Applying Generalized Pareto Curves to Inequality Analysis

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ASSA Conference, January 2018

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The Pareto distribution

- Goes back to Pareto (1896). Still the most common model of income and wealth distributions.
- For $\alpha > 1$ (the "Pareto coefficient") and $x \geq x_0 > 0$:

$$\mathbb{P}\{X > x\} = (x_0/x)^{-\alpha}$$

- Characterization (van der Wijk's law) :

$$\frac{\mathbb{E}[X|X > x]}{x} = \text{constant} = \frac{\alpha}{\alpha - 1}$$

- $b = \alpha/(\alpha - 1)$ is called the "inverted Pareto coefficient." Can be interpreted as a measure of inequality.

Beyond Pareto

- The Pareto distribution is a good first-order approximation. But in many practical settings, the constraints it imposes are too tight.
- Using "generalized Pareto curves" allows for more flexibility and precision.
    - Methodological improvements that underlie many of the recent empirical inequality research.
    - Useful to analyze patterns in the tail of income and wealth distributions.

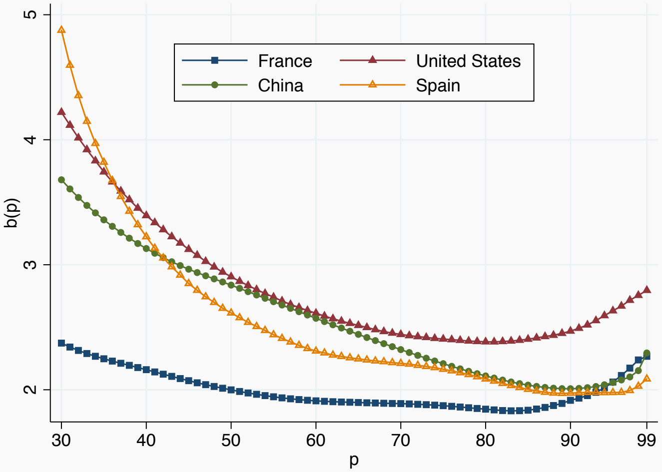
Generalized Pareto curves

- A constant Pareto coefficient means that inequality always remains the same within all top income groups (fractal inequality). What if that is not exactly true ?
- Let the inverted Pareto coefficient vary :

$$b(p) = \frac{\mathbb{E}[X|X > Q(p)]}{Q(p)}$$

- $p \mapsto b(p)$ is the generalized Pareto curve.

Generalized Pareto curves : pre-tax income (2000–2014)



Increasing $b(p)$ at the top $\Rightarrow$ increasing income concentration

Generalized Pareto interpolation

- Use for empirical inequality research.
- Tax data is typically available as :

Income bracket	Bracket size	Bracket average income
From 0 to 1000	300 000	500
From 1000 to 10 000	600 000	5 000
From 10 000 to 50 000	80 000	30 000
More than 50 000	20 000	200 000

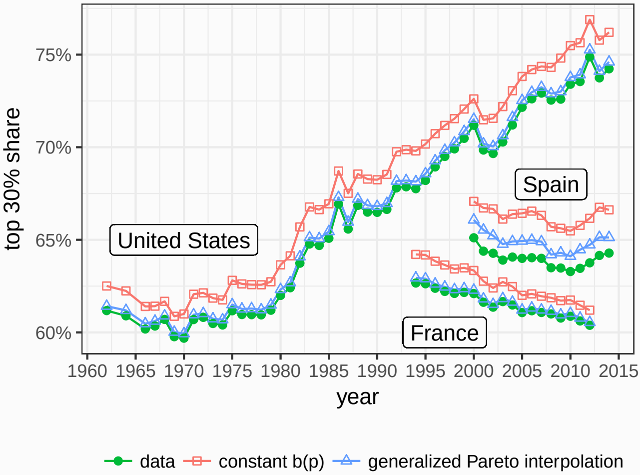
- We need to get the entire distribution sometimes based on a few brackets only.

Classical Pareto interpolation

- The standard Pareto model does not offer enough degrees of freedom.
- Piketty (2001), Piketty and Saez (2003) :
  - Use a piecewise constant $b(p)$.
  - Does not use all the information efficiently.
  - Does not yield a consistent distribution.
  - Other methods, but none fully satisfying.
- Blanchet, Fournier and Piketty (2017) approach : find the most regular curve $b(p)$ that properly interpolates the tabulation.

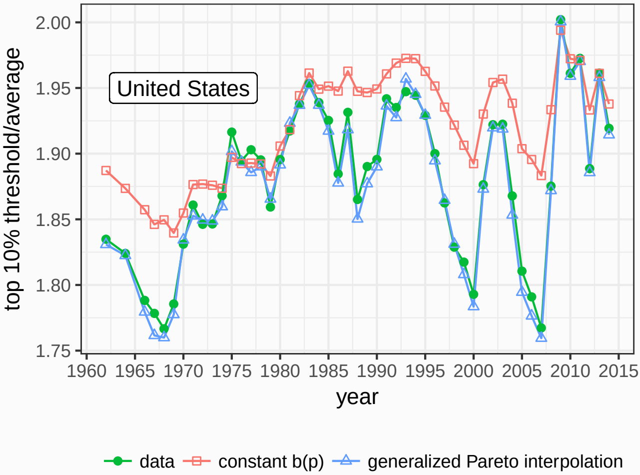
Comparison of interpolation methods (I)

Top 30% share from the top 50% and the top 10%.



Comparison of interpolation methods (II)

Top 10% threshold from the top 30% and the top 1%.



Usefulness of tax tabulations

- Even with coarse tabulations, we can recover the entire distribution quite well.
- Importance of having tax data for the top of the distribution, even in such censored form.
- Estimating the top 1% share from the top 10% and the top 0.1%, the average error in the US from 1962 to 2014 is 0.15 pp.
- Monte-Carlo simulations suggest that the average estimation error for the same quantity based on large random subsamples is higher :
    - 10^4 observations : ± 3.32 pp.
    - 10^5 observations : ± 1.63 pp.
    - 10^6 observations : ± 0.72 pp.

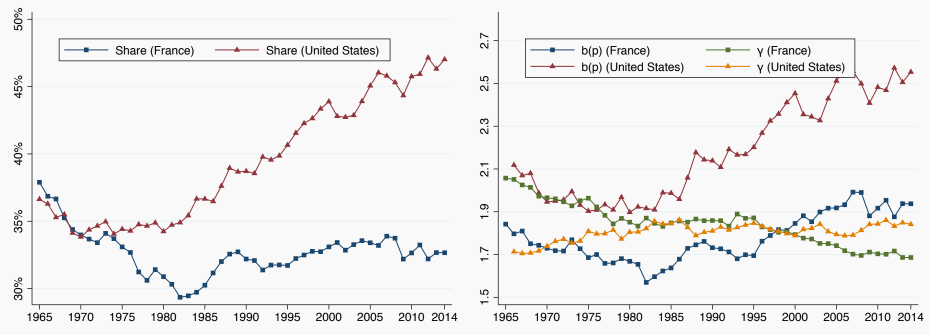
Interpreting the evolution of top shares

- Pareto coefficients are also useful to interpret changing patterns in the top tail of the income distribution.
- Disentangling forces behind the evolution of top shares. For example, decompose the top 10% share as :

$$\text{top 10\% share} = 0.1 \times b(\text{p90}) \times \gamma(\text{p90})$$

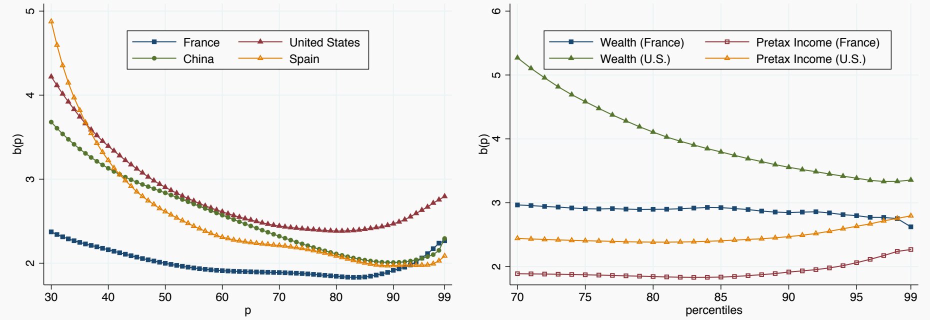
  where $\gamma(\text{p90})$ is the top 10% income threshold divided by the average.
- $b(\text{p90})$ is driven by what's happening within the top 10%, while $\gamma(\text{p90})$ corresponds to the evolution of the top 10% income threshold relative to the average.

Evolution of top shares in France and the United States



- In France : $b(\text{p90}) \nearrow$ and $\gamma(\text{p90}) \searrow$
  - $\Rightarrow$ relatively stable 10% share
- In the United States : $b(\text{p90}) \nearrow$ and $\gamma(\text{p90})$ stable
  - $\Rightarrow$ increasing 10% share

Shape of Pareto curves for income and wealth (2000–2014)



- U-shaped pattern for income but not so much for wealth.
- Gap between income and wealth inequality narrows at the top.

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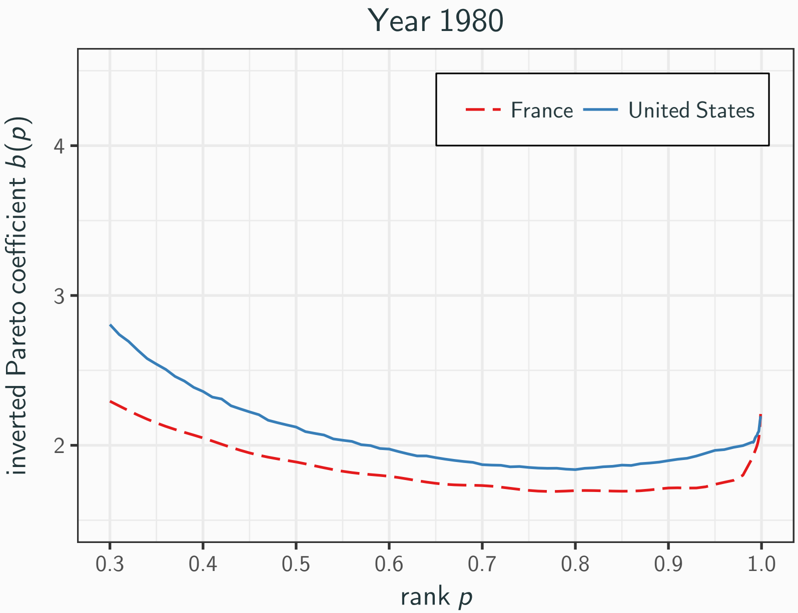
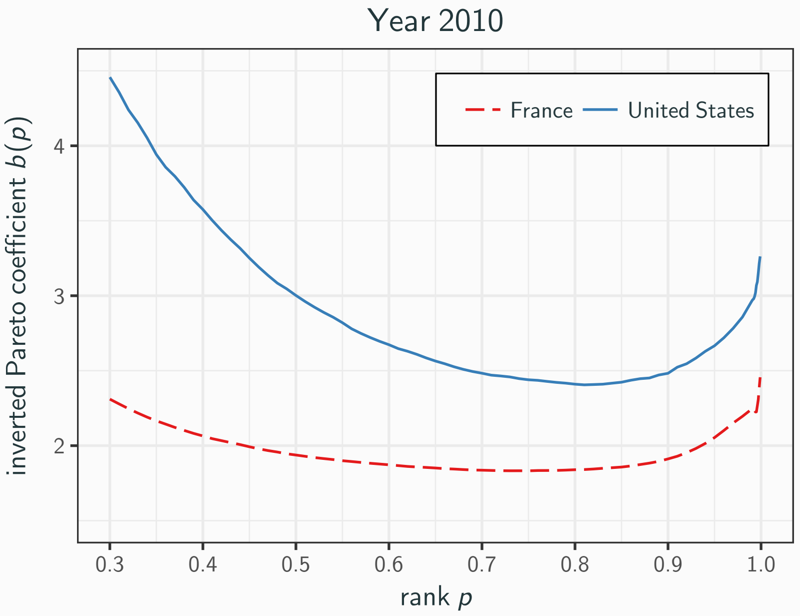
Additional slides

Pre-tax national income



Pre-tax national income



US pre-tax national income, 2010 : generalized Pareto curve