A Rationale for the Clientele Effect in Money Management[1]

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Abstract. This paper proposes a rational explanation for the existence of clientele effects under commonly used portfolio management contracts. Contrary to the common view that investors always benefit from a manager's market timing skill (private information about future market returns), we show that the value of a manager's private information to an investor can be negative when the investor is sufficiently more risk-averse than the manager. This suggests different clienteles for skilled and unskilled funds. Investors in skilled funds are uniformly more risk-tolerant than investors in unskilled funds. We examine the effects of the manager's skill level, contract parameters, and market conditions on an investor's fund choice. Our results suggest that the investors who are sufficiently more risk-averse than the manager should include fulcrum fees in the contract to benefit from the skilled manager's information advantage.

JEL Classification: G11, G12, G23

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1. Introduction

The money management industry plays a very important role in modern economies. For example, about half of all U.S. households delegate the management of their wealth to professional managers and the total net assets of U.S. mutual funds reached $18.7 trillion at

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year-end 2017 (Investment Company Institute, 2018). Given the size of the money management industry, studying the implications of portfolio management skills on investors' fund investment appears to be a critical task. A substantial literature finds empirical evidence that the mutual fund investors chase performance (e.g., Chevalier and Ellison, 1997; Sirri and Tufano, 1998). Berk and Green (2004) employ a model of competitive capital market and rational learning to explain the fund-performance relationship. They argue that fund managers with superior skills will manage more money but have the same fund returns as less-skilled managers because of decreasing return to scale. Furthermore, Berk and van Binsbergen (2015) use the value a mutual fund extracts from capital markets to measure the fund's skill and find the evidence of investment skill. They also find that investors appear to be able to identify the managers with superior abilities and invest more money in better funds. However, one thing that has been missing from the traditional performance evaluation and fund flow literature is the heterogeneity in investors' preferences and its effect on the segmentation of fund investors, i.e., the clientele effect. In markets where segmentation is caused by clientele effects, the fund's superior performance will not necessarily attract more money if stochastic dominance relations prevail among funds for certain groups of investors. In this case, certain risk-aversion types of investors will prefer not to invest in a skilled fund despite its superior performance.

A growing literature shows that fund investors appear to segment the market and demonstrates the importance of clientele effects in fund performance evaluation. Blackburn et al. (2009) find that there are different investor clienteles in value and growth funds and risk aversion is an important attribute to differentiate these two groups of investors. They document that investors in value funds are more risk-averse than investors in growth funds. Moreover, Chan et al. (2002) find that growth managers have better abilities to generate alpha than value managers. Given the empirical evidence for clientele effects based on the heterogeneity in investor's risk aversion and manager's abilities, it should be investigated whether there is a rational explanation for the clientele effects in the money management industry as an alternative for behavioral interpretations such as investor sophistication (Barber et al., 2016).

Our paper studies the value of a manager's market timing skill to fund investors with heterogeneous risk preferences.[2] The clientele effect emerges as an endogenous result. We

[2] Despite the widespread belief that mutual fund managers lack skill (e.g., Carhart, 1997; Fama and French, 2010), there is a growing number of studies that do find evidence of market timing skill (e.g., Mamaysky et al., 2008; Elton et al., 2012; Kacperczyk et al., 2008).

model a skilled fund manager as endowed with privately informed information about the future market returns whose content is unknown to the investors.[3] This paper focuses on market timing skill for ease of presentation. In unreported results, we consider both stock selection and market timing skills in a multi-asset setting, and the clientele effect results are qualitatively similar. The anticipative information is always valuable to the manager and increases in the information precision (i.e., skill). However, when the manager and the investors exhibit different risk preferences, the private nature of this information can be costly and even adverse to the investors. We show that the investors whose risk aversion lies above a threshold value would prefer the unskilled fund to the skilled one. Thus, there are two distinct clienteles to skilled and unskilled funds. Investors in skilled funds are uniformly more risk-tolerant than investors in unskilled funds.

We also analyze the impacts of commonly used portfolio management contracts on investors' fund investment. The management fee is typically a portion of the delegated wealth's value. In addition to this purely proportional fee contract, the compensation schemes including a performance-based fee that depends on the excess return of the managed portfolio relative to a benchmark are common in the money management industry. In the absence of performance fees, we find that investors whose relative risk aversion exceeds the relative prudence coefficient of the logarithmic fund managers always prefer the unskilled fund to the skilled fund irrespective of excess returns' measures like alpha and Sharpe ratios generated by the skilled fund. Conversely, investors with relative risk aversion smaller than the manager's relative prudence will choose the skilled funds. The clientele effect result still holds under the fulcrum and asymmetric performance fees. In contrast to the constant threshold under the purely proportional fee, in the presence of performance fees the relative risk aversion thresholds are affected by the skill, contract parameters, and market conditions. The comparative static analysis shows that the relative risk aversion threshold is substantially affected by the sensitivity of the contract with regard to the underperformance penalty. We find that including a fulcrum fee in the manager's compensation contract could lead to a higher value of manager's information to sufficiently risk-averse investors than that under option-like asymmetric fees. This suggests the use of fulcrum fee for investors who are much more risk-averse than the fund manager.

[3] This paper does not consider the effort incentive problem and thus abstract from moral hazard consideration. We assume that investors can observe managers' skill level and risk preference. Koijen (2014) shows that the manager's skill and risk preference parameters can be estimated using the volatility of fund returns.

Our analysis proceeds in two steps. First, we derive and analyze optimal portfolio choices of a fund manager with private information. The informed manager receives a private signal about future market excess returns with noise.[4] In the presence of management fees, the investment problem is no longer maximizing the manager's utility function of terminal fund portfolio, but rather a composed utility function of fund performance. In particular, with asymmetric performance fees, the composed utility function is neither concave nor differentiable in the terminal value of fund portfolio. We employ the concavification technique pioneered by Aumann and Perles (1965), Carpenter (2000), Cuoco and Kaniel (2011) and Bichuch and Sturm (2014) to solve the manager's maximization problem. In the second step of the analysis, we study the value of the manager's information to investors and the clientele effect. Following Detemple and Rindisbacher (2013) (henceforth DR), we show that the public state price density (SPD) second-order stochastically dominates the private SPD. The clientele effect result follows from the second-order stochastic dominance relationship and the fact that the composed utility function of investors (i.e., the utility derived from delegation to the manager as function of the SPD) may be concave or convex in the SPD depending on the investors' risk aversion is larger or smaller than a threshold value. We specialize our general results to the noisy return forecast model and conduct a comparative static analysis to understand the impacts of performance fees on investors' preference between skilled and unskilled funds in an empirically relevant setting. Finally, we discuss extensions of results to more general settings that fund managers have constant relative risk aversion (CRRA) preference and investors can invest in the market index, as an alternative to investing in the active funds.

Our paper contributes to the growing literature on the clientele effect in the money management industry. Clientele effects are of great interest to research in behavioral finance. Prior studies attribute the clientele effects to irrationality, investor sophistication or other psychological tendencies (e.g., Barberis and Shleifer, 2003; Del Guercio and Reuter, 2014). Compared to behavioral attributes that may or may not determine fund clienteles, to derive the emergence of clienteles endogenously based purely on risk aversion is a viable rational alternative and as such of first order importance. In this paper, we show that the investor clientele in a skilled fund is more risk-tolerant than the investor clientele in an unskilled fund. This result is consistent with some recent empirical findings. Bergstresser et al. (2009) detail

[4] The manager's skill is from his private information. Henceforth, informed is used interchangeably with skilled.

the difference between broker-sold investors and self-directed investors. They state that the broker-sold investors are a bit more risk-averse than self-directed investors. Del Guercio and Reuter (2014) document that the mutual fund market is a segmented market catering to two distinct types of investors: direct-sold investors and broker-sold investors. Their results also suggest that direct-sold fund managers are more skilled than broker-sold fund managers because direct-sold funds have stronger incentives to hire managers with superior abilities.

Second, our analysis is related to the delegated portfolio management literature. Existing theoretical research on delegated portfolio management focuses on two main areas. The first strand of literature studies how commonly observed compensation contracts affect manager' decisions (e.g., Grinblatt and Titman, 1989; Carpenter, 2000; Hugonnier and Kaniel, 2010). The second examines the optimal contract design problem (e.g., Admati and Pfleiderer, 1997; Li and Tiwari, 2009). We complement this literature by considering a different problem. Rather than solving for the optimal design of contracts in general, we analyze the impacts of performance fees on the value of the manager's private information to investors. Grinblatt and Titman (1989) argue that contracts should be designed with caps and have penalties for performance below the benchmark to mitigate the adverse risk incentives of managers. However, the manager is allowed to have a personal portfolio and hedge the management fees in their model. Even without the unrealistic assumption that the manager has personal accounts, our results suggest that the investors who are sufficiently more risk-averse than the manager may include a fulcrum fee component in the manager's compensation contract to realize higher value from the manager's superior information.

Finally, our paper is closely related to DR. They develop a structural dynamic model of market timing and find that individuals with relative risk aversion greater than the relative risk prudence of a log manager will never prefer the skilled fund. However, their model does not consider the presence of compensation contracts between investors and fund managers. Sotes-Paladino and Zapatero (2017) find that fulcrum fees are able to help align the risk preference of investors and managers, which may distort the clientele effect results. It is thus of importance to study how the performance fees would affect investors' fund choice. Our paper extends the result of DR by showing that the clientele effect still holds even in the presence of performance fees. Moreover, DR only consider log managers and investors need to make a binary choice between skilled and unskilled actively managed funds. In extensions to our basic setup, we consider settings that both investors and managers have CRRA utility and investors can choose to invest in a passive alternative. We show that the clientele effect still exists and it is affected by the fee structures, market conditions, and the manager's skill level. The comparative analysis shows that if the types of these contracts are not properly chosen, the manager's private information would be costly and even detrimental to some fund investors. Our results suggest that highly risk-averse investors should employ skilled fund managers with a linear performance-based contract in order to benefit from the manager's superior abilities.

The article is organized as follows. Section 2 describes the economic setup. It also presents the investment problems of managers and investors. Section 3 solves the portfolio optimization problem of an informed fund manager under commonly observed performance contracts. Section 4 analyzes the value of manager's private information and the clientele effect. Section 5 specializes the general results to the noisy return forecast model. Section 6 provides a detailed numerical analysis of the value of manager's information and investor's fund choice under fulcrum and asymmetric performance fees. Section 7 studies extensions of the basic model. Conclusions are in Section 8.

2. Model

This section describes the economic setup and the portfolio management problems of managers and investors. Financial markets and the information structure are described in Section 2.1. Section 2.2 introduces the private information price of risk. The agents and their risk preference are described in 2.3. The manager's optimization problem is described in Section 2.5. Section 2.6 describes the investor's problem.

2.1 FINANCIAL MARKETS AND INFORMATION STRUCTURE

Financial markets are represented by a risky market portfolio (or stock) and a riskless bond. We work with the following model of timing information considered by DR. The instantaneous market excess return (dR^m_v) and the gross market excess return ($S^m_{\tau_{i-1},\tau_i}$) over a period $[\tau_{i-1}, \tau_i)$ are given by

$$dR^m_v = \sigma^m_v(\theta^m_v dv + dW^m_v) \quad \text{and} \quad S^m_{\tau_{i-1},\tau_i} = \exp\left(\int_{\tau_{i-1}}^{\tau_i} dR^m_v - \frac{1}{2}\int_{\tau_{i-1}}^{\tau_i} (\sigma^m_v)^2 dv\right) \qquad (1)$$

where σ^m is positive and bounded away from zero. The volatility coefficient σ^m, the public market price of risk θ^m, and the interest rate r are stochastic process adapted to public information. The bond's price dynamics are given by $dB_v = r_v B_v dv$.

An informed agent has access to private information about future market excess returns. Her private information can be represented by the filtration

$$\mathcal{G}_{(\cdot)} = \mathcal{F}^m_{(\cdot)} \bigvee \mathcal{F}^Y_{(\cdot)} \tag{2}$$

where $\mathcal{F}^m_{(\cdot)}$ represents the public information generated by market excess returns dR^m, and $\mathcal{F}^Y_{(\cdot)}$ represents the filtration generated by a private signal Y. We assume that the private signal has the general anticipative form

$$Y_v \equiv \sum_{i=1}^{N} G_i \mathbb{1}_{[\tau_{i-1},\tau_i)}(v), \tag{3}$$

where τ_i is a sequence of deterministic dates with $\tau_0 = 0, \tau_N \leq T$, $\mathbb{1}_{[\tau_{i-1},\tau_i)}(v)$ equals 1 if $v \in [\tau_{i-1}, \tau_i)$ otherwise 0, and the private signal for the period $[\tau_{i-1}, \tau_i)$ is given by

$$G_i \equiv g\left(S^m_{\tau_{i-1},\tau_i}, \zeta_i\right) \tag{4}$$

for some function g and random variable ζ_i, independent of the public information $\mathcal{F}^m_T$. The independent random variable ζ_i introduces noise into the private signal G_i, thus ruling out arbitrage opportunities for the informed agent within the period $[\tau_{i-1}, \tau_i)$. The private signal works as follows. At time τ_i, the informed agent observes the signal realization $G_{i+1} \equiv g\left(S^m_{\tau_i,\tau_{i+1}}, \zeta_{i+1}\right)$ and obtains the anticipative information about the gross market excess return for the period $[\tau_i, \tau_{i+1})$. As time elapses, the informed agent learns from public information about realized market returns but the signal G_{i+1} remains valuable to her even if the time is very close to τ_{i+1}. At τ_{i+1}, a new signal realization is observed and the process repeats. In this way the informed agent maintains her information advantage against those who only have public information about market realized returns.

As in reality that active funds report their realized returns with a typically coarse schedule, we assume that informed agents only need to report their realized fund returns at τ_0 and τ_N for the period $[\tau_0, \tau_N]$. $\mathcal{F}_v = \mathcal{F}^m_v \otimes \mathcal{F}^a_{\tau_0}$ denotes the public information at time $v \in [\tau_0, \tau_N)$, where $\mathcal{F}^m_v$ is the filtration generated by realized market returns and $\mathcal{F}^a_{\tau_0}$ is the filtration generated by previously reported fund return. At time τ_N, $\mathcal{F}_{\tau_N} = \mathcal{F}^m_{\tau_N} \otimes \mathcal{F}^a_{\tau_N}$. We suppose that the investment evaluation period coincides with the time interval between the reporting dates, i.e., $T = \tau_N - \tau_0$.

2.2 INFORMATION PREMIUM

An agent with only public information has a premium per-unit risk denoted by the public market price of risk θ^m. The private (anticipative) information changes the price of risk by the private information price of risk (PIPR)

$$\theta_v^{\mathcal{G}} \equiv \frac{1}{\sigma_v^m} \lim_{\epsilon \downarrow 0} \frac{1}{\epsilon} E\left[\int_v^{v+\epsilon} dR_t^m \,\middle|\, \mathcal{G}_v \right] - \theta_v^m = \lim_{\epsilon \downarrow 0} \frac{1}{\epsilon} E\left[\int_v^{v+\epsilon} dW_t^m \,\middle|\, \mathcal{G}_v \right] \tag{5}$$

for all $v \in [0, T]$. PIPR represents the incremental price of risk, relative to θ_v^m, due to private information. The informed agent's total price of risk is thus given by $\theta_v \equiv \theta_v^m + \theta_v^{\mathcal{G}}$. When the agent has no private information, $\mathcal{G}_{(\cdot)} = \mathcal{F}_{(\cdot)}^m$ and the PIPR is null. If the agent is endowed with perfect foresight, the PIPR explodes and there exists an arbitrage opportunity. Noisy private information will lead to a PIPR with finite value and there is no arbitrage opportunity.

The market excess returns have representation $dR_v^m = \sigma_v^m((\theta_v^m + \theta_v^{\mathcal{G}})dv + dW_v^{\mathcal{G}})$ under $\mathcal{G}_{(\cdot)}$, where $dW_v^{\mathcal{G}} \equiv dW_v^m - \theta_v^{\mathcal{G}} dv$ is a Brownian motion relative to the private filtration $\mathcal{G}_{(\cdot)}$.

2.3 AGENTS

We consider an economy populated by three types of agents: investors, a skilled fund manager, and an unskilled fund manager. All agents are price-takers. The investors and the unskilled fund manager have only access to public information, while the skilled fund managers have private information about future market returns $\mathcal{G}_{(\cdot)}$. We assume that an investor at time 0 needs to make a choice between delegating the portfolio management of her wealth to either the skilled fund manager or the unskilled fund manager. No additional share purchases or redemptions are allowed during the whole investment period. The fund managers do not to have any private wealth and derive utility from the management fees received at terminal date.

We assume that both skilled and unskilled fund managers have logarithmic utility function $u^M(x) = \log(x)$ and fund investors have CRRA utility with different coefficients of relative risk aversion R. Let $\mathcal{U}$ be the class of CRRA utility functions

$$u(x) = \begin{cases} \frac{x^{1-R}}{1-R} & \text{if } R > 0, R \neq 1 \\ \log(x) & \text{if } R = 1 \end{cases}.$$

Section 7 also examines the case of managers who have CRRA utility function with $R \neq 1$.

2.4 MANAGER'S COMPENSATION CONTRACT

As is standard in practice, we assume that a manager is compensated at time T with a management fee F_T which depends on the end-of-period value of the fund portfolio and the end-of-period value of a benchmark portfolio. Let X_T^a represent the value assets under management (AUM) at the terminal date T. We assume that the compensation of the manager is of the general form as introduced by Cuoco and Kaniel (2011)

$$\begin{aligned} F_T &= F\left(X_T^a, X_T^b; \alpha, \beta_1, \beta_2, \delta, \pi^b\right) \\ &= \alpha X_T^a - \alpha\beta_1 X_0^a \left(\frac{X_T^a}{X_0^a} - \frac{X_T^b}{X_0^b}\right)^- + \alpha\beta_2 X_0^a \left(\frac{X_T^a}{X_0^a} - \frac{X_T^b}{X_0^b}\right)^+ \\ &= \alpha X_T^a - \alpha\beta_1 \left(X_T^a - \delta X_T^b\right)^- + \alpha\beta_2 \left(X_T^a - \delta X_T^b\right)^+, \end{aligned}$$

where α, β_1, β_2, π^b are exogenously given parameters, $\delta = X_0^a/X_0^b$. The management fee at time T consist of three parts: a regular fee αX_T^a which is proportional to the value of the fund portfolio at time T, a performance bonus $\alpha\beta_2 \left(X_T^a - \delta X_T^b\right)^+$ which depends on the excess return of the managed fund over the benchmark, and an underperformance penalty $\alpha\beta_1 \left(X_T^a - \delta X_T^b\right)^-$. We assume that $\alpha > 0$, $\beta_2 \geq \beta_1 \geq 0$. This ensures that F is increasing and convex in the fund portfolio's end-of-period value X_T^a and decreasing in the benchmark portfolio's end-of-period value X_T^b. The benchmark portfolio process X_T^b is generated by a dynamic trading strategy π_v^b, which is known to all market participants.

The contract specification is general enough to encompass typical fee structures for different types of investment companies. When the performance bonus is symmetric to the underperformance penalty, $\beta_1 = \beta_2$, the performance fee is linear in the excess return of the actively managed fund relative to the benchmark. It is known as a "fulcrum" fee. The 1970 Amendment to the Investment Advisers Act of 1940 rules that the U.S. mutual fund's performance fees must be the "fulcrum" type. The SEC approved the use of asymmetric performance fees in contracts for investment advisers of wealthy individuals in 1985. A recent study by Ma et al. (2016) argues that, even though the advisory contracts between the asset management companies and fund investors are prohibited from having asymmetric incentive fees, the compensation incentive contracts for portfolio managers are not subject to this regulatory restriction. They document that typical compensation contracts signed by the U.S. mutual fund managers are the asymmetric, option-like type. Hedge funds are not subject to the fulcrum fee requirement, and asymmetric performance fees $\beta_1 = 0, \beta_2 > 0$ are the norm. Performance-based fees were also allowed by the Labor Department for corporate pension funds in 1986 (see Cuoco and Kaniel, 2011).

2.5 MANAGER'S PROBLEM

We consider a portfolio optimization problem of a privately informed fund manager. The problem of an uninformed manager with only public information is a special case with the PIPR $\theta^{\mathcal{G}}$ is null. Managers receive an initial endowment X_0^a from investors and choose admissible trading strategies (written $\pi_v^a \in \mathcal{G}_v$) to maximize the expected utility function of her management fee. The informed manager's problem is given by

$$\max_{\pi_v^a \in \mathcal{G}_v} E\left[u^M\left(F\left(X_T^a, X_T^b\right)\right) \middle| \mathcal{G}_0\right] \tag{6}$$

subject to

$$dX_v^a = X_v^a r_v dv + X_v^a \pi_v^a \sigma_v^m ((\theta_v^m + \theta_v^{\mathcal{G}})dv + dW_v^{\mathcal{G}}), \tag{7}$$

$$X_v^a \geq 0, \quad \forall v \in [0, T]. \tag{8}$$

Note that all the coefficients are adapted to the to the private filtration $\mathcal{G}_{(\cdot)}$, the manager's investment problem collapses to a traditional portfolio optimization problem.

2.6 INVESTOR'S PROBLEM

We assume that investors cannot directly invest in the financial markets and need to employ a fund manager. Suppose that investors can observe the manager's skill level and risk preference. At time 0, an investor makes a choice between delegating his wealth to either the skilled manager or the unskilled manager based on only public information. The decision to delegate is exogenous. It captures in a reduced form the choice to abstain from direct investing because of participation constraint, transaction costs or other frictions. The composed utility function of an investor who delegates his wealth to a fund manager is $v\left(X_T^{a^*}, X_T^b\right) \equiv u\left(X_T^{a^*} - F\left(X_T^{a^*}, X_T^b\right)\right)$, where $X_T^{a^*}$ is the optimal fund value at time T chosen by the fund manager and $F\left(X_T^{a^*}, X_T^b\right)$ is the management fee paid to the manager. A fund investor maximizes the expected value of his derived utility by solving

$$\max_{\mathscr{C}\in\{s,u\}} E\left[v\left(X_T^{a,\mathscr{C}^*}, X_T^b\right)\right], \tag{9}$$

where X_T^{a,s^*} is the optimal terminal fund value chosen by the skilled fund manager and X_T^{a,u^*} is the optimal terminal fund value chosen by the unskilled fund manager. Extensions to the basic model consider a more general case that, alternatively to employing active managers, investors can choose a passively managed index fund.

3. Manager's Optimal Portfolio Policies

This section solves an informed manager's portfolio optimization problem under "fulcrum" and "asymmetric" performance fees.

The portfolio optimization problem (6) can be restated in the static form (see Pliska, 1986; Karatzas et al., 1987; Cox and Huang, 1989, 1991):

$$\max_{X_T^a\in\mathcal{G}_T} E\left[u^M\left(F\left(X_T^a, X_T^b\right)\right) \,\middle|\, \mathcal{G}_0\right] \tag{10}$$

subject to

$$E\left[\xi_T^{\mathcal{G}} X_T^a \,\middle|\, \mathcal{G}_0\right] \le X_0^a \tag{11}$$

and non-negativity constraints in (8).

Unless $\beta_1 = \beta_2$, the fund managers' objective function $u^M\left(F\left(X_T^a, X_T^b\right)\right)$ is neither concave nor differentiable in X_T^a; it cannot be solved using the usual approach. On the other hand, the fund manager's marginal utility at zero is negative infinity, which implies that the management fee $F\left(X_T^a, X_T^b\right)$ must be strictly positive at time T. It follows that $X_T^a > \underline{X}(X_T^b)$, where $\underline{X}(X^b) = \beta_1\delta X^b/(1+\beta_1)$. The objective function $u^M(F(\cdot, X^b))$ is piecewise concave and piecewise differentiable on the interval $[\underline{X}(X^b), \infty)$, we can follow Aumann and Perles (1965), Carpenter (2000), Cuoco and Kaniel (2011) and Bichuch and Sturm (2014) in constructing the concavification $v^M(\cdot, X^b)$ of $u^M(F(\cdot, X^b))$ (that is the smallest concave function that dominates $u^M(F(\cdot, X^b))$ for all $X^a \geq \underline{X}(X^b)$. Lemma 1 and 2 below are closely based on Lemma 1 and 2 in Cuoco and Kaniel (2011).

Lemma 1. *Suppose that* $X^b > 0$, $\alpha > 0$, $\beta_2 > \beta_1 \geq 0$, *there exist unique* $X_1(X^b)$ *and* $X_2(X^b)$ *with*

$$\underline{X}(X^b) < X_1(X^b) < \delta X^b < X_2(X^b)$$

that satisfy the system of equations

$$\begin{cases} \alpha(1+\beta_2)u_x^M(F(X_2(X^b), X^b) = \frac{u^M(F(X_2(X^b),X^b)) - u^M(F(X_1(X^b),X^b))}{X_1(X^b) - X_2(X^b)}, \\ (1+\beta_1)u_x^M(F(X_1(X^b), X^b)) = (1+\beta_2)u_x^M(F(X_2(X^b), X^b)). \end{cases}$$

In particular, if marginal utility is homogeneous of degree $-R$ $(R \neq 1)$, *letting* $\eta = \left(\frac{1+\beta_2}{1+\beta_1}\right)^{1-1/R}$, *direct computation shows that*

$$X_1(X^b) = \left(\frac{(\frac{\eta}{R} - 1)\frac{\beta_1}{1+\beta_1} + \eta(1 - \frac{1}{R})\frac{\beta_2}{1+\beta_2}}{\eta - 1} \right) \delta X^b,$$
$$X_2(X^b) = X_1(X^b) + \frac{1}{R}\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1} \right) \delta X^b.$$

For logarithmic utility

$$X_1(X^b) = \left(log\left(\frac{1+\beta_2}{1+\beta_1}\right)\right)^{-1} \left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1} \right) \delta X^b + \frac{\beta_1}{1+\beta_1}\delta X^b > \underline{X}(X^b),$$
$$X_2(X^b) = X_1(X^b) + \left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1} \right) \delta X^b.$$

Lemma 2. *Suppose that* $X^b > 0$, *let* $X_1(X^b)$ *and* $X_2(X^b)$ *be as in Lemma 1 if* $\alpha > 0$, $\beta_2 > \beta_1 \geq 0$ *and* $X_1(X^b) = X_2(X^b) = \delta X^b$ *if* $\alpha > 0, \beta_1 = \beta_2 \geq 0$. *The concavified objective function* $v^M(\cdot, X^b)$ *of* $u^M(F(\cdot, X^b))$ *on* $[\underline{X}(X^b), \infty)$ *is given by*

$$v^M(X^a, X^b) = \begin{cases} u^M(F(X^a, X^b)) & \text{if } X \in Y(X^b), \\ u^M(F(X_1(X^b), X^b)) & \\ +\alpha(1+\beta_2)u_x^M(F(X_2(X^b), X^b))(X^a - X_1(X^b)) & \text{otherwise}, \end{cases}$$

where $Y(X^b) = [\underline{X}(X^b), X_1(X^b)] \bigcup [X_2(X^b), \infty)$.

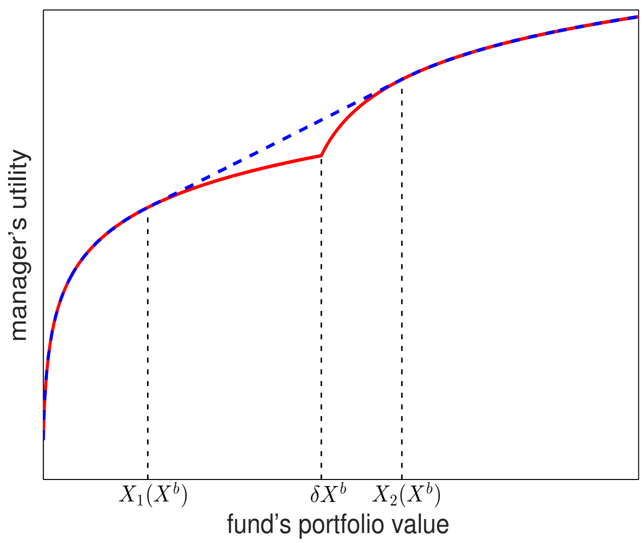

Fig. 1: Manager's composed utility function and the concavified function. The figure plots the manager's composed utility function $u^M(F(\cdot, X^b))$ (red solid line) and the corresponding concavified utility function $v^M(\cdot, X^b)$ (dashed blue line) with $\alpha > 0$, $\beta_2 > \beta_1 \geq 0$.

As illustrated in Figure 1, the concavified objective function $v^M(\cdot, X^b)$ in Lemma 2 replaces part of the original non-concave function $u^M(F(\cdot, X^b))$ with a chord between $X_1(X^b)$ and $X_2(X^b)$. The slope of the chord coincides with the slope of $u^M(F(\cdot, X^b))$ at $X_1(X^b)$ and $X_2(X^b)$, which makes the function $v^M(\cdot, X^b)$ concave. $Y(X^b)$ denotes the interval in which $v^M(\cdot, X^b)$ and $u^M(F(\cdot, X^b))$ coincide. The intuition is that because the chord between $X_1(X^b)$ and $X_2(X^b)$ lies above the true objective function $u^M(F(\cdot, X^b))$, a fund portfolio's value that takes on the value $X_1(X^b)$ in some states or $X_2(X^b)$ in other states will dominate a fund portfolio's value that takes on values in the interval $(X_1(X^b), X_2(X^b))$ in some states with positive probability. Thus, the manager will never choose a fund's asset value that lies in $(X_1(X^b), X_2(X^b))$.

Since the new objective function $v^M(\cdot, X^b)$ is concave, we can use the standard method to solve the portfolio choice problem. The solution to the concavified problem also solves the original portfolio optimization problem. The solution is described formally as follows.

Proposition 1. *Suppose that the performance fee is of the fulcrum type:*

$$F(X_T^a, X_T^b) = \alpha X_T^a + \alpha\beta_2 \left(X_T^a - \delta X_T^b\right) \quad \textit{with} \quad \alpha > 0, \beta_2 \geq 0.$$

The optimal weight invested in risky asset and optimal fund value at $v \in [0,T]$ are given by

$$\pi_v^{a,s^*} = \frac{\theta_v^m}{\sigma_v^m} + \frac{\beta_2}{1+\beta_2}\frac{\delta X_v^b}{X_v^{a,s^*}}\left(\pi_v^b - \frac{\theta_v^m}{\sigma_v^m}\right) + \left(1 - \frac{\beta_2}{1+\beta_2}\frac{\delta X_v^b}{X_v^{a,s^*}}\right)\frac{\theta_v^{\mathcal{G}}}{\sigma_v^m}, \tag{12}$$

$$= \frac{\theta_v^m + \theta_v^{\mathcal{G}}}{\sigma_v^m} + \frac{\beta_2}{1+\beta_2}\frac{\delta X_v^b}{X_v^{a,s^*}}\left(\pi_v^b - \frac{\theta_v^m + \theta_v^{\mathcal{G}}}{\sigma_v^m}\right), \tag{13}$$

$$X_v^{a,s^*} = \frac{1}{y^{s^*}\xi_v^{\mathcal{G}}} + \frac{\beta_2 \delta X_v^b}{1+\beta_2}, \tag{14}$$

where $y^{s^*} = (1+\beta_2)/X_0^a$ and $\theta_v^{\mathcal{G}}$ is the PIPR (5). Correspondingly, the manager's compensation at time T is

$$F(X_T^{a,s^*}, X_T^b) = \frac{\alpha X_0^a}{\xi_T^{\mathcal{G}}}. \tag{15}$$

Private information updates the informed manager's perceived price of risk from θ^m to $\theta^m + \theta^{\mathcal{G}}$. The first two components in (12) are motivated by public information and the remaining component in (12) is motivated by private information. If the private signal is uninformative about future market excess returns, $\theta^{\mathcal{G}} = 0$ and the optimal policy collapses to that of a manager with public information. Private signals could induce either positive or negative PIPR. When $E[dW_v^m|\mathcal{G}_v]$ is positive (negative), the PIPR is positive (negative) and the privately informed manager would invest more (less) in the market index than that of an uninformed manager. The instantaneous fund excess return generated by the informed manager is given by $d^- R_v^a = \pi_v^{a,s^*} d^- R_v^m$, where $d^- R_v^m$ represents forward integration (see Russo and Vallois, 1993). When private information induces a positive (negative) PIPR, the optimally managed fund return's volatility then increases (decreases) relative to that of a fund based only on public information. Since fund returns are only reported at the τ_0, τ_N and informed manager's trades are unobserved, this will not reveal private information to the uninformed manager.

The optimal policy can also be decomposed into two components as in Equation (13). The first component is the mean-variance demand and represents the manager's optimal risk taking absent the fulcrum performance fee. The second component is the benchmark-hedging demand. It could either be positive or negative, depending on whether the benchmark portfolio's weight in the risky asset π^b is higher or lower than the mean-variance demand $(\theta_v^m + \theta_v^{\mathcal{G}})/\sigma_v^m$. This component helps the manager perfectly hedge her risk exposure to the benchmark portfolio. As a result, the presence and composition of the benchmark are irrelevant to the manager's compensation at time T as given by (15). The manager's compensation is affected by the contract only through the proportional fee parameter α. Although the benchmark-linked incentive parameters β_2 and π^b do not affect the manager's compensation, they, together with the proportional fee parameter α, have an impact on the expected utility of fund investors' after-fee wealth. This will be analyzed in details in Section 4.

In line with Sotes-Paladino and Zapatero (2017), we find that the proportional fee parameter α does not affect the manager's portfolio choice. This is in contrast to the prior literature (e.g., Admati and Pfleiderer, 1997) with CARA utility function, in which the proportional fees could affect the manager's optimal risk exposure. Stronger benchmark-linked incentive β_2 leads to larger benchmark-hedging demand, which could be either a long or short position. A higher fraction of the benchmark portfolio invested in the market index increases the manager's optimal risk exposure.

Proposition 2. *Suppose that the performance fee is asymmetric:*

$$F\left(X_T^a, X_T^b\right) = \alpha X_T^a - \alpha\beta_1\left(X_T^a - \delta X_T^b\right)^- + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)^+ \quad \text{with } \alpha > 0, \beta_2 > \beta_1 \geq 0.$$

The optimal end-of-period fund value at T is given by

$$X_T^{a,s^*} = \frac{1}{y^{s^*}\xi_T^{\mathcal{G}}} + \frac{\beta_2 \delta X_T^b}{1+\beta_2}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} \leq \Psi(X_T^b)\}} + \frac{\beta_1 \delta X_T^b}{1+\beta_1}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} > \Psi(X_T^b)\}}, \tag{16}$$

where $\Psi\left(X_T^b\right) = \frac{\log\left(\frac{1+\beta_2}{1+\beta_1}\right)}{\delta X_T^b\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}$ *and y^{s^*} is a Lagrangian multiplier solving*

$$E\left[\xi_T^{\mathcal{G}} X_T^{a,s^*} \,\middle|\, \mathcal{G}_0\right] = X_0^a.$$

The fund manager's compensation at time T is given by

$$F(X_T^{a,s^*}, X_T^b) = \frac{\alpha(1+\beta_2)}{y^{s^*}\xi_T^{\mathcal{G}}}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} \leq \Psi(X_T^b)\}} + \frac{\alpha(1+\beta_1)}{y^{s^*}\xi_T^{\mathcal{G}}}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} > \Psi(X_T^b)\}}. \tag{17}$$

The first component in (16) corresponds to the optimal fund value absent performance fees ($\beta_1 = \beta_2 = 0$). The remaining two components in (16) are induced by the asymmetric performance fees, whose values depend on whether the normalized SPD $y^{s^*}\xi_T^{\mathcal{G}}$ is larger than $\Psi\left(X_T^b\right)$ or not. The optimal fund value at time T is thus a piecewise function of the normalized SPD $y^{s^*}\xi_T^{\mathcal{G}}$ and the end-of-period benchmark portfolio value X_T^b. Optimal end-of-period fund value X_T^{a,s^*} is greater than $X_2(X_T^b)$ and decreasing in $y^{s^*}\xi_T^{\mathcal{G}}$ until $y^{s^*}\xi_T^{\mathcal{G}}$ hits $\Psi\left(X_T^b\right)$, then X_T^{a,s^*} jumps from $1/(y^{s^*}\xi_T^{\mathcal{G}}) + \beta_2\delta X_T^b/(1+\beta_2)$ to $1/(y^{s^*}\xi_T^{\mathcal{G}}) + \beta_1\delta X_T^b/(1+\beta_1)$.

When the normalized SPD $y^{s^*}\xi_T^{\mathcal{G}}$ is smaller than $\Psi\left(X_T^b\right)$, the managed portfolio outperforms the given benchmark portfolio and the manager receives a relatively high compensation $\alpha(1+\beta_2)/(y^{s^*}\xi_T^{\mathcal{G}})$. Conversely, when the normalized SPD $y^{s^*}\xi_T^{\mathcal{G}}$ is larger than $\Psi\left(X_T^b\right)$, the managed portfolio underperforms the benchmark and the manager's compensation is then $\alpha(1+\beta_1)/(y^{s^*}\xi_T^{\mathcal{G}})$. In contrast to the fulcrum type fee case, the manager cannot completely hedge the risk induced by the asymmetric performance fees by moving up or down the risky asset in the portfolio. All the non-linear contract parameters will affect the manager's compensation as well as the derived utility of investors' after-fee wealth. As in the fulcrum type fee case, the proportional fee parameter α has no impact on the optimal fund value X_T^{a,s^*} as given by (16). Thus, it does not affect the optimal portfolio weight in the risky asset either.

4. Value of Information and the Clientele Effect

This section analyzes the incremental value of a manager's private information relative to public information and the clientele effect, which emerges as a result of investors' choices. Section 4.1 examines the case of fulcrum type contracts. Section 4.2 studies the asymmetric performance fees case.

4.1 FULCRUM PERFORMANCE CONTRACTS

We start our analysis of an uninformed investor's fund choice in the presence of symmetric fees, $F(X_T^a, X_T^b) = \alpha X_T^a + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)$ with $\alpha > 0$ and $\beta_2 \geq 0$. The performance-related component of the management fee $F(X_T^a, X_T^b)$ is linear in the excess return of the managed fund over a benchmark. These types of contracts are known as fulcrum performance contracts and commonly observed in practice. In 1970, the amendment to the Investment Advisers Act of 1940 rules that the U.S. mutual fund performance fees must be the fulcrum type.

The (ex ante) value of private information for the fund manager can be computed as the certainty equivalent return (CER) achieved with private information in excess of the certainty equivalent return without the information advantage. It is described next.

Proposition 3. *Let process* $p^G_{(\cdot)} \equiv \{p^G_v(z) : v \in [\tau_{i-1}, \tau_i)\}$ *be the conditional density process of the signal* G_i *given public information. In the presence of fulcrum performance fees* $F(X^a_T, X^b_T) = \alpha X^a_T + \alpha\beta_2\left(X^a_T - \delta X^b_T\right)$ *with* $\alpha > 0$ *and* $\beta_2 \geq 0$*, the (ex ante) incremental value of the private signal* $Y_v = \sum_{i=1}^N G_i \mathbb{1}_{[\tau_{i-1},\tau_i)}(v)$ *for the manager is*

$$V^{M,f} \equiv CER^{M,s} - CER^{M,u} = \frac{1}{2}\int_0^T E\left[\left(\theta^{\mathcal{G}}_v\right)^2\right] dv = \sum_{i=1}^N E\left[\mathcal{D}_{KL}\left(p^G_{\tau_i}(G_i) \,\middle|\, p^G_{\tau_{i-1}}(G_i)\right)\right] \quad (18)$$

where $\mathcal{D}_{KL}\left(p^G_{\tau_i}(G_i) \,\middle|\, p^G_{\tau_{i-1}}(G_i)\right) \equiv E\left[\log \frac{p^G_{\tau_i}(G_i)}{p^G_{\tau_{i-1}}(G_i)} \,\middle|\, \mathcal{F}_{\tau_i}\right]$ *is the relative entropy of the signal. The private signal has no value to the manager if and only if the PIPR is null.*

The value of information to the fund manager (18) is non-negative and increasing in the absolute value of the PIPR. Proposition 3 also shows that the source of private information value is the relative entropy between an uninformed individual's beliefs about the signal at time τ_{i-1} and τ_i. The relative entropy $\mathcal{D}_{KL}\left(p^G_{\tau_i}(G_i) \,\middle|\, p^G_{\tau_{i-1}}(G_i)\right)$ measures the information gained when one updates her beliefs from the prior probability distribution $p^G_{\tau_{i-1}}(G_i)$ to the posterior probability distribution $p^G_{\tau_i}(G_i)$. If the public information at time τ_i provides valuable information about the signal G_i relative to the public information at time τ_{i-1}, the relative entropy will be positive. If the signal is unrelated to the public information, the prior and posterior probability distributions will be the same, leading the relative entropy to be zero.

Neither the fulcrum performance fees nor the proportional fees affect the value of information to the fund manager. Since the manager is able to undo any benchmark-linked incentive implemented through linear contracts, the manager's excess CER is unaffected by the power of incentives β_2 or the benchmark composition π_b. The proportional fee parameter α does not affect the manager's excess CER either. This is because the skilled (respectively unskilled) manager's compensation at time T is $\alpha X^a_0/\xi^{\mathcal{G}}_T$ (respectively $\alpha X^a_0/\xi^m_T$), α vanishes when the excess CER is computed.

Let $I(y,b) = \frac{1-\alpha-\alpha\beta_2}{y} + \frac{\beta_2\delta b}{1+\beta_2}$. The derived utility function of a fund investor is

$$u\left(X^{a^*}_T - F\left(X^{a^*}_T, X^b_T\right)\right) = u\left(I(y^*\xi_T, X^b_T)\right) \equiv v^f\left(\xi_T, X^b_T\right),$$

where $y^* = (1+\beta_2)/X^a_0$. The associated value function is $U \equiv E\left[v^f\left(\xi_T, X^b_T\right)\right]$. An unskilled (respectively skilled) fund manager optimizes her portfolio based on the public SPD ξ^m

(respectively private SPD $\xi^{\mathcal{G}}$). Investors would prefer a skilled fund instead of an unskilled fund when $U^s > U^u$. Let $\mathcal{U}^s \subset \mathcal{U}$ be the subset of investors' utilities that find it better off investing in the skilled fund rather than the unskilled one

$$\mathcal{U}^s = \left\{u \in \mathcal{U} : \Delta \equiv U^s - U^u = E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right) - v^f\left(\xi_T^m, X_T^b\right)\right] > 0\right\}.$$

Let $F^{\xi^m,b}$ (respectively $F^{\xi^{\mathcal{G}},b}$) be the cumulative distribution function (CDF) of ξ_T^m (respectively $\xi_T^{\mathcal{G}}$) conditional on the σ-algebra generated by the benchmark portfolio at time T, $\sigma(X_T^b)$. As $E\left[\xi_T^{\mathcal{G}} \,\middle|\, \sigma(X_T^b)\right] = E\left[\xi_T^m \,\middle|\, \sigma(X_T^b)\right]$ (see the proof of Proposition 4), one does not dominate the other in the sense of first-order stochastic dominance. Let $T^{\xi,b}$ be the cumulative spread between the distributions of private and public SPDs conditional on $\sigma(X_T^b)$.

Proposition 4. *The public state price density second-order stochastically dominates (SSD) the private state price density*

$$\xi_T^m \ \mathcal{SSD}\ \xi_T^{\mathcal{G}} \quad \textit{and} \quad T^{\xi,b}(z) \equiv \int_{-\infty}^{z} \left(F^{\xi^{\mathcal{G}},b}(y) - F^{\xi^m,b}(y)\right) dy \geq 0 \quad \textit{for all} \quad z \in \mathbb{R}^+. \tag{19}$$

Proposition 4 generalizes the stochastic dominance result of DR, allowing the distribution of the SPDs to be conditional on the benchmark and signal realizations for the whole investment period. The second-order stochastic dominance result follows from the fact that the ratio of private and public SPDs corresponds to the product of reciprocal of the signal's density process, $\xi_T^{\mathcal{G}} = \xi_T^m \prod_{i=1}^N p_{\tau_{i-1}}^{\mathcal{G}}(G_i)/p_{\tau_i}^{\mathcal{G}}(G_i)$. As ξ_T^m and $\xi_T^{\mathcal{G}}$ have the same mean conditional on $\sigma(X_T^b)$, the private SPD $\xi_T^{\mathcal{G}}$ is a mean-preserving spread of the public SPD ξ_T^m.

The SSD result has the potential to formulate a second-order stochastic dominance test to evaluate whether actively managed funds have timing skills or not controlling the effects of management fees. For example, with a linear performance-based fee, the test can be implemented using a Kolmogorov-Smirnov statistic or other statistics to quantify the spread between the empirical distribution of $\left(X_T^{a,s^*} - \frac{\beta_2}{1+\beta_2} X_T^b\right)^{-1}$ and the known parametric distribution of $\left(X_T^{a,s^*} - \frac{\beta_2}{1+\beta_2} X_T^b\right)^{-1}$.

Proposition 5 describes the value of private information to investors and the clientele effect by making use of the SSD result.

Proposition 5. *In the presence of fulcrum performance fees* $F(X_T^a, X_T^b) = \alpha X_T^a + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)$ *with* $\alpha > 0$ *and* $\beta_2 \geq 0$*, the value of the manager's private signal* G_i (4)

to an investor with relative risk aversion R is

$$V^f \equiv CER^s - CER^u = \frac{1}{1-R}\log\left(\frac{E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right)\right]}{E\left[v^f\left(\xi_T^m, X_T^b\right)\right]}\right). \tag{20}$$

Let $\Delta \equiv U^s - U^u = E\left[\int_0^\infty \frac{\partial^2 v^f}{\partial z^2}(z, X_T^b) T^{\xi,b}(z) dz\right]$. *The set of investors who prefer the skilled fund* $\mathcal{U}^s = \{u \in \mathcal{U} : \Delta > 0\} = \left\{u \in \mathcal{U} : V^f > 0\right\}$ *is given by*

$$\mathcal{U}^s = \left\{ u \in \mathcal{U} : E\left[\int_0^\infty \frac{\left(\frac{1-\alpha(1+\beta_2)}{y^* z^2}\right)^2 \left[2\left(1 + \frac{\beta_2 \delta X_T^b}{1+\beta_2}\frac{y^* z}{1-\alpha(1+\beta_2)}\right) - R\right] T^{\xi,b}(z)}{\left(\frac{1-\alpha(1+\beta_2)}{y^* z} + \frac{\beta_2 \delta X_T^b}{1+\beta_2}\right)^{R+1}} dz\right] > 0 \right\}.$$

where $y^* = (1+\beta_2)/X_0^a$ *is the Lagrange multiplier,* R *is the relative risk aversion (RRA) of investors. The unskilled fund is preferred by investors with utility function in the set* $\mathcal{U}^u = \{u \in \mathcal{U} : \Delta < 0\} = \left\{u \in \mathcal{U} : V^f < 0\right\}$. *For any level of skill, there exists a value* $R^* > 0$ *and investors with coefficient of RRA exceeding* R^* *will choose the unskilled fund.*

In particular, with purely proportional fees $\beta_2 = 0$, *skilled fund returns are preferred by all investors with* $R < 2$. *Conversely, investors with relative risk aversion* $R \geq 2$ *will be better off choosing the unskilled fund irrespective of the skill level. The RRA threshold value 2 here corresponds to the relative risk prudence (RRP) of the manager with log utility.*

Expression (20) describes the (incremental) value of the manager's private signal to an investor. It is the analog of (18), except that all the contract parameters affect the investor's excess CER. The risk aversion misalignment between the manager and investors leads to a loss of the value of manager's information that investors can exploit. Proposition 5 shows that when the misalignment is large enough, the value of the manager's private information to investor might even be negative.

Proposition 5 also provides a characterization of potential investors in a skilled fund. It shows that investors with RRA less than the manager's RRP always prefer the skilled funds to unskilled ones. However, when investors' relative risk aversion is sufficiently high, they would choose the uninformed funds rather than the ones which have access to anticipative information. In particular, when the management fee is purely proportional to the end-of-period AUM ($\beta_2 = 0$), the manager's optimal portfolio collapses to the mean-variance demand. This leads to a fixed relative risk aversion threshold, equaling the manager's RRP, that divides the investors into different clienteles to skilled and unskilled funds. Investors with relative risk aversion less than the manager's RRP prefer the skilled fund return; conversely, investors with RRA exceeding the manager's RRP never prefer the skilled return independently of all the parameters. The result incorporates the special case of no management fee considered in DR.

In the presence of fulcrum performance fees, for any given skill level there always exist investors whose RRA exceeds a threshold value R^* will prefer the unskilled fund manager to the skilled one. R^* is determined by the equation $\Delta(R) = 0$. If the solution is not unique, we have an odd number of roots and there still exists a group of highly risk-averse investors whose RRA larger than the largest root will choose the unskilled fund. The threshold value R^* is no longer constant but depends on the manager's skill, contract parameters, and market conditions. The clientele effect is a direct result of the SSD relationship between public and private SPDs and the composed utility function of the investor $u \circ I$ is strictly concave (respectively convex) in the SPD ξ if and only if $R \geq R^*$ (respectively $R < R^*$).

4.2 ASYMMETRIC PERFORMANCE CONTRACT

Although the Investment Company Amendments Act of 1970 places restrictions that U.S. mutual funds' advisory contracts must be the fulcrum type, many U.S. hedge funds, institutional funds, and mutual funds outside the United States employ the asymmetric fees (Golec and Starks, 2004). Furthermore, a recent study by Ma et al. (2016) document that most of U.S. mutual fund managers are offered the option-like, performance-based compensation contracts.

We assume that both the skilled and unskilled fund managers receive asymmetric performance fees $F(X_T^a, X_T^b) = \alpha X_T^a - \alpha\beta_1 \left(X_T^a - \delta X_T^b\right)^- + \alpha\beta_2 \left(X_T^a - \delta X_T^b\right)^+$ with the same parameters $\alpha > 0, \beta_2 > \beta_1 > 0$. It is shown in (17) that the optimal fund value and manager's compensation are both piecewise functions of the normalized state price density and benchmark portfolio's value. Let

$$
\begin{aligned}
g(y,b) &= \frac{1}{y} + \frac{\beta_2 \delta b}{1+\beta_2} \mathbb{1}_{\{y \leq \Psi(b)\}} + \frac{\beta_1 \delta b}{1+\beta_1} \mathbb{1}_{\{y > \Psi(b)\}}, \\
g^c(y,b) &= \frac{\alpha(1+\beta_2)}{y} \mathbb{1}_{\{y \leq \Psi(b)\}} + \frac{\alpha(1+\beta_1)}{y} \mathbb{1}_{\{y > \Psi(b)\}},
\end{aligned}
$$

where $\Psi\left(b\right) = \frac{\log\left(\frac{1+\beta_2}{1+\beta_1}\right)}{\delta b\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}$. The skilled and unskilled fund managers' compensation at time T are given by

$$F(X_T^{a,s^*}, X_T^b) = g^c(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b),$$
$$F(X_T^{a,u^*}, X_T^b) = g^c(y^{u^*}\xi_T^{m}, X_T^b),$$

where the Lagrange multipliers y^{s^*} and y^{u^*} are determined by

$$E\left[\xi_T^{\mathcal{G}} g\left(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b\right) \mid \mathcal{G}_0\right] = X_0^a, \tag{21}$$
$$E\left[\xi_T^{m} g\left(y^{u^*}\xi_T^{m}, X_T^b\right)\right] = X_0^a. \tag{22}$$

The associated utility function of a fund investor is given by

$$v^a\left(y^*\xi_T, X_T^b\right) \equiv u\left(g\left(y^*\xi_T, X_T^b\right) - g^c\left(y^*\xi_T, X_T^b\right)\right) \tag{23}$$

Note that y^{s^*} is a random variable that depends on the realization of the private signal G_1 and solves the Equation (21). By contrast, in the case of the fulcrum type fee, y^{s^*} is independent of the signal and it equals y^{u^*}.

Proposition 6. *In the presence of asymmetric performance fees:*

$$F(X_T^a, X_T^b) = \alpha X_T^a - \alpha\beta_1\left(X_T^a - \delta X_T^b\right)^- + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)^+ \ \textit{with}\ \alpha > 0, \beta_2 > \beta_1 > 0,$$

the ex ante value of the private signals G_i with $i = 1, \ldots, N$ to a fund manager is

$$V^{M,a} \equiv CER^{M,s} - CER^{M,u} = E\left[\log\frac{g^c(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b)}{g^c(y^{u^*}\xi_T^{m}, X_T^b)}\right]$$

and the value of the private signals G_i with $i = 1, \ldots, N$ to a fund investor is

$$V^a \equiv CER^s - CER^u = \frac{1}{1-R}\log\left(\frac{E\left[v^a\left(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b\right)\right]}{E\left[v^a\left(y^{u^*}\xi_T^{m}, X_T^b\right)\right]}\right). \tag{24}$$

Proposition 6 describes the value of private information to fund manager and investors in the presence of asymmetric fees. Since the manager cannot completely undo the incentives implemented through non-linear contract, her excess CER is no longer independent of the contract parameters. The investor's excess CER is also affected by all the parameters.

Since the derived utility function of a fund investor (23) is neither concave nor convex in the normalized SPD, we cannot apply Jensen's inequality to compare the value functions U^s

and U^u as in Proposition 5. The intuition that risk-aversion misalignment may also lead to a negative value of information to investors with asymmetric performance fees is confirmed in the numerical examples described in Section 6.

5. Noisy Return Forecast Timing Model

In this section, we specialize to the case of a private signal with a linear-multiplicative form. To simplify the presentation, it is assumed that r, θ^m, σ^m, π^b are constant and there is only one signal received during the investment period. Thus, $N = 1$ and $\tau_0 = 0, \tau_N = T$.

Suppose that the skilled fund manager receives a private signal which informs the future market excess return with noise. We consider a signal (4) with the linear multiplicative form[5]

$$G \equiv g\left(S_T^m, \zeta\right) = S_T^m \zeta \quad \text{with} \quad \zeta \equiv \exp\left(\sigma^y W_T^\zeta - \frac{1}{2}(\sigma^y)^2 T\right), \tag{25}$$

where W^ζ is a standard Brownian motion process, independent of the market innovation W^m. Thus, $log(\zeta) \sim N(-(\sigma^y)^2 T/2, (\sigma^y)^2 T)$, and $E\left[\zeta \mid \mathcal{F}_t^m\right] = 1$ for all $t \in [0, T]$. The logarithm of the signal is the cumulative local excess return of the market plus a normally distributed noise term. Smaller volatility σ^y makes the signal more informative. The inverse volatility $(\sigma^y)^{-1}$ measures the information extraction skill of the fund manager. A more-skilled manager is able to extract more precise information than a less-skilled manager. An unskilled manager does not observe the signal or has a signal with pure noise.

Corollary 1. *Suppose that θ^m and σ^m are constant and the private information filtration is $\mathcal{G}_{(\cdot)} = \mathcal{F}_{(\cdot)}^m \bigvee \mathcal{F}_{(\cdot)}^Y$, where $\mathcal{F}_{(\cdot)}^m$ is the public information generated by the market returns and $\mathcal{F}_{(\cdot)}^Y$ is the filtration generated by the private signal (25). For all $t \in [0, T)$, the conditional density of the signal is given by*

$$p_t^G(x) = \frac{\phi(d(x,t))}{x\sqrt{\Sigma_{t,T}}}$$

and the PIPR is given by

$$\theta_t^{\mathcal{G}} = \mathcal{D}_t^m \log p_t^G(x)_{|x=G} = \sigma^m \left(\frac{\log G - E_t[\log G]}{VAR_t[\log G]}\right),$$

[5] The form is an extension of the return forecast model studied in DR section 2.1 by allowing the variance of the signal noise to increase as the timing interval T increases. Thus, the manager is not able to extract more precise information about the future market returns because of shorter investment period T.

where $\phi(\cdot)$ is the standard normal probability density distribution function and

$$d(x,t) = \frac{\log(x) - E_t[\log(G)]}{\sqrt{\Sigma_{t,T}}},$$
$$VAR_t[\log G] = (\sigma^m)^2(T-t) + (\sigma^y)^2 T \equiv \Sigma_{t,T},$$
$$E_t[\log G] = \left(\sigma^m\theta^m - \frac{(\sigma^m)^2 + (\sigma^y)^2}{2}\right)T + \sigma^m W_t^m.$$

Corollary 1 shows that the PIPR is linear in the innovation in the log signal $\log G - E_t[\log G]$ and inversely related to the log signal's conditional variance $\Sigma_{t,T}$. The sign of the PIPR is the same as that of the innovation in the log signal. As time elapses, the informed manager learns from market realized returns and revises her assessment of risk and PIPR. For a fixed signal realization and realized gross excess return S_t^m, an increase in the skill level s raises the absolute value of the PIPR. When $\sigma^y = 0$, the informed manager has perfect foresight about future returns, the PIPR explodes as the time approaches T, and an arbitrage opportunity emerges. When the signal is uninformative $s = 0$, the variance of the noise goes to infinity and the PIPR is null.

The optimal informed investment policy with the fulcrum fees is given by (13). When the PIPR is positive (negative), the informed manager invests more (less) in the risky stock, and the volatility of the informed fund portfolio is greater (smaller) than that of the uninformed fund portfolio. Since the optimal informed policy and the fund return volatility are linear in PIPR, they share the same structure and properties as PIPR's . For a given innovation $\log G - E_t[\log G]$, as time passes, a manager with positive news increases the share of risky asset in the portfolio as the variance of the forecast decreases.

Corollary 2 describes an explicit formula for the incremental value of a manager's information to a fund manager with a logarithmic utility function and fulcrum fees.

Corollary 2. *In the presence of fulcrum performance fees:*

$$F(X_T^a, X_T^b) = \alpha X_T^a + \alpha\beta_2\left(X_T^a - \delta X_T^b\right) \text{ with } \alpha > 0, \beta_2 \geq 0,$$

the ex ante value of the private signal G, as described in (25)*, to a fund manager with logarithmic utility function is*

$$V^{M,f} \equiv CER^{M,s} - CER^{M,u} = \frac{1}{2}\log\left(1 + (\sigma^m/\sigma^y)^2\right). \tag{26}$$

As shown by the expression (26), the manager's excess CER is positive and increasing in the skill level and the market volatility. A manager with greater skill level is able to extract more precise information about the future market excess returns. When the market is more volatile, the private signal is more valuable to the informed manager. The value of the private information does not depend on the public market price of risk θ^m or the timing interval T.

Corollary 3 gives an explicit formula for the value of information to a fund investor with relative risk aversion R and purely proportional fees.

Corollary 3. *Suppose managers' compensation at time T is purely proportional to the terminal value of the managed portfolio $F(X_T^a) = \alpha X_T^a$ with $\alpha > 0$ and investors' relative risk aversion coefficients $R < 1 + \sqrt{1 + (\sigma^y/\sigma^m)^2}$.* [6] *The incremental value of the log manager's private signal G (relative to public information), as described in* (25)*, to the investor and its first derivative with respect to the skill $s = 1/\sigma^y$ are*

$$V^p = \log \sqrt{1 + \left(\frac{\sigma^m}{\sigma^y}\right)^2} + \frac{\log \sqrt{1 - \frac{(R-1)^2(\sigma^m)^2}{(\sigma^m)^2+(\sigma^y)^2}}}{R-1} + \frac{(R-1)^2(R-2)(\theta^m)^2 T (\sigma^m)^2}{2\left(R(R-2)(\sigma^m)^2 - (\sigma^y)^2\right)},$$

$$\frac{\partial V^p}{\partial s} = \frac{(2-R)(\sigma^m)^2 \left[\left(R(\sigma^m)^2 + 1/s^2\right)\left(1 - \frac{(R-1)^2(\sigma^m)^2}{(\sigma^m)^2+1/s^2}\right) + (R-1)^2(\theta^m)^2 T/s^2 \right]}{s\left[R(R-2)(\sigma^m)^2 - 1/s^2\right]^2}.$$

*When $R < 2$, the value of private information V^p is positive and it is increasing in the skill. Conversely, when $R > 2$, the value of private information V^p becomes negative and it is decreasing in the skill. Consequently, investors with relative risk aversion $R < 2$ would choose the manager with the **highest level of skill** on the market, and investors with $R > 2$ would prefer the manager with the **lowest level of skill**.*

Corollary 3 extends the result in Proposition 5 by showing that investors could be categorized into two groups: one group who would choose the most-skilled manager and the other group who would choose the least-skilled manager. The private information is only valuable and increases in skill level for investors whose relative risk aversion is smaller than 2, the relative prudence of the log manager. These investors would choose managers with the highest skill level on the market. If the investor's relative risk aversion is greater than 2, the value of the manager's private signal to investors becomes negative due to its private nature, and the private information's negative effect on the investors' utility is more prominent as

[6] This condition guarantees the ex ante expected utility of a fund investor invests in the skilled manager is finite.

the manager's skill level increases. As a result, investors with $R > 2$ are better off choosing the least skilled manager.

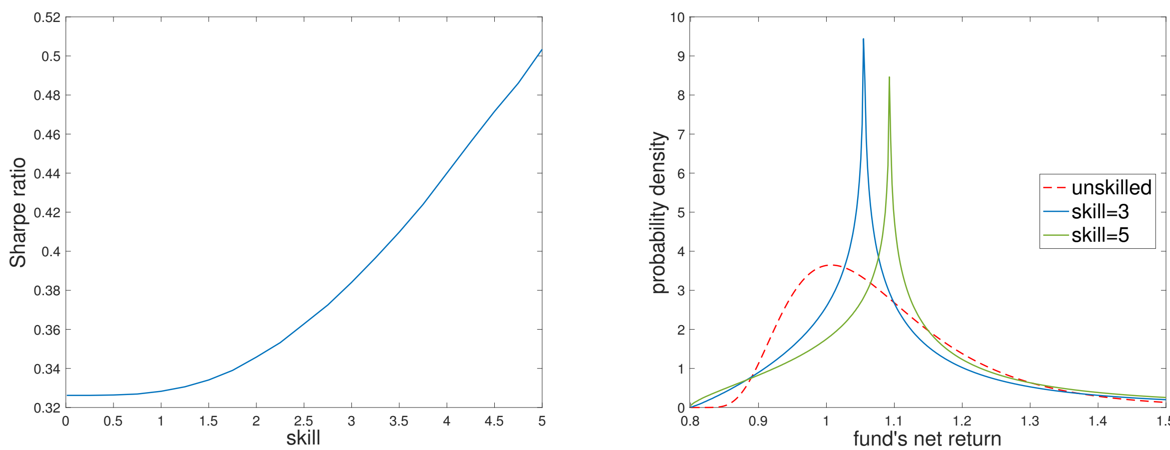

Fig. 2: Sharpe ratio and probability density of after-fee fund returns under public information in the noisy return forecast model. The left panel shows the Sharpe ratios generated versus the manager's skill under fulcrum fee contract. The right panel plots the probability density function of after-fee fund returns with three different skill levels under fulcrum fee contract. The fixed parameter values are $\alpha = 0.6\%, \beta_1 = \beta_2 = 2\%, \sigma^m = 0.155, \theta_m = 0.47, \delta = 1, \pi^b = 0, T = 1$.

We illustrate the intuition behind the clientele effect in Figure 2, which presents the properties of fund returns in the noisy return forecast timing model. From the perspective of an investor who has only public information, a higher skill level of manager increases the portfolio's downside tail risk as shown in the right panel. This directly follows from the fact that the manager's anticipative information is noisy. A more skilled fund manager may suffer from larger losses when her signal is misleading. This explains why investors with sufficiently high relative risk aversion will choose the least skilled fund.

The result that there exist two distinctive groups of investors is notable and has important implications. The left panel in Figure 2 shows that the Sharpe ratio of the skilled fund's net return is monotonically increasing in the manager's level of skill. Since Sharpe ratio is a commonly used criterion to consider when investors make investment decisions, Corollary 3 implies that a certain group of investors would not invest in the most skilled funds despite that a high Sharpe ratio is generated. The result thus highlights the importance of controlling heterogeneity in investors' risk preference when one evaluates investors' fund investment.

Corollary 4 gives explicit formulas for the optimal portfolio choices and fund value in the presence of asymmetric fees.

Corollary 4. *Define the constant* $\Delta_\beta = \frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}$. *With asymmetric performance fees:*

$$F\left(X_T^a, X_T^b\right) = \alpha X_T^a - \alpha\beta_1\left(X_T^a - \delta X_T^b\right)^- + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)^+ \text{ with } \alpha > 0, \beta_2 > \beta_1 > 0,$$

the optimal fund value based on public and private information at time $t \in [0, T]$ *are*

$$X_t^{a,u^*} = \frac{1}{y^{u^*}\xi_t^m} + \frac{\beta_2 \delta X_t^b}{1+\beta_2}\mathcal{N}(d_{1,t}) + \frac{\beta_1 \delta X_t^b}{1+\beta_1}\mathcal{N}(-d_{1,t}), \tag{27}$$

$$X_t^{a,s^*} = \frac{1}{y^{s^*}\xi_t^{\mathcal{G}}} + \frac{\beta_2 \delta X_t^b}{1+\beta_2}\left(\mathcal{N}(d_{2,t}^+) - \mathcal{N}(d_{2,t}^-)\right) + \frac{\beta_1 \delta X_t^b}{1+\beta_1}(\mathcal{N}(-d_{2,t}^+) + \mathcal{N}(d_{2,t}^-)) \tag{28}$$

and their optimal weights invested in stock are given by

$$\pi_t^{a,u^*} = \frac{\theta^m}{\sigma^m} + \frac{\beta_2}{1+\beta_2}\frac{\delta X_t^b}{X_t^{a,u^*}}\left(\pi^b - \frac{\theta^m}{\sigma^m}\right) - \frac{\Delta_\beta \delta X_t^b}{X_t^{a,u^*}}\left(\mathcal{N}(-d_{1,t})\left(\pi^b - \frac{\theta^m}{\sigma^m}\right) + \frac{\mathcal{N}'(d_{1,t})}{\sqrt{T-t}}\right) \tag{29}$$

$$\begin{aligned}\pi_t^{a,s^*} = {} & \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m} + \frac{\beta_2}{1+\beta_2}\frac{\delta X_t^b}{X_t^{a,s^*}}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right) \\ & - \frac{\Delta_\beta \delta X_t^b}{X_t^{a,s^*}}\left((\mathcal{N}(-d_{2,t}^+) + \mathcal{N}(d_{2,t}^-))\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right) - (\eta_t^+\mathcal{N}'(d_{2,t}^+) - \eta_t^-\mathcal{N}'(d_{2,t}^-))\right)\end{aligned} \tag{30}$$

where y^{u^*} *solves* $E\left[\xi_T^m X_T^{a,u^*}\right] = x^a$ *and* y^{s^*} *solves* $E\left[\xi_T^{\mathcal{G}} X_T^{a,s^*} \,\middle|\, \mathcal{G}_0\right] = x^a$, $\mathcal{N}(\cdot)$ *is the standard normal cumulative distribution function and*

$$d_{1,t} = \frac{\log\left(\frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^m X_t^b y^{u^*}\delta\Delta_\beta}\right) - \frac{(\pi^b\sigma^m - \theta^m)^2(T-t)}{2}}{\sqrt{(\pi^b\sigma^m - \theta^m)^2(T-t)}},$$

$$d_{2,t}^\pm = \frac{\pm\sqrt{\Sigma_{T,T}\left(2\log\left(\frac{\sqrt{\frac{\Sigma_{t,T}}{\Sigma_{T,T}}}\log\frac{1+\beta_2}{1+\beta 1}}{\xi_t^{\mathcal{G}} X_t^b y^{s^*}\delta\Delta_\beta}\right) + \Sigma_{t,T}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right)^2\right)} - \Sigma_{t,T}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right)}{\sqrt{\Sigma_{t,T} - \Sigma_{T,T}}}$$

$$\eta_t^\pm = \pm\sqrt{\frac{\frac{T(T-t)}{t^2}\left(\frac{(\sigma^m)^2 + (\sigma^y)^2}{\sigma^y}\left(\frac{\theta_t^{\mathcal{G}}}{\sigma^m} - \frac{1}{2}\right) + \frac{\sigma^m\theta^m}{\sigma^y}\right)^2}{2\log\left(\frac{\sqrt{\frac{\Sigma_{t,T}}{\Sigma_{T,T}}}\log\frac{1+\beta_2}{1+\beta 1}}{\xi_t^{\mathcal{G}} X_t^b y^{s^*}\delta\Delta_\beta}\right) + \Sigma_{t,T}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right)^2}} - \frac{1}{\sqrt{T-t}}.$$

The uninformed and informed managers' compensation at time T is

$$F(X_T^{a,u^*}, X_T^b) = \frac{\alpha(1+\beta_2)}{y^{u^*}\xi_T^m}\mathbb{1}_{\{y^{u^*}\xi_T^m \le \Psi(X_T^b)\}} + \frac{\alpha(1+\beta_1)}{y^{u^*}\xi_T^m}\mathbb{1}_{\{y^{u^*}\xi_T^m > \Psi(X_T^b)\}}, \tag{31}$$

$$F(X_T^{a,s^*}, X_T^b) = \frac{\alpha(1+\beta_2)}{y^{s^*}\xi_T^{\mathcal{G}}}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} \le \Psi(X_T^b)\}} + \frac{\alpha(1+\beta_1)}{y^{s^*}\xi_T^{\mathcal{G}}}\mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} > \Psi(X_T^b)\}}, \tag{32}$$

where $\Psi(b) = \log\left(\frac{1+\beta_2}{1+\beta_1}\right) / (\delta b \Delta_\beta)$.

As shown in (29) and (30), the manager's optimal portfolio is a sum of a standard mean-variance component plus additional components. The difference between the manager's optimal portfolio policy and the mean-variance demand can be interpreted as the hedging demands, motivated by the asymmetric performance fees. The second component is the hedging demand due to the performance bonus $\alpha\beta_2(X_T^a - \delta X_T^b)^+$. The last component is the hedging demand due to the performance penalty $-\alpha\beta_1(X_T^a - \delta X_T^b)^-$. Equations (31) and (32) show that in the presence of asymmetric fees the optimal trading strategies cannot fully hedge the manager's exposure to the benchmark portfolio. There is a jump in the manager's end-of-period compensation when the normalized SPD $y^*\xi_T$ hits the critical value $\Psi(X_T^b)$. As in the linear contract case, the proportional fee α does not affect the manager's optimal portfolio either. However, α and other contract parameters do affect the manager's compensation as well as the investor's after-fee wealth.

The informed agent's price of risk is changed from θ^m to $\theta^m + \theta^{\mathcal{G}}$ due to the private information. When the private signal is uninformative or the manager lacks true timing skill, $\sigma^y = \infty$, the PIPR is null and the optimal portfolio of the skilled manager as given by (30) collapses to that of his unskilled peer as given by (29).

In particular, expression (27) and (28), evaluated at $t = T$, identify the optimal fund value and can be used for the computation of the investor's and manager's CERs.

6. Numerical Example

This section conducts a numerical analysis of the value of information and investors' preference between skilled and unskilled funds in the noisy return forecast model. Section 6.1 examines the value of the manager's information to investors under the three commonly used contracts. Section 6.2 conducts a sensitivity analysis of the investor's relative risk aversion threshold and the key parameters.

We calibrate the parameters of risky and risk-free assets using quarterly U.S. data beginning in 1947 and ending in the first quarter of 2010. The risky asset is constructed using the CRSP value-weighted index, while the risk-free rate is constructed from real returns on 3-month Treasury bill. The market parameters are $\theta^m = 0.47, \sigma^m = 15.5\%$, $r = 3.5\%$.

We consider three performance fee structures: purely proportional fees, fulcrum performance fees, and asymmetric performance fees. In the last two cases, the performance fee is added on top of a non-zero proportional fee ($\alpha > 0$). Based on the evidence reported by Cuoco and Kaniel (2011) that "the value-weighted average proportional component across funds charging performance fee is 60 basis, and the typical fulcrum performance fee is 2%; both on an annual basis." We set $\alpha/T = 0.6\%$ and $\beta_1 = \beta_2 = 2\%/\alpha$ for the fulcrum fees, where T is the investment horizon. For asymmetric fees, we analyze the most commonly observed two-twenty hedge fund contract and set $\alpha/T = 2\%$, $\beta_1 = 0$, and $\beta_2 = 20\%/\alpha$. For ease of exposition, we set the benchmark portfolio's weight in the stock π^b to be 0.[7] Ma et al. (2016) document "the performance evaluation window in mutual fund industry ranges from one quarter to ten years, and the average evaluation window is three years." We consider an investment horizon T of three years. The skill level $s = 1/\sigma^y$ is calibrated to be in the range of $(0, 10)$ to deliver a range of $(0, 20\%)$ for excess certainty equivalent return under the setting of purely proportional fees. We set the parameter $\delta = X_0^a/X_0^b = 1$.

6.1 THE VALUE OF INFORMATION

Figure 3 presents the investor's excess CER from delegation under three different fee structures (purely proportional fees, fulcrum fees, and asymmetric fees) as a function of the investor's coefficient of relative risk aversion R, for various timing skill levels. It shows that in all three cases the investor's excess CERs are all decreasing in R and become negative as R hits a threshold value R^*.[8] It is notable that when investors are sufficiently risk-averse ($R > R^*$), the value of the manager's private information to investors is negative and investors prefer the unskilled fund to the skilled ones. This suggests that there are different groups

[7] Hedge funds usually use 0% return or treasury rates as the benchmark in the incentive scheme (see Brown et al., 1999). The results for $\pi^b \in (0, 1]$ are qualitatively similar.

[8] The threshold value R^* is constant under purely proportional fees, while it depends on the parameters under asymmetric and symmetric fees.

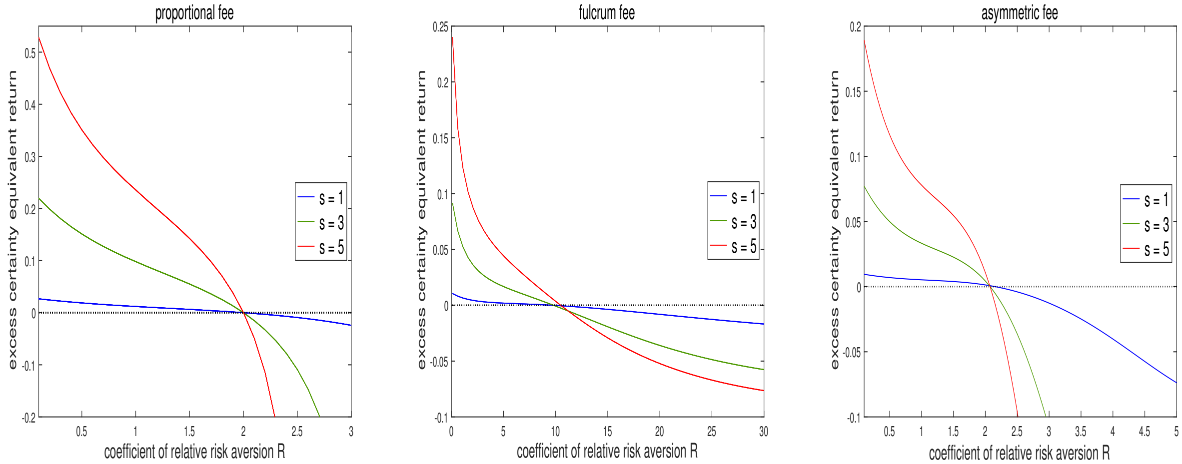

Fig. 3: The value of private information to investors in the noisy return forecast model. The left panel plots the investor's excess CER from delegation under purely proportional fee contract as a function of the investor's RRA coefficient, with different skill levels $s = 1, 3, 5$. Excess CERs are computed as the investors' CER from delegation to skilled funds in excess of the CER from delegation to an unskilled fund. The contract parameters are $\alpha/T = 0.6\%, \pi^b = 0$. The middle panel plots the investor's excess CER from delegation under fulcrum fee contract as a function of the investor's RRA coefficient, with different skill levels $s = 1, 3, 5$. The contract parameters are $\alpha/T = 0.6\%, \beta_1 = \beta_2 = 10/3, \pi^b = 0$. The right panel plots the investor's excess CER from delegation under asymmetric fee contract as a function of the investor's RRA coefficient, with different skill levels $s = 1, 3, 5$. The fixed parameters values are $r = 3.5\%, \theta^m = 0.47, \sigma^m = 0.155, \delta = 1, T = 3$.

of investors to skilled and unskilled funds under commonly observed portfolio management contracts. Under the same fee structure, investors in skilled funds are a more risk-tolerant clientele than investors in unskilled funds.

Notably, Figure 3 also shows that the higher the skill, the steeper the lines of investor's excess CERs. The value of the private information to the relatively risk-tolerant individuals ($R < R^*$) increases as the skill level of manager increases. On the contrary, relatively risk-averse individuals ($R > R^*$) suffer from larger losses when the manager is more skilled. This implies that some asset allocation or hiring decisions are inappropriate. If fund investors or owners are sufficiently more risk-averse than the managers, they should not delegate the portfolio management to the manager with higher skill level.

The intuition for these results is as follows. The private information adds value to investors because it helps investors better assess the investment opportunities. An increase in skill increases the private information precision. However, the investors do not just evaluate the benefits of the private information precision but also take into account the cost incurred by the noisy nature of this information. The downside tail risk of portfolios is also increasing in the manager's skill and has a negative effect on the value of private information. When the investor's risk aversion is sufficiently low, the information precision effect dominates and the investor would choose the fund manager with the highest skill. Conversely, when the investor's risk aversion is sufficiently high, the downside tail risk effect dominates and the investor would prefer the unskilled fund.

As shown in Proposition 1, and displayed in the left panel of the figure, the threshold value for the investor's coefficient of relative risk aversion is the relative prudence of the logarithmic fund manager irrespective of the manager's skill level in the purely proportional fee case. In the presence of fulcrum fees, the risk aversion threshold value is larger than that with purely proportion fees. The middle panel shows that the threshold value R^* is around 10, and investors with $R < R^*$ prefer the skilled fund and are able to extract positive value from the private signal under delegation. Conversely, investors with $R \geq 2$ will prefer the fund without any anticipative information. Interestingly, for the two-twenty asymmetric performance fees, the threshold value for the coefficient of the relative risk aversion is around 2, as displayed in the right panel. In contrast to the constant threshold in purely proportional fee case, the threshold value R^* in both the fulcrum and asymmetric fee cases is not constant and is affected by the manager's skill, the contract parameters, and the market conditions.

Table 1 shows investors' CER from delegation to a skilled fund manager and the value of manager's private information to the investors (excess CER) across the three commonly observed types of contracts: proportional-only fees ($\alpha > 0, \beta_1 = \beta_2 = 0$), asymmetric performance fees ($\alpha > 0, \beta_1 = 0, \beta_2 > 0$), and symmetric performance fees ($\alpha > 0, \beta_1 = \beta_2 > 0$). The excess CERs are computed as the investors' CER from delegation to the skilled funds in excess of the CER from delegation to the unskilled fund as given by (20) and (24). Table 1 shows that symmetric performance fee contract dominates proportional-only fee contract and asymmetric performance fee contract for investors with risk aversion larger than or equal 2 in the sense that the CER and excess CER are both higher under symmetric fee contract. For the relatively risk-averse investors with $R \geq 2$, the fact that CERs are higher indicates symmetric performance fee contract entails less welfare loss.[9] This is consistent with the finding of Sotes-Paladino and Zapatero (2017), in which they endogenize the contract parameters

[9] The welfare loss is due to the misalignment between the risk aversions of manager and investors. The manager will choose a portfolio that deviates from the investor's desired policy π^* that investors would choose if they had access to the same private information as the manager. The inclusion of symmetric performance fees might also be welfare-improving for relatively risk-tolerant investors with optimal contract parameters as in Sotes-Paladino and Zapatero (2017).

		CER, Excess CER (%)									
	Investor's risk aversion	1		2		3		4		5	
$s=1$	Proportional-only	44.82	1.19	10.50	0.00	-27.34	-4.71	-76.27	-20.50	-153.34	-64.44
	Asymmetric	26.03	-1.06	-18.91	-0.17	-68.14	-3.32	-113.60	-10.59	-158.04	-20.89
	Symmetric	17.61	-0.55	15.03	0.29	13.04	0.19	11.44	0.15	10.10	0.13
$s=3$	Proportional-only	53.42	9.79	10.50	0.00	-125.47	-102.83	-∞	-∞	-∞	-∞
	Asymmetric	34.81	9.85	-17.21	1.54	-129.26	-64.45	-293.35	-190.34	-385.59	-251.45
	Symmetric	21.43	4.36	17.20	2.47	14.57	1.72	12.68	1.39	11.19	1.23
$s=5$	Proportional-only	67.15	23.52	10.50	0.00	-1.45e6	-1.45e6	-∞	-∞	-∞	-∞
	Asymmetric	48.26	23.29	-19.21	-0.46	-186.88	-122.07	-335.82	-232.81	-419.85	-282.71
	Symmetric	27.23	10.16	20.79	6.06	17.36	4.51	15.04	3.75	13.30	3.33

Table 1: Investors' CER and Excess CER from Delegation. This table reports the investors' certainty equivalent returns (CER) from delegation to a skilled fund manager and excess CER, which measures the value of the manager's information to the investors, under proportional-only fees, asymmetric performance fees and symmetric fees for different skill levels and investors' relative risk aversion R. Excess CER are computed as the CER from delegation to a skilled fund in excess of the CER from delegation to an unskilled fund. The fixed parameter values are $r = 0.035, \theta^m = 0.47, \sigma^m = 0.155, \delta = 1, T = 3, s = 1, \alpha/T = 0.02, \pi_b = 0$. For proportional-only fees, $\beta_1 = \beta_2 = 0$. For asymmetric fees, $\beta_1 = 0, \beta_2 = 0.2/\alpha$. For symmetric fees, $\beta_1 = \beta_2 = 0.2/\alpha$.

and show that symmetric performance fee contract is optimal and welfare-improving for investors irrespective of the investors' risk aversion relative to the manager's. Since the excess CERs measure the value of the manager's information to investors, the results suggest that highly risk-averse investors may include a symmetric performance fee in the manager's compensation to realize higher value from the manager's anticipative information.

6.2 SENSITIVITY ANALYSIS

To analyze the sensitivity of the relative risk aversion thresholds under fulcrum fees, Figure 4 illustrates the effects of skill $s = 1/\sigma^y$, proportional fee α, fulcrum incentive β_2, market price of risk θ^m, and market volatility σ^m. The upper panels show that the relative risk aversion threshold is almost invariant to the proportional fee α but increases in the fulcrum incentive β_2. According to Proposition 5, the risk aversion threshold under fulcrum fees is larger than that under purely proportional fees. An increase in the fulcrum incentive β_2 diminishes the relative impact of proportional fee component on fund managers' compensation. Consequently, the risk aversion threshold is higher with more powerful incentive β_2. Similarly, since the relative impact of the proportional component is independent of the parameter α, there is little effect of the proportional fee α on the risk aversion threshold. The lower-left panel shows the clientele of skilled fund expands as the market improves (θ^m increases) and

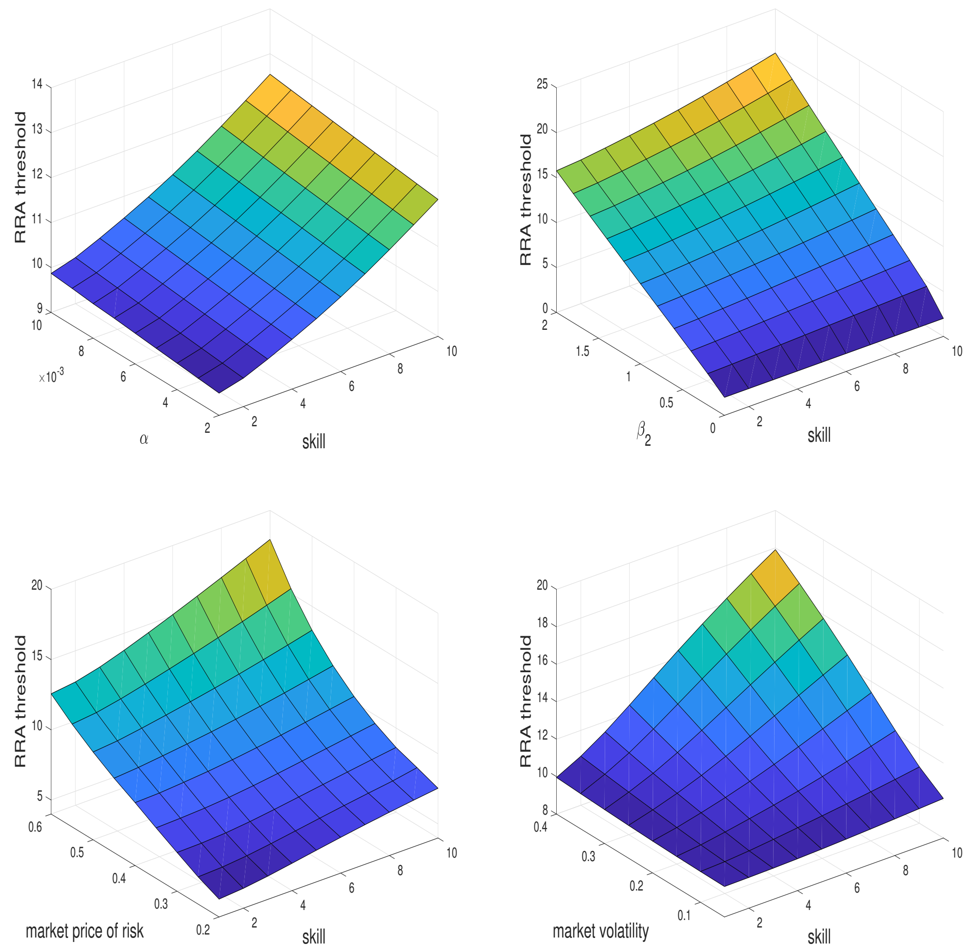

Fig. 4: Sensitivity of RRA threshold to model parameters under fulcrum fees contract. The figure presents the RRA threshold R^* of investors under fulcrum type contract versus proportional fee parameter α, performance bonus parameter β_2, (public) market price of risk θ^m, market volatility σ^m and the skill level s of fund manager. The fixed parameter values (where applicable) are $r = 3.5\%, \theta^m = 0.47, \sigma^m = 0.155, \delta = 1, T = 3, \alpha/T = 0.6\%, \beta_1 = \beta_2 = 10/9, \pi^b = 0$ and $s = 5$.

a one-unit increase in the skill would lead to a larger increase in the risk aversion threshold when the market price of risk is higher. This is because the negative effect of downside risk on investor's choice is alleviated in good states. By contrast, the value of the private information to a manager does not depend on the market price of risk θ^m as shown in Corollary 2. The lower-right panel of the figure displays that the set of investors in skilled fund is larger when the market is more volatile and this effect is more pronounced for high skill level. The higher market volatility the more valuable the information advantage. Therefore, more investors are investing in the skilled funds. As shown in all the panels of the figure, the investor's relative risk aversion threshold is increasing in skill. The intuition is as follows. The higher the manager's level of skill, the more advantageous the private information relative to public information, the higher the value of the private information to the investors in skilled funds.

Figure 5 plots the risk aversion threshold of investors as a function of key parameters with asymmetric performance fees. It shows that in the absence of penalty component ($\beta_1 = 0$) the risk aversion threshold with different parameters lies in a relatively small range $[1.8, 2.3]$. This indicates that the proportional fee α, bonus incentive β_2, market price of risk θ^m, market volatility σ^m, and skill s have little impact on the investor's preference between the skilled and unskilled fund. The risk aversion threshold seems to slightly increase in the market price of risk θ^m, market volatility σ^m, and manager's skill s. The qualitative relationships are similar to those under fulcrum fee contract. One major observation in Figure 5 is that the range of the risk aversion threshold as a function of the penalty sensitivity β_1 is much larger than that of other parameters. This implies that the value of market timing to investors is much more affected by the penalty sensitivity β_1 than the bonus incentive β_2 or other parameters. Increasing the penalty sensitivity β_1 leads to a wider investor clientele in skilled funds. This is because an increase in the penalty sensitivity β_1 causes the managers to reduce portfolio volatility and alleviates investors' concerns about the larger tail risk in the skilled fund relative to the unskilled fund.

The fulcrum fee contract can be regarded as the extreme case of the asymmetric fee contract with underperformance penalty sensitivity β_1 equals to the outperformance bonus incentive β_2. Comparing the Figure 4 and Figure 5 further illustrates that the qualitative impact of adding a penalty component into the manager's compensation scheme on the value of manager's information to investors. It shows that the risk aversion threshold is larger under fulcrum fees than under option-like asymmetric fees. Furthermore, the variations in the investor's risk aversion threshold as a function of different parameters are more pronounced with fulcrum fees compared to those with option-like asymmetric fees. The results suggest the important role of the underperformance penalty component in affecting the value of the manager's private information to investors and their fund investment. When investors are sufficiently more risk-averse relative to managers, the fulcrum fee contract serves the purpose of realizing positive value from the manager's private information better than the option-like

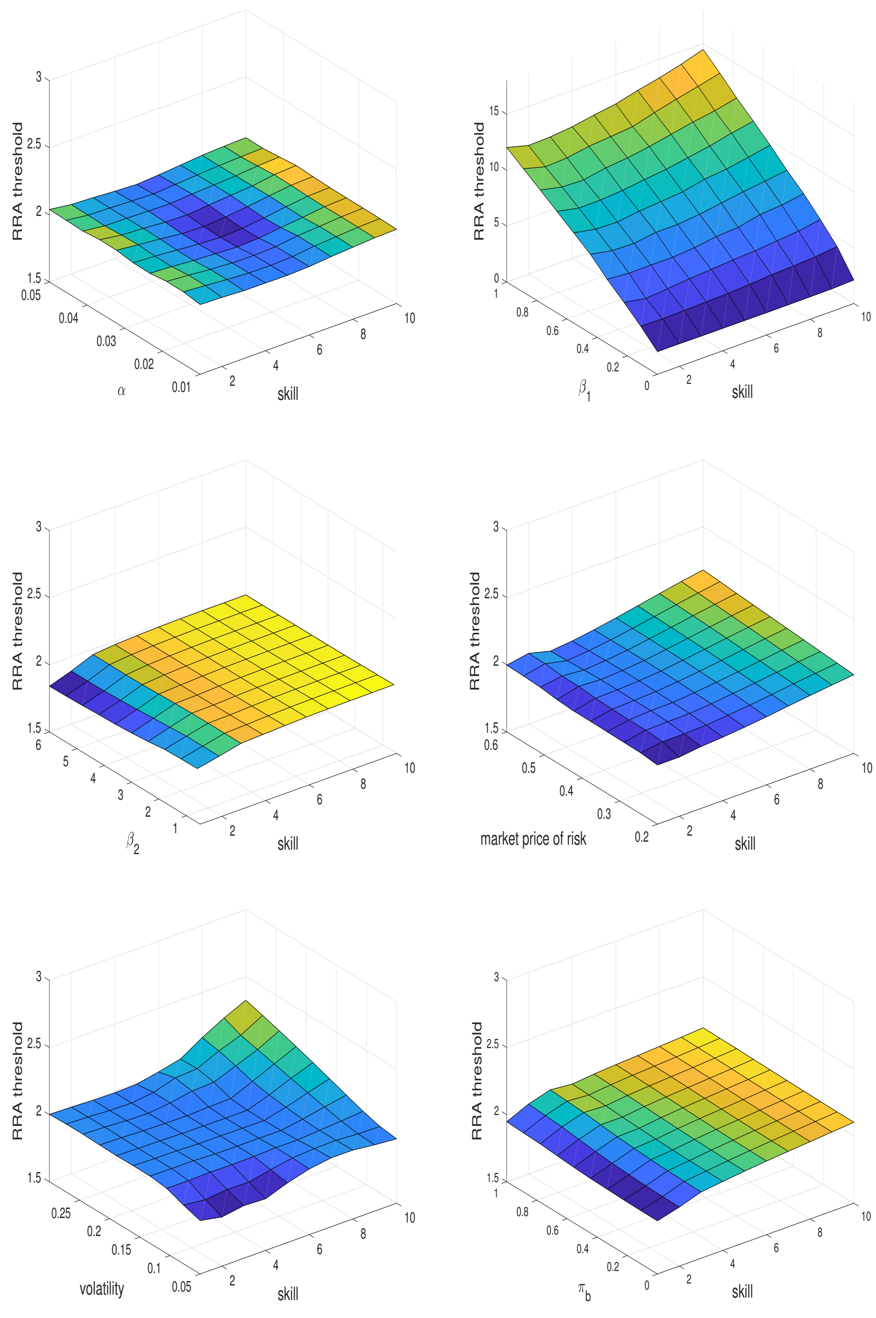

Fig. 5: Sensitivity of RRA threshold to model parameters under asymmetric fees contract. The figure presents the RRA threshold R^* of investors under asymmetric performance fees contract versus proportional fee parameter α, penalty parameter β_1, performance bonus parameter β_2, (public) market price of risk θ^m, market volatility σ^m, the skill level s and the benchmark portfolio's weight in the stock π^b. The fixed parameter values (where applicable) are $r = 3.5\%, \theta^m = 0.47$, $\sigma^m = 0.155, \delta = 1, T = 3, \alpha/T = 2\%, \beta_1 = 0, \beta_2 = 10/3, \pi^b = 0$ and $s = 5$.

asymmetric fee contract. On the other hand, if managers are able to dictate the fee structure, the unskilled managers may abstain from including an underperformance penalty in their contracts in order to expand their clientele when soliciting funds from potential investors.

7. Extensions of the Model

Suppose that θ^m, σ^m, r are constant and there is one signal received by the informed manager at the inception of investment period, namely $N = 1$, $\tau_0 = 0$, and $\tau_N = T$. For tractability, we assume that the manager observes the private signal with linear multiplicative form as described in (25) and takes prices as given.

7.1 MANAGERS WITH CONSTANT RELATIVE RISK AVERSION

We generalize the logarithmic manager assumption and consider an informed manager who has general CRRA utility with a coefficient of relative risk aversion equal to R^a. The manager maximizes the expected utility of her management fee, which is a fraction of the total asset under management at the terminal date, and solves

$$\begin{aligned} &\sup_{X_T^a \in \mathcal{G}_T} E\left[\frac{(\alpha X_T^a)^{1-R^a}}{1-R^a} \middle| \mathcal{G}_0 \right], \\ &\text{s.t.} \quad E\left[\xi_T^{\mathcal{G}} X_T^a\right] = X_0^a, X_T^a \geq 0, \end{aligned}$$

where α is the proportional fee parameter and $\xi_T^{\mathcal{G}}$ is the private state price density given by

$$\xi_T^{\mathcal{G}} = \exp\left(-\int_0^T \left(r + \frac{1}{2}\left(\theta^m + \theta_v^{\mathcal{G}}\right)^2 \right) dv - \int_0^T \left(\theta^m + \theta_v^{\mathcal{G}}\right) dW_v^{\mathcal{G}} \right).$$

The next proposition describes the manager's optimal investment strategies.

Proposition 7. *Suppose that θ^m and σ^m are constant and the private information filtration is $\mathcal{G}_{(\cdot)} = \mathcal{F}_{(\cdot)}^m \bigvee \mathcal{F}_{(\cdot)}^Y$, where $\mathcal{F}_{(\cdot)}^m$ is the public information generated by the market returns and $\mathcal{F}_{(\cdot)}^Y$ is the filtration generated by the private signal (25). Consider a manager with a coefficient of relative risk aversion equal to R^a. The manager's optimal fund value at time $t \in [0, T]$ is given by*

$$X_t^{a^*} = \left(\xi_t^{\mathcal{G}}\right)^{-1/R^a} X_0^a H_t^{\mathcal{G}} / H_0^{\mathcal{G}}$$

and the optimal risk exposure at time $t \in [0, T]$ is given by

$$\pi_t^* = \pi_t^m + \pi_t^h = \frac{\Sigma_{t,T}}{\Sigma_{t,T} + (R^a - 1)\Sigma_{T,T}} \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}, \tag{33}$$

$$\pi_t^m = \frac{\theta^m + \theta_t^{\mathcal{G}}}{R^a \sigma^m}, \tag{34}$$

$$\pi_t^h = \frac{(R^a - 1)(\Sigma_{t,T} - \Sigma_{T,T})}{\Sigma_{t,T} + (R^a - 1)\Sigma_{T,T}} \frac{\theta^m + \theta_t^{\mathcal{G}}}{R^a \sigma^m} \tag{35}$$

where

$$\begin{aligned} H_t^{\mathcal{G}} =& \sqrt{\frac{R^a \Sigma_{T,T}}{\Sigma_{t,T} + (R^a - 1)\Sigma_{T,T}}} \left(\frac{\Sigma_{T,T}}{\Sigma_{t,T}} \right)^{-\frac{1}{2R^a}} \\ & \times \exp\left(-\left(r + \frac{\Sigma_{t,T}(\theta^m + \theta_t^{\mathcal{G}})^2}{2(\Sigma_{t,T} + (R^a - 1)\Sigma_{T,T})} \right) \frac{(R^a - 1)(T - t)}{R^a} \right), \\ \Sigma_{t,T} =& (\sigma^m)^2 (T - t) + (\sigma^y)^2 T. \end{aligned}$$

The manager's compensation at time T is

$$F(X_T^{a^*}) = \alpha \left(\xi_T^{\mathcal{G}} \right)^{-1/R^a} X_0^a H_T^{\mathcal{G}} / H_0^{\mathcal{G}}$$

As shown by Equation (33), the optimal portfolio policy can be decomposed into the mean-variance demand π_t^m and the dynamic hedging demand π_t^h. By standard arguments, $\pi_t^m = \frac{\theta^m + \theta_t^{\mathcal{G}}}{R^a \sigma^m}$. Relative to the logarithmic case, the mean-variance demand is scaled by the manager's relative risk aversion, and the optimal portfolio choice has an additional term, reflecting the manager's hedging behavior. The dynamic hedging demand π_t^h is given by

$$\frac{d[H, W^{\mathcal{G}}]_t}{\sigma^m H_t dt} = \frac{(R^a - 1)(\Sigma_{t,T} - \Sigma_{T,T})}{\Sigma_{t,T} + (R^a - 1)\Sigma_{T,T}} \frac{\theta^m + \theta_t^{\mathcal{G}}}{R^a \sigma^m} = \frac{R^a - 1}{1 + \frac{T}{T-t} \left(\frac{\sigma^y}{\sigma^m} \right)^2 R^a} \frac{\theta^m + \theta_t^{\mathcal{G}}}{R^a \sigma^m}. \tag{36}$$

Since the public market price of risk θ^m is constant, the hedging demand is motived by the stochastic fluctuations in $\theta_t^{\mathcal{G}}$. As shown in (36), when the manager is more risk-averse than a log manager, her hedging demand is positive (negative) if the total price of risk $(\theta^m + \theta_t^{\mathcal{G}})$ is positive (negative). Moreover, as time passes, the magnitude of the hedging demand decreases, inducing the informed manager to adjust the share of stocks in the portfolio. $\pi_t^h \to 0$ when clock approaches the terminal date T. A longer investment horizon reduces the magnitude of the hedging demand and the horizon effect is weaker for a manager with higher skill. Finally, the magnitude of the hedging demand is decreasing in the variance ratio $(\sigma^y)^2/(\sigma^m)^2$. This implies that a more skilled manager will have a larger hedging demand.

With proportional only fees, the risk sharing is perfect and the proportional fee does not affect the manager's portfolio. The proportional fee parameter α only affects the manager's compensation as well as the investor's welfare.

Corollary 5 describes the value of the private information to a fund investor (Excess CER) when both the manager and investor have CRRA under purely proportional fees.

Corollary 5. *Suppose the manager's compensation at time T is purely proportional to the terminal value of the managed portfolio $F(X_T^a) = \alpha X_T^a$ with $\alpha > 0$ and the manager (respectively investor) has CRRA utility with a coefficient of relative risk aversion equal to R^a (respectively R). The incremental value of the manager's private signal G, as described in* (25)*, to the investor is*

$$V^p = \begin{cases} \log\sqrt{1 + \frac{(\sigma^m)^2}{R^a(\sigma^y)^2}} + \frac{\log\sqrt{1 - \frac{(R-1)(R-R^a)(\sigma^m)^2}{R^a((\sigma^m)^2 + R^a(\sigma^y)^2)}}}{R-1} + \frac{(R-R^a)^2(P^a-R)(\theta^m)^2(\sigma^m)^2 T}{2(R^a)^2(R(P^a-R)(\sigma^m)^2 + (R^a)^2(\sigma^y)^2)}, & \text{if } R < R^e \\ -\infty, & \text{if } R \geq R^e, \end{cases}$$

where P^a is the manager's relative risk prudence coefficient and $R^e = \frac{P^a + \sqrt{(P^a)^2 + 4(\sigma^y/\sigma^m)^2(R^a)^2}}{2}$.

Note that as the precision of the private signal goes to infinity, namely $\sigma^y \to 0$, the investors with relative risk aversion R smaller than the manager's relative risk prudence P^a will choose the skilled fund. Conversely, the investors whose relative risk aversion R lies above the manager's relative risk prudence P^a will prefer the unskilled fund.

The fact that the investors with $R > P^a$ will prefer the uninformed manager to the informed one though the informed manager has nearly perfect private information about the future market returns is remarkable. Corollary 5 implies that the clientele effect result still holds under the case of managers with CRRA preference. Assuming the manager has logarithmic utility $R^a = 1$ leads to a special case of the finding in Corollary 3.

Figure 6 illustrates the threshold of investor's RRA when the managers also have general CRRA utility. The threshold values can be obtained by finding the root of $V^p = 0$ numerically. R^e provides an upper bound for the RRA threshold. Contrary to the logarithmic manager case, the investor's RRA thresholds here are affected by the manager's skill $s = 1/\sigma^y$ as

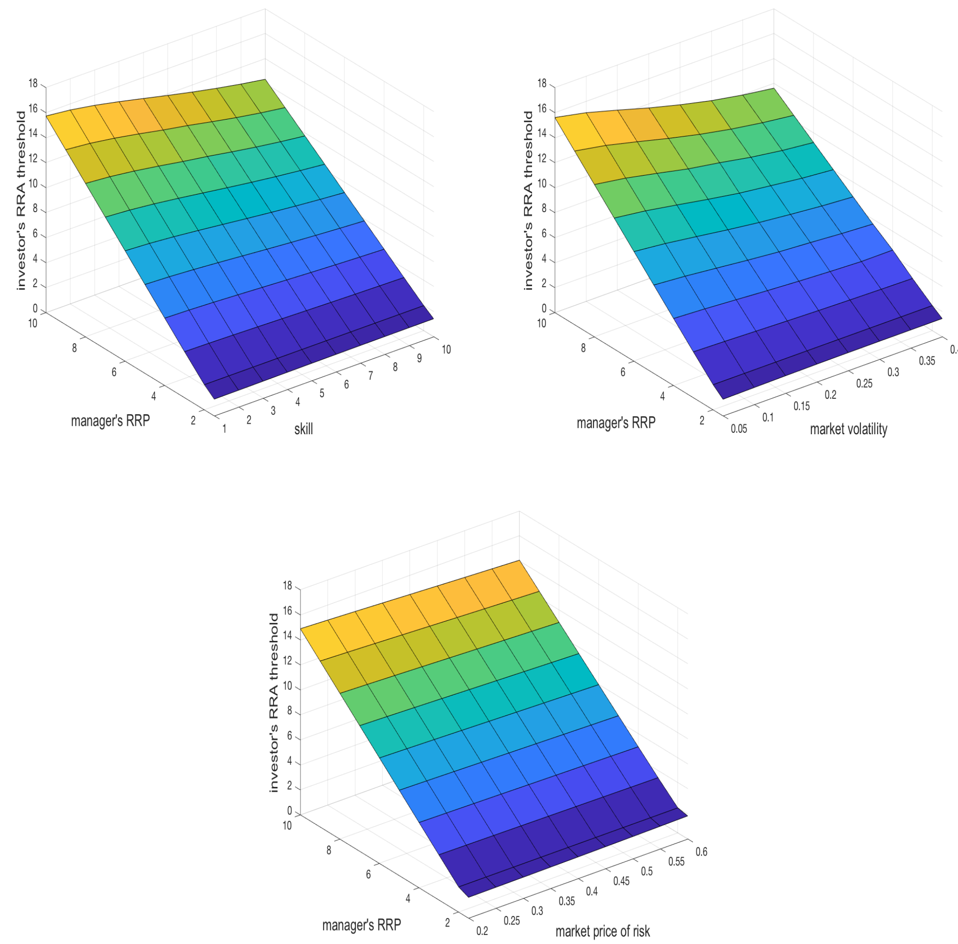

Fig. 6: Investor's RRA threshold versus manager's RRP under purely proportional fees. The figure presents the relative risk aversion threshold of investors under purely proportional fee contract versus manager's relative risk prudence coefficients and skill levels. The fixed parameter values (where applicable) are $r = 3.5\%, \theta^m = 0.47, \sigma^m = 0.155, s = 5, T = 3, \alpha/T = 2\%$.

well as the market conditions θ^m, σ^m. The upper left panel shows that the investor's RRA threshold is decreasing in the manager's skill when the manager's RRP is above 2, whereas the investor's RRA threshold is slightly increasing in the manager's skill level when the manager's RRP lies below than 2. Consistent with the theoretical finding in Corollary 5, it appears that the investor's RRA threshold converges to the manager's relative risk prudence as the skill level increases. The remaining panels show that the investor's RRA threshold is decreasing in both the market price of risk and market volatility when the manager's RRP is above 2. However, the pattern changes when the manager's RRP is below 2.

7.2 PASSIVE ALTERNATIVE

We further extend the model in Section 7.1 by allowing the fund investors to choose a passively managed index fund (e.g., market index) as an alternative to the actively managed funds. Suppose that investors choose among an actively managed skilled fund, an actively managed unskilled fund, and the market index. The fund investor's portfolio-choice problem can be characterized as

$$\max_{\mathscr{C}\in\{s,u,m\}} E\left[\frac{\left((1-\alpha^{\mathscr{C}})X_T^{\mathscr{C}}\right)^{1-R}}{1-R}\right], \tag{37}$$

where X_T^s (respectively X_T^u) is the optimal terminal fund value chosen by the skilled (respectively unskilled) fund manager with relative risk aversion R^a and X_T^m is the market index as given by Equation (1). The parameter $\alpha^{\mathscr{C}}$ with $\mathscr{C} \in \{s,u,m\}$ represents the proportional fee charged by these funds.

Figure (7) illustrates the fund investor's choice among a skilled fund manager, an unskilled fund manager, and the market index as a function of the investor's RRA and the manager's RRP. It highlights the impact of the manager's and investor's risk preference on fund investors' investment behaviors.

Although some investors turn to index investing, the clientele effect still exists in the active management industry. Unskilled fund investors (represented by the dark blue area) are generally more risk-averse than skilled fund investors (represented by the light blue area). Figure (7) illustrates this finding for different level of skills. A bit surprisingly, the unskilled fund is not completely dominated by the passive index fund for relatively risk averse investors, but depending on the RRP of the fund managers. When the fund managers are relatively risk-averse (e.g., $P^a > 5$), the investors whose risk aversion above the red solid line will choose the unskilled fund manager. The intuition is that as the manager's risk aversion increases, the misalignment between the risk preference of the unskilled manager and the relatively risk-averse investors becomes smaller than that between the risk exposure of the market index and the investors' optimal portfolios. In the same vein, when the risk preference misalignment between manager and investors $|R - R^a|$ is large enough, the alignment of the passive fund

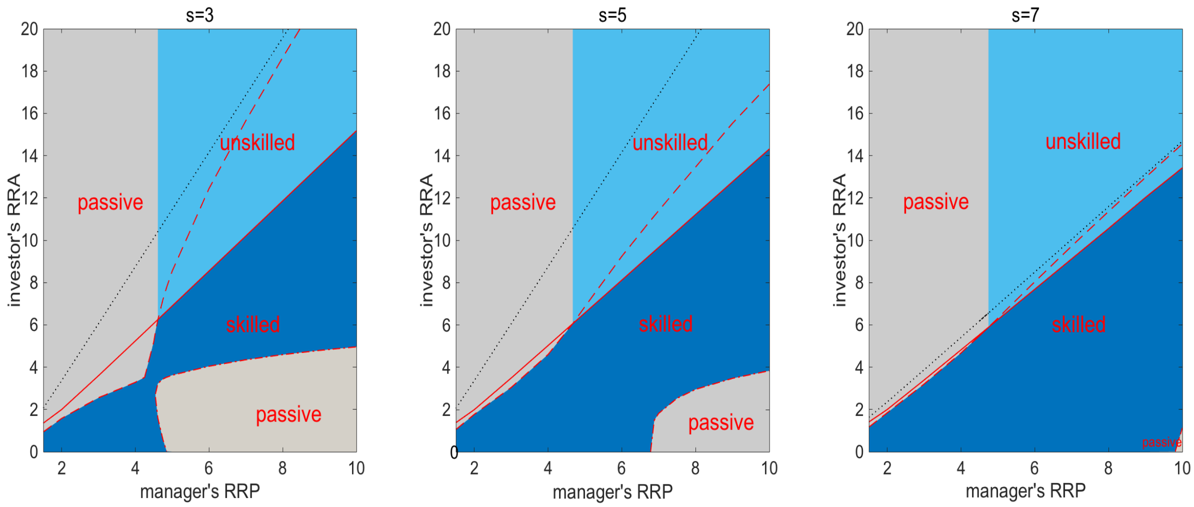

Fig. 7: Investor's choice among skilled, unskilled, and index funds under purely proportional fees. The figure presents the investor's choice among skilled, unskilled, and index funds under purely proportional fee contract. The dark blue area represents the region in which the investors will choose skilled fund. The light blue area represents the region in which the investors will choose unskilled fund. The grey areas represent the regions in which the investors will choose the index fund. The black dotted line represents R^e as a function of manager's RRP. Parameter values are $r = 3.5\%, \theta^m = 0.47, \sigma^m = 0.155, T = 3, \alpha/T = 2\%$ for active (skilled and unskilled) funds and $\alpha/T = 1\%$ for the passively managed index fund.

and the investors' optimal portfolios is better than that of the active fund managers' and investors' optimal portfolios. As a result, these investors will choose the passively managed index fund, as represented by the grey regions in the above plots.

Another insight derived from Figure (7) is that investor's choice between active and passive investing does not only depend on risk preference of manager and investor, but also on the manager's skill level. As seen in the figure, the grey regions expand as the manager's skill s decreases from 7 to 3. It implies that the investors in passively managed index fund are growing as the active managers become less skilled. This result is consistent with the findings of many studies that managers' abilities to beat the market declines as the active management industry size increases in recent decades and this could help explain the growing popularity of index funds (e.g., Berk and Green, 2004; Pástor and Stambaugh, 2012).

8. Conclusion

The clientele effects in the money management industry have been widely documented. Prior literature attributes these clientele effects to irrationality or psychological tendencies. In this paper, we establish a rational theory to explain the clientele effect in the money management industry and show that investors in skilled funds are uniformly more risk-tolerant than investors in unskilled funds.

Taking a general parametric class of contracts as given, we first derive and analyze the optimal trading strategies of the skilled fund manager who is endowed with private information about future market returns. Then we analyze the value of the private information to both managers and investors. Though the privation information is always valuable to the manager, it might not add value and may be harmful to the investors who are sufficiently more risk-averse than the manager. Investors with risk aversion exceeding a threshold value will never find it beneficial to delegate the management of their wealth to the skilled fund manager. As a result, the fund investor clientele is endogenously segmented. The relatively risk-tolerant investors will prefer skilled funds, whereas the highly risk-averse investors will prefer unskilled funds. This result provides theoretical justification for some recent empirical findings about the clientele effect in the mutual fund industry.

In the absence of performance fees, the relative risk aversion threshold that separates investors in skilled and unskilled funds equals to the log manager's RRP, irrespective of the skill or other parameters. When the fund manager receives performance-based fees, the relative risk aversion threshold is affected by the skill level, contract parameters, and market conditions. We specialize the general results to a parametric timing model. A comparative static analysis of the risk aversion threshold is carried out to analyze the impacts of symmetric and asymmetric performance fees on investors' fund investment. The results suggest that sufficiently risk-averse investors should include a fulcrum performance fee in the manager's compensation contract for the purpose of realizing positive value from the manager's private information. The intuition behind this is that lifting the penalty sensitivity to the same level of bonus incentive reduces the portfolio's volatility and alleviates highly risk-averse investors' concerns about the larger downside tail risk in the skilled fund relative to the unskilled fund.

Our qualitative results do not depend on the assumptions that managers have logarithmic utility function or investors are restricted to choose among actively managed funds. Extensions to the basic setup examine the cases of managers with general CRRA utility and investors who can invest in a passive alternative. There still exist two distinctive groups of investors for skilled and unskilled funds. Moreover, our results in Section 2 are easily generalizable to a multi-asset setting that managers have both the timing and selection skills.

Appendix

In order to prove the main results, the following regularity conditions are assumed.

ASSUMPTION

We consider the following assumptions as in DR. Let $\theta_v \equiv \theta_v^m + \theta_v^{\mathcal{G}}$ be an informed agent's price of risk. For $i = 1, \ldots, N$, the private information price of risk $\theta^{\mathcal{G}}$ and the informed trading strategy π satisfy the conditions

(i) (Finite quadratic variation) $\int_{\tau_{i-1}}^{\tau_i} \theta_v^2 dv < \infty$ P-a.s.

(ii) (Gains from trade bounded below) Let $H_i^\epsilon \equiv \frac{1}{\epsilon} \int_{\tau_{i-1}}^{\tau_i} \pi_v (R_{v,v+\epsilon}^m - E[R_{v,v+\epsilon}^m | \mathcal{G}_v]) dv$ be the innovations in the gains from trade. Then, $H_i^\epsilon > -\underline{H}_i$ for some positive $\mathcal{G}_0$-measureable random variable H_i with $E[\underline{H}_i | \mathcal{G}_0] < \infty$ P-a.s.

The first condition ensures that the private information price of risk is finite, ruling out arbitrage opportunities. The second condition rules out doubling strategies.

PROOF OF LEMMA 1

Proof. Suppose $X_b > 0$ and $\alpha > 0$, $\beta_2 > \beta_1 \geq 0$, there exist unique numbers $X_1(X_b)$ and $X_2(X_b)$ solve the system of equations

$$\begin{cases} \frac{u^M(F(X_2(X_b),X_b)) - u^M(F(X_1(X_b),X_b))}{X_1(X_b) - X_2(X_b)} = u_x^M(F(X_2(X_b), X_b)\alpha(1+\beta_2), \\ u_x^M(F(X_1(X_b), X_b))(1+\beta_1) = u_x^M(F(X_2(X_b), X_b))(1+\beta_2). \end{cases} \tag{A1}$$

In particular, if marginal utility is homogeneous of degree $-R$ ($R \neq 1$), letting $\eta \equiv \left(\frac{1+\beta_2}{1+\beta_1}\right)^{1-1/R}$ and $\rho \equiv \left(\frac{1+\beta_2}{1+\beta_1}\right)^{-1/R}$. In particular, if marginal utility is homogeneous of degree $-R$, the above equations imply that

$$\begin{cases} X_1(X_b) + \beta_1(X_1(X_b) - \delta X_b) = \rho(X_2(X_b) + \beta_2(X_2(X_b) - \delta X_b)), \\ X_2(X_b) = X_1(X_b) + \frac{u^M(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b) - u^M(\rho(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b)))}{u_x^M(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b))\alpha(1+\beta_2)}. \end{cases}$$

Equivalently, u^M is homogeneous of degree $1-R$, we have

$$\begin{aligned}
X_2(X_b) &= X_1(X_b) + (1-\rho^{1-R})\frac{u^M(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b))}{u_x^M(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b))\alpha(1+\beta_2)} \\
&= X_1(X_b) + \frac{1-\eta}{(1-R)\alpha(1+\beta_2)}(\alpha X_2(X_b) + \alpha\beta_2(X_2(X_b) - \delta X_b)),
\end{aligned}$$

Direct computation yields

$$\begin{cases}
X_1(X_b) = \left(\frac{(\frac{\eta}{R}-1)\frac{\beta_1}{1+\beta_1} + \eta(1-\frac{1}{R})\frac{\beta_2}{1+\beta_2}}{\eta-1}\right)\delta X_b, \\
X_2(X_b) = X_1(X_b) + \frac{1}{R}\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)\delta X_b.
\end{cases}$$

Moreover, $\underline{X}(X_b) = \frac{\beta_1 \delta X_b}{1+\beta_1} < X_1(X_b) < \delta X_b < X_2(X_b)$. In particular, for logarithmic utility

$$\begin{aligned}
X_1(X^b) &= \left(log\left(\frac{1+\beta_2}{1+\beta_1}\right)\right)^{-1}\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)\delta X^b + \frac{\beta_1}{1+\beta_1}\delta X^b > \underline{X}(X^b), \\
X_2(X^b) &= X_1(X^b) + \left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)\delta X^b.
\end{aligned}$$

PROOF OF LEMMA 2

Proof. This closely follows the proof of Lemma 2 from Cuoco and Kaniel (2011). The first equation in the system (A1) shows that $v^M(\cdot, X_b)$ is continuous at $X_2(X_b)$, while the second equation in the system (A1) show that $v^M(\cdot, X_b)$ is continuously differentiable at $X_1(X_b)$. Thus $v^M(\cdot, X_b)$ is continuously differentiable and concave on $[\underline{X}(X_b), \infty)$. Since $v^M(X_1(X_b), X_b) = u^M(F(X_1(X_b), X_b))$ and $u^M(F(\cdot, X_b))$ is strictly concave on the interval $(X_1(X_b), \delta X_b]$ while $v^M(\cdot, X_b)$ is linear, we must have that $v^M(\cdot, X_b) > u^M(F(\cdot, X_b))$ on the interval. Similarly, we have $v^M(\cdot, X_b) > u^M(F(\cdot, X_b))$ on the interval $[\delta X_b, X_2(X_b))$. Moreover, $v^M(\cdot, X_b) = u^M(F(\cdot, X_b))$ on $A(X_b)$, thus we have $v^M(\cdot, X_b) \geq u^M(F(\cdot, X_b))$ on the interval $[\underline{X}(X_b), \infty)$.

Suppose $\hat{v}^M(\cdot, X_b)$ is any concave function with $\hat{v}^M(\cdot, X^b) \geq u^M(F(\cdot, X^b))$ on the interval $[\underline{X}(X^b), \infty)$. It follows from the definition that $\hat{v}^M(\cdot, X^b) \geq u^M(F(\cdot, X^b)) = v^M(\cdot, X^b)$ on $A(X^b)$. In addition, $\hat{v}^M(\cdot, X^b)$ is concave on the interval $(X_1(B), X_2(B))$ while $v^M(\cdot, X^b)$ is linear, it follows that $\hat{v}^M(\cdot, X^b) > v^M(\cdot, X_b)$ on $(X_1(X_b), X_2(X_b))$. Thus, $v^M(\cdot, X^b)$ is the smallest concave function with $\hat{v}^M(\cdot, X^b) \geq u^M(F(\cdot, X_b))$ on the interval $[\underline{X}(X_b), \infty)$.

PROOF OF PROPOSITION 1

Proof. Suppose that $\alpha > 0, \beta_1 = \beta_2 \geq 0$. The first order condition of the fund manager's static problem is

$$\alpha(1+\beta_2)u_x^M(\alpha X_T^{a,s^*} + \alpha\beta_2(X_T^{a,s^*} - \delta X_T^b)) = y^{s^*}\xi_T^{\mathcal{G}}.$$

Direct computation yields

$$X_T^{a,s^*} = \frac{1}{\alpha(1+\beta_2)} I^M\left(\frac{y^{s^*}\xi_T^{\mathcal{G}}}{\alpha(1+\beta_2)}\right) + \frac{\beta_2 \delta X_T^b}{1+\beta_2}, \tag{A2}$$

where $I^M(\cdot)$ is the inverse function of $u_x^M(\cdot)$ and y^{s^*} is determined by the static budget constraint

$$\frac{x^a}{1+\beta_2} = E\left[\frac{\xi_T^{\mathcal{G}}}{\alpha(1+\beta_2)} I^M\left(\frac{y^{s^*}\xi_T^{\mathcal{G}}}{\alpha(1+\beta_2)}\right)\middle| \mathcal{G}_0\right] \equiv \chi(y^{s^*}).$$

The function $\chi(y)$ is continuous and strictly decreasing on $(0,\infty)$. Moreover, $\chi(y) \to 0$ as $y \to \infty$ and $\chi(y) \to \infty$ as $y \to 0$. Therefore, there exists a unique $y^{s^*} > 0$ such that $\chi(y^{s^*}) = x^a/(1+\beta_2)$. In particular, with $u^M(x) = \log(x)$, $y^{s^*} = (1+\beta_2)/x^a$. We can find the manager's optimal portfolio choice by applying Itô's Lemma on both sides of (A2) and matching the coefficients in front of $dW_v^{\mathcal{G}}$. The optimal portfolio choice is given by

$$\pi_v^{a,s^*} = \frac{\theta_v^m + \theta_v^{\mathcal{G}}}{\sigma_v^m} + \frac{\beta_2}{1+\beta_2}\frac{\delta X_v^b}{X_v^{a,s^*}}\left(\pi^b - \frac{\theta_v^m + \theta_v^{\mathcal{G}}}{\sigma_v^m}\right).$$

Substituting the optimal fund's end-of-period value (A2) into the contract yields the manager's compensation at time T, $F(X_T^{a,s^*}, X_T^b) = \alpha x^a/\xi_T^{\mathcal{G}}$.

PROOF OF PROPOSITION 2.

Proof. Suppose that $\alpha > 0$, $\beta_2 > \beta_1 \geq 0$. The concavified utility function v^M concave and continuously differentiable on the interval $[\underline{X}(X_b), \infty)$. Thus we can solve the concavified problem using standard optimization theory. The sufficient and necessary condition for X_T^{a,s^*} to be optimal is

$$\frac{\partial v^M}{\partial x^a}(X_T^{a,s^*}, X_T^b) = y^{s^*}\xi_T^{\mathcal{G}}.$$

We can define a function $g^I : (0,\infty) \times (0,\infty)$

$$g^I(y,b) = \begin{cases} \frac{1}{\alpha(1+\beta_2)} I^M\left(\frac{y}{\alpha(1+\beta_2)}\right) + \frac{\beta_2 \delta b}{1+\beta_2} > X_2(b) & \text{if} \quad y \leq \Psi(b)\,, \\ \frac{1}{\alpha(1+\beta_1)} I^M\left(\frac{y}{\alpha(1+\beta_1)}\right) + \frac{\beta_1 \delta b}{1+\beta_1} < X_1(b) & \text{if} \quad y > \Psi(b)\,, \end{cases}$$

where $I^M(\cdot)$ is the inverse function of $u_x^M(\cdot)$ and $\Psi(b) = \frac{\log\left(\frac{1+\beta_2}{1+\beta_1}\right)}{\delta b\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}$. The optimal end-of-period fund value is given by

$$\begin{aligned} X_T^{a,s^*} &= g^I\left(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b\right) \\ &= \frac{1}{\alpha(1+\beta_2)} I^M\left(\frac{y^{s^*}\xi_T^{\mathcal{G}}}{\alpha(1+\beta_2)}\right) + \frac{\beta_2 \delta X_T^b}{1+\beta_2} \mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} \leq \Psi(X_T^b)\}} + \frac{\beta_1 \delta X_T^b}{1+\beta_1} \mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} > \Psi(X_T^b)\}}, \end{aligned}$$

where y^{s^*} is the Lagrange multiplier solving the static budget constraint:

$$x^a = E\left[\xi_T^{\mathcal{G}} g^I\left(y^{s^*}\xi_T^{\mathcal{G}}, X_T^b\right) \,\middle|\, \mathcal{G}_0\right] \equiv \tilde{\chi}(y^{s^*}).$$

The function $\tilde{\chi}(y)$ is continuous and strictly decreasing on $(0,\infty)$. Moreover, $\tilde{\chi}(y) \to \beta_1 X_0^a/(1+\beta_1)$ as $y \to \infty$ and $\chi(y) \to \infty$ as $y \to 0$. Therefore, there exists a unique $y^{s^*} > 0$ such that $\chi(y^{s^*}) = X_0^a$. With $u^M(x) = \log(x)$, we immediately obtain (16) and the manager's compensation at time T:

$$F(X_T^{a,s^*}, X_T^b) = \frac{\alpha(1+\beta_2)}{y^{s^*}\xi_T^{\mathcal{G}}} \mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} \leq \Psi(X_T^b)\}} + \frac{\alpha(1+\beta_1)}{y^{s^*}\xi_T^{\mathcal{G}}} \mathbb{1}_{\{y^{s^*}\xi_T^{\mathcal{G}} > \Psi(X_T^b)\}}.$$

PROOF OF PROPOSITION 3

Proof. Define $H_v^\epsilon \equiv \frac{1}{\epsilon}\int_v^{v+\epsilon} dW_v^m$. For $v \in [\tau_{i-1}, \tau_i)$, using Bayes' rule $\frac{P(dW^m|\mathcal{F}_v, G_i = z)}{P(dW^m|\mathcal{F}_v)} = \frac{P(G_i \in dz|\mathcal{F}_{v+\epsilon})}{P(G_i \in dz|\mathcal{F}_v)}$ gives

$$\begin{aligned} E\left[H_v^\epsilon \,|\, \mathcal{G}_v\right] =& E\left[\frac{1}{\epsilon}\int_v^{v+\epsilon} dW_s^m \,\middle|\, \mathcal{F}_v, G_i = z\right]_{|z=G_i} \\ =& E\left[H_v^\epsilon \frac{P(dW^m|\mathcal{F}_v, G_i = z)}{P(dW^m|\mathcal{F}_v)} \,\middle|\, \mathcal{F}_v\right]_{|z=G_i} \\ =& E\left[H_v^\epsilon \frac{P(G_i \in dz|\mathcal{F}_{v+\epsilon})}{P(G_i \in dz|\mathcal{F}_v)} \,\middle|\, \mathcal{F}_v\right]_{|z=G_i}. \end{aligned}$$

Therefore, with $p^G_v(z) = P(G_i \in dz|\mathcal{F}_v)$, for $v \in [\tau_{i-1}, \tau_i)$ when $z = G_i$

$$\begin{aligned}\theta^{\mathcal{G}}_v &= \lim_{\epsilon\downarrow 0} E\left[H^\epsilon_v \,|\, \mathcal{G}_v\right] = \lim_{\epsilon\downarrow 0} E_v\left[\frac{p^G_{v+\epsilon}(z)}{p^G_v(z)}\int_v^{v+\epsilon} dW^m_s\right]_{|z=G_i} \\ &= \left(\frac{d[\log p^G_v(z), W^m]_v}{dv}\right)_{|z=G_i} = \mathcal{D}_v \log p^G_v(z)_{|z=G_i},\end{aligned}$$

where $E_v[\cdot] \equiv E[\cdot|\mathcal{F}_v]$. In the presence of fulcrum performance fees $\alpha X^a_T + \alpha\beta_2\left(X^a_T - \delta X^b_T\right)$ with $\alpha > 0$ and $\beta_2 \geq 0$, the managers compensation is $\alpha x^a/\xi_T$. Thus, the ex ante value functions for the skilled and unskilled managers are

$$\begin{aligned}E\left[\log\frac{\alpha x^a}{\xi^{\mathcal{G}}_T}\right] &= \log\left(\alpha X^a_0\right) + E\left[\int_0^T r_v dv\right] + \frac{1}{2}E\left[\int_0^T \left(\theta^m_v + \theta^{\mathcal{G}}_v\right)^2 dv\right] \\ E\left[\log\frac{\alpha x^a}{\xi^m_T}\right] &= \log\left(\alpha X^a_0\right) + E\left[\int_0^T r_v dv\right] + \frac{1}{2}E\left[\int_0^T \left(\theta^m_v\right)^2 dv\right].\end{aligned}$$

Using $E[\theta^m_v\theta^{\mathcal{G}}_v] = 0$, which follows from $E_v[\theta^{\mathcal{G}}_v] = E_v\left[\lim_{\epsilon\downarrow 0} E\left[H^\epsilon_v \,|\, \mathcal{G}_v\right]\right] = 0$, gives $E\left[\log\frac{\alpha X^a_0}{\xi^{\mathcal{G}}_T}\right] = E\left[\log\frac{\alpha X^a_0}{\xi^m_T}\right] + \frac{1}{2}E\left[\int_0^T\left(\theta^{\mathcal{G}}_v\right)^2 dv\right]$ and the ex ante value of information to the manager

$$V^{M,f} \equiv CER^{M,s} - CER^{M,u} = \frac{1}{2}\int_0^T E\left[\left(\theta^{\mathcal{G}}_v\right)^2\right] dv.$$

The process $p^G_v(z)$ is a martingale. By the Clark-Ocone formula,

$$\begin{aligned}p^G_{\tau_i}(z) &= p^G_{\tau_{i-1}}(z) + \int_{\tau_{i-1}}^{\tau_i} E_v[\mathcal{D}_v p^G_{\tau_i}(z)]dW^m_v = p^G_{\tau_{i-1}}(z) + \int_{\tau_{i-1}}^{\tau_i} \mathcal{D}_v E_v[p^G_{\tau_i}(z)]dW^m_v \\ &= p^G_{\tau_{i-1}}(z) + \int_{\tau_{i-1}}^{\tau_i} \mathcal{D}_v p^G_v(z)dW^m_v = p^G_{\tau_{i-1}}(z) + \int_{\tau_{i-1}}^{\tau_i} p^G_v(z)\mathcal{D}_v \log p^G_v(z)dW^m_v \\ &= p^G_{\tau_{i-1}}(z) + \int_{\tau_{i-1}}^{\tau_i} p^G_v(z)\theta^{\mathcal{G}}_v(z)dW^m_v.\end{aligned}$$

Solving this linear stochastic differential equation gives

$$\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)} = \exp\left(\int_{\tau_{i-1}}^{\tau_i}\theta^{\mathcal{G}}_v(z)dW^m_v - \int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\theta^{\mathcal{G}}_v(z)^2 dz\right). \tag{A3}$$

and

$$
\begin{aligned}
E_{\tau_{i-1}}\left[\log\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)}\middle| G_i=z\right] =& E_{\tau_{i-1}}\left[\int_{\tau_{i-1}}^{\tau_i}\theta^{\mathcal{G}}_v(z)dW^m_v-\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\theta^{\mathcal{G}}_v(z)^2dz\middle| G_i=z\right]\\
=& E_{\tau_{i-1}}\left[\int_{\tau_{i-1}}^{\tau_i}\theta^{\mathcal{G}}_v(z)(dW^m_v-\theta^{\mathcal{G}}_v(z)dv)+\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\theta^{\mathcal{G}}_v(z)^2dv\middle| G_i=z\right]\\
=& \frac{1}{2}E_{\tau_{i-1}}\left[\int_{\tau_{i-1}}^{\tau_i}\theta^{\mathcal{G}}_v(z)^2dv\middle| G_i=z\right]
\end{aligned}
$$

Using Bayes' rule $\frac{P(dW^m|\mathcal{F}_{\tau_{i-1}},G_i=z)}{P(dW^m|\mathcal{F}_{\tau_{i-1}})}=\frac{P(G_i\in dz|\mathcal{F}_{\tau_i})}{P(G_i\in dz|\mathcal{F}_{\tau_{i-1}})}=\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)}$ gives

$$
\begin{aligned}
\frac{1}{2}E_{\tau_{i-1}}\left[\int_{\tau_{i-1}}^{\tau_i}\left(\theta^{\mathcal{G}}_v\right)^2dv\right] &= \int_{-\infty}^{\infty}\frac{1}{2}E_{\tau_{i-1}}\left[\int_{\tau_{i-1}}^{\tau_i}\theta^{\mathcal{G}}_v(z)^2dv\middle| G_i=z\right]p^G_{\tau_{i-1}}(z)dz\\
&= \int_{-\infty}^{\infty}E_{\tau_{i-1}}\left[\log\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)}\middle| G_i=z\right]p^G_{\tau_{i-1}}(z)dz\\
&= \int_{-\infty}^{\infty}E_{\tau_{i-1}}\left[\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)}\log\frac{p^G_{\tau_i}(z)}{p^G_{\tau_{i-1}}(z)}\right]p^G_{\tau_{i-1}}(z)dz\\
&= \int_{-\infty}^{\infty}E_{\tau_{i-1}}\left[p^G_{\tau_i}(z)\log p^G_{\tau_i}(z)\right]dz-\int_{-\infty}^{\infty}\log p^G_{\tau_{i-1}}(z)p^G_{\tau_{i-1}}(z)dz\\
&= E_{\tau_{i-1}}\left[\log p^G_{\tau_i}(G_i)\right]-E_{\tau_{i-1}}\left[\log p^G_{\tau_{i-1}}(G_i)\right]\\
&= E_{\tau_{i-1}}\left[\log\frac{p^G_{\tau_i}(G_i)}{p^G_{\tau_{i-1}}(G_i)}\right]
\end{aligned}
$$

Using the definition $\mathcal{D}_{KL}(p^G_{\tau_i}(G_i)|p^G_{\tau_{i-1}}(G_i))=E_{\tau_{i-1}}\left[\log\frac{p^G_{\tau_i}(G_i)}{p^G_{\tau_{i-1}}(G_i)}\right]$ and the law of iterated expectations gives $V^{M,f}=\frac{1}{2}E\left[\int_0^T\left(\theta^{\mathcal{G}}_v\right)^2dv\right]=\sum_{i=1}^N E\left[\mathcal{D}_{KL}(p^G_{\tau_i}(G_i)|p^G_{\tau_{i-1}}(G_i))\right]$

PROOF OF PROPOSITION 4

Proof. The state price densities for the time interval $[\tau_{i-1},\tau_i]$ are

$$
\begin{aligned}
\xi^{\mathcal{G}}_{\tau_{i-1},\tau_i} &\equiv \exp\left(-\int_{\tau_{i-1}}^{\tau_i}\left(r_v+\frac{1}{2}\left(\theta^m_v+\theta^{\mathcal{G}}_v\right)^2\right)dv-\int_{\tau_{i-1}}^{\tau_i}\left(\theta^m_v+\theta^{\mathcal{G}}_v\right)dW^{\mathcal{G}}_v\right)\\
\xi^m_{\tau_{i-1},\tau_i} &\equiv \exp\left(-\int_{\tau_{i-1}}^{\tau_i}\left(r_v+\frac{1}{2}\left(\theta^m_v\right)^2\right)dv-\int_{\tau_{i-1}}^{\tau_i}\theta^m_v dW^m_v\right).
\end{aligned}
$$

where $\theta_v^{\mathcal{G}} = \theta_v^{\mathcal{G}}(G_i)$. Using $dW_v^{\mathcal{G}} = dW_v^m - \theta_v^{\mathcal{G}} dv$ and (A3) gives $\frac{\xi^{\mathcal{G}}_{\tau_{i-1},\tau_i}}{\xi^m_{\tau_{i-1},\tau_i}} = \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)}$. It follows that $\frac{\xi_T^{\mathcal{G}}}{\xi_T^m} = \prod_{i=1}^N \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)}$. Let $E\left[\cdot \,\middle|\, \mathcal{F}_{\tau_{i-1}} \bigvee \sigma(X_T^b)\right] = E_{\tau_{i-1},b_T}[\cdot]$. As

$$E\left[\frac{p^G_{\tau_{n-1}}(G_i)}{p^G_{\tau_n}(G_i)} \,\middle|\, \mathcal{F}_{\tau_n} \bigvee \sigma(X_T^b) \bigvee \sigma(\xi_T^m)\right] = E\left[\frac{p^G_{\tau_{n-1}}(G_i)}{p^G_{\tau_n}(G_i)} \,\middle|\, \mathcal{F}_{\tau_n}\right] = \int_{-\infty}^{\infty} \frac{p^G_{\tau_{n-1}}(z)}{p^G_{\tau_n}(z)} p^G_{\tau_n}(z)dz = 1.$$

The law of iterated expectation gives

$$\begin{aligned}
E_{\tau_0,b_T}\left[\prod_{i=1}^N \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)} \,\middle|\, \xi_T^m\right] &= E_{\tau_0,b_T}\left[\prod_{i=1}^{N-1} \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)} E\left[\frac{p^G_{\tau_{N-1}}(G_i)}{p^G_{\tau_N}(G_i)} \,\middle|\, \mathcal{F}_{\tau_N}\right] \,\middle|\, \xi_T^m\right] \\
&= E_{\tau_0,b_T}\left[\prod_{i=1}^{N-2} \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)} E\left[\frac{p^G_{\tau_{N-2}}(G_i)}{p^G_{\tau_{N-1}}(G_i)} \,\middle|\, \mathcal{F}_{\tau_{N-1}}\right] \,\middle|\, \xi_T^m\right] \\
&\;\;\vdots \\
&= E_{\tau_0,b_T}\left[E\left[\frac{p^G_{\tau_0}(G_i)}{p^G_{\tau_1}(G_i)} \,\middle|\, \mathcal{F}_{\tau_1}\right] \,\middle|\, \xi_T^m\right] = 1.
\end{aligned}$$

As $\frac{\xi_T^{\mathcal{G}}}{\xi_T^m} = \prod_{i=1}^N \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)}$., it follows that

$$\begin{aligned}
E_{\tau_0,b_T}\left[\xi_T^{\mathcal{G}} \,\middle|\, \xi_T^m\right] &= \xi_T^m \\
E_{\tau_0,b_T}\left[\xi_T^{\mathcal{G}}\right] &= E_{\tau_0,b_T}\left[\xi_T^m\right]
\end{aligned}$$

Let $\epsilon^\xi \equiv \xi_T^m \left(\prod_{i=1}^N \frac{p^G_{\tau_{i-1}}(G_i)}{p^G_{\tau_i}(G_i)} - 1\right)$ and note that $\xi_T^{\mathcal{G}} = \xi_T^m + \epsilon^\xi$ with $E_{\tau_0,b_T}\left[\epsilon^\xi \,\middle|\, \xi_T^m\right]$. Thus $\xi_T^m \; \mathcal{SSD} \; \xi_T^{\mathcal{G}}$ in the mean-preserving spread sense.

PROOF OF PROPOSITION 5

Proof. Suppose the performance fees are of fulcrum type: $F(X_T^a, X_T^b) = \alpha X_T^a + \alpha\beta_2\left(X_T^a - \delta X_T^b\right)$ with $\alpha > 0$ and $\beta_2 \geq 0$. The definition of certainty equivalent returns gives

$$\begin{aligned}
CER^s &= \frac{1}{1-R} \log\left((1-R)E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right)\right]\right) - \log(X_0^a), \\
CER^u &= \frac{1}{1-R} \log\left((1-R)E\left[v^f\left(\xi_T^m, X_T^b\right)\right]\right) - \log(X_0^a).
\end{aligned}$$

Thus, the value of the private signals G_i with $i = 1, \ldots, N$ within the period $[0, T]$ to the fund investor with relative risk aversion R is

$$V^f \equiv CER^s - CER^u = \frac{1}{1-R} \log \left(\frac{E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right)\right]}{E\left[v^f\left(\xi_T^m, X_T^b\right)\right]} \right).$$

Let $F^{\xi^m, b}$ (respectively $F^{\xi^{\mathcal{G}}, b}$) be the cumulative distribution function (CDF) of ξ_T^m (respectively $\xi_T^{\mathcal{G}}$) based on $\sigma(X_T^b)$, where $\sigma(X_T^b)$ is the filtration generated by the benchmark portfolio at time T. Define $\Delta^{\xi,b}(z) \equiv F^{\xi^{\mathcal{G}},b}(z) - F^{\xi^m,b}(z)$ and $T^{\xi,b}(z) \equiv \int_{-\infty}^{z} \left(F^{\xi^{\mathcal{G}},b}(y) - F^{\xi^m,b}(y)\right) dy$. We have

$$\begin{aligned}
\Delta =& E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right)\right] - E\left[v^f\left(\xi_T^m, X_T^b\right)\right] = E\left[E\left[v^f\left(\xi_T^{\mathcal{G}}, X_T^b\right) - v^f\left(\xi_T^m, X_T^b\right) \Big| \sigma(X_T^b)\right]\right] \\
=& E\left[\int_0^\infty v^f\left(z, X_T^b\right) d\Delta^{\xi,b}(z)\right] = E\left[v^f\left(z, X_T^b\right) \Delta^{\xi,b}(z)\Big|_0^\infty - \int_0^\infty \frac{\partial v^f\left(z, X_T^b\right)}{\partial z} \Delta^{\xi,b}(z) dz\right] \\
=& E\left[-\int_0^\infty \frac{\partial v^f\left(z, X_T^b\right)}{\partial z} \Delta^{\xi,b}(z) dz\right] = E\left[-\frac{\partial v^f\left(z, X_T^b\right)}{\partial z} T^{\xi,b}(z)\Big|_0^\infty + \int_0^\infty \frac{\partial^2 v^f\left(z, X_T^b\right)}{\partial z^2} T^{\xi,b}(z) dz\right] \\
=& E\left[\int_0^\infty \frac{\partial^2 v^f\left(z, X_T^b\right)}{\partial z^2} T^{\xi,b}(z) dz\right] \qquad \text{(A4)}
\end{aligned}$$

where the last equality follows from the fact $T^{\xi,b}(\infty) = 0$. Together with $T^{\xi,b}(z) \geq 0$ for all $z \in \mathbb{R}_+$, $\Delta = E\left[\int_0^\infty \frac{\partial^2 v^f\left(z, X_T^b\right)}{\partial z^2} T^{\xi,b}(z) dz\right] \leq 0$ (respectively > 0) if $v^f(\cdot, X_T^b)$ is concave (respectively convex). Using $v^f(z, X_T^b) = u(I(y^* z, X_T^b))$, we have

$$\begin{aligned}
& \frac{\partial^2 v^f\left(z, X_T^b\right)}{\partial z^2} \\
=& \frac{du}{dx}(I(y^* z, X_T^b)) \frac{\frac{\partial I}{\partial z}(y^* z, X_T^b)^2}{I(y^* z, X_T^b)} \left[\frac{\frac{d^2 u}{dx^2}(I(y^* z, X_T^b)) I(y^* z, X_T^b)}{\frac{du}{dx}(I(y^* z, X_T^b))} + \frac{\frac{\partial^2 I}{\partial z^2}(y^* z, X_T^b)}{\frac{\partial I}{\partial z}(y^* z, X_T^b)^2} I(y^* z, X_T^b)\right] \\
=& \frac{\frac{\partial I}{\partial z}(y^* z, X_T^b)^2}{I(y^* z, X_T^b)^{R+1}} \left[P^a \frac{\alpha(1+\beta_2) I(y^* z, X_T^b)}{(1-\alpha(1+\beta_2)) I^M\left(\frac{y^* z}{\alpha(1+\beta_2)}\right)} - R\right] \\
=& \frac{\left(\frac{1-\alpha(1+\beta_2)}{y^* z^2}\right)^2 \left[2\left(1 + \frac{\beta_2 \delta X_T^b}{1+\beta_2} \frac{y^* z}{1-\alpha(1+\beta_2)}\right) - R\right]}{\left(\frac{1-\alpha(1+\beta_2)}{y^* z} + \frac{\beta_2 \delta X_T^b}{1+\beta_2}\right)^{R+1}}
\end{aligned}$$

where $y^* = (1+\beta_2)/X_0^a$ and $P^a \equiv -\frac{d^3 u^M}{dx^3}(x) x / \frac{d^2 u^M}{dx^2}(x)$ is the relative risk prudence of the manager. The third equality follows because $P^a = 2$ for logarithmic utility and $I(y^* z, X_T^b) = \frac{1-\alpha(1+\beta_2)}{y^* z} +$

$\frac{\beta_2 \delta X_T^b}{1+\beta_2}$. Substituting the expression of $\frac{\partial^2 v^f\left(z, X_T^b\right)}{\partial z^2}$ into $(A4)$ yields

$$\Delta = E\left[\int_0^\infty \frac{\left(\frac{1-\alpha(1+\beta_2)}{y^* z^2}\right)^2 \left[2\left(1+\frac{\beta_2\delta X_T^b}{1+\beta_2}\frac{y^* z}{1-\alpha(1+\beta_2)}\right)-R\right] T^{\xi,b}(z)}{\left(\frac{1-\alpha(1+\beta_2)}{y^* z}+\frac{\beta_2\delta X_T^b}{1+\beta_2}\right)^{R+1}} dz\right]$$

PROOF OF PROPOSITION 6

Proof. The claims immediately follow from the definition of the value of the private signal.

PROOF OF COROLLARY 1

Proof. For $t \in [0, T)$, the conditional density of the signal is

$$p_t^G(x) \equiv \partial_x P_t(G \le x) = \partial_x P_t\left(\log\left(S_{0,T}^m\right) + \sigma^y \int_0^T dW_v^\zeta \le \log(x) + \frac{1}{2}(\sigma^y)^2 T\right).$$

Thus, we have

$$p_t^G(x) = \partial_x P_t\left(\frac{\sigma^m \int_t^T dW_v^m + \sigma^y \int_0^T dW_v^\zeta}{\sqrt{\Sigma_{t,T}}} \le d(x,t)\right) = \partial_x \Phi(d(x,t)) = \frac{\phi(d(x,t))}{x\sqrt{\Sigma_{t,T}}},$$

where

$$\begin{aligned} d(x,t) &= \frac{\log(x) - E_t[\log(G)]}{\sqrt{\Sigma_{t,T}}}, \\ VAR_t[\log G] &= (\sigma^m)^2(T-t) + (\sigma^y)^2 T \equiv \Sigma_{t,T}, \\ E_t[\log G] &= \log S_{0,t}^m + (\sigma^m\theta^m - \frac{1}{2}(\sigma^m)^2)(T-t) - \frac{1}{2}(\sigma^y)^2 T. \end{aligned}$$

PROOF OF COROLLARY 2

Proof. As in the proof of Proposition 3

$$\begin{aligned} &\frac{1}{2}E\left[\int_0^T \left(\theta_v^{\mathcal{G}}\right)^2 dv\right] \\ =&E\left[\log\frac{p_T^G(G_i)}{p_0^G(G_i)}\right] = E\left[\log\left(\frac{\phi(d(G_i,T))}{\phi(d(G_i,0))}\sqrt{\frac{\Sigma_{0,T}}{\Sigma_{T,T}}}\right)\right] = E\left[\log\frac{\phi(d(G_i,T))}{\phi(d(G_i,0))}\right] + \log\sqrt{\frac{\Sigma_{0,T}}{\Sigma_{T,T}}} \\ =&E\left[\frac{d(G_i,0)^2 - d(G_i,T)^2}{2}\right] + \frac{1}{2}\log\left(1+(\sigma^m/\sigma^y)^2\right) = \frac{1}{2}\log\left(1+(\sigma^m/\sigma^y)^2\right). \end{aligned}$$

Thus, in the presence of fulcrum performance fees, the ex ante value of the private signal G, as described in (25), to a fund manager is

$$V^{M,f} \equiv CER^{M,s} - CER^{M,u} = \frac{1}{2}\log\left(1 + (\sigma^m/\sigma^y)^2\right).$$

PROOF OF COROLLARY 3

Proof. Using $\xi_T^{\mathcal{G}} = \xi_T^m p_0^G(G)/p_T^G(G)$ and $p_t^G(x) = \phi(d(x,t))/(x\sqrt{\Sigma_{t,T}})$, we have

$$\begin{aligned}
&E\left[\left(\xi_T^{\mathcal{G}}\right)^{R-1}\right] \\
=&E\left[\left(\sqrt{\frac{(\sigma^y)^2}{(\sigma^m)^2+(\sigma^y)^2}}\exp\left(-\left(r+\frac{1}{2}(\theta^m)^2\right)T - \theta^m\sqrt{T}W_1^m + \frac{1}{2}(W_1^\zeta)^2 - \frac{(\sigma^m W_1^m + \sigma^y W_1^\zeta)^2}{2\left((\sigma^m)^2+(\sigma^y)^2\right)}\right)\right)^{R-1}\right] \\
=&\left(\frac{(\sigma^y)^2}{(\sigma^m)^2+(\sigma^y)^2}\right)^{\frac{R-1}{2}}\exp\left(-(R-1)\left(r+\frac{1}{2}(\theta^m)^2\right)T + \frac{1}{2}(R-1)^2(\theta^m)^2T\frac{(R-2)(\sigma^m)^2-(\sigma^y)^2}{R(R-2)(\sigma^m)^2-(\sigma^y)^2}\right) \\
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{2\pi}\exp\left(-\frac{1}{2}(w-\mu)^\intercal\Sigma^{-1}(w-\mu)\right)dw^m dw^\zeta \\
=&\left(\frac{(\sigma^y)^2}{(\sigma^m)^2+(\sigma^y)^2}\right)^{\frac{R-1}{2}}\exp\left(-(R-1)\left(r+\frac{1}{2}(\theta^m)^2\right)T + \frac{(R-1)^2(\theta^m)^2T(R-2)(\sigma^m)^2-(\sigma^y)^2}{2\left(R(R-2)(\sigma^m)^2-(\sigma^y)^2\right)}\right)|\Sigma|^{1/2} \\
=&\left(\frac{(\sigma^y)^2}{(\sigma^m)^2+(\sigma^y)^2}\right)^{\frac{R-1}{2}}\sqrt{\frac{(\sigma^m)^2+(\sigma^y)^2}{-R(R-2)(\sigma^m)^2+(\sigma^y)^2}}\exp\left(-(R-1)\left(r+\frac{1}{2}(\theta^m)^2\right)T\right. \\
&\left.+\frac{1}{2}(R-1)^2(\theta^m)^2T\frac{(R-2)(\sigma^m)^2-(\sigma^y)^2}{R(R-2)(\sigma^m)^2-(\sigma^y)^2}\right),
\end{aligned}$$

where

$$w = \begin{pmatrix} w^m \\ w^\zeta \end{pmatrix}, \mu = \begin{pmatrix} \theta^m(R-1)\sqrt{T}\frac{(R-2)(\sigma^m)^2-(\sigma^y)^2}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} \\ \theta^m(R-1)\sqrt{T}\frac{(R-1)\sigma^m\sigma^y}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} \end{pmatrix}, \Sigma = \begin{pmatrix} \frac{(R-2)(\sigma^m)^2-(\sigma^y)^2}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} & \frac{-(R-1)\sigma^m\sigma^y}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} \\ \frac{-(R-1)\sigma^m\sigma^y}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} & \frac{-R(\sigma^m)^2-(\sigma^y)^2}{R(R-2)(\sigma^m)^2-(\sigma^y)^2} \end{pmatrix}.$$

Direct computation yields

$$E\left[(\xi_T^m)^{R-1}\right] = \exp\left(-(R-1)\left(r+\frac{1}{2}(\theta^m)^2\right)T + \frac{1}{2}(R-1)^2(\theta^m)^2T\right).$$

Thus, with purely proportional fees $F(X_T^a) = \alpha X_T^a$, the value of a private signal G to investors with relative risk aversion R is

$$\begin{aligned}
V^p =&\frac{1}{1-R}\log\frac{E\left[\left(\xi_T^{\mathcal{G}}\right)^{R-1}\right]}{E\left[(\xi_T^m)^{R-1}\right]} \\
=&\frac{1}{2}\log\left(1+\left(\frac{\sigma^m}{\sigma^y}\right)^2\right) + \frac{1}{2(R-1)}\log\left(1-\frac{(R-1)^2(\sigma^m)^2}{(\sigma^m)^2+(\sigma^y)^2}\right) + \frac{(R-1)^2(R-2)(\theta^m)^2T(\sigma^m)^2}{2\left(R(R-2)(\sigma^m)^2-(\sigma^y)^2\right)}.
\end{aligned}$$

According to Proposition 5, $V^p > 0$ when $R < 2$ and $V^p < 0$ when $R > 2$. Differentiating V^p with respect to σ^y yields

$$\frac{\partial V^p}{\partial \sigma^y} = \frac{(R-2)(\sigma^m)^2 \left[\left(R(\sigma^m)^2 + (\sigma^y)^2\right)\left(1 - \frac{(R-1)^2(\sigma^m)^2}{(\sigma^m)^2 + (\sigma^y)^2}\right) + (R-1)^2(\theta^m)^2 T(\sigma^y)^2\right]}{\sigma^y \left[R(R-2)(\sigma^m)^2 - (\sigma^y)^2\right]^2}.$$

As $s = 1/\sigma^y$, it follows that

$$\frac{\partial V^p}{\partial s} = \frac{(2-R)(\sigma^m)^2 \left[\left(R(\sigma^m)^2 + 1/s^2\right)\left(1 - \frac{(R-1)^2(\sigma^m)^2}{(\sigma^m)^2 + 1/s^2}\right) + (R-1)^2(\theta^m)^2 T/s^2\right]}{s \left[R(R-2)(\sigma^m)^2 - 1/s^2\right]^2}.$$

Suppose $R < 1 + \sqrt{1 + (\sigma^y/\sigma^m)^2}$, which guarantees the ex ante expected utility of a fund investor who delegates his wealth to the skilled manager does not explode, $\frac{\partial V^p}{\partial s} < 0$ for $R > 2$ and $\frac{\partial V^p}{\partial s} > 0$ for $R < 2$. Thus, investors with relative risk aversion $R < 2$ would choose the manager with the highest skill level on the market, and investors with $R > 2$ would prefer the least skilled manager.

PROOF OF COROLLARY 4

Proof. The optimal fund value of the uninformed manager at time $t \in [0, T]$ is given by

$$\begin{aligned}
X_t^{a,u^*} =& E\left[\xi_{t,T}^m X_T^{a,s^*} \,\middle|\, \mathcal{F}_t\right] \\
=& \frac{1}{y^{s^*}\xi_t^m} + E\left[\frac{\beta_2\delta}{1+\beta_2}\xi_{t,T}^m X_T^b \mathbb{1}_{\{\xi_T^m < \Psi(X_T^b)\}} \,\middle|\, \mathcal{F}_t\right] + E\left[\frac{\beta_1\delta}{1+\beta_1}\xi_{t,T}^m X_T^b \mathbb{1}_{\{\xi_T^m > \Psi(X_T^b)\}} \,\middle|\, \mathcal{F}_t\right] \\
=& \frac{1}{y^{s^*}\xi_t^m} + \frac{\beta_2\delta}{1+\beta_2} X_t^b E\left[\xi_{t,T}^m X_{t,T}^b \mathbb{1}_{\left\{\xi_{t,T}^m X_{t,T}^b < \frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^m X_t^b y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}\right\}} \,\middle|\, \mathcal{F}_t\right] \\
&+ \frac{\beta_1\delta}{1+\beta_1} X_t^b E\left[\xi_{t,T}^m X_{t,T}^b \mathbb{1}_{\left\{\xi_{t,T}^m X_{t,T}^b > \frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^m X_t^b y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}\right\}} \,\middle|\, \mathcal{F}_t\right].
\end{aligned} \tag{A5}$$

Since $\int_t^T dW_v^m$ is normally distributed with mean 0 and variance $T - t$ under $\mathcal{F}_t$, replacing $\int_t^T dW_v^m = \sqrt{T-t}z$ yields

$$\xi_{t,T}^m X_{t,T}^b < \frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^m X_t^b y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)} \Rightarrow \quad z < \frac{\log\left(\frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^m X_t^b y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}\right) + \frac{\left(\pi^b\sigma^m - \theta^m\right)^2 (T-t)}{2}}{\sqrt{\left(\pi^b\sigma^m - \theta^m\right)^2 (T-t)}} \equiv \bar{d}_{1,t}.$$

The first expectation on the RHS of (A5) is

$$E\left[\xi^m_{t,T}X^b_{t,T}\mathbb{1}_{\left\{\xi^m_{t,T}X^b_{t,T}<\frac{\log(\frac{1+\beta_2}{1+\beta1})}{\xi^m_t X^b_t y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}\right\}}\middle|\mathcal{F}_t\right]$$
$$=\int_{-\infty}^{\bar{d}_{1,t}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\pi^b\sigma^m-\theta^m\right)^2(T-t)+\left(\pi^b\sigma^m-\theta^m\right)\sqrt{T-t}z-\frac{1}{2}z^2\right)=\mathcal{N}\left(d_{1,t}\right),$$

where

$$d_{1,t}=\frac{\log\left(\frac{\log(\frac{1+\beta_2}{1+\beta1})}{\xi^m_t X^b_t y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}\right)-\frac{1}{2}\left(\pi^b\sigma^m-\theta^m\right)^2(T-t)}{\sqrt{\left(\pi^b\sigma^m-\theta^m\right)^2(T-t)}}.$$

Similar computation applies to the second expectation of (A5). The Lagrange multiplier y^{u^*} can be obtained by solving $X^{a,u^*}_0=X^a_0$. Plugging the two expectation in (A5), we obtain

$$X^{a,u^*}_t=\frac{1}{y^{u^*}\xi^m_t}+\frac{\beta_2\delta X^b_t}{1+\beta_2}\mathcal{N}(d_{1,t})+\frac{\beta_1\delta X^b_t}{1+\beta_1}\mathcal{N}(-d_{1,t}).$$

In order to find the optimal fund value of informed manager, we need to find the distribution of W^m_t under private information $\mathcal{G}$. We notice that $W^m_t=W^{\mathcal{G}}_t+\int_0^t\theta^{\mathcal{G}}_v dv$ for $t\in[0,T]$, where

$$\begin{aligned}\theta^{\mathcal{G}}_v&=\sigma^m\left(\frac{\log(G)-E_v[\log G]}{VAR_v[\log G]}\right)\\&=\sigma^m\left(\frac{\log(G)-\left(\sigma^m\theta^m-\frac{1}{2}(\sigma^m)^2-\frac{1}{2}(\sigma^y)^2\right)T-\sigma^m W^m_v}{(\sigma^m)^2(T-t)+(\sigma^y)^2T}\right)\\&=m_vW^m_v+n_v,\end{aligned}$$

where

$$m_v=\frac{1}{v-\left(1+\left(\frac{\sigma^y}{\sigma^m}\right)^2\right)T},n_v=\frac{\left(\theta^m-\frac{(\sigma^m)^2+(\sigma^y)^2}{2\sigma^m}\right)T-\frac{log(G)}{\sigma^m}}{v-\left(1+\left(\frac{\sigma^y}{\sigma^m}\right)^2\right)T}.$$

We have $dW^m_t=(m_tW^m_t+n_t)dt+dW^{\mathcal{G}}_t$. The solution to the stochastic differential equation is

$$W^m_t=\int_0^t\frac{\left(1+\left(\frac{\sigma^y}{\sigma^m}\right)^2\right)T-t}{\left(1+\left(\frac{\sigma^y}{\sigma^m}\right)^2\right)T-v}dW^{\mathcal{G}}_v+\frac{\left(\frac{log(G)}{\sigma^m}-\left(\theta^m-\frac{(\sigma^m)^2+(\sigma^y)^2}{2\sigma^m}\right)T\right)t}{\left(1+\left(\frac{\sigma^y}{\sigma^m}\right)^2\right)T}.$$

This implies that $\int_t^T dW_v^m$ is normally distributed under private information $\mathcal{G}_t$ with mean $\mu_{t,T}$ and variance $\sigma_{t,T}^2$ where

$$\mu_{t,T} = \frac{\sigma^m(T-t)}{\sqrt{\Sigma_{t,T}}} d(G,t),\ \ \sigma_{t,T}^2 = \frac{\Sigma_{T,T}(T-t)}{\Sigma_{t,T}}.$$

The optimal fund value of the informed manager is given by

$$\begin{aligned}
X_t^{a,s^*} =& E\left[\xi_{t,T}^{\mathcal{G}} X_T^{a,s^*} \,\middle|\, \mathcal{G}_t\right] = \frac{1}{y^{s^*}\xi_t^{\mathcal{G}}} + E\left[\frac{\beta_2 \delta \xi_{t,T}^{\mathcal{G}} X_T^b}{1+\beta_2} \mathbb{1}_{\left\{\xi_T^{\mathcal{G}} < \Psi^{\mathcal{G}}\right\}} \,\middle|\, \mathcal{G}_t\right] + E\left[\frac{\beta_1 \delta \xi_{t,T}^{\mathcal{G}} X_T^b}{1+\beta_1} \mathbb{1}_{\left\{\xi_T^{\mathcal{G}} > \Psi^{\mathcal{G}}\right\}} \,\middle|\, \mathcal{G}_t\right] \\
=& \frac{1}{y^{s^*}\xi_t^{\mathcal{G}}} + \frac{\beta_2\delta}{1+\beta_2} X_t^b E\left[\xi_{t,T}^{\mathcal{G}} X_{t,T}^b \mathbb{1}_{\left\{\xi_{t,T}^{\mathcal{G}} X_{t,T}^b < \frac{\log\left(\frac{1+\beta_2}{1+\beta 1}\right)}{\xi_t^{\mathcal{G}} X_t^b y^{s^*} \delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}\right\}} \,\middle|\, \mathcal{G}_t\right] \\
&+ \frac{\beta_1\delta}{1+\beta_1} X_t^b E\left[\xi_{t,T}^{\mathcal{G}} X_{t,T}^b \mathbb{1}_{\left\{\xi_{t,T}^{\mathcal{G}} X_{t,T}^b > \frac{\log\left(\frac{1+\beta_2}{1+\beta 1}\right)}{\xi_t^{\mathcal{G}} X_t^b y^{s^*} \delta\left(\frac{\beta_2}{1+\beta_2} - \frac{\beta_1}{1+\beta_1}\right)}\right\}} \,\middle|\, \mathcal{G}_t\right]. \qquad \text{(A6)}
\end{aligned}$$

We find that

$$\begin{aligned}
&\xi_{t,T}^{\mathcal{G}} X_{t,T}^b = \xi_{t,T}^m \frac{p_t^{\mathcal{G}}(G)}{p_T^{\mathcal{G}}(G)} X_{t,T}^b \\
=& \sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}} \exp\left(-\frac{1}{2}\left(\pi^b\sigma^m - \theta^m\right)^2 (T-t) + \left(\pi^b\sigma^m - \theta^m\right)\int_t^T dW_v^m + \frac{1}{2}\left(d(G,T)^2 - d(G,t)^2\right)\right) \\
=& \sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}} \exp\left(-\frac{\left(\pi^b\sigma^m - \theta^m\right)^2 (T-t)}{2} - \frac{\left((\pi^b\sigma^m - \theta^m)\Sigma_{T,T} - \sigma^m\sqrt{\Sigma_{t,T}}d(G,t)\right)^2}{2(\sigma^m)^2\Sigma_{T,T}}\right. \\
&\left. + \frac{d(G,t)^2}{2}\left(\frac{\Sigma_{t,T}}{\Sigma_{T,T}} - 1\right) + \frac{(\sigma^m)^2}{2\Sigma_{T,T}}\left(\int_t^T dW_v^m + \frac{(\pi^b\sigma^m - \theta^m)\Sigma_{T,T} - \sigma^m\sqrt{\Sigma_{t,T}}d(G,t)}{(\sigma^m)^2}\right)^2\right) \\
=& \sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}} \exp\left(-\frac{\Sigma_{t,T}}{2}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)^2 + \frac{\left(\sigma^m\int_t^T dW_v^m + \left(\pi^b - \frac{\theta^m}{\sigma^m}\right)\Sigma_{T,T} - \sqrt{\Sigma_{t,T}}d(G,t)\right)^2}{2\Sigma_{T,T}}\right).
\end{aligned}$$

Let $\int_t^T dW_v^m = \mu_{t,T} + \sigma_{t,T} z$ and $\Omega = \frac{\log(\frac{1+\beta_2}{1+\beta 1})}{y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}$, we have

$$
\begin{aligned}
&\xi_{t,T}^{\mathcal{G}} X_{t,T}^b < \frac{\Omega}{\xi_t^{\mathcal{G}} X_t^b} \\
&\sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}} \exp\left(-\frac{\Sigma_{t,T}}{2}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)^2 + \frac{\left(\sigma^m \sigma_{t,T} z + \Sigma_{T,T}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)\right)^2}{2\Sigma_{T,T}}\right) < \frac{\Omega}{\xi_t^{\mathcal{G}} X_t^b} \\
&\left(\sigma^m \sigma_{t,T} z + \Sigma_{T,T}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)\right)^2 < \Sigma_{T,T}\left(2\log\left(\sqrt{\frac{\Sigma_{t,T}}{\Sigma_{T,T}}}\Omega\right) + \Sigma_{t,T}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)^2\right) \\
&\bar{d}_{2,t}^- < z < \bar{d}_{2,t}^+,
\end{aligned}
$$

where

$$
\bar{d}_{2,t}^{\pm} = \frac{\pm\sqrt{\Sigma_{T,T}\left(2\log\left(\frac{\sqrt{\frac{\Sigma_{t,T}}{\Sigma_{T,T}}}\log\frac{1+\beta_2}{1+\beta 1}}{y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}\right) + \Sigma_{t,T}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)^2\right)} - \Sigma_{T,T}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)}{\sigma^m \sigma_{t,T}}.
$$

The first expectation in (A6) is given by

$$
\begin{aligned}
&E\left[\xi_{t,T}^{\mathcal{G}} X_{t,T}^b \mathbb{1}_{\left\{\xi_{t,T}^{\mathcal{G}} X_{t,T}^b < \frac{\log(\frac{1+\beta_2}{1+\beta 1})}{\xi_t^{\mathcal{G}} X_t^b y^{s^*}\delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}\right\}} \middle| \mathcal{G}_t\right] \\
=&\int_{\bar{d}_{2,t}^-}^{\bar{d}_{2,t}^+} \sqrt{\frac{\Sigma_{T,T}}{2\pi\Sigma_{t,T}}} \exp\left(-\frac{1}{2}\left(\sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}} z - \sqrt{\Sigma_{t,T} - \Sigma_{T,T}}\left(\pi^b - \frac{\theta^m}{\sigma^m} - \frac{d(G,t)}{\sqrt{\Sigma_{t,T}}}\right)\right)^2\right) dz \\
=&\mathcal{N}\left(d_{2,t}^+\right) - \mathcal{N}\left(d_{2,t}^-\right),
\end{aligned}
$$

where

$$
d_{2,t}^{\pm} = \frac{\pm\sqrt{\Sigma_{T,T}\left(2\log\left(\frac{\sqrt{\frac{\Sigma_{t,T}}{\Sigma_{T,T}}}\log\frac{1+\beta_2}{1+\beta 1}}{y^{s^*}\xi_t^{\mathcal{G}} X_t^b \delta\left(\frac{\beta_2}{1+\beta_2}-\frac{\beta_1}{1+\beta_1}\right)}\right) + \Sigma_{t,T}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right)^2\right)} - \Sigma_{t,T}\left(\pi^b - \frac{\theta^m + \theta_t^{\mathcal{G}}}{\sigma^m}\right)}{\sqrt{\Sigma_{t,T} - \Sigma_{T,T}}}.
$$

Plugging the expectations in (A6), we get

$$
X_t^{a,s^*} = \frac{1}{y^{s^*}\xi_t^{\mathcal{G}}} + \frac{\beta_2 \delta X_t^b}{1+\beta_2}\left(\mathcal{N}(d_{2,t}^+) - \mathcal{N}(d_{2,t}^-)\right) + \frac{\beta_1 \delta X_t^b}{1+\beta_1}(\mathcal{N}(-d_{2,t}^+) + \mathcal{N}(d_{2,t}^-))
$$

The optimal trading strategies of fund managers can be obtained by taking derivatives on both sides of X_t^{a,u^*} (X_t^{a,s^*}) and matching the coefficients in front of dW_t^m.

PROOF OF PROPOSITION 7

Proof. The first order condition of the fund manager's static problem is

$$((1-\alpha)X_T^{a^*})^{-R^a} = y^*\xi_T^{\mathcal{G}} \Leftrightarrow X_T^{a^*} = \left(y^*\xi_T^{\mathcal{G}}\right)^{-R^a}/(1-\alpha)$$

where y^* is determined by the static budget constraint

$$E\left[\xi_T^{\mathcal{G}}\left(y^*\xi_T^{\mathcal{G}}\right)^{-R^a}\right] = X_0^a.$$

Thus, we have

$$X_T^{a^*} = \frac{X_0^a\left(\xi_T^{\mathcal{G}}\right)^{-1/R^a}}{E_t\left[\left(\xi_T^{\mathcal{G}}\right)^{1-1/R^a}\right]}, F(X_T^{a^*}) = \alpha\left(\xi_T^{\mathcal{G}}\right)^{-1/R^a} X_0^a H_T^{\mathcal{G}}/H_0^{\mathcal{G}}.$$

The manager's optimal fund value at time $t \in [0,T]$ is given by

$$X_t^{a^*} = E_t[\xi_{t,T}^{\mathcal{G}} X_T^{a^*}] = \left(\xi_t^{\mathcal{G}}\right)^{-1/R^a} X_0^a H_t^{\mathcal{G}}/H_0^{\mathcal{G}}$$

where $H_t^{\mathcal{G}} = E_t\left[\left(\xi_{t,T}^{\mathcal{G}}\right)^{1-1/R^a}\right]$.

Using $d(G,t) = \frac{\theta_t^{\mathcal{G}}}{\sigma^m}\sqrt{\Sigma_{t,T}}$ and $d(G,T) = \frac{\theta_t^{\mathcal{G}}\Sigma_{t,T}-(\sigma^m)^2W_{t,T}^m}{\sigma^m\sqrt{\Sigma_{T,T}}}$ we have

$$\begin{aligned}
\xi_{t,T}^{\mathcal{G}} =& \xi_{t,T}^m p_t^G(G)/p_T^G(G) \\
=& \sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}}\exp\left(-\left(r+\frac{1}{2}(\theta^m)^2\right)(T-t) - \theta^m W_{t,T}^m - \frac{d(G,t)^2}{2} + \frac{d(G,T)^2}{2}\right) \\
=& \sqrt{\frac{\Sigma_{T,T}}{\Sigma_{t,T}}}\exp\Bigg(-\left(r+\frac{1}{2}(\theta^m)^2\right)(T-t) - \frac{1}{2}\left(\frac{\theta_t^{\mathcal{G}}}{\sigma^m}\right)^2\Sigma_{t,T} + \frac{1}{2}\left(\frac{\theta_t^{\mathcal{G}}}{\sigma^m}\right)^2\frac{\Sigma_{t,T}^2}{\Sigma_{T,T}} \\
&+ \frac{(\sigma^m)^2}{2\Sigma_{T,T}}(W_{t,T}^m)^2 - \left(\frac{\Sigma_{t,T}\theta_t^{\mathcal{G}}}{\Sigma_{T,T}} + \theta^m\right)W_{t,T}^m\Bigg)
\end{aligned}$$

Since $W^m_{t,T}$ is normally distributed with mean 0 and variance $T-t$ under public information, direct computation yields

$$H^{\mathcal{G}}_t = \sqrt{\frac{R^a\Sigma_{T,T}}{\Sigma_{t,T}+(R^a-1)\Sigma_{T,T}}}\left(\frac{\Sigma_{T,T}}{\Sigma_{t,T}}\right)^{-\frac{1}{2R^a}} \exp\left(\left(r+\frac{\Sigma_{t,T}(\theta^m+\theta^{\mathcal{G}}_t)^2}{2(\Sigma_{t,T}+(R^a-1)\Sigma_{T,T})}\right)\frac{(1-R^a)(T-t)}{R^a}\right)$$

The manager's optimal trading strategies can be determined by

$$\pi^*_t = \frac{d[X^{a^*}, W^{\mathcal{G}}]_t}{\sigma^m X^{a^*}_t dt} = \frac{\Sigma_{t,T}}{\Sigma_{t,T}+(R^a-1)\Sigma_{T,T}}\frac{\theta^m+\theta^{\mathcal{G}}_t}{\sigma^m}.$$

Similarly,

$$\pi^h_t = \frac{d[H, W^{\mathcal{G}}]_t}{\sigma^m H_t dt} = \frac{(R^a-1)(\Sigma_{t,T}-\Sigma_{T,T})}{\Sigma_{t,T}+(R^a-1)\Sigma_{T,T}}\frac{\theta^m+\theta^{\mathcal{G}}_t}{R^a\sigma^m} = \frac{R^a-1}{1+\frac{T}{T-t}\left(\frac{\sigma^y}{\sigma^m}\right)^2 R^a}\frac{\theta^m+\theta^{\mathcal{G}}_t}{R^a\sigma^m}.$$

and $\pi^m_t = \pi^*_t - \pi^h_t$.

PROOF OF COROLLARY 5

Proof.

$$E\left[\frac{\left(X^{\mathcal{G},a}_T\right)^{1-R}}{1-R}\right] = E\left[\frac{\left(\left(\xi^{\mathcal{G}}_T\right)^{-\frac{1}{R^a}} x/H^{\mathcal{G}}_0\right)^{1-R}}{1-R}\right] = \frac{x^{1-R}}{1-R}E\left[\left(\left(\xi^{\mathcal{G}}_T\right)^{-\frac{1}{R^a}}/H^{\mathcal{G}}_0\right)^{1-R}\right]$$

$$= \frac{x^{1-R}}{1-R}E\left[\left(\sqrt{\frac{R^a(\sigma^y)^2}{(\sigma^m)^2+R^a(\sigma^y)^2}}\exp\left(-\left(r+\frac{1}{2}(\theta^m)^2\right)T - \frac{1}{R^a}\theta^m\sqrt{T}W^m_1 + \frac{1}{2R^a}(W^\zeta_1)^2 - \frac{(\sigma^m W^m_1+\sigma^y W^\zeta_1)^2}{2R^a(\sigma^y)^2} + \frac{\left(\sigma^m\sigma^y W^\zeta_1 + (\sigma^m)^2 W^m_1 - (R^a-1)(\sigma^y)^2\theta^m\sqrt{T}\right)^2}{2R^a(\sigma^y)^2\left((\sigma^m)^2+R^a(\sigma^y)^2\right)}\right)\right)^{R-1}\right]$$

$$= \frac{x^{1-R}}{1-R}\left(\frac{R^a(\sigma^y)^2}{(\sigma^m)^2+R^a(\sigma^y)^2}\right)^{\frac{R-1}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{2\pi}\exp\left(-\frac{1}{2}(w-\mu)^\intercal\Sigma^{-1}(w-\mu)\right)dw^m dw^\zeta$$
$$\exp\left(-(R-1)(r+\frac{1}{2}(\theta^m)^2)T + \frac{\left((R-1)(R^a+1-R)(\sigma^m)^2+(R+(R^a)^2-2R^a)(\sigma^y)^2\right)(R-1)(\theta^m)^2T}{2\left((1+R^a-R)R(\sigma^m)^2+(R^a)^2(\sigma^y)^2\right)}\right)$$

$$= \frac{x^{1-R}}{1-R}\left(\frac{R^a(\sigma^y)^2}{(\sigma^m)^2+R^a(\sigma^y)^2}\right)^{\frac{R-1}{2}}\sqrt{\frac{R^a((\sigma^m)^2+R^a(\sigma^y)^2)}{R(R^a+1-R)(\sigma^m)^2+(R^a)^2(\sigma^y)^2}}$$
$$\exp\left(-(R-1)(r+\frac{1}{2}(\theta^m)^2)T + \frac{\left((R-1)(R^a+1-R)(\sigma^m)^2+(R+(R^a)^2-2R^a)(\sigma^y)^2\right)(R-1)(\theta^m)^2T}{2\left((1+R^a-R)R(\sigma^m)^2+(R^a)^2(\sigma^y)^2\right)}\right)$$

where $w = (w^m, w^\zeta)^\intercal$ and

$$\mu = \begin{pmatrix} \frac{((R-R^a-1)(\sigma^m)^2 - R^a(\sigma^y)^2)\theta^m(R-1)\sqrt{T}}{R(R-R^a-1)(\sigma^m)^2-(R^a)^2(\sigma^y)^2} \\ \frac{(R-R^a)\sigma^m\sigma^y\theta^m(R-1)\sqrt{T}}{R(R-R^a-1)(\sigma^m)^2-(R^a)^2(\sigma^y)^2} \end{pmatrix}, \Sigma = \begin{pmatrix} \frac{-R(\sigma^m)^2-R^a(\sigma^y)^2}{(\sigma^m)^2+R^a(\sigma^y)^2} & \frac{(1-R)\sigma^m\sigma^y}{(\sigma^m)^2+R^a(\sigma^y)^2} \\ \frac{(1-R)\sigma^m\sigma^y}{(\sigma^m)^2+R^a(\sigma^y)^2} & \frac{(R-R^a-1)(\sigma^m)^2-(R^a)^2(\sigma^y)^2}{R^a((\sigma^m)^2+R^a(\sigma^y)^2)} \end{pmatrix}.$$

Direct computation yields

$$\begin{aligned} E\left[\frac{\left(X_T^{m,a}\right)^{1-R}}{1-R}\right] &= E\left[\frac{\left((\xi_T^m)^{-1/R^a}\, x/H_0^m\right)^{1-R}}{1-R}\right] = \frac{x^{1-R}}{1-R}\frac{E\left[(\xi_T^m)^{\frac{R-1}{R^a}}\right]}{(H_0^m)^{1-R}} \\ =&\frac{x^{1-R}}{1-R}\exp\left(-(R-1)(r+\frac{1}{2}(\theta^m)^2)T + \frac{1}{2}\frac{(R+(R^a)^2-2R^a)(R-1)}{(R^a)^2}(\theta^m)^2T\right) \\ =&\frac{x^{1-R}}{1-R}\exp\left(-(R-1)rT + \frac{1}{2}\frac{(R-2R^a)(R-1)}{(R^a)^2}(\theta^m)^2T\right) \end{aligned}$$

Thus, with purely proportional fees $F(X_T^a) = \alpha X_T^a$, the value of a private signal G to investors with relative risk aversion R is

$$V^p = \frac{1}{2}\log\left(1 + \frac{(\sigma^m)^2}{R^a(\sigma^y)^2}\right) + \frac{\log\left(\frac{R^a((\sigma^m)^2+R^a(\sigma^y)^2)}{R(R^a+1-R)(\sigma^m)^2+(R^a)^2(\sigma^y)^2}\right)}{2(1-R)} + \frac{(R-R^a)^2(R^a+1-R)(\theta^m)^2(\sigma^m)^2T}{2(R^a)^2\left(R(R^a+1-R)(\sigma^m)^2+(R^a)^2(\sigma^y)^2\right)}.$$

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