#### Credit-Implied Volatility

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AQR

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### This Paper

- Introduce "credit-implied volatility" (CIV) surface
- Organizes behavior of corporate credit prices into a handful of easy-to-visualize facts
  - Across wide range of firms (credit quality)
  - For short and long maturities
  - Dynamics
  - \* New stylized facts for credit pricing
- Simple visual diagnostic for candidate credit risk models
- Infer distribution of asset growth (firms and aggregate)

### Brief Review of Option-Implied Volatility

Q: "Given the Black-Scholes model, how much equity volatility is required to justify observed put price (given its strike price and maturity)?"

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- Expensiveness in terms of deviations from well understood benchmark
- Visualize prices for entire asset class in a single graph
- ► Read skewness and kurtosis directly off of curves (Backus, Foresi, and Wu, 2004)
- Options literature organized around this object (model diagnostic)

#### "Debt is a Short Put in a Gaussian World" Brief Review of Merton Model

- Assumptions of Merton (Black-Scholes) Model
  - 1. Asset growth through time T is  $N(\mu_A, \sigma_A^2)$
  - 2. Debt matures at T, face value of F, can only default at T

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- In other words,
- Debt is (short) put on  $A_T$  with strike F
- Credit spread simple translation of this put's price

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Data: Credit Spread  $(S_{i,\tau})$  & Firm Leverage  $(L_i = \frac{F_i}{A_i})$  & Debt Maturity  $(\tau)$ Merton Formula:  $S_{i,\tau} = f(\sigma_A, L_i, \tau)$  $\downarrow$ Invert:  $CIV_{i,\tau}$ 

▶ CIV Surface: Plot  $CIV_{i,\tau}$  against "moneyness" and maturity of debt

#### Data

- All CDS in Markit 2002-2014 (530 firms, 156 months)
- CDS: Standardized, no callability/optionality, liquid (bonds in "extensions")

Leverage and other supplements from Compustat and CRSP



- Scatter CIV vs. leverage (pooling all firm-months) for 1-year CDS
  - Fitted non-parametric curve in gray
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- In Merton model, firm's leverage ratio describes "moneyness" of the put option implicit in its debt
- Two basic features summarize the unconditional average shape of the CIV surface
- 1. **Moneyness smirk**: From POV of Merton model, needs disproportionately high asset vol to match CDS spreads for firms with low leverage (OTM) vs. firms with high leverage (ATM)



2. **CIV term structure**: Smirk is steepest for one-year CDS, monotonically flattens as maturity lengthens



- CIV surface a more complicated object than OIV surface
- For CDS, multiple maturities but only one "strike price" per firm
- Pooled scatter mixes many dimensions of heterogeneity
  - Industry, credit rating, asset risk
  - Time (crisis, great moderation)
- Why is CIV high for OTM contracts? Non-gaussianity? Different risk?

### Credit-Implied Volatility Smirk (Heterogeneity-Adjusted)

$$\begin{split} \mathsf{CIV}_{i,\tau,t} &= \delta_{0,\tau} + \delta'_{1,\tau} [\mathsf{Size},\mathsf{Beta},\mathsf{Vol},\mathsf{Skew},\mathsf{Kurt}]'_{i,t} \\ &+ \mathsf{Rating FE} + \mathsf{Sector FE} + \mathsf{Month FE} + \epsilon_{i,\tau,t}, \end{split}$$

- Regression run separately for each maturity, au
- Soak up all heterogeneity excluding leverage
  - Noting of course other measures correlated
- Plot the residual—"Heterogeneity-adjusted" CIV

 $\widehat{\mathsf{CIV}}_{i,\tau,t} = \epsilon_{i,\tau,t}$ 

Heterogeneity-Adjusted:  $\widehat{\text{CIV}}_{i,\tau,t} = \epsilon_{i,\tau,t}$ 



Credit Fact 1: IV from credit spreads possesses a steep moneyness smirk

- Differences in credit spreads in cross section explained by leverage and maturity
- ► Fact not explained by non-leverage firm heterogeneity
- Suggests (severe) non-normalities in asset growth distribution

Dynamics of Credit Spreads

#### **CIV Smirk Snapshot**



### Constant-Leverage and Constant-Maturity Portfolios



- Individual CDS unbalanced panel
- To track surface, want to track "constant-leverage" credits
- 1. Fit non-parametric moneyness curve each month (at each maturity)
- 2. Interpolate curve at grid points of 20%, 40%, 60%, and 80% leverage
- Portfolios are "local averages" around grid points. A firm's weight in this average is inversely proportional to the difference between its leverage and the grid point (E.g., firm with 77% leverage will have large contribution to 80% portfolio, small contribution to 60% portfolio, very small contribution to 20% portfolio)
- 20 portfolios: four leverage portfolios, five maturities (1Y, 3Y, 5Y, 7Y and 10Y)

## Surface Dynamics

- High degree of commonality in CIV fluctuations for portfolios sorted by leverage and maturity
- ▶ We first analyze the factor structure of CIV surface using PCA on panel of 156×20 month-by-portfolio CIV observations
- ► Five leading components explain 87.4%, 9.6%, 1.8%, 0.4%, and 0.3% of the panel variation in CIV, respectively
- We focus on first three PCs

#### First Component: Surface Level



Second Component: Term Structure Slope











Credit Fact 2: 99% of panel dynamics in spreads captured with

- 1. Variation in common level of CIV (~87%)
- 2. Variation in term structure slope ( $\sim 10\%$ )
- 3. Variation in moneyness slope ( $\sim 2\%$ )

Collectively, Facts 1 + 2 suggest that vast majority of credit spread heterogeneity, both across individual CDS and over time, is associated with the leverage of the reference entity, the maturity of the CDS contract, and a small number of common state variables

		× .		( )		
	(1)	(2)	(3)	(4)	(5)	(6)
Lev.	-0.397*	-0.402*	-0.201*	-0.21*	-0.130*	-0.191*
1Y			0.311*	0.31*	0.290*	0.311*
3Y			0.102*	0.10*	0.092*	0.101*
5Y			0.050*	0.05*	0.043*	0.049*
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Vol.					0.262*	0.128*
Skew.					-0.187*	-0.141*
Kurt.					-0.028*	-0.023*
Beta					-0.025	0.870*
AA					-0.003	-0.004
BBB					-0.001	0.008
BB					0.017*	0.026*
Cons. Prod.					0.005	0.002
Cons. Svc.					0.003	0.001
Energy					002	-0.017
Financials					-0.027	-0.016
Hlth.					0.014	0.021
Indust.					-0.011	-0.010
Tech.					0.023	0.071*
Telcom. Svc.					-0.001	0.001
Util.					-0.026	-0.025*
VIX	No	Yes	No	Yes	Yes	Yes
Ν	253,410	253,410	253,410	253,410	151,259	253,188
R <sup>2</sup>	0.49	0.68	0.65	0.84	0.88	0.87

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#### Patterns of Credit Risk Premia

- So far analysis focused on levels of spreads and CIV
  - Levels all but perfectly explained with two or three common factors interpretable via differences in leverage and maturity
- Next natural question: Do credit risk premia also align with its leverage and maturity?
- Study average returns of 20 leverage/maturity CDS portfolios

### CIV and Risk Premia

- Annualized Sharpe ratios of monthly returns on 20 CDS portfolios
- Returns to selling CDS—risk premia that accrue to insurance provider



 Differences in compensation for selling CDS closely align with leverage and maturity

#### CIV and Risk Premia

#### Why do credit risk premia align with leverage and maturity?

- PCA showed that credit risk for all firms well captured by small number of shocks
- Are these shocks risk factors?

Fama-MacBeth analysis using shocks to PC's

- 1. Betas of each portfolio on factor innovations (time series)
- 2. Do betas align with differences in mean portfolio returns?

#### CIV and Risk Premia

Expected Returns: Actual vs. Model Fit



Credit Fact 3: Differences in average CDS returns align with same dimensions of CIV surface (leverage and maturity)

- These differences are fully explained by differences in exposure to fluctuations in CIV surface
- Much of the credit risk premium is a variance risk premium. Similar behavior to equity VRP but larger magnitude

#### Model

# Background: Credit Spread Empirics

#### No-arbitrage Models

- "Structural" or "Reduced-form"
- Enforce cross-equation restrictions
- Typical estimation approach calibrates to
  - Average spread (no dynamics)
  - Representative credit in rating category (no cross section)

#### Unrestricted Models

- Regressions
- LHS: Spread levels or changes
- RHS: Firm characteristics
- Fit full panel (dynamics and cross section)

- We propose no-arbitrage ("structural") model
- Specification based on what we've learned from CIV surface
- Matches panel of spreads nearly as well as unrestricted models—across wide range of credit quality and throughout the credit cycle

# Choosing Specification

#### What a model should accomplish:

Steep moneyness smirk  $\rightarrow$  (Severe) Excess kurtosis

Smirk flattens with maturity  $\rightarrow$  Mean reversion

Panel dynamics entirely captured by 2-3 factors  $\rightarrow$  Exposure to aggregate shocks, Few state variables. No independent idiosyncratic risk variation Model: "1 Vol, 1 Jump"

Aggregate Asset Growth:

$$\frac{dA_{m,t}}{A_{m,t}} = rdt + \underbrace{\sqrt{\mathbf{v}_t} dW_t^{m,1}}_{Agg. \ Stochastic \ Vol} + \underbrace{\left(\left(e^{-q_m} - 1\right) dJ(\lambda_t) - \lambda_t \xi_m\right)}_{Agg. \ Jump \ Risk}$$
$$dv_{1,t} = \kappa_v \left(\theta_v - v_t\right) dt + \sigma_v \sqrt{v_{1,t}} dW_t^v$$
$$\lambda_t = av_t + \mathbf{z}_t \quad , \quad dz_t = \kappa_z \left(\theta_z - z_t\right) dt + \sigma_z \sqrt{z_{,t}} dW_t^z$$

Firm Asset Growth:

$$\frac{dA_{i,t}}{A_{i,t}} = rdt + \underbrace{\beta_i \left(\frac{dA_{m,t}}{A_{m,t}} - rdt\right)}_{Agg. \ Exposure} + \underbrace{\sqrt{v_{i,t}} dW_t^i + \left(\left(e^{-q_i} - 1\right) dJ(\lambda_t) - \lambda_t \xi_i\right)}_{Common \ Idios. \ Risk}$$

$$\mathbf{v}_{i,t} = \mathbf{v}_i + \gamma_i \mathbf{v}_t$$
,  $q_i \sim q_m$ 

### **Compare Specifications**

- Stochastic vol only (1-factor)
- Jump only (1-factor)
- Stochastic Vol + Jump (2-factor)
- Two Stochastic Vols + Jump (3-factor)

#### Results

No-arbitrage								Unrestricted		
	1 Vol, 0 Jump	0 Vol, 1 Jump	1	Vol, 1 Jun	np :	2 Vol, 1 Jump	PCA1	PCA2	PCA3	
$R^{2}$ (%)	10.6	70.0		77.9		83.5	88.6	98.6	99.4	
Parameters	4 + 12	5 + 12		10 + 12		14 + 12	40	60	80	
States/factors	1	1		2		3	1	2	3	

# **CIV Surface Dynamics**



### Why Do Estimates Land on Rare Disaster Specification?

Under RN measure:

- Aggregate jumps arrive on average once every 100+ years
- ▶ Expected log jump size is -71%



CIV 1-Year Maturity (Heterogeneity-adjusted)

#### What We Learn From the Model

Highly accurate fit of spreads based on

- Crash-risky aggregate asset growth
- 2-factor dynamics in higher moments (stochastic vol, crash risk)
- > All firms exposed to aggregate shock, inherit its higher moments
- By looking at CIV smirk, we read off the (risk-neutral) distribution of aggregate asset growth, despite not having a cross section of "strikes." This is not something we have "seen" before
- More work to be done here

#### Extension 1: Bond CIV



#### Extension 2: Sovereign CIV

- Sovereign CDS for 24 OECD countries
- What is "moneyness" of sovereign credit?

#### From OECD Consolidated National Balance Sheets for General Government

The difference between the financial assets and liabilities held by governments (also known as financial net worth or as a broad description of net government debt), gives an extensive measure of the government's capacity to meet its financial obligation. While financial assets reflect a source of additional funding and income available to government, liabilities reflect the debts accumulated by government. Thus, an increase in the financial net worth signals good financial health. Net worth may be depleted by debt accumulation, indicating a worsening of fiscal position and ultimately forcing governments to either cut spending or raise taxes.

We define sovereign leverage as the ratio of total financial liabilities (net of shares and other equity) to financial assets for the general government sector

#### Extension 2: Sovereign CIV



## Conclusion

- CIV surface as organizing framework for empirical analysis of credit
- Almost all variation in relative cost of a credit claim lines up with
  - Moneyness of contract (underlying firm's leverage)
  - Contract maturity
- Document steep CIV moneyness slope that implies large deviations from normality in the risk-neutral distribution of aggregate asset growth
- Dynamics of surface summarized with three interpretable factors
  - CIV level
  - Term structure slope
  - Moneyness slope
  - \* Provides compact and complete description of time-variation in the entire panel of firm-level credit spreads
- A parsimonious structural model with stochastic volatility and jumps provides an accurate description of CDS spreads for firms across the credit spectrum, at short and long maturities, and at all points throughout the credit cycle
- Our estimation suggests risk-neutral distribution of aggregate asset growth effectively described as a rare disaster model