Volatility Risk Pass-Through

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Main Question

Uncertainty in a one-country setting:

- Sizeable impact of volatility risks on growth and asset prices
- Typically, high aggregate volatility is "bad":
  - Lowers output and investment
  - Lowers asset valuations
  - Increases risk premia and marginal utility

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Open question:

- How are volatility risks shared internationally?
  - Novel empirical investigation on G17 countries
  - Novel theoretical insights on volatility risk-sharing

Main Findings

1. International pass-through of output vol shocks to consumption vol
   - Home output vol $\rightarrow$ home and foreign consumption vol
   - Trade channel: higher vol $\rightarrow$ lower net exports
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   - corr$(\sigma_t(\Delta e_{t+1}), \sigma_t(\Delta c_{t+1} - \Delta c^*_{t+1}))$=.30
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   - Beyond the Backus & Smith 93 puzzle
4. Assess these findings through a recursive risk sharing of output vol risks

Empirical Analysis

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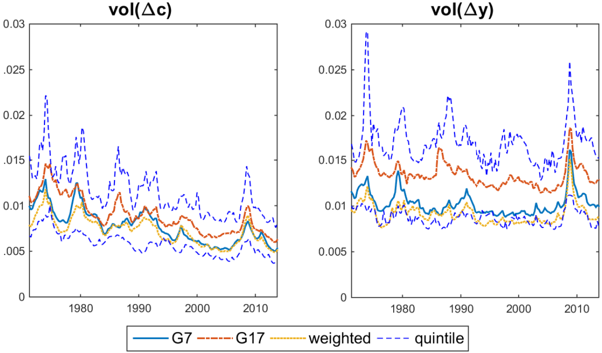
- Quarterly data for 17 major industrialized countries from 1971 to 2014
- For variable of interest in each country, run a filter:

$$z_t = \mu(1-\rho) + \rho z_{t-1} + e^{\sigma_t(z)/2}\eta_t$$
$$\sigma_t(z) = \mu_\sigma(1-\nu) + \nu\sigma_{t-1}(z) + \sigma_w w_t$$

  - z is real output, consumption, net exports, exchange rates

- $\sigma(z)$ is our estimate of the volatility

Macroeconomic Volatilities



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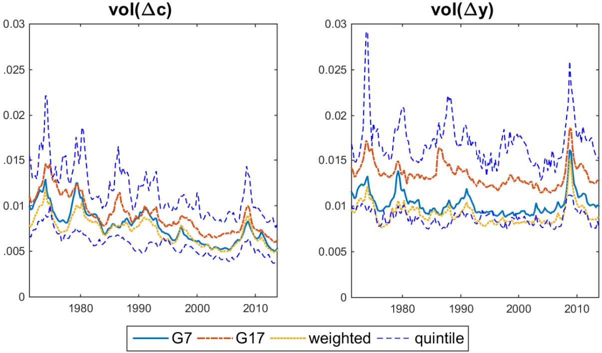

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- Across countries: $\rho(\sigma_t^y, \sigma_t^{y*}) = 0.30 < \rho(\sigma_t^c, \sigma_t^{c*}) = 0.50$

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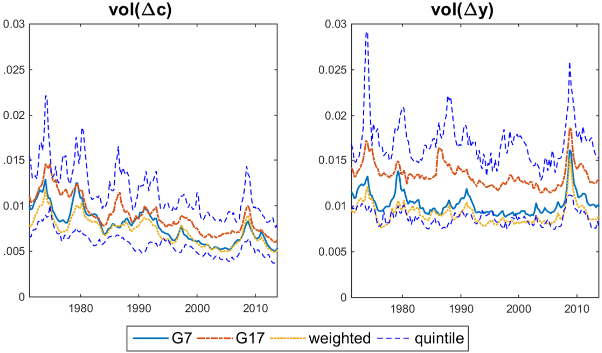

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- Across countries: $\rho(\sigma_t^y, \sigma_t^{y*}) = 0.30 < \rho(\sigma_t^c, \sigma_t^{c*}) = 0.50$
- Within countries: $\rho(\sigma_t^c, \sigma_t^y) = 0.70 < 1 \rightarrow$ international pass-through.

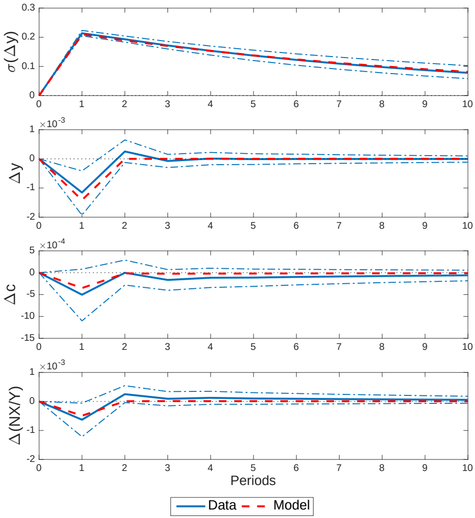
Measuring Relative Impulse Impact

- Identify impact of relative output vols on quantities
- In benchmark case, stack country variables, relative to US:

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix},$$

- Estimate a pooled VAR(1) across countries
- Trace impulse response of relative output vol shocks on consumption, net exports, and consumption volatility

Response to Volatility Shocks



Take-aways:

- High output volatility decreases the growth rate of output

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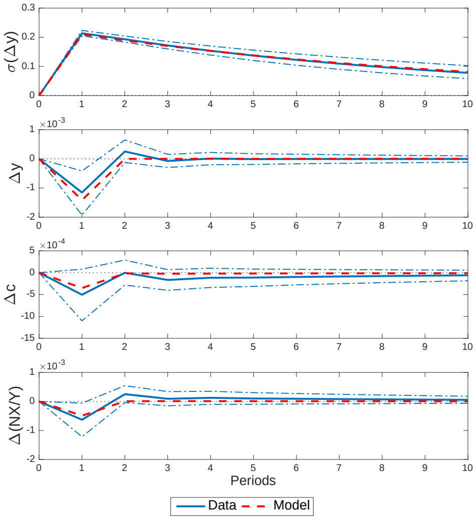
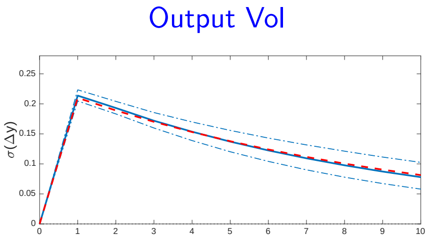
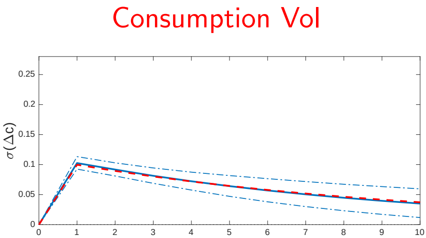


Take-aways:

- High output volatility decreases the growth rate of output
- However, net imports increase, and consumption falls by less
  Evidence of international risk-sharing

Volatility Pass-Through





High Output Vol increases Consumption Vol less than one-to-one

Volatility Pass-Through Index(VPI)

$$\text{Pass-through Index} \quad := \quad 1 - \frac{\partial(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}))}{\partial(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))}$$

- larger index implies better sharing of volatility risk
  - Interpretation of VPI with one good and CRRA
  - $0 \rightarrow$ no risk sharing, i.e., autarky ($\Delta c_{i,t} = \Delta y_{i,t}$)
  - $1 \rightarrow$ perfect risk sharing ($\Delta c_{i,t} = \Delta c_{j,t}$)

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- In the data:
  - G7 countries, VPI = 50%
  - Bottom-10 G17 countries, VPI = 70% w.r.t. shocks originating in small countries

Volatility Disconnect (Puzzle)

- If

$$\Delta cd_{t+1} := \Delta c_{h,t+1} - \Delta c_{f,t+1}$$

captures risk-sharing opportunities, FX-vol should depend on $\sigma_t(\Delta cd_{t+1})$.

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- This paper: empirical disconnect of **vols**

$$Corr(Var_t[\Delta e_{t+1}], Var_t[\Delta c_{h,t+1} - \Delta c_{f,t+1}]) \approx 0.20$$

  - Puzzle with CRRA
  - Puzzle with EZ and level shocks
    (e.g., Colacito Croce (JPE 2011,JF2013) resolve Backus-Smith)

Model

Model

- Two countries: home (h) and foreign (f)
- Recursive **EZ** utility over the consumption aggregate C_t

$$C_t^h = \left(x_t^h\right)^{\alpha} \left(y_t^h\right)^{1-\alpha}, \quad C_t^f = \left(x_t^f\right)^{1-\alpha} \left(y_t^f\right)^{\alpha}$$

- x^h, x^f, y^h, and y^f are allocations of each good to each country
- $\alpha > 1/2$ captures home bias

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- x^h, x^f, y^h, and y^f are allocations of each good to each country
- $\alpha > 1/2$ captures home bias
- Endowments are co-integrated, and feature long-run and volatility risks:

$$\begin{aligned}
\Delta \log X_t &= \mu_x + z_{1,t-1} - \tau \log\left(X_{t-1}/Y_{t-1}\right) + e^{\sigma_{x,t}/2}\sigma\varepsilon_{x,t} \\
\Delta \log Y_t &= \mu_y + z_{2,t-1} + \tau \log\left(X_{t-1}/Y_{t-1}\right) + e^{\sigma_{y,t}/2}\sigma\varepsilon_{y,t} \\
z_{j,t} &= \rho z_{j,t-1} + \sigma_z \varepsilon_{j,t}, \forall j \in \{1,2\}
\end{aligned}$$

- Focus on short-run volatilities of endowments, as in the data.
  - Can extend to accommodate long-run volatility risks

Equilibrium Allocations and Relative Size

- Under complete markets, compute efficient allocations by solving Pareto problem with time-varying weights

$$\Delta c_{t+1} = \Delta c_{t+1}^{aut} + f(S_{t+1}) - f(S_t),$$

- Optimal allocations depend on ratio of Pareto weights (country size) S_t :

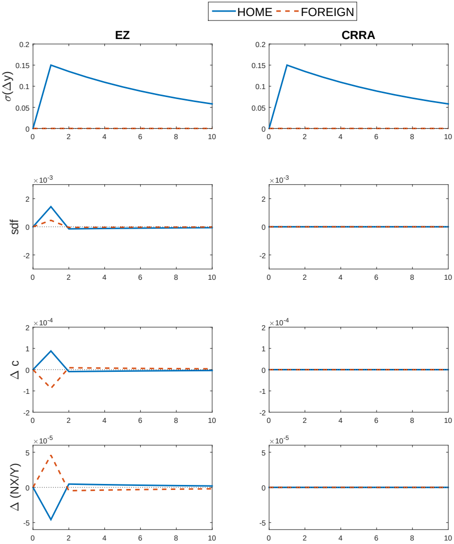
$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left(\frac{C_t^h / C_{t-1}^h}{C_t^f / C_{t-1}^f} \right), \quad \forall t \geq 1$$

- Evolution of S_t depends on pricing kernels M^h and M^f
- Under recursive preferences, volatility news are priced, and affect consumption allocations ▸ Details

Model Calibration

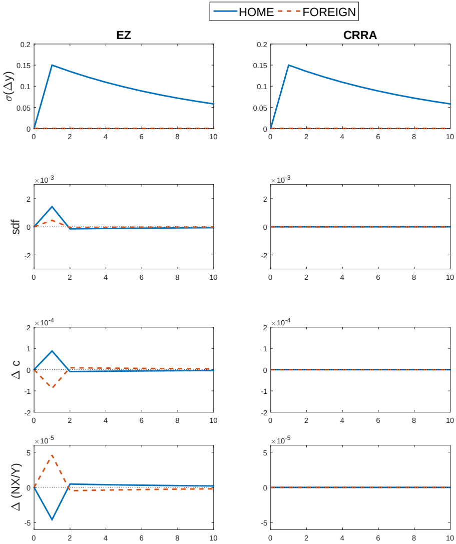
- Calibration for level shocks: similar to Colacito Croce (JPE 2011, JF 2013)
  - Risk aversion is 7
  - Intertemporal elasticity of substitution is 1.5
- Calibration for vol shocks: median estimates in our data
  - Output volatility shocks are persistent
  - Negatively correlated with endowment shocks (-0.12, as in the data)
  - Weakly correlated across countries (0.30)
- Same 'successes' of Colacito Croce (2013) + explains vol pass-through and vol disconnect

Risk Sharing



- Home country receives vol shock

Risk Sharing



- Home country receives vol shock
- Under EZ utility, vol shock is bad news
  - Home SDF ⇈

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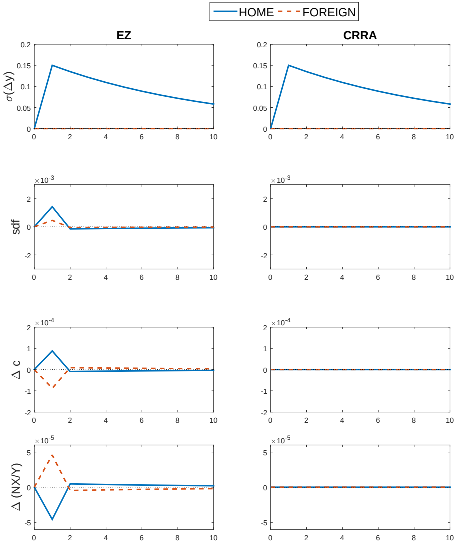

- Home country receives vol shock
- Under EZ utility, vol shock is bad news
  - Home SDF ⇈
- Under EZ utility, high vol country receives resources from abroad
  - Home Consumption ↑
  - Home NX ↓

Unconditional co-movements of volatilities

	Avg.	Quintiles [1^{st}; 4^{th}]	Bench-mark	No TVV ($\sigma_\sigma = 0$)	CRRA ($\gamma = 7$)
$corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta y_{t+1})$	0.65	[0.26; 0.80]	0.88	–	0.98
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- CRRA overshoots with both correlations

Pass-through and size

	SWC	US vol shock	Foreign vol shock
US/G7 Countries:			
Data	[0.44 0.51]	[0.43 0.54]	[0.51 0.63]
Model (EZ)	0.50	0.53	0.53
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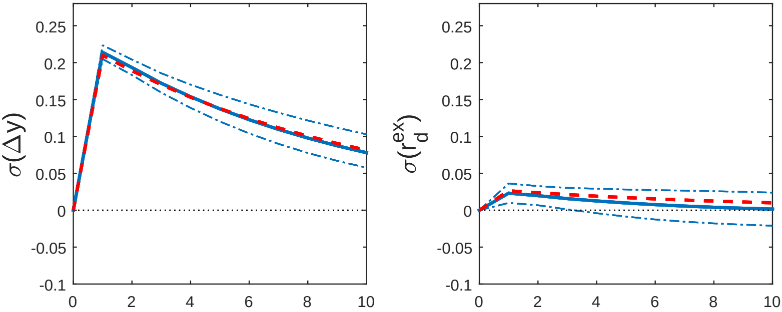
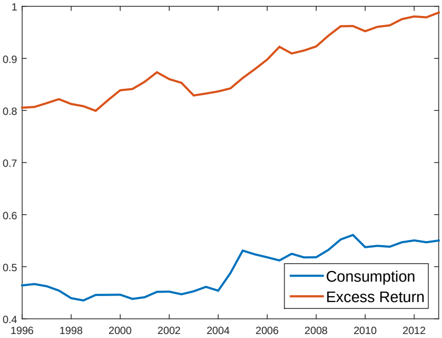
  - US unloads less vol to smaller countries
  - smaller countries unload a lot of vol risk to US

Return Vol Pass-through



- Excess return pass-through similar to the data (0.89)

Change in Pass-through



Panel C: Change in Pass-through

	Benchmark	CRRA
Consumption vol pass-through	0.40	0.20
Financial pass-through	0.57	0.00

FX and Consumption Disconnect in the Model

$$\Delta e_{t+1} = (\Delta c_{h,t+1} - \Delta c_{f,t+1}) - \Delta \log S_{t+1}$$

	G-17 Data		Model		
	Avg.	Quintiles [1^{st}; 4^{th}]	Bench-mark	No TVV ($\sigma_\sigma = 0$)	CRRA ($\gamma = 7$)
Levels Disconnect					
$corr(\Delta cd_{t+1}, \Delta e_{t+1})$	-0.13	[-0.19; -0.04]	-0.25	-0.27	1.00
$corr(\Delta \widehat{\ } cd_{t+4}, \Delta e\widehat{\ }_{t+4})$	-0.14	[-0.29; -0.05]	-0.21	-0.24	1.00

- good long-run risks and volatility shocks decrease relative consumption and size of country
- Produces weak negative correlation between the levels of FX and consumption differential, as in the data

FX and Consumption Disconnect in the Model

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Volatility Disconnect					
$corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$	0.20	[-0.01 0.42]	0.56	-0.75	1.00
$corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$	0.26	[-0.02 0.52]	0.47	-0.75	1.00

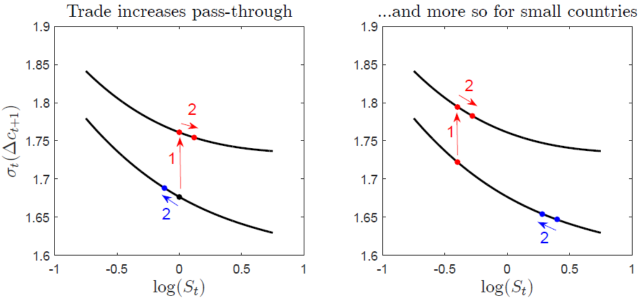
- Volatilities of consumption differential and consumption share:
  - Move in the same direction in response to volatility shocks
  - Move in the opposite direction in response to long-run shocks
- CRRA and model with no TVV cannot match this moment

Pass-through

$$= 1 - \frac{\partial(\sigma_t(\Delta c_h) - \sigma_t(\Delta c_f))}{\partial(\sigma_t(\Delta y_h) - \sigma_t(\Delta y_f))}$$

Positive shock to home endowment vol: $\sigma_{x,t} \uparrow$

1. Exogenous effect: $\sigma_t(\Delta c_h) \uparrow$
2. Endogenous effect: $M \uparrow \rightarrow NX \downarrow \rightarrow S_t \uparrow \rightarrow \sigma_t(\Delta c_h) \downarrow$



Conclusions

1. Domestic volatility risks are "passed through" internationally
2. Volatility pass-through is significant
   - Smaller countries better share volatility risks
3. FX-Vol Disconnect Puzzle
4. Resolve these puzzles with recursive risk sharing of vol shocks

Table 1: **Data Summary Statistics**

	G7 Avg.	G17 Avg.		G17 Quintile	
	Simple	Simple	Weighted	1^{st}	4^{th}
Consumption growth					
Mean	1.91	1.63	1.89	1.26	2.02
Std. Dev.	1.75	1.99	1.67	1.34	2.47
AR(1)	0.11	0.07	0.17	-0.16	0.31
Output growth					
Mean	1.94	1.71	1.93	1.43	2.00
Std. Dev.	2.21	2.97	2.02	2.01	4.43
AR(1)	0.00	-0.09	0.07	-0.26	0.09
Δ*Net Exports over Output:*					
Mean	0.03	0.08	0.04	-0.30	0.34
Std. Dev.	1.60	2.48	1.45	1.79	3.24
AR(1)	0.00	-0.09	0.07	-0.26	0.09
Within-Country Correlations:					
Consump. and output growth	0.67	0.51	0.71	0.35	0.72
Consump. and output vol	0.54	0.47	0.65	0.26	0.80
Across-Country Correlations:					
Consump. growth	0.27	0.24	0.25	0.13	0.33
Output growth	0.15	0.14	0.14	0.06	0.20
Consump. vol	0.51	0.47	0.45	0.35	0.66
Output vol	0.32	0.30	0.30	0.18	0.45

Table 2: Volatility Risk Pass-Through

Panel A: Contemporaneous adjustments to relative volatility shocks

$\sigma(\Delta y)$	Δy	$\sigma(\Delta c)$	Δc	$\Delta(NX/Y)$	Pass-through
US/G7 Countries:					
0.21	-0.46	0.10	-0.20	-0.25	0.52
[0.20 0.22]	[0.09 0.11]	[-0.44 0.03]	[-0.44 0.03]	[-0.49 -0.02]	[0.48 0.56]
US/Bottom-10 G17 Countries:					
0.21	-0.57	0.08	-0.16	-0.39	0.61
[0.21; 0.22]	[-0.95; -0.19]	[0.07; 0.09]	[-0.41; 0.09]	[-0.73; -0.06]	[0.56; 0.65]

Panel B: Pass-through and size

	Origin of Vol Shock:	
	U.S.	Foreign Country
US/G7 Countries:	0.49	0.57
	[0.43; 0.54]	[0.51; 0.63]
US/Bottom-10 G17 Countries:	0.51	0.72
	[0.45; 0.57]	[0.66; 0.78]

Table 3: Volatility Disconnect Puzzle

	G7 Avg.	G17 Avg.		G17 Quintile	
	Simple	Simple	Weighted	1^{st}	4^{th}
Levels Disconnect					
$corr(\Delta cd_{t+1}, \Delta e_{t+1})$	-0.14	-0.11	-0.13	-0.19	-0.04
$corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$	-0.14	-0.17	-0.14	-0.29	-0.05
Volatility Disconnect					
$corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$	0.20	0.21	0.20	-0.01	0.42
$corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$	0.27	0.25	0.26	-0.02	0.52

Table 4: Calibration

Description	Parameter	Value
Panel A: Standard Parameters		
Relative Risk Aversion	γ	7
Intertemporal Elasticity of Substitution	ψ	1.50
Subjective Discount Factor	δ^4	0.98
Degree of Home Bias	α	0.96
Mean of Endowment Growth	$\mu \cdot 4$	2.00%
Short-Run Risk Volatility	$\sigma \cdot \sqrt{4}$	1.87%
Long-Run Risk Autocorrelation	ρ^4	0.953
Relative Long-Run Risk Volatility	σ_z/σ	6.90%
Cross-Correlation of Short-Run Shocks	ρ_X	00.15
Cross-Correlation of Long-Run Shocks	ρ_z	00.92
Panel B: Time-Varying Short-Run Risk		
Persistence of Short-Run Volatility	ρ_σ	0.90
		[0.89–0.93]
Volatility of Short-Run Volatility	σ_{sr}	0.15
		[0.15–0.16]
Cross-Correlation of Short-Run Volatility	ρ_{σ,σ^*}	0.30
		[0.13–0.45]
Short-Run Volatility Correlation with Short-Run Shocks	$\rho_{\sigma,\Delta y}$	-0.12
		[-0.15 -0.05]

Table A1: Robustness of Pass-Through Results

Panel A: Contemporaneous adjustments to relative volatility shocks

$\sigma(\Delta y)$	Δy	$\sigma(\Delta c)$	Δc	$\Delta(NX/Y)$	Pass-through
Global Benchmark, G17 Countries:					
0.16	-0.44	0.06	-0.06	-0.37	0.61
[0.15; 0.16]	[-0.67; -0.21]	[0.06; 0.07]	[-0.20;0.09]	[-0.56; -0.18]	[0.57; 0.64]
US/Pooled G7:					
0.19	-0.52	0.09	-0.26	-0.26	0.53
[0.19; 0.20]	[-0.83; -0.23]	[0.08; 0.10]	[-0.50;-0.02]	[-0.49; -0.03]	[0.49; 0.56]
VAR(2) Model:					
0.21	-0.41	0.09	-0.11	-0.29	0.59
[0.20; 0.21]	[-0.71; -0.11]	[0.08; 0.09]	[-0.34; 0.13]	[-0.53; -0.06]	[0.55; 0.62]

Panel B: Pass-through and size

	Origin of Vol Shock: US	Origin of Vol Shock: Foreign Country
Global Benchmark/G17 Countries:	0.52	0.62
	[0.45; 0.59]	[0.58; 0.66]
US/Pooled G7:	0.47	0.64
	[0.43; 0.52]	[0.58; 0.70]
VAR(2):	0.55	0.63
	[0.50; 0.60]	[0.58; 0.68]

Table B1: Standard Unconditional Moments

	G-17 Data		Model		
	Avg.	Quintiles [1^{st}; 4^{th}]	Bench-mark	No TVV ($\sigma_\sigma = 0$)	CRRA ($\gamma = 7$)
$corr(\Delta c, \Delta c^*)$	0.25	[0.13; 0.33]	0.38	0.37	0.74
$\sigma(\Delta c)(\%)$	1.67	[1.34; 2.47]	1.85	1.82	1.64
$\sigma(\Delta c)/\sigma(\Delta y)$	0.88	[0.57; 0.82]	0.93	0.94	0.83
$ACF1(\Delta c)$	0.17	[-0.16; 0.31]	0.06	0.07	0.08
$\sigma(M)/E(M)(\%)$	–	–	47.86	47.85	11.49
$\sigma(\Delta e)(\%)$	10.50	[10.2; 11.4]	12.80	12.65	8.31
$E(r^f)(\%)$	1.35	[1.44; 2.41]	2.17	2.19	14.91
$\sigma(r^f)(\%)$	1.79	[1.61; 2.27]	0.33	0.33	3.47
$corr(r^f, r^{f*})$	0.51	[0.37; 0.56]	0.91	0.92	0.98
$\sigma(\Delta(NX/Y))/\sigma(\Delta y)$	0.70	[0.67; 0.97]	0.32	0.32	0.16

Volatility Pass-Through Index ▸ Back

- Using the VAR on

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix},$$

  the VPTI is

$$VPTI = 1 - \frac{\tilde{\Sigma}_{3,1}}{\tilde{\Sigma}_{1,1}}$$

Volatility Pass-Through Index (cont'd) Back

- Using the VAR on

$$\tilde{Y}_{i,t} = \Big[\underbrace{\sigma_t(\Delta y_{US})}_{1}, \underbrace{\sigma_t(\Delta y_i)}_{2}, \underbrace{\Delta y_{US}}_{3}, \underbrace{\Delta y_i}_{4}, \underbrace{\sigma_t(\Delta c_{US})}_{5}, \underbrace{\sigma_t(\Delta c_i)}_{6}\Big],$$

- VPTI from country i to US

$$VPTI = 1 - \frac{\tilde{\Sigma}_{6,2} - \tilde{\Sigma}_{5,2}}{\Sigma_{2,2}}$$

- VPTI from US to country i

$$VPTI = 1 - \frac{\tilde{\Sigma}_{5,2} - \tilde{\Sigma}_{6,2}}{\Sigma_{1,1}}$$

Volatility shocks are priced ▸ Back

- Consider the case of $\psi = 1$, then

$$U_t = (1-\delta)\log C_t + \delta\theta \log E_t \exp\left\{\frac{U_{t+1}}{\theta}\right\}, \quad \theta = 1/(1-\gamma) < 0$$

- A second order Taylor expansion about $E_t[U_{t+1}]$ yields

$$U_t \approx (1-\delta)\log C_t + \delta E_t[U_{t+1}] + \frac{\delta}{2\theta} Var_t[U_{t+1}]$$

- The SDF is

$$m_t - E_{t-1}[m_t] = -(\Delta c_t - E_{t-1}[\Delta c_t]) + \frac{U_t}{\theta}$$

- If $Var_t[U_{t+1}] \uparrow$ then $U_t \downarrow$ and $m_t \uparrow$