A Random Attention Model

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Limited Attention

- Abundance of Alternatives
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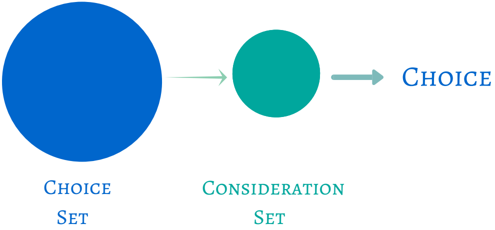
- Limited Attention: a serious critique for revealed preferences

Limited (Deterministic) Attention

- Masatlioglu, Nakajima, and Ozbay (2012) shows that inferring preference from choices is possible (Revealed Preferences).

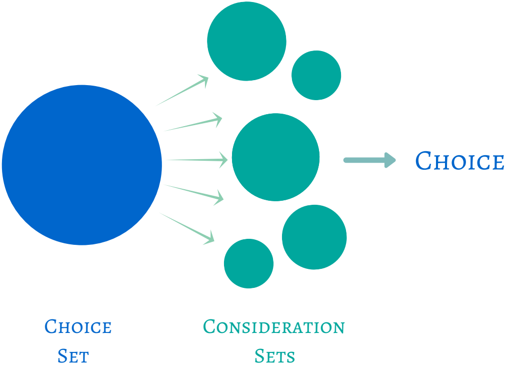
  ▷ Two-stage Choice



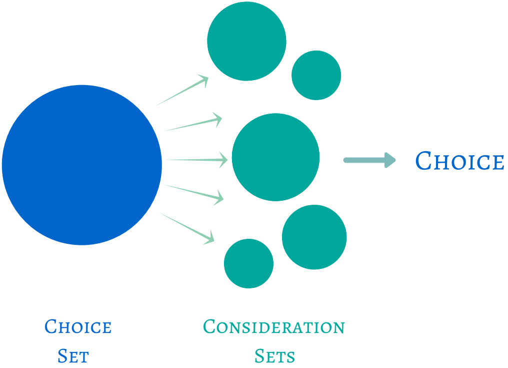
Random Consideration

- The revealed preferences result of Masatlioglu, Nakajima, and Ozbay (2012) is not applicable if the consumer utilizes
  - multiple E-commerces
  - and/or multiple platforms

Random Attention

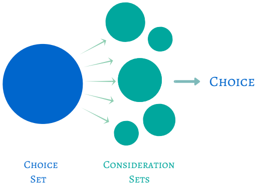


Random Attention



S $\mu(\cdot|S)$ $\pi(\cdot|S)$

Stochastic Choice



$$\pi(a|S) = \sum_{\substack{T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S)$$

- $\succ$ - complete and transitive

Two Approaches

- Two possible approaches

    1) Committing to a particular attention formation

    2) Imposing intuitive and nonparametric restrictions on μ

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- Two possible approaches
  1) Committing to a particular attention formation
  2) Imposing intuitive and nonparametric restrictions on μ
- We choose the second one
  - our revealed preference result is applicable for multiple attention formations as long as our restriction is satisfied.

Monotonic Attention

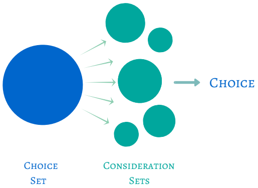
Monotonic Attention: *If* $a \notin T$*, then*

$$\mu(T|S) \leq \mu(T|S - a)$$

Some Examples of Monotonic Attention Formations

- **Fixed Independent Consideration (MM, 2014)**
- **Variable Independent Consideration (MM, 2014)**
- **Logit Attention (BR, 2017)**
- **Ordered Logit**
- **Elimination by Aspect**
- **Stochastic Satisficing**
- **Amazon versus Jet**
- ...

Random Attention Model (RAM)



$$\pi(a_k|S) = \sum_{\substack{T \subset S, \\ a_k \text{ is } \succ\text{-best in } T}} \mu(T|S)$$

- $\succ$ - complete and transitive
- μ - monotonic

Random Attention Model (RAM)

RAM accommodates well-documented and seemingly anomalous behaviors.

Attraction Effect

- Probabilistic Attraction Effect
  - a_1 and a_2 are equally chosen in a binary comparison,
  - a_3 is a decoy for a_1,

$\pi(a\|S)$	$\{a_1, a_2, a_3\}$	$\{a_1, a_2\}$	$\{a_1, a_3\}$	$\{a_2, a_3\}$
a_1	1	1/2	1	
a_2	0	1/2		1
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$$\pi(a_1|\{a_1, a_2, a_3\}) > \pi(a_1|\{a_1, a_2\})$$

Violation of Regularity

- Random Attention Model allows

$$\pi(a|S) > \pi(a|S - b)$$

  - Removing an alternative can decrease the choice probability

Prediction Power

- Is the model too general?
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- Is the model too general?
- The random attention model can be falsified.
  - For example, the following π is outside of the model whenever $\beta_1\beta_2\beta_3 > 0$,

$\pi(a\|S)$	$\{a_1, a_2, a_3\}$	$\{a_1, a_2\}$	$\{a_1, a_3\}$	$\{a_2, a_3\}$
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a_2	β_2	0		1
a_3	β_3		1	0

Revealed Preference

- How can we deduce preferences under random attention?

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- How can we deduce preferences under random attention?
- However, richness does not help us much
  - More degree of freedom
  - Allowing many possibilities
  - Less revelations

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  How?
  WTS: There exists at least one consideration set T such that

- $\mu(T|S) \neq 0$
- $b \in T$
- a is chosen from T

Revealed Preference

PROOF:

$$\begin{aligned}
\pi(a|S) &= \sum_{\substack{T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S) \\
&= \sum_{\substack{b \in T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S) + \sum_{\substack{b \notin T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S) \\
&\leq \sum_{\substack{b \in T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S) + \sum_{\substack{b \notin T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S-b) \quad \text{(by monotonicity)} \\
&\leq \sum_{\substack{b \in T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S) + \pi(a|S-b)
\end{aligned}$$

Revealed Preference

PROOF continues...

$$\pi(a|S) - \pi(a|S-b) \quad \leq \quad \sum_{\substack{b \in T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S)$$

Revealed Preference

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$$\pi(a|S) - \pi(a|S-b) \quad \leq \quad \sum_{\substack{b \in T \subset S, \\ a \text{ is } \succ\text{-best in } T}} \mu(T|S)$$

If $\pi(a|S) - \pi(a|S-b) > 0$ then there exists at least one T such that

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Hence, a is revealed to be preferred to b. DONE

Revealed Preference

- $a\mathcal{P}b$ if $\pi(a|S) > \pi(a|S-b)$
- Let $\bar{\mathcal{P}}$ be the transitive closure of $\mathcal{P}$
- While $\bar{\mathcal{P}}$ informs us about preference, do we miss some revelation?

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Theorem (Revealed Preference)

Let π have a RAM representation. Then a is **revealed to be preferred** to b if and only if $a\bar{\mathcal{P}}b$.

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Theorem (Revealed Preference)

Let π have a RAM representation. Then a is **revealed to be preferred** to b if and only if $a\bar{\mathcal{P}}b$.

- $\bar{\mathcal{P}}$ provides all the information we need to know.

Characterization

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A stochastic choice π has a RAM representation
iff
$\mathcal{P}$ has no cycle.

Improving Revealed Preference

- Currently, no regularity violation $\Rightarrow$ no preference revelation
- How can we improve revealed preference?

Improving Revealed Preference

- Consider an policy maker: Poly
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- ϕ: the degree of caution
- The corresponding assumption on attention rule is

$$\mu(\{a,b\}|\{a,b\}) \geq \frac{1-\phi}{\phi} \max\left\{\mu(\{a\}|\{a,b\}), \mu(\{b\}|\{a,b\})\right\}.$$

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- if $\phi = 0.5$
  then
  $$\mu(\{a,b\}|\{a,b\}) \geq \max\left\{\mu(\{a\}|\{a,b\}), \mu(\{b\}|\{a,b\})\right\}.$$
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  and
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- if $\phi = 0.75$
  then
  $$\mu(\{a,b\}|\{a,b\}) \geq \frac{1}{3}\max\left\{\mu(\{a\}|\{a,b\}), \mu(\{b\}|\{a,b\})\right\}.$$
  and
  $$a \text{ is revealed to preferred to } b \text{ if } \pi(a|\{a,b\}) > 0.75$$

Improving Revealed Preference

- Consider the following data

$\pi(a\|S)$	$\{a_1, a_2, a_3\}$	$\{a_1, a_2\}$	$\{a_1, a_3\}$	$\{a_2, a_3\}$
a_1	0.6	0.5	0.6	
a_2	0.2	0.5		0.2
a_3	0.2		0.4	0.8

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- $\pi(a_1 \mid \{a_1, a_2, a_3\}) > \pi(a_1 \mid \{a_1, a_2\}) \Rightarrow a_1 \succ a_3$
- Assume Poly's caution parameter is 0.75
- $\pi(a_3 \mid \{a_2, a_3\}) > 0.75 \Rightarrow a_3 \succ a_2$
- Full Revelation $a_1 \succ a_3 \succ a_2$

Related Literature

- Manzini and Mariotti (2014),
- Brady and Rehbeck (2016), Gul, Natenzon, and Pesendorfer (2014),
- Echenique, Saito, and Tserenjigmid (2014),
- Echenique and Saito (2017),
- Fudenberg, Iijima, and Strzalecki (2015) and
- Aguiar, Boccardi, and Dean (2016)
- Dogan and Yildiz (2018)
- Ahumada and Ulku (2018)
- Horan (2018)
- Yildiz (2016)
- Li and Tang (2016)

WRAP-UP

- Provides conditions under which the preference is partially identified from choice data, without observing consideration sets.
- Constructs test statistics facilitating estimation and inference:
  - Reformulates identification as testing moment inequalities.

    There is a large literature on testing moment inequalities and inference in partially identified models.

    Other test statistics and methods for critical values can be easily adapted.
  - Provides uniformly valid distributional approximations and critical values.
  - Implements in R and Matlab.
- Revealed Preference is a powerful tool:
  - both rational and boundedly rational behavior,
  - both deterministic and stochastic choice.