.2 Ownership and Control of Pension Plans: Legal Aspects

Corporate DB pension plans are typically set up as trusts. The beneficiaries are qualified current and retired employees. The trustees are in charge of pension administration, supposedly in the interest of the beneficiaries. In practice, trustees are appointed by firms sponsoring the pension plans. This means that plan sponsors have effective control of pension decisions.

Prior to the Employee Retirement Income Security Act (ERISA) of 1974, corporate pension plans are governed by the general trust laws. ERISA sets a comprehensive list of standards for pension vesting, funding, termination, and disclosure. It subjects pension trustees and pension asset managers to an explicit set of fiduciary duties, and establishes the Pension Benefit Guaranty Corporation (PBGC) to provide insurance to beneficiaries against pension failures. ERISA is amended by various subsequent legislations, such as the Pension Protection Act (PPA) of 2006 (which tightens pension funding and reporting requirements); but its major framework remains intact.

Several legal aspects of pension laws are of particular relevance of this study. First, Section 403(a) of ERISA explicitly requires pension assets to be held in a trust. This separates pension assets from the rest of a plan sponsor's assets. The same section further sets forth that "a fiduciary shall discharge his duties with respect to a plan solely in the interest of the participants and beneficiaries...". And Section 403(c) of ERISA requires that "the assets of a plan shall never inure to the benefit of any employer and shall be held for the exclusive purposes of providing benefits to participants in the plan ...". Combined, they suggest that pension decisions, including investment decisions, should be made in the interest of plan beneficiaries instead of plan sponsors. However, ERISA falls short of making specific requirements on how plan trustees should be appointed or how pension assets should be invested. With the exception of multi-employer plans (where labor unions have some controls over pension plans), corporate plan sponsors have effective pension control rights via the ability to appoint trustees.

Second, when a plan sponsor is in bankruptcy (either Chapter 7 or Chapter 11), because pension assets are held in a separate trust, they are protected from the claims of the plan sponsor's creditors. However, the plan sponsor in bankruptcy may choose to terminate an underfunded pension plan, leaving insufficient pension assets to cover pension liabilities. This distress termination (Section 4041(c) of ERISA) triggers PBGC to take over the pension plan and provide insurance to beneficiaries.[8] But firms are not completely off the hook for their pension obligations in distress termination. Section 4062 of ERISA sets forth that in distress termination, PBGC holds a claim against a plan sponsor for unfunded pension liabilities, which are treated typically as general unsecured debt (below the priority of senior unsecured debt) in bankruptcy proceedings.

PBGC's pension benefit coverage has limits. In 2017, the maximum insured benefit for a 65-year old retiree in a single-employer plan is $64,000 annually. Further, certain types of vested benefits are not covered. Finally, PBGC uses a different approach to value future benefits than the plan sponsors. As a result, in many distress terminations, the coverage received by pension beneficiaries from PBGC falls below the value of the expected benefits

[8] PBGC can also initiate "involuntary termination" of severely underfunded pensions.

from the pension plan.[9]

Third, the most uncertain legal aspect about pension plans is perhaps which party holds claim on pension surplus, i.e., the part of pension assets in excess of pension liabilities. Although ERISA requires pension assets to be managed in the exclusive benefit of plan participants and beneficiaries, firms have several ways to claw back pension surplus into corporate assets, which is known as "reversion". The standard termination procedure described by Section 4041(b) of ERISA allows plan sponsors to terminate a pension under the condition that the sponsor makes alternative arrangements to meet all plan liabilities (e.g., paying with lump sum cash or annuities). Outfight termination of overfunded pensions to claw back surplus was quite popular in the 1980s (e.g., VanDerhei 1987; Cather, Cooperman, and Wolfe 1991) – and generally survived court challenges – but was effectively stopped by a 50% exercise tax rate on such surplus reversion imposed by the Omnibus Budget Reconciliation Act of 1990, a rate still in effect today.[10] The rate is reduced to 20% if at least 25% of the surplus reversion is transferred into a qualified replacement plan (e.g., a defined contribution plan) or at least of 20% of the surplus is used to increase the pension benefits of qualified participants. This provides an incentive for surplus sharing between plan sponsors and plan participants.

Because plan sponsors have effective control over pension decisions, there are alternative ways for firms to recapture pension surplus. For example, a firm with a growth in future pension liabilities can reduce surplus by simply reducing future pension contributions. The exercise tax can be avoided by merging an over-funded plan with an under-funded plan, either within a firm or through a merger of two firms. Finally, pension regulations have some vague parts in treating health benefits and severance benefits, allowing firms to dip into pension surplus to pay for what are otherwise considered normal operating costs or restructuring costs. Thus, if is fair to say that plan beneficiaries generally are not entitled

[9]According to a PBGC (2008) analysis of 125 (still healthy) pension plans with 525,000 participants, 16% of the participants would see their benefits reduced if PBGC takes over, and their benefits on average would be reduced by 28%.

[10]Prior to that, the Tax Reform Act of 1986 initially introduced an exercise tax rate of 10%, and the Technical and Miscellaneous Revenue Act of 1988 increased the rate to 15%.

to all the pension surplus.

III. One-period Model

III.1 Basic Model Setup

Our baseline model has one period with two dates: time 0 and time T. At time 0, the firm hires an employee, contributes to a pension fund, and makes investment allocation decisions for the pension fund. At time T, the employee retires, the pension investment return is realized, the firm fulfills the pension contract obligations and the employee gets the pension payment.[11] The overall problem faced by the firm is how to set the promised pension benefit, make a initial contribution, and decide on investment allocation, so that to keep the employee satisfied in a cost efficient way.

We assume that at time 0, the firm has a certain amount of discretionary cash flows, denoted by H_0, to be invested in the pension fund. The firm can make incremental (non-negative) contributions to the pension fund at times 0 and T, denoted by h_0 and h_T. The contributions incur financing cost $C_0(h_0)$ and $C_T(h_T)$, with the cost functions $C_0(.)$ and $C_T(.)$ increasing and convex. The investment opportunity for the firm consists of a risk free asset with the gross rate for the period denoted by R_f and a risky asset, which may be viewed as the market portfolio, with the gross return denoted by R_m. The pension asset at time T is then

$$W_T = W_0(wR_m + (1-w)R_f) \tag{1}$$

where w is the portfolio weight of the pension fund on the risky asset, W_0 is the pension fund starting value, and W_T is the pension fund value at the terminal date. We denote the realized pension payment to the employee at time T by S. By simple accounting, the firm's cash flows are $CF_0 = -W_0 - C_0(h_0)$ and $CF_T = W_T - S - C_T(h_T)$ where $h_0 = \max(W_0 - H_0, 0)$ and $h_T = \max(S - W_T, 0)$. The pension contract dictates how the payment S depends on

[11] DB plans provide coverage on employees' longevity risk, so that retired employees get pension payments in annuities until death. For simplicity we do not explicitly consider longevity risk in our model.

the pension fund wealth W_T and other state variables, e.g., whether the firm default. We will provide further detail on those relations below.

The firm's objective is to maximize shareholder value subject to the employee's participation constraint. We assume that shareholders value firm cash flows via the risk-neutral probability approach. Specifically, we assume a market equilibrium implied risk neutral probability, denoted by Q, in contrast to the physical probability, denoted by P. Shareholders value the firm's time-T cash flows by discounting at the risk free rate the risk-neutral probability based expectation. We assume that the firm has a default probability of p, and the default risk is purely idiosyncratic. Therefore, the risk-neutral probability of default is also p. That is, $E(D) = E^Q(D) = p$.

The employee's preference is captured by the expected utility with an increasing and concave utility function $U(S)$, where S is the pension payoff to the employee at time T. In reality, the employee may combine the pension payment with other financial resources to form her total wealth at the retirement age. Our assumption is an approximation to the view that the employee sees the pension payment S as a major (if not the sole) source of funding for future consumption at the retirement age. At this stage, we can lay out the firm's optimization problem:

$$\max_{w,W_0,S} CF_0 + \frac{1}{R_f} E^Q(CF_T) \tag{2}$$

$$\text{subject to:} \quad E(U(S)) = \mathbb{U} \tag{3}$$

where $\mathbb{U}$ is the employee's reservation utility, and Equation (3) represents the employee's participation constraint. The other constraints omitted from the above specification include the wealth evolution according to Equation (1) and several additional constraints posed by the pension contract form, which will become explicit once we specify the pension payment schedule.

III.2 Analytical Results for Some Payment Specifications

In this paper, we focus on the defined benefit pension fund. We denote the promised pension payment at time T by F. While the promised payment is fixed, the real payment to the employee, as discussed in section II.2, may vary depending on a few factors, including whether the firm defaults before the retirement time and what is the pension's asset value relative to the promised pension payment. To sort out the precise payment to the employee under all the possible scenarios is not a trivial exercise. It depends on the contractual specification of the pension plan, the regulatory requirements, and potential outcomes following the legal procedures, among other things. To examine the problem in face of such complication, we start by examining some prototypes of the payment settlements. They serve as illustrations of the basic intuition and highlight some common underlying theme. We later examine additional variations.

We start with the case that is most susceptible to analytical examination. We assume that if the firm does not default at time T, the firm pays the fixed payment F to the employee. It takes the remainder pension fund value if there is a surplus or pay the difference if there is a deficit. In the view that the pension fund investment serves as the collateral of the contractual liability of the pension payment, in the case of the firm's default, the employee takes over the collateral, that is, the totality of the pension fund for the pension payment. That is:

$$S_B = F(1 - D) + W_T D. \tag{4}$$

We use the subscript B to indicate the base case. As noted before, the random variable D indicates the firm's default event. For this special case, we further assume the CRRA utility for the employee. That is, $U(S) = \frac{S^{1-\gamma}}{1-\gamma}$. We have the following results:

Proposition 1. *Consider the case of the pension payment specified in Equation (4)*

(1) In the case that the default probability is zero (i.e., $p = 0$), the pension fund will invest only in risk free asset (i.e., $w = 0$)

(2) In the case of positive default probability (i.e., $p > 0$), the pension fund's stock

allocation is strictly positive (i.e., $w > 0$), and is lower than the optimal allocation of an investor who shares the same utility as the employee, with the sole exception that the pension is fully funded and the cost function's slope at 0 is higher than some threshold (i.e., $C_T'(0) > C$, where C is specified in the appendix), in which case, the pension fund's stock allocation may be zero.[12]

Proof. See Appendix.

From the firm's stand point, investing in stocks generates zero net present value as an investment strategy, and then the convexity in the financing cost function makes the firm averse even to the systematic risk. When there is no default risk, the pension payment is then fixed at F in the current setting. Therefore, the employee's utility is not affected by the pension fund's asset allocation. The firm's aversion to risk leads to the full allocation of pension asset to the risk free asset.

In the case of firm default, the employee takes the full pension fund. From the employee's perspective, her utility, conditional on the event of the firm's default, will be maximized if the pension asset is allocated between the risk free asset and the risky asset in line with her risk aversion level to achieve the desirable risk and return trade-off. When the firm's default probability is considered, the employee's perspective of the risk-return trade-off is thus balanced with the firm's aversion to risk, leading to a compromised solution: the asset allocation on the risky asset will be strictly positive but lower than what is optimal purely from the employee's investment perspective. [13]

Proposition 1 can be extended to cover additional cases. We report results regarding two cases in the following corollary.

[12] We refer to the pension fund as over, under, or fully funded at time zero by comparing the fund asset W_0 with the PBO (i.e., the risk free rate discounted committed payment F).

[13] The sole exception in the proposition is when the pension fund is fully funded and the financing cost function has a high slope at zero. In this case, investing in the risk free will result in the time T fund value from the investment being equal to the committed payment. Consider increasing the risky asset allocation by an infinitesimal amount from 0. On the positive side, the expected return for the pension fund investment portfolio will increase proportional to w. On the negative side, the firm faces the chance of fund deficit, which incurs financing cost. Whether it is desirable to move away from the pure risk free investment depends on which of the above two effects dominates, the precise condition of which is given in the proposition.

Corollary 1. *(1) In the case the payment is specified as*

$$S = F(1 - D) + \min(W_T, F)D. \tag{5}$$

and the default probability is positive, if the pension is underfunded at time 0, the optimal stock market allocation is strictly positive (i.e. $w > 0$).

(2) In the case that there is zero probability of default and the payment is specified as

$$S = F + (1 - \alpha) \max(W_T - F, 0) \tag{6}$$

, where $0 \leq \alpha < 1$, if the pension is over-funded at time 0, the optimal stock market allocation is strictly positive.

Proof. See Appendix.

The corollary examines very different cases. It highlight the fact that our basic intuition can apply in a straightforward way to very different scenarios. In (1) of the corollary, the employee receives F when the firm does not default, the same as our baseline case. Different from the baseline case, when the firm default, the employee gets paid by the pension asset up to the level of the promised payment. In (2) of the corollary, we take out the consideration of firm default, and we assume that, in the case of asset surplus, the firm keeps α share of the surplus, and the employee gets the remaining $1 - \alpha$ shares. It is worth noting in the first case of the corollary, the employee never receives payment from the pension higher than the promised payment F. Yet, when the pension investment return is low, she receives less payment in the case the firm default. In effect, she bears the downside risk of the pension investment in the case of the firm default. In the second case, there is no default possibility. The pension payment is never below the promised payment F. Yet, the employee shares the upside of the investment outcome. Despite their apparent differences, in both cases, the optimal investment strategy involves positive allocation to the risky asset. So, as long as the employee is exposed to the investment risk, being it through upside profit sharing or downside risk bearing, it can be optimal for the pension fund to invest some portion of its asset in the stock market. To further examine the richness of the different variations, we resort to numerical analysis.

IV. Numerical Analysis

In this section, we numerically calibrate the one-period model. We consider a variety of specifications of the pension payment schedule. We focus on the optimal risk allocation for the pension asset, and examine how different parameters affect this solution.

IV.1 Calibration

We use the Fama-French equity market data from 1926 to 2016 and estimate the market risk premium at 6.1%, the risk free rate 3.3%, and the market volatility 18.5%. We assume that the employee retires in 30 years. For the default rate, we consider a BBB rated firm. The annual default probability is estimated to be 0.5% (Berk and DeMarzo, 2016). We take the rate as fixed in the thirty-year period and calculate the aggregated probability of the firm default within the 30 year window to be 13.93%. We assume the CRRA utility for the employee with the relative risk averse coefficient $\gamma = 6$ based on Constantinides (1990).

To calibrate the representative employee's reservation utility, we take the following approach. We consider a hypothetical pension commitment F and use the risk free rate to discount F to estimate the Projected Benefit Obligation (PBO). Note that all the variables calibrated so far are scale invariant. So, it is without lose of generality to unify this hypothetical PBO to 1. Then we assume the investor is handed this money to invest for herself for the 30-year period. We use the resulting utility as the reservation utility. According to Chen et al (2014), the average pension PBO is \$1,016.25 million. Therefore, our unit 1 represents approximately \$1 billion.

Following Hennessy and Whited (2007), we assume a quadratic form for the external financing cost function. Specifically, $C_t(h) = c_a + c_b h + c_c h^2$, where $t = 0, T$. In Hennessy and Whited (2007), the financing amount h is in millions. Given that our unit is in billions, we make the scale adjustment accordingly. We set the cost function parameters as $c_a = 5.98 \times 10^{-5}$, $c_b = 0.091$, $c_c = 0.4$. We assume that the firm has zero free cash flow at time 0.

For the purpose of cross-sectional comparison, we consider a more general form of the pension payment schedule as follows:

$$S = F(1 - D) + \min(W_T, F)D + (1 - \alpha) \times \max(W_T - F, 0). \tag{7}$$

In this specification, the employee gets the promised payment F when the firm does not default, thus the term $F(1-D)$. In the case of a default and if the pension asset falls below the promised payment, the employee only gets the pension asset, thus the term $\min(W_T, F)D$. Finally, if there is a surplus, the firm keeps α share of the surplus and the employee gets the remainder $(1 - \alpha)$ share, thus the term $(1 - \alpha) \times \max(W_T - F, 0)$. For the baseline setup, we assume that the firm keeps 80% of the surplus while the employee keeps the remaining 20%. That is, $\alpha = 0.8$. Overall, our baseline parameters are as in the following table. In the following analysis, we vary some of the parameters to examine their impact.

Table 1: Benchmark Parameters

μ	r_f	σ	c_a	c_b	c_c	p	γ	H_0	α
0.094	0.033	0.185	5.98×10^{-5}	0.091	0.4	0.005	6	0	0.8

IV.2 Comparative Results

Figure 1 plots the portfolio's risk allocation (w), the pension commitment (F) and the initial contribution (W_0), when we vary the default probability while keeping other parameters constant. In Panel (1), we see the portfolio's risk allocation decreases as the default probability increases, consistent with the empirical findings.

To see the intuition, it is helpful to pay attention to the sensitivity of the employee's pension payment to the pension asset value. We refer to this sensitivity as the delta of S to

W_T, or briefly, delta. Formally,

$$\delta = (\partial S)/(\partial W_T) = (W_T < F)D + (1-\alpha)(W_T > F).^{14} \tag{8}$$

Consider the case of default probability being 0. The pension payment takes an option-like form: $S = F + (1-\alpha) \times \max(W_T - F, 0)$. That is, the employee gets the fixed payment F in the case that the pension's investment asset falls below F. In the case the investment asset is higher, the employee gets $1-\alpha$ share of the surplus. Thus, $\delta = 0$ in the region $W_T < F$ and $\delta = 1-\alpha$ in the region $W_T > F$. The average delta is thus somewhere in between 0 and $1-\alpha$. With a relative low delta, it takes a high allocation of the pension fund to the risky asset for the employee to achieve desirable risk exposure from the pension payment. The firm invests thus aggressively in order to get closer to the employee's desired level. Furthermore, as shown in Panel (2) of the figure, the firm chooses a relatively low promised payment, which helps enlarge the relatively high delta region (i.e. the region of $\delta = 1-\alpha$ or equivalently $W_T > F$) and reduce the 0 delta region (i.e, the region of $W_T < F$), with the effect of a higher average delta. The higher average delta is desirable because it improves the effectiveness of passing the risk exposure of the pension fund investment to the pension payment that the employee gets. Furthermore, as part of the optimal strategy that involves aggressive risk allocation, the firm chooses to fund aggressively at time 0 which helps reduce the financing concern at time 1 and thus reduce the potentially high impact of the convex financing cost from the aggressively risky investment.

On the other extreme, consider the case that the firm defaults for sure. The pension payment is $S = \min(W_T, F) + (1-\alpha) \times \max(W_T - F, 0)$. The pension payment's delta to the pension investment asset is 1 in the region of $W_T < F$, and the delta is $1-\alpha$ in the region of $W_T > F$. The average delta is thus somewhere in between $1-\alpha$ and 1. The overall delta in the case is in a relative high comparing with the first case discussed above. The riskiness of the pension asset investment can be effectively transferred to the employee

[14] We adopt de Finetti's notation convention and not distinguish a random event with its indicate variable. Further, the partial derivative, conditional on D, is to be understood in the general sense, with a few indifferentiable points being ignored.

through the highly variable pension payment. Therefore, the firm needs only to allocate a modest amount of the pension asset in the risky asset to achieve optimal. The firm would choose a high promised payment F in order to enlarge the average delta by enlarging the region of delta being 1 (i.e., the region of $W_T < F$). There will be severe underfunding at time zero, in order to balance the effect of the high level of the promised payment F, so as to keep the employee's utility at the reservation level. Furthermore, doing so is optimal because, the future financing cost is a not a concern as it incurs only in the nondefault case. Once we understand the two extreme cases of the default probability, the firm's decisions for intermediary cases fall naturally somewhere in between the two extremes.

In Figure 2, we examine the effects of various factors on the optimal asset allocation and the initial funding. In Panel A, we look at the effect of the initial free cash flow. The intuition for this analysis is straightforward. With high free cash flow at time 0, naturally the firm contributes more at time 0, as shown on the right side chart of Panel A. Such contribution reduces the concern the potential deficit at time T. As a consequence, as shown on the left chart of the panel, the firm makes more aggressive risk allocation for the pension fund to be in line with the employee's preference on the risk-return trade-off. Consequently, the firm can promise a lower payment F while still keeping the employee at the reservation utility, as shown on the right chart of the panel.

In Panel B of Figure 2, we examine the effect of varying the ratio of surplus that the firm keeps. As shown in the left chart of Panel B, as the share of surplus the firm keeps (i.e., α)increases, the risk allocation initially increases and then decreases. As the firm keeps some surplus, the pension payment's delta in the region of $W_T > F$ decreases. Consequently, the overall average delta decreases as well. Naturally, it would take a more aggressive risk allocation in the pension fund to offset the decrease in delta, in order to keep the riskiness of the pension payment leveled. Indeed, the firm acts accordingly, as shown in the chart. As the firm's share of surplus further increases, the delta in the no-default case starts to approach zero. As a result, any reasonable level of risk allocation in the pension fund leads to only meager stock market exposure of the pension payment to the employee. That is, with extreme low values of delta, the channel of passing the risk exposure of the pension investment to the employee through the varying pension payment is by and large shut down, making it ineffective for the pension fund to provide the risk exposure to the stock market for the employee. Overall, when the firm takes large enough share of the surplus, further increasing in the firm's share of the surplus and thus reducing the employee's share of the surplus, the optimal risk allocation decreases.

As the firm's share of the surplus increases, the delta of the pension payment to the pension fund investment value decreases in the surplus region (i.e., the region of $W_T > F$). As shown in Plot B.2 , the firm chooses a high level of the PBO and accordingly the low level of the the initial funding ratio so as to reduce the probability of the surplus region, the region of the reduced delta. Overall, the firm chooses the funding ratio in an attempt to maintain a relatively high average delta.

The cases discussed above help establish the basic intuition of our model. We further examine other comparative statistics, including the effect of varying financing cost (Panels C and D of Figure 2), the effect of varying stock market condition (Panels E, F, and G). The results are largely consistent with our model's basic intuition: the optimal risky allocation decision is reached by negotiating between the demand of risky exposure from the employee and the convex financing cost.

IV.3 Cross-section of Pension Investments

In our model, both the initial funding ratio and the asset allocation are endogenous. Thus, the empirically observed cross-sectional relation between the funding ratio and the risk allocation should be interpreted with care. In this subsection, we examine various cross sectional relations that can result from the model. The results of the analysis are presented in Figure 3.

In the model, a simple way to vary the initial funding ratio is to vary the initial free cash. If a firm has less free cash at time 0, it is naturally costly for the firm to fund the pension generously. The pension is thus likely to be underfunded, ceteris paribus. How the initial free cash affects the firm's initial funding, PBO, and the asset allocation is examined in Panel A of Figure 2, in Subsection IV.2. Here, we present the cross-sectional relation of the initial funding ratio and the risky allocation in Plot 1 of Figure 3 (the dot-line). Basically, as the initial funding ratio due to higher initial free cash flow, the fund's risky allocation is also higher.

In the plot, we also examine the cross-section by letting the bankruptcy probability vary. Some properties of this case are examined in Figure 1, in Subsection IV.2. As the bankruptcy probability increases, the promised obligation increases and the initial funding decreases. At the same time, the risky asset allocation decreases. So, if we simply relate the funding ratio and the asset allocation, we again see a positive relation.

Finally, in the plot, we present the cross-section by varying the surplus sharing. Some properties of the case are examined in Figure 2, Panel B, in Subsection IV.2. As the firm's share in the upside increases, the employee's share in the upside decreases. The funding ratio decreases and the asset allocation increases initially and decreases at the end. The intuition is discussed in detail in Subsection IV.2. Overall, there is a negative relation between the funding ratio and the risky allocation in some range of the two variables when the firm's share of the surplus is low, but in other range of the two variables when the firm's share of the surplus is high.

In summary, we see that, with different choice of the underlying force that causes the cross-sectional variation, the relation between the funding ratio and the risky allocation is mostly positive, but can be negative.

IV.4 A "Variable Benefit" Plan

Our analysis so far suggests a general result that it is optimal for the employee to bear a large fraction of pension investment risk, i.e., having a high delta. The defined benefit feature, however, severely limits employee's exposure to investment risk by keep a low average delta. This leads naturally to the question we explore in this part: what if we let the employee bears the full risk of the investment? That is, let $S = W_T$. This assumption resembles the employee's payoff in a defined contribution plan, although we still assume that the investment asset is pooled among all the employees and is managed uniformly, and the firm insures the longevity risk. Thus, strictly speaking, it is a hybrid of the defined benefit plan and the defined contribution plan. We call it the variable benefit plan (the VB plan). Such a plan eliminates the firm's concern of financing cost at time T. More important, such a plan has the highest possible delta in our model, namely 1.

We compare the outcome of the defined benefit plan with that of the variable benefit plan. In particular, we examine how much the asset allocation differs across the two plans, and how much more the defined benefit plan costs the firm than the variable benefit plan while keeping the employee's reservation utility fixed. In Figure 4, Panel A, we study the comparison for firms with different bankruptcy risk. For this part of the analysis we keep the funding ratio exogenous. One can also interpret the result as if there is an exogenous shock that causes the funding ratio to vary. Alternatively, we can generate essentially the same chart by varying the time 0 free cash flow. As the funding ratio of the DB plan increases, similar to the results in Panel A of Figure 2, the risky allocation increases.

Interestingly, the DB plan in general invests much more aggressively than the VB plan, as shown in Chart 1 of the panel. In Chart 2, we see that the cost of the DB plan to the firm is much higher than that of a VB plan (about 40% higher for an investment-grade firm). It may also come as a surprise that the cost of the DB plan is the lowest for a low credit-rating firm. Thus, the results highlight that the efficiency of risk sharing plays a very important role in pension decisions. To minimize pension funding cost, it is optimal for the employee to bear all the investment risk.

In recent decades, there is a trend of firms shifting away from defined benefit plans into defined contribution plans. One reason often cited by firms making this shift is that defined benefit plans have much higher funding cost (Munnell and Soto 2008). The analysis performed here supports this notion.

IV.5 The Case with PBGC Insurance

In this section, we consider the case with PBGC pension insurance. The PBGC insurance provides guarantee to employees' retirement payment up to a ceiling — $64,000 a year for a retiree at age of 65 as of 2017. For highly paid employees such as pilots, PBGC insurance provides only a partial coverage. To capture the partial coverage feature of PBGC insurance, we assume that the employee receives θF, with $\theta \in (0, 1)$, from PBGC when the firm is bankrupt and when the pension is underfunded. We set $\theta = 0.7$ and investigate its variation in later analysis. Thus the employee's expected pension payment becomes:

$$S = F(1 - D) + \min(W_T, 0.7 \times F)D + 0.2 \times \max(W_T - F, 0). \tag{9}$$

We also assume that PBGC has access to the optimal risky portfolio, but is thinly capitalized and faces a convex financing cost similar to but less than the firm. We set $c_{ins,a} = 0$, $c_{ins,b} = 0.0353$, and $c_{ins,c} = 0.1333$. The insurance premium by PBGC is considered fairly priced if PBGC breaks even. Too low a premium (undervalued premium) will result in PBGC subsidizing the firm, and too high a premium will result in PBGC gains at the expense of the firm. To model the over-/under-pricing of PBGC insurance premium, we follow the insurance literature (i.e., Doherty and Schlesinger 1990) and set the PBGC insurance premium as the product of the risk-neutral expectation of cash flow to PBGC in case of bankruptcy and a loading factor. The insurance is overpriced if the loading factor m is greater than 1, underpriced if m is smaller than 1, and fairly priced if m equals to 1. Thus PBGC insurance premium can expressed as

$$I = mD\frac{1}{R_f}E^Q\left[\max(0.7 \times F - W_T, 0) + C_B(\max(0.7 \times F - W_T, 0))\right] \tag{10}$$

where C_B is the financing cost function of PBGC. Firms need to purchase PBGC insurance when forming the pension portfolio, which changes firm's cash flow at period 0 to $CF_0 = H_0 + h_0 - W_0 - I$. Again, initial contribution h_0 needs to be chosen such that CF_0 is non-negative.

Figure 5 displays the pension plan's risky allocation and funding decision in the presence of PBGC. In Panel A, we look at the effect of bankruptcy risk. When the bankruptcy risk is relatively small (less than 0.5), the risk allocation-bankruptcy risk relationship is similar to the case without PBGC insurance – corporate pension investment becomes more aggressive when the bankruptcy risk is higher. However, as the bankruptcy probability increases above 0.5, the relationship is reversed. On one hand, partial coverage of PBGC insurance reduces the employee's downside risk exposure in bankruptcy, therefore the employee requires more risk allocation as the perceived bankruptcy risk increases. On the other hand, higher bankruptcy risk increases insurance premium, which discourages the firm from investing in the risky asset. The pattern shown in Panel A (1) results from the trade-off of two forces. Pension funding-bankruptcy risk relationship in Panel A (2) is similar to that without PBGC –the pension becomes more underfunded when bankruptcy risk increases.

In Panel B, we investigate the effect of initial free cash. When firm has little initial free cash to fund pension plan, it takes advantage of the lower financing cost of PBGC and partial coverage of PBGC insurance through investing aggressively in the risky asset and contributing as little as possible. Essentially, the firm bets on its bankruptcy and PBGC insurance to meet the employee's reservation utility. As the firm's initial free cash increases, the above strategy becomes less attractive. The firm starts to contribute more, promise less, and allocate more to the risky asset.

The effect of surplus sharing, as shown in Panel C, is similar to the case without PBGC. Further, Panel D shows that the effect of PBGC's financing cost basically replicates the pattern of firm's own financing cost at time T.

As shown in Panel E, pension insurance encourages pension risk taking particularly when the insurance premium is under valued. Even with fair valued insurance, the risky allocation with PBGC insruance is still higher than the case without PBGC. This is because the greater financing capacity of PBGC alleviates the concern of the financing cost. Meanwhile, overpriced PBGC insurance makes the firm promise more to compensate for the reduction in risk exposure. Pension plan becomes less underfunded as a result of increased insurance premium.

Last, we look at the effect of PBGC coverage in Panel F. Higher PBGC coverage reduces the employee's downside risk exposure, which requires firm to make more risky investment. If the PBGC coverage is too low, the employee bears almost the same downside risk exposure as in the case without PBGC. And as a result, the risky investment in the presence of PBGC gets close to that without PBGC.

For robustness, we further examine the cross-sectional relations among key variables in a setting where the pension is fully funded at the initial stage. We thus focus only on the the firm's asset allocation decision, while the fund contribution decision is fixed by assumption. We repeat the above numerical exercises for both the cases of with and without the PBGC. The results are reported in Figure 6. It shows that all the results obtained earlier in this section remain qualitatively robust to this variation. It is noticeable that even witout PBGC insurance, firms with overly high bankruptcy probability when required to fully fund the pension plan will have a very aggressive investment policy. This is because for firms with high bankruptcy probability, it is optimal to underfund the pension plan, which, however, is not allowed in this setting. Thus firms lower the promised retirement payment and compensate employees with higher risky allocation.

IV.6 The Case with Reversion Tax

As discussed in section I and II, to prevent firms clawing back all the pension surplus, U.S. government put a punitive excise tax on firms' reversion of pension assets starting from 1990. We add the reversion tax feature to our model in this section.

According to the Omnibus Budget Reconciliation Act of 1990, the excise tax rate on reverting the pension surplus is 50% or 20% if the plan sponsor is in Chapter 7 bankruptcy liquidation or at least 20% of excess assets are distributed to plan participants' benefits or the firm transfers 25% or more of the excess assets to a qualified replacement plan.[15] In the case of no bankruptcy, the firm is motivated to share 20% pension surplus with its employee

[15]In the static model, firms are not allowed to adjust the sponsored plan hence we do not consider the case of 25% reversion tax on plan replacement here.

and pay 20% as reversion tax instead of suffering the 50% as deadweight loss. In the case of bankruptcy, the firm is no longer motivated to share pension surplus because it has to pay 20% reversion tax regardless of sharing or not. Therefore, the employee's pension payment becomes:

$$S = F(1-D) + \min(W_T, F)D + (1-\alpha) \times \max(W_T - F, 0)(1-D). \tag{11}$$

Parameterizing the reversion tax rate as τ which equals 20% in current regulation and is subject to change in later analysis, the firm's cash flow at time T turns to

$$CF_T = W_T + h_T - S - \max(W_T - F, 0) \times \tau(D + \alpha(1-D)). \tag{12}$$

The numerical analysis focuses on how the existence of reversion tax affects the two risk sharing channels - bankruptcy and surplus sharing, and how the reversion tax changes would affect the firm's risky allocation in pension portfolio and funding decision. In general, as shown in Figure 7, taxing the reversion of pension surplus reduces the risky allocation of the pension portfolio. It is because the reversion tax lowers the firm's cash flow at the plan termination date thus makes it less worthy for the firm at the beginning to plan on bearing the future financing cost due to the risky investment. Figure 7 also shows that the firm is more likely to underfund the pension plan with higher reversion tax. The firm's lower funding ratio is driven by the employee's demand of more promised retirement payment which stems from the reduced risk exposure in the pension portfolio. Besides the aforementioned endogenous reduction of risk exposure, the employee also experiences the loss of upside risk sharing in case of bankruptcy because the firm is not willing to share the surplus if it has to pay the reversion tax either way.

It is worthy to mention that, as illustrated in A.1 of Figure 7, with reversion tax, firms with very high bankruptcy risk will choose to implement a more aggressive investment policy than firms with moderate bankruptcy risk, which leads to a U-shape relationship of the bankruptcy risk and the risky allocation. This is because the employee loses too much upside risk exposure from the pension portfolio in case of bankruptcy, which in turn drives up its demand of risky allocation of the portfolio in general. This model prediction is consistent with the empirical finding by An, Huang, and Zhang (2013) that for finanically distressed firms, the higher default risk the firm has the more investment risk the firm will take.

V. Dynamic Model

While our one-period model delivers a rich set of results, not allowing firms to adjust pension investment allocations and to make fund contribution in time is unrealistic. In this section, we consider a dynamic model setup within which the firm can adjust the pension asset allocations and make fund contribution at interim stages.

V.1 Model Setup

The entire time span is from 0 to T. At time 0, the firm hires the employee. At time T, the employee retires and the pension fund is liquidated. The investment opportunity again consists of two assets, the risk free asset with the instantaneous rate r_f, and the risky asset its value process following a geometrical Brownian motion:

$$\frac{dP_m}{P_m} = (\mu + r_f)dt + \sigma dB \tag{13}$$

where B is a standard Brownian motion, μ the risk premium, and σ the asset volatility. We also assume that the market is dynamically complete, and there is the unique the risk neutral probability Q.

$$\frac{dP_m}{P_m} = r_f dt + \sigma dB^Q, \tag{14}$$

where B^Q is a standard Brownian motion under the risk neutral probability Q.

Let w_t be the portfolio weight on the market portfolio. We denote the cumulative pension contribution by the firm up to time t by H_t, which is a non-decreasing stochastic process.

Together, the pension value process is governed by

$$dW_t = W_t[(w_t\mu + r_f)dt + w_t\sigma dB] + dH_t. \tag{15}$$

To facilitate numerical solutions, we assume that the firm only assesses the situation and accordingly makes contribution to the fund or makes adjustments on its decision of portfolio weights in discrete time: at times $n = 0, 1, \ldots, T$. Specifically, the portfolio weight is kept constant between the discrete decision times, i.e., $w_t = w_n$ if $t \in [n, n+1)$ for integer n. Furthermore, the cumulative contribution process H is a stochastic process that increases only by steps at the discrete time points. That is, $H_t = H_n$ if $t \in [n, n+1)$, and $dH_n = h_n$ for some discrete process h_n. At the time when $dH_n > 0$, a financial cost is incurred. We denote the cumulative cost process by C_t. With an abuse of notation, we use $C(\cdot)$ to indicate the cost function. Formally, we can write $dC_t = C(dH_t)(dH_t > 0)$ for $t \in [0, T]$. Of course, such defined dC_t can be non-zero only at the discrete times n and only when there is a positive contribution. Specifically, $dC_n = C(h_n)(h_n > 0)$.

The firm's default indicator, D_t, is a dynamic process governed by the first arrival of a Poisson process with constant intensity δ. We denote the firm default time by τ, a stopping time defined by $\tau = \min(t|D_t = 1)$. The pension will be liquidated and payment is made to the employee either when the firm defaults at time τ ($\tau \leq T$) or at terminal time T. The payment schedule is in principle the same as in Equation (7) with some notation adjustment to accommodate the dynamic model structure. Specifically,

$$S_\tau = F(1 - D_\tau) + \min(W_{\tau-}, F)D_\tau + (1 - \alpha) \times \max(W_{\tau-} - F, 0) \tag{16}$$

The firm gets the difference of the pension payment and the pension asset. That is, $CF_\tau = W_\tau - S_\tau$. At the payment time, we further adopt the convention that $W_\tau \geq S_\tau$. That is, if the pension's asset falls short of the payment S_τ (i.e., $W_{\tau-} < S_\tau$), the firm's time τ contribution has to make up the shortfall. According to this convention, the CF_τ is non-negative.

The employee have a CRRA utility function on time-T wealth, $U(S) = \frac{S^{1-\gamma}}{1-\gamma}$. If at time

τ, the employee receives the pension payment as the consequence of firm bankruptcy, she will invest optimally with the weight on the market portfolio $w = \frac{\mu}{\gamma\sigma^2}$ for the remainder time interval $[\tau, T]$. Thus,

$$S_T = S_\tau e^{(r_f + w\mu - \frac{1}{2}w^2\sigma^2)(T-\tau) + w\sigma(B_T - B_\tau)} \tag{17}$$

The firm values future cash flows (including the non-negative cash flows, the contributions, and the financing cost) using the risk neutral probability and the risk free discount rate. The firm's optimization problem is:

$$\max E^Q \left[e^{-r_f\tau} CF_\tau - \int_0^\tau e^{-r_f t}(dH_t + dC_t) \right]. \tag{18}$$

$$\text{subject to:} \quad E(U(S_T)) = \mathbb{U}. \tag{19}$$

There are additional constraints to the problem, including CF_τ being non-negative per our convention, the investment value process governed by Equation (15), and the final pension payout to the employee by Equations (16) and (17).

We numerically solve the dynamic model. Most of the economic parameters, including the stock market information, and parameters in the financing cost function, the employee's risk aversion, the number of years till retirement, and the annual default rate are the same as in the one-period model. The only difference is that we assume the fixed cost of external financing as zero in the dynamic model to smooth the optimization solutions. Our one-period model results do not change under the zero fixed cost assumption.

V.2 Cross-section of Initial Pension Decisions

Within the dynamic model setup, we revisit some of the main cross section relations that we have examined in the one-period model setup. In particular, we examine how the firm's decision on the initial fund contribution and on the asset allocation varies when we vary the model's parameters. The results are reported in Figure 8.

Three sets of results regarding the cross-section of the firm's initial investment allocation

(w_0), cash contribution (h_0) and PBO (F), when we vary the probability of bankruptcy (δ), the firm's initial free cash (H_0), and the percentage of the pension surplus the firm keeps (α). The cross-sectional patterns observed in Figure 8 is qualitatively similar to those seen in Figures 1 and 2.

In plot A.1 of the figure, similar to what we find in Figure 1 for the one-period model, we see that a firm with high default probability tends to choose a relatively lower risk exposure portfolio, everything else the same. For a firm with no default concern, the risky asset weight of the portfolio is above 80%. In comparison, for a triple B rated firm, the weight drops to 62%. When comparing the results here with the results in Figure 1, the the general risk exposure is higher for the dynamic model. This is to be expected. As we explained in Section IV.3, in the dynamic model, the firm would optimally choose to invest more aggressively early on and gradually become more conservative as the time passes. In contrast, in the 2-period model, the firm chooses a fixed risk exposure for the whole period. So, naturally, the dynamic portfolio weight tends to be higher than the fixed rate early on and lower than it at the later stage. Furthermore, in the dynamic model, the firm can spread the fund contribution into multiple years when facing fund shortfall. Doing so reduces the effect of the convexity of the financing cost, which is the key counterweight that holds back the firm's risk exposure.

In Panels B and C, we vary the initial free cash (H_0) and the parameter of the surplus sharing (α), respectively. The results are qualitatively similar as those in Figure 2, Panels A and B. Specifically, when the firm has more free cash at time 0, it contributes more to the pension fund at the time. This further leads to more aggressive asset allocation because more initial contribution mitigates the concern of convex financing cost due to future fund shortfall. The relation between the investment strategy and the ratio of surplus that the firm keeps is non-monotonic, with the investment strategy being the most aggressive when the firm's surplus share is moderate. The investment strategy is less aggressive when the firm's surplus share is either very high or very low. The intuition is as discussed in Section III.2. It is worth noting that the portfolio weight on the risky asset stays above 55% through the range of the surplus share parameter.

Overall, we find that in the dynamic model, corporate pension continues to take substantial investment risk, and the relation of pension risk taking with some of the key parameters share the similar patterns with what we find in the one-period model. Thus, the ability of the firm to dynamically adjust pension contribution over time does not significantly remove the risk-sharing between shareholders and employees, nor does it significantly reduces optimal pension risk taking.

V.3 The Dynamics of Allocation and Contribution

The dynamic model further allows us to investigate how the pension fund's asset allocation and the firm's fund contribution vary in time. Figure 9 displays the pension fund's asset allocation in time for different sets of parameters. The default probability is set constant at 0.05%. In the two charts of Panel A, we start with the benchmark case with the base-line parameters and show the solution for the portfolio allocation on the left and the pension contribution on the right. We then change the default probability to a lower value of 0.1% and show the solution in the two charts in Panel B. In Panel C, we vary the surplus sharing parameter. We let the firm keep 60% of the surplus in both the default and non-default scenarios.

To varying degrees, all the charts share some common features. In the early years, that is, when the retirement time is far in the future, the stock allocation is relatively high and close to the employee's own optimal level. And as the retirement time draws closer, for most range of the funding ratio, the allocation level drops down. The intuition in Proposition 1 can help us understand this pattern, but there is also important differences due to the added richness of the dynamic model, as we explain below.

Unlike the one-period model where the asset allocation decision is made only once, in the dynamic model, asset allocation varies over time in response to funding level. The two main counterbalancing factors identified in Proposition 1 for optimal asset allocation are: (1) the employee's desire to have investment risk exposure; and (2) the firm's convex financing cost.

In the dynamic model, the firm chooses to have high risk exposure at times and states that such investment risk exposure can be effectively passed on to the employee and that the cost of such risk exposure due to the convex financing cost is relatively low.

Along the time dimension, the firm would choose to have high risk exposure when the retirement time is far away. A long investment horizon means that the optimal risk exposure has a bigger impact on the employee's utility, ceteris paribus. Furthermore, when the time horizon is long, the firm has the time to make the periodic contributions, and this would substantially reduces the impact of the convexity in the financing cost. Consider this simple thought experiment: there is a shortfall of \$10. If the firm has to make a one time contribution, the convex component of the financing cost is proportion to $10^2 = 100$. In contrast, if the firm breaks down the contribution into 10 equal pieces. Then the convex component of the financing cost adds up to an amount proportion to only $10 \times 1^2 = 10$, which is only one tenth of the one time contribution cost. In addition, the firm has the option to choose whether and when to make the contributions, as well as to adjustment portfolio allocations when needed. Such option has a higher value when the uncertainty is higher, that is, when the pension's risk allocation is high. All this point to the direction that the pension plan should have a high risk allocation early on, ceteris paribus.

In addition to the time dimension, when consider the state dimension of the time-state space, the firm should also allocation the pension fund's risk exposure across the state in an optimal fashion. There are a number of factors influence the decision. One can enhance the overall risk exposure, while keeping the average risk exposure across the state space fixed, by take up more risky positions in the up and down sides of the funding status. This idea is simple, as we make both ends of the state-space more volatile, the total volatility increases. Therefore, high risk allocations in the both the highly over-funding and the highly under-funding states, which make the pension asset more risky, help to expose the employee to the stock market risk. This consideration helps explain why the average risk allocation is lower in the dynamic model than in the one-period model.

Second, from a cost-effective stand point, when the under-funding is severe, the ex ante average delta of the pension payment to the pension asset approaches 1, the upper bound of the delta in the model. Therefore, it is the most effective in this range to expose the employee to the investment risk. In the middle range, when the pension plan is close to the fully funded or slightly over-funded, given the surplus sharing, the investment risk can still be passed to the employee, though at a lower delta. Yet, the convex cost is still a concern. The combination result is that the pension fund's risk allocation is relatively low in this range. Yet on the other end, when the pension plan is substantially over-funded, the decrease in the average delta reaches a plateau, and yet the financing cost become a lesser concern because the likelihood of future fund shortfall is low. The joint effect is that the risk allocation is high.

The patterns we observe from asset allocations across firms with different default probabilities help us gain further intuition on the risk allocation problem in the dynamic setting. When we reduce the default probability from that of a BBB firm in our benchmark case in Panel A to a AAA firm in Panel B, we see that overall the risk allocation is higher. This is consistent with what we find in Panel A of Figure 1. The pension payment's delta to the pension investment asset is higher in the default case than in the non-default case, because the employee is exposed to the downside risk in the default case and is protected by the promised obligation in the non-default case. Therefore, it takes a higher risk allocation in the pension investment to achieve the same level of risk exposure in the pension payment. That is, an AAA firm has a low pension payment to pension asset delta than a BBB firm, and therefore, the AAA firm should choose a higher risk allocation for the pension investment to make up the reduced delta. On surface, this statement seems to contradict what we stated above, that it is more effective for the firm to choose high investment risk allocation in the states where the payment delta is relative high, ceteris paribus. The key difference is that, when we are considering the dynamic problem for a single firm, there is a internal balancing across states and times. The firm decides to allocate more to the risky asset in some states and times to make up the low allocation in some other states and times. In contrast, there is no such balancing effect across firms, that is, the two firms are not dealing with each other to reach a mutually beneficial agreement to jointly solve the the pension optimization problem. Furthermore, we focus only on the comparison of the average allocation across firms.

As we reduce the firm's share of the surplus, we see the allocation varies from the charts of Panel A to the charts in C. The main result is that the allocation to the risky asset is reduced as the firm's share of the surplus reduces. This pattern is consistent with what we observe in Panel B of Figure 2 for the one-period model. As the firm's share of the surplus increases, the employee's share naturally decreases. Consequently, the pension payment's delta to the pension's investment value is lowered. To provide the desirable level of the risk-return trade-off, in other words, to provide the desirable exposure to the investment risk for the employee, the asset allocation has to increase according to compensate for the decline in delta.

VI. Conclusions

This paper provides a new perspective to understand the risky asset allocation policies pursued by corporate pensions. In our model, pension risk taking is driven by employees' preference for systematic risk exposure, while the firm balances employees' preference with its concern for reducing external financing cost. The pension investment risk is shared between shareholders and employees. The firm's decisions on pension benefits and pension funding are endogenous to such risk sharing. For a reasonable set of parameter values, the optimal pension investment risk and its relations with a firms' bankruptcy probability and pension funding ratio predicted by the model are consistent with empirical observations.

The stakeholder approach we take combines two polarized views on the objective function of pension decisions – the shareholder value maximization view and the beneficiary utility maximization view. The former has been a prevalent approach in recent academic studies to evaluate corporate pension decisions. The challenge for this view is the extreme implication that pension investment should be riskfree, which contrasts dramatically with the observed pension investment behavior. The beneficiary utility maximization view is recognized by Bodie (1990) as an alternative interpretation of the risky pension investment behavior. Perhaps implicitly, under this view a DB plan should invest in a way similar to what employees were to manage their portfolios outside a DB plan – a mean-variance efficient portfolio, for example. However, this does not take into account the convoluted cash flow rights (in particular, the benefit guarantee by firms) a DB plan offers to employees, which may substantially alter the investment decisions. The stakeholder approach appears to be successful in meshing the two views to deliver a reasonable interpretation of the observed investment policies by corporate pensions.

We conclude with a remark on the lack of efficiency of risk sharing in the DB plans. The defined benefit plans, perhaps to protect employees from firm-specific risk, make employees' pension payoffs relatively insensitive to systematic risk. In other words, from a systematic risk sharing perspective, defined benefit pension contracts are suboptimal. If pension beneficiaries are indeed severely wealth constrained, their pension cash flows should optimally be exposed to systematic risk. A more efficient contract would let employees to shoulder all the pension investment risk while keeping them off firm-specific risks. Interestingly, this arrangement resembles what a defined contribution plan offers (although DC plans lack the longevity risk sharing feature of DB plans). Our analysis shows that such an arrangement may substantially reduce firms' pension funding costs. Perhaps this is one of the reasons that many firms are shifting from DB plans to DC plans (Munnell and Soto 2008; Rauh and Stefanescu 2009).

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Appendix

Proof of Proposition 1

Proof. First we define pension plan funding ratio δ as the ratio of initial pension asset W_0 to the present value of the firm's pension liability F with discount rate R_f, thus we have $W_0 = FR_f^{-1}\delta$.

In the first case with no bankruptcy risk, the pension payment is $S = F$. Define function $\phi(x)$ as $\phi(x) = C_1(x)$ if $x > 0$, and $\phi(x) = 0$ otherwise. The firm's objective function is specified as

$$G(w,\delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}E^Q\left(\phi(z)\right)$$

where $z = F - W_T$.

For the case of $\delta \geq 1$, we have $G(0,\delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1}$. Since $\phi(z) \geq 0$, we have $G(w,\delta) \leq -C_0((W_0 - H_0)^+) - FR_f^{-1}$. Thus $(0,\delta) = \operatorname{argmax} G(w,\delta)$ for $\delta \geq 1$.

For the case of $\delta < 1$, $G(0,\delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}\phi(z_0)$ where $z_0 = F(1-\delta)$. Under the assumption that $\phi(z)$ is convex, we have $\phi(z) \geq \phi(z_0) + c(z - z_0)$ for some constant c.[16] Therefore,

$$\begin{aligned}
G(w,\delta) \leq& -C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}E^Q\left[(\phi(z_0) + c(z - z_0))\right] \\
=& G(0,\delta) - cR_f^{-1}E^Q[(z - z_0)] = G(0,\delta) - cR_f^{-1}E^Q\left[(F - W_T - F + F\delta)\right] \\
=& G(0,\delta) - cW_0R_f^{-1}E^Q\left[(R_f - R(w))\right] = G(0,\delta)
\end{aligned}$$

Given that $\phi(z)$ is strictly convex at least at one point (otherwise it would be constant 0), the above inequalities will be strict when $w \neq 0$. Hence $G(0,\delta)$ is the unique maximum for any δ.

Now turn to the case with bankruptcy risk. We need the following two lemmas for the proof.

Lemma 1. *Given any promised retirement benefit fixed, more risky asset investment will raise the present value of external financing cost regardless of the initial funding status.*

Need to show that

$$\frac{\partial E^Q\left[C_T\left((F - W_T)^+\right)\right]}{\partial w} > 0, \text{ for any given } F$$

Taking the derivative with respect to w, we have

$$\frac{\partial E^Q\left[C_T\left((F - W_T)^+\right)\right]}{\partial w} = -\int_{-\infty}^{R_m^c} C_T'(F - W_T)W_0R_ef^Q(R_m)dR_m$$

where R_m^c is the realized risky return such that $F = W_T$.

If the pension plan is initially overfunded, $\delta > 1$, $R_m^c < R_f$, the risky asset's excess return R_e is always negative in the integration. Thus the derivative is positive.

If the pension plan is initially underfunded, $\delta > 1$, $R_m^c > R_f$, the derivative can be written as

[16] Where c is the slope of a support of the convex function. If $\phi(x)$ is differentiable at z_0, then the support is unique and $c = \phi'(z_0)$.

$$\begin{aligned}\frac{\partial E^Q\left[C_T\left((F-W_T)^+\right)\right]}{\partial w} &= -\int_{-\infty}^{R_f} C_T'(F-W_T)W_0R_ef^Q(R_m)dR_m - \int_{R_f}^{R_m^c} C_T'(F-W_T)W_0R_ef^Q(R_m)dR_m \\ &> -\int_{-\infty}^{R_f} C_T'(F-F\delta)W_0R_ef^Q(R_m)dR_m - \int_{R_f}^{R_m^c} C_T'(F-F\delta)W_0R_ef^Q(R_m)dR_m \\ &= -\int_{-\infty}^{R_m^c} C_T'(F-F\delta)W_0R_ef^Q(R_m)dR_m > -C_T'(F-F\delta)W_0\int_{-\infty}^{+\infty} R_ef^Q(R_m)dR_m = 0\end{aligned}$$

The second inequality holds because $\frac{\partial C_T'(F-W_T)}{\partial R_m} = C_T''(F-W_T)(-W_0w) < 0$ as long as $w > 0$. Therefore, $C_T'(F-W_T) > C_T'(F-F\delta)$ for $R_m < R_f$, and $C_T'(F-W_T) < C_T'(F-F\delta)$ for $R_m > R_f$.

Next, consider the employees' participation constraint

$$(1-p)U(F) + pEU(W_T(w,\delta)) = \mathbb{U}$$

which implies $F = F(\mathbb{U}, w, \delta)$. We can prove the following lemma.

Lemma 2. *$\frac{\partial F(\mathbb{U},0,\delta)}{\partial w} < 0$, $\frac{\partial F(\mathbb{U},w_e,\delta)}{\partial w} = 0$, and $\frac{\partial F(\mathbb{U},\hat{w}_e,\delta)}{\partial w} > 0$, for all $\mathbb{U} > 0$ and $\delta > 0$. w_e is employee's optimal investment allocation of self-management portfolio, and $\hat{w}_e > w_e$.*

With the other parameters exogenously given, we can obtain the general form of $\frac{dF}{dw}$ using implicit function theorem.

$$\frac{dF}{dw} = \frac{\partial F}{\partial w} = -\frac{pE[U'(W_T)W_0R_e]}{(1-p)U'(F) + pE[U'(W_T)R_f^{-1}\delta R(w)]}$$

Now let's figure out the sign of $\frac{dF}{dw}$ evaluated at three critical w's: 0, w_e, and $\hat{w}_e$.

$$\left.\frac{dF}{dw}\right|_{w=0} = -\frac{pU'(F\delta)W_0E(R_e)}{(1-p)U'(F) + pU'(F\delta)\delta} < 0$$

The CRRA employee chooses w_e such that $\frac{dEU(W_T)}{dw} = E[U'(F,w_e)W_0R_e] = 0$ for any F. Therefore,

$$\left.\frac{dF}{dw}\right|_{w=w_e} = -\frac{pE[U'(F,w_e)W_0R_e]}{(1-p)U'(F) + pE[U'(F,w_e)R_f^{-1}\delta R(w_e)]} = 0$$

For any given F,

$$\frac{dE[U'(F,w)R_e]}{dw} = E[U''(F,w)R_e^2]W_0 < 0$$

which implies $E[U'(F,\hat{w}_e)R_e] < E[U'(F,w_e)R_e] = 0$. Further we are able to show

$$\begin{aligned}\frac{dE[U'(F,w)R(w)]}{dw} =& E[U''(F,w)W_0R_eR(w) + U'(F,w)R_e] \\ =& E[U''(W_T)W_TR_e + U'(W_T)R_e] = E[(1-\gamma)U'(F,w)R_e] \geq 0, \forall w \geq w_e\end{aligned}$$

Hence $E[U'(F,\hat{w}_e)R(\hat{w}_e)] > E[U'(F,w_e)R(w_e)] = EU'(F,w_e)R_f > 0$. So we have

$$\left.\frac{dF}{dw}\right|_{w=\hat{w}_e} = -\frac{pE[U'(F,\hat{w}_e)W_0R_e]}{(1-p)U'(F) + pE[U'(F,\hat{w}_e)R_f^{-1}\delta R(\hat{w}_e)]} > 0$$

Now, to finish the proof of Proposition 1, we need to sign $G'(w)$ for $w = 0$, w_e, and $\hat{w}_e$. Let's start from $w = w_e$ and $w = \hat{w}_e$.

$$
\begin{aligned}
G'(w_e) = &- \left.\frac{dF}{dw}\right|_{w=w_e} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=w_e} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1} \int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))(1 - R_f^{-1}\delta R(w_e)) \left.\frac{dF}{dw}\right|_{w=w_e} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1} \int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))W_0 R_e f^Q(R_m)dR_m \\
=&(1-p)R_f^{-1} \int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))W_0 R_e f^Q(R_m)dR_m < 0
\end{aligned}
$$

The last inequality comes from Lemma 1.

$$
\begin{aligned}
G'(\hat{w}_e) = &- \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1} \int_{-\infty}^{R_m^c(\hat{w}_e)} C_T'(F - W_T(\hat{w}_e))(1 - R_f^{-1}\delta R(\hat{w}_e)) \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1} \int_{-\infty}^{R_m^c(\hat{w}_e)} C_T'(F - W_T(\hat{w}_e))F R_f^{-1}\delta R_e f^Q(R_m)dR_m < 0
\end{aligned}
$$

For $w = 0$, we discuss the results separately with different δ:

1. If the pension plan is initially underfunded, $\delta < 1$, then $R_m^c|_{w=0} = +\infty$,

$$
\begin{aligned}
G'(0) = &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1} \int_{-\infty}^{+\infty} C_T'(F - F\delta)(1-\delta) \left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1} \int_{-\infty}^{+\infty} C_T'(F - F\delta)W_0 R_e f^Q(R_m)dR_m \\
= &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}\left[p\delta + C_0'(W_0 - H_0)\delta + (1-p)(1 + C_T'(F - F\delta)(1-\delta))\right] > 0
\end{aligned}
$$

2. If the pension plan is initially overfunded, $\delta > 1$, then $R_m^c|_{w=0} = -\infty$,

$$
\begin{aligned}
G'(0) = &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1} \int_{-\infty}^{-\infty} C_T'(F - F\delta)(1-\delta) \left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1} \int_{-\infty}^{-\infty} C_T'(F - F\delta)V_0 R_e f^Q(R_m)dR_m \\
= &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}\left[(p\delta + (1-p)) + C_0'(W_0 - H_0)\delta\right] > 0
\end{aligned}
$$

3. If the pension plan is initially fully funded, $\delta = 1$, then $R^c_m|_{w=0} = R_f$, $\frac{dF}{dw}\big|_{w=0} = -pFR_f^{-1}E[R_e]$,

$$\begin{aligned}
G'(0) = & -\left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1} \\
& - (1-p)R_f^{-1}\int_{-\infty}^{R_f} C_T'(F-F)(1-1)\left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\
& + (1-p)R_f^{-1}\int_{-\infty}^{R_f} C_T'(F-F)FR_f^{-1}R_e f^Q(R_m)dR_m \\
= & FR_f^{-1}\left\{pR_f^{-1}E[R_e]\left[1 + C_0'(FR_f^{-1} - H_0)\right] - (1-p)R_f^{-1}C_T'(0)E^Q[R_e^+]\right\}
\end{aligned}$$

If the cost function is in quadratic form, $C'_{0,T}(0) = 0$, and $G'(0) > 0$; otherwise $V'(0) > 0$ if and only if $C_T'(0) < \frac{pR_f^{-1}E[R_e]\left[1+C_0'(FR_f^{-1}-H_0)\right]}{(1-p)R_f^{-1}E^Q[R_e^+]} \equiv C$.

Given the sign of $G'(0)$, $G'(w_e)$, and $G'(\hat{w}_e)$, we can conclude that there exists global maximization solution $w^* \in (0, w_e)$ such that $G'(w^*) = 0$. □

Proof of Corollary I

Proof. The first pension payment schedule implies a positive bankruptcy risk and that the firm takes all the surplus if any.

The firm's objective function $G(w, \delta)$ becomes,

$$\begin{aligned}
G(w,\delta) = & -W_0(\delta) - C_0\left((W_0(\delta) - H_0)^+\right) + pR_f^{-1}E^Q\left[(W_T - F)^+\right] \\
& + (1-p)R_f^{-1}E^Q\left[W_T(w,\delta) - F - C_T\left((F - W_T(w,\delta))^+\right)\right]
\end{aligned}$$

Employee's participation constraint becomes,

$$\mathbb{U} = (1-p)U(F) + pEU(F + (W_T(w,\delta) - F)^-) \equiv EU(w,\delta)$$

The firm needs to maximize the following Lagrange function $\mathbb{L} = G(w,\delta) + \lambda(EU(w,\delta) - \mathbb{U})$. The first-order derivative w.r.t. w is

$$\frac{\partial \mathbb{L}}{\partial w} = \frac{\partial G(w,\delta)}{\partial w} + \lambda\frac{\partial EU(w,\delta)}{\partial w}$$

Look at the first term, the marginal utility of firm,

$$\frac{\partial G(w,\delta)}{\partial w} = R_f^{-1}\int_{-\infty}^{R^c_m} W_0R_e\left[C_T'(F - W_0R(w))(1-p) - p\right] f^Q(R_m)dR_m$$

where $R^c_m = \left(\frac{F}{W_0} - R_f\right)\frac{1}{w} + R_f$.

At $w = 0$, $R^c_m \to +\infty$ if underfunded; $R^c_m \to -\infty$ if overfunded, either way we have

$$\frac{\partial G(0,\delta)}{\partial w} = 0$$

Now look at the second term, the marginal utility of employee,

$$\frac{\partial EU}{\partial w} = p\int_{-\infty}^{R^c_m} U'(W_0R(w))W_0R_e f(R_m)dR_m$$

If pension is underfunded, $W_0R_f < F$, $R_m^c > R_f$,

$$\frac{\partial EU(0,\delta)}{\partial w} = p\int_{-\infty}^{\infty} U'(W_0R_f)W_0R_e f(R_m)dR_m = pU'(W_0R_f)E(R_m - R_f) > 0$$

Therefore, when pension is underfunded,

$$\left.\frac{\partial \mathbb{L}}{\partial w}\right|_{w=0} = \frac{\partial G(0,\delta)}{\partial w} + \lambda\frac{\partial EU(0,\delta)}{\partial w} > 0$$

The second pension payment schedule implies no bankruptcy risk and that the firm and the employee shares the surplus if any. The firm's objective function $G(w,\delta)$ becomes

$$G(w,\delta) = -W_0(\delta) - C_0\left((W_0(\delta) - H_0)^+\right) + R_f^{-1}E^Q\left[(W_T - F)^+\alpha + (W_T - F)^- - C_T((F - W_T)^+)\right]$$

Employee's participation constraint becomes

$$\mathbb{U} = EU\left(F + (W_T - F)^+(1-\alpha)\right)$$

Following the same procedure as above,

$$\frac{\partial G}{\partial w} = R_f^{-1}\int_{-\infty}^{R_m^c} W_0R_e\left[(1-\alpha) + C_T'(F - W_T)\right]f^Q(R_m)dR_m$$

where $R_m^c = \left(\frac{F}{W_0} - R_f\right)\frac{1}{w} + R_f$.

At $w = 0$, $R_m^c \to +\infty$ if underfunded; $R_m^c \to -\infty$ if overfunded, either way we have

$$\frac{\partial G(0,\delta)}{\partial w} = 0$$

Similarly,

$$\frac{\partial EU}{\partial w} = \int_{R_m^c}^{+\infty} U'(F\alpha + W_T(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m$$

If fully funded,$R_m^c = R_f$,

$$\frac{\partial EU(0,1)}{\partial w} = \int_{R_f}^{+\infty} U'(F\alpha + W_0R_f(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m > 0$$

If overfunded, $R_m^c \to -\infty$,

$$\frac{\partial EU(0,\delta)}{\partial w} = \int_{-\infty}^{+\infty} U'(F\alpha + W_0R_f(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m > 0$$

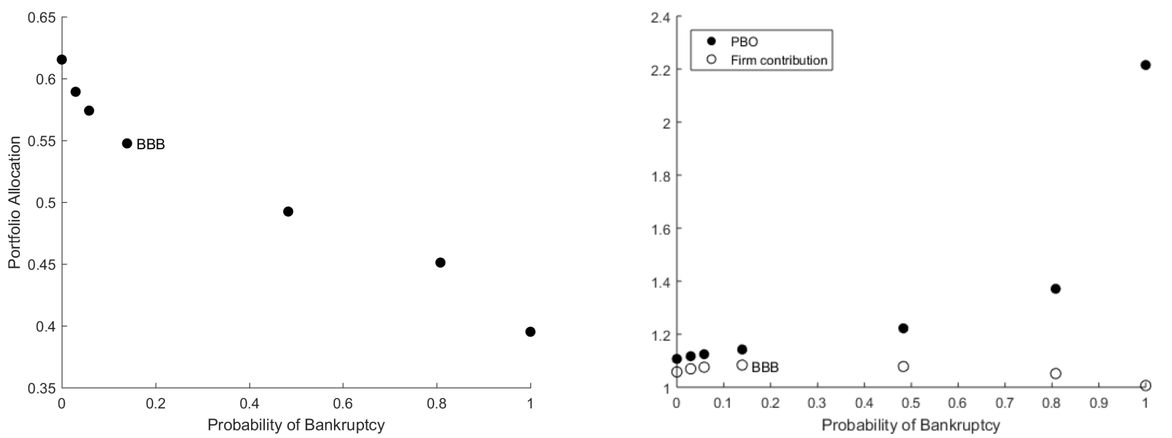
Therefore, when pension is not underfunded,

$$\left.\frac{\partial \mathbb{L}}{\partial w}\right|_{w=0} = \frac{\partial G(0,\delta)}{\partial w} + \lambda\frac{\partial EU(0,\delta)}{\partial w} > 0$$

□

Figure 1 Impact of Bankruptcy Risk on Pension Plan Investment and Funding

This figure illustrates the impact of bankruptcy risk on pension plan investment and funding decision under the surplus sharing model with 20% ($\alpha = 0.8$) pension surplus given to the employee. The value of the other parameters are listed in Table 1. Given different probabilities of bankruptcy, plot (1) shows the corporate pension fund's optimal asset allocation to risky assets and plot (2) shows the optimal pension PBO and firm's contribution. Plot (2) also shows the optimal PBO if employee is risk-neutral as the shareholder. From left to right, the probabilities of bankruptcy represent firms with credit rating AAA, AA, A, BBB, BB, B, respectively, and the last one represents firm in process of bankruptcy.

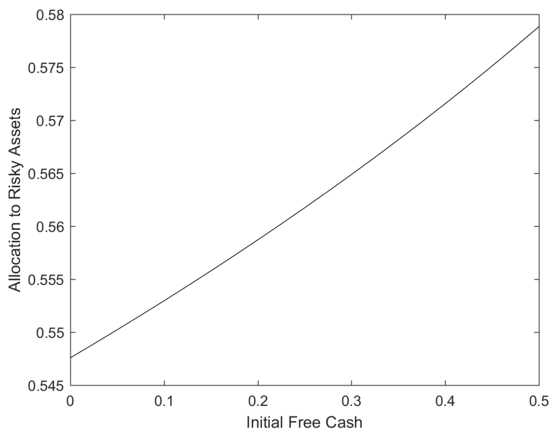


(1) (2)

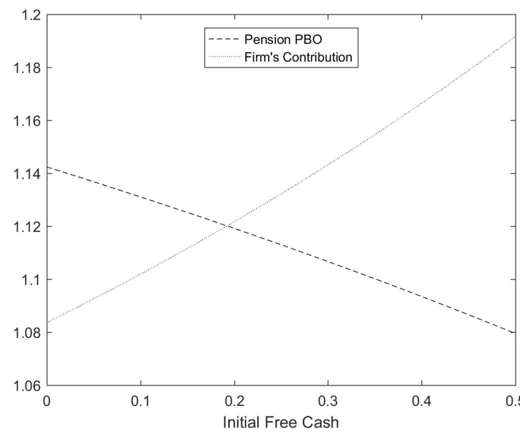
Figure 2 Impact of The Other Factors on Pension Plan Investment and Funding

This figure illustrates the impact of the other factors on pension plan investment and funding decision under the surplus sharing model. The benchmark value of the parameters are listed in Table 1. Panel A shows the impact of firm's initial free cash on allocation to risky assets (1) and optimal PBO and firm's contribution (2). Panel B shows the impact of pension surplus sharing between the firm and the employee. Panel C shows the impact of linear part of external financing cost. Panel D shows the impact of quadratic part of external financing cost. Panel E shows the impact of risk free rate. Panel F shows the impact of market risk premium. Panel G shows the impact of market volatility.

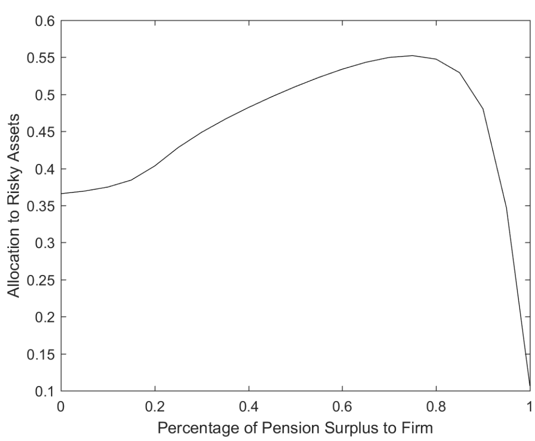
A.1 Impact of Initial Free Cash on Allocation



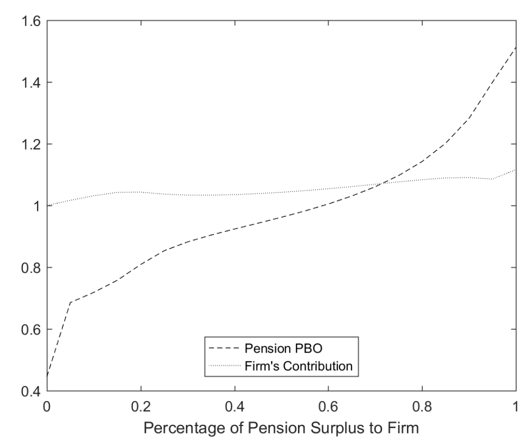
A.2 Impact of Initial Free Cash on PBO and Contribution



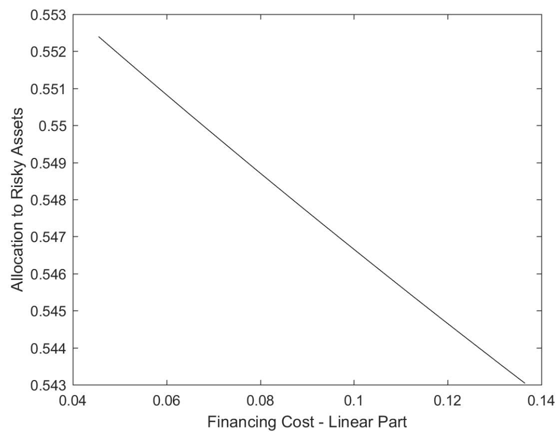
B.1 Impact of Surplus Sharing on Allocation



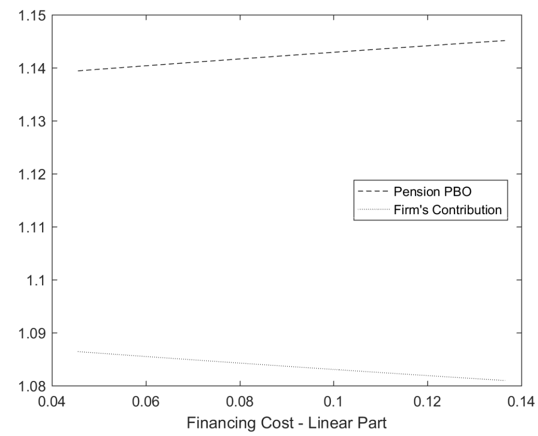
B.2 Impact of Surplus Sharing on PBO and Contribution



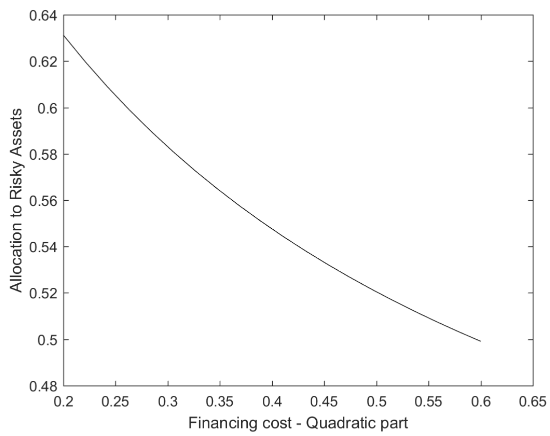
C.1 Impact of Linear Part of Financing Cost on Allocation



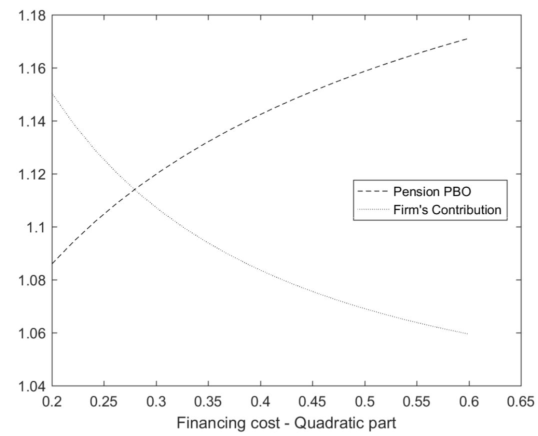
C.2 Impact of Linear Part of Financing Cost on PBO and Contribution



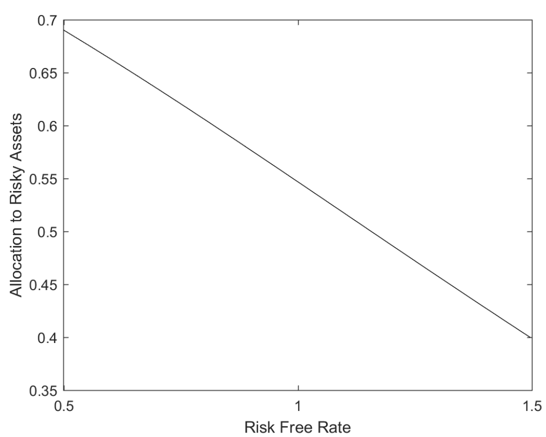
D.1 Impact of Quadratic Part of Financing Cost on Allocation



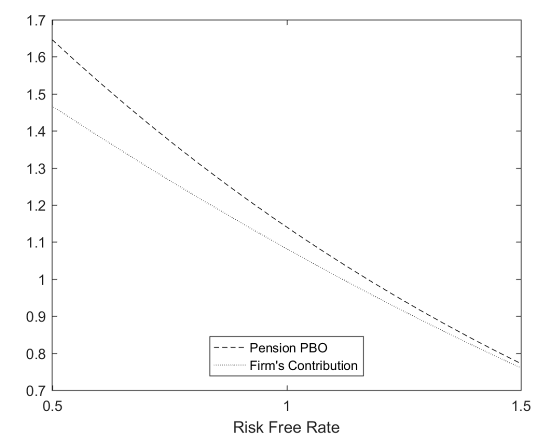
D.2 Impact of Quadratic Part of Financing Cost on PBO and Contribution



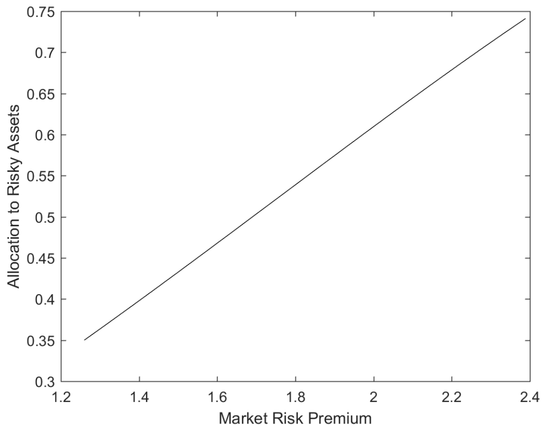
E.1 Impact of Riskfree Rate on Allocation



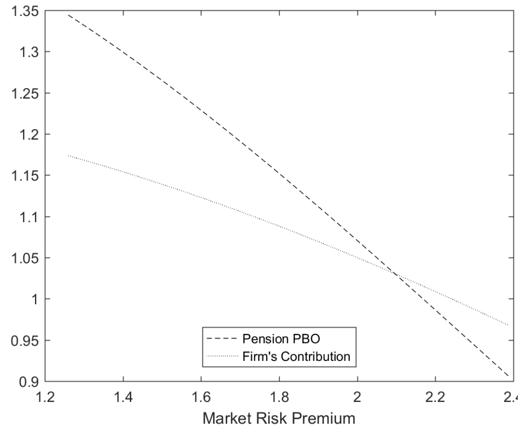
E.2 Impact of Riskfree Rate on PBO and Contribution



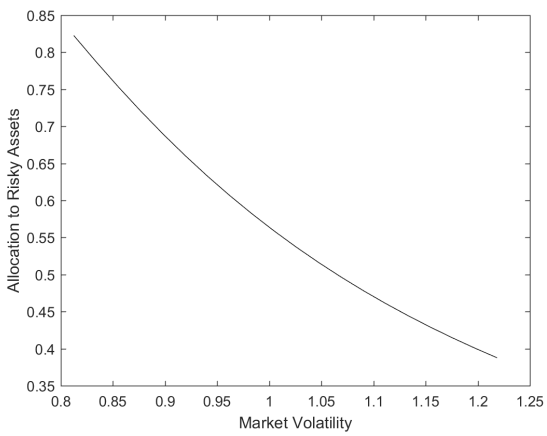
F.1 Impact of Market Risk Premium on Allocation



F.2 Impact of Market Risk Premium on PBO and Contribution



G.1 Impact of Market Volatility on Allocation



G.2 Impact of Market Volatility on PBO and Contribution

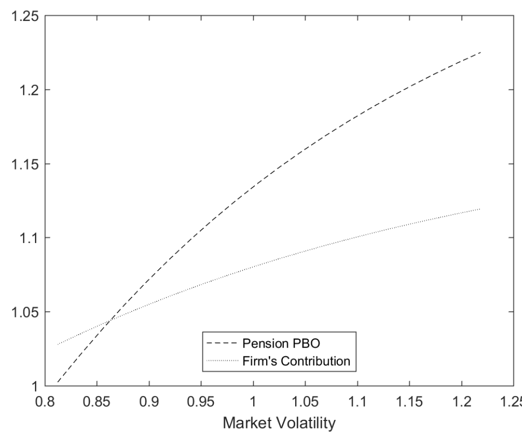
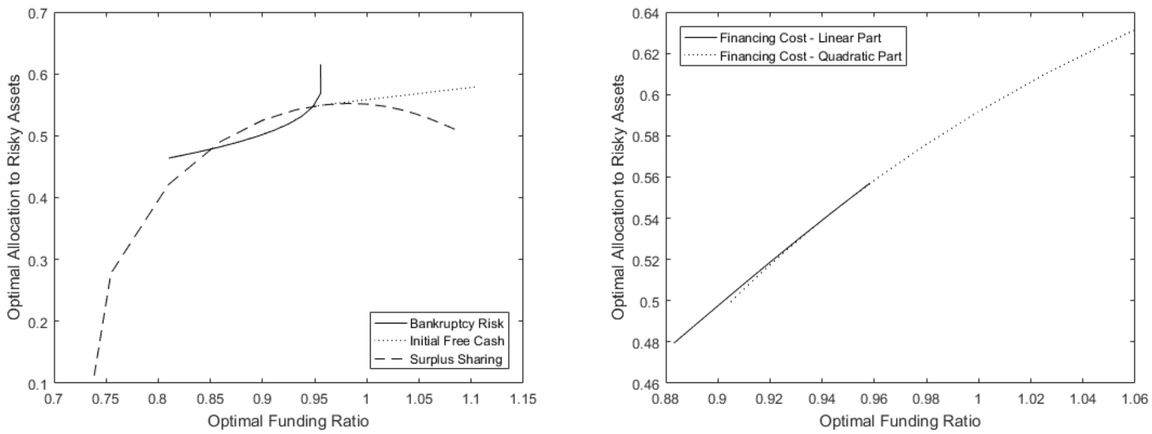


Figure 3 Relation of Optimal Allocation and Optimal Funding Ratio

This figure illustrates the relation of optimal pension fund allocation and optimal funding ratio with varying parameter values. The value of the other parameters are kept the same as listed in Table 1 when varying the specified parameter. Plot (1) shows this relation with varying values of bankruptcy risk, initial free cash, and surplus sharing. Plot (2) shows this relationship with varying values of time 0 linear cost and time T linear cost.

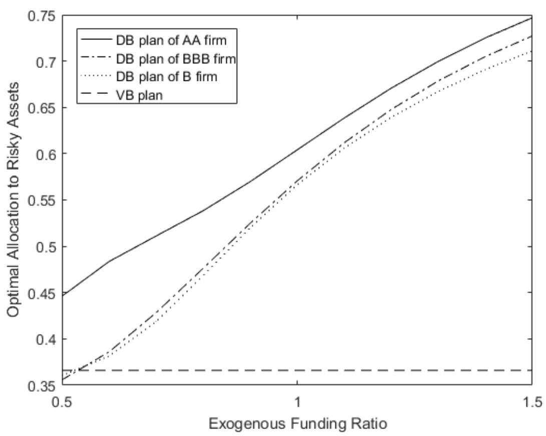


(1) (2)

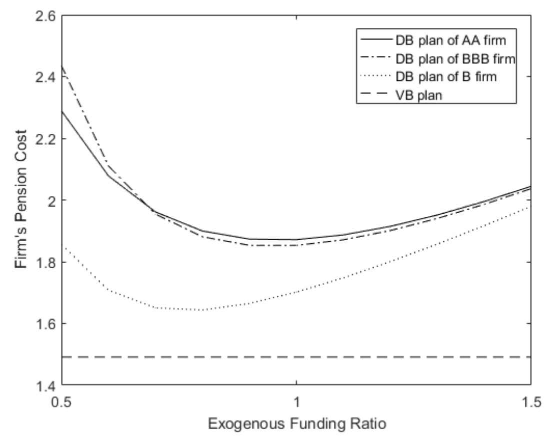
Figure 4 Firm's Pension Allocation and Cost with Exogenous Funding Ratio

This figure illustrates firm's pension fund allocation and cost with exogenous funding ratio. The firm's pension cost is the present value of cash outflow at time 0 plus expected cash outflow at time T, which is also the negative of firm's utility. The value of the other parameters are kept the same as listed in Table 1 when varying the specified parameter. Panel A shows firm's defined benefit plan asset allocation and pension cost with varying exogenous funding ratio under bankruptcy risks of AA firm, BBB firm, and B firm. Panel B shows firm's defined benefit plan asset allocation and pension cost with varying exogenous funding ratio under the schemes of 30% ($\alpha = 0.7$), 20%($\alpha = 0.8$), and 10%($\alpha = 0.9$) pension surplus given to employee. Both panels also include the asset allocation and cost of variable benefit plan. Variable benefit plan invests with asset allocation of employee's self-managed portfolio.

Panel A Bankruptcy Risk

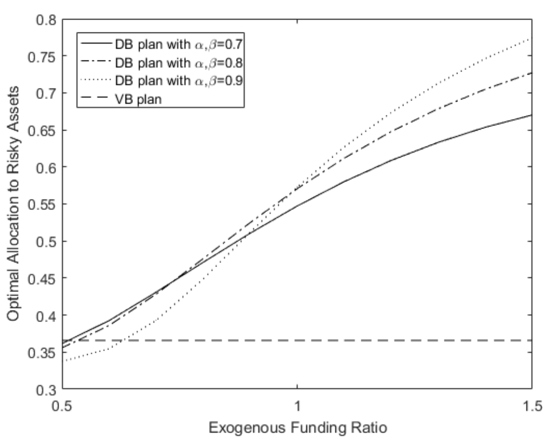


(1)

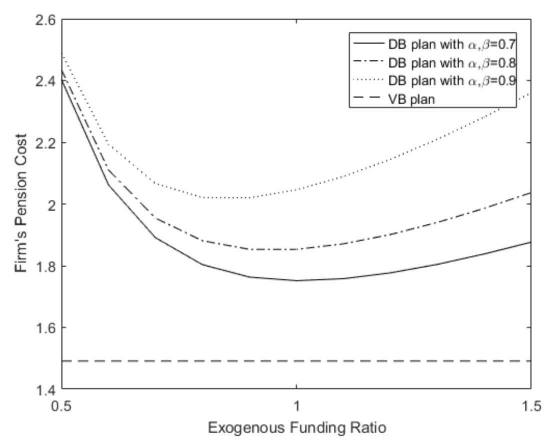


(2)

Panel B Surplus Sharing



(1)

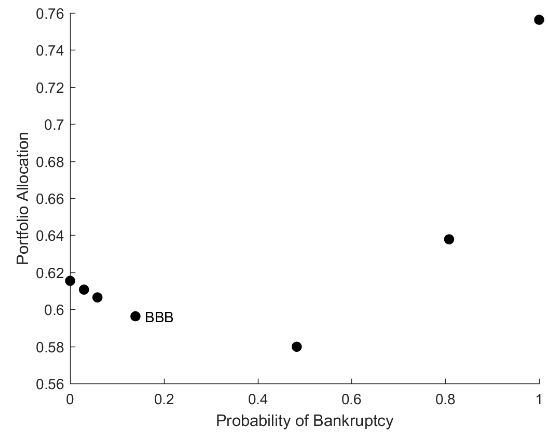


(2)

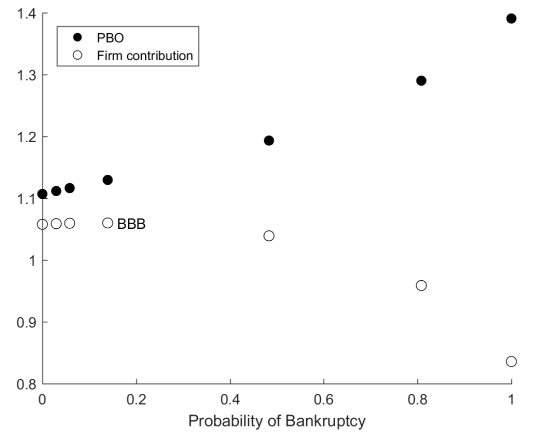
Figure 5 Pension Plan Investment and Funding with PBGC

This figure illustrates the impact of various factors on pension plan investment and funding decision under the PBGC model. In addition to the benchmark value of the parameters are listed in Table 1, we set insurance coverage ratio $\theta = 0.7$, and pricing factor $m = 1$. Panel A shows the impact of firm's bankruptcy risk on allocation to risky assets (1) and optimal PBO and firm's contribution (2). Panel B shows the impact of firm's initial free cash. Panel C shows the impact of pension surplus sharing between the firm and the employee. Panel D shows the impact of PBGC linear financing cost. Panel E shows the impact of PBGC insurance over-/under-pricing. Panel F shows the impact of PBGC insurance coverage.

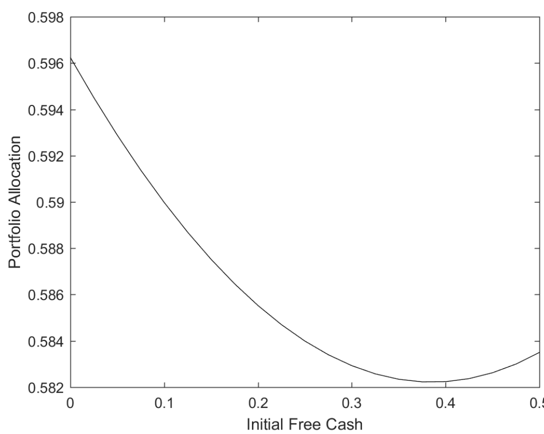
A.1 Impact of Bankruptcy Risk on Allocation



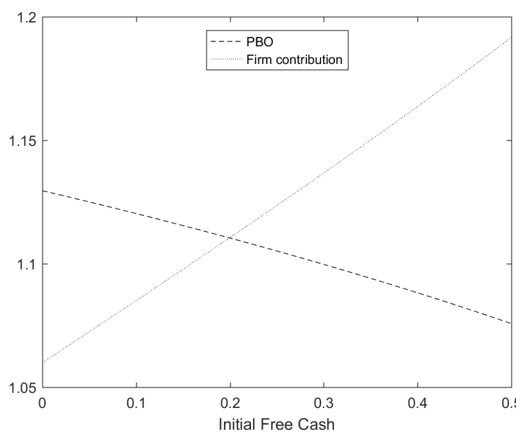
A.2 Impact of Bankruptcy Risk on PBO and Contribution



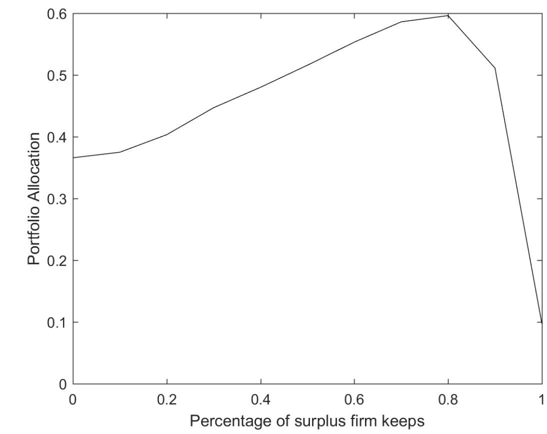
B.1 Impact of Initial Free Cash on Allocation



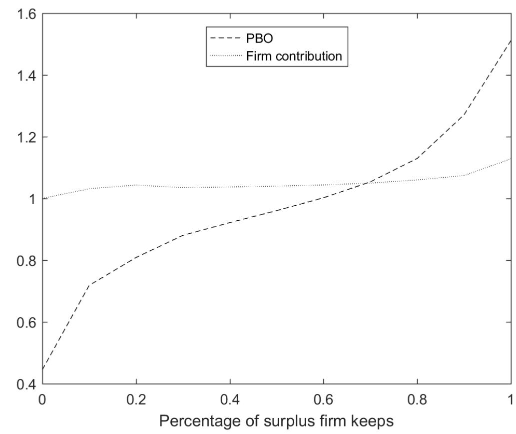
B.2 Impact of Initial Free Cash on PBO and Contribution



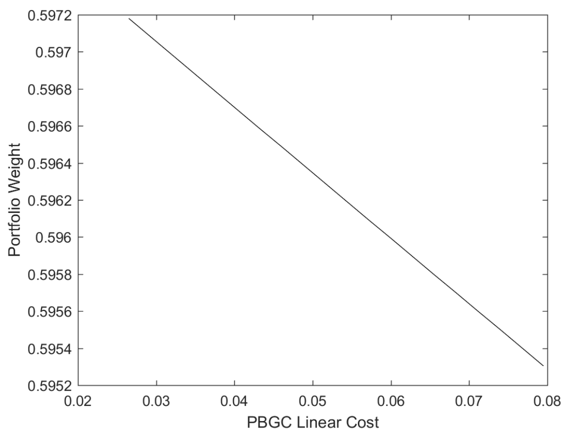
C.1 Impact of Surplus Sharing on Allocation



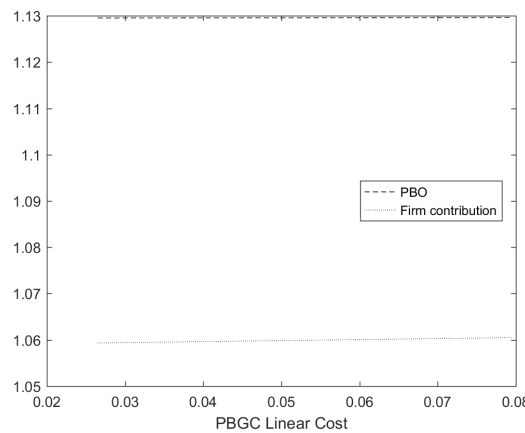
C.2 Impact of Surplus Sharing on PBO and Contribution



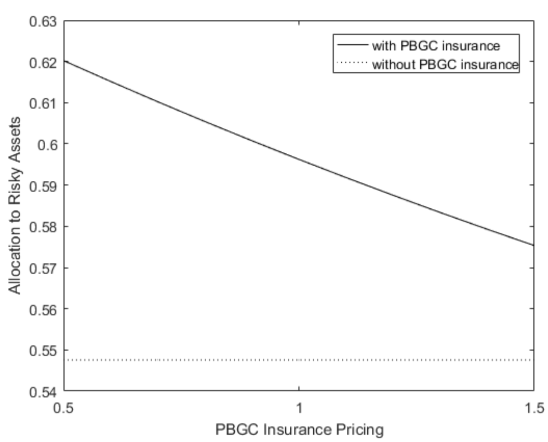
D.1 Impact of PBGC Linear Financing Cost on Allocation



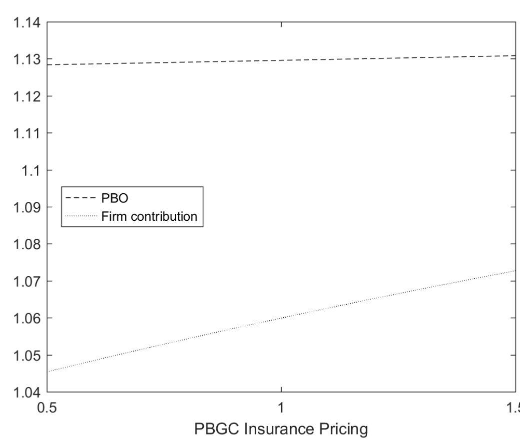
D.2 Impact of PBGC Linear Financing Cost on PBO and Contribution



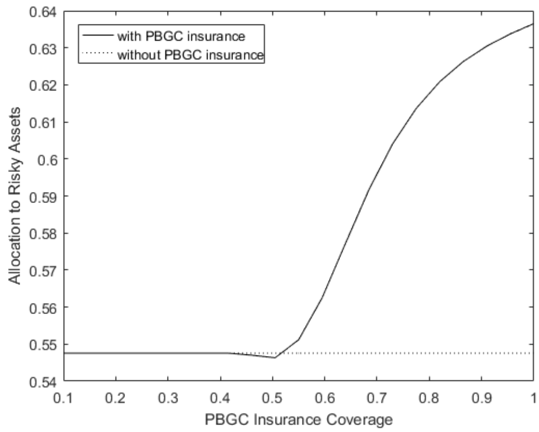
E.1 Impact of PBGC Insurance Pricing on Allocation



E.2 Impact of PBGC Insurance Pricing on PBO and Contribution



F.1 Impact of PBGC Insurance Coverage on Allocation



F.2 Impact of PBGC Insurance Coverage on PBO and Contribution

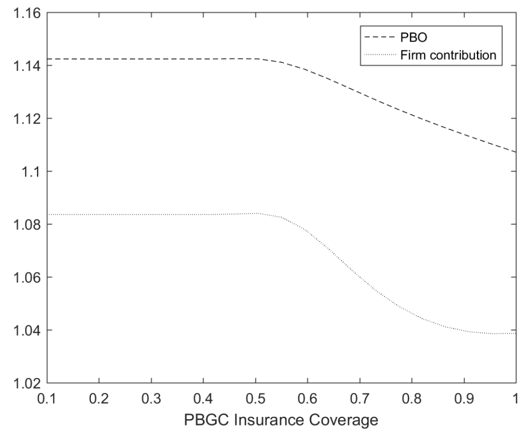
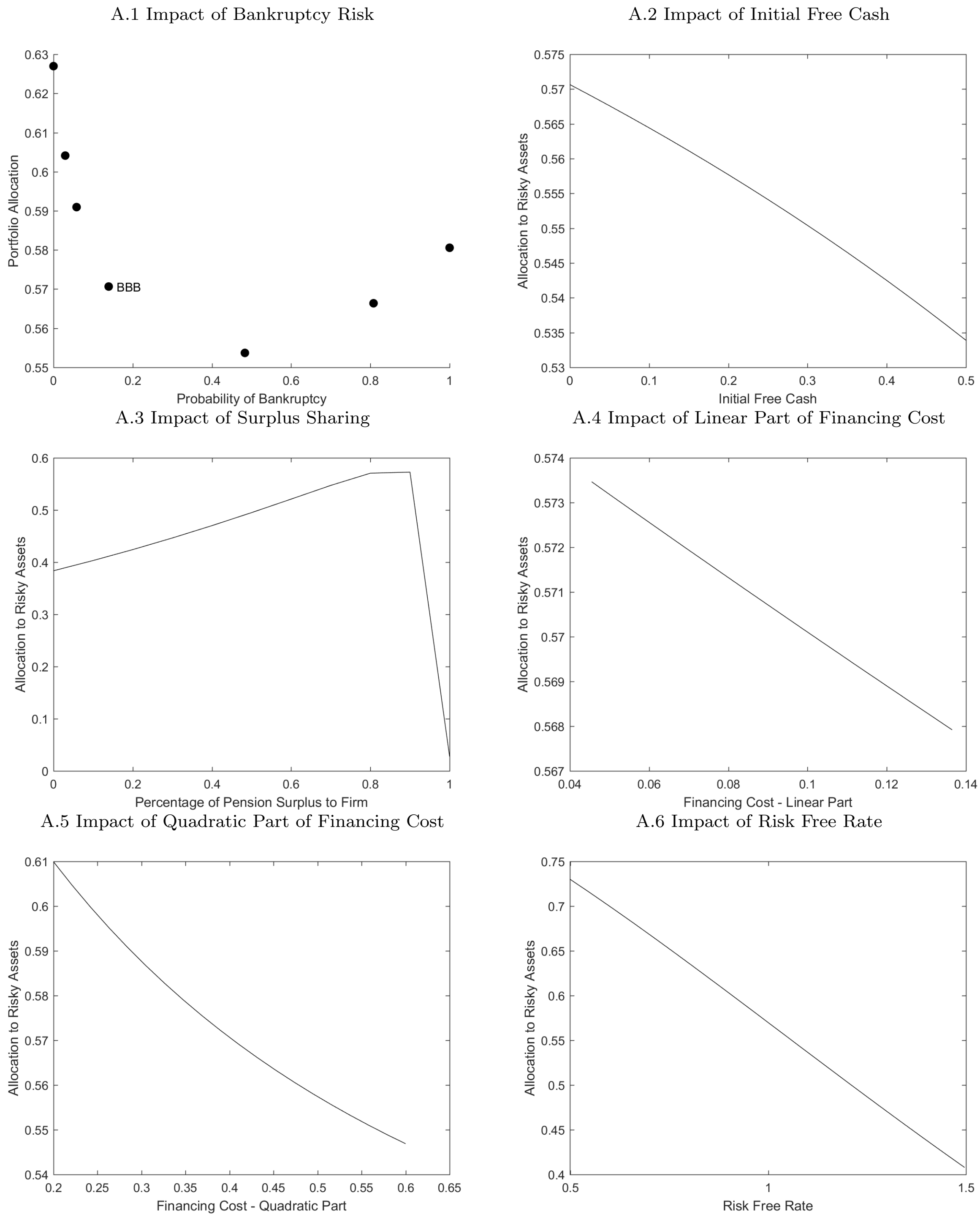
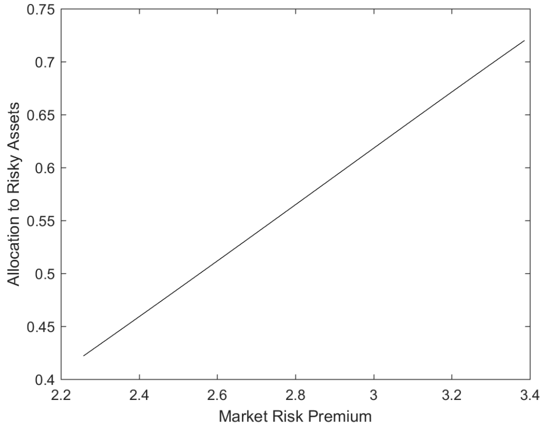


Figure 6 Initially Fully Funded Pension Plans

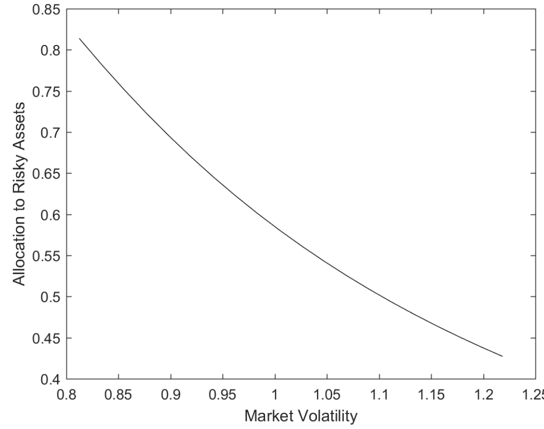
This figure illustrates the impact of various factors on pension plan asset application if the plan is initially fully funded. Panel A shows the asset allocation without PBGC. Panel B shows the asset allocation with PBGC. The parameter values are listed in Table 1.



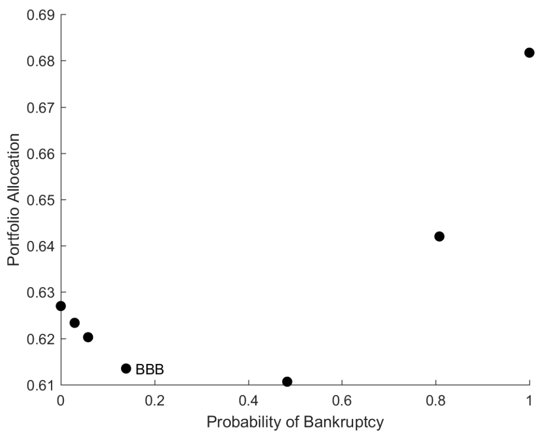
A.7 Impact of Market Risk Premium



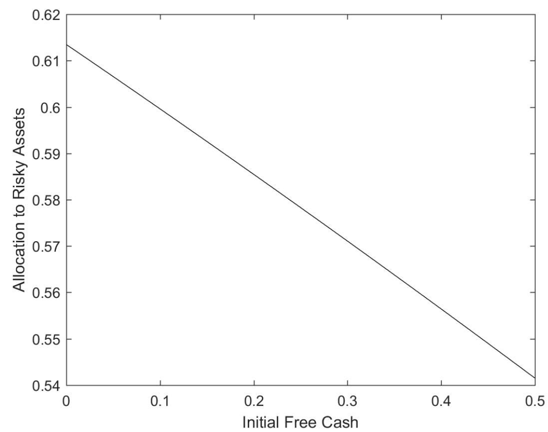
A.8 Impact of Market Volatility



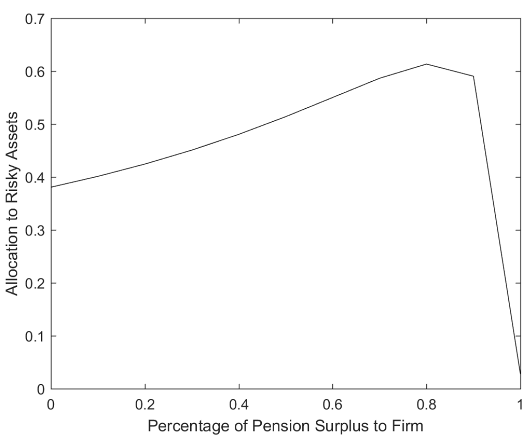
B.1 Impact of Bankruptcy Risk



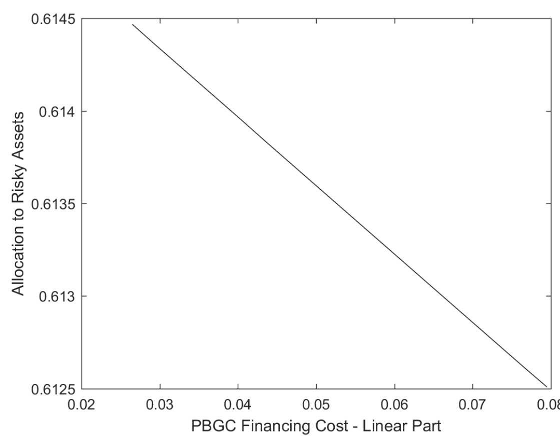
B.2 Impact of Initial Free Cash



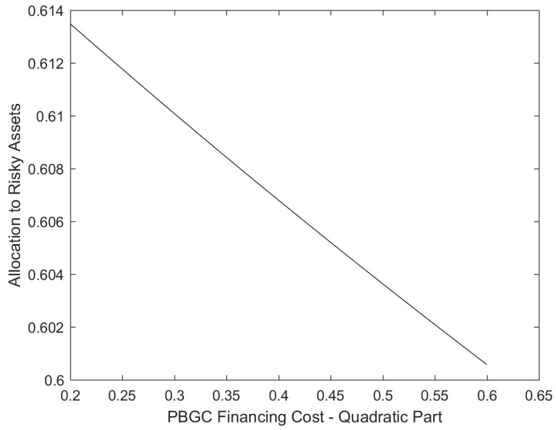
B.3 Impact of Surplus Sharing



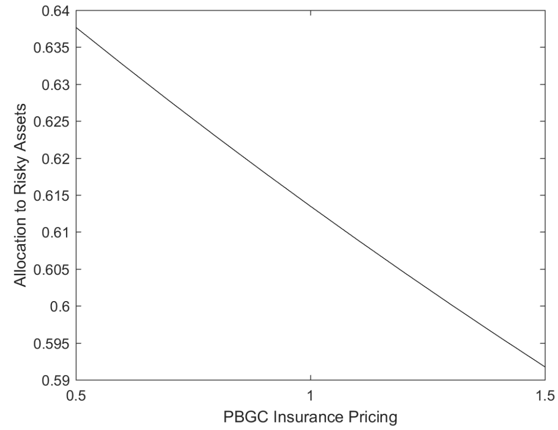
B.4 Impact of Linear Part of Financing Cost



B.5 Impact of Quadratic Part of Financing Cost



B.6 Impact of PBGC Insurance Pricing



B.7 Impact of PBGC Insurance Coverage

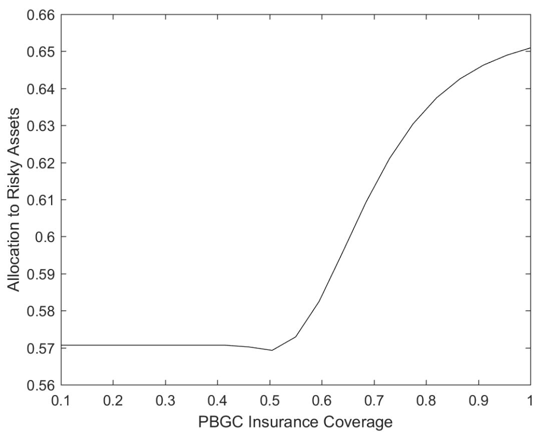


Figure 7 Pension Plan Investment and Funding with Reversion Tax

This figure illustrates the pension plan risky allocation and plan funding ratio (firm contribution divided by plan PBO) under different bankruptcy risks, pension surplus sharing schemes, and reversion tax rates. If bankruptcy occurs, firms keep all the pension surplus. Otherwise, firms keep 80% pension surplus. Firms always pay 20% tax on the pension surplus reversion. Panel A shows the impact of bankruptcy risk. Panel B shows the impact of pension surplus sharing. Panel C shows the impact of reversion tax.

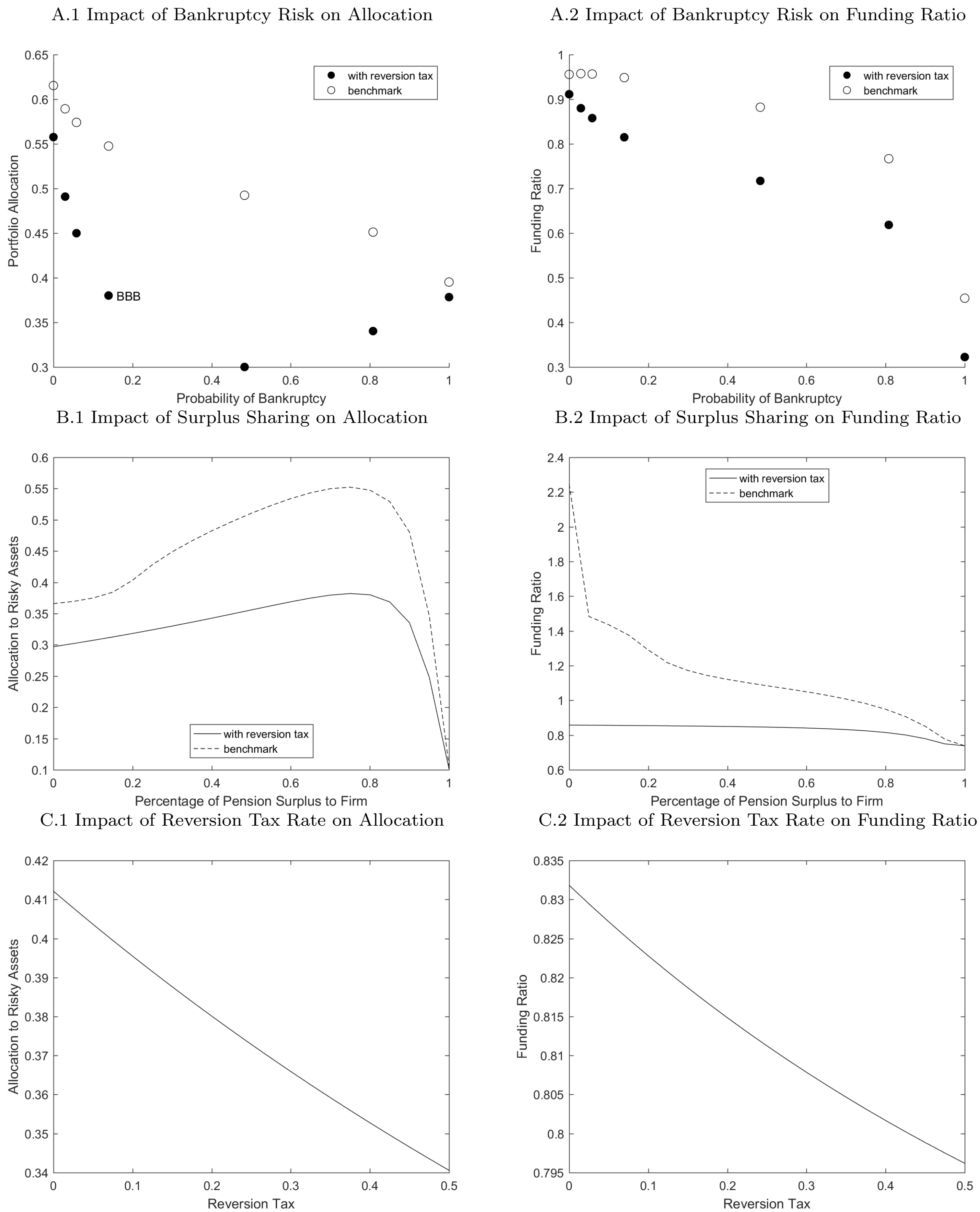
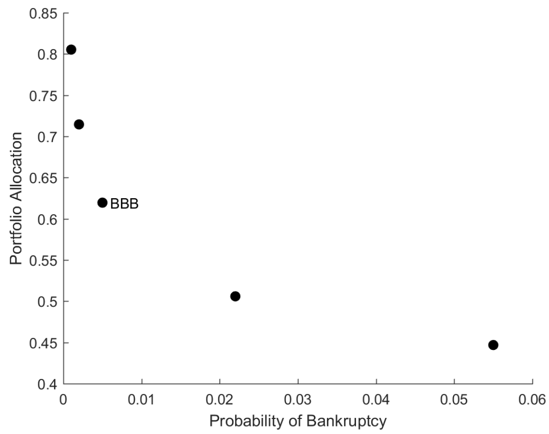


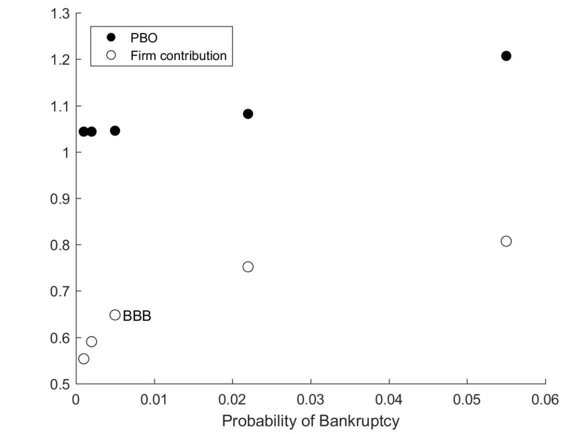
Figure 8 Initial Optimal Allocation and Cash Contribution of Dynamic Model

This figure illustrates firm's pension fund asset allocation and cash contribution at time 0 in dynamic model. The value of parameters is the same as the one listed in Table 1 unless other specified. Panel A shows the optimal allocation and cash contribution at time 0 under different bankruptcy risk. Panel B shows the optimal allocation and cash contribution at time 0 under different initial free cash. Panel C shows the optimal allocation and cash contribution at time 0 under different surplus sharing ratio.

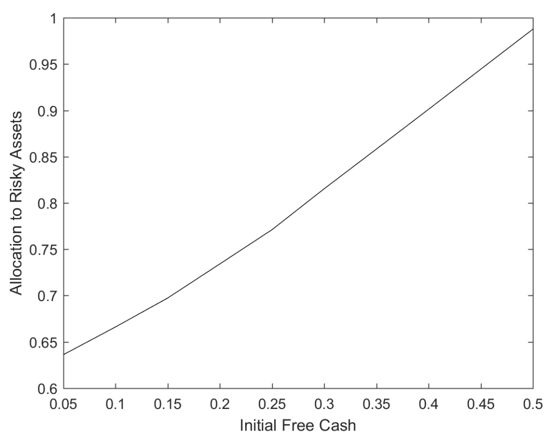
A.1 Impact of Bankruptcy Risk on Allocation



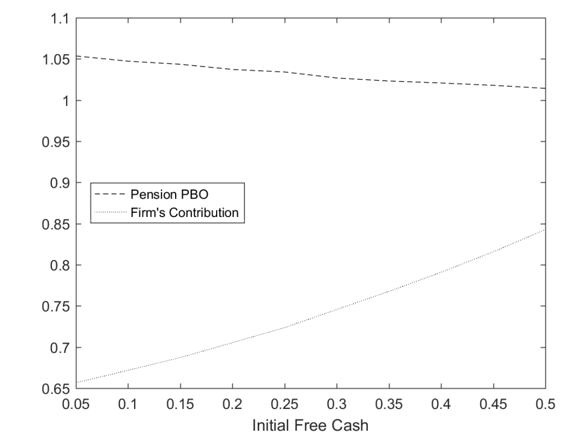
A.2 Impact of Bankruptcy Risk on PBO and Contribution



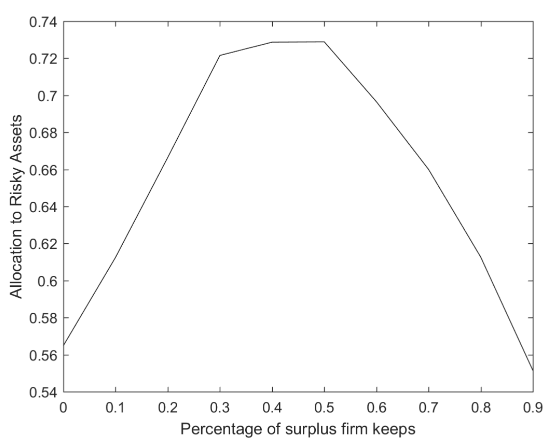
B.1 Impact of Initial Free Cash on Allocation



B.2 Impact of Initial Free Cash on PBO and Contribution



C.1 Impact of Surplus Sharing on Allocation



C.2 Impact of Surplus Sharing on PBO and Contribution

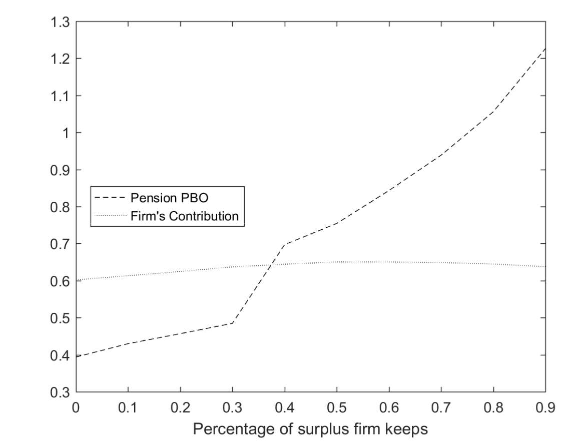
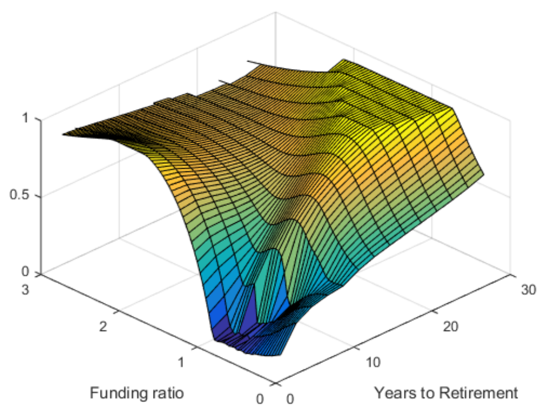


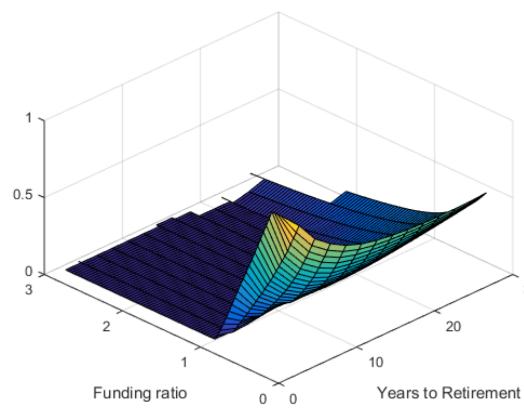
Figure 9 Optimal Allocation and Cash Contribution of Dynamic Model

This figure illustrates firm's pension fund asset allocation and cash contribution across time and states (funding ratio before cash contribution). The value of parameters is the same as the one listed in Table 1 unless other specified. Panel A shows the optimal allocation and cash contribution across time and states under benchmark parameters. Panel B shows the optimal allocation and cash contribution across time and states under smaller probability of bankruptcy as AAA firm. Panel C shows the optimal allocation and cash contribution across time and states under the scheme of more pension surplus (40%) given to employee.

Panel A Benchmark

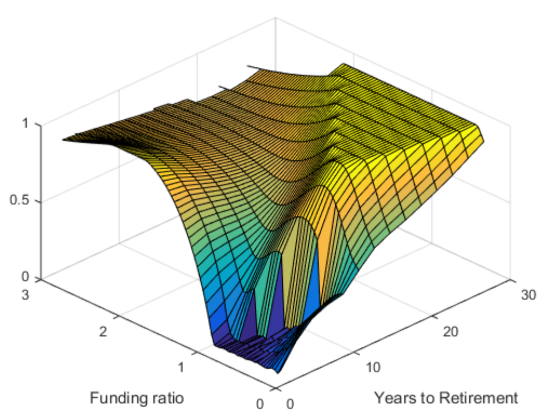


(1)



(2)

Panel B Lower Bankruptcy Risk (AAA firm)



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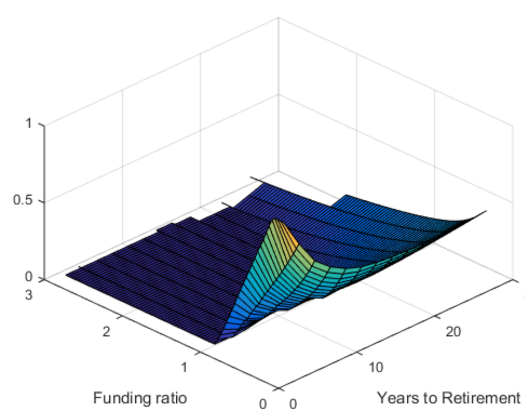
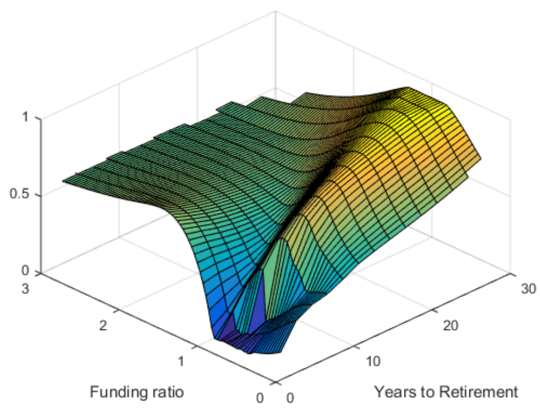
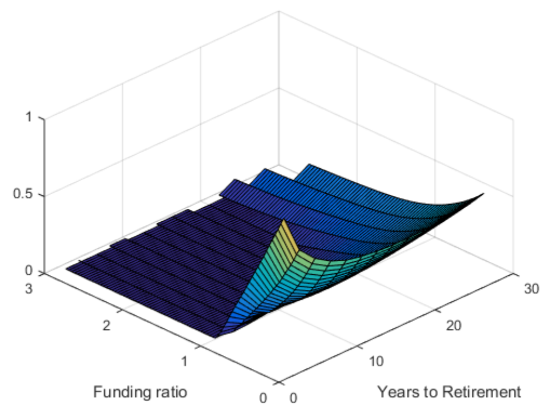


(2)

Panel C More Pension Surplus to Employee (firm keeps 60%, employee gets 40%)



(1)



(2)