An Equilibrium Model of the International Price System*

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October 3, 2018

Abstract

The currency in which international prices are set is a factor of fundamental importance in international economics: it determines the benefits of floating versus pegged exchange rates and the spillover effects of monetary policy across economies. However, the standard assumption in existing models — that all prices are set in a currency of either the producer or the consumer — is inconsistent with the dominant status of the dollar in global trade. In this paper, I develop a general equilibrium framework with *endogenous* currency choice and establish three main results. First, there are strategic complementarities in currency choice across exporters, which can lead to a dominant currency in international trade. The dollar is more likely to play this role because of the large size and relative stability of the U.S. economy and history dependence. Second, despite small private costs, the invoicing decisions of firms lead to large aggregate spillover effects between countries. I show that in contrast to the standard "currency war" logic, a depreciation of the U.S. exchange rate has a positive effect on output in other economies when international prices are set in dollars. Finally, there are general equilibrium complementarities between firms' currency choice and the optimal monetary policy: because of U.S. spillover effects arising under dollar pricing, it is optimal for other countries to partially peg their exchange rates to the dollar, which in turn stimulates firms to set prices in dollars.

*This paper is based on my PhD dissertation at Princeton. I am extremely grateful to my advisor Oleg Itskhoki, as well as to Mark Aguiar, Nobu Kiyotaki, and Esteban Rossi-Hansberg for continual guidance and support. I am also indebted to Michael Devereux and Povilas Lastauskas for insightful discussions, Fernando Alvarez, Manuel Amador, Costas Arkolakis, Philippe Bacchetta, Martin Beraja, Markus Brunnermeier, Ariel Burstein, Dan Cao, V.V. Chari, Javier Cravino, Konstantin Egorov, Charles Engel, Linda Goldberg, Gita Gopinath, Gregor Jarosch, David Nagy, Ezra Oberfield, Mikkel Plagborg-Møller, Richard Rogerson, Chris Sims, and Aleh Tsyvinski as well as numerous seminar/conference participants for helpful comments and suggestions, Julia Soloveva for the outstanding research assistance and the International Economics Section at Princeton University and the Cowles Foundation at Yale University for financial support.

1 Introduction

The currency in which international prices are set is crucial for the transmission of monetary shocks across countries. When sticky in the currency of the producer (PCP), the import prices move one to one with the exchange rate. On the contrary, the import prices are immune to exchange rate shocks in the short run when sticky in the local currency (LCP).[1] As a result, the answers to the fundamental questions in international economics can change dramatically, depending on the assumption about firms' currency choice. In particular, while the classical argument in favor of floating exchange rates (Friedman 1953) holds when the prices are set in the currency of the producer, it might be optimal to peg exchange rates when prices are set in the currency of the consumer (Devereux and Engel 2003). Similarly, the spillover effects of monetary policy on foreign output, which have been at the center of public debate during the global recession (Bernanke 2017), are negative under PCP, but positive under LCP (Betts and Devereux 2000).

The standard assumptions in the existing models are, however, inconsistent with two basic empirical facts about the "International Price System", a concept introduced recently by Gopinath (2016). First, while most of the theoretical literature has focused on the case of PCP and, to a lesser extent LCP, the empirical evidence shows that most of the international prices are set in just a few currencies, with the dollar playing the role of the main vehicle currency (see Figures A1) (Goldberg and Tille 2008).[2] This suggests that the transmission of shocks across countries might be more asymmetric than predicted by the existing models. Second, in contrast to the standard assumption in the literature, the data suggests that firms endogenously choose the currency of invoicing that maximizes their profits (Gopinath, Itskhoki, and Rigobon 2010). The models with exogenous currency choice are therefore subject to the Lucas critique and can potentially lead to inaccurate policy implications. In addition, they cannot be used to think about transition from one dominant currency to another — a question that has recently attracted much attention given the growing global role of China and the willingness of several developed and emerging economies to decrease their dependence on the dollar (see Eichengreen 2011).

This paper develops a tractable general equilibrium framework with *endogenous* currency choice that is consistent with the key stylized facts about international invoicing and shows that these facts have important positive and normative implications. I augment a conventional sticky-price open-economy model in the spirit of Gali and Monacelli (2005) with two additional ingredients. First, rather than taken as exogenous, the currency of invoicing is optimally chosen by individual exporters to bring their prices — that are sticky and cannot adjust in response to the shocks — closer to the optimal level (Engel 2006). Second, I add input-output linkages and complementarities in price setting. These two

[1] The empirical evidence shows that international prices are as sticky as producer prices with the median duration of eleven months (Gopinath and Rigobon 2008) and that the pass-through of monetary shocks into prices depends on the currency of invoicing (see e.g. Boz, Gopinath, and Plagborg-Møller 2017, Auer, Burstein, and Lein 2018).

[2] This fact holds even if one excludes commodities and considers only manufactured goods.

types of price linkages are strong in the data, especially for large firms that account for most of international trade (see e.g. Amiti, Itskhoki, and Konings 2014), and are important to explain several puzzles in international economics (Atkeson and Burstein 2008, Itskhoki and Mukhin 2017, Casas, Díez, Gopinath, and Gourinchas 2017). I show that because of these linkages, each firm wants to synchronize its price with the prices of suppliers and competitors, which generates strategic complementarities in currency choice across exporters and gives rise to an equilibrium with vehicle currency pricing.

Two fundamental factors — the large size of the U.S. economy and its relative stability — make the dollar the most likely candidate for a vehicle currency. Intuitively, when selling in the U.S. market, foreign firms compete with a large number of local producers, which set their prices in dollars. To avoid losing the market share because of unexpected movements in exchange rates, foreign suppliers prefer to use dollars to align their prices with the local competitors. The U.S. exporters then find the costs of both labor and intermediate goods stable in dollars and are more likely to use dollar currency pricing (DCP) when selling goods in other markets. This, in turn, increases the share of intermediate goods and competing products invoiced in dollars that firms in other economies face, making them more willing to use DCP as well. This mechanism is amplified by the fact that many developing countries have very volatile exchange rates, which makes their currencies less attractive for invoicing. With both the U.S. and emerging economies using dollars in global trade, the exporters from other developed countries have high incentives to set prices in dollars as well. This in particular applies to exporters to the U.S., which completes the argument.[3]

The international price system is shaped, however, not only by fundamental factors, but also by history. Because of the complementarities in currency choice, no firm wants to be the first to switch from the old vehicle currency to a new one. As a result, the incumbent currency can retain its global status even after its issuer looses the advantage in terms of economy size and stability. This explains the late transition from the pound to the dollar in the first half of the twentieth century and the dominant status of the U.S. dollar since then (see Krugman 1984). The model also predicts that a large rival country like China can speed up the transition by making local exporters and importers adopt its currency for invoicing: because of strategic complementarities, such policy increases the chances that the new currency is also used in trade between third countries and gains the status of the vehicle currency.

Armed with the model of the international price system, I then re-examine the classical positive and normative questions in monetary economics. In the spirit of Mankiw (1985), I show that despite only *second-order private* gains, the currency choice has *first-order aggregate* implications, i.e. a small perturbation of the fundamentals that makes firms switch from one invoicing regime to another, leads to discontinuous changes in how prices, output, and trade balance respond to exogenous shocks.

[3] I also show that domestic firms are less likely to set prices in foreign currency than exporters, but can switch to dollar pricing in response to large fundamental shocks, e.g. a volatile monetary policy. The complementarities in currency choice imply that the dollarization of emerging economies persists even after inflation is stabilized and contributes to the widespread use of the dollar in international trade.

First, I identify a novel source of U.S. monetary spillovers on foreign output that arises because of dollar pricing and has largely been ignored in the previous debates (see e.g. Bernanke 2017). In contrast to the PCP case, DCP implies that a depreciation of the U.S. exchange rate decreases the prices of *all* internationally traded goods, not only of goods exported from the U.S. (Gopinath 2016). This lowers import prices and consumer prices at the global level. With the aggregate nominal demand unchanged, a fall in prices increases the real demand and drives world consumption upwards (Goldberg and Tille 2009), stimulating production in the global economy.[4] This channel has an unambiguously positive effect on foreign output and outweighs the standard expenditure switching towards U.S. goods under the baseline calibration. Thus, the argument of policymakers from emerging economies that the depreciation of the U.S. exchange rate during the global recession is a zero-sum game of "currency wars" is less of a concern once dollar pricing is taken into account (*cf.* Caballero, Farhi, and Gourinchas 2016). At the same time, the model predicts that the depreciation of a non-vehicle currency has no additional positive spillover effects on other economies and is also less effective in stimulating local output.

Second, I show that some normative results from the previous literature are indeed subject to Lucas critique and no longer apply once firms are allowed to optimally choose the currency of invoicing. As has been demonstrated by Devereux and Engel (2003) in the context of a standard sticky-price open economy model, the optimal monetary policy implies a free floating exchange rate under PCP and a pegged one under LCP. I show, however, that in a standard model, PCP is the only type of invoicing that can arise in equilibrium under the optimal policy and hence, the floating exchange rates are always optimal. At the same time, while this result provides an important theoretical benchmark, it is hardly consistent with the dominance of DCP in the data. I next relax the (restrictive) assumptions underlying this result and show that dollar pricing can arise in an equilibrium with optimal monetary policy.

Third, I argue that DCP contributes to the "fear of floating" and the widespread use of the dollar as an anchor currency in the monetary policy of other economies (Calvo and Reinhart 2002). As discussed above, dollar pricing implies that the U.S. monetary policy affects output and consumption in other countries. These spillovers are distortionary as they do not reflect changes in productivities. The optimal policy in the rest of the world, therefore, "leans against the wind" and partially offsets movements in exchange rates against the dollar. This crawling peg generates a positive comovement of monetary policies across countries, which can contribute to the global financial cycle (Rey 2015). Importantly, however, while DCP worsens the trade-off that policymakers in other economies face, it does not fully eliminate the independence of their monetary policy: the peg to the dollar is only partial and it is still optimal to fully adjust the bilateral exchange rates between other countries even though the resulting expenditure switching is lower than under PCP (*cf.* Gopinath 2017).

[4]In contrast to the effect of dollar depreciation on global trade in Boz, Gopinath, and Plagborg-Møller (2017), the response of global output comes from the general equilibrium effects rather than the partial equilibrium expenditure switching and does not depend on the elasticity of substitution between goods.

The model also predicts strategic complementarities between exporters' currency choice and the optimal monetary policy. DCP makes it optimal for monetary authorities to peg exchange rates to the dollar, which, in turn, decreases the effective volatility of the U.S. exchange rate and makes dollar invoicing even more attractive to the firms. As a result, the evolution of the international monetary system can be closely related to the evolution of the international price system. In particular, promoting its currency in international trade can help a country change its status in the global financial system.

There are three main strands in the literature that use different types of frictions to explain the dominant status of the dollar in international trade. First, there is a long tradition in economics, going back to Krugman (1980), that emphasizes the *transaction costs* in exchange markets: coordination on a single currency raises the chances of a "double coincidence of wants" (Matsuyama, Kiyotaki, and Matsui 1993) and increases the "thickness" of markets (Rey 2001, Devereux and Shi 2013, Chahrour and Valchev 2017). These theories, therefore, explain the widespread use of the dollar as a medium of exchange but have little to say about its role as an invoicing currency. Second, the use of the dollar as a unit of account can be due to *financial frictions*: the firms try to synchronize the risks on their contracts (Doepke and Schneider 2013, Drenik, Kirpalani, and Perez 2018) and borrow in a cheaper currency (Gopinath and Stein 2017).

This paper belongs to the third strand in the literature, the one that emphasizes the role of *nominal frictions* (see e.g. Devereux and Engel 2001, Bacchetta and van Wincoop 2005, Bhattarai 2009, Cravino 2014, Goldberg and Tille 2008, Drenik and Perez 2017) and has two advantages over the other two alternatives. First, Gopinath, Itskhoki, and Rigobon (2010) provide direct empirical evidence in favor of this mechanism, which allows to discriminate it against alternative theories. Second, in most existing open-economy models, the monetary policy effects depend on exporters' currency choice only when prices are sticky. It is therefore natural to use nominal frictions as a starting point for the analysis of firms' invoicing decisions.

2 Baseline Model

2.1 Environment

I start with a simple framework that relies on conventional assumptions in the international macro literature and attains closed-form characterization. To think about a vehicle currency, I assume a continuum of symmetric regions $i \in [0, 1]$ as in Gali and Monacelli (2005). There is one large economy (the U.S.) that includes regions $i \in [0, n]$, $n < 1$, and can also be interpreted as a currency union or a set of dollarized countries. The other regions $i \in (n, 1]$ are small open economies, each with its own nominal unit of account, in which local wages and prices are expressed. Denote the bilateral nominal exchange rate between regions i and j with $\mathcal{E}_{ijt}$, which goes up when currency i devalues relative to currency j.

In each country, there is a representative household, a local government and a continuum of firms producing different varieties of tradable and non-tradable goods. The tradable sector is characterized by intermediate goods in production, strategic complementarities in price setting and the home bias towards domestically produced goods. The prices are set before the realization of shocks and stay rigid for one period with a given probability. While the structure of the tradable sector is crucial, the other details of the model are less important. I make specific assumptions about preferences, the structure of asset markets, and monetary policy to simplify exposition. I discuss below how they can be relaxed. The set of exogenous shocks includes changes in productivity, money supply, government spendings, preferences for imported goods and shocks in financial markets.

Households in region i have log-linear preferences over consumption and labor[5]

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big(\log C_{it}-L_{it}\Big), \tag{1}$$

where consumption bundle is a Cobb-Douglas aggregator of tradable and non-tradable goods:

$$C_{it}=\left(\frac{C_{Tit}}{\eta}\right)^{\eta}\left(\frac{C_{Nit}}{1-\eta}\right)^{1-\eta}. \tag{2}$$

Households earn labor income $W_{it}L_{it}$, get dividends from local firms Π_{it}, pay lump-sum tax T_{it} to the government, and spent $P_{it}^C C_{it}$ on consumption. The remaining income can be invested into internationally traded Arrow securities D_{it+1}:

$$P_{it}^C C_{it}+e^{\psi_{it}}\mathcal{E}_{i0t}\Big(\mathbb{E}_t\left[Q_{t+1}D_{it+1}\right]-D_{it}\Big)=W_{it}L_{it}+\Pi_{it}-T_{it}+\Omega_{it}, \tag{3}$$

where Q_{t+1} is the (normalized) price of Arrow security that pays one dollar in a given state of the world in the next period. Because markets are complete, the assumption that both prices and returns on the Arrow securities are in dollars is without loss of generality. I allow for cross-country wedges in asset prices and returns ψ_{it}, which can be interpreted as a shock in the local financial markets and may be an important source of exchange rate volatility.[6] The resulting profits (or losses) of the financial sector Ω_{it} are reimbursed lump-sum to local households.[7]

[5]This functional form has widely been used in macroeconomic literature in a context of both closed and open economy (see e.g. Golosov and Lucas 2007) and arises naturally when labor is indivisible (Rogerson 1988).

[6]See e.g. Itskhoki and Mukhin (2017), Lustig and Verdelhan (2016), Devereux and Engel (2002). The assumption that the shock is the same for prices and returns is not important.

[7]The profits of financial sector are $\Omega_{it}=(e^{\psi_{it}}-1)\mathcal{E}_{i0t}\Big(\mathbb{E}_t\left[Q_{t+1}D_{it+1}\right]-D_{it}\Big)$.

Non-tradable sector in each country is characterized by a continuum of monopolistically competitive firms producing different varieties ω using the same production technology:

$$Y_{Nit}(\omega) = e^{a_{Nit}} L_{Nit}(\omega). \tag{4}$$

The individual products are then combined into consumption basket C_{Nit} with a CES aggregator:

$$C_{Nit} = \left(\int_0^1 C_{Nit}(\omega)^{\frac{\theta-1}{\theta}} \mathrm{d}\omega \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

Within each period, firms preset prices in local currency before the realization of shocks and update them afterwards with a probability $\lambda < 1$.

Tradable sector differs from the non-tradable sector in three dimensions. First, production of a continuum of unique tradable products ω in country i requires both labor L_{Tit} and tradable intermediate goods X_{it}:

$$Y_{it}(\omega) = e^{a_{Tit}} \left(\frac{L_{Tit}(\omega)}{1-\phi} \right)^{1-\phi} \left(\frac{X_{it}(\omega)}{\phi} \right)^{\phi}, \qquad \phi < 1. \tag{6}$$

Second, the bundle of tradables Y_{Tit} used in consumption and production includes both local and foreign varieties, which are combined with a homothetic aggregator:

$$\Phi\left(\left\{ \frac{Y_{jit}(\omega)}{Y_{Tit}} \right\}_{j,\omega}, \xi_{it}, \gamma \right) = 1, \tag{7}$$

where $Y_{jit}(\omega)$ denotes exports of product ω from country j to country i, ξ_{it} is a relative demand shock for foreign versus domestic goods, and the home bias $1-\gamma$ reflects either trade costs or preferences for domestic goods, $\gamma \in (0,1)$. Note that when $n > 0$, a positive fraction of the global trade happens between the regions within the U.S. As a result, the home bias is effectively higher in the U.S. than in small economies. I assume $\Phi(\cdot)$ is the Kimball (1995) aggregator (see (A1) in Appendix A.2) to allow for complementarities in price setting, which as explained below, are important for firms' currency choice.

Finally, for each country of destination, exporters choose the currency of invoicing, in which they set the price before the realization of shocks. With a probability λ, the price can be updated after the uncertainty is resolved. While any currency can be used for invoicing in international trade, it is assumed that local firms set prices exclusively in domestic currency — perhaps due to legal reasons. Section 3.4 relaxes this assumption and derives additional results when domestic firms optimally choose the currency of invoicing.

Government in each country collects lump-sum taxes T_{it} from households to purchase G_{it} units of goods, which for simplicity have the same composition of products as the consumption bundle.

It also pays a production subsidy τ to producers to eliminate the markup distortion, which does not affect any results about currency choice below, but plays an important role for the welfare analysis in Section 5. The government runs a balanced budget, which is without loss of generality given the Ricardian equivalence holds in the model.

Following the previous literature (see e.g. Carvalho and Nechio 2011), I focus on a class of monetary rules that implement an exogenous stochastic process for nominal aggregate spendings $e^{m_{it}} \equiv C_{it}P^C_{it}$.[8] I make this assumption to simplify the exposition and remove it in Section 5 when solving for the optimal monetary policy.

Equilibrium conditions require that labor supply equals total demand of non-tradable and tradable sectors:

$$L_{it} = L_{Nit} + L_{Tit}. \tag{8}$$

Non-tradable goods are sold locally to households and to the government:

$$Y_{Nit}(\omega) = C_{Nit}(\omega) + G_{Nit}(\omega). \tag{9}$$

Similarly, tradable goods are used as intermediates in production and for final consumption by households and the government:

$$Y_{Tit} = X_{Tit} + C_{Tit} + G_{Tit}. \tag{10}$$

Finally, the market clearing in the international asset markets implies

$$\int_0^1 D_{it+1}\mathrm{d}i = 0. \tag{11}$$

Shocks are assumed to consist of a global component that is the same for all countries and an idiosyncratic country-specific component: e.g. for monetary shocks we have $m_{it} = \bar{m}_t + \tilde{m}_{it}$, where $\tilde{m}_{it}$ is uncorrelated across i. In addition, the volatility of country-specific shocks in the U.S. is potentially lower than in other countries by a factor of $\rho \leq 1$. This can be rationalized with a better diversification of regional risk in a large economy and weaker granularity forces a la Gabaix (2011), and results in a more stable exchange rate in the U.S. For simplicity, I do not impose any functional relation between n and ρ and treat these parameters as exogenous.

Definition 1 *Given shocks $\{a_{Nit}, a_{Tit}, m_{it}, \xi_{it}, g_{it}, \psi_{it}\}$, a monopolistically competitive equilibrium is defined as follows: a) households maximize utility over consumption of products, labor supply and asset holdings, b) each firm maximizes expected profits over labor and intermediate inputs, currency of invoicing and*

[8] As shown by Kehoe and Midrigan (2007), such policy can be implemented in an extension of the model with a cash-in-advance constraint without altering other equilibrium conditions.

prices in each market, taking the decisions of all other firms as given and setting domestic prices in local currency, c) the government collects taxes to satisfy budget constraint, d) all markets clear.

2.2 Firm's currency choice

This section describes the currency choice problem of an individual exporter. To obtain sharp analytical results, I approximate equilibrium conditions around the symmetric steady state (see Appendix A.2 for details). Let small letters denote the log-deviations of variables from the steady-state. To simplify the notation, I suppress time subscript and use $\mathbb{E}(\cdot)$ and $\mathbb{V}(\cdot)$ to denote the expectations and variances conditional on the information at the beginning of the period before the realization of shocks.

Let $\Pi_{ji}(p)$ denote the profit of exporter from country j to country i as a function of price p expressed in the currency of destination.[9] Define the optimal static price $\tilde{p}_{ji}$ that maximizes profits in a given state of the world:

$$\tilde{p}_{ji} = \underset{p}{\operatorname{argmax}} \ \Pi_{ji}(p).$$

Ideally, the firms would like to implement $\tilde{p}_{ji}$ in every state of the world. This is, however, not feasible in a general case because of price stickiness, and the firms preset price to maximize their expected profits:[10]

$$\bar{p}_{ji}^{k} = \underset{p}{\operatorname{argmax}} \ \mathbb{E}\Pi_{ji}(p + e_{ik}).$$

It can be shown that to the first-order approximation, the optimal preset price equals the *expected* value of $\tilde{p}_{ji}$ expressed in currency of invoicing k:

$$\bar{p}_{ji}^{k} = \mathbb{E}\big[\tilde{p}_{ji} + e_{ki}\big]. \tag{12}$$

Thus, the preset price allows firms to replicate the mean value of the optimal price. With the endogenous currency choice, however, firms can go one step further and target the second moment of $\tilde{p}_{ji}$ (see Engel 2006, Gopinath, Itskhoki, and Rigobon 2010, Cravino 2014). Using the second-order approximation to the profit function and noticing that expected movements in prices and exchange rates are fully absorbed by the preset price and have no effect on the currency choice, we get the following result.[11]

[9]Because of constant returns to scale technology, the marginal costs are independent of the quantity produced. Therefore, the profit function is separable across markets, and firms choose prices and currencies independently for each destination.

[10]I assume that profits are expressed in real discounted units, i.e. $\Pi_{ji}(\cdot)$ includes the stochastic discount factor (SDF). The variation in SDF does not affect the results under the approximation used below.

[11]One can show using the classical result from portfolio theory established first by Samuelson (1970) and applied recently in a general equilibrium setup by Devereux and Sutherland (2011) that the second-order approximation to the profit function and the first-order approximation to all other equilibrium conditions are sufficient to get consistent solutions.

Lemma 1 (Currency choice) *To the second-order approximation, the currency choice problem of exporter is equivalent to choosing the currency k, in which the optimal price $\tilde{p}_{ji} + e_{ki}$ is most stable:*

$$\max_{k\in[0,1]} \mathbb{E}\,\Pi_{ji}(\bar{p}^k_{ji} + e_{ik}) \quad \Leftrightarrow \quad \min_{k\in[0,1]} \mathbb{E}\Big[\bar{p}^k_{ji} + e_{ik} - \tilde{p}_{ji}\Big]^2 \quad \Leftrightarrow \quad \min_{k\in[0,1]} \mathbb{V}\Big[\tilde{p}_{ji} + e_{ki}\Big]. \tag{13}$$

As the second expression makes clear, exporters choose currency k to mitigate the effect of sticky prices and to bring ex post price $\bar{p}^k_{ji} + e_{ik}$ closer to the optimal state-dependent value $\tilde{p}_{ji}$. Equivalently, firms are minimizing the distance between preset price $\bar{p}^k_{ji}$ and the desired price expressed in currency k $\tilde{p}_{ji} + e_{ki}$. Since the former stays constant across the states of the world, it is optimal to choose currency that minimizes the volatility of the latter, i.e. the currency in which the optimal price is most stable.[12] To give an example, if the optimal static price is always $\$100$, then setting the price in dollars allows the firm to replicate $\tilde{p}_{ji}$ in every state of the world.

The choice is more nuanced when the optimal price is not fully stable in one currency, e.g. when $\tilde{p}_{ji}$ can be expressed as $\$50 + £50$. In this case, the firm would ideally like to set the price in terms of a basket of currencies.[13] I discuss this possibility in detail in Appendix A.4 and show that predictions of the model are inconsistent with the key stylized facts about the international price system in this case. In particular, the share of the dollar cannot exceed the share of the U.S. in global trade. I therefore assume that currency choice is discrete and that individual firms find it suboptimal to use baskets of currencies for invoicing, e.g. because of rational inattention (see e.g. Sims 2003, Mankiw and Reis 2002). In the spirit of Mankiw (1985), I show below that while small frictions are sufficient to rationalize dollar pricing, they lead to large aggregate effects.

Notice that the firm's invoicing problem, of choosing a basket of currencies that minimizes (13), resembles the classical portfolio problem a la Markowitz (1952). The assumption that currency choice is discrete is analogous to how financial frictions have been used to explain the global status of the dollar in asset markets (see e.g. Bruno and Shin 2015, Rey 2015). It is worth emphasizing, however, that despite these similarities, invoicing decisions of firms in the model are based on nominal frictions, not financial ones: exporters choose the currency of invoicing to bring ex-post prices closer to the optimal level and increase *average* profits, not to redistribute profits across states to hedge against risk.[14]

While the previous analysis is based on a one-period version of Calvo (1983) price setting, it also applies in other models of price rigidity. Appendix A.5 discusses four alternatives. In particular, I show

[12] In other words, it is optimal to set prices in currency k rather than in currency h if the pass-through of bilateral exchange rate shocks e_{kh} into the desired price $\tilde{p}_{ji} + e_{ki}$ is low: see e.g. Proposition 2 in Gopinath, Itskhoki, and Rigobon (2010).

[13] Notice this is not the same as using mixed strategies (lotteries) across currencies. Moreover, the optimal currency basket is firm-specific and there is no one-size-fits-all solution like the Special Drawing Rights (SDR).

[14] Abstracting from financial frictions might be a reasonable assumption given that most of the international trade is done by large firms, which have arguably better access to financial markets. At the same time, the model can be extended to incorporate effects of asset market imperfections on currency choice: e.g. if firms have to borrow in dollars to purchase inputs, the pass-through of dollar shocks into costs and optimal price $\tilde{p}_{ji}$ is high, which makes DCP more likely.

that the baseline results about currency choice can be derived analytically under staggered pricing. The benchmark model is also isomorphic to the model with Rotemberg (1982) pricing and have similar numerical predictions as the menu cost model with fixed costs of adjustment and idiosyncratic productivity shocks. Finally, I relax the assumption that currency choice is made unilaterally by suppliers and show that the same results can be obtained in a model with bargaining between buyers and sellers.

2.3 Partial equilibrium

Lemma 1 shows that firms' currency choice depends on the properties of the desired price $\tilde{p}_{ji}$, which is determined by the equilibrium conditions in the tradable sector. A constant returns to scale technology ensures that equilibrium prices depend only on the supply side of the economy and can be analyzed separately from the quantities. In contrast to the CES case, the Kimball demand generates strategic complementarities in price setting across firms, so that the optimal price of an exporter from j to i depends not only on its marginal costs but also on the prices of competitors in the destination market:

$$\tilde{p}_{ji} = (1-\alpha)(mc_j + e_{ij}) + \alpha p_i, \tag{14}$$

where parameter α depends on the curvature of $\Phi(\cdot)$ and is different from demand elasticity. In the limit $\alpha \to 0$, the Kimball aggregator converges to the CES, firms charge a constant markup, and cost shocks are the only source of variation in the optimal price. The marginal costs of production in country i are a weighted sum of local wages w_i and prices of intermediates p_i adjusted for the productivity:

$$mc_i = (1-\phi)w_i + \phi p_i - a_{Ti}. \tag{15}$$

The first-order approximation of the aggregate price index is the sum of prices of locally produced goods p_{ii} and imported ones p_i^I with the weight of the former determined by the home bias $1-\gamma$:

$$p_i = (1-\gamma)p_{ii} + \gamma p_i^I, \qquad \text{where} \qquad p_i^I = \int_0^1 p_{ji}\mathrm{d}j, \tag{16}$$

and the bilateral price index averages the prices of adjusting and non-adjusting firms:[15]

$$p_{ji} = \lambda\tilde{p}_{ji} + (1-\lambda)(\bar{p}_{ji}^k + e_{ik}). \tag{17}$$

A fraction λ of firms update prices after the realization of shocks and set them at the optimal level $\tilde{p}_{ji}$. The prices of other firms stay constant in the currency of invoicing k, which means they move one-to-one with the exchange rate e_{ik} in the currency of the customers. The currency choice therefore has

[15] To simplify the notation, I assume that all exporters from j to i use the same currency of invoicing k. The results in Section 3, however, apply even under mixed strategies if not noted otherwise.

the first-order effect on ex-post prices. At the same time, Lemma 1 implies that the invoicing decision of an individual firm is determined by optimal price $\tilde{p}_{ji}$, which depends on aggregate price indices p_i. Thus, the equilibrium price system can be defined as follows.

Definition 2 *Given $\{w_i, e_{ij}\}$, the equilibrium international price system consists of price indices $\{p_{ji}\}$ and firms' currency choice $\{k_{ji}\}$ such that: (a) given invoicing regime, $\{p_{ji}\}$ solve the system (14)-(17), (b) given prices, $\{k_{ji}\}$ solve problem (13).*

2.4 General equilibrium

Definition 2 implies that the only general equilibrium objects that matter for exporters' currency choice are the second moments of exchange rates and nominal wages. In the general case, there is also a feedback effect: the dynamics of w_i and e_{ij} depend on international prices, which in turn are shaped by the invoicing decisions of firms. This section shows, however, that under the assumptions made in the baseline model, exchange rates and nominal wages do not depend on currency of invoicing, and therefore, the model attains the block-recursive structure: one can solve for equilibrium currency choice taking the relevant general equilibrium moments as given. Importantly, this result does *not* mean that invoicing decisions of firms have no general equilibrium effects. As Section 4 makes clear, the aggregate consumption, output, exports and imports do change with the currency of invoicing even though wages and exchange rates do not.

Lemma 2 (Exchange rates) *The second moments of equilibrium nominal wages and exchange rates are independent of invoicing decisions of firms.*

The result follows from the combination of log-linear utility, complete asset markets and the monetary rule that targets nominal spendings. While the first two assumptions can be easily relaxed — e.g. the result still holds under incomplete asset markets with only one internationally traded bond (see Proposition A1 in the Appendix) — the monetary rule is crucial for the result. In particular, Lemma 2 does not apply under inflation targeting: the currency of invoicing determines the pass-through of shocks into import prices and CPI, which in turn affects the response of monetary policy and hence, the dynamics of nominal wages and exchange rates in the economy. Section 5 below analyzes such general equilibrium complementarities under the optimal monetary policy. To simplify the exposition, however, I first describe the equilibrium currency choice in the absence of general equilibrium feedback. Furthermore, motivated by the low correlation between exchange rates and nominal wages in the data — the "exchange rate disconnect puzzle" (Meese and Rogoff 1983) — I abstract from monetary shocks, $m_{it} = 0$, until Section 3.4, which provides additional results for economies with volatile monetary policy.[16]

[16]This approach contrasts with most of the previous models of the endogenous currency choice, which focus exclusively on monetary shocks and are not consistent with the exchange rate disconnect puzzle.

3 Equilibrium Currency Choice

Throughout most of the history of modern capitalism, the overwhelming share of global trade has been priced in one currency — first in the pound sterling and later in dollars. This section shows that the model is consistent with this observation. Because of strategic complementarities in currency choice that arise naturally across firms due to input-output and price-setting linkages, exporters prefer to coordinate on the same currency of invoicing. Two fundamental factors — the volatility of exchange rate and the country's share in global trade — then determine which currency is more likely to be used as a vehicle currency. I then combine these two results to analyze the transition from one vehicle currency to another: as fundamental advantages of the pound sterling deteriorate, exporters become more likely to use dollars instead. However, due to strategic complementarities, no firm wants to change the currency of invoicing before other firms do, generating path-dependence in currency choice. This result can account for the delayed transition from using the pound to the dollar in the twentieth century and the wide use of the dollar in the modern economy, despite increasing competition with the euro and renminbi. Section 3.4 concludes with two extensions that can further strengthen the incentives of firms to set prices in dollars.

3.1 Why vehicle currency?

While it is intuitive that firms might set prices in producer or customer currency, it is not immediately clear why they would use a third currency for invoicing. In this section, I show that a vehicle currency equilibrium (VCP), in which all international prices are set in one currency, can arise naturally when price linkages across firms from different countries are strong enough. The question of *which* currency is used as a vehicle currency is discussed in the next section.

According to Lemma 1, firms choose the currency of invoicing, in which their optimal price is more stable. The currency choice of individual exporter from j to i depends therefore on the properties of its desirable price $\tilde{p}_{ji}$, which is determined by the system of equilibrium conditions in a tradable sector (14)-(17) summarized in Figure 1. The optimal price depends on marginal costs and the prices of competitors, with the weight of the latter determined by strategic complementarities in price setting α. In turn, the marginal costs consist of labor costs and the prices of intermediate goods with the weights $1-\phi$ and ϕ respectively. The fraction $1-\gamma$ of intermediates is produced domestically, while the share γ is imported from other countries. Similarly, out of all competitors in the destination market, a fraction $1-\gamma$ are local producers, while importers from other countries account for the remaining share γ.[17]

To understand how the currency choice is determined in the model, I start with a few limiting cases.

[17]There are three additional parameters that affect currency choice. The frequency of price adjustment λ affects the prices of inputs and competing products. The size of the large economy n determines the share of goods in global trade coming from the U.S. The relative volatility of exchange rates ρ affects the probability distribution of $\tilde{p}_{ji}$.

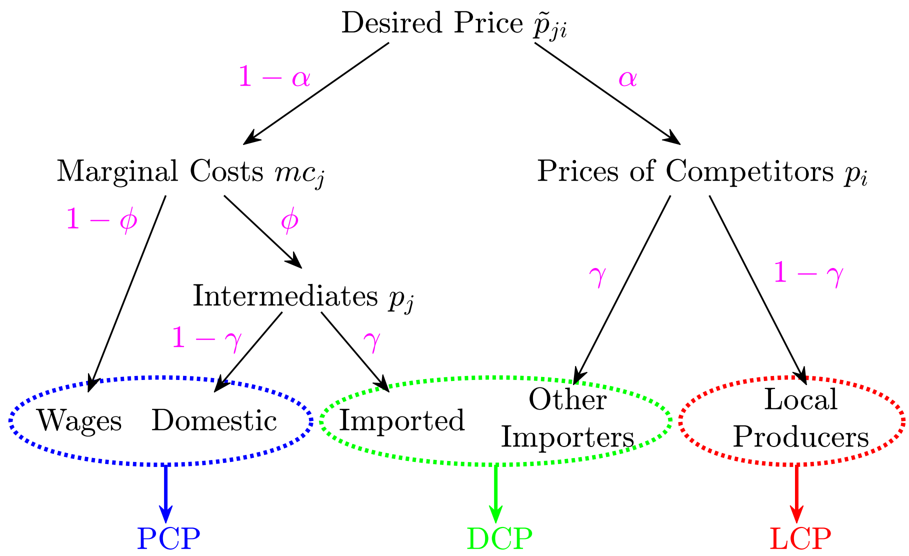

Figure 1: Optimal price and the currency choice of an individual firm

Consider first the conventional case of CES aggregator and no intermediates in production. With no complementarities in price setting under CES demand, the desired price is proportional to marginal costs (see Figure 1). The latter depends exclusively on nominal wages, which are by assumption stable in domestic currency. It follows that the optimal price of exporter $\tilde{p}_{ji}$ is constant in producer currency as well, and hence, PCP is always optimal.

Lemma 3 (No price linkages) *With no intermediates in production, $\phi = 0$, and CES aggregator, $\alpha = 0$, exporters always choose PCP, and no VCP equilibrium exists.*

Thus, the standard assumption of PCP in the open economy models with $\phi = \alpha = 0$ (see e.g. Obstfeld and Rogoff 1995, Clarida, Gali, and Gertler 2001, Gali and Monacelli 2005) is internally consistent: the equilibrium would not change if firms were allowed to choose optimally the currency of invoicing. The proposition also implies that price linkages across firms are a necessary condition to rationalize the use of vehicle currencies in the global trade.

Consider next the autarky limit $\gamma \to 0$. While the mass of exporters goes to zero, their currency choice is still well determined. As countries of origin and destination are (almost) closed, the marginal costs of exporters are stable in the producer currency and the prices of competitors are stable in the local currency. As a result, depending on the value of α, firms choose either PCP or LCP.

Lemma 4 (Autarky limit) *Near the autarky limit $\gamma \to 0$, exporters choose PCP if $\alpha \leq 0.5$ and LCP if $\alpha \geq 0.5$, and no VCP equilibrium exists.*

Figure 2a shows equilibria in the autarky limit in the coordinates α and λ. Because the share of exporters is zero, the equilibrium in the tradable sector does not depend on their invoicing decisions. This in turn implies that there is no room for strategic complementarities in currency choice across firms and the equilibrium invoicing is always uniquely pinned down.[18] Lemma 4 also suggests that for any values of other parameters, the existence of PCP/LCP equilibrium can be guaranteed when the openness of the tradable sector is low.

On the other hand, when openness γ is high, so that a significant fraction of suppliers and competitors are coming from the third countries, the optimal price $\tilde{p}_{ji}$ of an exporter is no longer stable in either producer or local currency, and using vehicle currency might be optimal. The prices of inputs and competing products that individual exporters faces in this case depend on invoicing decisions of other firms. In particular, if firm's suppliers and competitors set prices in dollars, its own optimal price becomes more stable in dollars and hence, the firm is more likely to choose DCP as well. Interestingly, while both input-output and price-setting linkages generate complementarities in currency choice, there are important differences between the two. A higher share of intermediates in production ϕ unambiguously increases the share of foreign inputs in marginal costs and always makes VCP more attractive. In contrast, the effect of price complementarities α is non-monotonic: a low α increases the share of producer currency in the optimal price, while a high α increases the share of the local currency (see Figure 1). On the other hand, neither producer nor local currency dominates when α takes intermediate values, which increases the chances of the VCP equilibrium. I summarize comparative static results in the next proposition.[19]

Proposition 1 (Price linkages) *The region of the VCP equilibrium in parameter space is non-empty and is increasing in the openness of economies γ and the share of intermediates in production ϕ, and can be non-monotonic in complementarities in price setting α.*

Interpreting empirical evidence through the lens of the model, one can argue that globalization has contributed to the widespread use of vehicle currency in international trade. In particular, the high participation of several Asian countries in global value chains can be interpreted as a rise in $\gamma\phi$, which increases the chances of the vehicle currency relative to PCP/LCP. The higher openness γ of other countries, including post-Soviet states, makes the use of vehicle currency in international trade more appealing as well. The model also suggests that the puzzlingly high use of dollar currency in imports and exports of advanced economies such as South Korea, Japan and Australia can be due to strategic complementarities in currency choice: with other countries in the region using DCP, it might be optimal for firms in these countries to set prices in dollars as well.

Complementarities in currency choice also imply that multiple equilibria can emerge despite unique

[18]Here and below I abstract from the knife-edge values of parameters, under which firms are indifferent between two invoicing options.

[19]I use the following definition throughout the paper: the region of equilibrium Z in parameter space is said to be increasing in parameter x if for any $x_2 > x_1$ the set of (other) parameters for which Z exists under $x = x_2$ includes the set for which Z exists under $x = x_1$.

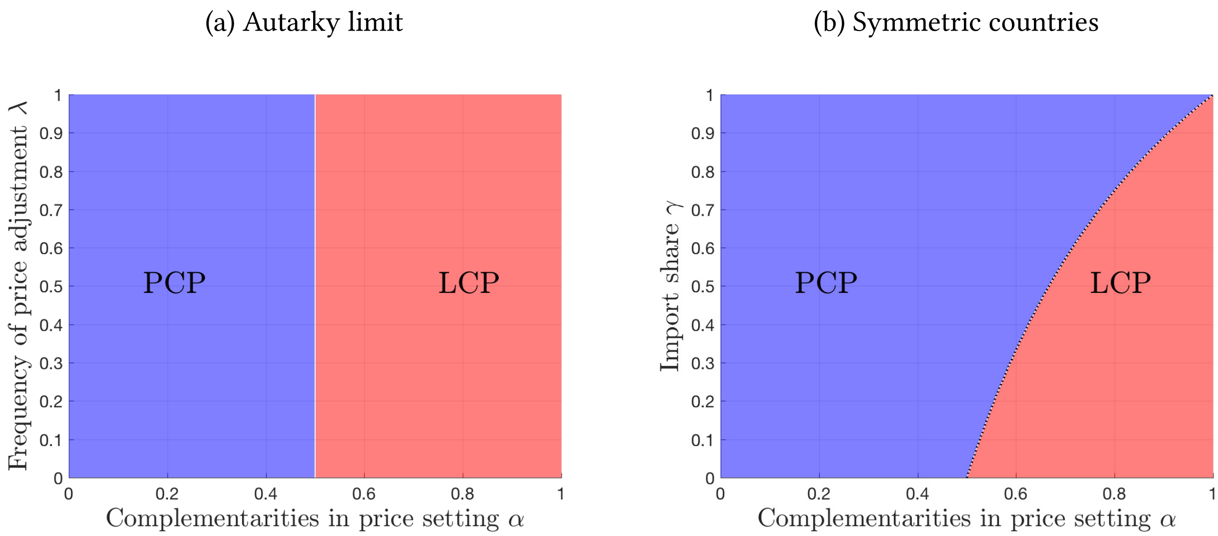

Figure 2: Two limiting cases with no DCP equilibrium

Note: the values of parameters are $\phi = 0.5$, $\rho = 1$, $n = 0$ in both plots and (a) $\gamma \to 0$, (b) $\lambda \to 1$. PCP and LCP denote respectively the areas where producer and local currency pricing equilibria can be sustained.

currency choice at the firm level. While the set of potential equilibria is quite rich in the general case, there is some discipline imposed by the complementarities in currency choice.

Definition 3 *An equilibrium is symmetric if all exporters in the world use either PCP, LCP or the same vehicle currency. The equilibrium is unstable if exogenous perturbation of currency choice of an arbitrarily small fraction of exporters makes a positive mass of other firms change their invoicing decisions.*

Proposition 2 (Multiple equilibria) *Assume that $n = 0$ and $\rho = 1$. Then*

1. *at least one symmetric equilibrium always exists,*
2. *if symmetric equilibrium is unique, then no other equilibria exist,*
3. *all non-pure-strategy equilibria are unstable.*

Intuitively, the complementarities in currency choice imply that if a given exporter does not choose a vehicle currency when *all* other firms are using it, then it cannot be optimal for the exporter to set prices in the vehicle currency when only *some* other firms are using it. The complementarities also imply that mixed-strategy equilibria are unstable: if for example, firms are indifferent between DCP and LCP in some market, a small exogenous increase in the share of importers pricing in dollars will make indifferent firms strictly prefer DCP to LCP.[20]

[20]Section 5.2 discusses the welfare ranking of coexisting equilibria under the optimal policy.

3.2 Why dollar?

While the previous section rationalizes the use of a vehicle currency in global trade, it does not tell us which currency plays this role. This section describes two fundamental advantages that make dollar pricing more attractive than pricing in any other currency.

In order to separate fundamental factors from the complementarity motive described above, I focus on the flexible-price limit $\lambda \to 1$, when almost all firms adjust prices ex-post and hence, the invoicing decision of a given exporter does not depend on currency choice of other firms and the equilibrium is always unique (see Appendix A.2 for details). Notice that the currency choice is still well-defined in the limit $\lambda \to 1$ with an arbitrary small price stickiness: exporter's invoicing decision depends only on the states of the world in which price remains unadjusted and is determined even when the probability of these states converges to zero. This contrasts with the case of fully flexible prices $\lambda = 1$, when currency choice is completely inconsequential and therefore, is not determined. I start with the case when no DCP equilibrium exists to outline necessary conditions for dollar invoicing.[21]

Lemma 5 (No-DCP benchmark) *If prices are almost flexible, $\lambda \to 1$, and countries are symmetric, $n = 0$, $\rho = 1$, exporters choose PCP when $\alpha \leq \frac{1}{2-\gamma}$, LCP when $\alpha \geq \frac{1}{2-\gamma}$, and no DCP equilibrium exists.*

When prices are (almost) flexible, the weights of different exchange rates in the optimal price $\tilde{p}_{ji}$ are determined solely by trade linkages. If countries are symmetric, $n = 0$, the market share of U.S. products is infinitely small and hence, the weight of the dollar in the optimal price of exporters from other countries is trivial. Given that all exchange rates have the same volatility, $\rho = 1$, firms unambiguously prefer to set prices in producer or local currency, which have a positive weight in $\tilde{p}_{ji}$. Figure 2b illustrates this result in the coordinates α and γ. The region of DCP is empty, while the choice between PCP and LCP depends on α and γ: using local currency is optimal only when complementarities in price setting are strong and the share of local firms in the destination market is sufficiently high. Next, I show that any deviation from the benchmark described in Proposition 5 is sufficient to sustain the DCP equilibrium for some values of other parameters.

Suppose first that countries are symmetric in terms of their size, $n = 0$, but the volatility of the dollar exchange rate is lower relative to other currencies because of higher diversification of the U.S. economy and smaller fundamental shocks, i.e. $\rho < 1$. To see the benefits of DCP in this case, consider a limiting case $\gamma = \alpha = 1$ when the optimal price of a given firm depends only on prices set by exporters from other economies (see Figure 1). It follows that all exchange rates enter symmetrically the optimal price $\tilde{p}_{ji}$ and the firm would ideally like to set prices in terms of a fully diversified basket of currencies.

[21] Even though the flexible-price limit might be not empirically relevant, it is still informative about the equilibrium invoicing under sticky prices because the DCP region changes continuously in λ, i.e. the correspondence is upper- and lower-hemicontinuous. Note also that the limit $\lambda \to 1$ provides a good approximation to some internationally traded goods, such as commodities.

This is, however, not feasible because of the discrete nature of the invoicing problem, and firms look for a currency that can replicate most closely this diversified portfolio. Since the dollar has the lowest idiosyncratic volatility, $\rho < 1$, it strictly dominates other alternatives.

Away from this limit, there is a trade-off between producer/local currency and the dollar: the prices of domestic inputs and local competitors are more stable in the former, while the dollar provides a better proxy for prices of goods coming from third countries. At the same time, DCP strictly dominates any other potential vehicle currency. Figure 3a shows equilibria for different values of ρ in the coordinates α and γ. The line separating the PCP and LCP equilibria remains the same as in Figure 2b since the value of ρ does not affect the trade-off between producer and local currencies. The region of DCP equilibrium is one point when $\rho = 1$ and increases continuously as dollar volatility goes down. Consistent with the discussion above, DCP equilibrium is more likely for higher import share γ and intermediate values of price complementarities α, while PCP and LCP are always optimal when import share γ is low.

Proposition 3 (Volatility advantage) *Assume $\lambda \to 1$ and $n = 0$. Then as long as the dollar has lower volatility than other currencies, $\rho < 1$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as ρ goes down.*

While this result alone is not sufficient to rationalize the global status of the dollar, it explains why the use of currencies with volatile exchange rates in international trade is very limited: for example, almost all imports and exports of Latin American and Eastern European countries are invoiced in foreign currencies (Casas, Díez, Gopinath, and Gourinchas 2017). In contrast to the previous literature, the model shows that the relative volatilities of exchange rates are important even when they are not driven by nominal shocks (*cf.* Devereux and Engel 2001, Bhattarai 2009).

Consider next the case when volatility of exchange rates is the same for all countries, $\rho = 1$, but the U.S. accounts for a non-trivial share of global trade, $n > 0$.[22] This implies that a positive fraction of inputs and competing products in small economies are coming from the U.S. and the dollar has a positive weight in the optimal price $\tilde{p}_{ji}$ of foreign exporters. As a result, the dollar is preferable to any other vehicle currency and can also dominate PCP/LCP depending on other parameters. Figure 3b shows that the DCP region consists of only one point when $n = 0$ and increases as n goes up.

Proposition 4 (Large economy advantage) *Assume $\lambda \to 1$. Then as long as the share of the U.S. economy in international trade is positive, $n > 0$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as n goes up.*

The figure also shows that equilibria with asymmetric invoicing can arise when $n > 0$.[23] In particular, firms might choose to use producer currency when trading between small economies, but set prices

[22] Note that PCP, LCP and DCP coincide for trade flows between regions within the U.S.

[23] Strictly speaking, the same is true in a model with $n = 0$ and $\rho < 1$, but since U.S. economy has zero mass, the effects are negligible at the global level.

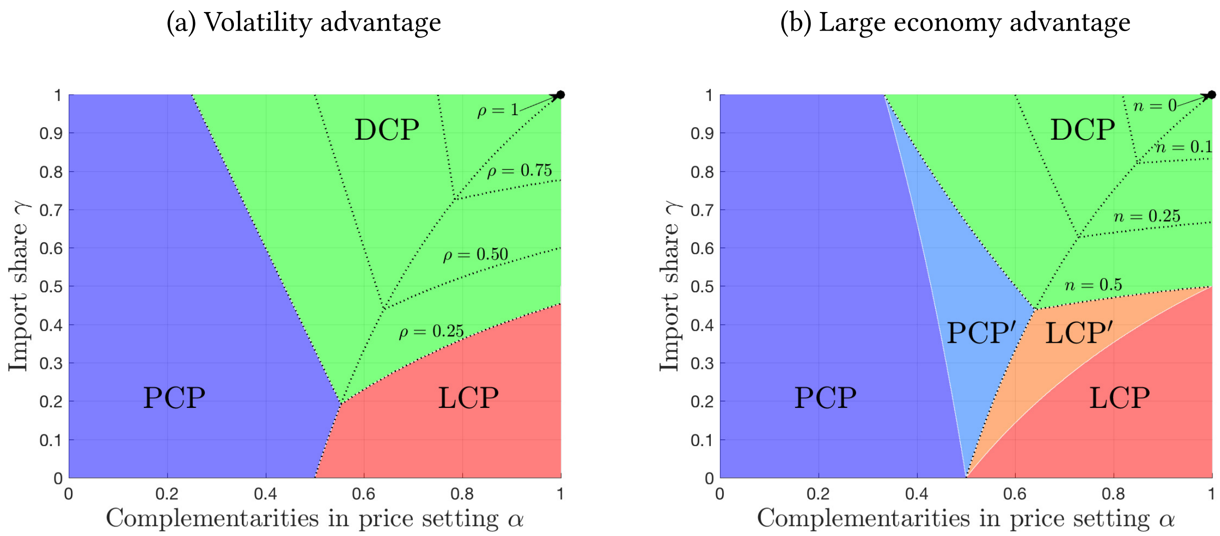

Figure 3: Currency choice: flexible price limit

Note: the figure shows equilibria for $\lambda \to 1$, $\phi = 0.5$ and (a) $n = 0$, different values of ρ, (b) $\rho = 1$, different values of n. PCP′ (LCP′) denotes the region where small countries set prices in producer (local) currency when trading with each other and use dollars when trading with the U.S.

in dollars when exporting to the U.S. This is because the home bias is larger in the U.S. than in other economies when $n > 0$, and more competitors in the destination market have prices stable in local currency, i.e. in dollars. Similarly, exporters from the U.S. have a higher share of their marginal costs that are stable in producer currency and they can use DCP even when other firms prefer LCP.

Calibration Are the price linkages and fundamental advantages described in the previous sections strong enough to sustain the DCP equilibrium? To answer this question, consider a simple calibration of the model. Clearly, there is almost no hope to get string linkages across exporters if one calibrates the openness γ to match the U.S. import-to-GDP ratio of 0.15. A large fraction of non-tradable goods in GDP, however, masks a high import share in the tradable (manufacturing) sector, which is about $\gamma = 0.6$ for small economies and 0.4 for the U.S. Both firm-level data and the aggregate input-output tables imply that the share of intermediate goods in production is around $\phi = 0.5$. The empirical evidence on the complementarities in price setting are more scarce, but the recent estimates by Amiti, Itskhoki, and Konings (2016) suggest $\alpha = 0.5$. This calibration implies that 45% of exporters' optimal price is determined by the prices of foreign suppliers and competitors and the number goes even higher if one takes into account the equilibrium effect of import prices on local firms (Tintelnot, Kikkawa, Mogstad, and Dhyne 2017) and the fact that both α and ϕ are higher for large firms that account for most of the global trade (Amiti, Itskhoki, and Konings 2014).

Assuming that one period corresponds to a year, I calibrate $\lambda = 0.5$, so that half of firms update

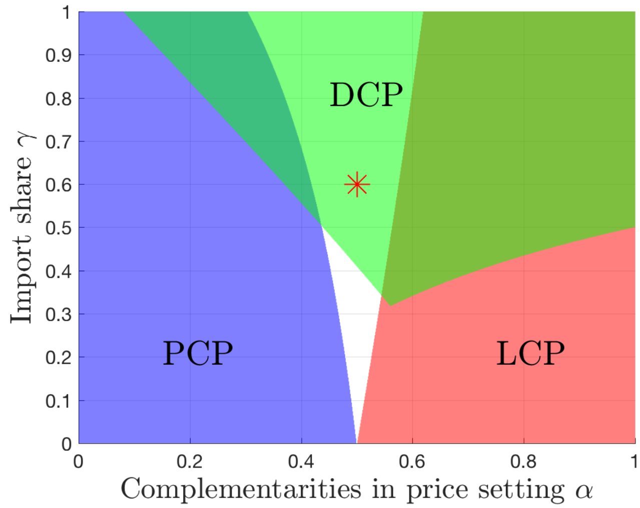

Figure 4: Currency choice under the baseline calibration

Note: the figures show the regions where the symmetric PCP, LCP and DCP equilibria can be sustained (there are no symmetric equilibria in the white region). Parameter values are from the benchmark calibration: $\phi = 0.5$, $\lambda = 0.5$, $\rho = 0.5$, $n = 0.3$ and the red star corresponds to $\gamma = 0.6$ and $\alpha = 0.5$.

prices by the end of the first year and the remaining ones adjust by the end of the second year. Given the volatility of the bilateral exchange rate between developing countries is on average higher than the volatility of exchange rate between a developing country and the U.S. by 30%, ρ is calibrated to 0.5. Finally, I use $n = 0.3$, which is a conservative value relative to the large share of dollarized economies in the world (see Ilzetzki, Reinhart, and Rogoff 2017). Figure 4 shows the resulting symmetric equilibria under the baseline calibration. The DCP equilibrium is indeed sustainable and no other symmetric equilibria — with PCP, LCP or an alternative vehicle currency – exist for these parameter values.

3.3 Transition

The previous section argues that both fundamental factors and the complementarities in currency choice contribute to the dominant status of the dollar in today's world. What happens when these factors work in the opposite direction? This is what happened to the pound sterling in the twentieth century and what might be relevant for the dollar in the future if China overtakes the U.S.

To answer this question, I extend the model to have two large countries, the U.S. and the U.K. (see Figure 5a). The global economy starts from the point when the U.K. has a fundamental advantage over the U.S. in terms of economy size or exchange rate volatility, which it gradually looses along the transition path. I make three simplifying assumptions as in Matsuyama, Kiyotaki, and Matsui (1993) and Rey (2001). First, all countries are assumed to trade either in dollars or in pounds, so no PCP/LCP is allowed. Second, since I am interested in the long-run changes, the focus is on the evolution of the steady state, while the transition between steady states is ignored. Third, with multiple equilibria in the model, there is a continuum of possible transition paths. For selection, I use the argument in the spirit of evolutionary game theory that as long as the old equilibrium exists, the firms do not coordinate to jump into a new one. This is equivalent to choosing among all possible transition paths, the one with the highest hysteresis.[24] Let n_X and σ^2_X denote the size and the volatility of shocks in economy X. The next proposition characterizes the transition.

Proposition 5 (Transition) *Let $T(x)$ denote the threshold of $\frac{\sigma^2_{UK}}{\sigma^2_{US}}$ or $\frac{n_{US}}{n_{UK}}$, at which trade flow x from Figure 5a switches from the pound to the dollar. Then*

1. *the share of pound invoicing in international trade is decreasing along the transition path,*
2. *the trade flows switch from the pound to the dollar in the following order:*
   - $T(a), T(b) \leq T(c) \leq T(f), T(g)$
   - $T(a) \leq T(d) \leq T(g)$
   - $T(b) \leq T(e) \leq T(f)$

Thus, as the U.K. economy becomes smaller or/and more volatile, the share of the pound in international trade monotonically decreases. Figure 5b shows the transition path for changes in union size under the baseline calibration, while Figure A2 in the Appendix shows transition driven by changes in volatilities. While equilibrium invoicing clearly evolves together with the fundamental factors, there is also a path-dependence due to strategic complementarities in currency choice.[25] In particular, when the size of the U.K. and U.S. is about the same, the share of the pound in global trade remains as high as 85%. At the same time, the transition is much faster in the limit of flexible prices $\lambda \to 1$ with no complementarities in currency choice.

The model has also clear predictions about the order in which trade flows switch from one currency to another. The trade between the U.S. and small economies is the first to become invoiced in dollars because of the prevalence of U.S. firms with costs stable in dollars. At the second stage, the small economies start using the dollar as a vehicle currency when trading with each other, and the trade flows between two unions also change the currency of invoicing. Finally, the trade between the U.K. and small economies switches to DCP as well. Interestingly, if complementarities in currency choice are strong enough, some flows might remain invoiced in the pound even as $n_{UK} \to 0$.

These predictions are broadly consistent with historical evidence — the transition from the pound to the dollar was sluggish, followed with the lag after the U.S. overtook the U.K. as the largest economy,

[24]While a dynamic model with staggered pricing can be used to select between "history" vs. "expectations", the equilibrium remains non-unique in the general case (see e.g. Matsuyama 1991, Krugman 1991). Alternatively, one can use a global game approach in the spirit of Morris and Shin (2001), but its application in dynamic settings is complex and goes beyond the scope of this paper.

[25]The standard caveat that there are also equilibria with fast adjustment applies here as well. See Figure A2 for the lower and upper boundaries on the transition paths.

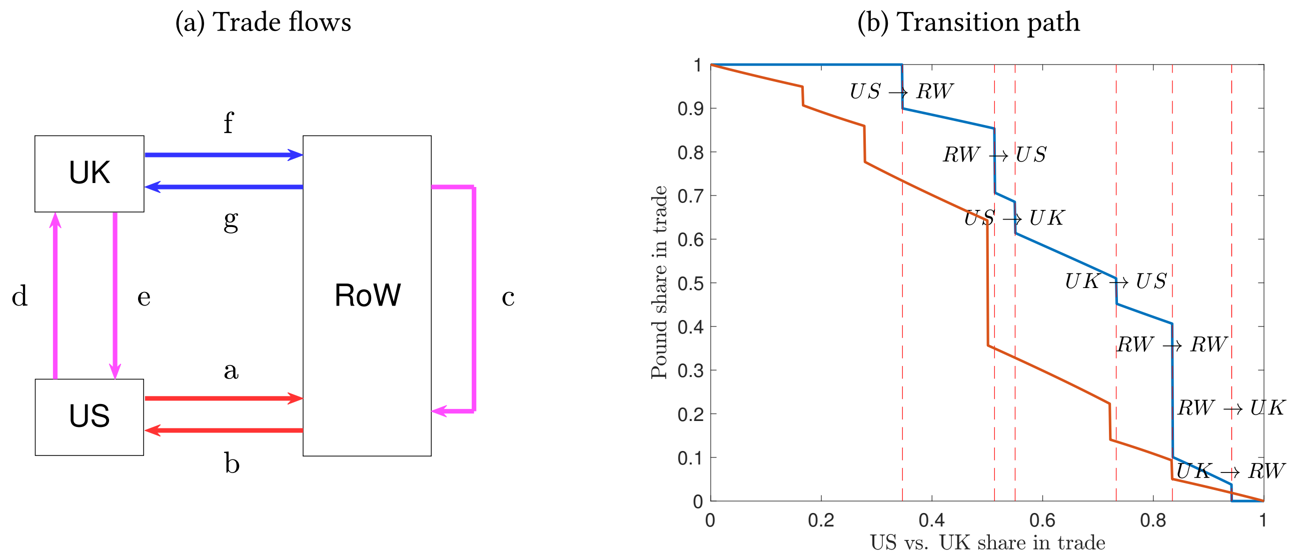

Figure 5: Transition from one vehicle currency to another

Note: plot (a) shows the structure of the economy with two large economies — the U.K. and the U.S. — and the rest of the world (RW) consisting of a continuum of small economies. The arrows correspond to the trade flows between countries. Plot (b) shows transition from the pound to the dollar as the relative size of the U.S. goes up. The blue line is the benchmark transition under $\gamma = 0.6$, $\alpha = 0.5$, $\phi = 0.5$, $\lambda = 0.5$, $n_{US} + n_{UK} = 0.5$ and $\sigma^2_{UK} = \sigma^2_{US}$. The red line uses the same values except for $\lambda \to 1$.

and was accelerated by high volatility of the U.K. exchange rate after World War I (see Eichengreen 2011). While the invoicing data is scarce for most of the twentieth century, the recent experience of the Eurozone also fits the predictions of the model. In particular, the euro is more commonly used in Eurozone trade with developing countries, much less so in trade with the U.S. and even more rarely as a vehicle currency (Kamps 2006).

3.4 Extensions

This section relaxes two assumptions from the baseline model and describes new mechanisms that further strengthen firms' incentives to use dollar pricing. I focus mostly on economic intuition leaving the formal results for Appendix A.3.3.

Monetary shocks While movements in exchange rates are largely disconnected from monetary shocks for most economies, the correlation is much higher for countries with unstable inflation. I therefore relax the assumption that $m_i = 0$ and allow for exogenous monetary shocks.[26]

Consider first the limiting case when m_i is the only shock in the economy and prices are almost flexible $\lambda \to 1$. The labor costs are no longer stable in producer currency and as a result, neither are the

[26] I focus on the second rather than the first moments of monetary shocks, which complements the effect of inflation rate on currency choice emphasized by the previous literature (see e.g. Drenik and Perez 2017).

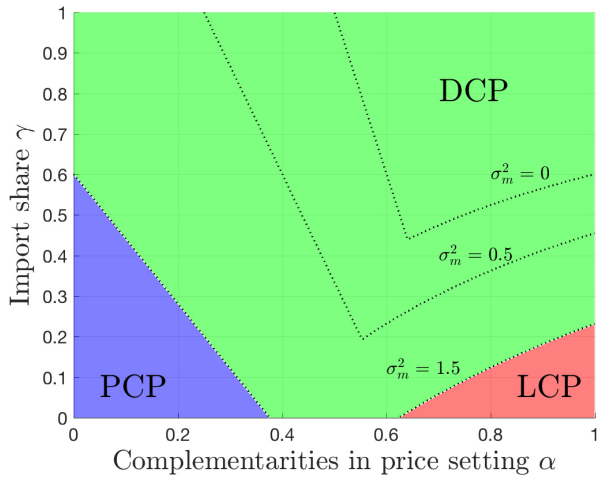

Figure 6: Currency choice under exogenous monetary shocks

Note: the figure shows equilibria in a model with exogenous monetary and financial shocks. The volatility of financial shocks is normalized to one, while the volatility of nominal shocks is shown in the figure, $\lambda \to 1$, $\phi = 0.5$, $n = 0$, $\rho = 0.5$.

prices of domestic intermediate goods. At the same time, a positive monetary shock is associated with a one-to-one depreciation of the local exchange rate, which implies that nominal wages can actually be more stable in foreign currency than in a local one. In particular, as long as $\rho < 1$, the volatility of nominal wages in dollars is lower than their volatility in producer currency, and firms unambiguously prefer DCP to PCP. A symmetric argument applies to LCP. DCP is therefore a unique equilibrium for arbitrary values of other parameters and can be sustained even in the limit of closed economy $\gamma \to 0$. This prediction of the model is consistent with the wide use of DCP during episodes of high and unstable inflation in Latin American countries during the 1980s and in Eastern Europe during the 1990s.

More generally, in the presence of other shocks, the higher volatility of money supply increases the correlation between wages and exchange rates and extends the region of DCP (see Figure 6). Importantly, this result holds even when volatility of monetary shocks in the U.S. increases proportionately with other countries. In contrast to the mechanism outlined in Devereux, Engel, and Storgaard (2004), a higher volatility of monetary shocks makes DCP more appealing to firms not because of increasing volatility of other currencies relative to the dollar, i.e. falling ρ, but because of lower stability of input and competitor prices in producer and local currencies respectively. The model thus suggests that periods of high *global* inflation — as observed during the 1970s — can actually increase the use of the dollar in international trade despite higher volatility of the U.S. exchange rate.

Dollarization In contrast to the assumption in the baseline model, it is not uncommon for *local* firms in developing countries to set prices in dollars (see e.g. Drenik and Perez 2017). I therefore extend the model allowing domestic producers in the tradable sector to choose optimally the currency of invoicing

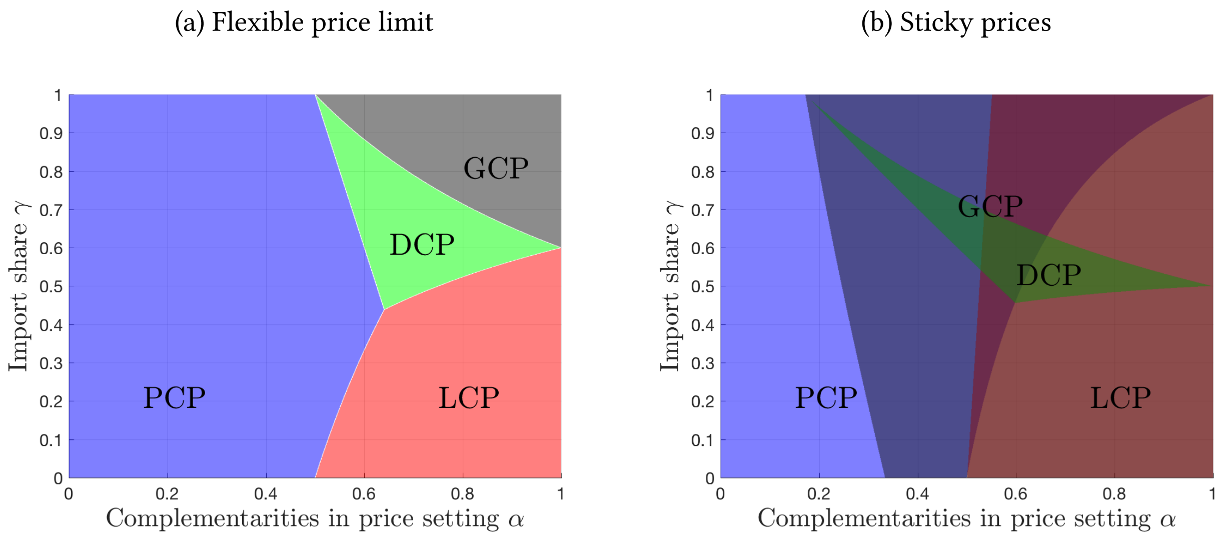

Figure 7: The optimal invoicing of domestic firms

Note: figure (a) shows equilibria in the flexible price limit $\lambda \to 1$ and $\rho = 0.5$, while figure (b) shows symmetric equilibria under sticky prices $\lambda = 0.5$ and $\rho = 1$. The grey area is the region of the global currency pricing (GCP) equilibrium with all firms including domestic ones using the dollar for invoicing. Other parameters: $\phi = 0.5$, $n = 0$.

and define the global currency pricing (GCP) equilibrium, in which all firms in the tradable sector including domestic ones set prices in dollars.

Consider first the flexible-price limit $\lambda \to 1$ when equilibrium prices are independent from firms' invoicing decisions (see Figure 7a). This implies that the currency choice of domestic producers has no effect on invoicing decisions of exporters, which remain the same as in the baseline model. Since producer and local currencies coincide for domestic firms, their total weight in the optimal price is higher. Local firms are therefore less likely to use dollar invoicing and the GCP equilibrium is a subset of the DCP equilibrium. The equilibrium invoicing looks very different when prices are sticky: in the limiting case of fully rigid prices, the DCP region is always a subset of the GCP region. Intuitively, when strategic complementarities in currency choice are strong, it is easier to support the equilibrium where all firms invoice in dollars than the equilibrium where only exporters use dollars and domestic firms set prices in local currency. As Figure 7b shows, even incomplete price rigidity is sufficient for the GCP region to dominate both the DCP and LCP regions.

Thus, the model predicts that while domestic firms might be less likely to switch to dollar invoicing than exporters, once they do so — e.g. because of the unstable monetary policy discussed above — the DCP equilibrium can be sustained more easily and can persist even after fundamental factors turn against the dollar. The wide use of the dollar in Latin American and some East European countries contributes therefore, to the status of the dollar in international trade.

4 Transmission of Monetary Shocks

This section shows that despite only *second-order* effects on firm's profits, the currency choice has *first-order* general equilibrium implications. In particular, a small perturbation of fundamentals that makes firms switch from one invoicing regime to another leads to a discontinuous change in how prices, output, consumption, and trade balance react to monetary shocks. I first take dollar pricing as given and argue that the stimulating effect of exchange rate depreciation on local output is higher in the U.S. than in other economies, and that spillover effects of dollar depreciation on foreign output are significantly higher than predicted by the standard models with PCP and LCP.[27] I then use a simple calibration of the model to show that small private costs of currency choice can lead to large differences in international business cycles (*cf.* Mankiw 1985). For simplicity of exposition, I focus exclusively on equilibria with symmetric invoicing and unexpected shocks. I also assume $n = 0$ to isolate the effects of DCP from the asymmetries arising from differences in size of the economies. For all proofs and derivations see Appendix A.6.1.

Local effects The effect of exchange rate depreciation on trade balance, consumption and output depends on how import and export prices respond to these shocks. As emphasized by the previous literature, the pass-through of exchange rates into customer prices is high under PCP and low under LCP, which implies that quantities respond much less under LCP (see e.g. Betts and Devereux 2000). Relative to this benchmark, invoicing in dollars introduces two types of asymmetries — between export and import prices, and between the U.S. and other economies. In particular, the price response resembles PCP on the export side and LCP on the import side for the U.S. and the opposite way for other countries. Thus, in response to a positive monetary shock, the trade balance adjusts more through higher exports in the U.S. and lower imports elsewhere.[28] Interestingly, despite asymmetries across countries, the elasticity of *net export* with respect to the trade-weighted exchange rate is the same for all economies including the U.S., i.e. the higher elasticity of exports and the lower elasticity of imports in the U.S. exactly offset each other. It follows that the trade-weighted rather than the invoicing-weighted exchange rate remains a sufficient statistic for net exports.

The differences in trade balance adjustment across countries under DCP translate into the asymmetric response of consumption and output. The depreciation of the exchange rate stimulates production more in the U.S. than in other countries because of larger expenditure switching towards exported

[27] A tractability of the model allows me not only to formalize several conjectures about the partial equilibrium adjustment of exports and imports from Gopinath (2016) and Goldberg and Tille (2006), but also to gain new insights about the general equilibrium effects of monetary policy on output and consumption.

[28] The total effect, however, is more than just a convex combination of the two due to input-output linkages. Consider an economy other than the U.S. Relative to the LCP case, imported intermediates become more expensive and hence, prices of adjusting exporters fall less, depressing exports even further. Relative to the PCP case, a weaker growth in exports implies lower demand for foreign intermediates, which amplifies contraction in imports.

goods and lower increases in prices of foreign intermediates. At the same time, the lower pass-through of exchange rate shocks into the CPI implies that the U.S. enjoys a smaller drop in consumption.

Proposition 6 (Transmission of monetary shocks) *Assume DCP and $n = 0$. Then relative to the effects of monetary shock in other economies, an expansionary monetary policy in the U.S. implies*

1. *higher exports and imports,*
2. *lower inflation and higher output,*
3. *the same net export.*

International spillovers The last decade has witnessed a lively debate about the spillover effects of the Fed's monetary policy on other countries (see e.g. Bernanke 2017). On the one hand, easy monetary policy increases U.S. demand for imported goods, stimulating production in all economies. On the other hand, such policy also leads to a depreciation of the national currency, which can potentially make U.S. goods cheaper relative to foreign products and have negative spillovers on other economies. The classic result in the literature is that the net effect is negative under PCP and positive under LCP: while the former effect does not depend on currency of invoicing, the latter effect is large under PCP and mild under LCP (see e.g. Corsetti and Pesenti 2005).[29] I next argue that DCP generates an additional channel with unambiguously positive spillovers that have been largely ignored by the previous literature.[30]

Consider a positive monetary shock in the U.S. To isolate the effect of dollar pricing, assume that the U.S. is small, $n = 0$, and symmetric to other economies. This implies that an increase in aggregate demand in the U.S. has only trivial effects on the global economy. Similarly, if prices were set in producer or local currency, the fraction of international trade flows invoiced in dollars would be infinitely small and hence, depreciation of the U.S. exchange rate would also have (almost) no effect on the global economy. The situation is different under DCP. A depreciation of the dollar exchange rate decreases prices of all internationally traded goods relative to domestic products. This generates expenditure switching towards foreign goods and raises the volume of international trade. The result is consistent with the conjecture put forward by Gopinath (2016) and the recent empirical evidence from Casas, Díez, Gopinath, and Gourinchas (2017) and Boz, Gopinath, and Plagborg-Møller (2017).

This partial equilibrium effect, however, is not sufficient to predict the spillover effects on global output: despite an increase in the volume of trade, the global net exports are always zero, i.e. an increase in exports due to the depreciation of the dollar exchange rate is fully offset by an increase in imports leaving the global GDP unchanged. Instead, it is a change in consumption that stimulates global production. Aggregating across all countries, one can write the global market clearing condition as $gdp = c = m - p^C$. A depreciation of the dollar exchange rate decreases import prices in all countries,

[29]The words "positive" and "negative" in this section refer to signs of the effects and not to their welfare implications.

[30]The important exception is the paper by Goldberg and Tille (2009), which shows in a context of a three-country model that U.S. shocks have larger effects on global consumption under DCP.

which lowers the global CPI, raises the real demand $m - p^C$ and stimulates production worldwide. Note that in contrast to the expenditure switching effect discussed above, the effect on global output is independent from the elasticity of substitution between domestic and foreign goods and depends exclusively on the pass-through of exchange rate shocks into the CPI.[31]

How is this increase in the global output divided between the U.S. and the rest of the world? Clearly, when $n = 0$, the global output coincides with the output in a representative economy other than the U.S. The spillover effects are therefore unambiguously positive in this case. When the U.S. accounts for a positive share of global trade, there is an additional expenditure switching effect towards U.S. goods. The latter is stronger when θn is high, decreasing the net spillovers on other economies.[32]

Proposition 7 (International spillovers) *Relative to the PCP/LCP benchmark, dollar invoicing implies that expansionary monetary policy in the U.S.*

1. *increases the volume of international trade,*
2. *increases global output and consumption, with the effect independent from elasticity θ,*
3. *decreases CPI in other economies and boosts consumption and production if θn is not too high.*

An interesting corollary of this result is that the spillover effects of dollar depreciation on foreign output can be positive even when monetary authorities are constrained by the zero lower bound and cannot stimulate the aggregate demand. This contrasts with the conclusions from Caballero, Farhi, and Gourinchas (2016) that a depreciation of exchange rate during the global liquidity trap is a zero-sum policy that "exports" recession to other countries and can potentially lead to "currency wars". It is still true, however, that the devaluation of *non-vehicle* currencies leads to a standard expenditure-switching effect and is closer to the beggar-thy-neighbor benchmark.

Private vs. aggregate effects While the model is intrinsically stylized and abstracts from both cross-country heterogeneity and several ingredients from the DSGE literature (e.g. capital accumulation, habit formation, wage rigidity, etc.), it is still informative to compare private costs of currency choice with the resulting aggregate effects. I use the values of parameters from the benchmark calibration (see Sections 3.2), the share of tradable sector $\eta = 0.15$ calibrated to the share of manufacturing in global GDP and the elasticity of substitution between goods $\theta = 2$ close to the values used in the previous literature (see e.g. Chari, Kehoe, and McGrattan 2002, Backus, Kehoe, and Kydland 1994, Feenstra, Luck, Obstfeld, and Russ 2014).

[31] The recent empirical evidence indicates that the currency of invoicing indeed affects the pass-through of exchange rate shocks into the *consumer* prices (see Auer, Burstein, and Lein 2018).

[32] The small previous literature that studied transmission of shocks under DCP has mostly assumed only two countries (see e.g. Canzoneri, Cumby, Diba, and López-Salido 2013, Corsetti and Pesenti 2007). In this case, all imports of the RoW come from the U.S., so that effectively $n = 1$ and there are no positive spillovers: the depreciation of the dollar generates expenditure switching exclusively towards U.S. goods instead of exports from other countries.

Table 1: Local and spillover effects of monetary shocks

		U.S. shock			Non-U.S. shock		
		DCP	PCP	LCP	DCP	PCP	LCP
U.S.	gdp	5.52	5.17	5.46	–	–	–
	c	5.41	4.91	5.36	–	–	–
Non-U.S.	gdp	0.69	0.12	0.00	4.82	5.05	5.46
	c	0.74	0.23	0.04	4.68	4.68	5.32

Note: the table shows the percentage change in GDP and consumption of the U.S. and other countries in response to a local and a foreign 10% monetary shock.

Table 1 shows the effects of a 10% increase in monetary supply. The first three columns correspond to the U.S. monetary shock, while the following columns show the effect of monetary expansion in another countries. Despite the large share of the U.S. in the global economy, $n = 0.3$, the results from Proposition 6 hold: under DCP, the stimulating monetary policy is significantly more efficient in the U.S. than in other countries as the GDP increases by around 5.5% in the U.S. and 4.8% in other economies. In line with Proposition 7, the spillover effect of the U.S. shock on foreign GDP is more than 5 times higher under dollar invoicing than under PCP/LCP.[33]

These large differences in monetary effects across alternative invoicing regimes contrast with relatively modest differences in exporters' profits. Assuming that the standard deviation of the bilateral exchange rate between countries other than the U.S. is 0.15, I calculate losses of an individual exporter from using dollar pricing instead of the optimal basket of currencies keeping the aggregate DCP equilibrium constant. The total costs across all exporters are only 0.02% of the global GDP, which is more than one magnitude lower than the spillover effects discussed above. The result resembles the classical argument of Mankiw (1985) and Ball and Romer (1990) that small menu costs can lead to large business cycles. In the case of an open economy, there is, however, an additional dimension as exporters choose in which currency to set their prices. These decisions are based on the second-order effects on firm's profits (Lemma 1), but have the first-order implications for the transmission of monetary shocks within and across countries (Propositions 6-7). Just like the real rigidities in a menu cost model, the complementarities in currency choice amplify the difference between private and aggregate effects: when other firms are using DCP, the share of the dollar in the optimal basket is high and hence, the private costs of setting prices in dollars are low.

[33] In contrast to the conventional model, the spillovers on foreign GDP are positive under PCP because of a high share of intermediate goods: dollar depreciation decreases costs of inputs in other countries and stimulates production and consumption. See Rodnyansky (2017) for the empirical evidence in favor of this mechanism.

5 Optimal Monetary Policy

The optimal monetary and exchange rate policy is one of the central questions in international economics. Should the policy focus on inflation targeting and output stabilization as in the closed economy, or movements in exchange rates that can be a separate concern for policymakers? Under which conditions is it optimal to peg exchange rates rather than let them float? While the previous literature has shown that the answers to these questions depend crucially on exporters' currency choice, the latter has predominantly been taken as exogenous.[34] The results are therefore potentially subject to Lucas critique: the models ignore the fact that firms might change their invoicing decisions in response to the monetary policy. In addition, the literature has predominantly focused on PCP and LCP rather than a more empirically relevant case of DCP.

This section fills in this gap. I first show that in a conventional open economy model augmented with the endogenous currency choice, the optimal monetary policy always results in a unique equilibrium with PCP and free floating exchange rates. Thus, the optimal policy results under LCP and DCP from the previous literature do not apply in equilibrium when firms are allowed to optimally choose the currency of invoicing. While standard in this literature, the assumptions underlying this result are restrictive and inconsistent with the data. I then relax these assumptions and argue that there are complementarities between exporters' invoicing decisions and the optimal monetary policy, which can explain the dominant status of the dollar in trade and its use as an anchor currency in exchange rate policy. Finally, I contrast the optimal policy with and without commitment.

To obtain sharp analytical results and make them directly comparable to the previous literature, I assume perfect risk-sharing, steady-state markups eliminated with a production subsidy, only tradable sector ($\eta = 1$), fully sticky prices ($\lambda = 0$), only productivity shocks, and focus on equilibria with symmetric invoicing (see Appendix A.6 for details). The framework itself is, however, much richer and can be used to study the optimal policy under a wide range of alternative assumptions.

5.1 Efficient benchmark

In a general case, when nominal prices are sticky, the relative international prices get distorted and the equilibrium allocation is not efficient. However, as has been famously argued by Milton Friedman, the movements in exchange rates allow countries to adjust their relative prices even if the nominal ones are fully rigid: "It is far simpler to allow one price to change, namely, the price of foreign exchange, than to rely upon changes in the multitude of prices that together constitute the internal price structure" (Friedman 1953). This argument was formalized fifty years later by Devereux and Engel (2003), who showed that the *first-best* allocation can be implemented with the *optimal* monetary policy under the

[34] To mention just a few: Clarida, Gali, and Gertler (2001), Benigno and Benigno (2003), Corsetti and Pesenti (2005), Monacelli (2005), Gali and Monacelli (2005), Devereux, Shi, and Xu (2007), Corsetti, Dedola, and Leduc (2010), Engel (2011).

floating exchange rates if firms set prices in *producer currency*. Under LCP, on the other hand, the efficient allocation cannot be achieved and keeping exchange rates constant might be optimal. I use the model with endogenous currency choice to reexamine conclusions of this literature. To make results directly comparable, I start with a case when price rigidity is the only source of distortions in the economy and the model is isomorphic to the models studied in the previous literature.

Proposition 8 (First best) *Assume (i) no complementarities in price setting, $\alpha = 0$, (ii) full commitment, and (iii) cooperative policy across countries. Then the efficient allocation can be implemented by the optimal monetary policy that allows for the free floating exchange rates and stabilizes producers' marginal costs. The equilibrium invoicing is PCP.*[35]

Intuitively, in the presence of nominal rigidities, the optimal monetary policy stabilizes the marginal costs, so that firms do not need to adjust their prices at all. The law of one price holds under PCP and hence, movements in exchange rates guarantee that product prices in the customers' currency adjust optimally in response to shocks even though prices remain fully stable in the currency of the producer. This summarizes the logic behind the result from Devereux and Engel (2003).

In contrast to their setup, the model with endogenous currency choice generates an additional constraint on the planner's problem. The key insight of Proposition 8 is that this constraint is not binding at the optimum: the firms always choose PCP under the optimal monetary policy. The assumption $\alpha = 0$ implies that exporters' optimal prices are proportional to their marginal costs, which in turn are stabilized by the monetary policy. As a result, the optimal price is stable in producer currency and firms unambiguously prefer PCP. Importantly, however, while the PCP constraint is not binding under the *optimal* policy, this is not true for an arbitrary monetary policy. In other words, condition $\alpha = 0$ alone is not sufficient to guarantee the PCP equilibrium — depending on parameter values, the DCP or multiple equilibria can arise. The fact that the planner commits to stabilize costs even off equilibrium — if firms set prices in dollars — is crucial to implement the first-best allocation.

Thus, Proposition 8 shows that the analysis of the optimal policy under exogenous LCP or DCP in a standard open economy model is subject to the Lucas critique: it is not possible to sustain such equilibria under the optimal policy. Interestingly, not only does the decentralized currency choice generate no additional inefficiencies per se in this setup, it actually makes the argument in favor of the free floating exchange rates more robust.

5.2 Discretionary policy

While Proposition 8 provides a useful theoretical benchmark, the conclusion that firms set prices in producer currency is inconsistent with the observed patterns in the data. I therefore relax the assumptions

[35]The result holds more generally for arbitrary isoelastic preferences, price stickiness and exogenous shocks (except for financial shocks and markup shocks).

and study the optimal monetary policy in a more realistic environment. First, I bring back complementarities in price setting, $\alpha > 0$.[36] Second, to capture the spillover effects from the U.S. to the rest of the world, I drop the assumption of a cooperative policy. Since solving for a fully non-cooperative policy is analytically challenging and requires restrictive assumptions on parameter values (see e.g. Benigno and Benigno 2003, Corsetti and Pesenti 2001, Farhi and Werning 2012), I consider the limiting case when the U.S. economy is closed and hence, its policy is fully inward-looking, while the monetary policy of all other countries remains cooperative.[37] Finally, I start with the discretionary policy that is chosen after the realization of shocks and takes the ex-ante invoicing decisions of exporters as given. In other words, the planner cannot make a credible threat to punish firms for choosing a "wrong" currency. I solve for the Nash equilibrium, in which firms simultaneously choose the currency of invoicing under the rational expectations about future monetary policy.

Proposition 9 (Discretion) *The optimal non-cooperative discretionary policy in the DCP region implies*

1. *partial peg of other currencies to the dollar:* $\mathbb{V}\left(e_{i0}^{DCP}\right) \leq \mathbb{V}\left(e_{i0}^{PCP}\right)$,
2. *free floating exchange rates between non-U.S. countries:* $\mathbb{V}\left(e_{ij}^{DCP}\right) = \mathbb{V}\left(e_{ij}^{PCP}\right)$, $i, j \neq 0$,
3. *the welfare of a non-U.S. economy in a region with multiple equilibria:* $\mathbb{W}^{DCP} \leq \mathbb{W}^{PCP}$,
4. *complementarities between firms' currency choice and the monetary policy (if θ is not too high).*

In contrast to the benchmark case described in Proposition 8, the monetary policy is no longer inward-looking under DCP. Instead, other countries respond to the U.S. monetary shocks by trying to smooth movements in their exchange rates against the dollar. Intuitively, as shown in Section 4, U.S. monetary policy affects international prices and affects consumption and production in other economies. These fluctuations are distortionary as they do not reflect changes in productivities. The optimal policy in other countries therefore "leans against the wind" and partially offsets movements in import prices by stabilizing exchange rate against the dollar.[38] At the same time, such policy is costly as it distorts relative prices *within* countries. As a result, U.S. shocks are only partially offset under optimal policy, and the equilibrium exchange rate is neither floating nor fixed. This prediction of the model is consistent with the empirical fact that more than 70% of countries in the world have a managed floating regime ("crawling peg", "dirty float") and use the dollar as an anchor currency in their exchange rate policy (Ilzetzki, Reinhart, and Rogoff 2017, Calvo and Reinhart 2002).

The fact that all economies respond to movements in the U.S. exchange rate also implies that monetary policy is correlated across countries even if the fundamental shocks are purely idiosyncratic. An

[36] The previous normative literature has largely ignored price complementarities and provides little guidance about their effect on the optimal policy even under an exogenous currency choice.

[37] Egorov and Mukhin (2018) extend the analysis to the fully non-cooperative case showing the robustness of the main predictions of the model and complementing them with new ones about the benefits of DCP for the U.S.

[38] This result contrasts with the optimal policy derived in Goldberg and Tille (2009) and Casas, Díez, Gopinath, and Gourinchas (2017), which rely on isoelastic preferences, $\theta = 1$, and no intermediates in production, $\phi = 0$.

expansionary monetary policy in the U.S. leads to a depreciation of the dollar exchange rate and makes central banks in other countries ease their policy as well. This is consistent with the evidence on the global financial cycles (Rey 2015) and shows that a positive comovement of monetary policy across countries can arise not only due to financial linkages, but also because of the dominant status of the dollar in international trade (*cf.* Aoki, Benigno, and Kiyotaki 2016). It also provides a theoretical explanation to the recent empirical finding in Zhang (2018) that countries with a higher share of dollar pricing have tighter peg and respond more aggressively to the U.S. shocks.

Does this mean that policymakers face a "Dilemma" rather than the "Trilemma" (Rey 2015)? As Proposition 9 makes clear, it is indeed the case that the trade-off for the monetary policy is worse under DCP than under PCP because of the distortionary effects of U.S. policy and inefficient expenditure switching. At the same time, the partially flexible exchange rates are still preferable to fully fixed rates. In fact, while all countries (partially) offset shocks from the U.S., they fully adjust their exchange rates in response to local shocks. Thus, it is optimal to have free floating bilateral exchange rates between non-U.S. countries, even though the resulting expenditure switching is lower than under PCP.

Finally, going back to the endogenous currency choice, there is feedback from the optimal monetary policy into firms' invoicing decisions. Even if the volatility of fundamental shocks is the same for the U.S. as for other countries, the optimal policy of pegging exchange rates to the dollar implies that the dollar is effectively more stable than other currencies and hence, exporters are more likely to use dollar pricing.[39] Thus, there are strategic complementarities between firms' invoicing decisions and the monetary policy: DCP makes it optimal to peg exchange rates to the dollar, which in turn increases incentives of exporters to set prices in dollars. These general equilibrium complementarities contribute to the multiplicity of equilibria and help to sustain the DCP equilibrium even when the U.S. has no fundamental advantage at all (see Figure 8a).[40] It follows that the evolution of the international monetary system (see Farhi and Maggiori 2017) is closely related to the evolution of the international price system, and by promoting its currency in international trade, a country can alter its status in the global financial architecture.

5.3 Policy with commitment

While it is arguably more realistic to assume that monetary authorities take firms' currency choices as given, it is still informative to study the optimal policy that internalizes its effect on the invoicing decisions of exporters. To this end, consider the problem of the Ramsey planner that is free to choose monetary policy in all countries and maximizes global welfare (see Appendix A.6.4 for details).

[39] At the same time, a peg also implies that nominal wages in all countries become positively correlated with the U.S. exchange rate, which makes dollar pricing less attractive. This effect is, however, small when θ is not too high.

[40] While the equilibrium exchange rates and the welfare implications depend on the type of exogenous shock, the results about partial peg to the dollar, the global monetary cycle, and the general equilibrium complementarities hold for non-productivity shocks as well.

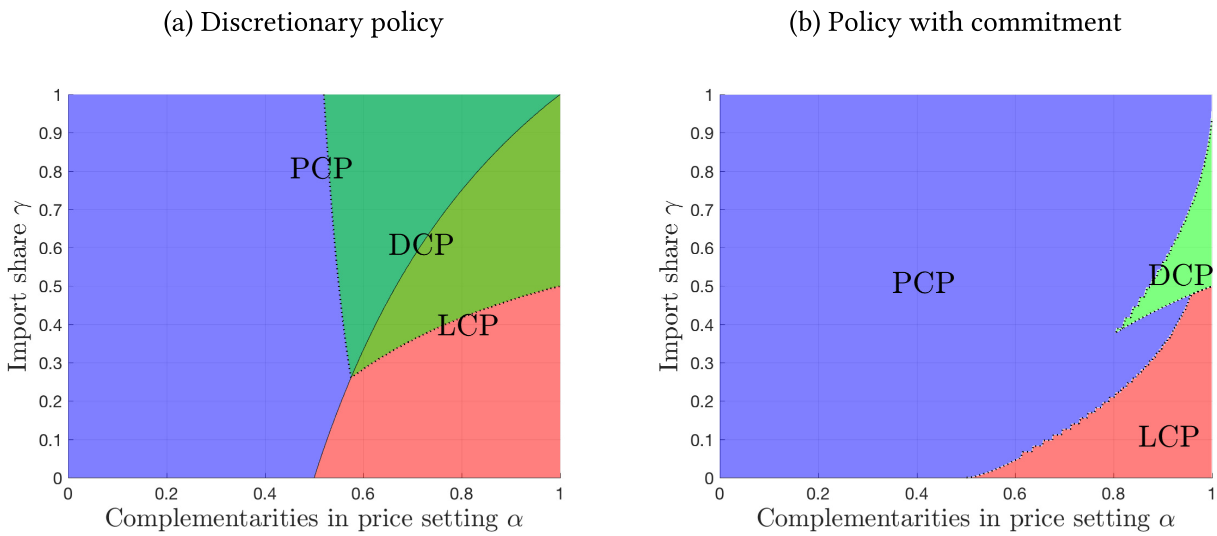

Figure 8: Currency choice under the optimal monetary policy

Note: the figure shows the equilibrium invoicing under the optimal monetary policy: plot (a) assumes discretionary policy, while plot (b) assumes full commitment. In plot (a), the solid line shows the boundary between PCP and LCP, while the dashed line shows the boundary of the DCP region. The values of parameters are: $\lambda = 0$, $\phi = 0.5$, $n = 0$, $\theta = 2$.

Figure 8b shows the equilibrium currency choice under the optimal policy. To see the intuition, note that the Ramsey planner can always implement any allocation that arises under the optimal discretionary policy from Figure 8a. According to Proposition 9, because of stronger expenditure switching and lower distortionary spillovers, the PCP equilibrium generates higher welfare than the DCP equilibrium. This motivates the planner to improve upon the discretionary allocation in two dimensions. First, in a region with multiple equilibria, the planner unambiguously chooses the equilibrium with PCP. Second, it deviates from the optimal ex-post policy to tilt firms towards PCP. To see this, consider the boundary of the PCP region from Figure 8a. Under these parameter values, the firms are indifferent between PCP and DCP/LCP, and an arbitrary small change in monetary policy makes them choose one over the other. At the same time, the global welfare changes discontinuously as firms switch from one currency to another because of the first-order effects on the transmission of shocks. Thus, a small change in the monetary policy at the boundary has only second-order costs for a given currency regime, but generates first-order benefits by changing exporters' invoicing decisions. As a result, the PCP region is significantly larger and the DCP region is much smaller under the Ramsey allocation than under the discretionary policy (see Figure 8).

Proposition 10 (Commitment) *The Ramsey allocation implies a larger PCP region and more stable exchange rates than the discretionary policy, and can be implemented via "sophisticated monetary policies".*

How does the planner implement the optimal allocation? In the region of the parameter space, in which the currency regime coincides with the regime under the discretionary policy, there is no reason for the planner to manipulate the exchange rates and hence, the optimal monetary policy is also the same as under the discretion. On the other hand, to make firms switch to PCP in the rest of the parameter space, the planner mutes the response of monetary policy to productivity shocks and makes exchange rates more stable relative to the discretionary case. This decreases the volatility of import prices and makes the optimal price of exporters more stable in the currency of the producer.

Interestingly, while this policy can sustain the optimal allocation, it is also consistent with other (suboptimal) equilibria.[41] To ensure the uniqueness of equilibrium, I therefore follow the approach from Atkeson, Chari, and Kehoe (2010) and allow for the "sophisticated monetary policies", which are a function not only of an exogenous state of the world, but also of the endogenous decisions of other agents. The planner can then use the off-equilibrium monetary policy to "punish" firms for choosing the "wrong" currency to ensure that only the optimal one is chosen in equilibrium.

6 Conclusion

This paper provides a tractable framework with endogenous currency choice that can be used to study simultaneously the determinants and the policy implications of the international price system. The model is broadly consistent with the key stylized facts, including the dominant status of the dollar in global trade and the delayed transition from the pound to the dollar in the twentieth century. Despite small private costs, the currency choice of exporters has large aggregate effects. In particular, the spillover effects of dollar depreciation on foreign output are much larger and can flip the sign when international prices are set in dollars rather than in producer or local currency. The optimal policy analysis, on the other hand, shows a close relation between the dominant status of the dollar in international trade and the wide use of the dollar as an anchor currency in exchange rate policy.

The tractability of the baseline model allows for several extensions and applications, which are left for future research. First, augmenting the model with a more realistic financial sector would allow analyzing the interactions between the dominant status of the dollar as a vehicle currency in international trade and as a reserve currency in global asset markets (see Gopinath and Stein 2017). Second, a quantitative version of the model with heterogeneous countries and sectors can be used to test the cross-sectional predictions of the model about the currency of invoicing, to do counterfactuals about the future changes in the international price system and to quantify the spillover effects for individual countries. The heterogeneity might also be useful to narrow down the set of potential equilibria. Finally, while this paper studies the optimal monetary policy, the same framework can be used to solve for the optimal policy when a planner has fiscal instruments that affect firms' invoicing decisions.

[41] The same situation arises in a standard New-Keynesian model with an interest rate rule, which has multiple equilibrium paths even when the Taylor principle is satisfied (see e.g. Woodford 2003). Notice, however, that the source of indeterminacy is completely different in my model, in which monetary policy is specified in terms of a money rule and the equilibrium is always unique for a given (exogenous) currency choice.

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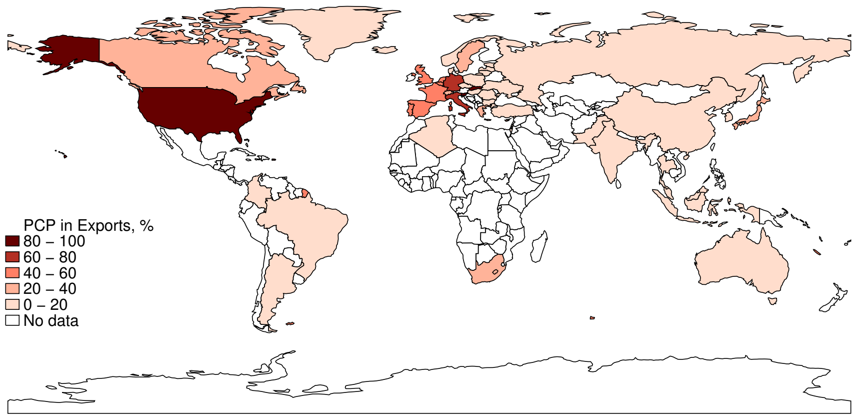
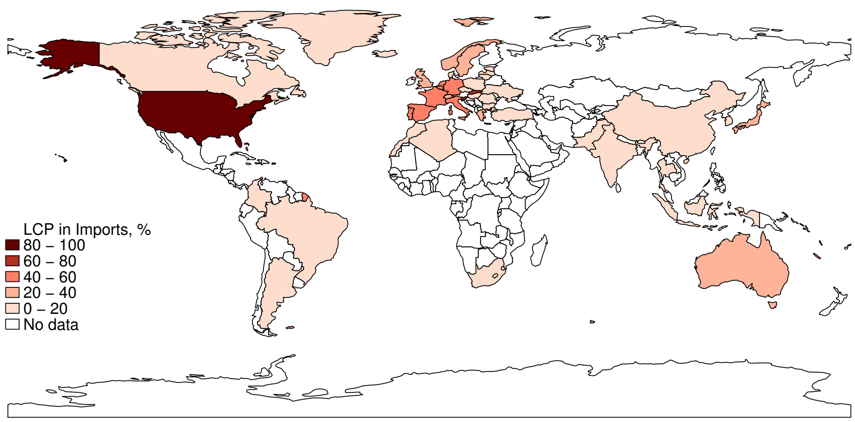
Appendix — for online publication

A.1 Additional figures

(a) Share of country's exports priced in producer currency (PCP)



(b) Share of country's imports priced in local currency (LCP)



(c) Share of country's exports priced in dollars (DCP)

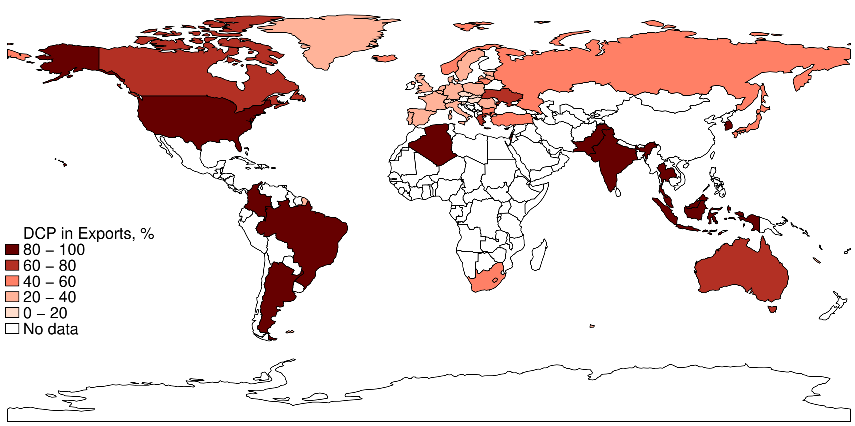


Figure A1: The use of producer currency, local currency and the dollar in global trade
Data sources: Gopinath (2016), Kamps (2006), Lai and Yu (2015), Sokolova (2015).

A.2 Equilibrium system

A.2.1 Equilibrium conditions

The Kimball aggregator for consumption bundle of tradable goods in region i is defined as

$$(1-\gamma)e^{-\gamma\xi_i}\int_0^1 \Upsilon\left(\frac{C_{ii}(\omega)}{(1-\gamma)e^{-\gamma\xi_i}C_{Ti}}\right)\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_i}\int_0^1\int_0^1 \Upsilon\left(\frac{C_{ji}(\omega)}{\gamma e^{(1-\gamma)\xi_i}C_{Ti}}\right)\mathrm{d}\omega\mathrm{d}j = 1, \quad \text{(A1)}$$

where $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$. I borrow expressions for price index and demand for individual goods under the Kimball aggregator from Itskhoki and Mukhin (2017) and Amiti, Itskhoki, and Konings (2016). The equilibrium system of the model consists of the following blocks:

1. Labor supply and labor demand:

$$C_{it} = \frac{W_{it}}{P_{it}^C}, \quad \text{(A2)}$$

$$L_{it} = (1-\phi)\left(\frac{P_{it}}{W_{it}}\right)^{\phi}\frac{Y_{it}}{A_{Tit}} + \frac{Y_{Nit}}{A_{Nit}}. \quad \text{(A3)}$$

2. Demand for non-tradables:

$$Y_{Nit} = (1-\eta)\int_0^1\left(\frac{P_{Nit}(\omega)}{P_{Nit}}\right)^{-\theta}\mathrm{d}\omega\left(\frac{P_{it}^N}{P_{it}^C}\right)^{-1}(C_{it}+G_{it}),$$

3. Price setting in non-tradable sector:

$$P_{it}^N(\omega) = \begin{cases} \bar{P}_{it}^N, & \text{w/p } 1-\lambda \\ \tilde{P}_{it}^N, & \text{w/p } \lambda \end{cases},$$

   where

$$\tilde{P}_{it}^N = \arg\max_P \left(P-(1-\tau)\frac{W_{it}}{A_{Nit}}\right)\left(\frac{P}{P_{it}^N}\right)^{-\theta}(C_{Nit}+G_{Nit}),$$

$$\bar{P}_{it}^N = \arg\max_P \mathbb{E}_{t-1}\left(P-(1-\tau)\frac{W_{it}}{A_{Nit}}\right)\left(\frac{P}{P_{it}^N}\right)^{-\theta}(C_{Nit}+G_{Nit}).$$

4. Demand for tradables:

$$\begin{aligned} Y_{it} =& (1-\gamma)e^{-\gamma\xi_{it}}\int_0^1 h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\mathrm{d}\omega\,(C_{Tit}+X_{it}+G_{Tit}) \\ &+\gamma\int_0^1 e^{(1-\gamma)\xi_{jt}}\int_0^1 h\left(\frac{D_{jt}P_{ijt}(\omega)}{P_{jt}}\right)\mathrm{d}\omega\,(C_{Tjt}+X_{jt}+G_{Tjt})\,\mathrm{d}j, \end{aligned} \quad \text{(A4)}$$

   with intermediate and final demand given by

$$X_{it} = \phi\left(\frac{W_{it}}{P_{it}}\right)^{1-\phi}\frac{Y_{it}}{A_{Tit}}, \quad \text{(A5)}$$

$$C_{Tit} + G_{Tit} = \eta \frac{P_{it}^C}{P_{it}} \left(C_{it} + G_{it}\right).$$

5. Price setting and currency choice in tradable sector:

$$P_{jit}(\omega) = \begin{cases} \bar{P}_{jit}^k, & \text{w/p } 1-\lambda \\ \tilde{P}_{jit}, & \text{w/p } \lambda \end{cases},$$

where

$$\tilde{P}_{jit} = \arg\max_P \left(P\mathcal{E}_{jit} - (1-\tau) MC_{jt}\right) \gamma e^{(1-\gamma)\xi_{it}} h\left(\frac{D_{it}P}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right),$$

$$\bar{P}_{jit}^k = \mathcal{E}_{ikt} \cdot \arg\max_{P,k} \mathbb{E}_{t-1}\left(P\mathcal{E}_{jkt} - (1-\tau) MC_{jt}\right) \gamma e^{(1-\gamma)\xi_{it}} h\left(\frac{D_{it}P\mathcal{E}_{ikt}}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right),$$

and marginal costs of production are

$$MC_{jt} = \frac{1}{A_{Tjt}} W_{jt}^{1-\phi} P_{jt}^{\phi}. \tag{A6}$$

6. Definition of price indices:

$$P_{it}^C = \left(P_{it}^N\right)^{1-\eta} P_{it}^{\eta},$$

$$P_{it}^N = \left[\int_0^1 \left(P_{it}^N(\omega)\right)^{1-\theta} \mathrm{d}\omega\right]^{\frac{1}{1-\theta}},$$

$$(1-\gamma) e^{-\gamma\xi_{it}} \int_0^1 \Upsilon\left(h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\right)\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_{it}} \int_0^1 \int_0^1 \Upsilon\left(h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\right)\mathrm{d}\omega \mathrm{d}j = 1,$$

$$(1-\gamma) e^{-\gamma\xi_{it}} \int_0^1 h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\frac{P_{iit}(\omega)}{P_{it}}\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_{it}} \int_0^1 \int_0^1 h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\frac{P_{jit}(\omega)}{P_{it}}\mathrm{d}\omega \mathrm{d}j = 1.$$

7. Risk-sharing: given the stochastic discount factor $\Theta_{it+1} = \frac{C_{it}P_{it}^C}{C_{it+1}P_{it+1}^C}$,

$$e^{\Delta\psi_{it+1}} \Theta_{it+1} \frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} = e^{\Delta\psi_{0t+1}} \Theta_{0t+1}. \tag{A7}$$

8. Country's budget constraint is a side equation under complete markets. The net export expressed in dollar terms is

$$\begin{aligned} NX_{it} = \int_0^1 \int_0^1 \Big\{ & \gamma e^{(1-\gamma)\xi_{jt}} \mathcal{E}_{0jt} P_{ijt}(\omega)\, h\left(\frac{D_{jt}P_{ijt}(\omega)}{P_{jt}}\right)\left(C_{Tjt} + X_{jt} + G_{Tjt}\right) \\ & - \gamma e^{(1-\gamma)\xi_{it}} \mathcal{E}_{0it} P_{jit}(\omega)\, h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right) \Big\} \mathrm{d}\omega \mathrm{d}j. \end{aligned} \tag{A8}$$

9. Monetary policy:
$$M_{it} = P^C_{it} C_{it}. \tag{A9}$$

A.2.2 Symmetric steady state and log-linearization

Consider symmetric steady state with zero net foreign asset positions, no government spendings and all shocks equal zero:
$$a_{Ni} = a_{Ti} = w_i = \xi_i = \psi_i = 0.$$

I assume that production subsidy eliminates monopolistic distortion $\tau = \frac{1}{\theta}$. This assumption has no effect on the first-order approximation of the equilibrium system discussed below, but is important for the welfare analysis. The symmetry implies that bilateral exchange rate between any countries is one, $\mathcal{E}_{ij} = 1$, and therefore, the prices for all products equal one as well:
$$P_i = P_{ii} = P_{ji} = P^N_i = P^C_i = 1.$$

Steady-state consumption can then be found from labor supply condition: $C_i = 1$. Combining market clearing in non-tradable and tradable sectors
$$\begin{aligned} Y_{Ni} &= C_{Ni} = (1-\eta)\, C_i \\ Y_i &= C_{Ti} + X_i = \eta C_i + \phi Y_i, \end{aligned}$$
one can solve for steady state level of labor and output: $L_i = 1$, $Y_{Ni} = 1-\eta$, and $Y_i = \frac{\eta}{1-\phi}$.

I next log-linearize the equilibrium system around the symmetric steady state. It is convenient to split the system into four blocks — prices, quantities, dynamic equations and currency choice, and solve them recursively. The time index is suppressed in static blocks to simplify the notation. Small letters denote log-deviations from the steady state, while small letters without subscript i denote the global averages, i.e. $x \equiv \int_0^1 x_i \mathrm{d}i$. I decompose bilateral exchange rates into country-specific components: $e_{ijt} = e_{it} - e_{jt}$. Such decomposition is non-unique: intuitively, in a world with N countries, there are only $N-1$ independent bilateral exchange rates. I therefore normalize the average exchange rate across countries to zero, i.e. $\int_n^1 e_{it} \mathrm{d}i = 0$. The country-specific exchange rate e_{it} can then be interpreted as an average of bilateral exchange rates against other countries.

A.2.3 Prices

The price index for non-tradable goods and consumer price index are
$$p^N_i = \lambda\,(w_i - a_{Ni}),$$
$$p^C_i = \eta p_i + (1-\eta)\, p^N_i. \tag{A10}$$

The price block in tradable sector includes marginal costs of production

$$mc_i = \phi p_i + (1-\phi)\, w_i - a_i, \tag{A11}$$

the optimal static price

$$\tilde{p}_{ji} = (1-\alpha)\,(mc_j + e_i - e_j) + \alpha p_i, \tag{A12}$$

the price indices

$$\begin{aligned} p_i^I &= \int_0^1 p_{ji} \mathrm{d}j, \\ p_i &= (1-\gamma)\, p_{ii} + \gamma p_i^I, \\ p_{ji} &= \lambda \tilde{p}_{ji} + (1-\lambda)\left(e_i - e_{k_{ji}}\right), \end{aligned} \tag{A13}$$

where k_{ji} denotes the currency choice of exporters from country j to i. For future use, define also the export price index as

$$p_i^E = \int_0^1 p_{ij} \mathrm{d}j, \tag{A14}$$

Assume that domestic firms set prices in local currency and invoicing is symmetric across countries. Combine next equations (A11)-(A13) to solve for p_i:

$$p_i = \chi e_i - \chi_0 e_0 + \chi_m m_i + \chi_{\bar{m}} m - \chi_a a_i - \chi_{\bar{a}} a, \tag{A15}$$

where μ^P and μ^D are dummy variables for exporters choosing respectively PCP and DCP and

$$\begin{aligned} \chi &= \frac{\gamma\left[\lambda(1-\alpha) + (1-\lambda)\left(\mu^P + \mu^D\right)\right]}{1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)}, \\ \chi_0 &= \frac{\gamma}{1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)}\left[\lambda(1-\alpha)\, n + (1-\lambda)\left(n\mu^P + \mu^D\right) + \frac{\lambda(1-\lambda)(1-\alpha)\gamma\phi\mu^D(1-n)}{1-\lambda(\alpha + (1-\alpha)\phi)}\right], \\ \chi_m &= \frac{\lambda(1-\gamma)(1-\alpha)(1-\phi)}{1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)}, \\ \chi_{\bar{m}} &= \frac{\lambda\gamma(1-\alpha)(1-\lambda\alpha)(1-\phi)}{[1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)]\,[1-\lambda(\alpha + (1-\alpha)\phi)]}, \\ \chi_a &= \frac{\lambda(1-\gamma)(1-\alpha)}{1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)}, \\ \chi_{\bar{a}} &= \frac{\lambda\gamma(1-\alpha)(1-\lambda\alpha)}{[1-\lambda(\alpha + (1-\gamma)(1-\alpha)\phi)]\,[1-\lambda(\alpha + (1-\alpha)\phi)]}. \end{aligned} \tag{A16}$$

Integrate across countries to obtain the global price index

$$p = (\chi n - \chi_0) e_0 + (\chi_m + \chi_{\bar{m}}) m - (\chi_a + \chi_{\bar{a}}) a, \tag{A17}$$

Finally, solve for import price index

$$
\begin{aligned}
p_i^I = & -\left[\lambda\left[(1-\alpha)(1-\phi\chi)n+((1-\alpha)\phi+\alpha)\chi_0\right]+(1-\lambda)\left(n\mu^P+\mu^D\right)\right]e_0 \\
& +\left[\lambda(1-\alpha+\alpha\chi)+(1-\lambda)\left(\mu^P+\mu^D\right)\right]e_i \\
& +\lambda\alpha\chi_m m_i+\lambda\left[(1-\alpha)(1-\phi+\phi\chi_m)+((1-\alpha)\phi+\alpha)\chi_{\bar{m}}\right]m \\
& -\lambda\alpha\chi_a a_i-\lambda\left[(1-\alpha)(\phi\chi_a+1)+\chi_{\bar{a}}(\alpha+(1-\alpha)\phi)\right]a
\end{aligned}
\tag{A18}
$$

and export price index

$$
\begin{aligned}
p_i^E = & \left[\lambda\left[(1-\alpha+\alpha\chi)n-((1-\alpha)\phi+\alpha)\chi_0\right]+(1-\lambda)\left(n\mu^P-(1-n)\mu^D\right)\right]e_0 \\
& -\left[\lambda(1-\alpha)(1-\phi\chi)+(1-\lambda)\mu^P\right]e_i \\
& +\lambda(1-\alpha)(1-\phi+\phi\chi_m)m_i+\lambda\left[\alpha\chi_m+((1-\alpha)\phi+\alpha)\chi_{\bar{m}}\right]m \\
& -\lambda(1-\alpha)(\phi\chi_a+1)a_i-\lambda\left[((1-\alpha)\phi+\alpha)\chi_{\bar{a}}+\alpha\chi_a\right]a
\end{aligned}
\tag{A19}
$$

A.2.4 Quantities

The market clearing conditions for labor and goods allow to express consumption, labor and output as functions of prices and shocks. The labor supply together with the money rule imply

$$
c_i = w_i - p_i^C = m_i - p_i^C. \tag{A20}
$$

Substitute final demand for tradables

$$
c_{Ti} = p_i^C - p_i + c_i + g_i \tag{A21}
$$

and intermediate demand for tradables

$$
x_i = mc_i + y_i - p_i \tag{A22}
$$

into the market clearing condition

$$
\begin{aligned}
y_i &= (1-\gamma)\,y_{ii}+\gamma y_i^E, \\
y_i^E &= \int_0^1 y_{ij}\mathrm{d}j, \\
y_{ii} &= -\gamma\xi_i-\theta(p_{ii}-p_i)+(1-\phi)\,c_{Ti}+\phi x_i, \\
y_{ij} &= (1-\gamma)\,\xi_j-\theta(p_{ij}-p_j)+(1-\phi)\,c_{Tj}+\phi x_j.
\end{aligned}
\tag{A23}
$$

Integrate across countries, use equation (A20) for consumption as well as equations (A11) and (A10) from the price block to solve for global production of tradable goods:

$$y = (1 + \phi)(w - p) + g - \frac{\phi}{1 - \phi} a. \tag{A24}$$

Substitute this expression back into the market clearing condition of a given country to solve for output:

$$\begin{aligned} y_i = \frac{\gamma\theta}{1 - (1-\gamma)\phi} \left[\left(p_i^I - p_i^E\right) - (p_i - p)\right] + \frac{(1-\gamma)\left(1-\phi^2\right)}{1-(1-\gamma)\phi}(w_i - p_i) + \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi}(w-p) \\ - \frac{\gamma(1-\gamma)}{1-(1-\gamma)\phi}(\xi_i - \xi) + \frac{(1-\gamma)(1-\phi)}{1-(1-\gamma)\phi} g_i + \frac{\gamma}{1-(1-\gamma)\phi} g \end{aligned} \tag{A25}$$

The total labor demand of tradable and non-tradable sectors is

$$\begin{aligned} l_i &= \eta l_{Ti} + (1-\eta) l_{Ni} \\ l_{Ti} &= mc_i + y_i - w_i \\ l_{Ni} &= y_{Ni} - a_{Ni}, \end{aligned}$$

where market clearing for non-tradable goods implies

$$y_{Ni} = c_{Ni} = p_i^C - p_i^N + c_i + g_i. \tag{A26}$$

Combine these equations together with tradable output (A25) to solve for labor:

$$l_i = (1-\eta)\left(p_i^C - (1-\eta)p_i^N\right) - (1-\eta)\eta p_i. \tag{A27}$$

The aggregate imports and exports of country i are

$$im_i = p_i^I + y_i^I, \qquad ex_i = p_i^E + y_i^E,$$

where volume of imports is $y_i^I = \int_0^1 y_{ji} \mathrm{d}j$. Use expressions for output (A25), consumption (A20) and bilateral trade flows (A23) to solve for exports

$$y_i^E = -\theta\left(p_i^E - p\right) + (1-\eta)\left(p^N - p\right) + \phi(m-p) + \left(m - p^C\right) + g + (1-\gamma)\xi - \frac{\phi}{1-\phi} a \tag{A28}$$

and imports

$$\begin{aligned} y_i^I = \frac{1-\phi}{1-(1-\gamma)\phi} \Big\{ -\theta\left(p_i^I - p_i\right) + (1-\eta)\left(p_i^N - p_i\right) + \phi(m_i - p_i) + \left(m_i - p_i^C\right) \\ + g_i + (1-\gamma)\xi_i - \frac{\phi}{1-\phi} a_i \Big\} + \frac{\gamma\phi}{1-(1-\gamma)\phi} y_i^E. \end{aligned} \tag{A29}$$

The linearized equation for net exports is

$$nx_i = ex_i - im_i + (e_i - ne_0) .$$

Substitute in expressions for exports (A28) and imports (A29) to get

$$\begin{aligned}
nx_i = {} & (e_i - ne_0) - (p_i - p) + \left[\frac{(1-\phi)\,\theta}{1-(1-\gamma)\,\phi} - 1\right]\left[\left(p_i^I - p_i\right) - \left(p_i^E - p\right)\right] \\
& - \frac{(1-\phi)}{1-(1-\gamma)\,\phi}\left\{\phi\left[(m_i - m) - (p_i - p)\right] + (1-\eta)\left[\left(p_i^N - p^N\right) - (p_i - p)\right] + \left[(m_i - m) - \left(p_i^C - p^C\right)\right]\right\} \\
& - \frac{(1-\phi)(1-\gamma)}{1-(1-\gamma)\,\phi}(\xi_i - \xi) - \frac{(1-\phi)}{1-(1-\gamma)\,\phi}(g_i - g) + \frac{\phi}{1-(1-\gamma)\,\phi}(a_i - a) .
\end{aligned} \tag{A30}$$

A.2.5 Equilibrium exchange rates

There are two types of dynamic equations in the model that pin down equilibrium exchange rates — the Euler equations and countries' budget constraints. I show in this section that results from Lemma 2 can be derived for both complete and incomplete asset markets. In all cases, exchange rate shocks are uncorrelated corr$(e_i, e_j) = 0$ for $\forall i \neq j$ and the relative volatility of exchange rates depends only on volatility of exogenous shocks $\frac{\mathbb{V}(e_0)}{\mathbb{V}(e_i)} = \rho$ for $\forall i \in (n, 1]$.

Proof of Lemma 2 When asset markets are complete, the risk-sharing condition (A7) implies

$$e_{it} - e_{0t} = (c_{it} - c_{0t}) + \left(p_{it}^C - p_{0t}^C\right) + (\psi_{it} - \psi_{0t}) .$$

Substitute in expressions for consumption (A20) to obtain

$$e_{it} - e_{0t} = (m_{it} - m_{0t}) + (\psi_{it} - \psi_{0t}) .$$

Integrate the risk-sharing condition across countries from n to 1, apply the law of large numbers for uncorrelated shocks and use normalization of exchange rates to get $e_{0t} = \tilde{m}_{0t} + \tilde{\psi}_{0t}$, where $\tilde{s}_{it}$ denotes the country-specific component of shock s_{it}. Substitute this condition back into the previous expression to get for any $i \in [0, 1]$

$$e_{it} = \tilde{m}_{it} + \tilde{\psi}_{it}. \tag{A31}$$

Thus, the second moments of exchange rates are independent from firms' currency choice. ■

Proposition A1 (Exchange rates under incomplete markets) *Suppose that the only internationally traded asset is a risk-free nominal bond denominated in arbitrary currency and that all shocks are integrated of the first order. Assume either (i) $\beta \to 1$, or (ii) all exporters in the world use either PCP, LCP or DCP. Then the second moments of exchange rates are independent from firms' currency choice.*

Proof Assume that the internationally traded bond pays in dollars, which is without loss of generality under the first-order approximation used to solve the model. The risk-sharing condition

$$\mathbb{E}_t \left\{ e^{\Delta\psi_{it+1}} \Theta_{it+1} \frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} - e^{\Delta\psi_{0t+1}} \Theta_{0t+1} \right\} = 0$$

can be log-linearized to get the UIP condition with the risk premium $\varsigma_{it} \equiv \mathbb{E}_t \Delta\psi_{it+1}$:

$$\mathbb{E}_t \left[\Delta e_{it+1} - \Delta e_{0t+1}\right] = \mathbb{E}_t \left[\left(\Delta c_{it+1} - \Delta c_{0t+1}\right) + \left(\Delta p^C_{it+1} - \Delta p^C_{0t+1}\right)\right] - \left(\varsigma_{it} - \varsigma_{0t}\right).$$

Integrate across countries from n to 1, apply the law of large numbers for uncorrelated shocks and use the normalization of exchange rates to get for any $i \in [0, 1]$

$$\mathbb{E}_t \Delta e_{it+1} = \mathbb{E}_t \Delta \tilde{m}_{it+1} - \tilde{\varsigma}_{it}. \tag{A32}$$

As explained in Itskhoki and Mukhin (2017), this condition determines future changes in the exchange rate, while its level is pinned down by the intertemporal budget constraint

$$\sum_{\tau=0}^{\infty} \beta^{\tau} NX_{it+\tau} = 0.$$

Rewrite it in log-linear form:

$$\sum_{t=0}^{\infty} \beta^t nx_{it} = 0$$

This can be decomposed into net export in the first period with sticky prices and in all other periods when prices are flexible:

$$\sum_{t=1}^{\infty} \beta^t nx_{it} + nx_{i0} = 0.$$

Expression (A30) together with price indices implies that under flexible prices the net export of country i can be written as

$$nx^{fp}_{it} = k_e \left(e_{it} - ne_{0t}\right) + k_s \left(s_{it} - ns_{0t}\right),$$

where s_{it} is the vector of shocks and (k_e, k_s) is a vector of constants independent from firms' currency choice. Combining the last two expressions, one obtains

$$\sum_{t=1}^{\infty} \beta^t \left[k_e \left(e_{it} - ne_{0t}\right) + k_s \left(s_{it} - ns_{0t}\right)\right] + nx_{i0} = 0.$$

Integrate across countries from n to 1, apply the law of large numbers and use the exchange rate normalization:

$$\sum_{t=1}^{\infty} \beta^t \left[k_e e_{it} + k_s \tilde{s}_{it}\right] + \widehat{nx}_{i0} = 0,$$

where $\widehat{nx}_{i0} \equiv nx_{i0} - \int_n^1 nx_{i0}\mathrm{d}i$. Rewrite the last equation in terms of initial values and growth rates

$$\sum_{t=1}^{\infty}\beta^t\left[k_e e_{i0} + k_s\tilde{s}_{i0} + \sum_{\tau=1}^{t}\left(k_e\Delta e_{i\tau} + k_s\Delta\tilde{s}_{i\tau}\right)\right] + \widehat{nx}_{i0} = 0,$$

change the order of summation and substitute in the UIP condition (A32):

$$\beta\left(k_e e_{i0} + k_s\tilde{s}_{i0}\right) + \beta\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[k_e\Delta\tilde{m}_{it+1} - k_e\Delta\tilde{\psi}_{it+1} + k_s\Delta\tilde{s}_{it+1}\right] + (1-\beta)\,\widehat{nx}_{i0} = 0.$$

Assume that all shocks are integrated of the first order and take the limit $\beta \to 1$ using the fact that coefficients (k_e, k_s) do not depend on β:

$$e_{it} = -\frac{k_s}{k_e}\tilde{s}_{it} - \mathbb{E}_t\sum_{\tau=0}^{\infty}\left[\Delta\tilde{m}_{it+\tau+1} - \tilde{\varsigma}_{it+\tau} + \frac{k_s}{k_e}\Delta\tilde{s}_{it+\tau+1}\right].$$

Since invoicing decisions of exporters have no effect on the coefficients in this expression, the (conditional) second moments of exchange rates are independent from firms' currency choice.

Alternatively, consider the case with symmetric invoicing across all countries. From (A15)-(A19), it follows that $p_i - p$ and $p_i^I - p_i^E$ are proportional to $\chi(e_i - ne_0)$ and

$$\left[\lambda\left((1-\alpha)\left(2-\phi\chi\right)+\alpha\chi\right) + (1-\lambda)\left(2\mu^P + \mu^D\right)\right](e_i - ne_0)$$

respectively. The expression for net exports (A30) implies then that the elasticity of nx_i with respect to $e_i - ne_0$ is the same for all countries. Since both the budget constraint and the UIP condition are the same for all countries, the elasticity of equilibrium exchange rates wrt (local) exogenous shocks is equal across countries as well. ■

A.3 Currency choice

A.3.1 Dollar pricing

Proof of Lemma 1 Suppress country indices and take the second-order approximation of the profit function at price p around the state-dependent optimal price $\tilde{p}_{ji}$:

$$\Pi(p) = \Pi(\tilde{p}_{ji}) + \Pi_p(\tilde{p}_{ji})(p - \tilde{p}_{ji}) + \frac{1}{2}\Pi_{pp}(\tilde{p}_{ji})(p-\tilde{p}_{ji})^2 + \mathcal{O}(p-\tilde{p}_{ji})^3,$$

The first term on the right hand side does not depend on currency of invoicing. From the first-order condition for optimal price, $\Pi_p(\tilde{p}_{ji}) = 0$. Finally, to the zero-order approximation, $\Pi_{pp}(\tilde{p}_{ji}) = \bar{\Pi}_{pp}(\bar{\tilde{p}}_{ji}) < 0$, where $\bar{\Pi}_{pp}(0)$ denotes the derivative in the deterministic steady state and $\bar{\tilde{p}}_{ji}$ is the corresponding optimal price. Therefore, to the second-order approximation, the currency choice problem is equivalent to minimizing $\mathbb{E}(p-\tilde{p}_{ji})^2$. Note that only the first-order approximation is required for p and

$\tilde{p}_{ji}$. In particular, the optimal preset price in currency k is $\bar{p}^k_{ji} = \mathbb{E}\left(\tilde{p}_{ji} - e_{ik}\right)$, so that ex post price is $p = \bar{p}^k_{ji} + e_{ik}$. Substitute this expression into the objective function to write the currency problem as

$$\min_k \mathbb{V}\left(\tilde{p}_{ji} + e_{ki}\right)^2, \tag{A33}$$

which completes the proof of the lemma. ■

Combining equations (A11)-(A13) and suppressing monetary and productivity shocks, we get the optimal price expressed in terms of currency k:

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)(1-\phi\chi)e_j - \alpha(1-\chi)e_i - (\alpha + (1-\alpha)\phi)\chi_0 e_0. \tag{A34}$$

It is easy to verify that the aggregate pass-through coefficients (A16) are positive and no greater than one, i.e. $0 \leq \chi, \chi_0 \leq 1$. It follows that the coefficients before e_j, e_i and e_0 are between 0 and 1 as well. Since exchange rates e_i are uncorrelated across countries, a firm is more likely to choose the currency with the higher weight in (A34). This result underlies the comparative statics analysis below.

Proof of Lemma 3 When $\alpha = \phi = 0$, we get $\tilde{p}_{ji} + e_{ki} = e_k - e_j$ and the minimum volatility is attained by setting $k = j$, i.e. exporters choose PCP. ■

Proof of Lemma 4 Expression (A16) implies that in the autarky limit $\gamma \to 0$, the pass-through coefficients are $\chi, \chi_0 \to 0$. Thus, $\tilde{p}_{ji} + e_{ki} \to e_k - (1-\alpha)e_j - \alpha e_i$ and $\mathbb{V}\left(\tilde{p}_{ji} + e_{ki}\right)^2$ is equal $2\alpha^2\sigma_e^2$ under PCP, $2(1-\alpha)^2\sigma_e^2$ under LCP and $(\rho + \alpha^2 + (1-\alpha)^2)\sigma_e^2$ under DCP. Hence, exporters choose $k = j$ when $\alpha \leq 0.5$ and $k = i$ when $\alpha \geq 0.5$. ■

Proof of Proposition 1 Consider for example the limit $\gamma, \alpha \to 1$, so that $\chi \to \mu^P + \mu^D$, $\chi_0 \to \mu^D$ and $\tilde{p}_{ji} + e_{ki} \to e_k - (1-\chi)e_i - \chi_0 e_0$. Conjecture that other firms choose DCP, so that $\mu^D = 1$. Hence, $\tilde{p}_{ji} + e_{ki} \to e_k - e_0$ and the firm finds it optimal to choose $k = 0$. The DCP equilibrium can therefore be sustained in the neighbourhood of $\gamma = \alpha = 1$ when prices are sticky.

Note that both χ and χ_0 are increasing in γ and ϕ. In addition, given χ and χ_0, the coefficient before e_j is decreasing in ϕ, while the coefficient before e_0 is increasing in ϕ. It follows that higher γ and ϕ decrease the weights of e_j and e_i and increase the weight of e_0 in (A34), which makes PCP and LCP less likely and raises the chances of DCP. The effect of α, on the other hand, is not monotonic. ■

Lemma A1 *In the flexible-price limit $\lambda \to 1$, the equilibrium exists and is generically unique. The invoicing is symmetric across small countries.*

Proof In the flexible-price limit $\lambda \to 1$, the pass-through coefficients from (A16) converge to $\chi \to \frac{\gamma}{1-(1-\gamma)\phi}$ and $\chi_0 \to \frac{\gamma n}{1-(1-\gamma)\phi}$ and do not depend on invoicing decisions of firms. The currency choice problem (A33)-(A34) then has a unique solution except for some borderline values of parameters. Finally, since coefficients before exchange rates are the same for exporters from all small economies and the volatility of exchange rates is also the same, the equilibrium invoicing is symmetric across them. ■

Proof of Proposition 5 When $n = 0$, the desired price of exporters is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big]. \tag{A35}$$

Since volatility of all exchange rates is the same when $\rho = 1$, the exporter chooses between producer and local currency based on their weights in (A35): $k = j$ when $1-\alpha \geq \alpha(1-\gamma) \Leftrightarrow \alpha \leq \frac{1}{2-\gamma}$ and $k = i$ otherwise. ■

Proof of Proposition 3 For simplicity, rewrite expression (A35) as $\tilde{p}_{ji} + e_{ki} = e_k - ae_j - be_i$. The volatility (A33) under DCP is then $(\rho + a^2 + b^2)\sigma_e^2$. Since ρ does not affect volatility under PCP and LCP, lower values of ρ unambiguously increase the chances of DCP. Note that in the limit $\phi \to 1$, we have $a = b = 0$ and under $\rho < 1$ DCP strictly dominates both PCP and LCP. ■

Proof of Proposition 4 The desired price in the flexible-price limit with $n > 0$ is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big] - \frac{\gamma(\alpha+(1-\alpha)\phi)}{1-(1-\gamma)\phi}ne_0.$$

As long as $n > 0$, choosing $k = 0$ is optimal for example in the limit $\phi \to 1$. Moreover, keeping the values of other parameters fixed, higher n increases the relative weight of e_0 in the optimal price, and therefore makes DCP more likely. ■

The proof of Proposition 2 requires a few additional lemmas. When $n = 0$ and $\rho = 1$, the currency choice of exporters is based on the following inequalities:

$$PCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\chi + \alpha\,(2-\chi) < 1, \tag{A36a}$$

$$PCP \succ DCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\,(\chi+\chi_0) + \alpha\,(1+\chi_0) < 1, \tag{A36b}$$

$$DCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)\,(1-\phi\chi_0) + \alpha\,[2-(\chi+\chi_0)] < 1. \tag{A36c}$$

where $\succ$ stays for "prefered to". I use χ^X and χ_0^X to denote the values of the corresponding pass-through coefficients in (A16) under symmetric invoicing X.

Lemma A2 *If DCP is prefered to PCP (LCP) under PCP (LCP) price index, then this ordering holds under DCP price index as well. Symmetrically, if PCP (LCP) dominates DCP under DCP price index, then this ordering holds under PCP (LCP) price index as well.*

Proof Since condition (A36b) gets tighter with χ and χ_0, and $\chi^P = \chi^D$, $\chi_0^P < \chi_0^D$, the relation $DCP \succ PCP$ under χ^P and χ_0^P implies the same ordering under χ^D and χ_0^D. Since condition (A36c) is relaxed by higher χ and χ_0 and $\chi^L < \chi^D$, $\chi_0^L < \chi_0^D$, the relation $DCP \succ LCP$ for χ^L and χ_0^L implies the same ordering for χ^D and χ_0^L. ■

Lemma A3 *It is impossible that for given parameter values, an exporter (i) chooses PCP when all others choose LCP, and (ii) chooses LCP when all others choose PCP.*

Proof Suppose that was the case. Then from (A36a) $\frac{1-\phi\chi^P}{2-\chi^P(1+\phi)} < \alpha < \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)}$. But this requires $\chi^L > \chi^P$, which can not be the case. ■

Lemma A4 *Consider a pure-strategy NE with a choice only between PCP and LCP. If the symmetric LCP equilibrium does not exist, the only possible pure-strategy NE is the symmetric PCP.*

Proof Pure-strategy equilibria can be parametrized by cdf $F(\cdot)$ for $\mu_i^P \in [0,1]$ across countries. PCP is chosen by exporter from country j to country i iff

$$(1-\alpha)\,\phi\chi_j + \alpha\,(2-\chi_i) < 1 \quad \Rightarrow \quad \mu_j < a + b\mu_i$$

for some positive constants a and b. Integrating across importers, we then derive the equilibrium condition: $\mu_i = \int_j \mathbb{I}\{\mu_j < a + b\mu_i\}\,dj$, or equivalently

$$\int_0^1 \mathbb{I}\{z < a + bx\}\,dF(z) = F(a+bx) = x$$

for any x with positive density. Suppose next that symmetric LCP equilibrium does not exist, i.e. $F(a) = 0$ is unattainable. This is possible only if $a > 1$. But then for any $x > 0$ with positive density we have $x = F(a+bx) \geq F(a) = 1$, i.e. symmetric PCP is the only PSE. ■

Proof of Proposition 2 (1) Suppose there are no symmetric equilibria for some combination of parameters. Note that since $\chi^P = \chi^D$, it follows from (A36a) that the preferences between PCP and LCP should be the same under PCP and DCP price indices. First, suppose that $PCP \succ LCP$ under DCP and PCP. Since there is no PCP equilibrium, we must have $DCP \succ PCP$ under PCP price index. But by Lemma A2, we have $DCP \succ PCP$ under DCP price index as well and hence, DCP equilibrium exists. Second, suppose that $LCP \succ PCP$ under DCP and PCP. Then from Lemma A3, we have $LCP \succ PCP$ under LCP price index. Non-existence of LCP equilibrium requires then $DCP \succ LCP$ under LCP price index. By Lemma A2, $DCP \succ LCP$ under DCP price index as well and hence, we obtain DCP equilibrium. In both cases we arrive to contradiction.

(2) First, suppose that DCP is a unique symmetric equilibrium. Then $DCP \succ LCP$ under LCP and $DCP \succ PCP$ under PCP price index. Since χ^i and χ_0^i can get only higher as one deviates from

symmetric LCP, constraint (A36c) implies that DCP dominates LCP in any PSNE. But then χ^i stays the same and χ_0^i can only increase relative to symmetric PCP and constraint (A36b) implies that DCP dominates LCP in any PSNE as well. Second, suppose that LCP is a unique symmetric equilibrium. Since χ^i and χ_0^i can only get lower as one deviates from symmetric DCP, constraint (A36c) implies that LCP dominates DCP in any PSE as well. The existence of symmetric LCP requires according to constraint (A36a) that $\alpha > \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)} > \frac{1}{2}$. This implies $\alpha > (1-\alpha)\,\phi$, so that constraint (A36a) relaxes as χ^i decreases. Therefore, there can be no PSNE with PCP. Finally, suppose that PCP is a unique symmetric NE. Since χ^i and χ_0^i can get only lower than under symmetric DCP, constraint (A36b) implies that DCP is dominated by PCP in PSNE. According to Lemma A4, there can also be no PSE with positive measure of LCP.

(3) Suppose there is market i, in which a positive mass of importers are indifferent between PCP and DCP and play mixed strategies. Take an arbitrary small share of firms pricing in the producer currency and exogenously switch their invoicing into dollars. The coefficient χ^i does not change, while χ_0^i increases. Condition (A36b) implies that the firms that were indifferent now strictly prefer DCP, while condition (A36c) implies that the share of LCP can only fall. Since firms (endogenously) switch to dollar in response to the perturbation, the initial equilibrium is not stable. Note there are no indirect effects coming from other markets: as country i is infinitely small, the changes in invoicing of its imports or exports has no impact on other countries. A symmetric argument applies for other types of mixed equilibria. ■

A.3.2 Transition

Proof of Proposition 5 It is convenient to use a slightly different notation than in other sections: two currency unions have masses n_1 and n_2 with $n \equiv n_1 + n_2$, the relative exchange rate volatility of pound is $\rho \equiv \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}$, μ_i^k denotes the share of country i imports invoiced in currency k ($\mu_i^1 + \mu_i^2 = 1$). I also define pass-through coefficients as follows: $p_i = \chi_0^i e_i - \chi_1^i e_1 - \chi_2^i e_2$. Vehicle currency 1 dominates vehicle currency 2 for exporter from j to i iff

$$(1-\alpha)\frac{\text{cov}\left(\phi p_j + e_1 - e_j, e_1 - e_2\right)}{\text{var}\left(e_1 - e_2\right)} + \alpha\frac{\text{cov}\left(p_i + e_1 - e_i, e_1 - e_2\right)}{\text{var}\left(e_1 - e_2\right)} < \frac{1}{2}.$$

Applying this formula for each bilateral trade flow, we get:

- RoW exports to RoW:

$$\left(\alpha + (1-\alpha)\,\phi\right)\chi_2^N + \left[1 - \left(\chi_1^N + \chi_2^N\right)\left(\alpha + (1-\alpha)\,\phi\right)\right]\rho < \frac{1}{2},$$

- RoW exports to currency unions:

$$(1-\alpha)\,\phi\chi_2^N+\alpha\chi_2^1+\left[(1-\alpha)\left(1-\phi\chi_1^N-\phi\chi_2^N\right)+\alpha\left(\chi_0^1-\chi_1^1-\chi_2^1\right)\right]\rho<\frac{1}{2},$$
$$(1-\alpha)\,\phi\chi_2^N+\alpha\left(1+\chi_2^2-\chi_0^2\right)+\left[(1-\alpha)\left(1-\phi\chi_1^N-\phi\chi_2^N\right)+\alpha\left(\chi_0^2-\chi_1^2-\chi_2^2\right)\right]\rho<\frac{1}{2},$$

- Currency union exporting to RoW:

$$(1-\alpha)\,\phi\chi_2^1+\alpha\chi_2^N+\left[(1-\alpha)\,\phi\left(\chi_0^1-\chi_1^1-\chi_2^1\right)+\alpha\left(1-\chi_1^N-\chi_2^N\right)\right]\rho<\frac{1}{2},$$
$$(1-\alpha)\left(1+\phi\chi_2^2-\phi\chi_0^2\right)+\alpha\chi_2^N+\left[(1-\alpha)\,\phi\left(\chi_0^2-\chi_1^2-\chi_2^2\right)+\alpha\left(1-\chi_1^N-\chi_2^N\right)\right]\rho<\frac{1}{2},$$

- One currency union exporting to the other:

$$(1-\alpha)\,\phi\chi_2^1+\alpha\left(1+\chi_2^2-\chi_0^2\right)+\left[(1-\alpha)\,\phi\left(\chi_0^1-\chi_1^1-\chi_2^1\right)+\alpha\left(\chi_0^2-\chi_1^2-\chi_2^2\right)\right]\rho<\frac{1}{2},$$
$$(1-\alpha)\left(1+\phi\chi_2^2-\phi\chi_0^2\right)+\alpha\chi_2^1+\left[(1-\alpha)\,\phi\left(\chi_0^2-\chi_1^2-\chi_2^2\right)+\alpha\left(\chi_0^1-\chi_1^1-\chi_2^1\right)\right]\rho<\frac{1}{2}.$$

(1) Parameter ρ is present only in the currency choice (CC) block (not in price index):

$$\begin{aligned}(1-\alpha)&\left[\phi\chi_2^j+\left(1-\phi\chi_1^j-\phi\chi_2^j\right)\rho-\left(1-\phi\chi_0^j\right)\frac{\text{cov}\left(e_j,e_1-e_2\right)}{\text{var}\left(e_1-e_2\right)}\right]\\&+\alpha\left[\chi_2^i+\left(1-\chi_1^i-\chi_2^i\right)\rho-\left(1-\chi_0^i\right)\frac{\text{cov}\left(e_i,e_1-e_2\right)}{\text{var}\left(e_1-e_2\right)}\right]<\frac{1}{2}.\end{aligned}$$

The derivative of each term with respect to ρ is clearly positive for all countries except for country 1, for which it is proportional to $\chi_0^1-\chi_1^1-\chi_2^1$. This term, however, is non-negative as well:

$$\gamma\lambda\,(1-\alpha)\,(1-n)\left[\frac{\lambda\,(1-\alpha)\,(1-\phi)+(1-\lambda)\,(1-\gamma\phi)}{1-\lambda\,(\alpha+(1-\alpha)\,\phi)}\right].$$

Thus, as ρ goes up, all constraints become more binding and everything else equal, can only decrease the use of the pound. Hence, μ_i^1 falls and μ_i^2 rises, which leaves χ_0^i unaffected, decreases χ_1^i and increases χ_2^i. This tightens the constraint for currency 1 even further in a monotonic way.

Consider next an increase in n_2, assuming that n_1+n_2 remains unchanged. Country sizes n_i are present only in price indices, but not directly in the currency choice inequalities. The second part of the proposition (proven below) implies that the share of dollar denominated imports from RoW to the first country is not smaller than the one to the second country. From the inequalities for the trade flows between the currency unions we get $\mu_1^1-\mu_2^1\geq n_1-n_1=0$. This inequality ensures that for a given currency choice, χ_1^i is monotonic in n_1 and χ_2^i is monotonic in n_2. This implies χ_1^i decreases and χ_2^i increases in n_2. The currency choice inequalities then tighten with n_2. The argument from above

(a) Changes in volatility (b) Bounds on transition path

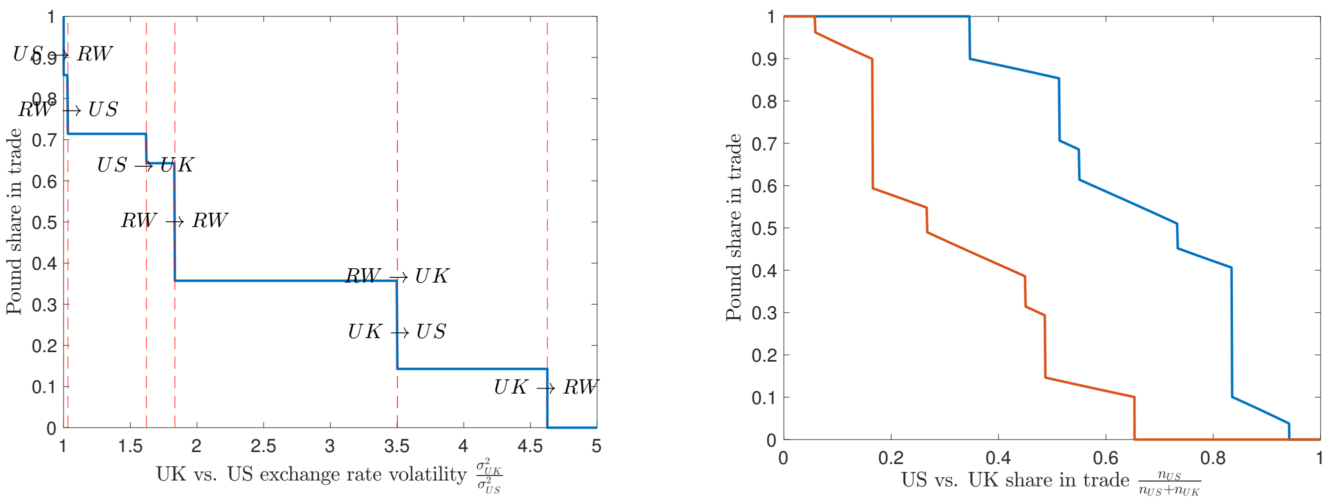


Figure A2: Transition from pound to dollar

Note: figure (a) shows transition from the pound to the dollar as the relative volatility of shocks in the U.K. goes up, while figure (b) shows lower and upper bounds of transition paths driven by changes in relative sizes of the economies. The parameter values are: $\gamma = 0.6$, $\alpha = 0.5$, $\phi = 0.5$, $\lambda = 0.5$ and $n_{US} = n_{UK} = 0.25$.

shows that endogenous change in invoicing patterns amplifies the fall in the global share of the pound.

(2) Consider an increase in n_2, which leaves n unchanged. First, note that price index for any country consists of three terms:

$$p_i \propto \lambda\gamma(1-\alpha)\phi\int_j p_j dj + \lambda\gamma(1-\alpha)\int(e_i - e_j)\,dj + (1-\lambda)\gamma\left[e_i - \mu_i^1 e_1 - \mu_i^2 e_2\right]$$

The first term is the same for all countries, while the second one does not depend on currency of invoicing. The last term, however, implies that in the initial equilibrium with all global trade denominated in currency 1, μ_i^2 is positive only for $i = 2$. Therefore, χ_2^j is higher and χ_1^j is lower for country 2. The currency choice inequalities from above imply then $T(b) \le T(c)$, $T(e) \le T(f)$ and $T(a) \le T(c)$, $T(d) \le T(g)$. This in turn implies $\chi_2^2 \ge \chi_2^j$ for any j, which confirms that the previous inequalities hold and the ordering of switches is correct. The symmetric argument can be made for country 1 with higher χ_1^j and lower χ_2^j implying $T(c) \le T(f)$, $T(b) \le T(e)$ and $T(c) \le T(g)$, $T(a) \le T(d)$. The comparative statics for ρ can be made in the similar way: the derivative of the LHS of currency choice inequality with respect to ρ is the same for all countries, so that only levels of χ_k^j matter. ■

A.3.3 Extensions

Proposition A2 *In a model with exogenous monetary shocks $\{m_i\}$,*

1. *if $\lambda \to 1$, $\rho < 1$, DCP is the only possible equilibrium in the limit $\sigma_m^2 \to \infty$,*
2. *if $n = 0$, a proportional increase in volatility of $\{m_i\}$ in all countries expends the DCP region.*

Proof Substitute the aggregate price index (A15) into the desired price (A34) to obtain

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)\left[(1-\phi\chi)\, e_j - (1-\phi+\phi\chi_m)\, m_j\right] \\ - \alpha\left[(1-\chi)\, e_i - \chi_m m_i\right] - (\alpha + (1-\alpha)\,\phi)\,(\chi_0 e_0 - \chi_{\bar{m}} n m_0)\,.$$

Consider first the limiting case when monetary shocks dominate any other shocks in the economy, and according to (A31) and (A.2.5), equilibrium exchange rate is $e_i \approx m_i$. This implies

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)\,\phi\,(1-\chi-\chi_m)\, e_j - \alpha\,(1-\chi-\chi_m)\, e_i - (\alpha + (1-\alpha)\,\phi)\,(\chi_0 - \chi_{\bar{m}} n)\, e_0,$$

where

$$1 - \chi - \chi_m = \frac{(1-\lambda)\left(1-\gamma\left(\mu^P+\mu^D\right)\right)}{1-\lambda\,(\alpha+(1-\gamma)\,(1-\alpha)\,\phi)} \geq 0,$$

$$\chi_0 - \chi_{\bar{m}} n = \frac{\gamma\,(1-\lambda)}{1-\lambda\,(\alpha+(1-\gamma)\,(1-\alpha)\,\phi)}\left[\left(n\mu^P+\mu^D\right) + \frac{\lambda\,(1-\alpha)\,\phi\left[n+\gamma\,(1-n)\,\mu^D\right]}{1-\lambda\,(\alpha+(1-\alpha)\,\phi)}\right] \geq 0.$$

In the flexible price limit $\lambda \to 1$, both coefficients converge to zero and $\tilde{p}_{ji} + e_{ki} = e_k$. While firms are indifferent between all currencies when $\rho = 1$, an arbitrary small volatility advantage is sufficient to guarantee that DCP is the only equilibrium for any values of other parameters. More generally, cov $(m_i, e_i) > 0$, so that higher volatility of monetary shocks decreases the effective weight of producer and local currencies in the optimal price and makes exporters more willing to choose DCP. ■

Define the global currency pricing (GCP) equilibrium as the one in which all firms in the world (including domestic ones) use dollars for invoicing. In contrast, in DCP equilibrium only exporters price in dollars, while domestic firms use local currency.

Proposition A3 *Assume that domestic firms optimally choose the currency of invoicing and $n = 0$. Then*

1. *in the flexible price limit $\lambda \to 1$, the region of GCP is the subset of DCP, is non-empty as long as $\rho < 1$ and is increasing in γ, ϕ and α,*
2. *in the limit of fully rigid prices $\lambda \to 0$, the region of DCP is a subset of GCP.*

Proof As before, the import price index is

$$p_i^I = \lambda\left[(1-\alpha)\,(\phi p + e_i) + \alpha p_i\right] + (1-\lambda)\left[\left(\mu^P+\mu^D\right) e_i - \mu^D e_0\right].$$

Denote the currency choice of domestic firms with $\hat{\mu}$. Note that PCP and LCP coincide for domestic firms and therefore it is sufficient to focus on $\hat{\mu}^D$. The price indices are therefore

$$p_i^D = \lambda\left[(1-\alpha)\,\phi + \alpha\right] p_i + (1-\lambda)\,\hat{\mu}^D\,(e_i - e_0)\,,$$

$$p_i = \frac{\gamma\lambda(1-\alpha) + \gamma(1-\lambda)\left(\mu^P + \mu^D\right) + (1-\gamma)(1-\lambda)\hat{\mu}^D}{1-\lambda(\alpha+(1-\alpha)(1-\gamma)\phi)} e_i - \frac{(1-\lambda)\left[\gamma\mu^D + (1-\gamma)\hat{\mu}^D\right]}{1-\lambda(\alpha+(1-\alpha)\phi)} e_0.$$

In the flexible price limit, the currency of invoicing of both exporters and domestic firms has no effect on equilibrium prices, so that the currency choice of exporters remain the same as in the baseline model. The volatility of the optimal price of local firms expressed in domestic currency and dollars is

$$V^{PCP/LCP} = \left[1 - \frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi}\right]^2, \qquad V^{DCP} = \rho + \left[\frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi}\right]^2.$$

It follows that local firms choose DCP if $2\frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi} < 1-\rho$, which is more likely when ρ is low and γ, ϕ and α are high.

Consider next the case with $\lambda = 0$. Assuming that exporters choose DCP, the volatility of the optimal price of local firms in different currencies is

$$\begin{aligned} V^{PCP/LCP} &= \left[1-(1-\alpha)(1-\phi\chi)-\alpha(1-\chi)\right]^2 + (\alpha+(1-\alpha)\phi)^2\chi_0^2\rho, \\ V^{DCP} &= \left[(1-\alpha)(1-\phi\chi)+\alpha(1-\chi)\right]^2 + \left[1-(\alpha+(1-\alpha)\phi)\chi_0\right]^2\rho. \end{aligned}$$

The GCP equilibrium exists if $\alpha + (1-\alpha)\phi > \frac{1}{2}$, which is always satisfied for $\alpha > 0.5$ or $\phi > 0.5$. At the same time, DCP equilibrium requires

$$\begin{aligned} (1-\alpha)\gamma\phi + \alpha(1+\rho\gamma) &> \frac{1+\rho}{2}, \\ (1-\alpha)(1-\gamma\phi)\rho + \alpha(1-\gamma)(1+\rho) &< \frac{1+\rho}{2}. \end{aligned}$$

I next argue that the last two conditions imply that $\alpha + (1-\alpha)\phi > \frac{1}{2}$ is satisfied as well. Prove by contradiction. Condition for GCP does not depend on γ, while conditions for DCP relax as γ becomes larger. Therefore, take $\gamma = 1$

$$(1-\alpha)\phi > \left(\frac{1}{2}-\alpha\right)(1+\rho),$$

$$(1-\alpha)\phi > \left(\frac{1}{2}-\alpha\right) - \frac{1}{2\rho}.$$

If $\alpha > 0.5$, the GCP equilibrium exists and we arrive to contradiction. If $\alpha < 0.5$, then conditions are relaxed for $\rho = 1$

$$(1-\alpha)\phi > (1-2\alpha),$$

$$(1-\alpha)\phi > -\alpha.$$

The second condition is always satisfied, while the first one implies $\alpha + (1-\alpha)\phi > 1-\alpha > 0.5$, so GCP equilibrium exists and we again arrive to contradiction. ■

A.4 Invoicing in terms of currency baskets

This section extends the baseline model by allowing firms to set prices in terms of an arbitrary basket of currencies, e.g. Apple sells iPhone 7 in Germany for 500 dollars plus 300 euros plus 200 Swiss francs. Importantly, this is not the same as using lotteries (mixed strategies) when choosing the currency of invoicing since the (ex-post) pass-through of exchange rate shocks is either zero or one in this case. Another wrong interpretation is that a firm sells some fraction of its products in one currency and some fraction in another currency. If this is the same product, customers will only buy with the lowest ex-post price. If the products are different, the firm will have suboptimal pricing for each of them.

Lemma A5 *Suppose that prices are set in terms of basket of arbitrary currencies. Then exporters can achieve the optimal pass-through of exchange rate shocks in every state of the world.*

Proof It is sufficient to check that the sum of exchange rate weights in the optimal price is one. Since bilateral exchange rates remain unchanged if all $\{e_i\}$ increase by the same constant, the sum of exchange rate weights in p_i is zero and the sum of weights in $\tilde{p}_{ji} + e_{ki}$ is one. ■

Thus, even if prices of firms are fully rigid, exporters can construct such invoicing baskets that their prices will move optimally with exchange rates. While this is a strong result, it is important to realize what it does not say:

- While the weights of all exchange rates are positive in the baseline model, in a more general environment, they might be negative. From an economic perspective, this means that firms are allowed to make transfers to the customers, e.g. a client pays 1200 dollars for the good and gets back 200 euros as a discount.
- While the pass-through of exchange rates into prices is optimal, the pass-through of other shocks is not. In particular, the pass-through is zero for idiosyncratic productivity shocks and even for the aggregate shocks as long as they are uncorrelated with movements in exchange rates.
- As long as domestic firms are obliged to set prices in local currency, the prices of importers are different from the ones under flexible prices.

Proposition A4 *Assume $m_i = a_{Ti} = 0$, domestic firms set prices in local currency, while exporters can use arbitrary baskets of currencies for invoicing. Then*

1. *equilibrium is always unique,*
2. *the share of the dollar in international trade cannot be higher than n,*
3. *relative dollar volatility ρ has no effect on dollar use in international trade,*
4. *high price rigidity $1 - \lambda$ decreases the use of the dollar and stimulates LCP,*
5. *the share of the dollar increases in γ and ϕ and might be not monotone in α.*

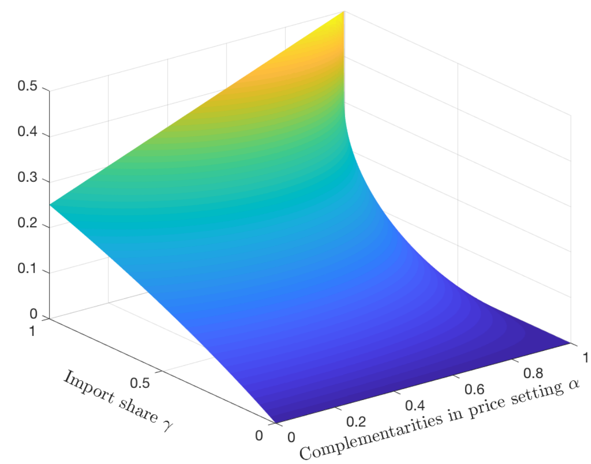


Figure A3: Dollar share in trade

Note: the figure shows the share of dollars in trade between non-U.S. countries when exporters can use arbitrary baskets of currencies. Parameter values: $\rho = 0.5$, $\phi = 0.5$, $\lambda = 0.5$, $n = 0.5$.

Proof The importers enjoy the optimal state contingent prices:

$$p_i = \frac{\gamma (1-\alpha)}{1 - \gamma\alpha - \lambda (1-\gamma)(\alpha + (1-\alpha)\phi)} (e_i - n e_0) .$$

All results except for the second one follow immediately from expressions for p_i and $\tilde{p}_{ji}$. Let $\bar{s}_i$ denote the share of local currency in the optimal basket of exporters. Since $\chi \leq 1$ and $\chi_0 = n\chi \leq n$, the dollar share in trade between third countries is $[\alpha + (1-\alpha)\phi]\,\chi_0 \leq n$ and the dollar share in international trade is

$$\frac{1}{1-n^2}\left[(1-n)^2 \bar{s}_0 + n(1-n)(\bar{s}_i + \bar{s}_0) + n(1-n)(\bar{s}_j + \bar{s}_0)\right] \leq n.$$

■

The intuition for results 1 and 3 is straightforward: the optimal share of the dollar depends on the fraction of suppliers and competitors from the U.S., which cannot be higher than n (see Figure A3). Thus, the model with complete basket cannot match empirical fact that the share of DCP is much higher than the share of the U.S. in international trade. In addition, the model predicts that the relative volatility of dollar ρ plays no role because it has zero effect on the optimal pass-through of exchange rate shocks. Also, in contrast to the baseline model, higher price rigidity actually reduces the international use of the dollar. This is because lower frequency of price adjustment has a direct effect only on domestic producers, while the effect on importers is indirect and decreases the pass-through of exchange rate shocks. Finally, the comparative statics with respect to import share γ and intermediate share ϕ remains the same as in the baseline model since their main effect comes from the weights of currencies in the optimal basket.

A.5 Alternative models of sticky prices

A.5.1 Calvo pricing

This section shows that the main results about currency choice from Section 3 hold under staggered pricing. As before, I abstract from monetary and productivity shocks. In addition, to simplify the analysis, the exchange rates are assumed to follow random walk, which requires that the process for ψ_{it} is also a random walk.

Assume that prices of all firms are set a la Calvo with the probability of adjustment $1-\lambda$ (note the difference in notation from the baseline model). Start with exporter from country j to country i. Since exchange rates follow random walk, the first order approximation to the adjusted price does not depend on the currency of invoicing (see Gopinath, Itskhoki, and Rigobon 2010) and can be written in destination currency as

$$\hat{p}_{jit} = (1-\beta\lambda)\,\tilde{p}_{jit} + \beta\lambda\mathbb{E}_t\hat{p}_{jit+1},$$

where the optimal static price $\tilde{p}_{jit}$ is the same as in the baseline model. The import price index from j to i aggregates across adjusting and non-adjusting firms

$$p_{jit} = (1-\lambda)\,\hat{p}_{jit} + \lambda\left(p_{jit-1} + \mu^P\Delta e_{ijt} + \mu^D\Delta e_{i0t}\right).$$

The standard manipulations lead to the NKPC:

$$\pi_{it} = \frac{(1-\beta\lambda)\,(1-\lambda)}{\lambda}\,(\tilde{p}_{it}-p_{it}) + \beta\mathbb{E}_t\pi_{it+1} + \gamma\left[\mu^P\left(\Delta e_{it} - n\Delta e_{0t}\right) + \mu^D\left(\Delta e_{it} - \Delta e_{0t}\right)\right],$$

where $\tilde{p}_{it} = (1-\gamma)\,(1-\alpha)\,\phi p_{it} + \gamma\,(1-\alpha)\,(\phi p_t + e_{it} - ne_{0t}) + \alpha p_{it}$.

I solve for p_{it} in two steps. First, denote deviations of local variables from global averages with bars:

$$-\beta\mathbb{E}_t\bar{p}_{it+1} + \left[1+\beta+(1-(1-\gamma)\,\phi)\,\kappa\right]\bar{p}_{it} - \bar{p}_{it-1} = \kappa\gamma\bar{e}_{it} + \gamma\left(\mu^P+\mu^D\right)\Delta\bar{e}_{it},$$

where $\kappa \equiv \frac{(1-\beta\lambda)(1-\lambda)(1-\alpha)}{\lambda}$ and $\bar{e}_{it} \equiv e_{it} - ne_{0t}$. Rewrite it in terms of lag operator $\mathbb{L}$ and factorize applying Vieta's formula:

$$-\left[\beta\mathbb{L}^{-1} - \left(1+\beta+(1-(1-\gamma)\phi)\,\kappa\right) + \mathbb{L}\right] = -\left(1-\beta\varphi\mathbb{L}^{-1}\right)\left(1-\varphi^{-1}\mathbb{L}^{-1}\right)\mathbb{L},$$

$$\varphi = \frac{1+\beta+\varsigma\kappa-\sqrt{(1+\beta+\varsigma\kappa)^2-4\beta}}{2\beta} = \frac{1+\beta+\varsigma\kappa-\sqrt{(1-\beta+\varsigma\kappa)^2+4\beta\kappa\varsigma}}{2\beta} > 0,$$

where $\varsigma \equiv 1-(1-\gamma)\,\phi$ and $\varphi\in(0,1)$. Substitute solution back into the difference equation:

$$\bar{p}_{it} = \varphi\bar{p}_{it-1} + \varphi\mathbb{E}_t\sum_{\tau=0}^{\infty}(\beta\varphi)^{\tau}\left\{\kappa\gamma\bar{e}_{it+\tau} + \gamma\left(\mu^P+\mu^D\right)\Delta\bar{e}_{it+\tau}\right\}.$$

Since exchange rates follow a random walk, we get

$$\bar{p}_{it} = \varphi \bar{p}_{it-1} + \frac{\varphi \kappa}{1-\beta\varphi} \gamma \bar{e}_{it} + \gamma \varphi \left(\mu^P + \mu^D\right) \Delta \bar{e}_{it}.$$

Second, integrate across countries to get the second-order difference equation for global price index:

$$-\beta \mathbb{E}_t p_{t+1} + \left[1 + \beta + (1-\phi)\kappa\right] p_t - p_{t-1} = -\gamma \mu^D (1-n) \Delta e_{0t}.$$

Using the same steps as above, obtain solution

$$p_t = \hat{\varphi} p_{t-1} - \gamma \hat{\varphi} \mu^D (1-n) \Delta e_{0t},$$

$$\hat{\varphi} = \frac{1 + \beta + \hat{\varsigma}\kappa - \sqrt{(1+\beta+\hat{\varsigma}\kappa)^2 - 4\beta}}{2\beta}, \qquad \hat{\varsigma} \equiv 1 - \phi.$$

Finally, back out dynamics of country i price index from $p_{it} = \bar{p}_{it} + p_t$.

To solve the currency choice problem, consider without loss of generality the case when initial values of all shocks are zero and the optimal preset prices in any currency is zero as well. The ex post price in period t conditional on non-adjustment is therefore e_{ikt} when currency k is used for invoicing. The second-order approximation to the currency choice problem of exporter from j to i is

$$\min_k \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\lambda)^t \left(\tilde{p}_{jit} + e_{kit}\right)^2.$$

Note that the interpretation that a firm chooses currency k to mimic dynamics of the optimal invoicing basket is still valid. It also follows that exporters prefer currency k to currency l iff

$$\sum_{t=0}^{\infty} (\beta\lambda)^t \mathbb{E}_0 \left(\tilde{p}_{jit} - e_{ikt}\right)^2 < \sum_{t=0}^{\infty} (\beta\lambda)^t \mathbb{E}_0 \left(\tilde{p}_{jit} - e_{ilt}\right)^2.$$

Following the steps from Gopinath, Itskhoki, and Rigobon (2010), the inequality can be rewritten as

$$(1-\beta\lambda) \sum_{t=0}^{\infty} (\beta\lambda)^t \frac{\text{cov}\left(\tilde{p}_{jit}^k, \Delta e_{kl0}\right)}{\text{var}\left(\Delta e_{kl0}\right)} < \frac{1}{2},$$

or after substituting the optimal price as

$$(1-\beta\lambda) \sum_{t=0}^{\infty} (\beta\lambda)^t \frac{\text{cov}\left[(1-\alpha)(\phi p_{jt} - e_{jt}) + \alpha (p_{it} - e_{it}) + e_{kt}, \Delta e_{kl0}\right]}{\text{var}\left(\Delta e_{kl0}\right)} < \frac{1}{2}$$

To find covariance terms, I normalize volatilities of non-dollar exchange rates to one and the volatility of dollar to ρ, and use the Yule-Walker equations to estimate the autocovariance function:

$$\begin{aligned}
\text{cov}\left(\bar{p}_{it}, \Delta e_{i0}\right) &= \varphi \text{cov}\left(\bar{p}_{it-1}, \Delta e_{i0}\right) + \frac{\varphi\gamma\kappa}{1-\beta\varphi}\text{cov}\left(e_{it}, \Delta e_{i0}\right) + \gamma\varphi\left(\mu^P + \mu^D\right)\text{cov}\left(\Delta e_{it}, \Delta e_{i0}\right), \\
\text{cov}\left(\bar{p}_{it}, \Delta e_{00}\right) &= \varphi \text{cov}\left(\bar{p}_{it-1}, \Delta e_{00}\right) - \frac{\varphi\gamma\kappa n}{1-\beta\varphi}\text{cov}\left(e_{0t}, \Delta e_{00}\right) - \gamma\varphi n\left(\mu^P + \mu^D\right)\text{cov}\left(\Delta e_{0t}, \Delta e_{00}\right), \\
\text{cov}\left(p_t, \Delta e_{00}\right) &= \hat{\varphi}\text{cov}\left(p_{t-1}, \Delta e_{00}\right) - \gamma\hat{\varphi}\mu^D\left(1-n\right)\text{cov}\left(e_{0t}, \Delta e_{00}\right).
\end{aligned}$$

The resulting IRFs are

$$v_{it} \equiv \text{cov}\left(p_{it}, \Delta e_{i0}\right) = \gamma\varphi^{t+1}\left[\frac{\kappa}{1-\beta\varphi} + \left(\mu^P + \mu^D\right)\right] + \frac{1-\varphi^t}{1-\varphi}\frac{\gamma\varphi\kappa}{1-\beta\varphi},$$

$$v_{0t} \equiv \text{cov}\left(p_{it}, \Delta e_{00}\right) = -\gamma\varphi^{t+1}\left[\frac{\kappa n}{1-\beta\varphi} + n\left(\mu^P + \mu^D\right)\right]\rho - \frac{1-\varphi^t}{1-\varphi}\frac{\gamma\varphi\kappa n}{1-\beta\varphi}\rho - \gamma\hat{\varphi}^{t+1}\mu^D\left(1-n\right)\rho,$$

and zero for all other exchange rates. Three inequalities determine invoicing decisions of firms:

$$\begin{aligned}
V^{PCP} < V^{LCP} \quad &\Leftrightarrow \quad \left[\left(1-\alpha\right)\phi - \alpha\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] < 1 - 2\alpha, \\
V^{DCP} < V^{PCP} \quad &\Leftrightarrow \quad \left[\alpha\rho n + \left(1-\alpha\right)\phi\left(1+\rho n\right)\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] \\
&\qquad + \left(\alpha + \left(1-\alpha\right)\phi\right)\frac{\gamma\hat{\varphi}\left(1-\beta\lambda\right)}{1-\beta\lambda\hat{\varphi}}\rho\mu^D\left(1-n\right) > \frac{1}{2}\left(1+\rho\right) - \alpha, \\
V^{DCP} < V^{LCP} \quad &\Leftrightarrow \quad \left[\alpha\left(1+\rho n\right) + \left(1-\alpha\right)\phi\rho n\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] \\
&\qquad + \left(\alpha + \left(1-\alpha\right)\phi\right)\frac{\gamma\hat{\varphi}\left(1-\beta\lambda\right)}{1-\beta\lambda\hat{\varphi}}\rho\mu^D\left(1-n\right) > \alpha - \frac{1}{2}\left(1-\rho\right).
\end{aligned}$$

Proposition A5 *All results about the currency choice from the benchmark model remain true in the model with staggered pricing:*

1. *there can be no DCP equilibrium in the closed economy limit $\gamma \to 0$,*
2. *there can be no DCP equilibrium in the limit $\lambda \to 0$, $n = 0$, $\rho = 1$,*
3. *the DCP region is increasing in ρ for $\lambda \to 0$, $n = 0$,*
4. *the DCP region is increasing in n for $\lambda \to 0$,*
5. *the DCP region is non-empty when prices are sticky $\lambda > 0$.*

Proof

1. In the limit $\gamma \to 0$, the processes for $\bar{p}_{it}$ and p_t have the same AR root, $\varphi \to \hat{\varphi} > 0$. Therefore,

the inequalities reduce to

$$\alpha < \frac{1}{2}, \qquad \alpha > \frac{1}{2}\left(1+\rho\right), \qquad \alpha < \frac{1}{2}\left(1-\rho\right).$$

The last two expressions imply there are no values of α, for which DCP dominates both PCP and LCP. From the first inequality, the equilibrium invoicing is PCP if $\alpha < \frac{1}{2}$ and LCP if $\alpha > \frac{1}{2}$.

2. In the limit $\lambda \to 0$, we obtain $\kappa \to \infty$, $\varphi, \hat{\varphi} \to 0$, $\kappa\varphi \to \frac{1}{1-(1-\gamma)\phi}$ and $\kappa\hat{\varphi} \to \frac{1}{1-\phi}$. Use conditions $n = 0$ and $\rho = 1$ and take the limit in the inequalities:

$$\frac{\gamma\left[\left(1-\alpha\right)\phi-\alpha\right]}{1-\left(1-\gamma\right)\phi} < 1-2\alpha \quad \Rightarrow \quad \left(1-2\alpha+\gamma\alpha\right)\left(1-\phi\right) > 0,$$

$$\frac{\gamma\left(1-\alpha\right)\phi}{1-\left(1-\gamma\right)\phi} > 1-\alpha \quad \Rightarrow \quad \left(1-\alpha\right)\left(1-\phi\right) < 0,$$

$$\frac{\gamma\alpha}{1-\left(1-\gamma\right)\phi} > \alpha \quad \Rightarrow \quad \alpha\left(1-\gamma\right)\left(1-\phi\right) < 0.$$

Thus, both PCP and LCP strictly dominate DCP. The only two points, for which firms are indifferent between three options are $\alpha = \gamma = 1$ and $\phi = 1$ as in the baseline model.

3. Note that κ, φ and $\hat{\varphi}$ do not depend on ρ. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt ρ is

$$\left(\alpha+\left(1-\alpha\right)\phi\right)\frac{\gamma\varphi n}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^{P}+\mu^{D}\right)+\frac{\kappa}{1-\beta\varphi}\right]+\left(\alpha+\left(1-\alpha\right)\phi\right)\frac{\gamma\hat{\varphi}\left(1-\beta\lambda\right)}{1-\beta\lambda\hat{\varphi}}\mu^{D}\left(1-n\right)-\frac{1}{2},$$

which is always negative for $\lambda \to 1$ and $n = 0$.

4. Note that κ, φ and $\hat{\varphi}$ do not depend on n. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt n is

$$\left(\alpha+\left(1-\alpha\right)\phi\right)\frac{\gamma\varphi\rho}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\mu^{P}+\frac{\kappa}{1-\beta\varphi}\right]-\left(\alpha+\left(1-\alpha\right)\phi\right)\gamma\rho\left[\frac{\hat{\varphi}}{1-\beta\lambda\hat{\varphi}}-\frac{\varphi}{1-\beta\lambda\varphi}\right]\left(1-\beta\lambda\right)\mu^{D},$$

where $\hat{\varphi} > \varphi$. The derivative is positive in the flexible price limit.

5. Suppose $n = 0$ and $\rho = 1$. Take the limit $\alpha, \gamma \to 1$, which implies $\kappa \to 0$, $\varphi, \hat{\varphi} \to 1$ and show that DCP equilibrium always exists for $\mu^{D} = 1$.

■

A.5.2 Rotemberg pricing

I argue next that the results from the baseline model remain robust under a state-dependent price adjustment, i.e. to the first-order approximation, the same equilibrium arises under Rotemberg pricing. To simplify the notation, I suppress below the indices of exporters' origin and destination.

There are two steps in firm's optimization. In the second one, after the shocks are realized, a firm decides how much to adjust its prices. Taking the second-order approximation to the (static) profit function with quadratic costs of price adjustment, the problem of the firm can be written as

$$\min_{p} \left\{ \varphi \left(p - \tilde{p}\right)^2 + \left(p - p_0^k\right)^2 \right\},$$

where $\varphi < 0$ is a constant determined at the point of approximation, $\tilde{p}$ is the optimal price in a given state of the world, p_0^k is the value of the preset price, which depends on the currency of invoicing k. The first-order condition implies then that firms choose a price as a weighted average of the optimal price and the preset price $p = \omega \tilde{p} + (1 - \omega)\, p_0^k$, where $\omega = \frac{\varphi}{1+\varphi}$. Therefore,

$$p - \tilde{p} = (1 - \omega) \left(p_0^k - \tilde{p}\right), \qquad p - p_0^k = -\omega \left(p_0^k - \tilde{p}\right),$$

and hence, the profit function is proportional to $\left(p_0^k - \tilde{p}\right)^2$. The first period problem of the firm is to choose the currency of invoicing, which to the second-order of approximation is equivalent to

$$\max_{k} \mathbb{E}\, \Pi \left(p^k\right) \quad \Leftrightarrow \quad \min_{k} \mathbb{E} \left(p_0^k - \tilde{p}\right)^2.$$

Thus, the currency choice problem is isomorphic to the one in the benchmark case.

A.5.3 Menu cost model

The baseline model predictions remain also robust in a model with menu costs. To simplify, I assume as before that firms can preset prices each period before the realization of shocks, but can now pay some fixed menu costs to update prices after the uncertainty is resolved. I use the second-order approximation to firm's profit function and the first-order approximation for price indices (see appendix in Gopinath and Itskhoki (2010) for the proof that such approximation is consistent). In addition to the aggregate exchange rate shocks, firms also experience idiosyncratic productivity shocks, which according to the previous studies, account for most price adjustments (see e.g. Golosov and Lucas 2007). As before, I abstract from the aggregate monetary and productivity shocks.

I solve the model numerically using the following algorithm. I first guess a (linear) price function $p_i = p(e_i, e_0)$ for a given currency of invoicing. I then estimate deviation of producer's ex-post price from the optimal level $\tilde{p}_{ji}$ in each state of the world and solve for the price adjustment decision. Integrating across both idiosyncratic productivity shocks and exchange rates e_i, I then update function $p(\cdot, \cdot)$ and iterate this procedure until convergence. Finally, I compute expected profits of a given exporter under the alternative invoicing regimes and check whether conjectured currency choice can be sustained in equilibrium. To implement this algorithm, I use a grid with 31 points for exchange rates and 51 points for idiosyncratic shocks. Following Gopinath and Itskhoki (2010), I calibrate the standard deviation of productivity shocks to be five times larger than the standard deviation of exchange rates.

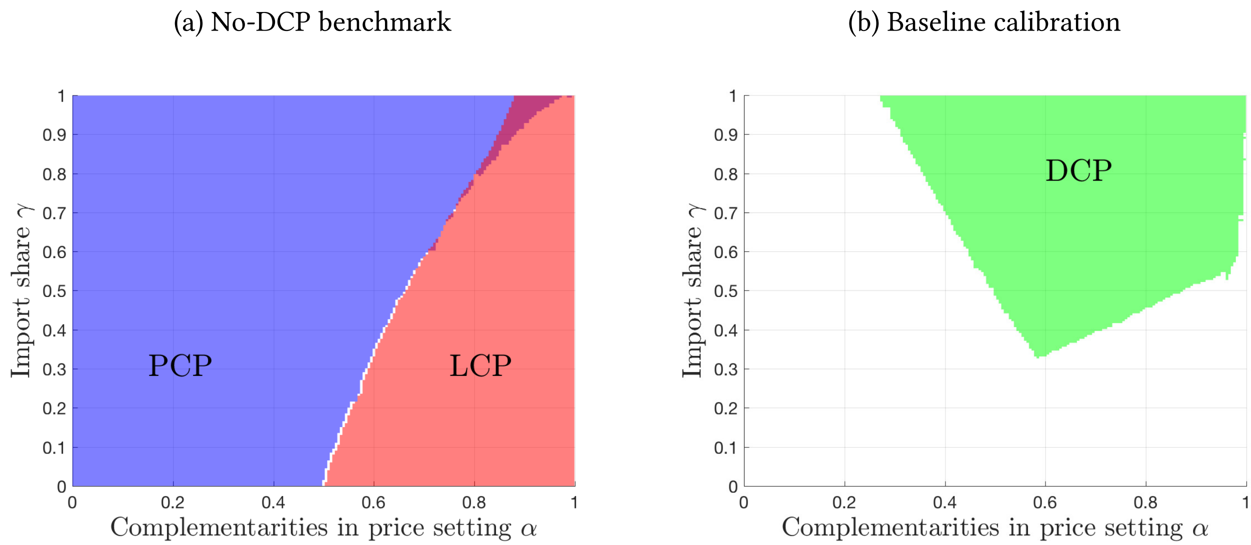


Figure A4: Currency choice in the menu cost model

Note: plot (a) shows DCP region is empty in the limiting case of almost zero menu costs and $n = 0$, $\rho = 1$. Plot (b) shows the region of symmetric DCP equilibrium (other equilibria are suppressed) under the baseline calibration: $n = 0.3$, $\rho = 0.5$, $\phi = 0.5$ and menu costs are calibrated in such way that the probability of price adjustment is 0.5 for $\alpha = 0.5$, $\gamma = 0.6$.

The menu costs are calibrated to generate the same probability of price adjustment $\lambda = 0.5$ as in the benchmark model.

Figure A4 reproduces two key results from the baseline model in the extension with menu costs. The left plot shows equilibrium invoicing when menu costs are close to zero and the dollar has no fundamental advantages. As in Figures 2b, the equilibrium is unique for most parameter values and no DCP equilibrium exists. The right figure shows instead that the region of DCP is large and close to the one from Figure 4 when prices are sticky and countries are asymmetric. Intuitively, with idiosyncratic shocks accounting for most of the variation in firms' optimal prices, the probability of price adjustment remains close to $\lambda = 0.5$ in the most of the region of the parameter space. This in turn implies that the currency choice does not change much relative to the benchmark model.

A.5.4 Model with bargaining

This section outlines a model with bargaining between suppliers and buyers based on Gopinath and Itskhoki (2011), and shows that the same equilibrium as in the baseline model can arise even when prices and invoicing currency are chosen jointly by two firms.

The general equilibrium setup is the same as in the benchmark model. The tradable sector is populated by two types of firms. As before, there is a continuum of manufacturing firms producing intermediate goods in each country. In addition, there are wholesale firms, which combine local and imported products using the Kimball aggregator and sell output to final consumers and to firms in the tradable sector as intermediate inputs. I use the Klenow and Willis (2007) specification, which is the most commonly used functional form for the Kimball aggregator. Wholesale firms set prices flexibly and charge a constant markup over marginal costs, i.e. demand for their output is $Q_i = P_i^{-\zeta} B_i$, where B_i is demand shifter taken as given by individual firms and $\zeta > 1$.

The wholesale firms and their suppliers bargain over prices and the currency of invoicing before the realization of shocks. After uncertainty is resolved, the sales are determined by the demand of the wholesale firms. With probability λ, firms experience large enough idiosyncratic shocks to renegotiate prices ex post. The assumption that contract specifies prices, but not quantities is motivated by the result from the optimal contract literature by Hart and Moore (2008): "The parties are more likely to put restrictions on variables over which there is an extreme conflict of interest, such as price, than on variables over which conflict is less extreme, such as the nature or characteristics of the good to be traded."

The marginal costs of production for manufacturers are the same as in the baseline model. The price index for a bundle of intermediate goods p_i remains also unchanged because of the combination of two assumptions: (i) prices of all wholesale firms are equal in equilibrium due to symmetry, (ii) wholesale firms charge a constant markup over marginal costs. Denote the marginal costs of a wholesaler with R_i. The profits of a wholesale firm for given costs are

$$\Pi_i = \frac{B_i}{\zeta^\zeta \left(\zeta - 1\right)^{\zeta - 1}} R_i^{1-\zeta}.$$

Lemma A6 *The marginal effect of signing a contract with an additional supplier j on the marginal costs of a wholesaler i is equal to*

$$dR_i = D_i P_{ji} h\left(\frac{D_i P_{ji}}{R_i}\right) - R_i \Upsilon\left(h\left(\frac{D_i P_{ji}}{R_i}\right)\right).$$

Proof The equilibrium values of R_i and D_i are characterized by a system of equations:[42]

$$\frac{1}{N}\int_0^n \Upsilon\left(h\left(\frac{D_i P_{ji}}{R_i}\right)\right) dj = 1,$$

$$\frac{1}{N}\int_0^n h\left(\frac{D_i P_{ji}}{R_i}\right)\frac{P_{ji}}{R_i} dj = 1.$$

Take a full differential of two equations and use $x_j \equiv \frac{D_i P_{ji}}{R_i}$ to simplify notation

$$\Upsilon\left(h\left(x_n\right)\right) \mathrm{d}n + \int_0^n \Upsilon'\left(h\left(x_j\right)\right) h'\left(x_j\right) x_j \mathrm{d}\log\left(\frac{D_i}{R_i}\right) dj = 0,$$

[42] For simplicity, I assume that demand shifter γ reflects the mass of varieties coming from different countries (extensive margin) rather than the trade flow of a given firm (intensive margin).

$$h(x_n)x_n\mathrm{d}n+\int_0^n\left[h'(x_j)x_j^2\mathrm{d}\log\left(\frac{D_i}{R_i}\right)-h(x_j)x_j\mathrm{d}\log R_i\right]dj=0.$$

Note that $\Upsilon'(h(x_j))=x_j$ in the first condition from definition of $h(\cdot)$ and that $\frac{1}{N}\int_0^n h(x_j)x_jdj=1$ in the second condition according to the initial equilibrium system. Using these equalities and substituting the first equation into the second one, we obtain

$$\mathrm{d}\log R_idj=[h(x_n)x_n-\Upsilon(h(x_n))]\frac{\mathrm{d}n}{N},$$

which proves the lemma. ■

The benefit of signing a contract for a given supplier is

$$(P_{ji}-MC_j^i)Q_{ji}=(P_{ji}-MC_j^i)h\left(\frac{D_iP_{ji}}{R_i}\right)R_i^{-\zeta}B_i,$$

where MC_j^i are marginal costs of producer j expressed in currency i. Nash bargaining solution can then be obtained from the following problem:

$$\max_{P_{ji}}\left[\left(P_{ji}-MC_j^i\right)h\left(\frac{D_iP_{ji}}{R_i}\right)R_i^{-\zeta}B_i\right]^{1-\tau}\left[\frac{B_i}{\zeta^\zeta(\zeta-1)^\zeta}R_i^{-\zeta}\left[D_iP_{ji}h\left(\frac{D_iP_{ji}}{R_i}\right)-R_i\Upsilon\left(h\left(\frac{D_iP_{ji}}{R_i}\right)\right)\right]\right]^\tau,$$

or equivalently

$$\max_{P_{ji}}\ (1-\tau)\log\left[(\frac{R_i}{D_i}x-MC_j^i)h(x)\right]+\tau\log\left[xh(x)-\Upsilon(h(x))\right],$$

where τ denotes the bargaining power of the wholesaler. The first-order condition is

$$\frac{(1-\tau)\frac{R_i}{D_i}}{\frac{R_i}{D_i}x-MC_j^i}+\frac{(1-\tau)h'(x)}{h(x)}+\frac{\tau\left[h(x)+xh'(x)-\Upsilon'(h(x))h'(x)\right]}{xh(x)-\Upsilon(h(x))}=0.$$

Multiply all terms by x, use the definition of $h(x)=\Upsilon'^{-1}(x)$, which implies $\Upsilon'(h(x))=x$, and definition of $\theta(x)\equiv-\frac{h'(x)x}{h(x)}$ to rewrite optimality condition as

$$(1-\tau)\left[\frac{P_{ji}}{P_{ji}-MC_j^i}-\theta(x)\right]=\tau\frac{h(x)}{\Upsilon(h(x))-xh(x)}.$$

Log-linearize equilibrium condition around symmetric deterministic point with all prices being equal $P_{ji}=P=R_i$, $x=D$, $\Upsilon'(1)=D$, $\Upsilon(1)=h(D)=1$:

$$(1-\tau)\left[\frac{P/MC}{(P/MC-1)^2}\left(mc_j^i-\tilde{p}_{ji}\right)-\varepsilon\theta\left(\tilde{p}_{ji}-p_i\right)\right]=\tau\frac{\theta D}{(1-D)^2}\left[\frac{\theta-1}{\theta}-D\right](\tilde{p}_{ji}-p_i)$$

where $\varepsilon\equiv\frac{\partial\log\theta(x)}{\partial\log x}$ and $p_i=r_i$. When suppliers have all bargaining power $\tau=0$, the optimal price is exactly the same as in the benchmark case. More generally, since equation is homogeneous in $\left(\tilde{p}_{ji}, mc^i_j, p_i\right)$, the optimal price $\tilde{p}_{ji}$ can be written as a weighted sum of marginal costs and the local price index as in the baseline model. Moreover, for the Klenow and Willis (2007) aggregator, we have $D = \frac{\theta-1}{\theta}$, and therefore the optimal price does not depend on the bargaining power τ.

Lemma A7 *The first-order approximation to the optimal price (17) is the same in the model with bargaining as in the baseline model.*

Finally, because contract is sticky and can be renegotiated only in extreme states of the world, suppliers and wholesalers choose the currency of invoicing to minimize deviations of ex post price from the optimal one. Under the second order approximation, this implies the same invoicing problem as in the benchmark model: $\min\limits_k \mathbb{E}\left[\tilde{p}_{ji} + e_{ki}\right]^2$. Thus, the equilibrium conditions for marginal costs, price index, optimal price and currency choice are the same to the first-order approximation as the ones in the baseline model, and therefore two models have the same equilibrium.

A.6 General equilibrium implications of currency choice

A.6.1 Transmission of shocks

Proof of Proposition 6 Consider a positive monetary shock m_i. The risk-sharing condition (A31) implies that the depreciation of exchange rate e_i in response to local monetary shock is the same in all countries. Moreover, conditional on exchange rates, the pass-through of m_i into prices and quantities (A15)-(A25) is independent from the currency regime and is the same for all countries when $n = 0$. The only difference between the U.S. and other economies is, therefore, coming from the effect of e_i on prices and quantities. Both the export and the import elasticities with respect to trade-weighted exchange rate e_i are different for the U.S. than for other countries because of the effect of e_0 on the global economy, which is summarized by the partial elasticity

$$\frac{\partial ex_i}{\partial e_0} = \frac{\partial im_i}{\partial e_0} = \left[(1-\gamma)(\theta-1) + \frac{\gamma\phi}{1-\lambda\left(\alpha+(1-\alpha)\phi\right)}\right](1-\lambda)\,\mu^D,$$

which is positive under DCP. The effect of e_i on CPI inflation is given by $p^C_i = \eta\chi e_i$ for non-U.S. economies and $p^C_0 = \eta\left(\chi-\chi_0\right)e_0$ for the U.S., which implies that inflation is lower in the U.S. From equation (A26), e_i has no effect on output in non-tradable sector: $y_{Ni} = p^C_i - p^N_i + c_i = m_i - p^N_i$. Equation (A25) implies that the relevant price terms in tradable production are

$$y_i = \frac{\gamma\theta}{1-(1-\gamma)\phi}\left[\left(p^I_i - p^E_i\right) - \left(p_i - p\right)\right] - \frac{(1-\gamma)\left(1-\phi^2\right)}{1-(1-\gamma)\phi}p_i - \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi}p.$$

Again, the asymmetries across countries come from the partial derivative with respect to e_0:

$$\frac{\partial\left[\left(p^I_i - p^E_i\right) - \left(p_i - p\right)\right]}{\partial e_0} = 0, \qquad \frac{\partial p_i}{\partial e_0} = \frac{\partial p}{\partial e_0} = -\chi_0.$$

Hence, the stimulating effect on local output is higher in the U.S. Finally, Proposition A1 implies that the effect of e_i on local net exports is the same for all countries when $n = 0$. ■

Proof of Proposition 7 Consider a positive monetary shock in the U.S. m_0. The risk-sharing condition (A31) implies that the depreciation of dollar exchange rate e_0 is the same under all invoicing regimes. Moreover, conditional on e_0, the pass-through of m_0 into prices and quantities (A15)-(A25) is independent from currency regime as well. The only difference in international spillovers under PCP/LCP and under DCP come from the effect of e_0 on foreign prices and quantities. From equations (A15)-(A19), the pass-through of the dollar exchange rate into prices is given by

$$p = \gamma p^I, \qquad p^C = \eta p + (1-\eta)p^N, \qquad p^I = -\frac{(1-\lambda)(1-n)\mu^D}{1-\lambda(\alpha+(1-\alpha)\phi)} e_0.$$

Substitute prices into (A28) and integrate across countries to show $y^I = -\left[\theta(1-\gamma)+\gamma(1+\phi)\right] p^I$. This proves result 1 that the volume of the global trade goes up. While production in the non-tradable sector does not change $y_{Ni} = p_i^C - p_i^N + c_i = m_i - p_i^N$, the global production of tradables (A24) goes up $y = -\gamma(1+\phi)p^I$ and so does consumption $c = m - \eta\gamma p^I$, which proves result 2.

The price index (A15) implies that higher e_0 decreases p_i and CPI in other economies, and the foreign consumption increases according to (A20). Equation (A25) implies that the relevant price terms in tradable production are

$$y_i = \frac{\gamma\theta}{1-(1-\gamma)\phi}\left[\left(p_i^I - p_i^E\right) - (p_i - p)\right] - \frac{(1-\gamma)\left(1-\phi^2\right)}{1-(1-\gamma)\phi} p_i - \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi} p,$$

where

$$\left(p_i^I - p_i^E\right) - (p_i - p) = \left[\chi - \lambda\left(\alpha\chi + (1-\alpha)(2-\phi\chi)\right) - (1-\lambda)\left(2\mu^P + \mu^D\right)\right] n e_0.$$

Thus, when $\theta n \to 0$, the first term in y_i drops out and since both p_i and p decrease in e_0 under DCP, the effect on foreign output is positive. ■

A.6.2 Efficient allocation

Proof of Proposition 8 Assume CES aggregator across tradable products, $\alpha = 0$, and no non-tradable sector, $\eta = 1$. The first-best allocation maximizes the global welfare state by state subject to the resource and technology constraints:

$$\max \int_0^1 \left(\log C_i - L_i\right) \mathrm{d}i$$

$$\text{s.t.} \qquad C_i + X_i + G_i \le \left[(1-\gamma)^{\frac{1}{\theta}} e^{-\frac{\gamma}{\theta}\xi_i} Y_{ii}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} e^{\frac{1-\gamma}{\theta}\xi_i} \int_0^1 Y_{ji}^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}},$$

$$Y_{ii} + \int_0^1 Y_{ij} \mathrm{d}j \leq A_i \left(\frac{L_i}{1-\phi} \right)^{1-\phi} \left(\frac{X_i}{\phi} \right)^{\phi}.$$

The first-order optimality conditions are

$$C_i = \frac{1-\phi}{\phi} \frac{X_i}{L_i}, \tag{A37}$$

$$\left[(1-\gamma)\, e^{-\gamma \xi_i} \frac{C_i + X_i + G_i}{Y_{ii}} \right]^{\frac{1}{\theta}} = \frac{1}{A_i} \left(\frac{1-\phi}{\phi} \frac{X_i}{L_i} \right)^{1-\phi}, \tag{A38}$$

$$\left(e^{-\xi_i} \frac{1-\gamma}{\gamma} \frac{Y_{ji}}{Y_{ii}} \right)^{-\frac{1}{\theta}} = \frac{A_i}{A_j} \left(\frac{X_i}{L_i} / \frac{X_j}{L_j} \right)^{\phi}. \tag{A39}$$

I show next that the equilibrium allocation under PCP and the monetary policy that stabilizes marginal costs in every country satisfies these conditions. First, note that with $\alpha = 0$ and constant marginal costs, both adjusting and non-adjusting firms keep their prices constant in producer currency at $P_{ii} = 1$, so that $P_{ij} = \mathcal{E}_{ji}$. Second, divide labor demand (A3) by demand for intermediate goods (A5) to get expression for real wages

$$\frac{W_i}{P_i} = \frac{1-\phi}{\phi} \frac{X_i}{L_i}.$$

Substitute it into the labor supply to show that the optimality condition (A37) is satisfied:

$$C_i = \frac{W_i}{P_i} = \frac{1-\phi}{\phi} \frac{X_i}{L_i}.$$

Third, using demand for local goods (A4), we obtain

$$Y_{ii} = (1-\gamma)\, e^{-\gamma \xi_i} \left(\frac{P_{ii}}{P_i} \right)^{-\theta} (C_i + X_i + G_i)$$

and

$$\left[(1-\gamma)\, e^{-\gamma \xi_i} \frac{C_i + X_i + G_i}{Y_{ii}} \right]^{\frac{1}{\theta}} = \frac{P_{ii}}{P_i}.$$

Combine stable marginal costs condition (A6) with the real wage from above to show

$$P_i = A_i \left(\frac{W_i}{P_i} \right)^{-(1-\phi)} = A_i \left(\frac{1-\phi}{\phi} \frac{X_i}{L_i} \right)^{-(1-\phi)}.$$

Together, the last two equations imply that the optimality condition (A38) is satisfied. Fourth, divide demand for local and foreign goods

$$Y_{ji} = \gamma e^{(1-\gamma)\xi_i} \left(\frac{P_{ji}}{P_i} \right)^{-\theta} (C_i + X_i + G_i)$$

to show

$$\left(e^{-\xi_i}\frac{1-\gamma}{\gamma}\frac{Y_{ji}}{Y_{ii}}\right)^{-\frac{1}{\theta}}=\frac{P_{ji}}{P_{ii}}=\mathcal{E}_{ij}.$$

Substitute expression for P_i from above into the risk-sharing condition (A7) to get equilibrium exchange rate:

$$\mathcal{E}_{ij}=\frac{C_iP_i}{C_jP_j}=\frac{A_i}{A_j}\left(\frac{X_i}{L_i}/\frac{X_j}{L_j}\right)^{\phi}.$$

Combining the last two equations, we get the optimality condition (A39). This completes the proof of efficiency of the allocation given that firms use PCP.

I next show this is indeed the only equilibrium currency choice. Given $\alpha=0$, the desired price of exporter from j to i in terms of currency k is

$$\tilde{p}_{ji}-e_{ik}=mc_j+e_k-e_j=e_k-e_j,$$

where the last equality follows from the stability of the marginal costs. It follows that producer currency unambiguously dominates any alternative for both exporters and domestic firms.

Finally, consider the optimal monetary policy. Complete risk sharing implies $e_{ij}=m_i-m_j$. With marginal costs fully stabilized and $\alpha=0$, the price index is

$$p_i=\gamma\int_0^1 e_{ij}\mathrm{d}j=\gamma\left(m_i-m\right).$$

Substitute this expression into the marginal costs and integrate across countries to show

$$m_i=\frac{1}{1-(1-\gamma)\phi}\left[a_i+\frac{\gamma\phi}{1-\phi}a\right].$$

It follows that equilibrium exchange rates are $e_i=\frac{1}{1-(1-\gamma)\phi}a_i$. ■

A.6.3 Loss function

This subsection derives the second-order approximation (SOA) to the loss function of a global planner. To economize on the notation, I suppress the higher order terms $\mathcal{O}(x^3)$ and focus exclusively on productivity shocks (more general results are available upon request).

Kimball price index To economize on indices, consider a general price index for Kimball demand that is implicitely determined by the following system:

$$\int_0^1\gamma_i\Upsilon\left(h\left(De^{x_i}\right)\right)\mathrm{d}i=1,$$

$$\int_0^1\gamma_ie^{x_i}h\left(De^{x_i}\right)\mathrm{d}i=D,$$

where $\int_0^1 \gamma_i \mathrm{d}i = 1$ and with some abuse of the notation, x_i is the log-deviation of $\frac{DP_i}{P}$ from symmetric deterministic point with $P_i = P$, normalized by the steady-state value of D. Take the SOA to this system. Start with the first equation:

$$\int_0^1 \gamma_i \Bigg[\Upsilon\left(h\left(D\right)\right) + \Upsilon'\left(h\left(D\right)\right) h'\left(D\right) D x_i \\ + \frac{1}{2}\Big(\frac{d\Upsilon'\left(h\left(X\right)\right)}{dX} h'\left(D\right) D^2 + \Upsilon'\left(h\left(D\right)\right) h''\left(D\right) D^2 + \Upsilon'\left(h\left(D\right)\right) h'\left(D\right) D\Big) x_i^2 \Bigg] \mathrm{d}i = 1.$$

From the properties of the functions, we have $\Upsilon\left(h\left(D\right)\right) = 1$, $\Upsilon'\left(h\left(D\right)\right) = D$ and $\frac{d\Upsilon'(h(X))}{dX} = \frac{dX}{dX} = 1$. From the definitions of elasticity and superelasticity of demand:

$$\theta\left(X\right) \equiv -h'\left(X\right)\frac{X}{h\left(X\right)} \ \Rightarrow \ h'\left(X\right) = -\theta\left(X\right)\frac{h\left(X\right)}{X},$$

$$\varepsilon\left(X\right) \equiv \frac{d\log\left(-h'\left(X\right)\frac{X}{h(X)}\right)}{d\log X} = h''\left(X\right)\frac{X}{h'\left(X\right)} + 1 + \theta\left(X\right) \ \Rightarrow \ h''\left(X\right) = \left(\theta\left(X\right) + 1 - \varepsilon\left(X\right)\right)\frac{\theta\left(X\right)h\left(X\right)}{X^2}.$$

Substitute these equalities into the SOA to obtain

$$\int_0^1 \gamma_i \left[x_i + \frac{1}{2}\left(1 - \theta + \varepsilon\right) x_i^2\right] \mathrm{d}i = 0.$$

Consider next the second equation of the system determining price indices:

$$\int_0^1 \gamma_i \left[h\left(D\right)D + \big(h'\left(D\right)D^2 + h\left(D\right)D\big)x_i + \frac{1}{2}\big(h''\left(D\right)D^3 + 3h'\left(D\right)D^2 + h\left(D\right)D\big)x_i^2\right]\mathrm{d}i, \\ = D\left[1 + d + \frac{1}{2}d^2\right].$$

Substitute steady-state values:

$$\int_0^1 \gamma_i \left[\left(1-\theta\right)x_i + \frac{1}{2}\left(\left(1-\theta\right)^2 - \varepsilon\theta\right)x_i^2\right]\mathrm{d}i = d + \frac{1}{2}d^2.$$

Note that to the FOA $d = 0$, which implies that all second-order terms with d are zero. Multiply the first equation by $1-\theta$ and subtract from the second one to show the following result.

Lemma A8 *The SOA to the Kimball price index is*

$$\int_0^1 \gamma_i \left[\left(p_i - p\right) + \frac{1}{2}\left(1-\theta\right)\left(p_i - p\right)^2 + \left(p_i - p\right)z_i + \frac{1}{2\left(1-\theta\right)}z_i^2\right]\mathrm{d}i = 0,$$

$$-\frac{1}{2}\varepsilon\int_0^1 \gamma_i\left(p_i - p\right)^2\mathrm{d}i = d.$$

Consider next the SOA to the relative demand $V_i \equiv h\left(e^{x_i}\right)$:

$$v_i + \frac{1}{2}v_i^2 = h'\left(D\right)Dx_i + \frac{1}{2}\left(h''\left(D\right)D^2 + h'\left(D\right)D\right)x_i^2 = -\theta x_i + \frac{1}{2}\left(\theta - \varepsilon\right)\theta x_i^2.$$

Therefore, using result from Lemma A8,

$$\int_0^1 \gamma_i\left(v_i + \frac{1}{2}v_i^2\right)\mathrm{d}i = -\theta\int_0^1 \gamma_i\left[\left(d + p_i - p\right) + \frac{1}{2}(\varepsilon - \theta)\left(d + p_i - p\right)^2\right]\mathrm{d}i = \frac{\theta}{2}\int_0^1 \gamma_i\left(p_i - p\right)^2\mathrm{d}i.$$

Lemma A9 *The integral of the SOA to the relative demand is equal to*

$$\int_0^1 \gamma_i\left(v_i + \frac{1}{2}v_i^2\right)\mathrm{d}i = \frac{\theta}{2}\int_0^1 \gamma_i\left(p_i - p\right)^2\mathrm{d}i.$$

Labor market and intermediates Both labor demand and labor supply equations are exact in logs:

$$c_i = w_i - p_i, \qquad l_i = -\phi(w_i - p_i) - a_i + y_i.$$

Demand for intermediate goods is also exact in logs

$$x_i = y_i - a_i + \left(1 - \phi\right)\left(w_i - p_i\right).$$

The sum of final and intermediate demand is therefore,

$$\begin{aligned}(1-\phi)\left(c_i + \frac{1}{2}c_i^2\right) + \phi\left(x_i + \frac{1}{2}x_i^2\right) &= \left(1 - \phi^2\right)\left(w_i - p_i\right) + \frac{1}{2}\left[\left(1-\phi\right) + \phi\left(1-\phi\right)^2\right]\left(w_i - p_i\right)^2 \\ &\quad -\phi\left(1-\phi\right)\left(w_i - p_i\right)a_i + \phi\left[\left(1-\phi\right)\left(w_i - p_i\right) - a_i\right]y_i + \phi\left(y_i + \frac{1}{2}y_i^2\right) - \phi\left(a_i - \frac{1}{2}a_i^2\right).\end{aligned}$$

Goods market The market clearing condition in tradable sector of country i can be written as

$$\begin{aligned}Y_i &= \left(1-\gamma\right)\int_0^1 h\left(\frac{D_iP_{ii}\left(\omega\right)}{P_i}\right)\mathrm{d}\omega\left(C_i + X_i\right) + \gamma\int_0^1\int_0^1 h\left(\frac{D_jP_{ij}\left(\omega\right)}{P_j}\right)\mathrm{d}\omega\left(C_j + X_j\right)\mathrm{d}j \\ &\equiv \left(1-\gamma\right)\int_0^1\left(V_{ii\omega}C_i + V_{ii\omega}X_i\right)\mathrm{d}\omega + \gamma\int_0^1\int_0^1\left(V_{ij\omega}C_j + V_{ij\omega}X_j\right)\mathrm{d}\omega\mathrm{d}j.\end{aligned}$$

The SOA to this equation is

$$\begin{aligned}y_i + \frac{1}{2}y_i^2 = &\left[\left(1-\gamma\right)\int_0^1\left(v_{ii\omega} + \frac{1}{2}v_{ii\omega}^2\right)\mathrm{d}\omega + \gamma\int_0^1\int_0^1\left(v_{ij\omega} + \frac{1}{2}v_{ij\omega}^2\right)\mathrm{d}\omega\mathrm{d}j\right] \\ &+ \left(1-\gamma\right)\left[\left(1-\phi\right)\left(c_i + \frac{1}{2}c_i^2\right) + \phi\left(x_i + \frac{1}{2}x_i^2\right)\right] + \gamma\int_0^1\left[\left(1-\phi\right)\left(c_j + \frac{1}{2}c_j^2\right) + \phi\left(x_j + \frac{1}{2}x_j^2\right)\right]\mathrm{d}j \\ &+ \left[\left(1-\gamma\right)\int_0^1 v_{ii\omega}\mathrm{d}\omega\left(\left(1-\phi\right)c_i + \phi x_i\right) + \gamma\int_0^1\int_0^1 v_{ij\omega}\mathrm{d}\omega\left(\left(1-\phi\right)c_j + \phi x_j\right)\mathrm{d}j\right].\end{aligned}$$

Integrate market clearing conditions across countries:

$$\int_0^1 \left(y_i + \frac{1}{2}y_i^2\right) \mathrm{d}i = \int_0^1 \left[(1-\gamma)\int_0^1 \left(v_{ii\omega} + \frac{1}{2}v_{ii\omega}^2\right)\mathrm{d}\omega + \gamma \int_0^1\int_0^1 \left(v_{ji\omega} + \frac{1}{2}v_{ji\omega}^2\right)\mathrm{d}\omega \mathrm{d}j\right] \mathrm{d}i$$
$$+ \int_0^1 \left[(1-\phi)\left(c_i + \frac{1}{2}c_i^2\right) + \phi\left(x_i + \frac{1}{2}x_i^2\right)\right]\mathrm{d}i$$
$$+ \int_0^1 \left[(1-\gamma)\int_0^1 v_{ii\omega} d\omega + \gamma\int_0^1\int_0^1 v_{ji\omega}\mathrm{d}\omega\mathrm{d}j\right]\left(\left(1-\phi\right)c_i + \phi x_i\right)\mathrm{d}i.$$

According to Lemma A9, $(1-\gamma)\int_0^1 v_{ii\omega}\mathrm{d}\omega + \gamma\int_0^1\int_0^1 v_{ji\omega}\mathrm{d}\omega\mathrm{d}j$ is of the second order and therefore, the last term is zero in the SOA. Substitute the expression from Lemma A9 into the first term:

$$\int_0^1 \left(y_i + \frac{1}{2}y_i^2\right) \mathrm{d}i = \frac{\theta}{2}\int_0^1 \sigma_{Pi}^2 \mathrm{d}i + \int_0^1 \left[(1-\phi)\left(c_i + \frac{1}{2}c_i^2\right) + \phi\left(x_i + \frac{1}{2}x_i^2\right)\right]\mathrm{d}i\mathrm{d}i,$$

where σ_{Pi}^2 denotes dispersion of prices in region i for brevity. Substitute next the expression for consumption and intermediate demand to obtain

$$\int_0^1 \left(y_i + \frac{1}{2}y_i^2\right) \mathrm{d}i = \int_0^1 \left[(1+\phi)(w_i - p_i) + \frac{1}{2}\left[1 + \phi(1-\phi)\right](w_i - p_i)^2 - \phi(w_i - p_i)a_i\right]\mathrm{d}i$$
$$+ \int_0^1 \left[\frac{\theta}{2}\frac{1}{1-\phi}\sigma_{Pi}^2 + \phi\left[(w_i - p_i) - \frac{1}{1-\phi}a_i\right]y_i - \frac{\phi}{1-\phi}(a_i - \frac{1}{2}a_i^2)\right]\mathrm{d}i.$$

Loss function The preferences in country i are given by

$$U_i = \log C_i - L_i.$$

The SOA to the objective function is

$$U_i = \log C - L + c_i - L\left(l_i + \frac{1}{2}l_i^2\right).$$

Use steady-state values $C = L = 1$ and suppress a constant term:

$$u_i = c_i - l_i - \frac{1}{2}l_i^2.$$

Next, substitute in consumption and labor from labor market clearing condition:

$$u_i = (1+\phi)(w_i - p_i) + \left(a_i - \frac{1}{2}a_i^2\right) - \frac{1}{2}\phi^2(w_i - p_i)^2 - \phi(w_i - p_i)a_i - \left(y_i + \frac{1}{2}y_i^2\right) + \left[\phi(w_i - p_i) + a_i\right]y_i.$$

Integrate across countries and use expression for total output from the goods market clearing to see several terms cancel out. Suppress exogenous terms to simplify the expression:

$$u = \int_0^1 \left[-\frac{1}{2} (1+\phi) (w_i - p_i)^2 - \frac{\theta}{2} \frac{1}{1-\phi} \sigma_{Pi}^2 + \frac{1}{1-\phi} a_i y_i \right] \mathrm{d}i.$$

The FOA to the output of an individual country in (A25) implies that the price terms in y_i are

$$y_i = \frac{\gamma\theta}{1-(1-\gamma)\phi} \left[\left(p_i^I - p_i\right) - \left(p_i^E - p\right) \right] + \frac{(1-\gamma)(1-\phi^2)}{1-(1-\gamma)\phi} (w_i - p_i) + \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi} (w-p).$$

Substitute this equation, use $w_i = m_i$ and change the signs to obtain the loss function:

$$\begin{aligned} \mathcal{L} = \int_0^1 \Bigg[& \frac{1}{2} (1+\phi) (m_i - p_i)^2 - \frac{1}{1-\phi} \frac{\gamma\theta}{1-(1-\gamma)\phi} \left[\left(p_i^I - p_i\right) - \left(p_i^E - p\right) \right] a_i \\ & + \frac{1}{1-\phi} \frac{\theta}{2} \sigma_{Pi}^2 - \frac{(1-\gamma)(1+\phi)}{1-(1-\gamma)\phi} (m_i - p_i) a_i \Bigg] \mathrm{d}i + \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi} \frac{1}{1-\phi} (m-p) a. \end{aligned} \tag{A40}$$

A.6.4 Optimal policy

This section proves the results about the optimal monetary policy. While I make several restrictive assumptions to simplify the analysis, most of the results can be extended for the case with two sectors in the economy, partially adjusting nominal prices, and multiple shocks (the additional results are available upon request).

Proof of Proposition 9 Because the U.S. is a closed economy $\eta \to 0$, its optimal policy does not depend on either the currency choice or the monetary policy of other countries and is simply $m_0 = a_0$. Since the U.S. is small $n = 0$, the total welfare maximized by other countries is given by (A40). The fact that loss function contains only second-order terms implies that the FOA to the pricing block and risk-sharing conditions is sufficient. Assuming that prices are fully sticky and invoicing is symmetric across countries and using $\int_0^1 e_i \mathrm{d}i = 0$, we obtain

$$\begin{aligned} p_{ji} &= \left(\mu^P + \mu^D\right) e_i - \mu^D e_0 - \mu^P e_j, \\ p_i^I &= \left(\mu^P + \mu^D\right) e_i - \mu^D e_0, \\ p_i^E &= -\mu^D e_0 - \mu^P e_i, \\ p_i &= \gamma \left(\mu^P + \mu^D\right) e_i - \gamma \mu^D e_0, \\ p &= -\gamma \mu^D e_0, \end{aligned} \tag{A41}$$

$$\sigma_{Pi}^2 = \gamma \int_0^1 p_{ji}^2 \mathrm{d}j - p_i^2 = \gamma \int_0^1 \left[\left(\mu^P + \mu^D\right) e_i - \mu^D e_0 - \mu^P e_j \right]^2 \mathrm{d}j - \gamma^2 \left[\left(\mu^P + \mu^D\right) e_i - \mu^D e_0 \right]^2.$$

The perfect risk-sharing and normalization $\int_0^1 e_i \mathrm{d}i = 0$ imply $e_i = m_i - m$, while from the labor supply condition the nominal wage is $w_i = m_i$.

If firms choose PCP, the optimal policy is:

$$m_i = e_i = \frac{1}{1-(1-\gamma)\phi} a_i,$$

$$\mathcal{L}^{PCP} = -\frac{1}{2}\left[\frac{\gamma(2-\gamma)\theta}{1-\phi} + (1-\gamma)^2(1+\phi)\right]\left(\frac{1}{1-(1-\gamma)\phi}\right)^2 \sigma_a^2.$$

If firms choose DCP, on the other hand, the loss function is

$$\begin{aligned}\mathcal{L} = \int_0^1 &\left[\frac{1}{2}(1+\phi)\left((1-\gamma)(e_i+m)+\gamma m_0\right)^2 - \frac{1}{1-\phi}\frac{\gamma(1-\gamma)\theta}{1-(1-\gamma)\phi} e_i a_i \right.\\ &\left. - \frac{(1-\gamma)(1+\phi)}{1-(1-\gamma)\phi}\left((1-\gamma)(e_i+m)+\gamma m_0\right) a_i + \frac{\gamma(1-\gamma)\theta}{1-\phi}\frac{\theta}{2}(e_i+m-m_0)^2\right]\mathrm{d}i,\end{aligned}$$

where e_0 is a function of endogenous m. The FOCs with respect to e_i and m imply

$$\begin{aligned}e_i &= \frac{1}{1-(1-\gamma)\phi} a_i,\\ m &= \frac{\gamma\left[\theta-(1-\phi^2)\right]}{\gamma\theta+(1-\gamma)(1-\phi^2)} m_0,\\ e_0 &= \frac{1-\phi^2}{\gamma\theta+(1-\gamma)(1-\phi^2)} m_0,\\ m_i &= \frac{1}{1-(1-\gamma)\phi} a_i + \frac{\gamma\left[\theta-(1-\phi^2)\right]}{\gamma\theta+(1-\gamma)(1-\phi^2)} m_0.\end{aligned}$$

Results 1 and 2 of the proposition then follow directly from the comparison of the equilibrium exchange rates under PCP and DCP. The welfare losses under DCP

$$\mathcal{L}^{DCP} = -\frac{1}{2}\frac{1-\gamma}{1-\phi}\frac{\gamma\theta+(1-\gamma)(1-\phi^2)}{(1-(1-\gamma)\phi)^2}\sigma_a^2 + \frac{1}{2}\frac{\gamma(1+\phi)\theta}{\gamma\theta+(1-\gamma)(1-\phi^2)}\sigma_{m_0}^2, \tag{A42}$$

are higher than losses under PCP, $\mathcal{L}^{PCP} \le \mathcal{L}^{DCP}$. The currency choice of an individual exporter under two described policies is determined respectively by

$$\begin{aligned}\tilde{p}_{ji} + e_{ki} &= e_k + (1-\alpha)(mc_j - e_j) + \alpha(p_i - e_i) = e_k - (1-\alpha)e_j - \alpha(1-\gamma)e_i,\\ \tilde{p}_{ji} + e_{ki} &= e_k - (1-\alpha)e_j - \alpha(1-\gamma)e_i - \gamma\left[1 - \frac{(1-\alpha)\theta}{1+\phi}\right]e_0.\end{aligned}$$

Thus, as long as θ is not too large, the DCP is more likely under the second policy, which proves result 4 from the proposition. ■

Proof of Proposition 10 The planner's problem is to minimize (A40) subject to price setting constraints (A41) and the individual currency choice (A33). Because of the SOA, it is sufficient to focus on the class of linear policy functions. Given that the U.S. accounts only for a trivial fraction of the global economy, it is never optimal to make monetary policy in other countries respond to U.S. shocks: the same exchange rates can be achieved by changing the U.S. monetary policy instead without generating additional distortions in other economies. Thus, one can focus on the policies $m_i = \delta_i a_i$, where due to symmetry, $\delta_i = \delta$ is the same for all countries, except possibly the U.S. I impose an additional constraint that $|\delta_0| \leq |\delta_i|$ to ensure that no other currency than the dollar is used as a vehicle currency. This is without loss of generality, given we can always relabel countries. I also assume that productivity shocks are symmetric and uncorrelated across countries for simplicity: the solution for global shocks is trivial and does not affect firms' currency choice. It follows from (A42) that even under the best discretionary policy with $\sigma^2_{m_0} = 0$ the global welfare is still lower under DCP than under PCP. The same is true for LCP. Hence, the region of PCP cannot be smaller under commitment than under discretion.

When the optimal currency choice does not change relative to the discretionary case, the optimal monetary policy is also the same. Consider next the subset of parameters, for which the planner finds it optimal to make firms switch from DCP to PCP. The optimal currency choice is based on

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)a_j - (1-\alpha)(1-\gamma)\phi e_j - \alpha(1-\gamma)e_i - (\alpha + (1-\alpha)\phi)\,\gamma\mu^D e_0.$$

Given the linear policy functions and the same volatility of productivity shocks in all countries, the firms choose DCP over PCP if

$$\left[1 - (\alpha + (1-\alpha)\phi)\,\gamma\right]^2 \delta_0^2 \leq \left[1 - 2(1-\alpha)\left(\frac{1}{\mu} + \phi(1-\gamma)\right)\right]\delta^2.$$

It follows that to make exporters switch to PCP, the planner sets $\delta_0 = \delta$ (upper bound) and lowers the value of δ relative to the discretionary case. The same argument applies in the case of LCP. Thus, the volatility of all exchange rates (except possibly for the U.S. one) is lower under commitment.

Finally, to ensure that the optimal allocation is implemented as a unique equilibrium, I extend the optimal policy out of equilibrium as follows. Consider an arbitrary trade flow between countries j and i, which is invoiced in some currency k instead of PCP. Notice that it is always the case that currency k has a nonzero weight in the optimal basket: for any $k \neq j$, $\left|\frac{\partial p_j}{\partial e_k}\right| < 1$, $\left|\frac{\partial (e_i - p_i)}{\partial e_k}\right| < 1$ and hence,

$$\frac{\partial\left(\tilde{p}_{ji} + e_{ki}\right)}{\partial e_k} = 1 - \left[(1-\alpha)\,\phi\left(-\frac{\partial p_j}{\partial e_k}\right) + \alpha\frac{\partial\left(e_i - p_i\right)}{\partial e_k}\right] > 0.$$

The planner can always make the volatility of $\tilde{p}_{ji} + e_{ki}$ arbitrarily high by increasing the volatility of monetary shocks m_k. This, in turn, will ensure that setting prices in currency k is suboptimal for exporters and such equilibrium does not arise. ■