Income Shares, Secular Stagnation, and the Long-Run Distribution of Wealth

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Abstract

Four alarming stylized facts have recently emerged in the United States: (i) a fall in the labor share of income; (ii) a fall in labor productivity; (iii) an increase in the capital-income ratio, and (iv) an increase in the wealth share owned by top income earners. In this paper, we offer an explanation for these facts that is diametrically opposed to the account provided in Piketty (2014), by drawing from the Pasinetti (1962) approach to differential saving propensities among classes as well as the theory of induced technical change (ITC) by Kennedy (1964). In a simple model with two types of households—high income and low income—and endogenous saving rates, we show that institutional changes such that lower the labor share—declining unionization, increasing monopsony power in the labor market, the so-called 'race to the bottom' in a hyper-competitive global environment, or the exhaustion of path-breaking scientific discoveries as argued by Gordon (2015)—can lower labor productivity growth because of the reduced incentives to innovate to save on labor costs. Combined with ITC, differential saving implies a direct relationship between the capitalist share of wealth and the capital-income ratio *independent of the elasticity of substitution* in production. These tendencies are not inevitable: taxation can be used to implement *any* wealth distribution; while worker-crushing institutional arrangements can be reversed through policy. However, neither change appears likely given the current institutional and global climate.

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1 Introduction

Two spectres are haunting macroeconomics: the specter of secular stagnation, and the specter of inequality. The recent economic history of the United States has been characterized by simultaneous

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occurrence of: (i) a falling labor share; (ii) a falling rate of labor productivity growth; (iii) a rising capital-income ratio, and (iv) an increase in the share of wealth held by the top 1% of wealth owners. Figure (1) plots each of these series for the United States.

[Figure 1 about here.]

Despite the amount of attention that economists have paid to these trends, prominent theoretical explanations of inequality lack not only a clear link between a rising capital-income ratio, a falling labor share, and growing wealth inequality, but also a link from these distributional phenomena to changes in the rate of labor productivity growth. In his best-selling *magnum opus*, Piketty (2014) has argued that a differential between the rate of return on capital and the rate of growth (the famous $r > g$ inequality) is responsible for rising wealth inequality, but provides only a tangential link from an increasing capital-income ratio to rising wealth inequality, via changes in the capital share in national income. Further strain is put on Piketty (2014)'s logic by the fact that within his theoretical framework—a standard one-sector Neoclassical growth model—increases in the capital-income ratio only increase the capital share in national income if the elasticity of substitution between labor and capital is greater than one. As Jones (2016) points out, this is only likely to hold when the capital input to the production function includes land (thus stretching the notion of "capital"). Additionally, the empirical evidence is mixed with respect to whether the elasticity of substitution between capital and labor is higher or smaller than one: Karabarbounis and Neiman (2013) find an elasticity of substitution around 1.25 using a cross-section of countries, while Oberfield and Raval (2014) and Semieniuk (2017) find elasticities of substitution below one. Finally, the Piketty inequality only makes sense when the growth rate g is exogenous and the rate of return r is endogenous: in Classical and Post Keynesian theories, for instance, the growth rate and the profit rate are related through the Cambridge equation $g = sr$ which establishes a causal link from the (exogenous) rate of return to the (endogenous) growth rate via the saving propensity. Since the latter is less than one, the Piketty inequality *always* holds, but is not useful in explaining the increase in the capital-income ratio.

Strict requirements on the degree of substitutability between factors of production as a means of explaining the simultaneous positive trends in wealth inequality and the capital share in national income are not unique to Piketty (2014). Recently, Zamparelli (2017) revisited the debate on the long-run distribution of wealth between classes initiated by Pasinetti (1962) and Samuelson and Modigliani (1966). In an a two-class, exogenously growing economy where "capitalists" save at higher rates than "workers" Pasinetti (1962) famously demonstrated the irrelevance of workers' saving for the determination of the rate of profit. The long-run of this economy is characterized by a distribution of wealth where both workers and capitalists own a positive share of total overall wealth. As a rejoinder, Samuelson and Modigliani (1966) established a "dual" result for the Pasinetti theorem: they showed that a second type of equilibrium exists where workers own all the wealth in the economy, and the capital-output ratio is exclusively determined by the workers' propensity to save. This equilibrium requires a saving rate on behalf of workers that exceeds the savings rate of capitalists. On the other hand, Zamparelli (2017) demonstrates the existence of an "anti-dual" Pasinetti result in a Neoclassical economy with factor substitution: a long-run equilibrium where capitalists own the entire stock of wealth is assured as long as the elasticity of substitution between capital and labor is high enough for the marginal product of capital to converge to a positive constant in the long-run.

In this paper, we offer a non-Neoclassical way of organizing the stylized facts presented above that draws from a number of staples in alternative traditions of economic thought. First, we show that the simultaneous rise of the capital-income ratio and wealth inequality can occur even without any substitutability between factors of production. To make this point, and fully siding with the winning Cambridge of the capital controversy of the 1960s, we adopt a Leontief aggregate production technology for the economy under consideration, so that at each moment in time capital and labor are used in fixed proportions in producing output. Second, instead of allowing instantaneous substitution between capital and labor, we draw from the *induced invention hypothesis* first formalized by Kennedy (1964) drawing from an idea by Hicks (1932) in allowing capitalist firms to change the available production technique by choosing from a menu of factor-augmenting technologies (the so-called *invention possibility frontier*) in order to maximize the rate of unit cost reduction. As is well-known, the hypothesis of induced invention delivers the result that changes in factor-augmenting technologies respond to factor shares. In particular, labor (capital) productivity growth will increase (decrease) following an increase in the share of labor in production.

[Figure 2 about here.]

The third distinctive element of our analysis is the recognition by Classical and Post Keynesian economists that households in different positions on the income ladder save at different rates. Saez and Zucman (2016) show that an important feature of rising wealth inequality is the existence of high savings inequality across wealth levels. Figure 2 reproduces Saez and Zucman (2016)'s depiction of savings rates by wealth class over time. From 1970 onward the savings rate of the top 1% of wealth holders has increased relative to all other wealth classes. This result is not unique to Saez and Zucman (2016). Kumar (2016) finds that the relative saving rate of the top 1 percent of the income distribution in the United States has been roughly 300 percent of the aggregate saving rate since 1980. While Pasinetti (1962), Samuelson and Modigliani (1966), and Zamparelli (2017) all posited differential savings rates between capitalists and workers, none of them offered a behavioral explanation for the difference. Our model differs from these contributions by grounding differential savings rates in the prevalence of other-regarding preferences, which increase consumption and reduce savings and wealth accumulation, at the lower end of the income distribution. In line with the evidence presented in Petach and Tavani (2018), the preferences of households at the bottom and middle of the income distribution are assumed to be negatively affected by the average consumption of similar households,[1] motivating increases in consumption through external habits or "keeping up with the Joneses" behavior. Conversely, and backed by the empirical evidence, top income earners'

[1] See equation (2) below.

consumption appears not to be affected by peer consumption. A saving rate differential between high- and low-income households—as well as a simple relation between the accumulation rates of the two classes—emerges endogenously as a result, ensuring that the savings rate of the latter is always lower, thereby ruling out the "dual" outcome in the long run.

Importantly for our analysis, the combination of induced bias in technology and class-based differential saving rates generates a downward-sloping, long-run relationship between wealth inequality and the income-capital ratio which we will refer to as the "Piketty schedule." This finding is important because, while *Capital in the XXI Century* is silent on the relationship between the capital-income ratio and wealth inequality, its very argument presupposes a direct link between the two: if wealth was equally distributed, there would be no room for the gloomy predictions about an increase in the capital-income ratio translating into class-stratified outcomes with respect to wealth ownership.

It remains to be seen how these distributional changes relate to changes in the growth rate of labor productivity. We capture this within our model via a catch-all shift parameter that affects the induced bias in innovation and directly affects the labor share in the long run. We argue that changes in this parameter can be interpreted in a consistent fashion as a variety of policy and/or institutional changes potentially related to secular stagnation, by which we mean the general slowdown in the rate of labor productivity growth. While one popular explanation for secular stagnation revolves around an excess supply of saving in the market for loanable funds (Summers, 2015), we find this explanation unsatisfactory in that: (i) it hinges on the questionable argument that there exists a value for the nominal interest rate that ensures full employment, and (ii) it ignores important long-run structural forces in the economy related to both income distribution and labor productivity. Such forces include: (a) a slowdown in the growth rate due to the exhaustion of path-breaking scientific discoveries à la Gordon (2015), (b) increasing monopsony power in the labor market (Krueger and Posner, 2018; Dube et al., 2018), (c) globalization and the "race to the bottom" in unit labor costs (Rada and Kiefer, 2015), (d) fiscal austerity (Wisman, 2013), and (e) financialization and growing financial fragility (Skott and Ryoo, 2008; Cynamon and Fazzari, 2016; Michl, 2017). In our model, an institutional shift that results from these forces causes a simultaneous fall in the labor share, a rise in the long-run capital-income ratio, and an increase in wealth inequality along the "Piketty schedule" via differential savings between low income and high-income households.

Thus, the present contribution provides a parsimonious organizing framework for thinking about both secular stagnation and rising wealth inequality in the United States. Over the past thirty years, political and institutional changes have put downward pressure on the labor share in income, resulting in a rising capital-income ratio and a slower rate of labor productivity growth via induced technical change. Empirically-supported differences in savings rates across households have translated the increased wealth accumulation for the top 1% of wealth holders—at the expenses of everyone else—into an increase in the capital-income ratio. It is worth observing once again the difference between our explanation and that by Piketty (2014), where the causal linkages go from $r > g$ to an increase in the capital-income ratio to the falling labor share via a high elasticity of substitution.

We conclude our analysis with a few questions with policy relevance: to what extent are these trends irreversible? First, can taxation be used to counter the observed rise in wealth inequality? And second, are these labor-crushing institutions inevitable? Piketty (2014) delineated an ambitious list of policy proposals to combat growing inequality, none more provocative than his suggestion of a global tax on wealth; demonstrating the economic feasibility of using tax policy to alter the distribution of wealth is a necessary first step if such a policy is to enter the realm of possibility. Pace Zamparelli (2017), we show that a tax on capital income can be used to implement *any* given distribution of wealth between classes. In a way, a similar answer applies to the institutional shifts affecting labor: because of their very institutional nature, these shifts are not inevitable. However, addressing their causes—with the possible exception of the Gordon (2015) argument—requires a degree of international cooperation on labor conditions that is simply not currently in the cards.

The rest of our paper is organized as follows. Section 2 describes the economic environment of the baseline version of our model. Section 3 details the dynamics of the model. Section 4 characterizes the steady-state, presents results from simulations, and examines the policy implications of the model. Sections 5 and 6 discuss versions of the model with constant saving propensities à la Pasinetti (1962) and differential saving motives between classes à la Michl (2009) respectively; while Section 7 illustrates how a potential mechanism for an interior solution for the long-run distribution of wealth to emerge in this model. Section 8 concludes. Most of the mathematical arguments behind our results are presented in the Appendix. Finally, throughout the paper we will focus on the *income-capital* ratio—as opposed to its inverse—because doing so makes the mathematical exposition much easier than the opposite.

2 Main Elements of the Model

2.1 Production and Income Distribution

Consider a one-sector economy in a closed economy without government. Time is continuous, and the total labor force is assumed to be constant and normalized to one for simplicity. The economy is populated by two types of households, which we will refer to as "low-income" (and denote by the superscript L) and "high-income" (labeled H) for simplicity. All households supply their labor services inelastically in exchange for a real wage w; low-income households earn also income from their capital stock (in per capita terms) k^L consume, and save in order to accumulate new capital. High-income households own capital stock (again, per-capita) k^H, also earn profit incomes and wage incomes, consume and save. For the sake of simplicity, assume that neither type of capital depreciates. Output per worker y, homogeneous with capital stock, is produced using fixed proportions of capital per-worker $k \equiv k^H + k^L$ and labor: $y = \min\{A, Bk\}$ where B denotes the output-capital ratio, and A is the stock of labor-augmenting technology. Let r be the uniform rate of return on capital, endogenous to the model (see above) but given to each economic agent. Then, given the economy-wide labor share $\omega \equiv w/A$, total profits in the economy can be written as

$$\Pi = rk = B(1-\omega)(k^H + k^L) \tag{1}$$

where $\omega \equiv w/A$ is the labor share. On the other hand, the total wage bill, given constant returns to scale in production, is $\omega B(k^H + k^L)$, so that the amount of capital owned by each type of households fully determines their income. In fact, total income available to households of type i is $Bk^i, i = \{H, L\}$. The growth rates of factor-augmenting technologies are endogenous to the model, and determined below.

2.2 Economic Classes and Preferences

Both types of households discount the future at the same rate $\rho > 0$. The difference lays in their respective instantaneous preferences. Empirical evidence using Consumer Expenditure Survey data from the United States suggests that consumption peer effects are large—over 30% in magnitude—for the bottom and middle quintiles of the income distribution, but vanish as top income earners are considered (Petach and Tavani, 2018). Thus, we assume that, while high-income households derive (logarithmic, for simplicity) utility from their own consumption c^H, low-income households have (logarithmic, again) preferences that reflect so-called "external habits", or their intent to "keep up with the Joneses" (Ljunqvist and Uhlig, 2000; Turnovsky *et al.*, 2004; Dynan and Ravina, 2007; Alvarez-Cuadrado and Van Long, 2011):

$$u^w(c^L; \bar{c}) = \ln(c^L - \theta\bar{c}) \tag{2}$$

where c^L denotes the low-income household's own consumption, $\bar{c}$ stands for average consumption of the reference group, and $\theta \in (0, 1)$ denotes the extent to which low-income households' preferences are other-regarding. Each of these households takes $\bar{c}$ as a given *at all times* in their decision-making.[2] As such, average consumption across low-income households has the nature of a pure externality: neither households take into account the fact that their decisions affect average consumption within their class, nor consider the effect of changes in average consumption on the (shadow-) value of their wealth.

In order to avoid unnecessary complications, we assume that neither type of household holds debt at any moment in time. Thus, the accumulation constraints are given respectively by

$$\dot{k}^H = w + rk^H - c^H = Bk^H - c^H \tag{3}$$
$$\dot{k}^L = w + rk^L - c^L = Bk^L - c^L \tag{4}$$

Appendix A shows that simple dynamic optimization problems deliver the following Euler equa-

[2] Petach and Tavani (2018) also offer a generalization that delivers similar results.

tions for high-income and low-income households respectively:[3]

$$\frac{\dot{c}^H}{c^H} = B - \rho \tag{5}$$

$$\frac{\dot{c}^L}{c^L} = \frac{c^L - \theta\bar{c}}{c^L}(B - \rho) \tag{6}$$

2.3 Wealth Accumulation

For both types of households, we look at a balanced growth path where their respective consumption and their capital stock grow at the same rate—that may be different between classes, however—$g^i, i = \{H, L\}$. A balanced growth path for high-income households is straightforward: it is a textbook result (see Foley and Michl, 1999, Chapter 5) that consuming a constant fraction of their wealth $c^H = \rho k^H$ ensures that

$$g^H = B - \rho \tag{7}$$

On the other hand, imposing balanced growth for low-income households is slightly more involved. Setting $\dot{c}^L/c^L = \dot{k}^L/k^L$ requires also to impose that the household's consumption equals average consumption: $c^L = \bar{c}$. This is justified observing that the saving rule captured by (6) is a best-response function to average consumption. Imposing $c^L = \bar{c}$ at all times is therefore equivalent to imposing a symmetric Nash equilibrium in a "keeping up with the Joneses" game between the representative low-income household and the average low-income household.[4] By so doing, we obtain the growth rate of capital stock for low-income households as

$$g^L = (1 - \theta)[B - \rho] = (1 - \theta)g^H \tag{8}$$

The accumulation rates (7) and (8) can be used in order to assess the relation between the two classes' saving rates. Noting that, for class $i = \{H, L\}$, the saving rate is defined as $s^i = g^i k^i/(c^i + g^i k^i)$, after simple algebra one obtains:

$$\frac{s^L}{s^H} = 1 - \theta \tag{9}$$

which rules out the result obtained in Samuelson and Modigliani (1966), where a dual equilibrium exists provided that workers save at higher rates than capitalists. Absent social preferences both classes would save at the same rate: it is the extent of social preferences at the bottom and middle of the income distribution that is responsible for differential saving rates between the two classes.

There is also a difference between the workers' saving rate arising in our model and its counterpart in Michl (2009, Chapter 3), where workers save for life cycle purposes, in that they save out

[3]The literature has downplayed the effect of consumption externalities in dynamic optimization models. Petach and Tavani (2018) argued that a proper account of such externalities requires to consider their dynamic effects also, and that CEX data supports this hypothesis. Such considerations are crucial in order to obtain equation (6).

[4]The consumption function implied by the Nash equilibrium is $c^L = (1 - \theta)\rho k^L + \theta B k^L$, and has the flavor of a Keynesian aggregate consumption function with 'marginal propensity to consume' equal to the extent of peer effects. See Petach and Tavani (2018) for a derivation.

of wages they earn during working years in order to finance consumption spending as retirees. We will discuss the implications of this difference for the long-run distribution of wealth in Section 6.

Using (7) and (8), the accumulation rate of this stylized economy will be a weighted average of the growth rates of capital stock of the two types of households, the weight being the share of wealth owned by each class. Denoting the top-income share of wealth by $\phi \equiv k^H/(k^H + k^L)$, we have:

$$g = \phi g^H + (1-\phi)g^L = [1-\theta(1-\phi)][B-\rho] \tag{10}$$

The only difference between the two classes is the extent of consumption peer effects: accordingly, if $\theta = 0$ in equation (10), the class-distinction with respect to accumulation behavior vanishes, and both types of households accumulate at the same rate. In this case, the wealth distribution is irrelevant for long-run growth. As soon as θ becomes positive, however, the accumulation rate is directly related to the top-income share of wealth, given the higher saving rate of high income earners in the economy.

2.4 Technical Change: the Induced Invention Hypothesis

Following Kennedy (1964); Drandakis and Phelps (1965); Funk (2002); Julius (2005), we suppose that firms have access to a menu of technological improvements that potentially can increase both the income-capital ratio (at a rate χ) and labor productivity (at a rate γ). However, there are trade-offs between improving along one technological dimension versus the other. Such trade-offs are summarized by a twice-continuously differentiable, strictly decreasing, strictly concave *invention possibility frontier* (Kennedy, 1964, IPF henceforth) which can be written in explicit form as

$$\gamma = f(\chi), \;\; f' < 0, f'' < 0 \tag{11}$$

Firms choose a profile of technological improvements to maximize the rate of reduction in unit costs, or equivalently the rate of change in the profit rate (see Tavani, 2012, for a duality result):

$$\begin{array}{lll} \text{Choose} \quad \chi & \text{to maximize} & \omega\gamma + (1-\omega)\chi \\ & \text{subject to} & \gamma = f(\chi) \end{array} \tag{12}$$

The solution to the problem yields a dependence on growth rates of factor-augmenting technologies on factor shares through the first-order condition $-f'(\chi) = (1-\omega)/\omega$. Inverting this condition yields a positive (negative) relation between labor (capital) productivity growth and the labor share. We also assume an exogenous intercept of the IPF, denoted by z, which could be interpreted in standard fashion as either as the exogenous 'natural' growth rate or—and this would be our preferred interpretation—as any institutional variable positively affecting the labor share in the long run. Thus, the growth rates of capital- and labor-augmenting technologies that solve (12) can be written

$$\chi = \chi(\omega; z); \quad \gamma = f(\chi(\omega; z)) \tag{13}$$

with $\chi_\omega < 0$—and correspondingly $\gamma_\omega > 0$. In what follows, we assume $\chi_z > 0$.

3 Dynamics of Wealth, Income Shares, and the Income-Capital Ratio

Consider first the share of wealth owned by high-income households. Its law of motion over time obeys the replicator-style equation (see Appendix B for a derivation):

$$\dot{\phi} = \phi(1-\phi)(g^H - g^L) \tag{14}$$

which, using (7) and (8), gives simply

$$\dot{\phi} = \phi(1-\phi)\theta\,[B-\rho] \tag{15}$$

To close the model, we need to specify the dynamic evolution of the labor share. Condensing the insights from Goodwin (1967) model into a single equation, we assume that the rate of change of the labor share increases with capital accumulation, and it decreases with labor productivity growth. The intuition is as follows: as investment takes place, the labor market tightens and the resulting pressure on wages relative to labor productivity determines an increase in the wage share. Conversely, for a given state of the labor market, an increase in labor productivity growth reduces labor requirements thus putting downward pressure on the labor share. With speed of adjustment $\lambda > 0$, we have that:

$$\begin{aligned} \dot{\omega} &= \lambda(g-\gamma)\omega \\ &= \lambda\left\{[B-\rho][1-\theta(1-\phi)] - \gamma(\omega)\right\}\omega \end{aligned} \tag{16}$$

Finally, the evolution of the income-capital ratio is governed by induced invention, and satisfies:

$$\dot{B} = \chi(\omega; z)B \tag{17}$$

Equations (15), (16), and (17) form a dynamical system describing the economy under consideration. We turn to characterizing its steady state and implications.

4 Steady State and Policy

In order to simplify the analysis in what follows, we will utilize a linearized version for both growth rates of factor-augmenting technologies: $\chi(\omega; z) = \delta(z - \beta\omega)$, where $\beta > z > 0$ and δ is a speed-of-adjustment parameter; moreover, we set $\gamma = \eta\omega$. Setting $\dot{B} = 0$ in equation (17) solves for the long-run share of labor as

$$\omega_{ss} = \frac{z}{\beta} \tag{18}$$

which, in turn, gives the long-run growth rate of labor productivity as $\gamma(\omega_{ss}) = \eta(z/\beta)$. Notice that, as pointed out by Julius (2005), the labor share evolves so as to ensure a Harrod-neutral profile of technical change in the long run. Then, setting $\dot{\omega} = 0$ in equation (16) and using (18) gives the following nullcline, or "Piketty schedule:"

$$B(\phi) = \frac{\eta z}{\beta[1 - \theta(1 - \phi)]} + \rho \tag{19}$$

which is downward sloping: an increase in the top wealth share determines a decrease in the income-capital ratio (or equivalently an increase in the capital-income ratio, as highlighted by Piketty, 2014). Finally, so long as the economy is growing (that is, so long as $B > \rho$, the evolution of the wealth composition only has the extreme solutions $\phi_{ss} = 0$ and $\phi_{ss} = 1$, and there is no intermediate steady state where wealth is split among the two classes. This result holds because of the absence of factor substitution due to the fixed-proportion technology: induced bias is not sufficient for a distribution featuring both classes owning wealth to emerge. In the Appendix, we also show that the only (conditionally) stable distribution involves all the wealth accruing to high-income households. In this respect, induced invention reinforces the distributive conflict, contrary to factor substitution *à la* Samuelson and Modigliani (1966) which dampens it so as to make it possible that a stable "dual" distribution is achieved where low-income households own all the wealth in the economy. At $\phi_{ss} = 1$, the steady state income-capital ratio reduces to $B_{ss} = \rho + \eta z/\beta$.

Note the stark difference between the implications of this model and the well-known Piketty argument according to which an increase in the capital-income ratio affects the distribution of income through the production technology via the elasticity of substitution. Here, income distribution is independent of the production technology, but the relationship goes from the top wealth share to the capital-income ratio, and not vice versa.

It is worthwhile also noting that Michl (2009) is able to solve for an interior value of the distribution of wealth in a model without technical change, both under a distributive closure and an exogenous labor supply closure. The difference lays in the assumptions about the drivers of saving for low-income households. As mentioned above, in Michl (2009) workers save for life cycle purposes; here, low-income households too save dynastically, but are subject to peer effects that lower their saving rate relative to the capitalists' saving rate. Consequently, the distribution of wealth evolves to the extreme two-class case in our model. We will work out a version of the model with dynastic as opposed to life-cycle savings below.

4.1 Parameter Calibration and Numerical Simulations

Figure 1 shows that the upward trends in both the top wealth share and the capital-income ratio begin roughly in the 1980s. While labor productivity growth and the labor share have been subject to more fluctuations over the whole period displayed in the Figure, for the purpose of this analysis we can parameterize the model so as to to match the average values of the various endogenous variables of the model between 1950 and 1980, with the qualification that these simulations are meant to provide a qualitative illustration of the processes at work in this model and not to replicate exactly the facts illustrated above. In so doing, we proceed as follows. First, we set the ratio z/β equal to .64, which is roughly the mean value for the labor share over that period. Fixing $z = .04$, this implies $\beta = .0625$. Second, we parameterize $\eta = .0390625$ so as to obtain a labor productivity growth rate of 2.5%—about the average labor productivity growth rate up to 1980. Third, we use the estimates presented in Petach and Tavani (2018) to parameterize the extent of consumption externalities θ at .32. Fourth, noting that the average top wealth share between 1950 and 1980 was about 25% and that the capital-income ratio up to 1980 was roughly 3, we can use the above values in the "Piketty schedule" (19) to calibrate the discount rate at about .267, corresponding to a capital-income ratio of 3.5 at the beginning of the period. Finally, we fix the adjustment speeds in the labor share equation and in the income-capital ratio equation λ and δ, which are inconsequential in determining the steady state of our model, at .05. and .1 respectively. Higher (lower) values for the adjustment speeds would accelerate (slow down) the convergence to the steady state. The left panel of Figure 3 plots the corresponding baseline dynamic trajectories for the three endogenous variables in the model. The trajectory for labor productivity growth is omitted from the plot since it mirrors that of the labor share given the linear specification of induced bias.

[Figure 3 about here.]

4.2 Institutional Change and Secular Stagnation

Consider the effect on labor productivity growth of a reduction in the policy parameter z, which is a catch-all parameter that could capture alternatively: (i) a fall in unionization, (ii) a downward push on real wages arising from globalization, (iii) increased monopsony power in the labor market, (iv) a decline in workers' bargaining power due to financialization, or (v) a reduction in the growth rate of labor productivity growth due to the exhaustion of path-breaking scientific discoveries in the spirit of Gordon (2015). The labor share falls, and labor productivity growth follows as a result of the lessened incentive to bias technological change toward labor. Further, the decline in the labor share has a level effect on the income-capital ratio in (19) which falls as a result. The reason is that the income-capital ratio adjusts so as to ensure that the accumulation rate be equal to labor productivity growth. A falling labor share lessens accumulation in the economy since investment in new capital stock needs to accommodate the lower growth rate of labor productivity in response to the decline in the share of labor. Thus, the output/capital ratio needs to fall (the capital-income ratio needs to rise) to restore balanced growth. The right panel of Figure 3 displays the dynamic trajectories corresponding to a shock to the institutional parameter z: both the labor share and the income-capital ratio reach a new long-run value that sits strictly below the initial steady state.

4.3 Redistribution and Labor Market Institutions

Zamparelli (2017) has shown that, in an "anti-dual" Pasinetti economy, tax policy can be used in order to implement any wealth distribution among the two classes. Suppose that high incomes are taxed proportionally at a rate τ, while the tax proceedings are rebated to low-income households in the form of subsidies. The accumulation rate for the high-income households becomes

$$g^H = B(1-\tau) - \rho \tag{20}$$

while, since the low-income households accumulation constraint is $\dot{k}^L = Bk^L + \tau Bk^H - c$, the corresponding accumulation rate remains (8). Accordingly, the wealth distribution evolves over time following

$$\dot{\phi} = \phi(1-\phi)[\theta(B-\rho) - \tau B] \tag{21}$$

Suppose that the wealth distribution starts at $\bar{\phi}$. Making use of equation (19), it is easy to show that setting a tax rate equal to

$$\tau^* = \theta\left(\frac{B-\rho}{B}\right) \tag{22}$$

is sufficient to keep the wealth distribution constant no matter its composition. Thus, tax policy can be used in order to crystalize the wealth distribution the economy starts off with, preventing it from evolving toward the class-stratified long-run equilibrium.

Consider next the question regarding whether the institutional changes that constitute the first link of the chain reaction described in this paper are irreversible. Clearly, the answer is negative—with the possible exception of the Gordon (2015) argument. During the period between the 1950s and the 1970s, characterized by the so-called "capital-labor accord," the US economy saw high labor productivity growth coexisting with strong labor market institutions. The accord has faltered under the pressures of globalization on real US wages, the decline in unionization, and the overall retreat of the labor movement that have characterized the neoliberal era. In principle, there is no reason to see these developments as inevitable. Institutional changes are not a mechanistic process. The main issue, then, becomes the creation of a broad enough consensus about a reversal of these developments, and a coordinated solution to the tendency to suppress labor in the "race to the bottom" highlighted by Rada and Kiefer (2015).

5 Capitalists and Workers: Constant Saving Propensities

For comparison with both the older literature (Pasinetti, 1962; Samuelson and Modigliani, 1966) and the more recent one (Zamparelli, 2017), we also analyze a version of the model with one class that only earns profit income and one class that earns both wage and profit income, and constant saving propensities. In this Section, we focus on the steady state only to show that the conclusions of the analysis—that is, the tendency of the economy to settle onto the class-stratified equilibrium—still hold in the present framework except in a knife-edge case that requires a specific restriction on the parameters. Suppose that s^H, s^L denote the propensity to save out of income of workers and capitalists respectively, both within zero and one. The accumulation rates are then given by $g^H = s^H B(1-\omega)$ and $g^L = s^L B$. The evolution of the capitalist share of wealth satisfies:

$$\dot{\phi} = \phi(1-\phi)B[s^H(1-\omega) - s^L] \tag{23}$$

while the law of motion of the labor share is

$$\dot{\omega} = \lambda \left\{ B \left[\phi s^H (1-\omega) + (1-\phi) s^L \right] - \eta\omega \right\} \tag{24}$$

and the change in the income-capital ratio is still given by (17), which pins down the steady state labor share as in (18). A glance at (23) shows that there exists a knife-edge parametric configuration that ensures that the wealth distribution remain constant at any interior value. Such would be the case, using again (18) if

$$s^L = \frac{\beta - z}{\beta} s^H \equiv \bar{s}$$

giving a steady state income-capital ratio equal to $B_{ss} = \gamma / s^L$, independent of the capitalist share of wealth and decreasing in the workers' saving rate. Provided that the saving rates satisfy this restriction, the long-run value for the wealth distribution remains constant at its initial level, say $\bar{\phi}$.

6 Dynastic vs. Life-Cycle Savings

Michl (2009) has analyzed the distribution of wealth in a two-class economy where low-income households save for life-cycle purposes while high-income households save in order to leave bequests, thus behaving dynastically. Such an alternative specification of saving motives makes for an interesting comparison with our model, because saving rates are endogenous but determined differently than in our framework. Yet, the conclusions of this section mirror quite closely the model with constant saving propensities outlined just above. Suppose that the peer-effect parameter θ is proportional to the marginal propensity to consume out of wages,[5] so that the accumulation rate for low incomes g^L is equal to $(1-\theta)\omega B$, while the accumulation rate for high income earners is equal to $B - \rho$ as before. The evolution of the wealth composition now satisfies:

$$\dot{\phi} = \phi(1-\phi)\{B[1 - \omega(1-\theta)] - \rho\} \tag{25}$$

while the labor share evolves according to:

$$\begin{aligned} \dot{\omega} &= \lambda\{\phi(g^H - g^L) + g^L - \eta\omega\}\omega \\ &= \lambda\{\phi[B(1-\omega(1-\theta) - \rho] + (1-\theta)B\omega - \eta\omega\}\omega \end{aligned} \tag{26}$$

Similarly to the previous section, the wealth distribution will remain constant provided that the following condition holds:

$$1 - \theta = \frac{B_{ss} - \rho}{\omega_{ss} B_{ss}} \tag{27}$$

If this is the case, then the wealth distribution remains constant at any interior value $\bar{\phi}$, while the evolution of the output-capital ratio and of the labor share determine the two remaining endogenous

[5] If 'workers' consume out of both wage and profit income, but are subject to peer effects, the model collapses to the baseline version analyzed above.

variables.

7 An Interior Long-run Solution for the Distribution of Wealth

Saez and Zucman (2016) have shown that different classes might earn different rates of return on wealth. To conclude the analysis, and for illustrative purposes, we consider the effect of externalities such that the income-capital ratios are class-specific, and increase in the share of wealth owned by each class. A justification of this hypothesis may be the following: as the share of wealth increases for households of a certain type, their ability to secure funding for more remunerative investment projects also improves. Suppose accordingly that $B^H = b^H\phi$ and $B^L = b^L(1-\phi)$. The wealth distribution, assuming away consumption peer-effects for simplicity, evolves according to:

$$\dot{\phi} = \phi(1-\phi)[b^H\phi - b^L(1-\phi)] \tag{28}$$

and has an interior steady state at $\phi_{ss} = b^L/(b^H + b^L) \in (0,1)$. Supposing that both types of capital are subject to the same choice of technique as per the induced invention hypothesis—so that $\dot{b}^L = \dot{b}^H = 0$ solves for the long-run labor share $\omega_{ss} = z/\beta$ as before—a version of the "Piketty schedule" for this model is found from the steady state of the corresponding version of equation (26) as $b^L = \eta z/[\beta(1-\phi)]$, again delivering an inverse relationship between the income-capital ratio of the low-income households and their share in total wealth $1-\phi$.

8 Conclusion

In this paper, we drew from a number of established alternative traditions in economic theory to present a stylized model that can provide a political-economy account of the recent stylized facts regarding the increase in wealth inequality, the increase in the capital-income ratio, the decline in the labor share of income, and the decline in labor productivity growth in the United States.

The main mechanisms at work can be summarized as follows. Either the erosion of labor market institutions or the rise of globalization is responsible for the fall in the labor share of income. The induced invention mechanism implies that the growth rate of labor productivity will fall as a result, because firms' incentives to innovate in order to save on unit labor costs are lessened by this process. Differential saving rates, which respond to class-specific degrees of emulation in consumption that are supported by empirical evidence, determine the progressive concentration of wealth in the hands of high-saving households. As wealth concentrates, the capital-income ratio increases: since the anchor to long-run growth is the growth rate of labor productivity, which has declined, restoring balanced growth requires an increase in the capital-income ratio—or equivalently a decrease in the income-capital ratio.

Our intuition for the ongoing transformations in the US economy is diametrically opposed to the technological explanation put forward by Piketty (2014) where the degree of substitutability between capital and labor is responsible for the fall in the labor share given the increase in the capital-income ratio. Conversely, our view is that institutions matter: declines in labor protection are the first—not the last—link of the chain reaction that set in motion the global economic conjuncture.

We also argued that these outcomes are not inevitable: on the one hand, tax policy can be used in order to stop the otherwise natural process of wealth accumulation in the hands of high-saving households. On the other hand, worker-crushing policies or global arrangements can be reversed provided that there is the political will to do so.

However, there is not much to be optimistic about the reversal of this process. First, and as is well-known after the recent literature on the 'race to the bottom' (Rada and Kiefer, 2015), individual countries do have incentives to suppress labor in order to increase (or at least not to decrease) their export share in the global economy. Therefore, labor-friendly policies require international coordination: but there are no global agreements or mechanisms in place to enforce a coordinated effort toward this aim. Second, even if the US were to act unilaterally to address anti-worker institutional arrangements (such as the recent growth of monopsony power in the labor market documented by Dube et al., 2018), capital moves quite easily around the globe. Thus, as long as there is policy competition between countries geared toward attracting wealth by redistributing away from wages, there is little room to hope for the kind of policy solutions discussed in this paper to take place.

A Dynamic Optimization

With co-state variables μ_H, μ_L for the two types of households respectively, the current-value Hamiltonian functionals for the high-income households and the low-income households are:

$$\mathcal{H}^H = \ln c^H + \mu_H[Bk^H - c^H] \tag{29}$$

$$\mathcal{H}^L = \ln(c^L - \theta\bar{c}) + \mu_L[Bk^L - c^L] \tag{30}$$

and the battery of first-order conditions is:

$$(c^H)^{-1} = \mu_H \tag{31}$$

$$\rho\mu_H - \dot{\mu}_H = \mu_H B \tag{32}$$

$$(c^L - \theta\bar{c})^{-1} = \mu_L \tag{33}$$

$$\rho\mu_L - \dot{\mu}_L = \mu_L B \tag{34}$$

plus the usual transversality conditions on both types of capital stocks. Differentiating (31) with respect to time and making use of (32) gives (5). Differentiating (33) and using (34), while keeping $\bar{c}$ as an externality throughout—so that low-income households do not consider the effect of its rate of change on the (shadow-) value of their own wealth—gives (6).

B On the Evolution of the Capitalist Share of Wealth

Start from the definition of $\phi \equiv k^H/(k^H + k^L)$, and differentiate with respect to time to obtain:

$$\begin{aligned}
\dot{\phi} &= \frac{\dot{k}^H(k^H + k^L)}{(k^H + k^L)^2} - \frac{k^H}{k^H + k^L}\left(\frac{\dot{k}^L + \dot{k}^H}{k^H + k^L}\right) \\
&= \frac{\dot{k}^H}{k^H}\left(\frac{k^H}{k^H + k^L}\right) - \frac{k^H}{k^H + k^L}\left[\left(\frac{k^H}{k^H + k^L}\right)\frac{\dot{k}^H}{k^H} + \left(\frac{k^L}{k^H + k^L}\right)\frac{\dot{k}^L}{k^L}\right] \\
&= \phi g^H - \phi\left[\phi g^H + (1 - \phi)g^L\right] \\
&= \phi(1 - \phi)(g^H - g^L)
\end{aligned}$$

C Stability Analysis

We start with linearizing the dynamical system around the steady state where all wealth is the hands of the high-income households ($\phi_{ss} = 1$). This yields a Jacobian matrix with the following sign structure:

$$J(\omega_{ss}, 1, B_{ss}) = \begin{bmatrix} - & + & + \\ 0 & - & 0 \\ - & 0 & 0 \end{bmatrix}$$

given that

$$\begin{array}{rcll} J_{11} & = & -\lambda\eta(z/\beta) & < & 0; \\ J_{12} & = & \lambda\theta(B_{ss}-\rho)(z/\beta) & > & 0; \\ J_{13} & = & \lambda z/\beta & > & 0; \\ J_{21} & = & J_{23} & = & 0; \\ J_{22} & = & -\theta(B_{ss}-\rho) & < & 0; \\ J_{31} & = & -\beta B_{ss} & < & 0; \\ J_{32} & = & J_{33} & = & 0. \end{array}$$

The Routh-Hurwitz conditions for local stability are as follows:

1. $TrJ < 0$, which is clearly satisfied.

2. $DetJ < 0$. We have $DetJ = -J_{31}J_{13}J_{22} < 0$ as required.

3. $PmJ > 0$, that is a positive value for the sum of the principal minors —the determinants of the sub-matrices obtained removing the first, second, and third row and column respectively. We have that $PmJ = -J_{13}J_{31} + J_{11}J_{22} > 0$ as required.

4. The final condition requires that $-TrJPmJ + DetJ < 0$. After some algebra, this boils down to checking whether $J_{11}(J_{31}J_{13} - J_{11}J_{22} - J_{22}^2) < 0$. This condition is not satisfied. In fact, we know that $J_{11} < 0$, that $J_{13}J_{31} - J_{11}J_{22} < 0$ because of condition 3 above, and that $-J_{22}^2 < 0$ always, so that we end up with a positive value for that product.

Thus, the equilibrium is in principle unstable. However, the forward-looking nature of consumption allows the corresponding initial value to be picked freely: consumption can function as a jump variable in this case to bring the dynamics onto the stable manifold converging to the steady state. If the number of unstable roots in the Jacobian is equal to the number of jump variables, then the system satisfies the well-known Blanchard and Kahn (1980) requirements for *conditional* (or saddle-path) stability.

We can then turn to a numerical evaluation of whether the condition holds. Under the baseline parameterization, the Jacobian matrix evaluated at $\phi_{ss} = 1$ has two negative (stable) eigenvalues $\varepsilon_1 \simeq -.0182, \varepsilon_2 = -.008$ and one positive (unstable) eigenvalue $\varepsilon_3 = .0032$. We conclude that the equilibrium with $\phi_{ss} = 1$ is conditionally stable.

At $\phi_{ss} = 0$, the J_{22} entry turns positive—it is equal to $\theta[B_{ss} - \rho] > 0$—thus making it more difficult to check the various conditions analytically given, for instance, the ambiguity in the sign of the trace of the Jacobian matrix. Thus, we resort to evaluating the eigenvalues numerically at the baseline parameterization. We find two unstable roots $\varepsilon_2 \simeq .022, \varepsilon_3 \simeq .012$ and one stable root $\varepsilon_1 \simeq -.019$, while the number of jump variables is again one—consumption. Therefore, in this case the corresponding equilibrium is fully unstable, as confirmed by a quick glance at Figure 3, where the dynamics clearly pulls away from the $\phi_{ss} = 0$ steady state.

References

Alvarez-Cuadrado, F., and Van Long, N. (2011): 'The Relative Income Hypothesis', *Journal of Economic Dynamics and Control*, 35, pp. 1489-1501.

Bertola, G. (1993): 'Factor Shares and Savings in Endogenous Growth', *American Economic Review* 83 (5), pp. 1184-1198.

Blanchard, O. J., and Kahn, C. M., (1980): 'The Solution of Linear Difference Models under Rational Expectations', *Econometrica*, 48 (5), pp. 1305-1311.

Cynamon, B., and Fazzari, S. (2016): 'Inequality, the Great Recession and Slow Recovery', *Cambridge Journal of Economics*, 40 (2), pp. 373-399.

Drandakis, E., and Phelps, E., (1965): 'A Model of Induced Invention, Growth and Distribution, *Economic Journal*, Vol. 76 (304), pp. 823-840.

Dube, A., Jacobs, J., Naidu, S., and Suri, S. (2018): 'Monopsony in Online Labor Markets', *NBER Working Paper*, No. 24416.

Duesenberry, J. (1949): *Income, Saving, and the Theory of Consumer Behavior*. Cambridge, MA: Harvard University Press.

Dynan, K. E., and Ravina, E., (2007): 'Increasing Income Inequality, External Habits, and Self-Reported Happiness', *American Economic Review* 97(2), pp. 226-231.

Foley, D. K., and Michl, T. R., 1999. *Growth and Distribution*. Cambridge, MA: Harvard University Press.

Funk, P., (2002): 'induced invention Revisited', *Economica* 69, pp. 155-171.

Goodwin, R.M. (1967): "A Growth Cycle." in Feinstein, C.H., ed. *Socialism, Capitalism, and Economic Growth*. Cambridge: Cambridge University Press.

Gordon, R. (2015): 'Secular Stagnation: A Supply-Side View', *American Economic Review: Papers and Proceedings*, 105(5), pp 54-59.

Hicks, J. R. (1932): *The Theory of Wages*, London: MacMillan.

Jones, C. (2016): 'Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality.' *Journal of Economic Perspectives*, 29(1), pp. 29-46.

Julius, A. J., (2005): 'Steady State Growth and Distribution with an Endogenous Direction of Technical Change', *Metroeconomica*, 56 (1), pp. 101-125.

Karabarbounis, L., and Neiman, B. (2013): 'The Global Decline in the Labor Share', *Quarterly Journal of Economics*, 129(1), pp. 61-103.

Kennedy, C., (1964): 'Induced Bias in Innovation and the Theory of Distribution', *Economic Journal* 74: 541-47.

Krueger, A., and Posner, E. (2018): 'A Proposal for Protecting Low-Income Workers from Monopsony and Collusion', *Hamilton Project*, Policy Proposal 2018-05.

Kumar, R. (2016): 'Personal Savings from Top Incomes and Household Accumulation in the United States', *International Journal of Political Economy*, (45), pp. 224-240.

Ljungqvist, L.,and Uhlig, H., (2000): 'Tax Policy and Aggregate Demand Management under Catching up with the Joneses', *American Economic Review* 90, pp. 356-366.

Michl, T. R. (2009) *Capitalists, Workers, and Fiscal Policy*, Cambridge MA: Harvard University Press.

Michl, T. R. (2017). 'Profit-led Growth and the Stock Market', *Review of Keynesian Economics*, 5(1), pp. 61-77.

Oberfield, E., and Raval, D. (2014): 'Micro Data and Macro Technology', *NBER Working Paper*, No. 20452.

Pasinetti, L. (1962): 'Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth', *Review of Economic Studies*, 29(4), pp. 267-279.

Petach, L., and Tavani, D., (2018): 'Consumption Externalities and Growth: Theory and Evidence for the United States', (February 22, 2018). Available at SSRN: `https://ssrn.com/abstract=3050836`.

Piketty, T., 2014. *Capital in the XXI Century*. Cambridge, MA and London, UK: Belknap Harvard.

Piketty, T, Saez, E., and Zucman, G. (2018): 'Distributional National Accounts: Methods and Estimates for the United States', *Quarterly Journal of Economics*, Forthcoming.

Rada, C., and Kiefer, D. (2015). 'Profit Maximising Goes Global: The Race to the Bottom', *Cambridge Journal of Economics*, 39(5), pp. 1333-1350.

Rognlie, M. (2015): 'Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?', *Brookings Papers on Economic Activity*, Spring 2015.

Saez, E., and Zucman, G. (2016): 'Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data', *Quarterly Journal of Economics*, 131(2), pp. 519-578.

Samuelson, P., and Modigliani, F. (1966): 'The Pasinetti Paradox in Neoclassical and More General Models', *Review of Economic Studies*, 33(4), pp. 269-301.

Semieniuk, G. (2017): 'Piketty's Elasticity of Substitution: A Critique', *Review of Political Economy*, 29(1), pp. 64-79.

Skott, P., and Ryoo, S. (2008). 'Macroeconomic Implications of Financialisation', *Cambridge Journal of Economics*, 32(6), pp. 827-862.

Summers, L., (2015): 'Demand Side Secular Stagnation', *American Economic Review*, 105(5), pp. 60-65.

Tavani, D., (2012): 'Wage Bargaining and Induced Technical Change in a Linear Economy: Model and Application to the US, 1963-2003' *Structural Change and Economic Dynamics* 23, pp. 117-126.

Tavani, D., and Zamparelli, L., (2017): 'Endogenous Technical Change in Alternative Theories of Growth and Distribution', *Journal of Economic Surveys* 31(5), pp. 1272-1303.

Turnovsky, S., Monteiro, G., and Alvarez-Cuadrado, F. (2004): 'Habit Formation, Catching up with the Joneses, and Economic Growth', *Journal of Economic Growth*, 9, pp. 47-80.

van der Ploeg, R., (1985): 'Classical Growth Cycles.' *Metroeconomica* 37 (2), pp. 221-230.

Veblen, T. (1899). *The Theory of the Leisure Class*. New York: MacMillan.

Wisman, J. (2013). 'Wage Stagnation, Rising Inequality and the Financial Crisis of 2008', *Cambridge Journal of Economics*, 37(4), pp. 921-945.

Zamparelli, L. (2017): 'Wealth Distribution, Elasticity of Substitution, and Piketty: An 'Anti-Dual' Pasinetti Economy', *Metroeconomica* 68(4), pp. 927-946.

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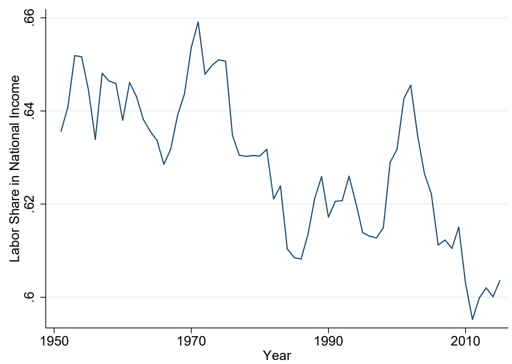
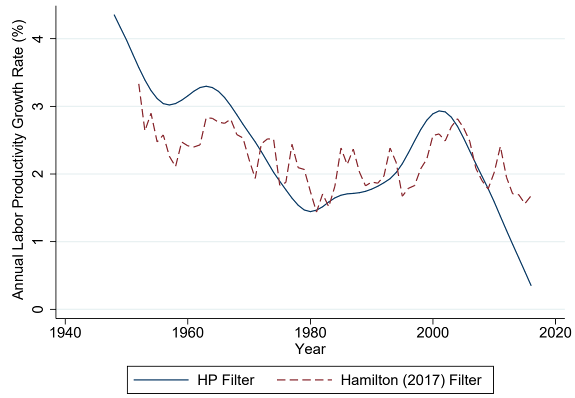
Figures

Figure 1: Secular Stagnation and Inequality in the United States: Stylized Facts

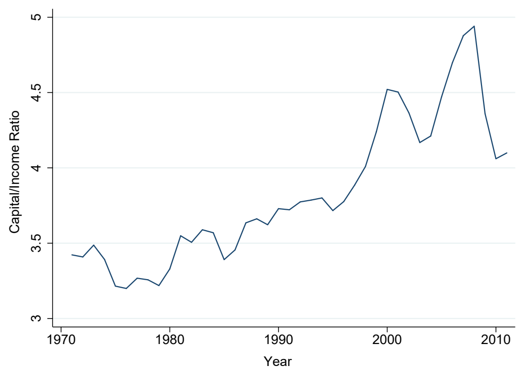
(a) Share of Labor Compensation in GDP, 1950 - 2014



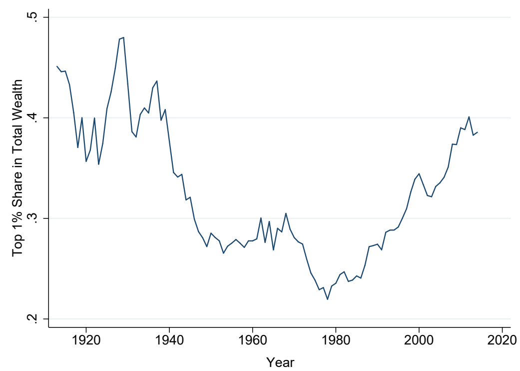
(b) Labor Productivity Growth, 1940-2016



(c) Capital-Income Ratio, 1970-2010

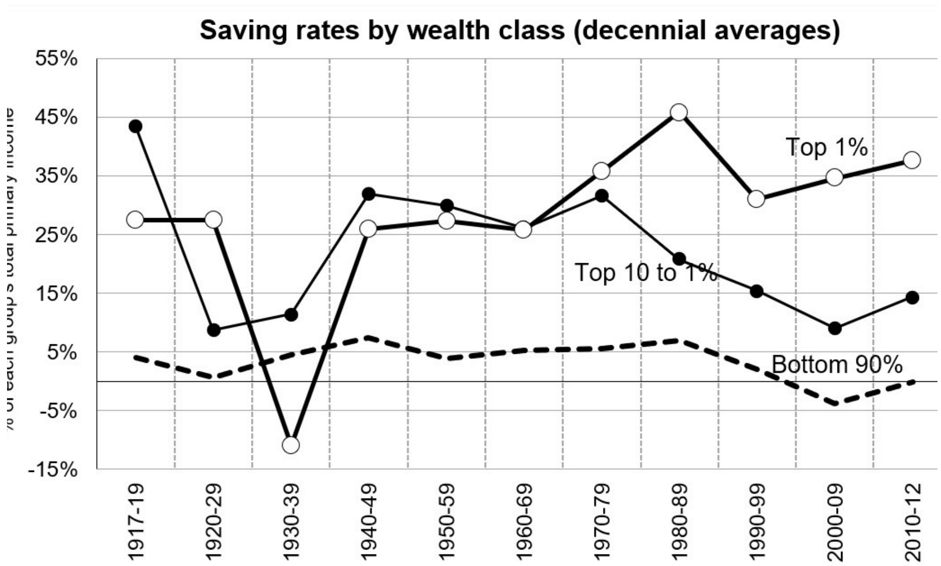


(d) Share of Top 1% in Total Wealth, 1913-2014



Notes: Data on the labor share, labor productivity, the top 1% wealth share, and the capital-income ratio are from the Federal Reserve, the Bureau of Labor Statistics, the World Top Incomes Database, and Piketty (2014), respectively. Figure (1b) plots the trend component of labor productivity growth.

Figure 2: Evolution of Savings in the United States by Wealth Class



Source: Saez and Zucman (2016)

Figure 3: Simulated trajectories for the labor share, the income-capital ratio, and the capitalist share of wealth. Parameter values: $\theta = .32, \rho = .267, z = .4$ (left panel), $z = .35$ (right panel), $\beta = .625 = \eta, \lambda = .05, \delta = .1$. The *Mathematica* code used for these simulations is available from the authors upon request.