PATHS OF IDEOLOGICAL CONFLICT: CLOSING THE GAP BETWEEN GAMSON'S LAW AND THEORY

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ABSTRACT

Gamson's law suggests that parties in a coalition tend to receive portfolio payoffs proportional to their relative seat share in parliament. Although evidence in favor of Gamson's law has been found in many empirical studies, its theoretical foundation is poor and even conflicts with standard bargaining theory. Specifically, the use of seat shares does not depict a party's ability to join *alternative* government coalitions. Both Gamson's Law and standard power indices ignore the impact of ideological conflicts within coalitions. This paper provides a power index based on both pivotal ability and ideological conflict analysis. We implicitly use content analysis of party programs to measure ideological closeness by conflict paths in the corresponding political network. An empirical analysis of portfolio allocation after German elections shows that our index confirms Gamson's law and therefore closes the gap to bargaining theory.

1 INTRODUCTION

Literature on government formation typically concentrates on governments in "minority legislatures", in which no political party controls a majority of seats, as widespread in the European Union. One of the most central issues in the formation process of such governments is the assignment of ministerial portfolios to parties (see, for example, Laver, 1998 and Laver and Schofield, 1998 for an extensive literature review). The most prominent landmark for portfolio allocation is *Gamson's law* (Gamson, 1961), which suggests that parties tend to receive portfolio payoffs proportional to the seats that each party contributed to the coalition, that is, the *coalitional* relative seat share. Although evidence in favor of Gamson's law has been found in many empirical studies (see Browne and Franklin, 1973; Laver and Schofield, 1998; Warwick and Druckman, 2001 and 2006 among others), its theoretical foundation is poor and even conflicts with standard bargaining theory (cf. Snyder et al., 2005, or Carroll and Cox, 2007 among others).

Specifically, the use of coalitional relative seats does not depict a party's ability to "pivot between alternative minimal-winning coalitions and its ability to propose governments" (cf. Carroll and Cox, 2007), that is, does not account for *outside options* - a central concern in the cooperative branch of bargaining theory. Economic theory suggests the use of power-indices as the popular Banzhaf-Power-Index (Banzhaf, 1952) [1]. However, approaches based on power indices and bargining theory, as, for example, Warwick and Druckman (2006) or Carroll and Cox (2007), stay behind Gamson's Law with respect to explanatory power (cf. Linhart et al., 2008). An explanation of the failure of power indices might be found in the lack of *factual content*: Albert (2003) argues that

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[1] also known as Banzhaf-Coleman index (cf. Coleman, 1971) or Penrose-Banzhaf index (cf. Penrose, 1946)

> "[The Power-index approach] should not (even) be considered as part of political science. Viewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists. [...] It has no factual content and can therefore not be used for purposes of prediction or explanation."

In this paper, we suggest portfolio allocation by relative weakness proportionality, where weakness of non-member parties is measured by *election specific ideological closeness* of potential winning coalitions. Therefore, our approach can be seen as an attempt towards factual content in theoretical indices.

Power indices as the Banzhaf-Power-Index or the Shapley-Index (Shapley and Shubik, 1969) suggest allocation by relative strength of parties. In contrast, we follow the idea of relative weakness proportionality, the interpretation of the coalitional Nash bargaining solution (Compte and Jehiel, 2010). In contrast to the original solution, we do not derive weakness by the analysis of unblocked coalitions in a game, but by measures of coalitional closeness. More precisely, we interpret ideological closeness of potential winning coalitions as a proportional measure for the corresponding materialization probabilities which, in turn, yields a measure of weakness for corresponding non-member parties. Since we consider all potential winning coalitions as credible, we account for the parties pivotal ability.

Whenever a parliament consists of three or more parties, any approach taking into account outside options needs to analyze coalitions of more than two parties. However, for coalitions of more than two parties, "operationalizing closeness presents fundamentally different challenges" (Kselman et al., 2017). We use different attempts on conflict paths in the corresponding political network, interpreting closeness as the difference between potential and actual conflict among election specific issues. For the latter, we use data from the "Wahl-O-Mat", a popular Voting Advice Application (VAA) for elections in Germany, established by the Federal Agency for Civic Education, where parties respond by self-placement. Hereby, we implicitly use content analysis of party programs which embeds factual content in our indices.

Voting Advice Applications are a commonly used tool in Europe (Garzia and Marschall (2012) found that 25 of 27 countries of the European Union offer at least one VAA; also see Marschall and Garzia, 2014) and " slowly but surely are gaining ground in other parts of the world" (cf. Van Camp et al., 2014). This paper uses data from the German "Wahl-o-Mat", but our approach could equivalently be used with data from other VAAs as "StemWijzer" (Netherlands), "Smartvote" (Switzerland) or "Vote Compass" (Canada, USA, Australia, New Zealand).

VAAs are meant to inform voters about political parties and their positions. Therefore, most research on VAAs concentrates on the effects of VAAs on voting behavior (e.g. Israel et al., 2017), voter turnout (e.g. Marschall and Schultze, 2012) or political knowledge about party positions (e.g. Schultze , 2014). Contrarily, we use the parties positions to measure political closeness *between* parties. Since VAAs provide data for an explicit election, we approach factual content regarding specific elections rather than "general closeness" based on statistics. This is especially important for state parliament governments, since "closeness" (by means of content regarding e.g. education, labor, or environment) is likely to vary considerably across states: for the 2017 elections, consensus between the two "big" German parties Christian Democratic Union (CDU) and Social Democratic Party (SPD) varies from almost 70% in North-Rhine-Westphalia to about 40% in Schleswig-Holstein.

We analyze the performance of our new approach with respect to both Gamson's law and the actual portfolio allocation by an empirical analysis of elections in Germany. In total, we analyze 31 elections from 2005 to 2018 in 13 states as well as the federal parliament elections. In 20 out of 31 elections, our approach confirms the allocation proposed by Gamson's law. In 9 out of the missing 11 elections, our approach performs better than Gamson's law regarding the actual portfolio allocation, leaving only 2 elections where we fail to confirm Gamson's law or to do better. Therefore, with a confirmation rate of 91% (20 out of 31-9=22) we provide an attempt to "fill the gap" between Gamson's Law and Bargaining Theory. Compared to both Gamson's Law and standard Power indices, our approaches yield the best proxy for portfolio allocation in 29 out of 31 cases.

The paper is structured as follows: Section 2 describes the relative weakness proportionality approach, section 3 is devoted to the "Wahl-o-Mat" and how VAA data can be used to formalize bilateral closeness, in section 4 we define closeness measures, and section 5 analyzes data from German elections.

2 Relative Weakness Proportionality

2.1 The coalitional Nash Bargaining Solution

Compte and Jehiel (2010) suggest the *coalitional Nash bargaining solution*, which maximizes the Nash product among all core allocations. Under certain assumptions, this solution is the equilibrium of a modified ultimatum game, the *bargaining game*: In any period t, each player $i \in N$ has an equal chance of being selected to propose an allocation. The proposer chooses a coalition $S \subseteq N$ and makes a proposal on how to share a specified surplus $v(S)$ among the players in S who, in turn, can accept or reject the proposal.

Per definition, the coalitional Nash bargaining solution is derived by solving a maximization problem (Nash-product) under core condition restrictions (no-blocking and feasibility):

$$\max \prod_{i=1}^{n} x_i \text{ w.r.t } \sum_{i \in N} x_i \leq v(N) - \Delta, \sum_{i \in S} x_i \geq v(S) + \Delta \text{ for all } S \subset N$$

where Δ is the core operator. However, we will not fix the surplus function v to derive allocation x, but use the interpretation of an alternative characterization: Compte and Jehiel (2010) show that, under certain conditions, there exists a vector of weights μ and a scalar $\alpha \in [0,1]$ such that coalitional Nash bargaining solution satisfies

$$(\alpha + m_i^{\mu})x_i = (\alpha + m_1^{\mu})x_1 \text{ where } m_i^{\mu} = \sum_{S \subseteq N \setminus \{i\}} \mu_S \tag{1}$$

where $\mu_S = 0$ for all coalitions that are *not credible*. In fact, μ coincides with the vector of Lagrange multipliers of the maximization problem. In contrast, we will not derive μ from a surplus function v, but follow its interpretation (cf. Compte and Jehiel, 2010):

> μ_S can be interpreted as the strength of coalition S. Specifically, a coalition S is credible if it has a strictly positive strength $\mu_S > 0$, and the strength of a coalition is proportional to the probability that it is proposed in equilibrium [...]. Finally, the parameter m_i^{μ} can be interpreted as a measure of the weakness of player i. A player, say i, who would belong to fewer credible subcoalitions than another player, say j, would be such that $m_i^{\mu} > m_j^{\mu}$ [...] [which] implies that x_i would be smaller than x_j.

In contrast to Compte and Jehiel (2010), we do not derive μ from the bargaining game with specified surplus function, but interpret it as a measure of closeness.

2.2 The relative weakness index

Let $N = \{1, \dots, n\}$ denote the parties in a parliament and let $\{w_i\}_{i \in N}$ denote their seat shares. Let $\{\mu_S\}_{S \subseteq N}$ be a measure of coalitional strength which satisfies $\mu_S \in [0,1]$ for all coalitions $S \subseteq N$ and $\mu_S = 0$ for all non-winning coalitions, that is, whenever $\sum_{i \in S} w_i < 0.5$. Consequently, $m_i^\mu = \sum_{S \subseteq N \setminus \{i\}} \mu_S$ denotes a party i's weakness. Then, we define bargaining power x_i to be proportional to i's relative weakness:

Case 1 There exists a pivotal party, i.e. there exists $i \in N$ such that $\sum_{j \in S} w_j < 0.5$ for all $S \subseteq N \setminus \{i\}$. Then, for any party $j \in N$ we have

$$x_j = \frac{1}{1 + m_j^\mu} x_i \quad \Leftrightarrow \quad (\alpha + m_i^\mu) x_i = (\alpha + m_j^\mu) x_j \text{ with } \alpha = 1$$

that is, $x_j < x_i$ and the weaker j, the higher m_j^μ and the lower x_j.

Case 2 There exists no pivotal party. Then, for all parties i, j we have

$$x_j = \frac{m_i^\mu}{m_j^\mu} x_i \quad \Leftrightarrow \quad (\alpha + m_i^\mu) x_i = (\alpha + m_j^\mu) x_j \text{ with } \alpha = 0$$

that is, if j is weaker than i (i.e. $m_j^\mu > m_i^\mu$), we have $x_j < x_i$ and the weaker j compared to i, the lower x_j compared to x_i.

Normalizing bargaining power by $\sum_{i \in N} x_i = 1$ to obtain an index on the unit interval, we get

$$x_i = \frac{1}{\tilde{m}_i^\mu} \cdot \left(\sum_{l=1}^{|N|} \frac{1}{\tilde{m}_l^\mu} \right)^{-1} \text{ where } \tilde{m}_l^\mu = \begin{cases} 1 + m_l^\mu & \text{in case 1} \\ m_l^\mu & \text{in case 2} \end{cases}$$

by solving the corresponding system of equations. Note that this yields a power index for all parties. Portfolio allocation can be calculated by relative bargaining power within the government coalition.

We continue by providing approaches for deriving $\{\mu_S\}_{S \subseteq N}$ from VAA data.

3 Consensus and Conflict in VAA's

3.1 The "Wahl-o-Mat"

The "Wahl-o-Mat", a popular voting advice application (VAA) for state and government elections in Germany, has been established by the Federal Agency for Civic Education (*Bundeszentrale für politische Bildung*, bpb) in 2002. It is based on the dutch "StemWijzer", established by "ProDemos - Huis voor democratie en rechtsstaat". The "StemWijzer" is used in the Netherlands since 1989 (online versions since 1998).

The "Wahl-o-Mat" provides yes/no/neutral positions of potential parliament parties for the "most important" election statements. These statements are extracted from the election programs by a committee consisting of first-time voters, political scientists, statisticians, pedagogues, other experts and members of the Federal Agency for Civic Education. The statements cover topics from

- labor, social affairs, integration and asylum
- energy, environment, infrastructure, health and consumer protection
- family, education, culture and religion
- homeland and foreign affairs, democracy, federalism and the European Union

and are chosen by means of importance, controversy, discriminability and providing a broad range of topics. Once the statements are fixed, the parties provide their positions according to each statement by self-placement.

The first “Wahl-o-Mat” was introduced for the German 2002 Federal Parliament Election. It covered 27 statements, was answered by 5 parties and has been used 3.6 million times. Followed by versions for the European Parliament elections and the State Parliament elections in Baden-Wuerttemberg, Bavaria, Berlin, Bremen, Hamburg, Lower Saxony, North-Rhine-Westphalia, Rhineland-Palatinate, Saarland, Saxony, Saxony-Anhalt, Schleswig-Holstein, and Thuringia, the “Wahl-o-Mat” established as a commonly used pre-voting tool in Germany. Since 2009, all registered parties are allowed to participate in the “Wahl-o-Mat”. Nowadays, the “Wahl-o-Mat” is an established information tool used before almost all German elections and has been used about 47 million times in total. Each version now covers 38 statements, answered by more than 20 parties.

3.2 Measuring Consensus and Conflict

We derive a measure for (bilateral) ideological closeness based on how the “Wahl-o-Mat” provides advice to voters: For each statement, consensus is measured as described in Table 1. This

Table 1: Measuring Consensus

	agree	neutral	not agree
agree	2	1	0
neutral	1	2	1
not agree	0	1	2

is formalized as follows:

Definition 1 (Consensus and Bilateral Closeness)**.** *For each statement* $s = 1, \dots, S$*,* N *parties* $i = 1, \dots, |N|$ *self-position by choosing from* $p_i^s \in \{-1 = \text{“not agree”}, 0 = \text{“neutral”}, 1 = \text{“agree”}\}$*. Then, for each two parties* $i, j \in N$ *and each statement* s*, we define the* consensus value *by*

$$c_{ij}^s := 2 - |p_i^s - p_j^s| = \begin{cases} 2 & , \text{ if } |p_i^s - p_j^s| = 0 \\ 1 & , \text{ if } |p_i^s - p_j^s| = 1 \\ 0 & , \text{ if } |p_i^s - p_j^s| = 2 \end{cases} .$$

For each two parties $i, j \in N$ *and each statement* $s = 1, \dots, S$*,* bilateral closeness between parties i and j *is given by the overall value of consensus relative to the maximal consensus across* *statements (i.e., the consensus* potential*):*

$$bilclos_{ij} := \left(\sum_{s=1}^{S} c_{ij}^{s}\right) \frac{1}{2 \cdot S} \in [0,1].$$

Remark 1 (Weighting Schemes)**.** *Note that the "Wahl-o-Mat" allows voters to favor statements, that is, voters can neutralize statements they are not interested in and can double-weight favored statements. All measures presented in this paper can easily be adjusted for any kind of weighting scheme on the statements.*

4 Coalitional Closeness and Conflict

Coalitional Closeness of 2-party coalitions is intuitively given by the corresponding bilateral closeness. Coalitional closeness for larger coalitions is much more challenging. Intuitively, for each two parties $i, j \in N$ and each statement $s = 1, \ldots, S$, the *conflict value* is given by $2 - c_{\mathrm{ij}}^{s}$. This yields a measure of bilateral distance between each two parties which could be interpreted as weights of a *distance network*. Centrality approaches of such distance networks suggest to derive closeness by inverting the length of paths of least distance.[2] However, this might result in problems regarding scales and relative differences. Furthermore, distance = 1− bilclos is *cumulated* across statements. Therefore, we analyze statement specific conflict and interpret coalitional closeness as the *difference between possible and actual conflict*.

4.1 Average Conflict Closeness

A first intuitive approach is to analyze the difference between possible and actual *average* conflict:

Definition 2 (Average Conflict Closeness)**.** *Let $K \subseteq N$ be a coalition of parties. For any statement s, the average conflict within K is given by*

$$\sum_{\substack{i,j \in K: \\ i \neq j}} (2 - c_{ij}^{s}) \binom{|K|}{2}^{-1}$$

which yields a maximum possible value of 2.[3] *Accordingly,* average conflict closeness of K *is given by*

$$AVCclos_K := \sum_{s=1}^{S} \left(2 - \sum_{\substack{i,j \in K: \\ i \neq j}} (2 - c_{ij}^{s}) \binom{|K|}{2}^{-1} \right) (2S)^{-1}$$

where $(2S)^{-1}$ *is a normalization by maximal average conflict across statements to obtain an index on a normalized scale (unit interval).*

Lemma 1. *We have*

$$AVCclos_K = \sum_{\substack{i,j \in K: \\ i \neq j}} bilclos_{ij} \binom{|K|}{2}^{-1},$$

[2] See Freeman (1978)'s Closeness Centrality and its generalization to weighted networks, cf. Brandes (2001) and **?**.

[3] Note that the definition uses a double-indexed sum to select *pairs*, not orders as a double sum $\sum_{i \in K} \sum_{j \in K \setminus \{i\}}$. The number of pairs in K is given by $\binom{|K|}{2}$.

that is, average conflict across statements coincides with average bilateral closeness.

Proof.

$$\sum_{s=1}^{S}\left(2-\sum_{\substack{i,j\in K:\\ i\neq j}}(2-c_{\mathrm{ij}}^{s})\binom{|K|}{2}^{-1}\right)(2S)^{-1}=\sum_{s=1}^{S}\left(\sum_{\substack{i,j\in K:\\ i\neq j}}c_{\mathrm{ij}}^{s}\binom{|K|}{2}^{-1}\right)(2S)^{-1}=\sum_{\substack{i,j\in K:\\ i\neq j}}\binom{|K|}{2}^{-1}\underbrace{\left(\sum_{s=1}^{S}c_{\mathrm{ij}}^{s}(2S)^{-1}\right)}_{=\mathrm{bilclos}_{ij}}$$

□

The average approach is simple but does not take into account any negotiation process. As a next step, we interpret coalitional negotiation as sequential process of bilateral negotiation on each statement. Specifically, we analyze *conflict paths* instead of independent conflict points.

4.2 Conflict Graph and Conflict Potential

Let $K \subseteq N$ be a coalition of parties. Let $p^K := \{ij | i, j \in K, i \neq j$ be the link set of the complete graph with node-set K. p^K could be interpreted as the set of all (bilateral) negotiation possibilities within K. We consider the difference between possible and actual *overall conflict*, that is, conflict across the complete negotiation graph in coalition K. This can be interpreted as conflict *potential* within K. For this, first note that maximal overall conflict within coalition K is much lower than maximal bilateral conflict times the number of pairs in K:

Theorem 1. *For each statement $s = 1,\ldots,S$, the maximal overall conflict within a coalition $K \subseteq N$ is given by $\left\lfloor \frac{|K|^2}{2} \right\rfloor$ which is smaller than $2\cdot\binom{|K|}{2}$ for all $|K| > 2$ (and equal for $|K| = 2$).*

Proof. Let $K \subseteq N$ be a coalition and consider any statement $s = 1,\ldots,S$. Let K_+, K_-, and K_0 denote the subcoalitions of parties with position “yes”, “no”, and “neutral”, respectively. The conflict value between parties of the same position, i.e. within $K_+/K_-/K_0$, is 0. The conflict value and between parties with weakly opposite positions, i.e. between any member of K_0 and any member of K_+ or K_- is 1. The conflict value and between parties with strongly opposite positions, i.e. between any member of K_+ and any member K_- is 2. As we analyze the complete conflict graph, any party is connected to any other. Therefore, the conflict value of the complete graph (overall conflict) is given by $1\cdot|K_0|\cdot(|K_+|+|K_-|)+2\cdot|K_+|\cdot|K_-|$ which gets maximal for

- $|K_+| = |K_-| = \frac{|K|}{2}$ and $|K_0| = 0$ with value $\frac{|K|^2}{2}$, if $|K|$ is even
- $|K_+| = \frac{|K|}{2} \pm 0.5$, $|K_-| = \mp\frac{|K|}{2}$ and $|K_0| = 1$ with value $\frac{|K|^2-1}{2}$, if $|K|$ is odd

Now note that $2\cdot\binom{|K|}{2} = 2\cdot\frac{1}{2}|K|(|K|-1) = |K|^2-|K| > \frac{|K|^2}{2}$ for $|K| > 2$ (and equal for $|K| = 2$). □

The reason for the bound of overall conflict is that for more than two parties, there must always be at least a weak consensus. This can be interpreted as follows: on a specific issue, there are not more than two *completely* opposite positions and if there are more than two parties, some parties will have at least weakly similar opinions.

Definition 3 (Conflict Potential Closeness)**.** *Let $K \subseteq N$ be a coalition of parties. For any state-*

ment s, the overall conflict within K is given by

$$\sum_{\substack{i,j\in K:\\ i\neq j}} (2-c^s_{ij})$$

with maximum possible value of $\left\lfloor \frac{|K|^2}{2} \right\rfloor$*. Accordingly,* conflict potential closeness of K *is given by*

$$CPclos_K := \sum_{s=1}^{S} \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor - \sum_{\substack{i,j\in K\\ i\neq j}} \left(2-c^s_{ij}\right) \right) \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \cdot S \right)^{-1}$$

where we normalize by maximal overall conflict across statements to obtain an index on a normalized scale (unit interval).

Lemma 2. *We have*

$$CPclos_K = \left(\left\lceil \frac{|K|}{2} \right\rceil \right)^{-1} - 1 + \left(2 - \left(\left\lceil \frac{|K|}{2} \right\rceil \right)^{-1} \right) AVCclos_K$$

which specifically yields $CPclos_K = \frac{3}{2}AVCclos_k - \frac{1}{2}$ *for* $|K| \in \{3,4\}$.

Proof.

$$\begin{aligned}
\mathrm{CPclos}_K &= \sum_{s=1}^{S} \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor - \sum_{\substack{i,j\in K\\ i\neq j}} \left(2-c^s_{\mathrm{ij}}\right) \right) \left(\left\lfloor \frac{|K|^2}{2} \cdot S \right\rfloor \right)^{-1} \\
&= 1 - \sum_{\substack{i,j\in K\\ i\neq j}} \sum_{s=1}^{S} \frac{2}{S} \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \right)^{-1} + \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \right)^{-1} \cdot 2 \sum_{\substack{i,j\in K\\ i\neq j}} \frac{1}{2S} c^s_{\mathrm{ij}} \\
&= 1 - \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \right)^{-1} \cdot 2 \cdot \frac{1}{2} |K|\,(|K|-1) + \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \right)^{-1} \cdot 2 \cdot \tfrac{1}{2} |K|(|K|-1) \cdot \mathrm{AVCclos}_K \qquad (*) \\
&= 1 - |K|(|K|-1) \left(\left\lfloor \frac{|K|^2}{2} \right\rfloor \right)^{-1} (1 - \mathrm{AVCclos}_K) \\
&= \begin{cases} 1 - \frac{2|K|(|K|-1)}{|K|^2} (1 - \mathrm{AVCclos}_K) & , |K| \text{ even} \\ 1 - \frac{2|K|(|K|-1)}{|K|^2-1} (1 - \mathrm{AVCclos}_K) & , |K| \text{ odd} \end{cases} \\
&= \begin{cases} 1 - \frac{2(|K|-1)}{|K|} + \frac{2(|K|-1)}{|K|} \mathrm{AVCclos}_K & , |K| \text{ even} \\ 1 - \frac{2|K|}{|K|+1} + \frac{2|K|}{|K|+1} \mathrm{AVCclos}_K & , |K| \text{ odd} \end{cases} \\
&= \begin{cases} \frac{2}{|K|} - 1 + (2 - \frac{2}{|K|}) \mathrm{AVCclos}_K & , |K| \text{ even} \\ \frac{2}{|K|+1} - 1 + (2 - \frac{2}{|K|+1}) \mathrm{AVCclos}_K & , |K| \text{ odd} \end{cases} \\
&= \left(\left\lceil \frac{|K|}{2} \right\rceil \right)^{-1} - 1 + \left(2 - \left(\left\lceil \frac{|K|}{2} \right\rceil \right)^{-1} \right) \mathrm{AVCclos}_K
\end{aligned}$$

where $(*)$ uses Lemma 1. Specifically, this yields $\mathrm{CPclos}_K = \frac{3}{2}\mathrm{AVCclos}_k - \frac{1}{2}$ for $|K| \in \{3,4\}$ □

CPclos could be seen as a rather pessimistic approach as it considers all possible conflict points relative to the upper bound of overall conflict. First of all one could argue, that coalitional negotiation might not pass *all* conflict points as there might be a mediating party, that is, the use of the *complete* negotiation path is too pessimistic. Furthermore, embedding the upper bound of conflict in coalitions right at the outset could be argued as ignoring the existence of "weak consensus".

On the other hand, embedding upper bounds rules out imperatively positive values and, hence, provides a measure on the *whole* unit interval: note that

$$\text{AVCclos}_K \in \left[\frac{|K|-2}{2(|K|-1)}, 1\right] \text{ if } |K| \text{ even, and } \text{AVCclos}_K \in \left[\frac{|K|-1}{2|K|}, 1\right] \text{ if } |K| \text{ odd.}$$

4.3 Path of Least Conflict Closeness

We now interpret the negotiation process as a path of bilateral negotiations. Let $K \subseteq N$ be a coalition of parties and let $p^K := \{ij | i, j \in K, i \neq j$ be the link-set of the complete negotiation graph on K. Parties i and j are called connected in $p \subseteq p^K$, if there exists a sequence $\{ik_1, k_1k_2, \ldots k_mj\} \in p$. Such a p can be interpreted as a *(negotiation) path* which depicts negotiation between party i and k_1, then between k_1 and k_2, ..., and then between k_m and j. Any negotiation path $p \in p^K$ induces a partition of K consisting of connected (negotiating) components $\mathcal{C}(p) = \{C_1, C_2, \ldots C_m\}$ with $C_k \cap C_l = \emptyset \, \forall \, k, l = 1 \ldots, m$ and $\bigcup_{k=1}^{m} C_k = K$. Path $p \subseteq p^K$ is called a connecting path, if $\mathcal{C}(p) = \{K\}$. Such a connecting path can be interpreted as coalitional negotiation in K, since all $i \in K$ are part of the path.

Definition 4 (Path of Least Conflict)**.** *Let K be a coalition. For any statement $s \in S$, the set of* paths of least conflict (PLC) *is given by*

$$PLC_K^s := \underset{\substack{p \subseteq p^K: \\ p \text{ connects } K}}{argmin} \left\{ \sum_{ij \in p} (2 - c_{ij}^s) \right\}$$

and the (unique) conflict value of PLC_K^s is given by

$$plc_K^s := \min_{\substack{p \subseteq p^K: \\ p \text{ connects } K}} \left\{ \sum_{ij \in p} (2 - c_{ij}^s) \right\}.$$

Note that any such $p \in PLC$ must be of a certain form. More precisely, it must consist of subpaths connecting all parties with the same position to "merged blocks" and then connecting the blocks. Let $\{K_+, K_-, K_0\}$ denote the partition of coalition K into "merged blocks" of parties with position "yes", "no", and "neutral", respectively. We have

$$plc_K^s = \begin{cases} 0, & \text{if } K \in \{K_+, K_-, K_0\} \\ 1, & \text{if } (K_+, K_0 \neq \emptyset \wedge K_- = \emptyset) \vee (K_-, K_0 \neq \emptyset \wedge K_+ = \emptyset) \\ 2, & \text{otherwise, i.e. } K_+ \neq \emptyset \wedge K_- \neq \emptyset \end{cases}$$

Therefore, a path of least conflict (itself) is not unique but it's *conflict value* is uniquely determined.

Remark 2. *Note that PLC does not distinguish between a negotiation of two completely opposite blocks directly (conflict value of 2) and a negotiation where a neutral block mediates (conflict* *value of 1+1).*

Again, we interpret closeness as the difference between possible and actual conflict. However, in contrast to CPclos_K, we do not embed the upper conflict bound of a PLC of 2 (which yields a measure very close to CPclos_K). Instead, we use maximal conflict times the length of a minimal connecting path, that is, $2\,(|K|-1)$. This can be interpreted as maximal conflict in a minimal negotiation sequence.

Definition 5 (Path of Least Conflict Closeness)**.** *Let $K \subseteq N$ be a coalition of parties. The* path of least conflict closeness of K *is given by*

$$PLCclos_K := \sum_{s=1}^{S} \left(2\,(|K|-1) - plc_K^s\right)\left(2\,(|K|-1)\cdot S\right)^{-1}$$

where we normalize by the maximal conflict value on a minimal connecting path across statements to obtain an index on a normalized scale (unit interval).

PLC-Closeness could be seen as rather optimistic and does not seem suitable for large coalitions as its minimum possible value increases (and even approaches 1 for $|K| \to \infty$). However, for coalitions of size 3 it seems a suitable approach emphasizing the possibility of mediating parties.

Lemma 3. *For $|K| = 3$ we have*

$$PLCclos_K = \tfrac{1}{4} + \tfrac{3}{4} avclos_K.$$

Proof. Let $|K| = 3$, that is $K = \{i, j, k\}$. For any statement s we have

$$\mathrm{plc}_K^s = 2 - c_{\mathrm{ij}}^s + 2 - c_{\mathrm{ik}}^s + 2 - c_{\mathrm{jk}}^s - \max\left\{2 - c_{\mathrm{ij}}^s, 2 - c_{\mathrm{ik}}^s, 2 - c_{\mathrm{jk}}^s\right\} = 3 - \frac{c_{\mathrm{ij}}^s + c_{\mathrm{ik}}^s + c_{\mathrm{jk}}^s}{2}$$

which yields

$$\begin{aligned}\mathrm{PLCclos}_K &= \sum_{s=1}^{S} \left(4 - \mathrm{plc}_K^s\right)\frac{1}{4\cdot S} = \sum_{s=1}^{S}\left(1 + \frac{c_{\mathrm{ij}}^s + c_{\mathrm{ik}}^s + c_{\mathrm{jk}}^s}{2}\right)\frac{1}{4\cdot S} \\ &= \frac{1}{4} + \frac{3}{4}\sum_{s=1}^{S}\left(\frac{1}{3}c_{\mathrm{ij}}^s + \frac{1}{3}c_{\mathrm{ik}}^s + \frac{1}{3}c_{\mathrm{jk}}^s\right)\frac{1}{2\cdot S} = \frac{1}{4} + \frac{3}{4}\mathrm{avclos}_K.\end{aligned}$$

□

Note that the term $\frac{1}{4}$ could be seen as the value corresponding to the existence of weak consensus in coalitions with more than 2 parties.

Example 1. *Let $S = 4$ and consider a coalition of three parties $K = \{1, 2, 3\}$. Table 2 presents the positions for the four statements and the (statement-specific) coalitional consensus value according to the average-, path-of-least-conflict, and conflict-potential approach, respectively. For comparison, values are normalized on a $[0, 2]$ scale on statement level.*
We see that PLC yields the highest concensus values and CP, as the rather pessimistic approach, the lowest. PLC emphasizes consensus within a group of the same position (parties 1 and 2 in statement 3) or consensus due to mediating parties (party 2 in statement 4) while CP emphasizes possible conflict points (parties 1/2 against party 3 in statement 3 and party 1 against party 3 in

Table 2: Comparison on ([0, 2] scale)

Positions				Coalitional Consensus		
Statement	Party 1	Party 2	Party 3	AV	PLC	CP
1	agree	agree	agree	2	2	2
2	agree	agree	neutral	1.33	1.5	1
3	agree	agree	not agree	0.67	1	0
4	agree	neutral	not agree	0.67	1	0

statement 4). Coalitional closeness according the average, PLC and CP approach yields 58.33%, 68.75%, and 37.50%, respectively.

5 Performance: An analysis of German elections

5.1 Adjustments and Example

Political Spectrum in Germany
Most common parties in German federal and state parliaments (no special order):

- Union (federal elections only): alliance between
  CDU: Christian Democratic Union (liberal-conservative) and
  CSU: Christian Social Union in Bavaria (liberal-conservative, Bavaria only)
- SPD: Social Democratic Party (social-democratic)
- FDP: Free Democratic Party (liberal/classic liberal)
- Grüne: Alliance90/The Greens
- Linke: Left Party (democratic-socialist)
  merger of the former Party of Democratic Socialism (PDS) and the Electoral Alternative for Labour and Social Justice (WASG)
- AfD: Alternative for Germany (right-wing populist)
- Piraten: The Pirate Party (information society/social-liberal)

Incompatibility Adjustments
Our approach is based on the assumption, that every winning coalition S could form a potential government (i.e., every winning coalition is credible), where the probability of materialization is given by μ_S. However, there might be incompatibilities which rule out certain theoretically possible government formations. If there exist such incompatibilities, this can be taken into account by adjusting μ accordingly (i.e., set $\mu_S = 0$). If there exist global incompatibilities (i.e. certain parties which have been excluded as a coalitional partner from all other parties or that self-excluded a government position), this can also be taken into account by adjusting N (while seat shares must stay relative to the original overall seats).

In German parliaments, global incompatibilities are commonly observed. We have only adjusted the model if such incompatibilities have been common knowledge prior to election (e.g. due to coalitional statements). This has always been the case if either the Pirate Party (Piraten; information society/social-liberal) or Alternative for Germany (AfD; right-wing populist) have been part of the parliament. Furthermore, the Left Party (Linke, democratic-socialist) has been excluded for the 2017 federal parliament elections for the 2015 state parliament election in Hamburg.

We only adjusted for a bilateral incompatibility once: after the 2011 state parliament election in Berlin, exploratory talks between the Social Democratic Party (SPD) and Alliance90/The Greens (Grüne) failed due to disagreements on a certain topic. Therefore, we adjusted the model accordingly (i.e., set $\mu_S = 0$ for all coalitions containing aforementioned parties).

Minority Government

We had to adjust the model for the 2010 state parliament election in North-Rhine-Westphalia, as the government has been build by a minority of seats.

Ministerial Portfolio

Our analysis covers portfolio allocations of ministerial positions only and does *not* count the prime minister/chancellor position, as this is allocated to the party with highest seat share by established norm. However, there are cases where the prime minister position is taken in to account as, for example, in Saarland where prime minister Annegret Kramp-Karrenbauer also holds a ministerial position.

Example 1: 2005 Federal Parliament Election

Table 8 shows the distribution of seats after the 2005 Federal Parliament Election in Germany.

Table 3: Distribution of seats 2005 Federal Parliament Election, Germany

	Union	SPD	FDP	Grüne	PDS	Sum
Seats	226	222	61	51	54	614
in100%	**36.81**	**36.16**	**9.93**	**8.31**		91.21

Finally, the government was build by the "Grand Coalition" {Union, SPD} with Angela Merkel as the new chancellor, resulting in "Cabinet Merkel I" with 7 ministerial positions for Union and 8 for SPD - that is, equally many cabinet positions for both parties and (even) more ministerial positions for the "weaker" SPD.

Prior to election, the PDS self-excluded government participation (and has been excluded as a coalitional partner from all other parties). Therefore, we adjust for global incompatibility.

Then, potential winning coalitions are {Union, SPD}, {Union, SPD, FDP}, {Union, FDP, Grüne}, {SPD, FDP, Grüne} and {Union, SPD, FDP, Grüne} = $\tilde{N}$ (while the latter does not influence μ as all parties are part of $\tilde{N}$). We obtain

$$
\begin{aligned}
m^{\mu}_{\text{Union}} &= \mu_{\{\text{SPD,FDP,Grüne}\}} \\
m^{\mu}_{\text{SPD}} &= \mu_{\{\text{Union,FDP,Grüne}\}} \\
m^{\mu}_{\text{FDP}} &= \mu_{\{\text{Union,SPD}\}} + \mu_{\{\text{Union,SPD,Grüne}\}} \\
m^{\mu}_{\text{Grüne}} &= \mu_{\{\text{Union,SPD}\}} + \mu_{\{\text{Union,SPD,FDP}\}}
\end{aligned}
$$

VAA data (Wahl-o-Mat) can be found in the appendix (Table 15). Corresponding bilateral closeness and ideological coalitional closeness of credible coalitions are presented in Table 4 and Table 5, respectively. This yields bargaining power as presented in Table 6, where "NoWeight" simply uses a closeness of 1 for each coalition. Distribution of 7 + 8 = 15 positions can be found can be found in Table 7.

Table 4: Similarity Matrix - BilClos

	Union	SPD	FDP	Grüne
Union	1.0000	0.3667	0.5333	0.3000
SPD	0.3667	1.0000	0.5333	0.6333
FDP	0.5333	0.5333	1.0000	0.5000
Grüne	0.3000	0.6333	0.5000	1.0000

Table 5: Ideological coalitional closeness of credible coalitions

Coalition	AVCclos	CPclos	PLCclos
{Union, SPD}	0.3667	0.3667	0.3667
{Union, SPD, FDP}	0.4778	0.2167	0.6083
{Union, SPD, Grüne}	0.4333	0.1500	0.5750
{Union, FDP, Grüne}	0.4444	0.1667	0.5833
{SPD, FDP, Grüne}	0.5556	0.3333	0.6667

Table 6: Bargaining Power

	Union	SPD	FDP	Grüne
NoWeight	0.333	0.333	0.167	0.167
AVCclos	0.278	0.347	0.193	0.183
CPclos	0.237	0.474	0.153	0.136
PLCclos	0.283	0.323	0.2	0.193

We see that the closeness-approaches assign higher bargaining power to the SPD due to an ideologically more central position with respect to conflict analysis.

Note that the ConflictPotential approach does not perform well with respect to portfolio allocation in this example (which is not the case in general). However, it selects the "Grand Coalition" {Union, SPD} - the actual government - as the coalition with highest coalitional closeness (cf. Table 5) while both the Average and PathofLeastConflict approach select the "Traffic Light Coalition" {SPD, FDP, Grüne}.

Example 2: 2016 State Parliament Election Rhineland-Palatinate
Table 8 shows the distribution of seats after the 2016 State Parliament Election Rhineland-Palatinate in Germany.
Finally, the government was build by the "Traffic-Light Coalition" {SPD, FDP, Grüne} with Malu Dreyer (SPD) as ongoing prime minister and deputy Volker Wissing (FDP). The resulting "Cabinet Dreyer II" consisted of 5 ministerial positions for SPD and both 2 positions for FDP and Grüne.

Table 7: Portfolio Allocation

	Union	SPD
Actual	7	8
Gamson's Law	7.57	7.43
NoWeight	7.5	7.5
AVCclos	6.67	8.33
CPclos	5	10
PLCclos	7	8

Table 8: Distribution of seats 2016 State Parliament Election Rhineland-Palatinate

	CDU	SPD	FDP	Grüne	AfD	Sum
Seats	35	39	7	6	14	614
in100%	**34.65**	**38.61**	**6.93**	**5.94**		86.14

Prior to election, the AfD has been excluded as a coalitional partner from all other parties. Therefore, we adjust for global incompatibility.

Then, potential winning coalitions are $\{\text{CDU}, \text{SPD}\}$, $\{\text{CDU}, \text{SPD}, \text{FDP}\}$, $\{\text{CDU}, \text{SPD}, \text{Grüne}\}$, $\{\text{SPD}, \text{FDP}, \text{Grüne}\}$ and $\{\text{CDU}, \text{SPD}, \text{FDP}, \text{Grüne}\} = \tilde{N}$ (while the latter does not influence μ as all parties are part of $\tilde{N}$). Note that SPD is pivotal as there is no potential winning coalition without SPD. We obtain

$$
\begin{aligned}
&m^{\mu}_{\text{CDU}} = 1 + \mu_{\{\text{SPD,FDP,Grüne}\}} \\
&m^{\mu}_{\text{SPD}} = 1 \\
&m^{\mu}_{\text{FDP}} = 1 + \mu_{\{\text{CDU,SPD}\}} + \mu_{\{\text{CDU,SPD,Grüne}\}} \\
&m^{\mu}_{\text{Grüne}} = 1 + \mu_{\{\text{CDU,SPD}\}} + \mu_{\{\text{CDU,SPD,FDP}\}}
\end{aligned}
$$

VAA data (Wahl-o-Mat) can be found in the appendix (Table 16). Corresponding bilateral closeness and ideological coalitional closeness of credible coalitions are presented in Table 9 and Table 10, respectively. This yields bargaining power as presented in Table 11, where "NoWeight" again simply uses a closeness of 1 for each coalition. Distribution of $5+2+2=9$ positions can be found can be found in Table 12.

Table 9: Similarity Matrix RP - BilClos

	CDU	SPD	FDP	Grüne
CDU	1.0000	0.6447	0.6974	0.3947
SPD	0.6447	1.0000	0.5789	0.7500
FDP	0.6974	0.5789	1.0000	0.4079
Grüne	0.3947	0.7500	0.4079	1.0000

We see that - in contrast to Gamson's Law - our closeness-approaches assign the correct portfolio allocation. Furthermore, the approaches using VAA data assign slightly more bargaining

Table 10: Ideological coalitional closeness of credible coalitions RP

Coalition	AVCclos	CPclos	PLCclos
{CDU, SPD}	0.6447	0.6447	0.6447
{CDU, SPD, FDP}	0.6404	0.4605	0.7303
{CDU, SPD, Grüne}	0.5965	0.3947	0.6974
{SPD, FDP, Grüne}	0.5789	0.3684	0.6842

Table 11: Bargaining Power

	CDU	SPD	FDP	Grüne
NoWeight	0.231	0.462	0.154	0.154
AVCclos	0.252	0.397	0.177	0.174
CPclos	0.271	0.371	0.182	0.176
PLCclos	0.243	0.410	0.175	0.172

Table 12: Portfolio Allocation

	SPD	FDP	Grüne
Actual	5	2	2
Gamson's Law	6.75	1.21	1.04
NoWeight	5.40	1.80	1.80
AVCclos	4.78	2.13	2.09
CPclos	4.58	2.25	2.18
PLCclos	4.87	2.08	2.05

power to the FDP (who placed the prime minister's deputy). Note that standard bargaining approaches would not be able to distinguish between FDP and Grüne.

Note that only the "optimistic" PathofLeastConflict approach selects the "Traffic Light Coalition" {SPD, FDP, Grüne} as the coalition with highest coalitional closeness (cf. Table 10) while both the Average and ConflictPotential approach select the "Grand Coalition" {CDU, SPD}.

5.2 Performance: Whole Data

We analyze the performance of our new approach with respect to both Gamson's law and the actual portfolio allocation by an empirical analysis of 31 cabinets from 2005 to 2018. Our analysis covers Germany's most recent federal parliaments (cabinet Merkel I - IV from 2005, 2009, 2013 & 2017) as well as the most recent cabinets in Baden-Wuerttemberg (2006, 2011 & 2016), Bavaria (2018), Berlin (2006, 2011 & 2016), Bremen (2007, 2011 & 2015), Hamburg (2008 & 2015)[4], Lower Saxony (2008 & 2013)[5], North-Rhine-Westphalia (2005, 2010, 2012 & 2017), Rhineland-Palatinate (2011 & 2016), Saarland (2012 & 2017), Saxony (2014), Saxony-Anhalt (2016), Schleswig-Holstein (2012 & 2017), and Thuringia (2014). Portfolio allocation due to Gamson's Law, the Banzhaf-Index (for cmparison) and the new approaches AVClos, CPclos, PLCclos and NoWeight (closeness of 1) can be found in the appendix.

Table 13 presents cumulative errors: SAE presents the sum of absolute errors with respect to the actual distribution. # 1^{st}min presents the number of cases in which the minimal error (sum per election) occurred and # $1^{st}/2^{nd}/3^{rd}$min presents the number of cases in which at least the third minimal error (sum per election) occured.

Table 13: Absolute Errors (w.r.t. actual distribution)

	SAE1	# 1^{st}min	# $1^{st}/2^{nd}/3^{rd}$min
Gamson	34.85	11	17
Banzhaf	58.82	1	12
NoWeight	53.29	2	11
AVCclos	35.19	2	22
CPclos	38.39	9	18
PLCclos	37.35	6	17

Gamson's Law yields the lowest sum of absolute errors (accross all elections) and the best proxy with respect to (first) minimal error. The closeness-approaches yield a comparably low sum of absolute errors (even more in comparison with the standard bargaining approaches) and the ConflictPotential approach performs comparably with respect to (first) minimal error. In order to compare predictive power, we now analyze corresponding (rounded) portfolio allocation. Table 14 presents the number of cases, in which the correct portfolio allocation, respectively best proxy for the correct portfolio allocation, respectively equal allocation to Gamson's Law is found.

In total, the correct portfolio allocation is found in 27 out of 31 cases. The CPclos approach (19 correct, 21 best proxy) seems to perform equally good compared to Gamson's Law (18 correct, 20 best proxy), but also the other two conflict approaches perform comparably well and much better than standard approaches (Banzhaf and NoWeight) do. Merging the conflict approaches

[4] The 2011 state parliament election in Hamburg has been excluded due to single-party government.

[5] The 2018 state parliament election in Lower Saxony could not be analyzed as no Wahl-o-Mat has been provided.

Table 14: Proxy-Performance for Portfolio Allocation

	Correct Proxy	Best Proxy	Gamson's Law
Gamson	18	20	
Banzhaf	4	7	8
NoWeight	4	8	9
AVCclos	15	18	14
CPclos	19	21	15
PLCclos	16	20	12

yields the correct proxy of portfolio allocation in 25 and the best available proxy in 29 out of 31 cases.

Compared to the portfolio allocation suggested by Gamson's Law, the ConflictPotential approach confirms Gamson's Law in 15 cases and performs better with respect to actual allocation in 8 of the missing cases. The PathofLeastConflict approach confirms in 12 cases and performs better in 10 of the missing cases. The AverageConflict approach confirms in 14 cases and performs better in 8 of the missing cases. Merging the conflict approaches, we are able to confirm the allocation proposed by Gamson's law in 20 out of 31 elections. In 9 out of the missing 11 elections, our approach performs better than Gamson's law regarding the actual portfolio allocation, leaving only 2 elections where we fail to confirm (Saarland 2012 and Saxony-Anhalt 2016).

6 Conclusion

This paper studies the impact of content specific conflict on closeness of coalitions and presents an index to assign portfolio payoffs. We suggest portfolio allocation by relative weakness proportionality: interpreting ideological closeness of potential winning coalitions as a measure of materialization probability yields a measure of weakness for corresponding non-member parties. Then, bargaining power is proportional to a party's relative weakness. Since we consider all potential winning coalitions as credible, our approach accounts for the parties pivotal ability.

To derive (ideological) coalitional closeness, we present different attempts on conflict paths in the political network, interpreting closeness as the difference between potential and actual conflict among election specific issues. For the latter, we use data from the "Wahl-O-Mat", a Voting Advice Application (VAA) for elections in Germany. Therefore, our approach can be seen as an attempt towards factual content in theoretical indices.

An empirical analysis of portfolio allocation after German elections shows that our index confirms Gamson's law in 20 out of the 31 elections. In 9 out of the missing 11 elections, our approach performs better than Gamson's law regarding the actual portfolio allocation, leaving only 2 elections where we fail to confirm or do better. A confirmation rate of 91 % (20 out of 31-9=22) for Gamson's Law, respectively 93.5% (20+9=29 out of 31) for the best available proxy, might be seen as closing the gap between Gamson's Law and theory.

APPENDIX

Table 15: Self-placed positions for the 2005 Wahl-o-mat

Statement	Union	SPD	FDP	Grüne
1	-1	-1	-1	0
2	1	-1	1	-1
3	-1	1	-1	1
4	1	-1	1	-1
5	-1	1	1	1
6	1	-1	-1	-1
7	-1	1	-1	1
8	0	1	-1	1
9	-1	-1	1	-1
10	-1	1	-1	-1
11	1	-1	-1	1
12	-1	1	0	1
13	1	1	1	1
14	-1	1	-1	-1
15	1	-1	-1	-1
16	1	1	-1	1
17	0	1	1	-1
18	-1	1	-1	1
19	1	-1	-1	-1
20	-1	-1	-1	1
21	0	1	-1	-1
22	-1	0	-1	-1
23	1	0	-1	-1
24	-1	0	1	1
25	1	-1	0	-1
26	-1	1	1	1
27	-1	-1	-1	1
28	1	1	0	-1
29	-1	-1	-1	1
30	-1	1	1	1

Table 16: Self-placed positions for the 2016 RP Wahl-o-mat

These	CDU	SPD	FDP	Grüne
1	-1	0	-1	1
2	1	1	1	-1
3	1	-1	-1	-1
4	1	1	-1	1
5	1	0	1	-1
6	-1	1	-1	1
7	-1	1	-1	1
8	-1	1	-1	1
9	1	0	1	-1
10	-1	-1	-1	-1
11	1	-1	1	-1
12	-1	1	1	1
13	1	1	-1	0
14	-1	-1	-1	-1
15	0	1	-1	1
16	-1	1	0	1
17	-1	-1	1	-1
18	0	0	-1	1
19	1	1	1	1
20	1	1	1	1
21	1	1	1	1
22	-1	-1	0	-1
23	1	0	1	-1
24	1	1	1	-1
25	1	1	-1	1
26	1	1	1	-1
27	0	-1	0	-1
28	1	-1	-1	-1
29	-1	0	-1	1
30	1	1	0	1
31	1	-1	1	-1
32	1	1	1	1
33	-1	-1	-1	1
34	-1	1	1	1
35	1	1	1	1
36	1	1	-1	-1
37	1	1	1	-1
38	1	1	1	1

PORTFOLIO ALLOCATION: DATA

The next tables present the actual ministerial distribution as well as portfolio payoffs due to Gamson's Law, the Banzhaf-Index and the new approaches AVClos, CPclos, PLCclos and NoWeight (closeness of 1). For comparison, portfolio payoffs are also rounded.

	BW 16		BW 11		BW 06		Bayern 18	
	Grüne	CDU	Grüne	SPD	CDU	FDP	CSU	FW
Actual	6	5	6	7	9	2	10	3
Gamson	5.81	5.19	6.59	6.41	9.04	1.96	9.87	3.13
Banzhaf	5.50	5.50	6.50	6.50	8.25	2.75	11.38	1.63
NoWeight	5.50	5.50	6.50	6.50	8.25	2.75	11.38	1.63
AVCclos	5.71	5.29	5.94	7.06	7.58	3.42	10.16	2.84
CPclos	5.98	5.02	5.70	7.30	8.52	2.48	9.33	3.67
PLCclos	5.63	5.37	6.03	6.97	7.32	3.68	10.49	2.51
Rounded								
Actual	6	5	6	7	9	2	10	3
Gamson	6	5	7	6	9	2	10	3
Banzhaf	6	6	7	7	8	3	11	2
NoWeight	6	6	7	7	8	3	11	2
AVCclos	6	5	6	7	8	3	10	3
CPclos	6	5	6	7	9	2	9	4
PLCclos	6	5	6	7	7	4	10	3

	Berlin 06		Berlin 11		Berlin 16		
	SPD	Linke	SPD	CDU	SPD	Grüne	Linke
Actual	5	3	4	4	4	3	3
Gamson	5.58	2.42	4.37	3.63	4.13	2.93	2.93
Banzhaf	6.00	2.00	3.20	4.80	4.12	2.94	2.94
NoWeight	6.00	2.00	2.67	5.33	4.29	2.86	2.86
AVCclos	6.12	1.88	3.21	4.79	4.43	2.80	2.77
CPclos	6.69	1.31	3.58	4.42	4.65	2.70	2.65
PLCclos	5.89	2.11	3.06	4.94	4.31	2.86	2.84
Rounded							
Actual	5	3	4	4	4	3	3
Gamson	6	2	4	4	4	3	3
Banzhaf	6	2	3	5	4	3	3
NoWeight	6	2	3	5	4	3	3
AVCclos	6	2	3	5	4	3	3
CPclos	7	1	4	4	5	3	3
PLCclos	6	2	3	5	4	3	3

	Bremen 07		Bremen 11		Bremen 15	
	SPD	Grüne	SPD	Grüne	SPD	Grüne
Actual	6	2	6	3	6	3
Gamson	5.57	2.43	5.68	3.32	6.14	2.86
Banzhaf	5.00	3.00	5.63	3.38	6.19	2.81
NoWeight	5.00	3.00	6.00	3.00	6.43	2.57
AVCclos	5.18	2.82	5.98	3.02	6.62	2.38
CPclos	5.89	2.11	6.67	2.33	7.00	2.00
PLCclos	4.98	3.02	5.78	3.22	6.44	2.56
Rounded						
Actual	6	2	6	3	6	3
Gamson	6	2	6	3	6	3
Banzhaf	5	3	6	3	6	3
NoWeight	5	3	6	3	6	3
AVCclos	5	3	6	3	7	2
CPclos	6	2	7	2	7	2
PLCclos	5	3	6	3	6	3

	Bundestag 05		Bundestag 09		Bundestag 13		Bundestag 17	
	Union	SPD	Union	FDP	Union	SPD	Union	SPD
Actual	7	8	10	5	9	6	9	6
Gamson	7.57	7.43	10.80	4.20	9.26	5.74	9.25	5.75
Banzhaf	7.50	7.50	11.25	3.75	11.25	3.75	9.38	5.63
NoWeight	7.50	7.50	11.25	3.75	11.25	3.75	10.00	5.00
AVCclos	6.67	8.33	10.51	4.49	8.95	6.05	9.14	5.86
CPclos	5.00	10.00	10.50	4.50	8.63	6.38	8.60	6.40
PLCclos	7.00	8.00	10.46	4.54	9.07	5.93	9.38	5.62
Rounded								
Actual	7	8	10	5	9	6	9	6
Gamson	8	7	11	4	9	6	9	6
Banzhaf	8	8	11	4	11	4	9	6
NoWeight	8	8	11	4	11	4	10	5
AVCclos	7	8	11	4	9	6	9	6
CPclos	5	10	10	5	9	6	9	6
PLCclos	7	8	10	5	9	6	9	6

	Hamburg 08		Hamburg 15		Niedersachsen 08		Niedersachsen 13	
	CDU	GAL	SPD	Grüne	CDU	FDP	SPD	Grüne
Actual	5	4	7	3	7	2	5	4
Gamson	7.41	1.59	7.95	2.05	7.56	1.44	6.39	2.61
Banzhaf	6.75	2.25	8.75	1.25	7.88	1.13	4.50	4.50
NoWeight	6.75	2.25	8.00	2.00	7.88	1.13	4.50	4.50
AVCclos	5.53	3.47	7.56	2.44	7.36	1.64	5.30	3.70
CPclos	5.44	3.56	7.48	2.52	7.23	1.77	5.71	3.29
PLCclos	5.56	3.44	7.61	2.39	7.26	1.74	5.16	3.84
Rounded								
Actual	5	4	7	3	7	2	5	4
Gamson	7	2	8	2	8	1	6	3
Banzhaf	7	2	9	1	8	1	5	5
NoWeight	7	2	8	2	8	1	5	5
AVCclos	6	3	8	2	7	2	5	4
CPclos	5	4	7	3	7	2	6	3
PLCclos	6	3	8	2	7	2	5	4

	NRW 05		NRW 10		NRW 12		NRW 17	
	CDU	FDP	SPD	Grüne	SPD	Grüne	CDU	FDP
Actual	9	2	8	3	9	3	8	3
Gamson	9.69	1.31	8.16	2.84	9.28	2.72	7.92	3.08
Banzhaf	8.25	2.75	7.33	3.67	10.50	1.50	6.88	4.13
NoWeight	8.25	2.75	7.00	4.00	9.60	2.40	7.33	3.67
AVCclos	8.29	2.71	7.17	3.83	8.94	3.06	7.55	3.45
CPclos	9.69	1.31	7.83	3.17	8.81	3.19	8.07	2.93
PLCclos	7.88	3.12	6.96	4.04	9.00	3.00	7.36	3.64
Rounded								
Actual	9	2	8	3	9	3	8	3
Gamson	10	1	8	3	9	3	8	3
Banzhaf	8	3	7	4	11	2	7	4
NoWeight	8	3	7	4	10	2	7	4
AVCclos	8	3	7	4	9	3	8	3
CPclos	10	1	8	3	9	3	8	3
PLCclos	8	3	7	4	9	3	7	4

	RP 11		RP 16			Saarland 12		Saarland 17	
	SPD	Grüne	SPD	FDP	Grüne	CDU	SPD	CDU	SPD
Actual	6	3	5	2	2	4	3	4	3
Gamson	6.30	2.70	6.75	1.21	1.04	3.69	3.31	4.10	2.90
Banzhaf	4.50	4.50	6.43	1.29	1.29	3.50	3.50	5.25	1.75
NoWeight	4.50	4.50	5.40	1.80	1.80	3.50	3.50	4.67	2.33
AVCclos	4.94	4.06	4.78	2.13	2.09	2.77	4.23	4.16	2.84
CPclos	5.53	3.47	4.58	2.25	2.18	2.51	4.49	4.16	2.84
PLCclos	4.78	4.22	4.87	2.08	2.05	2.87	4.13	4.16	2.84
Rounded									
Actual	6	3	5	2	2	4	3	4	3
Gamson	6	3	7	1	1	4	3	4	3
Banzhaf	5	5	7	1	1	4	4	5	2
NoWeight	5	5	5	2	2	4	4	5	2
AVCclos	5	4	5	2	2	3	4	4	3
CPclos	6	3	5	2	2	3	4	4	3
PLCclos	5	4	5	2	2	3	4	4	3

	Schleswig-Holstein 12			Schleswig-Holstein 17		
	SPD	Grüne	SSW	CDU	FDP	Grüne
Actual	4	2	1	3	2	2
Gamson	4.40	2.00	0.60	3.98	1.43	1.59
Banzhaf	3.50	2.63	0.88	3.89	1.56	1.56
NoWeight	3.11	2.33	1.56	3.89	1.56	1.56
AVCclos	3.17	2.32	1.52	3.63	1.70	1.67
CPclos	3.56	2.05	1.39	3.57	1.76	1.67
PLCclos	3.04	2.40	1.57	3.62	1.71	1.67
Rounded						
Actual	4	2	1	3	2	2
Gamson	4	2	1	4	1	2
Banzhaf	4	3	1	4	2	2
NoWeight	3	2	2	4	2	2
AVCclos	3	2	2	4	2	2
CPclos	4	2	1	4	2	2
PLCclos	3	2	2	4	2	2

	Sachsen 14		SachsenAnhalt 16			Thüringen 14		
	CDU	SPD	CDU	SPD	Grüne	Linke	SPD	Grüne
Actual	7	3	6	2	1	4	3	2
Gamson	7.66	2.34	5.87	2.15	0.98	5.48	2.35	1.17
Banzhaf	8.75	1.25	6.43	1.29	1.29	3.86	3.86	1.29
NoWeight	8.00	2.00	5.40	1.80	1.80	3.32	3.32	2.37
AVCclos	7.12	2.88	4.13	2.43	2.44	3.11	3.51	2.39
CPclos	6.94	3.06	3.75	2.61	2.64	2.93	3.81	2.26
PLCclos	7.21	2.79	4.30	2.34	2.35	3.16	3.41	2.43
Rounded								
Actual	7	3	6	2	1	4	3	2
Gamson	8	2	6	2	1	5	2	1
Banzhaf	9	1	6	1	1	4	4	1
NoWeight	8	2	5	2	2	3	3	2
AVCclos	7	3	4	2	2	3	4	2
CPclos	7	3	4	3	3	3	4	2
PLCclos	7	3	4	2	2	3	3	2

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