Consumer Search with Imperfect Vertical-quality Information

Yijuan Chen[*], Xiangting Hu[†], Sanxi Li[‡]

January 3, 2019

Abstract

In online markets the vertical product quality is usually not perfectly observed by buyers even upon visiting a store. We incorporate this feature into a consumer search model and examine its impact on market outcomes. In a baseline model where the vertical quality is drawn from a binary support, we show that imperfect vertical-quality information leads to two lines between the market price and consumer's reservation utility, namely the "search value line" that characterizes the consumer's search behavior, and the "price incentive line" that describes the firm's pricing decision. Their intersection determines the equilibrium. In contrast to the literature on consumer search that often assumes perfect information of vertical quality, we show that a reduced search cost will result in a lower consumer surplus and higher market prices. We extend the baseline model to a setting where the support of product quality is a continuum, and consumers can adjust their search intensity to virtually alter the informativeness of the quality signal upon purchase. We show that when the search cost decreases, given that initially the quality signal is sufficiently informative, the consumer surplus will drop, and the market price will increase. When the initial quality signal becomes less informative, consumers can still be better off if they increase the search intensity.

[*]Australian National University; E-mail: yijuan.chen@anu.edu.au

[†]School of Economics and Management, Harbin Institute of Technology, Shenzhen; E-mail: huxiangting@gmail.com

[‡]School of Economics, Renmin University of China; E-mail: lisanxi@gmail.com

1 Introduction

Consumer search models are widely applied in studies on online markets (Bakos, 1997; Bar-Isaac et al., 2012; Fishman and Levy, 2015; Lal and Sarvary, 1999; Yang, 2013). Many studies explore the feature that consumers incur lower search costs online than they do in offline markets. However, online consumers can be uncertain about product quality because they cannot physically inspect the product before purchase. For example, a consumer cannot know whether an LED monitor sold online is blur-free and reader-friendly, as claimed,and she will learn little from browsing the product information page using her CRT monitor.

The uncertainty over product quality can be horizontal and/or vertical. Horizontal quality is a product's matched value to a consumer and is idiosyncratic across consumers. Examples include clothing sizes, music genres, the design of arts and crafts, and so on. Vertical quality is a product's quality that is largely identical across consumers. Examples include the effectiveness of noise-canceling headphones, the actual display resolution of an LED monitor, the materials usedin furniture, and so on.

However, while some recent studies on consumer search have considered the uncertainty over horizontal product information (Gamp, 2015) or price (Ellison and Wolitzky, 2012; Carlin,2009; Wilson, 2010), few investigate the impacts of imperfect (ex post) vertical quality information. In addition,the existing studies that incorporate the vertical product attributes into search modelsusually assume that there exists—ex ante but not ex post—uncertainty over vertical product quality upon visiting online stores because they focus mainly on other market features, such as market structure or product design (Bar-Isaac et al., 2012; Fishman and Levy, 2015).

In this paper, we examine the impacts of consumers' ex post uncertainty over vertical product quality in a consumer search model and study how search costs and informativeness can change market price, firms' profits, and consumer welfare. In the model, we assume that there are two product quality levels: high quality and low quality. Consumers conduct a sequential random search with perfect recall and without replacement; and, upon visiting a store, consumers can observe price, but cannot directly observe the vertical product quality. Instead, we assume that, upon visiting a store, consumers can observe a noisy signal that is distributed, conditional on the true product quality. We first assume, in the baseline model,that the conditional distributions of the quality signal have a binary support. In the extended model, we assume that the conditional distributions of the quality signal have a continuous support.

We use the Perfect Bayesian Equilibrium as our solution concept. With our setup, consumers' searching-and-stopping rule is a cutoff rule, as in the standard search literature: a consumer will stop searching and buy from the store that she is currently visiting if the expected utility from the purchase exceeds a cutoff utility level, and, if not, she will continue to search. There can exist three types of equilibrium: An equilibrium is separating if different types of firms charge different prices. In contrast, a pooling equilibrium entails different types of firms to charge different prices. In a partial-pooling / partial-separating equilibrium, each type of firm can randomize over a distribution of prices, with the two distributions partially overlapped. Basically, the existence of separating equilibrium requires the distributions of quality signals, conditional on different quality levels, to have different supports, such as in Wolinsky (1983). When we focus on the case in which the conditional distributions have the same support, we show that the separating equilibrium does not exist.

Next, we characterize the partial-pooling equilibrium. In a partial-pooling equilibrium, the high-quality firm chooses the high price with probability one, while the low-quality firm randomizes over the high and the low price. Consumers will buy upon seeing the high price and the high signal, or seeing the low price and any quality signal, and they will continue searching upon seeing the high price and the low signal[1]. Following the equilibrium path, consumers form their expectations of the value of search. Consequently, we can derive a relationship between price(s) and consumers' reservation utility from the consumer's searching behavior, which we call the Search Value Line. The Search Value Line implies that, when price increases, the consumer's surplus will decrease.

In addition, from the firm's side, the incentive-compatibility condition of the firm implies another negative relationship between the equilibrium price and consumer's reservation utility, which we call the Price Incentive Line. The equilibrium can be derived from these two lines: in particular, an outcome of a partial-pooling equilibrium is the intersection of the Search Value Line and the Price Incentive Line. A limit case of the partial-pooling equilibrium will be the pooling equilibrium. In a pooling equilibrium, high-quality and low-quality firms charge the same price; a visiting consumer will purchase upon seeing a high signal and continue searching upon seeing a low signal on the equilibrium path.

Based on equilibrium characterization, we conduct comparative statics with respect to search costs and the informativeness of the quality signal. We show that a decrease in search costs brings a higher value of search,shifts out the Search Value Line, but does not change the Price Incentive Line, which finally leads to higher prices and a lower consumer's reservation utility. We call this effect the search-value effect: the increase in the value of search can benefit firms while hurting consumers. As the total value of search increases,

[1]In the partial-pooling equilibrium, consumers will use both the price information and the signal information to make purchase decision, which is supported by empirical and experimental evidence which suggest that consumers infer product quality from prices as well as non-price signals, such as brand name, online reviews and other search attributes (Zeithaml, 1988; Degeratu, Rangaswamy and Wu, 2000; Chatterjee, 2001; Kardes et al., 2004). By contrast, in the separating equilibrium consumers use only the price information to determine purchase, regardless of any signal; and in the pooling equilibrium consumers use only the signal information to infer quality, regardless of price.

the firm can take an even greater share of the total value than before without violating the incentive-compatibility condition.

The traditional wisdom on consumer search models (Wolinsky, 1986; Anderson and Renault, 1999) indicates that a decrease in search costs encourages consumers to search more intensively, brings more severe competition among firms, and lowers market price. However, we demonstrate another possible channel that leads to a novel result through the impacts on firms' incentives when there exists ex post uncertainty over vertical product quality. To show this comparison, in our baseline model, the search intensity does not change in search costs. We will incorporate the impacts of search costs on search intensity in the extended model.

We next study the comparative statics regarding the informativeness of the quality signal. When the signal is more informative, there are two competing effects. On the one hand, the value of search increases as the informativeness improves, such that there exists a search-value effect. On the other hand, the increase in informativeness rotates the Price Incentive Line, asthe low-quality firm's incentive to mimic is weakened because a visiting consumer will detect the true product quality with a higher probability and will be more likely to leave. This effect is called the Price-incentive Effect: the increase in informativeness can increase consumers' reservation utility and lower prices. The change in the equilibrium, thus, depends on the net effect of the two.

Our results help incomparing offline and online markets and explaining the difference between them. When consumers switch from offline to online markets, not only the search costs, but also informativeness, decrease. We provide a case in which consumers' welfare decreases unambiguously when both search costs and informativeness decrease, implying that the decrease in informativeness may not be offset by the decrease in search costs.

While, in the baseline model, search intensity depends only on informativeness and not on search costs, our extended model provides a case in which search intensity increases as search costs decrease. We assume that the conditional distributions of quality signal have a continuous support and satisfy the Monotonic Likelihood Ratio Property (MLRP). When search costs are lower, consumers will search more intensively and will buy only when they observe higher signals; hence, they can distinguish the high-quality products with a higher probability upon purchase under MLRP. We call this endogenous informativeness—that is, consumers will endogenously choose their ability to be informed upon purchase in equilibrium.

With this setup, a decrease in search costs brings about both the search-value effect and the price-incentive effect. Since the two effects work in different directions, we show that, when the exogenous informativeness is sufficiently good,the search-value effect will dominate. Moreover, when the exogenous informativeness decreases, if consumers respond by increasing their search intensity and, thus, the endogenous informativeness, then consumers will be better off.

The rest of the paper is organized as follows. Section 2 reviews the literature related to this paper. Section 3 presents the setup of the baseline model and characterizes the equilibria. In Section 4, we examine the comparative statics with respect to the search costs and informativeness. In Section 5, we briefly discuss the impacts of return/refund policies as an application of our model. Section 6 extends the baseline model. Section 7 concludes.

2 Related literature

First, this paper is related to the theoretical literature on consumer search that incorporates vertical product attributes into search models. For example, Bar-Isaac et al. (2012) investigate how firms with different quality levels choose horizontal product attributes and show that, in equilibrium, the higher-quality firms will choose a board design, and the lower-quality firms will choose a niche design. Fishman and Levy (2015) consider vertically differentiated products in a search model and analyze how lower search costs affect firms' incentives to invest in quality. However, in these papers, the vertical attributes are usually fully observable to consumers upon visiting. By contrast, our paper focuses mainly on the case in which the vertical attributes are imperfectly observed, even upon visiting.

Second, a consumer-search literature investigates the cases in which consumers may not perfectly observe product or price information upon visiting a firm, and also studies firms' information disclosure or obfuscation problem (Ellison and Wolitzky, 2012; Carlin, 2009; Wilson,2010; Gamp, 2015; Simon and Lu, 2017). However, while this stream of literature studies homogeneous goods orhorizontally-differentiated products, our paper focuses on the uncertainty over vertical product quality. In the literature, for the horizontal attributes, the seller is usually regarded as having the commitment power to truthfully communicate what he knows to the buyer. By contrast, our model is based on price signaling and, thus, is more closely related to the theoretical literature that considers the settings in which the seller has a limited ability to commit to communicate what he knows to buyers.

Moreover, this paper is also related to the literature that studies the price-signaling effects on product quality in the models with imperfect competition (Wolinsky, 1983; Daughety and Reinganum, 2008; Janssen and Roy, 2015). The general result in this literature is that the imperfect information about vertical quality requires firms to signal quality via the prices that are higher than complete-information levels, which softens price competition, benefits firms, and hurts consumers. Our paper is particularly closely related to Wolinsky (1983), who also studies a consumer-search setup, in which consumers can visit many firms before purchase and will observe imperfect information about product quality. A difference between our paper and Wolinsky (1983) is that, in his model, the search costs and the consumer's reservation utility have no impact on price, whereas in our model, the consumer's reservation utility is related to price, which allows us to examine the relationship between search costs and price. In addition, Wolinsky (1983) follows different assumptions on information structure and studies only the separating equilibrium, while we characterize conditions under which there exist separating, pooling and partial-pooling equilibria.

In addition, a few recent papers on consumer search show that a reduction in search costs can increase market price. For example, both Choi, Dai, and Kim (2016) and Haan, Moraga, and Petrikaite (2017) investigate ordered search models in which consumers can costlessly observe a firm's price before search and need only pay positive search costs to further explore the horizontal product attributes. In their papers, since prices are observable before search, the sellers compete in prices to attract consumers to visit first. When the search costs are high, the consumers who first visit the store are less likely to leave. Therefore, the firms compete more vigorously to attract consumers to visit their store first, resulting in lower prices. Other papers show that when firms endogenously choose both a pricing strategy and non-pricing strategies, a decrease in search cost may result in higher prices. Kuksov (2004) and Larson (2013) study the search models in which firms can endogenously choose product differentiation. They show that lower buyer search costs may lead to higher product differentiation, which softens price competition and generates higher market prices. Rhodes and Zhou (2017) study a model in which single-product firms have to decide endogenously whether to merge with a multi-product firm. They show that, when search costs are high, all firms are multi-product. However, when search costs are low, the equilibrium market structure is asymmetric, with some firms being multi-product and others being single-product. As a result, a decrease in search costs changes the market structure, which softens price competition and increases market prices. Gamp and Kramer (2017) study a model in which firms can endogenously choose whether to deceive naive consumers with inferior products. They show that a decrease in search costs can harm consumers; and the result depends on the extent to which decreasing search costs change the proportion of candid firms that sell the high-quality products. Cunha et al. (2018) obtain similar results in a model in which firms can endogenously choose both product design and quality, and consumers are rationally inattentive. Different from these papers, our paper highlights a different channel that can harm consumers when search costs decrease: even if the total surplus increases with lower search costs, consumers can still be hurt because firms can take a greater share of the total value. Given a higher total value of search, a higher price turns out to be compatible with the firm's incentives and, thus, can be sustained in equilibrium.

3 Model

3.1 Set-up

There are a continuum of risk-neutral firms of measure 1 and a continuum of consumers of measure 1. Each consumer has a unitary demand and conducts sequential random search on the market with perfect recall and without replacement[2] at a search cost s per firm.

Firms produce differentiated products at two quality levels: the high-quality product or the low-quality product. Denote the value of a high-quality product as v_h and a low-quality product as v_l, with $v_h > v_l$. We assume that each firm is high-quality with probability μ. Without loss of generality, the marginal cost of production is zero. We assume that the products have no horizontal differentiation.

Firms set their prices after observing their qualities. At the beginning of each period a consumer can choose not to search and leave the market without any purchase, in which case she receives an outside utility r. Upon sampling an online store, a consumer observes the price but cannot directly observe the vertical product attributes. We assume that the consumer can observe only a signal $\eta \in \{\eta_l, \eta_h\}$ that is distributed conditional on product quality v. The conditional distribution is that

$$\begin{aligned} \Pr(\eta = \eta_h | v = v_h) &= \beta_h, \Pr(\eta = \eta_l | v = v_h) = 1 - \beta_h \\ \Pr(\eta = \eta_l | v = v_l) &= \beta_l, \Pr(\eta = \eta_h | v = v_l) = 1 - \beta_l \end{aligned}$$

with $1 > \beta_h > \frac{1}{2}$ and $1 > \beta_l > \frac{1}{2}$. In words, the larger the β_h, the more likely a high-quality product will generate a high signal, whereas the larger the β_l, the more likely a low-quality product will yield a low signal. In this sense, we can say that β_l and β_h represent the informativeness of quality signals. We also call β_l and β_h the *quality of search.*

A firm's pure strategy is to set its price given its quality. In other words, it is a function $P : \{v_l, v_h\} \rightarrow \mathbb{R}_+$. For ease of presentation below, denote $p_l = P(v_l)$, and $p_h = P(v_h)$.

Denote p the price obersved by a consumer when she visits a firm. Therefore the signal space for a consumer is

$$S = \{(\eta, p) \mid \eta \in \{\eta_l, \eta_h\}, p \in \mathbb{R}_+\}.$$

After observing a signal (η, p), the consumer has three choices: to buy at the observed price, or not to buy and search again, or not to buy and stop searching. In each period t, a consumer will be active only if upon t she has not stopped searching or made a purchase.

[2] Consistent with the literature such as Wolinsky (1986), Anderson and Renault (1999), and Bar-Isaac et al. (2012), we assume that the search has perfect recall and is without replacement, implying that a consumer will never sample the same store twice by paying s twice, and can go back and buy from any store she has visited before anytime without incurring any additional cost.

Consequently, a (Markovian) pure strategy of a consumer, denoted by f, can be described by two disjoint subsets of S, denoted by A_f and B_f, such that the consumer will buy at $(\eta, p) \in A_f$, will search again at $(\eta, p) \in B_f$, and will choose not to buy and stop searching at $(\eta, p) \in S/(A_f \cap B_f)$.

The observed signal (η, p) may allow the consumer to update her belief about the product quality v, which we denote by $\Pr\{v|(\eta, p)\}$ for $v \in \{v_l, v_h\}$. Consequently, given the strategies of the firms and the consumer, the consumer's expected payoff from the search process is

$$u(P, f) = \sum_{t=0}^{\infty} \left[(\Pr((\eta, p) \in B_f))^t \left[\sum_{(\eta,p)\in A_f} \Pr(\eta, p) \left[E[v|(\eta, p)] - p \right] - s \right] \right]. \tag{1}$$

with $E[v|(\eta, p)] = \Pr(v_l|(\eta, p)) \cdot v_l + \Pr(v_h|(\eta, p)) \cdot v_h$.

On the other hand, given the strategies of the firms and the consumer, a firm's expected profit from a visiting consumer is the price times the consumer's probability of purchase. Following the literature of consumer search, we assume no discouting across time, and consequently, a firm's total expected profit is the expected profit from one visiting consumer times the mearsure of visiting consumers in all periods.

3.2 Equilibrium

We use the Perfect Bayesian Equilibrium (PBE) as the solution concept. A PBE consists of a strategy profile $\{P^*,\ f^*\}$, and the consumer's belief system $\Pr\{v|(\eta, p)\}$ for every $(\eta, p) \in S$ and $v \in \{v_l, v_h\}$. The solution concept requires an equilibrium strategy be optimal with respect to the belief system and the belief system be consistent with the equilibrium strategy profile.

On the consumer side we will be focused on pure strategies. Denote U a consumer's value of search. In equilibrium U satisfies the Bellman equation

$$U = \max \left\{ r, \max_f \left\{ \sum_{(\eta,p)\in A_f} \Pr(\eta, p) \left[E[v|(\eta, p)] - p \right] - s + \Pr((\eta, p) \in B_f) \cdot U \right\} \right\}.$$

For ease of analysis we will maintain the assumption that $r < -s$, such that the consumer strictly prefers to participating in search. Hence,

$$U = \max_f \left\{ \sum_{(\eta,p)\in A_f} \Pr(\eta, p) \left[E[v|(\eta, p)] - p \right] - s + \Pr((\eta, p) \in B_f) \cdot U \right\},$$

which means in equilibrium

$$U = \frac{\sum_{(\eta,p)\in A_{f^*}} \Pr(\eta,p)\,[E[v|(\eta,p)]-p]-s}{1-\Pr((\eta,p)\in B_{f^*})}. \tag{2}$$

Upon visiting a firm, the consumer expects to have a utility $E[v|(\eta,p)]-p$ if she buys and to have a utility U if she continues to search. Therefore the equlibrium strategy entails that she buys if $E[v|(\eta,p)]-p \geq U$ and not to buy if $E[v|(\eta,p)]-p < U$.[3] In other words, the equilibrium strategy is a cutoff rule that is similar as that in a standard consumer search model. It is worth noting here, however, that the consumer's belief system should also specify her belief upon observing off-equilibrium-path signals. Consequently, off the equilibrium path the consumer's expectation about the product quality is solely based the off-equilibrium-path belief, i.e., independent of the firm strategy, yet the consumer will follow the same cutoff rule. As to be seen more clearly in the analysis below, this implies the consumer's implementation of the cutoff rule in term of the observed quality signal η may differ between on and off the equilibrium path.

On the firm's side, the assumption of a continuum of firms implies that each firm has zero measure. As a result, in equilibrium an individual firm's pricing strategy will not affect consumers' search behaviour before they visit the firm, and thus will not affect the measure of its visiting consumers. This means in equilibrium each firm's profit maximization problem is equivalent to maximizing its profit from each visiting consumer.

In the following analysis we will be focused on three types of equilibrium: a separating equilibrium, a pooling equilibrium, and a partial pooling / partial separating equilibrium. An equilibrium is separating if different types of firms charge different prices, i.e., $p_l \neq p_h$. In contrast, a pooling equilibrium entails $p_l = p_h$. In a partial-pooling / partial-separating equilibrium, each type of firm can randomize over a distribution of prices, with the two distributions partially overlapped.

Proposition 1 *There does not exist a separating equilibrium.*

Proof. In the Appendix. ■

The intuition of Proposition 1 is as follows. First, if the high-quality and low-quality firms charge different prices, $p_h \neq p_l$, then the consistency of consumer belief requires that $\Pr(v = v_h|\eta,\, p = p_h) = 1$ and $\Pr(v = v_l|\eta,\, p = p_l) = 1$. In other words, the quality signal η plays no role in consumers' inference of product quality because consumers can directly infer quality from prices. Now suppose, without loss of generality, $p_h > p_l$ and consumers will choose to buy upon seeing p_h. Then all the low-quality firms will be better off by deviating to setting their prices at p_h, causing the equilibrium to break down.

[3] This means the consumer will buy from the current store when she is indifferent between buying and continuing searching.

A key factor that leads to Proposition 1 is the assumption that $\beta_h < 1$ and $\beta_l < 1$. The assumption implies that at each quality level the consumer may observe both quality signals η_h and η_l. That is, the distribution of the quality signal η conditional on v_h has the same support as that conditional on v_l. Consequently, when, say, a v_l-type firm imitates the price of a v_h-type firm, the quality signal cannot help consumers identify the imitator, and thus the imitation cannot be deterred. This insight then allows us to link Proposition 1 with two well-known findings in the literature, namely the Diamond paradox and the price signalling result as in Wolinsky (1983).

First, if we allow $\beta_h = \beta_l = 1$, then there is a separating equilibrium with $p_l = v_l$ and $p_h = v_h$, which is essentially a Diamond paradox result: The firms will charge the monopoly price, i.e., $p_l = v_l$ and $p_h = v_h$, in equilibrium. The logic is the same as that in the Diamond paradox: If there exists an equilibrium at which $p_l < v_l$ or $p_h < v_h$, then if $v_h - p_h = v_l - p_l$ in the equilibrium, a firm can increase its profit by increasing the price to $p + \varepsilon$, with $\varepsilon < s$, and the visiting consumers will not continue to search given other firms' pricing strategy. If $v_h - p_h > v_l - p_l$ in the equilibrium, then a high-quality firm will benefit from a small increase in price, such that the increased amount is less than the search cost and the visiting consumer will not run away. Similar argument applies to the case in which $v_h - p_h > v_l - p_l$.

Second, suppose $\beta_h = 1$ and $\beta_l < 1$, then we can show that there is a separating equilibrium, which is a special case of the result in Wolinsky (1983). In the Appendix A we characterize a separating equilibrium when $\beta_h = 1$ and $\beta_l < 1$. In the equilibrium, the consumer will buy from the first store she visits. The consumer will get a higher surplus if they buy from the high-quality firms. If the consumer first visits a high-quality firm, she will strictly prefer to stopping searching and buying from the firm. If she first visits a low-quality firm, she is indifferent between buying from the low-quality firm and continuing searching by paying a search cost s and to meet a high-quality firm, from whom she will get a high surplus. In the Appendix A we also show that as $\beta_l \to 1$ and $s \to 0$, the separating equilibrium converges to the equilibrium under Bertrand competition.

We now move on to characterize a partial-pooling equilibrium as follows. The high-quality firm sets its price at p_h, while the low-quality firm's price will be p_h with probability σ and p_l with probability $1 - \sigma$. On the equilibrium path, the consumer will buy upon seeing (η_h, p_h), (η_l, p_l) or (η_h, p_l), and will continue searching upon seeing (p_h, η_l). Off the equilibrium path, consumers holds the belief that $\Pr(v = v_l | p \neq p_h, p \neq p_l) = 1$, and will buy if $p \leq v_l - U$, and continue searching if $p > v_l - U$.

To construct such an equilibrium, we start from characterizing a consumer's search behaviour on the equilibrium path and off the equilibrium path. First, given the aboved mentioned pricing strategy, the consumer's belief on the equilibrium path will be

$$
\begin{aligned}
\Pr(v = v_h|(p_l, \eta)) &= 0 \\
\Pr(v = v_h|(p_h, \eta_h)) &= \frac{\mu\beta_h}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} \\
\Pr(v = v_h|(p_h, \eta_l)) &= \frac{\mu(1-\beta_h)}{\mu(1-\beta_h) + (1-\mu)\beta_l\sigma}.
\end{aligned}
$$

When the consumer holds the reservation utility U, by visiting an additional store, the consumer will observe (p_h, η_h) with probability $\mu\beta_h + (1-\mu)(1-\beta_l)\sigma$ and observe p_l with probability $(1-\mu)(1-\sigma)$. In these two cases the consumer will buy so the incremental utility is the expected utility from the purchase, which is $E[v|(\eta, p)] - p$, less the reservation utility U. Hence, the consumer's reservation utility U is determined by

$$
\begin{aligned}
& [\mu\beta_h + (1-\mu)(1-\beta_l)\sigma] \cdot \left[\frac{\mu\beta_h v_h + (1-\mu)(1-\beta_l)\sigma v_l}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} - p_h - U\right] \\
& + (1-\mu)(1-\sigma) \cdot [v_l - p_l - U] \\
= \; & s, \qquad (3)
\end{aligned}
$$

where the left-hand side is the incremental utility of searching one additional store, which equals the search cost of searching an additional store. (3) is also equivalent to the previous equation (2), which generally characterize the consumer's reservation utility in equilibrium.

Off the equilibrium path, we assume that the consumer believes that the product has a low quality if the price is neither p_h nor p_l. That is, for a price p such that $p \neq p_h$ and $p \neq p_l$, $\Pr(v = v_l|(\eta, p)) = 1$, or, equivalently, $E[v|(\eta, p)] = v_l$. Following this belief, the cutoff rule for the search decision then dictates that the consumer will buy if $p \leq v_l - U$, and continue searching if $p > v_l - U$.

Now we examine the firm's pricing strategy. First, the low-quality's firm randomization between p_h and p_l requires that the firm be indiferent between the two prices. Given the consumer strategy, if the low-quality firm charges p_l, then a visiting consumer will always buy; if $p = p_h$, then a visiting consumer will buy with probability $1 - \beta_l$. Then the low-quality firm's indifference between p_h and p_l leads to

$$
p_l = (1-\beta_l)p_h. \qquad (4)
$$

Next, upon seeing p_l, a consumer can infer that the product quality is v_l. Given this belief, the cutoff search rule then implies that the consumer will buy as long as $p_l \leq v_l - U$. This then leads the low-quality firm to charge the highest possible price, which is

$$
p_l = v_l - U \qquad (5)
$$

Combining equation (4) with equation (5), we can derive the *Price Incentive Line*, which characterizes the high-quality firm's pricing strategy in equilibrium

$$U = v_l - (1 - \beta_l)p_h. \tag{6}$$

The Price Incentive Line implies a negative relationship between the price and consumer's reservation utility: As consumer's reservation utility decreases by one unit, the sure-sell price $v_l - U$ will increase by one unit; hence the price of mimicing should increase by $\frac{1}{1-\beta_l}$ unit to keep the incentive compatibility condition.

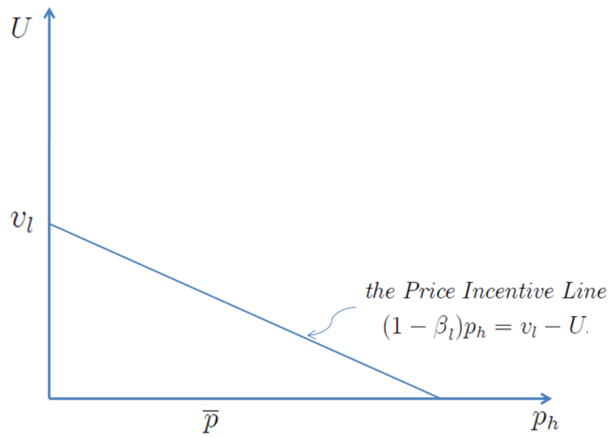

Figure 1a: The Price Incentive Line

Next, we can plug (5) into equation (3), which leads to

$$p_h + U = v_l + \phi(\beta_h, \beta_l, s; \sigma), \tag{7}$$

where

$$\phi(\beta_h, \beta_l, s; \sigma) \equiv \frac{\mu\beta_h(v_h - v_l) - s}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma}.$$

We refer to (7) as *the Search Value Line*, which characterizes consumer's searching behavior. Given the value of search, the Search Value Line implies a negative relationship between p_h and consumer's reservation utility: When p_h increases by one unit, consumer's reservation utility will decrease by one unit.

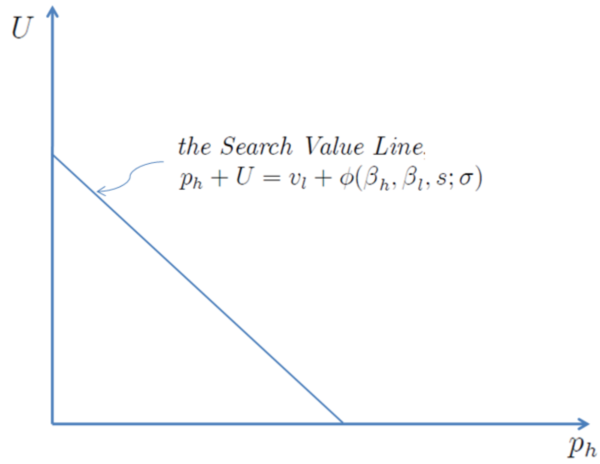

Figure 1b: The Search Value Line

The slope of the Search Value Line is -1; and the slope of the Price Incentive Line is $-(1-\beta_l)$, such that the Price Incentive Line is flatter. The equilibrium (p_h^*, U^*) is determined by the intersection of the Price Incentive Line and the Search Value Line. Proposition 2 describes the partial-pooling equilibrium.

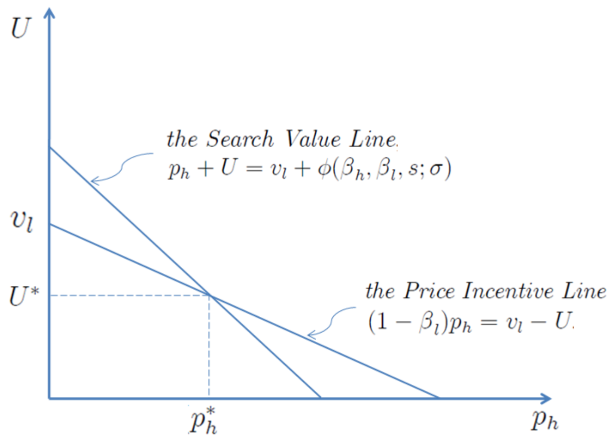

Figure 2: The partial-pooling equilibrium

Proposition 2 *If $\frac{s}{v_h - v_l} < \frac{\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l}$, then there exists $\bar{\sigma} \in (0,1)$, such that for $\sigma > \bar{\sigma}$, there is a partial pooling equilibrium at which:*

(1) The high-quality firm charges

$$p_h^* = \frac{\mu(v_h - v_l) - \frac{s}{\beta_h}}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} \tag{8}$$

with probability 1, while the low-quality firm charges p_h^* with probability σ and $p_l^* = (1-\beta_l)p_h^*$ with probability $1-\sigma$.

(2) The consumer hold the reservation utility $U^* = v_l - p_l^*$.

(3) On the equilibrium path, the consumer will buy upon seeing $p = p_l^*$ *or* (p_h^*, η_h)*; and continue to search upon seeing* (p_h^*, η_l)*.*

(4) Off the equilibrium path, the consumer holds the belief $\Pr(v = v_h | p \neq p_h^*, p \neq p_l^*) = 0$ *and will choose to buy if* $p \leq v_l - U^*$*, and continue searching if* $p > v_l - U^*$*.*

Proof. In the Appendix. ■

There exists multiple equilibria: Each $\sigma \in (\bar{\sigma}, 1)$ corresponds to an equilibrium. The proposition implies that the equilibrium can exist only when σ is sufficiently large. The intuition is as follows. As σ is low, by Bayesian updating, upon observing a high signal and a high price, the consumer will expect that the product is high-quality with a high probability. This effect allows the high-quality firm to charge a high p_h; but when p_h is high, the consumer will hold a low reservation utility U. Note that, the existence of the equilibrium requires consumer's reservation utility to be sufficiently high such that the consumer will prefer to continuing searching upon seeing a low signal and a high price. But this condition is violated when U is low.

Regarding the multi-equilibrium issues, there are three remarks as follows. First, note that as $\sigma \to 1$, the partial-pooling equilibrium becomes a pooling equilibrium. $\sigma = 1$ implies that the low-quality firm will mimic p_h with probability 1, such that p_h is the only price on the equilibrium path. In a pooling equilibrium, the consumer cannot infer product quality from price. Hence, on the equilibrium path, she will buy upon seeing a high signal η_h and continue searching upon seeing η_l. On the firm's side, given the off-equilibrium-path belief $\Pr(v = v_h | p \neq p_h) = 0$, the most profitable deviation for a low-quality firm is to charge $v_l - U$ such that the consumer will buy with probability 1 at this price. In equilibrium, the low-quality firm is indifferent between charging the sure-sell price $v_l - U$ and the equilibrium price p_h.

Second, even if there exist multiple partial-pooling equilibria, each partial-pooling equilibrium corresponds to a value of σ, such that in later analysis we can derive the comparative statics for each given σ.

Third, as a summary, the next figure shows the equilibrium types corresponding to each (β_h, β_l). In the rest of the paper, we will be focused on the partial-pooling equilibrium. To avoid redundant wordings, in the rest of the paper when we mention "equilibrium" we are referring to the partial-pooling equilibrium described in Proposition 2.

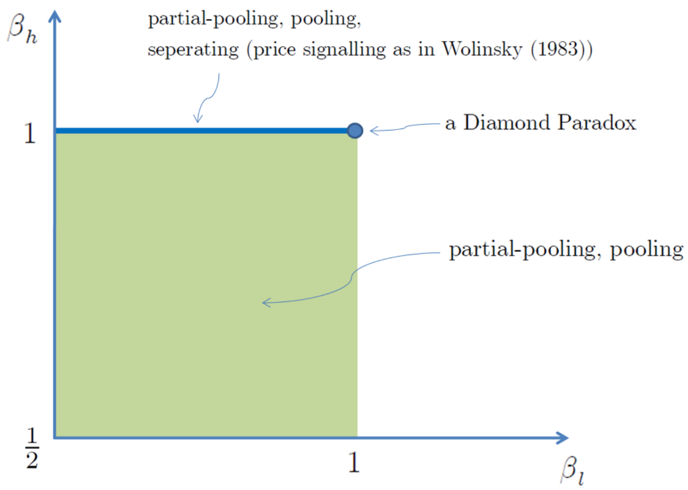

Figure 3: Summary of equilibrium types

4 Impact of search cost and search quality

4.1 Search cost

When the search cost s decreases, given σ, $\phi(\beta_h, \beta_l, s; \sigma)$ increases and the Search Value Line shifts outward. As a result, p_h^* increases while U^* decreases, as being shown by the following figure, and formally stated by the corollary below.

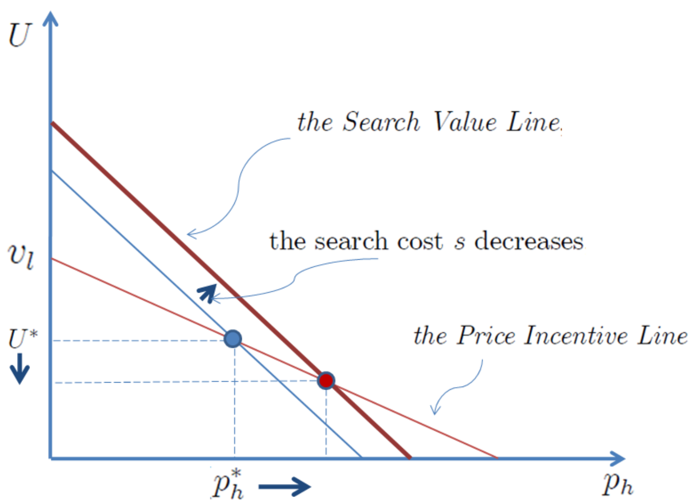

Figure 4: The impacts of a lower search cost on equilibrium

Corollary 1 *A decrease of search cost leads to a lower reservation utility U and a higher price p_h^*, and thus a higher p_l^*, in equilibrium.*

Proof. In the Appendix. ■

First, a lower search cost causes the value of search to increase. When the value of search changes, the equilibrium price and the reservation utility will shift *along* the Price Incentive Line. Recall that, the Price Incentive Line has a negative slope, implying that, as U increases, p_h needs to decrease to keep the low-quality firm being indifferent because the sure-sell price $v_l - U$ decreases. Moreover, to keep the firm being indifferent, the *decreased* amount in p_h needs to be even larger than the decrease in the sure-sell price (and the increase in U), since the low-quality firm can only sell the product at the high price with a probability less than one. But an increase in the total value of search means that the decrease in p_h should be dominated by the increase in U. Therefore, a higher U is in violation of the low-quality firm's incentive condition when the total value of search increases.

This mechanism, which is caused by the imperfect vertical-quality information, differs from the traditional wisdom that a decrease in search costs will lower the equilibrium price. In the literature, consumers will search more intensively as search costs are lower, which brings more severe competition among firms such that the equilibrium price will decrease due to the competition effect. By contrast, our baseline model shows a different story[4]: When search costs decrease, the value of search increase, but with imperfect vertical product information, an increase in the value of search also changes the incentive, which finally leads to higher prices in equilibrium.

In the rest of the paper, we will call the effect caused by the shifting of the Search Value Line the *search-value effect.*

4.2 Search quality

An increase of β_h shifts the Search Value Line outward, while has no impact on the price-incentive line. That is, there is only a search-value line effect. Hence, an increase in β_h leads to higher prices and a lower reservation utility.

Corollary 2 *An increase of β_h leads to higher prices and a lower reservation utility.*

When β_l increases, however, the Price Incentive Line also rotates outward. The consumer will have a lower probability to buy from a low-quality firm, such that at any p_h,

[4] To show the comparison, note that, in our baseline model, the search intensity does not change in search costs. Upon visiting a store, a consumer will buy with probability

$$\mu\beta_h + (1-\mu)(1-\beta_l)\sigma + (1-\mu)(1-\sigma)$$

and continue to search with probability

$$\mu(1-\beta_h) + (1-\mu)\beta_l\sigma.$$

The probabilities are independent of s. Hence, in the baseline model the result is not driven by the change in search intensity. In the extended model, we will allow the search intensity to change in search costs, but similar results can be derived with some restrictions on informativeness.

the low-quality firm's incentive to mimic is weakened. This means that, given any p_h, to keep the incentive compatibility condition, the sure-sell price $v_l - U$ needs to decrease and U needs to increase accordingly. We call the effect caused by the rotation of the Price Incentive Line the *price-incentive effect*.

In addition, as β_l increases, the total value of search increases, leading to the Search Value Line shifting outside. The following corollary characterizes the effects of the change in β_l.

Corollary 3 *(1)* $\frac{\partial U^*}{\partial \beta_l} > 0$*; and (2)* $\frac{\partial p_h^*}{\partial \beta_l} < 0$ *iff* $\beta_l < \frac{1}{2}(\frac{\mu}{\sigma(1-\mu)}\beta_h + 1)$.

Proof. In the Appendix. ■

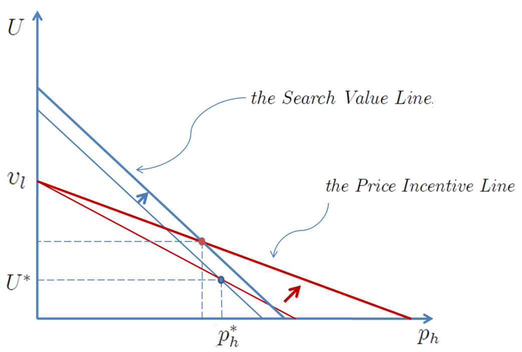

Figure 5a: $\beta_l < \frac{1}{2}(\frac{\mu}{\sigma(1-\mu)}\beta_h + 1)$

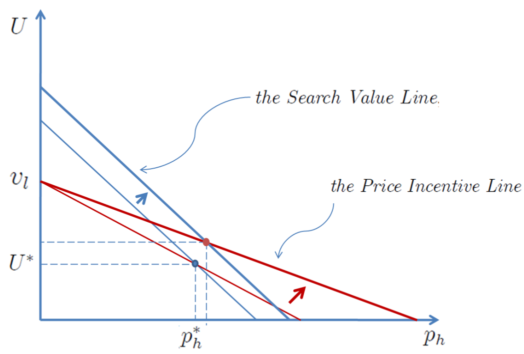

Figure 5b: $\beta_l > \frac{1}{2}(\frac{\mu}{\sigma(1-\mu)}\beta_h + 1)$

The impacts of an increase in β_l can be shown by Figure 5. The price-incentive effect always dominates the search-value effect in determining the change in the reservation utility. Regarding the prices, note that $\frac{\partial \phi(\beta_h, \beta_l, s; \sigma)}{\partial \beta_l}$ increases in β_l, which means that, the search-value effect increases as β_l is greater. Consequently, the search-value effect dominates the price-incentive effect when β_l is sufficiently large.

4.3 Simultaneous change of search cost and search quality

In this part we let $\beta_h = \beta_l = \beta$. We first apply the baseline model to study the case in which consumers switch from the offline market to the online market. Compared with the offline market, both the search costs and the informativeness decrease in the online market. We are concerned about whether and when the consumers and the firms may benefit from the online market. Second, we use this framework to discuss the impacts of the return / refund policies.

To avoid the Diamond Paradox, we suppose on the offline market the informativeness β is sufficiently close to but less than 1. When β decreases, the price-incentive line rotates inward, and the search-value line shifts inside. Simultaneously, the reducing search costs will lead the value line to shift out. The total effects then depend on the combination of the three. Suppose there is a function $s = g(\beta)$, with $g'(\beta) > 0$, that characterizes the path of the change in (β, s). The condition $g'(\beta) > 0$ implies that s and β are positively correlated, that is, s and β decrease at the same time. The next corollary describes the change in the prices and the consumer's reservation utility when s and β decrease at the same time.

Corollary 4 *Suppose* $\frac{s}{v_h - v_l} < \min\left\{\frac{\sigma\mu(1-\mu)(2\beta-1)}{\mu(1-\beta)+(1-\mu)\beta\sigma}, \frac{\beta^2\mu^2}{\mu+(1-\beta)^2[(1-\mu)\sigma-\mu]}\right\}$*, and the decrease in* s *is associated with the decrease in* β*, according to the path* $s = g(\beta)$*, where* $g'(\beta) > 0$*. Then when search costs and search quality simultaneously decrease:*

(1) U^* *decreases, and thus* p_l^* *increases, given any path.*

(2) There exists $\widehat{\beta}(\sigma) \in [1/2, 1]$*, such that if* $\mu > \frac{\sigma}{1+\sigma}$ *and* $\beta > \widehat{\beta}$*, then* p_h^* *increases for any path. Otherwise,* p_h^* *increases if* $g'(\beta) > \frac{s}{\beta} - \beta(\mu - (1-\mu)\sigma)p_h^*$*.*

Proof. In the Appendix. ■

Recall that, in the comparative statics we shows that both the decrease in the search costs and the decrease in the informativeness result in lower consumer's reservation utility. Therefore, U always decreases when the search costs and the informativeness decrease at the same time, regardless of the path. The corollary also implies that, the effect of decreasing informativeness cannot be offset by the decrease in search costs; and it can be the firms, rather than the consumers, who benefit from the online market.

5 Application: The impacts of return / refund policies

A widely-used online mechanism that may increase consumer's welfare is the return / refund policies, which allow consumer to return the purchased items and get refund during a period of time after purchase. In this part we will briefly discuss the impacts of the online return / refund policies and are concerned about whether and to which extent the return / refund policies can improve consumer's welfare.

For a simple analysis, suppose that, the consumers who purchase the high-quality products will never return the products, and the consumers who phachase the low-quality product will return the products and get a refund equal to the purchase price with a probability $\lambda < 1$. λ is less than 1 for several reasons as follows. First, in order to resell returned products, firm may impose stringent conditions on the eligibility for return. This applies to many personal and hygienic products, and makes it difficult for consumers to test and evaluate the product before returning. Moreover, before returning a product, consumers may have to incur the costs to unpack, assemble, re-pack, and transport. When such costs are sufficiently large, some consumers will keep the purchased items instead.[5]

Consider a similar partial-pooling equilibrium, in which the high-quality firm charges price p_h, while the low-quality firm charges p_h with probability σ and p_l with probability $1-\sigma$. The consumer will never return a product if it turns out to be a high-quality product, or a low-quality product but the price is at $p_l = v_l - U$. With the return / refund policies, the ultimate probability that a visiting consumer will purchase and keep the low-quality product is the probability of purchase times the probability of not returning. Thus, the incentive compatibility condition of the low-quality firm will be

$$(1-\beta)(1-\lambda)p_h = v_l - U. \tag{9}$$

With the return / refund policies, the incentive line rotates out.

On the equilibrium path, the consumer will buy and not return the product upon seeing $p_l = v_l - U$. The consumer will buy upon seeing p_h and $\eta = \eta_h$, and will keep the product if it turns out to be the high-quality product, while return it with probability λ if it is a low-quality product. The consumer will not buy and continue to search upon seeing p_h and $\eta = \eta_l$. If the consumer returns the product, she will continue to search the next store.

Thus, upon seeing p_h, the consumer will buy and keep the product with probability β if it is a high-quality product; and will buy and keep it with probability $(1-\beta)(1-\lambda)\sigma$ if it is a low-quality product. If the consumer returns the product it is equivalent to no purchase in the current period. Therefore, the consumer's reservation utility U is determined by

$$\mu \cdot \beta(v_h - p_h - U) + (1-\mu) \cdot (1-\beta)(1-\lambda)\sigma(v_l - p_h - U) = s. \tag{10}$$

The left-hand side is the incremental benefit of searching one additional store. The right-hand side is the cost to visit one more store. (10) further leads to

$$p_h + U = v_l + \psi(\beta, \lambda, s; \sigma), \tag{11}$$

where

$$\psi(\beta, \lambda, s; \sigma) = \frac{\mu\beta(v_h - v_l) - s}{\mu\beta + (1-\mu)(1-\beta)(1-\lambda)\sigma}.$$

It is straightforward to see that $\psi(\beta, \lambda, s; \sigma)$ is increasing in λ, i.e.,

$$\frac{\partial\psi(\beta, \lambda, s; \sigma)}{\partial\lambda} = \frac{[\mu\beta(v_h - v_l) - s](1-\mu)(1-\beta)\sigma}{[\mu\beta + (1-\mu)(1-\beta)(1-\lambda)\sigma]^2} > 0.$$

[5] See Chen et al. (2017) for several reasons to argue this problem on the demand side.

Hence, when the probability of returning is higher, the value of search increases.

The equations (9) and (11) determine the (p_h, U) in equilibrium. The following corollary formally states the partial-pooling equilibrium.

Corollary 5 *If* $\frac{s}{v_h - v_l} \in \left(\frac{\lambda(1-\mu)(1-\beta)\mu\beta\sigma}{\mu\beta+(1-\mu)(1-\beta)\sigma}, \frac{\mu(1-\mu)(2\beta-1)\sigma}{\mu(1-\beta)+(1-\mu)\beta\sigma} \right)$*, then there exists a partial-pooling equilibrium in which:*

(1) Both types of firm will provide the return / refund policies.

(2) The high-quality firm charges $p_h^\lambda = \frac{1}{1-(1-\beta)(1-\lambda)}\psi(\beta, \lambda, s)$*, while the low-quality firm charge* p_h^λ *with probability* σ *and* $p_l^\lambda = (1-\beta)(1-\lambda)p_h^\lambda$ *with probability* $1-\sigma$*.*

(3) On the equilibrium path, the consumer will buy and not return the product upon seeing p_l^λ*. The consumer will buy upon seeing* p_h^λ *and* $\eta = \eta_h$*, and will keep the product if it turns out to be a high-quality product; while keep the product with probability* $1-\lambda$*, or return it and continue searching with probability* λ *if it is a low-quality product. The consumer will not buy and continue to search upon seeing* p_h^λ *and* $\eta = \eta_l$*.*

(4) Off the equilibrium path, the consumer holds the belief $\Pr(v = v_h | p \neq p_h^\lambda, p \neq p_l^\lambda) = 0$ *and will choose to buy if* $p \leq v_l - U^\lambda$*, and continue searching if* $p > v_l - U^\lambda$*.*

Proof. In the Appendix. ∎

As we have mentioned, if we consider the possibility of return / refund policies, the price-incentive line rotates outwards and the Search Value Line shifts outward. The comparative statics with respect to λ further shows that, consumer's welfare increases in λ, that is, the return policies will benefit the consumer.

Corollary 6 *When* λ *increases, the consumer's reservation utility* U^λ *increases, that is,* $\frac{\partial U^\lambda}{\partial \lambda} > 0$*.*

Proof. In the Appendix. ∎

The result can be shown by the following Figure. As λ increases, the Price Incentive Line rotates out and the Search Value Line shifts out. In this case, the price-incentive effect dominates the search-value effect and the consumer's reservation utility increase.

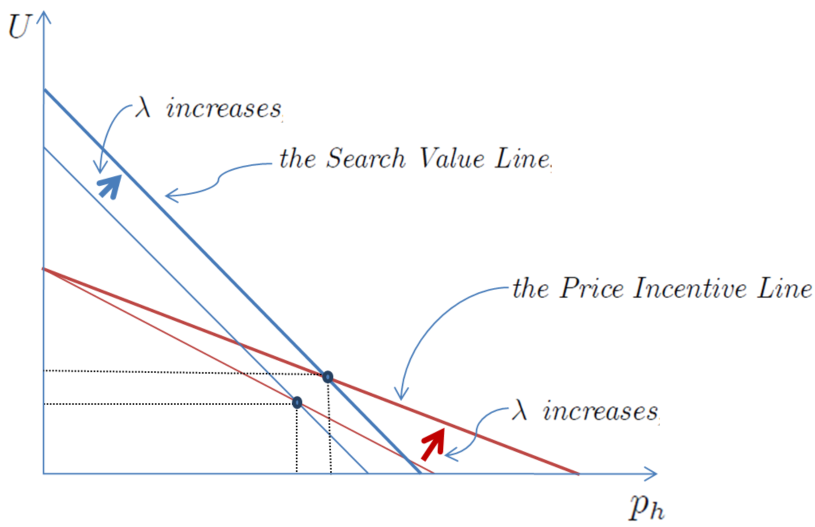

Figure 6: The impacts of return / refund policies on equilibrium

6 Extension: Continuous quality signals

6.1 Set-up and equilibrium

In the baseline model, the probability of buying from the high / low product is exogenously given. However, as search costs are lower, consumers may search more intensively and will endogenously choose their purchase probability. We incorporate this feature into the extended model in this section.

Suppose that, conditional on quality v_h, the signal η is distributed according to a continuous distribution with CDF $F_h(\eta)$ and PDF $f_h(\eta)$; and conditional on quality v_l, η is distributed according to a continuous distribution with CDF $F_l(\eta)$ and PDF $f_l(\eta)$ on $(-\infty, \infty)$. We assume that the conditional distributions satisfy the Monotonic Likelihood Ratio Property, i.e., $\frac{f_h(\eta)}{f_l(\eta)}$ is increasing in η. Other settings replicate those in the benchmark model.

Consider the possibility of the partial-pooling equilibrium as before. We first assume that the off-equlibrium-path belief is that $\Pr(v_h|p \neq p_h, p \neq p_l) = 0$.On the equilibrium path, suppose the high-quality firm will charge p_h with probability 1, while the low-quality firm will charge p_h with probability σ and p_l with probability $1-\sigma$. For the consumer, upon seeing $(\eta, p = p_h)$, she will expect the product quality to be high with probability

$$\Pr(v_h|\eta, p_h) = \frac{\mu f_h(\eta)}{\mu f_h(\eta) + (1-\mu) f_l(\eta)\sigma}.$$

Define

$$h(\eta) \equiv \Pr(v_h|\eta, p_h) = \frac{\mu f_h(\eta)}{\mu f_h(\eta) + (1-\mu) f_l(\eta)\sigma} = \frac{1}{1 + \frac{1-\mu}{\mu} \cdot \frac{f_l(\eta)}{f_h(\eta)}\sigma}.$$

Because the conditional distributions satisfy the Monotonic Likelihood Ratio Property, $h(\eta)$ is increasing in η. The expect product quality will be

$$E(v|\eta, p_h) = v_l + h(\eta)(v_h - v_l).$$

Apparently, $E(v|\eta, p_h)$ is increasing in η.

Suppose that, in equilibrium, there exists a cutoff value of η, $\hat{\eta}$, such that on the equilibrium path, upon seeing $(\eta, p = p_h)$, the consumer will buy if $\eta > \hat{\eta}$, and continue to search if $\eta < \hat{\eta}$. Suppose the consumer holds the reservation utility U. The cutoff rule of searching and stopping implies that, the consumer will buy iff $E(v|\eta, p_h) - p_h \geq U$ upon seeing $(\eta, p = p_h)$. Hence, $\hat{\eta}$ is determined by

$$E(v|\hat{\eta}, p_h) - p_h = U,$$

that is,

$$p_h + U = v_l + h(\hat{\eta})(v_h - v_l) \tag{12}$$

The reservation utility U is determined by

$$\mu(1 - F_h(\hat{\eta}))(v_h - p_h - U) + (1-\mu)\sigma(1 - F_l(\hat{\eta}))(v_l - p_h - U) = s. \tag{13}$$

The left-hand side is the incremental benefit of searching one more store, while the right-hand side is the cost of visiting one more store. Plug (12) into (13), $\hat{\eta}$ is implicitly determined by

$$\mu(1 - F_h(\hat{\eta}))(1 - h(\hat{\eta})) - (1-\mu)\sigma(1 - F_l(\hat{\eta}))h(\hat{\eta}) = \frac{s}{v_h - v_l}. \tag{14}$$

On the firm's side, the Incentive Compatibility Condition for the low-quality firm[6] is

$$(1 - F_l(\hat{\eta}))p_h = v_l - U. \tag{15}$$

The equilibrium is described by the following proposition.

Proposition 3 *Suppose the $\hat{\eta}$ determined by (14) is well-defined. Then there exists a partial-pooling equilibrium at which:*

(1) The high-quality firm charges $p_h^* = \frac{h(\hat{\eta})}{F_l(\hat{\eta})}(v_h - v_l)$, while the low-quality firm charges p_h^* with probability σ and $p_l^* = (1 - F_l(\hat{\eta}))p_h$ with probability $1 - \sigma$.

[6]For the high-quality firm, the IC condition is $(1 - F_h(\hat{\eta}))p^* \geq v_l - U$. Due to MLRP, if the IC condition for the low-quality firm is satisfied, then that for the high-quality firm is satisfied.

(2) The reservation utility is $U^* = v_l - \frac{(1-F_l(\hat{\eta}))h(\hat{\eta})}{F_l(\hat{\eta})}(v_h - v_l)$.

(3) On the equilibrium path, the consumer will buy from a visiting firm upon seeing $p = p_h^*$ *and* $\eta > \hat{\eta}$, *and continue to search if* $p = p_h^*$ *and* $\eta < \hat{\eta}$. *The consumer will buy upon seeing* p_l^*.

(4) Off the equilibrium path, the consumer holds the belief $\Pr(v_h|p \neq p_h^*, p \neq p_l^*) = 0$ *and will buy iff* $p \leq v_l - U^*$.

Proof. In the Appendix. ■

Similarly, we refer to (15) as the *Price Incentive Line* and (12) as the *Search Value Line.* The cutoff $\hat{\eta}$ represents the purchase probability in equilibrium. Further, $\hat{\eta}$ also represent the informativeness upon purchase. As the distributions of F_h and F_l denote the *exogenous* informativeness, $\hat{\eta}$ represents the *endogenous* informativeness of search: The consumer will endogenously adjust the search intensity and thus the ability to identify the high-quality product when she purchases.

6.2 Comparative statics

6.2.1 Search costs

First note that, the change of search costs affects the equilibrium outcome only through its impacts on $\hat{\eta}$. The following Lemma indicates that, with lower search costs, $\hat{\eta}$ will be higher and the consumer will buy only when she observes a better signal. Therefore, the consumer will search more intensively.

Lemma 1 *When s decreases, in equilibrium $\hat{\eta}$ will increase. That is,* $\frac{\partial \hat{\eta}}{\partial s} < 0$.

Proof. In the Appendix. ■

Since $h(\hat{\eta})$ increases in $\hat{\eta}$, when the search cost decreases, $\hat{\eta}$ increases and the Search Value Line shifts outwards. In addition, $1 - F_l(\hat{\eta})$, the purchase probability from the low-quality firm when it mimics, decreases, such that the Price Incentive Line rotates outwards. The net effect will depend on the maganitude of the two effects. The next corollary gives the condition under which a lower search cost will lead to higher prices and a lower reservation utility.

Corollary 7 *When search costs are lower, (1)* p_h^* *will increase if* $\frac{h(\eta)}{F_l(\eta)}$ *is increasing; and (2)* U^* *will decrease if* $\frac{h(\eta)(1-F_l(\eta))}{F_l(\eta)}$ *is increasing.*

Proof. In the Appendix. ■

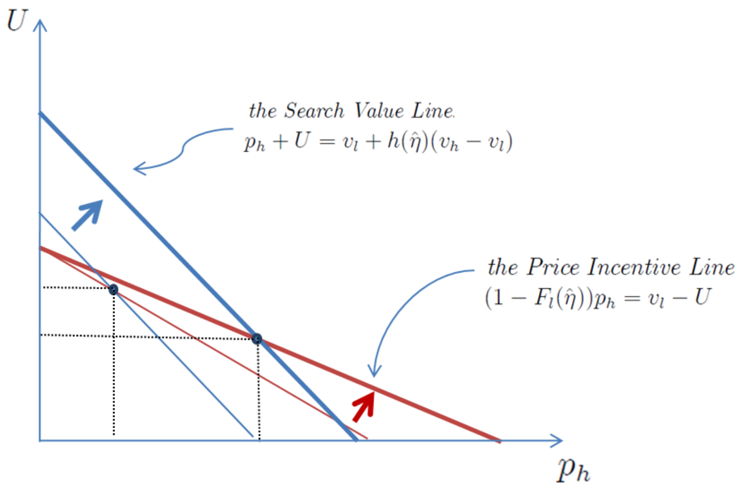

Figure 7: The impacts of a lower search cost on equilibrium

The result can be shown graphically by Figure 7. When s decreases, both the Price Incentive Line rotates outward and the Search Value Line shifts outward. The condition that $\frac{h(\eta)}{F_l(\eta)}$ and $\frac{h(\eta)(1-F_l(\eta))}{F_l(\eta)}$ are increasing functions guarantees that the search-value effect dominates the price-incentive effect.

6.2.2 Informativeness

The change of the exogenous informativeness, i.e., the distributions of the signals, changes the Price Incentive Line and the Search Value Line not only because the functional forms of $F_l(\cdot)$ and $h(\cdot)$ differ, but also because $\hat{\eta}$, i.e., the endogenous informativeness, is changed when the distributions differ. To capture the change of the exogenous informativeness, consider the assumption about f_h as follows:

Assumption 1 *Suppose there are a family of distributions of f_h^α associated with the index α, which satisfy MLRP, i.e., for $\alpha > \alpha'$, $\frac{f_h^\alpha(\eta)}{f_h^{\alpha'}(\eta)}$ is increasing in η.*

The decrease in the informativeness is corresponding to the decrease in α. In this part we assume that f_l does not change in α. Then $\hat{\eta}(\alpha)$, the cutoff signal in equilibrium that is associated with the distribution indexed by α, is determined by

$$\mu(1 - F_h^\alpha(\hat{\eta}(\alpha)))(1 - h^\alpha(\hat{\eta}(\alpha))) - (1-\mu)\sigma(1 - F_l(\hat{\eta}(\alpha)))h^\alpha(\hat{\eta}(\alpha)) = \frac{s}{v_h - v_l}, \tag{16}$$

where

$$h^\alpha(\eta) = \frac{1}{\mu f_h^\alpha(\eta) + (1-\mu)\sigma f_l(\eta)}.$$

The next corollary shows that p_h^* will increase while U^* will decrease as the imformativeness is improved if $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$. We can show that, when α increases, the value of search always increases such that the Search Value Line will always shift outward. However, the change in the Price Incentive Line is ambiguous. If $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$, then the Price Incentive Line will rotates inwards as α increases. As a result, p_h^* will increase while U^* will decrease.

Corollary 8 *Given Assumption 1, when α increases, p_h^* will increase while U^* will decrease if $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$.*

Proof. In the Appendix. ■

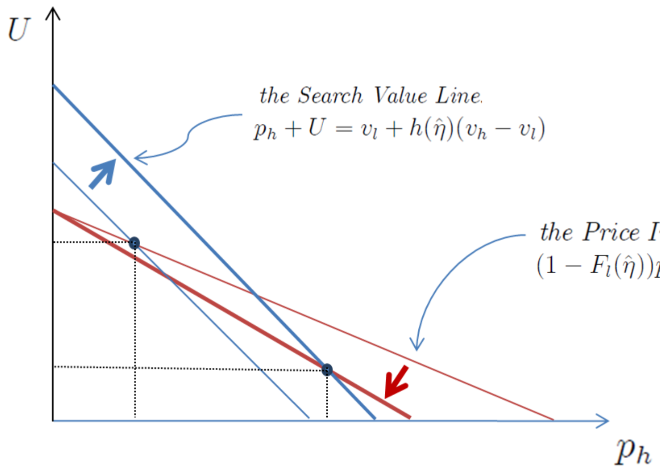

Figure 8: The impacts of better informativeness on equilibrium

The result of the corollary can by shown by the Figure above. The Price Incentive Line rotates inward while the Search Value Line shifts outward. As a result, the price increases while the reservation utility decreases.

Note that, the corollary also implies that, when the exogenous informativeness α decreases in online markets, if the consumer reacts by increasing the searching intensity and thus the endogenous informativeness, then consumer's welfare will be improved.

7 Conclusion

We incorporate consumers' uncertainty over vertical product quality into a consumer search model and examine how search costs and informativeness can affect both the market price and consumer surplus. We first build a baseline model in which we show that a separating perfect Bayesian equilibrium does not exist if the conditional distribution of the quality signal has full support. We define the Search Value Line that characterizes consumers' behavior and the Price Incentive Line that describes firms' incentives. We show that a partial-pooling equilibrium can be characterized by the intersection of the Search Value Line and the Price Incentive Line.

Accordingly, we show that the changes in the primitives can have two competing effects: the search-value effect and the price-incentive effect. Due to the search-value effect, the decrease in search costs can hurt consumers while benefiting firms. Due to the price-incentive effect, the increase in informativeness can benefit consumers but may lead to either higher or lower prices. The comparative statics depend on the net effect of the two. In particular, a decrease in search cost has a search-value effect but no price-incentive effect, and it will lead to a decrease in consumer surplus.

In the extended model, we introduce the concept of endogenous informativeness: consumers can endogenously choose the level of informativeness upon purchase by adjusting their search intensity. In the extended model, changes in search cost can result in both the search-value effect and the price-incentive effect. We show that, if the informativeness is good such that the search-value effect is dominant, then consumers still can be worse off when search costs are lower. However, when the exogenous informativeness decreases, consumers can be better off if they respond by increasing their search intensity.

There are several directions for future studies. First, for model clarity, we do not consider horizontal differentiation in this paper; however, our framework facilitates future research that can consider the information uncertainty over both vertical and horizontal product attributes. Second, while, in our model, the proportion of the high-quality sellers is exogenous, in future research, it would be natural to endogenize the proportion of high-quality sellers when the vertical product attributes are not fully observed. For example, one could consider incorporating a setup similar to Fishman and Levy's (2015), in which sellers can make investments to increase their probability of producinghigh-quality products. Moreover, while Chen et al. (2015, 2017) consider the imperfect vertical-quality information on the online market and study firms' market choices in monopolistic and duopolistic settings, future research could examine competitive firms' market choices in a search model when there exists uncertainty over vertical product quality.

References

[1] Anderson, Simon P., and Régis Renault, “Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model,” The RAND Journal of Economics, 1999, Vol. 30, No. 4, pp. 719-735.

[2] Bakos, 1997, "Reducing Buyer Search Costs: Implications for Electronic Marketplaces", Management Science

[3] Bar-Isaac, Heski, Guillermo Caruana, and Vicente Cuñat, “Search, Design, and Market Structure,” American Economic Review, 2012, Vol. 102, No. 2, pp. 1140-60.

[4] Board, Simon, and Jay Lu, “Competitive Information Disclosure in Search markets,” Journal of Political Economy, forthcoming.

[5] Carlin, Bruce I., “Strategic price complexity in retail financial markets,” Journal of Financial Economics, 2009, vol. 91, issue 3, 278-287.

[6] Chatterjee, P., 2001, “Online Reviews – Do Consumers Use Them?" ACR 2001 Proceedings, eds. M. C. Gilly and J. Myers-Levy, Provo, UT: Association for Consumer Research, 129-134.

[7] Chen, Pu, Yijuan Chen, Xiangting Hu, and Sanxi Li, 2015, "Can online markets attract high quality products?", Economic Modelling, Vol. 51, 65-71

[8] Chen, Yijuan, Xiangting Hu, and Sanxi Li, "Quality differentiation and firms' choices between online and physical markets", International Journal of Industrial Organization, 2017, Vol. 52, 96-132.

[9] Choi, Michael, Anovia Yifan Dai, and Kyungmin Kim (2016), “Consumer Search and Price Competition,” mimeo.

[10] Cunha, M., A. Osoio and R. Ribeiro, 2018, "Endogenous Product Design and Quality with Rationally Inattentive Consumers", working paper

[11] Daughety, A.F. and J.F. Reinganum, 2008, "Imperfect competition and quality signaling", RAND Journal of Economics, Vol. 39, No. 1, pp. 163–183

[12] Degeratu, A.M., A. Rangaswamy and J. Wu, 2000, "Consumer choice behavior in online and traditional supermarkets: The effects of brand name, price, and other search attributes", International Journal of Research in Marketing,17, 55–78.

[13] Ellison, Glenn, Alexander Wolitzky, “A Search Cost Model of Obfuscation,” The Rand Journal of Economics, 2012, Vol. 43, Issue 3, Page 417–441.

[14] Fishman, Arthur, and Nadav Levy, “Search Costs and Investment in Quality,” Vol. 63, Issue 4, 2015, Pages 625–641.

[15] Gamp, Tobias, “Search, Information Acquisition, and Obfuscation,” 2015, mimeo.

[16] Gamp, Tobias, and Daniel Krähmer, “Deception and Competition in Search Markets,” 2017, mimeo.

[17] Hann, Marco A., Jose L. Moraga-Gonzalez, and Vaiva Petrikaite, “A model of Directed Consumer Search,” Mimeo.

[18] Janssen, M.C.W. & Parakhonyak, A., “Consumer search markets with costly revisits,” Economic Theory (2014) 55: 481. https://doi.org/10.1007/s00199-013-0761-3.

[19] Kardes, F.R., M. Cronley, J. Kellaris and S. Posavac, 2004, "The Role of Selective Information Processing in Price-Quality Inference", Journal of consumer research, Vol.31, No.2, pp 368-374

[20] Kuksov, D., 2004, "Buyer Search Costs and Endogenous Product Design", Marketing Science, Vol. 23, No. 4, pp. 490–499

[21] Lal and Sarvary, 1999, "When and How is the Internet Likely to Decrease Price Competition?", Marketing Science

[22] Larson, N., 2013, "Niche products, generic products and consumer search." Economic Theory, 52(2), 793-832.

[23] Janssen M. and S. Roy, 2015, "Competition, Disclosure and Signalling", The Economic Journal, 125, 86–114.

[24] Moorthy, Sridhar, and Kannan Srinivasan, “Signaling Quality with a Money-Back Guarantee: The Role of Transaction Costs”, Marketing Science, 1995, Vol. 14(4), pp. 442-466.

[25] Petrikaite, V., “A search model of costly product returns”, International Journal of Industrial Organization (2017), http://dx.doi.org/10.1016/j.ijindorg.2017.06.003

[26] Rhodes, D. and J. Zhou, 2017, "Consumer Search and Retail Market Structure", TSE working paper,

[27] Wilson, Chris M., “Ordered Search and Equilibrium Obfuscation,” International Journal of Industrial Organization, 2010, 28, 496-506.

[28] Wolinsky, Asher, “Prices as Signals of Product Quality,” The Review of Economic Studies, 1983, Vol. 50, No. 4, pp. 647-658.

[29] Wolinsky, Asher. "True Monopolistic Competition as a Result of Imperfect Information," The Quarterly Journal of Economics, 1986, 101, 493-511.

[30] Yang, 2013, "Targeted Search and the Long Tail Effect", The Rand Journal of Economics.

[31] Zeithaml, V.A., 1988, "Consumer Perceptions of Price, Quality, and Value: A Means-End Model and Synthesis of Evidence", Journal of Marketing, Vol. 52, No. 3, pp. 2-22

8 Appendix A: The separating equilibrium

Consider the case in which $\beta_h = 1$ and $\beta_l < 1$. In any separating equilibrium, the product quality is fully revealed by the price. The Incentive Compatibility (IC) Condition will be

$$(1-\beta_l)p_h \leq p_l.$$

When $\beta_h = 1$, then a high-quality firm will never generate a low signal. This implies that, if the low-quality firm mimics the price of the high-quality firm then with positive probability its true type is revealed, and the visiting consumer will not buy. In this case, mimicing the higher price can result in less demand. The next lemma describes the separating equilibrium when the Incentive Compatibility Condition holds with equality.

Lemma 2 *Suppose $\beta_l \in (1/2, 1)$ and $\beta_h = 1$. In the separating equilibrium at which the IC condition holds with equality:*

(1) The high-quality firm charges price $p_h = \bar{p}_h \equiv \frac{1}{\beta_l}(v_h - v_l - \frac{s}{\mu})$, and the low-quality firm charges $p_l = \bar{p}_l \equiv \frac{1-\beta_l}{\beta_l}(v_h - v_l - \frac{s}{\mu})$.

(2) The consumer holds the reservation utility $\overline{U}_s \equiv v_l - \frac{1-\beta_l}{\beta_l}(v_h - v_l - \frac{s}{\mu})$.

(3) On the equilibrium path, the consumer will buy upon seeing $\bar{p}_h$ or $\bar{p}_l$, since $v_h - \bar{p}_h \geq \overline{U}_s$ and $v_l - \bar{p}_l \geq \overline{U}_s$.

(4) On the off-equilibrium path, the consumer holds the belief $\Pr(v = v_h | p \neq p_h, p \neq p_l) = 0$ and will buy if $p \leq v_l - \overline{U}_s$ and continue to search otherwise.

Proof. In a separating equilibrium, the product quality is fully revealed by the price. Conditional on this fact, a low-quality firm will charge a price no less than $v_l - U$. Hence, $p_l = v_l - U$, and the IC condition will be

$$(1-\beta_l)p_h \leq v_l - U.$$

Consider the separating equilibrium at $p_h = \frac{v_l - U}{1-\beta_l}$. Suppose the off-equilibrium path is that $\Pr(v = v_h | p \neq p_h, p \neq p_l) = 0$. The consumer's reservation utility is determined by

$$\mu[v_h - p_h - U] + (1-\mu)[v_l - p_l - U] = s.$$

Plug $p_l = v_l - U$ and $p_h = \frac{v_l - U}{1-\beta_l}$ into the above equation we will get $U = v_l - \frac{1-\beta_l}{\beta_l}(v_h - v_l - \frac{s}{\mu})$. Hence, $p_l = \frac{1-\beta_l}{\beta_l}(v_h - v_l - \frac{s}{\mu})$ and $p_h = \frac{1}{\beta_l}(v_h - v_l - \frac{s}{\mu})$. ■

In the equilibrium, the consumer will buy from the first store she visits. Note that $v_l - \bar{p}_l < v_h - \bar{p}_h$ so that the consumer will get a higher surplus if they buy from the high-quality firms. If the consumer first visits a high-quality firm, she will strictly prefer to stopping searching and buying from the firm. If she first visits a low-quality firm, she is indifferent between buying from the low-quality firm to get a surplus $v_l - p_l$, and continuing to search by paying a search cost s and to meet a high-quality firm, from whom she will get a surplus $v_h - p_h$, which is greater than $v_l - p_l$, with probability μ. The firms will have no incentive to increase price as in the Diamond paradox: For the high-quality firm, with information asymmetry, p_h is restricted by the IC condition. For the low-quality firm, if a low-quality firm charges a higher price, the visiting consumer will strictly prefer to running away because given that other firms charge the equilibrium price, the additional (expected) benefit of searching one more store strictly exceeds the search cost. In fact, as $\beta_l \to 1$ and $s \to 0$, the above equilibrium converges to $(p_l = 0, p_h = v_h - v_l)$, which is the equilibrium under Bertrand competition.

9 Appendix B: Proofs

9.1 Proof of Proposition 1

Proof. Suppose not, there exists a separating equilibrium in which $P(v_l) = p_l$ and $P(v_h) = p_h$, and $p_l \neq p_h$. On the equilibrium path, the belief of the consumer will be

$$\begin{aligned} \Pr(v_h|(\eta &= v_h, p_h)) = 1, \Pr(v_h|(\eta = v_l, p_h)) = 1, \\ \Pr(v_h|(\eta &= v_h, p_l)) = 0, \Pr(v_h|(\eta = v_l, p_l)) = 0. \end{aligned}$$

The visiting consumer expects $E[v|(\eta, p_h)] = v_h$ and $E[v|(\eta, p_l)] = v_l$ for any $\eta \in \{v_l, v_h\}$. First consider the scenario in which $p_h > p_l \geq 0$ in equilibrium. Regarding consumer's strategy there are four possible cases on the equilibrium path:

Case 1: Consumers buy upon seeing p_h ($v_h - p_h \geq U$), and not buy upon seeing p_l ($v_l - p_l < U$). In this case, the low-quality firm will surely be better off by deviating to p_h if $p_h > 0$.

Case 2: Consumers will buy upon seeing p_h or p_l, i.e., $v_h - p_h \geq U$ and $v_l - p_l \geq U$. In this case, the low-quality firm will be better off by deviating to p_h since $p_h > p_l \geq 0$.

Case 3: Consumers will buy from the low-quality firm but not the high-quality firm. In this case, $v_h - p_h < U$ and $v_l - p_l \geq U$. The high-quality firm will be strictly better off by deviating to p_l if $p_l > 0$. If $p_l = 0$, then according to the expression of U,

$$U = \frac{(1-\mu)(v_l - p_l) - s}{1-\mu} = v_l - \frac{s}{1-\mu}.$$

The low-quality firm can then be better off by deviating to any price $p \in (0, \frac{s}{1-\mu})$, since $E[v|(\eta, p)] \geq v_l$ for any p, and a visiting consumer will buy at any $p \in (0, \frac{s}{1-\mu})$ if she holds the reservation utility $U = v_l - \frac{s}{1-\mu}$.

Case 4: Consumers do not participate in the market. This case is excluded by the condition $r < -s$.

Follow similar argument, we can rule out the scenario in which $p_l > p_h \geq 0$ in equilibrium. ■

9.2 Proof of Proposition 2

Proof. First, it is obvious that the belief of the consumer is consistent with the strategy profile. Given the consumer's belief, since $(1-\beta_l)p_h^* = p_l^*$, the low-quality firm will not deviate because she will not be strictly better off by any deviation, given the off-equilibrium-path belief. Because $\beta_h > \frac{1}{2}$ and $\beta_l > \frac{1}{2}$, the high-quality firm will not deviate to any other price since $\beta_h p_h^* > \frac{1}{2}p_h^* > (1-\beta_l)p_h^* = p_l^*$.

For the consumer, on the equilibrium path, the consumer will buy upon seeing (p_h^*, η_h) or $p = p_l$, and will continue to search upon seeing (p_h^*, η_l). So we need to show

$$\begin{aligned} U &\leq v_l + \frac{\mu\beta_h(v_h - v_l)}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} - p_h^* & (17) \\ U &\leq v_l - p_l^* & (18) \\ U &> v_l + \frac{\mu(1-\beta_h)(v_h - v_l)}{\mu(1-\beta_h) + (1-\mu)\beta_l\sigma} - p_h^*. & (19) \end{aligned}$$

The equilibrium exists if the solution (p_l^*, p_h^*, U^*) satisfies (17)-(19).

It is straightforward to solve (6) and (7) to get (8) and

$$U^* = v_l - (1-\beta_l) \cdot \frac{\mu(v_h - v_l) - \frac{s}{\beta_h}}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma}. \tag{20}$$

By (3), since $s > 0$, (17) is satisfied. Since $p_l^* = v_l - U^*$, (18) is satisfied. The inequality (19) requires that

$$p_h^* + U^* = v_l + \frac{\mu\beta_h(v_h - v_l) - s}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} > v_l + \frac{\mu(1-\beta_h)(v_h - v_l)}{\mu(1-\beta_h) + (1-\mu)\beta_l\sigma},$$

implying $\frac{s}{v_h - v_l} < \frac{\sigma\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l\sigma}$. Note that $\frac{\sigma\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l\sigma} = \frac{\mu(1-\mu)(\beta_h+\beta_l-1)}{\frac{\mu(1-\beta_h)}{\sigma}+(1-\mu)\beta_l}$ is increasing in σ. Define $\bar{\sigma}$ to be the σ satisfying $\frac{s}{v_h - v_l} \equiv \frac{\bar{\sigma}\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l\bar{\sigma}}$. Since $\frac{s}{v_h - v_l} < \frac{\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l}$, $\bar{\sigma} \in (0, 1)$ and is well-defined. For any $\sigma > \bar{\sigma}$, $\frac{s}{v_h - v_l} < \frac{\sigma\mu(1-\mu)(\beta_h+\beta_l-1)}{\mu(1-\beta_h)+(1-\mu)\beta_l\sigma}$ is satisfied and the equilibrium can be sustained. ■

9.3 Proof of Corollary 1

Proof. (U^*, p_h^*) is determined by

$$
\begin{aligned}
U^* &= v_l + (1 - \frac{1}{\beta_l})\phi(\beta_h, \beta_l, s; \sigma) \\
p_h^* &= \frac{1}{\beta_l}\phi(\beta_h, \beta_l, s; \sigma).
\end{aligned}
$$

It is obvious that $\frac{\partial \phi(\beta_h, \beta_l, s;\sigma)}{\partial s} < 0$. Thus, $\frac{\partial p_h^*}{\partial s} < 0$ and $\frac{\partial U^*}{\partial s} > 0$. Since $p_l^* = v_l - U^*$, it follows that $\frac{\partial p_l^*}{\partial s} < 0$. ■

9.4 Proof of Corollary 2

Proof. An increase of β_h increases the IVS and has no impact on the price-incentive line. Hence, an increase in β_h leads to higher price and lower consumer surplus. For β_l, (U^*, p_h^*) is determined by

$$
\begin{aligned}
U^* &= v_l - \frac{1-\beta_l}{\beta_l} \cdot \frac{\mu\beta_h(v_h - v_l) - s}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma} = v_l - \frac{\mu\beta_h(v_h - v_l) - s}{\frac{\beta_l}{1-\beta_l}\mu\beta_h + (1-\mu)\beta_l\sigma} \\
p_h^* &= \frac{1}{\beta_l} \cdot \frac{\mu\beta_h(v_h - v_l) - s}{\mu\beta_h + (1-\mu)(1-\beta_l)\sigma}.
\end{aligned}
$$

For $\frac{\partial U^*}{\partial \beta_l}$, it is straightforward to calculate

$$
\frac{\partial}{\partial \beta_l}\left[\frac{\beta_l}{1-\beta_l}\mu\beta_h + (1-\mu)\beta_l\sigma\right] = \frac{\mu\beta_h}{1-\beta_l} + (1-\mu)\sigma + \frac{\mu\beta_l\beta_h\sigma}{(1-\beta_l)^2} > 0.
$$

Hence, $\frac{\partial U^*}{\partial \beta_l} > 0$.

In addition, $\frac{\partial p_h^*}{\partial \beta_l} < 0$ iff

$$
\frac{\partial}{\partial \beta_l}\left[\beta_l\left(\mu\beta_h + (1-\mu)(1-\beta_l)\sigma\right)\right] = \mu\beta_h + (1-\mu)(1-2\beta_l)\sigma > 0,
$$

i.e., iff $\beta_l < \frac{1}{2}(\frac{\mu}{\sigma(1-\mu)}\beta_h + 1)$. ■

9.5 Proof of Corollary 3

In the discussion part, we will have $\beta_h = \beta_l = \beta$. Thus $\phi(\beta_h, \beta_l, s; \sigma)$ can be written as

$$
\phi(\beta, s; \sigma) \equiv \frac{\mu\beta(v_h - v_l) - s}{\mu\beta + (1-\mu)(1-\beta)\sigma}.
$$

We first show the following two lemmas:

Lemma 3 *Suppose* $\frac{s}{v_h - v_l} < \frac{\sigma\mu(1-\mu)(2\beta-1)}{\mu(1-\beta)+(1-\mu)\beta\sigma}$ *such that the partial-pooling equilibrium with* σ *exists. Then (1)* $\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta} > 0$*; and (2)* $\frac{\partial^2\phi(\beta,s;\sigma)}{\partial\beta^2} < 0$ *if* $\mu > \frac{\sigma}{1+\sigma}$*, and* $\frac{\partial^2\phi(\beta,s;\sigma)}{\partial\beta^2} > 0$ *if* $\mu < \frac{\sigma}{1+\sigma}$*.*

Proof. It is straightforward to show that

$$\begin{aligned}\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta} &= \frac{\partial}{\partial\beta}(\frac{\mu\beta(v_h-v_l)-s}{\mu\beta+(1-\mu)(1-\beta)\sigma}) \\ &= \frac{1}{(\mu\beta+(1-\mu)(1-\beta)\sigma)^2}(s(\mu-\sigma(1-\mu))+\sigma\mu(1-\mu)(v_h-v_l)).\end{aligned}$$

Therefore, if $\mu > \frac{\sigma}{1+\sigma}$, apparently, $\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta} > 0$. If $\mu < \frac{\sigma}{1+\sigma}$, then $\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta} > 0$ if $\frac{s}{v_h-v_l} < \frac{\mu(1-\mu)}{\sigma(1-\mu)-\mu}$. Note that $\frac{\sigma\mu(1-\mu)(2\beta-1)}{\mu(1-\beta)+(1-\mu)\beta\sigma} < \frac{\mu(1-\mu)}{\sigma(1-\mu)-\mu}$. Hence, $\frac{s}{v_h-v_l} < \frac{\mu(1-\mu)}{\sigma(1-\mu)-\mu}$ holds when the equilibrium exists.

Note that

$$\begin{aligned}\frac{\partial^2\phi(\beta,s;\sigma)}{\partial\beta^2} &= \frac{-2(\mu-\sigma(1-\mu))(s(\mu-\sigma(1-\mu))+\sigma\mu(1-\mu)(v_h-v_l))}{(\mu\beta+(1-\mu)(1-\beta))^3} \\ &= \frac{-2(\mu-\sigma(1-\mu))}{(\mu\beta+(1-\mu)(1-\beta))}\cdot\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta}\end{aligned}$$

Since $\frac{\partial\phi(\beta,s;\sigma)}{\partial\beta} > 0$, $\frac{\partial^2\phi(\beta,s;\sigma)}{\partial\beta^2} < 0$ if $\mu > \frac{\sigma}{1+\sigma}$, and $\frac{\partial^2\phi(\beta,s;\sigma)}{\partial\beta^2} > 0$ if $\mu < \frac{\sigma}{1+\sigma}$. ■

Lemma 4 *Suppose* $\frac{s}{v_h-v_l} < a(\sigma) \equiv \min\left\{\frac{\sigma\mu(1-\mu)(2\beta-1)}{\mu(1-\beta)+(1-\mu)\beta\sigma}, \frac{\beta^2\mu^2}{\mu+(1-\beta)^2[(1-\mu)\sigma-\mu]}\right\}$*. Then: (1)* $\frac{\partial U^*}{\partial\beta} > 0$*. (2)* $\frac{\partial p_h^*}{\partial\beta} > 0$ *if* $\mu < \frac{\sigma}{1+\sigma}$*. If* $\mu > \frac{\sigma}{1+\sigma}$*, then there exists* $\widehat{\beta} \in [\frac{1}{2}, 1]$*, such that* $\frac{\partial p_h^*}{\partial\beta} > 0$ *if* $\beta < \widehat{\beta}$*, and* $\frac{\partial p_h^*}{\partial\beta} < 0$ *if* $\beta > \widehat{\beta}$*.*

Proof. (U^*, p_h^*) is determined by

$$\begin{aligned}U^* &= v_l + (1-\frac{1}{\beta})\phi(\beta,s;\sigma) = v_l - (1-\beta)\cdot\frac{\mu(v_h-v_l)-\frac{s}{\beta}}{\mu\beta+(1-\mu)(1-\beta)\sigma} \\ p_h^* &= \frac{1}{\beta}\phi(\beta,s;\sigma) = \frac{\mu(v_h-v_l)-\frac{s}{\beta}}{\mu\beta+(1-\mu)(1-\beta)\sigma}.\end{aligned}$$

It is straightforward to calculate $v\beta^2\mu^2 + s((\sigma(\mu-1)+\mu)(1-\beta)^2-\mu)$

$$\begin{aligned}\frac{\partial U^*}{\partial\beta} &= -\frac{\partial}{\partial\beta}(\frac{\mu(v_h-v_l)-\frac{s}{\beta}}{\mu\beta+(1-\mu)(1-\beta)\sigma}(1-\beta)) \\ &= \frac{s[(1-\beta)^2(\sigma(\mu-1)+\mu)-\mu]+\beta^2\mu^2(v_h-v_l)}{\beta^2(\mu\beta+(1-\mu)(1-\beta)\sigma)^2}.\end{aligned}$$

Note that $(1-\beta)^2(\sigma(\mu-1)+\mu)-\mu=-(1-\beta)^2(1-\mu)\sigma-\mu(1-(1-\beta)^2)<0$. Therefore, $\frac{\partial U^*}{\partial \beta}>0$ if

$$\frac{s}{v_h-v_l}<\frac{\beta^2\mu^2}{\mu+(1-\beta)^2[(1-\mu)\sigma-\mu]}.$$

Hence, $\frac{\partial U^*}{\partial \beta}>0$ when $\frac{s}{v_h-v_l}<\min\left\{\frac{\sigma\mu(1-\mu)(2\beta-1)}{\mu(1-\beta)+(1-\mu)\beta\sigma},\frac{\beta^2\mu^2}{\mu+(1-\beta)^2[(1-\mu)\sigma-\mu]}\right\}$.

In addition, for $\frac{\partial p_h^*}{\partial \beta}$,

$$\begin{aligned}
\frac{\partial p_h^*}{\partial \beta} &= \frac{\partial}{\partial \beta}\left(\frac{\mu(v_h-v_l)-\frac{s}{\beta}}{\mu\beta+(1-\mu)(1-\beta)\sigma}\right)\\
&= \frac{s}{\beta^2}\cdot\frac{1}{\mu\beta+(1-\mu)(1-\beta)\sigma}-\left(\mu(v_h-v_l)-\frac{s}{\beta}\right)\cdot\frac{\mu-(1-\mu)\sigma}{(\mu\beta+(1-\mu)(1-\beta)\sigma)^2}\\
&= \frac{s}{\beta^2}\cdot\frac{1}{\mu\beta+(1-\mu)(1-\beta)\sigma}-p_h^*\cdot\frac{\mu-(1-\mu)\sigma}{\mu\beta+(1-\mu)(1-\beta)\sigma}\\
&= \frac{1}{\mu\beta+(1-\mu)(1-\beta)\sigma}\cdot\left[\frac{s}{\beta^2}-(\mu-(1-\mu)\sigma)\cdot p_h^*\right].
\end{aligned}$$

Apparently, if $\mu<\frac{\sigma}{1+\sigma}$, then $\frac{\partial p_h^*}{\partial \beta}>0$. If $\mu>\frac{\sigma}{1+\sigma}$, $\frac{\partial p_h^*}{\partial \beta}$ can be written as

$$\frac{\partial p_h^*}{\partial \beta}=\frac{1}{\beta^2}\left[\frac{\partial \phi(\beta,s;\sigma)}{\partial \beta}\beta-\phi(\beta,s;\sigma)\right].$$

Note that when $\mu>\frac{\sigma}{1+\sigma}$,

$$\frac{\partial}{\partial \beta}\left(\frac{\partial \phi(\beta,s;\sigma)}{\partial \beta}\beta-\phi(\beta,s;\sigma)\right)=\frac{\partial^2\phi(\beta,s;\sigma)}{\partial \beta^2}\beta.$$

From the previous Lemma, $\frac{\partial^2\phi(\beta,s;\sigma)}{\partial \beta^2}<0$ when $\mu>\frac{\sigma}{1+\sigma}$. As a result, $\frac{\partial p_h^*}{\partial \beta}$ is decreasing in β. Hence, there exists $\widehat{\beta}$ such that $\frac{\partial p_h^*}{\partial \beta}>0$ if $\beta<\widehat{\beta}$, and $\frac{\partial p_h^*}{\partial \beta}<0$ if $\beta>\widehat{\beta}$. Finally, since $\beta\in(1/2,1)$, we can restrict the range of $\widehat{\beta}$ on $[1/2,1]$. ■

Note that following the path $s=g(\beta)$,

$$\frac{dU^*}{d\beta}=\frac{\partial U^*}{\partial \beta}+\frac{\partial U^*}{\partial s}\cdot g'(\beta).$$

By Corollary 1, $\frac{\partial U^*}{\partial s}>0$ and $\frac{\partial p_h^*}{\partial s}<0$. Since $\frac{\partial U^*}{\partial \beta}>0$ when $\frac{s}{v_h-v_l}<a(\sigma)$, $\frac{dU^*}{d\beta}>0$ holds for any path if $\frac{s}{v_h-v_l}<a(\sigma)$.

For p_h^*, we have

$$\frac{dp_h^*}{d\beta}=\frac{\partial p_h^*}{\partial \beta}+\frac{\partial p_h^*}{\partial s}\cdot g'(\beta).$$

From the previous lemma, $\frac{\partial p_h^*}{\partial \beta}>0$ if $\mu<\frac{\sigma}{1+\sigma}$ or if $\mu>\frac{\sigma}{1+\sigma}$ and $\beta<\widehat{\beta}$; and $\frac{\partial p_h^*}{\partial \beta}<0$ if $\mu>\frac{\sigma}{1+\sigma}$ and $\beta>\widehat{\beta}$.

If $\mu > \frac{\sigma}{1+\sigma}$ and $\beta > \widehat{\beta}$, then $\frac{\partial p_h^*}{\partial \beta} < 0$ and it is obvious that $\frac{dp_h^*}{d\beta} < 0$ since $\frac{\partial p_h^*}{\partial s} < 0$. If $\mu < \frac{\sigma}{1+\sigma}$ or if $\mu > \frac{\sigma}{1+\sigma}$ and $\beta < \widehat{\beta}$, then $\frac{dp_h^*}{d\beta} < 0$ if $-\frac{\frac{\partial p_h^*}{\partial \beta}}{\frac{\partial p_h^*}{\partial s}} < g'(\beta)$. We have shown in the previous proof that

$$\frac{\partial p_h^*}{\partial \beta} = \frac{1}{\mu\beta + (1-\mu)(1-\beta)\sigma} \cdot \left[\frac{s}{\beta^2} - (\mu - (1-\mu)\sigma) \cdot p_h^*\right].;$$

and it is straightforward to show that

$$\frac{\partial p_h^*}{\partial s} = -\frac{1}{\beta} \cdot \frac{1}{\mu\beta + (1-\mu)(1-\beta)\sigma}$$

Hence,

$$-\frac{\frac{\partial p_h^*}{\partial \beta}}{\frac{\partial p_h^*}{\partial s}} = \frac{s}{\beta} - \beta(\mu - (1-\mu)\sigma)p_h^*.$$

Therefore, $\frac{dp_h^*}{d\beta} < 0$ if $g'(\beta) > \frac{s}{\beta} - \beta(\mu - (1-\mu)\sigma)p_h^*$.

9.6 Proof of Corollary 4

Proof. We first check the consumer's incentive. Upon observing $(\eta = v_h, p = p_h^\lambda)$, the consumer expects the product value is

$$E(v|\eta = v_h, p = p_h^\lambda) = v_l + \frac{\mu\beta(v_h - v_l)}{\mu\beta + (1-\mu)(1-\beta)\sigma}.$$

and upon seeing $(\eta = v_l, p = p_h^\lambda)$, the consumer expects the product quality is

$$E(v|\eta = v_l, p = p_h^\lambda) = v_l + \frac{\mu(1-\beta)(v_h - v_l)}{\mu(1-\beta) + (1-\mu)\beta\sigma}$$

So the consumer will optimally choose to purchase when she observes a high signal and not to purchase when she observes a low signal if

$$\begin{aligned} v_l + \frac{\mu\beta(v_h - v_l)}{\mu\beta + (1-\mu)(1-\beta)\sigma} - p_h^\lambda &> U^\lambda, \\ v_l + \frac{\mu(1-\beta)(v_h - v_l)}{\mu(1-\beta) + (1-\mu)\beta\sigma} - p_h^\lambda &< U^\lambda. \end{aligned}$$

It is straightforward to show that, if $\frac{s}{v_h - v_l} > \frac{\lambda(1-\mu)(1-\beta)\mu\beta\sigma}{\mu\beta + (1-\mu)(1-\beta)\sigma}$,

$$p_h^\lambda + U^\lambda = v_l + \frac{\mu\beta(v_h - v_l) - s}{\mu\beta + (1-\mu)(1-\beta)(1-\lambda)\sigma} < v_l + \frac{\mu\beta(v_h - v_l)}{\mu\beta + (1-\mu)(1-\beta)\sigma};$$

and if $\frac{s}{v_h-v_l} < \frac{\mu(1-\mu)(2\beta-1)\sigma}{\mu(1-\beta)+(1-\mu)\beta\sigma}$, for any $\lambda \in (0,1)$,

$$p_h^\lambda + U^\lambda = v_l + \frac{\mu\beta(v_h-v_l)-s}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} > v_l + \frac{\mu(1-\beta)(v_h-v_l)}{\mu(1-\beta)+(1-\mu)\beta\sigma}$$

Hence, $E(v|\eta = v_h, p = p_h^\lambda) - p_h^\lambda > U^\lambda$ and $E(v|\eta = v_l, p = p_h^\lambda) - p_h^\lambda < U^\lambda$.

We then discuss firm's optimality. Apparently, it is optimal for the high-quality firm to provide the return / refund policy with probability 1, since consumers will not return a high-quality product. Hence, if the low-quality firm does not provide the return / refund policy, the belief will be that the product is low-quality with probability 1. Conditional on the provision of return / refund policy, suppose the off-equilibrium-path belief is $\Pr(v = v_h|p \neq p_h^\lambda, p \neq p_l^\lambda) = 0$. Hence, the low-quality firm will provide the return / refund policy and be indifferent between p_h^λ and p_l^λ when $(1-\beta)(1-\lambda)p_h^\lambda = v_l - U^\lambda$. The high-quality firm will optimally charge the p_h^λ if $\beta p_h^\lambda \geq v_l - U^\lambda$. Since $\beta > 1/2$, the inequality will hold.
■

9.7 Proof of Corollary 5

Proof. First note

$$\begin{aligned}\frac{\partial\psi(\beta,\lambda,s;\sigma)}{\partial\lambda} &= \frac{[\mu\beta(v_h-v_l)-s](1-\mu)(1-\beta)\sigma}{[\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma]^2} \\ &= \psi(\beta,\lambda,s;\sigma)\cdot\frac{(1-\mu)(1-\beta)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma}\end{aligned}$$

Since $U^\lambda = v_l + (1 - \frac{1}{1-(1-\beta)(1-\lambda)})\psi(\beta,\lambda,s;\sigma)$,

$$\begin{aligned}&\frac{\partial U^\lambda}{\partial\lambda} \\ =\ & -\frac{(1-\beta)(1-\lambda)}{1-(1-\beta)(1-\lambda)}\cdot\frac{\partial\psi(\beta,\lambda,s;\sigma)}{\partial\lambda} + \psi(\beta,\lambda,s;\sigma)\cdot\cdot\frac{1-\beta}{(1-(1-\beta)(1-\lambda))^2} \\ =\ & -\frac{(1-\beta)(1-\lambda)\psi(\beta,\lambda,s;\sigma)}{1-(1-\beta)(1-\lambda)}\cdot\frac{(1-\mu)(1-\beta)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} + \cdot\frac{\psi(\beta,\lambda,s;\sigma)\cdot(1-\beta)}{(1-(1-\beta)(1-\lambda))^2} \\ =\ & \frac{(1-\beta)\psi(\beta,\lambda,s;\sigma)}{1-(1-\beta)(1-\lambda)}\left[-\frac{(1-\lambda)(1-\mu)(1-\beta)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} + \frac{1}{1-(1-\beta)(1-\lambda)}\right].\end{aligned}$$

Therefore, $\frac{\partial U^\lambda}{\partial\lambda} > 0$ since $\frac{1}{1-(1-\beta)(1-\lambda)} > 1$ and $\frac{(1-\lambda)(1-\mu)(1-\beta)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} < 1$.

For p_h^*,

$$\begin{aligned}
\frac{\partial p_h^\lambda}{\partial \lambda} &= \frac{1}{1-(1-\beta)(1-\lambda)} \cdot \frac{\partial \psi(\beta,\lambda,s;\sigma)}{\partial \lambda} - \psi(\beta,\lambda,s;\sigma) \cdot \frac{1-\beta}{(1-(1-\beta)(1-\lambda))^2} \\
&= \frac{\psi(\beta,\lambda,s;\sigma)}{1-(1-\beta)(1-\lambda)} \cdot \frac{(1-\mu)(1-\beta)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} - \frac{\psi(\beta,\lambda,s;\sigma)\cdot(1-\beta)}{(1-(1-\beta)(1-\lambda))^2} \\
&= \frac{(1-\beta)\psi(\beta,\lambda,s;\sigma)}{1-(1-\beta)(1-\lambda)} \left[\frac{(1-\mu)\sigma}{\mu\beta+(1-\mu)(1-\beta)(1-\lambda)\sigma} - \frac{1}{1-(1-\beta)(1-\lambda)}\right] \\
&= \frac{(1-\beta)\psi(\beta,\lambda,s;\sigma)}{1-(1-\beta)(1-\lambda)} \left[\frac{1}{\frac{\mu\beta}{(1-\mu)\sigma}+(1-\beta)(1-\lambda)} - \frac{1}{1-(1-\beta)(1-\lambda)}\right]
\end{aligned}$$

Note that, $\frac{1}{\frac{\mu\beta}{(1-\mu)\sigma}+(1-\beta)(1-\lambda)} - \frac{1}{1-(1-\beta)(1-\lambda)}$ increases in λ. Then there exists $\hat{\lambda}$ such that $\frac{\partial p_h^\lambda}{\partial \lambda} > 0$ for $\lambda > \hat{\lambda}$. ■

9.8 Proof of Proposition 3

Proof. First, (12) and (15) imply that $p_h^* = \frac{h(\hat{\eta})}{F_l(\hat{\eta})}(v_h - v_l)$ and $U^* = v_l - \frac{(1-F_l(\hat{\eta}))h(\hat{\eta})}{F_l(\hat{\eta})}(v_h - v_l)$. On the consumer's side, if the consumer observes $\eta > \hat{\eta}$, then $E(v|\eta, p_h^*) - p_h^* > U^*$ and the consumer will prefer to buying upon seeing p_h^*. If $\eta > \hat{\eta}$, then $E(v|\eta, p_h^*) - p_h^* < U^*$ and the consumer will prefer to not buying upon seeing p_h^*, and will continue to search the next store. Assume that the off-equlibrium-path belief is that

$$\Pr(v_h|p \neq p_h^*, p \neq p_l^*) = 0.$$

Hence, on the off-equilibrium path, the consumer will buy iff $v_l - p \geq U^*$, i.e., $p \leq v_l - U^*$.

On the firm's side, since $(1-F_l(\hat{\eta}))p_h^* = v_l - U^*$, the low-quality firm will be indifferent between charging p_l^* and p_h^*. The high-quality firm will charge p_h^* since $(1-F_h(\hat{\eta}))p_h^* > v_l - U^*$. ■

9.9 Proof of Lemma 1

Proof. Recall that $\hat{\eta}$ is determined by (14), i.e.,

$$\mu(1-F_h(\hat{\eta}))(1-h(\hat{\eta})) - (1-\mu)\sigma(1-F_l(\hat{\eta}))h(\hat{\eta}) = \frac{s}{v_h - v_l}.$$

(14) can be written as

$$\mu(1-F_h(\hat{\eta})) - [\mu(1-F_h(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]h(\hat{\eta}) = \frac{s}{v_h - v_l}.$$

By the Implicit Function Theorem and the form of $h(\cdot)$,

$$\begin{aligned}
&\frac{\partial \hat{\eta}}{\partial s} \\
= \;& \frac{\frac{1}{v_h - v_l}}{\begin{array}{c}\mu(-f_h(\hat{\eta})) - h'(\hat{\eta})[\mu(1-F_h(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))] \\ +h(\hat{\eta})[\mu f_h(\hat{\eta})) + (1-\mu)\sigma f_l(\hat{\eta}))]\end{array}} \\
= \;& \frac{1}{v_h - v_l} \cdot \frac{1}{\mu(-f_h(\hat{\eta})) - h'(\hat{\eta})[\mu(1-F_h(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))] + \mu f_h(\hat{\eta}))} \\
= \;& \frac{1}{v_h - v_l} \cdot \frac{1}{-h'(\hat{\eta})[\mu(1-F_h(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]} \\
< \;& 0,
\end{aligned}$$

where the second equation comes from the definition of $h(\hat{\eta})$. ■

9.10 Proof of Corollary 6

Proof. From the expressions of p_h^* and U^*,

$$\begin{aligned}
\frac{\partial p_h^*}{\partial s} &= (v_h - v_l)\frac{\partial}{\partial \hat{\eta}}\left(\frac{h(\hat{\eta})}{F_l(\hat{\eta})}\right) \cdot \hat{\eta}'(s) \\
\frac{\partial U^*}{\partial s} &= -(v_h - v_l)\frac{\partial}{\partial \hat{\eta}}\left(\frac{(1-F_l(\hat{\eta}))h(\hat{\eta})}{F_l(\hat{\eta})}\right) \cdot \hat{\eta}'(s)
\end{aligned}$$

Since $\hat{\eta}'(s) < 0$, $\frac{\partial p_h^*}{\partial s} < 0$ and $\frac{\partial U^*}{\partial s} > 0$. ■

9.11 Proof of Corollary 7

Proof. Since $p_h^* = \frac{h^\alpha(\hat{\eta}(\alpha))}{F_l(\hat{\eta}(\alpha))}(v_h - v_l)$ and $U^* = v_l - \frac{(1-F_l(\hat{\eta}(\alpha))h^\alpha(\hat{\eta}(\alpha))}{F_l(\hat{\eta}(\alpha))}(v_h - v_l)$, it is straightforward to calculate

$$\begin{aligned}
\frac{1}{(v_h - v_l)} \cdot \frac{dp_h^*}{d\alpha} &= \frac{\partial}{\partial \eta}\left(\frac{h^\alpha(\hat{\eta})}{F_l(\hat{\eta})}\right) \cdot \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{\partial}{\partial \alpha}\left(\frac{h^\alpha(\hat{\eta})}{F_l(\hat{\eta})}\right) \\
&= \frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{\frac{\partial h^\alpha(\hat{\eta})}{\partial \eta}F_l(\hat{\eta}) - h^\alpha(\hat{\eta})f_l(\hat{\eta})}{(F_l(\hat{\eta}))^2} + \frac{\frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{F_l(\hat{\eta})} \\
&= -\frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{h^\alpha(\hat{\eta})f_l(\hat{\eta})}{(F_l(\hat{\eta}))^2} + \frac{\frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta} + \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{F_l(\hat{\eta}(\alpha))} \\
&= -\frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{h^\alpha(\hat{\eta})f_l(\hat{\eta})}{(F_l(\hat{\eta}))^2} + \frac{1}{F_l(\hat{\eta})} \cdot \frac{dh^\alpha(\hat{\eta})}{d\alpha}.
\end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{(v_h - v_l)} \cdot \frac{dU^*}{d\alpha} \\
= \quad & \frac{\partial}{\partial \eta}\left(\frac{(1-F_l(\hat{\eta}))h^\alpha(\hat{\eta})}{F_l(\hat{\eta})}\right) \cdot \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{\partial}{\partial \alpha}\left(\frac{(1-F_l(\hat{\eta}))h^\alpha(\hat{\eta})}{F_l(\hat{\eta})}\right) \\
= \quad & \left[h^\alpha(\hat{\eta})\frac{\partial}{\partial \eta}\left(\frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})}\right) + \frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})} \cdot \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta}\right] \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha} \cdot \frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})} \\
= \quad & h^\alpha(\hat{\eta})\frac{\partial}{\partial \eta}\left(\frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})}\right)\frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})}\left[\frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta} + \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}\right] \\
= \quad & -\frac{h^\alpha(\hat{\eta})f_l(\hat{\eta})}{(F_l(\hat{\eta}))^2} \cdot \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{(1-F_l(\hat{\eta}))}{F_l(\hat{\eta})} \cdot \frac{dh^\alpha(\hat{\eta})}{d\alpha}.
\end{aligned}
$$

Since $\frac{dh^\alpha(\hat{\eta}(\alpha))}{d\alpha} > 0$, apparently, $\frac{dp_h^*}{d\alpha} > 0$ and $\frac{dU^*}{d\alpha} < 0$ if $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$. ■

In addition, the next lemma shows that, when α increases, the value of search always increases. Consequently, the Search Value Line will shift outward.

Lemma 5 *When α increases, $h^\alpha(\hat{\eta}(\alpha))$ will increases. That is,*

$$
\frac{dh^\alpha(\hat{\eta}(\alpha))}{d\alpha} = \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta} \cdot \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha} > 0.
$$

Proof. Note that from (16), by the Implicit Function Theorem,

$$
\begin{aligned}
\frac{d\hat{\eta}(\alpha)}{d\alpha} \quad = \quad & -\frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta})) - [\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]\frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{\begin{array}{c}\mu(-f_h^\alpha(\hat{\eta})) - \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta}[\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))] \\ +h^\alpha(\hat{\eta})[\mu f_h^\alpha(\hat{\eta})) + (1-\mu)\sigma f_l(\hat{\eta}))]\end{array}} \\
= \quad & -\frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta})) - [\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]\frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{\mu(-f_h^\alpha(\hat{\eta})) - \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta}[\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))] + \mu f_h^\alpha(\hat{\eta})} \\
= \quad & \frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta})) - [\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]\frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{\frac{\partial h^\alpha(\hat{\eta})}{\partial \eta}[\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{d\hat{\eta}(\alpha)}{d\alpha} \cdot \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta} \quad = \quad & \frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta})) - [\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))]\frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}}{\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))} \\
= \quad & \frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta}))}{\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))} - \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha}.
\end{aligned}
$$

Thus,

$$
\frac{dh^\alpha(\hat{\eta}(\alpha))}{d\alpha} = \frac{\partial h^\alpha(\hat{\eta})}{\partial \eta} \cdot \frac{d\hat{\eta}(\alpha)}{d\alpha} + \frac{\partial h^\alpha(\hat{\eta})}{\partial \alpha} = \frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha}(1-h^\alpha(\hat{\eta}))}{\mu(1-F_h^\alpha(\hat{\eta})) + (1-\mu)\sigma(1-F_l(\hat{\eta}))}.
$$

By MLRP, $\frac{\partial F_h^\alpha(\hat{\eta})}{\partial \alpha} < 0$ and the numerator will be positive, implying that $\frac{dh^\alpha(\hat{\eta}(\alpha))}{d\alpha} > 0$. ■

Regarding the incentive line, note that the slope of the incentive line is $1 - F_l(\hat{\eta}(\alpha))$. If $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$, then the increase in α will induce less intensive search and the Price Incentive Line will rotate inward. If $\frac{d\hat{\eta}(\alpha)}{d\alpha} > 0$, then the Price Incentive Line will rotate outward. However, the sign of $\frac{d\hat{\eta}(\alpha)}{d\alpha}$ turns out to be ambiguous, which means that, better informativeness does not necessarily induce more intensive search.[7]

The intuition for the ambiguity is explained as follows. When α increases, on the one hand, because the signal is more informative, the consumer would expect a higher incremental benefit of searching one more store when she observes the same signal. On the other hand, the increasing informativeness can lead to a higher U; and when U increases, the additional benefit of searching one more store will decreases. As a total, the additional benefit of searching one more store can either increase or decrease. Consequently, the increase in informativeness can lead to either a higher or a lower cutoff signal.

[7] To see this, note that by the Implicit Function Theorem and the form of $h^a(\cdot)$, it follows that

$$\frac{d\hat{\eta}^\alpha}{d\alpha} = \frac{-\mu\frac{\partial F_h^\alpha(\hat{\eta}^\alpha)}{\partial\alpha}(1 - h^\alpha(\hat{\eta}^\alpha)) - [\mu(1 - F_h^\alpha(\hat{\eta}^\alpha)) + (1-\mu)\sigma(1 - F_l(\hat{\eta}^\alpha))]\frac{\partial h^\alpha(\hat{\eta}^\alpha)}{\partial\alpha}}{\frac{\partial h^\alpha(\hat{\eta}^\alpha)}{\partial\hat{\eta}^\alpha}[\mu(1 - F_h^\alpha(\hat{\eta}^\alpha)) + (1-\mu)\sigma(1 - F_l(\hat{\eta}^\alpha))]}.$$

$\frac{\partial F_h^a(\hat{\eta}^a)}{\partial a} < 0$ and $\frac{\partial h^a(\hat{\eta}^a)}{\partial \hat{\eta}^a} > 0$ by MLRP, but $\frac{\partial h^a(\hat{\eta}^a)}{\partial a} > 0$ iff $\frac{\partial f_h^a(\hat{\eta}^a)}{\partial a} > 0$. Hence, the sign of $\frac{d\hat{\eta}^a}{da}$ turns out to be ambiguous.