The Lost Capital Asset Pricing Model

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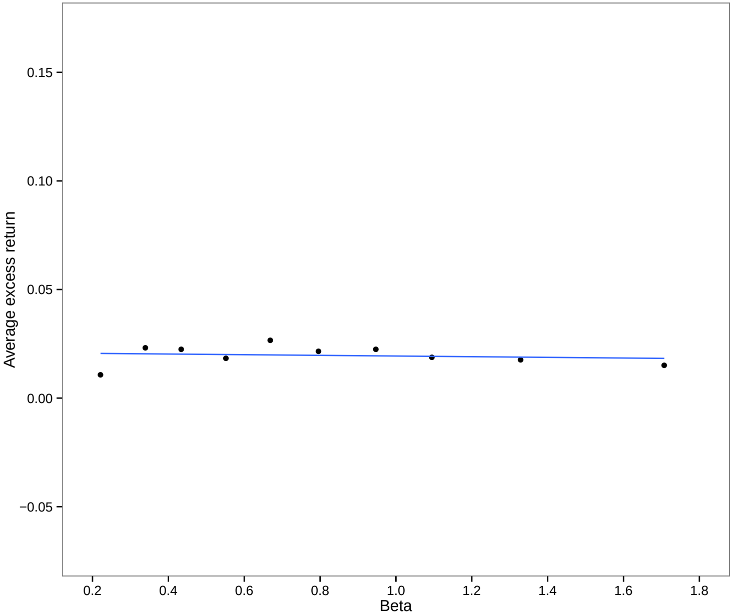
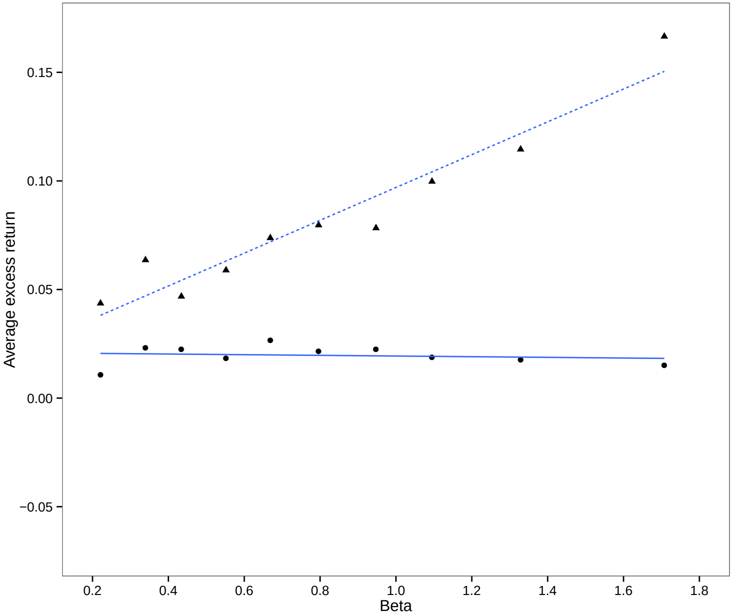
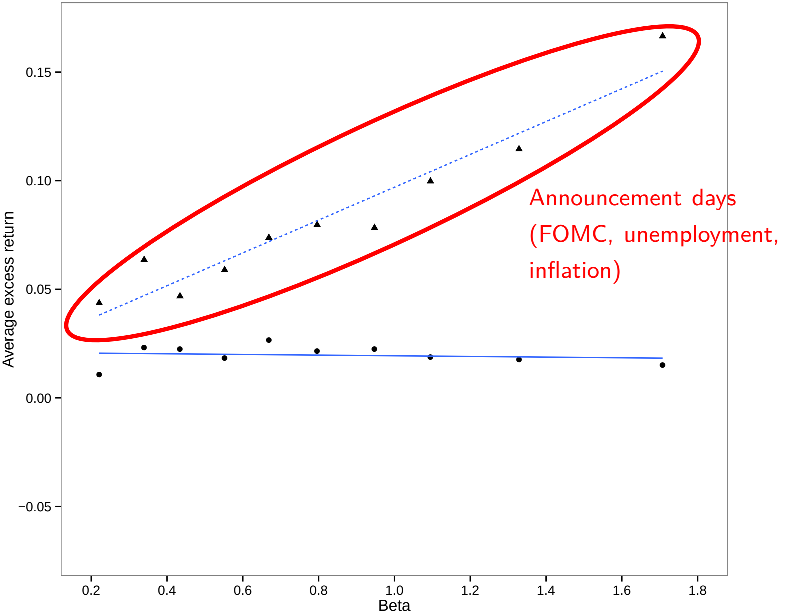
AFA, 2018


0.15
0.10
0.05
0.00
−0.05
Average excess return
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Beta



Source: (Savor and Wilson, 2014)



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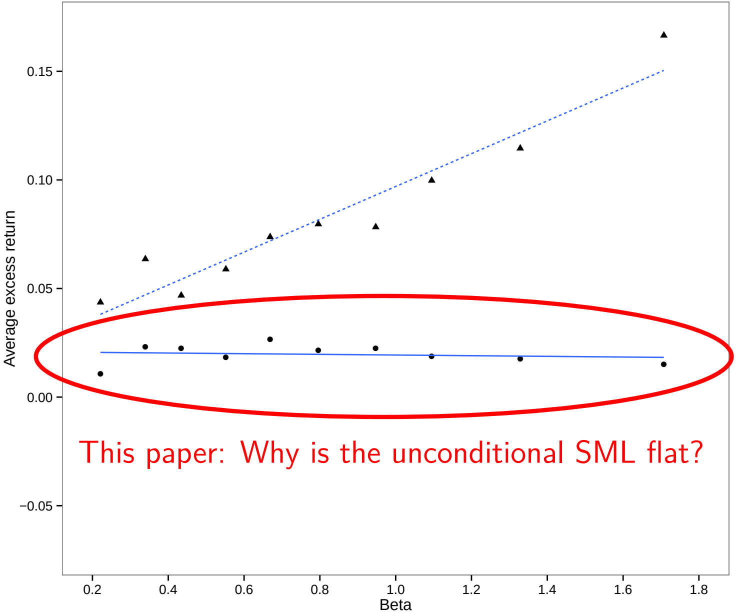

Source: (Savor and Wilson, 2014)

$$\mathbb{V}[R] = \mathbb{E}\left[\mathbb{V}[R \mid \mathscr{F}]\right] + \mathbb{V}\left[\mathbb{E}[R \mid \mathscr{F}]\right]$$

$$\boxed{\mathbb{V}[R]} = \mathbb{E}\left[\mathbb{V}[R \mid \mathscr{F}]\right] + \mathbb{V}\left[\mathbb{E}[R \mid \mathscr{F}]\right]$$

Measured by
the econometrician

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Equilibrium model

$$\mathbb{V}[R] = \mathbb{E}\big[\mathbb{V}[R \mid \mathscr{F}]\big] + \mathbb{V}\big[\mathbb{E}[R \mid \mathscr{F}]\big]$$

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Equilibrium model

Notation used for the rest of the talk: $\mathbb{E}[\cdot \mid \mathscr{F}] \equiv \overline{\mathbb{E}}[\cdot]$ and $\mathbb{V}[\cdot \mid \mathscr{F}] \equiv \overline{\mathbb{V}}[\cdot]$

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Informational distance: $\mathbb{V}[R] - \overline{\mathbb{V}}[R]$

- Payoffs:

$$\underbrace{\begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_N \end{bmatrix}}_{D} = \underbrace{\begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix}}_{\Phi} F + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}$$

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- Information:

$$\text{Private: } V_i = F + v_i$$

$$\text{Public: } G = F + v$$

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- Supply of assets $\underbrace{\left[M_1, M_2, ...M_N\right]'}_{M'}$ with mean $\underbrace{\left[\frac{1}{N}, \frac{1}{N}, ...\frac{1}{N}\right]'}_{\overline{M}'}$

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- Prices:

$$\begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} F + \begin{bmatrix} g_1 \\ \vdots \\ g_N \end{bmatrix} G + \begin{bmatrix} \xi_{11} & \cdots & \xi_{1N} \\ \vdots & \ddots & \\ \xi_{N1} & \cdots & \xi_{NN} \end{bmatrix} \begin{bmatrix} M_1 \\ \vdots \\ M_N \end{bmatrix}$$

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Roll critique

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Informational distance:

$$\begin{aligned}\mathbb{V}[R] &= \overline{\mathbb{V}}[R] + \mathbb{V}\left[\mathbb{E}[R \mid \mathscr{F}^i]\right] \\ &= \overline{\mathbb{V}}[R] + \mathbb{V}\left[\overline{\mathbb{E}}[R] + \Phi \frac{\tau_v}{\tau} v_i\right]\end{aligned}$$

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- Average agent's point of view:

$$\overline{\mathbb{V}}[R] = \frac{1}{\tau}\Phi\Phi' + \frac{1}{\tau_\epsilon}\mathbb{I}_N$$

$$\mathbb{E}[R] = \underbrace{\frac{\overline{\mathbb{V}}[R]\frac{1}{N}\mathbf{1}}{\overline{\mathbb{V}}[R_M]}}_{\beta}\mathbb{E}[R_M]$$

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- Econometrician's point of view:

  1. **Relationship 1 (general — statistics):**

$$\mathbb{V}[R] = \overline{\mathbb{V}}[R] + \mathbb{V}[\overline{\mathbb{E}}[R]] + \frac{\tau_v}{\tau^2}\Phi\Phi'$$

- Average agent's point of view:

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- Econometrician's point of view:

$\overline{\mathbb{E}}[R] = \gamma\overline{\mathbb{V}}[R]M$

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2. **Relationship 2 (specific to our model — economics):**

$$\mathbb{V}[R] = \overline{\mathbb{V}}[R] + \underbrace{\left(\kappa\tau_\epsilon + \frac{\gamma^2}{\tau_M\tau_\epsilon}\right)}_{\zeta}\overline{\mathbb{V}}[R] - \kappa\mathbb{I}_N$$

- Average agent's point of view:

$$\overline{\mathbb{V}}[R] = \frac{1}{\tau}\Phi\Phi' + \frac{1}{\tau_\epsilon}\mathbb{I}_N$$

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- **Roll critique: $\kappa > 0$ (without the Roll critique, the two views coincide)**

$$\widetilde{\beta} = \frac{\mathbb{V}[R]\frac{1}{N}\mathbf{1}}{\mathbb{V}[R_M]}$$

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$$= \frac{\left[(1+\zeta)\overline{\mathbb{V}}[R] - \kappa\mathbb{I}_N\right]\frac{1}{N}\mathbf{1}}{\mathbb{V}[R_M]}$$

$$
\begin{aligned}
\widetilde{\beta} &= \frac{\mathbb{V}[R]\frac{1}{N}\mathbf{1}}{\mathbb{V}[R_M]} \\
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&= \underbrace{(1+\zeta)\frac{\overline{\mathbb{V}}[R_M]}{\mathbb{V}[R_M]}}_{1+\delta}\beta - \frac{\kappa\frac{1}{N}\mathbf{1}}{\mathbb{V}[R_M]}
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- Take averages on both sides:

$$1 = 1 + \delta - \frac{\kappa/N}{\mathbb{V}[R_M]}$$

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$$1 = 1 + \delta - \frac{\kappa/N}{\mathbb{V}[R_M]} \quad \Rightarrow \quad \delta = \frac{1/N}{\mathbb{V}[R_M]}\left[\frac{\gamma^2}{\tau_M\tau_\epsilon}\left(\frac{1}{\tau_\epsilon} + \frac{\Phi'\Phi}{\tau}\right) + \frac{\tau_v}{\tau\tau_\epsilon}\right]$$

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\end{aligned}
$$

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$$

- Relationship between betas:

$$
(\widetilde{\beta} - \mathbf{1}) = (1+\delta)(\beta - \mathbf{1})
$$

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Roll critique

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(Hansen-Richard critique)

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Differential information

$$
1 = 1 + \delta - \frac{\kappa/N}{\mathbb{V}[R_M]} \quad \Rightarrow \quad \delta = \frac{1/N}{\mathbb{V}[R_M]}\left[\frac{\gamma^2}{\tau_M \tau_\epsilon}\left(\frac{1}{\tau_\epsilon} + \frac{\Phi'\Phi}{\tau}\right) + \frac{\boxed{\tau_v}}{\tau \tau_\epsilon}\right]
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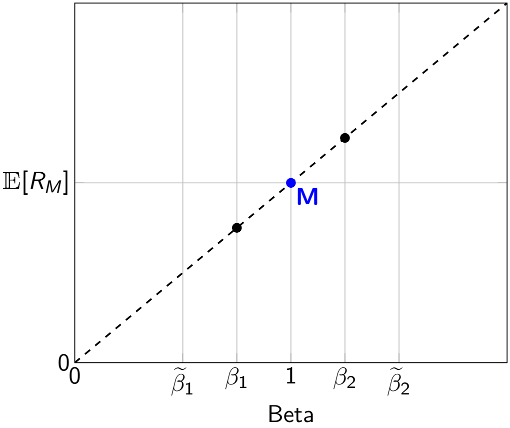

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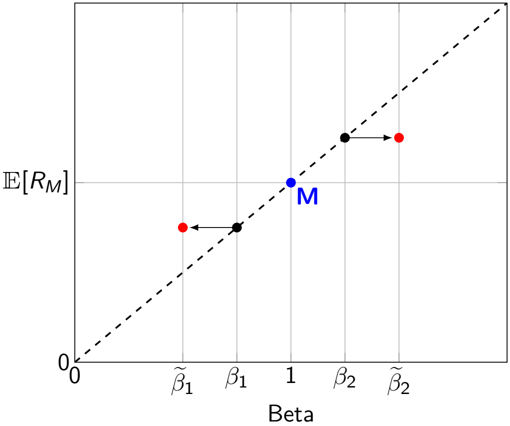

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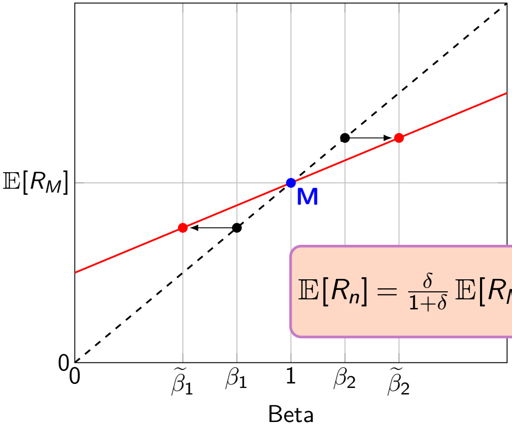

$$\mathbb{E}[R_n] = \frac{\delta}{1+\delta}\,\mathbb{E}[R_M] + \frac{\mathbb{E}[R_M]}{1+\delta}\,\widetilde{\beta}_n$$

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Vasicek (1973); Bodie, Kane, and Marcus (2007):

Merrill Lynch adjusts beta estimates in a simple way. They take the sample estimate of beta and average it with 1, using the weights of two thirds and one third:

$$\text{Adjusted beta} = 2/3 \text{ sample beta} + 1/3(1)$$

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Vasicek (1973); Berk and DeMarzo (2007):

are calculated by averaging the estimated beta with 1.0. For example, Bloomberg computes adjusted betas using the following formula:

$$\text{Adjusted Beta of Security } i = \tfrac{2}{3}\beta_i + \tfrac{1}{3}(1.0) \tag{12A.1}$$

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$$(\widetilde{\beta} - \mathbf{1}) = (1 + \delta)(\beta - \mathbf{1})$$

Back-of-the-envelope calculation

$$\widetilde{\beta}_n = \frac{\delta(1-\mathcal{R}^2)}{\mathcal{R}^2+\delta} + \left(1 - \frac{\delta(1-\mathcal{R}^2)}{\mathcal{R}^2+\delta}\right)\beta_{E,n}$$

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$$\delta = \frac{a\mathcal{R}^2}{1-a-\mathcal{R}^2}$$

Back-of-the-envelope calculation

$$\widetilde{\beta}_n = \underbrace{\frac{\delta(1-\mathcal{R}^2)}{\mathcal{R}^2+\delta}}_{a\in[0.63,0.87]} + \left(1 - \frac{\delta(1-\mathcal{R}^2)}{\mathcal{R}^2+\delta}\right)\beta_{E,n}$$

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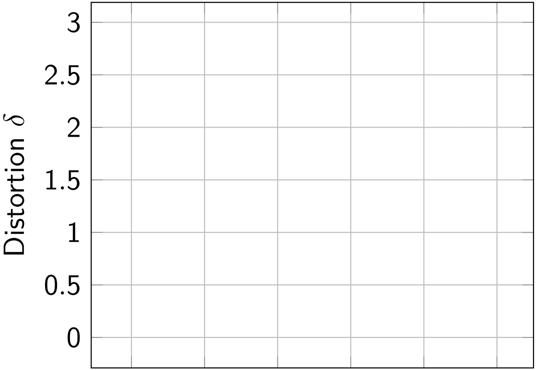

$$\mathcal{R}^2 = \frac{\mathsf{Var}[\overline{\mathbb{E}}[R_M]]}{\mathsf{Var}[R_M]}$$

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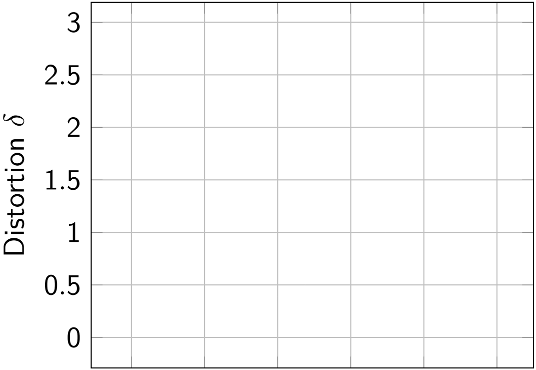

$$\mathcal{R}^2 = \frac{\mathrm{Var}[\overline{\mathbb{E}}[R_M]]}{\mathrm{Var}[R_M]} \approx 0.1$$ (Cochrane, 2011)

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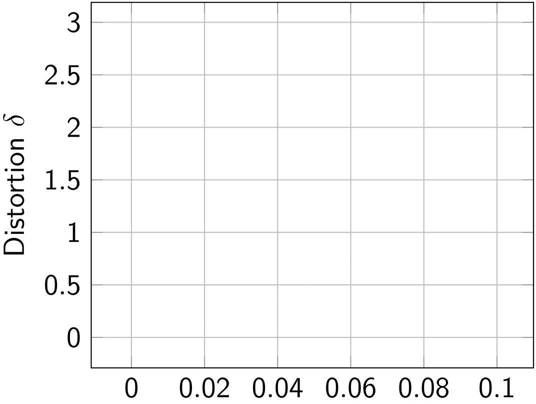

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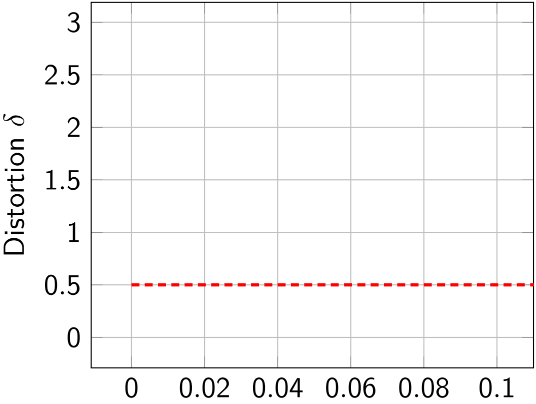

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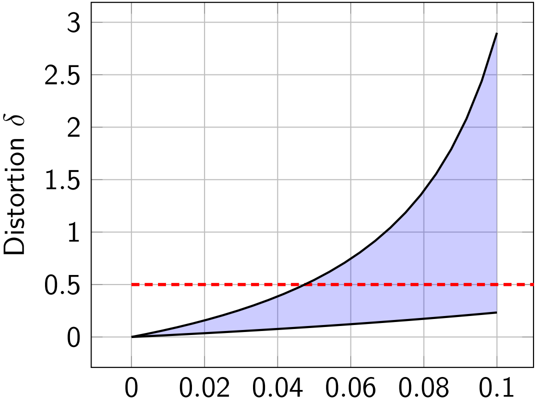

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