

# Pollution and Unemployment Over the Business Cycle

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### **Abstract**

Understanding how optimal environmental policy responds to adjustments in the economy over the business cycle is very important for policy makers and can aid in the design of more efficient and equitable policies. Environmental policies that operate by restricting current output or by changing the cost structure of firms may also distort firms' capital investment and hiring decisions. In this paper we introduce both a labor market search friction and a production process that releases harmful emissions as a by-product into a dynamic stochastic general equilibrium real business cycle model. Both search frictions and the modified production process generate externalities that are not internalized in a decentralized economy. We consider the interaction between labor market policies that either tax or subsidize vacancy creation with environmental policies that either impose a quantity restriction or an emission tax as an attempt to internalize all externalities present in the model economy.

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# 1 Introduction

The effects of environmental policy on unemployment are important, since much of the political opposition to environmental policy stems from its perceived effect on job creation. Also, how environmental policy interacts with the business cycle is important, as these cyclical fluctuations can have important real effects on the demand for environmental quality, the costs of reducing emissions, and the development of productive resources. While some papers in the literature have studied environmental policy over the business cycle, some have studied unemployment over the business cycle, and some have studied the unemployment effects of environmental policy, no other study examines these three issues - environmental policy, unemployment, and business cycles - together. In this paper, we consider the combined effect of labor market search frictions and a pollution externality within a dynamic stochastic general equilibrium (DSGE) model where fluctuations are driven by productivity shocks, as in the real business cycle (RBC) model.

A growing literature has included environmental policy in RBC models by modeling pollution as a by-product of production that can negatively affect productivity<sup>1</sup>. Current output is assumed to generate emissions, which if unabated will add to the economy's current stock of pollution. This stock of pollution limits the productive capacity of the economy in subsequent periods. While current emissions can be abated, the abatement technology is costly, requiring the use of current resources. These papers analyze optimal policy in this context. For example, Heutel (2012) shows that optimal environmental policy responds to business cycle fluctuations by dampening the procyclicality of emissions, with emissions taxes relaxing and output quotas tightening during economic downturns. While labor is included as an input in most of these models, none include labor market search frictions or involuntary unemployment. As such, the environmental RBC literature is unable to

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<sup>1</sup>For example, Fischer and Springborn (2011), Heutel (2012), Annicchiarico and Di Dio (2015), and Dissou and Karnizova (2016). See Fischer and Heutel (2013) for a literature review.

address the feedback that may exist between changes in environmental policy and hiring and employment dynamics in the economy. More recent papers have begun to consider the connection between employment and environmental regulation. For example, Hafstead and Williams (2018) consider the connection between employment and environmental regulation within a static general equilibrium model, and Castellanos and Heutel (2018) and Hafstead et al. (2018) develop computational general equilibrium (CGE) models of environmental policy allowing for unemployment.

Another literature incorporates involuntary unemployment into RBC models. In this paper, we merge the production process of Heutel (2012) with the RBC search model of Atolia, Gibson, and Marquis (2018a), which introduces involuntary unemployment through a modified Diamond-Mortensen-Pissarides (DMP) search and matching friction.<sup>2</sup> The inclusion of DMP search frictions within an RBC framework has roots in early works by Andolfatto (1996) and Merz (1995). However, we follow the more recent works of Atolia, Gibson, and Marquis (2018a and 2018b) and adapt the calibration strategy of Hagedorn and Manovskii (2008) within the RBC search environment. The inclusion of labor search frictions gives rise to an additional externality not considered in existing environmental RBC models. Specifically, in the decentralized search economy, firms may not produce the socially efficient level of vacancies. Therefore, we must also consider optimal employment policies, such as taxes or subsidies on vacancy creation, along with output quotas and emissions taxes that are needed to internalize the pollution externality. We calibrate our model to replicate key features of the U.S. economy, and we solve our model using standard linearization techniques. Simulations of our model are used to determine the efficiency and employment effects of environmental policies. We consider both static policies, where the vacancy creation and emissions tax rates and the quantity restriction are fixed over the cycle, and dynamic policies, where the

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<sup>2</sup>For a discussion of the basic search and matching friction, see Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994) and Pissarides (2000). The DMP model is applied to the context of environmental policy in Hafstead and Williams (2018) and Hafstead et al. (2018).

policy intensity is allowed to vary over the business cycle.

The paper proceeds as follows. In Section 2 we develop our model. In Section 3 we provide a discussion of our calibration strategy and the computational methods we use to approximate a solution of our model. In Section 4 we present our results, and in Section 5 we conclude.

## 2 Model

The economy consists of a continuum of identical households populated by employed and unemployed agents. Members of each household pool their resources and consume jointly at the end of each period. Labor markets are frictional, meaning that unemployed workers engage in time-costly search for new employment opportunities, while firms spend current resources to attract additional workers in the future. Along with labor search frictions, the production process also includes a pollution externality, with current production generating harmful emissions. These emissions add to the overall stock of pollution which, in turn, reduces the productive capacity of the economy in future periods. While current emissions can be mitigated through an abatement technology, this process is costly and requires current expenditures. We operationalize both labor search frictions and the pollution externality within an otherwise standard RBC model. The next several subsections provide details regarding the specific elements of our model.

### 2.1 Labor Market

Our labor market is subject to DMP-style search frictions where firms must use real resources to post vacancies in order to expand employment (see Diamond, 1982; Pissarides, 1985; Mortensen and Pissarides, 1994; Pissarides, 2000; and Hafstead and Williams, 2018). Furthermore, agents in our model exist in one of two states: employed or unemployed. Em-

employed agents supply labor inelastically to their firms while unemployed agents engage in time-costly search for new employment opportunities. As in Shimer (2010), unemployment agents are insulated against unemployment risk through their membership in the representative household.

In time  $t$ , a predetermined fraction of agents,  $n_t$ , are employed, and the remainder,  $1 - n_t$ , are unemployed. The amount of new employment matches formed in the current period,  $m_t$ , depends on the number of vacancies created by the firm,  $v_t$ , the number of unemployed agents searching for work,  $1 - n_t$ , and the following Cobb-Douglas matching function,

$$m_t = \gamma_1(1 - n_t)^{\gamma_2} v_t^{1-\gamma_2} \quad (1)$$

Using the matching function in equation (1), we can define labor market tightness,  $\Phi_t$ , the job-finding rate,  $f_t$ , and the vacancy-filling rate,  $q_t$ :

$$\Phi_t = \frac{v_t}{1 - n_t} \quad (2)$$

$$q_t = \frac{m_t}{v_t} = \gamma_1 \Phi_t^{-\gamma_2} \quad (3)$$

$$f_t = \frac{m_t}{1 - n_t} = \gamma_1 \Phi_t^{1-\gamma_2} = \Phi_t q_t \quad (4)$$

Equations (1), (3) and (4) specify how employment relationships are formed and how agents flow from unemployment into employment.

We must also specify how employment relationships dissolve and agents flow into unemployment. We follow the labor search literature and assume that a time-constant fraction,  $s$ , of all current employment matches will dissolve. This assumption of exogenous job destruction is consistent with that presented in Hall (2005) and Hagedorn and Manovskii (2008), and it is supported by Shimer (2012) which finds that the flow of labor from unemployment to employment is more important for explaining the dynamics of the U.S. unemployment

rate than the flow of labor into unemployment. Finally, using the exogenous separation rate and the aggregate matching function presented in equation (1), we can derive the aggregate evolution equation for employment as:

$$n_{t+1} = (1 - s)n_t + m_t \quad (5)$$

## 2.2 Production and Pollution

Our production process incorporates pollution and follows from Heutel (2012), though here labor is also included as a factor of production (as in Fischer and Springborn, 2011). The representative firm produces potential output using the following production technology:

$$y_t = \theta_t k_t^\alpha n_t^{1-\alpha} \quad (6)$$

where  $k_t$  and  $n_t$  denote aggregate capital and employment, which are rented/hired from households, and  $\theta_t$  is an exogenous total factor productivity shock, which follows:

$$\ln \theta_{t+1} = \rho_\theta \ln \theta_t + \epsilon_{t+1}, \epsilon \sim N(0, \sigma_\theta) \quad (7)$$

The total output produced in a given time-period,  $Y_t$ , depends positively on the level of potential output,  $y_t$ , and negatively on the stock of pollution present at the end of the period,  $x_{t+1}$ ,

$$Y_t = [1 - d(x_{t+1})] \cdot y_t \quad (8)$$

where  $d(\cdot)$  is a monotonically increasing function that takes on values between 0 and 1 and measures the amount of potential output lost from pollution.

Producing output also generates harmful emissions,  $e_t$ , as a by-product. These emissions are assumed to depend on the level of output being produced and the amount of abatement

conducted,

$$e_t = (1 - \mu_t) \cdot h(Y_t) \tag{9}$$

where  $h(\cdot)$  is a function determining how emissions are related to the current output level and  $\mu_t$  is the fraction of emissions that are abated. Abatement is costly, with  $z_t$  units of resources (output) being depleted in order to abate  $\mu_t$  of current emissions. This relationship is formalized by,

$$z_t = g(\mu_t) \cdot Y_t \tag{10}$$

where  $g(\cdot)$  relates the fraction of emissions abated to the fraction of output spent on abatement. The amount of unabated emissions feeds back on output through its effect on the stock of pollution, which evolves as,

$$x_{t+1} = \eta x_t + e_t + e_t^{\text{row}} \tag{11}$$

where  $\eta$  determines the rate of decay of the pollution stock and  $e_t^{\text{row}}$  is an exogenous term that stands in for the emissions from the rest of the world.

### 2.3 Planner's Problem

We first formalize our model from the perspective of a social planner. While planner's problems are often used in the RBC literature, standard RBC models lack externalities and through the first fundamental theorem of welfare economics, one can establish that the solution to the social planner's problem is equivalent to the competitive equilibrium allocations. This is not the case for our model as both the labor search friction and our production process generates externalities. By solving the planner's problem first, we are recovering the first-best solution where all externalities have been internalized. The planner's solution is compared to the competitive equilibrium allocation (described below) and used

to inform government policy which will be designed to address the inefficiencies arising from both the labor search friction and the pollution externality.

The social planner chooses consumption, capital investment, vacancy creation, and the amount of emissions to abate, in order to maximize the representative household's lifetime utility:

$$u(c, n) = \sum_{t=0}^{\infty} \beta^t [\ln c_t - \xi n_t] \quad (12)$$

where  $\beta$  denotes the agent's subjective discount factor and  $\xi$  denotes their disutility from work. The planner enters each period with a predetermined quantity of employed workers, capital, pollution, and knowledge of the current aggregate productivity in the economy. Their optimization problem is constrained by an aggregate resource constraint and the aggregate evolution of employment in the economy. This maximization problem can be written as the following dynamic program,

$$V(k_t, n_t, x_t; \theta_t) = \max_{c_t, v_t, k_{t+1}, x_{t+1}} [\ln c_t - \xi n_t + \beta E \{V(k_{t+1}, n_{t+1}, x_{t+1}; \theta_{t+1})\}]$$

s.t.

$$c_t + k_{t+1} - (1 - \delta)k_t + v_t G \leq (1 - g(\mu_t)) [1 - d(x_{t+1})] \theta_t k_t^\alpha n_t^{1-\alpha} \quad (13)$$

$$n_{t+1} = (1 - s)n_t + \gamma(1 - n_t)^{\gamma_2} v_t^{1-\gamma_2} \quad (14)$$

where, from equations (9) and (11):

$$\mu_t = 1 - [x_{t+1} - \eta x_t - e_t^{\text{row}}] \cdot h \left( (1 - d(x_{t+1})) \theta_t k_t^\alpha n_t^{1-\alpha} \right) \quad (15)$$

Equation (13) denotes the economy's aggregate resource constraint. The first two terms on the left-hand side denote consumption expenditures and the capital investment. The third term on the left-hand side is the economy's expenditures on vacancy creation, with  $G$  denoting per-vacancy posting cost, and the right-hand side denotes the total output less

abatement. Solving this problem yields the following three dynamic Euler equations:

$$\beta E \{V_{k_{t+1}}\} = \frac{1}{c_t} \quad (16)$$

$$\beta E \{V_{x_{t+1}}\} = \frac{1}{c_t} \left[ -(1 - g(\mu_t))d'(x_t)\theta k_t^\alpha n_t^{1-\alpha} + Y_t g'(\mu_t) \left[ \frac{h(Y_t) + e_t h'(Y_t) d'(x_t) \theta k_t^\alpha n_t^{1-\alpha}}{h(Y_t)^2} \right] \right] \quad (17)$$

$$\beta E \{V_{n_{t+1}}\} = \frac{1}{c_t} \left[ \frac{G}{\gamma_1(1 - \gamma_2)\Phi_t^{-\gamma_2}} \right] \quad (18)$$

where,

$$\begin{aligned} V_{k_{t+1}} &= \frac{1}{c_{t+1}}(1 - d(x_{t+1}))\theta_{t+1}\alpha \left(\frac{k_{t+1}}{n_{t+1}}\right)^{\alpha-1} \left[ 1 - g(\mu_{t+1}) - Y_{t+1}g'(\mu_{t+1}) \left(\frac{1 - \mu_{t+1}}{h(Y_{t+1})}\right) h'(Y_{t+1}) \right] \\ &\quad + \frac{1}{c_{t+1}}(1 - \delta) \\ V_{x_{t+1}} &= \frac{1}{c_{t+1}}g'(\mu_{t+1})Y_{t+1}\frac{\eta}{h(Y_{t+1})} \\ V_{n_{t+1}} &= \frac{1}{c_{t+1}}(1 - d(x_{t+1}))\theta_{t+1}(1 - \alpha) \left(\frac{k_{t+1}}{n_{t+1}}\right)^\alpha \left[ 1 - g(\mu_{t+1}) - g'(\mu_{t+1})Y_{t+1} \left[\frac{1 - \mu_{t+1}}{h(Y_{t+1})}\right] h'(Y_{t+1}) \right] \\ &\quad - \xi + \frac{1}{c_{t+1}} \left[ \frac{1 - s - \gamma_1\gamma_2\Phi_{t+1}^{1-\gamma_2}}{\gamma_1(1 - \gamma_2)\Phi_{t+1}^{-\gamma_2}} \right] G \end{aligned}$$

Equation (16) is the dynamic Euler on capital accumulation, while equation (17) is the dynamic Euler on emissions and pollution. Equations similar to (16) and (17) appear in Heutel (2012); here they depend on the current and future levels of employment in the economy. Equation (18) is unique to our model and defines the planner's vacancy creation condition. Inspection of equations (16)-(17) and the corresponding envelop conditions show that there is a feedback relationship between emissions and employment, with the current and future emissions impacting vacancy creation and current and future employment impacting the emissions decision. This connection is crucial for the current paper.

## 2.4 Decentralized Problem

As an alternative to the social planner's problem, in which all decisions are being made centrally to maximize social welfare in response to the exogenous TFP shocks, we also

consider a decentralized model. Here, a representative household chooses consumption and capital investment to maximize their utility, while the representative firm chooses how much capital and labor to rent from households, how many new vacancies to create, and how much of their current emissions to abate in order to maximize their profits. The firm is potentially subject to a pollution tax,  $\tau$ .

### 2.4.1 Household

The representative household chooses current consumption and next period's capital in order to maximize its net present value of discounted utility, taking the wage rate,  $w_t$ , rental rate on capital,  $r_t$ , and the job-finding rate,  $f_t$  as given. In solving this maximization problem, the household faces two constraints. Equation (19) is the household's budget constraint. The right-hand side of equation (19) is the household's expenditures on current consumption and capital investment, while the left-hand side is the household's sources of income which consist of their rental income,  $r_t k_t$ , labor income,  $w_t n_t$ , government transfers from emissions tax revenue,  $\tau_t e_t$ , and profits earned by the firm,  $\pi_t$ . Equation (20) is the evolution equation of labor from the perspective of the household. It differs from the aggregate evolution equation (5) in that it reflects the fact that the household does not internalize how their search activity influences the likelihood an unemployed worker meets a firm,  $f_t$ .

$$V(k_t, n_t) = \max_{c_t, k_{t+1}} [\ln c_t - \xi n_t + \beta E \{V(k_{t+1}, n_{t+1})\}]$$

s.t.

$$c_t + k_{t+1} - (1 - \delta)k_t \leq r_t k_t + w_t n_t + \tau_t e_t + \pi_t \tag{19}$$

$$n_{t+1} = (1 - s)n_t + f_t(1 - n_t) \tag{20}$$

### 2.4.2 Firm

The representative firm seeks to maximize profits given exogenous (to the firm) factor and output prices and the TFP shock. The firm also faces the exogenous policy variable  $\tau_t$ . The firm's profit function is

$$\pi_t = (1 - g(\mu_t))y_t - \tau_t(1 - \mu_t)h(y_t) - r_t k_t - w_t n_t - v_t G \quad (21)$$

The first term on the right-hand side of (21) is the firm's output net of abatement expenditure and the second term is the firm's tax bill on unabated emissions. The remaining three terms are the firm's capital rental bill, wage bill, and expenditure on vacancies.

The firm's problem can be written as the following dynamic program.

$$\begin{aligned} J(n_t, \theta_t) &= \max_{k_t, \mu_t, v_t} \left[ \pi_t + \beta E \left\{ \frac{U'_c}{U_c} J(n_{t+1}; \theta_{t+1}) \right\} \right] \\ &\text{s.t.} \\ n_{t+1} &= (1 - s)n_t + q_t v_t \end{aligned} \quad (22)$$

Though there is just one representative firm, we model pollution's effect on output as an externality by assuming that the firm treats  $x_t$  as exogenous. The firm chooses how much capital to rent, what fraction of emissions to abate, and how many vacancies to post in order to maximize the expected discounted value of lifetime profits, taking the wage rate,  $w_t$ , rental rate,  $r_t$ , and vacancy-filling rate,  $q_t$ , as given.<sup>3</sup> Equation (22) is the evolution of labor from the perspective of the firm. As in the household's problem, this evolution equation differs from the aggregate evolution equation as the firm does not internalize how their decision to create vacancies impacts the vacancy filling rate,  $q_t$ .

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<sup>3</sup>Note that we assume that firms discount future periods using the stochastic discount factor,  $\beta E \left\{ \frac{U'_c}{U_c} \right\}$ . As such, firms discount future values in a way that is consistent with household preferences.

### 2.4.3 Wage Bargaining

Wages in the decentralized problem are set through repeated Nash bargaining over the marginal surplus generated by a match. This is a common wage setting mechanism employed in the labor search and RBC-search literature and results in the following surplus sharing rule:

$$bU_c V_n = (1 - b)J_n \tag{23}$$

where  $b$  denotes workers' bargaining power and  $V_n$  and  $J_n$  denote the firm's and household's envelope conditions with respect to employment.

### 2.4.4 Government

The government chooses the policy variable  $\tau_t$  in each period. In principle, the government could dynamically optimize the pollution tax in response to business cycle shocks, as in Heutel (2012), or it could choose to keep it at the steady-state level that maximizes social welfare, as in Fischer and Springborn (2011). In our paper, we do not consider the government's maximization problem, and instead we simply solve the model of the firm's and the household's behavior for an arbitrary value of the tax. For simplicity and to make the most concrete comparison between the first-best planner's problem and the decentralized problem, we begin with a constant tax rate of zero; that is, no pollution policy.

### 2.4.5 Equilibrium

The equilibrium of the decentralized model consists of a set of prices,  $w_t$  and  $r_t$ , and a set of outcome variables such that households and firms are optimizing given those prices.

### 3 Calibration and Approximation

We set our model’s period length to a quarter and calibrate our model to replicate key features of the U.S. economy using both data and values reported in the existing literature. The next several sections detail our calibration strategy and provides a basic overview of the computational methods used to approximate our model.

#### 3.1 RBC Parameters

Given that our model is calibrated to a quarterly frequency, we follow the RBC literature and set the agent’s subjective discount factor,  $\beta$ , equal to 0.99. Also following convention, we set the quarterly depreciation rate of private capital,  $\delta$ , to 0.025, thereby targeting an annual depreciation rate of approximately 10 percent. We also set capital’s share of income,  $\alpha$ , equal to 0.36, following the RBC literature. The persistence in the productivity shock process,  $\rho_\theta$ , and the standard deviation of the innovation in the shock process,  $\sigma_\theta$  are set to 0.95 and 0.007, respectively. These values allow the first-order autocorrelation and volatility of output and average labor productivity to match the data reasonably well.

#### 3.2 Labor Market Parameters

Several calibration strategies exist for models with DMP search frictions, with the primary difference being how vacancy postings costs,  $G$ , agents’ disutility of work,  $\xi$ , and worker’s bargaining power,  $b$ , are determined. Shimer (2005) presents a calibration strategy where  $\xi$  is set to target the replacement rate of unemployment insurance for the U.S. (approximately 40%) and worker’s bargaining power is set equal to the elasticity of matches with respect to unemployment following Hosios (1990). The flow cost of vacancy creation,  $G$ , is not explicitly targeted under this strategy, but is instead left free to adjust so the model can reach other desired targets. In contrast, Hagedorn and Manovskii (2008) state that vacancy

costs are significantly lower in the data and, as such, they set  $G$  to match their empirical target. Workers' bargaining power is set to target the elasticity of wages with respect to average labor productivity observed in the data.

We adapt a calibration strategy similar to that presented in Hagedorn and Manovskii (2008). Specifically, vacancy posting costs,  $G$ , are very small, on the order of 2 percent of output. The matching function parameters,  $\gamma_1$  and  $\gamma_2$  are set to 1 and 0.5 respectively, while the exogenous separation rate,  $x$ , is set to target a steady state unemployment rate of 4.4%. For the decentralized economy, we consider two values for workers' bargaining power,  $b$ , 0.25 and 0.65. This allows us to consider both the case when the bargaining power of workers exceeds the elasticity of matches with respect to unemployment and when it is less than this value.

### 3.3 Environmental Parameters

The parameters related to pollution are calibrated in the same manner as in Heutel (2012), which draws predominantly from the DICE model (Nordhaus, 2017). The relationship between baseline pollution and output is given by  $h(y_t) = y_t^{1-\gamma}$ , where  $1 - \gamma$  is set at 0.696. The pollution stock decay rate  $\eta$  is set to 0.9979, indicating that carbon dioxide is a very long-lived pollutant (half-life of 83 years). The damage function  $d(x)$  is set to be a quadratic function and calibrated to the DICE-2007 model in Nordhaus (2008), and the abatement cost function  $g(\mu)$  is taken directly from Nordhaus (2008). More details of these parameterizations are available in Heutel (2012).

### 3.4 Solution Method

While there is a long-standing tradition of approximating RBC models using linearization methods, there is currently an ongoing debate regarding the importance of approximation ac-

curacy in models with labor market search frictions.<sup>4</sup> We currently approximate the solution to our planner’s problem using standard linear methods and we approximate the solution to the decentralized economy using the deterministic extended path (DEP) algorithm. We are currently in the process of revising our solution procedures in order to achieve a higher degree of accuracy. Specifically, we are adopting the generalized stochastic simulation algorithm (GSSA) presented in Judd, Maliar, and Maliar (2011), Maliar and Maliar (2014), and Atolia, Gibson, and Marquis (2018a) as this method has been shown to be highly accurate when applied to RBC-search models.

## 4 Results

### 4.1 Social Planner’s Problem

We begin by presenting impulse response functions (IRFs) of the response of several endogenous variables to a one-unit one-time innovation in the TFP shock, for the full social planner’s problem (presented in section 2.3) in which the frictions and distortions from labor search and from pollution are accounted for.

In Figure 1, we present IRFs for several variables, broken up into three panels for ease of interpretation (though all IRF curves come from the same model). The top panel presents the value of TFP over 100 periods, plus the IRFs for variables that are included in nearly all RBC models: output, capital, investment, and consumption. All of these IRFs demonstrate that a positive TFP shock leads to an expansion, with an immediate rise in output and investment and a slightly lagged rise in consumption and capital (due to the stock nature of capital and thus wealth).

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<sup>4</sup>A series of recent papers investigates the importance of approximation accuracy in models with DMP search frictions. While Petrosky-Nadeau and Zhang (2017) find that solving the models accurately can result in sizable adjustments to the models second moments Lan (2018), and Atolia, Gibson, and Marquis (2018a), find their models’ conclusions are less sensitive to the specific approximation method that is chosen.

The middle panel presents variables that are unique to the labor search literature: labor market tightness, unemployment, and vacancies. As in Atolia, Gibson, and Marquis (2018a), we find that an expansion is associated with a decrease in unemployment, with a one-time positive TFP shock of 0.7% leading to a drop in unemployment of  $-0.4\%$ . We also replicate their earlier result that labor market tightness and vacancies increase during the expansion.

The bottom panel of Figure 1 presents variables unique to the addition of pollution into the RBC model. We find results comparable to those in Heutel (2012) and other environmental RBC papers. Emissions increases during an expansion, and as a result so does the pollution stock. But, since the pollution stock is so slow to decay, the effect of a relatively short spike in emissions on pollution is very small in scale. This increase in emissions and pollution comes despite there being an increase in the fraction of pollution that is abated and in the resources spent on abatement. This implies that some of the increase in output from the positive TFP shock is spent on preventing pollution from increasing as much as it otherwise would have.

Figure 2 simulates an actual business cycle pattern by generating random draws of the TFP innovation and allowing the model to determine the resulting time paths of the endogenous variables. It shows that investment and abatement spending are the most volatile of all of the variables. Unemployment is moderately volatile, staying within about plus or minus 3 percent of its steady-state level. Pollution is almost entirely flat and constant at the steady-state level.

## 4.2 Constrained Simulations

Next we compare simulations of the social planner's problem to those of models that are constrained in some way, to assess the importance of cyclical fluctuations in either emissions or labor search.

### 4.2.1 No Fluctuations in Labor Search

We compare the full model to one in which the cyclical nature of the labor market is eliminated. That is, we assume that the vacancy cost is constant over the cycle rather than being allowed to endogenously respond. Figure 3 shows these simulations, where the black curves are for the full model. The green curves are the model under the assumption that the unemployment rate and search costs are kept constant rather than being allowed to endogenously fluctuate over the business cycle. The figure omits the IRFs for the labor market variables – vacancies, unemployment, and labor market tightness – since these are kept fixed in the simulations with no labor search variation. Figure 3 shows that both curves are almost perfectly coincidental, indicating that the omissions of cyclical nature or fluctuations in labor search does not have an observable effect on the response of these outcomes to TFP shocks. However, in the panels presenting outcomes related to pollution, we see that pollution is slightly (barely) lower and abatement is slightly higher in the full model than it is in the model omitting labor cyclical nature.

Next, we present business cycle simulations. In Figure 4, we compare the full model (in black curves) to the model omitting labor search fluctuations (in green curves), again looking at the same eight variables as in Figure 3. As in the IRFs, the two pairs of curves are indistinguishable from each other, indicating that omitting the cyclical nature of labor market search does not affect the behavior of the other feature of the model.

### 4.2.2 No Fluctuations in Pollution

Next we present solutions where the pollution part of the model is kept fixed at the steady state level rather than be allowed to optimally fluctuate in response to shocks. There are three different ways that we choose to do this constrained simulation. The first is analogous to how we modeled the constrained system with no fluctuations in labor search. We do not simply drop pollution from the model, because under that case there would be a substantial

difference in the levels of output and other variables due to the lack of pollution damages. Instead, we replace the pollution damages and abatement costs in the model with a fixed reduction in output, set to equal the steady-state pollution level damages to output, and a fixed cost in the resource constraint, set to equal the steady-state abatement cost.

Figure 5 presents these simulations. The black curves represent the IRFs from the full model described in the paper, and the green curves are IRFs from the constrained model. We omit presenting the variables related to the environment – emissions, pollution, and abatement – since those are all constant in the simulation with no pollution variation. The two curves are nearly identical to each other in each panel. In Figure 6, we compare the full model (in black curves) to the model omitting pollution and abatement fluctuations (in green curves), and again these business cycle simulations are nearly coincidental.

The second way that we constrain the pollution side of the model is by including both abatement spending and pollution in the model, but fixing the level of emissions each period to be at the steady-state level. This also fixes the level of the pollution stock to be at the steady-state level. However, the fraction of emissions abated is no longer fixed – when TFP spikes, to keep the steady-state emissions level, the fraction of emissions abated must increase (since baseline emissions increase). This is reflected in the IRFs presented in Figure 7. In the panel presenting emissions, emissions spikes in the full model in response to a TFP shock, but stays flat in the constrained model. The same is true for pollution. However, the opposite pattern holds for the fraction abated and abatement spending. In the full model, it stays relatively constant, though in the constrained model it must spike after a TFP shock to keep the emissions level constant. For the labor market variables, there is little difference between the full and constrained models. Figure 8 presents the business cycle simulations of these comparisons.

The third way that we constrain the pollution side of the model is to again include both abatement and emissions in the model, but fix the fraction of emissions abated at its

steady-state level. With a positive TFP shock, baseline emissions will increase, and so total emissions will increase even though the fraction of emissions abated is kept constant. These simulation results are presented in Figure 9. When abatement is kept fixed, there is a very slight difference between the full model and the constrained model in terms of the labor market variables. Unemployment and vacancies response very slightly more to the TFP shock under the constrained model where abatement is fixed than they do under the full model where abatement is allowed to vary. The business cycle simulations are presented in 10.

### 4.3 Decentralized Model

Here we present simulation results from the decentralized model presented in section 2.4. We simulate the business cycle outcomes for the endogenous variables under two different assumptions about the bargaining power given to workers. The first is that they have a low bargaining power (0.25), and the second is that they have a high bargaining power (0.65). For each value of the bargaining parameter, we present the first-best outcomes and compare them to the outcomes under the decentralized case, with no pollution policy and no labor market policy.

Figure 11 presents the simulations when the workers have a low bargaining power of 0.25. This value is lower than the elasticity of matches with respect to unemployment. Because of this, the number of vacancies is much higher in the decentralized model than in the first-best planner's problem. Labor market tightness is also higher, and as a result unemployment is lower, in the decentralized model. Because there is inefficiently-low unemployment, there is also a higher level of output in the decentralized case than in the planner's problem, and higher levels of capital and of investment. Because there is no pollution policy, abatement spending and the fraction of emissions abated are constant at zero in the decentralized model. Emissions and pollution are inefficiently high in the decentralized model for two

reasons. First, there is inefficiently low abatement, and second, there is inefficiently high output.

Figure 12 presents the simulations when the workers have a high bargaining power of 0.65, which is higher than the elasticity of matches with respect to unemployment. Because of this, in this figure the decentralized model diverges in a different direction compared to the results presented in Figure 11 – unemployment is higher and vacancies and tightness are lower in the decentralized model than in the first best. The higher bargaining power of the workers leads to less vacancies being posted and as a result more unemployment. Because there is more unemployment, there is less output and less capital in the decentralized case than in the first best, which is also opposite the result from the case where the bargaining power was low. However, the results related to the environmental are still qualitatively the same as they are in Figure 11 – emissions and pollution are higher in the decentralized case. But, emissions and pollution are not as high in the decentralized case in Figure 12 as they are in Figure 11. When bargaining power is lower, both the pollution externality and the labor market friction lead to higher than optimal emissions. When bargaining power is higher, though, the pollution externality leads to higher than optimal pollution but the labor market friction pushes in the other direction, suppressing emissions by suppressing employment and output. In Figure 12, the two market failures are working in opposite directions. Nevertheless, the overall impact is that there is still an inefficiently high level of emissions and pollution, albeit not as high as in the case of low bargaining power.

These figures demonstrate the crucial importance of the bargaining power parameter in determining the direction of the market failures and thus the appropriate policy tool. Emissions are inefficiently high for either bargaining parameter, but the direction that unemployment differs from its optimal level depends on the parameter value.

## 5 Conclusion

We develop a dynamic stochastic general equilibrium model that includes autocorrelated productivity shocks, labor market search frictions, and a stock pollution externality to examine the relationship between environmental policy, unemployment, and business cycles. We incorporate labor search frictions using the Diamond-Mortensen-Pissarides search and matching model, in which there are exogenous job destructions each period and a costly matching process to fill new jobs. We model pollution as a byproduct of output, which can be reduced through costly abatement spending and which has a negative effect on productivity. In the first-best planner's problem, the costs of pollution abatement are optimally traded off against the benefits in terms of reduced externalities, and the optimal amount of labor market search intensity is conducted. We demonstrate the cyclical nature of both unemployment and emissions in the first best. We compare the first best to a decentralized model, where firms do not internalize the negative effects of pollution and workers do not necessarily search optimally. In the decentralized case, we show how policies, including an emissions tax, can bring about the first best.

Like other DSGE models, ours uses many simplifying assumptions to yield our main results. The model could be extended in many ways by exploring the importance of these assumptions. For example, one extension to this paper would be to incorporate multiple sectors rather than just a representative firm, as in Dissou and Karnizova (2016). Or, one could add new Keynesian price stickiness, as in Annicchiarico and Di Dio (2015).

Our research has important policy implications. Policy makers are concerned over both efficiency and equity, and many models of optimal environmental policy ignore distributional concerns like the effects of policies on unemployment. Furthermore, even absent any concerns about equity and distributional outcomes, we show how the incorporation of labor frictions and unemployment into a model of optimal environmental policy can affect efficiency, given

that the labor frictions create distortions. The Great Recession has made clear the fact that policy makers care about the cyclical effects of policies, and it is important to consider how various policies, like environmental and labor policies, interact over the business cycle.

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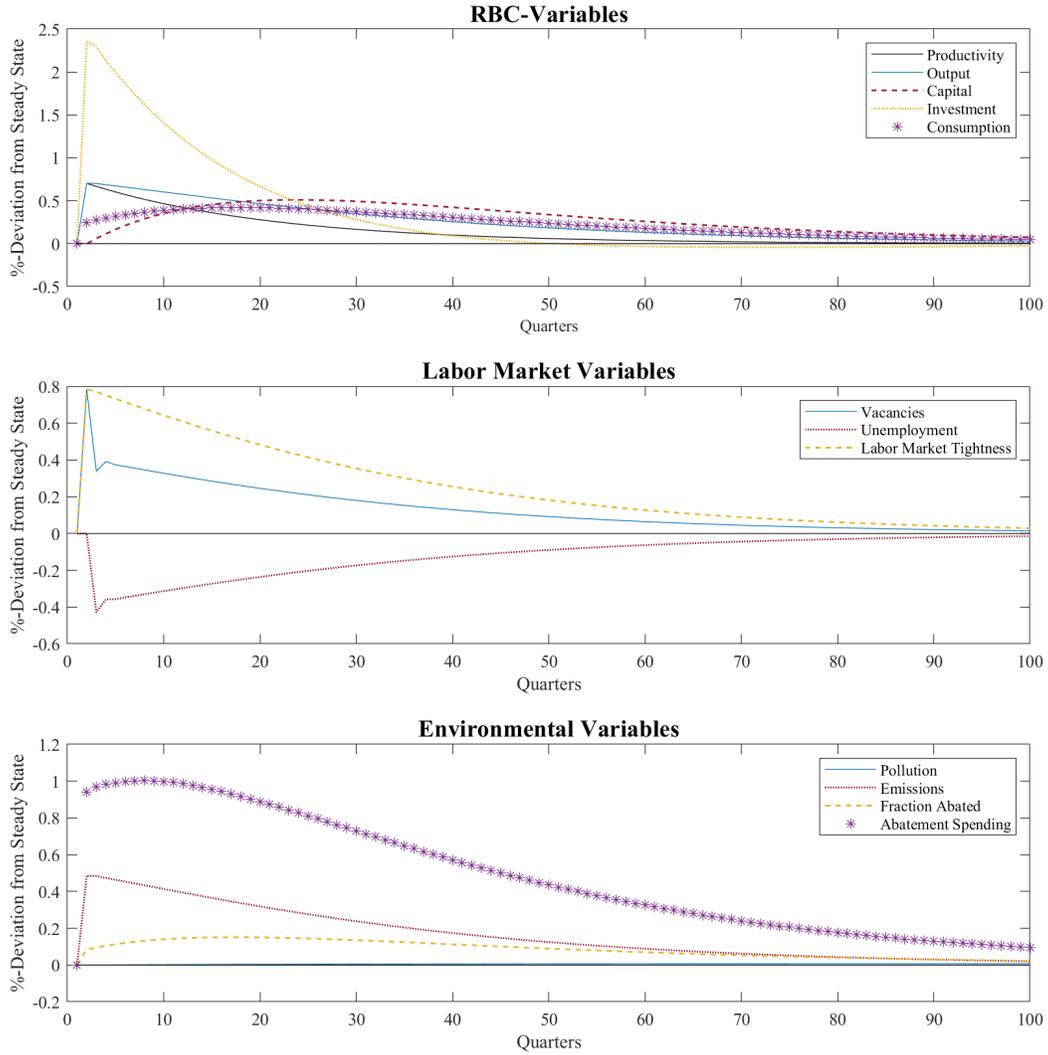
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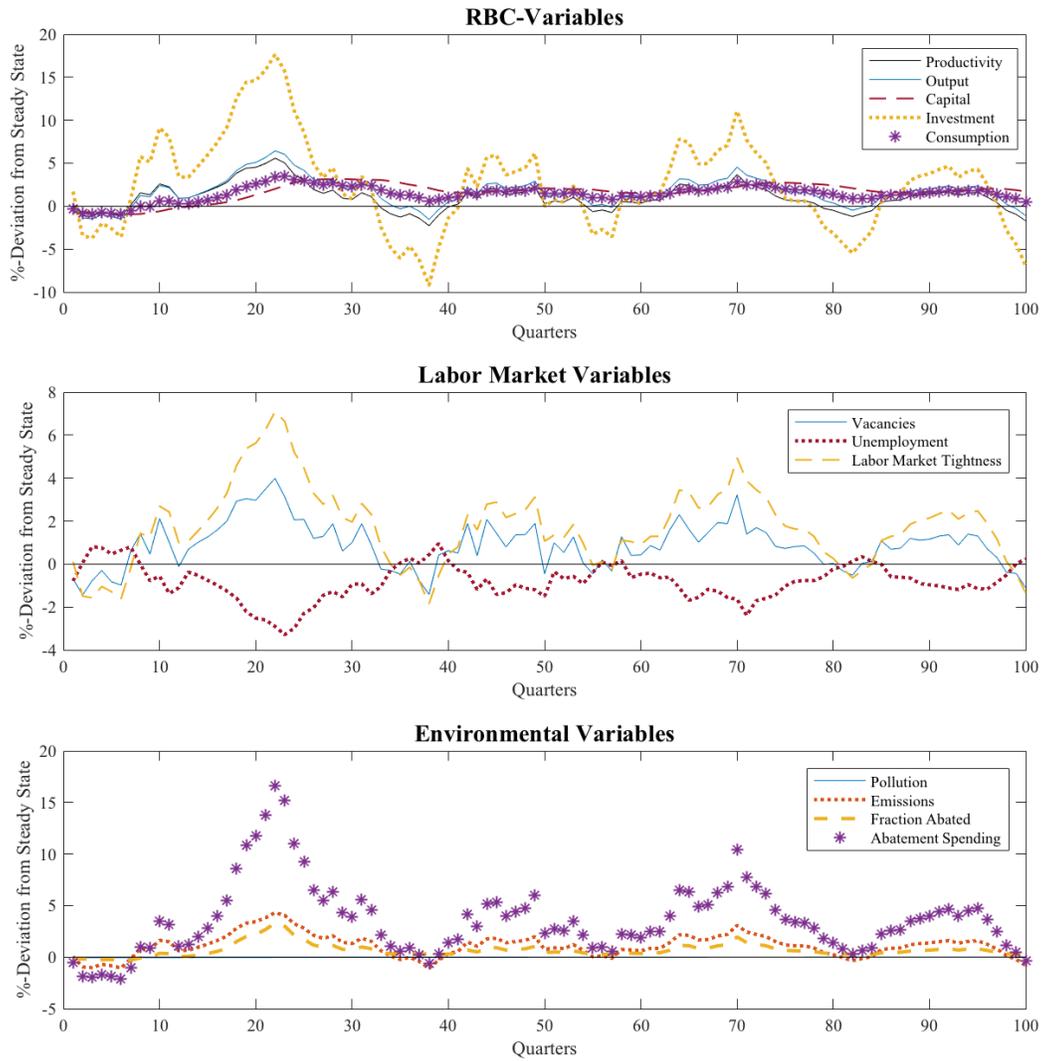
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Figure 1: Impulse Response Functions - Base Case



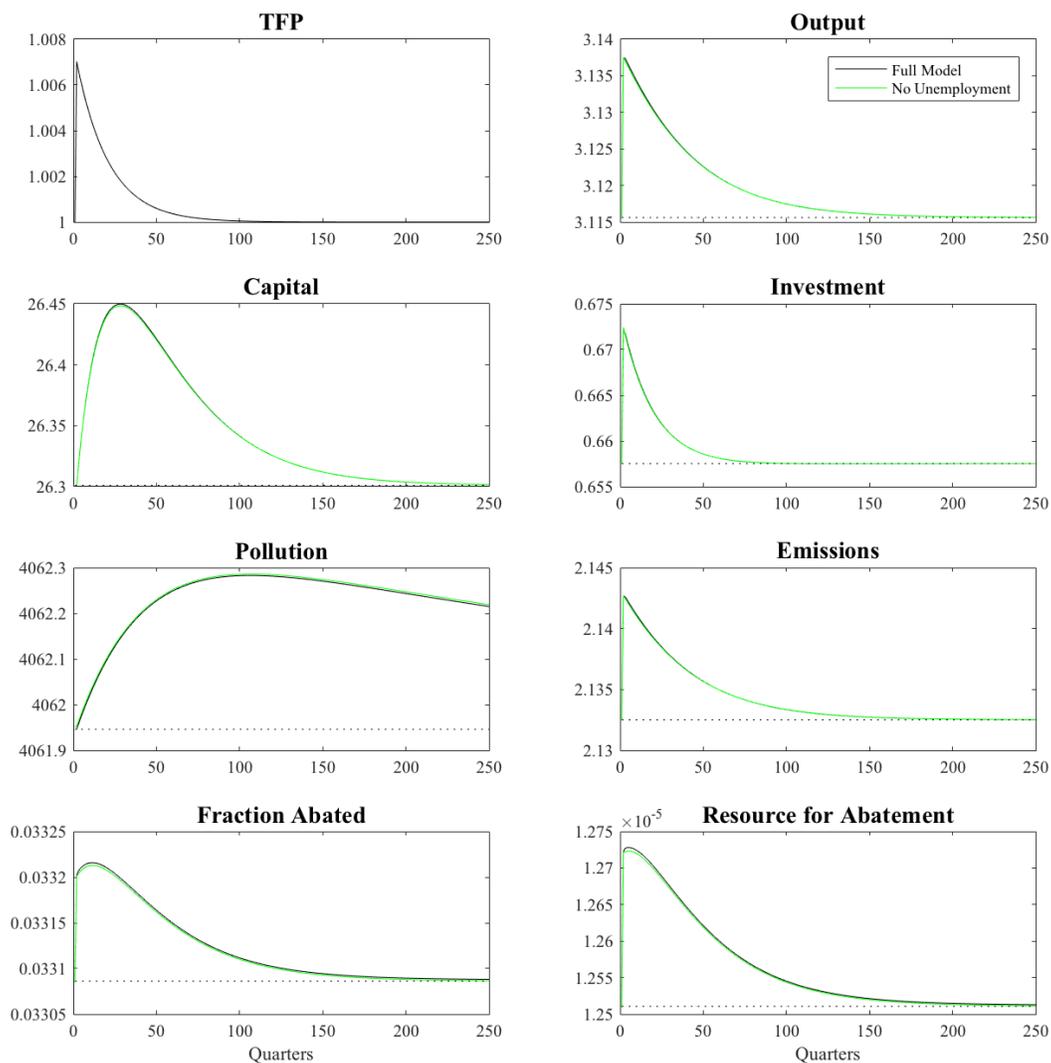
Notes: All simulations are from a one-unit one-time productivity innovation.

Figure 2: Business Cycle Simulations - Base Case



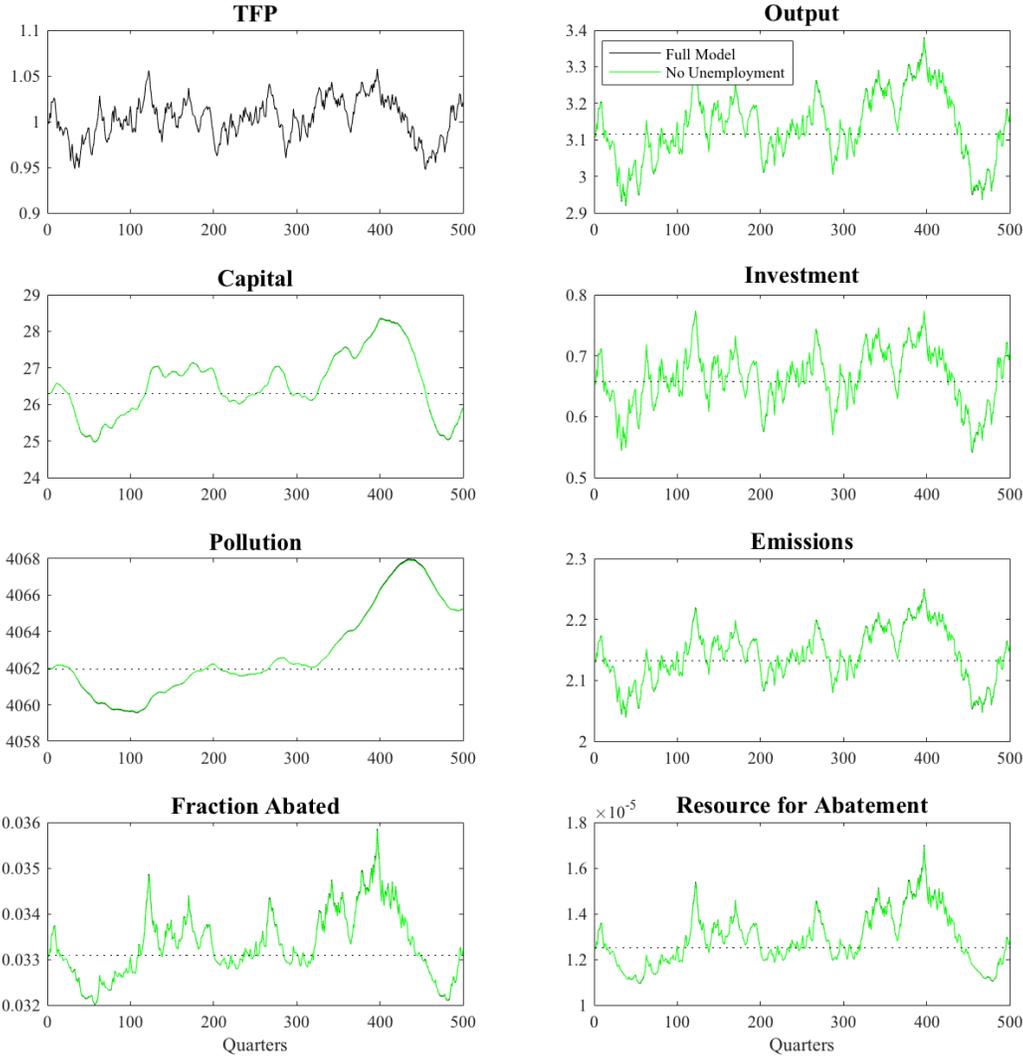
Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

Figure 3: Impulse Response Functions - Full Model vs. No Labor Variation



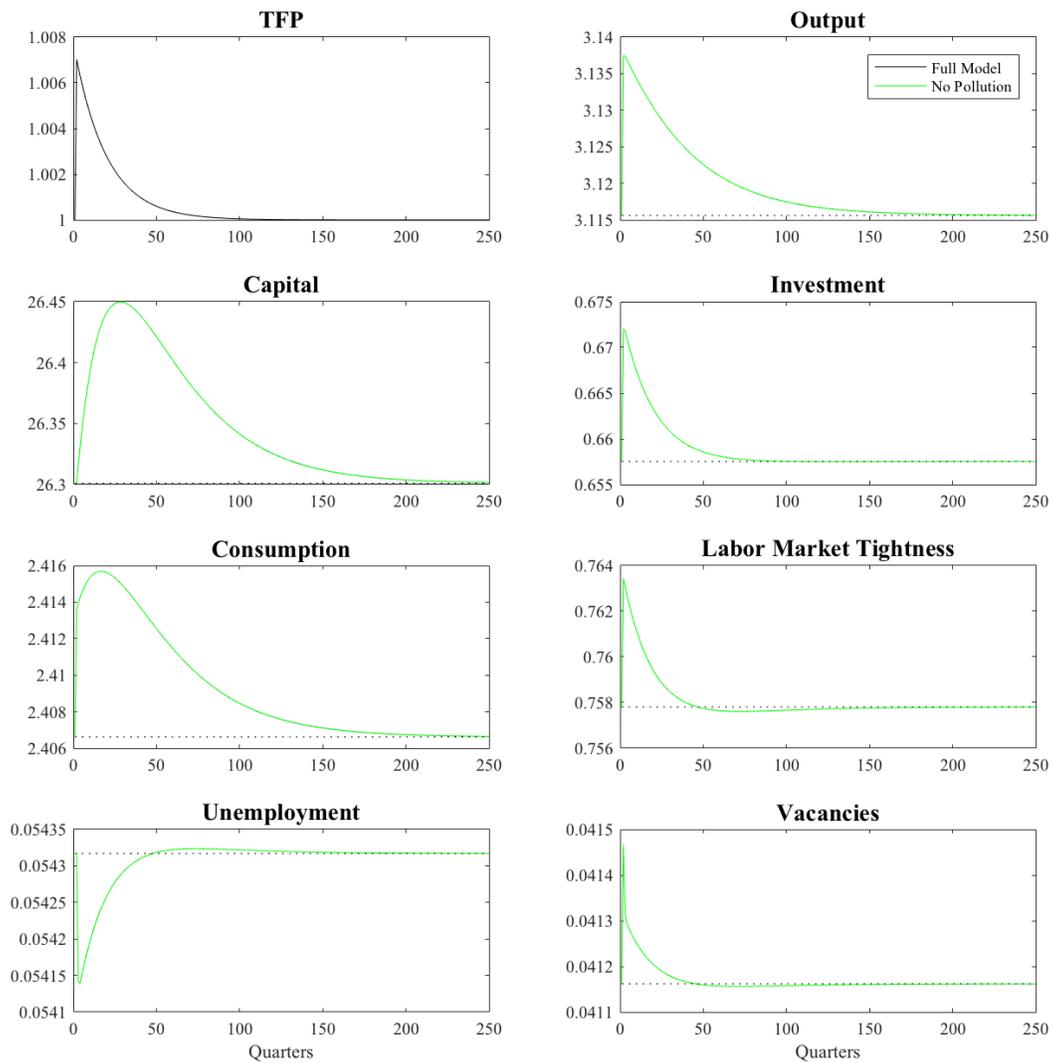
Notes: The black lines are the IRFs for the full model, and the green lines are the IRFs for the model in which labor market search variables are fixed at their steady-state levels. The dotted line indicates the steady-state level.

Figure 4: Business Cycle Simulations - Full Model vs. No Labor Variation



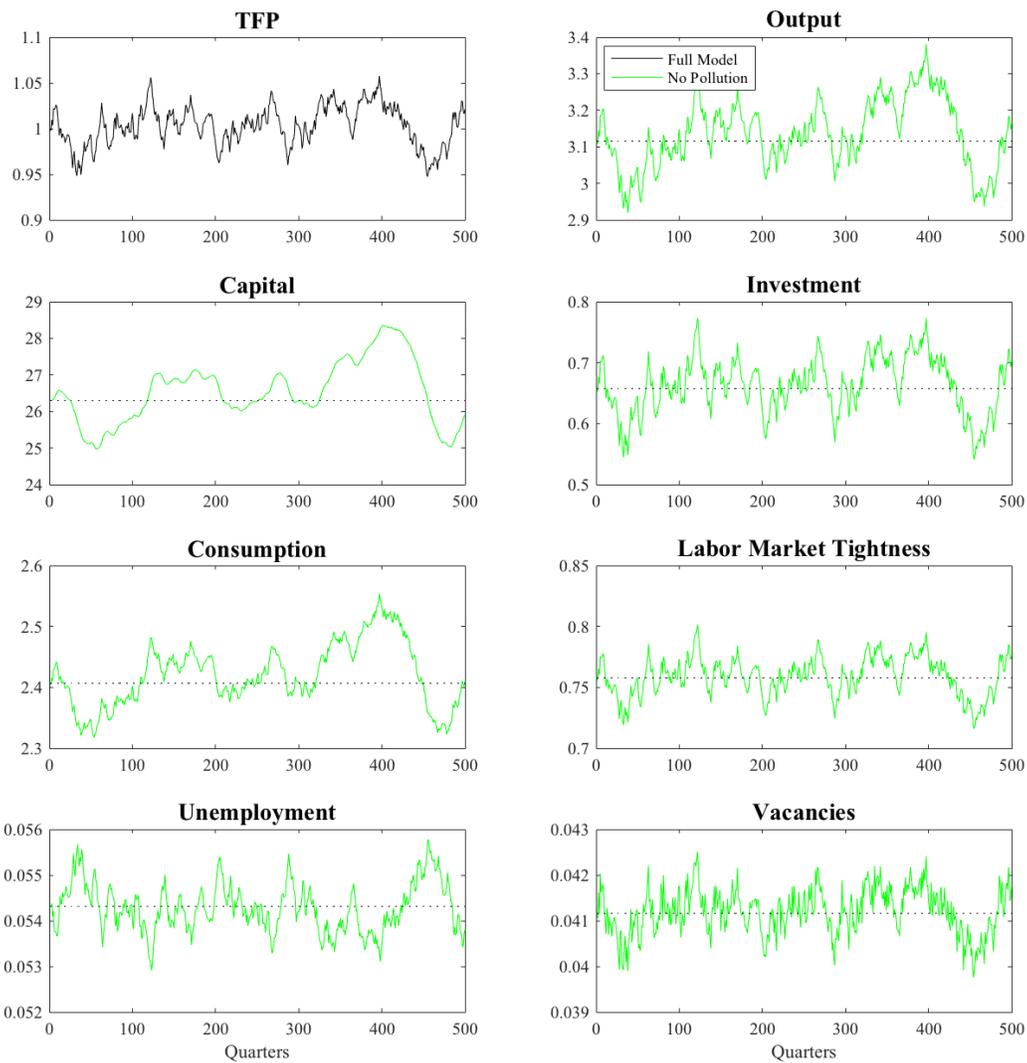
Notes: The black lines are for the full model, and the green lines are for the model in which labor market search variables are fixed at their steady-state levels. The dotted line indicates the steady-state level.

Figure 5: Impulse Response Functions - Full Model vs. No Pollution Variation



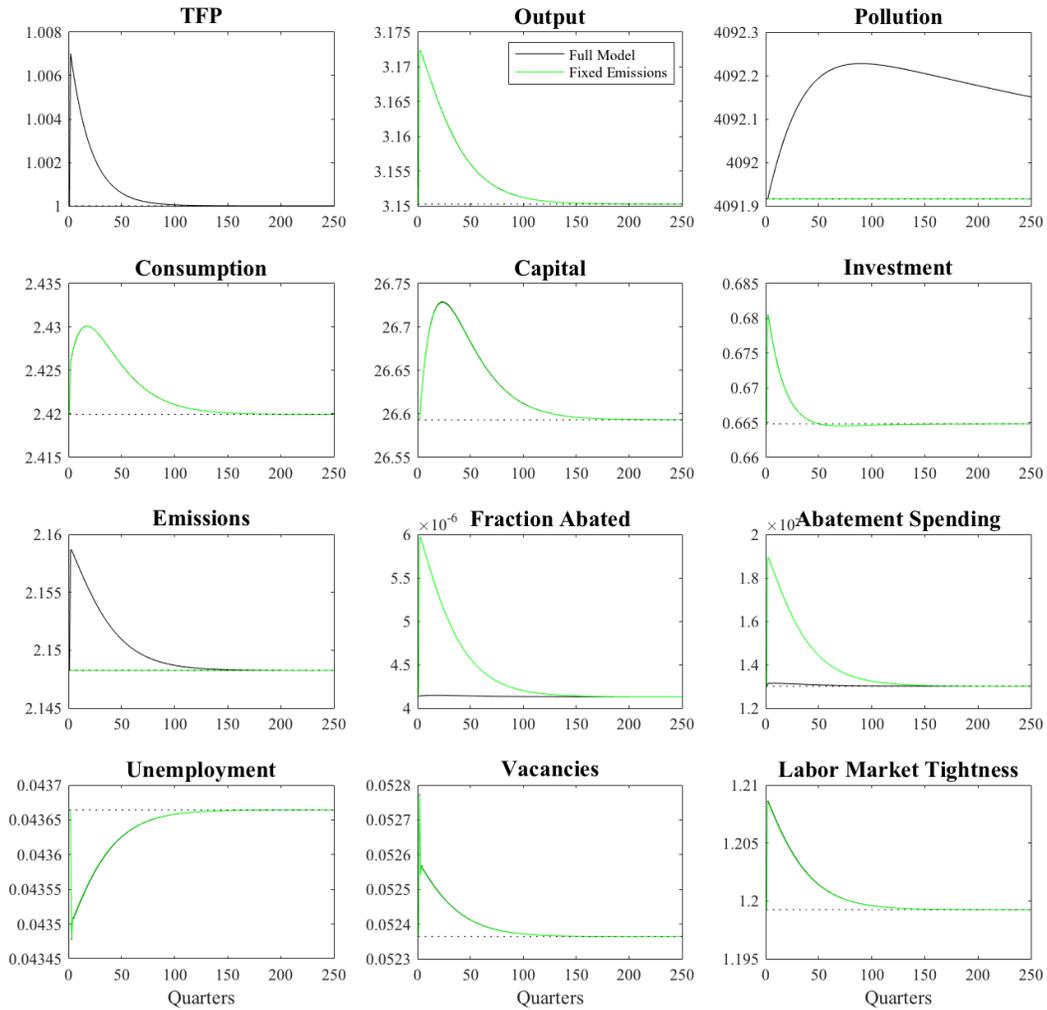
Notes: The black lines are the IRFs for the full model, and the green lines are the IRFs for the model in which pollution is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 6: Business Cycle Simulations - Full Model vs. No Pollution Variation



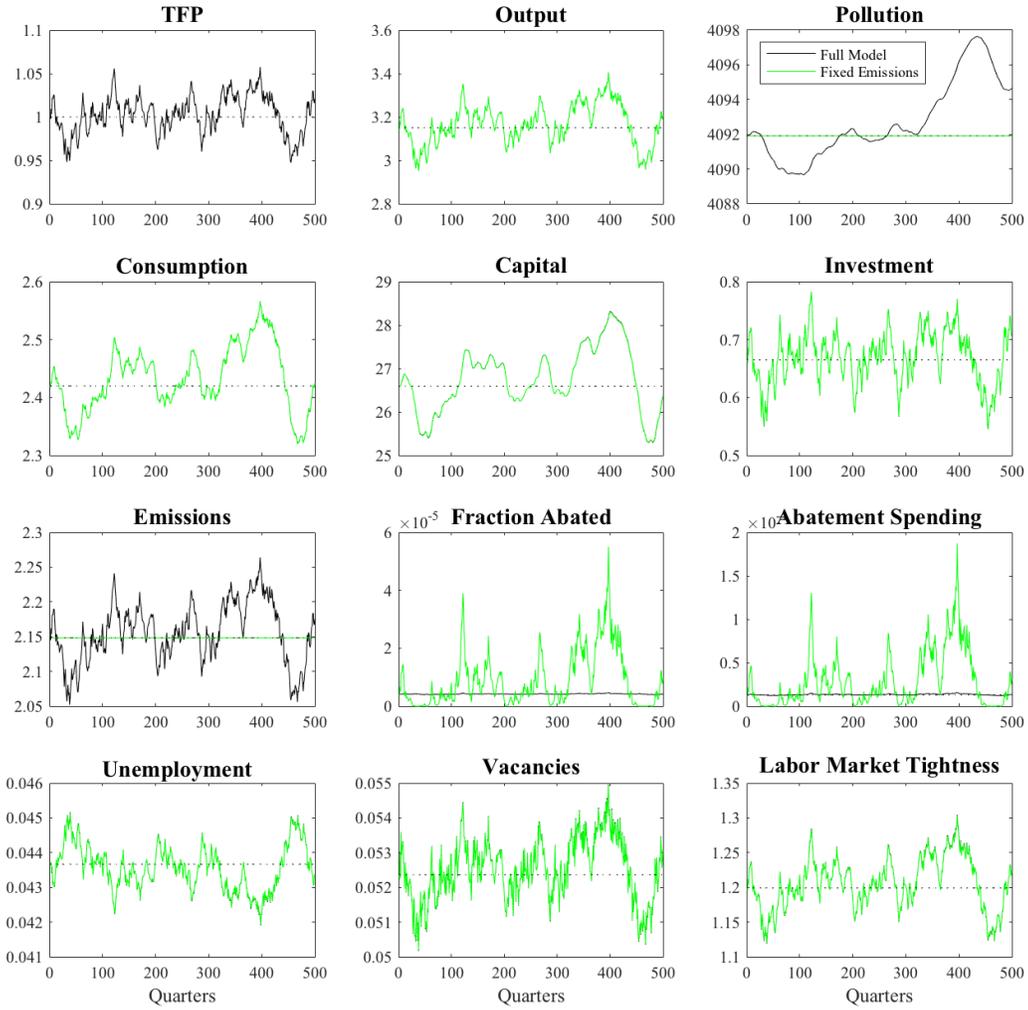
Notes: The black lines are for the full model, and the green lines are for the model in which pollution is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 7: Impulse Response Functions - Full Model vs. Fixed Emissions



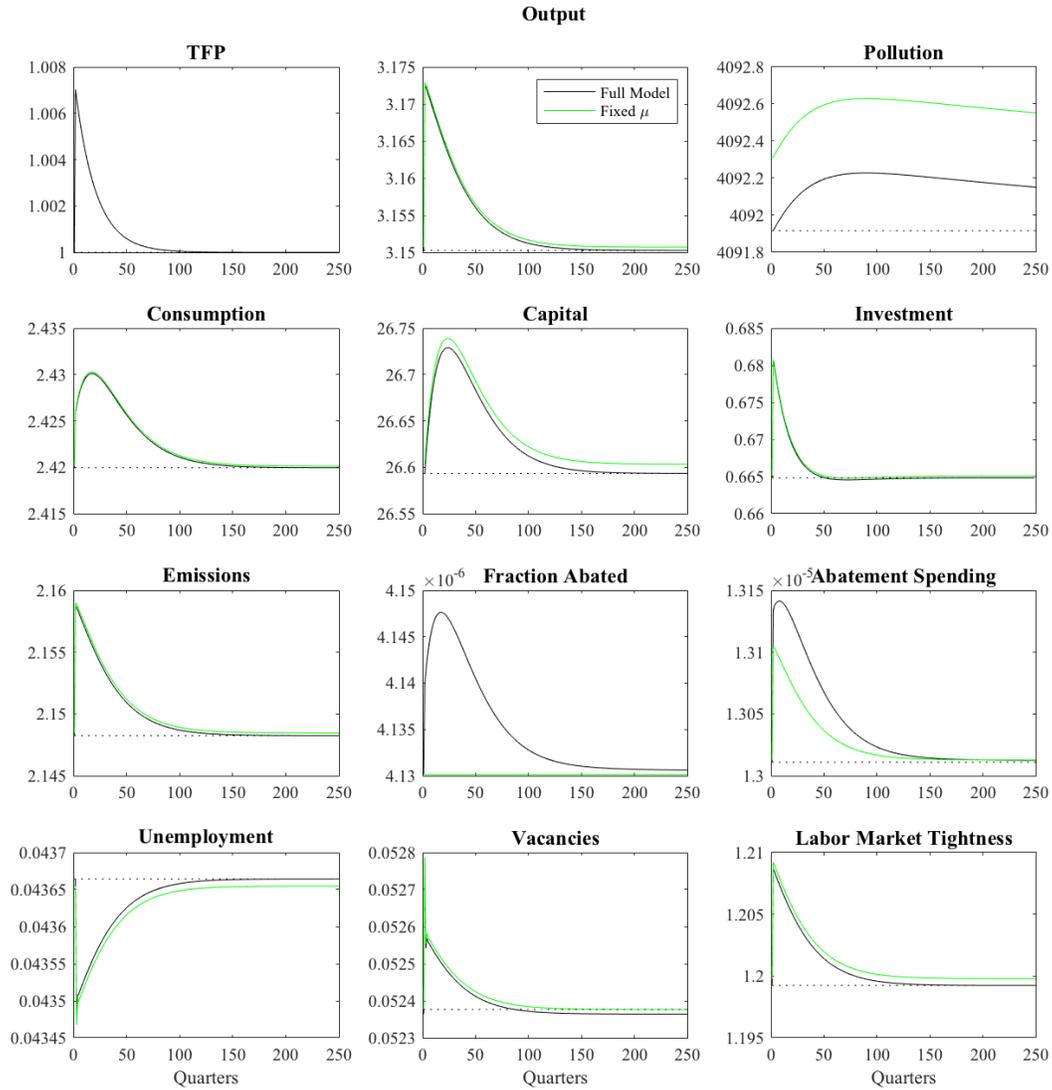
Notes: The black lines are the IRFs for the full model, and the green lines are the IRFs for the model in which emissions is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 8: Business Cycle Simulations - Full Model vs. Fixed Emissions



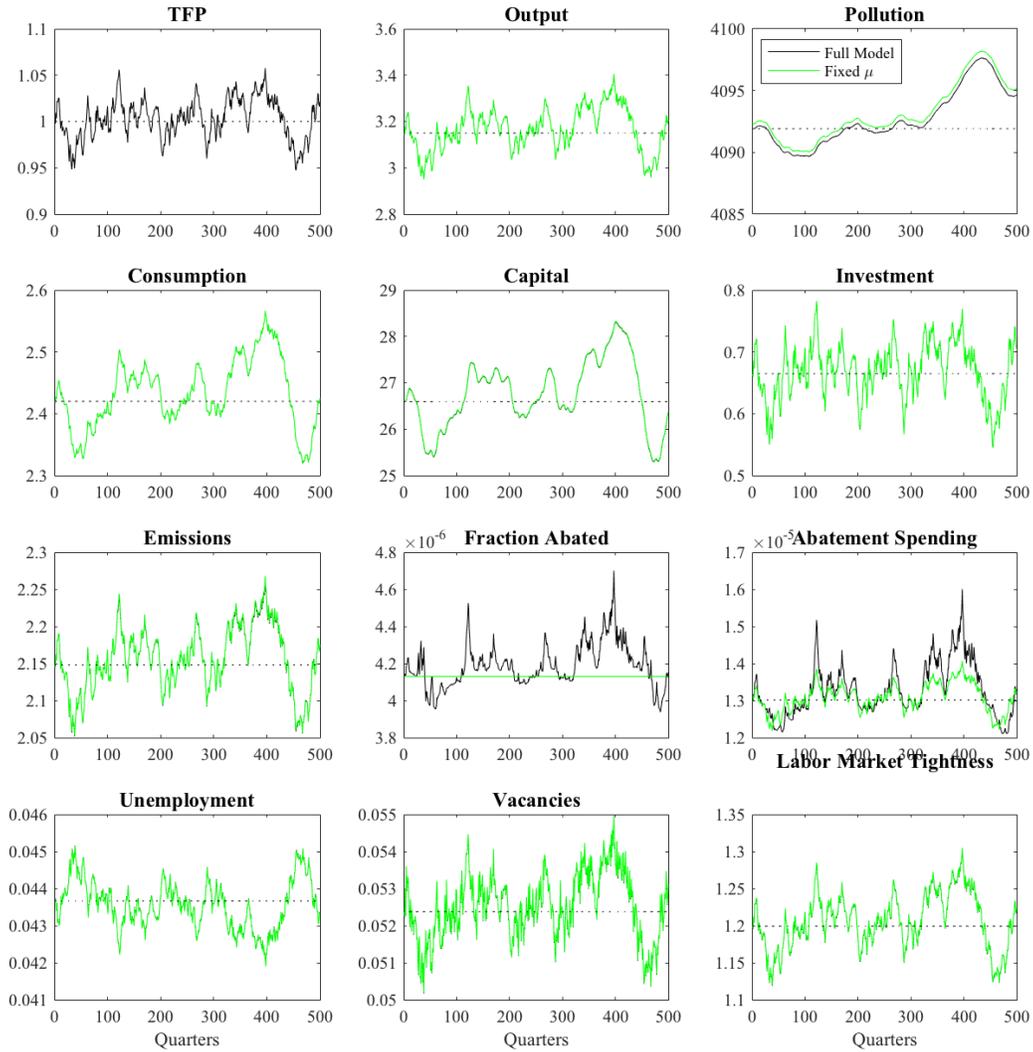
Notes: The black lines are for the full model, and the green lines are for the model in which emissions is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 9: Impulse Response Functions - Full Model vs. Fixed Abatement



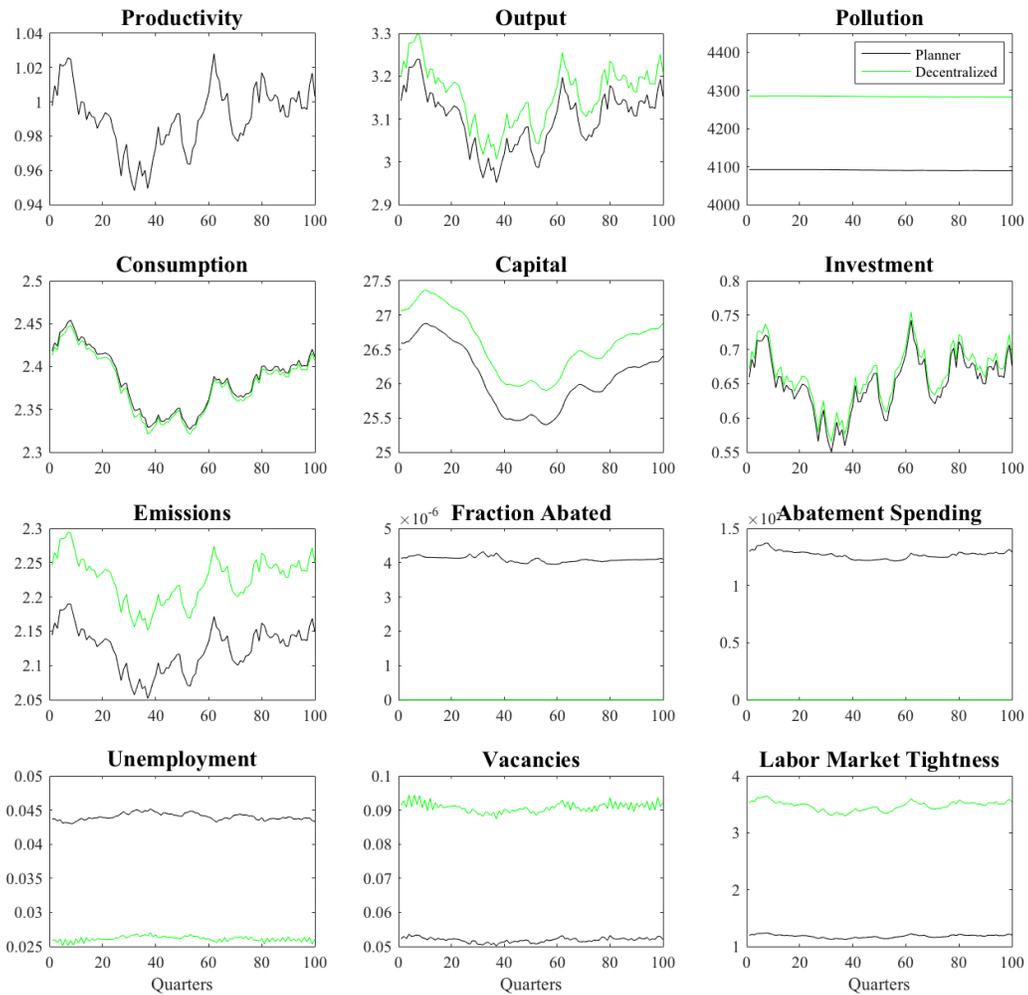
Notes: The black lines are the IRFs for the full model, and the green lines are the IRFs for the model in which the abatement fraction is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 10: Business Cycle Simulations - Full Model vs. Fixed Abatement



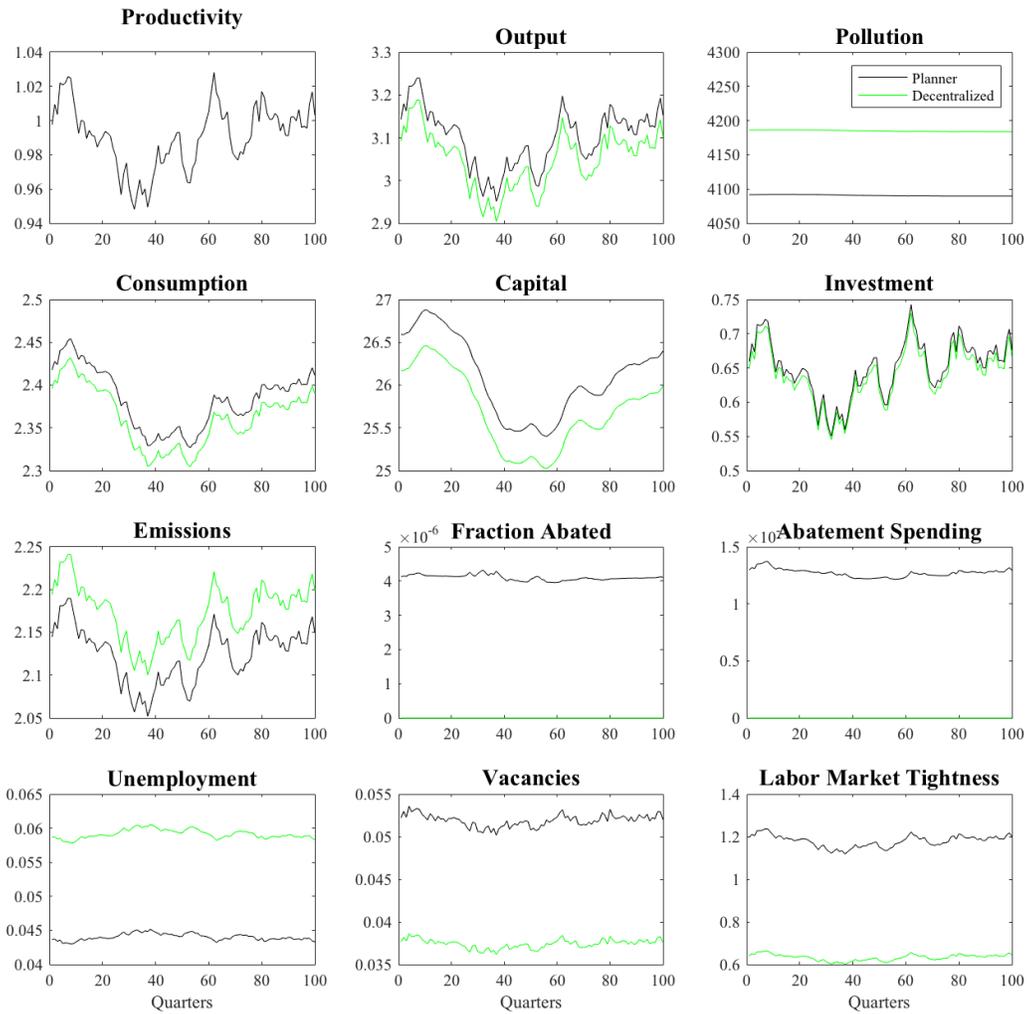
Notes: The black lines are for the full model, and the green lines are for the model in which emissions is fixed at the steady-state level. The dotted line indicates the steady-state level.

Figure 11: Business Cycle Simulations - First-Best vs. Decentralized - Low Bargaining Power



Notes: The black lines are for the first-best planner's problem, and the green lines are for the decentralized model.

Figure 12: Business Cycle Simulations - First-Best vs. Decentralized - High Bargaining Power



Notes: The black lines are for the first-best planner's problem, and the green lines are for the decentralized model.