Employer-Paid Parking, Mode Choice, and Suburbanization

by

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Abstract

This paper constructs a theoretical model that facilitates analysis of the effects of employer-paid parking on mode choice, road investment and suburbanization. The model simplifies urban space by dividing it into two zones (islands), center and suburbs, which are connected by a congested road and a public-transit line. Each road commuter requires an allotment of CBD land for parking, and because the central zone's area is fixed, parking land reduces the amount available for central residences and CBD production. The model characterizes optimal resource allocation from the perspective of a social planner. The planning solution can be decentralized, which requires employee- rather than employer-paid parking, congestion tolls, and a tax (subsidy) to offset the road capacity deficit (surplus). The analysis then considers the effect of switching to employer-paid parking, with the burden of parking costs shifting from road users to employers, thus reducing the wage for all workers. This switch inefficiently increases road usage and capacity investment, while spurring an inefficient increase in suburbanization.

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1. Introduction

According to US Census data, 86 percent of all commuters drove to work in 2013 (McKenzie 2015). In addition to creating unwelcome road congestion, this massive automobile commute flow affects land-use in the CBD, with employee parking in the employment centers of US cities consuming substantial amounts of land and other resources. Even though underground parking can limit the loss of land, parking in some CBDs covers more than 20 percent of the total land area. For example, in downtown Los Angeles, CA nearly 24% of land is allocated to industrial and commercial surface parking lots.[1] This loss of land area hurts the city, reducing urban vitality and competitiveness by reducing the CBD's production potential.[2] By contrast, public-transit usage does not create the same land demands as parking. Bus users require no land beyond the city streets on which buses travel, and while commuter rail users require a train station or subway stop, these facilities can be fairly compact and are often underground.

Although a shift toward public transit would thus reduce parking's pressure on available CBD land, the choice between automobile and transit commuting is subject to a major distortion caused by the way parking is provided to workers. In particular, in attempting to attract and retain the best employees, nearly three-fourth of all firms in the U.S. provide free parking for their workers, offering an estimated 85 million free commuter parking spaces with a net worth of nearly $31.5 billion (EPA 2005).[3] Since the practice is more common in large firms, 95 percent of all commuters who drive to work receive free parking. Even in the CBDs of large cities like New York and Los Angeles, where land is most scarce, over 50 percent of automobile commuters receive free parking paid for by their employer (Willson and Shoup 1990, Schaller Consulting 2007).

The fringe benefit of employer-paid parking, which is also nontaxable, is viewed as free by employees, even though its cost is borne, almost invisibly, through lower wages for all workers. With no explicit parking cost to bear, workers misperceive the cost of automobile commuting relative to the cost of public transit, producing effects that extend far beyond the confines of the individual firm. Free parking inefficiently discourages transit usage, reduces land availability for productive uses and residential development in the city center, and creates road congestion and air pollution (especially since the vast majority of auto commuters are solo drivers).[4]

As argued by Shoup (1997, 2005) and others, this distortion would be eliminated if employers charged workers for the cost of the parking they provide. With employers freed from the burden of parking costs, wages would rise. But the main effect would be to raise perceived auto commuting costs relative to the cost of public transit, encouraging some workers to switch to the transit mode. Therefore, charging employees for parking or letting them cash out their free parking space would lead to more efficient commuting choices.

The purpose of this paper is to construct a model that facilitates analysis of the effects of employer-paid parking on mode choice, road investment, and suburbanization. Following Brueckner and Helsley (2011), the model simplifies urban space by dividing it into two zones (islands), center and suburbs, which are connected by a congested road and a public-transit line. Each road commuter requires an allotment of CBD land for parking, and because the central zone's area is fixed, parking land reduces the amount available for central residences and CBD production. In addition, each road user requires a fixed amount of parking land at their suburban residence. The model characterizes optimal resource allocation from the perspective of a social planner.

The planning solution can be decentralized, which requires employee- rather than employer-paid parking, congestion tolls, and a tax (subsidy) to offset the road capacity deficit (surplus) and the fixed cost of the transit system. The analysis then considers the effect of a switch to employer-paid parking, with the burden of parking costs shifting from auto commuters to employers, thus reducing the wage for all workers. The analysis shows that this switch increases road usage and road capacity, while reducing the center's residential land area but leaving production unaffected. With less central residential land, the result is greater suburbanization of the population along with an overall increase in the suburban commute flow to the CBD.

Since all these changes are welfare reducing, the paper thus shows that employer-paid parking leads to *inefficiently high road usage and capacity investment along with an excessive degree of suburbanization*.

Since these results are derived under the assumption of quasi-linear preferences, the wage reduction's income effect on land consumption is absent, potentially masking important impacts on urban structure. To provide an analysis that captures income effects, the paper also carries out a numerical simulation with Cobb-Douglas preferences, which mainly replicates the patterns seen in the analytical results.

Even though the goal of the paper is to analyze the effects of employer-paid parking on resource allocation rather than fully explaining the genesis of this policy, some introductory analysis explores tax incentives as a likely motivating force. Partial-equilibrium analysis in section 2 shows that firms offering a nontaxable fringe benefit of employer-paid parking may compete more effectively for workers than those not offering it. Sections 3 and 4 then transition to a general-equilibrium framework that analyzes the effects of an exogenous switch to employer-paid parking by all employers in a setting without income taxes.

An empirical literature has emerged showing that employer-paid parking encourages automobile commuting (usually in solo fashion) over other modes. The literature contains case studies showing how the employee modal split differs across firms with and without employer-paid parking, as well as before-and-after studies showing how the modal split responds to cessation of this parking policy (see the survey by Willson and Shoup 1990 and Shoup 2005). While public transit is one of the chosen modes in these studies, another is carpooling, which employees adopt in response to higher parking costs. For reasons of analytical tractability, this mode is absent from the current model. The literature also explores the effects of cash-out policies, where employees can either take a free parking space or a wage supplement equal to the parking cost (see Shoup 1997). The welfare cost associated with parking's status as a nontaxable fringe benefit is computed by van Ommeren and Wentink (2012).

This empirical focus on employer-paid parking has not been matched on the theoretical side, with only a few analytical studies devoted to exploring the effects of employer-paid parking on commuting patterns and urban structure.[5] Borck and Wrede (2008) provide a foundation for such work by studying the distributional effects of commuting subsidies among city residents and absentee landowners in a monocentric city with two transport modes, two income groups, and fixed housing consumption.[6] Unpublished work by Franco (2014) finds results similar to those in Borck and Wrede (2008), although the focus of her study is on the spatial effects of employer-paid parking modeled as a subsidy for auto users. As in Borck and Wrede (2008), her setup is a closed monocentric model with two transport modes, although it assumes homogenous incomes while allowing for endogenous housing consumption. With suburban (central) residents commuting by auto (public transit), the study shows that a higher auto subsidy leads to a shift toward auto commuting at the expense of transit use and greater suburbanization of the population, as in the present paper (but without the current effects on the wage and business land use).[7]

Voith (1998) analyzes the effect of a parking tax used to subsidize transit commuting on total CBD employment, using a model that shares some of the present framework's features, including a brief extension where parking and CBD production compete for land, as in the present framework.[8] Employer-paid parking appears obliquely in De Borger and Wuyts' (2009) study of the gains from recycling congestion-toll revenue. They show that its presence raises the gain from revenue recycling through higher public-transit subsidies relative to the gain from reducing labor taxes.

The plan of the paper is as follows. Section 2 presents an introductory analysis of tax incentives in the adoption of employer-paid parking. Section 3 develops the model, while section 4 uses it to analyze the effect of employer-paid parking. Section 5 presents the numerical example, while section 6 explores some extensions to the model, including the presence of agglomeration economies and different assumptions on the public-transit technology. Section 7 presents conclusions.

2. The Tax Code and Employer-Paid Parking

Several forces could underlie the genesis of employer-paid parking. The practice could partly represent a societal norm, something that workers have grown to expect from their employers. But tax incentives may also help to explain its existence. As explained above, by making this fringe benefit nontaxable, the tax code encourages firms to pay their workers' parking costs. The purpose of this section is to demonstrate this point in a nonspatial, partial-equilibrium setting. The discussion will help to motivate the general-equilibrium analysis in the rest of the paper, where the effects of an exogenous switch from employee-paid to employer-paid parking are explored. To begin, let w denote the fixed wage earned by workers at the CBD and let κ be the (constant) income-tax rate. Let the cost of CBD parking be denoted B, and let the costs of commuting for public transit and auto users (exclusive of any parking cost) be denoted g_{auto} and $g_{transit}$.

At a firm that requires employees to pay their own parking costs, after-tax income net of commuting and CBD parking costs for an individual who drives to work equals $(1-\kappa)w - B - g_{auto}$. At this firm, the disposable income of a transit user is $(1-\kappa)w - g_{transit}$. Suppose a different firm adopts employer-paid parking, paying the cost B instead of expecting the worker to pay it. Since the firm incurs this new cost, the wage paid to all workers must fall to cover it. Suppose the firm expects its work force to consist of a fraction ρ of auto users and a fraction $1-\rho$ of transit users. Then the wage (which realistically does not depend on a worker's transport mode) falls to $w - \rho B$, dropping by the expected parking cost per worker (note that the share $1-\rho$ of transit workers generate zero parking costs).[9]

Incorporating the reduced wage, disposable after-tax income for auto users at the firm with employer paid parking equals $(1-\kappa)(w-\rho B) - g_{auto}$, which exceeds the income expression $(1-\kappa)w - B - g_{auto}$ at a firm without employer-paid parking. The reason is that the reduction in the after-tax wage ($(1-\kappa)\rho B$) is less than B, the forgone cost of parking.[10] Firms would therefore find it easier to attract auto users to their work force if they offer employer-paid parking. Transit users, however, would shun such a firm given that their disposable income falls from $(1-\kappa)w - g_{transit}$ to $(1-\kappa)(w-\rho B) - g_{transit}$, a consequence of subsidizing the parking costs of auto users. But since transit users are a small minority of workers in most US cities, firms can be expected to gravitate toward employer-paid parking when competing for employees. In the end, firms not offering this benefit would be few in number, with transit users having little choice but to work for firms where they subsidize the parking costs of drivers. The share $1-\rho$ of transit workers in each firm will be close to their overall share in the workforce.[11]

The effects of employer-paid parking on resource allocation could be analyzed in a model with income taxation, where a change in the tax treatment of the fringe benefit would induce a switch from employee-paid to employer-paid parking. However, to avoid inessential complexity, the model supresses income taxation and instead analyzes the impact of an *exogenous* switch to employer-paid parking. This approach is superior because, aside from inducing the switch, income taxes would play little further role in the model while requiring additional notation that would obscure the effects of interest.[12]

3. Model

The city has two islands, with the central island (denoted c) containing residences, land used for worker parking, and land used in production (see Figure 1). The total central land area is normalized to 1. Commuting cost within both zones is zero, with residents of the central island walking to work.[13] Access to the center from the suburban island (denoted s) comes from a road (mode 1) or public transit (mode 2), both of which use bridges. In contrast to parking, public transit requires no central land. The road capacity is k_1 (which costs γ_1 per unit), and the cost of using the road, which includes time and vehicle costs, is $t(n_{s1}, k_1)$, where n_{s1} is the number of suburban residents using it (t's derivatives satisfy $t^n > 0,\ t^k < 0,\ t^{nn}, t^{kk} > 0$). The transit user-cost curve is horizontal at height α, which gives the fixed time cost of public-transit use, as long as n_{s2} is less than capacity, denoted k_2. The user-cost curve becomes vertical when n_{s2} reaches k_2, indicating unbounded costs from exceeding capacity, or "infinite" public-transit crowding.[14] As a result, capacity (which has a constant marginal cost of γ_2 per unit) will always be set equal to n_{s2}. Provision of transit capacity also involves a fixed cost F, so that the transit technology exhibits economies of scale.[15] A different formulation with declining marginal cost and no fixed cost is explored in section 6.

It should be noted that this framework assumes that commuters do not have an innate preference for driving or transit use, which would introduce heterogeneity into the model with no effect on the main results. Without heterogeneity, commuters in a decentralized setting would choose the cheapest mode, a rule that the planner would also follow. Finally, workers walking to work or commuting by public transit are assumed not to own a car.[16]

Production in the CBD depends on land and labor inputs, with the CRS production function given by $F(\ell, L)$, where ℓ is the land input and L is the number of workers, equal to the city population, which is fixed at unity. Output can then be written as $LF(\ell/L, 1) = Lf(\ell/L) = f(\ell)$, setting $L = 1$ and letting f denote the intensive production function (f' is thus the marginal product of land, and $f'' < 0$ holds). Agglomeration economies in CBD production are absent, being introduced as an extension in section 6.

The land input in CBD production satisfies $\ell = 1 - \beta P - n_c q_c$, where P is the number of parking spaces, β is land area per space, and n_c and q_c are the number of central residents and their land consumption. The suburban values are n_s and q_s, and in addition, βP worth of suburban land is needed for residential parking. Consumer preferences are assumed be quasi-linear, being given by $e_i + v(q_i)$, for $i = c, s$, where e_i is nonland consumption and $v' > 0,\ v'' < 0$ hold.

3.1. The social planner's problem

The planner's problem is to maximize the city's uniform utility level u subject to various constraints. Letting r_a denote the exogenous opportunity cost of land (agricultural rent), the Lagrangean expression for the planning problem can be written as follows:

$$\begin{aligned} u \ &+\ \lambda[1 - (n_c + n_s)] \\ &+\ \theta_c(e_c + v(q_c) - u)\ +\ \theta_s(e_s + v(q_s) - u) \\ &+\ \delta\{f(1 - \beta P - n_c q_c) - [n_s e_s + n_c e_c + r_a(1 + n_s q_s + \beta P) + \gamma_1 k_1 + F + \gamma_2 n_{s2} + n_{s1} t(n_{s1}, k_1) + n_{s2}\alpha]\} \\ &+\ \phi(P - n_{s1}) \\ &+\ \psi(n_{s1} + n_{s2} - n_s). \qquad (1) \end{aligned}$$

The first constraint (with multiplier λ) is the overall population constraint, and the two constraints on the second line of (1) are the utility constraints. The next constraint (with multiplier δ) is the resource constraint. The first term involving f is the city's output. This output must cover non-land consumption (the next two terms), land costs (inclusive of suburban parking land), the costs of transport capacity, including transit fixed cost (the next three terms; recall $k_2 = n_{s2}$), and commuting costs (the last two terms). The last two constraints say that the number of parking spaces equals n_{s1} and that users on the two modes add up to n_s, the suburban population.[17]

The first-order conditions for the planning problem (which are shown in Appendix A1) can be manipulated to yield a set of necessary conditions for an optimum. The first two of these conditions are[18]

$$v'(q_c) \quad = \quad f' \tag{2}$$

$$v'(q_s) \quad = \quad r_a. \tag{3}$$

Note that since f', the marginal product of land in CBD production, will equal land rent in the central zone in a decentralized equilibrium, (2) and (3) equate marginal utilities to decentralized land rents.

The next condition is[19]

$$t \ + \ n_{s1}t^n \ + \ \beta(f' + r_a) \quad = \quad \alpha \ + \ \gamma_2, \tag{4}$$

which says that the social cost of an extra user should be the same on the two modes, holding k_1 fixed. The social cost of an extra road user includes t along with congestion cost $n_{s1}t^n$ and the cost $\beta(f' + r_a)$ of parking land at both ends of the commute trip (recall that f' represents central land rent), while the social cost of an extra transit user is the commuting cost incurred (α) plus the marginal capacity cost (γ_2). Since the LHS of (4) is increasing in n_{s1}, the equation yields a unique solution for n_{s1} conditional on k_1. For n_{s1} values below this solution, the road usage is less costly than transit usage, making an increase in n_{s1} optimal, while a decrease is optimal for n_{s1} values above the solution. While (4) assumes an interior solution for n_{s1}, a corner solution is possible, as explained further below.

The condition for optimal road capacity is

$$-n_{s1}t^k \quad = \quad \gamma_1 \tag{5}$$

(recall $t^k < 0$). This condition says that the marginal benefit from a marginal capacity expansion should equal the cost. The next condition is[20]

$$e_c + f'q_c \;=\; e_s \,+\, r_a q_s \,+\, \alpha \,+\, \gamma_2. \tag{6}$$

This condition characterizes the division of the population between the central and suburban zones, which is optimal when the resource consumption of an extra person is equal in the two zones. Although the public-transit cost appears on the RHS of (6), the fact that this cost equals the road cost via (4) means that (6) pertains to users of both modes.

The e_i variables in (6) can be eliminated using the utility constraints, with $e_i = u - v(q_i)$, $i = c, s$, substituted on the two sides of (6). After cancellation of u and substitution of $v'(q_c)$ and $v'(q_s)$ in place of f' and r_a, respectively, (6) becomes

$$q_c v'(q_c) \,-\, v(q_c) \;=\; q_s v'(q_s) \,-\, v(q_s) \,+\, \alpha \,+\, \gamma_2. \tag{7}$$

Since q_s is independently determined by equating $v'(q_s)$ to the exogenous r_a, the RHS of (7) can be treated as fixed. As a result, (7) determines q_c. With q_c given, (2), (4) and (5) then constitute three equations to determine n_{s1}, k_1 and n_c (recall that f's argument is $1 - \beta n_{s1} - n_c q_c$). The remaining unknowns, u, n_{s2}, n_s, e_c, e_s, and P, are then determined via the constraints in (1).

Note that since $qv'(q) - v(q)$ is decreasing in q (with derivative $qv'' < 0$), $q_c < q_s$ must hold for (7) to be satisfied, so that land consumption is smaller in the central zone than in the suburbs, as expected. In addition, (2) and (3) then imply that $f' > r_a$ must hold, so that the center commands the expected rent premium.

3.2. Decentralizing the planning solution

The planning solution can be decentralized by setting road capacity optimally (satisfying (5)), adjusting transit capacity to equal ridership, imposing a road congestion toll equal to $n_{s1}t^n$, setting the transit fare equal to the marginal capacity cost γ_2, and requiring road users to acquire their own parking land in the center as well as in the suburbs. With these charges, the equilibrium condition determining the split between transit and road usage is the same as the optimality condition (4).[21] In addition, the optimality conditions (2) and (3) are satisfied under decentralization, condition (6) holds given the common incomes of central and suburban residents, and (7) (derived from (6) by imposing the equal-utility restriction) also holds since equal utilities is a condition of equilibrium.

For both the road and public transit line, the difference between revenues and costs must be offset through the tax system. Since the transit fare only covers marginal cost γ_2, the fixed cost F must be financed by a tax levied on all urban residents, a standard transit financing setup under increasing returns. For the road, the difference between toll revenue and capacity cost can be either positive or negative depending on the nature of the t function (see below), and the gap is offset by a lump-sum tax or transfer.

With land rent in the central zone equal to f', the wage earned by workers equals $f - \ell f' = f - (1 - \beta n_{s1} - n_c q_c) f'$, a residual equal to output minus land cost, on a per-worker basis. In addition, the decentralized city must have internal land ownership (being fully closed), with differential rent (land rent in excess of r_a) accruing to residents as income on an equal per-capita basis. It should be noted that, although workers are viewed as renting their parking land directly, parking could be provided by competitive firms that rent land and resell it as parking spaces (while incurring no capital cost). Appendix A3 shows the aggregate and individual budget constraints under decentralization.

4. Analysis

4.1. The effect of a switch to employer-paid parking with k_1 fixed

The model can be used to analyze how a city with employer-paid parking differs from one with employee-paid parking. To address this question, start with the decentralized planning solution, and suppose that employers now provide parking without charge to workers, renting the required land and providing it as a fringe benefit. Auto commuters still need to pay, however, for their residential parking in the suburbs. As will be be seen shortly, commuters will shift from public transit to road use in response to the switch to employer-paid parking. But even before this shift occurs, the switch will lead to a reduction in the wage for all workers, with the employer's parking expenses reducing compensation for road plus transit users as well as central residents. This change can be seen in the aggregate budget constraint in appendix (A3), with the aggregate parking-cost term $n_{s1}\beta f'$ on the RHS of the expenditure expression ($a15$) set to zero and this term instead subtracted from the wage in the income expression ($a14$) (note that with $L = 1$, both total and individual wages decline by $n_{s1}\beta f'$). It is crucial to recognize that the wage is not adjusted on an individual basis depending on whether or not a particular worker uses employer-paid parking; all wages decline to cover its costs.

To analyze the shift from transit to road usage, consider first the case where road capacity is held fixed at the first-best level but that transit capacity is adjusted in step with ridership. In addition, suppose that a road congestion toll continues to be levied, with its magnitude adjusting to changes in traffic. Since the switch to employer-paid parking eliminates $\beta f'$ from the mode-choice condition (4), road usage becomes cheaper than public transit, encouraging some commuters to shift to the road. With n_{s1} thus increasing, $t + n_{s1}t^n$ increases as well, rising until it equals $\alpha + \gamma_2 - \beta r_a$, at which point a new mode-choice equilibrium is established. As a result, an increase in road congestion from the traffic shift completely eliminates the auto commuter's cost savings from employer-paid parking, with total cost returning to its original level. It is important to note that, in making his mode choice, the worker views the wage as parametric, not recognizing that a decision to drive to work would reduce total (and individual) wages by the extra parking cost $\beta f'$ he would generate.

With n_{s1} larger, more central parking land is needed, exerting downward pressure on the land available for production and residences in the center. However, since the shift to employer-paid parking has no effect on q_c, which is still determined by (7), it follows from (2) that f' (and thus its argument $1 - \beta n_{s1} - n_c q_c$) must also be unchanged. The upshot is that the increase in required parking land must be exactly offset by a decrease in residential land, which occurs via a decline in n_c. This decline has no effect on n_{s1}, which is determined solely by the new (4). As a result, the new suburban commuters created by the lower n_c all use public transit. Whether this traffic rebound is sufficient to offset the initial decline in transit users is unclear. Overall, therefore, the switch to employer-paid parking leads to increased road usage and greater decentralization, with population shifting toward the suburbs. Even though the change in transit usage is ambiguous, greater suburbanization implies that the switch leads to a larger overall commute flow to the CBD.

It is easy to see that the previous conclusions hold in a more realistic situation where k is fixed at some nonoptimal level, and they also hold for arbitrary k in the absence of congestion tolls. In both cases, n_{s1} must rise to equate road and transit costs, with n_c then falling. Another point to note is that, because of quasi-linear preferences, the income effect on land consumption from the lower wage is absent from the previous results, with only the e_i's being affected. Thus, one possible channel by which the switch can affect suburbanization and urban structure is neutralized.

4.2. The effect of a switch to employer-paid parking with k_1 adjusted

These results can be derived formally by multiplying $\beta f'$ in (4) by the factor μ, which equals zero with employer-paid parking. This change also appears in the aggregate decentralized budget constraint, with $n_{s1}\beta f'$ in (9) multiplied by μ and $(1-\mu)n_{s1}\beta f'$ subtracted from the income expression (8). Comparative-static analysis of μ's effects is then carried out using (2) and (4), with a reduction of μ from 1 to 0 corresponding to the switch to employer-paid parking (μ is thus the employee's parking cost share).

While this exercise is straightforward and confirms the logic from above, a less straightforward exercise analyzes the shift to employer-paid parking when road capacity k_1 adjusts according to the first-best optimality condition (5). As seen below, satisfaction of the first-best capacity condition may not be desirable under the distortion of employer-paid parking, but real world decisions could well follow such a rule. The required analysis makes use of the three-equation system consisting of (2), (4), and (5), and the same setup can also be used to derive the effect of the parking efficiency parameter β on the first-best allocation, with μ set equal to 1.

Letting $\overline{q}_c$ denote the fixed value of q_c from (7), totally differentiating (2), (4) and (5) yields

$$\begin{pmatrix} \beta f'' & 0 & \overline{q}_c f'' \\ 2t^n + n_{s1}t^{nn} - \mu\beta^2 f'' & t^k + n_{s1}t^{kn} & -\mu\overline{q}_c\beta f'' \\ t^k + n_{s1}t^{kn} & n_{s1}t^{kk} & 0 \end{pmatrix} \begin{pmatrix} dn_{s1} \\ dk_1 \\ dn_c \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -\beta f' \\ 0 \end{pmatrix} d\mu \; + \; \begin{pmatrix} -n_{s1} f'' \\ -(f' + r_a - \beta n_{s1} f'') \\ 0 \end{pmatrix} d\beta. \tag{8}$$

Let M denote the 2×2 matrix in the bottom right corner of the 3×3 matrix in the first line of (8), once the term $\mu\beta^2 f''$ has been suppressed. The resulting matrix is the Hessian matrix of $n_{s1}t(n_{s1}, k_1)$, which is positive definite under the assumption that this expression, equal to total road commuting cost, is strictly convex in n_{s1} and k_1.[22] As result, the determinant of M, denoted H, must be positive along with the matrix's diagonal elements (the latter requirement follows from $t^{nn}, t^{kk} > 0$). It can be shown that, after simplification, the determinant of the 3 $\times$ 3 matrix then reduces to $D \equiv \overline{q}_c f'' H < 0$.

Using Cramer's rule (with $d\beta = 0$) and letting $x = -\beta f' f'' > 0$ and $z = x n_{s1} t^{kk} > 0$, the comparative-static derivatives with respect to μ are

$$\frac{\partial n_{s1}}{\partial \mu} = \frac{\overline{q}_c z}{D} \;<\; 0, \qquad \frac{\partial n_c}{\partial \mu} = -\frac{\beta z}{D} \;>\; 0, \qquad \frac{\partial k_1}{\partial \mu} = -\frac{\overline{q}_c x (t^k + n_{s1} t^{kn})}{D} \;>(<)\; 0. \tag{9}$$

Therefore, when μ decreases, moving toward the zero value corresponding to employer-paid parking, n_{s1} increases and n_c decreases, as in the simpler fixed-k_1 case above. The adjustment in k_1 depends on the sign of $-(t^k + n_{s1}t^{kn})$, which is the derivative of the marginal benefit of capacity ($-n_{s1}t^k$) with respect to n_{s1}. This expression is naturally assumed to be positive, indicating that extra capacity helps more when n_{s1} is high and the road is heavily congested. Then $t^k + n_{s1}t^{kn} < 0$ holds and $\partial k_1/\partial\mu$ is negative like $\partial n_{s1}/\partial\mu$, indicating that both n_{s1} and k_1 increase in the switch to employer-paid parking.[23]

Note also that since

$$\frac{\partial n_{s2}}{\partial \mu} \;=\; \frac{\partial n_s}{\partial \mu} - \frac{\partial n_{s1}}{\partial \mu} \;=\; -\frac{\partial n_c}{\partial \mu} - \frac{\partial n_{s1}}{\partial \mu} \;=\; \frac{(\beta - \overline{q}_c) z}{D} \;>(<)\; 0, \tag{10}$$

the switch to employer-paid parking has an ambiguous effect on n_{s2}, public-transit ridership, as argued above. With both road usage and capacity increasing, the switch also can be shown to have an ambiguous effect on congestion, as measured by the magnitude of t.

The changes in n_{s1} and n_c when k_1 is optimally adjusted are larger than those when k_1 is held fixed at the first-best level. This conclusion follows because, with $t^k + n_{s1}t^{kn} < 0$ assumed to hold, the expression $t + n_{s1}t^n$ falls as k_1 increases. As a result, compared to the fixed-k case, n_{s1} must increase by more to equate this expression to $\alpha + \gamma_2 - \beta r_a$ when μ is set at zero. The decline in n_c is then also larger, implying a greater suburbanization response to the employer-paid parking switch. A crucial point, though, is that regardless of whether or not k_1 increases, the shift to road usage completely eliminates the auto commuter's cost savings from employer-paid parking, with costs still equal to the cost of public transit, $\alpha + \gamma_2$.

Finally, since the optimality conditions are not satisfied when $\mu = 0$, the value of the objective function (consumer utility) is lower with the switch to employer-paid parking. Since both q_s and q_c are unaffected by the switch, the source of lower utility is a reduction in e_s and e_c. Auto commuting costs (inclusive of suburban parking costs for auto users) are anchored by the cost $\alpha+\gamma_2$ of public-transit and are thus unchanged, and central land rent (and thus rental income) is unaffected, so that the decline in the e's comes from other sources. One source of the e reductions is the lower wage. The residual wage expression $f - (1 - \beta n_{s1} - n_c q_c)f'$ stays constant with the switch to employer-paid parking, but the residual now must also include subtraction of per-capita (same as total) parking costs, $\beta n_{s1} f'$. The wage is then $f - (1 - n_c q_c)f'$, a smaller value that contributes to the decline in the e_i's and in utility. Note, however, that the road surplus or deficit will also be altered by the switch to employer-paid parking, providing another source of income change through the tax system beyond the effect on the wage. The overall decline in e_c and e_s from these two sources can be verified from the resource constraint, as seen in Appendix A2. The decline establishes the harm from switching to employer-paid parking.

Summarizing yields

Proposition 1. *Under the assumptions of the model, a switch from the first-best allocation to employer-paid parking leads to an increase in road usage and road capacity along with greater suburbanization of the city's population. There is no change in land consumption, central rent or CBD output, but utility falls due to a reduction in nonland consumption. When road capacity is held fixed, the increases in road usage and suburbanization are smaller, while the other qualitative effects continue to hold.*

Since all these changes disrupt the first-best allocation, the proposition implies that the switch to employer-paid parking leads to *inefficiently high road usage and capacity investment along with an inefficiently high degree of suburbanization*.[24]

The comparative-static analysis so far assumes that (4) yields an interior solution for n_{s1}, so that each of commute modes is used by some suburban residents. However, the first-best allocation may involve a corner solution, with all commuters using one mode or the other. If the corner solution has all commuters using the road, then a switch to employer-paid parking obviously has no effect. If the corner solution instead has all commuters using public transit, then the switch could have no effect (leaving the corner solution in place) or it could shift the outcome to an interior solution or possibly to the other corner solution. In the two latter cases, the effects of the switch on road use and capacity and on suburbanization will be the same as in Proposition 1.

It is interesting to note that, when the $t(n_{s1}, k_1)$ function takes a commonly assumed form, an interior solution for n_{s1} is ruled out, with one of the two corner solutions instead being optimal. This outcome emerges when $t(n_{s1}, k_1) \equiv T(n_{s1}/k_1)$, so that congestion depends only on the traffic-to-capacity ratio n_{s1}/k_1. This conclusion is demonstrated by noting that, with the ratio form of t, (5) becomes $(n_{s1}/k_1)^2 T'(n_{s1}/k_1) = \gamma_1$, which determines a solution for n_{s1}/k_1. Substituting this solution into (4), the expression $t + n_{s1}t^n = T(n_{s1}/k_1) + (n_{s1}/k_1)T'(n_{s1}/k_1)$ is then a constant. Since f' is also constant by (2) and (7), the LHS of (4) is then constant. If the constant's value is larger than $\alpha + \gamma_2$, then all commuters use transit, while all commuters use the road if the inequality is reversed.[25]

The necessity of a corner solution disappears if the marginal capacity cost γ_2 for public transit is realistically declining instead of constant, in which case the previous argument no longer applies. While this alternate assumption rules out derivation of analytical results, the declining marginal-cost case is considered as extension of the main numerical exercise, with results presented in section 6.

The main numerical exercise itself is designed to relax the assumption of quasi-linear preferences, which is crucial in deriving the previous analytical results. Without the quasi-linear assumption, the only path to concrete conclusions is a numerical one, and section 5 presents such results using a different utility function. The numerical exercise in section 6 further relaxes the assumption that marginal capacity cost in public transit is constant.

4.3. The effects of a cash-out policy

As explained in Shoup (1997), a 1992 California law requires large employers offering paid parking to also offer the option of a wage supplement equal to the parking cost, which is available to all workers (drivers plus users of other modes). Shoup's empirical evidence shows that this "cash-out" policy reduces solo driving, with auto commuters shifting to car pools and public transit. Using the current framework, it is easily shown that a cash-out policy restores efficiency, leading to the first-best outcome, as follows.

Starting from a situation with employer-paid parking, adoption of a cash-out policy can be decomposed into two separate steps. First, employer-paid parking is terminated, reducing firms' costs and raising the wage, eliminating the earlier wage loss caused by the policy. Second, each employee (including central residents who walk to work) is given a wage supplement of $\beta f'$, the cost of a parking space. However, since this supplement raises costs, the base wage declines by exactly the amount of the supplement, leaving the effective wage unchanged. The upshot is that workers face the same wage as they would in the absence of employer-paid parking while needing to pay their own parking costs if they drive, thus leading to the first-best outcome. In actuality, this parking payment is not explicitly made, since workers wanting to park get a space in lieu of the wage supplement. But the outcome is the same as if workers take the supplement and use it to pay for parking. Summarizing yields[26]

Proposition 2. *In the model, a cash-out policy eliminates the inefficiency of employer-paid parking, leading to the socially optimal outcome.*

It is important to note that tax effects can undermine the efficiency of a cash-out policy. As explained above, the policy can be viewed as giving all workers a wage supplement equal to the parking cost, which can be returned to the employer in return for a parking space. With income taxation, the wage supplement is taxable, but the tax disappears if the supplement is exchanged for parking. As a result, the additional tax can be viewed as a cost of not using the auto mode (using transit or walking to work in the center). Since this cost will distort mode choice, leading to road usage beyond the first-best level, the cash-out policy does not have the same efficiency benefit as in a world without income taxes. In order to restore policy's efficiency, the fringe benefit of employer-paid parking would need to be taxable, in which case there would be no adverse tax effect from not choosing the auto mode.

4.4. The effects of a change in parking efficiency

Consider now the effects of an improvement in parking efficiency, which corresponds to a reduction in β, the land required per parking space. An improvement in parking efficiency could arise from replacement of parking lots with parking structures, which use less land per parking space than surface lots (their capital cost, however, is ignored). Alternatively, a lower β could come from a shift in household preferences toward smaller vehicles. For example, a standard parking space size in Los Angeles County is 8.6-by-18 feet, while the space size for a compact car is 7-by-15 feet.[27] Mobility through cities with heavy traffic and a shortage of parking may create a need for small vehicles due to their agility, ease of parking and lower fuel consumption, leading to a decline in β.

The effect on the first-best allocation of a reduction in β can be found by setting $\mu = 1$ and $d\mu = 0$ in (10) and using Cramer's rule to solve for the comparative-static derivatives with respect to β. Letting $w = -(f' + r_a)f'' > 0$ and $y = wn_{s1}t^{kk} > 0$, these derivatives are

$$\frac{\partial n_{s1}}{\partial \beta} = \frac{\overline{q}_c y}{D} \;<\; 0, \qquad \frac{\partial n_c}{\partial \beta} = -\frac{\beta y + n_{s1} f'' H}{D} \;>(<)\; 0, \qquad \frac{\partial k_1}{\partial \beta} = -\frac{\overline{q}_c w(t^k + n_{s1}t^{kn})}{D} \;<\; 0. \tag{12}$$

Therefore, an improvement in parking efficiency (a reduction in β) raises n_{s1} and k_1, while having an ambiguous effect on n_c. The reason for this ambiguity is that the parking land βn_{s1} required to accommodate the larger group of road commuters could rise or fall since β itself has decreased. As a result, the required change in residential land (and thus the effect on suburbanization) is unclear. Summarizing yields

Proposition 3. *Under the assumptions of the model, an improvement in parking efficiency leads to an increase in road usage and road capacity, no change in land consumption, central rent or CBD output, an increase in utility from an increase in nonland consumption, and ambiguous changes in suburbanization and public-transit ridership.*

The utility effect follows from applying the envelope theorem to (1), which shows that the derivative of maximized utility with respect to β equals $-\delta n_{s1} f' < 0$. While the proposition again applies to an interior mode-choice outcome, the conclusions can be adjusted to the case of a corner solution, as above.

5. Numerical Example

5.1. The setup

This section presents a numerical example to see if use of a different utility function alters the conclusions of section 4. Because of the highly stylized nature of the model, attempting a realistic calibration seems inappropriate, with the example attempting to be realistic only in its broadest features.

Rather than assuming quasi-linear preferences, utility is instead assumed to take the Cobb-Douglas form $\sigma_0 q^{\sigma_1} e^{1-\sigma_1}$, as shown in Table 1, with $\sigma_1 = 0.4$. The CBD production function is also assumed to be Cobb-Douglas, so that the intensive form $f(\ell)$ is the power function $\rho_0 \ell^{\rho_2}$, with $\rho_2 = 0.2$ indicating a realistically low land share in production. The values for the road capacity cost (γ_1), the transit user cost (α), the transit capacity cost (γ_2), the parking efficiency parameter (β), and agricultural rent (r_a) are also shown in Table 1. Transit fixed cost F, which plays an inessential role, is set zero.

The choice of the transit user-cost function requires additional discussion. A ratio form for this function led to a corner solution for n_{s1} in the quasi-linear case, as noted above, and a similar issue arises with more-general preferences. Even though f' in (4) is no longer a constant in this case (making an interior solution possible in principle), the traffic split is highly unstable with a ratio form for t. To avoid this outcome, the numerical example adopts a t function of the form $\tau_0 \, n_{s1}^{\tau_1} k_1^{-\tau_2}$, with τ_1 much larger than τ_2 (the ratio form would have $\tau_1 = \tau_2$). As seen in Table 1, τ_1 is set at 4.0, with τ_2 equal to 1.0. Under these assumptions, which indicate that road capacity investment has a relatively weak effect on congestion, interior mode-choice outcomes occur with both employee-paid and employer-paid parking.

A ratio form of t exhibits homogeneity of degree zero, yielding the well-known self-financing theorem given the maintained assumption of constant road-capacity costs. The t function in

Table 1, by contrast, is homogenous of degree 3, which can be shown to imply that toll revenue at the social optimum exceeds the cost of road capacity. As a result, the capacity tax seen in ($a14$) will be negative, indicating a partial rebate of toll revenue.

5.2. Results

The numerical results are shown in Table 2, and they reflect the same general pattern of impacts from employer-paid parking seen in section 4. The discussion first focuses on the comparison between the two parking regimes when k_1 is adjusted optimally, contrasting the first and second columns of numbers. With employee-paid parking, about 80% of the city's unitary population resides in the suburbs ($n_s = 0.7961$), with an appreciable number of these residents (0.3324) using public transit. Following the switch to employer-paid parking, commuters shift toward road usage, with transit ridership (whose overall change was ambiguous a priori) dropping all the way to 0.0247 and road usage rising from 0.4638 to 0.8251, a case of low transit usage like that seen in many US cities.[28] These changes lead to a greater overall suburban commute flow, with the suburban population rising to 0.8498 as a result of the switch. In addition, road capacity increases substantially from 0.2072 to 0.8746, which offsets some of the congestion that would otherwise occur with a surge in road usage. Therefore, the main conclusions of Proposition 1 (an increase in road traffic and capacity accompanied by greater suburbanization) are reaffirmed by the numerical results.

The mode-choice shift from Table 1 is illustrated in Figure 2, which shows how the mode choices vary as a function of μ, the employee's parking cost share from the comparative-static analysis of section 4. The employer-paid and employee-paid parking outcomes in the table correspond to $\mu = 0$ and $\mu = 1$, but outcomes for intermediate μ values are shown as well. The "foot" choice in the figure indicates a central residence, which entails the mode choice of walking to work.

While land consumption was unaffected by the switch to employer-paid parking under quasi-linear preferences, q_c and q_s decline slightly in the numerical example. Central parking land rises from 0.0937 to 0.1650 in step with the rise in n_{s1}, but the changes in q_c and n_c now fail to exactly offset this increase, with production land (which accounts for about 2/3 of central use) falling slightly from 0.6602 to 0.6559, which reduces output. Given $f'' < 0$, the lower land input raises central land rent from 1.5333 to 1.5414 and thus the cost of an employee parking space, which rises from 0.3067 to 0.3083. The employee-paid parking cost represents about 8% of the pre-switch wage of 4.0494, while after the switch, the higher cost is deducted from the wage, which falls to 3.7358. The road-capacity tax, which is negative as predicted, becomes more negative after the switch, falling from -0.0621 to -0.2624 (from 1.4% to 6.0% of the wage plus differential land rent).

Net income for each worker, given by the expression in (8), falls from 4.6449 to 4.5935 after the switch, with the larger toll rebate mostly offsetting the decline in the wage (the rental-income component, equal to $f' - 1$, rises slightly). This 1.1% drop in income translates into reductions in nonland consumption, with e_c falling from 2.7869 to 2.7561 and e_s falling from 2.3489 to 2.3181. Reinforced by the declines in q_c and q_s noted above, the upshot is a decline in utility from 5.2929 to 5.2837.

Even though resource reallocation is fairly dramatic in response to the switch to employer-paid parking, the net effect on consumer welfare, reflected in the 1.1% net income loss, is modest. Other major urban distortions, however, produce welfare effects with similar orders of magnitude. For example, Brueckner (2007) shows that imposition of congestion tolls in a realistically calibrated monocentric city produces a welfare gain worth only 0.7% of income. Borck and Brueckner (2016) find similarly modest welfare gains from imposing optimal emissions taxes in a monocentric city. It is worth noting, however, that the welfare loss is much larger when expressed as a percentage of the original expenditure on parking. Dividing the income loss by the initial parking cost of 0.3063 shows that the welfare loss relative to parking cost is almost 17%.

As noted in Proposition 1, the mode-choice effects of the switch to employer-paid parking are much less dramatic when k_1 is held fixed at the first-best level instead of being optimally adjusted, as seen in the third column of Table 2. Rather than dropping to 0.0247, transit ridership only declines to 0.2371. In addition, the increase in suburbanization is only about one-third as large, with n_s rising to 0.8127 rather than to 0.8498. The decline in net income is also smaller. However, since urban road investments in the US have been made conditional on the mode choices spurred by employer-paid parking, the comparison that allows adjustment of

k_1 would appear to be the more relevant one in gauging the effects of this policy. Hence, the proper focus is on the more-dramatic impacts in the second column of Table 2.

As for sensitivity analysis, a main finding is that a reduction in the size of the n_{s1} exponent in the t function can produce a mode-choice corner solution under employer-paid parking, with all commuters using the road. Other less notable changes occur in response to other parameter changes, while preserving the main features of the outcomes shown in Table 1.

An interesting aspect of the results is that the utility level with employer-paid parking is smaller when k_1 is adjusted than when it is fixed (5.2837 vs. 5.2932). This relationship may at first appear surprising, but it illustrates the principle that, in the presence of a distortion, satisfaction of the remaining conditions for first-best optimality (namely, the road capacity condition (5)) may not be desirable. In effect, adjustment of k_1 reinforces the inefficient switch to auto use rather than restraining it. As a result, adjusting k_1 to satisfy (5) is worse than keeping it fixed when switching to employer-paid parking. However, continued satisfaction of this standard cost-benefit condition would appear to be a realistic description of decision-making in response to employer-paid parking, hence the relevance of this case.

6. Extensions

6.1. Introducing agglomeration economies

The analysis of section 4 showed that the switch to employer-paid parking had no effect on CBD output, with parking-induced pressure on central land fully offset by a decline in the size of the residential area via greater suburbanization. As a result, the loss of CBD "vitality" mentioned in the introduction did not arise, and a similar conclusion emerged in the simulation of section 5, where CBD output fell only slightly with a switch to employer-paid parking.

This section of the paper asks whether these conclusions are altered in the presence of agglomeration economies, which are absent from the basic model. To capture agglomeration, the production function $f(\ell)$ is replace by $a(\ell)f(\ell)$, where the factor $a(\ell)$ captures external economies of scale ($a'(\ell) > 0$ holds). The resulting agglomeration effects can be viewed as arising from the interplay of two forces. Greater CBD output via a higher ℓ makes all firms more productive, but a higher ℓ means a lower employment density (fewer workers per acre), which could reduce output by lowering the spatial concentration of production. With $a'(\ell) > 0$, the first effect is assumed to dominate.

Let $h(\ell) \equiv a(\ell)f(\ell)$, with $h' = af' + a'f$ and $h'' < 0$ assumed to hold. Then, it is easy to see that the planning solution with agglomeration economies is characterized by the previous first-order conditions with f' replaced by h'. As a result, (2) is replaced by $v'(q_c) = h'$, indicating that h' represents central land rent under decentralization, given that it is equated to the marginal utility of q. For h' to represent land rent, decentralization of the optimum requires subsidization of land-use by firms, correcting their failure to account for agglomeration effects. To see this conclusion, let r_c denote central land rent, with the choice of ℓ by firms yielding $f' = r_c$ in the previous model without agglomeration economies. Letting ν denote a subsidy per unit of land, the equivalent condition with agglomeration effects would become $af' = r_c - \nu$, recognizing that firms ignore agglomeration effects in choosing their land inputs. For h' to represent land rent, the subsidy must satisfy $\nu = a'f$ (after substituting h' for r_c), so that ν captures the external benefit of greater land usage.

Thus, decentralization of the first-best allocation with agglomeration economies requires all the previous steps (employee-paid parking, congestion tolls, etc.) along with subsidization of firm land inputs. Aside from the replacement of f' with h', the only other change in the previous equilibrium conditions is the need for a tax to finance the central land-rent subsidy. The total outlay on the subsidy, equal the required tax revenue, is $\ell a'f$, land area times the subsidy per unit of land. But once this tax is incorporated in the aggregate budget constraint, that constraint becomes identical to the previous one, with h replacing f. This conclusion follows because the wage is now equal to $h - \ell(h' - a'f)$, or output h minus $\ell(h' - a'f)$, which equals the land-rent outlay less the subsidy. When the tax $a'f$ is subtracted to get the net-of-tax wage, this expression cancels, so that the wage minus the tax is $h - \ell h'$, the same expression that would result if h rather than f were the production function. With f' in the other equilibrium conditions replaced by h', the upshot is that the decentralized first-best allocation with agglomeration economies can be generated by replacing f in the no-agglomeration case with h. In other words, with the agglomeration externalities internalized via the land-rent subsidy, the outcome is the same as the one where firms view h rather than

f as their own production function.

Given this equivalence, the analysis of the switch to employer-paid from section 4 (including Proposition 1) is unaffected, so that it applies equally well to the agglomeration case, provided that a land-rent subsidy is paid. Note that since ℓ remains constant in this quasi-linear case, the switch to employer-paid parking has no effect on the generation of agglomeration economies.

Analogous conclusions apply to the previous simulation analysis. For example, suppose that $a(\ell) = \ell^{0.1}$ and $f(\ell) = 5.5\ell^{0.1}$. Then $af = h = 5.5\ell^{0.5}$, so that h is the same as the previous f function (see Table 1). Given the previous discussion, the simulation results in Table 2 then show the employee- and employer-paid parking outcomes in the presence of agglomeration economies, assuming that $a(\ell)$ and $f(\ell)$ take the given forms and that a land-rent subsidy is present.

In a model with intercity migration, employer-paid parking could have larger effects on CBD production and agglomeration. In this setting, the reduction in utility caused by the switch to employer-paid parking would lead to outmigration and a reduction in the city's work force, amplifying the small output loss seen in Table 2 and generating a loss of agglomeration economies. However, if the switch to employer-paid parking occurs in all cities, reducing utility everywhere, no migration is generated and the current effects apply.

6.2. Alternative modeling of public transit scale economies

In the model of section 3, economies of scale in public transit arose through the presence of a fixed cost. An alternative approach to generating scale economies would suppress the fixed cost but assume a declining marginal cost of capacity. Capacity costs would be written $\Gamma(k_2)$, with $\Gamma'' < 0$ and n_{s2} again substituted for k_2. The RHS of the mode-choice condition (4) would be replaced by $\alpha + \Gamma'(n_{s2})$, and the tax to cover F would be replaced by a tax of $\Gamma(n_{s2}) - n_{s2}\Gamma'(n_{s2})$ to cover the deficit resulting from marginal-cost pricing, which is required for an optimum (the transit fare equals Γ').

Since the RHS of (4) is no longer a constant under this modification, the simplicity of the analysis of section 4 disappears despite the presence of quasi-linear preferences. Numerical simulation is then the only path to investigating the effects of employer-paid parking. But with the RHS of (4) no longer constant in the quasi-linear analysis, the mathematical conditions that prevented an interior mode-choice outcome under a ratio form for the $t(\cdot)$ function no longer prevail, possibly reducing the need for a large gap between the exponents of the function in the numerical example.

Even though a mode-choice corner solution may seem easier to avoid, falling marginal costs in public transit should lead to more-exaggerated swings in the numbers of road and transit users under the switch to employer-paid parking. The reason is that, as commuters shift from transit to the road, transit marginal cost (and hence the fare) rises, increasing the incentive to abandon transit relative to the case where the fare stays the same. Since this effect tends to increase the likelihood of mode-choice corner solutions at the same time as the mathematical pressure for such a solution is relieved, the net effect on the feasibility of a ratio form for t is unclear.

It turns out, however, that interior mode-choice solutions can indeed be achieved with a smaller τ_1 value than before (1.5 as opposed to the previous 4.0 value), which gives the $t(\cdot)$ function an approximate ratio form (recall from Table 1 that the other exponent is -1.0). In specifying public-transit costs, the user-cost α is raised from 0.5 to 0.7, and the $\Gamma(n_{s2})$ function is set equal to $0.00005n_{s2}^{0.05}$, indicating strong economies of scale (capacity rises slowly with ridership). A final parameter change is that road capacity cost γ_1 is raised from 0.1 to 0.23.

The numerical example based on these assumptions is shown in Table 3, with the employer-paid parking case shown only with k_1 adjusted. As can be seen, the mode-choice shift with the switch to employer-paid parking is more dramatic than before, as predicted. The number of road users is below 0.02 with employee-paid parking (in contrast to the earlier value of 0.4638) but grows to 0.7563 after the switch, with transit users falling from 0.7123 to 0.0780. Suburban population growth in this new example, near 0.1, is about double that in Table 2, and the drop in income (which measures the welfare cost of the switch) is 3.3%, triple the previous value. All the other effects of the switch to employer-paid parking are qualitatively the same as in Table 2. Thus, the assumption of declining marginal capacity cost in public transit leaves the main qualitative conclusions of the previous example unchanged, while altering some of the quantitative magnitudes.

7. Conclusion

This paper has analyzed the effect of employer-paid parking on aspects of urban form. The analysis shows that, to generate the social optimum, auto commuters should pay their own parking costs while also paying congestion tolls. In switching from this first-best regime of employee-paid parking to one of employer-paid parking, commuters dramatically shift from public transit to road usage. This sizable shift spurs a large increase in road capacity, but the cost of road usage ultimately rises by enough to eliminate the gain from the absence of parking costs. Since the mode shift from transit to auto use requires more parking land, leaving less for other uses, some central residents relocate to the suburbs. All of these changes are inefficient, leading to a city with too much road usage, too much road investment, and too much suburbanization, effects that reduce the urban utility level. The paper argues that these distortions are encouraged by nontaxability of the fringe benefit of employer-paid parking, which has spurred its adoption. Therefore, this seemingly innocuous feature of the tax code may underlie the major distortions in the urban economy identified by the analysis.

The analysis shows that, in the absence of income taxes, employer-paid parking's undesirable effects can be exactly reversed by a cash-out policy like the one mandated in California. Such a policy is equivalent to an arrangement where employers give all workers a stipend that can be used to cover parking costs or spent elsewhere, effectively restoring the employee-paid parking regime and thus the efficiency of the urban equilibrium.

Extensions of the model could make the provision of central parking more realistic. One easy extension would replace surface parking with underground parking, which uses no land but entails a high capital cost per parking space. In the model of section 4, it is easy to see that a switch to employer-paid parking in the underground case leads to familiar inefficient increases in road usage and capacity while having no effect on suburbanization. Following Brueckner and Franco (2016), the case of structural parking could also be analyzed, recognizing that a parking structure of a greater height corresponds to an increase in parking efficiency, but at the cost of greater capital investment, which would be chosen optimally.

The absence of public-transit crowding is another assumption that could be relaxed. Like the assumption of decreasing transit marginal costs, this change would prevent the derivation of analytical results. Intuitively, however, the presence of crowding would attenuate the increase in road use with a switch to employer-paid parking, given that transit would become less congested and more attractive as its usage falls. Nevertheless, the main qualitative conclusions of the analysis are likely to be unaffected.

Finally, the mode choice between transit and road usage could be modified by including idiosyncratic consumer preferences for particular modes. Following Borck (2017), a logit model could then capture the split between the two modes, eliminating the knife-edge feature of the current choice setup.

Appendix

A1. First-order conditions

The first-order conditions for the planning problem are as follows:

$$u: \quad 1 \; - \; (\theta_c + \theta_s) \; = \; 0 \tag{a1}$$

$$e_c: \quad \theta_c \; - \; \delta n_c \; = \; 0 \tag{a2}$$

$$e_s: \quad \theta_s \; - \; \delta n_s \; = \; 0 \tag{a3}$$

$$q_c: \quad \theta_c v'(q_c) \; - \; \delta n_c f' \; = \; 0 \tag{a4}$$

$$q_s: \quad \theta_s v'(q_s) \; - \; \delta r_a n_s \; = \; 0 \tag{a5}$$

$$n_c: \quad -\lambda \; - \; \delta(q_c f' + e_c) \; = \; 0 \tag{a6}$$

$$n_s: \quad -\lambda \; - \; \delta(e_s + r_a q_s) \; - \; \psi \; = \; 0 \tag{a7}$$

$$n_{s1}: \quad -\delta(t_1 + n_{s1} t^n) \; - \; \phi \; + \; \psi \; = \; 0 \tag{a8}$$

$$n_{s2}: \quad -\delta(\gamma_2 + \alpha) \; + \; \psi \; = \; 0 \tag{a9}$$

$$k_1: \quad -\delta(\gamma_1 + n_{s1} t^k) \; = \; 0 \tag{a10}$$

$$P: \quad -\delta(\beta f' + r_a \beta) \; + \; \phi \; = \; 0. \tag{a11}$$

A2. Sign of $\partial e/\partial \mu$

To maintain equal utilities, e_c and e_s must change by the same amount as μ is altered, with the common change denoted by $\partial e/\partial \mu$. Differentiation of the resource constraint with respect to μ, yields, after cancelling terms,

$$\begin{aligned} -\frac{\partial e}{\partial \mu} &= (t + n_{s1} t^n - \alpha - \gamma_2)\frac{\partial n_{s1}}{\partial \mu} \; - \; (\alpha + \gamma_2 + e_s - e_c + r_a q_s)\frac{\partial n_c}{\partial \mu} \\ &= (-\mu \beta f')\overline{q}_c z/D \; + \; (\alpha + \gamma_2 + e_s - e_c + r_a q_s)\beta z/D, \end{aligned} \tag{a12}$$

where (9) is used to substitute for the derivatives of n_{s1} and n_c, and where (4) (with μ appended) is used to substitute for the first term in the first line of (a12). Using (6), (a12) reduces to

$$
\begin{aligned}
-\frac{\partial e}{\partial \mu} &= (\alpha + \gamma_2 + e_s - e_c + r_a q_s - \mu f' \overline{q}_c)\beta z/D \\
&= (1-\mu) f' \overline{q}_c \beta z/D \quad < \quad 0. \qquad (a13)
\end{aligned}
$$

Thus, $\partial e/\partial \mu > 0$ holds, so that a reduction in μ reduces e_c and e_s.

A3. Budget constraints under decentralization

The income side of the budget constraint equals total wages plus differential land rent minus a tax to pay for the road capacity deficit (the tax is negative in the case of a surplus) and a tax to cover the fixed cost F of the transit system. Recalling that the labor force size equals 1, total income is then

$$f \;-\; (1 - \beta n_{s1} - n_c q_c) f' \;+\; f' \;-\; r_a \;-\; (\gamma_1 k_1 - n_{s1}^2 t^n) \;-\; F, \qquad (a14)$$

where $f' - r_a$ is differential land rent (recall that the central land area equals 1) and $n_{s1}^2 t^n$ is toll revenue. Note that since profit of CBD producers is zero, there is no corresponding income term.

Total expenditures are given by

$$r_a(q_s n_s + \beta n_{s1}) + f' n_c q_c + n_{s1}\beta f' + n_{s1}(t + n_{s1} t^n) + (\alpha + \gamma_2) n_{s2} + F + n_c e_c + n_s e_s, \quad (a15)$$

where the second and third terms are rent outlays and total parking costs in the center, and where the fourth term includes congestion tolls. Setting income equal to expenditures, and rearranging, it is easily seen that the resulting equality reduces to the aggregate resource constraint from (1).

To write the individual budget constraints, note that since the population size L equals 1, (a14)) also gives individual income. The expenditure sides of the individual constraints are

$e_c + f'q_c$ for central residents, $e_s + r_a q_s + \alpha + \gamma_2$ for suburban transit users, and $e_s + r_a(q_s + \beta) + t + n_{s1}t^n + \beta f'$ for suburban road users.

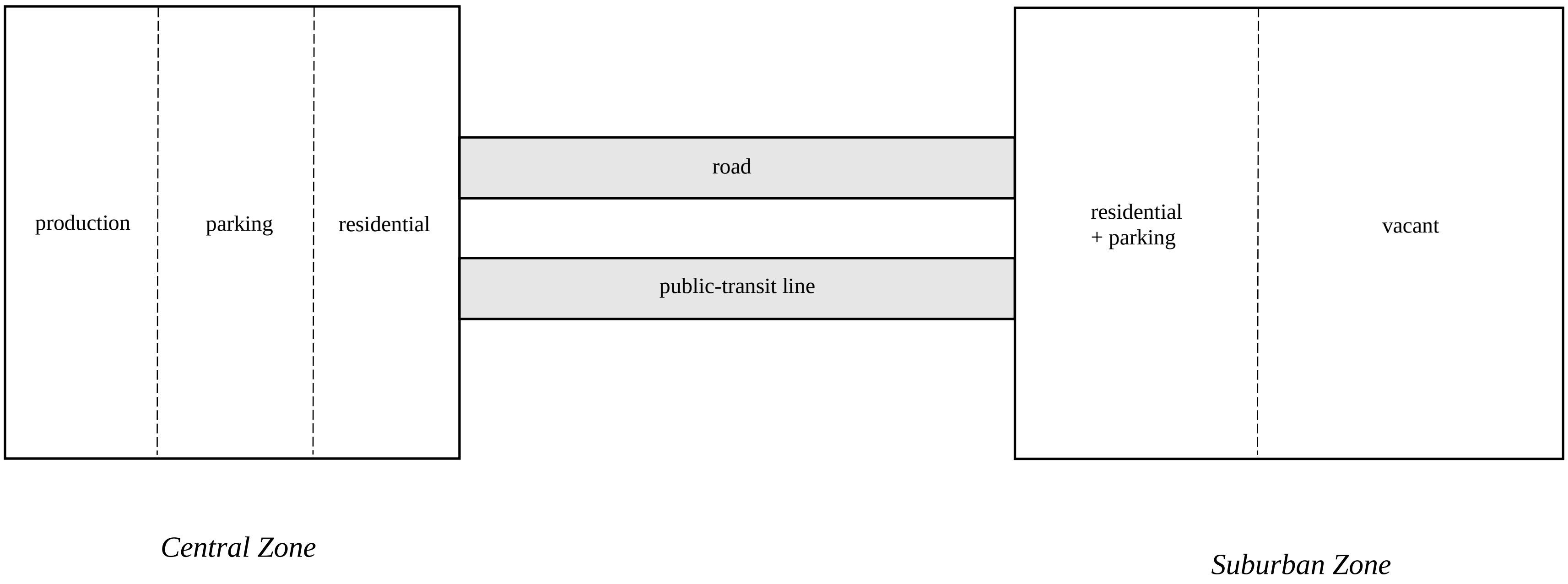

Figure 1: City Map

Figure 2: Transportation mode choice for different values of the employee's parking cost share (μ)

Table 1: Functional Forms and Parameter Values

Utility function	$\sigma_0\, q^{\sigma_1} e^{1-\sigma_1}$
σ_0	100.0
σ_1	0.4
Production function	$\rho_0\, \ell^{\rho_1}$
ρ_0	5.5
ρ_1	0.2
Road user cost	$\tau_0\, n_{s1}^{\tau_1} k_1^{-\tau_2}$
τ_0	0.2
τ_1	4.0
τ_2	1.0
Road capacity cost (γ_1)	0.1
Transit user cost (α)	0.5
Transit capacity cost (γ_2)	0.23
Parking efficiency (β)	0.2
Agricultural rent (r_a)	1.0

Table 2: Effects of Employer-Paid Parking

	Parking cost paid by		
	Employee ($\mu = 1$)	Employer ($\mu = 0$)	
		k_1 *adjusted*	k_1 *fixed*
Variable			
Road users (n_{s1})	0.4638	0.8251	0.5756
Transit users (n_{s2})	0.3324	0.0247	0.2371
Suburban population (n_s)	0.7961	0.8498	0.8127
Road capacity (k_1)	0.2072	0.8746	0.2072
Central land consumption (q_c)	1.2117	1.1921	1.2060
Suburban land consumption (q_s)	1.5660	1.5454	1.5600
Parking land (βn_{s1})	0.0928	0.1650	0.1151
CBD production land (ℓ)	0.6602	0.6559	0.6590
Central land rent (f')	1.5333	1.5414	1.5356
Central parking cost ($\beta f'$)	0.3067	0.3083	0.3071
Wage (after subtraction of any employer-paid parking cost	4.0494	3.7898	3.8711
Road-capacity tax	−0.0621	−0.2624	−0.2234
Net income (wage + diff. rent − capacity tax)	4.6449	4.5935	4.6301
Central nonland consumption (e_c)	2.7869	2.7561	2.7781
Suburban nonland consumption (e_s)	2.3489	2.3181	2.3401
Utility	5.2969	5.2837	5.2932

Table 3: Effects with Declining Marginal Capacity Cost in Transit

	Parking cost paid by	
	Employee	Employer (k_1 *adjusted*)
Variable		
Road users (n_{s1})	0.0183	0.7563
Transit users (n_{s2})	0.7123	0.0780
Suburban population (n_s)	0.7306	0.8343
Road capacity (k_1)	0.0063	0.6577
Central land consumption (q_c)	1.2090	1.1516
Suburban land consumption (q_s)	1.5508	1.4909
Parking land (βn_{s1})	0.0036	0.1513
CBD production land (ℓ)	0.6706	0.6579
Central land rent (f')	1.5143	1.5377
Central parking cost ($\beta f'$)	0.3029	0.3075
Wage (after subtraction of any employer-paid parking cost	4.0621	3.8139
Road-capacity tax	−0.0007	−0.0756
Public-transit tax	0.00005	0.00004
Net income (wage + diff. rent − capacity tax)	4.5770	4.4272
Central nonland consumption (e_c)	2.7462	2.6563
Suburban nonland consumption (e_s)	2.3262	2.2363
Utility	5.2872	5.2478

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Footnotes

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[1]This value comes from GIS calculations with data provided by the Los Angeles County GIS Data Portal for the boundaries in 2014 of county parking lots 5,000 square feet or larger associated with commercial, industrial and government properties.

[2]See Shin, Vuchic and Bruun (2009).

[3]More recent data from the Society for Human Resource Management (2013) shows 87% of employers offering free parking.

[4]The 2013 share of solo drives was 77 percent (McKenzie 2015a).

[5]Most theoretical work on the pricing of parking focuses on the efficiency of second-best pricing of parking spots in the absence of congestion tolls (Arnott et al. 1991, Arnott and Rowse 1999, Verhoef et al. 1995, Calthrop et al. 2000, Anderson and de Palma 2004). See Inci (2015) for a survey. Brueckner and Franco (2016) present a spatial analysis of residential as opposed to employee parking, focusing on the choice among different parking technologies (surface, structural and underground) and on the effect of minimum parking requirements. In work more closely related to the present paper, Ersoy, Hasker and Inci (2016) provide an analysis of the interaction of mode choice and provision of parking in the shopping-center context, with some shoppers accessing the mall by car and others by public transit.

[6]See also Borck and Wrede (2005).

[7]While auto users living in the suburbs benefit from the reduction in commuting costs, public-transit riders living near the CBD also benefit because of the subsidy's negative impact on central land rents.

[8]The city in Voith's model is open, nonspatial, and includes agglomeration economies in the CBD, but it has an auto/transit mode choice that resembles the present one, with all commuters indifferent between the modes in equilibrium. The effects of the parking tax are generally ambiguous.

[9]If payroll taxes were present, then the reduction in the wage under employer-paid parking would cause the firm's payroll-tax liability to fall, a benefit that would make the required reduction in the wage smaller, as follows. Letting the payroll tax rate equal ξ, the labor cost per worker with employer-paid parking is $\widetilde{w} + \xi\widetilde{w} + \rho B$, where $\widetilde{w}$ is the wage with employer-paid parking and $\xi\widetilde{w}$ is the payroll tax liability. The wage must adjust to make this labor-cost expression equal to the original expression, $w + \xi w$. Setting the two expressions equal yields $\widetilde{w} = w - \rho B/(1+\xi)$, so that the wage falls by less than ρB.

[10]If employer-paid parking were taxable, then the first of these after-tax income expressions would be reduced by $-\kappa B$. If $\rho = 1$, then two income expressions would be identical, indicating that taxing the fringe benefit removes the advantage it offers. But the expression with employer-paid parking remains larger in the presence of $-\kappa B$ when $\rho < 1$.

[11]Although no data on this point are available, firms may target the provision of employer paid parking to the highest paid workers, who value the nontaxable nature of the fringe benefit most. Incorporating different income groups in the model would be impractical, however.

[12]van Ommeren and Wentink (2012) estimate the welfare loss from overconsumption of parking due to the nontaxability of the fringe benefit. They estimate the demand and supply curves for parking and compute the area of the surplus triange lost due to overconsumption, a result of the price being depressed by a factor equal to one minus the marginal tax rate. By contrast, our numerical approach computes the general-equilibrium welfare loss under the realistic assumption that employees perceive employer-paid parking as free, rather than available at a reduced price.

[13]This assumption means that residential parking space in the center is unrealistically absent. The paper also does not include parking space used by shoppers and other drivers making nonwork trips. Incorporating these elements, however, would make the model intractable.

[14]For a comprehensive recent discussion of crowding in public transit, see Tirachini, Hensher and Rose (2013). For an analytical model, see de Palma, Lindsey and Monchambert (2015).

[15]Note that congestion spillovers between the modes are absent. This assumption may be unrealistic for bus transit, which shares congested roads with cars.

[16]This pattern is likely in cities with efficient public-transit systems, with households renting or borrowing a car for occasional trips out of the city. For example, New York City is distinguished from other American cities by low private auto ownership and heavy use of public transit. While nearly 86 percent of American workers drive to work, 4 out of every 5 rush-hour commuters to New York City's CBDs avoid traffic congestion by taking transit service (Metropolitan Transportation Authority (2016), McKenzie (2015b)). In cities with

less-efficient public-transit systems, both central and suburban households may own a car, but because of congestion, gas prices or even parking costs, households may still decide to commute to work by an alternative less-expensive transportation mode. Such a scenario can be easily accommodated in our framework by assuming that central residents and suburban transit users would also need βP worth of land for residential parking. However, none of our qualitative results would change.

[17]The planning problem assumes common consumption levels for suburban residents regardless of their mode choice. Allowing these levels to differ in the problem's setup would introduce additional notation without any effect, since the first-order conditions would imply common consumption levels.

[18]Adding appendix equations ($a2$) and ($a3$) and using $n_c+n_s = 1$, it follows that $\delta = 1$, $\theta_c = n_c$, and $\theta_s = n_s$. Substituting, ($a4$) and ($a5$) reduce to (2) and (3).

[19]Eliminating ϕ and μ in ($a8$) using ($a9$) and ($a11$), the condition reduces to (4).

[20]Combining (a6) and (a7) to eliminate λ and substituting for μ from (a9) yields (6).

[21]Note that if the toll amount were added to the cost of central parking, there would be no need for explicit congestion tolls.

[22]Convexity is required in the usual road-use optimization problem, where the goal is to maximize consumer benefit (equal to $\int_0^n D(z)dz$, with D giving demand) minus user costs $nt(n,k)$ and capacity costs γk. Convexity of $nt(n,k)$ is required for concavity of this objective function.

[23]Given $t^k < 0$, a sufficient condition for this outcome is $t^{kn} < 0$, which says the effect of a higher n_{s1} on the individual user cost is smaller the larger is k.

[24]In another scenario that may be more realistic, tolls are absent but road capacity is adjusted to satisfy the optimality condition (5). In this case, $n_{s1}t^n$ disappears from (4), leading to disappearance of the t^{nn} and t^{kn} terms in the second row of the matrix in (10) (the 2 factor in the first term also becomes 1). These changes make the determinant of the 2×2 matrix analogous to M ambiguous in sign, implying that none of the comparative-static derivatives analogous to those in (11) can be signed (the determinant equals $n_{s1}t^n t^{kk} - t^k(t^k + n_{s1}t^{kn})$). This ambiguity applies beyond the context of this particular model, arising in any situation where $t(n,k)$ equals a constant a due to the availability of another fixed-cost mode and $-nt^k$ equals a constant capacity cost b. Comparative statics of this two-equation system with respect to either a or b are ambiguous because the determinant of the system's matrix

of derivatives (equal to the above expression, with n replacing n_{s1}) has an ambiguous sign.

[25] If the two sides of (4) happen to be equal, then any division of commuters between the modes (including the corner solutions) is optimal. It should be noted that, with a ratio form for t, the Hessian determinant H and thus the determinant D can be shown to equal zero, yielding degenerate derivatives in (9).

[26] It can be shown that, in the presence of income taxes, the cash-out policy is fully efficient only if employer-paid parking is a taxable fringe benefit. In the absence of this feature, a cash-out will still be welfare improving.

[27] In some areas, the normal size can be as high as 9-by-19 feet. The size of parking spaces is mandated by local zoning or land-development ordinances and is based on typical use. The lower the turnover, or the more urban the location, the smaller are the spaces that can be tolerated by users. On the other hand, areas with high turnover that are less urban will generally have larger spaces.

[28] The negative sign of the change in transit ridership (which was ambiguous by (12)) follows because the chosen β is smaller than q_c.