Welfare effects of dynamic matching: An empirical analysis

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Abstract

Assignment mechanisms that do not rely on monetary transfers lead to equitable access but may yield inefficient allocations. Theory suggests that introducing rationing when resources are allocated repeatedly over time can mitigate this issue, while the magnitude of the resulting efficiency gains is an empirical question in most settings. We study a dynamic assignment mechanism used by the Michigan Department of Natural Resources to allocate bear hunting permits and find that it yields a more efficient allocation than static mechanisms both by inducing participants to be more selective and by allowing participants with a higher preference for hunting to obtain permits more frequently. Our empirical analysis also highlights the importance of heterogeneity across participants and across allocated resources for determining the efficiency of a dynamic allocation mechanism.

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1 Introduction

Government agencies frequently face the task of regulating access to publicly provided resources while satisfying their mandate of offering these resources in an equitable way, i.e. guaranteeing that low income users are not barred from access, so that money cannot be used as a medium of exchange. Examples include amenities such as public housing, which is by definition offered at prices below market clearing prices; transplant organs, for which payment is ruled out for ethical and legal reasons; and access to public schooling, which is considered an essential right that should be tax-funded.

Resource allocation without monetary transfers is generally inefficient, in the sense of failing to maximize total surplus, when individual valuations for the resources being allocated are private information.[1] For example, when access to one resource is being allocated and no information is available on individual valuations for this resource, assignment mechanisms without transfers will necessarily lead to the same expected total surplus as a simple lottery that gives equal probability of access to all individuals who desire access. This allocation will fail to maximize total surplus since it provides individuals with the same probability of access regardless of their personal values for the resource. In contrast, if monetary transfers were admissible, a Vickrey auction is predicted to yield an efficient allocation where only individuals with the highest value for the resource are granted access, thus maximizing total surplus.

In settings where several types of resources are assigned, or where a resource is allocated repeatedly over time, assignment mechanisms can use opportunity costs instead of monetary transfers to increase average surplus per assignment and, hence, increase total surplus. Here we study how successful a particular assignment mechanism has been at reducing the difference between the total surplus of an allocation obtained without relying on monetary transfers and the maximized total surplus that would be obtained if monetary transfers were

[1] In this paper we call an allocation efficient if it maximizes total surplus, i.e. the sum of valuations across all assignments.

admissible.

We study an allocation problem where access to several types of resources (positions) is being allocated, where each position can be assigned to a certain number of individuals, but where each individual can be assigned to at most one position. In addition, positions can be reallocated periodically among individuals. The assignment mechanism we study, which we call a dynamic lottery, endows participants with a stock of "preference points" that indexes their seniority. Every period, participants can apply to be assigned to a particular position. After all applications have been submitted, assignment is done separately for each position by reverse order of applicants' stocks of preference points, with random tie-breaking. New participants enter the lottery with zero preference points. A participant's stock of preference points increases by one if she is not assigned to a position in a given period and is reset to zero upon being assigned to a position. A participant can also apply directly for an increase in her stock of preference points instead of applying for a position.

The Michigan Department of Natural Resources (DNR) has used this dynamic assignment mechanism to allocate approximately 12,000 bear hunting permits for 22 different hunting sites every year since 2000. Because part of the mission of the DNR is to guarantee that natural resources are accessible to all, these hunting permits are offered at very low prices.[2] The DNR therefore faces significant excess demand for permits, with around 55,000 participants in its lottery every year.

Using data on participants and applications obtained from the DNR and a dynamic model of applicants' choices, we find that this dynamic lottery leads to a significant increase in total surplus compared to static alternatives, while total surplus also remains significantly below the maximized total surplus obtained by an efficient auction. We estimate that the average annual social surplus per applicant is $193 greater with the dynamic lottery than

[2]Another part of the DNR's mission is to manage natural resources, including wildlife, which leads to the use of permits in order to regulate access to hunting and guarantee that the bear population in the state is stable.

under random serial dictatorship,[3] and \$119 greater with the dynamic lottery than it would have been with a static lottery that requires applicants to simultaneously choose one position for which to apply but does not track seniority. An efficient auction would lead to an average surplus \$278 greater than the dynamic lottery.

We estimate a large degree of heterogeneity across hunters in their persistent preference for hunting. Hunters with high or low persistent preference for hunting both benefit from a greater average surplus under the dynamic lottery than under either static assignment mechanism without money, but this increase in total surplus originates from two different sources.

First, hunters with a relatively low preference for hunting are assigned to a permit less frequently under the dynamic lottery, but their surplus per permit assigned is greater. This is explained by the intertemporal opportunity cost of obtaining a permit elicited by the dynamic lottery which does not appear in a static assignment mechanism. The dynamic lottery imparts value to seniority by prioritizing applicants with higher stocks of preference points when allocating permits. Participants in the dynamic lottery are then faced with either obtaining a permit but seeing their seniority reset to zero or abstaining in order to benefit from an increase in seniority and a potentially more advantageous assignment in the future. This creates an option value of waiting for a future assignment which mimics the effect of market clearing prices for access, forcing applicants to be more selective about when and where to apply for assignment.

Second, hunters with a relatively high preference for hunting have an average surplus per permit approximately equal to their average surplus per permit under the static lottery or random serial dictatorship. However, these high-preference hunters are assigned to permits more frequently under the dynamic lottery. This is explained by the way in which applicants' increased selectivity and short waiting times interact with heterogeneity across participants

[3]Under random serial dictatorship, every year applicants are randomly assigned to a place in a queue and choose in an unrestricted way among positions that are still available after all applicants ahead of them have made their choices.

and across types of resources. In the dynamic lottery, waiting times even for the most desirable sites are kept relatively short by prioritizing applicants with the highest seniority. As a result, applicants with low preference for hunting increase the concentration of their applications to high desirability sites, preferring to wait for such an assignment rather than being assigned a permit at a less desirable site immediately. This “frees up” less-desirable sites, which are still highly valuable to applicants who have a high preference for hunting.

Using a real world application and a centralized assignment mechanism which is already being implemented, our results provide empirical evidence on the magnitude of efficiency gains obtained by a dynamic assignment mechanism which rations applicants’ assignments by using a dynamically evolving budget in an artificial currency (preference points). A large body of theoretical results points to the usefulness of leveraging opportunity costs when facing allocation problems without monetary transfers and in the presence of private information. Hylland and Zeckhauser (1979) show that introducing an artificial currency can transform the static allocation of many individuals to several positions into a pseudo-market and lead to a Pareto optimal assignment.[4] Jackson and Sonnenschein (2007) show that linking a large number of assignment problems and introducing rationing can lead to an efficient allocation in a simplified setting.[5] Guo and Hörner (2017) consider the allocation of a good by a principal to an agent whose valuation of the good is private information and evolves stochastically over time. They show that an optimal allocation mechanism in their setting provides the agent with a dynamically evolving budget in an artificial currency which decreases when the agent obtains the good and increases if the agent abstains. This feature of their proposed mechanism is remarkably similar to the dynamic lottery we study where a participant’s stock of preference points increases by one if the participant is not assigned and is reset to zero otherwise. In our application we see a rich market with several participants estimated to be of different types and several types of resources being allocated. We show

[4]See also He et al. (2017) for additional results and their review of existing results on no-transfer allocation mechanisms that account for intensity of preferences.

[5]See also Radner (1981), Townsend (1982), Rubinstein and Yaari (1983). See also Fang and Norman (2006) for a similar result in the context of bundling.

that these two dimensions of heterogeneity have a first order effect on the features of the allocation obtained by the dynamic lottery.

The repeated allocation of several resources to several individuals has been studied as a dynamic one-sided matching problem with private information in Arnosti and Shi (2017), Bloch and Cantala (2017), Leshno (2017), Schummer (2016), and Thakral (2016). This prior work also shows that assignments that create intertemporal trade-offs (or, in other words, increase the option value of waiting for a future assignment) can induce applicants to be more selective, leading to an increase in match quality. Arnosti and Shi (2017) find in their setting that there is necessarily a trade-off between what they refer to as matching and targeting. In our application this would translate into mechanisms either ensuring that participants of each type of persistent preference for hunting extract a higher average surplus per permit (matching), or ensuring that hunters with a higher preference for hunting obtain permits more frequently (targeting). As described above, we find in our application that the dynamic lottery improves upon a static lottery or random serial dictatorship along both matching and targeting. We show that this difference is explained by hunting sites being heterogenous in our application, leading to a sorting of applicants by type across hunting sites, and this sorting is more effective with a dynamic lottery than with a static lottery.

A related empirical literature studies one-sided and two-sided static matching mechanisms. Abdulkadiroğlu, Agarwal, and Pathak (2017), Agarwal and Somaini (2017), Agarwal (2015), Calsamiglia, Fu, and Güell (2017), Fack, Grenet, and He (2017), Hastings, Kane, and Staiger (2009), He (2017), and Narita (2016) evaluate empirically two-sided matching mechanisms used for school and medical residency assignment. Li (2017) compares the welfare gains under a static lottery and an auction when allocating automobile licenses in the presence of negative externalities.

Although to our knowledge no other empirical work evaluates the potential of an existing dynamic assignment mechanism to mitigate the problem of inefficient resource allocation when money cannot be used as a medium of exchange, related empirical results on dynamic matching are found in work contemporary to ours. Thakral (2016) studies the allocation of affordable housing and estimates a static model of choices as in Geyer and Sieg (2013) with data originating from assignments by a unique centralized waitlist with limited deferrals. He predicts that implementing an assignment mechanism where agents are placed on a centralized waitlist and can choose between the first assignment offered to them and switching to a housing site specific waitlist would lead to a significant increase in total surplus. Waldinger (2018) also studies the allocation of affordable housing, and uses a dynamic model of applicants' choices. He studies the importance of his mechanism having an intermediary stage where information is revealed to applicants; this consideration is absent in our study. Finally, Agarwal et al. (2018) study the allocation of donor transplant organs using a unique centralized waitlist with deferrals. The allocation of donor transplant organs creates several medical and ethical considerations which are considered carefully in Agarwal et al. (2018) but are not present in our application where the only welfare function considered is total surplus (with equal weights).[6]

In the following section, we provide background on bear hunting in Michigan. We then describe our model of application choice under a dynamic lottery for bear permits and our estimation approach. Finally we compare the dynamic lottery to alternative allocation mechanisms.

[6] Apart from methodological differences, these empirical studies differ from ours because they consider applications where new units arrive continuously over time rather than periodically. The dynamic lottery we study here could be modified by tracking seniority continuously rather than discretely and allowing participants to update their application choice continuously rather than periodically, while still restricting them to apply for only one type of resource at a time. It would be interesting to evaluate the extent to which the insights of our paper would apply in other applications. For instance, allowing participants to update their choice of housing site at any time could increase the option value of waiting and lead to higher surplus per assignment relative to housing site-specific waitlisting if participant valuations of different housing sites are at least partially transient. Likewise, allowing participants to queue for organs from donors of different ages could improve sorting and decrease waiting times relative to a unique centralized waitlist by allowing high-need patients to queue for organs from older donors while patients who can afford to wait longer queue for organs from younger donors.

2 Black Bear Hunting in Michigan

Bears (and other wildlife) in the US are held in the public trust. An important implication of managing resources under this doctrine is ensuring equitable access, regardless of socioeconomic status. Accordingly, resource management agencies sell permits for accessing these resources at very low prices. Permit quotas are fixed and determined independently of price to avoid excessive exploitation and ensure healthy resource stocks. Low prices generally result in excess demand for permits, and many agencies rely on lotteries to allocate permits.

Since 2000, the Michigan DNR has allocated permits for black bear hunting via a dynamic lottery. Hunts take place in late summer through mid-autumn. The black bear range in Michigan is divided into ten bear management units (BMUs; Figure 1). The Lower Peninsula BMUs each host a single hunt lasting one week in mid-September. Drummond Island, on the eastern tip of the Upper Peninsula, hosts a single hunt that lasts six weeks over September and October. The remaining Upper Peninsula BMUs each host three hunts over the course of the autumn. There are 22 total hunts each year (Table 2). Hunt quality characteristics vary across each hunt and BMU; Table 1 summarizes several key characteristics of each BMU, including population, total forest land open to hunting (the sum of private commercial forest and state forest and wildlife area acreage), the number of hunts each BMU hosts, the season duration, and mean success rate (i.e., the proportion of hunters who took a bear).

Applicants for hunting permits pay an application fee and a license fee up front. The application fee is $4 for all applicants except Comprehensive Lifetime License holders, for whom the application fee is waived. License fees are $15 for Michigan residents and $150 for nonresidents. Unsuccessful applicants are refunded the license fee, but not the application fee. Every year, an applicant enters the drawing with a stock of preference points. Applicants have zero preference points when entering the drawing for the first time. An unsuccessful application increases an applicant's stock by one, and a successful application resets this stock to zero. Greater preference point stocks lead to greater success probabilities; we describe this mechanism in detail in the next section. Applicants can also apply for the "preference point-only" option. Those who take this option are automatically awarded a preference point for use in future drawings and pay only the $4 application fee, but cannot hunt in the current season. With the preference-point only option, an applicant's choice set comprises 23 options.[7]

The permit quota for each hunt ranges from only 6 per year in the Drummond Island BMU to 1,850 for the Red Oak BMU (Table 2). For BMUs with multiple hunts, the quota increases for seasons that open later in the year, and application numbers tend to decrease for these later hunts. This is likely because bears are less active later in the autumn, and because bears become more easily "spooked," or more difficult to hunt, as the season wears on. The license quota for each hunt is made available on the DNR's website before the drawing, along with the previous year's drawing success rate for each hunt conditional on one's stock of preference points.

A total of 55,454 and 56,762 individuals participated in Michigan's preference point lottery for bear permits in 2008 and 2009, respectively. Data describing each applicant—including hunt choices, addresses, and their preference point stock—were provided by the DNR. Figure 2a shows a histogram of applicants' preference point stocks in 2009; most applicants have fewer than three preference points, and only two applicants have ten—the maximum possible. Of those who applied for the 2008 hunt, 16,021 (28.89 percent) did not apply in 2009.[8]

We estimate round-trip travel costs to each BMU for each applicant using US Census data

[7]The bear permit drawing in Michigan is actually divided into two rounds; applicants apply for a first and second choice of hunt before the drawing takes place. If an applicant is not drawn for her first choice, then she is entered in the drawing for her second choice if any permits for that hunt remain after the first round. We model only a single round here. Including a second round makes estimation infeasible due to large number of choice alternatives it implies. (The choice set balloons from 23 alternatives if considering the first round only to 485 if considering both rounds). However, fewer than half of applicants even entered a second choice on the 2009 application, and approximately 2 percent of those that did were awarded a permit. Hence, ignoring the second round is unlikely to have a significant effect on our results.

[8]Applicants who do not apply for three consecutive years forfeit their stock of preference points. We assume applicants who do not apply in 2009 after applying in 2008 never wish to hunt again. This is because it is inexpensive to apply for the preference point-only option and, hence, maintain one's stock of preference points.

on median annual income for each applicant's ZIP code. We assume the opportunity cost of travel time to be one third of each applicant's inputed hourly wage, following Parsons (2003). This opportunity cost is then multiplied by the travel time to the BMU, which was calculated using PC*Miler (ALK Technologies, Inc., 2015).[9] To this cost were added the application fees and mileage costs calculated for a four-wheel drive truck (American Automobile Association, 2009). Figure 2b shows a histogram of applicants' round-trip travel costs to each BMU. The vast majority are less than \$750 trip per trip, although the distribution is skewed by out-of-state hunters whose costs total more than \$1,500 per trip in some cases.

3 Equilibrium Application Choices

In this section, we develop a model for applicants' choices under a dynamic lottery. Before defining our model formally, we discuss the modeling choices we made and how they were informed by four main patterns observed in the data (summarized in Table 3):

1. Forward looking behavior: At low levels of preference points, we observe a significant share of applicants who could obtain a permit in the current drawing but instead apply either for the preference-point only option or for a permit at an "impossible" site, i.e. a site where they have no chance of obtaining a permit. We interpret this as evidence that applicants are willing to trade current hunting opportunities for future opportunities.

2. Hunting site heterogeneity: Some sites require several years of waiting before applicants can obtain a permit, while other sites have an excess supply of permits. This remains true even when only considering sites with similar numbers of allocated licenses. We interpret this as evidence that some hunting sites may be systematically more desirable

[9]Our data are limited in that we do not have information on where each applicant hunted within a given BMU. We therefore calculate travel time based on the distance between the applicant's address and the BMU centroid. Many of the BMUs are very large (e.g., the Red Oak BMU in the Lower Peninsula; Figure 1), and hence there is likely to be considerable error in our travel cost estimates. That said, we discretize the travel cost data as part of our estimation procedure (described below). This should negate some (although likely not all) of this error.

than others.

3. Persistent heterogeneity in applicants' preference for hunting: The share of applicants applying for the preference point-only option increases with the stock of preference points. This result may seem surprising since as preference points increase, so do success probabilities, and most likely the quality of hunting sites that can be accessed. However, this pattern is consistent with some hunters being persistently less interested in hunting than others. Hunters with a lower preference for hunting would rarely apply for a permit and thus accumulate preference points. Hunters with a higher preference for hunting would apply for and obtain a permit more often, thus resetting their stock of preference points to zero more frequently.[10]

4. Equilibrium sorting: At low levels of preference points, a significant share of applicants applies for permits at sites that have low demand and, hence, a positive probability of success. At high levels of preference points, these sites draw a very small share of applications. We interpret this as evidence that applicants evaluate both the desirability of a site and their probability of success when applying. In particular, applicants apply more frequently for less-desirable permits when having no chance of winning other, more desirable permits, but tend to shun these less desirable permits when they have a positive success probability for other sites.

We model persistent heterogeneity in preferences for hunting by classifying applicants into discrete "types," denoted $\tau = 1, \ldots, \bar{\tau}$.[11] The utility that a type-τ applicant i receives from hunt $j \in \{1, ..., J\} = \mathcal{J}$ in year t is assumed to be

$$\chi_{0j} + \eta_{0\tau} + \mu_0 TC_{ij} + \epsilon_{ijt} \tag{3.1}$$

[10]Note that since the cost of staying in the lottery is very small, and exiting the lottery would reset an applicant's stock of preference points to zero, it can be optimal for a hunter to stay in the lottery without applying for a permit for a long period of time.

[11]The use of discrete types to capture persistent unobserved heterogeneity in dynamic discrete choice models has been advocated for in Heckman and Singer (1984) and subsequent papers.

where χ_{0j} represents the baseline utility from hunt j; $\eta_{0\tau}$ is the type-specific utility from hunting, where η_{01} is normalized to zero; $-\mu_0$ is the marginal utility of income; TC_{ij} is the travel cost of applicant i to site j; ϵ_{ijt} is an independent and identically distributed (i.i.d.) type-1 extreme value preference shock for hunting at site j.

In order to simplify notation, and since we will assume that the environment is stationary over time, we suppress the indexing by time and write the utility of type-τ applicant i for hunt j as $\chi_{0j} + \eta_{0\tau} + \mu_0 TC_{ij} + \epsilon_{ij}$.

An applicant choosing to apply for a permit at site j faces a probability of obtaining a permit that is determined by her stock of preference points p; we denote this probability ϕ_{jp}. We collect success probabilities across all sites and preference point stocks and define $\boldsymbol{\phi} = \{\phi_{jp}\}_{j\in\mathcal{J},\forall p}$. The expected flow utility that applicant i derives from applying for hunt j is

$$\phi_{jp}\left(\chi_{0j} + \eta_{0\tau} + \mu_0 TC_{ij} + \epsilon_{ij}\right) = v_{i\tau jp}\left(\boldsymbol{\theta}_0\right) + \phi_{jp}\epsilon_{ij} \tag{3.2}$$

where $\boldsymbol{\theta}_0 = \left\{\{\chi_{0j}\}_{j\in\mathcal{J}}, \{\eta_{0\tau}\}_{\tau=2,\ldots,\bar{\tau}}, \mu_0\right\}$ collects the unknown parameters of the model and we define $v_{i\tau jp}\left(\boldsymbol{\theta}_0\right) = \phi_{jp}\left(\chi_{0j} + \eta_{0\tau} + \mu_0 TC_{ij}\right)$.

Once applications have been submitted by all participants, the success probability ϕ_{jp} is determined by an allocation of permits by reverse order of preference points among applicants for hunt j, with random tie-breaking if needed. Formally, let N_p be the number of applicants with p preference points and σ_{jp} be the share of participants with p preference points who apply for site j. Success probabilities are then

$$\begin{aligned}\phi_{jp} =& 1\left(\sum_{p'\geq p} N_{p'}\sigma_{jp'} \leq q_j\right)\\ &+ 1\left(\sum_{p'=p} N_{p'}\sigma_{jp'} > q_j, \sum_{p'\geq p+1} N_{p'}\sigma_{jp'} \leq q_j\right)\frac{q_j - \sum_{p'\geq p+1} N_{p'}\sigma_{jp'}}{N_p\sigma_{jp}}\end{aligned} \tag{3.3}$$

where q_j is the permit quota for hunt j and $1(\cdot)$ is an indicator function that takes a value of one if the argument is true and zero otherwise. Consider an applicant with p preference points. If the number of applicants with p or more preference points, given by $\sum_{p' \geq p} N_{p'}\sigma_{jp'}$, is less than the quota for hunt j, then the first right-hand side term in (3.3) evaluates to one and the second indicator function evaluates to zero, so that the applicant receives a permit with certainty. If the number of applicants with p or more preference points is greater than the quota, but the number of applicants with more than p preference points is less than the quota (so that the second right-hand side indicator function evaluates to one and the first indicator function evaluates to zero), then tie-breaking is performed by a simple lottery for all permits that remain after the applicants with more than p points are issued permits. The probability of winning a permit is then $\left(q_j - \sum_{p' \geq p+1} N_{p'}\sigma_{jp'}\right)/N_p\sigma_{jp}$. If neither of these conditions hold, then the probability the applicant wins a permit is zero.

Since in our application we observe many applicants ($> 55,000$ in any year) choosing over relatively few sites (22), the success probabilities calculated using realized application shares in (3.3) are approximately equal to the ex-ante success probabilities that are obtained by taking the expectation of the success probabilities in (3.3) with respect to the distribution of application shares. This allows us to treat the success probabilities defined in (3.3) as the success probabilities used by each agent to inform their choice, and simplifies the estimation procedure described in the next section. Observing many participants across relatively few sites also allows us to treat applicants as "probability takers," i.e. making their application choice taking ϕ_{jp} as given.

We assume that the environment is stationary over time. This implies that quotas $q_j \; \forall j$, application shares $\sigma_j \; \forall \, j, p$, the number of applicants per preference points stock $N_p \; \forall p$, and success probabilities $\boldsymbol{\phi}$ are constant over time and known by applicants. To evaluate the assumption of stationarity, we study the change in success probabilities across the two years observed in our data (2008 and 2009). To do so, we define preference-point "cut-offs" for each site as the expected minimum level of preference points required to obtain a permit at each site. For instance if a site j only yields a positive probability of success starting at three preference points, i.e. $\phi_{j2} = 0$ and $\phi_{j3} > 0$ and $\phi_{j4} = 1$, its cut-off would be given by

$3 \cdot \phi_{j3} + 4 \cdot (1 - \phi_{j3})$, reflecting the fact that for a fraction of applicants with three preference points the cut-off is three preference points, while for the rest of these applicants the cut-off is four preference points (and hence above their current stock of preference points, so that they will not be successfully drawn this year). These cut-offs contain all of the information contained in $\boldsymbol{\phi}$ and can heuristically be interpreted as the expected minimum "price" in preference points of each site. Table 4 shows these expected cut-offs for each site and each year in our data. We see that the cut-offs remain approximately constant across both years, supporting the assumption of stationarity. The assumption that applicants know their success probabilities for each site also seems realistic given that the DNR posts results from past drawings every year.

We normalize the flow utility from applying for the preference point-only option to zero; we denote this choice as $j = 0$.[12] The applicant's full choice set is then $\bar{\mathcal{J}} = \{0\} \cup \mathcal{J}$.

We assume that each applicant chooses the application path that maximizes the present value of expected utility. Formally, a type-τ applicant with p preference points chooses site j in the current period if $V_{i\tau jp}(\cdot) \geq V_{i\tau j'p}(\cdot)\ \forall j' \in \bar{\mathcal{J}}$, where

$$V_{i\tau jp}(\boldsymbol{\theta}_0) = v_{i\tau jp}(\boldsymbol{\theta}_0) + \rho E\left(\max_{j' \in \bar{\mathcal{J}}} V_{i\tau j'p'}(\boldsymbol{\theta}_0)\right) + \phi_{jp}\epsilon_{ij} \tag{3.4}$$

where the expected value is taken with respect to the joint distribution of future preference shocks $\left\{\epsilon'_{ij'}\right\}_{j' \in \mathcal{J}}$ and the distribution of next year preference points, p', conditional on having a stock of preference points p and on having chosen option j, and ρ is a discount factor.

In order for the definition of the value functions in (3.4) to be notationally correct for $j = 0$, we set $\phi_{0p} = 0$ so that $v_{i\tau 0p}(\boldsymbol{\theta}_0) + \phi_{0p}\epsilon_{ij} = 0$. This corresponds to the probability of obtaining a permit being zero if the applicant chooses the preference-point only option.

An applicant's preference points either (i) increase by one if she chooses the preference point-only option ($j = 0$) or is unsuccessful in her application for a permit ($j \in \mathcal{J}$) or (ii)

[12] Note that the flow utility from not hunting for a type-τ applicant could equivalently be denoted $-\eta_\tau$ — as long this term is then not included in the value of hunting — so that our model allows for applicants to have heterogeneous outside options.

are reset to zero if she receives a permit. Let $\Pi_{jpp'}$ denote the probability that an applicant with p preference points who chooses option j transitions to having p' preference points the following year, we have:

$$\Pi_{jpp'} = \begin{cases} \phi_{jp} & \text{for } p' = 0 \\ 1 - \phi_{jp} & \text{for } p' = p + 1 \\ 0 & \text{otherwise.} \end{cases} \tag{3.5}$$

Define $R_{i\tau p}(\boldsymbol{\theta}_0) = E\left(\max_{j \in \bar{\mathcal{J}}} V_{i\tau jp}(\boldsymbol{\theta}_0)\right)$, where the expectation is now only taken with respect to the joint distribution of $\{\epsilon_{ij}\}_{j \in \mathcal{J}}$. Then we can write

$$V_{i\tau jp}(\boldsymbol{\theta}_0) = v_{i\tau jp}(\boldsymbol{\theta}_0) + \rho \sum_{p'} \Pi_{jpp'} R_{i\tau p'}(\boldsymbol{\theta}_0) + \phi_{jp}\epsilon_{ij} \tag{3.6}$$

The second right-hand side term in (3.6) captures the present value of expected future utility for each choice j. The term $R_{i\tau p'}$ is the conditional value term, which measures the expected maximized value of i's lifetime utility from hunting conditional on her updated stock of preference points, p'.

Define $V^b_{i\tau jp}(\boldsymbol{\theta}_0)$ to be the baseline value function associated with choice j:

$$V^b_{i\tau jp}(\boldsymbol{\theta}_0) = v_{i\tau jp}(\boldsymbol{\theta}_0) + \rho \sum_{p'} \Pi_{jpp'} R_{i\tau p'}(\boldsymbol{\theta}_0) \tag{3.7}$$

Given $V^b_{i\tau jp}(\boldsymbol{\theta}_0)$, the probability an applicant chooses j is:

$$\begin{aligned} \Pr\left(V_{i\tau jp} \geq V_{i\tau j'p} \,\forall\, j'\right) &= \Pr\left(V^b_{i\tau jp} + \phi_{jp}\epsilon_{ij} \geq V^b_{i\tau j'p} + \phi_{j'p}\epsilon_{ij'} \,\forall\, j'\right) \\ &= \Pr\left(V^b_{i\tau jp} - V^b_{i\tau j'p} \geq \phi_{j'p}\epsilon_{ij'} - \phi_{jp}\epsilon_{ij} \,\forall\, j'\right) \end{aligned} \tag{3.8}$$

where we suppress the value function arguments for conciseness.

Multiplying the ϵ_{ij} terms in (3.8) by the success probabilities implies a form of structural heteroscedasticity in our model. Hence, the composite error terms $\phi_{jp}\epsilon_{ij}$ are not i.i.d. type-1 extreme value, and the choice probability (3.8) cannot be written using the standard closed-form expression for a conditional logit.[13] Under our distributional assumption on the ϵ_{ij} terms, the probability that a type-τ applicant i with p preference points applies for a permit at site j with $\phi_{jp} > 0$ is

$$\begin{aligned} P_{i\tau jp}(\boldsymbol{\theta}_0) &= \Pr\left(V^b_{i\tau jp} - V^b_{i\tau j'p} \geq \phi_{j'p}\epsilon_{i\tau j'p} - \phi_{jp}\epsilon_{i\tau jp}\, \forall\, j'\right) \\ &= \int_{\epsilon \geq \frac{V^b_{i\tau 0p} - V^b_{i\tau jp}}{\phi_{jp}}} \prod_{j' \in \mathcal{J}: j' \neq j, \phi_{j'p} > 0} \Lambda\left(\frac{V^b_{i\tau jp} - V^b_{i\tau j'p} + \phi_{jp}\epsilon}{\phi_{j'p}}\right) \lambda(\epsilon)\, d\epsilon \end{aligned} \tag{3.9}$$

where $\Lambda(x) = e^{-e^{-x}}$ and $\lambda(x) = e^{-x}e^{-e^{-x}}$ are the cumulative distribution function and probability density function of the type-1 extreme value distribution.

Note that for any stock of preference points p, sites for which $\phi_{jp} = 0$ yield the same present value of expected utility as the preference point-only option ($j = 0$). Hence, we treat these choices as corresponding to the same alternative and write the probability of applying for the preference point-only option or for a permit at a site with $\phi_{jp} = 0$ as

$$P_{i\tau 0p}(\boldsymbol{\theta}_0) = \prod_{j \in \mathcal{J}: \phi_{jp} > 0} \Lambda\left(\frac{V^b_{i\tau 0p} - V^b_{i\tau jp}}{\phi_{jp}}\right) \tag{3.10}$$

Finally, we can write the conditional value term as

$$\begin{aligned} R_{i\tau p}(\boldsymbol{\theta}_0) &= E\left(\max_{j \in \bar{\mathcal{J}}} V^b_{i\tau jp} + \phi_{jp}\epsilon_{ij}\right) = V^b_{i\tau 0p} P_{i\tau 0p} + \\ &\int_{r > V^b_{i0p}} r \sum_{j \in \mathcal{J}: \phi_{jp} > 0} \frac{1}{\phi_{jp}} \lambda\left(\frac{r - V^b_{i\tau jp}}{\phi_{jp}}\right) \prod_{j' \in \mathcal{J}: j' \neq j, \phi_{j'p} > 0} \Lambda\left(\frac{r - V^b_{i\tau j'p}}{\phi_{j'p}}\right) dr \end{aligned} \tag{3.11}$$

Equations (3.7) and (3.9)–(3.11) comprise our model of applicants' choices in the dynamic lottery. The unknown parameters that enter the functions $P_{i\tau jp}$, $V^b_{i\tau jp}$, and $R_{i\tau p}$ are those contained in $\boldsymbol{\theta}_0$: $\chi_{0j}\ \forall j$, μ_0, and $\eta_{0\tau}\ \forall \tau$. The observed data entering these functions are

[13] Bhat (1995) obtains similar formulae for choice probabilities. In his setting the variances of shocks across alternatives are parameters to be estimated. Here, it is agents choosing over uncertain lotteries that leads to a heteroscedastic formulation.

individual travel costs and the success probabilities $\boldsymbol{\phi}$, which are calculated from application shares, quotas, and the number of applicants at each level of preference points using equation (3.3). We take the discount factor ρ as known. Brookshire, Eubanks, and Randall (1983) find evidence of discount factors for big game hunting opportunities ranging from $\rho \in [0.95, 0.99]$. We therefore assume $\rho = 0.975$. The next section describes estimation of these parameters and of the distribution of unobserved types in the population.

3.1 Estimation

Let $\pi_{0\tau}$ denote the probability that applicants are of type τ, $\tau = 1, ..., \bar{\tau}$ and define $\boldsymbol{\pi}_0 = \{\pi_{0\tau}\}_{\tau=1,...,\bar{\tau}}$. We make three more assumptions in order to propose an estimation method for the unknown parameters of the model $\boldsymbol{\theta}_0$ and the distribution of applicant types in the population $\boldsymbol{\pi}_0$. First, we assume that the probability that an applicant is of type τ is independent of her travel costs: $P(\text{i's type} = \tau | TC_{ij} \, \forall \, j) = \pi_{0\tau}$. Second, we assume that the distribution of applicants' types across stocks of preference points corresponds to the steady state of the environment. Finally, recall that we observe some applicants leaving the lottery between 2008 and 2009 and approximately the same number of applicants entering the lottery in 2009 for the first time. We model this phenomenon as attrition and renewal at random and define the attrition rate α as

$$\alpha = \frac{\text{\# of applicants present in 2008 but not in 2009}}{\text{\# of applicants present in 2008}}.$$

Section A of the online appendix shows that the model of applicants' choices developed in Section 3 can be used to define the likelihood function corresponding to each agent i's choices in the lottery in 2008 and 2009 conditional on her travel costs and stock of preference points in 2008. This likelihood function is composed of the choice probabilities (3.9) and (3.10), for which the value functions are calculated using a fixed-point algorithm as in Rust (1987), a law of motion for preference points from 2008 to 2009, and the probability that an applicant is of each type $\tau = 1, ..., \bar{\tau}$ given her travel costs and stock of preference points in 2008. We therefore estimate the parameters $\boldsymbol{\theta}_0$ and $\boldsymbol{\pi}_0$ by maximum likelihood estimation. All details are provided in Section A of the online appendix.

Our parameter estimates are presented in Table 5. We assume three different types of applicants, denoted $\tau = 1, 2, 3$. The type with the highest preference for hunting is type $\tau = 1$, for which η_{01} is normalized to zero. For this type, the baseline site utility χ_{0j} is estimated to be positive for all sites. Type $\tau = 2$ has a lower preference for hunting and for this type the baseline site utility $\chi_{0j} + \eta_{02}$ is positive for the most desirable sites and negative for the less desirable sites. Type $\tau = 3$ has a very low preference for hunting, indeed guaranteeing that the estimated probability that an applicant of type $\tau = 3$ applies for a permit is approximately zero. The marginal utility of income is estimated to be positive — the expected sign — and statistically significant at the 1% level. We estimate that the majority (79.8%) of applicants are of type $\tau = 2$, with 6.6% of applicants having a high preference for hunting (type $\tau = 1$) and 13.6% of applicants having a very low preference for hunting (type $\tau = 3$).

The estimated heterogeneity in hunting sites aligns well with observed site characteristics that are expected to make some hunting sites more desirable than others. For instance, the sites that are estimated to yield the greatest baseline utility tend to have the greatest hunters' success rates (see Table 1). Early seasons also seem to be favored by hunters, as well as sites with more land available and with larger human populations (likely because these areas offer more amenities to visiting hunters). In ongoing work, we use a model similar to the one developed here to estimate applicants' willingness-to-pay for sites to remain open rather than closing given that permits are allocated by the dynamic lottery, and we study the importance of different features of our model for calculating these measures accurately. Here we are interested in the welfare implication of using the dynamic lottery to allocate permits instead of a static assignment mechanism. Before presenting our results on the efficiency of the dynamic lottery, we discuss the advantages and shortcomings of our model to study this question.

3.2 *Discussion of the Model*

The main advantage of the model outlined above for the analysis of a dynamic matching mechanism is that it separates transitory heterogeneity (ϵ_{ij} in the equations above), which is drawn every period, from persistent heterogeneity (η_τ for applicants of type τ in the equations above), which is constant over time. The next section discusses in detail how intertemporal trade-offs leveraged in dynamic matching mechanisms induce participants to only request access when their valuation for the resource being allocated is high, while abstaining when it is low. A simple example of this phenomenon is found in Jackson and Sonnenschein (2007). They consider a setting where there is only one type of agent (i.e. no persistent heterogeneity), and where transitory heterogeneity is binary. With only one type of resource and no discounting, they show that agents endowed with a budget for access over relatively many time periods would approximately only request the resource when their valuation for it is high. The presence of persistent unobserved heterogeneity across participants would lead to an inefficient allocation since participants with a low valuation for the resource would have an incentive to report being of a high preference type in order to obtain more frequent access. Therefore using an empirical model with both persistent and transitory unobserved heterogeneity is key for evaluating the efficiency of dynamic matching mechanisms, particularly given the observed patterns discussed above that are indicative of persistent heterogeneity among participants. This is discussed in more details in the next section.

From an econometric perspective, the distribution of persistent heterogeneity is identified with our data because i) applicants are observed making choices repeatedly over two time periods, and ii) we observe the applicants' stock of preference points, which provides information on their unobserved preference type. An applicant's stock of preference points evolves according to a known and simple law of motion: the stock increases by one if an applicant is not assigned a permit and is reset to zero otherwise. Therefore applicants with a high preference for hunting, who will tend to apply for a license more frequently, will see their stock of preference points reset to zero more frequently, while applicants with a low preference for hunting will tend to accumulate more preference points. Consequently comparing choice probabilities at high stocks of preference points with choice probabilities at low stocks of preference points will allow one to learn both about the choice probability of each type and the probability of participants being of each type in the population. The assumption that the environment is stationary and in its steady state is important for our ability to estimate a dynamic discrete choice model using only two time periods, but seems to be supported by the data as discussed previously.[14] Section A of the online appendix discusses identification in more detail. The sources of identification here highlight the importance of using data from an existing dynamic matching mechanism to evaluate its efficiency, or the efficiency of alternative dynamic matching mechanisms, rather than extrapolating from static matching mechanisms.

Another advantage of our model is that it accounts for heterogeneity across the resources being allocated. Arnosti and Shi (2017) derive theoretical results in a setting with heterogenous participants but homogenous resources. The next section discusses the interaction between heterogeneity across resources and across participants, and the importance of accounting for both dimensions of heterogeneity simultaneously.

On the other hand, our model makes many simplifying assumptions. We assume that the disutility of traveling to a hunting site is equal to the disutility of income decreasing by the monetary cost of traveling to the site, calculated as the sum of the direct travel cost and of the opportunity cost of travel time. In the next section, we use this assumption to calculate total surplus in dollars by calculating compensating variations as in Small and

[14]As discussed in Section A of the online appendix, assuming that the environment is in its steady-state allows us to infer the probability of each applicant type conditional on the applicant's stock of preference points solely from the choice probabilities of each type and the unconditional probability of applicant type in the entire population. See e.g. Kasahara and Shimotsu (2009), Hu and Shum (2012) and Arcidiacono and Miller (2017) for a discussion of identification in dynamic discrete choice models without stationarity.

Rosen (1981). This assumption that participants could be compensated for traveling to a hunting site by receiving a payment equal to their travel cost is standard in the valuation of natural resources (see .e.g. Haab and McConnell (2002)), so that we adopted it here as well.[15]

From an econometric standpoint, the disutility of travel cost is identified separately from site heterogeneity because applicants are distributed geographically, so that we can compare the choice probabilities of applicants across space. As discussed above, we estimate a negative and statistically significant disutility of travel cost, reflecting that applicants' choices vary geographically and tend to favor nearby sites.

The assumption that travel cost is independent of preference type is made for simplicity. Correlation between travel cost and preference type could be accommodated by allowing type probabilities to vary across space. Modeling attrition as unexpected and at random is also done for simplicity. Results when attrition is expected are similar to the results presented here.

4 Efficiency of the Dynamic Lottery as an Assignment Mechanism

In this section we evaluate the relative efficiency of the dynamic lottery as an assignment mechanism by comparing its equilibrium allocation with the equilibrium allocation obtained by two alternative static assignment mechanisms without money and with the equilibrium allocation obtained by an efficient auction.

Section B of the appendix discusses how our estimated choice model can be used to calculate characteristics of the equilibrium allocation of the dynamic lottery such as total surplus, or probabilities that applicants of each type obtain a permit. The next three subsections introduce the alternative assignment mechanisms we consider, and we conclude this section

[15]In contrast Abdulkadiroğlu, Agarwal, and Pathak (2017) reported welfare results in terms of "willingness to travel" rather than willingness to pay when evaluating a two-sided static matching mechanism used to assign students to schools. It is possible that, among high-school students, the willingness to pay for shorter travel times may not be accurately captured by foregone income since these students are not active in the labor force. Reporting welfare results in terms of willingness to pay seems more plausible in our empirical application where the majority of applicants (85%) is of working age.

with a comparison of the four resulting equilibrium allocations.

4.1 Efficient Allocation by Auction

We start by defining an efficient auction which leads to an equilibrium allocation that maximizes total surplus. Given (3.2) and our marginal utility estimates from Section 3.1, we can write applicant i's willingness to pay (WTP) for a permit to hunt j as:

$$w_{ij} = \frac{\chi_{0j} + \eta_{0\tau_i} + \epsilon_{ij}}{-\mu_0} - TC_{ij} \tag{4.1}$$

where τ_i denotes the applicant's type. Let $\{a_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$ denote some sequence of assignments of applicants to permits such that $a_{ij} = 1$ if applicant i obtains a permit for hunt j and $a_{ij} = 0$ otherwise. This assignment is efficient if it solves:[16]

$$\max_{\{a_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}} \sum_{i,j\in\mathcal{N}\times\mathcal{J}} a_{ij} w_{ij} \tag{4.2}$$

$$\text{s.t.} \sum_{j\in\mathcal{J}} a_{ij} \leq 1 \qquad \text{and} \qquad \sum_{i\in\mathcal{N}} a_{ij} \leq q_j \tag{4.3}$$

A social planner cannot solve for this optimal allocation directly since applicants' valuations w_{ij} are private information. Leonard (1983) shows that an assignment mechanism that is incentive compatible and solves problem (4.2)–(4.3) can be achieved with an extension of a Vickrey (second bid price) auction. Demange, Gale, and Sotomayor (1986) show that the efficient allocation can be approximated arbitrarily well through a sequential procedure whereby prices are gradually increased until the market clears. We use simulations to obtain results for the equilibrium allocation of the efficient auction as well as for the other two alternative static mechanisms introduced below. We provide the details of our calculations

[16] We impose the constraint that each participant receive at most one permit as this constraint is imposed by the Michigan DNR when allocating bear hunting permits. (A participant cannot enter the dynamic lottery twice.) Similarly the number of permits assigned to each site may not exceed the quota set for this site by the Michigan DNR. Koopmans and Beckmann (1957) show that the solution to the constrained maximization problem (4.2)–(4.3) necessarily sets $a_{ij} = 0$ or $a_{ij} = 1$ so that we do not need to explicitly impose the constraint that assignment be a binary variable.

in Section B of the appendix.

4.2 Allocation by Random Serial Dictatorship

As a benchmark for evaluating allocation mechanisms without monetary transfers, we consider an assignment mechanism that has been widely studied in theoretical work called random serial dictatorship, see e.g. Abdulkadiroğlu and Sönmez (1998) for a review. This assignment mechanism is similar to the random assignment of one type of resource discussed in Section 1 but takes into account that several types of resources are allocated in our application and that each participant can be assigned at most one permit. Every year, all participants are assigned a place in a centralized queue at random. The first participant chooses any permit among all the permits being allocated. Subsequent participants choose any permit among those that remain after participants ahead of them have made their choice. This process continues until all permits have been allocated. A participant can also choose not to receive a permit, in which case the assignment mechanism simply moves on to the next participant. This mechanism is a static mechanism, i.e. the allocation of permits in any given year with this mechanism is independent of the allocation in other years.

4.3 Allocation by Static Lottery

As another alternative to the dynamic lottery used by the DNR, we consider a static lottery that does not track seniority among applicants. As in the dynamic lottery, applicants decide whether to apply for a permit for hunt j or whether to abstain from entering a drawing. In contrast to the dynamic lottery, however, the problem faced by applicants is static, i.e. their choice environment is not affected by past outcomes.[17]

For each hunt j, applicants face a probability of success ϕ_j which depends on the share of applicants that apply for this permit, σ_j. The success probability at each hunt is determined

[17]This assignment mechanism is similar to the mechanism described in Hylland and Zeckhauser (1979) except that here participants are required to choose one site for which to apply instead of being endowed with a budget of application weights that they can divide between several sites.

by $\phi_j = \min\left\{\frac{q_j}{\sigma_j N}, 1\right\}$, where q_j is the quota for hunt j (as before) and N is the total number of participants, so that $\sigma_j N$ is the number of participants who apply for site j.

Applicant i will apply for hunt j if it maximizes her expected welfare, i.e. if $\phi_j w_{ij} \geq \phi_{j'} w_{ij'} \,\forall\, j'$, where w_{ij} is applicant i's WTP for a permit to hunt at site j. Let

$$b_{ij}(\boldsymbol{\phi}) = \begin{cases} 1 & \text{if } \phi_j w_{ij} \geq \phi_{j'} w_{ij'} \,\forall j' \\ 0 & \text{otherwise} \end{cases}$$

denote applicant i's choice to enter the lottery for permit j given the success probabilities $\boldsymbol{\phi} = \{\phi_j\}_{j \in \mathcal{J}}$.

We can use applicants' best responses to success probabilities to define equilibrium success probabilities in the static lottery as

$$\boldsymbol{\phi}^\star = \left\{\frac{q_j}{\sigma_j^\star N}\right\}_{j \in \mathcal{J}} \tag{4.4}$$

$$\sigma_j^\star = \frac{1}{N}\sum_i b_{ij}(\boldsymbol{\phi}^\star) \,\forall\, j \in \mathcal{J} \tag{4.5}$$

from which the equilibrium allocation is obtained via an iterative procedure described in the appendix.

4.4 Comparison of Equilibrium Allocations

Table 6 compares expected total surplus for each assignment mechanism. It also compares average total surplus by applicant type, the probability of obtaining a permit unconditionally and by applicant type, average surplus per permit unconditionally and by applicant type, average waiting time before obtaining a permit unconditionally and by applicant type, and the probability of winning a permit for different quality sites under each mechanism. We summarize our results by categorizing each hunt into one of three tiers. The first tier includes permits for hunting sites that we estimate to have the highest baseline utility (the first season in the Newberry, Red Oak, and Baldwin BMUs). The third tier includes the permits we estimate to be the least desirable (the second and third seasons in the Bergland, Baraga, Amasa, Gwinn, and Carney BMUs and the third season in the Newberry and Gladwin BMUs). The second tier contains the remaining permits.

Note from Table 6 that type-3 applicants (with the lowest taste for hunting) neither apply for nor obtain permits for any hunt regardless of the assignment mechanism. Hence, the remainder of our discussion concentrates on type-1 and type-2 applicants.

Random serial dictatorship leads to the equilibrium allocation with the lowest annual average total surplus ($575), followed by the static lottery ($649) and the dynamic lottery ($768). The maximized total surplus achieved by the efficient auction is $1,046. The probability that a participant obtains a permit is slightly lower in the dynamic lottery (0.21) than in the other assignment mechanisms (0.22) because a few permits at the least desirable sites go unclaimed in the dynamic lottery. This difference is small enough that comparing assignment mechanisms in terms of average surplus or in terms of average surplus per permit is approximately equivalent. Average surplus per permit is $2,603 under random serial dictatorship, $2,942 in the static lottery, $3,578 in the dynamic lottery, and $4,742 in the efficient auction.

Average surplus per permit is obtained by weighting average surplus per permit for each type of applicant by the fraction of permits allocated to each type. The average surplus per permit of type-1 participants will generally be greater than that of type-2 participants. Hence, an allocation mechanism can increase average surplus per permit—and thus average surplus—compared to an alternative mechanism by yielding an allocation with either (i) a higher average surplus per permit for participants of a given type (improved selectivity) or (ii) a larger fraction of permits assigned to type-1 participants (improved targeting).[18] In the next sections we compare the equilibrium choice environments of each mechanism. In

[18]We choose to use the terms selectivity and targeting to refer to these two sources of efficiency gains. Arnosti and Shi (2017) use the term matching instead of selectivity, while we use the term matching to denote the general problem of allocating several individuals to several positions.

particular, we explain how the allocation efficiency of each mechanism is determined by the selectivity and targeting originating from these choice environments.

4.4.1 Improved Selectivity via Equilibrium Sorting

Compared to random serial dictatorship, the static lottery allocates a smaller fraction of permits to type-1 participants (0.21 instead of 0.22), but nevertheless yields a higher average surplus by allocating permits to type-2 participants for a higher surplus per permit ($2,278 instead of $1,710).[19] This efficiency-improving increase in selectivity follows from the static lottery eliciting a trade-off between site desirability and equilibrium success probability. With random serial dictatorship, the first applicant in the queue simply chooses the site that provides her with the highest welfare. The rest of the applicants face the same choice but only across sites for which permits remain. This leads to participants towards the front of the queue selecting sites with high baseline utility too frequently, crowding out participants further down the queue who might have a higher value for these sites but do not have a chance to obtain access. The static lottery mitigates this inefficiency by associating more desirable sites with lower success probabilities than less desirable sites. This leads applicants to be more selective about applying for more desirable sites than under random serial dictatorship.

To understand this phenomenon more precisely, consider the simplified case where only two sites are available and ignore all sources of persistent heterogeneity (i.e. applicant types and travel costs). Set the baseline utility of the first site to zero, and the baseline utility of the second site to $\chi_{0,2} > 0$. The willingness to pay of participant i for each site is then given in this simplified example by:

$$w_{i1} = \frac{1}{-\mu_0}\epsilon_{i1}, \qquad\qquad w_{i2} = \frac{1}{-\mu_0}\chi_{0,2} + \frac{1}{-\mu_0}\epsilon_{i2}$$

where as before ϵ_{i1} and ϵ_{i2} are independent of each other and i.i.d. type-1 extreme value

[19]The average surplus per permit for type-1 participants is lower under the static lottery than under random serial dictatorship because they avoid high quality sites, with all of their applications concentrated on third tier sites.

distributed. Suppose for simplicity that the permit quotas are equal at both sites and take value $q_1 = q_2 = P(max(\epsilon_{i1}, \epsilon_{i2}) \geq 0)\frac{N}{2}$ where N is the total number of applicants and is considered to be relatively large.

The efficient auction prices $p_1^\star$ and $p_2^\star$ in this simplified example satisfy approximately $p_2^\star - p_1^\star = \frac{1}{-\mu_0}\chi_{0,2}$ and $p_1^\star = 0$ so that a participant chooses site $j = 2$ if $\epsilon_{i2} - \epsilon_{i1} \geq 0$ and $\epsilon_{i2} \geq 0$.

Under random serial dictatorship, as long as permits at both sites are still available, applicants choose site $j = 2$ if $\epsilon_{i2} - \epsilon_{i1} \geq -\chi_{0,2}$ and $\epsilon_{i2} \geq -\chi_{0,2}$. This will lead to an allocation with a lower total surplus than the allocation obtained by the efficient auction since we are likely to find an applicant i who chose site $j = 2$ and who has $0 \leq \epsilon_{i2} < \epsilon_{i1}$ and an applicant $i^{'}$ further down the queue who has $\epsilon_{i^{'}2} \geq \epsilon_{i^{'}1} \geq 0$ but for whom no site 2 permits remain, leading applicant $i^{'}$ to obtain a permit at site $j = 1$. Reassigning applicant i to site $j = 1$ and applicant $i^{'}$ to site $j = 2$ would increase total surplus by $\frac{1}{-\mu_0}(\epsilon_{i1} - \epsilon_{i2} + \epsilon_{i^{'}2} - \epsilon_{i^{'}1}) > 0$.[20]

The static lottery mitigates this issue to some extent by requiring that all applicants submit an application for either site simultaneously. Applicant i chooses which site to apply for in order to maximize her expected welfare. She chooses site $j = 2$ if $\epsilon_{i2} - \epsilon_{i1} \geq (\frac{\phi_1}{\phi_2} - 1)\epsilon_{i1} - \chi_{0,2}$ and $\epsilon_{i2} \geq -\chi_{0,2}$, where ϕ_1 and ϕ_2 are the equilibrium success probabilities of each site. Because the baseline utility of site $j = 2$ ($\chi_{0,2}$) is strictly positive, there will be a higher demand for permits at site $j = 2$, so that $\frac{\phi_1}{\phi_2} > 1$. As a result, among participants who have $\epsilon_{i1} > 0$, an applicant will only apply for site $j = 2$ if $\epsilon_{i2} - \epsilon_{i1} \geq (\frac{\phi_1}{\phi_2} - 1)\epsilon_{i1} - \chi_{0,2} > -\chi_{0,2}$, i.e. applicants will be more selective about applying to from site $j = 2$ than they would be under random serial dictatorship.[21]

[20] An additional source of inefficiency arises from the possibility that applicant i with $-\chi_{0,2} \leq \epsilon_{i2} < 0$ might choose site $j = 2$ while an applicant $i^{'}$ further down the queue might have $\epsilon_{i2} > 0$ but might not have access to permits for site $j = 2$, leading her to choose the outside option $j = 0$ either because no permit remains or because $\epsilon_{i^{'}1} < 0$. Reassigning applicant i to the outside option and applicant $i^{'}$ to site $j = 2$ would increase total surplus by $\frac{1}{-\mu_0}(\epsilon_{i^{'}2} - \epsilon_{i2}) > 0$. This source of inefficiency is not addressed by the static lottery but is addressed by the dynamic lottery by using intertemporal opportunity costs as discussed in the next subsection.

[21] Participants who have $\epsilon_{i1} \leq 0$ face the same choice under random serial dictatorship and the static lottery of choosing to apply for site $j = 2$ if $\epsilon_{i2} \geq -\chi_{0,2}$ or choosing the outside option $j = 0$ otherwise.

4.4.2 Intertemporal Opportunity Costs as Prices

The dynamic lottery improves selectivity compared to the static lottery, with average surplus per permit being greater in the dynamic lottery for both types of participants.[22] Average surplus per permit for type-2 participants is \$2,887 under the dynamic lottery instead of \$2,278 under the static lottery, and average surplus per permit for type-1 participants is \$5,597 under the dynamic lottery instead of \$5,421 in the static lottery. The dynamic lottery yields an allocation with higher average surplus per permit for both types of active participants by rationing assignment over time, so that obtaining a permit today leads to a smaller chance of obtaining a permit in the future. This intertemporal trade-off results in participants being more selective about when and where to apply for a permit, preferring to abstain from applying for a permit if their value for hunting today is relatively low compared to their expected value of hunting in the future.

The increase in selectivity elicited by the dynamic lottery can be understood by considering first a simplified case. Suppose there is a single hunt. Set the baseline utility of this hunting site to zero and ignore all sources of persistent heterogeneity (i.e. applicant types and travel costs). Then applicant i's value for a permit is $w_i = \frac{\epsilon_i}{-\mu_0}$, where $\{\epsilon_i\}_{i \in \mathcal{N}}$ is i.i.d. type-1 extreme value. Let q be the permit quota for this single hunt.

The efficient auction in this simplified setting becomes a Vickrey auction which would lead to an equilibrium price for access of $p^* = w_{(q+1)}$, where $w_{(1)}, \ldots, w_{(N)}$ are the order statistics of $\{w_i\}_{i \in \mathcal{N}}$ and are assumed to be known to the auctioneer (since, in a Vickrey auction, participants are predicted to reveal their true valuations for the resource being allocated). This auction is efficient since applicants who obtain a permit have the greatest values for the hunt, $w_{(1)}, ..., w_{(q)}$.[23]

In the static lottery, applicants will apply for the permit as long as $w_i \geq 0$. Winning

[22] The dynamic lottery also obtains efficiency gains by improving targeting, i.e. increasing the fraction of permits allocated to type-1 participants. We discuss this second source of efficiency in the next subsection.

[23] Here we consider the case where more than q applicants have $w_i \geq 0$. Doing so implies excess demand, which is the only interesting case. If fewer than q applicants have value $w_i \geq 0$ for the permit, then all applicants interested in hunting can obtain a permit and any assignment mechanism would be efficient.

applicants will be randomly drawn from the pool $w_{(1)}, ..., w_{(r)}$, where r is such that $w_{(r)} \geq 0$ and $w_{(r+1)} < 0$. This assignment mechanism will therefore lead to inefficiency by possibly assigning permits to applicants with value for hunting less than $w_{(q)}$.

The dynamic lottery mitigates this issue by creating a "price" for obtaining a permit in the form of lost continuation value. An applicant with p preference points chooses between applying for a permit, with expected present value utility $\phi_p(\epsilon_i + \rho R_0) + (1 - \phi_p)\rho R_{p+1}$, or waiting to apply in the following period, with expected present value utility ρR_{p+1}. Here, ϕ_p is the success probability with p preference points and R_p is the applicant's continuation value with p preference points (as before). For simplicity, assume that no applicant with zero preference points can obtain a permit. There will be some preference point stock $p > 0$ such that $\phi_p > 0$. Applicants with p preference points can obtain a permit, but they will only apply for a permit if $w_i \geq \frac{\rho}{-\mu_0}(R_{p+1} - R_0) > 0$. The last inequality holds as long as $\rho > 0$ since $\phi_{p+1} \geq \phi_p > 0$. Hence, a dynamic lottery will necessarily lead to a more selective assignment than a static lottery. The degree of selectivity depends on the loss of continuation value associated with having a stock of preference points reset to zero, $\frac{\rho}{-\mu_0}(R_{p+1} - R_0)$. This selectivity will translate into a welfare-improving sorting where applicants with large draws of ϵ_i will be assigned to the resource while applicants with lower draws of ϵ_i will exclude themselves from consideration, preferring to wait for a future assignment.[24]

In our application the dynamic lottery allocates permits to several sites, which are estimated to have large differences in baseline utility. The option value of waiting for a future assignment in the dynamic lottery is therefore combined with a sorting of types of resources across levels of seniority to yield heterogeneous opportunity costs for different types of resources.

[24]As discussed in Section 3.2, note that the efficiency gain of a dynamic assignment mechanism relies on at least part of the preference heterogeneity being transitory. For instance, in this simplified example, ϵ_i is assumed to be drawn independently every year. In order to quantify the efficiency gains of a dynamic assignment mechanism it is therefore important to use models which account for persistent heterogeneity in preferences (the unobserved types in our choice model in Section 3) and to use data from a natural choice experiment that allow for separately identifying persistent and transitory preference heterogeneity, i.e. data that contain information on applicants' choices repeatedly, such as the dynamic lottery we study here.

With different types of resources, an efficient allocation is obtained by having participants choose to obtain a permit for a hunting site (or choose to abstain) in order to maximize their net welfare $w_{ij} - p_j^\star$ across choices $j \in \bar{\mathcal{J}}$, where as before w_{ij} is the value of a permit at site j to applicant i, $p_j^\star$ is the equilibrium price of a permit for site j in the efficient auction, and $\bar{\mathcal{J}}$ is the augmented choice set $\bar{\mathcal{J}} = \mathcal{J} \cup \{0\}$ which includes all sites $j \in \mathcal{J}$ and the outside option $j = 0$.[25].

As discussed above, the static lottery does not charge any price for obtaining a permit, but requires that applicants submit applications for permits simultaneously, so that applicants choose the site that maximizes their expected utility $\phi_j w_{ij}$ across choices $j \in \bar{\mathcal{J}}$ where ϕ_j are equilibrium success probabilities.

Using the notation developed in Section 3, an applicant i of type τ in the dynamic lottery will incur an intertemporal opportunity cost $c_{ip} = \frac{\rho}{-\mu_0}(R_{i\tau p+1} - R_{i\tau 0})$ for hunting today rather than abstaining and waiting for a future assignment. This opportunity cost will generally still lead to some excess demand, so that an applicant will choose the site that maximizes her expected net welfare $\phi_{jp}(w_{ij} - c_{ip})$ across all choices $j \in \bar{\mathcal{J}}$, given equilibrium success probabilities ϕ_{jp}.[26]

Figure 3 plots the average intertemporal opportunity cost of hunting (c_{ip}) for each type of participant against the average auction prices of the permits allocated by the dynamic lottery at each level of preference point stock. The intertemporal opportunity cost of hunting elicited by the dynamic lottery is significant for both types of participants under the dynamic lottery, ranging from \$727 to \$3,412 for type-2 participants and from \$1,477 to \$5,417 for type-1 participants. More desirable sites are in higher demand, so that they become available at higher levels of preference points than less desirable sites in the dynamic lottery. Figure 3 provides a representation of this sorting of sites by preference point stocks by showing that the average auction price of sites increases with preference point stock. This sorting also results

[25]The value of abstaining is normalized to zero, $w_{i0} = 0$, and the price of abstaining is zero as well, $p_0 = 0$

[26]As before, the success probability of abstaining is set to zero, $\phi_{j0} = 0$, so that this representation is notationally correct for $j = 0$.

in the intertemporal opportunity cost of hunting increasing with preference point stock. This is because hunting in the current period always leads to being deprived of access to the most desirable sites for several years. A participant with a high stock of preference points who chooses to abstain in a given year would have immediate access to these sites in the following year (and hence a large opportunity cost of hunting today), whereas an applicant with zero preference points would have to wait several years before gaining access to the most desirable sites regardless of whether she hunts today or not (and hence a small opportunity cost of hunting today). The dynamic lottery is therefore able to associate higher opportunity costs with more desirable sites in a similar way as the efficient auction associates higher prices with more desirable sites. Having participants choosing to maximize $\phi_{jp}(w_{ij} - c_{ip})$ across all choices $j \in \bar{\mathcal{J}}$ in a dynamic lottery mimics to some extent their choice to maximize $w_{ij} - p_j^\star$ across all choices $j \in \bar{\mathcal{J}}$ in an efficient auction and leads to improved selectivity compared to static assignment mechanisms.

4.4.3 Improved Targeting in the Dynamic Lottery

The relative efficiency of the dynamic lottery also originates from the fraction of permits assigned to type-1 participants being higher in the dynamic lottery than in the static lottery (0.27 instead of 0.21). This originates from a separation of hunter types across preference point stocks and hunting site quality. Compared to a static lottery, type-2 applicants can aim for the more desirable sites and still benefit from low waiting times because a dynamic lottery results in success probabilities that increase sharply with waiting time (i.e. with preference point stock). Therefore type-2 applicants concentrate on obtaining permits for the more desirable first and second tier sites. This frees up less-desirable third tier sites, allowing type-1 applicants to obtain permits for these sites in larger proportions. Under both mechanisms, type-1 participants obtain virtually all of their permits from third tier sites, but in the dynamic lottery they benefit from a greater probability of obtaining a permit to one of these sites (0.87 under the dynamic lottery versus only 0.71 in the static lottery).

Figure 3 provides a representation for this efficiency gain by showing the maximized value of hunting for sites that are available under the dynamic lottery at each level of preference point stock. We find that type-1 participants have a value of hunting that is significantly above their opportunity cost, so that they apply for hunting with a probability that is approximately one. On the other hand, a significant share of the distribution of value of hunting for type-2 applicants is below their intertemporal opportunity cost of hunting. This leads these applicants to apply for the preference-point only option in the dynamic lottery, "freeing up" permits for type-1 participants.

4.4.4 Inefficiency of the Dynamic Lottery

While the dynamic lottery achieves efficiency gains compared to a static lottery through the two channels described above, it leads to a total surplus that is still significantly lower than the total surplus achieved by an efficient auction.

With one type of participant and one type of resource, Jackson and Sonnenschein (2007) show that an efficient allocation can be obtained by rationing access over time when agents value their future utility without discounting. In the same environment but with agents who discount their future utility, Guo and Hörner (2017) show that the total surplus achieved by the optimal mechanism in the class of mechanisms that do not rely on monetary transfers is lower than the maximized total surplus. In our application, discounting and attrition will prevent assignment mechanisms without money from achieving an efficient allocation. This can also be understood by considering the extreme case where there is full discounting (i.e. where agents are myopic), and where the attrition rate is one (i.e. all applicants are replaced every year), since in this case rationing access over time would be impossible.

Additionally, the heterogeneity estimated to exist across participants implies that mechanisms that rely on opportunity costs rather than monetary transfers to elicit selectivity will necessarily reach either over-selectivity among type-1 participants or under-selectivity among type-2 participants. Figure 3 shows that type-1 applicants are roughly as selective as they would be in an efficient auction (their opportunity cost of hunting is approximately equal to the auction price of the corresponding permits), but type-2 applicants have an opportunity cost that is significantly lower than the efficient prices. An applicant's opportunity cost for obtaining a permit today originates from a decrease in the probability of obtaining a permit in the future. Therefore the magnitude of the resulting opportunity cost depends on an applicant's expected future utility from obtaining a permit. Consequently this opportunity cost will necessarily be lower among type-2 applicants than among type-1 applicants. This leads type-2 applicants to apply for permits more frequently than they would in an efficient auction, which in turn leads to a lower surplus per permit assigned than would arise in an efficient auction. This also crowds out type-1 applicants, with fewer permits at fewer sites being allocated to type-1 applicants in a dynamic lottery compared to an efficient auction.

Finally, we also estimate that there exists a large degree of heterogeneity across sites, but at a given stock of preference points the dynamic lottery charges a uniform "price" for hunting across all sites. The dynamic lottery charges an applicant with p preference points a price for hunting, denominated in preference points, of $p+1$. This price is independent of the site from which the applicant chooses to obtain a permit. In contrast, an efficient auction leads to very heterogeneous prices across sites, estimated to range from \$82 to \$10,122.[27] As discussed above, the dynamic lottery still leads to heterogenous opportunity costs across sites because access to sites of different desirability and scarcity is distributed across different levels of seniority, but at high levels of seniority the dynamic lottery results in the effective choice set of applicants being needlessly narrow. Indeed at high levels of preference points the choices of applicants are concentrated around the few most desirable sites and the choice probability for sites with low baseline desirability are approximately zero. This is a symptom of an inefficient allocation since applicants that might have relatively low draws of idiosyncratic preference shocks ϵ_{ij} for sites j with high baseline utility χ_{0j} receive assignments for these

[27]While a predicted auction price of \$10,000 for a single permit at the most desirable sites may seem high, we note that other U.S. states have implemented auctions for big game hunting permits, yielding similar or significantly higher prices, see e.g. Branch (2017).

sites, crowding out other participants who have higher draws of preference shocks for these sites.

We provide additional details on the role of these two dimensions of heterogeneity in determining the relative efficiency of the dynamic lottery in Section C of the online appendix, where in particular we explore the potential of a simple modification to the dynamic lottery to mitigate the last source of inefficiency raised above.

5 Conclusion

Our results show that dynamically evolving budgets in an artificial currency (preference points) can lead to allocations that are more efficient than allocations obtained with static mechanisms. More generally, our results point to the importance of two sources of efficiency gains. First, mechanisms that increase the option value of waiting can yield efficiency gains by making participants more selective. Second, the heterogeneity in the resources being allocated can be leveraged to yield a separation by applicants' types, so that applicants with a high value for the resource can benefit from obtaining an assignment with a shorter waiting time, which in turn allows applicants with lower value to concentrate on more desirable resources (and vice versa). It is likely that the design of many assignment mechanisms for repeated allocation of heterogenous resources could benefit from incorporating these aspects, as we noted in our introduction for the examples of affordable housing or transplant organ donation.

On the other hand we find that the degrees of heterogeneity among participants and across the resources being allocated are also key in determining the social efficiency of the dynamic lottery relative to an efficient auction. While simple recommendations can be made to address heterogeneity across resources, which we discuss in the online appendix, the efficiency frontier of allocation mechanisms without monetary transfers and with persistently heterogeneous participants is, to our knowledge, unknown. We consider this to be evidence that additional empirical and theoretical results are needed to form a more complete picture of how specificities of each application should guide the design of assignment mechanisms.

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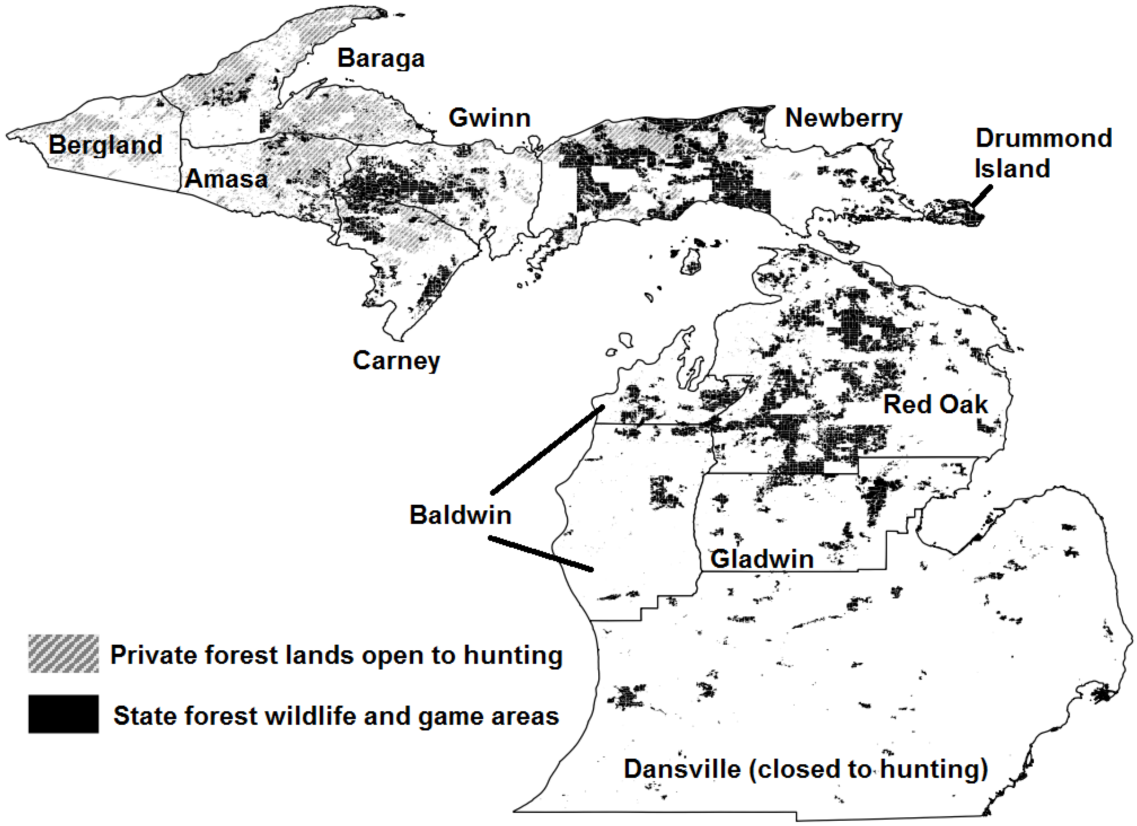


Figure 1: Michigan bear management units (source: DNR 2015a; 2015b)

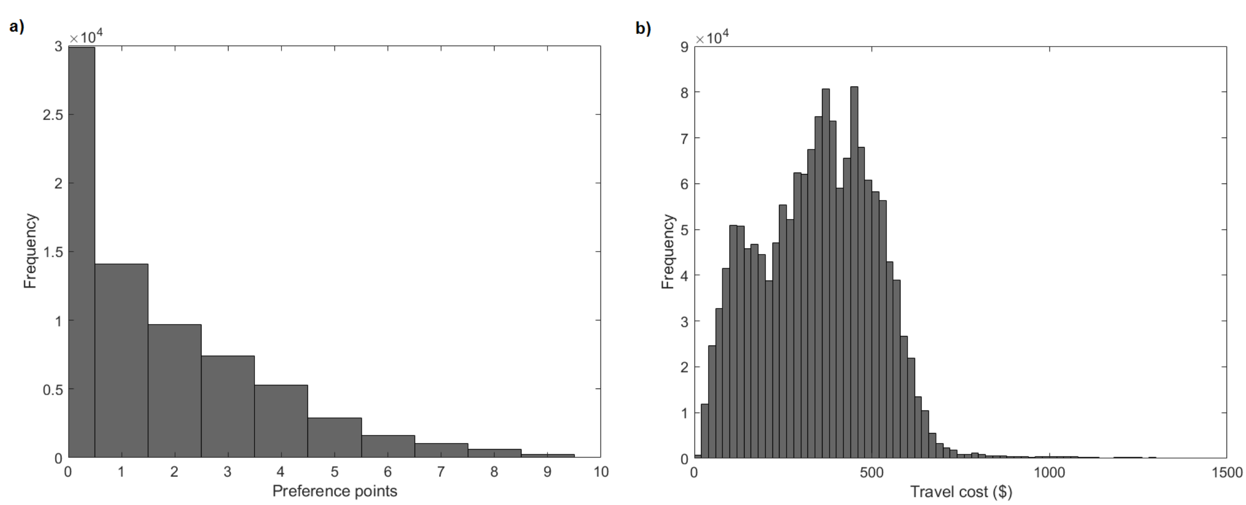
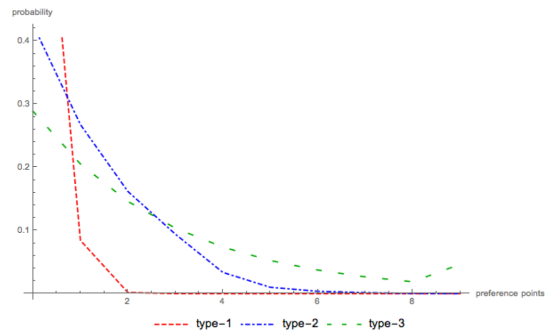
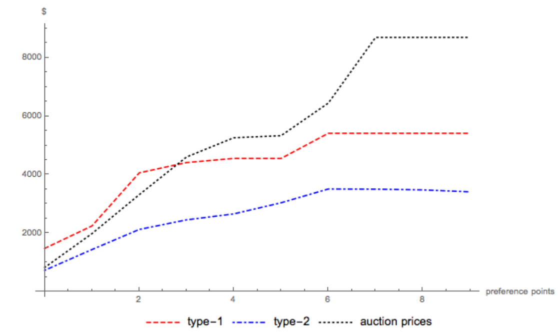


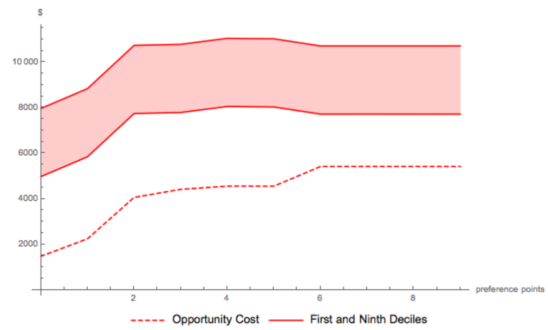
Figure 2: Histograms of applicants' a) preference point stocks and b) travel costs in dollars



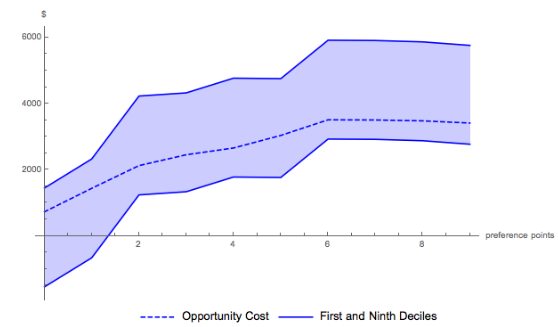
(a) Probability distribution of preference points for participants of each type



(b) Intertemporal opportunity cost of hunting for participants of each type and auction prices of allocated permits



(c) Intertemporal opportunity cost of hunting and first and ninth deciles of the maximized value of hunting for participants of type-1



(d) Intertemporal opportunity cost of hunting and first and ninth deciles of the maximized value of hunting for participants of type-2

Figure 3: Characteristics of the equilibrium allocation with dynamic lottery

Table 1: Bear Management Unit Characteristics and Summary Statistics (Source: DNR 2009; 2015a)

BMU	Population	Forest land (ac)	Hunts /yr	Mean success rate	Season duration (days)
Bergland	16,452	340,020	3	0.28	47
Baraga	78,327	974,399	3	0.26	47
Amasa	23,636	500,823	3	0.33	47
Gwinn	48,954	574,025	3	0.24	47
Carney	57,362	436,796	3	0.21	47
Newberry	60,591	1,333,215	3	0.26	47
Drummond Island	457	22,550	1	0.65	42
Red Oak	294,981	1,394,083	1	0.28	7
Baldwin	467,081	288,297	1	0.50	7
Gladwin	303,693	231,582	1	0.17	7

Table 2: Michigan Bear Hunting Seasons and Quotas, 2008–9 (Source: DNR 2010)

Hunt	BMU	Season	Avg. quota	Avg. applicants
1	Bergland	9/10–10/21	480	1348
2		9/15–10/26	613	680
3		9/25–10/26	675	528
4	Baraga	9/10–10/21	440	2073
5		9/15–10/26	770	1172
6		9/25–10/26	1415	1159
7	Amasa	9/10–10/21	170	1220
8		9/15–10/26	235	593
9		9/25–10/26	420	681
10	Carney	9/10–10/21	215	1250
11		9/15–10/26	388	593
12		9/25–10/26	545	422
13	Gwinn	9/10–10/21	250	1682
14		9/15–10/26	375	809
15		9/25–10/26	800	774
16	Newberry	9/10–10/21	388	4274
17		9/15–10/26	495	1992
18		9/25–10/26	1480	2027
19	Drummond Is.	9/10–10/21	6	285
20	Red Oak	9/18–9/26	1850	12337
		(10/2–10/8 archery)		
21	Baldwin	9/18–9/26 (all)	63	2647
		9/11–9/26 (north area)		
22	Gladwin	9/18–9/26	163	848

Table 3: Application shares and success probabilities by levels of preference points

preference points		0	1	2	3	4	5	6	7	8	9
Share of applicants	p.p.-only	0.20	0.30	0.34	0.36	0.40	0.51	0.59	0.63	0.76	0.89
	tier-1 sites	0.31	0.34	0.36	0.40	0.43	0.35	0.29	0.26	0.17	0.05
	tier-2 sites	0.17	0.19	0.20	0.18	0.14	0.11	0.10	0.08	0.06	0.06
	tier-3 sites	0.32	0.17	0.10	0.06	0.03	0.02	0.02	0.02	0.01	0.00
Shares excluding p.p.-only	tier-1 sites	0.39	0.48	0.55	0.63	0.72	0.72	0.71	0.72	0.70	0.46
	tier-2 sites	0.22	0.27	0.30	0.28	0.23	0.23	0.23	0.23	0.25	0.50
	tier-3 sites	0.40	0.25	0.16	0.09	0.06	0.05	0.06	0.05	0.04	0.04
Average success probability	tier-1 sites	0.00	0.00	0.00	0.06	0.33	0.56	0.67	0.68	1.00	1.00
	tier-2 sites	0.00	0.03	0.24	0.57	0.79	0.86	0.86	0.86	0.93	1.00
	tier-3 sites	0.57	0.86	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00

p.p.-only stands for the preference point-only option. In order to summarize information, the 22 hunting sites were divided into three tiers. These tiers are defined in Section 3.1. Tier-1 contains sites which are estimated to be the most desirable. Tier-3 contains sites which are estimated to be the least desirable.

Table 4: Expected preference-points cut-offs for each hunt in years 2008 and 2009

Hunt	cut-off 2008	cut-off 2009
1	1.70	1.89
2	0.08	0.17
3	0.00	0.00
4	2.91	3.17
5	0.40	0.60
6	0.00	0.00
7	4.14	4.75
8	1.75	1.73
9	0.52	0.77
10	2.94	3.37
11	0.62	0.42
12	0.00	0.00
13	3.78	3.83
14	0.90	0.99
15	0.01	0.00
16	5.24	5.42
17	2.40	2.23
18	0.30	0.44
19	7.83	8.94
20	3.70	3.93
21	7.63	8.40
22	2.38	2.62

Cut-offs correspond to the expected minimum stock of preference points required to obtain a permit at each hunt in each year. For instance in 2008, 30% of applicants with one preference point could obtain a permit for hunt 1, while the cut-off for 70% of these applicants was two preference points (i.e. they were not successfully drawn).

Table 5: Maximum Likelihood Estimates of Marginal Utility Parameters

Parameter	Estimate	Std. Error
Marginal utility parameters		
μ[a]	–0.206	0.0063
η_2	–6.725	0.0122
η_3	–21.987	744.42
Type probabilities		
π_2	0.798	0.0016
π_3	0.136	0.0015
Baseline utility from hunt j		
χ_1	5.30	0.0302
χ_2	3.41	0.0286
χ_3	2.85	0.0313
χ_4	6.69	0.0375
χ_5	4.63	0.0213
χ_6	3.58	0.0236
χ_7	6.94	0.0531
χ_8	4.38	0.0455
χ_9	4.24	0.0263
χ_{10}	5.97	0.0522
χ_{11}	3.85	0.0297
χ_{12}	2.56	0.0363
χ_{13}	6.60	0.0408
χ_{14}	4.32	0.0258
χ_{15}	3.14	0.0274
χ_{16}	8.62	0.0358
χ_{17}	5.88	0.0331
χ_{18}	4.77	0.0176
χ_{19}	5.25	0.2168
χ_{20}	8.65	0.0165
χ_{21}	10.49	0.0493
χ_{22}	4.85	0.0566
Log-likelihood		–87239.8

[a] Travel costs are discretized into units of \$200. The marginal utility of income in unscaled dollars per util is –0.206/200 = –0.00103.

Table 6: Welfare Comparison across Different Assignment Mechanisms

		Random serial dictatorship	Static lottery	Dynamic lottery	Auction
Expected total surplus ($)[a]		31.9M	36.0M	42.6M	58.0M
Average total surplus per applicant ($)		575	649	768	1,046
Average applicant surplus		575	649	768	526
State revenue per applicant		—	—	—	520
Average total surplus by applicant type ($)[b]	1	4,277	3,846	4,914	7,947
	2	367	496	557	656
	3	0	0	0	0
Probability of obtaining a permit		0.221	0.221	0.212	0.221
by applicant type	1	0.746	0.710	0.878	1
	2	0.215	0.218	0.193	0.194
	3	0	0	0	0
Average surplus per permit ($)		2,603	2,942	3,578	4,742
by applicant type	1	5,731	5,421	5,597	7,947
	2	1,710	2,278	2,887	3,383
Average years before obtaining a permit		2.93	2.89	1.74	3.31
by applicant type	1	0.34	0.41	0.10	0
	2	3.65	3.59	1.88	4.15
Probability of winning permit to tier-1 site		0.041	0.041	0.041	0.041
by applicant type	1	0.048	0	0	0.189
	2	0.048	0.052	0.052	0.036
Probability of winning permit to tier-2 site		0.037	0.037	0.037	0.037
by applicant type	1	0.052	0	0.006	0.168
	2	0.042	0.046	0.046	0.033
Probability of winning permit to tier-3 site		0.142	0.142	0.133	0.142
by applicant type	1	0.646	0.709	0.872	0.644
	2	0.124	0.120	0.095	0.125

[a] We normalize total surplus absent hunting to $0.
[b] The proportions of type-1, -2, and -3 applicants are 0.066, 0.798, and 0.136, respectively.