Greed versus Fear: Optimal Time-Consistent Taxation with Default*

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Abstract

This paper studies the optimal time-consistent distortionary taxation when the repayment of government debt is not enforceable. The government taxes labor income or issues non-contingent debt in order to finance an exogenous stochastic stream of fiscal shocks. Debt can be repudiated subject to some default costs. Optimal policy is characterized by two opposing incentives: an incentive to postpone taxes by issuing more debt for the future ("greed"), and an incentive to tax more currently in order to avoid punishing default premia ("fear"). A Generalized Euler Equation (GEE) captures these two effects and determines the optimal back-loading or front-loading of tax distortions. Even if default risk is small, tax-smoothing is severely limited. The same mechanisms operate also in environments with long-term debt.

Keywords: Labor tax, sovereign default, Markov-perfect equilibrium, time-consistency, generalized Euler equation, long-term debt.

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1 Introduction

This paper studies the optimal time-consistent allocation of tax distortions and the optimal issuance of debt in an environment where government debt can be defaulted on. We consider a government that has to finance an exogenous stream of stochastic government expenditures and maximizes the utility of the representative household. The government can use distortionary labor taxes or issue non-contingent debt. The government can default on its debt subject to a default cost. Our setup is fully time-consistent; neither tax nor debt promises are honored.

Our analysis builds on the notion of Markov-perfect equilibrium (MPE) of Klein et al. (2008). Optimal policy is time-consistent in the payoff-relevant state variables, which for our case are government debt and government expenditures. Furthermore, we model default as in the work of Arellano (2008) and Aguiar and Gopinath (2006), that builds on the debt repudiation setup of Eaton and Gersovitz (1981). This setup allows us to observe default in equilibrium.[1]

In most of the sovereign default literature, government debt is assumed to be held only by foreigners whereas domestic households are hand-to-mouth consumers. However, Reinhart and Rogoff (2011) find that, on average, domestic debt accounts for nearly two-thirds of total public debt for a large group of countries. We consider a closed economy in which domestic households hold government debt. Thus, our model takes into account that default events often involve default on debt held by domestic households. This assumption is supported by the empirical literature. While domestic default events are more difficult to identify than external default events, Reinhart and Rogoff (2011) document 68 cases of overt default on domestic debt since 1800. Moreover, often even when default is only on external debt (which we do not model), a significant portion of the external debt is held by domestic investors (Sturzenegger and Zettelmeyer 2006). For these reasons, it is of interest to understand the tradeoffs governments face when considering whether to default on domestic households.

Our purpose it to analyze optimal tax-smoothing and debt issuance in such an environment. The lack of state-contingent insurance markets hinders the ability of the government to smooth taxes. Default can in principle make debt partially state-contingent. This state-contingency does not come for free: default risk is reflected in equilibrium prices, altering the tradeoffs that the government is facing in smoothing taxes over states and dates.

The government has an incentive to default when either debt or expenditures are high. By defaulting the government can avoid high distortionary taxation. However, default entails either direct costs in terms of output losses, or indirect costs, in terms of a limited functioning of the market of government debt after a default event. In particular, we follow Arellano (2008) and assume that the market for government debt pauses to function for a random number of periods after a default event.

[1] See for further references the survey of Aguiar and Amador (2014).

Optimal policy is analyzed through the lens of a Generalized Euler Equation (GEE), which summarizes the dynamic costs and benefits that the government is facing when it issues debt. The average welfare loss that is incurred by an increase in debt issuance (higher debt is repaid in the future with higher distortionary taxes) has to be balanced with the benefits of relaxing the government budget and allowing less taxes today. The GEE reflects two opposite forces that reflect the lack of commitment to old tax promises and to debt repayment. Facing a fiscal shock, the policymaker can either increase taxes or issue more debt that is due next period. The current policymaker realizes that, by increasing debt, will make the future policymaker tax more and reduce future consumption, conditional on repaying. This raises future marginal utility and increases the current price of debt. Debt becomes cheaper, creating the incentive to back-load tax distortions. We dub this effect "greed" in order to capture the desire of the current government to manipulate interest rates against the future government. This price manipulation is in the heart of the problem of a sophisticated policymaker who realizes that next period he will not be bound by past promises.

However, this desire to postpone taxes and issue more debt is counteracted by the "fear" of default. Higher debt issuance increases the probability of default, decreasing the price of debt and raising default premia. Debt becomes expensive, forcing the planner to use more current taxes to absorb fiscal shocks. Consequently, default risk creates an incentive to front-load tax distortions and issue less debt. Ultimately, the dynamics of the tax rate are determined by the relative strength of these opposite forces, which depend on the curvature of the pricing kernel and the size of the region at which government debt is subject to default risk.

We consider also the optimal taxation, debt management and default plan when the government can issue long-term debt as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Long-term debt has been shown to capture better the mean and volatility of spreads in the sovereign default literature and has allowed to sustain more debt in equilibrium. Furthermore, the volatility in prices due to the long duration of debt provides an additional tool for absorbing fiscal risks, which is of particular interest for our optimal taxation exercise. We show that a richer version of our GEE goes through. Reflecting the longer maturity of debt, the GEE encapsulates now the *present value* of the two opposing price effects ("greed" versus "fear") that stem from the lack of commitment to old policies and from the option to default.

We take a first pass on the properties of the optimal taxation plan in a series of quantitative exercises, comparing the properties of taxes and debt to economies with full commitment as in Aiyagari et al. (2002) or to economies with time-consistent policies but no option to default. The Barro (1979) random walk results about the tax rate are substantially altered. Even if default risk is small and default *rare*, tax-smoothing is severely limited: governments rely mostly on taxes to absorb fiscal shocks and issue limited amounts of debt.

Related literature. A short, though undoubtedly *incomplete*, list of references should include Aiyagari et al. (2002), the basic paper that analyzes optimal taxation in incomplete markets. They solve for optimal policy under commitment and without default. In the time-consistent literature, Krusell et al. (2004) and Debortoli and Nunes (2013) analyze time-consistent taxation and debt in deterministic setups without default. Martin (2009) analyzes the joint determination of time-consistent fiscal and monetary policy. Chari and Kehoe (1993) and Aguiar and Amador (2016) study environments with sustainability constraints and Dovis (2018) considers also informational frictions.

The closest paper to ours is Pouzo and Presno (2016), which has been the first to consider the possibility of debt repudiation à la Arellano in the optimal taxation problem. These authors alter the Aiyagari et al. (2002) setup only in one dimension; they allow the government to default but they retain a notion of commitment.[2] In their setup, the government cannot commit to repay debt, but as long as the government decides to repay, it honors the marginal utility promises of the plan devised in previous periods and commits therefore to the tax sequence and the evolution of interest rates. In contrast, we treat debt and taxes symmetrically and derive the fully time-consistent policy in terms of the payoff -relevant endogenous state variable which is debt. Another paper of interest is D'Erasmo and Mendoza (2018), who study optimal domestic sovereign default in a quantitative model with heterogenous agents. They focus on the redistributional effects of default, but do not treat the optimal taxation problem.

Organization. Section 2 lays out the economy. Section 3 sets up the fiscal policy problem. The main forces of price manipulation and back-loading and front-loading of tax distortions are analyzed through the lens of a Generalized Euler Equation in section 4. Section 5 extends the analysis to environments with long-term debt. Section 6 provides various numerical exercises and section 7 concludes. An Appendix with the proofs of various propositions and the details of the numerical method follows.

2 Economy

Consider an infinite horizon model economy, where the government can default on issued debt. I use an economy similar to Lucas and Stokey (1983) and Aiyagari et al. (2002). The uncertainty is coming from exogenous government expenditures shocks g_t that take values in the set G. Let $g^t \equiv (g_0, g_1, ..., g_t)$ denote the partial history of shocks up to t and assume that the initial shock g_0 is given.

The economy is populated by a representative household with preferences over consumption

[2] Pouzo and Presno (2016) consider also the possibility of secondary markets in the event of default, a feature we do not share.

and leisure and an endowment of one unit of time every period. Let $c_t(g^t)$ denote private consumption and let $h_t(g^t)$ denote labor, so leisure is $l_t(g^t) = 1 - h_t(g^t)$. Let $d_t(g^t)$ denote an indicator variable that captures the decision of the government to default, so $d_t(g^t) = 1$ if there is default and $d_t(g^t) = 0$ if the government repays.

The resource constraint in the economy when the government repays is

$$c_t(g^t) + g_t = h_t(g^t). \tag{1}$$

If the government defaults, there are direct default costs that are modelled effectively as an adverse technology shock. The resource constraint in the event of default is

$$c_t(g^t) + g_t = zh_t(g^t), \tag{2}$$

where $z < 1$. In the numerical section I explore more elaborate default costs as in Arellano (2008).

Household. The household's preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t),$$

where u increasing and concave, and $\mathbb{E}_0$ the expectation operator with respect to the physical measure of uncertainty, conditional on the initial shock g_0. The household trades with the government a discount bond that gives one unit of consumption next period at any state of the world at which the government is not defaulting and zero in the event of default. The price of the bond is $q_t(g^t)$. The household pays a linear tax $\tau_t(g^t)$ on labor income $w_t(g^t)h_t(g^t)$. The household's budget constraint when the government is not defaulting reads

$$c_t(g^t) + q_t(g^t)b_{t+1}(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + b_t(g^{t-1}). \tag{3}$$

Note that the bond position b_{t+1} is function of the partial history g^t, capturing the non-contingency of payoffs in the event of repayment. The household has also initial debt b_0 and is also subject to some borrowing limits that we assume that are large enough so that they do not bind.

Default. Default entails direct and indirect costs. The direct costs were captured in the resource constraint (2). The indirect costs come from exclusion from government debt markets. When the government defaults at time t, b_t is wiped out and the government cannot issue new debt at the same period, so it runs a balanced budget. This can be thought of as a collapse of the market. At every period after default, the government can reenter with probability α the market with zero initial debt or stay excluded with probability $1-\alpha$.[3] There are still direct output costs z, whenever the government stays excluded. When $\alpha = 1$, the indirect cost of default is small, since the exclusion lasts only one period, whereas when $\alpha = 0$, the cost is large, since the government has to run a balanced budget forever and also incurs an output cost in doing so, compounding the default cost.

The specification of the consequences of default is standard in the open economy literature and follows directly Arellano (2008). In those setups, α is calibrated in order to match the average duration of exclusion from international markets. The international market justification is not obviously relevant for the closed economy, thus our market "collapse" interpretation.

Consequently, the household's budget constraint in the event of default, or for any exclusion period, is

$$c_t(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t). \tag{4}$$

Wages. Competitive factor markets lead to an equilibrium wage rate that is $w_t(g^t) = 1$ if $d_t(g^t) = 0$ and $w_t(g^t) = z$ if $d_t(g^t) = 1$ or if there is exclusion.

Government. The budget constraint of the government in the event of repayment is

$$B_t(g^{t-1}) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + q_t(g^t)B_{t+1}(g^t). \tag{5}$$

$B_t(g^{t-1}) > 0$ means that the government borrows and $B_t(g^{t-1}) < 0$ that the government lends.

If there is default or for any period after a default event for which there is exclusion, the government runs a *balanced* budget,

$$\tau_t(g^t)w_t(g^t)h_t(g^t) = g_t. \tag{6}$$

[3]Note that I am abusing notation by not including the history of exclusion shocks ε^t in the description of uncertainty and write all relevant variables as functions of $s^t \equiv (g^t, \varepsilon^t)$. The numerical exercises in this version of the draft have $\alpha = 1$, so the exclusion shock is not currently relevant.

Equilibrium. A competitive equilibrium with taxes and default is a price-tuple $\{q, w\}$, a government policy $\{\tau, B, d\}$, and a household's allocation and bond holdings $\{c, h, b\}$ such that: 1) Given prices and government policies, the household maximizes its utility subject to the budget constraint, 2) Given wages, firms maximize profits, 3) Prices and government policies are such so that the resource constraint and the government budget constraint hold. Furthermore, the bond market clears, $b_t = B_t$.[4]

2.1 Optimality conditions

The household's labor supply condition equates the marginal rate of substitution between consumption and leisure to the after-tax wage,

$$\frac{u_{lt}}{u_{ct}} = (1 - \tau_t) w_t. \tag{7}$$

The Euler equation for government bonds is

$$q_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{ct}} (1 - d_{t+1}). \tag{8}$$

Equation (8) is the standard asset pricing equation that prices the risky payoff $1 - d_{t+1}$. The equilibrium price of debt is zero, if the government defaults with certainty. If the government repays with certainty, then (8) reduces to the standard Euler equation without default.

Default premium. We contrast at a later stage the return of government debt to the return of a risk-free asset in order to get a measure of the default *premium*.[5] Assume that there is a private risk-free asset that is in *zero* net supply, so that the equilibrium allocation is not affected. This asset would have a price $q_t^F = \beta \mathbb{E}_t u_{c,t+1}/u_{ct}$. It is easy to see that the default premium can be expressed as

$$\frac{r_t - r_t^F}{1 + r_t^F} = \frac{\mathbb{E}_t x_{t+1} d_{t+1}}{\mathbb{E}_t x_{t+1}(1 - d_{t+1})} = \frac{\text{Prob}_t^x(\text{default at } t+1)}{\text{Prob}_t^x(\text{repayment at } t+1)}, \tag{9}$$

where $x_{t+1} \equiv u_{c,t+1}/\mathbb{E}_t u_{c,t+1}$, the risk-adjusted change of measure and $r_t \equiv 1/q_t - 1, r_t^F \equiv 1/q_t^F - 1$, the respective net returns. Therefore, the default premium at t is proportional to the ratio of the conditional *risk-adjusted* probability of default next period over the respective risk-adjusted

[4]Given $b_t = B_t$ and the rest of the equilibrium conditions, the government budget constraint is redundant.

[5]In open economy models the premium is measured against an international risk-free asset.

probability of repayment.

3 Markov-perfect policy

The policymaker chooses tax, debt and default policies to maximize the utility of the representative household. His choice of (τ, B, d) is subject to the constraints of the competitive equilibrium, that is, optimality conditions, budget and resource constraints. I use the primal approach of Lucas and Stokey (1983) and eliminate tax rates and prices (τ, q) from budget constraints by using the respective optimality conditions (7) and (8).

I am assuming a Markov-perfect timing protocol as in Klein et al. (2008), so the solution to the policy problem will be time-consistent in the payoff-relevant state variables.[6] The Markov-perfect equilibrium (MPE) has two "natural" state variables, government debt B, and the exogenous shock g, which is also assumed to be Markov. Let $V^r(B,g)$ denote the value function if the government decides to repay and $V^d(g)$ the value function if the government defaults. The value function of the government is

$$V(B,g) = \max\{V^r(B,g), V^d(g)\}.$$

Value of default. When the government defaults, the consumption and labor allocation at g, $(c^d(g), h^d(g))$, is determined by the resource constraint, the labor supply condition and the balanced budget requirement. Using (7) to eliminate the tax rate from the household's budget (4), the balanced budget requirement becomes

$$\Omega(c,h) = 0 \tag{10}$$

where

$$\Omega(c,h) \equiv u_c(c, 1-h)c - u_l(c, 1-h)h, \tag{11}$$

i.e. consumption net of after tax labor income, in marginal utility units. Ω is exactly equal to the primary *surplus* in marginal utility units.[7] So (c^d, h^d) has to satisfy the resource constraint (2)

[6] See Bassetto (2005) for a careful analysis of the timing protocols underlying optimal policy design.

[7] Note that if there are multiple feasible allocations that satisfy the balanced budget requirement (10), we should pick the one with the highest consumption. This allocation should correspond to a tax rate that is at the increasing portion of the tax Laffer curve.

and (10). Given (c^d, h^d) we can immediately deduce the default tax rate as, $\tau^d(g) = 1 - u_l^d/(zu_c^d)$. So the value of default is[8]

$$V^d(g) = u(c^d(g), 1 - h^d(g)) + \beta \mathbb{E}_{g'|g}\big[\alpha V(0, g') + (1-\alpha) V^d(g')\big].$$

Default decision. Define the default and its complement repayment set as

$$D(B) \equiv \{g \in G | V^d(g) > V^r(B, g)\} \quad \text{and} \quad A(B) \equiv D(B)^c = \{g \in G | V^d(g) \leq V^r(B, g)\}.$$

Given an amount of debt B at the beginning of the period, the default set denotes the set of values of g for which the government decides to default, so $d(B, g) = 1$ and $V(B, g) = V^d(g)$ if $g \in D(B)$. Similarly, $d(B, g) = 0$ and $V(B, g) = V^r(B, g)$ if $g \in A(B)$. Define also the upper and lower debt limit as

$$\bar{B} \equiv \inf\{B | D(B) = G\} \quad \text{and} \quad \underline{B} \equiv \sup\{B | D(B) = \emptyset\}.$$

$\bar{B}$ is the lowest amount of debt for which the government defaults with certainty, whereas $\underline{B}$ is the highest amount of debt for which the government repays with certainty.

Value of repayment. In a Markov-perfect equilibrium optimal policy has two features: a) the planner is not bound by *past* promises, a fact which is reflected in the choice of the state variables (B, g), and b) the planner takes into account at the current period that he will follow an *optimal* policy from next period onward, given the value of the natural state variable next period. To capture this requirement, let $\mathcal{C}(B', g')$ and $\mathcal{H}(B', g')$ denote the repayment consumption and labor policy functions next period when the state is (B', g'). We obviously have $\mathcal{C}(B', g') + g' = \mathcal{H}(B', g'), \forall (B', g')$. The "current" policymaker takes into account that by choosing debt B', he affects the consumption-labor choice of the "future" policymaker though $\mathcal{C}$ and $\mathcal{H}$. The value of repayment is

$$V^r(B, g) = \max_{c \geq 0, h \in [0,1], B'} u(c, 1-h) + \beta \mathbb{E}_{g'|g} V(B', g') \tag{12}$$

[8]Note that if $\alpha = 0$, i.e. if the market for government debt seized to exist forever after a default event, and if the set G were finite, we could calculate immediately the value of "autarky" as $\mathbf{V^d} = (I - \beta\Pi)^{-1}\mathbf{u^d}$, where boldface variables denote vector columns, I the identity matrix and Π the transition matrix of the shocks.

subject to

$$u_c(c,1-h)B \leq \underbrace{\Omega(c,h)}_{\text{surplus in } u_c \text{ units}} + \underbrace{\beta\mathbb{E}_{g'\in A(B')|g}u_c(\mathcal{C}(B',g'),1-\mathcal{H}(B',g'))}_{Q(B',g)} \cdot B' \tag{13}$$

$$c+g = h \tag{14}$$

The planner chooses a consumption-labor allocation and issues new debt subject to the implementability constraint (13) and the resource constraint (14). The implementability constraint is just the government's (or household's) budget constraint in current marginal utility units, after (7) and (8) are used to eliminate τ and q. In particular, let $Q_t \equiv u_{ct} \cdot q_t$ denote the price of government debt in current marginal utility units. From (8) we know that Q_t is equal to average future marginal utility, conditional on repaying. This is exactly the object that shows up in (13).

So the planner maximizes utility by choosing effectively the current tax rate and new debt (i.e. future taxes) subject to the government budget constraint. What matters in this dynamic decision is the surplus Ω and the revenue that the planner can achieve from new debt issuance, $Q(B',g) \cdot B'$. Moreover, the current planner cares for the optimal policy functions of next period $(\mathcal{C},\mathcal{H})$, only as long as there is curvature in the utility function, and, consequently, a margin for affecting interest rates. In other words, if utility were linear in consumption, so if there was no room for interest rate manipulation through the pricing kernel, $\mathcal{C}$ and $\mathcal{H}$ would be irrelevant, and the commitment solution would be time-consistent in the natural state variables (B,g).

MPE requirement. Let $c(B,g)$, $h(B,g)$ and $B'(B,g)$ be the policy functions that solve (12). The Markov-perfect requirement is that $c(B,g) = \mathcal{C}(B,g)$ and $h(b,g) = \mathcal{H}(B,g)$.[9]

4 Greed versus fear

To prepare the ground for the main analysis, consider the following lemma about default sets:

Lemma 1. *("Default sets")*

1. *The value of repayment is decreasing in B, so $B_1 > B_2 \Rightarrow D(B_2) \subseteq D(B_1)$.*

2. *Let $\omega(g)$ denote the amount of debt such that the government is indifferent between defaulting and repaying at g, $V^d(g) = V^r(\omega(g),g)$. The government defaults if $B > \omega(g)$ and repays if $B \leq \omega(g)$.*

[9] A more precise MPE requirement would be that $\mathcal{C}(B,g)$ and $\mathcal{H}(B,g)$ are maximizers of the stated problem in order to account for the existence of multiple solutions. This is for example what Klein et al. (2008) do.

3. $V(0,g) = V^r(0,g), \forall g$. *If the government has no debt, it does not default. Thus,* $D(0) = \emptyset$ *and* $\underline{B} \geq 0$.

Proof. This is obvious since for $B_1 < B_2$ the constraint correspondence increases, and therefore the repayment value is larger at B_1, $V^r(B_1, g) \geq V^r(B_2, g)$. This implies the threshold policy in terms of $\omega(g)$. The third part is to be written. □

Assumptions. Assume a continuous distribution of shocks with bounded support $[\underline{g}, \bar{g}]$ and a conditional density $f(g'|g)$. Furthermore, assume differentiability of the value functions, the policy functions of the future planner $(\mathcal{C}, \mathcal{H})$ and the default threshold, in order to derive a Generalized Euler Equation (GEE). This is *only* to develop intuition for the tradeoffs that the government is facing. Any differentiability holds *at most* locally.[10] I am *not* making any smoothness assumption in the numerical treatment of the problem and I work directly with the Bellman equation.

The non-differentiabilities arise from two sources: a) the discreteness of the default decision b) the time-consistency of policy. We will analyze these issues in more detail in the numerical section of the paper. For the derivation of the GEE we will also assume that the repayment and default sets are "nice". It is easy to see that *if* the government defaults more in bad times (high g), then the threshold $\omega(g)$ is monotonically decreasing.[11] If ω is *strictly* decreasing, then we can invert it and interpret $\omega^{-1}(B)$ as the value of government spending that makes the government indifferent between repaying or defaulting given B. Thus, the repayment and default sets become "nice" intervals, $A(B) = [\underline{g}, \omega^{-1}(B)]$ and $D(B) = (\omega^{-1}(B), \bar{g}]$.

4.1 Optimal debt issuance

We are finally at the stage where we can derive the optimal debt issuance of the government in problem (12). Let Φ denote the multiplier on the implementability constraint (13). This multiplier denotes the welfare cost of the absence of lump-sum taxes and captures essentially *tax distortions*. We will call Φ the *excess burden of taxation*.

The first-order necessary condition with respect to B' at an interior optimum takes the form

$$-\beta \frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g') = \Phi \cdot [Q(B', g) + \frac{\partial Q(B', g)}{\partial B'} \cdot B']. \tag{15}$$

where $Q(B', g) \equiv \beta \mathbb{E}_{g' \in A(B')|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))$, that is, average marginal utility conditional on repaying.

[10] Note that non-differentiability issues emerge in the infinite horizon economy– the analysis a two-period economy would not face this type of problems.

[11] By defaulting more in bad times I mean if $g' > g$ and $g \in D(B)$ then $g' \in D(B)$. This statement is equivalent to $\omega(g') \leq \omega(g)$ for $g' > g$.

Optimality condition (15) captures the dynamic tradeoffs of issuing debt. The left-hand side denotes the marginal *cost* of issuing debt. Debt is costly in this second-best world because the government has to raise revenues through distortionary taxation in order to repay. The right-hand side of (15) denotes the marginal benefit of issuing debt, which is equal to the *marginal revenue* from debt issuance, multiplied by the excess burden of taxation. By issuing more debt, the government *relaxes* the government budget, which has shadow value Φ. The incentives to back-load or front-load tax distortions come from the manipulation of prices through debt, $\partial Q/\partial B'$. To see that, we rewrite (15) as follows.

Proposition 1. *("GEE")*

1. *The Generalized Euler Gquation in an environment with incomplete markets and default takes the form*

$$-\int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B',g')}{\partial B'} f(g'|g)dg' = \Phi \cdot \Big\{ \int_{\underline{g}}^{\omega^{-1}(B')} u_c(\mathcal{C}(B',g'), 1-\mathcal{H}(B',g')) f(g'|g)dg'$$
$$+ \Big[\underbrace{u_c(\mathcal{C}(B',\omega^{-1}(B')), 1-\mathcal{H}(B',\omega^{-1}(B'))) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'}}_{\text{"Fear": } Q\downarrow \text{ due to default risk}}$$
$$\underbrace{+\int_{\underline{g}}^{\omega^{-1}(B')} [u'_{cc} - u'_{cl}] \frac{\partial \mathcal{C}}{\partial B'} f(g'|g)dg'}_{\text{"Greed": } Q\uparrow \text{ due to an increase of } u_c} \Big] \cdot B' \Big\} \tag{16}$$

2. *Let $m_{t+1}(B_{t+1}, g_{t+1}) \equiv \frac{(1-d_{t+1})u_{c,t+1}}{\mathbb{E}_t(1-d_{t+1})u_{c,t+1}}$ denote the default-and-risk-adjusted change of measure ($\mathbb{E}_t m_{t+1} = 1$), with induced conditional density $f^m(g_{t+1}, B_{t+1}|g_t) \equiv m_{t+1}(B_{t+1}, g_{t+1}) \cdot f(g_{t+1}|g_t)$. Using the envelope condition $\frac{\partial V^r}{\partial B} = -\Phi u_c < 0$, we can rewrite (16) in sequential notation as*

$$\mathbb{E}_t m_{t+1}\Phi_{t+1} = \Phi_t \Big\{ 1 + \big[\underbrace{f^m(\omega^{-1}(B_{t+1}), B_{t+1}|g_t) \frac{d\omega^{-1}}{dB'}}_{\text{"Fear"}} + \underbrace{\mathbb{E}_t m_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}}\big]}_{\text{"Greed"}} \cdot B_{t+1} \Big\} \tag{17}$$

Proof. See Appendix. □

Greed. Consider first the pricing effect coming from the assumption that policy is Markov-perfect, so there is lack of commitment to to tax and debt policies designed in the past, and assume that $u_{cl} \geq 0$. An increase in debt B' leads to a decrease in the consumption of the future policymaker ($\partial \mathcal{C}/\partial B' < 0$), since, conditional on repaying, the future policymaker will have to tax more in a distortionary way. This fall in future consumption raises marginal utility and, therefore, *increases* the price Q, $(u'_{cc} - u'_{cl})\partial \mathcal{C}/\partial B' > 0$. In other words, by bequeathing higher debt to the future government, the current policymaker reduces the cost of new debt. So higher debt leads to lower interest rates due to the manipulation of the pricing kernel inherent in a setup without commitment. We dub this effect "greed".

Fear. The Markov-perfect effect through the pricing kernel is naturally accompanied with an opposite effect when there is default risk. Higher debt issuance B' increases the default region and raises the probability of default. This is reflected in the GEE (16) or (17) by the term that reflects the risk-adjusted mass at the threshold value of government expenditures *times* the change in the threshold, which we expect to be negative, $d\omega^{-1}/dB' < 0$. As a result, the price of debt falls, $\partial Q/\partial B' < 0$ due to default risk. We call this force "fear," referring to the fear of no repayment, which is reflected in equilibrium prices and default premia.

Tax-smoothing. The GEE in (17) makes clearer the connection to the tax-smoothing literature and helps contrast our analysis to Aiyagari et al. (2002), Krusell et al. (2004), Debortoli and Nunes (2013) and Pouzo and Presno (2016).[12] At first, note that the left-hand side denotes the *average* tax distortions with respect to the default-and-risk-adjusted measure. The right-hand side denotes the change in revenue from debt issuance due to a larger position and to the pricing effects that we previously identified. Thus, (17) just says that the planner should issue more debt and tax more on average for tomorrow, if the interest cost of debt is low. To understand better the mechanism, consider the setup of Aiyagari et al. (2002), who analyze optimal taxation without state-contingent debt and with full commitment to old policies and debt repayment. The respective tax-smoothing condition is $\mathbb{E}_t x_{t+1} \Phi_{t+1} = \Phi_t$, where $x_{t+1} \equiv u_{c,t+1}/\mathbb{E}_t u_{c,t+1}$, the *risk-adjusted* change of measure.[13] Thus, the planner tries to make tax distortions on *average* (with respect to the risk-adjusted measure) *constant* and the pricing effects $\partial Q/\partial B'$ are absent.

Back-loading versus front-loading. In contrast, in this paper the planner does not try to make average distortions constant due to the lack of commitment to the two dimensions of policy. If the reduction in interest rates due to the marginal utility channel ("greed") is larger than the increase in interest rates due to default risk, then it is optimal for the planner to tax less today, issue more debt and *postpone* on average distortions for the future. This translates to a positive drift in the excess burden according to the default-and-risk-adjusted measure, $\mathbb{E}_t m_{t+1} \Phi_{t+1} \geq \Phi_t$,

[12] Krusell et al. (2004) and Debortoli and Nunes (2013) analyze interest rate manipulation in deterministic setups through the marginal utility channel, so they do not consider uncertainty and default. Their respective GEE is the deterministic version of (17), and they differ in terms of the sign of $\partial \mathcal{C}/\partial B'$. See for further discussion the numerical section 6. In Pouzo and Presno (2016), the manipulation of prices through the marginal utility channel is not present due to the commitment to previously designed tax policies, so their respective GEE exhibits only the default term.

[13] Φ is still the multiplier on the respective implementability constraint, but the state variable in the commitment problem is a measure of both debt and marginal utility. See the Appendix.

when $B_{t+1} > 0$. The opposite will happen if the default force is stronger, i.e. if default premia increase so much that they dominate the decrease in interest rates due to high marginal utility. Then the planner on average wants to *decrease* the excess burden over time and to front-load tax distortions, $\mathbb{E}_t m_{t+1}\Phi_{t+1} \leq \Phi_t$. This negative drift (with respect to the measure m_{t+1}) in the excess burden should also materialize as a negative drift in debt.[14]

4.2 Optimal tax rate

Consider now the optimal tax rate of problem (12).

Proposition 2. *1. ("Optimal tax rate") The optimal tax rate takes the form*

$$\tau = \frac{\Phi\left[(\epsilon_{cc} + \epsilon_{hc})(1 - B/c) + \epsilon_{ch} + \epsilon_{hh}\right]}{1 + \Phi(1 + \epsilon_{hh} + \epsilon_{hc}(1 - B/c))}, \tag{18}$$

where $\epsilon_{cc} \equiv -u_{cc}c/u_c, \epsilon_{ch} \equiv u_{cl}h/u_c$ the own and cross elasticity of the marginal utility of consumption and $\epsilon_{hh} \equiv -u_{ll}h/u_l, \epsilon_{hc} \equiv u_{cl}c/u_l$, the own and cross elasticity of the marginal disutility of labor.

2. ("Constant Frisch elasticity") Assume the utility function $U = \frac{c^{1-\rho}-1}{1-\rho} - a_h \frac{h^{1+\phi_h}}{1+\phi_h}$. Then τ becomes

$$\tau = \frac{\Phi(\rho(1 - B/c) + \phi_h)}{1 + \Phi(1 + \phi_h)}. \tag{19}$$

Proof. See Appendix. □

Discussion. The optimal tax rate (18) depends on the excess burden of taxation $\Phi > 0$, the elasticities of the period utility function and the amount of initial debt, B. Debt enters the problem though the lack of commitment of the planner to past promises. To see that, consider the tax rate for the constant Frisch case, where we make the elasticity channel constant. An increase in initial debt B leads to a decrease in the tax rate, *ceteris paribus*. This is the typical time-inconsistency issue in Lucas and Stokey (1983), and it refers to the incentive of the planner to devalue initial debt.[15] At $t = 0$, the planner has an incentive to have a low tax rate, leading

[14]The discussion above is silent about drifts with respect to the physical measure. It is still possible to have a positive drift with respect to m_{t+1} (or x_{t+1} in the case without default) and still have a negative drift with respect to the physical measure. A thorough numerical analysis is necessary to determine that.

[15]The optimal tax rate at $t = 0$ in Lucas and Stokey (1983) is given by (18), if we set $B = B_0$ and keep the excess burden of taxation constant.

to high consumption. This is useful, because the value of initial debt in marginal utility units falls, since $u_c(c_0)$ falls. In other words, the interest rate at the initial period falls.

In the time-consistent solution though, these comparative statics analysis is not valid, since debt has *two* effects: a direct one through B and an indirect one though Φ, $\Phi = \Phi(B, g)$. In other words, the planner is aware of the incentives to devalue, a fact which leads typically to an increase in the excess burden of taxation.[16]

5 Long-term debt

Consider now the possibility of long-term debt with exponentially decaying coupons as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).[17] Coupons decay at the rate δ, so the coupon payments are $(1-\delta)^i, i \geq 0$. Assume that the government chooses every period to default or not on all current and future debt obligations and that the same costs (direct and indirect in form of the exclusion from debt markets) apply as in the case with short-term debt ($\delta = 1$). Under this assumption the definition of the default value function remains the same as previously.

To calculate the value of repayment, we need to modify the policy problem. Let $B_t(g^{t-1})$ denote the debt obligation of the government at the beginning of the period, which consists of the coupon payments that correspond to debt issued in all previous periods. This takes the form of

$$B_t(g^{t-1}) = b_t(g^{t-1}) + (1-\delta)b_{t-1}(g^{t-2}) + (1-\delta)^2 b_{t-2}(g^{t-1}) + ... = b_t(g^{t-1}) + (1-\delta)B_{t-1}(g^{t-2}),$$

where b_t are the holdings of the security at the end of period $t-1$, b_{t-1} at the end of $t-2$ (so the coupon is $1-\delta$) and so on and so forth. Thus, the household's budget constraint in the event of repayment reads

$$c_t(g^t) + q_t(g^t)(B_{t+1}(g^t) - (1-\delta)B_t(g^{t-1})) = (1-\tau_t(g^t))w_t(g^t)h_t(g^t) + B_t(g^{t-1}). \tag{20}$$

The Euler equation with defaultable long-term debt becomes

$$q_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{ct}}(1-d_{t+1})\big[1 + (1-\delta)q_{t+1}\big]. \tag{21}$$

[16] We will see in the numerical section that the tax rate is an increasing function of B.

[17] See also Woodford (2001) for the analysis of non-Ricardian fiscal regimes with this type of long-term debt.

It is useful for later purposes to rewrite the Euler equation as

$$Q_t = \beta \mathbb{E}_t (1 - d_{t+1}) \big[u_{c,t+1} + (1-\delta) Q_{t+1} \big], \tag{22}$$

where Q_t is defined as the price of long-term debt in *marginal utility* units, $Q_t \equiv u_{ct} \cdot q_t$.

5.1 Value of repayment

Let $\mathcal{C}(B', g')$, $\mathcal{H}(B', g')$ and $\mathcal{K}(B', g')$ denote the consumption, labor and debt policy function next period. The value of repayment is defined as

$$V^r(B, g) = \max_{c \geq 0, h \in [0,1], B'} u(c, 1-h) + \beta \mathbb{E}_{g'|g} V(B', g') \tag{23}$$

subject to

$$\begin{aligned} [u_c(c, 1-h) + (1-\delta) Q(B', g)] B &\leq \Omega(c, h) + Q(B', g) B' & (24) \\ c + g &= h & (25) \end{aligned}$$

where the price of long-term debt satisfies the following recursion,

$$Q(B', g) = \beta \mathbb{E}_{g' \in A(B')|g} \Big[u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1-\delta) Q\big(\mathcal{K}(B', g'), g'\big) \Big] \tag{26}$$

Debt is long-lived, so we need to specify next period's debt policy function, in order to determine the price of the long-term asset. The Markov-perfect requirement is obviously $c(B, g) = \mathcal{C}(B, g)$, $h(B, g) = \mathcal{H}(B, g)$ and $B'(B, g) = \mathcal{K}(B, g)$ for all (B, g).

5.2 Analysis

Let Φ denote again the multiplier on the implementability constraint (24). It is easy to see that the formula for the optimal tax rate with long-term debt is the same as in proposition 2. Turning to the optimal debt issuance, we have:

$$-\beta \frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g') = \Phi \cdot [Q(B', g) + \frac{\partial Q}{\partial B'} (B' - (1-\delta) B)] \tag{27}$$

As previously, the left-hand side denotes the cost of issuing more debt, whereas the right-hand side the welfare benefit of the marginal revenue from debt issuance. Note that a change in price affects also the price of the remaining coupons that are to be paid in the future, due to the long-term nature of debt. The respective envelope condition with long-term debt is

$$\frac{\partial V^r(B,g)}{\partial B} = -\Phi[u_c + (1-\delta)Q]. \tag{28}$$

To get a simpler expression, assume as previously that default sets have a "nice" structure and that the conditional density of shocks is $f(g'|g)$. The derivative of price with respect to debt is

$$\begin{aligned}\frac{\partial Q}{\partial B'} &= \beta\Big[\underbrace{\big\{u_c\big(\mathcal{C}(B',\omega^{-1}(B')),1-\mathcal{H}(B',\omega^{-1}(B'))\big) + (1-\delta)Q(\mathcal{K}(B',\omega^{-1}(B')),\omega^{-1}(B'))\big\}f(\omega^{-1}(B')|g)\frac{d\omega^{-1}}{dB'}}_{\text{“Fear”: decrease in prices due to default risk}} \\ &+ \underbrace{\int_{\underline{g}}^{\omega^{-1}(B')}\big(u'_{cc}-u'_{cl}\big)\frac{\partial C}{\partial B'}f(g'|g)dg'}_{\text{“Greed”: increase in price due to an increase in } u_c} + (1-\delta)\underbrace{\int_{\underline{g}}^{\omega^{-1}(B')}\frac{\partial Q'}{\partial B''}\frac{\partial \mathcal{K}}{\partial B'}f(g'|g)dg'}_{\text{future price change}}\Big]\end{aligned} \tag{29}$$

The derivative of the price with respect to debt exhibits again the two forces that we previously identified: the decrease in prices due to an increased default probability ("fear") and the increase in prices due to increased marginal utility ("fear"), if, as we expect, $\partial\mathcal{C}/\partial B' < 0$. Debt though is long-lived so there are capital gains and losses and the derivative of the current price is determined also by the (properly discounted) derivative of the future price Q' with respect to debt, taking into account how current debt issuance will affect future debt issuance, $\partial\mathcal{K}/\partial B'$.

As in the case with short-term debt, I will rewrite the GEE in terms of the excess burden.

Proposition 3. *("GEE with long-term debt")*

1. *Define the default-and-long-term-debt-adjusted change of measure as*

$$n_{t+1}(B_{t+1},g_{t+1}) \equiv \frac{(1-d_{t+1})\big[u_{c,t+1}+(1-\delta)Q_{t+1}\big]}{\mathbb{E}_t(1-d_{t+1})\big[u_{c,t+1}+(1-\delta)Q_{t+1}\big]} \geq 0 \quad \textit{with} \quad \mathbb{E}_t n_{t+1} = 1.$$

Define also the price semi-elasticity with respect to debt as $\eta_t \equiv \frac{\partial Q_t}{\partial B_{t+1}}\frac{1}{Q_t}$. The GEE takes the form

$$\mathbb{E}_t n_{t+1}\Phi_{t+1} = \Phi_t \cdot \big[1 + (B_{t+1}-(1-\delta)B_t)\eta_t\big] \tag{30}$$

2. *The semi-elasticity η_t follows the recursion*

$$\eta_t = \left[f^n(\omega^{-1}(B_{t+1}), B_{t+1}|g_t)\frac{d\omega_{t+1}^{-1}}{dB_{t+1}} + \mathbb{E}_t n_{t+1}\frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1} + (1-\delta)Q_{t+1}}\frac{\partial \mathcal{C}_{t+1}}{\partial B_{t+1}}\right] + (1-\delta)\mathbb{E}_t n_{t+1}\frac{Q_{t+1}}{u_{c,t+1} + (1-\delta)Q_{t+1}}\frac{\partial \mathcal{K}_{t+1}}{\partial B_{t+1}}\eta_{t+1} \quad (31)$$

where $f^n(g_{t+1}, B_{t+1}|g_t) \equiv n_{t+1}(B_{t+1}, g_{t+1})\cdot f(g_{t+1}|g_t)$, the default-and-long-term-debt-adjusted conditional density. Thus, η_t captures the <u>*present value*</u> *of the two opposing price effects of default risk and the lack of commitment ("fear" and "greed"),*

$$\eta_t = \mathbb{E}_t^n \sum_{i=1}^{\infty}(1-\delta)^{i-1}\left(\prod_{j=1}^{i-1}\frac{Q_{t+j}}{u_{c,t+j} + (1-\delta)Q_{t+j}}\cdot\frac{\partial \mathcal{K}_{t+j}}{\partial B_{t+j}}\right)\cdot\left[f^n(\omega^{-1}(B_{t+i}), B_{t+i}|g_{t+i-1})\frac{d\omega_{t+i}^{-1}}{dB_{t+i}} + \frac{u_{cc,t+i} - u_{cl,t+i}}{u_{c,t+i} + (1-\delta)Q_{t+i}}\frac{\partial \mathcal{C}_{t+i}}{\partial B_{t+i}}\right],$$

where $\mathbb{E}_t^n$ denotes expectation according to the default-and-long-term-debt-adjusted measure.

Proof. See Appendix. □

Thus, with long-term debt the entire stream of the future opposing pricing effects affect the elasticity of prices with respect to debt and therefore the revenue schedule of the government. Obviously, for $\delta = 1$ the above formulas reduce to the case with short-term debt in proposition 1.

6 Numerical results

For the basic numerical exercise I use the constant Frisch elasticity utility function that delivers the tax rate (19) in proposition 2. At this stage I use some standard parameter values in order to illustrate the main forces of the model. A future goal is to have a calibration that considers particular characteristics of a country.

We set $(\beta, \phi_h, \rho) = (0.9, 1, 2)$. The labor disutility parameter is set to $a_h = 19.2901$, so that the household works 40% of its time at the first-best. For the government expenditure shocks we assume that they are i.i.d. and that they follow a uniform distribution, $g \sim U[\underline{g}, \bar{g}]$, where $\underline{g} = 0$ and $\bar{g} = 0.08$. The maximum amount of g corresponds to 20% of first-best output.

The probability of re-entry is set to unity, $\alpha = 1$. We allow asymmetric default costs, i.e. it is less costly to default when government expenditures are high. We use a linear specification, $z(g) = z(\underline{g}) + \lambda_z(g - \underline{g})$, where $(z(\underline{g}), \lambda_z) = (0.98, 0.2375)$.

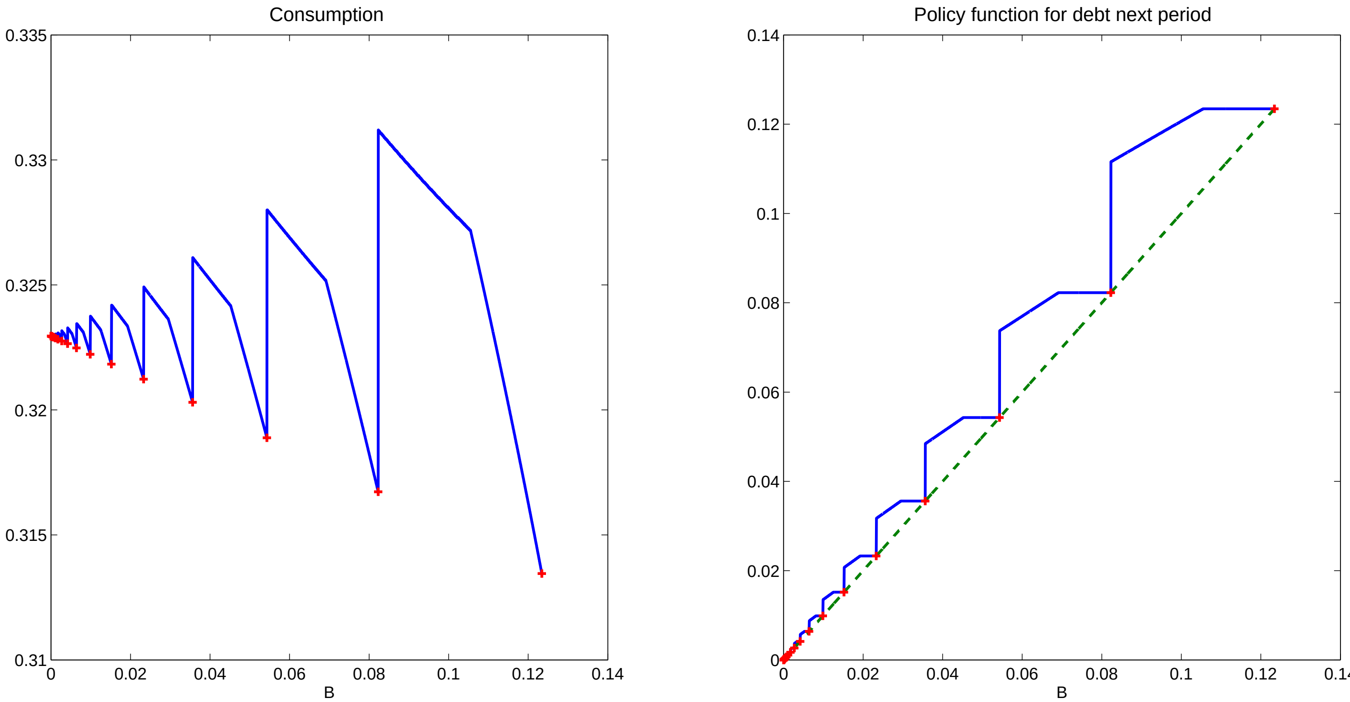

Figure 1: Consumption and debt policy function in the deterministic Krusell et al. (2004) economy. The crosses '+' correspond to steady states. At the right of each of these points, there is a jump upwards in the debt and consumption policy function.

The government is not allowed to lend so the lower debt limit is zero. For the upper limit of the debt grid, I use the maximum surplus that can be sustained by the static tax Laffer curve. This corresponds to about 30% of output. Given that upper limit, the model generates an endogenous debt limit that is smaller. Moreover, I focus on the MPE that is the limit of the finite horizon problem.

There are several issues with the numerical computation of the problem. These issues have to do with non-convexities which lead to discontinuous policy functions and touch upon the Markov-perfect nature of policy.

6.1 Deterministic setup: non-convexities and discontinuities

If we shut off uncertainty and the option to default, the setup reduces to a deterministic Lucas and Stokey (1983) economy. In contrast to Lucas and Stokey (1983), the policymaker has no commitment. The GEE (17) reduces to

$$\Phi_{t+1} = \Phi_t \cdot [1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}} \cdot B_{t+1}].$$

Krusell et al. (2004) have analyzed this problem and have shown that the non-convexities associated with the Markov-perfect assumption in this economy introduce serious discontinuities in the policy functions for consumption and debt.[18] The GEE is then valid only locally. Figure 1 displays the severity of these discontinuities. The debt policies are continuous from the left. At each jump, if current debt increases marginally, the current planner has an incentive to issue a large amount of debt for next period, reducing a lot the interest rate (since the consumption of the *future* policymaker will jump downwards and therefore marginal utility will jump upwards) and allowing therefore to tax less currently– which is why also current consumption jumps upwards as well.

6.2 Stochastic setup

When we turn into a stochastic setup, there is a possibility for elimination of the jumps in the policy functions, due to the *smoothing* effect of uncertainty. To see that in a heuristic way, drop for simplicity the option to default, assume that $u_{cl} = 0$ and consider the average marginal utility over the entire support of government expenditures, $\mathbb{E}_t u_c(\mathcal{C}(B_{t+1}, g_{t+1}))$. This object captures equilibrium prices and is the source of discontinuity in the implementability constraint, through the actions of the future policymaker $\mathcal{C}$. Assume for the sake of the argument that each policy functions of consumption for next period (indexed by g'), is discontinuous in B'. As long as the points where the policy functions have jumps are countable and not the same across shocks g', average marginal utility will smooth out the jumps, leading to a continuous price function of debt. At a more fundamental level, uncertainty partially "convexifies" the constraint set.[19]

6.2.1 No option to default

To see the smoothing effect of uncertainty numerically consider first the Markov-perfect policy without the option to default. The GEE (whenever valid) takes the form

$$\mathbb{E}_t x_{t+1}\Phi_{t+1} = \Phi_t \cdot \left[1 + \mathbb{E}_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}} \cdot B_{t+1}\right], \quad x_{t+1} \equiv \frac{u_{c,t+1}}{\mathbb{E}_t u_{c,t+1}}.$$

In that case, we have only the *average* interest-rate effects emerging from the Markov-perfect policy assumption through $\mathcal{C}$ ("greed"). More debt for tomorrow is reducing the interest rate

[18] Debortoli and Nunes (2013) allow government spending to provide utility, making the incentives for interest rate manipulation opposite to Krusell et al. (2004). See also Occhino (2012). Endogenous government spending seems to make the discontinuity issue less prevalent.

[19] I conjecture that a formal introduction of lotteries in debt as in Chatterjee and Eyigungor (2016) would in principle perform the same role. Otherwise, an additional shock as in Chatterjee and Eyigungor (2012) could also smooth the constraint set. I am currently abstaining from these approaches.

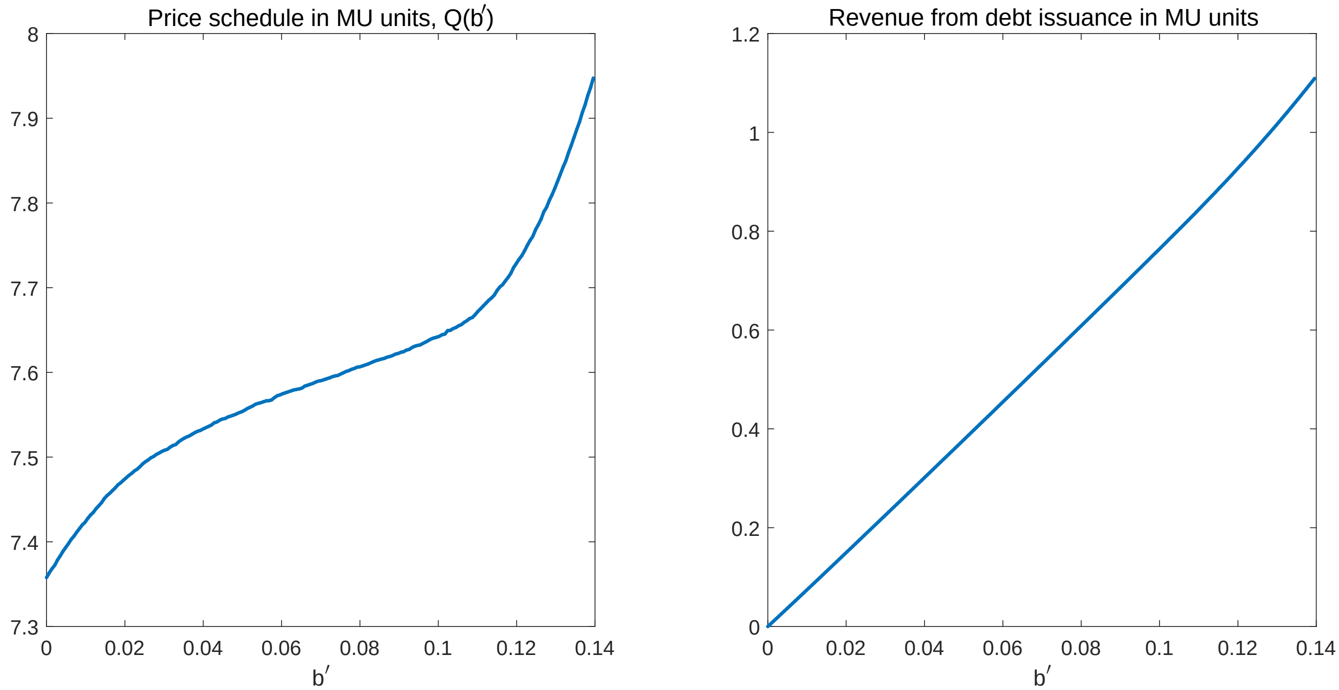

Figure 2: The left panel depicts average marginal utility ("price"), in a world with Markov-perfect policy under uncertainty and without default. The right panel plots the respective revenue from debt issuance.

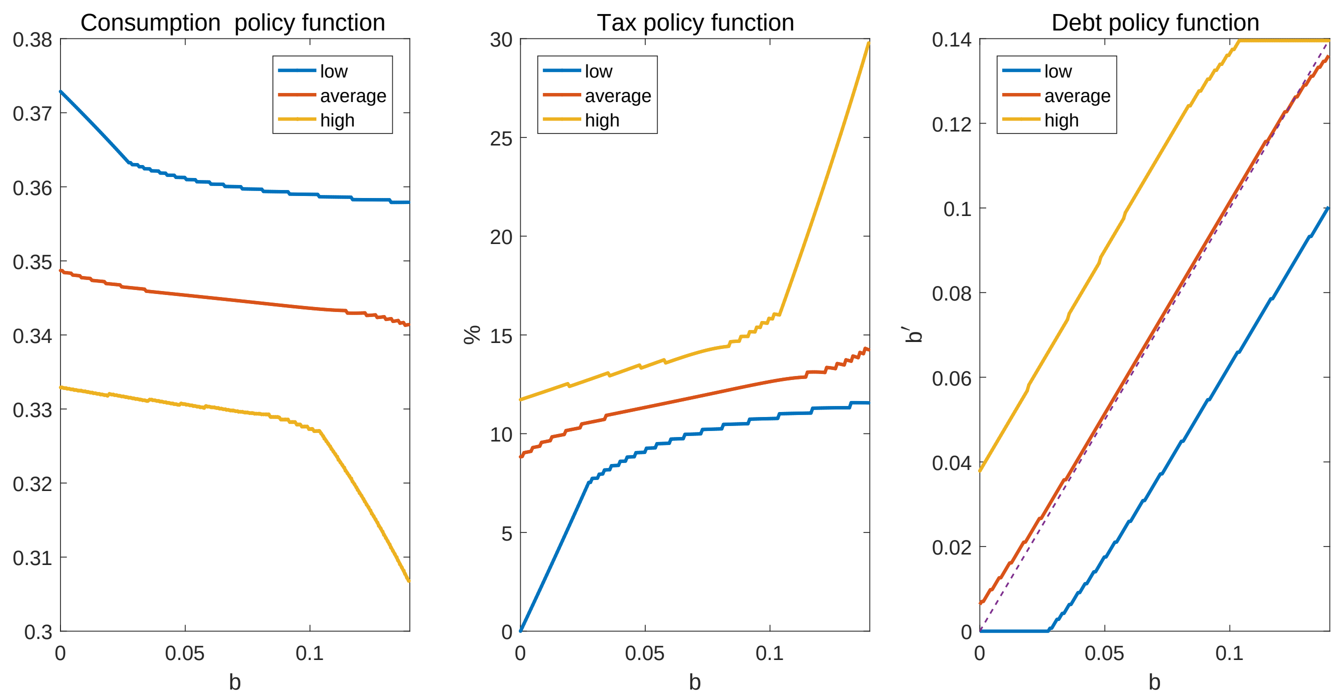

Figure 3: Policy functions for consumption, taxes and debt in a world with Markov-perfect policy under uncertainty and without default. The policy functions are drawn for three different level of government expenditures.

by increasing next period's average marginal utility, giving an incentive to the planner to increase average distortions tomorrow relative to today, i.e. giving an incentive to back-load tax distortions.

The left panel in figure 2 depicts the effect of Markov-perfect policy on equilibrium prices. An increase in debt for next period, reduces consumption and increases average marginal utility, reducing the interest rate. Note that uncertainty has smoothed out the jumps in policy functions, leading to a continuous and increasing pricing schedule. The right panel depicts the respective revenue from debt issuance, which, since there is not default risk, is always increasing. Figure 3 depicts the respective policy functions for consumption, taxes and debt. Note the incentive of the government to do precautionary savings when the government expenditure shock is low, which is similar qualitatively to the case with commitment of Aiyagari et al. (2002) (albeit different quantitatively). To see that, consider the low government expenditure shock schedules (blue lines - corresponding to $g = 0$) in the three panels of figure 3. When debt is low, the government is financing this debt by taxing labor income and abstaining from issuing new debt. Only when initial debt starts being large enough, the government starts using also new debt to pay back inherited debt. In contrast, when the fiscal shock is average (red schedules in the panels) the government is slightly increasing the tax rate when debt increases, and uses tax revenues to finance government expenditures and interest payments to the public (debt issuance is almost on the 45 degree line). For high government expenditure shocks, the government is using much more new debt issuance than taxes to pay back higher inherited debt, as long as the upper bound on debt issuance is not hit.

6.2.2 Default option

We turn now to the full-blown model with the option to default. Figure 4 plots the default and repayment sets. Both of our claims are valid, i.e. the default set is increasing in debt and the country defaults more in "bad" times (high government expenditure shocks). Figure 5 plots the price schedule and the debt Laffer curve in an economy with Markov-perfect policy and default. In contrast to the findings of figure 2, the price schedule starts to decrease when we enter the region where there is positive probability of default, i.e. the negative price effect of an increased default probability is larger than the positive price effect through the reaction function of the future policymaker, $\mathcal{C}$. Furthermore, the amount of debt that corresponds to the maximum of the debt Laffer curve is *larger* than the maximum amount of debt for which the government repays with certainty. As a result, there is a region for which there is risky debt in equilibrium.[20] Lastly, figure 6 patches together the respective policy functions for consumption, taxes and debt

[20]To obtain this region, it was necessary to assume asymmetric default costs that vary with the shock g. See Arellano (2008) for a discussion of the importance of the maximum of the debt Laffer curve in an open economy without distortionary taxation.

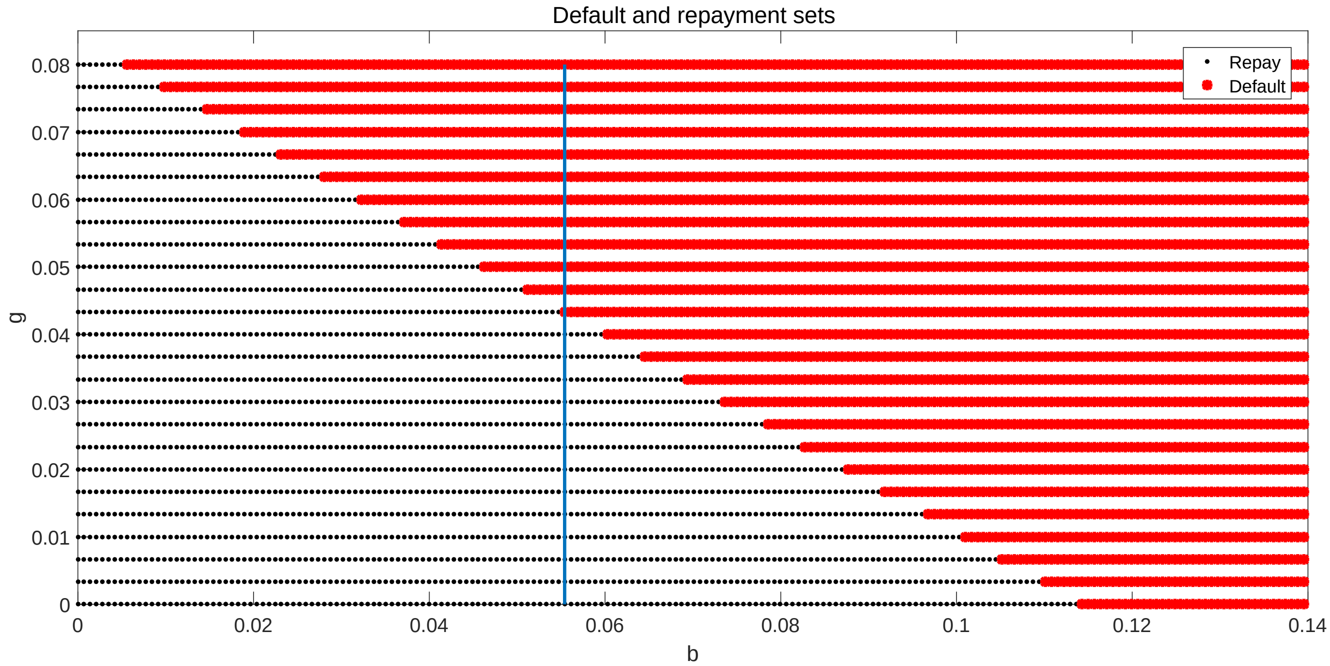

Figure 4: Default and repayment sets in the infinite horizon economy. The vertical line is plotted at the maximizer of the debt Laffer curve.

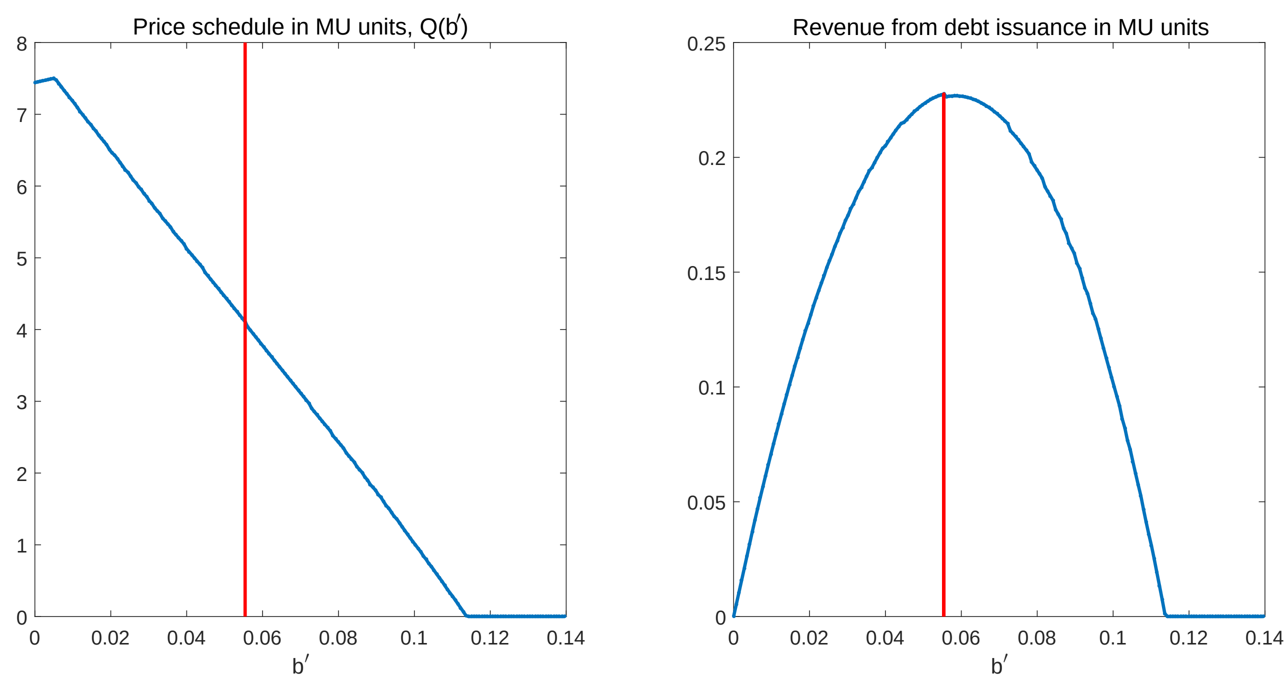

Figure 5: Price schedule and debt Laffer curve in the economy with default. Note that the price is increasing initially in debt (as in the economy without default) and then it starts decreasing when we enter the region where there is positive probability of default.

in the event of repayment and default.

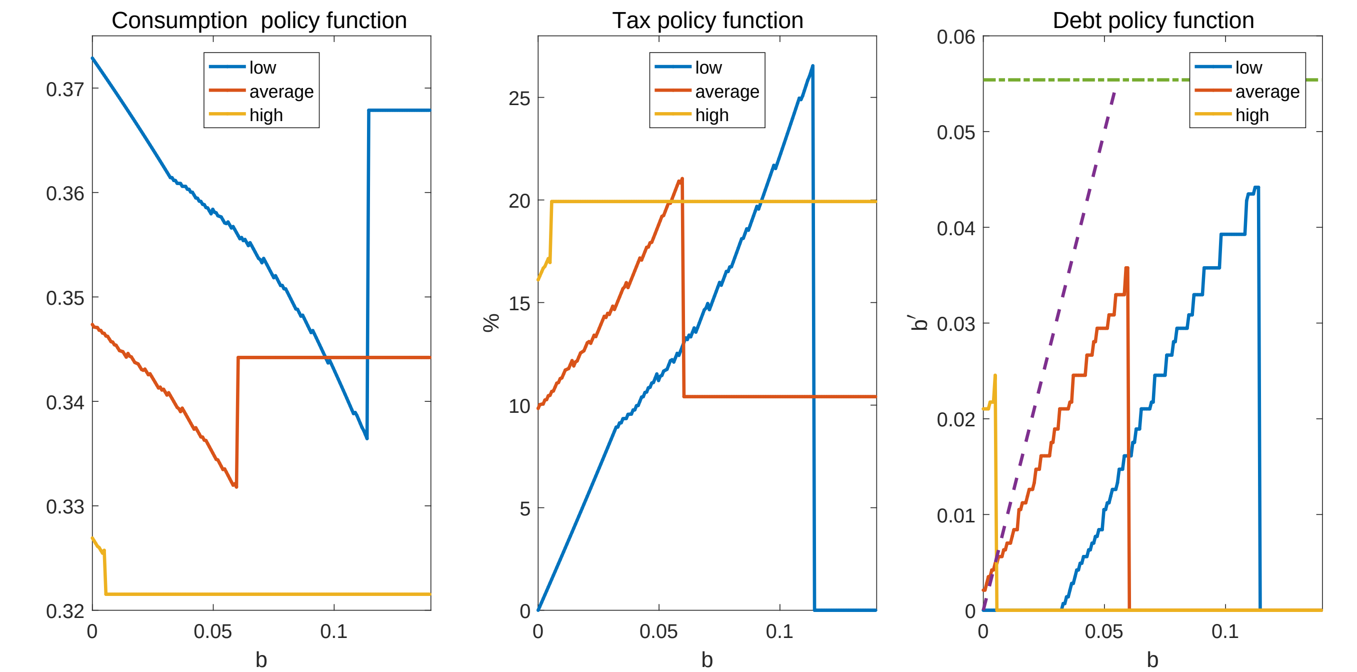

Figure 6: Policy functions for consumption, taxes and debt in the full-blown model with default. The policy functions are drawn for three different levels of government expenditures.

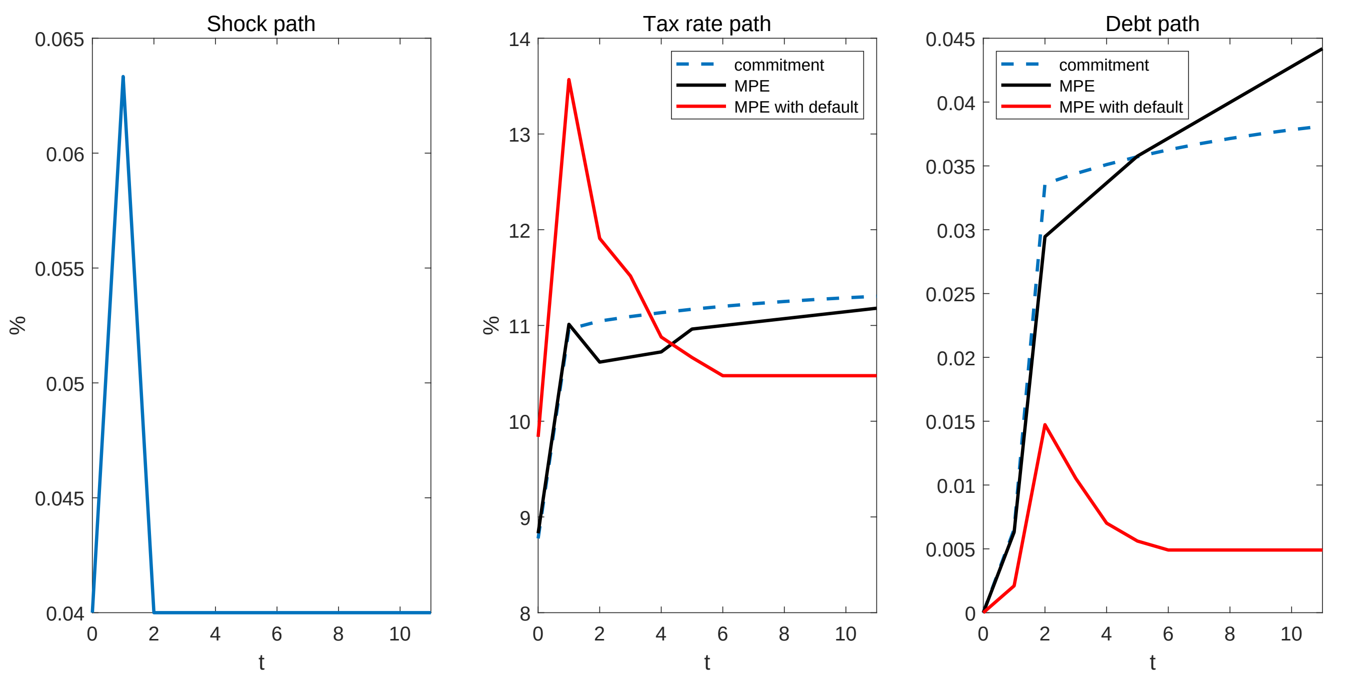

Figure 7: The left panel plots the path for government expenditures. The middle and right panel plot the respective tax rate and debt path when the planner follows the commitment solution of Aiyagari et al. (2002), the time-consistent policy with no option to default, and the time-consistent policy with the option to default.

6.3 Some sample paths

How does the government absorb a fiscal shock with taxes and new debt issuance? In figure 7 we answer this question by considering a one standard deviation shock above the mean at $t = 1$ and plotting the respective tax rate and debt issuance for the commitment case of Aiyagari et al. (2002) and the time-consistent case without or with default risk.[21] The government starts with zero initial debt. In all cases, when the government expenditure shock increases, both taxes and new debt increase. Figure 7 shows that in the absence of default risk, the government is increasing partially the tax rate in face of an adverse fiscal shock, but also increases substantially debt. Furthermore, the tax rate remains at high levels even if the fiscal shock goes back to the mean, confirming the persistence result of Aiyagari et al. (2002), and debt increases over time. It is of interest to observe that without commitment but no default option, the increase in debt is larger over time in comparison to the commitment case.

In contrast, the presence of default risk reduces the ability of the government to absorb fiscal shocks with new debt issuance, since debt becomes more expensive. As a result, the government absorbs the shock with a much larger increase of the tax rate at $t = 1$ (almost two and half percentage points higher than in the case without default) and a much smaller increase in new debt issuance. Moreover, the increase in the tax rate and debt is *not* persistent. After the second period, taxes and debt return to a slightly higher level than the initial one. Overall, default risk *limits* the fiscal hedging of the government.

To get a sense of how default occurs, figures 8 and 9 consider a random path of fiscal shocks of 30 periods and considers the respective debt, consumption tax and default decision paths. Note that the government defaults for the first time at $t = 10$, despite the fact that government expenditures happened to be higher at $t = 4$. The reason is obvious; debt was not as high at $t = 4$, which made it worthwhile for the government to repay.

6.4 Stationary moments

Consider now a long simulation of $100,000$ periods. Table 1 depicts the moments from the commitment problem and the Markov-perfect problem without or with default risk. Note that at the commitment solution the tax rate has high persistence (as expected from Aiyagari et al. (2002)). Moving to the case of no commitment but without the default option, the persistence of the tax rate is slightly reduced, but other than that, the mean and standard deviation of taxes remain similar. There is a noticeable change though in debt. In the no commitment solution, the policymaker is issuing on average more debt. This reflects the desire of the policymaker to manipulate interest rates by anticipating the reaction of the future policymaker. Moving to the case of default risk, we note that equilibrium debt is much smaller. As we noted in the analysis of

[21] See the Appendix for the formulation of the commitment problem.

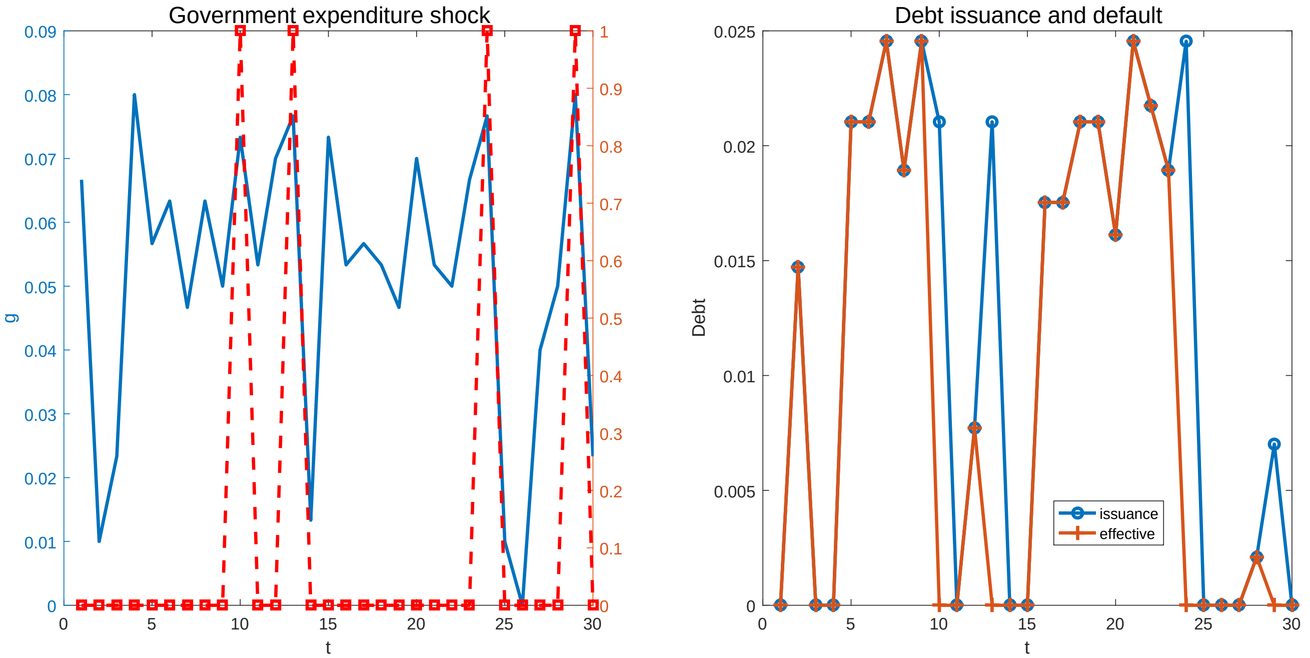

Figure 8: The default decision is depicted as an indicator variable that takes the value unity when there is default (right axis in each graph). *Effective* debt is debt in the beginning of the period *after* the decision to default (so it becomes zero when the government defaults).

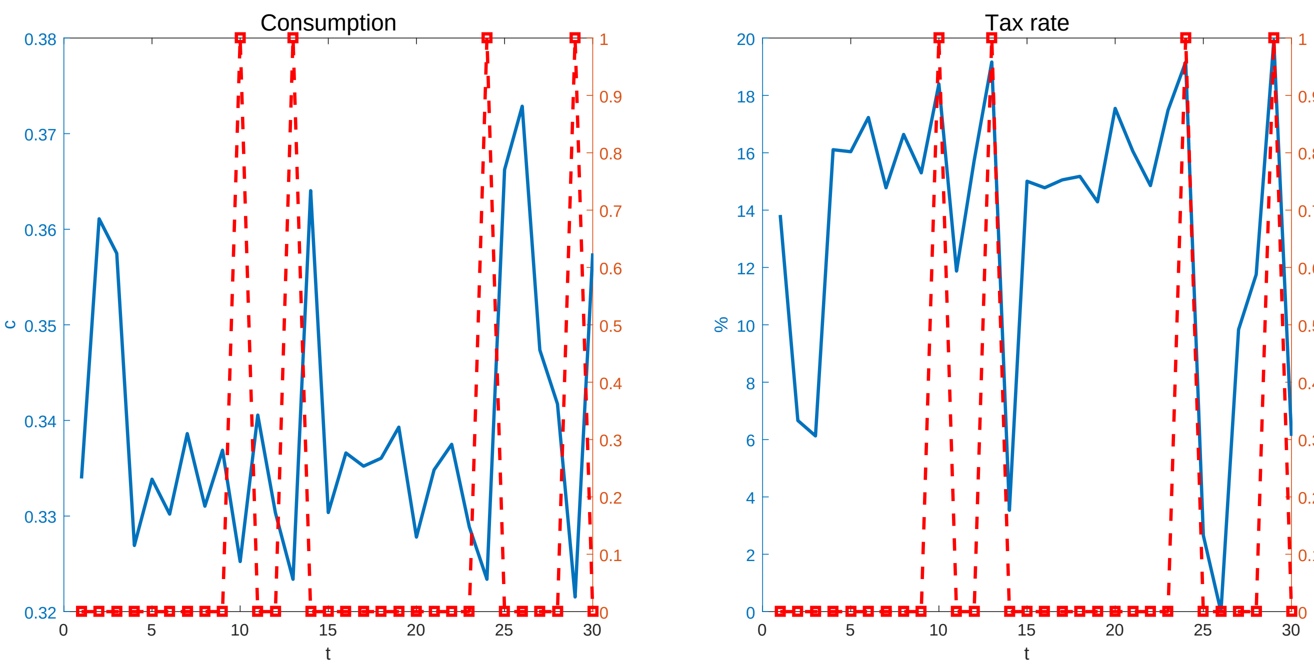

Figure 9: The paths of consumption and tax rates that correspond to the sample path of fiscal shocks in figure 8.

figure 7, the tax rate – relatively to debt– is used much more as a fiscal hedging tool when there

	Commitment (AMSS)	MPE	MPE with default
Tax rate in %			
Mean	11.95	12.64	10.44
St. Dev	3.2	3.8	4.82
Corr with g	0.5075	0.6033	0.9369
Autocorrelation	0.77	0.54	0.2316
Debt-output-ratio in %			
Mean	14.03	21	1.62
St. Dev	9.65	11.4	2
Autocorrelation	0.79	0.85	0.2

Table 1: Stationary moments from a $100,000$ period simulation, starting from zero initial debt. The calibration of the shocks implies a mean share of government expenditures that is 10% with a standard deviation of 6 percentage points. The probability of default is 5%.

is the option to default. The mean tax rate is smaller (because the debt that can be sustained is smaller), but the standard deviation of the tax rate is larger, close to 5 percentage points. Furthermore, the correlation of the tax rate with government expenditure shocks is much larger when there is default risk (0.93), and its autocorrelation small. This reveals the fact that the behavior of the tax rate reflects the behavior of the iid fiscal shock when there is the option to default, since the government budget is effectively balanced due to high debt “intolerance”.

6.5 Spreads, taxes and default costs

Consider now a different calibration of shocks with a mean and standard deviation equal to 17.5% and 4.5% of output and a subjective discount factor of 0.96 so that the frequency is annual. In order to make a meaningful comparison between commitment and default, we have to be explicit about the upper debt limits in the commitment case, since in the default case the upper debt limit of the government is endogenously determined. Table 2 shows the stationary moments for the commitment and the default case for varying debt limits and varying default costs.

The first column of the table reports moments from the commitment solution (AMSS) that correspond to an exogenously set upper debt limit that corresponds to 200% of output.[22] The second column repeats the same exercise for an upper debt limit that corresponds to the maximum of the static Laffer curve, and is about 30% of output. Higher upper debt limits under commitment allow the government to sustain more debt in equilibrium. Furthermore, higher

[22]As in the baseline computation, the lower limit for all experiments is set to zero; the government is not allowed to lend to the private sector.

	Commitment *high debt limit*	Commitment *low debt limit*	Default *low cost*	Default *high cost*
Tax rate in %				
Mean	21	19.23	18.56	18.69
St. Dev	2.39	1.79	3.67	2.96
Corr with g	0.08	0.45	0.9676	0.86
Autocorrelation	0.94	0.79	0.19	0.39
Debt-output-ratio in %				
Mean	53.75	12.89	0.93	3
St. Dev	41.13	8.09	1.01	2.3
Autocorrelation	0.98	0.84	0.16	0.42
Default probability in %	0	0	2.19	0.53
Spread in %				
Mean	0	0	1.97	0.075
St. Dev	0	0	3.63	0.33
Autocorrelation	-	-	0.15	0.14

Table 2: Stationary moments from a 2 million period simulation. Shocks are i.i.d. $U[0.04, 0.10]$ with lower and upper bounds that correspond to 10% and 25% of first-best output. The spending ratio has mean 17.5% and standard deviation of 4.5% in terms of first-best output. The high and low debt limit for the commitment case correspond to 200% or 30% of output respectively. Preference parameters are the same like before except for $(\beta, \alpha) = (0.96, 22.9568)$, so the frequency is annual. Default costs are linear in g with $(z(\underline{g}), \lambda_z) = (0.98, 0.3167)$ and $(z(\underline{g}), \lambda_z) = (0.96, 0.5833)$ for the low and high cost case.

debt limits allow the government to make the tax rate even more persistent and less correlated with spending, confirming the ad hoc analysis of Barro (1979). Naturally, with higher upper debt limits, the contrast between commitment and default is even starker.

Turning to the case of default risk, the third and fourth column in table 2 report the respective moments for an economy with low and high default costs, implying respectively high and low default risk. The average output cost is 1% and 2.25% in the low and high cost case respectively. As in table 1, default risk limits tax smoothing. The default probability in the low cost case is about 2.2%, so the government defaults twice every one hundred years. The default probability is higher than the implied default probability in the Reinhart and Rogoff (2011) data, which is about 1%.[23] The spread is about 2% with a high standard deviation of 3.6%. If we increase the default costs (last column) the default probability becomes just 0.5%, so the government defaults once every 200 years. This is even less frequent than in the data, a fact that is reflected in the very low spread of 7.5 basis points. Nonetheless, the standard deviation of the tax rate

[23] See D'Erasmo and Mendoza (2018).

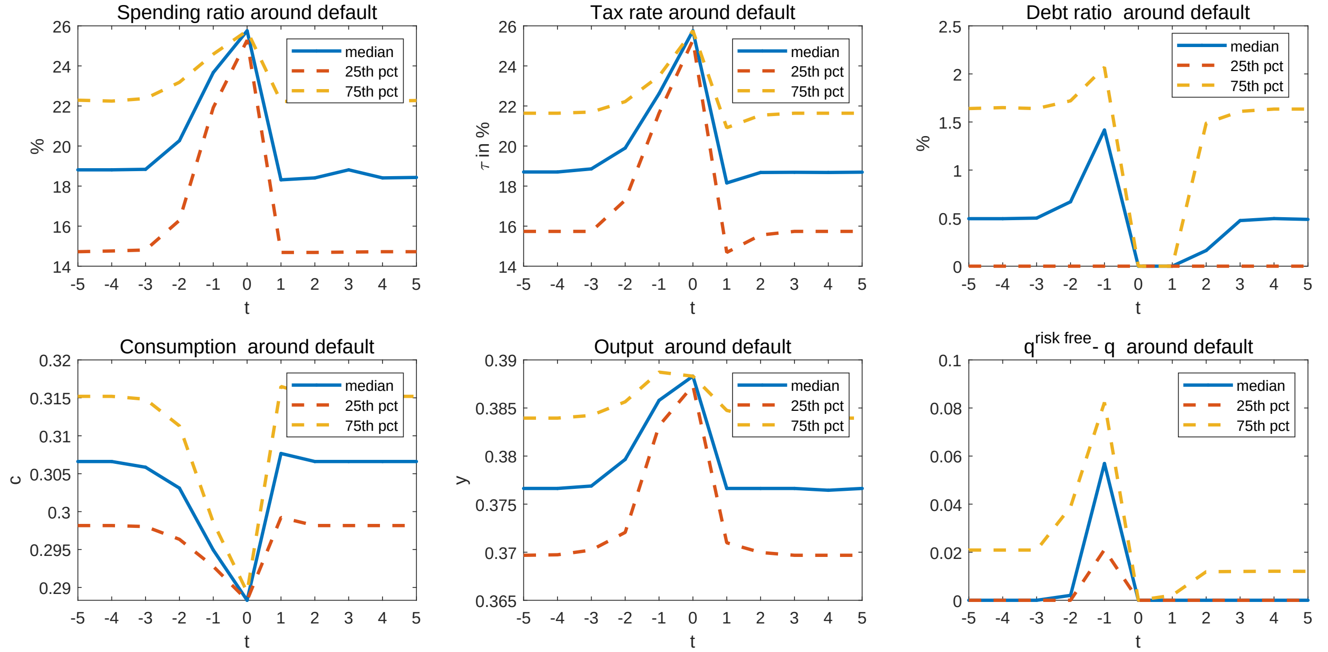

Figure 10: Dynamics 5 periods before and 5 periods after default events. The economy simulated is the low default cost economy of Table 2 which exhibited around 44,000 default events in 2 million years.

and its correlation with spending remain high and the debt that the government can sustain is just 3% of output, so the government still absorbs fiscal shocks mainly with increases in tax revenues. Thus, even if default is *rare*, tax smoothing is *severely* limited.

6.6 Dynamics around default events

Figure 10 displays the median dynamics (together with the 25th and 75th percentile) of various variables of interest around default events for the low default cost economy of Table 2. Recall that the default probability for this economy is 2.2% and that equilibrium debt is low. Overall, the dynamics are as we expect them. Default events are associated with a rise in the median spending about three periods before the default event (upper left panel). Government starts raising taxes and debt (upper center and right panel). Higher spending and debt increase the probability of default, which is reflected in the premium. The premium jumps one period before the default (lower right panel). New debt issuance becomes prohibitively expensive and the government defaults at $t = 0$, leading to a drop in consumption (lower left panel). Taxes and output return almost immediately to the same median value before the default event started (i.e. three periods before the event at $t = 0$). Debt ratios take a longer time to recover.

6.7 Future steps

Further quantitative analysis is necessary to flesh out the properties of the optimal taxation problem. In particular, we would like to introduce a) persistent shocks, b) solve the problem with a larger state space, and c) solve the model with long-term debt (proposition 3) in order to be able to sustain more debt in equilibrium like in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). All these features exacerbate the discontinuity issues, making computation challenging.

Furthermore, we would like to explore in more detail the optimal front-loading versus back-loading of taxes and debt. Ultimately, we want to use our optimal policy exercise in order to evaluate various fiscal rules that have been proposed to deal with the sovereign debt crisis. "Austerity" measures cannot be evaluated without considering both the default risk and the distortionary taxation implications from an optimal policy perspective.

7 Concluding remarks

In this paper we analyze optimal distortionary taxation in a setup where policymakers can commit to neither repaying debt nor to the taxation and debt scheme devised in the past. We want to understand how this double absence of commitment alters the basic tax-smoothing and debt issuance prescriptions. We are motivated by the fact that domestic sovereign default is an empirically relevant phenomenon, as Reinhart and Rogoff (2011) showed. Two opposite forces shape optimal policy, manifesting themselves as a desire to back-load or front-load tax distortions. We find that even when default is rare, tax smoothing is severely limited.

A Proofs of propositions 1, 2 and 3

A.1 GEE for short-term debt

Given the assumptions about uncertainty and the repayment and default sets, the expected value function next period and the price Q are

$$\begin{aligned}\mathbb{E}_{g'|g}V(B',g') &= \int_{\underline{g}}^{\omega^{-1}(B')} V^r(B',g')f(g'|g)dg' + \int_{\omega^{-1}(B')}^{\bar{g}} V^d(g')f(g'|g)dg' \\ Q(B',g) &= \beta\int_{\underline{g}}^{\omega^{-1}(B')} u_c(\mathcal{C}(B',g'),1-\mathcal{H}(B',g'))f(g'|g)dg'.\end{aligned}$$

Thus, using Leibnitz's rule, we get

$$\begin{aligned}\frac{\partial}{\partial B'}\mathbb{E}_{g'|g}V(B',g') &= f(\omega^{-1}(B')|g)\frac{d\omega^{-1}}{dB'}[V^r(B',\omega^{-1}(B')) - V^d(\omega^{-1}(B'))] + \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B',g')}{\partial B'}f(g'|g)dg' \\ &= \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B',g')}{\partial B'}f(g'|g)dg',\end{aligned}$$

since $V^d(\omega^{-1}(B)) = V^r(B,\omega^{-1}(B))$.

Similarly, we have

$$\frac{\partial Q}{\partial B'} = \beta\big[u_c(\mathcal{C}(B',\omega^{-1}(B')),1-\mathcal{H}(B',\omega^{-1}(B')))f(\omega^{-1}(B')|g)\frac{d\omega^{-1}}{dB'} + \int_{\underline{g}}^{\omega^{-1}(B')}(u'_{cc}-u'_{cl})\frac{\partial\mathcal{C}}{\partial B'}f(g'|g)dg'\big],$$

where I already used the fact that $\partial\mathcal{C}/\partial B' = \partial\mathcal{H}/\partial B'$. The GEE (16) follows.

To get (17), update one period the envelope condition and rewrite (16) in sequential notation. The derivative of the expected value function becomes $\mathbb{E}_t(1-d_{t+1})u_{c,t+1}\Phi_{t+1}$. Divide all terms in the GEE with $\mathbb{E}_t(1-d_{t+1})u_{c,t+1}$ and use the definition of m_{t+1} to get (17).

A.2 Optimal tax rate

Proof. Assign multiplier λ on the resource constraint (14). First-order necessary conditions are

$$c: \quad u_c + \Phi[\Omega_c - u_{cc}B] = \lambda \tag{A.1}$$

$$h: \quad -u_l + \Phi[\Omega_h + u_{cl}B] = -\lambda \tag{A.2}$$

Combining (A.1-A.2) and eliminating λ gives

$$1 = \frac{u_l}{u_c} \cdot \frac{1 - \Phi[\frac{\Omega_h}{u_l} + \frac{u_{cl}}{u_l} B]}{1 + \Phi[\frac{\Omega_c}{u_c} - \frac{u_{cc}}{u_c} B]} = \frac{u_l}{u_c} \cdot \frac{1 + \Phi\big[\epsilon_{hh} + \epsilon_{hc}(1 - \frac{B}{c})\big]}{1 + \Phi\big[1 - \epsilon_{cc}(1 - \frac{B}{c}) - \epsilon_{ch}\big]},$$

where the second line follows by expressing the derivatives of $\Omega_i, i = c, h$ in terms of elasticities, $\Omega_c/u_c = 1 - \epsilon_{cc} - \epsilon_{ch}$, and $\Omega_h/u_l = -1 - \epsilon_{hh} - \epsilon_{hc}$. Use now $u_l/u_c = 1 - \tau$ and rewrite the above expression in terms of τ. Note that the denominator of (18) is positive, since condition (A.2) and $\lambda > 0$ imply $1 + \Phi(1 + \epsilon_{hh} + \epsilon_{hc}(1 - B/c)) > 0$.

□

A.3 GEE for long-term debt

Proof. As with short-term debt, the derivative of the expected value function takes the form

$$\frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g') = \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B', g')}{\partial B'} f(g'|g) dg' = \mathbb{E}_t (1 - d_{t+1}) \frac{\partial V^r_{t+1}}{\partial B_{t+1}}.$$

Update the envelope condition (28) one period and rewrite (27) in sequential form as

$$\beta \mathbb{E}_t (1 - d_{t+1})[u_{c,t+1} + (1 - \delta) Q_{t+1}] \Phi_{t+1} = \Phi_t \Big[Q_t + (B_{t+1} - (1 - \delta) B_t) \frac{\partial Q_t}{\partial B_{t+1}} \Big]$$

Divide over Q_t and remember that Q_t satisfies the recursion (22). Use that fact in order to express the expectation in term of n_{t+1} to finally get (30). To get the recursion for η_t, divide (29) over Q_t and write it in sequential form as

$$\begin{aligned} \eta_t \quad = \quad & \beta \Big[\frac{\big(u_{c,t+1}(B_{t+1}, \omega^{-1}(B_{t+1})) + (1 - \delta) Q_{t+1}(\mathcal{K}(B_{t+1}, \omega^{-1}(B_{t+1})), \omega^{-1}(B_{t+1}))\big)}{Q_t} f(\omega^{-1}(B_{t+1})|g_t) \frac{d\omega^{-1}}{dB_{t+1}} \\ & + \mathbb{E}_t \frac{(1 - d_{t+1})}{Q_t} [u_{cc,t+1} - u_{cl,t+1}] \frac{\partial \mathcal{C}}{\partial B_{t+1}} + (1 - \delta) \mathbb{E}_t \frac{(1 - d_{t+1})}{Q_t} Q_{t+1} \frac{\partial \mathcal{K}_{t+1}}{\partial B_{t+1}} \eta_{t+1} \Big] \end{aligned}$$

Use again (22) and the definition of n_{t+1} to finally get (31). □

B Commitment problem

This section illustrates the problem under commitment as in Aiyagari et al. (2002) and Bhandari et al. (2017). The implementability constraint with non-contingent debt $b_{t+1}(g^t)$ and no risk of default takes the form

$$u_{ct}b_t = u_{ct}c_t - u_{lt}h_t + \beta \mathbb{E}_t u_{c,t+1} b_{t+1}.$$

Define $\tilde{B}_t \equiv \mathbb{E}_t u_{c,t+1} \cdot b_{t+1}$, which is a function of information at t and captures (besides debt) the commitment of the planner to marginal utility promises. The implementability constraint becomes

$$\frac{u_{ct}}{\mathbb{E}_{t-1} u_{ct}} \tilde{B}_{t-1} = u_{ct}c_t - u_{lt}h_t + \beta \tilde{B}_t.$$

Bellman equation. The commitment problem from period one onward is formed recursively as follows:

$$W(\tilde{B}_-, g_-) = \max_{c_g, h_g, \tilde{B}_g} \sum_g \pi(g|g_-) \big[u(c_g, 1-h_g) + \beta W(\tilde{B}_g, g) \big]$$

subject to

$$\begin{aligned}
&\frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) u_c(c_g, 1-h_g)} \tilde{B}_- = \Omega(c_g, h_g) + \beta \tilde{B}_g, \forall g \\
&c_g + g = h_g, \forall g \\
&c_g \geq 0, h_g \in [0,1] \\
&\underline{B_g} \leq \tilde{B}_g \leq \bar{B}_g
\end{aligned}$$

The value function W denotes the optimal average utility. Note that the debt limits are formulated directly in terms of the state variable $\tilde{B}_t$.

Initial period problem. As in Kydland and Prescott (1980), the initial value of the pseudo-state variable is chosen optimally at $t = 0$. The planner chooses $c_0 \geq 0, h_0 \in [0,1]$ and $\tilde{B}_0$ within the debt limits to maximize

$$u(c_0, 1-h_0) + \beta W(\tilde{B}_0, g_0)$$

subject to

$$
\begin{aligned}
u_c(c_0, 1-h_0)b_0 &= u_{c0}c_0 - u_{l0}h_0 + \beta \tilde{B}_0 \\
c_0 + g_0 &= h_0
\end{aligned}
$$

where (b_0, g_0) given.

C Computation

[To be completed]

References

Aguiar, Mark and Manuel Amador. 2014. Sovereign Debt. *Handbook of International Economics* 4:647–687.

———. 2016. Fiscal policy in debt constrained economies. *Journal of Economic Theory* 161:37–75.

Aguiar, Mark and Gita Gopinath. 2006. Defaultable debt, interest rates and the current account. *Journal of International Economics* 69 (1):64–83.

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppala. 2002. Optimal Taxation without State-Contingent Debt. *Journal of Political Economy* 110 (6):1220–1254.

Arellano, Cristina. 2008. Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review* 98 (3):690–712.

Barro, Robert J. 1979. On the Determination of the Public Debt. *Journal of Political Economy* 87 (5):940–71.

Bassetto, Marco. 2005. Equilibrium and govenrment commitment. *Journal of Economic Theory* 124 (1):79–105.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent. 2017. Fiscal Policy and Debt Management with Incomplete Markets. *Quarterly Journal of Economics* 132 (2):617–663.

Chari, V.V. and Patrick J. Kehoe. 1993. Sustainable plans and mutual default. *The Review of Economic Studies* 60 (1):175–195.

Chatterjee, Satyajit and Burcu Eyigungor. 2012. Maturity, Indebtedness, and Default Risk. *American Economic Review* 102 (6):26742699.

———. 2016. Continuous Markov equilibria with quasi-geometric discounting. *Journal of Economic Theory* 163:467–494.

Debortoli, Davide and Ricardo Nunes. 2013. Lack of commitment and the level of debt. *Journal of the European Economic Association* 11 (5):1053–1078.

D'Erasmo, Pablo and Enrique G. Mendoza. 2018. History Remembered: Optimal Sovereign Default on Domestic and External Debt. Mimeo, University of Pennsylvania.

Dovis, Alessandro. 2018. Efficient sovereign default. *The Review of Economic Studies* .

Eaton, Jonathan and Mark Gersovitz. 1981. Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies* 48 (2):289–309.

Hatchondo, Juan Carlos and Leonardo Martinez. 2009. Long-duration bonds and sovereign defaults. *Journal of International Economics* 79:117–125.

Klein, Paul, Per Krusell, and José-Víctor Ríos-Rull. 2008. Time-Consistent Public Policy. *The Review of Economic Studies* 75 (3):789–808.

Krusell, Per, Fernando M. Martin, and José-Víctor Ríos-Rull. 2004. Time-consistent debt. Mimeo, Institute for International Economic Studies.

Kydland, Finn E. and Edward C. Prescott. 1980. Dynamic optimal taxation, rational expectations and optimal control. *Journal of Economic Dynamics and Control* 2:79–91.

Lucas, Robert Jr. and Nancy L. Stokey. 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12 (1):55–93.

Martin, Fernando M. 2009. A Positive Theory of Government Debt. *Review of Economic Dynamics* 12 (4):608–631.

Occhino, Filippo. 2012. Govenrment debt dynamics under discretion. *The B.E. Journal of Macroeconomics* 12 (1).

Pouzo, Demian and Ignacio Presno. 2016. Optimal taxation with endogenous default under incomplete markets. Mimeo, UC Berkeley.

Reinhart, Carmen M. and Kenneith S. Rogoff. 2011. The Forgotten History of Domestic Debt. *The Economic Journal* 121 (552):319–350.

Sturzenegger, F. and J. Zettelmeyer. 2006. *Debt Defaults and Lessons from a Decade of Crises*. The MIT press.

Woodford, Michael. 2001. Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking* 33:669–728.