Reshaping the Financial Network: Externalities of Central Clearing and Systemic Risk

Olga Briukhova *[1,2], Marco D'Errico[1], and Stefano Battiston[1,2]

[1]Department of Banking and Finance, University of Zurich
[2]Swiss Finance Institute

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Abstract

Meant to limit systemic risk and ensure transparency, the mandate to clear specific transactions via central clearing counterparties (CCP) is at the heart of the following the crisis regulatory reforms and leads to the reshaping of the over-the-counter (OTC) network. Taking a network perspective, we show how the transition to central clearing effects the expected value of a derivative contract and thereby redistributes wealth among market participants. We find that the realistic assumptions of counterparty risk and costly collateral create distortions in the expected value of a symmetric derivative contract, which depend on the structure of the market and arise on three levels: i) credit quality distortion of netting, ii) loss-mutualisation, and iii) funding costs. Because of mutualisation of risks and funds in a CCP, these distortions lead to externalities between CCP members. Moreover, we find that even though a CCP interposes itself between bilateral counterparties, the real insulation from the counterparty risk does not always hold. We derive a threshold value on a CCP's "skin in the game" capital, below which expected exposures between members form a fully connected network. The threshold is hit precisely in times of distress, that might further increase systemic risk. Our work offers a simple network framework for further assessments of the policy implications of mandatory central clearing, including its impact on the relations between market participants, their incentives, and systemic risk.

Keywords: Central Clearing Counterparty (CCP); OTC derivatives; financial networks; systemic risk; market structure

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*Corresponding author: olga.briukhova@bf.uzh.ch; Phone: +41765395099; Address: Andreasstrasse 15, 8050, Zurich, Switzerland.

1 Introduction

Over-the-counter (OTC) derivatives played a significant role in the global financial crisis of 2007-2009 (Stulz, 2010; Cont, 2010). In addition to their opacity (Duffie, 2012), OTC derivatives were largely unregulated. This led to a systemic wide uncertainty which brought markets to a standstill (Haldane, 2009). As a response to this, the G20 leaders committed in 2009 to increase the safety and transparency of derivatives markets. The reforms have been implemented through the Dodd-Frank Wall Street Reform in the US and the European Market Infrastructure Regulation (EMIR). New regulation requires reporting of all OTC transactions to trade repositories and mandates central clearing of certain types of standardized OTC derivatives through central clearing counterparties (CCPs). This paper aims to shed a light on the effects of the introduction of central clearing on different participants of the derivatives market and stability of the financial system.

The reforms have led to a more diverse set of actors in the financial system and reshaped the OTC network by making the role of CCPs predominant in the intermediation structure of derivatives markets. However, the concept of central clearing is fairly new and some specifics of the CCPs functioning remains unknown (Coeure, 2014). CCPs have different from other intermediaries business models, face different risks, and evolve in different environments. Policymakers express concerns regarding the concentration of risks in the few CCPs (Powell, Jerome H and others, 2015) and catastrophic consequences of a failure of a CCP (Tucker, 2011). Recognizing the growing systemic importance of CCPs and the increasing complexity of cleared contracts, it is important to understand and quantify the impact of central clearing on the core aspects of the OTC market.

This paper takes a network perspective to analyse how transition from a fully bilateral market to a single CCP effects expected value of a derivative contract, thereby redistributes wealth among market participants and influences financial stability. We show that under realistic assumptions expected value of a symmetric derivative contract is not zero and depends on the market structure. We derive expected value function in the bilateral and centrally cleared settings and show that counterparty risk and costly collateral result in distortions on three levels: i) credit quality distortion of netting, ii) loss-mutualisation distortion, and iii) funding costs distortion. While credit quality distortion of netting and funding costs distortion occur in both market structures, loss-mutualisation distortion is specific to the centrally cleared setting. Moreover, we find that in a CCP each distortion is associated with externalities between CCP members, which emerge due to mutualisation of funds and risks. Externalities mean that, despite its position in a CCP is unchanged, member's risk and wealth could change as a result of other members' actions. Presence of externalities gives a rise to a new space of strategies that were not available to market participants in the bilateral setting. We analyse each type of distortions constituting expected value of a derivative contract.

First, we investigate how netting redistributes wealth between counterparties of different credit quality. Extending the result of Duffie and Huang (1996), we compare an effect of bilateral netting in case of multiple counterparties with an effect of netting via a single CCP. We show that, in general, netting is more beneficial for relatively high quality counterparties. In contrast to the bilateral netting, netting via a CCP produces credit quality effect of netting together with the counterparty effect. Combination of these two effects makes it possible for both counterparties to increase expected payoffs by transferring part of the negative counterparty effect to other members. Moreover, even though CCPs are intended to facilitate additional multilateral netting, due to higher credit quality of a CCP achieved through mutualisation and additional CCP's capital, members do not have incentives to net. This could incentivise a build-up of larger net positions and increase systemic risk. While the main focus in the previous literature on central clearing is put on the efficiency of multilateral netting provided by CCPs in comparison to the bilateral netting (Jackson and Manning, 2007; Duffie and Zhu, 2011; Cont and Kokholm, 2014; Heath et al., 2013), we contribute to it by studying the redistributive effect of netting between counterparties of different credit quality.

Second, we find that even though a CCP interposes itself between initial counterparties of a derivative contract, CCP members are potentially exposed to each other via default fund contributions. Insulation from the counterparty risk is achieved only if "skin in the game" capital posted by a CCP is sufficiently large. We derive the minimum threshold on CCP's capital which depends on the credit quality of CCP members and the share of CCP's exposure towards its members that is not covered by individual initial margin and default fund contributions. The interplay between these elements may engender procyclicality during distress periods, when credit quality deteriorates and collateral scarcity impairs the possibility of members to post additional resources. Our findings are in line with Arnsdorf (2012), Armakola and Laurent (2015), and Pirrong (2011) who point out that the original counterparty risk is transformed into the risk via loss-sharing mechanisms and Cont (2017) who shows that central clearing does not fully eliminate the counterparty risk but transforms solvency risk into the liquidity risk. Our findings contribute to a general discussion of CCP's "default waterfall" defense scheme (ISDA, 2015; Duffie, 2014; Cont, 2015).

Third, we contribute to the analysis of the "cover 2" regulatory requirement by investigating its impact on the distribution of funding costs and profitability among CCP members. We find that the growth of a member that further increases concentration of a CCP leads to higher funding costs for all CCP members and we propose a possible solution to avoid these negative externalities. We show that application of the "cover 2" standard leads to the distortions in funding costs. This complements previous analysis of the prudence of the "cover 2" standard, which is shown to be highly dependent on the size of a CCP and distribution of risk among CCP members (Murphy and Nahai-Williamson, 2014).

Joint analysis of these distortions allows us to investigate how central clearing changes incentives of market participants and what type of participants is better-off or worse-off with the transition. In particular, we show that higher credit quality market participants benefit more from the transition to central clearing. Due to the decomposition of the

expected value function, we identify trade-offs associated with the transition to central clearing and their determinants. In particular, we show that for a particular agent effect of transition on both counterparty risk and funding costs of collateral is ambiguous. Central clearing could potentially reduce losses due to default of a direct counterparty but makes members of a CCP exposed to indirect losses of mutualisation. Effect of the transition on funding costs depends on the relation between reduction in initial margin requirements due to multilateral netting and additional default fund contributions. However, since counterparty risk has mainly a redistributive effect, transition to central clearing has a non-negative effect on the counterparty risk component of the total welfare.
The paper is organised as follows. In Section 2 we review related works; Section 3 introduces the general framework of the model; in Section 4 we discuss distortions occurring in the expected value of a derivative contract and externalities associated with them; Section 5 presents trade-offs each market participant faces with the transition; Section 6 reviews the main contributions.

2 Related works

A large stream of literature focuses on a CCPs' ability to reduce systemic risk by increasing netting efficiency. Since the introduction of central clearing substitutes bilateral netting with a multilateral one, there is a trade-off between two types of netting opportunities in terms of the total exposure reduction. Jackson and Manning (2007) show that benefits of moving from bilateral to multilateral clearing arrangements increase with a number of participating agents but at a decreasing rate. Duffie and Zhu (2011) consider the relative benefits of netting schemes in the market of OTC derivatives. The result depends on the number of CCPs, the proportion of cleared contracts, and the number of contract types cleared by each CCP. The authors consider the average expected exposure of clearing members under different netting schemes as an indicator of the level of systemic risk, not taking into account capitalization and collateralisation of CCPs and clearing members. Cont and Kokholm (2014) extend the model by allowing for the heterogeneity of asset classes in terms of riskiness and correlation of exposures across them. Garratt and Zimmerman (2015) apply the basic methodology to more realistic scale-free and core-periphery networks, additionally considering the effect of the introduction of CCPs on the variance of net exposures. The authors find that a CCP is unlikely to be beneficial in a network with a small number of highly-connected nodes. This result is in line with the finding of Heath et al. (2013) that bilateral netting across products may, in some circumstances, deliver greater benefits than multilateral netting across counterparties and that multilateral netting benefits available to investors that constitute the periphery are limited disproportionately to the core. Lewandowska (2015) uses a simulation approach to provide a quantitative analysis of the netting efficiency and loss concentration under different types of clearing arrangements in the OTC market. The largest reduction in systemic risk can be achieved in case of central clearing via an exclusive CCP but only if a critical mass of asset classes is cleared and the critical mass of members participate in the CCP.

Even though multilateral netting might reduce total notional in the system, new margin requirements has significantly increased system-wide collateral demand (Duffie et al., 2015). High collateralisation of cleared deals leads to the transformation of the counterparty solvency risk into the liquidity risk (Cont, 2017; Heath et al., 2016). As stressed by Cont (2017), an important distinction between two types of losses is that, in contrast to solvency losses that appear in accounting terms, liquidity losses lead to the actual exchange of cash flows. The author argues that stress tests involving CCPs should focus on liquidity stress testing and adequacy of liquidity resources. Heath et al. (2015) find netting efficiency to be a critical determinant of the shape of the trade-off between solvency and liquidity losses. The authors apply liquidity and solvency shocks to the reconstructed network of exposures between the 41 largest bank participants in global OTC derivative markets. The authors consider allocation of uncovered losses via variation margin haircut as a channel for transmitting stress back to members. Potential exposures of clearing members to each other via risk-sharing mechanisms are discussed by Armakola and Laurent (2015). The original counterparty risk is transformed into the risk of losses due to the mutualisation of the default fund. Arnsdorf (2012) shows that the risk each member faces in a CCP is mainly driven by performance of all the other members rather than by the exposure on a member's own portfolio. Some members may incur even larger losses than they would suffer on their own contracts with a defaulter in a bilateral setting Pirrong (2011). Participating in a CCP, each member provides insurance on the tail losses of all other members without receiving full information on the portfolios they clear and collateral they post. Thus, the transparency of a CCP's default management is crucial to persuade institutions to participate and share risks. Biais et al. (2016) show that risk-sharing via derivatives can lead to excessive risk-taking by financial institutions and explain how margin deposits and clearing arrangements should be designed in order to preserve risk-prevention incentives.

In order to protect itself against the potential losses, a CCP maintains prefunded resources in the form of members' initial margins and default fund contributions collected from its members and CCP's own "skin in the game" capital. The resources are used in the specified sequence known as a "default waterfall". The optimal balance of resources between initial margin and default fund is studied by Haene and Sturm (2009) and depends on collateral costs, participants' default probability, and the extent to which margin requirements are associated with risk-mitigating incentives. Cumming and Noss (2013) provide a framework that allows to assess the adequacy of a CCP's financial resources and quantify the trade-off between initial margin and default fund. Capponi et al. (2018) derive an optimal default fund level taking into account a trade-off between ex-post risk-sharing and ex-ante risk-shifting. In extreme scenarios, when a "default waterfall" and CCP's own capital are exhausted, a CCP becomes insolvent. The processes of resolution and recovery of failing CCPs should take into account their "too-big-to-fail" nature. Discussions regarding resolution and recovery mechanisms of CCPs can be found in ISDA (2015), Duffie (2014), and Cont (2015).

3 General model framework

In order to assess potential externalities arising from the main aspects of central clearing, we introduce a simple network framework. As a counterfactual to the centrally cleared market, we consider a fully bilateral setting. While keeping the characteristic of the OTC derivative contract very general, we consider a single type of a swap contract. All contracts are either traded bilaterally or cleared via a single CCP. The market consists of M participants that constitute a given and fixed network of bilateral contracts, which then is transformed into a centrally cleared system. Those agents are heterogeneous in terms of their size, measured by the total notional value of contracts they trade, and their credit quality. Credit quality of agent m is characterized by probability p_m with which it fulfills its liabilities. This probability is exogenous, fixed and independent from credit quality of other agents in the system. Moreover, we assume a zero recovery rate (in excess to posted collateral) in case of default, that is a realistic assumption for quick asset liquidation in times of distress.
As an example of a derivative contract, we consider a swap contract that is originated at time 0 and matures at time 1, paying a difference between a floating rate r^{float} and a fixed rate r^{fixed} per unit of notional value of the contract. A notional value of a bilateral contract between counterparties i and j is denoted as x_{ij} ($x_{ij} = -x_{ji}$). A positive notional value $x_{ij} > 0$ represents a situation when counterparty i receives a floating rate and pays a fixed rate, while $x_{ij} < 0$ means that counterparty i pays a floating rate and receives a fixed rate. The time 1 payoff of i without accounting for credit risk of counterparties is $(r^{float} - r^{fixed})x_{ij} = Sx_{ij}$, where S is a random variable that represents a spread between the rates. Since realisation of the spread is positive in some states and negative in other states, ex-ante an agent does not know whether it would be a net payer or a net receiver. The swap contract is treated as a contingent asset if it produces a positive payoff and as a contingent liability in case of a negative payoff. We assume a symmetric distribution of the spread, meaning that $\mathbb{P}(S \geq 0) = \mathbb{P}(S < 0) = \frac{1}{2}$ and $\mathbb{E}(S|S \geq 0) = -\mathbb{E}(S|S < 0) = s$. This assumption makes the contract fair in a sense that, when counterparty risks are not taken into account, both counterparties in expectation pay and receive the same amount. Since agents are not risk-free, they ask each other to post collateral to mitigate counterparty credit risk. We account for specifics of risk-management schemes corresponding to the two market structures, introducing requirements on initial margins (IM) and, in case of centrally cleared deals, default fund contributions (DF). Posting collateral is associated with costs to raise funds and with opportunity costs of investing them. We assume that costs of posting one unit of collateral (ρ) are constant and are the same for all agents

regardless of their size and credit quality.
The same framework can be applied to the analysis of variation margin calls. For instance, in case of a CDS contract the spread would represent a change in the probability of default of an underlying asset.

3.1 Expected value of a derivative contract under different market structures

This section characterizes the effect of market structure, i.e. bilateral and centrally cleared settings, on a derivative contract's expected value for different market participants taking into account counterparty credit risk and specifics of risk-management schemes.
We perform valuation of a swap contract that is arranged at time 0 and matures at time 1. Value of the contract at time 0 is determined as an expected value of uncertain payoffs the swap gives at time 1 (for simplicity we take a discount factor of 1) minus the cost of investment at time 0. In this case the costs of investment correspond to funding costs that arise from the need to post initial margin and default fund contributions.
In the following subsections we derive a value function for a swap contract in bilateral and centrally cleared settings for a particular agent m. This allows us to identify specific factors determining the value under different market structures and to characterize market participants that benefit or lose from a transfer from bilateral to a centrally cleared market.
Since typically agents interact with multiple counterparties, we focus not on the value of a single contract but on the value of a total position traded. In the bilateral case, value of the position is the sum of values of contracts with individual counterparties. In the centrally cleared setting, deals with multiple counterparties are substituted with a single contract with a CCP; value of this contract determines value of the whole position.

3.1.1 Bilateral market

We start with an example of a bilateral contract between agent m and its counterparty i. Without loss of generality we assume that after bilateral netting m is a net receiver of a float rate and a net payer of a fixed rate, i.e. $x_{mi} > 0$; counterparty i holds an opposite position, i.e. $x_{im} = -x_{mi} < 0$.
Even though before the crisis bilateral OTC trades were allowed to be uncollateralized, the EMIR mandates an exchange of margins on non-centrally cleared OTC derivatives. Initial margins are aimed to cover potential future exposure and, under a standard predefined approach, are based on the notional value of the contracts and the underlying market risk.[1] Following these standards, we assume that an agent posts initial margin calculated as a fixed share of potential exposure towards that agent ($\alpha < 1$). Regardless of the direction of a position, a contract can realize to be a liability for an agent, thereby

[1] https://www.eba.europa.eu/documents/10180/1398349/RTS+on+Risk+Mitigation+Techniques+for+OTC+contracts+(JC-2016-+18).pdf

creating an exposure towards that agent. Therefore, agents from both sides of a deal have to post margins:

$$IM_m = -\alpha\mathbb{E}(S|S<0)x_{mi} = \alpha s x_{mi} = -\alpha\mathbb{E}(S|S\geq 0)x_{im} = IM_i.$$

We calculate expected value of the contract for agent m that is a net receiver when realisation of the spread is positive. Agent m receives the full payment as long as its counterparty i does not default, i.e. with probability p_i, and the margin posted by i otherwise. Since in this case m does not have a liability, it cannot default and gets back the margin it has posted. When the spread is negative, the contract is a liability for m, which m pays in full when it does not default i.e. with probability p_m, or covers with the margin when it defaults. Therefore, m gets back its margin only if it does not default. Moreover, posting initial margin at time 0 is associated with additional funding costs ρ per unit of resources. The value of the contract is then given by

$$\begin{aligned} V^b_{mi} &= \mathbb{P}(S\geq 0)[\mathbb{E}(S|S\geq 0)x_{mi}p_i + IM_i(1-p_i) + IM_m] \\ &\quad +\mathbb{P}(S<0)[\mathbb{E}(S|S<0)x_{mi}p_m + IM_m p_m] - IM_m(1+\rho) \\ &= \tfrac{1}{2}s(1-\alpha)x_{mi}(p_i-p_m) - s\alpha x_{mi}\rho. \end{aligned} \tag{1}$$

The same payoff structure holds, if agent m is a net receiver when the spread is negative and a net payer when the spread is positive. Due to the symmetry of spread distribution, agent's position is characterized by its notional value rather than by its direction. Therefore, a general expression for a fair value of a portfolio with multiple bilateral counterparties is given in Proposition 1.

Proposition 1. *[Expected value in the bilateral setting] Expected value of a total position of agent m in the bilateral setting is given by*

$$\begin{aligned} V^b_m &= \tfrac{1}{2}s(1-\alpha)X^G_m(\bar{p}^b_m - p_m) - s\alpha X^G_m\rho, \text{ where} \\ X^G_m &= \sum_i \max[x_{im}, x_{mi}] \\ \bar{p}^b_m &= \sum_i \tfrac{\max[x_{im}, x_{mi}]}{X^G_m} p_i. \end{aligned} \tag{2}$$

Proof. See Appendix A. □

Total position of an agent in the bilateral market is determined by its gross notional (X^G_m). Expected value of the contract consists of two components. The first component arises due to the counterparty risk and difference in credit quality of counterparties. We refer to the first component as *credit quality distortions*. This effect is zero-sum on the level of a single deal, meaning that in expectation one counterparty loses exactly the same amount as another counterparty gains. If an institution deals with multiple counterparties, it gains if its quality is worse than the weighted average quality of its counterparties ($\bar{p}^b_m$). The second component represents funding costs of posting initial margin and, in contrast to the first component, is a deadweight loss for the system. We refer to the second component as *funding costs distortions*.

3.1.2 Centrally cleared market

In this subsection we investigate how valuation of contracts changes once they are novated to a CCP or once new contracts are originated in a centrally cleared environment. We consider a market that is fully cleared via a single CCP that includes M members ($m \in M$). The first difference to the bilateral market is that instead of keeping a position with multiple counterparties each member has a single position in a CCP, whose size is determined by a net position of the member in the bilateral market. When market is fully cleared through a single CCP, net position of each member in a CCP is always not larger than its gross position in the bilateral market ($X_m^N = \max[\sum_i x_{im}, \sum_i x_{mi}]$). The decrease in the position, that is achieved due to multilateral netting across counterparties, is denoted by ΔX_m^N ($X_m^G - X_m^N = \Delta X_m^N > 0$ by Jensen's inequality). Total notional cleared in the CCP is denoted by X ($X = \sum_{m \in M} X_m^N$).

As a hedge against counterparty risk, CCPs ask their members to post collateral typically in the form of initial margins and contributions to the default fund, however CCPs do not post any margins to their members. As in the case of bilateral market, we assume that both sides of a deal are required to post collateral that is determined as a fixed share of potential exposure towards a member (α and β for initial margin and default fund contribution correspondingly; for simplicity we assume the same level of initial margin α as in the bilateral setting) and does not fully cover the expected exposure ($\alpha + \beta < 1$). Both types of collateral are associated with the same funding costs per unit of it (ρ).

$$IM_m = s\alpha X_m^N \ , \ DF_m = s\beta X_m^N \ , \ IM_m + DF_m = K_m < sX_m^N$$

As in the case of the bilateral market, we calculate expected value of a derivative contract with a CCP as member's expected payoff at time 1 net from funding costs at time 0.

Proposition 2. *[Expected value in the centrally cleared setting] Expected value of a position of agent m in the centrally cleared setting is given by*

$$\begin{aligned} V_m^{ccp} &= \tfrac{1}{2}[sX_m^N p^{ccp} + IM_m + DF_m - DF_m^{loss}] \\ &\quad + \tfrac{1}{2}[-sX_m^N p_m + p_m(IM_m + DF_m - DF_m^{loss})] - (IM_m + DF_m)(1+\rho) \\ &= \tfrac{1}{2}sX_m^N((1-\alpha-\beta)(1-p_m) - (1-p^{ccp})) - sX_m^N(\alpha+\beta)\rho - \mathbb{E}(DF_m^{loss}), \end{aligned}$$

where

$$\begin{aligned} p^{ccp} &= 1 - \max[(1-\alpha)(1-\bar{p}^m) - 2\tfrac{E^{skin}}{sX} - 2\beta, 0] \\ \mathbb{E}(DF_m^{loss}) &= \tfrac{1}{2}(1+p_m)X_m^N \min[\max[\tfrac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m) - \tfrac{E^{skin}}{X}, 0], s\beta]. \end{aligned}$$

Proof. See appendix B. □

Expected value of a swap consists of three components. The first two components are of the same nature as in the bilateral market. The difference from a bilateral case is that a CCP becomes a new counterparty whose credit quality is determined by the credit quality of its members and level of margins. When realisation of the spread is such that member m is a net receiver, it receives a full payment with probability p^{ccp} that is a probability that a CCP would fulfill its liabilities after loss-mutualisation. As an additional layer of defence, a CCP provides its own "skin in the game" capital (E^{skin}). The losses arise when the amount of prefunded resources is not sufficient to cover all expected losses. In this case, we assume that remaining losses are shared between all net receivers proportionally to their claims. Expected losses due to mutualisation are represented by the third component of the expected value function, which is specific to the centrally cleared environment ($\mathbb{E}(DF_m^{loss})$). We refer to this component as a *loss-mutualisation distortion*. Under assumption of full segregation of margins, absence of legal and operational risks, a member gets back posted initial margin whenever it does not default. A distinguish feature of DF contribution, in contrast to IM, is that when a member does not default its DF can be used to cover losses caused by default of other CCP members. Mutualisation losses can occur whenever a member does not default, regardless of whether the member is a net receiver or a net payer. Losses in the DF depend on the average quality of all CCP members ($\bar{p}^m$).

4 Distortions in the expected value of a derivative contract

In the previous section we have derived expected value functions for a derivative contract traded in the bilateral market or cleared through a CCP. Counterparty risk and costly collateralization result in a fact that a symmetric derivative contract has a non-zero expected value, which depends on the market structure. Conditions under which expected value of a derivative contract is zero for each market participant in both market structures are summarized in Lemma 1.

Lemma 1. *Expected value of a symmetric derivative contract is zero for each market participant in both bilateral and centrally cleared settings if one of the following conditions holds:*

- *contract is fully-collateralized ($\alpha + \beta = 1$) and there is no cost of collateral ($\rho = 0$),*
- *all members have the same credit quality ($p_m = \bar{p}^m = p \leq 1, \forall m \in M$) and no collateral is posted.*

Proof. See Appendix C. □

When those conditions do not hold, expected value of a symmetric contract is not zero and depends on the market structure. Expected value of a contract is determined by three types of distortions: (i) credit quality distortion, (ii) loss-mutualisation distortion,

and (iii) funding costs distortion. The first two types are associated with counterparty risk and occur because not risk-free market participants differ in their credit quality and a contract is not fully collateralized. While credit quality distortion appears in both market structures, loss-mutualisation distortion is specific to a centrally cleared structure. The third type of distortion is associated with funding costs of collateral and occurs in both market structures.
In this section we discuss each type of distortions and analyse how it changes with the transition from bilateral to centrally cleared structure.

4.1 Credit quality distortion

In this subsection we examine how netting, performed bilaterally or through a single CCP, effects credit quality distortion in the expected value function. Even though elimination of redundant positions is also associated with the reduction in collateral requirements, in order to focus on netting effects caused by differences in credit quality, in this subsection we do not account for collateral ($\rho = 0$) and losses due to mutualisation. In this case, change in the credit quality component corresponds to a change in the expected value of a contract.

Definition 4.1. *For an agent m,* ***credit quality distortion*** *in the expected value of a contract with counterparty i (CQD_{mi}) is determined by the difference in counterparties' credit qualities and size of exposure between them*

$$CQD_{mi} = \frac{1}{2} s \max[x_{mi}, x_{im}](p_i - p_m). \tag{3}$$

The credit quality distortion is proportional to the size of position between counterparties and difference in their credit qualities. When we keep probabilities to fulfill liabilities fixed and exogenous, the effect is determined by the size of position, therefore it highly depends on whether netting is executed or not.

4.1.1 Bilateral netting

In this subsection we investigate how expected value of a derivative contract changes due to bilateral netting. Bilateral netting substitutes two gross positions with a net one, determined as a difference between the two initial positions. If agents differ in their credit quality, netting leads to a change in the expected value of the contract for each counterparty. Even though the counterparties offset the same amount of notional, they face different changes in their expected cash flows.
We start with an example of a single bilateral contract between agent m and its counterparty i. Let $x^G_{mi} > 0$ represent a notional value of the contract according to which m receives a floating rate and pays a fixed rate, while $x^G_{im} > 0$ is a notional value of the

contract by which m receives a fixed rate and pays a floating rate. By Proposition 1, expected value of the contract before bilateral netting is determined as

$$CQD^g_{mi} = V^g_{mi} = \tfrac{1}{2}s(x^G_{mi} + x^G_{im})(p_i - p_m). \tag{4}$$

Without loss of generality, we assume that $x^G_{mi} > x^G_{im}$. After bilateral netting, m becomes a net receiver of a floating rate with the position of $x_{mi} = x^G_{mi} - x^G_{im}$. Total amount of notional eliminated from the system is $(x^G_{mi} + x^G_{im}) - (x^G_{mi} - x^G_{im}) = 2x^G_{im}$. By Proposition 1, expected value after bilateral netting is

$$CQD^b_{mi} = V^b_{mi} = \tfrac{1}{2}sx_{mi}(p_i - p_m).$$

Bilateral netting leads to a change in the credit quality distortion and expected value of a contract, that we would refer to as a *credit quality effect of netting*:

$$\Delta CQD^b_{mi} = \Delta V^b_{mi} = V^b_{mi} - V^g_{mi} = sx^G_{im}(p_m - p_i). \tag{5}$$

The credit quality effect of netting is equal to the part of expected exposure, which is eliminated due to netting, multiplied by the difference in credit qualities of the counterparties. The effect of netting for agent m is positive as long as m fulfills its liabilities with higher probability than its counterparty i. Riskier counterparty i loses exactly the same amount as safer counterparty m gains. Thus, whether netting is beneficial or not depends on the relative credit worthiness of counterparties, rather than on their absolute quality. Absolute value of netting benefits or losses is proportional to the amount that can be netted and to the difference in counterparties' probabilities to fulfill their liabilities. This leads us to the more general result that regardless of whether an agwnt is a net payer or a net receiver of a floating rate, it benefits from netting with riskier counterparties. The total effect of netting with multiple counterparties is determined by the sum of individual effects.

Proposition 3. *When there is no cost of collateral, the effect of bilateral netting on expected value of a contract is equal to*

$$\begin{aligned}
\Delta CQD^b_m = \Delta V^b_m &= V^b_m - V^g_m = \tfrac{1}{2}s\Delta X^B_m(p_m - \bar{p}^b_m), \text{ where} \\
\Delta X^B_m &= 2\sum_{i\neq m}\min[x^G_{mi}, x^G_{im}] \\
\bar{p}^b_m &= \sum_{i\neq m}\frac{2\min[x^G_{mi}, x^G_{im}]}{\Delta X^b_m}p_i.
\end{aligned} \tag{6}$$

Bilateral netting is beneficial for those agents whose credit quality is higher than the weighted average quality of their counterparties.

Proof. See Appendix D. □

In case of contracts with multiple counterparties, total effect of bilateral netting for agent m is equal to the netted out total expected exposure (ΔX_m^B) multiplied by the difference in the credit quality of the considered agent m and the weighted average quality of its bilateral counterparties ($\bar{p}_m^b$). In the bilateral setting, the effect of netting depends only on counterparties an agent deals with. If part of the exposure is covered by initial margin, it does not change the threshold credit quality above which netting is beneficial but proportionally reduces the credit quality effect of netting.

4.1.2 Transition to a CCP

In case of central clearing, instead of dealing with multiple counterparties, an agent has a single position with a CCP. This position is calculated as a difference between all incoming and outgoing committed payments, i.e. applying multilateral netting. When netting is performed through a CCP, credit quality effect of netting comes along with the counterparty effect. Counterparty effect of netting occurs because introduction of a CCP changes not only the size of the position but also the probability to receive a payment.
Credit quality distortion in the expected value of a derivative contract cleared through a CCP is determined as

$$CQD_m^{ccp} = V_m^{ccp} = \frac{1}{2} s X_m^N (p^{ccp} - p_m). \tag{7}$$

The total net notional cleared by member m in the CCP is X_i^N, and the probability to receive a payment from the CCP is p^{ccp}.
Due to mutualisation of risks and funds, probability to get a payment from a CCP depends on the credit quality of all CCP members. Therefore, in contrast to the bilateral setting, credit quality distortion in a CCP depends not only on the quality of initial bilateral counterparties but also on the quality of all other CCP members. Central clearing changes the set of counterparties to whom and by what amount each agent is exposed. Risk profile of each member, meaning the size and direction of dependencies between members, is effected by positions of all members with all other members and could completely change because of the transition.
With a single type of a derivative contract, transition to a single CCP leads to a higher netting efficiency ($X_m^N \leq X_m^B$, $\Delta X_m^N \geq 0$) (Duffie and Zhu, 2011). Effect of the transition to a CCP on the expected value of a contract consists of credit quality effect of netting and counterparty effect of netting:

$$\begin{aligned} \Delta CQD_m^{ccp} = \Delta V_m^{ccp} &= V_m^{ccp} - V_m^b \\ &= \tfrac{1}{2} s X_m^N (p^{ccp} - p_m) - \tfrac{1}{2} s X_m^B (\bar{p}_m^b - p_m) \\ &= \tfrac{1}{2} s \Delta X_m^N (p_m - p^{ccp}) + \tfrac{1}{2} s X_m^B (p^{ccp} - \bar{p}_m^b) \\ \text{where } X_m^B &- \text{ total position of } m \text{ in the bilateral market after bilateral netting,} \\ \Delta X_m^N &= X_m^B - X_m^N \text{ , additional amount that can be netted multilaterally.} \end{aligned} \tag{8}$$

Table 1: Effects of the transition to central clearing on the credit quality distortion of the expected value.

	credit quality effect	**counterparty effect**	**total effect**
1. $p_m > p^{ccp} > \bar{p}^b_m$	>0	>0	>0
2. $p_m > \bar{p}^b_m > p^{ccp}$	>0	<0	?
3. $\bar{p}^b_m > p_m > p^{ccp}$	>0	<0	?
4. $p^{ccp} > p_m > \bar{p}^b_m$	<0	>0	?
5. $p^{ccp} > \bar{p}^b_m > p_m$	<0	>0	?
6. $\bar{p}^b_m > p^{ccp} > p_m$	<0	<0	<0

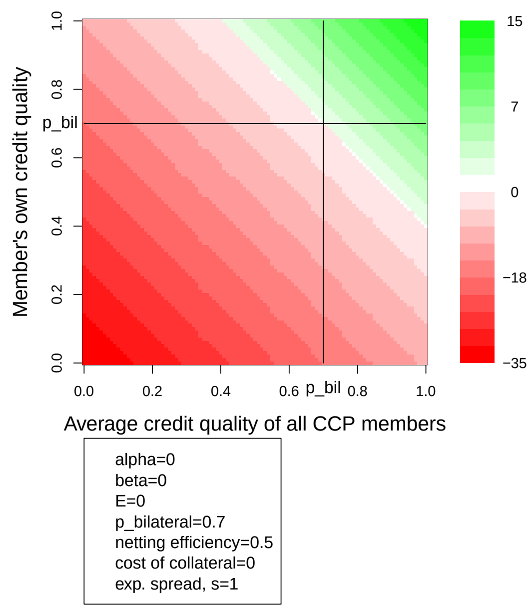
Note: *credit quality effect* $= \frac{1}{2} s \Delta X^N_m (p_m - p^{ccp})$, *counterparty effect* $= \frac{1}{2} s X^B_m (p^{ccp} - \bar{p}^b_m)$, *total effect* $=$ *credit quality effect* $+$ *counterparty effect.*

Credit quality effect of netting is of the same nature as in the bilateral case. Credit quality effect is proportional to the amount eliminated due to multilateral netting and difference between member's own credit quality and quality of its counterparty after the transition, i.e. a CCP. Counterparty effect of netting is specific to central clearing. *Counterparty effect of netting* is the amount to be received by the agent before multilateral netting multiplied by the difference in probabilities to receive it, i.e. by the difference in credit quality of a CCP and average credit quality of bilateral counterparties.

Decomposition of a change in the expected value of a contract into credit quality effect and counterparty effect allows us to analyse what types of institutions benefit from the introduction of central clearing and how an institution might adjust its strategic behaviour to make central clearing more favourable. Whether transition to a CCP is beneficial to a particular market participant depends on the relation between its own credit quality, credit quality of its bilateral counterparties, and credit quality of a CCP. Table 1 summarizes effects of the transition on the expected value of a contract for different types of institutions.

Transition from a bilateral market to a CCP is certainly beneficial for institutions with high credit quality, whose bilateral counterparties are on average worse than the CCP (case 1), since both credit quality and counterparty effects are positive. If opposite holds (case 6), i.e. a low quality institution deals with bilateral counterparties of higher credit quality than a CCP, central clearing is wealth destroying for the institution. In all other cases (cases 2-5) total effect of the transition is ambiguous and depends on the relation between credit quality and counterparty effects of netting. This relation is determined by differences in credit qualities and efficiency of multilateral netting. Since counterparty effect is proportional to the full exposure in the bilateral market while credit quality effect is proportional only to the netted out part, the counterparty effect can often dominate. In this case, only institutions, whose bilateral counterparties are of lower credit quality than the CCP, would benefit from entering the CCP. In order to make central clearing more attractive for high quality institutions, a CCP should be structured in a way that ensures large multilateral netting efficiency. Figure 1 illustrates the effect of the transition on the expected value for institutions of different credit quality in case without collateral, when credit quality of a CCP corresponds to the weighted average quality of all its members.

Figure 1: Change in the expected value of a contract with the transition to a CCP, no collateral



Note: The diagram shows how effect of the transition on the expected value of a contract for a particular member depends on the member's own credit quality and average quality of all CCP members, given a certain credit quality of the member's bilateral counterparties. Areas with positive and negative effect of the transition are coloured in green are red correspondingly. Intensity of the colour increases with the absolute value of the transition effect. In the right upper corner both netting and counterparty effects are positive, while in the left lower corner both effects are negative. In the left upper corner the credit quality effect is positive and counterparty effect is negative, opposite holds for the right lower corner. Slope of the separating line depends on the efficiency of multilateral netting. The scale represents the effect per 100 units of notional.

4.1.3 Netting through a CCP

In the previous subsection we have shown how transition to a CCP and multilateral netting change expected value of a contract relative to the bilateral setting. However, multilateral netting matters not only at the stage of transition from the bilateral market to a CCP but also when new deals are arranged through the CCP. In this subsection, we consider a credit quality distortion with the presence of collateral and discuss how mutualisation of risks and funds effects expected value of contracts in a CCP, changes incentives of members to arrange new contracts, and creates externalities between the members.

There are several crucial differences in how arrangement of a new contract between counterparties changes expected value of their total positions depending on whether the counterparties interact in the bilateral market or through a CCP. In the bilateral setting, a new contract increases gross position between counterparties but can increase or decrease absolute value of a net position between them. Since two counterparties are directly exposed to each other, change in the expected value of the position is determined by the credit quality effect of netting, which also equals to the expected value of the new deal (see Equation 5). Both counterparties experience the same change in the net position, therefore credit quality components are of the same size but have opposite signs. This implies that one counterparty gains exactly the same amount in expected value as another counterparty loses. Situation changes when a new contract is arranged through a CCP. First, since positions with all members are aggregated into a single position with a CCP, it is possible to arrange such a deal between two CCP members that increases absolute value of a net position of one counterparty and decreases absolute value of a net position of the other one. Second, the credit quality effect depends on the difference in credit quality of a member and a CCP, therefore for the two members involved in a new deal the total credit quality effect could be different from zero. Third, in addition to the credit quality effect, there is a counterparty effect, since a new deal has an impact on the credit quality of a CCP. All these differences make it possible to arrange a deal that creates value to the both counterparties, that is not the case in the bilateral setting.

In order to illustrate this effect let us consider an example of member m arranging a new deal with member i. This deal decreases absolute value of a net position of member m and increases absolute value of a net position of member i by a notional value $\Delta X = \Delta X_m = -\Delta X_i > 0$. Due to the combination of credit quality and counterparty effects, a new contract not only adds its own value but also changes the value of the initial position, thereby creating negative externalities to all other CCP members:

$$\begin{aligned} \Delta CQD^{ccp}_m = \Delta V^{ccp}_m &= \tfrac{1}{2}s\Delta X_m(p_m - p^{ccp'}) + \tfrac{1}{2}sX^N_m\Delta p^{ccp}, \text{ where} \\ p^{ccp'} &- \text{new credit quality of a CCP} \\ \Delta p^{ccp} &= p^{ccp'} - p^{ccp}. \end{aligned} \tag{9}$$

Proposition 4. *When a deal is arranged through a CCP, it could be Pareto-improving for involved counterparties m and i in terms of credit quality component of expected value of their positions.*

- *Case 1. If $\frac{E}{sX} < \frac{1}{2}(1-\alpha)(1-\bar{p}^m) - \beta$, if a deal transfers part of net position from member m that is safer than a CCP to a member i that is riskier than the CCP*

($p_m > p^{ccp} > p_i$) and the size of the transfer lies in the following range

$$\max[0, X_m^N - \frac{p_m - p^{ccp}}{(1-\alpha)(p_m - p_i)} X] < \Delta X_m^N < \min[\frac{p^{ccp} - p_i}{(1-\alpha)(p_m - p_i)} X - X_i^N, X_m^N]$$

,

it is beneficial to both counterparties.

- *Case 2. If $\frac{E}{sX} \geq \frac{1}{2}(1-\alpha)(1-\bar{p}^m) - \beta$, if a deal increases net position of each counterparty, it is beneficial to both of them.*

Proof. See Appendix E. □

Difference to the bilateral market that allows both counterparties to benefit from a deal through a CCP lies in the mutualisation of risks and funds. Such mutualisation is reflected in the calculation of credit quality of a CCP (p^{ccp}), which represents the probability that the CCP would fulfill its liabilities after mutualisation. According to the typical CCP's "default waterfall", when individual resources of a defaulted member are exhausted, prefunded default fund resources of survived members are used. When prefunded resources are not sufficient to cover all losses, a CCP can apply several loss-allocation mechanisms (e.g. haircutting of variation and initial margins, complete or selective termination of open contracts) or ask for additional contributions from members (rights of assessment). We assume that uncovered after mutualisation losses are divided between members proportionally to their share in a CCP, which is in line with margin haircut approach. Then CCP's credit quality after mutualisaion is determined as

$$p^{ccp} = 1 - \max[(1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta, 0]. \tag{10}$$

Due to the mutualisation of funds (DF and "skin in the game" equity), credit quality of a CCP is a piecewise-linear function that consists of two regions, which depend on the level of CCP's capitalization. If a CCP is not well-capitalized, a deal could be Pareto-improving for the involved counterparties when it increases net position of a relatively risky member and decreases net position of a relatively safe member. This implies that both members experience positive credit quality effect of netting (for a riskier member the effect is opposite to netting, since it builds up a net position). However, positive credit quality effect comes at the cost of negative counterparty effect. A Pareto-improving deal is possible, since positive credit quaity effect benefits only the involved counterparties while negative counterparty effect is shared among all CCP members. Due to mutualisation of risks, all CCP members experience negative externalities in the form of a decreased quality of a CCP. Counterparty effect and, therefore, externalities do not occur when a CCP is well-capitalized, i.e. $p^{ccp} = p^{ccp'} = 1$. In this case, both counterparties are of worse credit quality than a CCP, thus Pareto-improvement is possible only if both counterparties increase their net positions. This implies, that even though CCPs are intended to facilitate additional multilateral netting, members do not have incentives to net once they are in a CCP. This could encourage additional risk-taking and a build-up of net positions. Therefore, it is important to design CCPs in a way that guarantees netting, for example higher margins or mandatory clearing so that agents cannot optimize their positions between bilateral and cleared markets. Moreover, the lower is credit quality of a member relative to a CCP, the more it loses on the credit quality effect per unit of notional and the stronger incentives it has to increase its net position. It makes a CCP more exposed to lower credit members in case of a negative shock, that might increase systemic risk. The lower is member's credit quality, the larger negative impact per unit of notional its growth has on the credit quality of a CCP, and larger negative externalties it creates.

The main source of externalities between members is mutualisation of funds and risks. Mutualisation of funds increases credit quality of a CCP, since default fund of all survived members could be used to cover losses regardless of whether a member is a net payer or a net receiver. From one side, a larger buffer of potentially available prefunded resources prevents emergence of the externalities. From the other side, the higher is the credit quality of a CCP, the lower incentives members have to net their positions. In a special case, when a CCP does not collect any collateral, its credit quality is determined by the weighted average quality of all its members. Therefore, there are always some members who benefit and who lose from netting. Since there is no collateral buffer, there is always a counterparty effect and, therefore, externalities. Thus, mutualisation of risks is sufficient to create a possibility of Pareto-improving deals and externalities between members even without mutualisation of funds (see example in Appendix E).

Overall, in this section we show that members of relatively high quality benefit from netting, while riskier members have incentives to increase their ratio of net to gross positions. In contrast to the bilateral market, mutualisation of risks by a CCP enables deals, which are Pareto-improving for the involved members. These deals lead to a decreasing credit quality of the CCP and negative externalities to all CCP members. With mutualisation of funds, CCP's credit quality increases and members are less willing to net. Therefore, members have incentives to accumulate larger net positions that implies larger exposures and margin calls in times of distress.

4.2 Loss-mutualisation distortion and the insulation threshold

In the previous subsection we have discussed the credit quality distortion, which arises in both market structures due to the counterparty risk and difference in the counterparties' credit qualities. In contrast to the bilateral setting, in a CCP due to mutualisation of risks and default fund resources counterparty risk results in a specific additional type of distortion - *loss-mutualisation distortion*. Loss-mutualisation distortion appears when in expectation a CCP is not sufficiently capitalized. Our framework allows us to find

a specific threshold, which depends on the interplay between the credit quality of CCP members, level of margining, and CCP's "skin in the game" capital, for the network of expected interdependencies between members to become fully connected.
Risk-management schemes typically applied by CCPs are significantly different from the schemes used in the bilateral market. Losses caused by member's default are covered according to the "default waterfall" sequence of prefunded resources. First, initial margin and default fund of the defaulted member are used. When individual resources of the defaulter are exausted, a CCP has to step in and put part of its own equity, so called "skin in the game" capital (E^{skin}). After that level of the "default waterfall", mutualisation starts and default fund contributions of solvent members could be employed to cover remaining losses. We say that a member is insulated from other members' counterparty risk, if its resources are not needed to cover expected losses of a CCP caused by default of other members.
Following the structure of the typical "default waterfall", we calculate total expected losses of a CCP that need to be mutualised and divide them between members proportionally to their default fund contributions, i.e. their size (see Appendix B). Losses due to mutualisation can occur regardless of whether a member is a net payer or a net receiver, as long as the member does not default. We assume that there are no rights of assesment, i.e. a CCP cannot make additional capital calls, thus member's losses cannot exceed the initial default fund contribution.

Definition 4.2. *For a member m, **loss-mutualisation distortion** in the expected value of a contract with a CCP is expected loss in the default fund of m due to default of other CCP members*

$$\begin{aligned}\mathbb{E}(DF_m^{loss}) &= \tfrac{1}{2}(p_m+1)DF_m^{loss} \\ &= \tfrac{1}{2}(p_m+1)X_m^N \min[\max[\tfrac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m)-\tfrac{E^{skin}}{X},0],s\beta].\end{aligned} \tag{11}$$

There are no losses in default funds of non-defaulted members, i.e. insulation between members is achieved, when CCP's "skin in the game" capital exceeds the part of CCP's exposure that is expected to not be paid by defaulted members and is not covered by their individual resources. In other words, the insulation holds when the average probability of members' survival is higher than the amount of "skin in the game" capital per unit of uncollateralized expected exposure. However, even though a CCP interposes itself between bilateral counterparties, the insulation from the counterparty risk is not always achieved. When a CCP is not sufficiently capitalized, there are positive expected losses in the default fund due to mutualisation that are determined by the sum of expected losses coming from each CCP member. This implies that, once the insulation threshold is hit, the network of expected interdependencies between members becomes fully connected.

Proposition 5 (Threshold for a fully-connected network)**.** *The network of expected interdependencies between members becomes fully connected, if the amount of the CCP's "skin* *in the game" is lower than the total expected uncovered exposure*

$$
\begin{aligned}
E^{skin} &< \tfrac{1}{2} sX(1-\alpha-\beta)(1-\bar{p}^m) \ , \ \textit{where} \\
\bar{p}^m &= \frac{\sum\limits_m X_m^N p_m}{X}.
\end{aligned}
$$

Proof. The threshold value follows directly from the Equation 11. The network is fully connected, since once the threshold is hit, expected losses of each member directly depend on credit quality of every other CCP member. □

In contrast to the bilateral setting, in a CCP each member is potentially exposed to all other CCP members and not only to the actual bilateral counterparties. The size of exposure between members is not determined by bilateral derivative contracts between them but by transactions and credit quality of all CCP members, therefore it cannot be fully controlled and managed by any single member and risk-management functions are transferred to a CCP. When a network is fully connected, expected default fund losses of each member directly depend on the credit quality of all other CCP members. This leads to the fact, that deterioration of quality of any CCP member creates negative externalities for all other members. These externalities can be amplified in the system, if probability of members' survival is decreasing in expected default fund losses, that could cause further growth of expected losses and deterioration of members' credit quality.

The network of expected exposures is determined by the interplay between the credit quality of CCP members, level of margining, and CCP's "skin in the game" capital and can change from full insulation to a fully connected graph. Insulation and, therefore, sufficient amount of "skin in the game" capital are required to prevent emergence of externalities. The minimum amount of "skin in the game" required for insulation depends on the amount of collected prefunded resources. Minimum amount of default fund contributions (β) is determined by the "cover 2" regulatory standard, therefore it depends on the concentration of a CCP. The more diversified is CCP's membership base, the more "skin in the game" capital is needed to guarantee insulation. While high diversification level is motivated by CCPs' funding requirements, it reduces efficiency of the "cover 2" regulatory standard. Loss-mutualisation distortion is limited by the size of default fund contribution, therefore for this distortion to appear, mutualisation of resources and not only mutualisation of risks is required.

The exposures arise when volatility of the underlying asset increases or credit quality of CCP members falls. This means that precisely in times of financial distress when insulation from the counterparty risk is needed the most, a CCP further connects all the members. It has been argued in the previous literature on financial contagion that a more densely connected financial network does not always enhance financial stability (Acemoglu et al. (2015), Gai and Kapadia (2010), Battiston et al. (2012)). As shown in Acemoglu et al. (2015), financial contagion is characterized by a form of phase transition: a more diversified network leads to less fragility in case of small shocks but is prone to contagion when shocks are sufficiently large. Increased connectivity and risk sharing may lower the probability of contagious default but increases the potential for contagion to spread more widely (Gai and Kapadia, 2010). Diversification is beneficial for an individual institution but it increases the chances that the institution will act as channel to spread the distress in the system (Battiston et al., 2012). Overall, mutualisation, by creating direct interlinkages between members, can reduce the effect of idiosyncratic shocks but, in case of large aggregate shocks, can intensify propagation of contagion and distress, thus increase systemic risk. Mutualisation shifts the risk of contagion towards tails and the need to assist even larger and more complex institutions in case of a severe financial distress. A belief that a CCP would be "too-big-to-fail" could stimulate an excessive risk-taking by members.

In order to keep insulation in times of a severe distress, a CCP could put more "skin in the game" capital that is costly or ask members for additional contributions that is highly procyclical, since could further increase members' probabilities of default. Higher levels of collateralization enhance insulation but increase sensitivity to collateral haircut changes. Since a distress is often associated with collateral's quality deterioration and increased volatility, higher haircuts could serve as another source of procyclicality.

4.3 Funding costs distortion

Counterparty risk and cost of capital are attributed both to bilateral and centrally cleared markets, thus funding costs arise in both settings. Since clearing through a single CCP provides more netting opportunities than a bilateral market, it reduces total notional in the system, therefore, given the same level of margining, could be more efficient in terms of absolute amount of funds that should be posted as collateral. However, since the principles of calculation of requirements are different in centrally cleared and bilateral settings, the transition could be not equally beneficial to all market participants. While in the bilateral market agents ask each other only initial margins that are based on individual bilateral transactions between them, total collateral requirements in the centrally cleared setting are determined on the level of the whole CCP. We define funding costs as an amount prefunded resources that a member needs to post to a CCP per unit of created expected exposure. In this section we analyse an effect of mutualisation on funding costs. In particular, we show that the current regulatory "cover 2" standard leads to the externalities in funding costs and profitability of market participants. Moreover, we show that negative externalities remain if the "cover 2" standard is substituted with an alternative concentration measure, Herfindahl index, but the same principle of sharing requirements between members is applied. Finally, we suggest a way to avoid negative externalities between CCP members. For derivation of total and individual funding requirements as well as of funding costs under the three standards see Appendix F.

4.3.1 Negative externalities of funding costs

In order to limit exposures to clearing members, a CCP is required to maintain a minimum amount of prefunded resources. The combination of margins, default fund contributions and other prefunded financial resources shall be sufficient to cover the default of at least the two clearing members to which a CCP has the largest exposures under extreme but plausible market conditions, so called "cover 2" standard. Although the exact criteria for calculation of contributions to default fund by a single member are established by CCPs themselves, they shall be proportional to the exposure of each clearing member [2].
In line with the above described regulatory standard, we assume that a CCP's total funding requirement is determined by the expected exposure towards its two largest members and is divided between members proportionally to their size in the CCP. This approach gives funding costs (k_m) that depend on the level of initial margin and concentration of a CCP (see a more detailed derivation in Appendix F).

$$\begin{aligned} k_m^C &= \alpha + (1-\alpha)k_2 \text{ , where} \\ k_2 &= \tfrac{X_1^N + X_2^N}{X}\text{, a share of exposure towards the two largest members.} \end{aligned} \tag{12}$$

The "cover 2" standard leads to the following characteristics of funding costs: i) when total requirement is based on the concentration measure, funding costs depend on the positions of all CCP members ii) when individual requirements are set proportionally to member's share in a CCP, funding costs are the same for all members iii) given the initial margin level, contribution to the default fund is proportional to the share of the two largest CCP members. Therefore, the more concentrated is the CCP on the two largest members, the higher funding costs are beared by all CCP members, that implies negative externalities between members.
Since by construction CCPs are flat on market risk, regulation aims to limit concentration of CCP's membership base by accounting for it in the calculation of funding requirements. A straightforward limitation of the "cover 2" (or in general "cover k") measure is that it does not satisfy the Pigou-Dalton principle of transfers, i.e. the measure does not account for transitions in concentration that occur outside of the largest two members. For instance, the transition within these two largest members or within the smaller members is not taken into account. Therefore, in addition to the regulatory "cover 2" standard, we consider an alternative concentration measure that is sensitive to these transfers, the Herfindahl index. We assume, that a CCP holds an amount of prefunded resources that is sufficient to cover a share of total exposure that is not covered by individual initial margins, and the share is determined by the Herfindahl index of that CCP. We show that under such approach, based on the Herfindahl index, total CCP funding requirement is lower than under the "cover 2" standard (see Lemma 3, Appendix F). Then, we attribute individual requirements proportionally to members' contributions to the index. The resulting funding costs

[2] https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32012R0648&from=EN (EMIR, Article 42)

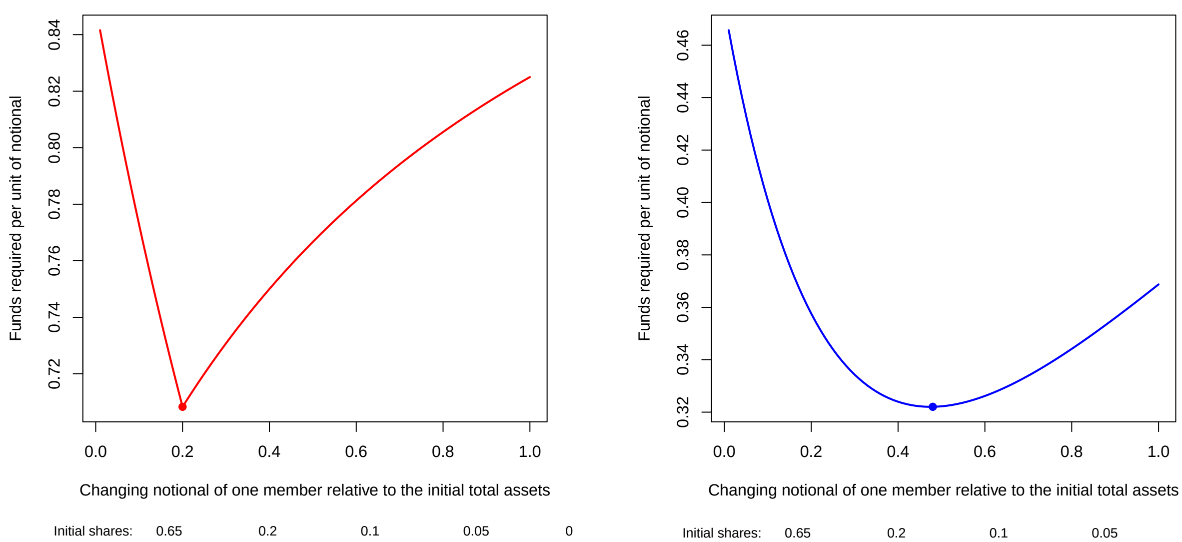
have the same form as under the “cover 2” standard but the share of the two largest members is substituted with a new concentration measure, the Herfindahl index:

$$
\begin{aligned}
k_m^H &= \alpha + (1-\alpha) H_{ccp} \text{ , where} \\
H_{ccp} &= \sum_{m \in M} (X_m^N / X)^2 \text{ , the Herfindahl index of a CCP.}
\end{aligned}
\tag{13}
$$

Application of the alternative concentration measure does not change the three characteristics of funding costs that emerge under the “cover 2” standard, thus might potentially give a rise to negative externalities between the members. When total requirements are based on the concentration of a CCP but shared between members proportionally to their size, funding costs are the same for all members, depend on positions cleared by all members, and increase in concentration of the CCP.

Figure 2: Funding costs of clearing

a) “Cover 2” standard (Case 1) b) Herfindahl index (Case 2)



Note: The diagram shows how funding costs of all members change with a growing size of one of the members. The threshold size of the member which determines negative externalities of further growth, could be higher or lower under the “cover 2” standard than under the Herfindahl index, depending on positions of other CCP members.

Figure 2 represents how funding costs change depending on the size of exposure created by one of the CCP members. Regardless of the concentration measure, member’s funding costs function is convex in the amount cleared by the member, i.e. the member can achieve the same funding costs by clearing a small or a large position. When clearing of a unit of notional is profitable, the member would choose a larger position, increasing CCP’s exposure and potentially systemic risk.

The lowest funding cost is achieved when a CCP is least concentrated, i.e. when positions are evenly distributed between all members. If members are heterogeneous in terms of their share in the CCP, change in the position of a “small” and a “large” member has different impact on the funding cost. We define “large” and “small” members as members whose growth increases and decreases the concentration measure correspondingly. In case of the “cover 2” standard, “large” members are the ones with the largest and the second largest positions in a CCP. If a Herfindahl index is applied, a member is “large” when its share in the CCP is larger than the Herfindahl index of the CCP. The threshold share can be higher or lower than in case of the “cover 2” standard depending on the concentration of positions of all CCP members.
When all members increase their positions proportionally, funding costs do not change. Whenever a “large” member increases its position, funding costs grow for each member. This happens because total CCP requirement grows by more than individual requirement for the “large” member, therefore part of the cost is shifted to other members. Other CCP members not only become more exposed to the “large” member with their prefunded resources but also have to contribute additional funds. When a “small’ member increases its position, in case of the “cover 2” standard, the same amount of resources is required to clear more notional. Growth of a “small’ member decreases funding costs but shares benefits with other members. Since funding costs are the same for all members, relative to the created exposure, smaller members pay disproportionally more.

Proposition 6. *When total funding requirement is based on a concentration measure but divided among members proportionally to their share in a CCP, increased positions of “large” members lead to higher funding costs for all CCP’s members.*

- ***Case 1.*** *Under the “cover 2” standard,*

$$\frac{\partial K_m^C}{\partial X_i^N} = s(1-\alpha)S_m(1-k_2) > 0 \ , \ X_i^N \in \{X_1^N, X_2^N\}.$$

- ***Case 2.*** *Under the Herfindahl index,*

$$\frac{\partial k_m^H}{\partial X_i} = 2(1-\alpha)\frac{S_i - H_{ccp}}{X} > 0 \ , \ \textit{if } S_i > H_{ccp}.$$

Proof. See Appendix G. □

Severity of funding costs distortions depends on the share of expected exposure not covered by initial margin. On the one hand, higher initial margin requirements reduce funding cost distortions. On the other hand, while funding cost distortions are mainly associated with redistribution of wealth between smaller and larger members, costs to fund higher levels of initial margins are deadweight losses for the system. Moreover, larger total amount of collateral would have to be posted to a CCP to provide the same level of exposure coverage, that could potentially lead to higher liquidity risks and lower quality of collateral. Therefore, there is a trade-off in the optimal levels of initial margin and default fund contributions.

A "large" member has an incentive to further increase its position, as long as increased funding cost is compensated by additional benefits received from clearing. Member's profit is determined by the benefits received from clearing of one unit of notional, for example fees collected from clients (f), and the cost of posting one unit of prefunded resources (ρ).

$$\pi_m = (f - \rho s k_m) X_m^N \tag{14}$$

When a member increases the amount it clears, it gets marginal benefits in terms of fees and bears marginal costs that consist of the funding cost associated with an additional cleared unit and change in the funding cost applied to all cleared units. The "large" member gets full profit but bears only part of the funding cost. The further growth of the "large" member creates negative externalities to small members, since they face increased funding costs without receiving additional benefits. The growth beyond the point of minimum funding costs is beneficial for the "large" profit-maximizing member only if fees that it receives from clearing of a unit of notional are sufficiently high. Given the initial, i.e. before the member's growth, composition of a CCP, we find the threshold fees under the "cover 2" and the Herfindahl index approaches. Under the Herfindahl index approach, lower benefits from clearing are sufficient to lead to the potential emergence of negative externalities.

Proposition 7. *If fees received from clearing of a unit of notional are higher than a certain threshold, a profit-maximizing "large" member i has incentives to grow beyond the point of minimum funding costs, thereby creating negative externalities to other members. The threshold fee is determined as*

- *Case 1:* $f^C = \rho s(\alpha + (1-\alpha)(s_1' + s_2') + (1-\alpha)(1-s_1')s_2')$ *, where*

  s_1'*,* s_2' *are shares of the first and second largest positions when i becomes the second large member.*

- *Case 2:* $f^H = \rho s(\alpha + (1-\alpha)\frac{H_{ccp}}{1+H_{ccp}})$*, where*

  H_{ccp} *is the Herfindahl index of the initial CCP.*

The threshold fee is lower applying the Herfindahl index approach (Case 2), than under the "cover 2" standard (Case 1).

Proof. See Appendix H. □

Due to increasing funding costs, the growth of each member within a single CCP is limited. When the CCP becomes too concentrated, clearing might be not profitable anymore. A "large" member would have incentives to split its position among multiple CCPs. By limiting CCP's concentration, regulation encourages common membership.

4.3.2 How to avoid negative externalities

In the previous subsection we show that, regardless of the concentration measure, the negative externalities emerge as long as total and individual funding requirements are based on different measures. In this subsection we show that negative externalities could be prevented by setting individual requirements according to members' contributions to the total capital requirement, i.e. to the concentration measure. In case of the "cover 2" standard, only the two largest members would be required to contribute to the default fund. Discontinuity of cost function implies much higher funding cost to the second largest member than to the third largest one, even if they clear almost the same amount of notional. The issue of discontinuity does not arise when the Herfindahl index is used as a measure of concentration. In this case (we refer to it as Case 3), funding costs are determined as (see Appendix F for a detailed derivation)

$$
\begin{aligned}
k_m^{HP} &= \alpha + (1-\alpha) S_m \text{ , where} \\
S_m &= \tfrac{X_m^N}{X} \text{ , a share of member } m \text{ in the total notional of a CCP.}
\end{aligned} \tag{15}
$$

When the total requirement is calculated on the base of the Herfindahl index and shared among members proportionally to their contributions to the index, individual relative default fund requirement is member-specific and equals to member's share in the CCP. Such approach leads to a more fair distribution of funding costs in the sense that, per unit of created exposure, funding costs associated with default fund contributions are the same for all members ($\frac{\beta_m^{HP}}{X_m^N} = \frac{1-\alpha}{X}$).

Depending on the share of a member and CCP's concentration, member's growth can lead to higher or lower total funding requirements. However, any member that increases its position in a CCP increases its own funding requirements but reduces requirements for all other members (see Figure 3a):

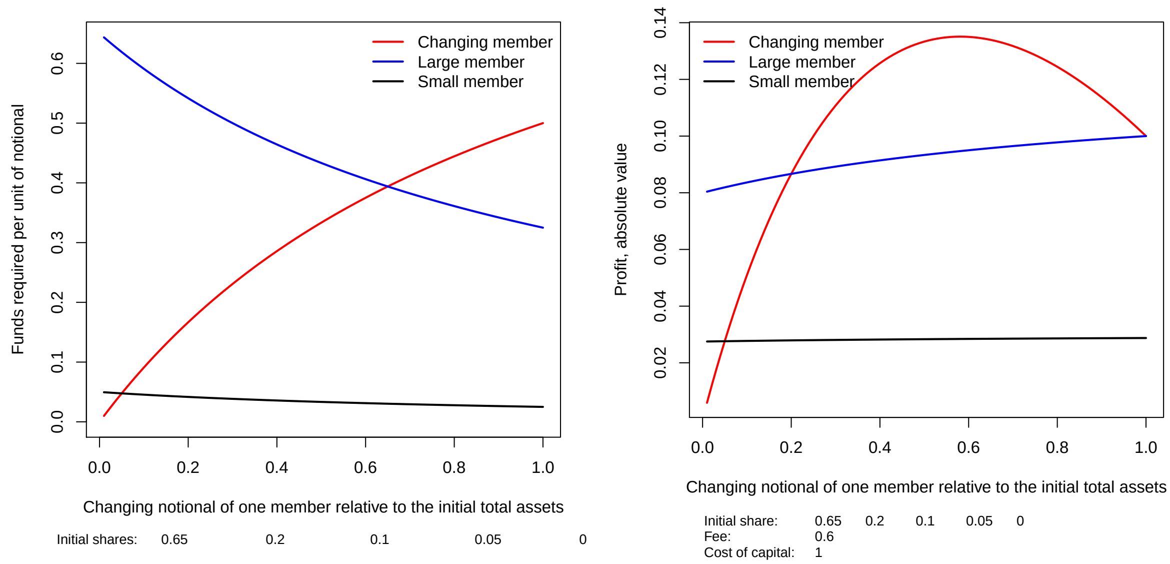
$$\frac{\partial k_i^{HP}}{\partial X_i} = (1-\alpha)\frac{1-S_i}{X} > 0, \forall i \in M, \tag{16}$$

$$\frac{\partial k_m^{HP}}{\partial X_i} = -(1-\alpha)\frac{S_m}{X} < 0, \forall i \in M. \tag{17}$$

Members with fixed positions experience positive externalities in the form of lower funding costs. These positive externalities can be considered as a compensation for an increased exposure towards the growing member. Moreover, since lower funding costs result in higher profits for members with fixed positions, further growth of a given member leads to a Pareto-improvement in terms of profits as long as the member's benefits from clearing an additional unit of notional compensate for increasing funding costs (see Figure 3b).

Figure 3: Redistribution effects under the approach based on the Herfindahl index and division proportionally to contributions to the index

a) Funding costs b) Profits



5 Transition from the bilateral to the centrally cleared setting

In this section we discuss the joint effect of the distortions on the expected value of the contract under different market structures, i.e. whether, when and for whom transition from the bilateral structure to a CCP is beneficial.

Effect of the transition for member m is defined as a total change in the expected value of its position due to move to a CCP and is determined as

$$\begin{aligned} \Delta V_m = V_m^{ccp} - V_m^b \quad = \quad & -\tfrac{1}{2}s(X_m^N(1-p^{ccp}) - X_m^G(1-\alpha)(1-\bar{p}_m^b)) \\ & +\tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha))(1-p_m) \\ & -\tfrac{1}{2}(1+p_m)DF_m^{loss} \\ & -s\rho(X_m^N(\alpha+\beta) - \alpha X_m^G). \end{aligned} \tag{18}$$

The total change can be decomposed into four components which reflect the considered distortions. The first three components are related to the counterparty risk and the forth component occurs due to the funding costs of collateral. Such decomposition allows us to identify the trade-offs associated with the transition and their determinants.

5.1 Trade-offs of the transition

Transition to a CCP changes both the counterparty risk and the cost of funding faced by market participants. However, effect of the transition on both components of the value function is ambiguous and is associated with trade-offs, which we describe in this subsection.

First trade-off is stemming from the counterparty risk component of the value function. Change in the counterparty risk component consists of three parts: (i) change in the expected losses due to default of the direct counterparty, (ii) change in the expected benefits of the member's own default, and (iii) change in the expected losses due to mutualisation of funds.

(i) Expected losses due to default of the direct counterparty change because transition to a CCP effects both the size of exposures and the quality of the direct counterparty. While transition to a single CCP definitely decreases the size of exposure, its effect on the counterparty's credit quality is uncertain. Though due to mutualisation of funds and additional CCP equity, credit quality of a CCP is likely to be higher than the average quality of bilateral counterparties, it is not necessarily the case. Credit quality of a CCP could be lower than the average quality of bilateral counterparties, for example when a member bilaterally deals only with a few high quality counterparties but then, due to the clearing mandate, it has to join a CCP with lower quality members.

(ii) The second part of the counterparty effect is associated with member's own risk of default. It represents change in the expected amount that an agent would not pay to its direct counterparty in case of its own default. Since nominal value of its position is reduced due to multilateral netting, the position is additionally collateralized by the individual default fund contribution and we assume probability of the member's default to be fixed, expected benefits of the agent's own default decrease with the transition.

(iii) Additional losses in a centrally cleared setting occur due to the mutualisation of funds and decrease expected value of the position.

The total effect of the transition to a CCP on the counterparty component of the expeceted value is ambiguous. Decomposition of the counterparty component reveals that transition to central clearing could increase or decrease expected losses due to direct counterparty default but it decreases expected benefits of member's own default and leads to additional expected losses due to mutualisation. Therefore, transition to central clearing could be beneficial for a member only if it leads to a sufficient reduction in direct counterparty losses. This reduction is achieved when a CCP is well-capitalized and provides high netting efficiency.

Second trade-off of the transition to central clearing is associated with funding costs of collateral. On the one hand, in a CCP initial margin requirements are reduced due to multilateral netting. On the other hand, a larger share of exposure is collateralized due to additional default fund contributions. Therefore, transition to a CCP has an ambiguous effect on collateral demand and funding costs.

Effect of the transition depends on the member's own quality. Losses in expected benefits of agent's own default decrease with the agent's credit quality, while incurred indirect losses of mutualisation increase with its credit quality. Since the first channel always dominates, transition to central clearing is more beneficial for higher quality agents.

Lemma 2. *Transition to a CCP is beneficial for a market participant m, if its credit quality is sufficiently high, i.e.*

$$p_m \geq 1 - \frac{\frac{1}{2}s(X_m^G(1-\alpha)(1-\bar{p}_m^b) - X_m^N(1-p^{ccp})) + s\rho(\alpha X_m^G - X_m^N(\alpha+\beta)) - DF_m^{loss}}{\frac{1}{2}s(X_m^G(1-\alpha) - X_m^N(1-\alpha-\beta)) - \frac{1}{2}DF_m^{loss}}. \tag{19}$$

Proof. See Appendix I. □

Analysis of the expected value function shows that whether transition from bilateral to the centrally cleared setting for a particular member is beneficial or not crucially depends on its own credit quality as well as multilateral netting efficiency and level of loss-mutualisation in a CCP. We discuss separately how those parameters resolve the counterparty risk trade-off assuming zero funding costs and then focus on the trade-off associated with the collateral demand.

5.2 Effect of the average credit quality of CCP members ($\bar{p}^m$)

In the subsection 4.1.2 we have discussed how, for a particular member, the effect of transition to a CCP, measured by the change in the expected value of its position, depends on the weighted average quality of the CCP members in case without any collateral. When no collateral is posted and a CCP does not contribute its "skin in the game" capital, average quality of CCP members corresponds to the credit quality of the CCP (p^{ccp}), and it is the only channel, how average quality of CCP members influences effect of the transition.

In case with mutualisation of funds, there are two differences. First, DF contributions and CCP's capital create a buffer in credit quality of the CCP after mutualisation, thus increasing average quality of CCP members improves CCP's credit quality only when average quality is low relative to capitalization of the CCP, i.e until the CCP is risk-free. Second, average quality of CCP members impacts the effect of transition also through mutualisation losses in the DF.

Average quality of CCP members has a non-negative impact on the effect of transition through both channels. However, while change in credit quality of a CCP has an equal effect on the expected value of a contract per unit of notional for all members, change in DF losses has larger effect on high quality members, since they face higher risk of mutualisation (Figure 4a).

5.3 Effect of the level of mutualisation (β)

Level of mutualisation influences each of the parts of the counterparty risk component of the expected value in a following way:

(i) Higher level of mutualisation increases total default fund that can be used to prevent default of a CCP, thereby it has a non-decreasing effect on credit quality of the CCP (p^{ccp}). Larger default fund contributions increase quality of a CCP, if the CCP is poorly capitalized or average quality of its members is low. Hihger quality of a CCP implies lower losses due to direct counterparty default, which increases benefits of the transition.

(ii) Since additional contribution to a DF increases individual collateral loss in case of default, benefits of member's own default decrease, therefore, expected value of position in a CCP decreases. The effect is larger when member's own quality is low, so its probability of default is high.

(iii) The effect of level of mutualisation on expected losses caused by mutualisation is ambiguous. If level of mutualisation is very high, then losses are mainly covered by individual collateral and, if a CCP is sufficiently capitalized, there is no need for mutualisation and expected losses are zero. If level of mutualisation is very low, losses also cannot be high, since they are bounded by the initial DF contribution. The largest expected DF losses correspond to the medium level of mutualisation, since members are not largerly covered against default of other members by their individual collateral but have big contributions to the DF. The effect of expected DF losses is stronger for high quality members, since they with higher probability participate in mutualisation.

Figure 4b represents how level of mutualisation effects a change in the counterparty component of the expected value function for market participants of different credit quality. Low quality institutions prefer medium level of mutualisation, since they do not face high risk of mutualisation but benefit from high credit quality of a CCP. When level of mutualisation is medium, expected losses in the DF are the highest and low quality institutions benefit from the transition the most (Figure 4b). High quality institutions benefit from a higher level of mutualisation, as long as it decreases mutualisation losses by making a defaulted member cover larger share of exposure with its individual resources. If a CCP is well-capitalized with its own resources and provides high netting efficiency, then, regardless of the level of mutualisation, only institutions who have higher credit quality than their bilateral counterparties benefit from the transition. When a CCP follows the rules discussed in the subsection 4.3, level of mutualisation can also be considered as CCP's concentration.

5.4 Effect of the netting efficiency ($\gamma = 1 - \frac{X_m^N}{X_m^G}$)

The level of netting efficiency determines the size of net position in a CCP, therefore influences expected value via the following channels:

(i) Netting decreases expected losses due to direct counterparty default, so it has a positive impact on the effect of transition.

(ii) Netting decreases potential gains of member's own default, since it decreases CCP's exposure towards the member. This effect is more pronounced for riskier members.

(iii) Higher netting efficiency decreases potential losses due to mutualisation, since DF contribution is proportional to the notional value of the position. This effect is more noticeable for high quality members, since they participate in loss-mutualisation with higher probability.

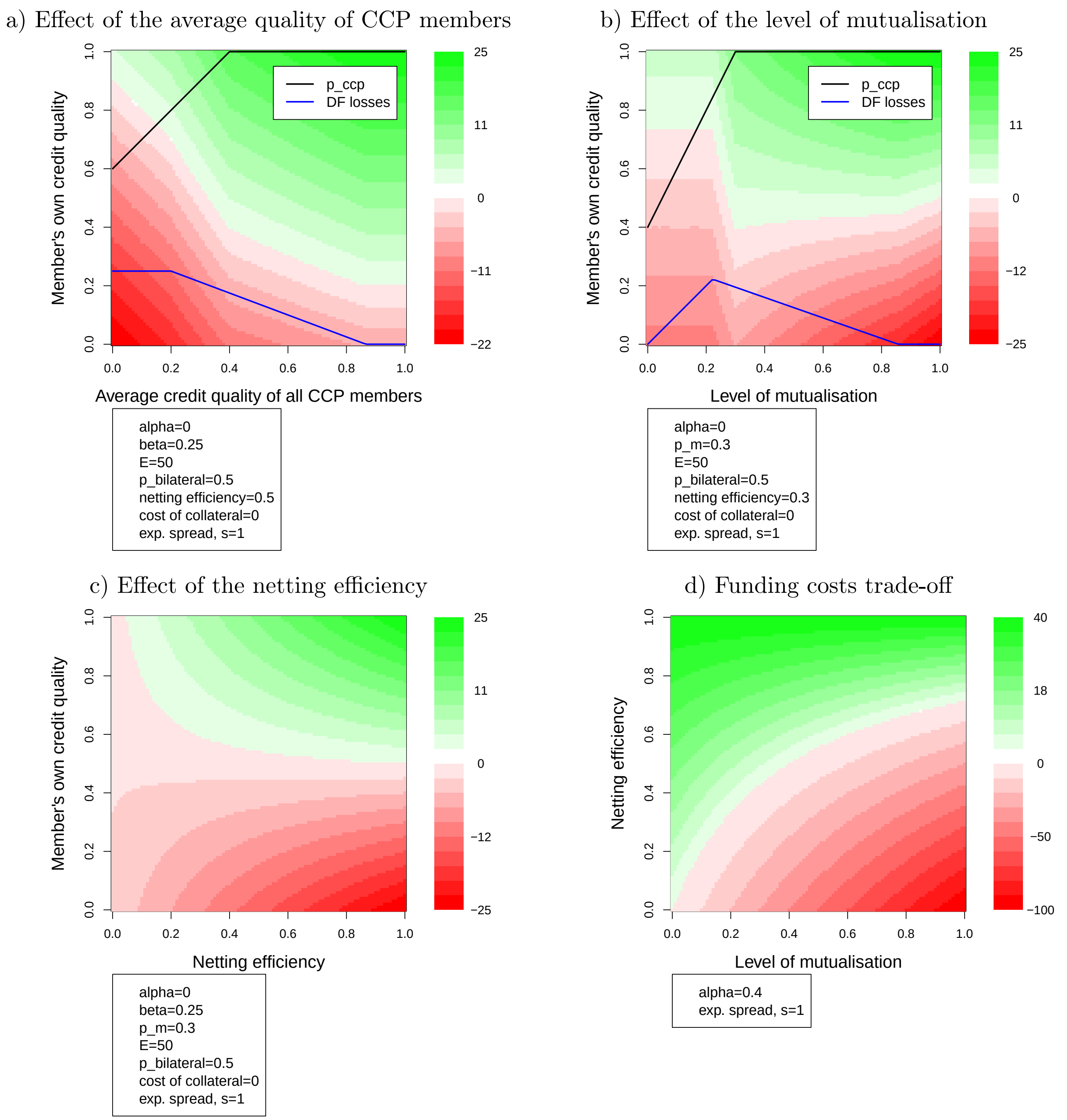
Figure 4c illustrates how effect of the transition to central clearing depends on the efficiency of multilateral netting. Since netting reduces expected benefits of member's own default, netting is beneficial for relatively high quality members and is value-destroying for relatively low quality members. Netting is beneficial for a member m if

$$p_m > \frac{(p^{ccp} - \alpha - \beta) - \min[\max[\frac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{sX}, 0], \beta] - 2\rho(\alpha+\beta)}{(1-\alpha-\beta) + \min[\max[\frac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{sX}, 0], \beta]}. \quad (20)$$

5.5 Funding costs trade-off

The effect of central clearing on collateral demand, and therefore funding costs, depends on the interplay between efficiency of multilateral netting and level of mutualisation (Figure 4d). Higher levels of mutualisation imply larger default fund contributions, thus increase funding costs of collateral. Higher efficiency of netting reduces collateral demand, since it is proportional to the notional value of the position. If positive effect of multilateral netting opportunities outweighs negative effect of additional contributions to the default fund, then central clearing results in lower funding costs. Effect of the transition on funding costs is proportional to a member's gross position but is independent of its own credit quality and credit quality of its counterparties.

Figure 4: Determinants of the effect of transition to a CCP, measured as a change in the expected value of member's position.



Note: The diagrams show how effect of the transition to central clearing, measured as a change in the expected value of member's position, depends on different characteristics. Areas with positive and negative effect of the transition are coloured in green are red correspondingly. Intensity of the colour increases with the absolut value of the transition effect. The scale represents the effect per 100 units of notional.

5.6 Welfare analysis

In the previous subsections we have shown that effect of the transition to central clearing for a particular market participant, determined as a change in the expected value of its position, is ambiguous and depends on member's own credit quality relative to the credit quality of its counterparties. In this subsection we analyse effect of the transition on the total expected value of all derivative contracts in the system. Since expected value of a position could be considered as expected utility of a risk-neutral agent, change in the total expected value of contracts corresponds to the change in total welfare of the members.
Expected value function consists of two components: counterparty risk component and funding costs component. Important difference between the two components is that while funding costs depend only on participant's own characteristics, change in the counterparty risk component depends on the relation between its own credit quality and credit quality of its counterparties. Therefore, if central clearing results in the same netting efficiency for all market participants, then all participants have either a decrease or increase in their funding costs. In contrast, in case of the conterparty risk component, some participants gain what other participants lose and transition to central clearing leads to redistribution of value between the members.

Proposition 8. *Transition to central clearing has a non-negative effect on the total welfare of the members in terms of the counterparty risk component of the expected value function.*

Proof. See Appendix J. □

Counterparty risk component arises on the part of exposure which is not covered with individual resources, while funding costs occur due to the collateralized part of exposure. Therefore, there is a trade-off between the two components. In terms of welfare analysis it might be optimal to minimize collateral requirements, since counterparty risk component mainly results in the redistribution of wealth among market participants, however this might distort incentives of the participants.

6 Discussion and conclusions

In this work we study how market structure effects expected value of a derivative contract and redistributes wealth among OTC derivatives market participants. The study is motivated by a series of regulatory reforms proposed by the G20 in the aftermath of the crisis, including mandatory central clearing, that has reshaped the OTC landscape. As these reforms are being implemented by regulators and market participants, the role central clearing counterparties play in the intermediation of derivative contracts and systemic stability is becoming increasingly more important.

We find that under realistic assumptions of counterparty risk and costly collateral, there are distortions in the expected value of a symmetric derivative contract, which depend on the market structure. Therefore, transition to central clearing results in the redistribution of wealth among market participants. Specific design of CCPs, in particular mutualisation of risks and funds, implies that distortions in the expected value are also associated with externalities between members. The analysis of the three types of distortions leads to the following results.

First, when netting is performed between agents of different credit quality, it is beneficial to the less risky counterparty. A distinctive feature of netting through a CCP is that it can be beneficial to both counterparties, since credit quality effect of netting comes along with the counterparty effect. Therefore, members with relatively high quality have incentives to minimize their net-to-gross position ratio, while riskier members keep larger net positions. If due to mutualisation and additional "skin in the game" equity, a CCP has higher probability to fulfill liabilities than any single member, then members do not have incentives to net. This increases exposure of a CCP to its members, especially to riskier ones, and might increase systemic risk.

Second, even though a CCP interposes itself between counterparties of a derivative contract, real insulation from the counterparty risk is not always achieved. When a CCP is not sufficiently capitalized, expected dependencies between members form a fully connected network. The threshold value of CCP's "skin in the game" capital depends on the level of margining and credit quality of members. The exposures arise when credit quality of the members deteriorates, i.e. precisely in times of distress. In further research, it is important to understand how a CCP and members would adjust their strategies to deal with arised exposures. Being in a CCP, members cannot close individual bilateral deals but would need either to run on a CCP or to buy protection on each other.

Third, current regulatory standards could lead to distortions in funding costs. Under the "cover 2" regulatory requirement, small members pay disproportionally more per one unit of cleared notional. This externality arises, since funding requirements associated with an additional unit cleared by a "large" member are shared among all members. In order to prevent negative externalities in funding costs, individual requirements should be assigned according to member's contribution to the concentration index.

Taking into account all components of the expected value function, we show that relatively high quality market participants benefit more from the transition to central clearing. Moreover, there is a trade-off between distortions associated with counterparty risk and funding costs, since the former one arises on uncollateralized part of exposure while the latter one occurs due to the collateralized part. An important difference between the two types of distortions is that credit quality difference mainly leads to the redistribution of wealth between counterparties while the need to post costly collateral generates deadweight losses to the system. When transactions are moved to a single CCP, less collateral is required to achieve the same level of margining. However, potential reduction in collateral requirements comes along with externalities and distortion of funding costs.

Our findings indicate that netting and mutualisation are the crucial concepts in CCP's operation that need to be better understood. Netting does not only reduce notional but could change counterparties' risk profiles and expected payoffs. While mutualisation increases credit quality of a CCP but loosen incentives to net and engender externalities.

A Proof of Proposition 1

Proof. Following the same intuition as in Equation 1, we consider a deal with counterparty j, in which agent m is a net receiver in case of a negative spread and a net payer in case of a positive spread ($x_{mj} \leq 0$):

$$\begin{aligned} V_{mj}^b &= \mathbb{P}(S<0)[\mathbb{E}(S|S<0)x_{mj}p_j + IM_j(1-p_j) + IM_m] \\ &\quad + \mathbb{P}(S \geq 0)[\mathbb{E}(S|S\geq 0)x_{mj}p_m + IM_m p_m] - IM_m(1+\rho) \\ &= -\tfrac{1}{2}s(1-\alpha)x_{mj}(p_j - p_m) + s\alpha x_{mj}\rho \\ &= \tfrac{1}{2}s(1-\alpha)x_{jm}(p_j - p_m) - s\alpha x_{jm}\rho. \end{aligned} \tag{21}$$

Since in the bilateral setting netting across counterparties is not possible, value of the portfolio of agent m can be decomposed into a sum of values of contracts with counterparties from whom agent m receives a floating rate ($x_{mi} > 0$) and counterparties to whom it pays a floating rate ($x_{mj} \leq 0$). Therefore, it follows from equations 1 and 21, that value of the portfolio is calculated as

$$\begin{aligned} V_m^b &= \sum_{i,x_{mi}>0} V_{mi}^b + \sum_{j,x_{mj}\leq 0} V_{mj}^b = \sum_{i,x_{mi}>0} [\tfrac{1}{2}s(1-\alpha)x_{mi}(p_i - p_m) - s\alpha x_{mi}\rho] \\ &\quad + \sum_{j,x_{mj}\leq 0} [\tfrac{1}{2}s(1-\alpha)x_{jm}(p_j - p_m) - s\alpha x_{jm}\rho] \\ &= \tfrac{1}{2}s(1-\alpha)[\sum_{i,x_{mi}>0} x_{mi}p_i + \sum_{j,x_{mj}\leq 0} x_{jm}p_j - p_m(\sum_{i,x_{mi}>0} x_{mi} + \sum_{j,x_{mj}\leq 0} x_{jm})] \\ &\quad - s\alpha(\sum_{i,x_{mi}>0} x_{mi} + \sum_{j,x_{mj}\leq 0} x_{jm})\rho \\ &= \tfrac{1}{2}s(1-\alpha)(\sum_i p_i \max[x_{im}, x_{mi}] - p_m \sum_i \max[x_{im}, x_{mi}]) - s\alpha \sum_i \max[x_{im}, x_{mi}]\rho \\ &= \tfrac{1}{2}s(1-\alpha)X_m^G(\bar{p}_m^b - p_m) - s\alpha X_m^G \rho, \text{ where} \end{aligned}$$

$$\begin{aligned} X_m^G &= \sum_i \max[x_{im}, x_{mi}] \\ \bar{p}_m^b &= \sum_i \frac{\max[x_{im}, x_{mi}]}{X_m^G} p_i. \end{aligned}$$

□

B Proof of Proposition 2

Following the same logic as in the bilateral case, we derive expected value of agent's m position, when it is moved to a CCP. The main differences are (i) after multilateral netting, total size of the position is X_m^N, (ii) multiple bilateral counterparties are substituted with a single CCP, (iii) part of collateral (DF) can be used even if a member itself does not default.

Since distribution of spread is symmetric, member m is a net receiver from a CCP and a net payer to a CCP with the same probability $\frac{1}{2}$. When m is a net receiver, it gets full payment from the CCP with probability p^{ccp}, which represents credit quality of the CCP after mutualisation. Being a net receiver, m cannot default on its payment, thus it gets back its IM and part of DF contribution which is not used for mutualisation. When m is a net payer, with probability p_m it does not default, thus pays its liability in full and participates in mutualisation. Therefore, expected value of a position in the CCP is calculated as

$$\begin{aligned}
V_m^{ccp} &= \tfrac{1}{2}[sX_m^N p^{ccp} + IM_m + DF_m - DF_m^{loss}] \\
&\quad + \tfrac{1}{2}[-sX_m^N p_m + p_m(IM_m + DF_m - DF_m^{loss})] - (IM_m + DF_m)(1+\rho) \\
&= \tfrac{1}{2}sX_m^N((1-\alpha-\beta)(1-p_m) - (1-p^{ccp})) - sX_m^N(\alpha+\beta)\rho - \tfrac{1}{2}(1+p_m)DF_m^{loss} \\
&= \tfrac{1}{2}sX_m^N((1-\alpha-\beta)(1-p_m) - (1-p^{ccp})) - sX_m^N(\alpha+\beta)\rho - \mathbb{E}(DF_m^{loss}).
\end{aligned}$$

We assume that calculating expected losses coming from mutualisation, each member can estimate a size of all other members, i.e. notional value of a position, but does not know whether a member is a net receiver or a payer of a floating rate. For simplicity, let p^{ccp} be the same for all members, i.e. we do not exclude a considered member from the calculation of CCP's quality.

Expected quality of a CCP is calculated on the base of the "default waterfall" structure:

1. Expected losses not covered by individual resources of a defaulter i
   $\frac{1}{2}s(1-\alpha-\beta)X_i^N(1-p_i) + \frac{1}{2}0 = \frac{1}{2}s(1-\alpha-\beta)X_i^N(1-p_i)$.

2. Total expected losses that remain to be mutualised after CCP contributes its "skin in the game" capital (E^{skin})
   $\max[\frac{1}{2}s(1-\alpha-\beta)\sum_{i\in M} X_i^N(1-p_i) - E^{skin}, 0]$.

   These losses are paid from the default fund contributions of non-defaulted members proportionally to their contributions:

$$\begin{aligned}
DF_m^{loss} &= \min[\tfrac{X_m^N}{X}\max[\tfrac{1}{2}s(1-\alpha-\beta)\sum_{i\in M} X_i^N(1-p_i) - E^{skin}, 0], \beta s X_m^N] \\
&= sX_m^N \min[\max[\tfrac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \tfrac{E^{skin}}{sX}, 0], \beta],
\end{aligned}$$

where $X = \sum_{i\in M} X_i^N$, total notional cleared in the CCP.

Total loss in the default fund is

$$DF^{loss} = sX \min[\max[\frac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{sX}, 0], \beta].$$

Member m participates in mutualisation, when it is realized to be a net receiver and when it does not default being a net payer:

$$\begin{aligned}
\mathbb{E}(DF_m^{loss}) &= \tfrac{1}{2}(1+p_m)DF_m^{loss} \\
&= \tfrac{1}{2}(1+p_m)sX_m^{net} \min[\max[\tfrac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \tfrac{E^{skin}}{sX}, 0], \beta].
\end{aligned}$$

Total DF available for mutualisation is $\sum\limits_{i\in M} \frac{1}{2}(1+p_i)DF_i = \frac{1}{2}s\beta \sum\limits_{i\in M}(1+p_i)X_i^N$.

3. Expected losses that remain uncovered after the usage of DF contributions that are expected to be available from non-defaulted members (UL):

$$\begin{aligned}
UL &= \max[\tfrac{1}{2}s(1-\alpha-\beta)\sum_{i\in M} X_i^N(1-p_i) - E^{skin} - \tfrac{1}{2}s\beta\sum_{i\in M}(1+p_i)X_i^N, 0] \\
&= sX \max[\tfrac{1}{2}(1-\alpha)(1-\bar{p}^m) - \tfrac{E^{skin}}{sX} - \beta, 0] \text{ , where} \\
\bar{p}^m &= \frac{\sum\limits_{i\in M} p_i X_i^N}{X} \text{ , average quality of the CCP members.}
\end{aligned} \tag{22}$$

4. Remaining losses are shared between members proportionally to their expected claim to the CCP:
   $\frac{1}{2}p^{ccp}sX_m^N = \frac{1}{2}sX_m^N - \frac{X_m^N}{X}UL$
   Therefore, probability of default of the CCP is

$$1 - p^{ccp} = \frac{2UL}{sX} = \max[(1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta, 0] \tag{23}$$

   This implies $0 \leq p^{ccp} \leq 1$.

In the case when there is no collateral, credit quality of a CCP is determined as a weighted average quality of all its members: $p^{ccp} = \bar{p}^m$.

C Proof of Lemma 1

In Lemma 1 we derive conditions under which expected value of a symmetric contract is zero in both bilateral and centrally cleared settings.

1. **Bilateral market.**

   Necessary condition for individual expected values of each market participant to be zero is that the total sum of expected values across participants is zero:
   $\sum_m V_m^b = \frac{1}{2}(1-\alpha)\sum_m X_m^G(\bar{p}_m^b - p_m) - s\alpha\rho\sum_m X_m^G = -s\alpha\rho\sum_m X_m^G = 0.$
   Thus, either a contract is not collateralized ($\alpha = 0$) or there is no cost of posting collateral ($\rho = 0$).

   This implies that counterparty component of the expected value for each participant also has to be zero $\frac{1}{2}(1-\alpha)X_m^G(\bar{p}_m^b - p_m) = 0$.

   If credit quality of each market participant equals weighted average quality of its counterparties, then all participants must have the same credit quality (we assume that network of exposures is connected). To show this let us assume that participants are of different credit quality and consider a participant with the lowest credit quality (without loss of generality, let it be participant l with credit quality $p_l \leq p_m, \forall m \in M$). Credit quality of other participants is determined as $p_m + \epsilon_m, \epsilon_m \geq 0, \forall m \in M$. Then, $p_l = \sum_m \frac{X_{lm}(p_l+\epsilon_m)}{X_l} = p_l + \sum_m \frac{X_{lm}\epsilon_m}{X_l}$. Since exposures between participants are non-negative, this condition holds if $\epsilon_m = 0, \forall m, X_{lm} > 0$. If ϵ_m is not determined for all participants after the first stage (since $X_{lm} = 0$), continue the same procedure by considering another participant for whom ϵ_m is already determined. Assumption that the network is connected, allows to show that $\epsilon_m, \forall m \in M$.

   If all counterparties have the same credit quality, there is no need to post initial margin to mitigate counterparty risk ($\alpha = 0$).

   Expected value of a contract is zero in the following cases:

   - contract is fully-collateralized ($\alpha = 1$) and there is no cost of collateral ($\rho = 0$)
   - all market participants have the same credit quality ($p_m = \bar{p}_m^b = p \leq 1, \forall m \in M$).

2. **Centrally cleared setting.**

   First, we find conditions for $V_i^{ccp} = V_j^{ccp} = 0$.

$$\begin{aligned} V_i^{ccp} &= \tfrac{1}{2}sX_i^N((1-\alpha-\beta)(1-p_i) - (1-p^{ccp})) - sX_i^N(\alpha+\beta)\rho - \tfrac{1}{2}(1+p_i)DF_i^{loss} \\ &= \tfrac{1}{2}sX_j^N((1-\alpha-\beta)(1-p_j) - (1-p^{ccp})) - sX_j^N(\alpha+\beta)\rho - \tfrac{1}{2}(1+p_j)DF_j^{loss} \\ &= V_j^{ccp} = 0 \end{aligned}$$

Thus, $s(1-\alpha-\beta)(p_j-p_i) = \frac{DF^{loss}}{X}(p_i-p_j)$. Necessary conditions for this to hold are either that a contract is fully-collateralized with individual resources ($\alpha+\beta = 1, DF^{loss}=0$) or that all members have the same credit quality p.

Expected value of a contract is zero in the following cases:

- contract is fully-collateralized ($\alpha+\beta=1$) and there is no cost of collateral ($\rho=0$)
- all members have the same credit quality ($p_m = \bar{p}^m = p \leq 1, \forall m \in M$).

Considering the situation when $p<1$, there are four cases depending on the values of DF losses and credit quality of a CCP:

$DF^{loss} = X \min[\max[\frac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{X}, 0], s\beta]$,

$p^{ccp} = 1-\max[(1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta, 0]$.

(a) $DF^{loss}=0, p^{ccp}=1$:
$V_m^{ccp} = \frac{1}{2}sX_m^N(1-\alpha-\beta)(1-p) - sX_m^N(\alpha+\beta)\rho = 0.$
In this case, for expected value to be zero for any $p<1$, a contract should be fully-collateralized with individual resources ($\alpha+\beta=1$) and cost of collateral should be zero ($\rho=0$). Full collateralization is consistent with the assumption that $DF^{loss}=0, p^{ccp}=1$.

(b) $DF^{loss} = X(\frac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{X}), p^{ccp}=1$:

$$\begin{aligned}V_m^{ccp} &= \tfrac{1}{2}sX_m^N(1-\alpha-\beta)(1-p) - \tfrac{1}{2}(1+p)X_m^N(\tfrac{1}{2}s(1-\alpha-\beta)(1-p) - \tfrac{E^{skin}}{X}) \\ &\quad -sX_m^N(\alpha+\beta)\rho \\ &= \tfrac{1}{4}sX_m^N(1-\alpha-\beta)(1-p)^2 + \tfrac{1}{2}(1+p)\tfrac{E^{skin}}{X} - sX_m^N(\alpha+\beta)\rho = 0.\end{aligned}$$

This holds for any $p<1$, if the contract is fully-collateralized with individual resources ($\alpha+\beta=1$), thus there is no need for CCP's capital ($E^{skin}=0$). This is consistent with assumptions $DF^{loss} = X(\frac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{X}) = 0, p^{ccp}=1$.

(c) $DF^{loss} = X(\frac{1}{2}s(1-\alpha-\beta)(1-\bar{p}^m) - \frac{E^{skin}}{X}) > 0$,
$p^{ccp} = 1-((1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta) < 1$:

$$\begin{aligned}V_m^{ccp} &= \tfrac{1}{2}sX_m^N((1-\alpha-\beta)(1-p) - (1-\alpha)(1-p) + 2\tfrac{E^{skin}}{sX} + 2\beta) \\ &\quad -\tfrac{1}{2}(1+p)sX_m^N(\tfrac{1}{2}(1-\alpha-\beta)(1-p) - \tfrac{E^{skin}}{sX}) - sX_m^N(\alpha+\beta)\rho \\ &= \tfrac{1}{2}sX_m^N(-\beta(1-p) - \tfrac{1}{2}(1-\alpha-\beta)(1-p^2) + \tfrac{1}{2}(1+p)\tfrac{E^{skin}}{sX}) \\ &\quad +sX_m^N(\tfrac{E^{skin}}{sX} + \beta) - sX_m^N(\alpha+\beta)\rho = 0.\end{aligned}$$

Value of a contract is independent of p only if it is fully-collateralized with individual resources ($\alpha + \beta = 1$), but this is not consistent with assumptions of the case ($DF^{loss} > 0, p^{ccp} < 1$).

(d) $DF^{loss} = s\beta X, p^{ccp} = 1 - ((1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta)$:
$V_m^{ccp} = \frac{1}{2}sX_m^N(1-\alpha-\beta)(1-p) - \frac{1}{2}(1+p)X_m^N s\beta - \frac{1}{2}sX_m^N((1-\alpha)(1-p) - 2\frac{E^{skin}}{sX} - 2\beta) - sX_m^N(\alpha+\beta)\rho = \frac{E^{skin}X_m^N}{X} - sX_m^N(\alpha+\beta)\rho = 0.$
Value of the contract does not depend on credit quality of members, thus there is no need to post collateral to deal with the counterparty risk. Value of the contract zero, when no collateral is posted ($\alpha = 0, \beta = 0, E^{skin} = 0$).

Only in case d) zero-value does not require full collateralization of a contract. Sufficient condition for a symmetric contract to have zero value when all members are of the same credit quality is that no collateral is posted.

D Proof of Proposition 3

Proof. First, we consider a simple case when m has contractual obligations only with one counterparty i.
From Equation 1 and Equation 4, difference in the value of the contract is determined as

$$\begin{aligned} \Delta V^b_{mi} &= V^b_{mi} - V^g_{mi} \\ &= \tfrac{1}{2} s \max[x^G_{mi} - x^G_{im}, x^G_{im} - x^G_{mi}](p_i - p_m) - \tfrac{1}{2} s (x^G_{mi} + x^G_{im})(p_i - p_m) \\ &= s \min[x^G_{mi}, x^G_{im}](p_m - p_i). \end{aligned} \tag{24}$$

Second, if m deals with multiple counterparties of different credit quality, the total change in the fair value of contracts is determined as a sum over all counterparties:

$$\begin{aligned} \Delta V^b_m &= \sum_{i \neq m} V^b_{mi} - \sum_{i \neq m} V^g_{mi} \\ &= \sum_{i \neq m} \tfrac{1}{2} s \max[x^G_{mi} - x^G_{im}, x^G_{im} - x^G_{mi}](p_i - p_m) - \sum_{i \neq m} \tfrac{1}{2} s (x^G_{mi} + x^G_{im})(p_i - p_m) \\ &= s \sum_{i \neq m} \min[x^G_{mi}, x^G_{im}](p_m - p_i) = \tfrac{1}{2} s \Delta X^b_m (p_m - \bar{p}^b_m), \text{ where} \\ \Delta X^b_m &= 2 \sum_{i \neq m} \min[x^G_{mi}, x^G_{im}], \\ \bar{p}^b_m &= \sum_{i \neq m} \tfrac{2 \min[x^G_{mi}, x^G_{im}]}{\Delta X^b_m} p_i. \end{aligned} \tag{25}$$

Total netting effect is positive if probability of i to pay its obligations is larger than the weighted average probability of repayment of its counterparties. If all counterparties are of the same quality, netting does not produce effects.
Introduction of initial margin does not change the threshold probability above which netting is beneficial but proportionally reduces size of the netting effect:
$\Delta V^b_m = s(1 - \alpha) \Delta X^b_m (p_m - \bar{p}^b_m)$. □

E Proof of Proposition 4

We consider a new deal that is arranged between members m and i within a CCP. Notional value of the deal is ΔX. The deal increases net position of one of the involved members and decreases net position of the other member, so that total notional of the CCP does not change ($\Delta X = \Delta X_m = -\Delta X_i > 0$).
Change in the value of the position of member m is determined by Equation 9

$$\begin{aligned}
\Delta V_m &= \tfrac{1}{2} s \Delta X_m (p_m - p^{ccp'}) + \tfrac{1}{2} s X_m^N \Delta p^{ccp}, \text{ where} \\
p^{ccp'} &- \text{ new credit quality of a CCP} \\
\Delta p^{ccp} &= p^{ccp'} - p^{ccp}.
\end{aligned}$$

Credit quality of a CCP after loss-mutualisation is determined by Equation 23:

$$p^{ccp} = 1 - \max[(1-\alpha)(1-\bar{p}^m) - 2\frac{E^{skin}}{sX} - 2\beta, 0].$$

Since credit quality of a CCP is a non-linear function, two cases are possible:

- **Case 1.** A CCP is not well-capitalized $p^{ccp} < 1$ and $p^{ccp'} < 1$.

  In this case credit quality of a CCP is determined as

  $$p^{ccp} = 1 - (1-\alpha)(1-\bar{p}^m) + 2\frac{E^{skin}}{sX} + 2\beta.$$

  Even a small deal changes credit quality of a CCP:

  $$\Delta p^{ccp} = (1-\alpha)(\bar{p}^{m'} - \bar{p}^m) = (1-\alpha)\frac{\Delta X}{X}(p_i - p_m).$$

  The value of positions of the involved in the deal members changes in the following way:

  $$\begin{aligned}
  \Delta V_m &= \frac{1}{2} s \Delta X (p_m - p^{ccp}) + \frac{1}{2} s (X_m^N - \Delta X) \Delta p^{ccp}, \\
  \Delta V_i &= -\frac{1}{2} s \Delta X (p_i - p^{ccp}) + \frac{1}{2} s (X_i^N + \Delta X) \Delta p^{ccp}.
  \end{aligned}$$

  The total effect to both members is positive only if net position is transferred from safer to a riskier member:

  $$\Delta V_m + \Delta V_i = \frac{1}{2} s \Delta X (p_m - p_i)(1 - (1-\alpha)\frac{X_m^N + X_i^N}{X}) > 0 \text{ , iff } p_m > p_i.$$

  Since counterparty effect is negative, necessary condition for each member to benefit from the deal is that net position is transferred from a member, which is safer than a CCP, to member, which is riskier than the CCP, i.e. $p_m > p^{ccp} > p_i$.

To ensure that the deal should be beneficial to each counterparty, the following conditions should hold:

$$\begin{cases} \Delta V_m > 0 \text{ , if } \max[0, X_m^N - \frac{p_m - p^{ccp}}{(1-\alpha)(p_m - p_i)} X] < \Delta X < X_m^N \\ \Delta V_i > 0 \text{ , if } 0 < \Delta X < \frac{p^{ccp} - p_i}{(1-\alpha)(p_m - p_i)} X - X_i^N. \end{cases}$$

A deal is Pareto-improving for the involved members if its notional value lies in the following range:

$$\max[0, X_m^N - \frac{p_m - p^{ccp}}{(1-\alpha)(p_m - p_i)} X] < \Delta X < \min[\frac{p^{ccp} - p_i}{(1-\alpha)(p_m - p_i)} X - X_i^N, X_m].$$

However, such deal is associated with negative counterparty effect and decreasing quality of a CCP. Therefore, all other CCP members that are not involved in the deal experience negative externalities:

$$\Delta V_j = \frac{1}{2} s X_j^N \Delta p^{ccp} = \frac{1}{2} s X_j^N (1-\alpha) \frac{\Delta X}{X} (p_i - p_m) < 0.$$

- **Case 2.** A CCP is well-capitalized before and after the deal $p^{ccp'} = p^{ccp} = 1$, $\Delta p^{ccp} = 0$.

  Since there is only netting effect and each individual member is of worse credit quality than a well-capitalized CCP, a deal is beneficial to a member only if it increases its net position:

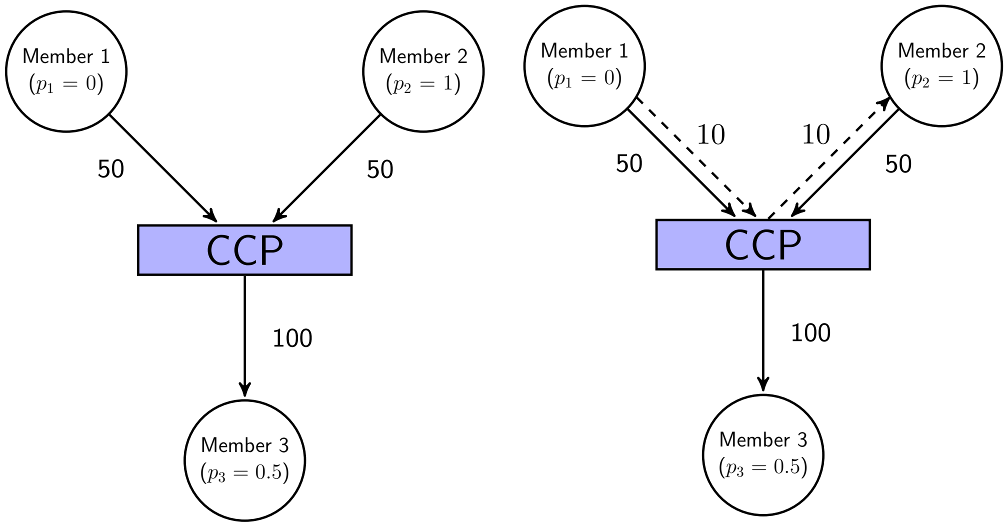
$$\Delta V_m = \frac{1}{2} s \Delta X_m (p_m - 1) < 0.$$

  There is no counterparty effect, thus no externalities:

$$\Delta V_j = \frac{1}{2} s X_j^N \Delta p^{ccp} = 0.$$

Example of redistribution of net positions in a CCP.
For simplicity we consider a case without any collateral, i.e. $\alpha = \beta = \frac{E}{sX} = 0$. The example shows how a deal that transferrs a net position from a safe Member 2 to a risky Member 1 is a Pareto-improvement for the involved counterparties but creates negative externalities to Member 3 through a decreased credit quality of the CCP.

Figure 5: Example of a Pareto-improving deal between member 1 and member 2



Initial CCP

CCP after a new deal is arranged

$$p^{ccp} = \bar{p}_m = \frac{50*0+50*1+100*0.5}{50+50+100} = \frac{1}{2}$$

$$V_1 = \tfrac{1}{2}50p^{ccp} - \tfrac{1}{2}50p_1 = \tfrac{25}{2}$$

$$V_2 = \tfrac{1}{2}50p^{ccp} - \tfrac{1}{2}50p_2 = -\tfrac{25}{2}$$

$$V_3 = \tfrac{1}{2}100p^{ccp} - \tfrac{1}{2}100p_3 = 0$$

$$V_1 + V_2 + V_3 = 0$$

$$p^{ccp'} = \bar{p}'_m = \frac{60*0+40*1+100*0.5}{60+40+100} = \frac{9}{20}$$

$$V'_1 = \tfrac{1}{2}60p^{ccp'} - \tfrac{1}{2}60p_1 = \tfrac{27}{2}$$

$$V'_2 = \tfrac{1}{2}40p^{ccp'} - \tfrac{1}{2}40p_2 = -11$$

$$V'_3 = \tfrac{1}{2}100p^{ccp'} - \tfrac{1}{2}100p_3 = -\tfrac{5}{2}$$

$$V'_1 + V'_2 + V'_3 = 0$$

$$\Delta V_1 = V'_1 - V_1 = 1 > 0$$

$$\Delta V_2 = V'_2 - V_2 = \tfrac{3}{2} > 0$$

$$\Delta V_3 = V'_3 - V_3 = -\tfrac{5}{2} < 0$$

Note: An arrow represents direction of payment of a float rate, while the weight indicates notional value of a deal.

F Funding requirements under different concentration measures

We define member's funding cost of clearing as an amount of required prefunded resources per unit of expexted exposure created by the member. The total amount of prefunded resources that a CCP is required to hold is denoted K_{ccp}, whereas the resources that member m must hold is K_m and the funding requirement per unit of created exposure is k_m.

First, we derive funding costs that arise under the current regulatory "cover 2" standard (Case 1) and under an alternative concentration measure (Herfindahl index) but keeping individual requirements proportional to members' shares in a CCP (Case 2). Then we show that negative externalities can be avoided by setting individual requirements proportionally to members' contributions to the concentration measure (Case 3). Superscripts C, H and HP indicate Case 1, Case 2 and Case 3 correspondingly.

- **Case 1:** based on the regulatory "cover 2" standard, funding requirements are divided between members proportionally to their share in the CCP.

  Therefore, the total requirement on the CCP's prefunded resources is determined by the expected exposure towards its two largest members $K^C_{ccp} = s(X^N_1 + X^N_2)$, where X^N_1, X^N_2 are positions of two largest members of the CCP. The resources that can be used to cover default of the two largest members are their individual contributions and default fund contributions of all other members. The amount of prefunded resources is just sufficient to satisfy the requirement when

  $s(X^N_1 + X^N_2) = s((\alpha + \beta^C)(X^N_1 + X^N_2) + \beta^C(X - X^N_1 - X^N_2))$.

  The proportionality requirement implies that β^C is the same for all members. Given the fixed level of initial margin α, the required share of the member's expected exposure that should be covered by default fund contribution is $\beta^C = (1 - \alpha)k_2$, where $k_2 = \frac{X^N_1 + X^N_2}{X}$ - the share of two largest members in the total notional cleared by the CCP. Required contributions to the default fund are proportional to the concentration of a CCP.

  The total amount of collateral required from member m is $K^C_m = sX^N_m(\alpha + \beta^C) = sX^N_m(\alpha + (1 - \alpha)k_2)$.

  The cost of clearing in terms of resources required per unit of created exposure is the same for all members and is determined by the share of two largest members:

  $k^C_m = \frac{K^C_m}{sX^N_m} = \alpha + (1 - \alpha)k_2$.

- **Case 2:** total funding requirement is calculated taking Herfindahl index as a measure of concentration and divided between members proportionally to their share in the CCP.

The Herfindahl index is determined as $H_{ccp} = \sum\limits_{m \in M} (X_m^N/X)^2 = \sum\limits_{m \in M} S_m^2$. Since the Herfindahl index accounts for potential default of each member, we determine the total funding requirement on the base of the exposure that is not covered by initial margins and should be mutualised through the common default fund:

$K_{ccp}^H = sX(1-\alpha)H_{ccp} = s\beta^H X$.

Thus, the minimum required share of the expected exposure that should be covered by default fund contribution is $\beta^H = (1-\alpha)H_{ccp}$.

This leads to the following individual funding requirements and funding costs correspondingly: $K_m^H = sX_m^N(\alpha + (1-\alpha)H_{ccp})$ and $k_m^H = \frac{K_m^H}{sX_m^N} = \alpha + (1-\alpha)H_{ccp}$.

Lemma 3. *Funding costs are not smaller under the "cover 2" standard than under the Herfindahl index standard ($k_2 \geq H_{ccp}$).*

Proof. Funding costs under the "cover 2" standard and under the Herfindahl index standard are correspondingly $k_m^C = \alpha + (1-\alpha)k_2$ and $k_m^H = \alpha + (1-\alpha)H_{ccp}$. Therefore, in order to show the statement it is sufficient to compare a share of the two largest members with the Herfindahl index.

$$\begin{aligned}
k_2 &= \frac{X_1^N + X_2^N}{X} \\
k_2 X^2 &= \frac{X_1^N + X_2^N}{X} X^2 = (X_1^N + X_2^N)(X_1^N + X_2^N + ... + X_M^N) \\
&= (X_1^N)^2 + 2X_1^N X_2^N + (X_2^N)^2 + (X_1^N + X_2^N)(X_3^N + ... + X_M^N) \\
H_{ccp} &= \sum_{m \in M} (X_m^N/X)^2 \\
H_{ccp} X^2 &= (X_1^N)^2 + (X_2^N)^2 + (X_3^N)^2 + ... + (X_M^N)^2 \\
&\leq (X_1^N)^2 + (X_2^N)^2 + (X_1^N + X_2^N)(X_3^N + ... + X_M^N) \\
&\leq (X_1^N)^2 + 2X_1^N X_2^N + (X_2^N)^2 + (X_1^N + X_2^N)(X_3^N + ... + X_M^N) \\
&\leq k_2 X^2
\end{aligned}$$

The first inequality follows from $(X_1^N + X_2^N) > X_i^N, \ i > 2$
Thus, $k_2 \geq H_{ccp}$ and $k_m^C \geq k_m^H$. □

- **Case 3:** both total and individual funding requirements are based on the Herfindahl index.

  The total CCP requirement is the same as in Case 2, however the difference is that the share of exposure covered by the default fund contribution is specific for each member: $K_{ccp}^{HP} = sX(1-\alpha)H_{ccp} = \sum\limits_{m \in M} s\beta_m^{HP} X_m^N$.

  In order to have individual contributions to the total required funds proportional to the contribution to the concentration measure, the following should hold: $\frac{s\beta_m^{HP} X_m^N}{sX(1-\alpha)H_{ccp}} = \frac{(X_m^N/X)^2}{H_{ccp}}$. This gives $\beta_m^{HP} = (1-\alpha)\frac{X_m^N}{X}$.

Therefore, individual requirements are determined as

$K_m^{HP} = (\alpha + \beta^{HP}) s X_m^N = (\alpha + (1-\alpha)\frac{X_m^N}{X}) s X_m^N$

and funding costs as

$k_m^{HP} = \frac{K_m^{HP}}{s X_m^N} = \alpha + (1-\alpha)\frac{X_m^N}{X} = \alpha + (1-\alpha) S_m$, where S_m - share of member m in the total notional of a CCP.

G Proof of Proposition 6

Proof. We derive an effect of changing position of one member on funding costs of other CCP members under the three approaches described in the previous section.

- **Case 1:** In case of the "cover 2" standard, the threshold after which further growth leads to an increase in funding costs is the size of the second largest position.

  When a "large" member further increases its position, the total change in funding requirements is

$$\frac{\partial K^C_{ccp}}{\partial X^N_i} = \frac{\partial [s(X^N_1 + X^N_2)]}{\partial X^N_i} = s \ , \ X^N_i \in \{X_1, X_2\}$$

  However, the "large" member bears only part of increased funding requirements and transfers the rest to other CCP members, creating negative externalities:

$$\frac{\partial K^C_i}{\partial X^N_i} = \frac{\partial [sX^N_i(\alpha + (1-\alpha)\frac{(X^N_1+X^N_2)}{X})]}{\partial X^N_i} = s(\alpha{+}(1{-}\alpha)(S_i(1{-}k_2){+}k_2)) < s \ , \ X^N_i \in \{X^N_1, X^N_2\}$$

$$\frac{\partial K^C_m}{\partial X^N_i} = \frac{\partial [sX^N_m(\alpha + (1-\alpha)\frac{(X^N_1+X^N_2)}{X})]}{\partial X^N_i} = s(1{-}\alpha)S_m(1{-}k_2) > 0 \ , \ X^N_i \in \{X^N_1, X^N_2\}$$

  Therefore, increase in the position of a "large" member leads to higher funding costs of clearing for all members:

$$\frac{\partial k^C_m}{\partial X^N_i} = \frac{\partial [\alpha + (1-\alpha)\frac{(X^N_1+X^N_2)}{X}]}{\partial X^N_i} = (1{-}\alpha)(-\frac{k_2}{X}{+}\frac{1}{X}) = (1{-}\alpha)\frac{1-k_2}{X} > 0 \ , \ X^N_i \in \{X^N_1, X^N_2\}$$

  The growth of a "small" member does not change the total funding requirement and decreases funding costs for all members:

$$\frac{\partial k^C_m}{\partial X^N_i} = \frac{\partial [\alpha + (1-\alpha)\frac{(X^N_1+X^N_2)}{X}]}{\partial X^N_i} = -(1-\alpha)\frac{k_2}{X} < 0 \ , \ X^N_i \notin \{X^N_1, X^N_2\}$$

  Change in the size of position of any member has an effect on funding costs of all other members. This effect is the same for all members. Closer to the threshold position, positive externalities of a growing "small" member are smaller, while negative externalities of further growth of a "large" member are higher.

- **Case 2:** When concentration is measured by the Herfindahl index, there is no difference in the functional form of the effect of "large" and "small" members. The total change in funding requirements is

$$\frac{\partial K^H_{ccp}}{\partial X^N_i} = s(1{-}\alpha)(H_{ccp}{+}X\frac{\partial H_{ccp}}{\partial X^N_i}) = s(1{-}\alpha)(H_{ccp}{+}2(S_i{-}H_{ccp})) = s(1{-}\alpha)(2S_i{-}H_{ccp})$$

The first component reflects change in funding requirements due to increased notional, while the second component arises due to the change in concentration measure and is shared among all members:

$$\frac{\partial K_i^H}{\partial X_i^N} = \frac{\partial[sX_i^N(\alpha + (1-\alpha)H_{ccp})]}{\partial X_i^N} = s(\alpha+(1-\alpha)(H_{ccp}+2S_i(S_i-H_{ccp}))) \lesseqgtr (s\alpha+\frac{\partial K_{ccp}^H}{\partial X_i^N})$$

$$\frac{\partial K_m^H}{\partial X_i^N} = \frac{\partial[sX_m^N(\alpha + (1-\alpha)H_{ccp})]}{\partial X_i^N} = s(1-\alpha)2S_m(S_i - H_{ccp}) \lesseqgtr 0$$

The effect on funding costs created by each member is different and is determined by positions of all other CCP members:

$$\frac{\partial k_m^H}{\partial X_i^N} = \frac{\partial(\alpha + (1-\alpha)H_{ccp})}{\partial X_i^N} = 2(1-\alpha)\frac{S_i - H_{ccp}}{X} \lesseqgtr 0$$

A member increases concentration, i.e. it is classified as a "large" member, when its share in the CCP is larger than the Herfindahl index of the CCP ($S_i > H_{ccp}$). Growth of a "large" member creates additional funding costs for all members. If a "small' member increases its position, it decreases funding costs but shares benefits with other members. Each member has its own threshold position, above which it starts to produce negative externalities.

The threshold share can be higher or lower than in case of the "cover 2" standard depending on the concentration of positions of all members. Applying the Herfindahl index, more members have an effect of "large" members if a share of the second largest member in a CCP is larger than the Herfindahl index ($S_2 > H_{ccp}$).

In contrast to the case of "cover 2" standard, with application of the Herfindahl index the negative effect of large member's growth increases when a CCP moves towards more concentrated structure.

□

Additionally, we describe a way to avoid negative externalities by dividing total funding requirement according to the contribution to the concentration measure.

- **Case 3:** An effect on total funding requirement is the same as in Case 2:

$$\frac{\partial K_{ccp}^{HP}}{\partial X_i^N} = s(1-\alpha)(H_{ccp}+X\frac{\partial H_{ccp}}{\partial X_i^N}) = s(1-\alpha)(H_{ccp}+2(S_i-H_{ccp})) = s(1-\alpha)(2S_i-H_{ccp}).$$

However, when the total requirement is divided proportionally to a member's contribution to the Herfindahl index, an increase in the individual requirement of a growing member is larger than an increase in the total requirement, that leads to lower individual fuding requirements for other members:

$$\frac{\partial K_i^{HP}}{\partial X_i^N} = \frac{\partial[sX_i^N(\alpha + (1-\alpha)\frac{X_i^N}{X})]}{\partial X_i^N} = s(\alpha + (1-\alpha)(2S_i - S_i^2)) > (s\alpha + \frac{\partial K_{ccp}^{HP}}{\partial X_i^N})$$

$$\frac{\partial K_m^{HP}}{\partial X_i^N} = \frac{\partial[sX_m^N(\alpha + (1-\alpha)\frac{X_m^N}{X})]}{\partial X_i^N} = -s(1-\alpha)S_m^2 < 0.$$

Therefore, a growth of a member increases its own funding costs but decreases funding costs of other members, thereby creating positive externalities:

$$\frac{\partial k_i^{HP}}{\partial X_i^N} = \frac{\partial(\alpha + (1-\alpha)\frac{X_i^N}{X})}{\partial X_i^N} = (1-\alpha)\frac{1-S_i}{X} > 0$$

$$\frac{\partial k_m^{HP}}{\partial X_i^N} = \frac{\partial(\alpha + (1-\alpha)\frac{X_m^N}{X})}{\partial X_i^N} = -(1-\alpha)\frac{S_m}{X} < 0.$$

H Proof of Proposition 7

Proof. Member's profit is expressed as

$$\pi_m = fX_m^N - \rho K_m = (f - \rho s k_m)X_m^N$$

Changing the amount it clears, each member influences not only its own profit but profits of all CCP members.

$$\frac{\partial \pi_i}{\partial X_i^N} = f - \rho s k_i - \rho s \frac{\partial k_i}{\partial X_i^N} X_i^N$$
$$\frac{\partial \pi_m}{\partial X_i^N} = -\rho s \frac{\partial k_m}{\partial X_i^N} X_m^N$$

If position is increased by a "large" member ($\frac{\partial k_i}{\partial X_i^N} > 0$), all other CCP members experience losses in profits due to increased funding costs. Members with fixed positions get the highest profits, when funding costs are the lowest. However, when clearing is sufficiently profitable, each member has incentives to grow more than it is optimal in terms of funding costs, creating negative externalities to other members.
We consider a situation when there are members with fixed positions. A new member enters a CCP and choses a size of position that is optimal to clear given initial structure of a CCP. We derive a threshold value of benefits received from clearing one unit of notional, i.e. fees f, above which the member has incentives to get a larger position than it is optimal for other members. Is threshold member-specific?

- **Case 1:** Under the "cover 2" standard, the lowest funding costs are achieved when a "small' member increases its position till the second largest one. The member has incentives for further growth, if derivative of its profits function is positive at that point:

$$\begin{aligned}
\left.\frac{\partial \pi_i^C}{\partial X_i^N}\right|_{X_i^N = X_2^N} &= f - \rho s (k_i^C)' - \rho s (1-\alpha) \frac{X + X_2^N - (X_1^N + X_2^N)}{(X + X_2^N)^2} X_2^N \\
&= f - \rho s \left((k_i^C)' + (1-\alpha) \frac{X - X_1^N}{X + X_2^N} \frac{X_2^N}{X + X_2^N}\right) \\
&= f - \rho s (\alpha + (1-\alpha)(s_1' + s_2') + (1-\alpha)(1 - s_1')s_2') > 0 \text{ , where} \\
X &- \text{ total notional in the initial CCP} \\
X_2^N &- \text{ position of the second largest member in the initial CCP} \\
s_1' \text{ , } s_2' &- \text{ shares of the first and second largest positions when } X_i^N = X_2^N \\
(k_i^C)' &= \alpha + (1-\alpha)(s_1' + s_2') \text{ , funding costs when } X_i^N = X_2^N.
\end{aligned}$$

  Under the "cover 2" standard, a profit-maximizing "large" member would have incentives to increase its position above the point of minimum funding costs and, thereby, create negative externalities to other members if

$$f > \rho s ((k_i^C)' + (1-\alpha)(1 - s_1')s_2') = f^C.$$

- **Case 2:** If funding requirements are calculated on the base of the Herfindahl index, funding costs are the lowest when $\frac{\partial k_i^H}{\partial X_i^N} = 0$. A "large" member has incentives to clear more if clearing remains profitable at the point of the minimum funding costs ($f > \rho s(k_i^H)'$). In order to calculate $(k_i^H)'$, we derive member's i position that minimizes funding costs for all members.

$$\begin{aligned}
(k_i^H)' &= \alpha + (1-\alpha)H'_{ccp} \\
H'_{ccp} &= \frac{\sum\limits_{m\in M}(X_m^N)^2 + (X_i^N)^2}{(X+X_i^N)^2} \\
\frac{\partial H'_{ccp}}{\partial X_i^N} &= \frac{2(X_i^N X - \sum\limits_{m\in M}(X_m^N)^2)}{(X+X_i^N)^3} = 0 \\
(X_i^N)' &= \frac{\sum\limits_{m\in M}(X_m^N)^2}{X} = H_{ccp}X \\
H'_{ccp} &= \frac{\sum\limits_{m\in M}(X_m^N)^2 + (H_{ccp}X)^2}{(X+H_{ccp}X)^2} = \frac{H_{ccp}}{1+H_{ccp}}
\end{aligned}$$

Thus, a "large" member benefits from further growth and creates negative externalities, if

$$\begin{aligned}
f &> \rho s(\alpha + (1-\alpha)\tfrac{H_{ccp}}{1+H_{ccp}}) = \rho s(k_i^H)' = f^H \text{ , where} \\
H_{ccp} &- \text{Herfindahl index of the initial CCP.}
\end{aligned}$$

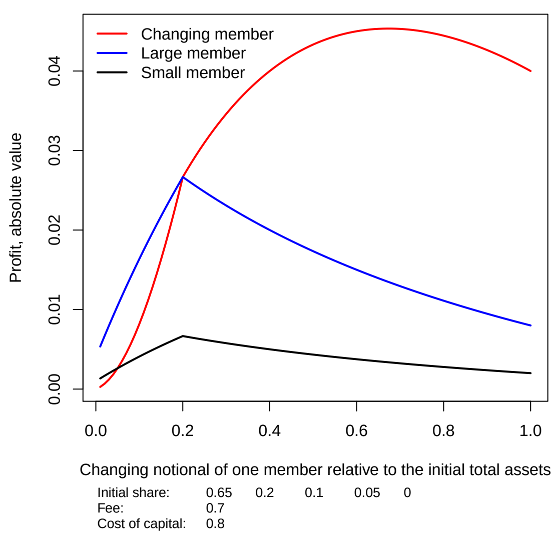
The threshold value of fees is lower applying the Herfindahl index approach than the "cover 2" standard. This follows from the fact that by Lemma 1 given the same CCP structure, i.e. $X_i^N = X_2$, funding costs under the "cover 2" standard are higher than applying the Herfindahl index and, when X_i^N is chosen to minimize the Herfindahl index, the funding costs are even lower:

$$f^C > \rho s(k_i^C)' \geq \rho s(k_i^H | (X_i^N = X_2^N)) \geq \rho s(k_i^H)' = f^H.$$

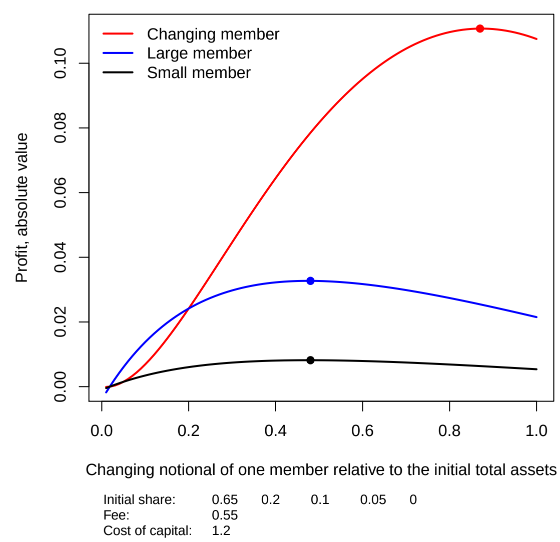
□

Figure 6: Dynamics of profits, individual funding requirements are proportional to members' shares in a CCP

“Cover 2” standard (Case 1)



Herfindahl index approach (Case 2)



I Proof of Lemma 2

$$\begin{aligned} \Delta V_m = V_m^{ccp} - V_m^b \quad = \quad & -\tfrac{1}{2}s(X_m^N(1-p^{ccp}) - X_m^G(1-\alpha)(1-\bar{p}_m^b)) \\ & +\tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha)) - \tfrac{1}{2}DF_m^{loss} \\ & -s\rho(X_m^N(\alpha+\beta) - \alpha X_m^G) \\ & -p_m(\tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha)) + \tfrac{1}{2}DF_m^{loss}). \end{aligned} \tag{26}$$

Change in the expected value increases in p_m, since

$$\begin{aligned} & \tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha) + \tfrac{1}{2}DF_m^{loss} \\ & = \tfrac{1}{2}s(X_m^G(1-\gamma)(1-\alpha) - X_m^G(1-\gamma)\beta - X_m^G(1-\alpha)) + \tfrac{1}{2}DF_m^{loss} \\ & = -\tfrac{1}{2}sX_m^G(1-\alpha)\gamma - \tfrac{1}{2}s\beta X_m^N + \tfrac{1}{2}DF_m^{loss} \leq 0 \text{ , since} \\ & DF_m^{loss} \leq s\beta X_m^N. \end{aligned} \tag{27}$$

This leads directly to the result that $\Delta V_m \geq 0$, when

$$p_m \geq 1 - \frac{\tfrac{1}{2}s(X_m^G(1-\alpha)(1-\bar{p}_m^b) - X_m^N(1-p^{ccp})) + s\rho(\alpha X_m^G - X_m^N(\alpha+\beta)) - DF_m^{loss}}{\tfrac{1}{2}s(X_m^G(1-\alpha) - X_m^N(1-\alpha-\beta)) - \tfrac{1}{2}DF_m^{loss}}. \tag{28}$$

Agents who could benefit from the transition exist, when potential benefits from reduction in direct counterparty losses and in cost of collateral exceed incurred indirect losses of mutualisation (i.e. $\tfrac{1}{2}s(X_m^G(1-\alpha)(1-\bar{p}_m^b) - X_m^N(1-p^{ccp})) + s\rho(\alpha X_m^G - X_m^N(\alpha+\beta)) \geq DF_m^{loss}$).

J Proof of Proposition 8

Proof. As stated in the Equation 18, change in the expected value of the position of member m due to transition to a CCP is determined as

$$\begin{aligned}\Delta V_m = V_m^{ccp} - V_m^b \;=\; & -\tfrac{1}{2}s(X_m^N(1-p^{ccp}) - X_m^G(1-\alpha)(1-\bar{p}_m^b)) \\ & +\tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha))(1-p_m) \\ & -\tfrac{1}{2}(1+p_m)DF_m^{loss} \\ & -s\rho(X_m^N(\alpha+\beta) - \alpha X_m^G).\end{aligned}$$

We focus on the first three elements which constitute a change in the total counterparty risk component:

$$\begin{aligned}\Delta V_m^{cpty} \;=\; & -\tfrac{1}{2}s(X_m^N(1-p^{ccp}) - X_m^G(1-\alpha)(1-\bar{p}_m^b)) \\ & +\tfrac{1}{2}s(X_m^N(1-\alpha-\beta) - X_m^G(1-\alpha))(1-p_m) \\ & -\tfrac{1}{2}(1+p_m)DF_m^{loss}.\end{aligned}$$

In order to find a change in total welfare, we sum up changes in expected values over all CCP members:

$$\begin{aligned}\sum_m \Delta V^{cpty} \;=\; & -\tfrac{1}{2}s(1-p^{ccp})\sum_m X_m^N + \tfrac{1}{2}s(1-\alpha)\sum_m X_m^G(1-\bar{p}_m^b) \\ & +\tfrac{1}{2}s((1-\alpha-\beta)\sum_m X_m^N - \tfrac{1}{2}s(1-\alpha)\sum_m X_m^G)(1-p_m) \\ & -\tfrac{1}{2}\sum_m(1+p_m)DF_m^{loss} \\ =\; & \tfrac{1}{2}s(p^{ccp}-\alpha-\beta)\sum_m X_m^N \\ & -\tfrac{1}{2}s(1-\alpha-\beta)\sum_m p_m X_m^N \\ & -\tfrac{1}{2}\sum_m(1+p_m)DF_m^{loss}.\end{aligned} \tag{29}$$

Second equation follows from the fact that

$$\begin{aligned}\sum_m X_m^G(1-\bar{p}_m^b) \;=\; & \sum_m X_m^G\left(1-\frac{\sum_i p_i X_{im}^G}{X_m^G}\right) = \sum_m X_m^G - \sum_i p_i \sum_m X_{im}^G = \sum_m X_m^G - \sum_i p_i X_i^G \\ =\; & \sum_m X_m^G(1-p_m).\end{aligned}$$

From Equation 10,

$$\begin{aligned}p^{ccp}-\alpha-\beta \;=\; & 1-\alpha-\beta-\max[(1-\alpha)(1-\bar{p}^m) - 2\tfrac{E^{skin}}{sX} - 2\beta, 0] \\ =\; & \min[(1-\alpha)\bar{p}^m + 2\tfrac{E^{skin}}{sX} + \beta, 1-\alpha-\beta].\end{aligned} \tag{30}$$

From Equation 11,

$$
\begin{aligned}
&-\tfrac{1}{2}s(1-\alpha-\beta)\sum_m p_m X_m^N - \tfrac{1}{2}\sum_m (1+p_m) DF_m^{loss} = \\
&-\tfrac{1}{2}sX((1-\alpha-\beta)\bar{p}^m + (1+\bar{p}^m)\min[\max[\tfrac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m) - \tfrac{E^{skin}}{sX}, 0], \beta]) = \\
&-\tfrac{1}{2}sX\min[\max[\tfrac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) - \tfrac{E^{skin}}{sX}(1+\bar{p}^m), (1-\alpha-\beta)\bar{p}^m], \beta + (1-\alpha)\bar{p}^m].
\end{aligned} \tag{31}
$$

By substituting Equation 30 and Equation 31 into Equation 29, we obtain that change in the total counterparty risk component is determined as

$$
\begin{aligned}
\sum_m \Delta V^{cpty} \;=\; & \tfrac{1}{2}sX(\min[(1-\alpha)\bar{p}^m + 2\tfrac{E^{skin}}{sX} + \beta, 1-\alpha-\beta] \\
& - \min[\max[\tfrac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) - \tfrac{E^{skin}}{sX}(1+\bar{p}^m), (1-\alpha-\beta)\bar{p}^m], \beta + (1-\alpha)\bar{p}^m].
\end{aligned}
$$

Next we show that change in the total counterparty risk component is non-negative:

1. $(1-\alpha)\bar{p}^m + 2\frac{E^{skin}}{sX} + \beta \geq \beta + (1-\alpha)\bar{p}^m$, since $E^{skin} \geq 0$
2. $1-\alpha-\beta \geq (1-\alpha-\beta)\bar{p}^m$, since $\bar{p}^m \leq 1$
3. $1-\alpha-\beta \geq \frac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) - \frac{E^{skin}}{sX}(1+\bar{p}^m)$, since
   $(1-\alpha-\beta) - \frac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) + \frac{E^{skin}}{sX}(1+\bar{p}^m) =$
   $= \frac{1}{2}(1-\alpha-\beta)(1-\bar{p}^m)^2 + \frac{E^{skin}}{sX}(1+\bar{p}^m) \geq 0$
4. $(1-\alpha-\beta) \geq \max[\frac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) - \frac{E^{skin}}{sX}(1+\bar{p}^m), (1-\alpha-\beta)\bar{p}^m]$, follows from (2) and (3)
5. $\min[(1-\alpha)\bar{p}^m + 2\frac{E^{skin}}{sX} + \beta, 1-\alpha-\beta] \geq$
   $\geq \min[\max[\frac{1}{2}(1-\alpha-\beta)(1+2\bar{p}^m - \bar{p}_m^2) - \frac{E^{skin}}{sX}(1+\bar{p}^m), (1-\alpha-\beta)\bar{p}^m], \beta + (1-\alpha)\bar{p}^m]$,
   follows from (1) and (4).

Thus, $\sum_m \Delta V^{cpty} \geq 0$.

□

J.1 Special cases of Proposition 8

1. No mutualisation of funds ($\beta = 0$, $E^{skin} = 0$, $\alpha \geq 0$)

$$
\begin{aligned}
&\sum_m \Delta V^{cpty} = \\
&= \tfrac{1}{2}sX(\min[(1-\alpha)\bar{p}^m, 1-\alpha] - \min[\max[\tfrac{1}{2}(1-\alpha)(1+2\bar{p}^m - \bar{p}_m^2), (1-\alpha)\bar{p}^m], (1-\alpha)\bar{p}^m] \\
&= \tfrac{1}{2}sX((1-\alpha)\bar{p}^m - (1-\alpha)\bar{p}^m) = 0
\end{aligned}
$$

   When no additional funds are posted, transition to a CCP just redistributes value associated with counterparty risk component among market participants.

2. CCP contributes its "skin in the game" capital but no mutualisation of default fund ($\beta = 0$, $E^{skin} \geq 0$, $\alpha = 0$)

$$\begin{aligned}\sum_m \Delta V^{cpty} &= \tfrac{1}{2} sX(\min[\bar{p}^m + 2\tfrac{E^{skin}}{sX}, 1] - \min[\max[\tfrac{1}{2}(1 + 2\bar{p}^m - \bar{p}_m^2) - \tfrac{E^{skin}}{sX}(1 + \bar{p}^m), \bar{p}^m], \bar{p}^m] \\ &= \tfrac{1}{2} sX \min[2\tfrac{E^{skin}}{sX}, 1 - \bar{p}^m] = \min[E, \tfrac{1}{2} sX(1 - \bar{p}^m)] \geq 0.\end{aligned}$$

When additional CCP's capital is contributed, the total counterparty risk component increases by the amount equal to the contributed capital or by total expected CCP losses, if they are lower than the contributed CCP capital.

3. Default fund contributions are mutualised, but no CCP capital is provided ($\beta \geq 0$, $E^{skin} = 0$, $\alpha = 0$)

$$\sum_m \Delta V^{cpty} = \tfrac{1}{2} sX(\min[\bar{p}^m + \beta, 1 - \beta] - \min[(1 - \beta)\bar{p}^m, \bar{p}^m + \beta]) \geq 0.$$

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