Electoral Incentives and the Allocation of Public Funds*

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Abstract

Politicians allocate public resources in ways that maximize political gains, and potentially at the cost of lower welfare. In this paper, we quantify these welfare costs in the context of Brazil's federal legislature, which grants its members a budget to fund public projects within their states. Using data from the state of Roraima, we estimate a model of politicians' allocation decisions and find that 25 percent of the public funds allocated by legislators are distorted relative to a social planner's allocation. We then use the model to simulate two potential policy reforms to the electoral system: the adoption of approval voting and imposing a one-term limit. We find that both policies are effective at reducing distortions. The one-term limit policy, however, increases corruption, which makes it a welfare-reducing policy.

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1 Introduction

A central function of governments is the provision of public goods and services. In 2017, governments throughout the world spent on average more than 34 percent of their countries' GDP on these goods and services.[1] When allocated efficiently, these expenditures can be important drivers of economic development and key determinants of quality of life. But, public expenditures are allocated by politicians who, as an extensive theoretical literature has argued, care about being elected. This creates incentives that can distort how public funds are spent relative to the social optimum. While the existence of these distortions is rarely disputed, there is no empirical evidence on their magnitude or whether appropriate electoral policies can reduce them.

The main contribution of this paper is to address the following two questions: What is the size of the distortions generated by electoral incentives? And, can electoral policies reduce these distortions? To answer these questions, we develop a model that characterizes the allocation decisions of federal legislators in Brazil, who are granted an annual budget to fund local public goods across regions within the state they represent.

Our model starts with the basic premise that citizens value public goods and vote for the politicians who they believe will provide their region with more resources in the future to fund these goods. With this feature, the model can explain the strong correlation we observe in our data between the votes politicians receive and the amount of funds they allocated to a particular region during their previous term.[2] In the model, voters' decisions are also affected by a politician's electoral appeal. Thus, at the time of the elections, politicians with higher electoral appeal fare better at the polls, all else equal.

In most models of distributive politics, politicians care exclusively about getting elected (e.g. Myerson (1993), Lizzeri and Persico (2001)). In our model, politicians care not only about their election probabilities, but also about the welfare of the people in their state. Moreover, the relative weight politicians assign to welfare is allowed to vary across them. This heterogeneity in the degree of a politician's altruism enables us to account for the following pattern in our data: while some politicians target mostly places with a lot of voters, as has been documented in other settings (e.g. Atlas et al. (1995), Rodden (2002)), others politicians target predominantly regions that are less populated and less developed.

[1] This statistic comes from the 2017 Index of Economic Freedom.

[2] Evidence that voters reward politicians for transfers has been documented in other contexts as well (e.g. Levitt and Snyder (1997), Manacorda, Miguel, and Vigorito (2011)).

At the end of a political term, we model an incumbent's decision to run for reelection. By modeling this choice, we can account for the observation that incumbents who run, and thus have stronger electoral incentives, allocate their public funds differently than those who do not. This last pattern is also consistent with a large literature documenting the importance of reelection incentives on policy choices (e.g. Besley and Case (1995), List and Sturm (2006), Ashworth (2012), Lim (2013)).

Our model also allows for the funds that politicians allocate to a region to not fully translate into pure local public goods. This can happen for two reasons. First, politicians can divert some of the funds, as the media frequently reports. Second, the public goods may have some degree of rivalry, in that the welfare value of the goods may decrease as more people use them.

Lastly, we consider a setting in which multiple politicians are simultaneously elected to represent their state. Thus, when incumbents decide where to allocate their funds and whether to run for reelection, they must consider the decisions of the other incumbents within the state. To account for this institutional feature, in the model politicians make their decisions simultaneously and with incomplete information about the degree of altruism and electoral appeal of their opponents.

We estimate the model by Simulated Method of Moments (SMM) using data on the universe of public funds allocated from 1996 to 2013 by federal legislators representing the state of Roraima in Brazil. Brazil's federal legislature, and the state of Roraima in particular, provide an ideal setting to estimate a model in which politicians allocate funds across regions. Each year the Brazilian Constitution grants each federal legislator a budget of BRL$1.5 million (US$750,000) to fund public projects in the state where the legislator is elected. This constitutional provision allows us to investigate the effect of electoral incentives on the politicians' allocation decisions without worrying about the endogeneity of whom has access to these funds, which is an important issue in other contexts, such as the U.S. Congress. Moreover, these funds are commonly used for large-scale development projects that have important welfare consequences.

We chose the state of Roraima because it is a poorer and less populated state, where the welfare consequences of the allocation decisions of politicians are likely to be more important. Additionally, the computational burden of estimating our model increases exponentially with the number of incumbent politicians who compete for votes. In Brazil, each state is an individual district that elects a fixed number of legislators to the federal government. Roraima elects only 8 legislators, which makes the estimation of our model challenging but feasible.

Using the estimated model, we then proceed to answer our first question: What is the size of the distortions generated by electoral incentives? We find that about 25 percent of public funds are distorted relative to a social planner's allocation. Our estimates indicate that, to explain the observed allocation patterns, we need two groups of politicians: a group composed of egoistic individuals, who care almost exclusively about electoral incentives and represent 71 percent of the candidate pool; and a group composed of altruistic individuals, who also value the welfare implications of their decisions. The egoistic politicians, who target regions with more votes at the cost of poorer and more productive places, are responsible for a large fraction of the distortions. Had the pool of politicians only consisted of altruistic individuals, the distortions would have declined by 51 percent. These findings highlight the importance of unobserved heterogeneity in the allocation of public funds and the need to account for it when trying to understand allocation decisions.

To answer the second question about whether electoral policies can reduce these distortions, we simulate two possible policy reforms. We first consider the effects of adopting approval voting, which is an electoral system that allows people to vote for multiple candidates. We find that approval voting has the desired effect of reducing distortions, but the size of the decline is limited. For example, if Brazil's government replaced its current system, where residents can vote for only one candidate, with one in which voters can vote for 6 candidates, distortions would decrease by 3.3 percent.

Although this represents an improvement in aggregate welfare, there are two countervailing forces that limit the efficacy of approval voting. On the one hand, under approval voting, incumbents are rewarded for transferring resources to the region with higher political gains even if they are not ranked at the top of the voters' preferences. The incentives to target those regions are therefore weaker. If places with more voters tend also to be less productive, then as politicians shift their resources to other more productive municipalities, the distortions decrease. On the other hand, approval voting increases the probability that weaker politicians – mostly politicians who run for the first time – win the elections. For instance, going from the current system to a 6-person voting system increases the probability that a non-incumbent wins by 24.6 percent. As elections become more competitive, incumbents have stronger incentives to target regions with more voters, which increases the distortions if these places are less productive.

We also examine the effects of limiting politicians to a single term. Brazil currently allows legislators to be elected indefinitely. But several countries have argued for, and in some cases implemented, term limits as a way to improve representation and reduce politicians' pandering. In our model, the advantage of a one-term-limit policy is that electoral incentives would no longer influence the way public funds are allocated. The disadvantage is that, according to our model, incumbents who forgo reelection divert 17 percent more funds than those who still face reelection incentives. The ability to determine which of these two effects dominates is an important contribution of our model. When we compare our results to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions decrease by 64 percent (from 25 to 9 percent). However, because of the increase in corruption, welfare as a whole goes down by 1.9 percent, suggesting that approval voting should be favored over a one-term limit policy.

Overall, our findings contribute to two broad strands of the literature. First, our study relates to an extensive literature in both economics and political science that investigates the causes and consequences of distributive politics. As Golden and Min (2013) report in a comprehensive review of this vast literature, numerous studies have documented the importance of electoral incentives in the allocation of public goods and services. Yet our study is, to our knowledge, the first to quantify the welfare consequences of electoral incentives, and to show how electoral rules can help reduce potential deviations from a social planner's allocation.

Our focus on electoral rules naturally relates to a more specific literature within distributive politics that examines the importance of the electoral system for public goods provision.[3] The empirical studies have been almost entirely reduced-form (e.g. Milesi-Ferretti, Perotti, and Rostagno (2002), Besley and Case (2003), Persson and Tabellini (2005), and Beath et al. (2014)). This paper instead uses a structural approach to understand the effects of electoral institutions on the allocation of public funds. In this regard, our paper is related to a growing literature on the structural estimation of political economy models. For instance, Stromberg (2008) structurally estimates how U.S. presidential candidates allocate their campaign resources across states to maximize their election chances. We complement this study by examining the allocation of public resources, which besides providing electoral returns introduces important welfare considerations. Diermeier, Keane, and Merlo (2005), Lim (2013), Aruoba, Drazen, and Vlaicu (2015) and Sieg and Yoon (2017) also estimate structural models of political choices to understand how electoral institutions, such as term limits, affect politicians' behavior. While these studies model many of the dynamic aspects of politicians' decisions, in our paper we account for them only through the incumbents' choice to run for reelection. But we complement these studies in two important ways. First, differently from these studies, politicians in our

[3]See Persson and Tabellini (2000) for a general review of the literature. The implications of different vote-counting schemes for candidate behavior is also reviewed in a 1995 JEP issue, see for example Levin and Nalebuff (1995).

model allocate public resources across regions that differ in the number of voters and demand for public funds. Second, we consider the interactions among the politicians' decisions by estimating a strategic game with incomplete information, whereas in those studies politicians make independent decisions.

The paper proceeds as follows. Section 2 provides background on Brazil's federal legislature and presents the reduced-form findings that motivate the model. Section 3 describes the model. Section 4 discusses our estimation approach and the identification of the model's parameters. Section 5 presents estimation results and policy simulations. Section 6 concludes the paper.

2 Background and Data

In this section, we discuss Brazil's political system, highlighting some of the institutional features that facilitate our analysis. We then describe our data, and present some of the patterns that guide our modeling choices.

2.1 Brazil's Federal Legislature

Brazil's federal legislature, also referred to as the Chamber of Deputies (we will use the terms "Deputy" and "Legislator" interchangeably), consists of 513 seats allocated across 26 states according to population size. Each state is a multi-member district. During elections, incumbents face competition from both new potential challengers, as well as from the other incumbents.

Nationwide elections for the legislature are held on a four-year cycle and incumbents can be reelected indefinitely. Candidates are elected based on a D'hondt open-list proportional representation method, which determines how the available seats are allocated. Specifically, seats are allocated to parties based on the total number of votes their candidates receive. Given a party's allotment of seats, candidates within the party are elected based on their vote total. Voting in Brazil is mandatory, and although citizens can vote for a political party, they almost always vote directly for a single candidate. It is also not unusual for several elected officials to change parties during their electoral terms. In the 49^{th} parliamentary session, for example, 55 percent of deputies switched parties during their term. With such a low degree of party loyalty, both from the standpoint of the politician as well as the electorate, party objectives tend not to play an important role in the allocation of public funds.

The primary responsibility of federal deputies is to allocate public funds. Brazil's legislature is comparatively weak and seldom legislates on issues of national concern (Ames 1995). As a

Federal Deputy from Ceará stated in the Brazilian newspaper *Folha de São Paulo* on February 21, 1988: "A political career in Brazil is closely connected to success in bringing home material benefits ... Especially in the poorest regions, communities judge their deputies on what they bring home". A similar opinion was expressed by Federal Deputy Joaquim Haickel: "The primary function of a deputy is getting resources; legislating comes second." (Mainwaring 2002).

Unlike other settings, access to these public funds is exogenous. Starting from 1996, the first year in our sample period, Brazil's constitution has granted a fixed yearly budget of BRL$1.5 million to each deputy.[4,5] In total, these budgetary amendments amount to 0.2 percent of total discretionary spending in a given year. Although this is a fairly small fraction of Brazil's GDP, these projects often represent an important injection of public goods for small and medium-sized states. For example, in Roraima the average municipality has a yearly budget of BRL$6.18 millions and received BRL$1.51 million in budgetary amendments.

2.2 Data

For this study, we combine administrative data from three sources. The budgetary amendment data come from the Chamber of Deputies. These data describe each budgetary amendment issued from 1996 to 2013, including the author's name, the amount, type, and location of the public investment. Based on the author's name, we merge these data with election results from the Tribunal Superior Eleitoral (TSE). The election data include vote totals for each candidate by municipality, along with various individual characteristics, including gender, education, occupation, and party affiliation. We use this information to construct our primary measure of political support – municipal vote share – as well as various other measures of electoral performance and competition, such as the candidate's rank and vote total. Our final data source comes from the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística (IBGE)). The 2000 and 2010 population censuses provide several socioeconomic and demographic characteristics such as poverty rates, income inequality, and population size.

In Table 1, we present summary statistics for three different samples. The sample used in column 1 is for Brazil's 50^{th} legislature, which issued budgetary amendments during 1996-1999 and faced reelection in 1998. We use this sample for the reduced-form analysis, presented below.

[4]In other settings, such as the U.S. Congress, access to public funds are endogenously determined by committee assignments and logrolling.

[5]The total amount has increased over time to adjust for inflation.

The restriction to one term is simply for convenience.[6] In column 3, we restrict the sample to the state used in the estimation, Roraima, as a point of comparison. The sample in column 4 is also only for Roraima, but covers the period 1996-2013. We use this sample to estimate our model.

From 1996-1999, federal deputies across Brazil issued, on average, approximately 16 budgetary amendments per year, totaling $1.3 million reais.[7] There is considerable geographic variation in the distribution of these public works (see Figure A.1). More than 10 percent of municipalities did not receive a single public work during the 1996-1999 term, with the median municipality only receiving BRL$280,000 per year in budgetary amendments. In contrast, the top one percent of municipalities receive BRL$10,000,000 per year.

Elections in Brazil are highly contested. For example, during the 1998 elections, over 6 candidates per seat competed and 68 percent of incumbents ran for reelection. Conditional on running, 72 percent were subsequently reelected.

Brazil has over 26 political parties, which resulted in about 1.4 political parties per seat during the 1998 elections. Despite Brazil's weak party system, with a proportional representation (PR) system, a small fraction of candidates will occasionally be elected because of their party. This occurs when a party has earned a sufficient number of seats to elect members from its list whose vote totals would not have been otherwise sufficient to elect them. In 1998, this occurred for 13 percent of the seats. Although this fraction is relatively small, it does create some uncertainty when candidates try to assess their ex-ante probabilities of winning. We will account for this type of electoral uncertainty in the model.

The average municipality within Roraima is comparable to the average municipality in Brazil in terms of municipal GDP per-capita, but is less populated. In terms of political environment, Roraima is similar to the average state.

[6] The match between the budgetary data and the election data has to be done by hand, which is why we limit the reduced-form analysis to a single term. The study of Brazil's budgetary amendments has a long tradition in the comparative politics literature. Since Ames (1995) classic study, there have been numerous empirical papers investigating the allocation of these budget appropriations and their electoral returns (e.g. Samuels (2003); Pereira and Renno (2003), Firpo, Ponczek, and Sanfelice (2012)).

[7] As mentioned previously, federal deputies were allowed outlays totaling up to 1.5 million per year. The limit is generally reached. However, in our analysis, we only consider outlays targeted to a municipality and exclude the ones that are designed to benefit either the state or the country as a whole.

2.3 Descriptive Evidence

Several of our modeling choices are guided by features observed in the data. In the model, we assume that voters care about the public funds they received and reward the politicians who supply them. We base these assumptions on two pieces of evidence. First, we observe a strong association across municipalities between the number of votes deputies receive and where they allocated their funds. We depict this association in Figure 1 using data from the 50^{th} legislature. In Panel (a), we see a clear positive relationship between the share of votes deputies received in a municipality during the 1998 elections and the amount of public funds they had allocated during the previous term. The simple correlation implies that a BRL$100,000 increase in the amount of public funds is associated with a 2.8 percentage point increase in a deputy's vote share.

The second piece of evidence is based on the following idea. Residents can only vote for a single candidate. So, if they care about public funds, they should be more likely to vote for the incumbent who transferred the largest amount, or one of the largest amounts, independent of the actual value. To test this hypothesis, we first compute how deputies rank within a municipality in terms of the amount of funds they had allocated. We then examine how this ranking relates to their vote share within the municipality. Panel (b) describes that relationship by plotting the coefficients from a regression of the share of votes within a municipality on a set of indicators that measure the deputy's ranking in the municipality. The regression controls for municipal fixed-effects. Deputies ranked first receive vote shares that are 27 percentage points higher than those ranked above 22 (the excluded category). The electoral returns to coming in second fall considerably, as second-place finishers only have a 7 percentage point advantage. The vote shares of deputies ranked third and above are statistically indistinguishable from the excluded category. These results indicate that when casting their ballots, voters prefer the candidate who provides them with the most public funds. Given the relationships depicted in panels (a) and (b), politicians have incentives to target municipalities with more voters. This hypothesis is consistent with what we see in panel (c): politicians do in fact allocate more money to places with more voters.[8]

Another key assumption in our model is that politicians who run for reelection allocate

[8]The correlations presented in Figure 1 are robust to unobserved deputy and municipal characteristics. Because each state is a multi-member district, we can estimate these correlations controlling for the fixed characteristics of the deputy (e.g. valence, education levels) as well as the fixed characteristics of the municipality (e.g. poverty levels, party affiliation). As we see in Table A.1, considering deputy and municipal fixed effects has little impact on the estimated relationship between electoral performance and public spending in the municipality.

resources differently from those who do not. We document evidence of this in panel (a) of Figure 2, where we plot the distribution of public funds by poverty levels for the two groups of incumbents. Panel (b) plots a similar figure using the municipality's human development index, which is a commonly used composite index of a country's (or in our case a municipality's) life expectancy, educational attainment, and income level. As both graphs indicate, incumbents who do not run for reelection are, on average, much more likely to target poorer and less developed municipalities, relative to those with electoral concerns. The graphs also suggest that politicians might care about the impact of their funds on the municipal welfare, given that they allocate a large share of their resources to poorer places, many of which have few voters.

In sum, the descriptive evidence suggests that deputies may have both electoral and non-electoral motives when allocating their public funds, and that politicians are rewarded at the polls based on whether voters receive these public resources. These patterns are present for Brazil as a whole, but also for Roraima in particular, as we will detail below when displaying the moments we use to estimate the model.

3 Model

Consider a model in which, in term t, J deputies decide how to allocate a fixed amount of resources $\bar{Q}$ among M municipalities and whether to run for reelection at the end of the term. Let $q^{j,m}$ denote the amount of resources deputy j allocates to municipality m, $q^j = \left\{q^{j,1}, \dots, q^{j,M}\right\}$ the collection of allocations chosen by deputy j, $q = \left\{q^1, \dots, q^J\right\}$ the allocations of all deputies, and $q^{-j} = \left\{q^1, \dots, q^{j-1}, q^{j+1}, \dots, q^J\right\}$ the allocations of all deputies except j. Finally, let $Q^m = \sum_{j=1}^{J} q^{j,m}$ denote the total amount of funds received by municipality m.

3.1 Preferences

Voters' Preferences. The resources voters receive from the J deputies, Q^m, fund a local public good. Voters have municipality-specific preferences over that good or, equivalently, over the allocated resources Q^m. Their preferences also depend on a variable K^m that accounts for all other factors that affect the residents' welfare. We represent these preferences with the welfare function $w_m\left(Q^m, K^m\right)$, where w_m is increasing and concave in Q^m. The total welfare of municipality m can then be calculated by multiplying w_m by the number of people living in the municipality, N^m:

$$W_m = N^m w_m\left(Q^m, K^m\right).$$

The welfare function varies across municipalities. Thus, it can account for productivity differences across municipalities in the use of public funds. With this feature, we can accommodate an environment in which politicians target places that have more voters because they are more productive and not just for political reasons.

The amount of public funds a municipality receives may not translate entirely into welfare gains. Deputies may decide to steal some of the funds. Additionally, the public good may not be fully non-rival, in the sense that the amount of welfare an individual derives from the public good may depend on the number of people who use it. For example, the benefits an individual derives from going to a park decline when more people visit it. Therefore, we redefine Q^m as the actual amount of resources used by municipality m to fund the local public good after taking into account the diversion of resources and rivalry concerns. Formally, we define Q^m as follows:

$$Q^m = \phi^{'} \left[\sum_{j=1}^{J_R} \phi^{'}_{R} q^{j,m} + \sum_{j=1}^{J_{NR}} \phi^{'}_{NR} q^{j,m} \right], \tag{1}$$

where the parameter $\phi^{'}_{R} \in [0,1]$ measures the fraction of total resources not stolen by the J_R deputies who run for reelection, the parameter $\phi^{'}_{NR} \in [0,1]$ represents the analogous fraction for the J_{NR} deputies who choose not to run, and the parameter $\phi^{'} \in [0,1]$ measures the degree of rivalry.

Because only two of the parameters in Equation (1) can be identified, we rewrite Q^m as follows:

$$Q^m = \phi^{'} \phi^{'}_{R} \left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \frac{\phi^{'}_{NR}}{\phi^{'}_{R}} q^{j,m} \right] = \phi \left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \phi_{NR} q^{j,m} \right], \tag{2}$$

where ϕ measures the degree of rivalry times the fraction actually transferred by incumbents who run and ϕ_{NR} measures the fraction of funds not diverted by incumbents who do not run relative to those who do.

To estimate the model, we will assume the following functional form for the individual welfare function:

$$w_m = \rho_m \log \left(y^m + Q^m \right),$$

where y_m is per-capita income of municipality m. Despite its parsimony, this welfare function captures several important features. With the logarithmic specification we can allow for decreasing returns. The municipality per-capita income y_m enables us to account for the existence of other goods and services that may affect the individual's welfare. Lastly, with the coefficient ρ_m

we account for two types of heterogeneity across municipalities: (i) productivity differences and (ii) differences in the preferences deputies might have for different regions. These two sources of heterogeneity cannot be separately identified without additional data on the productivity of projects located in different municipalities, which currently do not exist. This identification issue is a general result that is independent of our functional form assumption.

Deputies' Preferences. We first describe the preferences of deputies who choose to run for reelection. The preferences of deputies who forgo reelection can be derived as a special case. If deputies choose to run, their utility function is composed of four parts: (i) the expected utility from staying in power; (ii) the welfare of the people living in their state; (iii) the utility cost of running for reelection; and (iv) preference shocks.

Specifically, let v_p^j be deputy j's utility from being in power in the next term, v_{np}^j the utility if deputy j is not in power in the next term, $\bar{\bar{C}}_R$ the cost of running for reelection, ϵ_R^j a preference shock, and α^j the weight that deputy j assigns to the welfare component of the utility function. Lastly, let p^j denote the probability that deputy j wins the election at the end of the term, which will be derived in the next subsection. For a particular allocation of resources by all incumbents $q = \{q^1, \ldots, q^J\}$, deputy j's utility can be written as follows:

$$\bar{U}_R^j(q) = p^j(q)\, v_p^j + \left(1 - p^j(q)\right) v_{np}^j + \alpha^j \sum_{m=1}^{M} W_m - \bar{\bar{C}}_R + \epsilon_R^j\left(q^j\right).$$

The first part of the utility function, $p^j(q)\, v_p^j + (1 - p^j(q))\, v_{np}^j$, measures the expected utility of running for reelection and represents the egoistic motive of politician j. Provided that $v_p^j \geq v_{np}^j$ – the only case in which the deputy will choose to compete in the next election – this part captures the fact that politicians tend to allocate more resources to municipalities with higher electoral returns. The second part of the utility function, $\alpha^j \sum_m W_m$, describes the altruistic motive of a politician and it enables us to explain why politicians who run for reelection transfer a large fraction of their budget to poor municipalities with few voters.

To allow for sufficient flexibility, we assume that the preference shock ϵ_R^j comprises two parts. The first part depends on the allocation chosen by deputy j, which we will denote by $\bar{\varepsilon}_R^j(q)$. The second part, which we will denote by $\bar{\nu}_R^j$, does not vary by allocation, but is specific to the decision of running for reelection. Using simple calculations, we can then rewrite deputy j's utility in the following form:

$$\bar{U}_R^j(q) = p^j(q)\left(v_p^j - v_{np}^j\right) + \alpha^j \sum_{m=1}^{M} W_m - \bar{C}_R + \bar{\varepsilon}_R^j\left(q^j\right) + \bar{\nu}_R^j,$$

where $\bar{C}_R^j = \bar{\bar{C}}_R - v_{np}^j$. Because v_p^j and v_{np}^j do not vary with the allocation chosen by deputy j, we can divide the politician's utility by $\alpha^j + v_p^j - v_{np}^j$ and obtain

$$U_R^j(q) = (1-\beta_j)\, p^j(q) + \beta_j \sum_{m=1}^{M} W_m - C_R^j + \varepsilon_R^j\left(q^j\right) + \nu_R^j,$$

where $\beta_j = \frac{\alpha^j}{\alpha^j + v_p^j - v_{np}^j}$. We assume that $\varepsilon_R^j(q^j)$ and ν_R^j are independently distributed, with $\varepsilon_R^j(q^j) \sim N(0, \sigma_{\varepsilon,R}^2)$ and $\nu_R^j \sim N(0, \sigma_{\nu_R}^2)$.

From this alternative formulation of the politician's utility, we can clearly see the trade-off deputies face when choosing how to allocate their budget across municipalities. They can allocate their resources to increase the probability of remaining in power at the cost of reducing welfare or they can make a choice that favors welfare over the probability of winning the elections. How much a deputy is willing to trade off between these two considerations depends on the parameter β_j. We interpret β_j as the degree of altruism of deputy j. In the estimation, we assume that the degree of altruism can take on two values: β_L for egoistic politicians and β_H for altruistic politicians. The probability that a politician is altruistic will be denote by π_β.[9]

The utility function of a deputy who decides not to run has the same form as the utility of a politician who chooses to run, except that the probability of winning the elections and the cost of running are now equal to zero. Thus, it takes the following form:

$$\bar{U}_{NR}^j(q) = v_{np}^j + \alpha^j \sum_{m=1}^{M} W_m + \bar{\varepsilon}_{NR}^j\left(q^j\right) + \bar{\nu}_{NR}^j,$$

where $\bar{\varepsilon}_{NR}^j(q^j)$ and $\bar{\nu}_{NR}^j$ denote the corresponding preference shocks for deputies who do not run.

If we divide the utility by the same value used for a deputy who participates in the election,

[9]We have experimented with more than two degrees of altruism, but found that having two degrees was sufficient to explain the allocation patterns we observe in the data.

$\alpha^j + v_p^j - v_{np}^j$, we have a utility function that depends on the degree of altruism β_j:

$$U_{NR}^j(q) = \bar{v}_{np}^j + \beta_j \sum_{m=1}^{M} W_m + \varepsilon_{NR}^j\left(q^j\right) + \nu_{NR}^j,$$

where $\bar{v}_{np}^j = \frac{v_{np}^j}{\alpha^j + v_p^j - v_{np}^j}$, $\varepsilon_{NR}^j(q^j)$ and ν_{NR}^j are independently distributed, $\varepsilon_{NR}^j(q^j) \sim N(0, \sigma_{\varepsilon,NR}^2)$, and $\nu_{NR}^j \sim N(0, \sigma_{\nu_{NR}}^2)$. Here, political incentives do not play a role, and only welfare considerations affect the allocation decisions of deputies who do not run. This feature of the model allows us to generate the observed pattern that deputies who choose not to run are more likely to allocate resources to poorer municipalities with fewer votes. Even though these incumbents care only about welfare, they will not necessarily maximize aggregate municipal welfare because they can divert part of their funds.

Note that diverted funds only enter a deputy's utility through the welfare function. This simplification is without loss of generality. Because the fraction diverted does not vary across regions, the amount stolen is the constant $\left(1 - \phi_R'\right)\bar{Q}$ for deputies who run and $\left(1 - \phi_{NR}'\right)\bar{Q}$ for deputies who do not. Therefore, the diversion of resources only affects the allocation decisions through W_m.

3.2 Voting Decisions and Strategic Interactions

Voting Decisions. Citizens vote based on three factors: the amount of future resources they expect to receive from a given candidate; the candidate's ability to appeal to voters during the elections δ_j; and a voting preference shock, $\xi^{i,j,m}$.

Voters form expectations over future levels of public funds differently depending on whether a politician is an incumbent or a challenger. For incumbents, we assume that voters use past allocation decisions to predict the amount of funds they will receive in the next term.[10] Specifically, the amount $q^{j,m\,\prime}$ a voter in municipality m expects to receive from deputy j in the next term takes the following form:

$$E_j\left(q^{j,m\,\prime}\left|q^{j,m}, q^{-j,m}\right.\right) = f_j\left(q^{j,m}, q^{-j,m}\right).$$

In the estimation of the model, the expected allocation function f_j satisfies the following

[10] The data support this assumption. Conditional on deputy fixed-effects, a region that received public funds in the previous term is 45.8 (robust standard error = 0.011) percentage points more likely to receive public resources in the next term.

conditions. First, f_j is independent of $q^{-j,m}$ and linear in $q^{j,m}$.[11] Second, the constant term in f_j varies between incumbents (R) and challengers (C), but not within the two groups. Thus, we can account for possible incumbency effects in the estimation of the model. Third, the coefficient on the current allocation $q^{j,m}$ varies across municipalities. Consequently, conditional on $q^{j,m}$, voters will have different expectations about future allocations depending on where they reside. These conditions imply the following form for $f_j\left(q^{j,m}, q^{-j,m}\right)$:

$$f_j\left(q^{j,m}, q^{-j,m}\right) = \gamma_{0,j} + \gamma_{1,m} q^{j,m},$$

where $\gamma_{0,j} = \gamma_{0,R}$ if j is an incumbent and $\gamma_{0,j} = \gamma_{0,C}$ otherwise. We normalize $\gamma_{0,C} = 0$.

Since the J_C challengers are not in power during the current term, voters cannot condition on their previous decisions. Voters form their expectations by using the probability with which incumbents choose each feasible allocation in the current term. This assumption guarantees consistency of the deputies' choices across terms.

The voters' decisions also depend on the candidate's ability to appeal to voters during the elections, δ_j. We assume δ_j is revealed at the time of the elections. It is therefore known to voters when they cast their ballot, but not to deputies when they make their allocation decisions.[12] In the estimation, we assume δ_j can take two values: δ_H with probability π_δ for candidates with high electoral appeal and δ_L with probability $1 - \pi_\delta$ for candidates with low electoral appeal. We normalize $\delta_L = 0$. We therefore have four types of candidates: (i) high-altruism and high-appeal; (ii) high-altruism and low-appeal; (iii) low-altruism and high-appeal; and (iv) low-altruism and low-appeal. Finally, voters' choices are also affected by a preference shock $\xi^{i,j,m}$, which we assume is drawn from a type I extreme-value distribution.

We can now formalize the voters' decisions. Let $J_E = J_R + J_C$ denote the number of politicians running for office. Individual i in municipality m votes for politician j, if

$$j = \operatorname{argmax}_{j \in J} \left\{ f_1\left(q^{1,m}, q^{-1,m}\right) + \delta_1 + \xi^{i,1,m}, \ldots, f_J\left(q^{J_E,m}, q^{-J_E,m}\right) + \delta_{J_E} + \xi^{i,J_E,m} \right\}. \tag{3}$$

[11] We also experimented with an increasing and concave transformation of $q^{j,m}$ by taking the log of $q^{j,m}$, which resulted in similar findings. Also, from various explorations, we found that once we controlled for $q^{j,m}$, the other terms, such as the maximum amount of resources allocated by deputy j's opponents, did not predict subsequent allocations.

[12] Computationally, it makes little difference to assume that deputies observe the δ_j's of their rivals. We have opted for the alternative assumption because we believe that, for most politicians, the ability to connect with voters varies depending on the economic and social conditions. Therefore, it becomes public knowledge only during the electoral campaign. For instance, Bernie Sanders had a message that clearly resonated with voters during the 2016 Presidential Elections, but his message was not as effective in previous elections.

This voting decision is consistent with the residents' preferences. Because voters' welfare is increasing in public funds, it is optimal for residents to vote for the politician who is expected to transfer the largest amount of resources to their municipality, all else equal. With this voting rule, we can rationalize the relationship between vote shares and the candidate's ranking in terms of the amount allocated to a municipality, as highlighted in panel (b) of Figure 1.

Using Equation (3), we can calculate the total number of votes each candidate receives. Let ζ_j^i equal 1 if resident i plans to vote for candidate j and 0 otherwise. Also, let $\theta_j \sim U[0, \sigma_\theta]$ be a state-level voting shock that determines the share of politician j's supporters who abstain from voting, with $\sigma_\theta \leq 1$. This shock can be interpreted as the arrival of news about the candidate that leads his or her supporters not to vote. The total number of votes for politician j can then be computed as

$$nv(j) = (1 - \theta_j) \sum_{i=1}^{N} \zeta_j^i,$$

where N is the total number of citizens in the state.

As we discussed in Section 2, Brazil's PR system does not necessarily elect all of the top vote getters. This can happen when a candidate receives a lot of votes, but her or his party did not receive enough votes to earn a seat. In Roraima, which is represented by 8 deputies, one elected candidate per term was not ranked among the top 8 in terms of vote totals during our sample period. To account for these party effects in calculating the probability of winning an election, we first compute in the data the probabilities that a candidate loses a seat if ranked 1 through 8 based on the vote total $nv(j)$. Since these probabilities are similar for adjacent positions we compute three probabilities: the probability of losing a seat if ranked first or second; the corresponding probability if ranked third through sixth; and the analogous probability if positioned seventh or eight. These probabilities are: $p_{out} = [0.0, 0.0, 0.05, 0.05, 0.05, 0.05, 0.40, 0.40]$. We then compute the probability a candidate gains a seat if positioned 9 through J_E to obtain $p_{in} = [0.40, 0.40, 0.05, 0.05, 0.05, 0.05, 0.0, \ldots, 0.0]$. We then proceed in three steps: (i) we rank-order all the candidates based on their vote totals $nv(j)$; (ii) with the probabilities given by p_{out} we move one person out of the top 8 places, and replace this candidate with one person not in the top 8 according to the probabilities given by p_{in}; (iii) we use the new ranking to determine the probability that a candidate wins the election. Although this approach is reduced-form, it is consistent with the data and obviates the complications of having to add parties to the model, which would make the estimation infeasible.

Strategic Interactions. In our model, the choices of deputy j depend on the decisions of the other deputies. To deal with these strategic interactions, we make two assumptions. First, deputies decide simultaneously. Second, deputies do not observe the degree of altruism and electoral appeal of the other legislators. They only know the probabilities π_β and π_δ with which the types are independently drawn.

Given these assumptions, when deputies choose their optimal allocation q^j and whether to run d_R^j, they cannot calculate the optimal decisions of the other incumbents. Let $\sigma\left(q^h, d_R^h \,\middle|X^h\right)$ denote the probability with which incumbent h decides to run and allocate q^h given his characteristics X^h. Under the assumption that types are drawn independently from the same distribution, the probability that j's rivals choose the sequence of choices $q^{-j} = \left\{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\right\}$ and the sequence of decisions to participate $d_R^{-j} = \left\{d_R^1, \ldots, d_R^{j-1}, d_R^{j+1}, \ldots, d_R^J\right\}$ can be written as:

$$\sigma^{-j}(q^{-j}, d_R^{-j}|X^{-j}) = \Pi_{h\neq j}\sigma\left(q^h, d_R^h \,\middle|X^h\right).$$

3.3 Deputies' Optimal Decisions

Given the preferences, the voting rule, and the nature of the strategic interactions, we can describe the optimal decisions of the politicians. We proceed in two steps. We first determine their optimal allocation decisions conditional on whether they run for reelection. Given these choices, we then describe whether it is optimal to run.

Conditional on running, politician j chooses the allocation $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ that solves the following problem:

$$V_R^j\left(X^m, \beta_j\right) = \max_{q^j} \int \left[\left(1-\beta_j\right)p^j\left(q\right) + \beta_j \sum_{m=1}^{M} W_m\right] d\sigma^{-j}(q^{-j}, d_R^{-j}|X^{-j}) - C_R + \varepsilon_R^j\left(q^j\right) + \nu_R$$

$$s.t. \sum_{m=1}^{M} q^{j,m} \leq \bar{Q}, \tag{4}$$

where $V_R^j\left(X^m, \beta_j\right)$ denotes the value of running.

Conditional on not running, politician j chooses the allocation $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ that solves the following problem:

$$V_{NR}^{i}\left(X^{m},\beta_{j}\right)=\max_{q^{j}}\int\left[\bar{v}_{np}^{j}+\beta_{j}\sum_{m=1}^{M}W_{m}\right]d\sigma^{-j}(q^{-j},d_{R}^{-j}|X^{-j})+\varepsilon_{NR}^{j}\left(q^{j}\right)+\nu_{NR}$$
$$s.t.\sum_{m=1}^{M}q^{j,m}\leq\bar{Q}, \tag{5}$$

where $V_{NR}^{j}\left(X^{m},\beta_{j}\right)$ denotes the value of not running.

Deputy j will compete in the election if

$$V_{R}^{j}\left(X^{m},\beta_{j}\right)\geq V_{NR}^{j}\left(X^{m},\beta_{j}\right).$$

Timing and Equilibrium. The timing of the game is as follows: (i) nature privately reveals β_j and δ_j to the politicians; (ii) the deputies' preference shocks are realized and incumbents simultaneously decide their budget allocations and whether to run for reelection; (iii) the voting preference shocks, the electoral appeal of candidates, and the state-level shock are revealed to the voters, who then cast their vote.

The equilibrium that characterizes our model is a Bayesian-Nash equilibrium, which can be defined as follows.

Definition 1 *Allocations $q^{1*},\ldots,q^{J*}$ and the deputies' decisions to run for reelection $d_R^{1*},\ldots,d_R^{J*}$ are a Bayesian-Nash equilibrium if, for each deputy j, conditional on q^{-j*} and d_R^{-j*}, the decisions q^{j*} and d_R^{j*} maximize deputy j's expected utility.*

The following Proposition shows that, given our assumptions, a Bayesian-Nash equilibrium exists.

Proposition 1 *The model considered in this paper has a Bayesian-Nash equilibrium in mixed strategies.*

Proof. In the Appendix. ■

3.4 The Social Planner's Problem

Before we discuss how we estimate the model, it is useful to define the benchmark we will use to measure the size of the distortions. We define our benchmark as the allocation $Q^{sp}=$

$\{Q^1, \ldots, Q^M\}$ of aggregate funds $J \times \bar{Q}$ that maximizes aggregate welfare, i.e.

$$Q^{sp} = \underset{Q^1,\ldots,Q^M}{\arg\max} \sum_{m=1}^{M} W_m \tag{6}$$
$$s.t. \sum_{m}^{M} Q^m \leq \phi \left(J \times \bar{Q}\right),$$

where ϕ accounts for the degree of rivalry of the local public good. We will use deviations from this benchmark in the allocation of public funds as our measure of distortions.[13]

4 Model Estimation and Identification Discussion

4.1 Model Estimation.

To make the estimation of the model computationally tractable, we impose the following additional assumptions. First, as it is common in the estimation of games, we will assume that only one equilibrium is observed in the data (Draganska et al. 2008; Bajari et al. 2010).[14] Second, we discretize the provision of public goods into four choices. Specifically, a deputy can choose to give 0 percent, 33.33 percent, 66.66 percent, or 100 percent of their budget to a given municipality, subject to the constraint that the allocations must add up to BRL$1.5 million. Third, we aggregate Roraima's 15 municipalities into 4 macro-regions.[15] Region 1, which contains the capital city, is the wealthiest and most populated of the four regions. It has a population of 80,293 inhabitants and its GDP per capita is BRL$5,833. Region 2 is the least populated with 9,658 inhabitants, followed by region 4 (10,495) and region 3 (10,820). These other regions are similar in terms of their GDP per capita. Besides easing the computational burden, the aggregation is also important for mitigating the spillover effects that may arise from a municipality's public project benefiting the other surrounding municipalities.

The final simplification relates to the electoral appeal of politicians. To choose their optimal allocations, deputies have to compute the probability of winning the elections for any

[13] In the social planner's problem, the parameter ϕ measures the degree of rivalry only if the fraction of resources actually transferred by deputies who run, ϕ_R, is equal to one. If in reality $\phi_R < 1$, we underestimate the distortions because in this case the social planner can allocate more funds than the deputies.

[14] Bajari et al. (2010) argue that in static games with incomplete information, the number of equilibria tend to decrease as the number of actions and players increase. In their numerical example of a static entry game, they find that with only 5 players a unique equilibrium occurred in 93% of the models they considered.

[15] See Figure A.2 for a map of Roriama and its macro-regions.

possible combination of electoral appeal of all the other candidates, both incumbents and non-incumbents. With two types of electoral appeal, eight incumbents, and twenty challengers, allowing for this form of heterogeneity increases the number of iterations required to compute the optimal choices of each incumbent by a factor of $2^{(7+20)} = 134,217,728$. Thus, it is simply not feasible to estimate this model without additional assumptions.[16] To simplify the calculations, we assume that for a given set of parameter values, incumbents compute the probability of winning the elections using as the electoral appeal of their rivals the average $\pi_\delta \delta_H + (1 - \pi_\delta)\,\delta_L$. With this assumption we have to perform only one iteration for each type of incumbent.[17]

Despite these simplifying assumptions, the estimation of our model is nevertheless computationally demanding due to the strategic interactions among 8 players. To see why, consider that in each term we have 8 deputies. With 4 regions and 4 possible choices per region, each of the 8 deputies can select among 20 feasible allocations. To compute the expected utility of a single deputy, we have to consider for each one of the deputy's possible choices, all possible combinations of allocations by the seven rivals. In total, this involves computing $20^8 = 2.56e^{10}$ possible combinations. Even with the use of Fortran, MPI, and between 96 and 148 processors, a single iteration of the model takes on average more than 200 seconds.

We estimate the model's parameters by simulated method of moments (SMM) using data on allocation choices, the decision to run, and electoral outcomes. To compute the simulated moments, we need to iterate over the deputies' beliefs. We proceed as follows. For any given set of parameters, we simulate the deputies' decisions for an initial set of beliefs. Given these optimal decisions, we then compute the set of beliefs consistent with those decisions. If the distance between the two sets of beliefs is large, we re-simulate the model using as initial beliefs the newly generated beliefs. We repeat this procedure until the distance between the initial and simulated beliefs is sufficiently small. Once this fixed point has been achieved, we compute the simulated moments used in the estimation and compare them with the corresponding data moments. We compute the standard errors using the asymptotic distribution of the estimated parameters.

[16]Using Fortran and Message Passing Interface with 128 processors, without additional assumptions, it takes approximately 12 hours to solve the model for one set of parameter values.

[17]We can test how sensitive our results are to this assumption by computing the full problem at the final set of estimated parameters. We find that the differences are negligible and do not affect any of the main results of the paper.

4.2 Moments and Identification

We estimate a vector of 20 parameters. In this section, we discuss the identification of these parameters. Given the model's complexity, a formal proof of identification is not possible. Instead, we provide a heuristic argument for the variation we use to identify each one of the parameters.

Productivity Parameters: ρ_1, ρ_2, ρ_3, ρ_4. The welfare function consists of four parameters: ρ_1, ρ_2, ρ_3, and ρ_4. Because allocations must sum up to BRL\$1.5 million per deputy, we can only identify three out of the four parameters. We therefore normalize their sum to 1 and estimate only ρ_1, ρ_2, and ρ_3. In our model, deputies who do not run for reelection allocate resources based only on welfare considerations. Thus, we exploit their allocation decisions to identify the productivity parameters. Accordingly, we use as moments the average share of resources allocated to regions 1, 2, and 3 by incumbents who do not run.

In principle, incumbents who do not run may have other electoral motives that affect their allocation decisions. In practice, however, these motives are limited for the deputies of Roraima. Among the deputies who do not run for reelection, 65 percent have remained out of politics. The electoral motives for these deputies are likely to be nonexistent, or at best minimal. Of the remaining 35 percent, 85 percent sought an elected office in the capital city, such as vice mayor or vice governor, and 15 percent ran for the state legislature or for a federal seat in the Senate. These politicians do have electoral motives and, given their career choices, had an incentive to target region 1 where a majority of the voters reside and the capital city is located. Our estimate of the size of the political distortions should therefore be interpreted as a lower bound.

Another possibility is that deputies who choose not to run allocate more resources to their hometown for personal reasons, such as to fund projects that will benefit themselves and their neighbors. All the deputies from Roraima come from region 1. Thus, if these hometown motives are present, we again estimate a lower bound for political distortions.

Altruism Parameters: β_L, β_H, π_β. To identify the altruism parameters, we rely on differences in allocations between incumbents who run versus those who do not. To see how this variation identifies these parameters, suppose first that all deputies have the same degree of altruism. In this case, the difference in allocations between those who run and those who do not identifies the parameter β. If there is no difference, the degree of altruism will be identified to be 1. If deputies who run transfer a larger fraction of resources to municipalities with higher political gains, β will be less than 1 and will approach 0 as this difference increases.

With two degrees of altruism, the model can account for situations in which there are two distinct groups of deputies who run for reelection. The first allocates a larger fraction of their funds to regions with high political gains when compared to deputies who do not run. The second group also targets their funds to regions with high political gains, but to a lesser extent, again when compared to deputies who do not run.

The fraction of deputies with high degree of altruism π_β is identified by the difference between the average allocation that deputies who run give to the region with the highest political gain and the average of their allocations given to the remaining regions. If the difference is large the model requires a high fraction of egoistic deputies to rationalize the data, and a low fraction if the difference is small.

Based on this rationale, to identify the altruism parameters, we add to the three moments used in the identification of the welfare parameters, the average share of resources allocated to regions 1, 2, and 3 by incumbents who decide to run for reelection.

Voting Function Parameters: γ_0, $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, $\gamma_{1,4}$, σ_θ, δ, π_δ. To identify the incumbency effect γ_0, we use the difference between incumbents and challengers in the average probability of getting elected. A larger difference implies a higher value for γ_0, all else equal.

To identify the effect of public funds on votes $\gamma_{1,m}$, $m = 1, \ldots, 4$, we use two sets of moments: (i) the probability that an incumbent wins conditional on transferring at least 2/3 of their funds to region 1 and the corresponding probability conditional on allocating at least 2/3 of their funds to region m and zero to region 1, for $m = 2, \ldots, 4$;[18] (ii) the difference in the average share of resources allocated to region m, for $m = 1, 3, 4$, between reelected and non-reelected politicians.[19] Once we condition on the number of voters in region m and a deputy's electoral appeal, large values in both sets of moments indicate that funds allocated to that region translate into a high number of votes. We should therefore identify a larger $\gamma_{1,m}$.

The first set of moments is also used to identify the support of the district-specific shock σ_θ. If σ_θ is large then the electoral shock will play an important role in determining a deputy's probability of winning. In that case, we should expect to see small differences in the probability of winning between deputies who allocated most of their resources to region 1 and deputies who allocated most of their budget to the other regions. If instead, σ_θ is small, then transfers will

[18]For regions 2-4, we condition on transferring zero funds to region 1 in order to isolate the choices in which deputies allocated all of their resources to the poor regions with minimal political gains.

[19]The corresponding moment for region 2 is a linear combination of the moments for the other 3 regions.

have a larger impact on vote totals, and the difference in the probabilities will be larger.

The electoral appeal parameter δ is identified using the probability of winning conditional on allocating zero funds to region 1. If deputies who ignore the high political-return region have a significant probability of reelection, then the parameter measuring high electoral appeal should be large. Similarly, we can identify the proportion of deputies with high electoral appeal in the population π_δ using the fraction of incumbents who ran for reelection but did not allocate to region 1. If the fraction is large, then the proportion of incumbents with high appeal in the population should also be large.

Rivalry and Diversion of Resource Parameters: ϕ, ϕ_{NR}. To identify ϕ and ϕ_{NR}, we first compute the difference in per-capita GDP between the region with the highest per-capita GDP and every other region and the corresponding difference in the amount of funds received, conditional on running and not running. We then use how these two variables covary for deputies who run and those who do not to identify the two parameters. Specifically, after having computed the two variables, we calculate their ratio conditional on running and not running for each term and $m = 2, 3, 4$. We then derive the two moments used in the estimation by averaging the ratios over terms and $m = 2, 3, 4$, for deputies who run and do not run for reelection.

We use these moments because of the concavity of the welfare function. Deputies without electoral incentives will transfer public funds to the regions with lower per-capita GDP until its residents' marginal welfare equals the marginal welfare of residents of the region with the highest per-capita GDP. The same argument holds for deputies with electoral incentives and some degree of altruism, except that these deputies will only narrow the distance between the marginal welfare of the regions, where the distance depends on the relevance of the electoral incentives and the degree of altruism. Now consider the case in which ϕ is low, and hence there is a high degree of rivalry in Q_m. In this environment, conditional on the productivity parameters, the transfers to the regions with lower per-capita GDP have to be significantly larger to reduce the difference in marginal welfare with the region that has the highest GDP. If instead, ϕ is high, a smaller difference in transfers is sufficient to generate the required outcome. We can therefore pin down the parameter ϕ using the described variation in per-capita GDP and allocated funds.

The identification of the diversion parameter ϕ_{NR} requires a similar type of variation for incumbents who do not run. If these deputies divert more funds than incumbents who run, a small difference in per-capita GDP requires a larger difference between the funds transferred to region 1 and to region m, for $m = 2, \ldots, 4$, to equate marginal welfare.

Cost of Running Parameter and Variance Parameters: ν, σ_ν, $\sigma_{\varepsilon,R}$, $\sigma_{\varepsilon,NR}$. We identify the cost of running ν using the fraction of deputies who run for reelection: the higher the cost of running, the lower the fraction of deputies who choose to run.

To identify the variance of the shocks to the decision to run σ_ν, we use the probability of running conditional on allocating at least 2/3 of one's budget to region 1 and the probability of running conditional on allocating at least 2/3 of resources to region m, for $m = 2, 3$, and 0 to region 1. In our model, the decision to run for reelection depends mainly on two variables: (i) the allocation of funds, which determines the probability of winning a seat and, therefore, the expected benefits of running; and (ii) the size of the shock to the decision to run, which affects directly the utility value of running. If σ_ν is low, the running shocks are generally small and the decision to run is mostly affected by the allocation of funds. In this case, deputies who allocate most of their funds to region 1 have probabilities of running that are much larger than deputies who allocate to other regions. For higher values of σ_ν, the running shocks are generally large and the allocation decisions have smaller effects on the decision to run. The difference between the probability of running for deputies who allocate most of their resources to region 1 and the corresponding probability for deputies who allocate all of their resources to other regions is therefore smaller.

Finally, to identify the variances of the preference shocks $\sigma_{\varepsilon,R}$ and $\sigma_{\varepsilon,NR}$, we use the variances in allocations conditional on running and conditional on not running.

5 Results

5.1 Parameter Estimates

Productivity Parameters: ρ_1, ρ_2, ρ_3, ρ_4. Recall that the allocation decisions of deputies who forgo reelection identify our productivity parameters. We plot these allocations in Panel (a) of Figure 3. Region 1 receives only 10% of the funds, regions 2 and 3 receive more than twice that amount, and region 4 receives the largest proportion of funds at 40%. Our estimates follow a similar pattern: region 1 has the lowest productivity parameter at 0.032, regions 2 and 3 have significantly larger estimated productivity parameters at 0.274 and 0.220, and region 4 has the highest at 0.474 (see Table 2). With these estimates, our model matches the observed allocations quite well. As displayed in Panel (a) of Figure 3 and reported in the first three rows of Table 3, the largest difference between the simulated data (lighter bars) and the actual data (darker bars) is only 0.9 percentage points.

To provide an economic interpretation of their magnitudes, we compute the change in welfare generated by reallocating one dollar from the lowest productivity region (region 1) to one of the other three regions. The welfare effect from such a reallocation is 3.2 times larger for region 4 than for region 2 and 5.8 times larger for region 4 than for region 3.

Altruism Parameters: β_L, β_H, π_β. Conditional on the allocation decisions of incumbents who do not run, our altruism parameters are identified using the allocation decisions of those who do run. The darker bars in Panel (b) of Figure 3 plot these allocation decisions. Regions 1 and 4 receive the largest fractions of resources, 30% and 29% respectively. Region 2 receives 24% of the funds, and region 3 receives the lowest amount with only 17% of the funds. To match this U-shaped pattern, the model needs two groups of deputies with different degrees of altruism: (i) an egoistic group who cares almost exclusively about reelection incentives ($\beta_H = 0.054$), to explain the large fraction allocated to region 1; (ii) an altruistic group who cares both about altruism and electoral incentives ($\beta_L = 0.140$), to explain the large fraction allocated to region 4, which has the highest productivity, but fewer voters than region 1. Based on these estimates, an altruistic deputy is willing to substitute 1 vote for 63 dollars of welfare, whereas an egoistic deputy requires a welfare improvement of 181 dollars in order to trade off one vote.

We estimate that 29% of candidates are altruistic. This proportion mostly reflects the difference in the amount of funds region 1 receives relative to the other regions. In the data, region 1 receives about 30% of the funds, whereas the other three regions receive on average around 23%. If the difference of approximately 7 percentage points had been larger then, given our estimates of β_L and β_H, the model would have required a larger fraction of egoistic types in the candidate pool. With these parameter estimates, we match the U-shaped pattern in the data quite well. The largest difference between the simulated and actual data is only 2.8 percentage points, for region 3.

Voting Parameters: γ_0, $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, $\gamma_{1,4}$, σ_θ, δ, π_δ. On average, incumbents enjoy a 32.9 percentage point advantage over challengers in the probability of getting elected. Our model matches this difference reasonably well with $\gamma_0 = 0.017$. Based on this estimate, we can compute the incumbency advantage by calculating the average probability that a deputy wins reelection over all possible allocations in our model, and compare it to the same probability when $\gamma_0 = 0$. The incumbency advantage is substantial. When we set $\gamma_0 = 0$, the average probability of winning for an incumbent declines from 53.1 to 34.7 percent.

Our estimates of the effect of public funds on votes by region are: $\gamma_{1,1} = 0.071$, $\gamma_{1,2} = 0.236$,

$\gamma_{1,3} = 0.000$, and $\gamma_{1,4} = 0.234$. The mapping between these estimates and the moments we used to identify them is quite intuitive. In the data, the incumbents that allocate most of their budget to region 3, while ignoring region 1, are never re-elected; hence, an estimate for $\gamma_{1,3}$ that is statistically equal to zero.

A similar argument applies to the estimation of the parameters $\gamma_{1,1}, \gamma_{1,2}$, and $\gamma_{1,4}$ once we account for the number of voters in the regions. Region 1 has the highest number of voters. Thus, the model does not require a large coefficient for $\gamma_{1,1}$ to explain the high electoral return for funds allocated to that region. In contrast, regions 2 has only 1/8 of the population of region 1. And yet, if deputies allocate 2/3 of their budget to that region without allocating any resources to region 1, they still have a 25 percent chance of winning, which is slightly below 1/3 of the probability of winning if they trasfer all their resources to region 1. A similar argument applies to region 4, which also has only 1/8 of the population of region 1. To match those moments, our model requires $\gamma_{1,2}$ and $\gamma_{1,4}$ to be approximately 3.3 times the size of $\gamma_{1,1}$.

With these estimates, we also match relatively well the differences in allocations between the elected versus non-elected deputies. Analogously to the data, the model generates a large and positive difference for region 1, a negative difference for region 4, and a larger negative difference for region 3. The biggest discrepancy between the data and simulated moments (4.5 percentage points) is in region 3, due to the lower bound of zero imposed on $\gamma_{1,3}$.

To provide an economic interpretation of the parameters $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, and $\gamma_{1,4}$, we compute how the probability of getting elected changes as an incumbent shifts resources from one region to another. Region 1 is the most attractive in terms of electoral returns: if a deputy were to shift all of his resources from Region 3 (the least attractive) to Region 1, the probability of reelection would increase by 98 percentage points. By comparison, transferring those resources to Region 2 or Region 4 would increase the likelihood of reelection by 35 and 37 percentage points, respectively.

We estimate the upper bound of the district-level shock σ_θ to be 0.024. In the data, the difference between the probability of winning conditional on allocating at least 2/3 of one's budget to region 1 and the corresponding probability conditional on allocating at least 2/3 of the budget to region 3 and nothing to region 1 is quite large at 85.7 percentage points. This suggests that the allocation choices have large effects on voters' decisions and that the model does not need large district-level shocks to explain the observed patterns. Our estimates imply that politicians lose at most 2.4% of their votes because of the arrival of district-level shocks.

We estimate the electoral appeal parameter δ to be 0.004, which is about 1/4 of the size of the parameter that determines the incumbency effect γ_0. This is consistent with a relatively high observed probability of winning the elections if zero resources are allocated to region 1, which is equal to 20%. Our estimate of δ implies that individuals with high electoral appeal have a 6.6 percentage point advantage over low-appeal types in the probability of winning, all else equal. We also estimate that approximately 17.5 percent of politicians have high electoral appeal, which enables us to match well the fraction of deputies who choose to transfer zero funds to region 1 and run for reelection.

To see how electoral appeal and altruism affect the allocation of public funds in our model, we plot in Figure 4 the simulated share of funds allocated to each region by our four types of politicians, conditional on running. There is a stark difference between altruistic and egoistic politicians in the allocation of funds. Egoistic deputies allocate approximately 35 percent of their funds to region 1 compared to only 20 percent for altruistic politicians. In contrast, altruistic politicians give twice as much to region 3, where the electoral returns are the smallest. Both types of politicians give a substantial share of their funds to region 4, where the welfare impacts are the highest and the political gains are substantial. These differences in allocations across deputies with different degrees of altruism allow the model to explain the U-shaped pattern we observe in Panel (b) of Figure 3. The distinction between high and low electoral appeal types is significant but less pronounced. Given their electoral advantage, high appeal types can afford to allocate a smaller fraction of their funds to region 1 and shift some of their resources to region 4. This pattern is more pronounced for egoistic politicians, whose decisions are influenced to a larger degree by electoral incentives.

Rivalry and Diversion Parameters: ϕ, ϕ_{NR}. Recall that the rivalry parameter ϕ is identified by how the following two variables covary for deputies who run for reelection: (i) the difference in per-capita GDP between region 1 and any other region and (ii) the corresponding difference in allocated funds. In the data, this moment is negative and equal to -0.19, indicating that incumbents who run transfer more funds to the region with the highest per-capita GDP. This is to be expected since these deputies care about being elected and the region with the highest per-capita GDP also has the most votes. At the same time, this number is substantially closer to zero than it would be absent welfare considerations. To match this number, our model requires a $\phi = 0.008$, which indicates that for each dollar spent in a municipality slightly less than one cent of it represents a pure public good. This estimate implies that the allocated resources fund projects that have a high degree of non-rivalry. Given that the average population across municipalities in the state of Roraima is 27,817, for the funded projects to be a private good, ϕ would have to be equal to 0.000036 (\$1/27,817).

To identify the diversion parameter ϕ_{NR}, we employ the same variation used to identify ϕ, but for deputies who forgo reelection. In the data, this moment is equal to 0.158. A positive moment indicates that the region with the highest per-capita GDP receives fewer funds from them than poorer regions. This should be expected if incumbents who do not run have no or only limited electoral incentives, as we assume in the model. The moment is also not large, suggesting that the model needs a relatively small fraction of diverted funds to explain the data. We estimate $\phi_{NR} = 0.826$, which implies that incumbents who forgo reelection divert 17.4% more funds than those who run. Overall, the estimated ϕ_{NR} highlights an important tradeoff: although deputies who do not run place more weight on welfare considerations, they also divert more resources. We explore this tradeoff further when we consider the policy of a one-term limit.

With the estimated ϕ and ϕ_{NR}, we match relatively well the data moment for deputies who forgo reelection, which is just below the value obtained using the simulated data. The model has more difficulties matching the data moment for incumbents who run for reelection. As with the data moment, the simulated moment is positive and large. But its value is about 32% lower than what we observe in the data. To better match this moment, the model would have to increase the share of resources allocated to the region with high GDP, region 1, for incumbents who run for reelection. But the share allocated to this region by these politicians in the model is already high (0.316) relative to the value observed in the data (0.3).

Cost of Running and Variances: ν, σ_ν, $\sigma_{\varepsilon,R}$, $\sigma_{\varepsilon,NR}$. We estimate the cost of running parameter and the variance parameters as a fraction of the value of running to simplify their interpretation. The cost of running parameter is identified using the fraction of deputies who run for reelection. In the data, 71% of incumbents run for reelection. To match this moment, we estimate a cost of running equal to about 0.9% of the total utility value of running. With this estimated parameter we match the data moment reasonably well: our simulated fraction of incumbents who choose to run is 68%. The estimated variances are all relatively small, with the largest one being equal to 3.7% of the utility value of running. This suggests that the shocks in our model are not major factors in explaining the variation we see in politicians' decisions.

Specification Tests. In Figure 5, we test how well our model matches data moments not used in the estimation. We consider the following 11 moments: the probability of running conditional on allocating at least 1/3 of the budget to region m, for $m = 1, \ldots, 4$; the probability of winning conditional on allocating at least 1/3 of the budget to region m, for $m = 1, \ldots, 4$; the share of funds allocated to region m by deputies who ran but did not win, for $m = 1, 2, 3$.[20]

Despite its parsimony, the model matches these additional moments reasonably well. Out of the 11 moments, only 3 cases have a difference larger than 5 percentage points between the simulated and data moments. The model struggles with the probability of running conditional on allocating at least 1/3 to region 3 (too low), the probability of winning for region 1 (too low), and the probability of winning for region 3 (too low). But for the last two moments, the model is still able to account for the ranking across regions.

5.2 Political Distortions and Policy Evaluations

To what extent do political incentives affect the allocation of public funds? To answer this question, we compare the distribution of public funds generated by the estimated model to the social planner's allocation, as defined in problem (6). Note that the social planner's allocation differs from the allocations of deputies who do not run. Unlike the social planner's allocation, both stealing and the beliefs over the actions of the other incumbents affect the decisions of deputies who forgo reelection.

Figure 6 plots the allocation of public funds relative to the social planner's allocation. We estimate that 25% of the allocations are distorted compared to the social optimum. All of the distortions occur towards region 1 and away from the other poorer, less-populated regions. Two factors cause these distortions. The first is electoral incentives, which induce politicians to target the region that has the most votes, but is also the least productive. The second is the diversion of public resources. To determine how much these factors contribute to the total amount of distortions, we simulate the estimated model under the assumption that incumbents cannot run for reelection. We therefore eliminate electoral incentives, but preserve the diversion of resources. In this case, the distortions drop to 9%, indicating that electoral incentives explain 64% of the total and the rest is due to corruption.[21]

Given the central role that electoral incentives play in distorting the allocation of public funds, an important question is whether changes to the electoral rules can help reduce these distortions. We focus on two alternative electoral rules that have the potential of reducing the electoral distortions: approval voting and a one-term-limit policy. For each policy simulation, we

[20] Because of the budget constraint, only 3 out of the 4 moments are independent.

[21] To confirm that corruption accounts for the residual part of the distortions, we have also simulated the model under the assumption that deputies cannot run for reelection and the fraction of resources they can divert, $1 - \phi_{NR}$, is equal to zero. In this environment, the distortions go to zero.

recompute the deputies' beliefs to be consistent with their choices under the new environment. We do this by iterating over the beliefs until convergence.

Approval Voting

Under Brazil's current system, residents can only vote for a single candidate. This system is commonly known as plurality voting. Approval voting, by contrast, allows citizens to vote for multiple candidates. This alternative system has been proposed by economists and political scientists as an improvement over plurality voting (see Laslier and Sanver (2010) for a series of papers discussing the properties and virtues of approval voting). For example, approval voting encourages voters to vote sincerely and it produces electoral outcomes that better represent voters' preferences. Moreover, approval voting generally assures victory to candidates with the greatest overall support, which is not the case with the standard plurality voting system (Brams and Fishburn 2005).[22] Our paper is the first attempt to evaluate whether it also has the additional benefit of reducing the distortions created by electoral incentives.

To formally describe approval voting, consider J candidates who compete in the elections over S seats. Approval voting with residents that are allowed to vote for K candidates can be defined as a vector of scores $(s_1, \ldots, s_J)$, with $s_j = 1$ for $j \leq K$ and $s_j = 0$ for $j > K$. The S seats are won by the politicians with the S highest total scores. Plurality voting is a special case of approval voting in which $s_1 = 1$ and $s_j = 0$ for $j > 1$. In our simulations, we compare plurality voting with an approval voting system in which residents can vote for K candidates with $2 \leq K \leq 6$.[23]

In Figure 7, we report the simulation results. We plot the allocation of public funds across the four regions generated by the various approval voting rules relative to the social planner's allocation. Based on these simulations, we find that the deviations from the social optimum decrease as we allow voters to rank more candidates. For instance, if the government was to adopt a 6-person rule, the distortions in the allocation of public funds are predicted to decrease by 3.3 percent.

Although these reductions are significant, our findings raise the question as to why approval

[22] Ahn and Oliveros (2016) show that in elections with common values, approval voting outperforms other scoring rules including plurality voting and Borda count in aggregating information.

[23] In addition to finding a fixed point over beliefs, approval voting requires the computation of an exploded logit for each simulation in order to determine the total number of votes each candidate receives. As we increase the number of candidates a voter can list, the computation of the exploded logit becomes exponentially more burdensome. Due to these computational constraints, we are only able to simulate the model up to a 6-person rule, which requires 5 days to be completed using MPI, Fortran, and 128 processors.

voting does not reduce the distortions even further. The answer is that there are two countervailing forces at play. On the one hand, approval voting provides an incentive for politicians to reallocate their funds away from region 1. In contrast to plurality voting, politicians receive votes even if voters do not rank them at the top. Thus, the necessity to be ranked first in region 1 diminishes, allowing politicians to allocate more funds to other regions.

On the other hand, this policy makes the elections more competitive, which can be seen in Figure 8. As we increase the number of candidates voters can rank, the probability that weaker candidates – typically challengers – win a seat increases substantially. Under the current system, the probability that a challenger wins an election is only 13.7 percent. But going from a 1-person rule to a 6-person rule increases that probability by 24.6 percent. As the elections become more competitive, incumbents want to target region 1 more, which increases distortions. This second effect limits the efficacy of approval voting to further reduce distortions in the allocation of public funds.

The results depicted in Figure 8 illustrate another important effect of approval voting. In our model, voters believe that challengers will distribute public funds similarly to the current incumbents. Because of this consistency, the major difference between incumbents and challengers in the probability of winning is the incumbency advantage. The fact that challengers win more, as we increase the number of candidates voters can vote for, indicates that approval voting can be effective in reducing the incumbency advantage. It does so, however, at the unexpected cost of increasing electoral distortions.

Selection. A key assumption underlying the simulation of the approval-voting policy is that the parameters of the model are invariant to the electoral reforms. Because we do not model the decision to become a politician, one might be concerned that the proportion of types in the candidate pool might vary with changes to the electoral rules. In particular, since approval voting improves the electoral chances of politicians with lower probabilities of winning a seat, one might expect an increase in the proportion of politicians who are either altruistic or have low electoral appeal. The extent to which this selection concern affects our policy results depends on two opposing forces. On the one hand, if approval voting induces the entry of more altruistic types, we would expect the policy to reduce distortions even further. On the other hand, approval voting may also induce those with low electoral appeal to participate more frequently in the elections. Since these politicians allocate relatively more resources to region 1, as reported in Figure 4, we would expect this type of selection to increase the distortions.

In Figure 9, we document the robustness of our policy simulations to increases in the proportion of candidates who are either altruistic or of low electoral appeal. For both the 3-person and 5-person electoral rules, we measure the distortions in the allocation of public funds (y-axis), as we increase by 2 percentage points at a time the proportion of both altruistic and low-appeal types in the pool of candidates (x-axis). In both cases, we observe a steady decrease in the misallocation of public funds as the representation of those politicians in the candidate pool increases. For example, if the adoption of approval voting leads to a 10 percentage point increase in both altruistic and low-appeal candidates, a 5-person scoring rule would decrease distortions by 4.9 percent compared to a 2.2 percent decline for the base case. These results indicate that, if selection is an issue, our policy simulations under-estimate the impact of approval voting by not fully accounting for the effects of the policy on candidate entry.

Strategic Voting. We have also assumed that citizens vote sincerely for the candidates from whom they expect to receive the most public funds. This is a reasonable assumption given our reduced-form finding that voters only reward politicians who transfer the largest or second largest amount of resources to their municipality (see Figure 1). If strategic voting was important, we would expect to see a significant share of votes for candidates who do not transfer a substantial fraction of resources to the municipality. Nevertheless, using the estimated model, we can assess the robustness of our approval-voting results to the possibility that some residents vote strategically. In Figure 10, we report the size of the distortions computed by re-simulating the base case and the approval-voting policies for the 3-person and 5-person scoring rules under the assumption that residents do not vote for candidates with less than a 10 percent probability of winning the election, and instead cast their votes in favor of one of the remaining candidates.[24] Figure 10 documents that, in the base case, the distortions decrease slightly from 25% to 24% with strategic voting, as weaker contenders are eliminated. Because these candidates tend to be on average the challengers, strategic voting generally increases the incumbency effect. Incumbents can therefore allocate a larger share of their public funds away from Region 1. Figure 10 also reveals that the effects of the approval-voting policies are similar to those obtained without strategic voting. For instance, the adoption of a 5-person rule would reduce distortions by 1.1% in the presence of strategic voting.[25]

[24]As is common in the political science literature, we define strategic voting as the act of choosing a less-preferred candidate due to the fact that one's most preferred option has little to no chance of winning (e.g. Riker and Ordeshook (1968).

[25]The assumption of a 10 percent threshold is not critical for the results. We have experimented with various thresholds ranging from 5 to 20 percent and have found similar results.

Term Limits

The second policy we simulate limits incumbents to a single term. Brazil currently allows deputies to be elected indefinitely, but several countries have argued for, and in some cases implemented, term limits as a way to improve representation. It is noteworthy that the Chamber of Deputies recently voted in favor of this policy for some of Brazil's executive branches.[26]

In standard political economy models, single term limits are never optimal compared to either multiple term limits or no term limits (e.g. Smart and Sturm (2013)). In these models, politicians only care about private gains and the possibility of reelection gives them the incentives to choose policies in accordance with the voters' preferences. In our model, incumbents also care about aggregate welfare, which creates the following interesting tradeoff between having no term limits and a limit of a single term. On the one hand, a single term limit should reduce distortions because electoral incentives would no longer influence the way public funds are allocated. On the other hand, deputies who do not run divert significantly more funds than deputies who still face reelection incentives. The ability to determine which of these two effects dominates is an important contribution of our model. When we compare the results of our model to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions decrease from 25 to 9 percent. However, because of the increase in corruption, welfare as a whole goes down by 1.9 percent, indicating that approval voting should be favored over a one-term-limit policy.

6 Conclusions

In this paper, we have developed and estimated a model of how politicians allocate public funds in an environment in which incumbents play a strategic game. We have applied the model to the Brazilian context, where federal legislators are entitled to approximately R$1.5 million per year to fund projects within their state. Based on the budgetary allocation decisions of federal legislators representing the state of Roraima, our estimates suggest that electoral incentives distort the allocation of 25 percent of the value of these public funds. These distortions are driven mainly by the behavior of non-altruistic politicians, who put relatively little weight on the welfare of municipalities, and thus target less productive regions with more voters. We also

[26]On 5/27/2015, the Chamber of Deputies voted to eliminate reelection for the office of President, Governor, and Mayor. For more information see: http://noticias.uol.com.br/politica/ultimas-noticias/2015/05/27/camara-vota-o-fim-da-reeleicao.htm

find that while approval voting does reduce the distortions associated with electoral incentives, the effects can be muted because of two countervailing forces: approval voting reduces the incentives politicians have to target regions with more voters, but it also reduces their incumbency advantage, which in turn heightens the electoral-incentive effects.

Although our model captures tradeoffs common to politicians throughout the world, one should be careful to extrapolate our empirical findings to other settings. There is a lot of regional inequality in Roraima, which can deepen the tradeoff between targeting places with more voters versus places in higher need of public goods. Our estimated distortions would likely be lower in states where this tradeoff is less pronounced. We should also not necessarily expect the same level of distortions in the allocation of different types of public funds. Given that the primary responsibility of federal legislators is the provision of public works, the electoral incentives underlying their allocations are particularly acute.

These caveats aside, our study highlights the importance of political institutions for the allocation of public expenditures and the type of distortions that can arise when the incentives that affect politicians' decisions are different from those that characterize a social planner. Our paper is also one of the first attempts to determine whether changes to the electoral rule can help align these incentives. Our results suggest that some emphasis should be placed on attracting better types of politicians (Dal Bó, Finan, and Rossi (2013); Ferraz and Finan (2009)). But more work is needed in this area of research.

Although our model fits the data relatively well, it is quite parsimonious and can be extended and generalized in several interesting directions. One possible extension would be to make the game dynamic. As Diermeier, Keane, and Merlo (2005) correctly emphasize, politicians are forward-looking agents whose career choices are dynamic in nature. We model these intertemporal aspects to some degree through the decision to run for reelection, but it would be interesting to model the dynamic dimension more explicitly, for instance by taking into consideration that politicians can run for reelection for multiple terms. Other directions of future research will ultimately depend on the collection of new data. For example, with data on campaign spending, one could easily extend our model to examine whether budgetary amendments complement or substitute campaigning (Kang (2016), Da Silveira and De Mello (2011)). One could then investigate the impact of campaign financing laws on not only electoral performance, but also public funds allocation.

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7 Tables and Figures

Table 1: Summary Statistics

	Brazil 1996-1999		Roraima 1996-1999	Roraima 1996-2013
	Mean	S.D.	Mean	Mean
Budgetary Amendments				
Amount per deputy per year (millions)	1.32	2.47	1.21	1.17
Number issued per deputy per year	16.13	5.32	12.93	8.98[a]
Average number of candidates per seat	6.35	1.71	4.75	8.25
Number of parties per seat	1.44	0.71	1.75	2.50
Elected by the party	0.13	0.33	0.13	0.13
Share of incumbents who ran for reelection	0.68	0.18	1.00	0.71
Reelection rate	0.72	0.15	0.75	0.58
Average municipal GDP per capita	3,643.76	4,030.76	4,335.06	4,752.32
Average population	30,111.42	180,597.60	21,418.00	24,560.00

Notes: Columns 1 and 2 report summary statistics averaged across states for the period 1996-1999 (i.e. 50^{th} legislature). Column 3 reports summary statistics for the state Roraima during the same period. Column 4 reports summary statistics for the state of Roraima during the period 1996-2013, which is the period used in the estimation. Four legislative elections took place during this period: 1998, 2002, 2006, 2010.
[a] In column 4, we compute the average number of amendments issued for the period 1996-2003. After 2003, we only have data on the total amount issued by deputy per year.

Table 2: Parameter Estimates

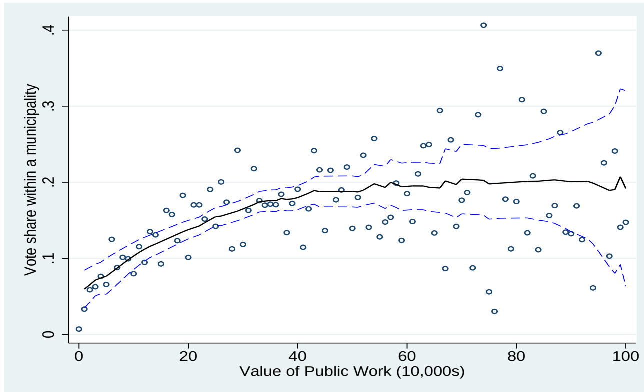
	Parameter	Estimate	Std Err
	Welfare Function		
Productivity in region 1	ρ_1	0.032	0.015
Productivity in region 2	ρ_2	0.274	0.020
Productivity in region 3	ρ_3	0.220	0.028
Productivity in region 4	ρ_4	0.474	0.046
Degree of rivalry	ϕ	0.008	0.003
Diversion of funds if not running	ϕ_{NR}	0.826	0.072
Egoistic type	β_H	0.054	0.004
Altruistic type	β_L	0.140	0.013
Proportion of altruistic politicians	π_β	0.292	0.030
	Voting Decisions		
Incumbency advantage	γ_0	0.017	0.001
Effects of public funds on vote shares, region 1	$\gamma_{1,1}$	0.071	0.005
Effects of public funds on vote shares, region 2	$\gamma_{1,2}$	0.236	0.025
Effects of public funds on vote shares, region 3	$\gamma_{1,3}$	0.00001	0.0001
Effects of public funds on vote shares, region 4	$\gamma_{1,4}$	0.234	0.014
High electoral appeal	δ	0.004	0.002
Proportion of high appeal politicians	π_δ	0.175	0.103
Upper bound of support of state-level shock	σ_θ	0.024	0.002
	Decision to Run and Shocks		
Cost of running	ν_L	0.009	0.001
Std. dev. of shocks on decision to run	σ_ν	0.037	0.003
Std. dev. of preference shocks if running	$\sigma_{\varepsilon,R}$	0.026	0.001
Std. dev. of preference shocks if not running	$\sigma_{\varepsilon,NR}$	0.015	0.001

Notes: This table presents the model's parameter estimates. The standard errors are computed using the asymptotic distribution of the estimated parameters.

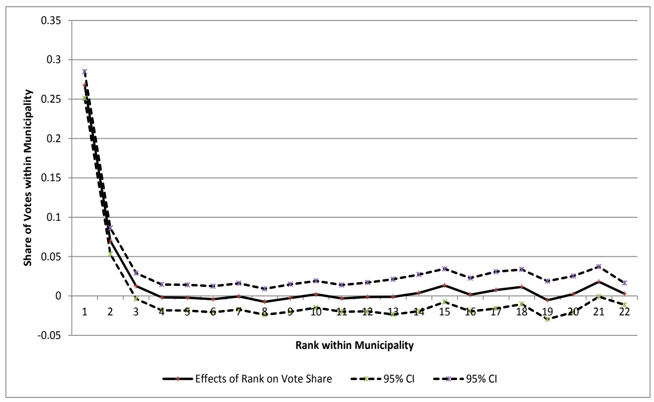
Table 3: Moments Used in the Estimation

Moments	Model (1)	Data (2)
Welfare Function		
Share allocated to region 1 if not running	0.106	0.104
Share allocated to region 2 if not running	0.234	0.229
Share allocated to region 3 if not running	0.259	0.250
Share allocated to region 1 if running	0.316	0.300
Share allocated to region 2 if running	0.219	0.242
Share allocated to region 3 if running	0.139	0.167
Voting Function		
Pr(Winning) for incumbents - Pr(Winning) for challengers	0.378	0.329
Pr(Winning) if share allocated $\geq 2/3$ to region 1	0.868	0.857
Pr(Winning) if share allocated $\geq 2/3$ to region 2 & 0 to region 1	0.276	0.250
Pr(Winning) if share allocated $\geq 2/3$ to region 3 & 0 to region 1	0.022	0.000
Pr(Winning) if share allocated $\geq 2/3$ to region 4 & 0 to region 1	0.302	0.333
Difference between winners and losers in share allocated to region 1	0.169	0.200
Difference between winners and losers in share allocated to region 3	-0.078	-0.033
Difference between winners and losers in share allocated to region 4	-0.037	-0.025
Covariance between difference in allocations and GDP if running	-0.130	-0.190
Covariance between difference in allocations and GDP if not running	0.148	0.158
Pr(Winning) if share allocated to region 1 = 0	0.234	0.200
Pr(Running) if share allocated to region 1 = 0	0.533	0.556
Decision to Run and Shocks		
Pr(Running)	0.679	0.714
Pr(Running) if share allocated $\geq 2/3$ to region 1	0.924	0.875
Pr(Running) if share allocated $\geq 2/3$ to region 2 & 0 to region 1	0.593	0.571
Pr(Running) if share allocated $\geq 2/3$ to region 3 & 0 to region 1	0.434	0.333
Variance of allocation shocks if running	0.006	0.003
Variance of allocation shocks if not running	0.011	0.012

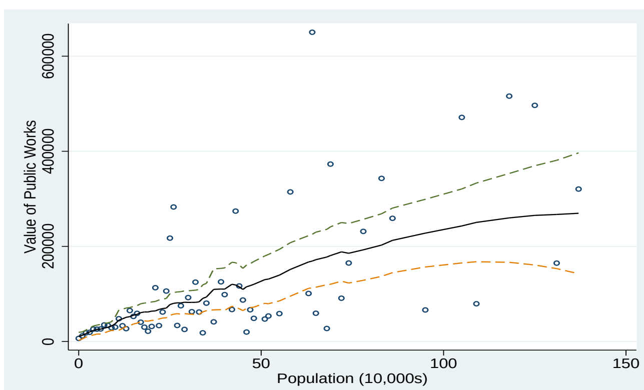
Notes: This table presents the moments used to estimate the model's parameter. Column 1 reports simulated moments based on 5,000 simulations. Column 2 reports the data moments.



(a) Public Funds and Share of Votes



(b) Vote Share and Rank by Amount of Public Funds



(c) Public Funds and Population Size

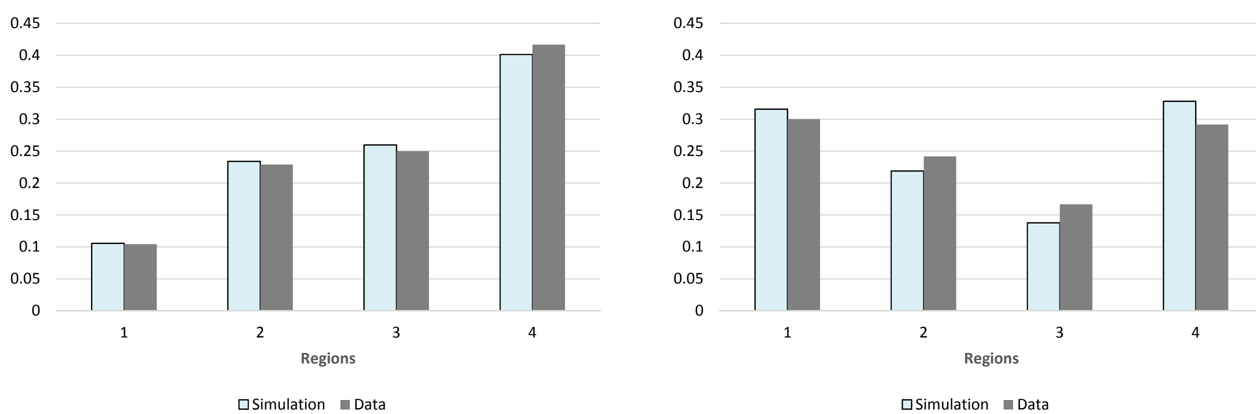
Figure 1: Public Expenditures and Outcomes

Notes: Panel A depicts the association between the amount of public funds deputies allocated to the municipality during the 1996-1999 term and the share of votes they received in 1998 elections. Panel B plots coefficient estimates from a regression of the share of votes an incumbent received in a municipality and a set of dummies indicating the incumbent's rank within the municipality based on spending. Incumbents ranked above 22 are the excluded category. Panel C depicts the association between population size and the amount of public funds a deputy allocated to the municipality during the 1996-1999 term. The solid line was computed using lowess. The dashed lines are the corresponding 95 percent confidence intervals. Each dot represents the mean of the dependent variable computed based on equally-sized bins.

(a) **(b)**

Figure 2: Distribution of Public Funds by Welfare Levels

Notes: Panel A depicts kernel density plots of the allocation public funds by poverty level of the municipality. Panel B depicts kernel density plots of the allocation public funds by the municipality's Human Development Index. These density plots are estimated separately for incumbents who ran for reelection and those that did not. These figures were computed based on a sample of 5,550 municipalities.



(a) Did not run for reelection **(b)** Ran for reelection

Figure 3: Comparison Between Model's Prediction and Actual Allocations

Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. Panel A plots by region, the share of public funds allocated by incumbents who did not run for reelection. Panel B plots by region, the share of public funds allocated by incumbents who ran reelection. The simulated allocations are based on 5,000 simulations.

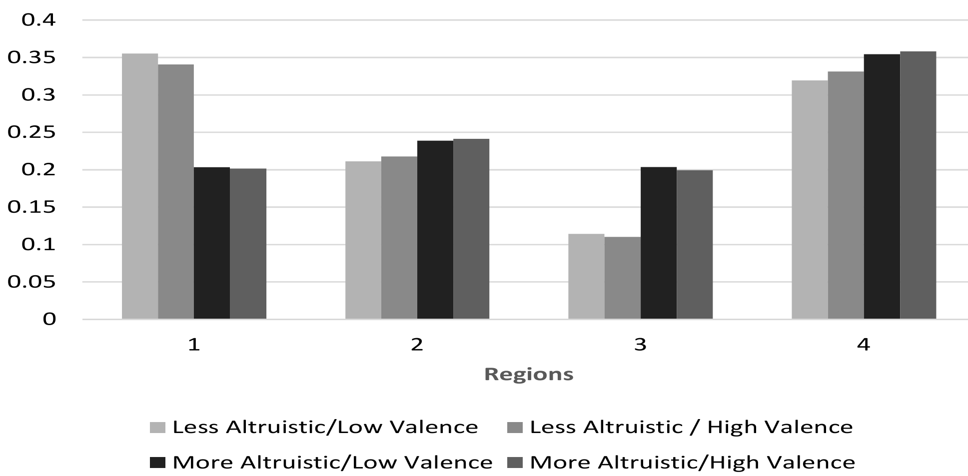
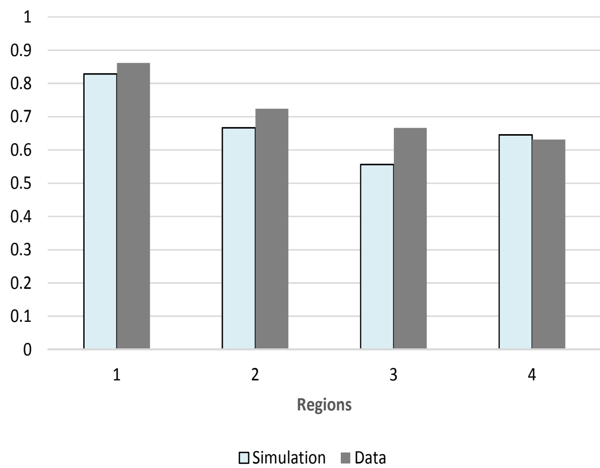
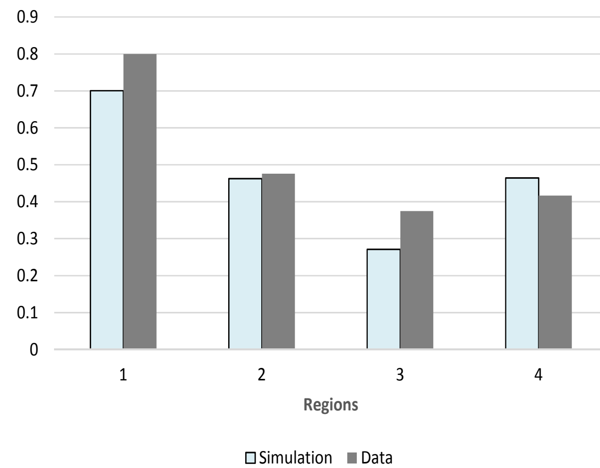


Figure 4: Distribution of Allocations By Politician Type

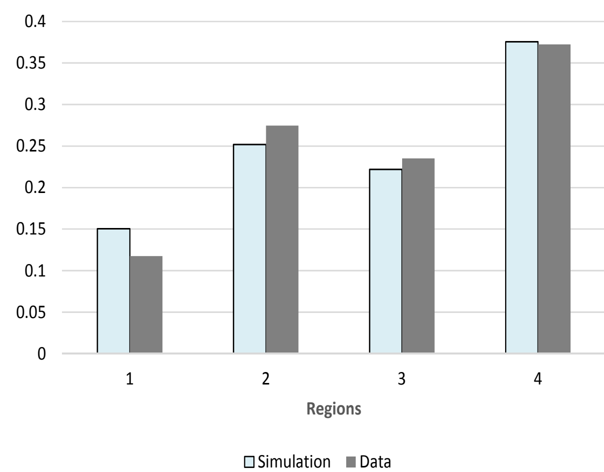
Notes: This figure plots the share of public funds allocated by incumbents who ran reelection, by politician type. The simulated allocations are based on 5,000 simulations.



(a) $Pr(\text{Running}|\text{Allocation} \geq 1/3)$



(b) $Pr(\text{Winning}|\text{Allocation} \geq 1/3)$



(c) Allocations among those who ran but were not elected

Figure 5: Comparison Between Model's Prediction and Actual Allocations - Moments not Matched

Notes: Panel A plots the probability of running conditional on allocating more than 1/3 of one's budget to a particular region. Panel B plots the probability of winning conditional on allocating more than 1/3 of one's budget to a particular region. Panel C plots the allocation decisions of incumbent who ran but were not elected.

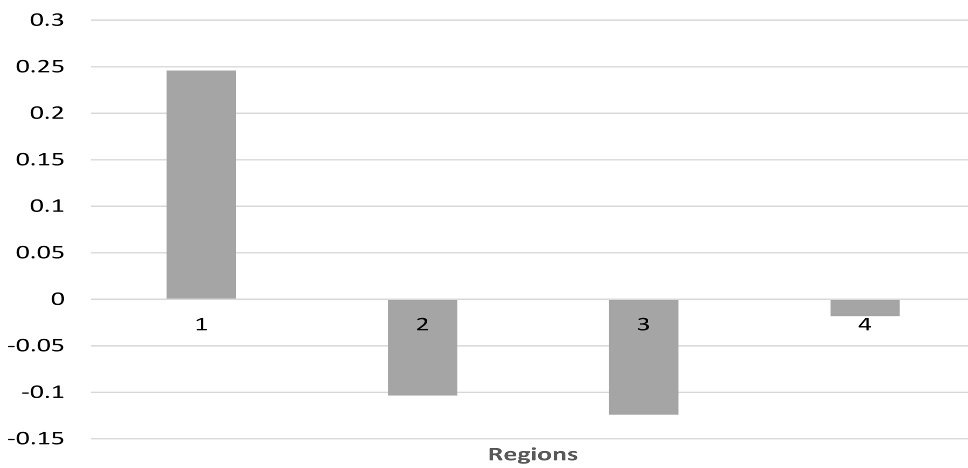


Figure 6: Deviation from the Social Planner

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations.

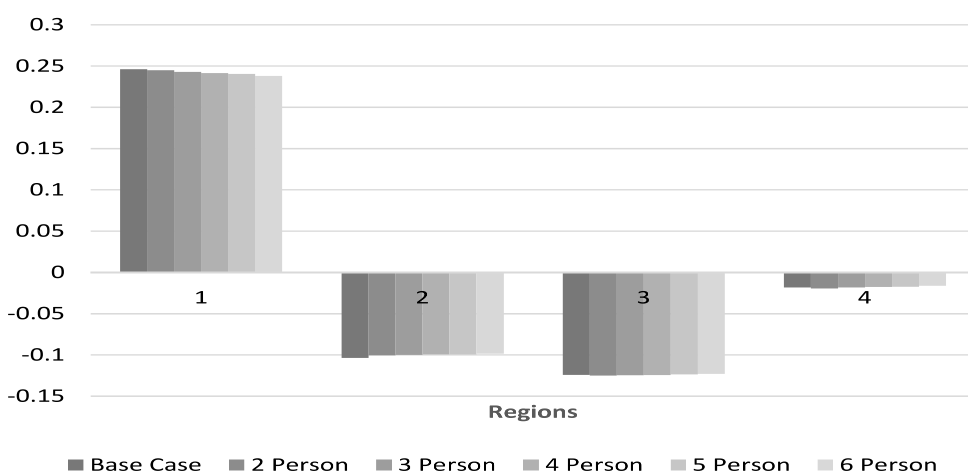


Figure 7: Policy Simulations: Approval Voting

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations. The 2-person policy refers to the scoring rule: $(1, 1, 0, \ldots, 0)$. The 3-person policy refers to the scoring rule: $(1, 1, 1, 0, 0, \ldots, 0)$, etc. The base case policy refers to our original results.

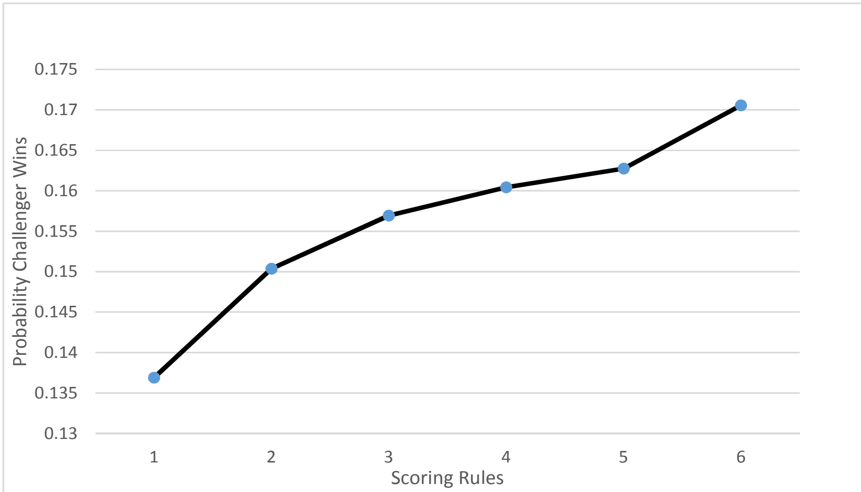


Figure 8: Probability a Challenger Wins

Notes: See note in Figure 7.

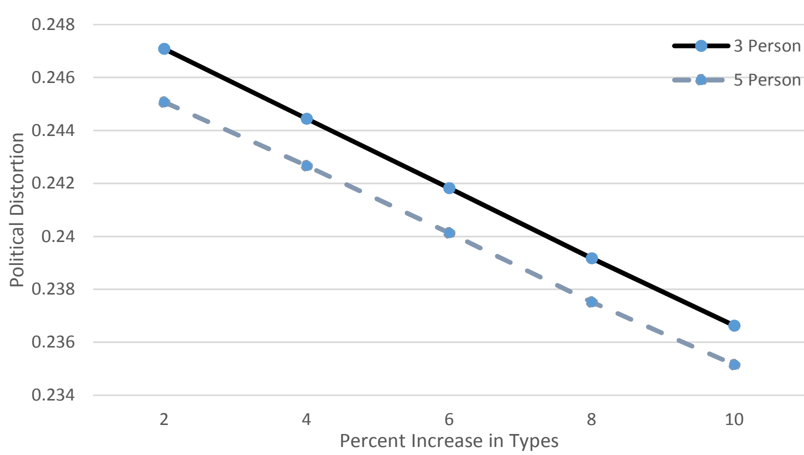


Figure 9: Policy Simulations: Approval Voting with Selection

Notes: This figure plots the allocation of public funds relative to the social planner allocation for a 3-person and 5-person scoring rule. The x-axis denotes the percentage point increase in the proportion of low altruistic types and low electoral appeal types in the candidate pool.

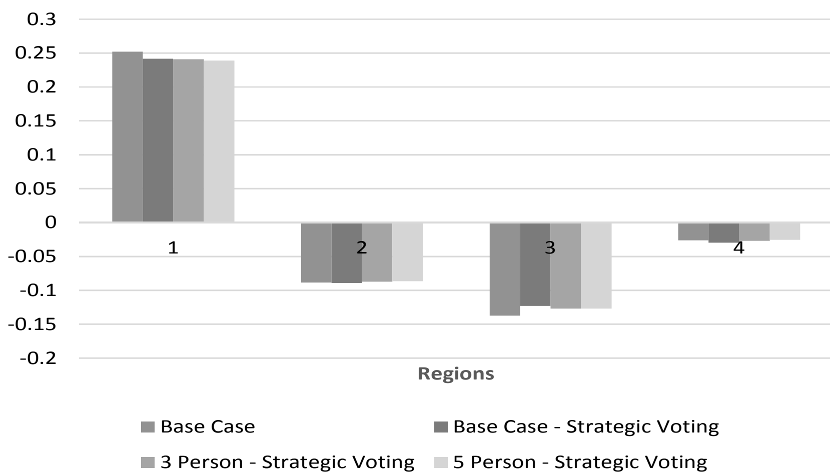


Figure 10: Policy Simulations: Approval Voting with Strategic Voting

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations, and assuming voters vote strategically. The share of public funds is computed by region and averaged over 5,000 simulations.

On-line Appendix

A Appendix: Tables and Figures

Table A.1: Relationship between Electoral Performance and Allocation of Public Funds

Dependent variable	Number of Votes			Vote Share		
	(1)	(2)	(3)	(4)	(5)	(6)
Public Funds ($100,000s)	677.491	679.28		0.028	0.028	
	[50.198]	[49.705]		[0.001]	[0.001]	
Rank within the municipality			-452.763			-0.027
			[35.468]			[0.001]
Municipal intercepts	N	Y	Y	N	Y	Y
Deputy intercepts	N	Y	Y	N	Y	Y
Observations	154,139	154,139	154,139	154,139	154,139	154,139
R-squared	0.47	0.47	0.36	0.2	0.16	0.12

Notes: Column 1 reports the unadjusted correlation between the amount of public funds a deputy allocated to the municipality during the 1996-1999 term and the number of votes he received from the municipality during the subsequent election. Column 2 reports the same relationship as in Column 1 but adjusts for both deputy and municipal fixed-effects. Column 3 reports the relationship between the number of votes a deputy received and his ranking in the municipality with respect to the amount of public goods he provided. Columns 4-6 replicate the regressions in columns 1-3 but use the deputy's vote share in the municipality as the dependent variable. The estimation has been restricted to only those incumbents that ran for reelection. Robust standard errors in brackets.

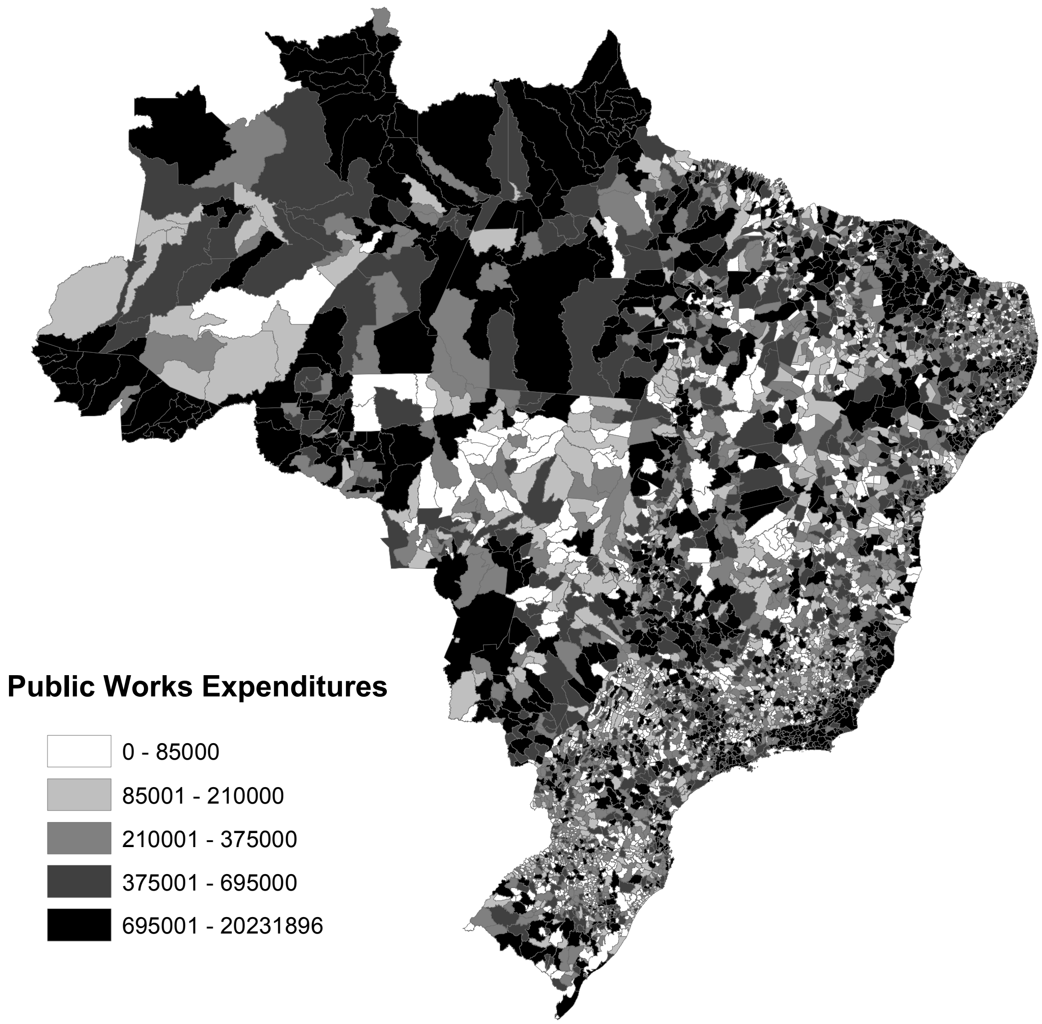


Figure A.1: Distribution of Budgetary Amendments

Notes: The map depicts the distribution of public funds during the 1996-1999 term by municipality.

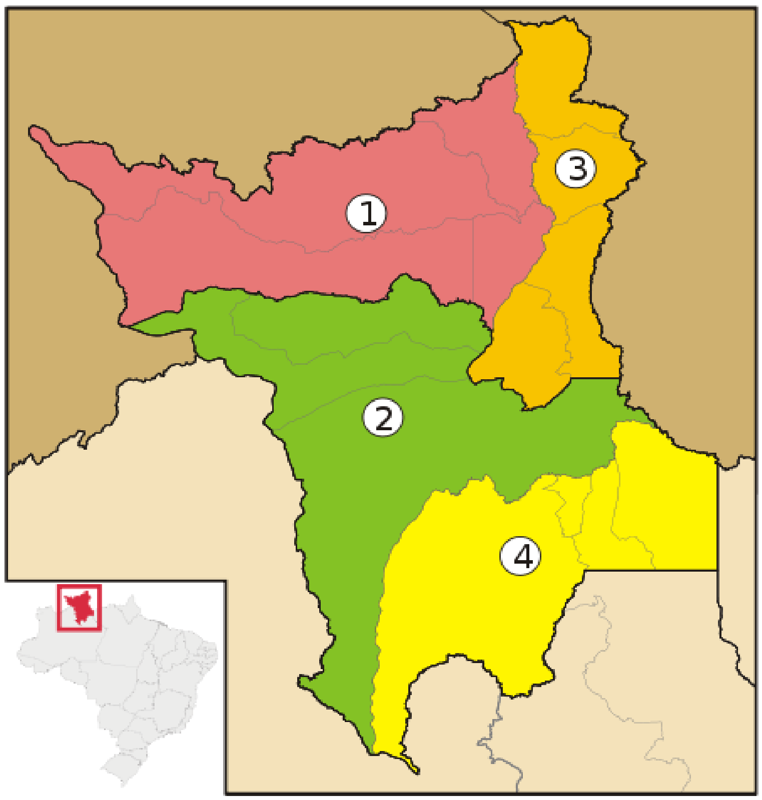


Figure A.2: State of Roraima

Notes: The map depicts the state of Roraima and its macro-regions.

B Appendix: Proof of Proposition 1

The proof is based on the existence results established in Milgrom and Weber (1985). To use their results is helpful to rewrite our model in the following way. Let

$$U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = \left(1-\beta^j\right) p^j\left(q, d_R\right) + \beta^j \sum_{m=1}^{M} W_m$$

and

$$U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = \bar{v}_{np}^j + \beta^j \sum_{m=1}^{M} W_m.$$

Then, deputy j chooses the optimal allocation and whether to run according to the following problem:

$$\max_{d_R^j}\left\{\max_{q^j}\int\left[U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right), \max_{q^j}\int U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\right]\sigma\left(d\beta^{-j}\right)\right\}$$

The problem can alternatively be written in the following form:

$$\max_{q^j, d_R^j} d_R^j\int U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right) + \left(1-d_R^j\right)\int U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right).$$

Or equivalently,

$$\max_{q^j, d_R^j}\int d_R^j U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) + \left(1-d_R^j\right)U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)d\sigma\left(\beta^{-j}\right).$$

We can therefore redefine the utility of deputy j as

$$U^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = d_R^j U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) + \left(1-d_R^j\right)U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right).$$

Deputy j's problem can therefore be written as follows:

$$\max_{q^j, d_R^j}\int U^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right).$$

We can now define a pure-strategy and a mixed-strategy Bayesian Nash equilibrium for this setting.

Definition 2 *A pure-strategy Bayesian Nash equilibrium is a vector of strategies* $s = \left(s^1, \ldots, s^J\right)$ *such that for every* $j \in J$*:*

$$s^j\left(\beta^j\right) = \underset{q^j, d_R^j}{\arg\max} \int U^j\left(q^j, d_R^j, s^{-j}\left(\beta^{-j}\right); \beta^j, \beta^{-j}\right) \sigma\left(d\beta^{-j}\right).$$

To define a mixed-strategy Bayesian Nash equilibrium, for every type β^j, let $m^j\left(s^j; \beta^j\right)$ be a probability measure over the strategy space S^j, and M^j player j's set of such mixed strategies. Then, we can extend the deputy j's utility to the set of mixed strategies by an expected utility calculation:

$$U^j\left(m^j, m^{-j}; \beta^j, \beta^{-j}\right) = \int_{S_1} \ldots \int_{S_J} U^j\left(s^j, s^{-j}; \beta^j, \beta^{-j}\right) m^1\left(ds^1; \beta^1\right) \ldots m^J\left(ds^J; \beta^J\right).$$

We can now introduce the mixed extension of the initial game in pure strategy $G = \left(S^j, U^j\right)_{j=1}^J$ as $\bar{G} = \left(M^j, U^j\right)_{j=1}^J$.

Definition 3 *A mixed-strategy* m^* *is a mixed-strategy Bayesian Nash equilibrium of the initial game* G *if* m^* *is a pure-strategy Bayesian Nash equilibrium of the extended game* $\bar{G}$.

Theorem 1, Propostion 1, and Proposition 3 in Milgrom and Weber (1985) establish that a game of incomplete information of the type considered in this paper has a mixed-strategy Bayesian Nash equilibrium if two conditions are satisfied: (i) the set of actions available to each player S^j is finite and (ii) the types of the players, $\beta^1, \ldots, \beta^J$, are drawn from independent distributions. In the model we estimate, each player has a finite set of actions since she can choose among four possible allocations of resources. Moreover, by assumption, types are drawn independently from the same distribution. Hence, a mixed-strategy Bayesian Nash equilibrium exists.