Time Series Forecasting using Functional Partial Least Square Regression with Stochastic Volatility, GARCH, and Exponential Smoothing

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Abstract

Our approach is based on the functional partial least squares (FPLS) model, which is capable of avoiding multicollinearity in regression by efficiently extracting information from the high-dimensional market data.

We provide an empirical application of our proposed methodology in terms of its ability to predict the conditional average log return and the volatility of crude oil prices via exponential smoothing, Bayesian stochastic volatility, and GARCH models.

We find evidence that the standard models with FDA traces significantly outperform our competing models. Our empirical results show that our new methodology significantly improves predictive ability of standard models in forecasting the latent average log return and the volatility of financial time series.

Introduction

Many financial data are also characterized by high frequency so that functional representations naturally arise from repeated observations. This feature substantially complicates econometric modeling and statistical analysis. Functional data analysis (FDA) recently has gained considerable importance in the literature (see Ramsay and Silverman 2005) for a comprehensive introduction to FDA to reduce the dimensionality.

Our proposed method enables researchers to take advantage of auxiliary variables in estimation. The multicollinearity issue that might be caused by the new method can be overcame by using the FPLS regression which is one of dimension reduction methods. We extract traces or smooth representations via the technique from raw observations of the return series frequently displaying large fluctuations. The extracted traces are themselves represented as a functional data set and called "FDA traces" in this article.

Our proposed method explores the performance of volatility forecasting in two crude oil markets: Brent and WTI. In particular, exchange rates for the 40 most actively traded currencies are used as auxiliary variables in forecasting.

The volatility forecasts obtained from the Bayesian SV and GARCH models with FDA traces are compared to those acquired from the same SV and GARCH models with the observed oil returns, PCR and LASSO models.

Table 1: Descriptive statistics of the log-return series

	Mean	Median	Minimum	Maximum	St.D	Skewness	Kurtosis
Brent	0.040	0.034	-3.827	3.953	1.079	0.111	3.687
WTI	-0.004	0.054	-4.172	6.072	1.244	0.282	4.624

Economic Models

The formulation of the SV model in its centered parameterization is given by

$$\begin{aligned} y_t|h_t &\sim \mathcal{N}(0, \exp(h_t)), \\ h_t|h_{t-1}, \mu, \phi, \sigma_\eta &\sim \mathcal{N}(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2), \\ h_0|\mu, \phi, \sigma_\eta &\sim \mathcal{N}(\mu, \sigma_\eta^2/(1-\phi^2)), \end{aligned}$$

A GARCH(p,q) with constant in mean model is given by

$$\begin{aligned} \epsilon_t &= \sigma_t e_t \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{aligned}$$

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Empirical Results

Table 6: Volatility prediction performance via Bayesian SV and GARCH models

		SV				GARCH	
		MSE_1					
	Forecasting horizons	FDA trace	PCR	LASSO	Observed returns	FDA traces	Observed returns
Brent	$h=3$	36.341	26.295	**21.654**	37.071	32.925	34.222
	$h=5$	22.761	19.182	**18.963**	23.125	21.205	21.742
	$h=10$	12.999	12.759	14.809	13.156	**12.416**	12.569
	$h=20$	8.863	8.902	11.119	8.974	**8.417**	8.563
	$h=50$	6.678	7.814	10.711	6.730	**6.523**	6.543
	$h=117$	7.066	9.520	13.176	7.069	7.304	**7.054**
		MSE_2					
	Forecasting horizons	FDA trace	PCR	LASSO	Observed returns	FDA traces	Observed returns
Brent	$h=3$	9.300	5.182	**4.589**	9.764	7.711	8.326
	$h=5$	5.805	**4.900**	6.520	6.039	5.102	5.337
	$h=10$	3.304	4.091	6.855	3.410	**3.070**	3.110
	$h=20$	2.243	3.227	6.080	2.318	**2.079**	2.113
	$h=50$	1.678	3.302	6.516	1.716	1.667	**1.626**
	$h=117$	**1.772**	4.135	7.754	1.777	1.981	**1.772**
		MSE_1					
	Forecasting horizons	FDA trace	PCR	LASSO	Observed returns	FDA traces	Observed returns
WTI	$h=3$	1.220	11.635	14.115	1.262	**0.741**	0.753
	$h=5$	3.066	12.880	15.188	3.094	**2.674**	2.682
	$h=10$	**2.927**	7.692	9.269	2.965	3.058	3.0578
	$h=20$	**4.981**	8.453	9.815	5.025	5.254	5.257
	$h=50$	**6.539**	11.797	13.404	6.549	6.631	6.632
	$h=117$	5.392	13.977	16.082	5.331	5.133	**5.111**
		MSE_2					
	Forecasting horizons	FDA trace	PCR	LASSO	Observed returns	FDA traces	Observed returns
WTI	$h=3$	0.180	7.475	9.488	0.222	**0.037**	0.040
	$h=5$	0.662	7.683	9.591	0.686	**0.560**	0.561
	$h=10$	**0.766**	5.301	6.838	0.792	0.928	0.926
	$h=20$	**1.313**	5.236	6.657	1.341	1.546	1.547
	$h=50$	**1.659**	6.504	8.057	1.669	1.795	1.798
	$h=117$	1.292	7.814	9.628	1.271	1.248	**1.244**

Discussion

The results exhibit better forecasting accuracy of our method than the alternative, especially in volatility forecasting over longer time horizons, such as twenty or fifty days.

Our empirical results provide strong empirical evidence that the use of FDA traces yields an improvement on the volatility models currently employed in the empirical literature.

We conduct more robust tests in order to evaluate whether an exhibited superiority in a forecasting performance of our method. Our proposed method significantly improve the predictive ability of the standard forecasting model currently being used in the literature.

Table 7: Tests for reality check and superior predictive ability

	Brent		WTI	
Forecasting horizon	RC (p-value)	SPA (p-value)	RC (p-value)	SPA (p-value)
$h=3$	0.031	0.038	1.000	1.000
$h=5$	0.047	0.064	1.000	1.000
$h=10$	0.113	0.114	1.000	1.000
$h=20$	0.303	0.304	1.000	1.000
$h=50$	0.269	0.282	1.000	1.000
$h=117$	0.258	0.304	1.000	1.000

Conclusions

We proposed a new method to improve the volatility forecasts by adopting the FPLS regression that allows us to incorporate useful auxiliary variables.

To do this, we extracted FDA traces via the FPLS regression from crude oil prices itself as well as exchange rates. Then, we used standard models such as an exponential smoothing method, Bayesian SV and GARCH models with FDA traces.

Our results indicated that standard forecasting models with FDA traces were statistically superior out-of-sample accuracy in terms of goodness-of-fit measures and a loss function.

References

1. Ramsay, J. and Silverman, B. Functional Data Analysis, Springer, New York, (2005).