A Theory of Liquidity Spillover Between Bond and CDS Markets

Batchimeg Sambalaibat*

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I build a dynamic search model of bond and credit default swap (CDS) markets and show that shorting bonds through CDS contracts increases liquidity of the underlying bonds. This result departs from existing theory, which argues that derivatives attract traders and liquidity away from the underlying market. I conclude the opposite by endogenizing the aggregate number of traders. My result suggests that by banning CDS during the 2010–2012 European debt crisis, regulators in Europe decreased liquidity in sovereign bond markets, reduced bond prices, and, as a result, increased sovereigns' borrowing costs when they intended to achieve the opposite and quell the debt crisis.

Keywords: credit default swaps, funding liquidity, search costs, over-the-counter markets.

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1 Introduction

This paper proposes a novel theory on how short positions through derivatives affect the underlying asset. Existing theories predict that derivatives fragment investors across the derivative and underlying markets and, as a result, attract liquidity away from the underlying market.[1] They predict this keeping the aggregate number of investors fixed. I build a dynamic search model of derivative and underlying markets and show that when the aggregate number of investors is instead endogenous, the result reverses. Short positions through derivatives increase liquidity of the underlying asset. I refer to this as a liquidity spillover effect.

I show the result in the context for bond and credit default swap (CDS) markets. It works as follows. Introducing short positions through CDS contracts attracts into credit markets not only investors who want to short the underlying credit risk and buy CDS, but also investors who want to take the opposite side and long the underlying credit risk. In turn, long investors—for whom buying bonds and selling CDS are economically similar positions—search and trade at the same time as bond buyers. They do this to expand their trading opportunities and to alleviate their search frictions. The result is an increase in bond market liquidity. The number of bond buyers, the bond turnover, the trading volume, and the speed it takes to sell all increase. The increase in liquidity, in turn, increases the bond price.

The data supports my theory. Sambalaibat (2014) documents that when the European Union banned in 2011 naked CDS purchases referencing European sovereigns, liquidity of the underlying bonds deteriorated.[2] In my model, banning naked CDS positions is equivalent to shutting down CDS trading, which in the model just reverses the spillover effect. Investors can no longer sell CDS because their counterparties are banned from buying CDS. Long investors exit the CDS market, but by exiting the CDS market, they also pull out from the bond market. The result is a decrease in bond market liquidity as observed after the ban. Thus, preventing investors from shorting ultimately ends up banning investors who want to take the opposite side and long the asset.

I show the liquidity spillover effect with a model that builds on Duffie, Garleanu, and Pedersen (2005, 2007) and, in particular, on Vayanos and Weill (2008). A fraction of bond owners

[1] Using Kyle (1985) framework, Subrahmanyam (1991) and Gorton and Pennacchi (1993) show that stock index futures and security baskets, respectively, lower liquidity of the underlying stock market because some traders migrate to the derivative markets. John, Koticha, Subrahmanyam, and Narayanan (2003) show that options have a similar effect on stock market liquidity using Glosten and Milgrom (1985) framework.

[2] Naked CDS purchases refer to CDS purchases in which the CDS buyer does not own the underlying bonds. They constitute a short position on the underlying bonds.

experience a liquidity shock and subsequently have to sell their bonds. They search for a buyer, and when they find one, the difficulty of finding another one forces them to sell their bond at a discount. Search frictions, as a result, create an illiquidity discount in the bond price. The illiquidity discount, the expected search times, and the volume of trade depend on the relative number of sellers and buyers. The number of buyers and sellers are, in turn, endogenous.

I extend this standard search framework in two ways. First, I add CDS contracts. CDSs pay when the underlying bond defaults. A CDS buyer—who benefits if the bond defaults—is short the underlying credit risk. A CDS seller has the opposite long exposure. CDSs are in zero net supply, while bonds are in fixed supply. As with trading bonds, investors search and bargain with a CDS counterparty. Second, I endogenize the aggregate number of investors by endogenizing their entry.[3]

With this model, I offer three contributions. First, I provide a novel insight on how derivatives affect the market for the underlying asset. The insight applies, beyond derivatives, to any mechanism that expands the set of feasible allocations in the economy (e.g., tradable securities and contracts, trading mechanisms and venues, private currencies).

Second, I provide the first theoretical framework of over-the-counter (OTC) trading in both the underlying and derivative markets. In existing microstructure models of derivatives, illiquidity arises from asymmetric information.[4] We lack models of derivatives in which illiquidity arises from a key friction in trading assets over-the-counter: search costs. In the context of OTC traded assets, search models are the current workhorse environment of endogenous liquidity frictions and asset prices. But so far, they feature either a single asset or multiple assets with identical cash flows.[5] We lack models of multiple OTC traded assets in which one asset is a derivative of the other. Models specific to CDS either feature exogenous trading costs, as in Oehmke and Zawadowski (2015), or abstract from bond trading, as in Atkeson, Eisfeldt, and Weill (2015). We thus need a model that features trading in both bonds and CDS, endogenous trading costs, and an endogenous feedback between the two assets. My framework fills each of these gaps.

Third, I shed light on naked CDS purchases and fill a gap in the CDS literature that focuses

[3]Afonso (2011) and Lagos and Rocheteau (2009) also endogenize entry but in a single market search model.

[4]In addition to Subrahmanyam (1991), Gorton and Pennacchi (1993), and John, Koticha, Subrahmanyam, and Narayanan (2003), Back (1993) studies how options affect the volatility of the underlying asset, while Biais and Hillion (1994) study how options affect the price informativeness of the underlying asset.

[5]Single asset frameworks include Duffie, Garleanu, and Pedersen (2005, 2007), Lagos and Rocheteau (2009), Neklyudov (2012), Hugonnier, Lester, and Weill (2014), Shen, Wei, and Yan (2015), Neklyudov and Sambalaibat (2016), and Uslu (2016). Multiple asset frameworks include Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008).

on covered CDS purchases—CDS purchases in which the protection buyer owns the underlying bonds (see, for example, Thompson (2007), Arping (2014), Bolton and Oehmke (2011), Sambalaibat (2012), and Parlour and Winton (2013)). Allowing investors to trade the issuer's credit risk without trading the bonds is what defines CDS, why they proliferated, and why they were controversial.

The paper is organized as follows. Section 2 presents the model environment, Section 3 characterizes the equilibrium, prices, and liquidity measures, and Section 4 presents the main result. Section 5 shows that the spillover effect is robust to two alternative specifications (in one, investors can short-sell; in another, investors optimally choose their search efforts). I also contrast the spillover effect with existing results on short sales. Proofs are relegated to the appendices.

2 Model Environment

Time is continuous and goes from zero to infinity. Agents are risk-averse, live infinitely, have idiosyncratic stochastic endowments, and can invest in a risk-free asset with return $r > 0$. They hold and trade bilaterally a risky bond and a CDS contract with a cash flow based on the risky bond. Finding someone to trade with involves search. Agents enter the economy if doing so makes them better off. This is the model in a nutshell; the rest of this section elaborates.

2.1 Assets

The bond is a perpetuity that occasionally comes short of its promised cash flow. I define such occasions as a default. In particular, the bond has supply S, trades at price p_b, and has a cumulative cash flow process $D_{b,t}$ satisfying

$$dD_{b,t} = \delta dt - JdN_t, \tag{1}$$

In (1), $\delta > 0$ is the promised rate of the coupon flow, $\{N_t, t \geq 0\}$ is a Poisson counting process with an intensity parameter $\eta > 0$, and $J > 0$ is the size of the default. The process N_t counts the number of defaults in $[0, t]$, and its increment, dN_t, is 0 or 1. Thus, (1) says, in a small interval $[t, t + dt]$, with probability ηdt, the bond defaults and its cash flow decreases by J. Otherwise, it pays the coupon at the promised rate. Agents can hold 0 or 1 unit of the bond, and I denote their bond position with $\theta_b \in \{0, 1\}$. I assume that agents cannot short bonds, but

I relax this in Section 5.

In a CDS contract, the buyer pays a premium flow p_c to the seller; the seller, in return, pays the buyer J if the bond defaults. The CDS buyer's cumulative cash flow $D_{c,t}$, as a result, follows

$$dD_{c,t} = JdN_t. \tag{2}$$

Since this is perfectly negatively correlated with the bond cash flow, the CDS buyer has a short exposure to the underlying credit risk. Conversely, the CDS seller has a cash flow that is positively correlated with the bond $(-JdN_t)$ and is thus long credit risk. Herein, when I refer to a long or a short position, I will mean with respect to the underlying credit risk.[6] I denote an agent's CDS position with $\theta_c \in \{-1, 0, 1\}$, where each denotes a short, a neutral, and a long position, respectively. I restrict the net asset position to $0 \leq |\theta_b + \theta_c| \leq 1$, which rules out simultaneous long positions in both assets.

An investor terminates a CDS contract by paying their counterparty a fee. The fee is endogenous and is such that the nonterminating side is indifferent between (a) continuing the contract and (b) accepting the fee, searching for a new counterparty, and, upon a match, entering a new position. I assume that when the nonterminating side is indifferent, she accepts the fee and starts the process again. I denote with T_{S} and T_{B} the fees the seller and the buyer pay their respective counterparties.

2.2 Agents

Agents have time preference rate β and CARA utility preferences with risk aversion parameter α: $u(C) = -e^{-\alpha C}$. Agent i's cumulative endowment process $e_{i,t}$ follows

$$de_{i,t} = \mu_e \rho_{i,t} dt + \rho_{i,t} \sigma_e (-dN_t) + \sqrt{1 - \rho_{i,t}^2} \sigma_e dZ_t, \tag{3}$$

where $\mu_e > 0$ and $\sigma_e > 0$ are constants, Z_t is a standard Brownian motion, and $\rho_{i,t}$ is the instantaneous correlation process between the bond cash flow and the agent's endowment process. The processes $\{Z_t, \rho_{i,t}, N_t\}$ are pairwise independent. The correlation process $\rho_{i,t}$ is independent across agents and is a three-state Markov chain with states $\rho_{i,t} \in \{-\rho, 0, \rho\}$ where $\rho > 0$. Agents switch from the negative and positive correlation states to the uncorrelated state with

[6] Thus, a long position through the CDS market, for example, does not mean an investor has bought CDS but means she has sold CDS and is thus long exposed to the underlying default risk.

Poisson intensities γ_d and γ_u, respectively. The intensity of switching from the uncorrelated state to either the positive or negative correlation state is zero (the uncorrelated state is thus an absorbing state).[7]

The different correlation realizations across agents generate heterogenous private valuations for the underlying credit risk. As I show later in Proposition 1, an investor whose endowment is currently negatively correlated with the bond ($\rho_{i,t} = -\rho$) has the highest private valuation for the bond (hence, the most willing to buy it); those with an uncorrelated endowment ($\rho_{i,t} = 0$) have an intermediate valuation; and those with a positively correlated endowment ($\rho_{i,t} = \rho$) have the lowest valuation. This difference in valuations creates a motive for trade. In particular, a random change in an agent's valuation (due to a random change in her correlation) generates a need to trade and rebalance her portfolio. From hereon, I will refer to an agent with $\rho_{i,t} = -\rho$ as a high-valuation agent or "h" for short, with $\rho_{i,t} = 0$ as an average-valuation ("a") agent, and with $\rho_{i,t} = \rho$ as a low-valuation ("l") agent. I will denote the valuations with i where $i \in \{h, a, l\}$. Referring to agents according to their valuations is simpler than referring to their correlations.

2.3 Agents' Decisions

Agents first decide whether to enter the economy. At any point in time, fixed flows of agents F_h and F_l are born as high- and low-valuation agents, respectively. An endogenous fraction ν_i of them enter according to

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i, \end{cases} \tag{4}$$

where $i \in \{h, l\}$, $V_{i[0,0]}$ is the investor's continuation value upon entry, the subscript $[0,0]$ captures the fact that investors enter without an existing position, and O_i is the investor's fixed entry cost.[8] Thus, investors enter if the continuation value of doing so is at least greater than their

[7] In Section 3, as I describe how agents trade in equilibrium, I explain why one of the correlations is an absorbing correlation. In short, I do so to model both short positions and entry and exit.

[8] We can ignore the entry decision of average-valuation agents because, in equilibrium and as a result of an additional parameter condition, the continuation value of an average-valuation agent is zero: $V_{a[0,0]} = 0$. Thus, for any positive entry cost, O_a, their entry rate is zero. Moreover, the results depend not on the absolute levels of O_h and O_l but on their magnitudes relative to O_a (i.e., the model can be recast in terms of $O_h - O_a$ and $O_l - O_a$). Thus, without loss of generality, I set $O_a = 0$. As for O_h and O_l, the results hold for any O_h and O_l including when $O_h = O_l$. I denote them separately to allow for general values of O_h and O_l and to later show where the effects come from.

entry cost. The total flows of high- and low-valuation entrants, as a result, are $\nu_h F_h$ and $\nu_l F_l$, and their steady state masses are $\frac{\nu_h F_h}{\gamma_d}$ and $\frac{\nu_l F_l}{\gamma_u}$, respectively. I explain in Section 3.2 how agents exit.

Second, once in the economy, agents choose their consumption, C, and their bond and CDS position, $[\theta_b, \theta_c]$. I categorize agents into types $\tau \in \mathcal{T}$—where a type $\tau = i[\theta_b, \theta_c]$ specifies the agent's valuation $i \in \{h, a, l\}$ and asset position $[\theta_b, \theta_c]$—and recast their choice over asset positions as a choice over types. The entire set of feasible positions is $[\theta_b, \theta_c] \in \{[1,0],[0,1],[0,0],[0,\text{-}1],[1,\text{-}1]\}$. Due to search frictions, however, only a subset of this is feasible to any one agent type, and the subset changes if, for example, the agent finds a counterparty. So, I summarize the events affecting a type τ_t agent with a counting process $\hat{N}_t(\tau_t)$ and denote its dimension with $K(\tau_t)$ and the intensity associated with dimension k with $\gamma(k, \tau_t)$.[9] When an event associated with dimension k arrives, the agent chooses between types $\tau_t^{'} \in \mathcal{T}(\tau_t, k) \subset \mathcal{T}$. Thus, denoting with $U(W_t, \tau_t)$ the indirect utility of type τ_t agent with wealth W_t, the agent solves

$$U(W_0, \tau_0) = \max_{\{C_t \in \mathbb{R}, \tau_t^{'} \in \mathcal{T}(\tau_t, k)\}} \mathbb{E}\left[\int_0^\infty e^{-\beta t} u(C_t) dt\right], \tag{5}$$

subject to the wealth process,

$$dW_t = (rW_t - C_t)\, dt + de_t + dD_t^b \theta_{b,t} - p_b d\theta_{b,t} + (p_c dt - dD_t^c)\theta_{c,t}, \tag{6}$$

and the transversality condition, $\lim_{T\to\infty} \mathbb{E}[e^{-\beta T} e^{-\alpha r W_T}] = 0$.

2.4 Search and Matching

An agent wishing to rebalance her position initiates a match with another agent at Poisson arrival times with intensity parameter $\lambda/2$. The total volume of matches between any two agent types τ and τ', as a result, is $\lambda \mu_\tau \mu_{\tau'}$ (half of it initiated by τ and the other half by τ'), where μ_τ and $\mu_{\tau'}$ are their respective masses. Given the total volume, a type τ agent matches with a type τ' agent with total intensity $\frac{\lambda \mu_\tau \mu_{\tau'}}{\mu_\tau} = \lambda \mu_{\tau'}$. The corresponding expected search time, $\frac{1}{\lambda \mu_{\tau'}}$, as a result, has both an exogenous (λ) and an endogenous component ($\mu_{\tau'}$). Upon a match, if trading either the bond or CDS yields positive gains from trade and the resulting positions are feasible, they trade at mutually agreeable terms of trade (to be described in Section 3.3). The

[9] Appendix A explains the counting process in detail. It is a notation that helps characterize the agents' optimization problem. Later, as I characterize the equilibrium, I incorporate the events affecting an agent (hence, this process) directly into the equilibrium conditions.

matching intensity λ is exogenous for now, but I endogenize it in Section 5.3 and allow it to differ for bond versus CDS matches.

3 Equilibrium, Prices, and Liquidity

I start this section by characterizing the agents' continuation values. Then, as standard in the literature, I conjecture agents' optimal trading strategies. Doing so helps characterize who are the bond and CDS buyers and sellers and their masses. Then, using the continuation values and the conjectured trading strategies, I characterize prices, define the steady state equilibrium, and prove its existence in Theorem 1. Since this section is tedious, readers wishing to see the main result may skim it and proceed to Section 4 which contains the main result of the paper.

3.1 Continuation Values

As Proposition 1 shows, the continuation values, V_τ, arise from the optimization problem (5).

Proposition 1. *Solutions for $U(W,\tau)$ are of the form $U(W,\tau) = -e^{-r\alpha(W+V_\tau+\bar{a})}$, where $\bar{a} \equiv \frac{1}{r}\left(\frac{log(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2\right)$. The term V_τ is given by*

$$\begin{aligned} rV_\tau = &(\delta - \eta J + x_\tau)\,\theta_b - y|\theta_b| + (p_c - \eta J + x_\tau)\,\theta_c - y|\theta_c| \\ &+ \sum_{k=1}^{K(\tau)} \gamma(k,\tau) \max_{\tau' \in \mathcal{T}(\tau,k)} \frac{1}{r\alpha}\left(1 - e^{-r\alpha(V_{\tau'} - V_\tau + P(\tau,\tau'))}\right), \end{aligned} \tag{7}$$

where $x_\tau = x$ for a high, $x_\tau = 0$ for an average, and $x_\tau = -x$ for a low-valuation investor,

$$x \equiv r\alpha\rho\sigma_e\eta J, \tag{8}$$

$$y \equiv \frac{r\alpha}{2}\eta J^2 \tag{9}$$

and $P(\tau,\tau')$, given in (B8), is the instantaneous payoff of switching from τ to τ'.

Equation (7) illustrates how private valuations differ across agents. A long position (either buying the bond, $[\theta_b,\theta_c] = [1,0]$, or selling CDS, $[\theta_b,\theta_c] = [0,1]$) yields a high-valuation investor an extra flow utility of x compared to an average-valuation investor and $2x$ compared to a low-valuation investor. Similarly, a short position (buying CDS, $[\theta_b,\theta_c] = [0,-1]$) yields the most utility to a low-valuation investor and the least utility to a high-valuation investor. This difference in valuations results in the equilibrium trading strategies discussed in the next section.

The difference in valuations, $x = r\alpha\rho\sigma_e\eta J$, captures the benefit of sharing the endowment risk. It increases in the agents' risk aversion (α), the correlation between the agents' endowment and the bond (ρ), the endowment volatility (σ_e), and the bond default risk (both the default intensity, η, and the size of the default, J). The term $y = \frac{r\alpha}{2}\eta J^2$ affects both long and short exposures and in the same direction; thus, it captures a holding cost. It increases in the agents' risk aversion and the default risk (again, both the default intensity and the size of the default).

I assume that the risk aversion parameter α is small and linearize (7). See Duffie, Garleanu, and Pedersen (2007) and Vayanos and Weill (2008) for similar approximations.

3.2 Optimal Trading Strategies

I describe now and illustrate in Figure 1 the stages investors go through and the optimal actions at each stage (Theorem 1 proves that they are indeed optimal).

Investors enter the economy as high- or low-valuation investors and subsequently search for a counterparty. Upon finding a counterparty with whom trading is profitable, they bargain over the price, trade, and reach their optimal asset position. At any point, high- and low-valuation investors may experience a valuation shock. If they do, they optimally exit the economy. If the shock occurs before they were able to reach their optimal position, they exit immediately. But if it occurs after they have established a position, they unwind their position and then exit.

In particular, those entering as a high-valuation investor, $h[0,0]$, seek to long credit risk by either buying the bond or selling CDS. They search for both a bond seller and a CDS buyer and trade with whomever they find first. The population of high-valuation investors, as a result, consist of investors who are at different stages in their search: those who have not established a position and are still searching ($h[0,0]$), those who have purchased the bond ($h[1,0]$), and those who have sold CDS ($h[0,1]$). The investors with the latter two positions have reached their optimal position. I will interchangeably refer to high-valuation investors as long investors.

Those entering as a low-valuation investor seek to short credit risk by buying CDS. They are the naked CDS buyers in the model (low-valuation investors do not own bonds in equilibrium). The population of low-valuation investors, as a result, consist of investors who are searching to buy CDS ($l[0,0]$) and investors who bought CDS ($l[0,\text{-}1]$). The latter have reached their optimal position. I will interchangeably refer to low-valuation investors as short investors.

Investors unwind exposures and exit as follows. Upon a valuation shock, investors with CDS exposures terminate their contract immediately (by paying a fee) and exit. Their counterparties accept the fee and start the search process again. Bond owners, on the other hand, first have to search for a bond buyer and, as a result, become one of the bond sellers in the economy. Upon finding a buyer, they sell and exit.

For unwinding and exiting to be optimal, the valuation that high- and low-valuation investors revert to has to be such that the optimal position for investors with that valuation is no position. This is the role average-valuation investors play in the model. First, the optimal position for average-valuation investors is no position, $[\theta_b, \theta_c] = [0, 0]$. Assumption 1 ensures this in equilibrium.[10] Second, when high- and low-valuation investors get a valuation shock, they revert to an average-valuation investor. Put together, it is optimal for high- and low-valuation investors to unwind their positions upon a valuation shock. Moreover, once an investor becomes an average-valuation investor, her valuation does not change again (recall from page 5 that the uncorrelated state is an absorbing state). This ensures that investors exit instead of waiting to switch back to a different valuation.

Assumption 1. $2y > x - (r + \gamma_d)O_h > -(x - 2y - (r + \gamma_u)O_l) > 0$.

As Appendix C explains further, this setup with entry and exit and three valuations is a simple way to endogenize the aggregate masses of investors with different valuations and to model short positions. See Vayanos and Wang (2007), Vayanos and Weill (2008), Rocheteau and Weill (2011), and Afonso (2011) for similar setups with entry and exit (though not necessarily all with endogenous entry or short positions as in my model).

Given the optimal positions and trading strategies, the equilibrium agent types are $\mathcal{T} \equiv \{h[0,0], h[1,0], h[0,1], a[1,0], l[0,0], l[0,\text{-}1]\}$. Of these, $a[1,0]$ and $h[0,0]$ are the actively searching bond sellers and buyers and thereby make up the bond market; $l[0,0]$ and $h[0,0]$ trade as CDS buyers and sellers and make up the CDS market. Note that $h[0,0]$-type investors are both a bond buyer and a CDS seller at the same time. The bond trading volume, as a result, is

$$M_b \equiv \lambda \mu_{a[1,0]} \mu_{h[0,0]}, \tag{10}$$

while the CDS trading volume is $M_c \equiv \lambda \mu_{l[0,0]} \mu_{h[0,0]}$.

[10] Appendix D explains Assumption 1 in detail. Suppose O_h and O_l are small. Then, the assumption bounds the default size, J, between 1 and 2 units of $\rho\sigma_e$, which is the part of the endowment risk that can be hedged by bonds or CDS. Otherwise, if the default risk is too small, then even the average-valuation investors also want to hold CDS positions. If it is too large, then no investor wants to enter CDS contracts.

3.3 Bargaining and Terms of Trade

Buyers and sellers Nash-bargain over the price so that each gets half of the total gains from trade. The marginal benefit of buying the bond (i.e., the buyer's reservation value) is the difference between the expected utility of owning versus not owning the bond: $V_{h[1,0]} - V_{h[0,0]}$. The buyer's gains from trade are then $V_{h[1,0]} - V_{h[0,0]} - p_b$. Similarly, the seller's reservation value is $V_{a[1,0]}$, and her gains from trade are $p_b - V_{a[1,0]}$. The total gains from trade are $V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}$. The bond price, as a result, is the average between the buyer's and the seller's reservation values:

$$p_b = \frac{1}{2}V_{a[1,0]} + \frac{1}{2}(V_{h[1,0]} - V_{h[0,0]}). \tag{11}$$

The CDS spread (i.e., the CDS premium), p_c, and termination fees are characterized similarly in Appendix B.

Prices capture, through the continuation values, the tradeoffs investors face as they negotiate with a counterparty. If an investor lets a counterparty go without trading, she (a) postpones hedging benefits a trade would have yielded and (b) while waiting for another counterparty, risks getting a valuation shock and losing altogether future trading opportunities. But if she trades, she foregoes other counterparties that she could have traded with. The price that the two parties negotiate—and hence the surplus each extracts—balances these incentives to reach a deal.

3.4 Equilibrium

I analyze the steady state equilibrium. It is continuation values $\{V_\tau\}_{\tau\in\mathcal{T}}$, population measures $\{\mu_\tau\}_{\tau\in\mathcal{T}}$, prices $\{p_b, p_c\}$, termination fees $\{T_{\text{B}}, T_{\text{S}}\}$, and entry rates $\{\nu_h, \nu_l\}$ such that (i) the continuation values $\{V_\tau\}_{\tau\in\mathcal{T}}$ solve the agents' optimization problem (5), (ii) population masses equate the flow of agents switching into type $\tau \in \mathcal{T}$ to the flow of agents switching out of τ and solve (B13)-(B18), (iii) market clearing conditions (B19) and (B20) hold, (iv) bond and CDS prices $\{p_b, p_c\}$ arise from bargaining and solve (11) and (B9), (v) entry decisions $\{\nu_h, \nu_l\}$ solve (4), and (vi) termination fees $\{T_{\text{B}}, T_{\text{S}}\}$ solve (B10) and (B11).

Combining the equilibrium conditions, the bond price and the CDS spread depend on the continuation values. The continuation values depend on the expected search times, which, in turn, depend on the masses of buyers and sellers. The masses of buyers and sellers and the entire distribution of agent types depend on the entry rates of high- and low-valuation investors.

The entry rates, in turn, depend on the continuation values.

Theorem 1. *Under conditions (E5) and (E8), a steady state equilibrium exists in which the entry rates are given by an interior solution:* $\nu_h \in (0,1)$ *and* $\nu_l \in (0,1)$*.*

Appendix E outlines the proof, while online Appendix F contains the full proof. Conditions (E5) and (E8) ensure that the entry rates of high- and low-valuation investors have an interior solution.

3.5 Equilibrium Prices and Liquidity

Later, I analyze the bond trading volume (10) and the illiquidity discount in the bond price as measures of bond market liquidity. To define the latter, Lemma 1 characterizes the bond price.

Lemma 1. *The bond price is*

$$p_b = \frac{(\delta - \eta J) + x - y}{r} - \underbrace{\frac{(r+\gamma_d)O_h}{r}}_{\substack{\textit{discount due to} \\ \textit{funding cost}}} - \underbrace{\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda\mu_{h[0,0]}\frac{1}{2}}}_{\textit{illiquidity discount}}. \tag{12}$$

The first two terms in (12) capture the bond price in the absence of search frictions ($\lambda \to \infty$). In the absence of frictions, a bond owner can sell her bond to a high-valuation (i.e. long) investor the moment she experiences a valuation shock. Only long investors, as a result, own the bond. Since the bond price is the average between the marginal valuations of different bond owners, and long investors are the only bond owners, the bond price depends on their valuation only. In particular, $\delta - \eta J$ is the expected cash flow of the bond, and $\delta - \eta J + x - y$ is the long investors' utility valuation of this cash flow. The additional discount in the bond price, $(r+\gamma_d)O_h$, compensates a long investor for her entry cost (or, equivalently, for her funding cost).

The third term in (12) captures the illiquidity discount created by search frictions. In the presence of frictions, a bond owner wishing to sell her bond has to search for a buyer. When she finds one, she sells her bond at a discount accounting for the difficulty of locating another buyer. Similarly, a buyer, anticipating the difficulty of reversing positions, negotiates a discount in the price. Thus, let d_b denote the illiquidity discount:

$$d_b \equiv \frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda\mu_{h[0,0]}\frac{1}{2}}. \tag{13}$$

Lemma 2. *The CDS premium (or, equivalently, the CDS spread) can be characterized as*

$$p_c = (\eta J - x + y) + \underbrace{(r+\gamma_d)O_h}_{\text{premium due to funding cost}} + \underbrace{(r+2\gamma_d)\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda\mu_{h[0,0]}\frac{1}{2}}}_{\text{premium due to CDS market illiquidity}}. \tag{14}$$

Eq. (14) expresses the CDS spread from the perspective of a CDS seller, who in equilibrium is a long investor. It reflects the cost of selling CDS. It increases in the default risk—both the default intensity, η, and the size of the default, J—and hence in the expected default payment. The entire term $\eta J - x + y$ is the long investor's utility valuation of the expected payment. The second term, $(r+\gamma_d)O_h$, shows that the long investors' entry cost (or, equivalently, their funding cost) gets passed on to buyers as a higher CDS premium. The third term is zero in a frictionless environment ($\lambda \to \infty$). Otherwise, it is positive. Thus, the third term captures CDS market illiquidity, and it increases the CDS premium.

4 The Main Result

Theorem 2 presents the main result of the paper: Shorting bonds through naked CDS purchases increases bond market liquidity.

Theorem 2 (The Spillover Effect)**.** *Suppose $\lambda < \infty$. In the equilibria of Theorem 1 in which investors trade CDS, the bond market has fewer sellers ($\mu_{a[1,0]}$) and more buyers ($\mu_{h[0,0]}$), the illiquidity discount (d_b) is smaller, the bond price (p_b) is higher, the volume of trade (M_b) is larger, and high-valuation investors enter at a higher rate than in the environment without CDS trading.*

The next paragraphs give the intuition. The introduction of CDS expands the pool of counterparties that long (i.e. high-valuation) investors can trade with. Before they only traded with bond sellers, but now they can also search for and trade with naked CDS buyers. The larger pool of counterparties has two benefits. First, it shortens a long investor's expected search time. She values this because she (a) discounts and (b) risks getting a valuation shock and losing future trading opportunities, both of which make her impatient. Second, the ease of finding a counterparty improves her bargaining position and, as a result, the surplus she extracts from a counterparty. Put together, the introduction of CDS improves the long investor's continuation value, $V_{h[0,0]}$.

The benefit of entering and trading as a long investor now exceeds the cost: $V_{h[0,0]} > O_h$. Long investors, as a result, enter at a higher rate and do so until the increase in $V_{h[0,0]}$ is reversed and the marginal entrant is again indifferent: $V_{h[0,0]} = O_h$. The result is an increase in the aggregate number of long investors in the economy.

The increase in the number of long investors, in turn, increases bond market liquidity. The additional long investors first spend time searching for a counterparty and, as a result, expand the mass of long investors looking for a counterparty, $h[0,0]$. Since $h[0,0]$-type investors search for both bond sellers and CDS buyers, for bond sellers, a larger mass of $h[0,0]$-type investors means a larger pool of potential counterparties. The result is an increase in bond market liquidity: a shorter expected search time for bond sellers, fewer bond sellers (hence, less misallocation), a larger bond turn over, and a larger trading volume. If the model had dealers, the increase in liquidity would also manifest as a decrease in the bond bid-ask spread.

The increase in bond market liquidity increases the bond price. Sellers, who now find a buyer more quickly, raise their reservation value for the bond. Buyers are also willing to pay a higher price knowing they can sell quickly should the need arise. Put together, buyers and sellers negotiate and trade at a higher price.

4.1 A Naked CDS Ban

Using a difference-in-difference analysis, Sambalaibat (2014) documents that following the European Union naked CDS ban, liquidity of sovereign bonds affected by the ban deteriorated.[11] My model predicts the same. Since the CDS buyers in my model are naked CDS buyers (they do not own bonds), a naked CDS ban in the model is equivalent to shutting down CDS trading. Shutting down CDS trading reverses the spillover effect. Long investors can no longer sell CDS because their counterparties, the naked CDS buyers, are banned from buying CDS. Long investors, as a result, scale back their credit market operations and, in doing so, pull out from their bond trading. The result is a decrease in bond market liquidity. Thus, preventing investors from shorting ultimately drives away investors who want to take the opposite side and long the underlying asset.

[11]In October 2011, the European Union banned naked CDS purchases referencing EU government bonds. It did so by allowng investors to buy CDS only if they held the underlying bonds. It thus prevented investors from purchasing CDS either to speculate or to hedge positions correlated with the sovereign. In the model, consistent with the actual ban, both would be considered a naked CDS purchase because the CDS buyer in the model does not hold the underlying bonds.

4.2 Key Ingredients

Four ingredients generate the liquidity spillover effect. The first is the endogenous entry of long investors. Long investors are the counterparty to both bond sellers and CDS buyers. If their entry rate and hence their mass are fixed, the introduction of naked CDS buyers just crowds out bond sellers and, as a result, exacerbates their search costs. Thus, introducing CDS but keeping the entry rate of long investors fixed reverses the spillover effect. Bond market liquidity deteriorates.

The opposite results with fixed versus endogenous entry rates suggest that bonds and CDS are complements when investors adjust their entry rates but are substitutes when they do not. We can interpret the results with fixed versus endogenous entry rates as partial versus general equilibrium effects of CDS (or as short- versus long-run effects). Most models predict the partial equilibrium or the substitution effect. Generating the complementarity is, therefore, nontrivial.

The second ingredient is condition (E5). It is related to the first ingredient. It ensured that the entry rate of high-valuation investors, ν_h, is given by an interior solution both before and after CDS introduction.[12] It implies that a sufficient number of long investors exist on the sideline who can enter and absorb the short interest. Otherwise, the demand for short positions (captured by the total mass of low-valuation agents, $\frac{\nu_l F_l}{\gamma_u}$) is too large relative to the supply of long capital (which is at most $\frac{F_h}{\gamma_d}$), and introducing short positions just crowds out bond sellers. This occurs if the equilibrium entry rate of long investors hits the corner value $\nu_h = 1$ before a sufficient number of them could enter and absorb the short interest. The short interest, in turn, is a function of the entry cost of short investors (O_l), the holding cost (y), and the intensity of experiencing a valuation shock (γ_u).[13]

Proposition 2. *In a frictionless environment ($\lambda \to \infty$), the introduction of CDS does not affect the illiquidity discount (d_b), the bond price (p_b), nor the volume of trade (M_b).*

As Proposition 2 shows, the third ingredient for the spillover effect is search frictions. In a frictionless environment ($\lambda \to \infty$), CDS attracts additional long investors as before. But the increase in the aggregate mass of long investors does not affect bond market liquidity. The illiquidity discount is already zero, and the bond volume is the maximum possible. CDS

[12] Condition (E5) simplifies the proof of the spillover effect. It is a sufficient but not a necessary condition for the spillover effect (see Appendix E for further discussion).

[13] If the entry cost of short investors, O_l, is small, the benefit of entering as a short investor is more likely to exceed it. A small holding cost y implies larger gains from CDS trade. If γ_u is small, short investors expect to hold their position longer, implying larger gains from trade. Each implies a larger entry of short investors.

contracts, as a result, are redundant. Thus, the broader message of the paper is that, in the presence of trading frictions, the introduction of securities that complete markets complements existing assets. In the absence of frictions, they are redundant. Similar results should arise with other frictions. Goldstein, Li, and Yang (2013) and Goldstein and Yang (2015), for example, highlight a similar complementarity theme in the context of multiple markets and multiple dimensions of information, respectively, using asymmetric information environments.

Proposition 3. *Suppose long investors cannot search for both bond and CDS counterparties at the same time but, upon entering, have to choose which one to search for (i.e., direct their search effort to). Then, with the introduction of CDS, bond market liquidity either deteriorates or remains unaffected.*

As Proposition 3 shows, the last ingredient is the ability to search for both bond and CDS counterparties at the same time. Recall that the ability to also search for short investors increased the probability of trade and the bargaining position of long investors. Removing this ability (and thereby segmenting bond and CDS markets) cancels these effects and, with them, the reasons that long investors increased their entry rate in the first place. As a result, the spillover effect does not arise.

5 Robustness Results and Discussion

In Section 5.1, I relax the assumption that investors cannot short-sell and show that the spillover effect remains intact. In Section 5.2, I show that covered CDS positions do not arise in equilibrium. In Section 5.3, I show that the spillover arises even if investors optimally choose the intensities with which they search for a bond versus CDS counterparty. In Section 5.4, I contrast the spillover effect with existing results on short sales.

5.1 If Investors Short-Sell

In this section, I relax the assumption that investors cannot short-sell and compare bond market liquidity between two environments: (1) a benchmark environment in which short-selling is feasible, but CDS trading is not and (2) an environment in which both short-selling and CDS trading are feasible. Online Appendix I presents the full model. The results of this section are numerical and are illustrated in Figure 2 in online Appendix I.[14]

[14]The model with both CDS and short-selling is complicated and involves solving, at minimum, a system of 23 equations and variables (10 value functions, 9 population masses, 2 entry rates, the CDS premium, and the

The short-selling part of the model follows Vayanos and Weill (2008) and works as follows. After purchasing the bond, long investors now lend their bond in a repo (i.e. a security lending) market and, as a result, earn a lending fee. On the other side of the repo transaction, short investors ($l[0,0]$)—in addition to searching for a CDS seller—search to borrow the bond to short-sell it in the spot market. Parties meet in the repo market through search and, upon a match, negotiate over the lending fee. I denote with λ_r the exogenous search intensity in the repo market. An investor unwinds the short sale by first buying the bond in the spot market and then delivering it back to the bond lender. To unwind a bond loan, if her counterparty has not yet (short-) sold the bond, the lender recalls the bond, sells it, and exits. If the counterparty has already sold the bond, the lender walks away with the collateral that the short seller puts aside.

In this section, I allow the search intensity, λ, to differ for bond versus CDS matches. Let λ_b and λ_c denote the search intensities governing the volume of bond and CDS transactions, respectively. Section 2 environment is a special case, where $\lambda_b = \lambda_c = \lambda$. The volume of bond and CDS transactions are then $M_b = \lambda_b \mu_{b,\text{B}} \mu_{b,\text{S}}$ and $M_c = \lambda_c \mu_{c,\text{B}} \mu_{c,\text{S}}$, where $\mu_{b,\text{B}}$ and $\mu_{b,\text{S}}$ are the masses of bond buyers and sellers, and $\mu_{c,\text{B}}$ and $\mu_{c,\text{S}}$ are the masses of CDS buyers and sellers.

Lemma 3. *The bond price in the presence of short sales is given by:*

$$p_b = \frac{(\delta - \eta J) + x - y - (r+\gamma_d)O_h}{r} - \underbrace{\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda_b\mu_{b,\text{B}}\frac{1}{2}}}_{\textit{illiquidity discount}} + \underbrace{\frac{(r+\lambda_b\mu_{b,\text{B}})}{r}\frac{1}{2}\frac{\lambda_r\mu_{l[0,0]}\frac{1}{2}\omega_r}{r+\gamma_d+\lambda_b\mu_{b,\text{B}}\frac{1}{2}}}_{\textit{lending fee effect}}, \tag{15}$$

where ω_r, defined by (I29), is the total gains from trade from a repo contract.

Lemma 3 shows that in the presence of short sales, two characteristics of the bond determine its price. The first is its liquidity as before. The second is the lending fee that the bond generates. Bond owners now earn an additional cash flow by lending their bond to short-sellers. The additional cash flow, captured by the third term in (15), increases the bond price.

The introduction of CDS affects both components. First, it creates the liquidity spillover effect as it did when investors could not short-sell. The intuition is the same. The ability to also trade with CDS buyers attracts additional long investors into the economy. Once they enter, the

lending fee). Thus, analytically showing any results is intractable.

additional long investors search simultaneously for a bond seller, creating the spillover of long investors and liquidity into the bond market. The result is a decrease in the illiquidity discount, an increase in the bond price, and an increase in the trading volume. This result arises even if the entry rate of low-valuation investors were to remain fixed. In addition, CDS also changes the participation incentives of low-valuation investors. As a result and for the same reasons as on the long side, CDS attracts additional low-valuation investors. The increase in the mass of low-valuation agents creates another layer of the spillover effect. Long investors react to the increase in the mass of low-valuation agents and enter at an even higher rate than if the entry rate of low-valuation agents were to remain fixed. The result is a further increase in the bond price and trading volume. Thus, allowing short-selling just changes the benchmark environment, and relative to this benchmark, the marginal effect of CDS remains the same.

Second, CDS affects the bond price by changing the bond's lending fee cash flow. The direction of the change depends on the matching efficiency of the CDS market, λ_c, relative to that of the bond and repo markets. If λ_c is small, the additional inflow of low-valuation agents translates to a large increase in the number of investors looking for a short position, $\mu_{l[0,0]}$. Since $l[0,0]$-type agents search to borrow the bond at the same time, an increase in their mass increases the bond borrowing demand, the lending fee, and, thereby, the bond price. As λ_c increases and investors start to buy CDS quickly, the number of investors still searching for a short position decreases. As a result, the borrowing demand decreases, increasing the supply of lendable bonds. This drives down the lending fee, the cash flow the bond generates through the lending fee, the buyers' reservation value for the bond, and hence the bond price.

The net impact on the bond price depends on the above two effects. For a relatively large λ_c, the lending fee decreases sufficiently that the resulting downward pressure on the bond price dominates the opposite pressure from the liquidity spillover effect. The net effect is a decrease in the bond price. For a relatively small λ_c, either the lending fee increases, or even if it decreases, the decrease does not dominate the spillover effect. The net effect is an increase in the bond price. Thus, when investors already short-sell, the net effect of CDS on the bond price is ambiguous. However, it still creates the liquidity spillover effect, and the effect of this channel on the bond price is unambiguous.

5.2 Covered CDS

Buying CDS as a hedge on bonds that one owns ($[\theta_b, \theta_c] = [1, \text{-}1]$) can be referred to as a covered CDS position. Since investors with this position are both long and short on the underlying credit risk, they also proxy arbitrageurs or CDS-bond basis traders. Although the position is feasible in the model, Lemma 4 shows that it does not arise in equilibrium.

Lemma 4. *The mass of agents with a covered CDS position,* $[\theta_b, \theta_c] = [1, \text{-}1]$, *is zero.*

It is a corollary of Proposition 1. The intuition is as follows. Regardless of the investors' bond position, the gains from CDS trade exist only between high and low-valuation investors (in particular, with the high-valuation investor as the CDS seller and the low-valuation investor as the CDS buyer). The gains from CDS trade between high- and average-valuation investors and between average and low-valuation investors are both negative.[15] So only low-valuation investors buy CDS in equilibrium. But low-valuation investors at no point find it optimal to buy bonds. Put together, a covered CDS position does not arise in equilibrium.

In Author (2015), I extend the environment of this paper so that covered CDS positions arise in equilibrium. I show that doing so just changes the benchmark environment and—relative to this benchmark—the marginal effect of naked CDS positions remains the same as in Section 4. The marginal effect of the covered CDS position itself is also an increased bond market liquidity. This is intuitive. The ability to hedge bonds with CDS makes buying bonds more attractive.

5.3 Endogenous Search Intensities

In this section, I endogenize the search intensity, λ, as follows. A high-valuation investor, $h[0, 0]$, searches for a counterparty in bond and CDS markets with search efforts λ_b and λ_c, respectively. As a result, she meets a bond seller with total intensity $\lambda_b\mu_{a[1,0]}$ and a CDS buyer with intensity $\lambda_c\mu_{l[0,0]}$. Since a total mass $\mu_{h[0,0]}$ of long investors does the same thing, the total volume of bond matches is $M_b = \lambda_b\mu_{a[1,0]}\mu_{h[0,0]}$, while the volume of CDS matches is $M_c = \lambda_c\mu_{l[0,0]}\mu_{h[0,0]}$. For simplicity, I endogenize the search effort of only long investors and set the search effort of investors on the opposite side (i.e., of bond sellers and CDS buyers) to zero.[16]

[15] For example, if an average-valuation investor buys CDS from a high-valuation investor, the gains from CDS trade are proportional to $x - 2y - (r + \gamma_d)O_h$. But, by Assumption 1, this is negative. That is, when high- and average-valuation investors enter a CDS contract, the holding cost both sides incur together with the entry cost outweigh the total hedging benefit.

[16] A model in which both sides of the market choose their search effort is intractable. Nevertheless, in online Appendix J, I endogenize the search effort of all investors and discuss the parameter conditions under which the spillover effect arises. Numerically, the results with one- versus two-sided endogenous search intensities are analogous.

Investors choose $\{\lambda_b, \lambda_c\}$ accounting for a flow search cost

$$c(\lambda_b, \lambda_c) \equiv c_0 \left((\lambda_b)^g + (\lambda_c)^g \right)^a , \tag{16}$$

where $c_0 > 0$, g, and a are constants. Investors thereby internalize the cost of searching in multiple markets at the same time. The steady state equilibrium now includes λ_b and λ_c as additional endogenous variables and the first order conditions with respect to them, (J5) and (J6), as the equilibrium conditions they satisfy.

Lemma 5. *The characterization of the bond price and the illiquidity discount are the same as (12) and (13).*

Proposition 4. *Suppose $g > 1$ and $a > \frac{1}{g}$ so that the cost function (16) is a strictly convex function. Then, with the introduction of CDS, the illiquidity discount (d_b) decreases and the bond trading volume (M_b) increases. Long investors lower their search effort in the bond market (λ_b) but without changing their total search cost, $c(\lambda_b, \lambda_c)$.*

The introduction of CDS creates the same liquidity spillover effect as in the environment with exogenous search efforts. The intuition is the same. Naked CDS buyers expand the pool of long investors in the economy. Long investors, in turn, trade with bond sellers due to the substitutability between bond and CDS positions. The result is an increase in the number of bond buyers, a decrease in the number of sellers, and an increase in bond market liquidity.

The CDS introduction, at the same time, reduces the long investors' incentive to search in the bond market. To see this, consider how long investors choose their search effort. For small α and for $g = 2$ and $a = 1$, for example, the first order condition with respect λ_b simplifies to

$$\lambda_b = \frac{1}{2c_0} \mu_{a[1,0]} \left(V_{h[1,0]} - V_{h[0,0]} - p_b \right) . \tag{17}$$

The right-hand-side is the long investor's expected rents from trading in the bond market: the mass of bond sellers ($\mu_{a[1,0]}$) times the gains from trade upon a match. The gains from trade, as an implicit function of $\mu_{a[1,0]}$, increase in the mass of bond sellers. The whole right-hand-side, as a result, increases in $\mu_{a[1,0]}$. The mass of sellers, however, captures bond market illiquidity (in particular, the extent of asset misallocation due to search frictions). Put together, long investors search more intensely for a bond seller if liquidity in the bond market is low. As liquidity improves, long investors reduce their search effort in the bond market and allocate it to the CDS market instead. They do so, in particular, without changing their total search cost, $c(\lambda_b, \lambda_c)$. Thus, in Proposition 4, the reduced search effort of long investors and the greater bond market liquidity are both byproducts of CDS introduction and the change it induces in the bond market.

These results highlight which ingredients in the main environment are crucial and which are not. In the main environment, I assumed that investors do not face explicit search costs. This assumption is not crucial. The spillover effect arises even if investors internalize the cost of searching in multiple markets and reshuffle their search efforts between the two markets. The crucial ingredients are instead (1) the substitutability between bond and CDS trades, (2) search frictions, and (3) costly and endogenous entry. Each matters because of the other ingredients.

5.4 Contribution to the Short Sale Literature

Now I explain how my results contribute to our understanding of the effect of short positions.

First, I shed light on the effect of synthetic short positions—positions that do not require trading the underlying asset for neither the short side nor for their counterparty, the long side. Prior results on short-selling arise because short-selling requires trading the underlying asset. Boehmer, Jones, and Zhang (2013) and Beber and Pagano (2013), for example, document that short-selling bans during the financial crisis deteriorated stock market liquidity. These were bans on regular short sales, not on short positions through derivatives. Consistent with this, Vayanos and Weill (2008) show that short-selling increases liquidity of the underlying asset. They emphasize this is because short sellers have to trade in the underlying spot market: first as a seller to establish the short position, then as a buyer to unwind the position. Since these results arise because short-selling requires trading the underlying asset, they do not generalize to synthetic short positions. Thus, the novelty of my paper lies in showing the effect of synthetic short positions.

Second, the spillover effect will arise not only with synthetic short positions but also with short-selling. In particular, the introduction of short sellers will have the same spillover effect on regular bond sellers that naked CDS buyers had in the context of CDS. They will expand the pool of sellers that long investors can trade with and, as a result, attract additional long investors into the economy. In turn, the additional long investors will trade not only with short sellers but also with regular bond sellers, creating the liquidity spillover to regular bond sellers. Thus, the spillover effect is a novel, general effect of short positions.

Third, in contrast to the literature started by Miller (1977), I highlight a general equilibrium effect of short positions. Miller (1977) and the subsequent literature propose that short-sale bans increase asset prices by keeping pessimists (who are equivalent to low-valuation investors in my model) out of the markets.[17] The participation rates of different investors are fixed in these environments. In contrast, I endogenize them and show that banning short positions keeps out not only investors who want to short but also investors who want to take the opposite side and long the asset. The result is a decrease in asset prices. Thus, endogenizing participation rates reverses the main result of this literature. As mentioned earlier, we can interpret the results with fixed versus endogenous participation rates as partial versus general equilibrium effects (or as short- versus long-run effects). The opposite conclusion of my paper shows that we tend to focus on partial equilibrium effects of financial innovations when their general equilibrium effects may be the opposite and more important.

Lastly, using asymmetric information environments, a body of work analyzes the effect of short sales on price informativeness (see, for example, Diamond and Verrecchia (1987), Bai, Chang, and Wang (2006), and Cornelli and Yılmaz (2013)). In contrast, I study how short positions affect asset prices through their effect on liquidity of the asset. For OTC traded assets (such as bonds and CDSs), illiquidity is a key asset pricing component. Moreover, in a large class of OTC markets (e.g., currency, sovereign bonds, municipal bonds, off-the-run Treasuries, and various derivatives), the illiquidity arises more from search costs than asymmetric information about the quality of the asset. Thus, my paper's contribution is to build a model of trading in derivative and underlying assets in which illiquidity arises from search costs.

6 Conclusion

The point I make in this paper is simple. If we want to model and understand the effect of new financial instruments and mechanisms on existing ones, the number of investors that could potentially trade and use the instruments should be endogenous.

I make this point in the context of bond and CDS markets. I build a search model of bond and CDS markets and show that introducing short positions through CDS contracts attracts into credit markets not only investors who want to short the underlying credit risk but also

[17] See, for example, Harrison and Kreps (1978), Jarrow (1980), Chen, Hong, and Stein (2002), Scheinkman and Xiong (2003), and Hong, Scheinkman, and Xiong (2006). In these environments, investors agree to disagree about asset fundamentals. Heterogenous beliefs, in turn, generate heterogenous private valuations as in my environment.

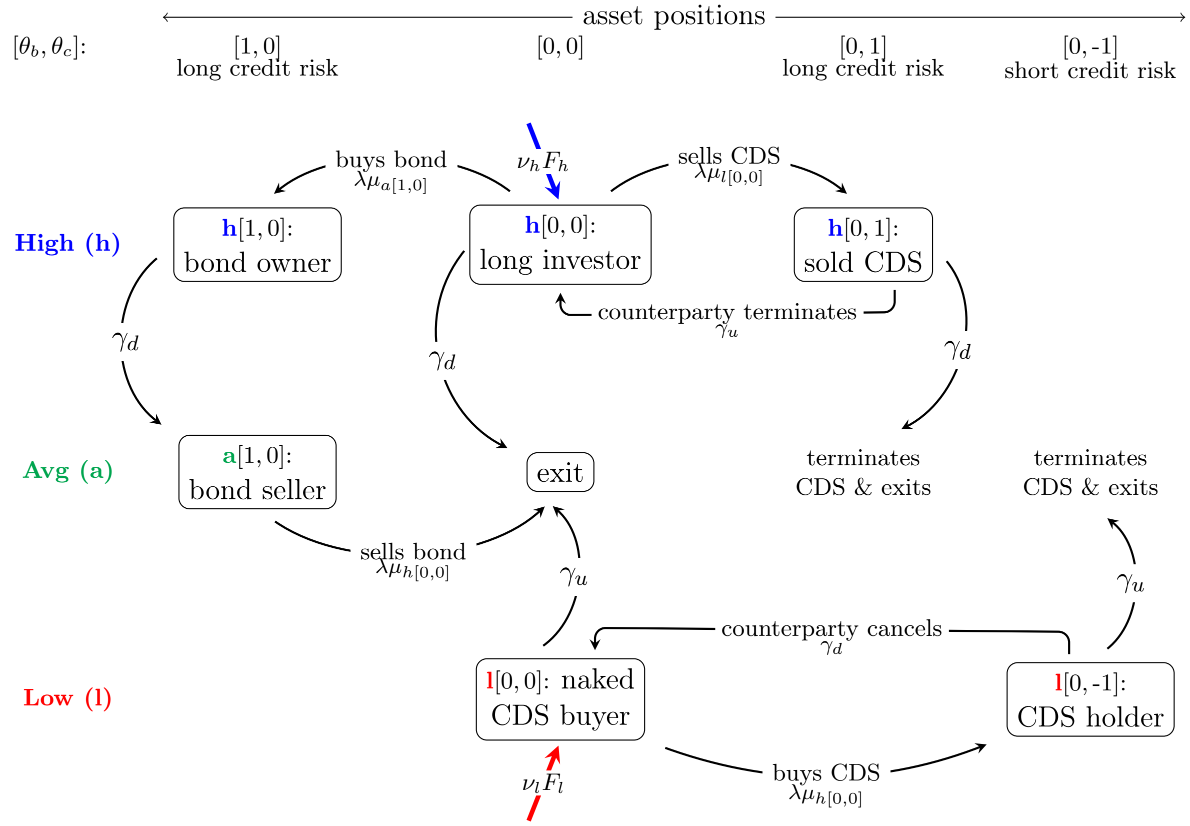
investors who want to take the opposite side and long the underlying credit risk. In turn, long investors—for whom bond and CDS positions are economically similar positions—search and trade at the same time in the bond market. They do this to expand their trading opportunities and to alleviate their search frictions. The result is an increase in the number of bond buyers, bond market liquidity, and the bond price. I refer to this effect as a liquidity spillover effect.

This insight applies not only to derivatives but also to any mechanism that expands the set of feasible allocations in the economy (e.g., tradable securities and contracts, trading mechanisms and venues, private currencies).

Shutting down naked CDS positions in the model reverses the spillover effect and, as a result, decreases bond market liquidity. This result suggests that by banning naked CDS positions on sovereign bonds in 2011, regulators in Europe inadvertently decreased bond market liquidity, reduced bond prices, and thereby increased sovereigns' borrowing costs when they intended to achieve the opposite and quell a sovereign debt crisis.

Figure 1: A Snapshot of Transitions Between Agent Types

The figure shows the transitions between agent types. Flows of $\nu_h F_h$ and $\nu_l F_l$ agents enter the economy as new high- and low-valuation investors. High- and low-valuation investors revert to average-valuation with intensities γ_d and γ_u, respectively. An investor seeking a long position ($h[0,0]$) finds a counterparty in the bond and CDS markets with intensities $\lambda\mu_{a[1,0]}$ and $\lambda\mu_{l[0,0]}$, respectively. A bond seller ($a[1,0]$) finds a buyer with intensity $\lambda\mu_{h[0,0]}$. A trader seeking a short position by buying CDS ($l[0,0]$) finds a counterparty with intensity $\lambda\mu_{h[0,0]}$.



Appendix

A The Counting Process $\hat{N}_t(\tau)$

The counting process $\hat{N}_t(\tau)$ captures the different ways an agent's type can change. Consider, for example, agent type $\tau = h[0,0]$. The dimension of $\hat{N}_t(\tau)$ is $K(\tau) = 3$, where the three possible events are: (1) the agent's valuation changes, (2) the agent finds a counterparty in the bond market, and (3) the agent finds a counterparty in the CDS market. Intensities of these events are $\gamma(1,\tau) = \gamma_d$, $\gamma(2,\tau) = \lambda\mu_{a[1,0]}$, and $\gamma(3,\tau) = \lambda\mu_{l[0,0]}$, respectively. Similarly, consider agent $\tau = h[0,1]$ (an agent who has sold CDS). Then, $K(\tau) = 2$, and the two possible events are (1) the agent himself gets a valuation shock or (2) his counterparty's valuation changes. The intensities are: $\gamma(1,\tau) = \gamma_d$ and $\gamma(2,\tau) = \gamma_u$. It is analogous for the other agent types.

B Value Functions, Terms of Trade, Population Masses

Substituting in (11) and (B9), the continuation values simplify to

$$
\begin{aligned}
rV_{l[0,0]} &= \gamma_u(0 - V_{l[0,0]}) + \frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c && \text{(B1)}\\
rV_{h[0,0]} &= \gamma_d(0 - V_{h[0,0]}) + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c && \text{(B2)}\\
rV_{h[1,0]} &= (\delta - \eta J) + x - y + \gamma_d(V_{a[1,0]} - V_{h[1,0]}) && \text{(B3)}\\
rV_{a[1,0]} &= (\delta - \eta J) - y + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b && \text{(B4)}\\
rV_{h[0,1]} &= p_c - (\eta J - x) - y + \gamma_d(-T_{\text{S}} - V_{h[0,1]}) && \text{(B5)}\\
rV_{l[0,\text{-}1]} &= -p_c + (\eta J + x) - y + \gamma_u\left(-T_{\text{B}} - V_{l[0,\text{-}1]}\right), && \text{(B6)}
\end{aligned}
$$

where ω_c is the total gains from a CDS transaction:

$$\omega_c \equiv \left(V_{h[0,1]} - V_{h[0,0]}\right) + \left(V_{l[0,\text{-}1]} - V_{l[0,0]}\right), \tag{B7}$$

and ω_b is the total gains from a bond transaction:

$$\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}.$$

The instantaneous payoff from a transition from τ to τ' is given by:

$$P(\tau,\tau') = \begin{cases} -p_b & \text{if } \tau = i[0,\theta_c] \text{ and } \tau' = i[1,\theta_c] \\ p_b & \text{if } \tau = i[1,\theta_c] \text{ and } \tau' = i[0,\theta_c] \\ 0 & \text{else.} \end{cases} \tag{B8}$$

The CDS buyer's surplus is $V_{l[0,\text{-}1]} - V_{l[0,0]}$, while the seller's is $V_{h[0,1]} - V_{h[0,0]}$. Thus, the CDS premium is implicitly defined by

$$V_{h[0,1]} - V_{h[0,0]} = \frac{1}{2}\left(V_{l[0,\text{-}1]} - V_{l[0,0]} + V_{h[0,1]} - V_{h[0,0]}\right). \tag{B9}$$

Consider the fees the CDS counterparties pay to terminate their contracts. If a buyer terminates, the seller goes from being a $h[0,1]$ type to $h[0,0]$, and the seller's utility decreases by $(V_{h[0,1]} - V_{h[0,0]})$. To make the seller indifferent, the buyer has to pay a fee equal to the decrease in the seller's utility:

$$T_{\text{B}} = V_{h[0,1]} - V_{h[0,0]}. \tag{B10}$$

Analogously, a CDS seller (the long side) has to pay the short side:

$$T_{\mathrm{s}} = V_{l[0,\text{-}1]} - V_{l[0,0]}. \tag{B11}$$

The right-hand sides coincide with the gains from trade to each side. Hence, both equal $\frac{1}{2}\omega_c$.

Inflow-Outflow Equations

Given the conjectured trading strategies, the steady state masses are such that the flow of agents switching into a type equals the flow of agents switching out of that type. For example, the mass of $h[0,0]$ agents evolves as

$$\frac{\partial \mu_{h[0,0]}}{\partial t} = \overbrace{\nu_h F_h + \gamma_u \mu_{h[0,1]}}^{\text{inflow}} - \overbrace{\left(\gamma_d \mu_{h[0,0]} + \left(\lambda \mu_{a[1,0]} + \lambda \mu_{l[0,0]}\right) \mu_{h[0,0]}\right)}^{\text{outflow}}. \tag{B12}$$

In (B12), the flow of agents turning into $h[0,0]$-type are (1) the new high-valuation entrants, $\nu_h F_h$, and (2) long investors who had previously sold CDS but are now searching again because their counterparty terminated the contract, $\gamma_u \mu_{h[0,1]}$. The agents switching out of type $h[0,0]$ are those who (1) experience a valuation shock, $\gamma_d \mu_{h[0,0]}$, (2) match with a bond seller, $\lambda \mu_{a[1,0]} \mu_{h[0,0]}$, and (3) match with a CDS buyer, $\lambda \mu_{l[0,0]} \mu_{h[0,0]}$. The steady state mass is characterized by $\frac{\partial \mu_{h[0,0]}}{\partial t} = 0$. That is, $\mu_{h[0,0]}$ is constant, and the inflow equals the outflow.

The inflow-outflow equations for the other agent types are analogous:

$$\begin{aligned}
\text{long investor } h[0,0]: \quad & \nu_h F_h + \gamma_u \mu_{h[0,1]} = \gamma_d \mu_{h[0,0]} + M_b + M_c && \text{(B13)}\\
\text{naked CDS buyer } l[0,0]: \quad & \nu_l F_l + \gamma_d \mu_{l[0,\text{-}1]} = \gamma_u \mu_{l[0,0]} + M_c && \text{(B14)}\\
\text{bond owner } h[1,0]: \quad & M_b = \gamma_d \mu_{h[1,0]} && \text{(B15)}\\
\text{bond seller } a[1,0]: \quad & \gamma_d \mu_{h[1,0]} = M_b && \text{(B16)}\\
\text{sold CDS } h[0,1]: \quad & M_c = \gamma_d \mu_{h[0,1]} + \gamma_u \mu_{h[0,1]} && \text{(B17)}\\
\text{bought CDS } l[0,\text{-}1]: \quad & M_c = (\gamma_u + \gamma_d)\mu_{l[0,\text{-}1]}. && \text{(B18)}
\end{aligned}$$

Market Clearing

For the bond market to clear, the total mass of bond owners has to equal the bond supply:

$$\mu_{h[1,0]} + \mu_{a[1,0]} = S. \tag{B19}$$

For CDS market clearing, the number of CDSs sold has to equal the number of CDSs purchased:

$$\mu_{h[0,1]} = \mu_{l[0,\text{-}1]}. \tag{B20}$$

Equations (B19) and (B20) show that bonds and CDSs are, effectively, inside and outside money: bonds are in fixed supply, while CDSs are created within the economy and are in zero net supply.[18]

[18]Bonds and CDSs are money in the sense that they serve as a store of value and expand the set of feasible allocations in the economy. For discussion on financial innovation as inside money, see, for example, Gennaioli, Shleifer, and Vishny (2012) and Brunnermeier and Sannikov (2016). For discussion on inside money (particularly, as a medium of exchange), see, for example, Cavalcanti and Wallace (1999a), Cavalcanti and Wallace (1999b), and Lagos (2010). The latter literature starting with Cavalcanti and Wallace (1999a) addresses when inside and outside money co-exist and whether they are complements or substitutes. In contrast, I shed light on how the creation of inside money (CDS) affects the liquidity of outside money (bonds).

C Three Valuations

I explain further why I need three valuations.

In an environment with just two valuations (say, high- and low-valuation agents) and no entry and exit, introducing CDS deteriorates bond market liquidity. In such environment and in the absence of CDS, the optimal position for the low-valuation investor is no position: Investors buy the bond as a high-valuation investor and sell when they become a low-valuation investor. When we introduce CDS, the optimal position for the low-valuation investor turns into a short position. They, as a result, go one asset position further and buy CDS after they sell their bond. But because the number of investors of each valuation is fixed, allowing CDS and hence short positions deteriorates bond market liquidity.

The point of the paper is to, instead, show that investors' participation incentives change in response to CDS. I thus need to endogenize the aggregate number of investors of each valuation. One way to do that is to endogenize their entry. But to model investors' entry, I have to model their exit also. Otherwise, entries without exits result in infinite masses.

Exit is optimal under two conditions. First, investors cannot exit with an existing position. So, to ensure that investors unwind their existing positions, their valuation has to change to a valuation whose terminal optimal position is no position. Second, once their valuation changes, they cannot have an incentive to wait to switch to another valuation instead of exiting. That is, the valuation they switch to has to be an absorbing valuation.

A model with just two valuations but with entry and exit also does not generate the result I want to show. Suppose investors enter the economy as a high-valuation investor, switch to a low-valuation investor at some point and, once they switch, remain forever a low-valuation investor. In the absence of short positions, this model works fine. Investors buy the bond after they enter; when they switch to a low-valuation, they sell and exit forever. In the presence of CDS, high-valuation investors still go long, but low-valuation agents now want to short (as their terminal optimal position). This implies that exiting is not optimal for low-valuation agents. They, instead, want to remain in the economy and short.

Thus, an environment with two valuations allows short positions in the absence of entry and exit, allows entry and exit in the absence of short positions, but does not allow both short positions and entry and exit.

D Intuition for Assumption 1

The intuition for Assumption 1 is as follows. The gains from CDS trade between high- and average-valuation investors are proportional to $x-2y-(r+\gamma_d)O_h$. This is negative by Assumption 1. Intuitively, the difference in their valuations—hence, the total hedging benefit $(x-0)$—is too small relative to the holding cost both sides incur $(2y)$ and the entry cost, $(r+\gamma_d)O_h$. The lack of gains from trade ensures that (a) an average-valuation investor does not buy CDS from a high-valuation investor, and (b) once a CDS buyer (initially, a low-valuation investor) switches to an average-valuation, she prefers to unwind the short position that she has with a high-valuation investor than to remain a CDS buyer.[19] The gains from CDS trade exist only between high- and low-valuation investors: $2x-2y-(r+\gamma_d)O_h-(r+\gamma_u)O_l>0$. Their valuations are far apart enough that the total hedging benefit, $2x=x-(-x)$, outweighs the holding cost, $2y$, and the costs of entry, $(r+\gamma_d)O_h+(r+\gamma_u)O_l$.

On the bond side, the gains from trade between high- and average-valuation investors are proportional to the difference in valuations, $x-0$, minus the entry cost: $x-(r+\gamma_d)O_h$.[20] This

[19]It is analogous between average- and low-valuation agents. The gains from CDS trade between them are proportional to $x-2y-(r+\gamma_u)O_l$, which is negative. This ensures that (a) an average-valuation investor does not sell CDS to a low-valuation investor, and (b) once a CDS seller (a high-valuation investor) switches to an average-valuation investor, she prefers to unwind her long position than to remain a CDS seller.

[20]When high and average-valuation investors trade a bond, the total holding cost does not change because,

is positive by Assumption 1. Thus, a bond owner who switches to an average-valuation investor prefers to unwind and sell her bond to a high-valuation investor.

Consider how these parameter conditions relate to the original parameters. To gain intuition, let us ignore O_h and O_l.[21] Then, Assumption 1 simplifies to $2x > 2y > x$ or $x > y > \frac{1}{2}x$. Substituting in the definitions of x and y,

$$2r\alpha\rho\sigma_e\eta J > 2\frac{r\alpha}{2}\eta J^2 > r\alpha\rho\sigma_e\eta J.$$

Canceling terms,

$$2\rho\sigma_e > J > \rho\sigma_e.$$

Thus, Assumption 1 bounds the default size, J, between 1 and 2 units of $\rho\sigma_e$, which is the part of the endowment risk that can be hedged by trading bonds or CDSs.

E Proofs

Because the proofs of Proposition 1 and Theorem 1 are long, I relegate them to online Appendix F and G.

In the body of the paper, the same search intensity, λ, governs matches among bond counterparties as CDS counterparties. However, the results in the paper hold even if the search intensity differs for bond versus CDS matches. Thus, herein, I adobt the general specification: $M_b = \lambda_b\mu_{h[0,0]}\mu_{a[1,0]}$ and $M_c = \lambda_c\mu_{h[0,0]}\mu_{a[1,0]}$. The specification in the paper is a special case: $\lambda_b = \lambda_c = \lambda$. I adopt this simpler specification in the paper body for exposition.

I summarize the main steps of Theorem 1 proof. In step 1 of the proof, I show that the equilibrium conditions narrow down to a set of five equations and five unknowns $\{\mu_{h[0,0]}, V_{h[0,0]}, V_{l[0,0]}, \nu_h, \nu_l\}$:

$$\begin{aligned}(r+\gamma_d)V_{h[0,0]} - \lambda_b\frac{\gamma_d S}{\left(\gamma_d+\lambda_b\mu_{h[0,0]}\right)}\frac{1}{2}\frac{x-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}}& \\ - \lambda_c\frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} &= 0\end{aligned} \tag{E1}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda_c\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} \tag{E2}$$

$$\nu_h F_h = \gamma_d\mu_{h[0,0]} + \gamma_d\frac{\lambda_b\mu_{h[0,0]}S}{\left(\lambda_b\mu_{h[0,0]}+\gamma_d\right)} + \gamma_d\lambda_c\mu_{h[0,0]}\frac{\frac{\nu_l F_l}{\gamma_u}}{\gamma_u+\gamma_d+\lambda_c\mu_{h[0,0]}} \tag{E3}$$

$$\nu_i = \begin{cases}1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i\end{cases} \qquad \text{for } i \in \{h,l\}. \tag{E4}$$

In step 2, I show that $V_{h[0,0]}$ decreases in ν_h. This result and the assumption that

$$\frac{S\lambda_b}{2\left(r+\gamma_d\right)+S\lambda_b}x > (r+\gamma_d)O_h > \frac{\frac{\lambda_c(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}\frac{1}{2}(2x-2y)}{(\gamma_d+\gamma_u+\lambda_c\bar{\mu})\left(r+\gamma_d+\gamma_u+\lambda_c\bar{\mu}\frac{1}{2}\right)} + \frac{\lambda_b\gamma_d S\frac{1}{2}x}{(\gamma_d+\lambda_b\bar{\mu})\left(r+\gamma_d+\lambda_b\bar{\mu}\frac{1}{2}\right)}}{1+\frac{\lambda_c(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}\frac{1}{2}}{(\gamma_d+\gamma_u+\lambda_c\bar{\mu})\left(r+\gamma_d+\gamma_u+\lambda_c\bar{\mu}\frac{1}{2}\right)} + \frac{\lambda_b\gamma_d S\frac{1}{2}}{(\gamma_d+\lambda_b\bar{\mu})\left(r+\gamma_d+\lambda_b\bar{\mu}\frac{1}{2}\right)}} \tag{E5}$$

unlike CDS transactions, investors do not create new positions. Only the ownership of the bond changes.

[21] If, for example, $x-y>0$, $x-2y<0$, and $(x-2y)+[x-(r+\gamma_d)O_h-(r+\gamma_u)O_l]>0$, then Assumption 1 holds.

for $\nu_l \in [0,1]$ and where $\bar{\mu}$ is given by (G21) ensure that the solution for ν_h is unique, positive, and interior.

Ensuring that the entry rate of high-valuation investors is given by an interior solution both before and after CDS introduction simplifies the analysis. In search models with exogenous entry, given the conjectured trading strategies, the system of equations characterizing the population masses does not depend on the value functions and can be solved on its own. Then, the value functions are a linear system of equations of the population masses. The conjecture-and-verify method, as a result, simplifies the analyses and proofs by decoupling the system of equations into two sets. Endogenizing entry, however, reverses this decoupling. The population masses depend on the entry rates, but the entry rates depend on the value functions, which, in turn, depend on the population masses. All three sets of variables have to be solved simultaneously. Thus, the model with endogenous entry is significantly more complicated. Focusing on the interior solution helps simplify the analysis.

In step 3, using the result from step 2 that ν_h is given by an interior solution, (E1) and (E2) become

$$\begin{aligned}(r+\gamma_d)O_h - \lambda_b \frac{\gamma_d S}{\left(\gamma_d + \lambda_b \mu_{h[0,0]}\right)} \frac{1}{2} \frac{x-(r+\gamma_d)O_h}{r+\gamma_d+\lambda_b \mu_{h[0,0]}\frac{1}{2}} \\ - \lambda_c \frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}} \frac{1}{2} \frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} = 0\end{aligned} \tag{E6}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda_c\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}}. \tag{E7}$$

Equations (E6) and (E7) together define $V_{l[0,0]}$ as an implicit function of ν_l: $V_{l[0,0]}(\nu_l)$. I show that $V_{l[0,0]}(\nu_l)$ strictly increases in ν_l. This result and the condition that

$$0 < \frac{2\gamma_u\left(r+\gamma_d+\gamma_u+\frac{1}{2}\lambda_c\mu_h^*\right)\left(\gamma_d+\gamma_u+\lambda_c\mu_h^*\right)\left[O_h\left(r+\gamma_d\right) - \frac{S\gamma_d(x-O_h(r+\gamma_d))\lambda_b}{\left(\gamma_d+\lambda_b\mu_h^*\right)\left(2(r+\gamma_d)+\lambda_b\mu_h^*\right)}\right]}{F_l\left(2x-2y-O_h\left(r+\gamma_d\right)\right)\left(\gamma_d+\gamma_u\right)\lambda_c} < 1, \tag{E8}$$

where

$$\mu_h^* \equiv \frac{2O_l\left(r+\gamma_u\right)\left(r+\gamma_d+\gamma_u\right)}{\lambda_c\left(2x-2y-\left(r+\gamma_d\right)O_h-\left(r+\gamma_u\right)O_l\right)},$$

ensure that a unique interior solution exists for ν_l. They also imply that two corner solutions exist: $\nu_l = 0$ and $\nu_l = 1$.

In step 4, I show that—taking the entry rates as given—the rest of the equilibrium variables are uniquely determined, and the population masses and the gains from trade are, in addition, positive. Finally, in step 5, I show that all the conjectured optimal trading strategies are indeed optimal.

The above discussion shows that multiple equilibria exist, each with a different entry rate of low-valuation investors: a unique interior solution $\nu_l \in (0,1)$ and two corner solutions ($\nu_l = 0$, $\nu_l = 1$). For a given level of ν_l, the entry rate of high-valuation investors, however, has a unique solution. Condition (E5) ensures that it is, in particular, an interior solution. The fact that $\nu_l = 0$ is one of the solutions shows that even if CDS trading is feasible, investors may not trade CDS in equilibrium. Since the paper is about the effect of CDS, I contrast the equilibria with CDS (i.e., $\nu_l > 0$, whether it is an interior or a corner solution) to the environment in which I shut down the CDS market (or, equivalently, to the equilibrium with $\nu_l = 0$). The marginal effect of CDS is qualitatively the same for both the interior, $\nu_l \in (0,1)$, and the corner, $\nu_l = 1$, levels of the entry rate. Thus, the equilibrium multiplicity due to the different entry rates of low-valuation agents is unimportant.

Proof of Lemma 1. Substituting the value functions of $h[1,0]$, $h[0,0]$ and $a[1,0]$ into the bond price, (11), and simplifying,

$$rp_b = \delta - \eta J + \frac{1}{2}x - y - \frac{1}{2}\gamma_d\omega_b - \frac{1}{2}\left(\lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda_c\mu_{l[0,0]}\frac{1}{2}\omega_c - \lambda_b\mu_{h[0,0]}\frac{1}{2}\omega_b\right). \tag{E9}$$

Combining the value functions for $h[0,0]$, $h[1,0]$, and $a[1,0]$, substituting in M_b and M_c, using $V_{h[0,0]} = O_h$, and simplifying, we get

$$(r+\gamma_d)\omega_b = x - (r+\gamma_d)O_h + \lambda_b\mu_{h[0,0]}\frac{1}{2}\omega_b. \tag{E10}$$

Using (B2), $V_{h[0,0]} = O_h$, and (E10), (E9) becomes

$$\begin{aligned} rp_b &= \delta - \eta J + \frac{1}{2}x - y - \frac{1}{2}\gamma_d\omega_b - \frac{1}{2}\left((r+\gamma_d)O_h - x + (r+\gamma_d)O_h + (r+\gamma_d)\omega_b\right) \\ &= \delta - \eta J + x - y - (r+\gamma_d)O_h - \frac{1}{2}\left(r+2\gamma_d\right)\omega_b. \end{aligned} \tag{E11}$$

From (E10),

$$\omega_b = \frac{x - (r+\gamma_d)O_h}{r + \gamma_d + \lambda_b\mu_{h[0,0]}\frac{1}{2}}. \tag{E12}$$

Substituting this into (E11), we get

$$p_b = \frac{\delta - \eta J + x - y - (r+\gamma_d)O_h}{r} - \frac{(r+2\gamma_d)}{2r}\frac{x - (r+\gamma_d)O_h}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}}. \tag{E13}$$

The proof of Proposition 2 in online Appendix H shows that as $\lambda_b \to \infty$, $\lambda_b\mu_{h[0,0]} \to \infty$. Hence, the bond price in the absence of bond market search frictions is given by the first term in (E13). □

Proof of Lemma 2. Combining (B2), (B5), and the termination fees, we get

$$(r+\gamma_d)\left(V_{h[0,1]} - V_{h[0,0]}\right) = p_c - (\eta J - x) - y - \gamma_d\frac{1}{2}\omega_c - \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b - \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c.$$

Using the fact that $V_{h[0,1]} - V_{h[0,0]} = \frac{1}{2}\omega_c$, the CDS premium is given by

$$\begin{aligned} p_c &= (\eta J - x) + y + \frac{1}{2}\left(r+2\gamma_d\right)\omega_c + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c \\ &= \eta J - x + y + \frac{1}{2}\left(r+2\gamma_d\right)\omega_c + (r+\gamma_d)O_h, \end{aligned} \tag{E14}$$

where the second equality uses (B2) and $V_{h[0,0]} = O_h$. Combining the value functions,

$$\omega_c = \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}}. \tag{E15}$$

Substituting this into (E14), we get (14). □

Proof of Theorem 2. Herein, I denote with hats the variables in the counterfactual environment without CDS.

The expected rents a long investor extracts from trading in the bond market, $\lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b$, have to be smaller in the equilibrium with CDS than in the equilibrium without CDS. To see this, the value function of a long investor is given by

$$(r+\gamma_d)V_{h[0,0]} = \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b + \lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c. \tag{E16}$$

The first term is the expected rents a long investor extracts from trading in the bond market. It is the probability of finding a counterparty in the bond market times the gains from trade from a bond transaction. The second term is the analogous expected gains from trade in the CDS market. Since the high-valuation agents' entry rate is an interior solution with and without CDS, (E16) with and without CDS is

$$\begin{aligned}
(r+\gamma_d)O_h &= \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b + \lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c \\
(r+\gamma_d)O_h &= \lambda_b \hat{\mu}_{a[1,0]} \frac{1}{2}\hat{\omega}_b,
\end{aligned}$$

respectively. Since $\lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c > 0$, and the left hand sides are the same, it has to be that: $\lambda_b \hat{\mu}_{a[1,0]} \frac{1}{2}\hat{\omega}_b > \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b$.

Combining (B19) and (B15), we get

$$\lambda_b \mu_{a[1,0]} \mu_{h[0,0]} = \gamma_d (S - \mu_{a[1,0]}). \tag{E17}$$

Equations (E12) and (E17) define $\mu_{h[0,0]}$ and ω_b as implicit functions of $\mu_{a[1,0]}$. Using (E12) and (E17), $\mu_{h[0,0]}$ and ω_b change with $\mu_{a[1,0]}$ as

$$\frac{\partial \mu_{h[0,0]}}{\partial \mu_{a[1,0]}} = -\frac{\gamma_d + \lambda_b \mu_{h[0,0]}}{\lambda_b \mu_{a[1,0]}}$$

$$\frac{\partial \omega_b}{\partial \mu_{a[1,0]}} = \frac{(\gamma_d + \lambda_b \mu_{h[0,0]})\frac{1}{2}\omega_b}{\mu_{a[1,0]}(r+\gamma_d+\lambda_b \mu_{h[0,0]}\frac{1}{2})}.$$

Thus, $\mu_{h[0,0]}$ decreases in $\mu_{a[1,0]}$, while ω_b increases in $\mu_{a[1,0]}$. Then, the expected rents a long investor extracts from trading in the bond market, $\lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b$, as an implicit function of $\mu_{a[1,0]}$, increase in $\mu_{a[1,0]}$. As a result, $\mu_{a[1,0]}$ has to be smaller in the equilibrium with CDS than in the equilibrium without CDS: $\mu_{a[1,0]} < \hat{\mu}_{a[1,0]}$. In turn, this implies that: $\omega_b < \hat{\omega}_b$ and $\mu_{h[0,0]} > \hat{\mu}_{h[0,0]}$. Since the illiquidity discount d_b just depends on $\mu_{h[0,0]}$, we have: $d_b < \hat{d}_b$. From (E17), a decrease in $\mu_{a[1,0]}$ implies an increase in the bond volume: $M_b > \hat{M}_b$. From (E3), an increase in $\mu_{h[0,0]}$ requires an increase in ν_h especially since ν_l changes from zero to a positive value in the presence of CDS. □

This proof relies on condition (E5), which is a sufficient but not a necessary condition for the spillover effect to arise. It ensured that the entry rate of high-valuation investors is an interior solution with and without CDS. The effect of CDS introduction before the entry rate of long investors adjusts is a decrease in the mass of long investors searching for a counterparty, $\mu_{h[0,0]}$, and an increase in their expected utility, $V_{h[0,0]}$. Both effects decrease the bond price. As long investors start to respond to short investors and enter at a higher rate, these two effects start to reverse: $\mu_{h[0,0]}$ starts to increase (this drives the bond price up), and $V_{h[0,0]}$, in turn, starts to decrease (this also drives the price up). At some point, which I will denote with $\widetilde{\nu}_h$, enough long investors enter so that (a) $\mu_{h[0,0]}$ is now higher than in the environment without CDS, (b) even though the increase in $V_{h[0,0]}$ is not reversed yet and $V_{h[0,0]}$ is still higher than without CDS, the increase in the bond price from the increase in $\mu_{h[0,0]}$ dominates the opposite pressure from the increase in $V_{h[0,0]}$. With a further entry of long investors, $\mu_{h[0,0]}$ increases so much so that the increase in $V_{h[0,0]}$ is fully reversed, and $V_{h[0,0]}$ is the same as in the absence of CDS. This is the new equilibrium entry rate of high-valuation investors. This level of the entry rate is higher than necessary because even at the lower entry rate, $\widetilde{\nu}_h$, the bond price and volume start to surpass their levels in the absence of CDS.

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Online Appendix: A Theory of Liquidity Spillover Between Bond and CDS Markets

F Proof of Proposition 1

Proof of Proposition 1. First, I derive the Hamilton-Jacobi-Bellman (HJB) equations.

Eq. (5) can be written recursively as:[22]

$$U(W_t,\tau_t) = \max_{C_t\in\mathbb{R},\tau_{t+\Delta t}\in\mathcal{T}(\tau_t,k)} u(C_t)\Delta t + (1-\beta\Delta t)\,\mathbb{E}U(W_{t+\Delta t},\tau_{t+\Delta t}). \tag{F1}$$

Subtract $(1-\beta\Delta t)\,U(W_t,\tau_t)$ from both sides and divide by Δt:

$$\beta U(W_t,\tau_t) = \max_{C_t\in\mathbb{R},\tau_{t+\Delta t}\in\mathcal{T}(\tau_t,k)} u(C_t) + (1-\beta\Delta t)\,\mathbb{E}\left[\frac{U(W_{t+\Delta t},\tau_{t+\Delta t}) - U(W_t,\tau_t)}{\Delta t}\right]. \tag{F2}$$

In the limit as $\Delta t\to 0$, (F2) becomes

$$\beta U(W_t,\tau_t) = \max_{C_t\in\mathbb{R},\tau_{t+dt}\in\mathcal{T}(\tau_t,k)} u(C_t) + \mathbb{E}\left[\frac{dU(W_t,\tau_t)}{dt}\right]. \tag{F3}$$

Consider the expectation of $dU(W_t,\tau_t)$. Applying a Taylor series expansion to $U(W_t,\tau_t)$ and taking its expectation, we get[23]

$$\begin{aligned}\mathbb{E}dU(W_t,\tau_t) =& U_W(W_t,\tau_t)\mathbb{E}\left[dW_t\right] + \frac{1}{2}U_{WW}(W_t,\tau_t)\mathbb{E}\left[dW_t^2\right] \\ &+ \sum_{k=1}^{K}\gamma_t(\tau_t,k)dt\left[U(W_t+P(\tau_t,\tau_{t+dt}),\tau_{t+dt}) - U(W_t,\tau_t)\right].\end{aligned} \tag{F4}$$

Consider dW_t in the first term. Using (1), (2), (3), and (6) and rearranging, we get

$$\begin{aligned}dW_t =& \left(rW_t - C_t + \mu_e\rho_t + \delta\theta_{b,t} + p_c\theta_{c,t}\right)dt + \left(\sigma_e\rho_t + J\left(\theta_{b,t}+\theta_{c,t}\right)\right)(-dN_t) \\ &+ \sqrt{1-\rho_t^2}\sigma_e dZ_t - p_b d\theta_{b,t}.\end{aligned}$$

Using $E[dN] = \eta dt$,

$$\mathbb{E}[dW_t] = \left(rW_t - C_t + \left[\mu_e\rho_t - \sigma_e\rho_t\eta\right] + (\delta-\eta J)\theta_{b,t} + (p_c-\eta J)\theta_{c,t}\right)dt. \tag{F5}$$

Using $E[dN^2] = \eta dt$,

$$\begin{aligned}\mathbb{E}[dW_t^2] &= \left(\sigma_e\rho_t + J\left(\theta_{b,t}+\theta_{c,t}\right)\right)^2\eta dt + \left(1-\rho_t^2\right)\sigma_e^2 dt \\ &= \Big(J^2\left(\theta_{b,t}+\theta_{c,t}\right)^2\eta + 2\sigma_e\rho_t J\left(\theta_{b,t}+\theta_{c,t}\right)\eta + \left[(\sigma_e\rho_t)^2\eta + \left(1-\rho_t^2\right)\sigma_e^2\right]\Big)dt.\end{aligned} \tag{F6}$$

[22] This comes from observing that over a small time interval $[0,\Delta t]$, (5) can be written as:

$$U(W_0,\tau_0) = \mathbb{E}\int_0^\infty e^{-\beta t}u(c_t^*)dt = u(c_0^*)\Delta t + e^{-\beta\Delta t}\mathbb{E}\left[\int_{\Delta t}^\infty e^{-\beta(t-\Delta t)}u(c_t^*)dt\right]$$

where $\{c_t^*\}$ is the optimal consumption path. The term inside the expectations operation is $U(W_{\Delta t},\tau_{\Delta t})$, thus $U(W_0,\tau_0) = \max_{c_0} u(c_0)\Delta t + e^{-\beta\Delta t}\mathbb{E}U\left(W_{\Delta t},\tau_{\Delta t}\right)$. Similarly, if we start at $\{W_t,\tau_t\}$ and approximate $e^{-\beta\Delta t}\approx 1-\beta\Delta t$, we get (F1).

[23] $dU(W_t,\tau_t) = U_W(W_t,\tau_t)dW_t + \frac{1}{2}U_{WW}(W_t,\tau_t)dW_t^2 + U_\tau(W_t,\tau_t)d\tau_t + \frac{1}{2}U_{\tau\tau}(W_t,\tau_t)d\tau_t^2$.

Thus, substituting (F5) and (F6) back into (F4), we get

$$
\begin{aligned}
&\mathbb{E}dU(W_t,\tau_t) = \\
&U_W(W_t,\tau_t)\left[(rW_t - C_t + [\mu_e\rho_t - \sigma_e\rho_t\eta] + (\delta-\eta J)\theta_{b,t} + (p_c-\eta J)\theta_{c,t})\,dt\right] \qquad \text{(F7)}\\
&+\frac{1}{2}U_{WW}(W_t,\tau_t)\left((J\theta_{b,t}+J\theta_{c,t})^2\eta + 2\sigma_e\rho_t\eta\,(J\theta_{b,t}+J\theta_{c,t}) + \left[(\sigma_e\rho_t)^2\eta + \left(1-\rho_t^2\right)\sigma_e^2\right]\right)dt\\
&+\sum_{k=1}^{K}\gamma_t(\tau_t,k)dt\left[U(W_t+P(\tau_t,\tau_{t+dt}),\tau_{t+dt}) - U(W_t,\tau_t)\right].
\end{aligned}
$$

Substituting (F7) back into (F3), the HJB in the steady state is given by

$$
\begin{aligned}
&\beta U(W,\tau) = \\
&\max_{C\in\mathbb{R},\tau'\in\mathcal{T}(\tau,k)} u(C) + U_W(W,\tau)\left[rW - C + [\mu_e\rho_\tau - \sigma_e\rho_\tau\eta] + (\delta-\eta J)\theta_b + (p_c-\eta J)\theta_c\right] \qquad \text{(F8)}\\
&+\frac{1}{2}U_{WW}(W,\tau)\left[(J\theta_b+J\theta_c)^2\eta + 2\sigma_e\rho_\tau\eta\,(J\theta_b+J\theta_c) + \left[(\sigma_e\rho_\tau)^2\eta + \left(1-\rho_\tau^2\right)\sigma_e^2\right]\right]\\
&+\sum_{k=1}^{K}\gamma(\tau,k)\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right].
\end{aligned}
$$

Using the guessed functional form $U(W,\tau) = -e^{-r\alpha(W+V_\tau+\bar{a})}$ and the FOC of (F8) with respect to C, the optimal consumption rate for agent τ is

$$
C_\tau = -\frac{log\,(r)}{\alpha} + r\left(W + V_\tau + \bar{a}\right), \qquad \text{(F9)}
$$

where[24]

$$
\bar{a} \equiv \frac{1}{r}\left(\frac{log\,(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2\right).
$$

Inserting (F9) back into (F8) and using $U(W,\tau) = -e^{-r\alpha(W+V_\tau+\bar{a})}$, $U_W = r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}$, and $U_{WW} = -r^2\alpha^2 e^{-r\alpha(W+V_\tau+\bar{a})}$, we get

$$
\begin{aligned}
&-\beta e^{-r\alpha(W_t+V_\tau+\bar{a})} = -e^{log(r) - r\alpha(W+V_\tau+\bar{a})} + \qquad \text{(F10)}\\
&r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}\left[\frac{log\,(r)}{\alpha} - r\left(V_\tau+\bar{a}\right) + [\mu_e\rho_\tau - \sigma_e\rho_\tau\eta] + (\delta-\eta J)\theta_b + (p_c-\eta J)\theta_c\right]\\
&-\frac{1}{2}r^2\alpha^2 e^{-r\alpha(W+V_\tau+\bar{a})}\left[(J\theta_b+J\theta_c)^2\eta + 2\sigma_e\rho_\tau\eta\,(J\theta_b+J\theta_c) + (\sigma_e\rho_\tau)^2\eta + \left(1-\rho_\tau^2\right)\sigma_e^2\right]\\
&+\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right].
\end{aligned}
$$

Divide both sides of (F10) by $-\frac{1}{r\alpha}e^{-ra(W+V_\tau+\bar{a})}$ and rearrange to get

$$
\begin{aligned}
0 = {}& rV_\tau - e^{ra(W+V_\tau+\bar{a})}\frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right] + r\bar{a}\\
&- r\frac{1}{r}\left[\frac{log\,(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2 + [\mu_e\rho_\tau - \sigma_e\rho_\tau\eta] - \frac{1}{2}r\alpha\left[(\sigma_e\rho_\tau)^2\eta - \sigma_e^2\rho_\tau^2\right]\right]\\
&-\left[(\delta-\eta J)\theta_b - \frac{1}{2}r\alpha\left((J\theta_b+J\theta_c)^2\eta + 2\sigma_e\rho_\tau\eta\,(J\theta_b+J\theta_c)\right) + (p_c-\eta J)\theta_c\right].
\end{aligned}
$$

[24]The FOC with respect to C is: $0 = \alpha e^{-\alpha C} - U_W(W_t,\tau_t)$. Using $U_W = r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}$, $re^{-r\alpha(W+V_\tau+\bar{a})} = e^{-\alpha C}$. Rewrite it as: $e^{log(r)}e^{-r\alpha(W+V_\tau+\bar{a})} = e^{-\alpha C}$.

Using the definition of $\bar{a}$, $b_\tau \equiv \mu_e\rho_\tau - \sigma_e\rho_\tau\eta - \frac{1}{2}r\alpha\left[(\sigma_e\rho_\tau)^2\eta - \sigma_e^2\rho_\tau^2\right]$, and $\theta_b\theta_c = 0$, we get

$$
\begin{aligned}
rV_\tau = {} & b_\tau + \left[\delta - \eta J - r\alpha\sigma_e\rho_\tau\eta J\right]\theta_b - \frac{1}{2}r\alpha J^2\eta\left[(\theta_b)^2 + (\theta_c)^2\right] + \left[p_c - (\eta J + r\alpha\sigma_e\rho_\tau\eta J)\right]\theta_c \\
& + e^{ra(W+V_\tau+\bar{a})}\frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right].
\end{aligned}
$$

I assume that $\mu_e\rho_\tau - \sigma_e\rho_\tau\eta - \frac{1}{2}r\alpha\left(\sigma_e\rho_\tau\right)^2(\eta-1) = 0$ so that $b_\tau = 0$. Using x_τ and y defined in (8) and (9) and the guessed functional form for $U(W,\tau)$,

$$
\begin{aligned}
rV_\tau = {} & \left((\delta - \eta J) - x_\tau\right)\theta_b - y|\theta_b| + \left[p_c - (\eta J + x_\tau)\right]\theta_c - y|\theta_c|. \qquad \text{(F11)} \\
& + \frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[1 - e^{-r\alpha(P(\tau',\tau)+V_{\tau'}-V_\tau)}\right].
\end{aligned}
$$

□

G Equilibrium Existence

Proof of Theorem 1. I prove in five steps. In step 1, I narrow down the equilibrium conditions into a set of five equations and five unknowns. In step 2, I show that the solution for the entry rate of high-valuation investors is unique and interior. In step 3, I characterize the solution for the entry rate of low-valuation investors and show that three solutions exist. In step 4, I show that—taking the entry rates as given—the population masses, value functions, the gains from trade, and prices are uniquely determined. The population masses and the gains from trade are also positive. Finally, in step 5, I show that the conjectured optimal trading strategies are indeed optimal. □

Step 1

Proof. From the market clearing conditions, the masses of agents who have reached their optimal asset position are

$$\mu_{h[1,0]} = S - \mu_{a[1,0]} \qquad \text{(G1)}$$

$$\mu_{l[0,\text{-}1]} = \mu_{h[0,1]} \qquad \text{(G2)}$$

$$\mu_{h[0,1]} = \frac{1}{\gamma_d + \gamma_u}M_c. \qquad \text{(G3)}$$

They depend only on the masses of active searchers.

I simplify the rest of the equilibrium conditions, first, into a set of nine equations of nine unknowns, $\mu_{a[1,0]}$, $\mu_{l[0,0]}$, $\mu_{h[0,0]}$, ω_b, ω_c, $V_{l[0,0]}$, $V_{h[0,0]}$, ν_h, and ν_l:

$$\lambda_b\mu_{a[1,0]}\mu_{h[0,0]} = \gamma_d(S - \mu_{a[1,0]}) \qquad \text{(G4)}$$

$$(r+\gamma_d)\omega_b = x - \lambda_b\left[\mu_{a[1,0]} + \mu_{h[0,0]}\right]\frac{1}{2}\omega_b - \lambda_c\mu_{l[0,0]}\frac{1}{2}\omega_c \qquad \text{(G5)}$$

$$(r+\gamma_d+\gamma_u)\omega_c = (2x - 2y) - \lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b - \lambda_c\left(\mu_{l[0,0]} + \mu_{h[0,0]}\right)\frac{1}{2}\omega_c \qquad \text{(G6)}$$

$$\frac{\nu_l F_l}{\gamma_u} = \mu_{l[0,0]} + \frac{1}{\gamma_d + \gamma_u}M_c \qquad \text{(G7)}$$

$$(r+\gamma_d)V_{h[0,0]} = \lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda_c\mu_{l[0,0]}\frac{1}{2}\omega_c \qquad \text{(G8)}$$

$$(r+\gamma_u)V_{l[0,0]} = \lambda_c \mu_{h[0,0]} \frac{1}{2}\omega_c \tag{G9}$$

$$\nu_h F_h + \gamma_u \frac{1}{\gamma_d+\gamma_u} M_c = \gamma_d \mu_{h[0,0]} + M_b + M_c \tag{G10}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i \end{cases} \qquad \text{for } i \in \{h, l\}. \tag{G11}$$

Eq. (G4) comes from combining (B19) and (B15). Eq. (G5) comes from combining the value functions for $h[0,0]$, $h[1,0]$, and $a[1,0]$, substituting in M_b and M_c, and simplifying. Combining the value functions for $h[0,0]$, $h[0,1]$, $l[0,0]$, and $l[0,\text{-}1]$ and substituting in M_b and M_c yields (G6). Eq. (G7) comes from (B14). Substituting M_b and M_c into (B2), we get (G8). Substituting M_b into (B1), we get (G9). Combining (B13) and (G3), we get (G10). Eq. (G11) is the entry condition for high and low-valuation agents, given in (4).

Next, I simplify the nine equations further into a set of five equations of five unknowns. Combining (G5) and (G8),

$$\omega_b = \frac{x-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}}. \tag{G12}$$

Combining (G6) and (G8),

$$\omega_c = \frac{2x-2y-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}}. \tag{G13}$$

From (G4), solve for $\mu_{a[1,0]}$ as

$$\mu_{a[1,0]} = \frac{\gamma_d S}{\left(\lambda_b \mu_{h[0,0]} + \gamma_d\right)}. \tag{G14}$$

From (G7), solve for $\mu_{l[0,0]}$ as

$$\mu_{l[0,0]} = \frac{\frac{\nu_l F_l}{\gamma_u}}{1+\frac{1}{\gamma_d+\gamma_u}\lambda_c\mu_{h[0,0]}}. \tag{G15}$$

Plugging these expressions back into (G8)-(G11) gives five equations of five unknowns $\{\mu_{h[0,0]}, V_{h[0,0]}, V_{l[0,0]}, \nu_h, \nu_l\}$:

$$\begin{aligned}(r+\gamma_d)V_{h[0,0]} - \lambda_b \frac{\gamma_d S}{\left(\gamma_d+\lambda_b\mu_{h[0,0]}\right)} \frac{1}{2} \frac{x-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}} \\ - \lambda_c \frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}} \frac{1}{2} \frac{2x-2y-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} = 0\end{aligned} \tag{G16}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda_c\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)V_{h[0,0]}}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} \tag{G17}$$

$$\nu_h F_h = \gamma_d\mu_{h[0,0]} + \gamma_d \frac{\lambda_b\mu_{h[0,0]}S}{\left(\lambda_b\mu_{h[0,0]}+\gamma_d\right)} + \gamma_d\lambda_c\mu_{h[0,0]}\frac{\frac{\nu_l F_l}{\gamma_u}}{\gamma_u+\gamma_d+\lambda_c\mu_{h[0,0]}} \tag{G18}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if } & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i \end{cases} \qquad \text{for } i \in \{h, l\}. \tag{G19}$$

□

Step 2

Proof. In the second step, I establish that, taking ν_l as given, a unique positive solution exists for ν_h.

I start by showing the sign of $\mu_{h[0,0]}$. The right-hand side of (G18) strictly increases in $\mu_{h[0,0]}$, is zero at $\mu_{h[0,0]} = 0$, and goes to ∞ as $\mu_{h[0,0]} \to \infty$. Thus, from (G18), $\mu_{h[0,0]}$ is positive and uniquely determined for any $\nu_h > 0$ and $\nu_l \geq 0$.

Next, I establish the existence of a solution for ν_h. When $\nu_h = 0$, $\mu_{h[0,0]} = 0$ from (G18). Also, $V_{l[0,0]} = 0$ from G17, and hence $\nu_l = 0$. Solving for $(r+\gamma_d)V_{h[0,0]}$ from (G16) and evaluating the solution at $\mu_{h[0,0]} = 0$ and $\nu_l = 0$ gives the left-hand-side of Assumption (E5). Thus, by Assumption (E5), $V_{h[0,0]} > O_h$ when $\nu_h = 0$. For a given level of ν_l, solving for $(r + \gamma_d)V_{h[0,0]}$ and $\mu_{h[0,0]}$ from (G16) and (G18), respectively, and evaluating them at $\nu_h = 1$, we get

$$(r+\gamma_d)V_{h[0,0]} = \frac{\frac{\lambda_c(\gamma_d+\gamma_u)\nu_l F_l \frac{1}{2}(2x-2y)}{\gamma_u(\gamma_d+\gamma_u+\lambda_c\bar{\mu})\left(r+\gamma_d+\gamma_u+\lambda_c\bar{\mu}\frac{1}{2}\right)} + \frac{\lambda_b\gamma_d S\frac{1}{2}x}{(\gamma_d+\lambda_b\bar{\mu})\left(r+\gamma_d+\lambda_b\bar{\mu}\frac{1}{2}\right)}}{1 + \frac{\lambda_b\gamma_d S\frac{1}{2}}{(\gamma_d+\lambda_b\bar{\mu})\left(r+\gamma_d+\lambda_b\bar{\mu}\frac{1}{2}\right)} + \frac{\lambda_c(\gamma_d+\gamma_u)\nu_l F_l\frac{1}{2}}{\gamma_u(\gamma_d+\gamma_u+\lambda_c\bar{\mu})\left(r+\gamma_d+\gamma_u+\lambda_c\bar{\mu}\frac{1}{2}\right)}}, \tag{G20}$$

where

$$\bar{\mu} = \left\{q + \left[q^2 + \left(\pi - p^2\right)^3\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} + \left\{q - \left[q^2 + \left(\pi - p^2\right)^3\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} + p, \tag{G21}$$

$p = -\frac{b}{3a}$, $q = p^3 + \frac{bc-3ad}{6a^2}$, $\pi = \frac{c}{3a}$,

$$a = -\lambda_b\lambda_c,$$

$$b = \left(\frac{F_h}{\gamma_d} - \frac{\nu_l F_l}{\gamma_u} - S\right)\lambda_c\lambda_b - \gamma_d\left(\lambda_b + \lambda_c\right) - \gamma_u\lambda_b,$$

$$c = (\gamma_d + \gamma_u)\left(\left(\frac{F_h}{\gamma_d} - S\right)\lambda_b - \gamma_d\right) + \left(\frac{F_h}{\gamma_d} - \frac{\nu_l F_l}{\gamma_u}\right)\gamma_d\lambda_c,$$

and

$$d = F_h\left(\gamma_d + \gamma_u\right).$$

By Assumption (E5), the right-hand-side of (G20) is less than $(r + \gamma_d)O_h$ for any $\nu_l \in [0, 1]$ implying that $V_{h[0,0]} < O_h$ when $\nu_h = 1$. Thus, a solution for ν_h exists, and it is an interior solution: $\nu_h \in (0, 1)$.

Next, I establish uniqueness. Equations (G16) and (G18) together define $V_{h[0,0]}$ as an implicit function of ν_h and ν_l: $V_{h[0,0]}(\nu_h, \nu_l)$. Applying the Implicit Function Theorem to (G16) and using the fact that $\mu_{h[0,0]}$ is positive, $\frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}}$ evaluated at an interior solution for ν_h (i.e., at $V_{h[0,0]} = O_h$) is negative. Applying the Implicit Function Theorem to (G18) and using the fact that $\mu_{h[0,0]}$ is positive, $\mu_{h[0,0]}$ increases in ν_h for any $\nu_l \geq 0$. Put together, for ν_h^* such that $V_{h[0,0]} = O_h$, $V_{h[0,0]}$ is strictly decreasing in ν_h:

$$\frac{\partial V_{h[0,0]}(\nu_h^*, \nu_l)}{\partial \nu_h} = \frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}}\frac{\partial \mu_{h[0,0]}(\nu_h^*, \nu_l)}{\partial \nu_h} < 0.$$

Thus, ν_h is given by a unique interior solution. □

Step 3

Proof. In this step, I characterize the solution for the entry rate of low-valuation agents.

First, I establish that $V_{l[0,0]}$ strictly increases in ν_l. Using the result in step 2 that the solution for ν_h is an interior solution, (G16) and (G17) become

$$(r+\gamma_d)O_h - \lambda_b \frac{\gamma_d S}{\left(\gamma_d + \lambda_b \mu_{h[0,0]}\right)} \frac{1}{2} \frac{x-(r+\gamma_d)O_h}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}} \\ - \lambda_c \frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}} \frac{1}{2} \frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} = 0 \tag{G22}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda_c\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}}. \tag{G23}$$

Equations (G22) and (G23) define $V_{l[0,0]}$ as an implicit function of ν_l: $V_{l[0,0]}(\nu_l)$. Applying the Implicit Function Theorem to (G22), $\mu_{h[0,0]}$ strictly increases in ν_l. The right-hand-side of (G23) increases in $\mu_{h[0,0]}$ and hence with ν_l. Put together, $V_{l[0,0]}$ strictly increases in ν_l.

Solving (G22) and (G23) for ν_l and $\mu_{h[0,0]}$ and evaluating them at $V_{l[0,0]} = O_l$, we get

$$\nu_l^* = \frac{2\gamma_u\left(r+\gamma_d+\gamma_u+\frac{1}{2}\lambda_c\mu_h^*\right)\left(\gamma_d+\gamma_u+\lambda_c\mu_h^*\right)\left[O_h\left(r+\gamma_d\right) - \frac{S\gamma_d(x-O_h(r+\gamma_d))\lambda_b}{\left(\gamma_d+\lambda_b\mu_h^*\right)\left(2(r+\gamma_d)+\lambda_b\mu_h^*\right)}\right]}{F_l\left(2x-2y-O_h\left(r+\gamma_d\right)\right)\left(\gamma_d+\gamma_u\right)\lambda_c}, \tag{G24}$$

where

$$\mu_h^* \equiv \frac{2O_l\left(r+\gamma_u\right)\left(r+\gamma_d+\gamma_u\right)}{\lambda_c\left(2x-2y-\left(r+\gamma_d\right)O_h-\left(r+\gamma_u\right)O_l\right)}.$$

If $0 < \nu_l^* \leq 1$ (which is Assumption (E8)), an interior solution exists and is uniquely given by (G24). Since $V_{l[0,0]}$ as an implicit function of ν_l strictly increases in ν_l, two corners solutions ($\nu_l = 0$ and $\nu_l = 1$) also exist when $0 < \nu_l^* < 1$. □

Table 1 characterizes the solutions for ν_l for all possible parameter conditions, not just (E8). The last row, for example, says that if the low-valuation investors' entry cost is too high, then none of them enter in equilibrium. By assuming (E8), I rule out such condition to ensure that the CDS market can exist in equilibrium.

Table 1:
ν_l^* is given by (G24).

Parameter condition	The solution
$\nu_l^* < 0$	$\nu_l = 1$
$\nu_l^* = 0$	$\nu_l = 0$, $\nu_l = 1$
$0 < \nu_l^* < 1$	$\nu_l = 0$, $\nu_l \in (0,1)$, $\nu_l = 1$
$\nu_l^* = 1$	$\nu_l = 0$, $\nu_l = 1$
$\nu_l^* > 1$	$\nu_l = 0$

Step 4

Proof. Equations (G1)-(G3), (G14), (G15), and (G22) uniquely determine the masses of agent types. In turn, ω_b and ω_c are uniquely given by (G12) and (G13), where $V_{h[0,0]} = O_h$. Given the agent masses, (B1)-(B6), (11)-(B9), (B10), and (B11) uniquely determine the value functions, prices, and termination fees.

The solution, moreover, is positive. Using $V_{h[0,0]} = O_h$, Assumption 1, and the fact that $\mu_{h[0,0]} > 0$, we have that $\omega_b > 0$ and $\omega_c > 0$. The result that $\mu_{h[0,0]} > 0$ implies that $\mu_{a[1,0]} > 0$ and $\mu_{l[0,0]} > 0$. Nonsearcher masses are similarly positive. □

Step 5

Proof. In this step, I verify that the conjectured trading strategies are in fact optimal.

From step 4, the gains from a bond transaction are positive, $\omega_b > 0$. The trading strategies of the bond buyer and seller, as a result, are optimal. This also shows that an average-valuation investor prefers to not hold a bond.

Analogously, since the total gains from a CDS transaction are positive: $\omega_c > 0$, it is optimal for $l[0,0]$ type agent to buy CDS, and for $h[0,0]$ type agent to sell CDS as conjectured.

Consider a CDS seller's decision to terminate her contract if she reverts to an average-valuation investor. Upon reverting to an average-valuation agent, if she pays the termination fee and terminates the CDS contract, her utility changes by $-T_{\text{S}} + 0 - V_{h[0,1]}$. If she instead remains a CDS seller, her utility changes by $V_{a[0,1]} - V_{h[0,1]}$, where

$$rV_{a[0,1]} = p_c - \eta J - y + \gamma_u(T_{\text{B}} + 0 - V_{a[0,1]}),$$

and $a[0,1]$ is an off-equilibrium asset position. She prefers to terminate if $V_{a[0,1]} < -T_{\text{S}}$, that is, if

$$\begin{aligned} V_{a[0,1]} + T_{\text{S}} &= V_{a[0,1]} + \frac{1}{2}\omega_c \\ &= \frac{p_c - \eta J - y + \gamma_u T_{\text{B}}}{(r+\gamma_u)} + \frac{1}{2}\omega_c \\ &= \frac{(r+\gamma_u+\gamma_d)(x-2y) - (x-(r+\gamma_d)O_h)\lambda_c\mu_{h[0,0]}\frac{1}{2}}{(r+\gamma_u)\left(r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}\right)} \end{aligned} \tag{G25}$$

is negative. The third equality uses (14) and (G13). For an interior solution of ν_l (i.e., using $V_l[0,0] = O_l$ and (G23)), the numerator of (G25) simplifies to

$$\frac{[r+\gamma_d+\gamma_u]\,[2x-2y-(r+\gamma_d)O_h]\,[x-2y-(r+\gamma_u)O_l]}{2x-2y-(r+\gamma_d)O_h-(r+\gamma_u)O_l}.$$

By Assumption 1, the second squared bracket is positive, the third bracket is negative, and the denominator is positive. The whole term, as a result, is negative. For a corner solution $\nu_l = 1$, since $\mu_{h[0,0]}$ increases in ν_l and the numerator of (G25) decreases in $\mu_{h[0,0]}$, the numerator of (G25) will remain negative. Thus, $V_{a[0,1]} < -T_{\text{S}}$: once a CDS seller switches to an average-valuation investor, she prefers to pay the fee and exit the market than to remain a CDS seller and wait until her counterparty terminates the contract. This shows that an average-valuation investor prefers no position than a long position through the CDS market.

Consider now a CDS buyer's decision to terminate. Upon reverting to an average-valuation agent, if she pays the termination fee and exits, her utility changes by: $-T_{\text{B}} + 0 - V_{l[0,\text{-}1]}$. If she remains a CDS buyer, her utility changes by: $V_{a[0,\text{-}1]} - V_{l[0,\text{-}1]}$, where

$$rV_{a[0,\text{-}1]} = -p_c + \eta J - y + \gamma_d(T_{\text{S}} + 0 - V_{a[0,\text{-}1]}).$$

She prefers to pay the fee if

$$V_{a[0,\text{-}1]} < -T_{\text{B}},$$

where $T_{\text{B}} = \frac{1}{2}\omega_c$. The difference is

$$V_{a[0,\text{-}1]} + T_{\text{B}} = \frac{x - 2y - (r+\gamma_d)O_h}{r+\gamma_d}.$$

The right-hand-side is negative by Assumption 1. Thus, a CDS buyer, upon a valuation shock, prefers to pay the fee and exit the market than to remain a CDS buyer. That is, an average-valuation agent prefers no position than a short position through the CDS market. □

H Proofs of Propositions 2-3

Proof of Proposition 2. For this proof, I set $\lambda_c = \lambda_b = \lambda$. I prove the result for two parameter regions separately: $\frac{F_h}{\gamma_d} < S$ and $\frac{F_h}{\gamma_d} \geq S$.

Consider the region $\frac{F_h}{\gamma_d} < S$ and what $\lambda\mu_{h[0,0]}$ limits to in this region. Suppose $\lim_{\lambda\to\infty} \lambda\mu_{h[0,0]} = \infty$. This implies that $\lim_{\lambda\to\infty} \mu_{h[0,0]} = 0$. To see this, if $\lim_{\lambda\to\infty} \mu_{h[0,0]} > 0$, then λ and $\lambda\mu_{h[0,0]}$ limit to ∞ at the same rate. The right-hand-side of (H3), as a result, limits to zero, implying $\nu_h \to 0$. But, using (G18), this contradicts that $\lim_{\lambda\to\infty} \mu_{h[0,0]} > 0$. Thus, $\lim_{\lambda\to\infty} \mu_{h[0,0]} = 0$ whenever $\lambda\mu_{h[0,0]}$ limits to ∞. Then, using $\lim_{\lambda\to\infty} \mu_{h[0,0]} = 0$ and $\lim_{\lambda\to\infty} \lambda\mu_{h[0,0]} = \infty$, (G18) becomes

$$\nu_h \frac{F_h}{\gamma_d} = S + \frac{\nu_l F_l}{\gamma_u}. \tag{H1}$$

From (H1), $\nu_h = 0$ cannot be a solution. If ν_h is an interior solution $\nu_h \in (0,1)$, taking the limit of (G17) as $\lim_{\lambda\to\infty} \lambda\mu_{h[0,0]} \to \infty$ and using $V_{h[0,0]} = O_h$, we get

$$\lim_{\lambda\to\infty} (r+\gamma_u)V_{l[0,0]} = 2x - 2y - (r+\gamma_d)O_h. \tag{H2}$$

Using Assumption 1, this implies $V_{l[0,0]} > O_l$, and hence $\nu_l = 1$. Plugging $\nu_l = 1$ in (H1),

$$\nu_h \frac{F_h}{\gamma_d} = S + \frac{F_l}{\gamma_u} > S.$$

This is a contradiction in the parameter region $\frac{F_h}{\gamma_d} < S$. If $\nu_h = 1$, (H1) becomes

$$\frac{F_h}{\gamma_d} = S + \frac{\nu_l F_l}{\gamma_u} \geq S,$$

which is again a contradiction in the parameter region $\frac{F_h}{\gamma_d} < S$. Thus, in the parameter region $\frac{F_h}{\gamma_d} < S$, it has to be $\lim_{\lambda\to\infty} \lambda\mu_{h[0,0]} < \infty$.

Now using $\lim_{\lambda\to\infty} \lambda\mu_{h[0,0]} < \infty$, I derive the limits of the entry rates. Solving for $V_{h[0,0]}$ from (G16),

$$(r+\gamma_d)V_{h[0,0]} = \frac{\frac{\lambda_c(\gamma_d+\gamma_u)\nu_l F_l \frac{1}{2}(2x-2y)}{\gamma_u\left(\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\right)\left(r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}\right)} + \frac{\lambda_b\gamma_d S\frac{1}{2}x}{\left(\gamma_d+\lambda_b\mu_{h[0,0]}\right)\left(r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}\right)}}{1 + \frac{\lambda_b\gamma_d S\frac{1}{2}}{\left(\gamma_d+\lambda_b\mu_{h[0,0]}\right)\left(r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}\right)} + \frac{\lambda_c(\gamma_d+\gamma_u)\nu_l F_l\frac{1}{2}}{\gamma_u\left(\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\right)\left(r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}\right)}}. \tag{H3}$$

The limit of the right-hand-side is

$$\lim_{\lambda\to\infty}(r+\gamma_d)V_{h[0,0]}=\frac{\frac{(\gamma_d+\gamma_u)\tilde{\nu}_l F_l \frac{1}{2}}{\gamma_u(\gamma_d+\gamma_u+m)\left(r+\gamma_d+\gamma_u+m\frac{1}{2}\right)}(2x-2y)+\frac{\gamma_d S\frac{1}{2}}{(\gamma_d+m)\left(r+\gamma_d+m\frac{1}{2}\right)}x}{\frac{(\gamma_d+\gamma_u)\tilde{\nu}_l F_l \frac{1}{2}}{\gamma_u(\gamma_d+\gamma_u+m)\left(r+\gamma_d+\gamma_u+m\frac{1}{2}\right)}+\frac{\gamma_d S\frac{1}{2}}{(\gamma_d+m)\left(r+\gamma_d+m\frac{1}{2}\right)}}, \tag{H4}$$

where $m \equiv \lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}$ and $\tilde{\nu}_l \equiv \lim_{\lambda\to\infty}\nu_l$. The right-hand-side is a weighted average between $(2x-2y)$ and x. By Assumption 1, both $2x-2y$ and x are larger than O_h. Thus, $\lim_{\lambda\to\infty}V_{h[0,0]} > O_h$, which implies that $\lim_{\lambda\to\infty}\nu_h = 1$. Plugging (H4) into (G17),

$$\lim_{\lambda\to\infty}(r+\gamma_u)V_{l[0,0]}=\frac{m\frac{1}{2}}{r+\gamma_d+\gamma_u+m\frac{1}{2}}\frac{\frac{\gamma_d S\frac{1}{2}}{(\gamma_d+m)\left(r+\gamma_d+m\frac{1}{2}\right)}(x-2y)}{\frac{\gamma_d S\frac{1}{2}}{(\gamma_d+m)\left(r+\gamma_d+m\frac{1}{2}\right)}+\frac{(\gamma_d+\gamma_u)\tilde{\nu}_l F_l \frac{1}{2}}{\gamma_u(\gamma_d+\gamma_u+m)\left(r+\gamma_d+\gamma_u+m\frac{1}{2}\right)}}, \tag{H5}$$

The coefficient in front of $(x-2y)$ is less than or equal to 1, the right-hand-side, as a result, is less than $x-2y$. But, by Assumption 1, $x-2y<(r+\gamma_u)O_l$. This implies that $V_{l[0,0]}<O_l$ and $\lim_{\lambda\to\infty}\nu_l = 0$. Thus, if $\frac{F_h}{\gamma_d}<S$, then $0<\lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}<\infty$, $\lim_{\lambda\to\infty}\nu_h=1$, and $\lim_{\lambda\to\infty}\nu_l=0$. Since $\lim_{\lambda\to\infty}\nu_l=0$, the environment converges to the environment without CDS, and hence the result that CDS is redundant.

Now consider the region $\frac{F_h}{\gamma_d}\geq S$. Suppose $m\equiv\lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}<\infty$. Then, using arguments as above, $\lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}<\infty$ implies that $\lim_{\lambda\to\infty}\nu_h=1$ and $\lim_{\lambda\to\infty}\nu_l=0$. Using these entry rate limits, $\lim_{\lambda\to\infty}\mu_{h[0,0]}=0$ (implied by $\lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}<\infty$), and (G18), we get

$$\frac{F_h}{\gamma_d}=\frac{m}{m+\gamma_d}S \tag{H6}$$

For any $\infty>m\geq 0$, the right-hand-side is less than S, which is a contradiction in the parameter region $\frac{F_h}{\gamma_d}\geq S$. Thus, it has to be $\lim_{\lambda\to\infty}\lambda\mu_{h[0,0]}\to\infty$.

The illiquidity discount just depends on the limit of $\lambda\mu_{h[0,0]}$. Since $\lambda\mu_{h[0,0]}\to\infty$, $d_b\to 0$. Moreover, the bond volume is

$$M_b=\frac{S\gamma_d\lambda\mu_{h[0,0]}}{\gamma_d+\lambda\mu_{h[0,0]}}.$$

As $\lambda\mu_{h[0,0]}\to\infty$, M_b limits to $S\gamma_d$. Thus, in a frictionless environment, the illiquidity discount and the bond trading volume are the same in the environments with and without CDS trading.

The above completes the proof. As an aside, I derive the limits of the entry rates for the parameter space, $\frac{F_h}{\gamma_d}\geq S$, in particular, in two subregions: $\frac{F_h}{\gamma_d}>S+\frac{F_l}{\gamma_u}$ and $S+\frac{F_l}{\gamma_u}\geq\frac{F_h}{\gamma_d}\geq S$. Taking the limit of the right-hand-side of (G18) as $\lambda\mu_{h[0,0]}\to\infty$ and using $\lim_{\lambda\to\infty}\mu_{h[0,0]}=0$,

$$\nu_h\frac{F_h}{\gamma_d}=S+\frac{\nu_l F_l}{\gamma_u}. \tag{H7}$$

Consider the first subregion: $\frac{F_h}{\gamma_d}>S+\frac{F_l}{\gamma_u}$. For (H7) to hold, it has to $\lim_{\lambda\to\infty}\nu_h\in(0,1)$ for any $\nu_l\in[0,1]$. Then, $\lim_{\lambda\to\infty}V_{h[0,0]}=O_h$. The limit of the right-hand-side of (G17) as $\lambda\mu_{h[0,0]}\to\infty$ is

$$\frac{1}{r+\gamma_u}\left(2x-2y-(r+\gamma_d)O_h\right). \tag{H8}$$

Using Assumption 1, this is greater than O_l. Thus, in the limit, $\nu_l=1$.

Consider the second subregion: $S+\frac{F_l}{\gamma_u}\geq\frac{F_h}{\gamma_d}\geq S$. Suppose $\nu_h\in(0,1)$. Then, from (H7),

$S+\frac{\nu_l F_l}{\gamma_u}=\nu_h\frac{F_h}{\gamma_d}<\frac{F_h}{\gamma_d}$, which is a contradiction. Thus, it has to be $\nu_h=1$ and becomes:

$$\frac{F_h}{\gamma_d}=S+\frac{\nu_l F_l}{\gamma_u}. \tag{H9}$$

From here, $\nu_l\in(0,1]$. $\square$

Proof of Proposition 3. I denote with $V^m_{h[0,0]}$ the long investor's expected utility associated with a market choice $m\in\{b,c\}$ and with ν^m_h the fraction of long investors that choose m, where b and c stand for entering the bond market and the CDS market, respectively. The equilibrium entry rates $\{\nu^m_h\}_{m\in\{b,c,bc\}}$ solve

$$\nu^m_h=\begin{cases}1 & V^m_{h[0,0]}>\left\{V^b_{h[0,0]},V^c_{h[0,0]},O_h\right\}/V^m_{h[0,0]}\\ [0,1]\text{ if} & V^m_{h[0,0]}=\left\{V^b_{h[0,0]},V^c_{h[0,0]}O_h\right\}/V^m_{h[0,0]}\\ 0 & V^m_{h[0,0]}<\left\{V^b_{h[0,0]},V^c_{h[0,0]},O_h\right\}/V^m_{h[0,0]}.\end{cases} \tag{H10}$$

The value functions of investors participating in the bond market are characterized by

$$rV^b_{h[0,0]} = \gamma_d(0-V^b_{h[0,0]})+\lambda\mu_{a[1,0]}\frac{1}{2}\omega_b \tag{H11}$$
$$rV_{h[1,0]} = (\delta-\eta J)+x-y+\gamma_d(V_{a[1,0]}-V_{h[1,0]}) \tag{H12}$$
$$rV_{a[1,0]} = (\delta-\eta J)-y+\frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b. \tag{H13}$$

The population masses $\{\mu_{h[1,0]},\,\mu^b_{h[0,0]},\,\mu_{a[1,0]}\}$ are given by

$$\nu^b_h F_h=\gamma_d\mu^b_{h[0,0]}+M_b$$
$$M_b=\gamma_d\mu_{h[1,0]} \tag{H14}$$
$$\mu_{h[1,0]}+\mu_{a[1,0]}=S. \tag{H15}$$

The entry rate into the bond market, ν^b_h, solves (H10).

The value functions of investors in the CDS market are characterized by

$$rV^c_{h[0,0]} = \gamma_d(0-V^c_{h[0,0]})+\frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c \tag{H16}$$
$$rV_{l[0,0]} = \gamma_u(0-V_{l[0,0]})+\frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c \tag{H17}$$
$$rV_{h[0,1]} = p_c-(\eta J-x)-y+\gamma_d(-T_{\mathrm{S}}-V_{h[0,1]}) \tag{H18}$$
$$rV_{l[0,\text{-}1]} = -p_c+(\eta J+x)-y+\gamma_u\left(-T_{\mathrm{B}}-V_{l[0,\text{-}1]}\right). \tag{H19}$$

The population masses $\{\mu_{h[0,1]},\,\mu^c_{h[0,0]},\,\mu_{l[0,\text{-}1]},\,\mu_{l[0,0]}\}$ of investors in the CDS market are characterized by

$$\nu^c_h F_h+\gamma_u\mu_{h[0,1]}=\gamma_d\mu^c_{h[0,0]}+M_c$$
$$\nu_l F_l+\gamma_d\mu_{l[0,\text{-}1]}=\gamma_u\mu_{l[0,0]}+M_c \tag{H20}$$
$$M_c=\gamma_d\mu_{h[0,1]}+\gamma_u\mu_{h[0,1]}$$

$$\mu_{h[0,1]}=\mu_{l[0,\text{-}1]}.$$

The entry rates into the CDS market, ν^c_h and ν_l, solve (H10) and (4), respectively.

Combining the equations characterizing the population masses, we get

$$\frac{\nu_h^b F_h}{\gamma_d} = \mu_{h[0,0]}^b + \frac{\lambda_b \mu_{h[0,0]}^b}{\left(\lambda_b \mu_{h[0,0]}^b + \gamma_d\right)} S.$$

Applying the Implicit Function Theorem, $\mu_{h[0,0]}^b$ increases in ν_h^b.

I describe next how bond market liquidity depends on the mass and hence the entry rate of high valuation investors. Combining the value functions for $h[0,0]$, $h[1,0]$, $a[1,0]$, the bond price is

$$p_b = \frac{\delta - \eta J - y}{r} + \frac{1}{r} \frac{(r + \lambda_b \mu_{h[0,0]}^b) \frac{1}{2} x}{r + \gamma_d + \lambda_b \mu_{h[0,0]}^b \frac{1}{2} + \lambda_b \mu_{a[1,0]} \frac{1}{2}} \tag{H21}$$

$$= \frac{\delta - \eta J - y}{r} + \frac{1}{r} \frac{x(r + \lambda_b \mu_{h[0,0]}^b)(\gamma_d + \lambda_b \mu_{h[0,0]}^b)}{2\gamma_d(r + \gamma_d) + \gamma_d \lambda_b S + (2r + 3\gamma_d + \lambda_b \mu_{h[0,0]})\lambda_b \mu_{h[0,0]}}, \tag{H22}$$

where the last equality uses (H14) and (H15). The right-hand side and, thus, the bond price is increasing in $\mu_{h[0,0]}$. Combining the equations characterizing the population masses,

$$M_b = \gamma_d \mu_{h[1,0]} \tag{H23}$$

$$= \gamma_d (S - \mu_{a[1,0]}) \tag{H24}$$

$$= \gamma_d \left(S - \frac{\gamma_d S}{\left(\lambda_b \mu_{h[0,0]} + \gamma_d\right)}\right) \tag{H25}$$

$$= S \frac{\lambda_b \mu_{h[0,0]} \gamma_d}{\lambda_b \mu_{h[0,0]} + \gamma_d}. \tag{H26}$$

The bond volume, as a result, increases in $\mu_{h[0,0]}$. Thus, the bond market is more liquid if high-valuation investors enter at a higher rate.

Combining the value functions and the population masses of the bond market investors,

$$(r + \gamma_d) V_{h[0,0]}^b = \frac{\lambda_b \gamma_d S}{\lambda_b \gamma_d S + 2(r + \gamma_d + \lambda_b \mu_{h[0,0]}^b \frac{1}{2}) \left(\lambda_b \mu_{h[0,0]}^b + \gamma_d\right)} x.$$

Thus, $V_{h[0,0]}^b$ is decreasing in $\mu_{h[0,0]}^b$ and hence in ν_h^b. To simplify the analysis, I assume that, prior to the CDS introduction, the entry rate into the bond market, ν_h^b, is given by an interior solution.

When CDS trading is feasible, we can rule out $V_{h[0,0]}^b < O_h$ as the equilibrium outcome. $V_{h[0,0]}^b < O_h$ implies $\nu_h^b = 0$. But a decrease in ν_h^b implies $V_{h[0,0]}^b > O_h$, not $V_{h[0,0]}^b < O_h$. Thus, in equilibrium when CDS is feasible: $V_{h[0,0]}^b \geq O_h$.

Table 2:

				Combined
1	$V^b_{h[0,0]} > O_h$	$V^c_{h[0,0]} > O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > V^c_{h[0,0]}$
2			$V^b_{h[0,0]} = V^c_{h[0,0]}$	$V^b_{h[0,0]} = V^c_{h[0,0]} > O_h$
3			$V^c_{h[0,0]} > V^b_{h[0,0]}$	$V^c_{h[0,0]} > V^b_{h[0,0]} > O_h$
4		$V^c_{h[0,0]} = O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > O_h = V^c_{h[0,0]}$
5		$V^c_{h[0,0]} < O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > O_h > V^c_{h[0,0]}$
6	$V^b_{h[0,0]} = O_h$	$V^c_{h[0,0]} > O_h$	$V^c_{h[0,0]} > V^b_{h[0,0]}$	$V^c_{h[0,0]} > V^b_{h[0,0]} = O_h$
7		$V^c_{h[0,0]} = O_h$	$V^c_{h[0,0]} = V^b_{h[0,0]}$	$V^c_{h[0,0]} = V^b_{h[0,0]} = O_h$
8		$V^c_{h[0,0]} < O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} = O_h > V^c_{h[0,0]}$

To figure out the equilibrium outcomes, Table 2 lists possible orderings of $V^b_{h[0,0]}$, $V^c_{h[0,0]}$, and O_h. The first row cannot be an equilibrium. $V^b_{h[0,0]} > V^c_{h[0,0]}$ would mean that $\nu^b_h = 1$ and $\nu^c_h = 0$. At $\nu^c_h = 0$, however, low-valuation agents do not enter either ($\nu_l = 0$), and hence $V^c_{h[0,0]} = 0$. This contradicts $V^c_{h[0,0]} > O_h$. The fourth and fifth rows cannot be an equilibrium. $V^b_{h[0,0]} > O_h \geq V^c_{h[0,0]}$ implies that $\nu^c_h = 0$ and $\nu^b_h = 1$, but since $V^b_{h[0,0]}$ is decreasing in ν^b_h, $V^b_{h[0,0]}$ evaluated at $\nu^b_h = 1$ should be lower than O_h. Hence, it is a contradiction. The sixth row is not an equilibrium. $V^c_{h[0,0]} > V^b_{h[0,0]} = O_h$ implies that $\nu^c_h = 1$ and $\nu^b_h = 0$. However, $V^b_{h[0,0]}$ evaluated at $\nu^b_h = 0$ does not equal O_h.

Four equilibria exist. In two of them, the entry rate into the bond market, ν^b_h, stays the same as in the environment without CDS. In one of these two, the entry rate into the CDS market is positive: $\nu^c_h + \nu^b_h \leq 1$ and $V^c_{h[0,0]} = V^b_{h[0,0]} = O_h$, while in the other, none of the high-valuation investors enter the CDS market, and $V^b_{h[0,0]} = O_h > V^c_{h[0,0]} = 0$. In the other two equilibria, the entry rate into the bond market, ν^b_h, decreases. In one of these, shown in row 3, $V^c_{h[0,0]} > V^b_{h[0,0]} > O_h$ implying that $\nu^b_h = 0$ and $\nu^c_h = 1$. This means that all high-valuation investors enter the CDS, and the trading activity in the bond market disappears. In the other, shown in row 2, $V^b_{h[0,0]} = V^c_{h[0,0]} > O_h$ implying that the entry rate into the bond market decreases (so that $V^b_{h[0,0]}$ is higher).[25]

Thus, with the introduction of the CDS market, high valuation investors enter the bond market either at the same or at a lower rate. If the entry rate remains the same, the illiquidity discount and bond volume also remain unaffected by CDS introduction. If the entry rate decreases, bond market liquidity deteriorates. The bond volume decreases, and the illiquidity discount increases. □

I Short-Selling and CDS

This section presents the model with both short-selling and CDS trading. The short-selling part of the model follows Vayanos and Weill (2008). To be self-containing, I describe the main features and mechanism of the Vayanos and Weill (2008) framework, but for more details I refer the reader to their paper.

I denote an agent's portfolio with $[\theta_b, \theta_r, \theta_c]$ where the CDS position is now denoted on the third position, $\theta_r = 1$ denotes that an agent has lent her bond, $\theta_r = -1$ denotes that the agent has borrowed a bond, and θ_b and θ_c denote bond and CDS positions as before. If an agent has

[25] The entry rate could decrease to another, smaller, rate $\nu^b_h > 0$, or all the way to zero, $\nu^b_h = 0$.

no CDS position, I denote her position with just $[\theta_b, \theta_r]$. Combined, we have seven feasible asset portfolios: $[1,0]$ is a bond owner, $[0,1]$ is an investor who has lent out her bond, $[1,\text{-}1]$ is an investor who has borrowed a bond but has not yet sold it short, $[0,\text{-}1]$ is an investor who has borrowed a bond and has sold it short, $[0,0,1]$ is an investor who has sold CDS, $[0,0,\text{-}1]$ is an investor who has bought CDS, and $[0,0]$ is an investor with no asset position.

Short-selling changes the equilibrium agent types and their optimal trading strategies as follows. A high-valuation bond owner ($h[1,0]$) now searches for a bond borrower in the repo market (i.e., for a short investor) to lend her bond to. After lending her bond, her position changes to $h[0,1]$. If she reverts to an average-valuation while her counterparty has not yet sold the bond, she demands back the bond and becomes a bond seller ($a[1,0]$). If she reverts to an average-valuation after her counterparty has already (short-) sold the bond, she seizes the collateral her counterparty has put aside and exits the market.

A low-valuation investor without a position ($l[0,0]$), in addition to searching in the CDS market, now searches at the same time for a bond lender in the repo market. If she first meets a bond lender, she borrows a bond and her type changes to $l[1,\text{-}1]$. She becomes a short seller in the bond market. If she reverts to an average valuation before she could short-sell the bond, she delivers back the bond and exits the market. But if she sells, her type changes to $l[0,\text{-}1]$. This is her terminal optimal position. If she gets a valuation shock and becomes an average-valuation agent ($a[0,\text{-}1]$), she searches to buy back the bond. After buying back, she delivers the bond back to the lender, unwinds her short position, and exits.

Define the total mass of bond sellers as

$$\mu_{b,\mathrm{S}} \equiv \mu_{l[1,\text{-}1]} + \mu_{a[1,0]},$$

where $\mu_{l[1,\text{-}1]}$ is the mass of short sellers and $\mu_{a[1,0]}$ is the mass of regular bond sellers (that is, those with a long position looking to unwind their long position). Analogously, the total mass of bond buyers is

$$\mu_{b,\mathrm{B}} \equiv \mu_{h[0,0]} + \mu_{a[0,\text{-}1]},$$

where $h[0,0]$ are the long investors seeking a long position by buying a bond, and $a[0,\text{-}1]$ are the investors looking to buy back the bond to deliver it to its repo counterparty and unwind their short position. In the repo market, the active searchers are the bond borrowers, $l[0,0]$, and bond lenders, $h[1,0]$.

Define with q's the meeting intensities:

$$q_{\mathrm{B}} \equiv \lambda_b \mu_{b,\mathrm{B}}$$

$$q_{\mathrm{S}} \equiv \lambda_b \mu_{b,\mathrm{S}}$$

$$q_{\mathrm{LE}} \equiv \lambda_r \mu_{h[1,0]}$$

$$q_{\mathrm{BO}} \equiv \lambda_r \mu_{l[0,0]}$$

$$q_{c,\mathrm{B}} \equiv \lambda_c \mu_{l[0,0]}$$

$$q_{c,\mathrm{S}} \equiv \lambda_c \mu_{h[0,0]}.$$

The inflow-outflow equations are

$$\nu_h F_h + \gamma_u \mu_{l[0,0,\text{-}1]} = \gamma_d \mu_{h[0,0]} + (q_{\mathrm{S}} + q_{c,\mathrm{B}})\, \mu_{h[0,0]} \tag{I1}$$

$$q_{\mathrm{S}} \mu_{h[0,0]} + \gamma_u \mu_{l[1,\text{-}1]} + q_{\mathrm{S}} \mu_{a[0,\text{-}1]} = q_{\mathrm{BO}} \mu_{h[1,0]} + \gamma_d \mu_{h[1,0]} \tag{I2}$$

$$q_{\mathrm{BO}} \mu_{h[1,0]} = \gamma_d \mu_{h[0,1]} + \gamma_u \mu_{l[1,\text{-}1]} + q_{\mathrm{S}} \mu_{a[0,\text{-}1]} \tag{I3}$$

$$\nu_l F_l + \gamma_d \mu_{l[1,\text{-}1]} + \gamma_d \mu_{l[0,\text{-}1]} + \gamma_d \mu_{l[0,0,\text{-}1]} = \gamma_u \mu_{l[0,0]} + (q_{\mathrm{LE}} + q_{c,\mathrm{S}})\, \mu_{l[0,0]} \tag{I4}$$

$$q_{\mathrm{LE}}\mu_{l[0,0]} = \gamma_u\mu_{l[1,\text{-}1]} + q_{\mathrm{B}}\mu_{l[1,\text{-}1]} + \gamma_d\mu_{l[1,\text{-}1]} \tag{I5}$$

$$q_{\mathrm{B}}\mu_{l[1,\text{-}1]} = (\gamma_u + \gamma_d)\,\mu_{l[0,\text{-}1]} \tag{I6}$$

$$\gamma_u\mu_{l[0,\text{-}1]} = q_{\mathrm{S}}\mu_{a[0,\text{-}1]} + \gamma_d\mu_{a[0,\text{-}1]} \tag{I7}$$

$$\gamma_d\mu_{h[1,0]} + \gamma_d\mu_{l[1,\text{-}1]} = q_{\mathrm{B}}\mu_{a[1,0]} \tag{I8}$$

$$q_{c,\mathrm{B}}\mu_{h[0,0]} = (\gamma_u + \gamma_d)\,\mu_{h[0,0,1]} \tag{I9}$$

$$q_{c,\mathrm{B}}\mu_{h[0,0]} = (\gamma_u + \gamma_d)\,\mu_{l[0,0,\text{-}1]}. \tag{I10}$$

The market clearing conditions are

$$\mu_{h[1,0]} + \mu_{a[1,0]} + \mu_{l[1,\text{-}1]} = S \tag{I11}$$

$$\mu_{h[0,1]} = \mu_{l[1,\text{-}1]} + \mu_{l[0,\text{-}1]} + \mu_{a[0,\text{-}1]} \tag{I12}$$

$$\mu_{h[0,0,1]} = \mu_{l[0,0,\text{-}1]}. \tag{I13}$$

To characterize the value functions of agents who have lent their bond, $h[0,1]$, we have to keep track of their counterparty. The subscript on the value functions of an $h[0,1]$ investor, as a result, denotes both the agent's own type and the type of her counterparty. For example, $V_{h[0,1]l[1,\text{-}1]}$ denotes the value function of an investor who has lent her bond ($h[0,1]$) whose counterparty is $l[1,\text{-}1]$ type.

The value functions are

$$rV_{h[0,0]} = \gamma_d\left(0 - V_{h[0,0]}\right) + q_{\mathrm{S}}\left(V_{h[1,0]} - V_{h[0,0]} - p_b\right) + q_{c,\mathrm{B}}\left(V_{h[0,0,1]} - V_{h[0,0]}\right) \tag{I14}$$

$$rV_{h[1,0]} = \delta - \eta J + x - y + q_{\mathrm{BO}}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) + \gamma_d\left(V_{a[1,0]} - V_{h[1,0]}\right) \tag{I15}$$

$$\begin{aligned} rV_{h[0,1]l[1,\text{-}1]} = {} & \delta - \eta J + x - y + fee + \gamma_d\left(V_{a[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right) \\ & + \gamma_u\left(V_{h[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right) + q_{\mathrm{B}}\left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) \end{aligned} \tag{I16}$$

$$rV_{h[0,1]l[0,\text{-}1]} = \delta - \eta J + x - y + fee + \gamma_d\left(z - V_{h[0,1]l[0,\text{-}1]}\right) + \gamma_u\left(V_{h[0,1]a[0,\text{-}1]} - V_{h[0,1]l[0,\text{-}1]}\right) \tag{I17}$$

$$rV_{h[0,1]a[0,\text{-}1]} = \delta - \eta J + x - y + fee + \gamma_d\left(z - V_{h[0,1]a[0,\text{-}1]}\right) + q_{\mathrm{S}}\left(V_{h[1,0]} - V_{h[0,1]a[0,\text{-}1]}\right) \tag{I18}$$

$$rV_{l[0,0]} = \gamma_u\left(0 - V_{l[0,0]}\right) + q_{\mathrm{LE}}\left(V_{l[1,\text{-}1]} - V_{l[0,0]}\right) + q_{\mathrm{CS}}\left(V_{l[0,0,\text{-}1]} - V_{l[0,0]}\right) \tag{I19}$$

$$rV_{l[1,\text{-}1]} = -fee + \gamma_u\left(0 - V_{l[1,\text{-}1]}\right) + q_{\mathrm{B}}\left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b\right) + \gamma_d\left(V_{l[0,0]} - V_{l[1,\text{-}1]}\right) \tag{I20}$$

$$rV_{l[0,\text{-}1]} = -fee - (\delta - \eta J - x) - y + \gamma_u\left(V_{a[0,\text{-}1]} - V_{l[0,\text{-}1]}\right) + \gamma_d\left(V_{l[0,0]} - z - V_{l[0,\text{-}1]}\right) \tag{I21}$$

$$rV_{a[0,\text{-}1]} = -fee - (\delta - \eta J) - y + q_{\mathrm{S}}\left(0 - p_b - V_{a[0,\text{-}1]}\right) + \gamma_d\left(0 - z - V_{a[0,\text{-}1]}\right) \tag{I22}$$

$$rV_{a[1,0]} = \delta - \eta J - y + q_{\mathrm{B}}\left(p_b + 0 - V_{a[1,0]}\right) \tag{I23}$$

$$rV_{h[0,0,1]} = p_c - \eta J + x - y + \gamma_d\left(-T_{\mathrm{S}} - V_{h[0,0,1]}\right) \tag{I24}$$

$$rV_{l[0,0,\text{-}1]} = \eta J + x - y - p_c + \gamma_u\left(-T_{\mathrm{B}} - V_{l[0,0,\text{-}1]}\right). \tag{I25}$$

where z is the collateral that the bond lender seizes if the bond borrower cannot deliver the bond, and fee is the lending fee the security borrower pays the security lender throughout the repo contract. Following Vayanos and Weill (2008), I set $z = V_{a[1,0]}$. The entry decisions of high- and low-valuation investors are as before and are given by (4).

The terms of trade arise from bilateral bargaining as before. In a bond transaction, the bond buyer $h[0,0]$ and the seller $a[1,0]$ bargain over price. The bond price, as a result, is characterized by

$$p_b = \frac{1}{2}\left(V_{h[1,0]} - V_{h[0,0]}\right) + \frac{1}{2}V_{a[1,0]}. \tag{I26}$$

I assume, following Vayanos and Weill (2008), that the other bond buyer ($a[0,\text{-}1]$) and seller ($l[1,\text{-}1]$) transact at the same price and focus on the parameter conditions where it is optimal to do so. The bond lender and the borrower negotiate over the lending fee so that each gets half of the total gains from trade:

$$V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]} = \frac{1}{2}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]} + V_{l[1,\text{-}1]} - V_{l[0,0]}\right). \tag{I27}$$

As in the main environment, the CDS spread is given by

$$V_{h[0,0,1]} - V_{h[0,0]} = \frac{1}{2}\left(V_{h[0,0,1]} - V_{h[0,0]} + V_{l[0,0,\text{-}1]} - V_{l[0,0]}\right). \tag{I28}$$

Define the gains from a repo transaction as

$$\omega_r \equiv \left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) + V_{l[1,\text{-}1]} - V_{l[0,0]}. \tag{I29}$$

Proof of Lemma 3. Take the difference between (I15) and (I14) and get

$$r\left(V_{h[1,0]} - V_{h[0,0]}\right) = \delta - \eta J + x - y + q_{\text{BO}}\frac{1}{2}\omega_r - \gamma_d\omega_b - q_{\text{S}}\frac{1}{2}\omega_b - q_{c,\text{B}}\frac{1}{2}\omega_c. \tag{I30}$$

Using (I14),

$$r\left(V_{h[1,0]} - V_{h[0,0]}\right) = \delta - \eta J + x - y + q_{\text{BO}}\frac{1}{2}\omega_r - \gamma_d\omega_b - (r+\gamma_d)V_{h[0,0]}. \tag{I31}$$

Combine this with (I23) and $V_{h[0,0]} = O_h$ and solve for ω_b as

$$\omega_b = \frac{x - (r+\gamma_d)O_h + q_{\text{BO}}\frac{1}{2}\omega_r}{\left(r+\gamma_d+q_{\text{B}}\frac{1}{2}\right)}. \tag{I32}$$

Combine (I23) and (I31), simplify, and get

$$rp_b = \delta - \eta J - y + \frac{1}{2}x + \frac{1}{2}\left(-\gamma_d\omega_b - (r+\gamma_d)V_{h[0,0]} + q_{\text{B}}\frac{1}{2}\omega_b\right) + \frac{1}{4}q_{\text{BO}}\omega_r. \tag{I33}$$

From (I32), $q_{\text{B}}\frac{1}{2}\omega_b = x - (r+\gamma_d)V_{h[0,0]} - (r+\gamma_d)\omega_b + q_{\text{BO}}\frac{1}{2}\omega_r$. Substitute this and $V_{h[0,0]} = O_h$ into (I33) and simplify to get

$$rp_b = \delta - \eta J + x - y - (r+\gamma_d)\,O_h - \left(\frac{r+2\gamma_d}{2}\right)\omega_b + q_{\text{BO}}\frac{1}{2}\omega_r. \tag{I34}$$

Plugging (I32) into (I34), and simplifying, we get

$$p_b = \frac{(\delta-\eta J) + x - y - (r+\gamma_d)O_h}{r} - \underbrace{\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda_b\mu_{b,\text{B}}\frac{1}{2}}}_{\text{illiquidity discount}} \tag{I35}$$
$$+ \underbrace{\frac{(r+\lambda_b\mu_{b,\text{B}})}{r}\frac{1}{2}\frac{\lambda_r\mu_{l[0,0]}\frac{1}{2}\omega_r}{r+\gamma_d+\lambda_b\mu_{b,\text{B}}\frac{1}{2}}}_{\text{lending fee effect}}.$$

The bond price without short-selling is

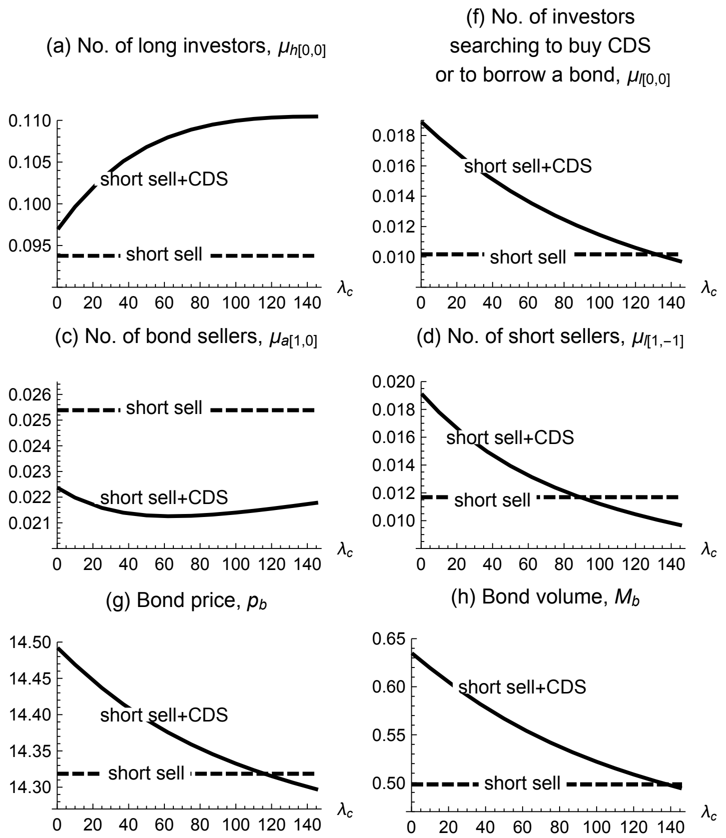
$$\hat{p}_b = \frac{\delta - \eta J + x - y - (r+\gamma_d)\, O_h}{r} - \left(\frac{r+2\gamma_d}{r}\right) \frac{1}{2} \frac{x - (r+\gamma_d) O_h}{\left(r + \gamma_d + \hat{q}_{\mathrm{B}} \frac{1}{2}\right)}. \tag{I36}$$

□

I compare the two environments without CDS and with CDS based on what the entry rates converge to once CDS is introduced. In particular, I use the following algorithm. First, I let the entry rate of low-valuation agents adjust to the CDS introduction but keeping the entry rate of high-valuation agents fixed. Next, I let the entry rate of high-valuation agents adjust to both the CDS introduction and the change in the low-valuation agents' entry rate that the CDS introduction induces in the first step. I repeat this process until both entry rates are at equilibrium. Whether low-valuation agents adjust their entry rate first then high-valuation agents adjust or vice versa gives the same result. The result is as follows. The CDS introduction increases the value of entering for both long and short investors. Short investors, as a result, enter at a higher rate (assuming that their entry rate was given by an interior solution before) until their entry rate converges to the maximum possible: the corner solution ($\nu_l = 1$). In response to both the CDS introduction itself and the resulting increase in the low-valuation agents' entry rate, long investors also enter at a higher equilibrium rate.

Figure 2: The Marginal Effect of CDS When Investors Already Short-Sell

The figures illustrate the results discussed in Section 5.1. The dashed lines plot the masses of actively searching investors (plots (a)-(e)), the bond price, the bond volume, and the repo contract lending fee for an environment in which investors short-sell but do not trade CDS. The solid lines do the same for an environment in which investors both short-sell and trade CDS. They are plotted as functions of CDS market matching efficiency (λ_c). The relevant model is in online Appendix I. The parameter values used to generate the plots are $r = 0.04$, $\alpha = 0.05$, $\sigma_e = 540$, $\rho = 1$, $\delta = 1$, $\eta = 0.0012$, $J = 500$, $F_h = 2.5$, $F_l = 0.18$, $\gamma_d = 0.35$, $\gamma_u = 0.44$, $\lambda_b = 120$, $\lambda_r = 17$, $O_h = 0.4$, and $O_l = 0.925$. I focus on parameter conditions such that short-selling exists in equilibrium both before and after CDS introduction. For example, as $\lambda_c \to 145$, the gains from a repo contract go to zero. So, for $\lambda_c > 145$, investors stop short-selling and start using CDS only.



J Endogenous Search Efforts

I setup the environment so that all investors who want to rebalance their asset position choose their search effort optimally. In particular, in addition to long investors, bond sellers, $a[1,0]$, and CDS buyers, $l[0,0]$, also choose their search effort. As a result, the total bond volume is $M_b = (\lambda_{a[1,0]} + \lambda_{b,h[0,0]})\mu_{a[1,0]}\mu_{h[0,0]}$, while the CDS volume is $M_c = (\lambda_{c,h[0,0]} + \lambda_{l[0,0]})\mu_{l[0,0]}\mu_{h[0,0]}$. Then, only for the proof of Proposition 4, I set the search efforts of the short side to zero: $\lambda_{l[0,0]} = 0$ and $\lambda_{a[1,0]} = 0$. I assume throughout that the parameter conditions are such that the entry rate of high-valuation investors is given by an interior solution with and without CDS.

In the paper, I focus on the simpler environment with only the long side choosing search efforts because endogenizing search efforts on both sides of the market complicates derivations significantly. Given the intractability, in Proposition J.1, I only characterize the parameter conditions under which introducing CDS still increases bond market liquidity. Numerically, the results when both sides choose search intensities versus when only one side chooses are analogous.

The HJB equations are derived analogously as in the environment with exogenous search intensities. As a result, the value functions are characterized by

$$
\begin{aligned}
rV_\tau =& -c(\lambda_{b,\tau},\lambda_{c,\tau}) + (\delta-\eta J - x_\tau)\,\theta_b - y|\theta_b| + (p_c - \eta J - x_\tau)\,\theta_c - y|\theta_c| \\
&+ \sum_{k=1}^{K(\tau)} \gamma(k,\tau) \max_{\tau'\in\mathcal{T}(\tau,k)} \frac{1}{r\alpha}\left(1-e^{-r\alpha(V_{\tau'}-V_\tau+P(\tau,\tau'))}\right),
\end{aligned} \tag{J1}
$$

where $c(\lambda_{b,\tau},\lambda_{c,\tau})$ is the agent's total search cost. Simplifying (J1) further, for the nonsearcher agent types, the value functions are identical to (B3), (B5), and (B6). The value functions of the searcher agents include the costs of their search efforts:

$$
rV_{l[0,0]} = \gamma_u(0-V_{l[0,0]}) - c(0,\lambda_{l[0,0]}) + \frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c \tag{J2}
$$

$$
rV_{h[0,0]} = \gamma_d(0-V_{h[0,0]}) - c(\lambda_{b,h[0,0]},\lambda_{c,h[0,0]}) + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c \tag{J3}
$$

$$
rV_{a[1,0]} = (\delta-\eta J) - y - c(\lambda_{a[1,0]},0) + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b. \tag{J4}
$$

The first order conditions with respect to the search efforts are:

$$
\frac{\partial c(\lambda_{b,h[0,0]},\lambda_{c,h[0,0]})}{\partial \lambda_{b,h[0,0]}} = \mu_{a[1,0]}\left(-p_b + V_{h[1,0]} - V_{h[0,0]}\right) \tag{J5}
$$

$$
\frac{\partial c(\lambda_{b,h[0,0]},\lambda_{c,h[0,0]})}{\partial \lambda_{c,h[0,0]}} = \mu_{l[0,0]}\left(V_{h[0,1]} - V_{h[0,0]}\right) \tag{J6}
$$

$$
\frac{\partial c(0,\lambda_{c,l[0,0]})}{\partial \lambda_{c,l[0,0]}} = \mu_{h[0,0]}\left(V_{l[0,\text{-}1]} - V_{l[0,0]}\right) \tag{J7}
$$

$$
\frac{\partial c(\lambda_{b,a[1,0]},0)}{\partial \lambda_{b,a[1,0]}} = \mu_{h[0,0]}\left(p_b - V_{a[1,0]}\right). \tag{J8}
$$

The optimal search efforts equate the marginal cost (the left-hand side) with the marginal benefit (the right-hand side) of an additional unit of search effort. The equilibrium equations are analogous to the baseline environment plus (J5)-(J8) that pin down the optimal search efforts.

Proof of Lemma 5. Combining (B3) and (J3), the reservation value of the buyer is

$$
r\left(V_{h[1,0]} - V_{h[0,0]}\right) = (\delta-\eta J + x - y) - \gamma_d\omega_b + c(\lambda_{b,h[0,0]},\lambda_{c,h[0,0]}) - q_{\text{bs}}\frac{1}{2}\omega_b - q_{\text{cB}}\frac{1}{2}\omega_c,
$$

where $q_{\text{bs}} = \frac{M_b}{\mu_{h[0,0]}}$ and $q_{\text{cB}} = \frac{M_c}{\mu_{h[0,0]}}$. Using (J3), we can write it as

$$
r\left(V_{h[1,0]} - V_{h[0,0]}\right) = (\delta-\eta J + x - y) - (r+\gamma_d)\,V_{h[0,0]} - \gamma_d\omega_b.
$$

Next, consider the seller's reservation value. Combining (B3), (B4), and (J3), we get

$$
(r+\gamma_d)\,\omega_b = x + c(\lambda_{b,h[0,0]},\lambda_{c,h[0,0]}) + c(\lambda_{a[1,0]},0) - q_{\text{bs}}\frac{1}{2}\omega_b - q_{b\text{B}}\frac{1}{2}\omega_b - q_{\text{cB}}\frac{1}{2}\omega_c. \tag{J9}
$$

where $q_{b\text{B}} = \frac{M_b}{\mu_{a[1,0]}}$. Using (J3), this becomes

$$
(r+\gamma_d)\,\omega_b = x - (r+\gamma_d)\,V_{h[0,0]} + c(\lambda_{a[1,0]},0) - q_{b\text{B}}\frac{1}{2}\omega_b. \tag{J10}
$$

From (J10),

$$q_{b\mathrm{B}}\frac{1}{2}\omega_b - c(\lambda_{a[1,0]},0) = x - (r+\gamma_d)\,\omega_b - (r+\gamma_d)\,V_{h[0,0]}.$$

Plug it into the reservation value of the seller and get

$$rV_{a[1,0]} = \delta - \eta J + x - y - (r+\gamma_d)\,V_{h[0,0]} - (r+\gamma_d)\,\omega_b.$$

Combining the reservation values of the bond buyer and the seller and using $V_{h[0,0]} = O_h$,

$$rp_b = \delta - \eta J + x - y - (r+\gamma_d)\,O_h - \frac{1}{2}\,(r+2\gamma_d)\,\omega_b. \tag{J11}$$

From (J10) and the fact $c(\lambda_{a[1,0]},0) = 0$ when $\lambda_{a[1,0]} = 0$,

$$\omega_b = \frac{x - (r+\gamma_d)\,O_h}{\left(r+\gamma_d+q_{b\mathrm{B}}\frac{1}{2}\right)}. \tag{J12}$$

Plugging it into (J11), the bond price and the illiquidity discount have the same characterization as in the environment with exogenous search efforts. □

The results so far apply to both the environment in which only the long side chooses their search effort (i.e., $\lambda_{b,h[0,0]}$ and $\lambda_{c,h[0,0]}$ are endogenous, and $\lambda_{a[1,0]} = 0$ and $\lambda_{l[0,0]} = 0$) and the environment in which both sides of the market choose their search effort (i.e., $\lambda_{b,h[0,0]}$, $\lambda_{c,h[0,0]}$, $\lambda_{a[1,0]}$, and $\lambda_{l[0,0]}$ are endogenous). In Proposition 4, only the long side chooses their search efforts. Then, Proposition J.1 allows both sides to choose their search efforts and characterizes the parameter conditions under which the liquidity spillover effect arises.

Proof of Proposition 4. I denote the variables in the counterfactual environment without CDS trading with hats. To simplify the notation, I denote $\lambda_{b,h[0,0]}$ as λ_b and $\lambda_{c,h[0,0]}$ as λ_c.

I first show that long investors keep their total search cost, $c(\lambda_b,\lambda_c)$, the same with the introduction of CDS. Using (J3) and $V_{h[0,0]} = O_h$,

$$(r+\gamma_d)O_h = -c_0\left((\lambda_b)^g + (\lambda_c)^g\right)^a + \lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda_c\mu_{l[0,0]}\frac{1}{2}\omega_c.$$

Using the first order conditions (J5) and (J6), this simplifies to

$$(r+\gamma_d)O_h = (ag-1)c_0\left((\lambda_b)^g + (\lambda_c)^g\right)^a$$

in the environment with CDS trading and

$$(r+\gamma_d)O_h = (ag-1)c_0\left(\left(\hat{\lambda}_b\right)^g\right)^a$$

in the environment without CDS. The right-hand-sides are the $(ag-1)$ times the total search cost incurred by long investors, $h[0,0]$, in the respective environments. Since the left-hand-sides are the same across the two environments, long investors incur the same total search cost in the environments with and without CDS trading: $c_0\left((\lambda_b)^g + (\lambda_c)^g\right)^a = c_0\left(\left(\hat{\lambda}_b\right)^g\right)^a$. This implies

$$(\lambda_b)^g + (\lambda_c)^g = \left(\hat{\lambda}_b\right)^g. \tag{J13}$$

Now consider how CDS introduction affects λ_b. Since $\lambda_c > 0$ in the environment with CDS trading and $g > 0$, from (J13), $\lambda_b < \hat{\lambda}_b$. Thus, investors lower their search effort in the bond market.

Now consider how CDS affects the marginal cost of an additional unit of λ_b, $\frac{\partial c(\lambda_b,\lambda_c)}{\partial \lambda_b}$. It is

$$\frac{\partial c(\lambda_b, \lambda_c)}{\partial \lambda_b} = agc_0 (\lambda_b)^{g-1} \left[((\lambda_b)^g + (\lambda_c)^g)^{a-1}\right]$$

in the environment with CDS trading and

$$\frac{\partial c(\lambda_b, \lambda_c)}{\partial \lambda_b} = agc_0 \left(\hat{\lambda}_b\right)^{g-1} \left[\left(\left(\hat{\lambda}_b\right)^g\right)^{a-1}\right]$$

in the environment without CDS trading. From (J13), the terms in the square brackets are equal. Since $\lambda_b < \hat{\lambda}_b$ and $g > 1$, we have that $(\lambda_b)^{g-1} < \left(\hat{\lambda}_b\right)^{g-1}$. Thus, the marginal cost, $\frac{\partial c(\lambda_b,\lambda_c)}{\partial \lambda_b}$, is lower in the environment with CDS than in the environment without CDS.

Now, consider how CDS affects bond market liquidity. Using (J5) and (J12),

$$\frac{\partial c(\lambda_b, \lambda_c)}{\partial \lambda_b} = \mu_{a[1,0]} \frac{1}{2} \frac{x - (r + \gamma_d)\, O_h}{\left(r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}\right)}. \tag{J14}$$

Combining (B19) and (B15),

$$\lambda_b \mu_{h[0,0]} = \frac{\gamma_d (S - \mu_{a[1,0]})}{\mu_{a[1,0]}}.$$

Plugging this in (J14), the right-hand-side of (J14) as an implicit function of $\mu_{a[1,0]}$ strictly increases in $\mu_{a[1,0]}$. Then, using the above result that the marginal cost (i.e. the left-hand-side of (J14)) decreases with CDS introduction, it has to be that $\mu_{a[1,0]} < \hat{\mu}_{a[1,0]}$. In turn, $\mu_{a[1,0]} < \hat{\mu}_{a[1,0]}$ implies that $\omega_b < \hat{\omega}_b$, $d_b < \hat{d}_b$, and $p_b > \hat{p}_b$. The bond volume is given by: $M_b = \gamma_d (S - \mu_{a[1,0]})$. Since the mass of bond sellers decreases, the bond volume increases. □

Proposition J.1. *Consider the environment where both sides of the market choose their search efforts (i.e., $\lambda_{b,h[0,0]}$, $\lambda_{c,h[0,0]}$, $\lambda_{a[1,0]}$, and $\lambda_{l[0,0]}$ are endogenous). Suppose (J18) holds. Then, with the introduction of CDS, the illiquidity discount (d_b) is smaller, the bond volume (M_b) is higher, and the bond price (p_b) is higher.*

Proof. The following three equations characterize $\mu_{a[1,0]}$, $\mu_{h[0,0]}$, and ω_b as implicit functions of A:

$$\frac{1}{4c_0} (\mu_{a[1,0]} + \mu_{h[0,0]}) \mu_{a[1,0]} \mu_{h[0,0]} \omega_b = \gamma_d (S - \mu_{a[1,0]}) \tag{J15}$$

$$(r + \gamma_d) \omega_b = x - \frac{\left(\left(\mu_{a[1,0]} + \mu_{h[0,0]}\right)^2 + 2\mu_{h[0,0]} \mu_{a[1,0]}\right) \omega_b^2}{16 c_0} - A \tag{J16}$$

$$(r + \gamma_d) O_h = \frac{\mu_{a[1,0]} \left(\mu_{a[1,0]} + 2\mu_{h[0,0]}\right) {\omega_b}^2}{16 c_0} + A, \tag{J17}$$

where

$$A = \frac{M_c}{\mu_{h[0,0]}} \frac{1}{2} \omega_c - c_0 (\lambda_{c,h[0,0]})^2.$$

Eq. (J15) comes from combining the inflow-outflow equations with (J5), (J16) comes from combining (J9) with (J5), and (J17) combines (J3) with (J5)-(J6).

Applying the Implicit Function Theorem,

$$\frac{\partial \mu_{a[1,0]}}{\partial A} = - \frac{4c_0 \left(8c_0 \left(\mu_{a[1,0]} + 2\mu_{h[0,0]}\right) (r + \gamma_d) + \mu_{h[0,0]} \left(\mu_{a[1,0]}^2 + 3\mu_{a[1,0]} \mu_{h[0,0]} + \mu_{h[0,0]}^2\right) \omega_b\right)}{B}$$

$$\frac{\partial \omega_b}{\partial A} = -\frac{4c_0\left(4c_0\left(\mu_{a[1,0]}+\mu_{h[0,0]}\right)\gamma_d + \mu_{h[0,0]}\left(\mu^2_{a[1,0]}+\mu_{a[1,0]}\mu_{h[0,0]}+\mu^2_{h[0,0]}\right)\omega_b\right)}{B},$$

where

$$B \equiv -16c_0^2(r+\gamma_d)\gamma_d\omega_b + 2c_0\left(\mu^2_{a[1,0]}+\mu_{a[1,0]}\mu_{h[0,0]}+\mu^2_{h[0,0]}\right)(2r+3\gamma_d)\,\omega_b^2$$
$$+\,\mu_{h[0,0]}\left(\mu_{a[1,0]}+\mu_{h[0,0]}\right)\left(\mu^2_{a[1,0]}+\mu_{a[1,0]}\mu_{h[0,0]}+\mu^2_{h[0,0]}\right)\omega_b^3.$$

Thus, $\frac{\partial \omega_b}{\partial A} < 0$ and $\frac{\partial \mu_{a[1,0]}}{\partial A} < 0$ if

$$B > 0. \tag{J18}$$

In turn, $\frac{\partial \omega_b}{\partial A} < 0$ and $\frac{\partial \mu_{a[1,0]}}{\partial A} < 0$ imply that bond market liquidity and the bond price are higher in the presence of the CDS market. □