Dynamic Inattention, the Phillips Curve and Forward Guidance

Hassan Afrouzi
Columbia U.

Choongryul Yang
UT Austin

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Motivation: Forward Guidance and Inflation Dynamics

> Monetary policy presumably plays a key role in shaping [inflation] expectations ... by providing guidance about the FOMC's objectives for inflation in the future. Even so, economists' understanding of exactly how and why inflation expectations change over time is limited.
>
> Janet Yellen (Sept. 2017)

- Objective of forward guidance:
  - affect the economy today through news about future policy.
- Two natural questions:
  - Do price setters pay attention to the news about future policy?
    - ⋆ Yes
  - If so, do their prices respond to such news?
    - ⋆ Yes
- More generally, how are price setters' expectations formed and how do they affect inflation dynamics?

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Overview

- Two types of Phillips curves for inflation dynamics:
  - Sticky/Noisy information Phillips curves:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t,$$

    - ⋆ criticized for not being forward looking.
  - Sticky price models:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \gamma y_t,$$

    - ⋆ ciricized for forward guidance puzzle.
- We micro-found a new Phillips curve that
  - captures the effects of expectations about future on inflation,
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Environment

- A measure of firms $i \in [0, 1]$.
- Firms' flow nominal profit depends on their own price, aggregate price and output:

$$\Pi_{i,t} = \Pi(P_{i,t}, P_t, Y_t)$$
$$\approx -(p_{i,t} - mc_t)^2 + \text{terms independent of } p_{i,t}$$

where small letters are log-deviations from steady state and

$$mc_t = p_t + \alpha y_t.$$

- Here:
  - y_t is output gap.
  - p_t is the aggregate price: $p_t = \int_0^1 p_{i,t} di$.
- Firms are rationally inattentive.

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Firms' Problem Given an Information Structure

- For any t, let S_i^t be i's information set at time t.
- i's pricing problem is

$$L_0^i \equiv \min_{p_{i,t}:S_i^t \to \mathbb{R}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (p_{i,t} - mc_t)^2 | S_i^0\right].$$

- Solution:

$$p_{i,t}(S_i^t) = \mathbb{E}[mc_t | S_i^t]$$

and

$$L_0^i = \sum_{t=0}^{\infty} \beta^t \mathrm{var}(mc_t | S_i^t).$$

- Kalman filtering:

$$\Delta p_{i,t} = \mathbb{E}[\Delta mc_t | S_i^{t-1}] + k_t^i (s_{i,t} - \mathbb{E}[s_{i,t} | S_i^{t-1}])$$

- Need to characterize what kind of signals firms choose to see.

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Endogenous Information Acquisition

- Firm i wakes up at time t with S_i^{t-1}.
- Chooses $S_{i,t} \subset \mathcal{S}_t$ subject to cost $C(S_{i,t}|S_i^{t-1})$ and forms $S_i^t = S_i^{t-1} \cup S_{i,t}$.
- Chooses $p_{i,t} : S_i^t \to \mathbb{R}$.

$$L_t(S_i^{t-1}) = \min_{S_{i,t} \subset \mathcal{S}_t} \{ \underbrace{var(mc_t|S_i^t)}_{\text{gain from information}} + \underbrace{C(S_{i,t}|S_i^{t-1})}_{\text{cost of information}} + \beta L_{t+1}(S_i^t) \}$$

$$\text{s.t. } S_i^t = S_i^{t-1} \cup S_{i,t}$$

- Today's signals end up in tomorrow's information set:
  - information has continuation value.

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Signals

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- A forward looking firm cares about mc_t, mc_{t+1}, mc_{t+2}, ...
- These are subject to shocks that might not have been realized at time t.
- So if $\mathbb{E}_t^f[.]$ captures availability of infromation at t, firms can learn about

$$mc_t, \ \mathbb{E}_t^f[mc_{t+1}], \ \mathbb{E}_t^f[mc_{t+2}], \ \ldots$$

- It is optimal for firm to consider signals of the following form at time t:

$$s_{i,t} = \underbrace{\sum_{j=0}^{\infty} b_{j,t} \mathbb{E}_t^f[mc_{t+j}]}_{\text{choose weights } (b_{j,t})_{j=0}^{\infty}} + \underbrace{\sigma_{s,t}^i}_{\text{std. dev. of inattention error}} e_{i,t}$$

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Properties of the Cost of Information

- Data Processing Inequality (DPI) in information theory:
  - for $\{s_1, s_2\} \subset \mathcal{S}_t$, seeing a combination of them is less costly than seeing both

$$C(as_1 + bs_2 | S^{t-1}) \leq C(s_1, s_2 | S^{t-1})$$

Proposition

Every firm observes only one signal at any time.

- Intuition:
  - Price is a linear combination of signals.
  - So instead of seeing signals separately and paying a high cost, the firm would like to see the combination.

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Properties of the Cost of Information

- The marginal cost of learning more about any $\mathbb{E}_t^f[mc_{t+\tau}]$ is increasing.

Proposition

Optimal signals are forward looking $(b_{\tau>0} \neq 0)$

$$s_{i,t} = \sum_{j=0}^{\infty} \beta^j b_j \mathbb{E}_t^f[mc_{t+j}] + \sigma_s^i e_{i,t}$$

- The agent is forward looking and wants to know about

$$mc_t, \mathbb{E}_t^f[mc_{t+1}], \mathbb{E}_t^f[mc_{t+2}], \dots$$

Marginal benefit is decreasing with horizon while marginal cost is increasing with precision $\Rightarrow$ Information smoothing.

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Results: the Phillips Curve

- Recall, $mc_t = p_t + \alpha y_t$.
- In sticky/noisy information models:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t$$

  - sticky information: λ is the fraction that update their information.
  - noisy information: λ is the Kalman gain.

- Under dynamic inattention:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t - \sum_{\tau=1}^{\infty} \beta^{\tau} \delta_{\tau} \tilde{\mathbb{FE}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}]$$

where $\tilde{\mathbb{FE}}[x] \equiv \tilde{\mathbb{E}}_t[x] - \mathbb{E}_t^f[x]$ is the forecast error of firms relative to a fully informed agent.

Results: the Phillips Curve

- Recall, $mc_t = p_t + \alpha y_t$.
- In sticky/noisy information models:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t$$

  - sticky information: λ is the fraction that update their information.
  - noisy information: λ is the Kalman gain.

- Under dynamic inattention:

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$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t - \sum_{\tau=1}^{\infty} \beta^\tau \delta_\tau \mathbb{F}\tilde{\mathbb{E}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}]$$

Remarks:

1. This imbeds the noisy information Phillips curve when $\beta = 0$.
2. Inflation is affected by expectations about future, but in a different way than sticky price models:
   1. $\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t$: inflation is increasing in expected inflation.
   2. dynamic inattention: inflation is decreasing in forecast errors.

Results: the Phillips Curve

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Results: Estimating the Phillips curve

- Estimate the Phillips curve using GMM

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t - \sum_{\tau=1}^{T} \beta^\tau \delta_\tau \mathbb{F}\tilde{\mathbb{E}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}] + \xi_t$$

1. Use Survey of Professional Forecasters as proxy for firms' forecasts.
2. Instrument forecast revisions for forecast errors. (Coibion Gorodnichenko (2015))
3. Null hypothesis: $\delta_\tau \neq 0$ for $\tau > 0$.

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Results: Estimating the Phillips curve

	π_t	
	(1)	(2)
	GDP Deflator (72Q1-16Q4)	CPI (81Q3-16Q4)
$\tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha\Delta y_t]$	1.00 ***	1.01 ***
	(0.01)	(0.14)
αy_t	1.28 **	0.67 ***
	(0.50)	(0.10)
$\tilde{\mathbb{FE}}_t[\pi_{t+1} + \alpha\Delta y_{t+1}]$	0.42 ***	0.16 ***
	(0.05)	(0.03)
$\tilde{\mathbb{FE}}_t[\pi_{t+2} + \alpha\Delta y_{t+2}]$	0.21 ***	-0.31 ***
	(0.05)	(0.03)
$\tilde{\mathbb{FE}}_t[\pi_{t+2} + \alpha\Delta y_{t+2}]$	0.11 ***	-0.17 ***
	(0.02)	(0.04)

Newey-West robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Example: One Period Ahead News

- Suppose $mc_t = mc_{t-1} + u_{t-1}$
- Shocks are announced one period ahead.
- How much do agents pay attention to this news and react?
- Under myopic inattention $(\beta = 0)$:

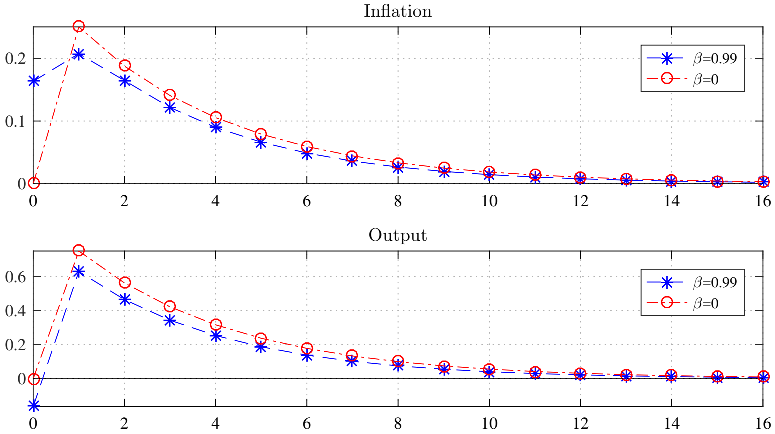
$$s_{i,t} = mc_t + e_{i,t}$$

- Notice that in this case $u_t \perp S_i^t$: myopic firms completely ignore news about future.
- Under dynamic inattention $(\beta > 0)$:

$$s_{i,t} = mc_t + \gamma mc_{t+1} + e_{i,t}$$

Example: One Period Ahead News

- Under dynamic inattention inflation responds to the news shock.
- Output falls on impact because marginal cost is fixed by assumption, which is relaxed in GE.



Example: General Equilibrium

A Three Equation Model

- Dynamic Phillips curve:

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- Dynamic IS curve:

$$y_t = \mathbb{E}_t^f[y_{t+1}] - \sigma^{-1}(i_t - \mathbb{E}_t^f[\pi_{t+1}])$$

- Taylor rule:

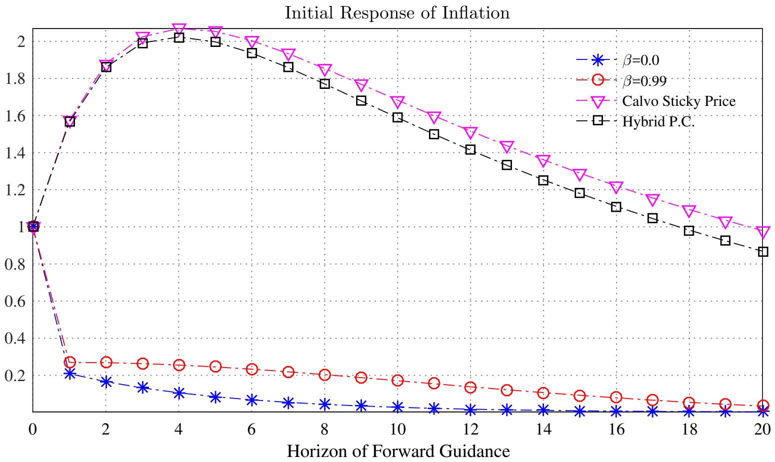
$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + u_{t-k}$$

where k is the horizon of forward guidance.

Example: General Equilibrium

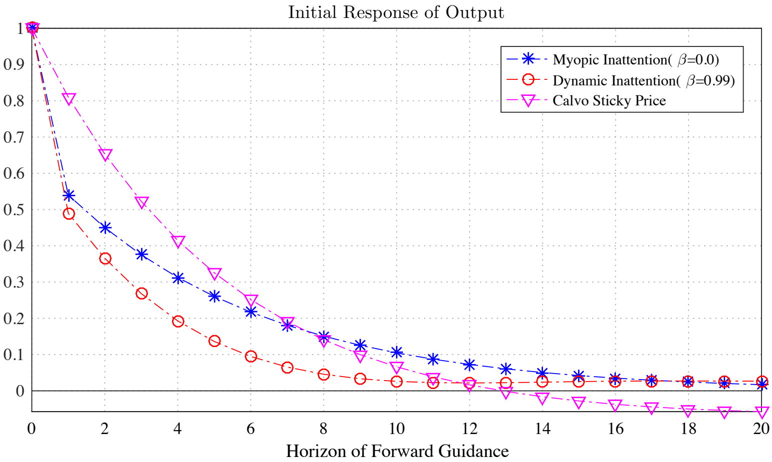
Forward Guidance Puzzle

- Impact response of inflation is decreasing in horizon of forward guidance



Example: General Equilibrium

4-period ahead Forward Guidance Shock



Conclusion

- Showed that firms have information smoothing incentives:
  - they pay attention to news about future,
  - and incorporate such news in their current prices.
- Derived and estimated a new micro founded Phillips curve:
  - inflation is forward looking in contrast to other models of information rigidity.
  - no forward guidance puzzle despite inflation being forward looking.

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Example: General Equilibrium

4-period ahead Forward Guidance Shock