Should We Do More When We Know Less? Optimal Risk Reduction under Technological Uncertainty*

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Abstract

Technological uncertainty (TU) arises whenever the effects of risk mitigation depend on exogenous factors or are subjectively perceived to be uncertain. It is a widespread condition in decision-making under risk and uncertainty. We study the effects of TU on self-insurance and self-protection. TU reduces the willingness to pay for self-insurance and has no effect on that for self-protection. Its impact on the optimal demand for both activities is jointly determined by the decision-maker's preferences and risk reduction effectiveness. We identify conditions for TU, FSD improvements and increases of TU to have unambiguous comparative statics. These conditions involve prudence, relative risk aversion, and relative prudence. We highlight cases where TU raises the optimal investment consistent with the precautionary principle. Our theory is rich in empirical predictions and our results have implications in the areas of safety, loss control, insurance demand under nonperformance risk, and climate change.

Keywords: self-insurance · self-protection · risk aversion · prudence · technological uncertainty

JEL-Classification: D61 · D81 · D91 · G22

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1 Introduction

Risk-averse people dislike risk and may benefit from risk mitigation. Economists commonly assume that the benefits of risk mitigation are precisely known at the time a decision is made. While entailing tractability, this assumption is hardly justifiable and diminishes the normative and descriptive value of existing analyses. We argue in this paper that many, if not all forms of risk reduction are characterized by *technological uncertainty* because the benefits are only imperfectly known by the decision-maker.[1]

For example, the effectiveness of crime-prevention expenditures for one's home might critically depend on the future development of the neighborhood. A sprinkler system might operate more or less reliably when a fire breaks out (see Briys et al., 1991). The benefits of climate change mitigation depend on a variety of environmental factors, over which there is considerable scientific disagreement; as illustrated by, in part, strongly differing predictions from various climate models (see Berger et al., 2016). In insurance, nonperformance risk is a model example for technological uncertainty (see Doherty and Schlesinger, 1990; Peter and Ying, 2016; Bourgeon and Picard, 2014); it refers to the uncertainty whether and to what extent the insurer will honor its indemnity promise in case of a loss and is a central concern of insurance regulation. Even when environmental factors are irrelevant for the benefits of risk reduction, individuals might simply *perceive* them as uncertain. Possible reasons include a lack of, inattention to, or ignorance of information (see Golman et al., 2017). Behavioral factors such as lack of confidence or sophistication (see Neumuller and Rothschild, 2017; Li et al., 2018) or limited financial literacy (see Lusardi and Mitchell, 2011) may also contribute to technological uncertainty. We provide the first systematic analysis of the effects of technological uncertainty on risk mitigation.

Ehrlich and Becker (1972) introduced the distinction between self-insurance and self-protection.[2] Self-insurance or loss reduction refers to a costly activity that reduces the size of loss, whereas self-protection or loss prevention refers to a costly activity that reduces the probability of loss.[3] Each form of risk mitigation reduces the expected loss, but besides

[1] The usage of the term technological uncertainty has been used differently in other fields. In the management literature, it often describes the uncertainty associated with a firm's application of knowledge or skills in its product or service development (see Song and Montoya-Weiss, 2001; Fleming, 2001; MacMinn and Holtmann, 1983). Some studies in environmental economics use technological uncertainty to refer to the random arrival time of future technological progress (see Dasgupta and Stiglitz, 1981; Fuss and Szolgayová, 2010)

[2] See Courbage et al. (2013) for a recent survey.

[3] Footnote 1 in Chiu (2000) or the introduction of Berger (2016) provide many examples of both activities.

this commonality, there is a broad array of dissimilarities.[4] This is why we maintain this distinction.

The implications of technological uncertainty *per se* have not been studied yet although some researchers have conducted comparative statics in the presence of technological uncertainty. Hiebert (1989) finds that an increase in risk aversion raises the investment in self-insurance when the loss size is uncertain, but not necessarily when the effectiveness of self-insurance is random. Similarly, Briys et al. (1991) show that, at an actuarially fair price, more or less than full coverage can be optimal when market insurance is non-reliable. In this case, even when a strong increase in risk aversion in the sense of Ross (1981) does not necessarily increase the demand for insurance or self-insurance. [5] These papers determine the effect of risk aversion while taking technological uncertainty as given in some parametric form. We in turn study the dual question and isolate the effects of technological uncertainty on optimal behavior. We make no parametric assumptions on how technological uncertainty affects risk reduction but rather identify conditions for unambiguous comparative statics as part of our analysis.

Another related stream of literature studies risk reduction in the presence of ambiguity. When probabilities are not uniquely assigned, the benefits of risk reduction are also uncertain. Relying on the Klibanoff et al. (2005, 2009) framework, Huang (2012) identifies conditions under which a more ambiguity-averse individual raises her investment to improve her initial wealth distribution. Alary et al. (2013) determine the effects of ambiguity aversion on the marginal willingness to pay and the optimal demand for self-insurance and self-protection in a single-period model, which has been extended to two periods by Berger (2016). Etner and Spaeter (2010) study prevention of health risks in the presence of complications that are perceived as ambiguous. They determine conditions under which an increase in ambiguity aversion raises optimal prevention. Snow (2011) finds that both self-insurance and self-protection increase with greater ambiguity aversion. His results extend to a probability weighting model in the spirit of Quiggin (1982).

These papers assume ambiguity and compare the behavior of an ambiguity-neutral and an ambiguity-averse agent, or more generally vary the agent's degree of ambiguity aversion. We show the relevance of technological uncertainty even without assuming behavioral preferences such as ambiguity aversion. Our results represent the relevant normative benchmark for

[4] Ehrlich and Becker (1972) find that self-insurance and market insurance are substitutes whereas self-protection and market insurance can be substitutes or complements. Dionne and Eeckhoudt (1985) show that increased risk aversion à la Pratt (1964) increases the demand for self-insurance but may or may not increase the demand for self-protection. As a result, wealth effects on self-insurance can be directly inferred from how wealth affects risk aversion (Lee, 2010b), which is not the case for self-protection (Sweeney and Beard, 1992).

[5] Similarly, the relationship between risk aversion and the optimal level of self-insurance is not necessarily monotonic with more than two states of the world (see Lee, 2010a). However, Eeckhoudt et al. (2017) showed that a restricted increase in risk aversion leads to unambiguous comparative statics in the case of insurance with nonperformance risk.

the effects of technological uncertainty on optimal behavior. Technological uncertainty and increases in technological uncertainty compromise the attractiveness of risk reduction and lower the willingness to pay for self-insurance. An FSD improvement, in turn, makes self-insurance more attractive. In case of self-protection, technological uncertainty and increases in technological uncertainty do not affect the willingness to pay because expected utility is linear in probabilities. FSD improvements have the same effect on the willingness to pay for self-protection – a rare case in which both mechanisms of risk reduction behave similarly (see Footnote 4).

For the optimal level of self-insurance and self-protection, matters become increasingly more complex. Technological uncertainty still compromises the effectiveness of self-insurance, which undermines the propensity of a risk-averse agent to invest in it. On the other hand, the additional uncertainty constitutes a precautionary motive, which induces a prudent agent to use more self-insurance. To resolve this trade-off, we identify conditions that allow for clear predictions on the effects of technological uncertainty and its stochastic changes. These conditions involve the agent's preferences including prudence, relative risk aversion, relative prudence, as well as a variety of elasticity measures that capture how uncertainty operates on the self-insurance technology. In many cases, we find behavior consistent with the precautionary principle: Being less certain about the future should induce us to do more to mitigate risk.

Our paper makes several contributions with implications for a variety of settings. We provide the first systematic analysis of the effects of technological uncertainty on optimal risk mitigation. Second, we determine conditions on the agent's preferences and on risk reduction effectiveness under technological uncertainty that yield clear comparative statics. We thus derive new hypotheses about the optimal use of self-insurance and self-protection. A lot is known about the empirical validity of certain preference traits such as prudence or relative risk aversion, but there is a gap in the empirical literature when it comes to our proposed technology measures. This motivates their measurement in the lab and in the field. Third, technological uncertainty can increase or decrease the optimal level of the available activity. It can thus serve as an explanatory variable in those cases where conventional theories have a hard time explaining why observed demand for risk mitigation deviates from predicted demand. Our analysis makes the simple wisdom precise that the benefits of risk reduction are *"in the eye of the observer"*, so that differences in behavior can be attributed to heterogeneity in perception.

2 Willingness to Pay

2.1 Self-Insurance

We consider an agent with a three times differentiable vNM utility function u, which represents her preferences over consumption. We assume non-satiation and risk aversion, $u' > 0$ and

$u'' < 0$, where primes denote derivatives of univariate functions. The agent has initial wealth of w and faces the risk of losing an amount $L < w$ with probability $p \in (0,1)$. Self-insurance refers to a decrease in the size of loss from L to L_0 with $L_0 < L$. The agent's willingness to pay (WTP) to pursue such a change, denoted by v_0, is implicitly defined by

$$(1-p)\cdot u(w) + p\cdot u(w-L) = (1-p)\cdot u(w-v_0) + p\cdot u(w-L_0-v_0). \tag{1}$$

Under TU the effect of self-insurance is no longer certain but varies. We represent this by replacing the reduced loss L_0 by $\widetilde{L}_1$ in Eq (1) with $\widetilde{L}_1 < L$. Each realization of $\widetilde{L}_1$ is less than the initial loss to preserve the meaning of self-insurance. For now we additionally assume $\mathbb{E}\widetilde{L}_1 = L_0$ for the sake of comparability, but relax this later on. In the presence of TU the WTP for self-insurance, denoted by v_1, is defined via

$$(1-p)\cdot u(w) + p\cdot u(w-L) = (1-p)\cdot u(w-v_1) + p\cdot \mathbb{E}u(w-\widetilde{L}_1-v_1). \tag{2}$$

Throughout the paper, we organize our results around three interrelated questions:

1) What is the effect of TU?

2) What is the effect of a first-order stochastically dominant (FSD) improvement in TU?

3) What is the effect of an increase in TU, modeled as an increase in risk?

We obtain the answer to the first question by comparing a situation with TU to one without. The second and third question pertain to cases where TU is present but changes, for example due to a change in environmental conditions, available technologies or the agent's perception. The first question is a special case of the third one and allows for simpler conditions.

We summarize the effect of TU on the WTP for self-insurance in the following proposition. We provide a proof in Appendix A.1.

Proposition 1. *TU and an increase in TU reduce the WTP for self-insurance. An FSD improvement in TU increases the WTP for self-insurance.*

TU introduces risk into the agent's endowment, and an increase in TU increases this risk. Both changes make the agent worse off due to risk aversion. As a result, self-insurance increases her welfare by less than if TU was absent. TU therefore compromises the effectiveness of self-insurance, which is reflected in a lower WTP. On the other hand, an FSD improvement of TU reduces the expected loss, which is desirable due to monotonicity. So unlike an increase in TU, an FSD improvement is a favorable change, which explains its positive effect on WTP.[6] As we will see in Section 3 the consideration of the optimal level of self-insurance introduces additional trade-offs.

[6] Proposition 1 continues to hold when the initial loss size L_0 is random as long as both sources of uncertainty, the one associated with the initial loss and the one resulting from the technology, are independent.

2.2 Self-Protection

Self-protection refers to a reduction of the probability of loss from an initial level p to p_0 with $p_0 < p$. The size of the loss is unaffected and is given by L independent of the agent's engagement in self-protection. Her WTP is defined by

$$(1-p)\cdot u(w) + p\cdot u(w-L) = (1-p_0)\cdot u(w-v_0) + p_0\cdot u(w-L-v_0). \qquad (3)$$

Under TU the effect of self-protection is no longer certain, which we represent by replacing p_0 with $\widetilde{p}_1$ in Eq. (3) with $\widetilde{p}_1 < p$. Each realization of $\widetilde{p}_1$ is smaller than p to maintain the meaning of self-protection. As before we assume initially that $\mathbb{E}\widetilde{p}_1 = p_0$ for reasons of comparability. This results in the following condition for WTP:

$$(1-p)\cdot u(w) + p\cdot u(w-L) = \mathbb{E}\left\{(1-\widetilde{p}_1)\cdot u(w-v_1) + \widetilde{p}_1\cdot u(w-L-v_1)\right\}. \qquad (4)$$

Since expected utility is linear in probabilities, TU and increases in TU do not affect the agent's WTP for self-protection because they preserve the mean of the loss probability. Only FSD improvements have non-trivial consequences.

Proposition 2. *Neither TU nor an increase in TU affect the WTP for self-protection. An FSD improvement in TU increases the WTP for self-protection.*

We omit the proof for its simplicity. If $\widetilde{p}_1$ undergoes an FSD improvement, the expected probability of loss decreases, which makes the agent better off and raises the value of self-protection.[7] For self-protection, any stochastic change that reduces the expected probability of loss has this effect on WTP whereas any stochastic change that leaves the expected probability of loss unchanged does not affect WTP. The linearity of expected utility in probability broadens the set of applicable risk changes because they occur "outside" the utility function. As we will see in Section 4, the null effect of second-order changes carries over to optimal demand whereas the first-order effect become more intricate.

Propositions 1 and 2 show that FSD improvements in TU have a positive effect on WTP for both self-insurance and self-protection. This is a rare occasion in which both mechanisms of risk reduction have identical comparative statics properties. In many other cases their economic behavior differs, as we explained in Footnote 4. These differences also prevail in our model when it comes to the effect of TU and of increases in TU on the WTP.

[7] p is the probability of loss, so an improvement is an FSD improvement that *lowers* the expected loss probability.

3 Optimal Demand for Self-Insurance

3.1 Preliminaries

The analysis of WTP provides a "take-it-or-leave-it" perspective on risk reduction: The agent can either maintain her endowment or switch to a modified situation with lower risk. A more subtle approach is to study the agent's optimal demand for risk reduction. This requires her to trade off costs against benefits to determine an optimal level of the available activity. We will investigate the role of TU in the agent's cost-benefit analysis. We first assume that she can invest in self-insurance to reduce the size of the loss, and denote by $y \geq 0$ the level of the self-insurance activity. We distinguish a monetary (also known as tangible or non-separable) and a non-monetary (also known as non-tangible or separable) cost of self-insurance.[8] In both cases we denote the cost of self-insurance by $c(y)$, which is assumed to be strictly increasing and non-concave, $c' > 0$ and $c'' \geq 0$.

To introduce TU, we allow the loss size to depend on both y and an exogenous variable $\kappa \in [\underline{\kappa}, \overline{\kappa}]$ so that $L = L(y, \kappa)$. The technology variable κ can represent, among other things, an environmental factor that is beyond the agent's control, or lack of information regarding the effectiveness of self-insurance. For any $\kappa \in [\underline{\kappa}, \overline{\kappa}]$ we assume that $L_y < 0$ and $L_{yy} \geq 0$ so that self-insurance reduces the loss size at a decreasing rate for any potential technology. We also suppose $L_\kappa < 0$ so that higher values of κ represent a better technology in the sense that losses are lower.[9] For the cross-derivative we distinguish three cases according to the following definition.

Definition 1. We speak of:

- Increasing difference (ID) if $L_{y\kappa} < 0$;
- Constant difference (CD) if $L_{y\kappa} = 0$;
- Decreasing difference (DD) if $L_{y\kappa} > 0$.

This terminology is due to Hoy (1989) who uses it for self-protection. We can explain it by studying the so-called "difference function": For $\kappa_1, \kappa_2 \in [\underline{\kappa}, \overline{\kappa}]$ with $\kappa_1 < \kappa_2$, we denote by $\delta(y) = L(y, \kappa_1) - L(y, \kappa_2)$ the difference between the loss size for two different technologies. How does this difference depend on the level of self-insurance? $\delta'(y) = L_y(y, \kappa_1) - L_y(y, \kappa_2)$, which is uniformly positive (zero, negative) if and only if $L_{y\kappa} < 0$ $(= 0, > 0)$. So the marginal product of self-insurance increases (stays constant, decreases) as the technology improves

[8] Two-period models without saving (e.g., Menegatti, 2009) are a special case of a separable cost function whereas two-period models with endogenous saving revert to their single-period counterpart with a non-separable cost, see Peter (2017). So all our results apply to intertemporal models of risk reduction.

[9] This assumption is without loss of generality. All our results hold if we assume $L_\kappa > 0$ instead so that higher values of κ imply larger losses. In this case, the signs of the cross-derivatives in Definition 1 need to be reversed for the definition to remain meaningful.

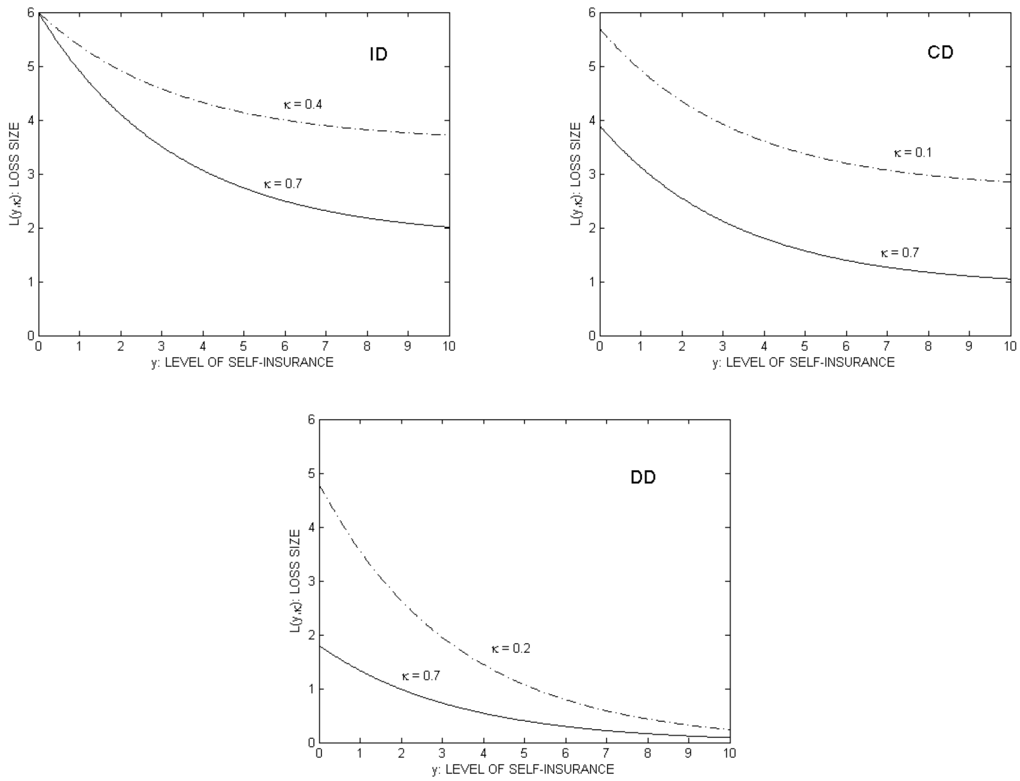

Figure 1: Increasing, constant and decreasing difference.
The underlying loss functions are:
- $L(y,\kappa) = l_0(1-\kappa h(y))$ with $l_0 = 6$, $h(y) = 1 - e^{-0.3y}$ and $\kappa \in \{0.4, 0.7\}$ for ID;
- $L(y,\kappa) = l_0(1-h(y)) + 3(1-\kappa)$ with $l_0 = 3$, $h(y) = 1 - e^{-0.3y}$ and $\kappa \in \{0.1, 0.7\}$ for CD;
- $L(y,\kappa) = l_0(1-\kappa)(1-h(y))$ with $l_0 = 6$, $h(y) = 1 - e^{-0.3y}$ and $\kappa \in \{0.2, 0.7\}$ for DD.

when self-insurance exhibits ID (CD, DD). As illustrated in Figure 1, a given increase in self-insurance reduces small losses by more than large losses under ID whereas it reduces large losses by more than small losses under DD. Many would argue for CD and DD to be more plausible than ID (e.g., Doherty and Posey, 1998; Crainich and Eeckhoudt, 2017) but we will consider all three possibilities in our analysis. We provide some specific examples from the literature.

Example 1. Hiebert's (1989) case of random loss size can be written as $L(y,\kappa) = (1-\kappa)\cdot l(y)$ in our notation with $l' < 0$, $l'' \geq 0$ and $\kappa \in (0,1)$. Then, $L_\kappa = -l(y) < 0$ and $L_{y\kappa} = -l' > 0$, so this is an example of DD.

Example 2. Hiebert's (1989) case of random effectiveness is given by $L(y,\kappa) = l_0\cdot(1-\kappa h(y))$ with $l_0 > 0$, $h' > 0$, $h'' \leq 0$ and $\kappa \in (0,1)$. Then, $L_y = -\kappa l_0 h' < 0$ and $L_{y\kappa} = -l_0 h' < 0$, so this is an example of ID.

Example 3. Briys et al. (1991) study risky self-insurance in the form of $L(y,\kappa) = l(\kappa y)$ with $l' < 0$, $l'' \geq 0$ and $\kappa \in (0,1)$. Then, $L_\kappa = yl' < 0$ and $L_{y\kappa} = l' + \kappa y l''$ is indeterminate. As a result, L can serve as an example of ID, CD or DD depending on the elasticity of the marginal loss size.[10]

Hiebert (1989) and Briys et al. (1991) both study the effect of an increase in risk aversion on optimal behavior while taking TU as given. We will answer the dual question by showing how TU affects behavior at a given level of risk aversion. For a separable cost, the agent's objective function is given by

$$\max_{y\geq 0} U(y) = (1-p)u(w) + p \cdot \mathbb{E}u\left(w - L(y,\widetilde{\kappa})\right) - c(y), \tag{5}$$

where the expectation is taken with respect to TU. The associated first-order condition is

$$U'(y) = -p \cdot \mathbb{E}L_y(y,\widetilde{\kappa})u'\left(w - L(y,\widetilde{\kappa})\right) - c'(y) = 0, \tag{6}$$

and we denote the agent's optimal level of self-insurance by y^*. The second-order condition is satisfied under the assumptions made. The marginal cost is the disutility of exerting a self-insurance effort whereas the marginal benefit results from the expected increases of expected consumption utility due to incurring a lower loss, where the first "expected" refers to the agent's beliefs over the productivity of self-insurance. Due to separability only the marginal benefit of self-insurance is affected by TU whereas its marginal cost is not.

For a non-separable cost we obtain the agent's objective function as follows:

$$\max_{y\geq 0} U(y) = (1-p)u(w - c(y)) + p \cdot \mathbb{E}u\left(w - c(y) - L(y,\widetilde{\kappa})\right). \tag{7}$$

The associated first-order condition is

$$\begin{aligned} U'(y) \;=\; & -p \cdot \mathbb{E}L_y(y,\widetilde{\kappa})u'\left(w - c(y) - L(y,\widetilde{\kappa})\right) \\ & - c'(y)(1-p)u'(w-c(y)) - c'(y)p \cdot \mathbb{E}u'\left(w - c(y) - L(y,\widetilde{\kappa})\right) = 0, \end{aligned} \tag{8}$$

and the second-order condition is satisfied. In the non-separable case both the marginal cost and the marginal benefit of self-insurance are affected by TU, which will turn out to play a role for some of our results.

3.2 TU and Optimal Demand for Self-Insurance

In this section, we will determine how TU affects the demand for self-insurance. In other words, we will provide an answer to question 1) posed in Section 2.1. Based on the analysis of WTP, we might suspect that TU compromises the attractiveness of self-insurance, thus lowering demand. On the other hand, TU could raise the investment in self-insurance for

[10] If we denote this elasticity as $E_{l',\kappa y} = \kappa y l''(\kappa y)/l'(\kappa y)$, then L exhibits DD (CD, ID) whenever the marginal loss is elastic (unit elastic, inelastic), i.e., whenever $E_{l'',\kappa y} < -1$ $(= -1, > -1)$. CD prevails if l is a negative affine transformation of the natural logarithm.

reasons of precaution. Our next proposition solves this tradeoff. The agent's prudence, defined as a positive third derivative of utility (Kimball, 1990), is relevant for whether precautionary behavior prevails.

Proposition 3. *TU:*

a) Raises the optimal level of self-insurance for a prudent agent under CD and DD.

b) Raises (leaves unchanged, lowers) the optimal level of self-insurance for an agent with quadratic utility under DD (CD, ID).

c) Lowers the optimal level of self-insurance for an imprudent agent under ID and CD.

	$u''' < 0$	$u''' = 0$	$u''' > 0$
ID	$-$	$-$	$+/-$
CD	$-$	0	$+$
DD	$+/-$	$+$	$+$

Table 1: The effect of TU on the demand for self-insurance

We give a proof in Appendix A.2. Table 1 visualizes the result in Proposition 3. Our finding does not depend on whether the cost is separable or not. When the agent is prudent and the technology exhibits non-increasing difference (CD or DD), we find a precautionary increase in the optimal level of self-insurance to cope with TU. The reverse holds whenever the agent is imprudent and the technology exhibits non-decreasing difference (CD or ID). TU may increase or decrease the optimal demand for self-insurance, and the direction of the effect depends on the combination of the agent's prudence and how uncertainty affects the self-insurance technology.

To illustrate our result, we provide a simple numerical example where the agent raises the level of self-insurance in response to TU. We assume $L(y,\kappa) = (1-\kappa)l(y)$ with $\kappa \in (0,1)$ and $l(y) = l_0 e^{-0.1y}$ (Example 1, DD). We set $w = 10$, $l_0 = 6$, $p = 0.1$, and use a separable cost function, $c(y) = 0.005y$. Risk preferences are represented by an iso-elastic utility function $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma = 0.5$, so the agent is risk-averse and prudent. We introduce TU by assuming $\widetilde{\kappa}$ to follow a symmetric beta distribution, denoted by $\text{Beta}(\alpha,\beta)$, with $\alpha = \beta = 0.01$.[11] The mean is $\overline{\kappa} = 0.5$, which represents the reference case without TU because $L(y,\kappa)$ is linear in κ.[12]

[11] We show in Appendix A.3 how the beta distributions parameterizes first-order and second-order risk changes. This identification makes it useful for illustrations.

[12] We choose all parameters to avoid corner solutions and facilitate illustration. We do not claim these parameters to be empirically supported.

Figure 2 shows expected utility with and without TU. If y^0 denotes the agent's optimal level of self-insurance in the absence of TU, then her demand for self-insurance increases from $y^0 = 7.19$ to $y^* = 7.83$ when we introduce TU, as predicted by Proposition 3*a)*. This increase is consistent with the agent's prudence and the technology exhibiting DD. We also see how TU reduces expected utility at any level of self-insurance due to risk aversion. The agent is better off under TU at her new level of self-insurance than if she had maintained the one in the absence of TU, but she can never be as well off as without TU due to risk aversion.

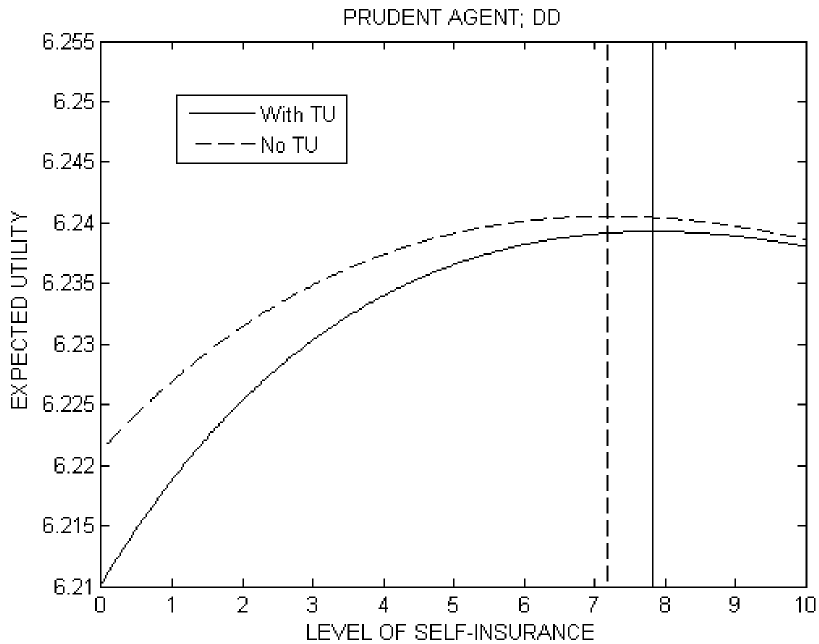

Figure 2: The effect of TU on the demand for self-insurance. The underlying parameters are $L(y, \kappa) = (1 - \kappa)l(y)$ with $l(y) = l_0 e^{-0.1y}$, $w = 10$, $l_0 = 6$, $p = 0.1$, a separable cost function of $c(y) = 0.005y$, and iso-elastic utility $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma = 0.5$. $\widetilde{\kappa}$ is distributed according to Beta$(0.01, 0.01)$ with mean $\overline{\kappa} = 0.5$.

We develop the economic intuition behind Proposition 3 for the case of prudence and DD with a separable cost function. To do so we need to understand how the marginal benefit of self-insurance is affected by TU. We obtain the following decomposition:

$$\begin{aligned} & -p \cdot \mathbb{E}L_y(y, \widetilde{\kappa})u'\left(w - L(y, \widetilde{\kappa})\right) + p \cdot \mathbb{E}L_y(y, \widetilde{\kappa}) \cdot u'\left(w - \mathbb{E}L(y, \widetilde{\kappa})\right) \\ = \; & -p \cdot \mathbb{E}L_y(y, \widetilde{\kappa})u'\left(w - L(y, \widetilde{\kappa})\right) + p \cdot \mathbb{E}L_y(y, \widetilde{\kappa})\mathbb{E}u'\left(w - L(y, \widetilde{\kappa})\right) \\ & -p \cdot \mathbb{E}L_y(y, \widetilde{\kappa}) \cdot \mathbb{E}u'\left(w - L(y, \widetilde{\kappa})\right) + p \cdot \mathbb{E}L_y(y, \widetilde{\kappa}) \cdot u'\left(w - \mathbb{E}L(y, \widetilde{\kappa})\right). \end{aligned} \tag{9}$$

The last line is a *precautionary effect*. At a given level of self-insurance, the loss size is random under TU whereas it is certain in the absence of TU. This consumption uncertainty gives rise to a precautionary motive, which induces a prudent agent to use self-insurance more in order to raise her expected consumption as a means of precaution. We reorganize the other terms as follows:

$$
\begin{aligned}
& -p \cdot \mathbb{E}L_y(y,\widetilde{\kappa})u'\left(w - L(y,\widetilde{\kappa})\right) + p \cdot \mathbb{E}L_y(y,\widetilde{\kappa})\mathbb{E}u'\left(w - L(y,\widetilde{\kappa})\right) \\
= \quad & -p\mathbb{E}u'\left(w - L(y,\widetilde{\kappa})\right) \cdot \left(L_y(y,\widetilde{\kappa}) - \mathbb{E}L_y(y,\widetilde{\kappa})\right) \qquad (10) \\
= \quad & \mathrm{Cov}\left(u'\left(w - L(y,\widetilde{\kappa})\right), -pL_y(y,\widetilde{\kappa}) + p\mathbb{E}L_y(y,\widetilde{\kappa})\right).
\end{aligned}
$$

For low values of κ losses are larger, resulting in lower consumption and higher marginal utility. Under DD, low values of κ correspond to high values of $-pL_y(y,\kappa)$. Consequently, an additional dollar invested in self-insurance reduces the expected loss by more when marginal utility is high than when marginal utility is low. This reinforces the use of self-insurance. We call this a *covariance effect* because it is driven by how marginal utility covaries with the expected productivity of self-insurance.

The comparison of Propositions 1 and 3 shows that TU has more subtle effects on optimal demand than on WTP. While TU always reduces the WTP for self-insurance, it may increase or decrease the optimal demand for self-insurance, depending on the relative strength of the precautionary effect and the covariance effect. Take the case of an agent with quadratic utility; this mutes the precautionary effect so that TU raises self-insurance if and only if the technology exhibits DD. Similarly, take the case of CD, which mutes the covariance effect; then TU raises self-insurance if and only if the agent is prudent. We also point out that the plausible combination of DD and prudence leads to higher levels of self-insurance under TU whereas WTP is lowered.

Two of the combinations in Table 1 are not conclusive because the precautionary effect and the covariance effect are conflicting. Under additional assumptions, we can also resolve these cases. Let $P(w) = -wu'''(w)/u''(w)$ denote the agent's relative prudence and let $w^{\ell}(y,\kappa)$ be shorthand for final wealth in the loss state, that is, $w^{\ell}(y,\kappa) = w - L(y,\kappa)$ in the separable case and $w^{\ell}(y,\kappa) = w - c(y) - L(y,\kappa)$ in the non-separable case. We obtain the following remark, which we prove in Appendix A.4.

Remark 1. *Assume the cost is separable.*

a) For $L(y,\kappa) = (1-\kappa)\cdot l(y)$ with $l' < 0$ and $l'' \geq 0$ (Example 1, DD) and an imprudent agent, TU raises optimal self-insurance if relative prudence is bounded by -2 and losses do not exceed loss state wealth.

b) For $L(y,\kappa) = l_0 \cdot (1-\kappa h(y))$ with $h' > 0$ and $h'' \leq 0$ (Example 2, ID) and a prudent agent, TU lowers optimal self-insurance if relative prudence does not exceed 2.

For *a)*, imprudence leads to a negative precautionary effect whereas DD results in a positive covariance effect. The bound on relative prudence ensures that the agent is not too imprudent so that the covariance effect dominates. This is why we obtain an increase in self-insurance. For *b)*, prudence implies a positive precautionary effect while ID induces a negative covariance effect. Prudence is bounded so that the covariance effect dominates, resulting in less self-insurance.

The assumption that final wealth in the loss state increases in the level of self-insurance is trivial for separable cost because it is precisely the meaning of self-insurance that losses are reduced, resulting in higher consumption when a loss happens. Matters are more intricate for a non-separable cost function. In the absence of TU, the same argument applies as for separable cost. At an optimal level of self-insurance, final wealth in the loss state must be upward sloping, otherwise self-insurance would not be in demand (see Dionne and Eeckhoudt, 1985, Eq. (2)). In the presence of TU, the level of self-insurance that is right "on average" may, however, be too high for some particular technologies so that $c'(y)$ could possibly preponderate $L_y(y, \kappa)$ for some values of κ. If it did, final wealth in the loss state would no longer be uniformly increasing in self-insurance at an optimal choice. This is what assumption $w^\ell_y(y^0, \kappa) > 0$ rules out.

3.3 An FSD Improvement in TU

So far, we have contrasted behavior with and without TU. We will now move on to situations where TU is present but may be subject to change. We first start with an FSD improvement, corresponding to question 2) posed in Section 2.1. Possible reasons for such a improvement include the advent of good news about the effectiveness of self-insurance or a technological improvement in available safety features. It can also identify heterogeneity in the perception of self-insurance effectiveness between different individuals. Some people may be more optimistic than others about the impact of their self-insurance efforts, and such differences in perception will affect their demand behavior.

Technically, we consider an FSD improvement of $\widetilde{\kappa}$. Due to $L_\kappa < 0$, this induces an FSD improvement in the distribution of final wealth in the loss state, making self-insurance less attractive due to diminishing marginal utility. A second effect arises from the associated change of the marginal product. To sign it, we introduce an elasticity that measures the impact of self-insurance on final wealth in the loss state.

Definition 2. $E_{w^\ell,y}(y, \kappa) = y \cdot w^\ell_y(y, \kappa)/w^\ell(y, \kappa)$ denotes the self-insurance elasticity of loss-state wealth.

The usual interpretation applies. A one percent increase in self-insurance raises final wealth in the loss state by $E_{w^\ell,y}$ percent. We also introduce relative risk aversion, defined as $R(w) = -wu''(w)/u'(w)$, which is sometimes interpreted as the wealth elasticity of marginal utility. We are now in a position to formulate our next result.

Proposition 4. *Assume $w^\ell_y(y^*, \kappa) > 0$. An FSD improvement in TU:*

a) Raises optimal self-insurance if relative risk aversion is less than 1 and the self-insurance elasticity of loss-state wealth is non-decreasing in κ.

b) Raises (leaves unchanged, lowers) optimal self-insurance if relative risk aversion is equal to 1 and the self-insurance elasticity of loss-state wealth increases (stays constant, decreases) in κ.

c) Lowers optimal self-insurance if relative risk aversion exceeds 1 and the self-insurance elasticity of loss-state wealth is non-increasing in κ.

	$R < 1$	$R = 1$	$R > 1$
$\partial_\kappa E_{w^\ell,y} > 0$	$+$	$+$	$+/-$
$\partial_\kappa E_{w^\ell,y} = 0$	$+$	0	$-$
$\partial_\kappa E_{w^\ell,y} < 0$	$+/-$	$-$	$-$

Table 2: The effect of an FSD improvement of TU on the demand for self-insurance

We provide a proof of Proposition 4 in Appendix A.5. So unlike WTP, which increases for all risk-averse agents following an FSD improvement in TU (see Proposition 1), the optimal demand for self-insurance may rise or fall, and Table 2 summarizes the exact conditions in compact form. The relevant preference criterion is how relative risk aversion compares to 1. Unity as a threshold for relative risk aversion is well established in the literature. Cheng et al. (1987) and Hadar and Seo (1990) study the effect of an FSD improvement in the return distribution of a risky asset on portfolio choice. Relative risk aversion below unity then ensures a larger investment in the risky asset. Eeckhoudt and Schlesinger (2008) analyze the effect of an FSD improvement in the interest rate on saving. Relative risk aversion then needs to be bounded by unity for optimal saving to increase.[13] The relevant technology criterion is how the self-insurance elasticity of loss-state wealth changes as the technology improves. We can connect this criterion to Definition 1. Direct computation reveals that

$$\partial_\kappa E_{w^\ell,y}(y,\kappa) \leq 0 \quad \iff \quad L_{y\kappa}(y,\kappa) \geq \frac{w^\ell_y(y,\kappa) L_\kappa(y,\kappa)}{w^\ell(y,\kappa)}. \tag{11}$$

The right hand side of (11) is negative, so as long as $L_{y\kappa}(y,\kappa) \geq 0$ the inequality is satisfied. This shows that DD and CD technologies have a self-insurance elasticity of loss-state wealth that is always decreasing in κ. Then the bottom row of Table 2 applies, and an FSD improvement in TU lowers optimal self-insurance as long as relative risk aversion is greater or equal to 1. This appears to be the most important case from an empirical perspective. (11) also reveals that ID is necessary to have constant or increasing self-insurance elasticity of loss-state wealth.

[13] Meyer and Meyer (2005) consolidate empirical results on relative risk aversion and find broad support for relative risk aversion *exceeding* one, in which case many comparative statics results remain indeterminate.

We provide two numerical examples to illustrate Proposition 4. In the first one we assume $L(y,\kappa) = l_0(1-\kappa h(y))$ with $l_0 > 0$, $h' > 0$, $h'' \leq 0$ and $\kappa \in (0,1)$ (Example 2, ID). This particular specification yields a self-insurance elasticity of loss-state wealth which is increasing in the technology parameter.[14] We set $w = 10$, $l_0 = 6$, $p = 0.1$, $h(y) = 1 - e^{-0.3y}$, and use a separable cost function, $c(y) = 0.005y$. Risk preferences are represented by an iso-elastic utility function $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma = 0.5$ so that relative risk aversion is below 1. Therefore, Proposition 4*a)* applies, corresponding to the top left corner of Table 2, and we expect an increase in self-insurance upon FSD improvements. We assume $\widetilde{\kappa}$ to follow a Beta$(\alpha, 1)$ distribution with α ranging from 0.01 to 1.5. As α increases, the distribution improves in the FSD sense (see Appendix A.3 for a proof). The upward sloping line in Figure 3 confirms that self-insurance increases from $y = 0$ to $y = 6.86$.

For the second numerical example we use $L(y,\kappa) = (1-\kappa)l_0(1-h(y))$ with $l_0 > 0$, $h' > 0$, $h'' \leq 0$ and $\kappa \in (0,1)$ (Example 1, DD). We take the same parameters as in the previous example except for relative risk aversion, which we set at $\gamma = 4$. DD implies that the self-insurance elasticity of loss-state wealth is decreasing in the technology parameter and relative risk aversion exceeds unity so that Proposition 4*c)* applies, corresponding to the bottom right corner of Table 2. We therefore expect a decrease in self-insurance upon FSD improvements. The downward sloping line in Figure 3 shows this effect. Self-insurance decreases from $y = 2.21$ to $y = 0$ as α increases.

To obtain economic intuition for Proposition 4, we examine the separable case where TU only affects the marginal benefit of self-insurance. For two different distributions $\widetilde{\kappa}_1$ and $\widetilde{\kappa}_2$ of the technology parameter, we obtain the following decomposition:

$$\begin{aligned} & -p\mathbb{E}L_y(y,\widetilde{\kappa}_2)u'(w-L(y,\widetilde{\kappa}_2)) + p\mathbb{E}L_y(y,\widetilde{\kappa}_1)u'(w-L(y,\widetilde{\kappa}_1)) \\ = \; & -p\mathbb{E}L_y(y,\widetilde{\kappa}_2)\left[u'(w-L(y,\widetilde{\kappa}_2)) - u'(w-L(y,\widetilde{\kappa}_1))\right] \\ & +p\mathbb{E}\left[L_y(y,\widetilde{\kappa}_1) - L_y(y,\widetilde{\kappa}_2)\right]u'(w-L(y,\widetilde{\kappa}_1)). \end{aligned} \tag{12}$$

If $\widetilde{\kappa}_2$ has FSD over $\widetilde{\kappa}_1$, losses under $\widetilde{\kappa}_2$ are lower than under $\widetilde{\kappa}_1$ in an FSD sense. But then loss-state wealth under $\widetilde{\kappa}_2$ has FSD over loss-state wealth under $\widetilde{\kappa}_1$ so that expected marginal utility is lower under $\widetilde{\kappa}_2$ than under $\widetilde{\kappa}_1$. This is due to diminishing marginal utility, which is why we label it a *risk aversion effect.* It implies for the first square bracket in Eq. (12) to have a negative expected value. Intuitively, with lower expected marginal utility, reducing losses increases expected utility by less than if marginal utility were high. So the risk aversion effect diminishes the demand for self-insurance. On the other hand, if $\widetilde{\kappa}_2$ has FSD over $\widetilde{\kappa}_1$, there is an effect on the marginal product of self-insurance. Under DD for example, better technologies have a lower marginal product so that $-L_y(y,\widetilde{\kappa}_1)$ has FSD over $-L_y(y,\widetilde{\kappa}_2)$. We call this a *productivity effect.* It implies for the second square bracket in Eq. (12) to

[14] The self-insurance elasticity of loss-state wealth is $E_{w^\ell,y} = yl_0\kappa h'/(w - l_0(1-\kappa h(y)))$. The numerator of $\partial_\kappa E_{w^\ell,y}$ is given by $yl_0h'(w-l_0)$, which is positive as long as some self-insurance is in demand.

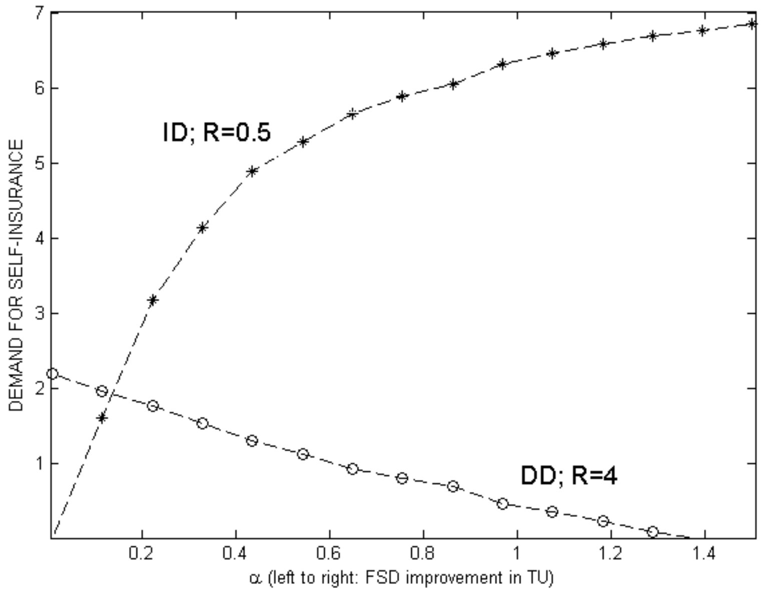

Figure 3: The effect of an FSD improvement in TU on optimal self-insurance. The underlying parameters are $L_{ID}(y,\kappa) = l_0(1-\kappa h(y)$ with $h(y) = 1 - e^{-0.3y}$,$L_{DD}(y,\kappa) = (1-\kappa)l(y)$ with $l(y) = l_0 e^{-0.3y}$, $w = 10$, $l_0 = 6$, $p = 0.1$, a separable cost function of $c(y) = 0.005y$, and iso-elastic utility $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma_{DD} = 4$ and $\gamma_{ID} = 0.5$. $\widetilde{\kappa}$ is distributed according to Beta$(\alpha, 1)$ with α ranging from 0.01 to 1.5.

have a negative expected value and exerts a negative effect on the demand for self-insurance. However, the net effect also depends on how the risk aversion effect covaries with the marginal product and how the productivity effect covaries with marginal utility. The conditions stated in Proposition 4 allow for a compact way of resolving all associated trade-offs.

In the following remark, we provide some further conditions that allow us to resolve the inconclusive combinations in Proposition 4.

Remark 2. *Assume the cost is separable.*

a) Under CD and DD, an FSD improvement of technological uncertainty always reduces the optimal level of self-insurance.

b) For $L(y,\kappa) = l_0 \cdot (1-\kappa h(y))$ with $h' > 0$ and $h'' \leq 0$ (Example 2, ID), an FSD improvement of the technological uncertainty raises the optimal level of self-insurance if relative risk aversion does not exceed 2 and $w > 2l_0$.

3.4 An Increase in TU

We now move on to the effect of an increase in TU on optimal self-insurance. Our analysis will provide an answer to question 3) posed in Section 2.1. People may differ in the perceived uncertainty of a self-insurance effort, and our results inform about the behavioral implications of such heterogeneity. We can also speak to cases where new information, scientific discovery or technological improvements reduce TU without eliminating it. This is an important generalization of Proposition 3 because risk reduction activities target uncertain outcomes in the future, making it hardly conceivable for TU ever to be absent at all.

From a modeling perspective, we cannot simply use a Rothschild and Stiglitz (1970) increase in risk of $\widetilde{\kappa}$ because such a change may or may not induce an increase in risk of $L(y, \widetilde{\kappa})$ for reasons of non-linearity in the technology parameter. We address this challenge explicitly in the strategy of our proof and consider changes that affect the riskiness of losses in a clearly defined way. The additional risk on consumption raises the incentive to use self-insurance for prudent agents. At the same time, there is an effect on the marginal product of self-insurance. To determine net effects, we require an additional measure of how self-insurance affects wealth.

Definition 3. $E_{w_\kappa^\ell, y}(y, \kappa) = y \cdot w_{y\kappa}^\ell(y, \kappa)/w_\kappa^\ell(y, \kappa)$ denotes the self-insurance elasticity of technological improvement.

This terminology is motivated by the following reasoning. Higher values of κ correspond to better technologies in the sense that losses are lower. In other words, loss-state wealth is increasing in κ, and w_κ^ℓ measures by how much it increases when moving to the next best technology. This technological improvement is sensitive to the use of self-insurance, and we know from Definition 1 that self-insurance may lower or raise it, depending on whether the technology has DD or ID. The elasticity in Definition 3 measures the magnitude of this effect. A one percent increase in self-insurance changes w_κ^ℓ by $E_{w_\kappa^\ell, y}$ percent. Under DD, for example, $E_{w_\kappa^\ell, y}$ is negative so if we increase self-insurance by one percent, w_κ^ℓ declines by $|E_{w_\kappa^\ell, y}|$ percent. Then, the favorable effect on loss-state wealth from moving to the next best technology is dampened by this factor due to the raise in self-insurance.[15]

Proposition 5. *Assume $w_y^\ell(y^*, \kappa) > 0$. An increase in TU:*

a) *Raises optimal self-insurance if relative prudence exceeds 2, the self-insurance elasticity of loss-state wealth is non-increasing in κ and the self-insurance elasticity of technological improvement is non-decreasing in κ.*

b) *Raises (leaves unchanged, lowers) optimal self-insurance if relative prudence is equal to 2, the self-insurance elasticity of loss-state wealth is decreasing (constant, increasing) in κ and the self-insurance elasticity of technological improvement is non-decreasing (constant, non-increasing) in κ.*

c) *Lowers optimal self-insurance if relative prudence is less than 2, the self-insurance elasticity of loss-state wealth is non-decreasing in κ and the self-insurance elasticity of technological improvement is non-increasing in κ.*

[15] Unlike the self-insurance elasticity of loss-state wealth, the self-insurance elasticity of technological improvement does not depend on initial wealth. An alternative notation is $E_{L_\kappa, y}$, and the reasoning would then be in terms of loss sizes instead of loss-state wealth.

We illustrate Proposition 5 in Table 3, where $P = -w\frac{u'''(w)}{u''(w)}$ indicates the agent's degree of relative prudence. Roughly speaking, in those cases where prudence is large, the precautionary effect dominates and the increase in technological uncertainty induces the agent to optimally increase her level of self-insurance. For this intuitive result to hold, certain requirements on the technology need to be satisfied. On the contrary, if prudence is small, the precautionary effect is less pronounced and we can think of the increase in technological uncertainty to further compromise the efficiency of self-insurance. Coupled with the appropriate elasticity conditions, this induces the decision-maker to optimally lower her investment in self-insurance.

	$P < 2$	$P = 2$	$P > 2$
$\partial_\kappa E_{w^\ell,y} > 0, \quad \partial_\kappa E_{w^\ell_\kappa,y} \leq 0$	$-$	$-$	$+/-$
$\partial_\kappa E_{w^\ell,y} = 0, \quad \partial_\kappa E_{w^\ell_\kappa,y} = 0$	$-$	0	$+$
$\partial_\kappa E_{w^\ell,y} < 0, \quad \partial_\kappa E_{w^\ell_\kappa,y} \geq 0$	$+/-$	$+$	$+$

Table 3: The effect of an increase in TU on the demand for self-insurance

We illustrate Proposition 5 with the following two examples whose numerical results are shown in Figure 4. Consider again an agent with iso-elastic utility function $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where γ is the degree of relative risk aversion, which also implies that the degree of relative prudence equals $\gamma + 1$. For the first example, assume $L(y, \kappa) = l_0(1 - \kappa h(y))$ where $h(y) = 1 - e^{-0.3y}$. (Example 2, ID) Simple calculation shows that this loss function implies $\partial_\kappa E_{w^\ell,y}(y, \kappa) > 0$ and $\partial_\kappa E_{w^\ell_\kappa,y}(y, \kappa) = 0$, which is why, according to Proposition 5, the optimal self-insurance should be reduced by an increase of technological uncertainty when the agent's degree of relative prudence does not exceed 2. To observe this numerically, we let $\widetilde{\kappa}$ follow a symmetric beta-distribution Beta(α, β) where α ranges from 0.01 to 0.5 and $\beta = \alpha$. As we show in the appendix, a decrease in α renders a mean-preserving spread of the Beta distribution. We let $\gamma = 0.9$ so that the agent's degree of relative prudence equals 1.9. As shown by Figure 4, an increase in technological uncertainty indeed reduces the optimal level of self-insurance from $y = 3.83$ to $y = 3.45$, which is in line with our theoretical prediction.

For the second example, $L(y, \kappa) = (1 - \kappa)l(y)$ where $l(y) = l_0 e^{-0.3y}$. (Example 1, DD) Obviously, $\partial_\kappa E_{w^\ell,y}(y, \kappa) < 0$ and $\partial_\kappa E_{w^\ell_\kappa,y}(y, \kappa) = 0$, therefore Proposition 5 predicts higher demand for self-insurance upon an increase in technological uncertainty when the degree of relative prudence exceeds 2. We set $\gamma = 1.5$ so that the degree of relative prudence is 2.5. The prediction is confirmed by the upward trend in Figure 4, where the agent's optimal self-insurance is increased from $y = 2.44$ to $y = 2.73$.

Again, the two ambiguous combinations of Proposition 5 can be resolved once we know further details about the loss function. The following remark provides such an example, whose proof can be found in the appendix.

Remark 3. *Assume separable cost. An increase in technological uncertainty:*

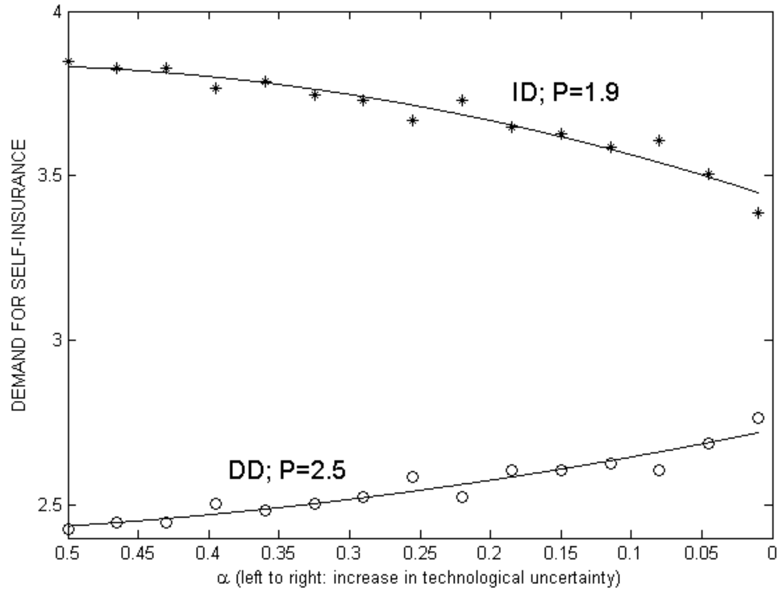

Figure 4: The effect of increases in technological uncertainty on the demand for self-insurance. The underlying parameters are $L_{ID}(y,\kappa) = l_0(1-\kappa h(y)$ with $h(y) = 1 - e^{-0.3y}$,$L_{DD}(y,\kappa) = (1-\kappa)l(y)$ with $l(y) = l_0 e^{-0.3y}$, $w = 10$, $l_0 = 6$, $p = 0.1$, a separable cost function of $c_{ID}(y) = 0.005y$, $c_{DD}(y) = 0.002y$ and iso-elastic utility $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma_{ID} = 0.9$ and $\gamma_{DD} = 1.5$. $\widetilde{\kappa}$ is distributed according to Beta(α,α) with α ranging from 0.01 to 0.5.

a) raises the optimal level of self-insurance for a prudent agent when $L(y,\kappa) = (1-\kappa)l(y)$, $l_y < 0,\ l_{yy} \geq 0$ *(Example 1, DD)*

b) lowers the optimal level of self-insurance for an agent with relative prudence below 4 when $L(y,\kappa) = l_0(1-\kappa h(y))$, $h_y > 0,\ h_{yy} \leq 0$ *(Example 2, ID) and* $w > 2l_0$.

In the first case, the marginal productivity of self-insurance is small when the loss is large (ID). Similar to our earlier observation in Proposition 3, this effect encourages a reduction of self-insurance upon an increase in technological uncertainty, which works against the precautionary effect and therefore relaxes the upper threshold of relative prudence for lower self-insurance from 2 to 4. In the second case, the loss function (DD) generates an effect in line with the precautionary effect such that it allows any prudent agent - including those whose degree of relative prudence is below 2 - to raise the level of self-insurance upon an increase of technological uncertainty.

4 Optimal Demand for Self-Protection

4.1 Preliminaries

We will now study self-protection activities. Unlike self-insurance, self-protection has no effect on the loss size but reduces the probability of loss instead. We denote by $x \geq 0$ the level of self-protection. We maintain the distinction between separable and non-separable cost, and denote the cost function by $c(x)$ with $c' > 0$ and $c'' \geq 0$. TU enters the analysis by letting the probability of loss depend on both x and an exogenous variable $\kappa \in [\underline{\kappa}, \overline{\kappa}]$ so that $p = p(x, \kappa)$. This technology variable has the same interpretation as for self-insurance. For any $\kappa \in [\underline{\kappa}, \overline{\kappa}]$, we assume $p_x < 0$ and $p_{xx} \geq 0$, that is, any possible self-protection technology reduces the probability of loss at a decreasing rate. We also assume $p_\kappa < 0$, so high values of κ identify better technologies in the sense of a lower loss probability. We use the taxonomy analogous to Definition 1 for the sign of the cross-derivative.

The agent's objective function for separable cost is then given by

$$\max_{x \geq 0} U(x) = (1 - \mathbb{E}p(x, \widetilde{\kappa}))u(w) + \mathbb{E}p(x, \widetilde{\kappa})u(w - L) - c(x), \tag{13}$$

where the expectation is taken with respect to the agent's beliefs over TU. The associated first-order condition is

$$U'(x) = -\left(u(w) - u(w - L)\right) \mathbb{E}p_x(x, \widetilde{\kappa}) - c'(x) = 0, \tag{14}$$

and we denote the optimal level of self-protection by x^*. The second-order condition holds under the assumptions made. The marginal cost is the disutility of exerting a self-protection effort whereas the marginal benefit results from the expected increase of expected consumption utility due to a lower loss probability. Only the marginal benefit is affected by TU because of the separability of cost.

In the non-separable case, the agent's objective function is

$$\max_{x \geq 0} U(x) = (1 - \mathbb{E}p(x, \widetilde{\kappa}))u(w - c(x)) + \mathbb{E}p(x, \widetilde{\kappa})u(w - L - c(x)), \tag{15}$$

with corresponding first-order condition

$$\begin{aligned} U'(x) &= -\left(u(w - c(x)) - u(w - L - c(x))\right) \mathbb{E}p_x(x, \widetilde{\kappa}) \\ &\quad - c'(x)(1 - \mathbb{E}p(x, \widetilde{\kappa}))u'(w - c(x)) - c'(x)\mathbb{E}p(x, \widetilde{\kappa})u'(w - L - c(x)) = 0. \end{aligned} \tag{16}$$

We assume the second-order condition to hold.[16] Without separability, TU enters both the marginal cost and the marginal benefit of self-protection.

An increase in TU does not affect the demand for self-protection whether the cost is separable or not. This null effect generalizes to any stochastic change that preserves the mean self-protection technology. In all these cases the objective function remains unaffected because TU simply washes out. Questions 1) and 3) from Section 2.1 therefore have a trivial answer. The effect of TU and of increases in TU is the same for WTP for self-protection and optimal demand for self-protection (see Proposition 2), and while these effects are complex in case of self-insurance (see Propositions 3 and 5), they are trivial for self-protection. The reason is

[16] The objective function in the standard self-protection problem may or may not be concave. Jullien et al. (1999) provide sufficient conditions for concavity in the absence of TU.

the linearity of expected utility in probabilities. So to argue for the relevance of TU in case of self-protection, deviations from expected utility are a prerequisite.

4.2 An FSD Improvement in TU

We study the effect of FSD changes in TU on optimal self-protection because such changes alter the expected probability of loss. This corresponds to question 2) in Section 2.1. Reasons for FSD improvements include new information about the effectiveness of self-protection, technological improvements or changes in the agent's perception. We summarize our findings in the following proposition, which we demonstrate in Appendix A.7

Proposition 6. *An FSD improvement in TU:*

a) Increases (leaves unchanged, decreases) self-protection if the cost is separable and the technology has ID (CD, DD).

b) Increases self-protection if the cost is non-separable and the technology has ID or CD.

Whereas the WTP for self-protection always increases following an FSD improvement in TU (see Proposition 2), the optimal demand for self-protection may increase or decrease. The FSD improvement affects the marginal product of self-protection in a way akin to the productivity effect in Section 3.3. Under DD, better technologies have lower marginal products, which reduces the marginal benefit of self-protection. When the cost is separable, this is the only effect of an FSD improvement, and demand for self-protection goes down. Under a non-separable cost, we also have to take into account that the FSD improvement lowers the average probability of loss. Then there is less weight on the state with high marginal utility, which reduces the marginal cost of self-protection. This explains why the DD case remains inconclusive under non-separable cost because the effect on marginal benefit conflicts with that on marginal cost. When the technology exhibits ID or CD, the productivity effect is positive, and the effects of the FSD improvement on marginal benefit and marginal cost are aligned.

In Figure 5, we provide two numerical examples that show the demand for self-protection as the technology undergoes an FSD improvement. The downward sloping curve represents a DD technology $p(x,\kappa) = (1-\kappa)g(x)$, $g(x) = e^{-0.05x}$ and a separable cost function $c_(x) = 0.004x$, whereas for the upward sloping curve, we have an ID technology with $p(x,\kappa) = 1 - \kappa h(x)$, $h(x) = 1 - e^{-0.05x}$ and a separable cost function $c(x) = 0.0015x$. The agent has iso-elastic utility $u(z) = z^{1-\gamma}/(1-\gamma)$ with $\gamma = 1.5$. $\widetilde{\kappa}$ is distributed according to Beta$(\alpha, 1)$ with α ranging from 0.01 to 1.5 so that an increase in α corresponds to an FSD improvement. As predicted by Proposition 6 a), DD (ID) induces the agent to decrease (increase) her effort when there is an FSD improvement of the technology.

While Proposition 4 requires restrictions on the intensity of risk aversion for FSD improvements and self-insurance, Proposition 6 holds for all risk-averse agents. This is a rare

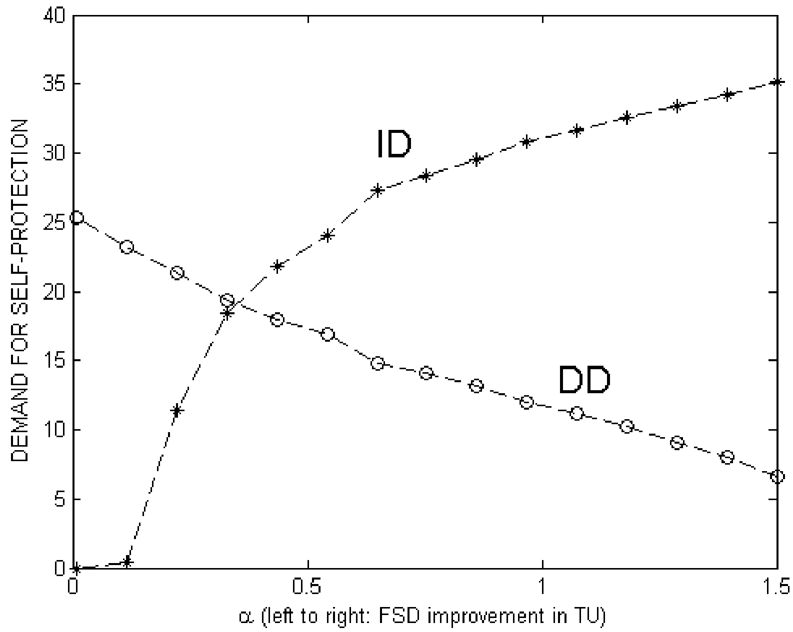

Figure 5: The effect of an FSD improvement in technological uncertainty on the demand for self-protection. The underlying parameters are $p_{ID}(x,\kappa) = 1 - \kappa h(x)$ with $h(x) = 1 - e^{-0.05x}$,$p_{DD}(x,\kappa) = (1-\kappa)g(x)$ with $g(x) = e^{-0.05x}$, $w = 10$, $L = 6$, a separable cost function of $c_{ID}(x) = 0.0015x$, $c_{DD}(x) = 0.004x$ and iso-elastic utility $u(z) = z^{1-\gamma}/(1-\gamma)$ with $\gamma = 1.5$. $\widetilde{\kappa}$ is distributed according to Beta$(\alpha, 1)$ with α ranging from 0.01 to 1.5.

occasion where self-protection admits much simpler comparative statics than self-insurance. Given that DD is considered more plausible than ID or CD (Doherty and Posey, 1998; Crainich and Eeckhoudt, 2017), we will develop conditions that allow to resolve the trade-off under non-separable costs. We define the utility premium (see Friedman and Savage, 1948; Eeckhoudt and Schlesinger, 2009) associated with loss L as

$$v(w;x) = u(w - c(x)) - u(w - L - c(x)), \tag{17}$$

and denote its derivative with respect to w as $v'(w;x)$. Monotonicity of u renders v positive. It measures the pain of incurring loss L in units of utility. Risk aversion of u implies that v is decreasing in wealth because at high wealth levels it is less painful to incur loss L than at low wealth levels. We also state a definition pertaining to the self-protection technology.

Definition 4. Assume that $p(x,\kappa)$ exhibits DD. We call $\rho(x,\kappa) = -p_{x\kappa}(x,\kappa)/p_{\kappa}(x,\kappa)$ the decay rate of technological improvement.

Higher values of κ correspond to better technologies because the associated loss probability is lower. One way to think of DD is that it dampens the gain from better technologies the more the agent invests in self-protection. ρ measures the magnitude of this effect.[17] We are now in a position to state our last proposition, which we prove in Appendix A.8.

Proposition 7. *Assume a non-separable cost and a technology with DD. Then, an FSD improvement in TU increases optimal self-protection if*

$$\rho(x^*, \kappa) < -c'(x^*)\frac{v'(w;x^*)}{v(w;x^*)} \quad \textit{for all } \kappa \in [\underline{\kappa}, \overline{\kappa}]. \tag{18}$$

It decreases optimal self-protection if the reverse inequality holds.

Condition (18) has an easy interpretation. The left hand side measures by how much the marginal product of self-protection deteriorates when moving to better technologies. If this measure is bounded, self-protection is still sufficiently productive for the agent to raise her investment in self-protection following the FSD improvement. In other words, the decrease in the marginal cost due to lower loss probabilities outweighs the decrease in the marginal benefit resulting from DD. The right hand side of (18) is related to the agent's level of risk aversion, which we can see most clearly with the help of a specific example.

Example 4. Consider an agent with exponential utility, $u(w) = 1 - e^{-Aw}$, with absolute risk aversion $A > 0$. Also assume a constant per-unit cost of self-protection, $c(x) = kx$ with $k > 0$, and a technology of $p(x, \kappa) = (1-\kappa)e^{-\nu x}$ with $\kappa \in (0, 1)$ and $\nu > 0$ (Example 1, DD). It is easy to see that $\rho(x, \kappa) = \nu$. Then, condition (18) becomes $\nu < kA$. An FSD improvement in TU increases optimal self-protection if and only if risk aversion exceeds ν/k.

Risk aversion determines the wedge between marginal utility in the loss state versus the no-loss state. Therefore, it measures by how much the marginal cost of self-protection decreases when moving to better technologies. Even outside the specific class of exponential utility functions, the decay rate of the utility premium can be related to the agent's degree of risk aversion, as is shown by the following remark, which we prove in the appendix.

Remark 4. *If u is risk vulnerable (Gollier and Pratt, 1996), we can express a lower bound for $-v'(w;x^*)/v(w;x^*)$ in terms of the agent's absolute risk aversion. Then the following conditions is sufficient for* (18)*:*

$$\rho(x^*, \kappa) < c'(x^*)\mathcal{A}(w - c(x^*) - L/2) \quad \textit{for all } \kappa \in [\underline{\kappa}, \overline{\kappa}], \tag{19}$$

where $\mathcal{A}(\cdot) = -u''(\cdot)/u'(\cdot)$ is Arrow-Pratt risk aversion.

We have seen that for high enough risk aversion, an FSD improvement in TU increases optimal self-protection under DD for a non-separable cost, while it decreases it for any degree of risk aversion under separable cost. This discrepancy illustrates how the distinction between separable and non-separable cost can lead to diametrically different results.

[17] Measures of this form occur in the comparative statics analysis of multivariate self-protection decisions (Hofmann and Peter, 2015) and for self-protection against multiple risks (Courbage et al., 2017).

5 Conclusion

In this paper, we study risk reduction in the presence of technological uncertainty. This is motivated by situations in which environmental factors influence the effectiveness of risk reduction, as well as where the lack of information or heterogeneity in individual perceptions make the benefits of risk reduction only imperfectly predictable. Consistent with prior literature, we conduct our analysis by differentiating between self-insurance and self-protection. Specifically, we study the effects of technological uncertainty as well as those of FSD improvements and increases in technological uncertainty on the willingness to pay and the optimal demand for both types of activities.

The effects on willingness to pay are very straightforward and can be subsumed by the observation that technological uncertainty compromises the attractiveness of risk reduction. In contrast, the effects on the optimal demand for self-insurance and self-protection are more intricate and give rise to interesting trade-offs that have not been identified in previous literature. In particular, the optimal demand for self-insurance may increase or decrease depending on how the channel through which uncertainty operates on the technology interacts with the agent's prudence. A prudent agent will have a precautionary demand for self-insurance in the presence of technological uncertainty, consistent with the precautionary principle, and this effect prevails as long as potentially countervailing technology effects are bounded. We also notice that in the case of self-insurance, technological uncertainty reduces the individual's welfare regardless of its behavioral implications, which is a direct consequence of risk aversion. It is thus impossible to use changes in the demand for self-insurance as an indicator of the agent's welfare unless additional preference and technology parameters are identified along the lines of the measures proposed in this paper.

Our findings have descriptive and normative implications. First, they may help explain why individuals use certain forms of risk reduction to a lesser extent than would be expected. Our propositions identify cases where technological uncertainty and increases thereof induce individuals to utilize less self-insurance, as well as those where FSD improvements in technological uncertainty lower the demand for self-insurance and self-protection. If technological uncertainty arises from a lack of information or from the agent's biased beliefs that underestimate the true effectiveness of risk mitigation, individuals might fail to maximize expected utility and make sub-optimal choices. Finally, our results inform public policy in the many areas where scientific knowledge about the precise cause-effect relationships is incomplete. In many of these cases, such as fighting against climate change, we are called to act today although our knowledge of how our actions are going to mitigate future risk is subject to considerable uncertainty. Proponents of higher investments to fight climate change argue based on the precautionary principle. Opponents argue that we might be wasting resources if we cannot even be sure how much future generations will benefit from our endeavor. Our results show that both sides have their point and that, in order to arrive at a solution, we need to reach an agreement on the intensity of our precautionary motive and the way we believe information to alter the effectiveness of risk mitigation. We are confident that our results will spur empirical tests in that direction and that our measures will find fruitful applications in many other areas of decision-making under risk and uncertainty.

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A Mathematical Proofs

A.1 Proof of Proposition 1

Combining Eq. (1) and (2), we obtain

$$\begin{aligned} &(1-p)\cdot u(w-v_0)+p\cdot u(w-L_0-v_0)=(1-p)\cdot u(w-v_1)+p\cdot \mathbb{E}u(w-\widetilde{L}_1-v_1) \\ &<(1-p)\cdot u(w-v_1)+p\cdot u(w-\mathbb{E}\widetilde{L}_1-v_1)=(1-p)\cdot u(w-v_1)+p\cdot u(w-L_0-v_1), \end{aligned} \tag{20}$$

where the inequality holds due to risk aversion. The claim then follows because expected utility is decreasing in v_0. The proofs for the FSD improvement and an increase in TU are analogous and utilize well-known links between changes in risk and expected utility, see, for example, Theorem 2 in Eeckhoudt et al. (2009).

A.2 Proof of Proposition 3

To remove TU, we solve the agent's problem if the technology is given by the expected technology, i.e., $\bar{L}(y)=\mathbb{E}L(y,\widetilde{\kappa})$, and then compare the optimal level of self-insurance to the one obtained in the presence of TU. We start with a separable cost function. The agent's objective in the absence of TU is given by

$$\max_{y\geq 0} V(y)=(1-p)u(w)+pu(w-\bar{L}(y))-c(y), \tag{21}$$

with associated first-order condition

$$-pu'(w-\bar{L}(y^0))\bar{L}_y(y^0)-c'(y^0)=0. \tag{22}$$

y^0 is shorthand for the optimal level of self-insurance in the absence of TU (i.e., when TU is set to zero). We insert this level into the agent's first-order expression in the presence of TU and utilize condition (22) to obtain

$$\begin{aligned} U'(y^0) &= -p\cdot \mathbb{E}L_y(y^0,\widetilde{\kappa})u'(w-L(y^0,\widetilde{\kappa}))-c'(y^0) \\ &= -p\cdot \mathbb{E}L_y(y^0,\widetilde{\kappa})u'(w-L(y^0,\widetilde{\kappa}))+pu'(w-\bar{L}(y^0))\bar{L}_y(y^0) \\ &= p\cdot \mathrm{Cov}\left[-u'(w-L(y^0,\widetilde{\kappa})),L_y(y^0,\widetilde{\kappa})\right] \\ &\quad -p\bar{L}_y(y^0)\left(\mathbb{E}u'(w-L(y^0,\widetilde{\kappa}))-u'(w-\bar{L}(y^0))\right). \end{aligned} \tag{23}$$

The last equality follows from the covariance rule and the fact that $\mathbb{E}L_y(y^0,\widetilde{\kappa})=\bar{L}_y(y^0)$. The sign of the covariance depends on how $u'(w-L(y^0,\kappa))$ and $L_y(y^0,\kappa)$ covary in κ. We obtain

$$\partial_\kappa(-1)\cdot u'(w-L(y^0,\kappa))=L_\kappa(y^0,\kappa)u''(w-L(y^0,\kappa))>0. \tag{24}$$

The sign of $L_{y\kappa}(y^0,\kappa)$ is directly determined by whether the technology exhibits DD, CD or ID. As a consequence, the covariance term is positive (zero, negative) under DD (CD, ID).

The term $-p\bar{L}_y(y^0)$ is always positive whereas $\mathbb{E}u'(w - L(y^0, \widetilde{\kappa})) - u'(w - \bar{L}(y^0))$ is positive (zero, negative) if the agent is prudent (has quadratic utility, is imprudent). Combining these signs accordingly completes the proof for separable cost.

For the non-separable case, the agent's objective function in the absence of TU is

$$\max_{y \geq 0} V(y) = (1-p)u(w - c(y)) + pu(w - \bar{L}(y) - c(y)), \tag{25}$$

with associated first-order condition

$$-(1-p) \cdot u'(w - c(y^0))c'(y^0) - p \cdot u'(w - \bar{L}(y^0) - c(y^0))(\bar{L}_y(y^0) + c'(y^0)) = 0. \tag{26}$$

Necessary for the first-order condition to be satisfied is that $\bar{L}_y(y^0) + c'(y^0) < 0$. We insert y^0 into the agent's first-order expression in the presence of TU and utilize (26) to obtain

$$\begin{aligned} U'(y^0) &= -(1-p) \cdot u'(w - c(y^0))c'(y^0) - p \cdot \mathbb{E}u'(w - L(y^0, \widetilde{\kappa}) - c(y^0))(L_y(y^0, \widetilde{\kappa}) + c'(y^0)) \\ &= -p \cdot \mathbb{E}u'(w - L(y^0, \widetilde{\kappa}) - c(y^0))(L_y(y^0, \widetilde{\kappa}) + c'(y^0)) \\ &\quad + p \cdot u'(w - \bar{L}(y^0) - c(y^0))(\bar{L}_y(y^0) + c'(y^0)) \\ &= p \cdot \mathrm{Cov}\left[-u'(w - L(y^0, \widetilde{\kappa}) - c(y^0)), L_y(y^0, \widetilde{\kappa}) + c'(y^0)\right] \\ &\quad - p\left(\bar{L}_y(y^0) + c'(y^0)\right)\left(\mathbb{E}u'(w - L(y^0, \widetilde{\kappa}) - c(y^0)) - u'(w - \bar{L}(y^0) - c(y^0))\right). \end{aligned} \tag{27}$$

The last equality follows from the covariance rule and the fact that $\mathbb{E}L_y(y^0, \widetilde{\kappa}) = \bar{L}_y(y^0)$. As before, we sign the covariance term depending on how its arguments covary in κ. It is positive (zero, negative) under DD (CD, ID). The term $-p\left(\bar{L}_y(y^0) + c'(y^0)\right)$ is positive due to the optimality of y^0 and the term $\mathbb{E}u'(w - L(y^0, \widetilde{\kappa}) - c(y^0)) - u'(w - \bar{L}(y^0) - c(y^0))$ is positive (zero, negative) if the agent is prudent (has quadratic utility, is imprudent). The proposition then follows by combining these signs accordingly.

A.3 First-order and second-order risk changes with the beta distribution

Technology parameter κ ranges between 0 and 1 so the beta distribution is a natural choice for illustration. It turns out to parameterize first-order and second-order risk changes, which we will show rigorously here. For parameters $\alpha > 0$ and $\beta > 0$, its density is

$$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha, \beta)}, \ x \in [0,1] \text{ or } x \in (0,1), \tag{28}$$

where $\mathrm{B}(\alpha, \beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$ denotes the beta function and $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}\mathrm{d}x$ the gamma function. Mean and variance are given by

$$\mu = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \tag{29}$$

The mean is increasing in α and decreasing in β, and the variance is decreasing in α if we choose β such as to keep the mean constant.[18]

Lemma 1. *Let $F(x;\alpha,\beta)=\int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}\mathrm{d}t$ be the cumulative distribution function of the beta distribution with parameters α and β.*

a) An increase in α or a decrease in β improve F in the sense of FSD.

b) At a constant mean, an increase in α induces a mean-preserving contraction in F.

Proof. For *a)* we need to show $\partial F(x;\alpha,\beta)/\partial\alpha \leq 0$ for all $x\in[0,1]$ and $\partial F(x;\alpha,\beta)/\partial\alpha < 0$ for some $x\in[0,1]$. We obtain

$$\begin{aligned}
\frac{\partial F(x;\alpha,\beta)}{\partial \alpha} &= \int_0^x \frac{\partial}{\partial\alpha}\left[\frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}\right]\mathrm{d}t \qquad (30)\\
&= \int_0^x \frac{B(\alpha,\beta)(\ln t)\cdot t^{\alpha-1}(1-t)^{\beta-1}-t^{\alpha-1}(1-t)^{\beta-1}B(\alpha,\beta)\left(\psi_0(\alpha)-\psi_0(\alpha+\beta)\right)}{\left(B(\alpha,\beta)\right)^2}\mathrm{d}t\\
&= \underbrace{\int_0^x \left[\ln t\cdot\frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}\right]\mathrm{d}t + \left(\psi_0(\alpha+\beta)-\psi_0(\alpha)\right)\cdot\int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}\mathrm{d}t}_{g(x)},
\end{aligned}$$

where $\psi_0(\omega)=\Gamma'(\omega)/\Gamma(\omega)$ is the digamma function.[19]. Furthermore,

$$g'(x)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}\cdot\left[\ln x+\psi_0(\alpha+\beta)-\psi_0(\alpha)\right]. \qquad (31)$$

Since $x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha,\beta) > 0$, the sign of $g'(x)$ coincides with the sign of the square bracket in (31). Now $\psi_0(\alpha+\beta)-\psi_0(\alpha)>0$ (see Footnote 19) and $\ln x$ is strictly increasing on $(0,1]$ from $-\infty$ to 0. Therefore, $x^*=\exp(\psi_0(\alpha)-\psi_0(\alpha+\beta))$ is the unique zero of $g'(x)$ on $(0,1)$, where it switches from negative to positive. Also $g(0)=g(1)=0$ as a result of $F(0;\alpha,\beta)\equiv 0$ and $F(1;\alpha,\beta)\equiv 1$. So $g(x)$ starts at zero, decreases until $x=x^*$ where it reaches a global minimum, and then increases back to 0 as x approaches 1. We conclude that $\partial F(x;\alpha,\beta)/\partial\alpha\leq 0$ for all $x\in[0,1]$ and $\partial F(x;\alpha,\beta)/\partial\alpha<0$ for $x\in(0,1)$ as required. The proof for β is analogous.

To show *b)*, we fix the mean at $\mu_0\in(0,1)$. Then, per Eq. (29), we obtain $\beta=\alpha(1-\mu_0)/\mu_0$ so that we can rewrite the cumulative distribution function as $J(x;\alpha)=F(x;\alpha,\alpha(1-\mu_0)/\mu_0)$. Its derivative with respect to α is

$$\frac{\partial J(x;\alpha)}{\partial\alpha} = \frac{\partial F(x;\alpha,\beta)}{\partial\alpha}+\frac{1-\mu_0}{\mu_0}\cdot\frac{\partial F(x;\alpha,\beta)}{\partial\beta}, \qquad (32)$$

[18] See Johnson et al. (1995) for a textbook introduction to the beta distribution.

[19] $\psi_0(\omega)=\int_0^\infty\left(\frac{e^{-z}}{z}-\frac{e^{-z\omega}}{1-e^{-z}}\right)\mathrm{d}z$, which is increasing in ω.

which we compute to be

$$\int_0^x (\ln t) \cdot \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} \mathrm{d}t + (\psi_0(\alpha+\beta) - \psi_0(\alpha)) \int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} \mathrm{d}t$$
$$+ \frac{1-\mu_0}{\mu_0} \left\{ \int_0^x \ln(1-t) \cdot \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} \mathrm{d}t + (\psi_0(\alpha+\beta) - \psi_0(\alpha)) \int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} \mathrm{d}t \right\}. \tag{33}$$

We abbreviate this as $k(x)$ and note that $k(1) = k(0) = 0$ since $J(0;\alpha) \equiv 0$ and $J(1;\alpha) \equiv 1$. If we set

$$\chi(x) = \frac{1}{\mu_0} \left[\mu_0 \ln x + (1-\mu_0) \ln(1-x) + \psi_0(\alpha+\beta) - \psi_0(\alpha) \right], \tag{34}$$

it follows that

$$k'(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \cdot \chi(x). \tag{35}$$

Since $x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha,\beta) > 0$, $k'(x)$ and $\chi(x)$ must have the same sign. To determine this sign, we note that $\chi(0) = \chi(1) = -\infty$ and

$$\chi'(x) = \frac{\mu_0 - x}{\mu_0 x(1-x)}. \tag{36}$$

So $\chi(x)$ is strictly increasing for $x < \mu_0$, obtains a maximum at $x = \mu_0$, and is strictly decreasing for $x > \mu_0$. At this maximum $\chi(x)$ is positive because if we had $\chi(\mu_0) \leq 0$ instead, $k(x)$ would be strictly decreasing on $(0,\mu_0) \cup (\mu_0,1)$, contradicting with $k(1) = k(0) = 0$. But if $\chi(\mu_0) > 0$, we can find $0 < x' < x'' < 1$ such that $\chi(x)$ is negative on $(0,x') \cup (x'',1)$ and positive on (x',x''). In other words, $k(x)$ starts at zero, decreases on $[0,x')$, obtains a minimum for $x = x'$, increases on (x',x''), obtains a maximum for $x = x''$ and decreases back to zero on $(x'',1]$. To maintain continuity, there must then be a $x^{**} \in (x',x'')$ with $k(x^{**}) = 0$ such that $k(x)$ is negative on $(0,x^{**})$ and positive on $(x^{**},1)$. But this implies that an increase in α lowers $J(x;\alpha)$ for $x \in (0,x^{**})$ and raises $J(x;\alpha)$ for $x \in (x^{**},1)$ while leaving the mean unchanged. Said differently, an increase in α concentrates probability mass around the mean resulting in a mean-preserving contraction. $\square$

A.4 Proof of Remark 1

We assume a separable cost function. For $L(y,\kappa) = (1-\kappa) \cdot l(y)$ we obtain

$$\begin{aligned} U'(y^0) &= -p\mathbb{E}L_y(y^0,\widetilde{\kappa})u'(w - L(y^0,\widetilde{\kappa})) + pu'(w - \overline{L}(y^0))\overline{L}_y(y^0) \\ &= -pl'(y^0)\mathbb{E}(1-\widetilde{\kappa})u'(w - (1-\widetilde{\kappa})l(y^0)) + pl'(y^0)(1-\overline{\kappa})u'(w - (1-\overline{\kappa})l(y^0)), \end{aligned} \tag{37}$$

with $\overline{\kappa} = \mathbb{E}\widetilde{\kappa}$. Let $f(\kappa) = (1-\kappa)u'(w^\ell)$ with $w^\ell = w - (1-\kappa)l(y^0)$; then,

$$f''(\kappa) = u''(w^\ell)l(y^0) \left[-2 - \frac{(1-\kappa)l(y^0)}{w^\ell} \left(-w^\ell \frac{u'''(w^\ell)}{u''(w^\ell)} \right) \right], \tag{38}$$

which is positive under the assumptions made. Therefore, $\mathbb{E}f(\widetilde{\kappa}) > f(\overline{\kappa})$ so that $U'(y^0) > 0$ indicating the optimality of a higher level of self-insurance under TU. This shows *a)*.

For *b)* we use $L(y,\kappa) = l \cdot (1-\kappa h(y))$ so that

$$\begin{aligned} U'(y^0) &= -p\mathbb{E}L_y(y^0,\widetilde{\kappa})u'(w - L(y^0,\widetilde{\kappa})) + pu'(w - \overline{L}(y^0))\overline{L}_y(y^0) \\ &= plh'(y^0)\mathbb{E}\widetilde{\kappa}u'(w - l(1-\widetilde{\kappa}h(y^0))) - plh'(y^0)u'(w - l(1-\overline{\kappa}h(y^0))). \end{aligned} \tag{39}$$

If we set $f(\kappa) = \kappa u'(w^\ell)$ with $w^\ell = w - l(1-\overline{\kappa}h(y^0))$, then

$$f''(\kappa) = lh(y^0)u''(w^\ell)\left[2 - \frac{\kappa l h(y^0)}{w^\ell}\left(-w^\ell \frac{u'''(w^\ell)}{u''(w^\ell)}\right)\right]. \tag{40}$$

This is negative under the stated assumptions. Then, $\mathbb{E}f(\widetilde{\kappa}) < f(\overline{\kappa})$, implying $U'(y^0) < 0$. As a result, TU lowers the optimal level of self-insurance.

A.5 Proof of Proposition 4

$\widetilde{\kappa}_1$ and $\widetilde{\kappa}_2$ represent the relevant distributions of the technology variable, and we assume $\widetilde{\kappa}_2$ to dominate $\widetilde{\kappa}_1$ in the sense of FSD. Let $V(y)$ denote the agent's objective function under $\widetilde{\kappa}_2$ and let y_1^* denote the optimal level of self-insurance under $\widetilde{\kappa}_1$. To compare the demand for self-insurance before and after the FSD improvement, we insert y_1^* into the first-order expression of $V(y)$ and determine its sign. For both the separable and the non-separable cost we obtain

$$V'(y_1^*) = p\mathbb{E}u'\left(w^\ell(y_1^*,\widetilde{\kappa}_2)\right)w_y^\ell(y_1^*,\widetilde{\kappa}_2) - p\mathbb{E}u'\left(w^\ell(y_1^*,\widetilde{\kappa}_1)\right)w_y^\ell(y_1^*,\widetilde{\kappa}_1) \tag{41}$$

after inserting the first-order condition for y_1^*. If we define

$$\Phi(\kappa) = pu'\left(w^\ell(y_1^*,\kappa)\right)w_y^\ell(y_1^*,\kappa), \tag{42}$$

we can rewrite $V'(y_1^*) = \mathbb{E}\Phi(\widetilde{\kappa}_2) - \mathbb{E}\Phi(\widetilde{\kappa}_1)$ and use the result in Ekern (1980) to rank both expectations by signing Φ'. We obtain that

$$\begin{aligned} \Phi'(\kappa) &= pu''\left(w^\ell(y_1^*,\kappa)\right)w_\kappa^\ell(y_1^*,\kappa)w_y^\ell(y_1^*,\kappa) + pu'\left(w^\ell(y_1^*,\kappa)\right)w_{y\kappa}^\ell(y_1^*,\kappa) \\ &= pu'\left(w^\ell(y_1^*,\kappa)\right)w_\kappa^\ell(y_1^*,\kappa)\left[\frac{w_{y\kappa}^\ell(y_1^*,\kappa)}{w_\kappa^\ell(y_1^*,\kappa)} - R\left(w^\ell(y_1^*,\kappa)\right)\cdot\frac{w_y^\ell(y_1^*,\kappa)}{w^\ell(y_1^*,\kappa)}\right], \end{aligned} \tag{43}$$

where $R(\cdot)$ abbreviates the agent's degree of relative risk aversion. Due to $w_y^\ell(y^*,\kappa) > 0$, the sign of $\Phi'(\kappa)$ coincides with the sign of the square bracket. Since

$$\partial_\kappa E_{w^\ell,y}(y,\kappa) = y \cdot \frac{w_{y\kappa}^\ell(y,\kappa)w^\ell(y,\kappa) - w_\kappa^\ell(y,\kappa)w_\kappa^\ell(y,\kappa)}{\left(w^\ell(y,\kappa)\right)^2}, \tag{44}$$

$E_{w^\ell,y}(y,\kappa)$ is increasing (constant, decreasing) in κ if $\frac{w^\ell_{y\kappa}(y_1^*,\kappa)}{w^\ell_\kappa(y_1^*,\kappa)} > (=,<) \frac{w^\ell_y(y_1^*,\kappa)}{w^\ell(y_1^*,\kappa)}$ for all κ. Combining this with the appropriate restriction on relative risk aversion completes the proof.

Proof of Remark 2

If the cost of self-insurance is separable, we have

$$\Phi'(\kappa) = pu''\left(w - L(y_1^*,\kappa)\right) L_\kappa(y_1^*,\kappa) L_y(y_1^*,\kappa) - pu'\left(w - L(y_1^*,\kappa)\right) L_{y\kappa}(y_1^*,\kappa).$$

The first term is negative because u'', L_κ and L_y are. Under CD and DD, the second term is non-positive because $L_{y\kappa} \geq 0$. As a result $\Phi' < 0$ such that $\mathbb{E}\Phi(\widetilde{\kappa}_2) < \mathbb{E}\Phi(\widetilde{\kappa}_1)$ per Ekern (1980) and $V'(y_1^*) < 0$. Consequently, $y_2^* < y_1^*$. For b), we have Eq (43) is positive if:

$$\begin{aligned} R < \frac{w^\ell_{y\kappa} w^\ell}{w^\ell_\kappa w^\ell_y} &= 1 + \frac{w - l_0 + l_0\kappa h(y)}{l_0 \kappa h(y)} \\ &\geq 2 \text{ if } w \geq 2l_0. \end{aligned}$$

Therefore, $R < 2$ is sufficient for Eq (43) > 0 if $w \geq 2l_0$.

A.6 Proof of Proposition 5

Let $\widetilde{\kappa}_1$ and $\widetilde{\kappa}_2$ be two different distributions of the technology parameter. If y_1^* denotes the optimal level of self-insurance under $\widetilde{\kappa}_1$, we assume that $L(y_1^*,\widetilde{\kappa}_2)$ is a Rothschild and Stiglitz (1970) increase in risk over $L(y_1^*,\widetilde{\kappa}_1)$. $V(y)$ denotes the objective function after the risk change (i.e., under $\widetilde{\kappa}_2$). To determine the effect of greater TU on self-insurance, we insert y_1^* into the first-order expression for $V(y)$ and determine the sign.

At a fixed level of self-insurance, say $y = y_1^*$, the loss function maps technology parameters into loss levels, that is,

$$L(y_1^*,\cdot) : [\underline{\kappa},\overline{\kappa}] \to [\underline{l},\bar{l}], \kappa \mapsto L(y_1^*,\kappa). \tag{45}$$

Due to monotonicity ($L_\kappa < 0$), we can define the inverse function $h : [\underline{l},\bar{l}] \to [\underline{\kappa},\overline{\kappa}]$ such that $L(y_1^*, h(l)) = l$ for all $l \in [\underline{l},\bar{l}]$. Its first and second derivative are given by

$$h'(l) = \frac{1}{L_\kappa(y_1^*,h(l))} \quad \text{and} \quad h''(l) = -\frac{L_{\kappa\kappa}(y_1^*,h(l)) \cdot h'(l)}{L_\kappa(y_1^*,h(l))^2} = -\frac{L_{\kappa\kappa}(y_1^*,h(l))}{L_\kappa(y_1^*,h(l))^3}. \tag{46}$$

For separable cost, we can use the first-order condition for y_1^* to rewrite the first-order expression under $\widetilde{\kappa}_2$ evaluated at y_1^* as

$$V'(y_1^*) = \mathbb{E}\Psi(L(y_1^*,\widetilde{\kappa}_2)) - \mathbb{E}\Psi(L(y_1^*,\widetilde{\kappa}_1)) \tag{47}$$

with $\Psi(l) = -pu'(w-l)L_y(y_1^*,h(l))$. We performed a change of variables because Ψ is parameterized by the codomain of $L(y_1^*,\cdot)$. This allows us to exploit that $L(y_1^*,\widetilde{\kappa}_2)$ is a Rothschild and Stiglitz (1970) increase in risk over $L(y_1^*, \widetilde{\kappa}_1)$, because if we determine the curvature of Ψ in l, we can then sign (47). Direct computation and utilization of (46) yields

$$\begin{aligned}
\Psi'(l) &= pu''L_y - pu'L_{y\kappa}h'(l) = pu''L_y - pu'\frac{L_{y\kappa}}{L_\kappa}, \\
\Psi''(l) &= -pu'''L_y + pu''L_{y\kappa}h'(l) + \left[pu''L_{y\kappa} - pu'L_{y\kappa\kappa}h'(l)\right]h'(l) - pu'L_{y\kappa}h''(l) \\
&= -pu'''L_y + 2pu''\frac{L_{y\kappa}}{L_\kappa} - pu'\frac{L_{y\kappa\kappa}}{L_\kappa^2} + pu'\frac{L_{y\kappa}L_{\kappa\kappa}}{L_\kappa^3}, \qquad (48)
\end{aligned}$$

where we omit the arguments to compress notation. We also use $w_y^\ell = -L_y$, $w_\kappa^\ell = -L_\kappa$, and so on, to rewrite $\Psi''(l)$ as follows:

$$\begin{aligned}
\Psi''(l) &= pu'''w_y^\ell + 2pu''\frac{w_{y\kappa}^\ell}{w_\kappa^\ell} + pu'\frac{w_{y\kappa\kappa}^\ell}{(w_\kappa^\ell)^2} - pu'w_{y\kappa}^\ell\frac{w_{\kappa\kappa}^\ell}{(w_\kappa^\ell)^3} \\
&= \frac{pu''}{w^\ell w_\kappa^\ell}\left[2w^\ell w_{y\kappa}^\ell - P(w^\ell)\cdot w_y^\ell w_\kappa^\ell\right] + \frac{pu'}{(w_\kappa^\ell)^3}\left[w_\kappa^\ell w_{y\kappa\kappa}^\ell - w_{y\kappa}^\ell w_{\kappa\kappa}^\ell\right]. \qquad (49)
\end{aligned}$$

$P(\cdot)$ abbreviates the agent's degree of relative prudence. $E_{w^\ell,y}$ is increasing (constant, decreasing) in κ if and only if $w^\ell w_{y\kappa}^\ell - w_y^\ell w_\kappa^\ell > 0\ (= 0, < 0)$ for all κ. Similarly, $E_{w_\kappa^\ell,y}$ is increasing (constant, decreasing) in κ if and only if $w_\kappa^\ell w_{y\kappa\kappa}^\ell - w_{y\kappa}^\ell w_{\kappa\kappa}^\ell > 0\ (= 0, < 0)$ for all κ. Combining these signs accordingly and using the appropriate restriction on relative prudence completes the proof.

Proof of Remark 3

It follows immediately from the loss function in a) that $L_{\kappa\kappa}(y,\kappa) = w_{\kappa\kappa}^\ell(y,\kappa) = 0$ and $L_{y\kappa\kappa}(y,\kappa) = w_{y\kappa\kappa}^\ell(y,\kappa) = 0$. We can therefore rewrite (49) as follows:

$$\Psi''(l) = \frac{pu''}{w^\ell w_\kappa^\ell}\left[2w^\ell w_{y\kappa}^\ell - Pw_y^\ell w_\kappa^\ell\right]. \qquad (50)$$

Obviously, (50) > 0 if and only if $2w^\ell w_{y\kappa}^\ell - Pw_y^\ell w_\kappa^\ell < 0$, which is equivalent to

$$\begin{aligned}
P > \frac{2w^\ell w_{y\kappa}^\ell}{w_y^\ell w_\kappa^\ell} &= \frac{2(w - (1-\kappa)l(y))l'(y)}{-(1-\kappa)l'(y)l(y)} \\
&= 2(1 - \frac{w}{(1-\kappa)l(y)}) \\
&< 0.
\end{aligned}$$

Hence, an increase of technological uncertainty leads to an increase of self-insurance as long as $P > 0$.

Similarly, in b), we have $w^\ell_{\kappa\kappa}(y,\kappa) = w^\ell_{y\kappa\kappa}(y,\kappa) = 0$ and therefore 50 holds as well. Furthermore, we have $\Psi''(l) < 0$ if and only if

$$\begin{aligned} P < \frac{2w^\ell w^\ell_{y\kappa}}{w^\ell_y w^\ell_\kappa} &= \frac{2l_0 h'(y)(w - l_0 + l_0\kappa h(y))}{l_0\kappa h'(y) l_0 h(y)} \\ &= 2(1 + \frac{w - l_0}{\kappa l_0 h(y)}) \\ &\geq 4 \text{ if } w \geq 2l_0. \end{aligned}$$

Therefore, $P < 4$ is sufficient for $\Psi''(l) < 0$ given $w \geq 2l_0$.

A.7 Proof of Proposition 6

Let $\widetilde{\kappa}_1$ and $\widetilde{\kappa}_2$ be the two distributions of the technology parameter with $\widetilde{\kappa}_2$ dominating $\widetilde{\kappa}_1$ in the sense of FSD. $V(x)$ denotes the agent's objective function under $\widetilde{\kappa}_2$ and x_1^* denotes the optimal level of self-protection under $\widetilde{\kappa}_1$. We insert x_1^* into the first-order expression under $\widetilde{\kappa}_2$ and determine the sign.

With a separable cost we obtain

$$\begin{aligned} V'(x_1^*) &= -\left(u(w) - u(w-L)\right)\mathbb{E}p_x(x_1^*, \widetilde{\kappa}_2) - c'(x_1^*) \\ &= \left(u(w) - u(w-L)\right)\cdot\left[\mathbb{E}p_x(x_1^*, \widetilde{\kappa}_1) - \mathbb{E}p_x(x_1^*, \widetilde{\kappa}_2)\right], \end{aligned} \tag{51}$$

where the last equality follows from substituting the first-order condition for x_1^*. The square bracket is positive (zero, negative) if $p_{x\kappa} < 0$ $(= 0, > 0)$ per Ekern (1980). As a result, $V'(x_1^*) > 0$ $(= 0, < 0)$ under ID (CD, DD) so that a higher (identical, lower) level of self-protection is optimal. This proves *a)*.

To show *b)*, we use a non-separable cost function. This yields

$$\begin{aligned} V'(x_1^*) &= -\left(u(w - c(x_1^*)) - u(w - L - c(x_1^*))\right)\mathbb{E}p_x(x_1^*, \widetilde{\kappa}_2) \\ &\quad - c'(x_1^*)(1 - \mathbb{E}p(x_1^*, \widetilde{\kappa}_2))u'(w - c(x_1^*)) - c'(x_1^*)\mathbb{E}p(x_1^*, \widetilde{\kappa}_2)u'(w - L - c(x_1^*)) \\ &= \left(u(w - c(x_1^*)) - u(w - L - c(x_1^*))\right)\cdot\left[\mathbb{E}p_x(x_1^*, \widetilde{\kappa}_1) - \mathbb{E}p_x(x_1^*, \widetilde{\kappa}_2)\right] \\ &\quad + c'(x_1^*)\left(u'(w - L - c(x_1^*)) - u'(w - c(x_1^*))\right)\cdot\left[\mathbb{E}p(x_1^*, \widetilde{\kappa}_1) - \mathbb{E}p(x_1^*, \widetilde{\kappa}_2)\right]. \end{aligned} \tag{52}$$

The last equality holds if we solve the first-order condition under $\widetilde{\kappa}_1$ for $c'(x_1^*)u'(w - c(x_1^*))$ and substitute. As in the separable case, the first square bracket in Eq. (52) is positive (zero, negative) if $p_{x\kappa} < 0$ $(= 0, > 0)$. The second square bracket in Eq. (52) is always positive because $p_\kappa < 0$. Hence, we obtain $V'(x_1^*) > 0$ for ID and CD technologies, indicating that more self-protection is optimal under $\widetilde{\kappa}_2$ than under $\widetilde{\kappa}_1$.

A.8 Proof of Proposition 7

Let $F_1(\kappa)$ and $F_2(\kappa)$ be the cumulative distribution function of $\widetilde{\kappa}_1$ and $\widetilde{\kappa}_2$, respectively. According to the definition of FSD, $F_1(\kappa) \geq F_2(\kappa)$ for all $\kappa \in [\underline{\kappa}, \overline{\kappa}]$ with a strict inequality for some κ. Using the definition of ρ, we can rewrite condition (18) as

$$p_{x\kappa}(x_1^*, \kappa) - c'(x_1^*)\frac{v'(w; x_1^*)}{v(w; x_1^*)}p_\kappa(x_1^*, \kappa) < 0 \quad \text{for all } \kappa \in [\underline{\kappa}, \overline{\kappa}]. \tag{53}$$

Integration preserves the sign, so we obtain

$$\int_{\underline{\kappa}}^{\overline{\kappa}} \left[p_{x\kappa}(x_1^*, \kappa) - c'(x_1^*)\frac{v'(w; x_1^*)}{v(w; x_1^*)}p_\kappa(x_1^*, \kappa)\right] \cdot [F_1(\kappa) - F_2(\kappa)]\, \mathrm{d}\kappa < 0. \tag{54}$$

If we integrate by parts, condition (54) becomes

$$\int_{\underline{\kappa}}^{\overline{\kappa}} p_x(x_1^*, \kappa)\ \mathrm{d}\,[F_1(\kappa) - F_2(\kappa)] - c'(x_1^*)\frac{v'(w; x_1^*)}{v(w; x_1^*)} \int_{\underline{\kappa}}^{\overline{\kappa}} p(x_1^*, \kappa)\ \mathrm{d}\,[F_1(\kappa) - F_2(\kappa)] > 0. \tag{55}$$

Using the definition of F_1 and F_2 yields

$$\int_{\underline{\kappa}}^{\overline{\kappa}} p_x(x_1^*, \kappa)\ \mathrm{d}\,[F_1(\kappa) - F_2(\kappa)] = \mathbb{E}p_x(x_1^*, \widetilde{\kappa}_1) - \mathbb{E}p_x(x_1^*, \widetilde{\kappa}_2) \tag{56}$$

and

$$\int_{\underline{\kappa}}^{\overline{\kappa}} p(x_1^*, \kappa)\ \mathrm{d}\,[F_1(\kappa) - F_2(\kappa)] = \mathbb{E}p(x_1^*, \widetilde{\kappa}_1) - \mathbb{E}p(x_1^*, \widetilde{\kappa}_2). \tag{57}$$

Using the definition of v then shows that (55) rearranges to $V'(x_1^*) > 0$. So under $\widetilde{\kappa}_2$ more self-protection is optimal than under $\widetilde{\kappa}_2$. The argument is analogous if the reverse of inequality (18) holds.

A.9 Proof of Remark 4

The fundamental theorem of calculus yields

$$v(w; x^*) = \int_{-L}^{0} u'(w - c(x^*) + t)\mathrm{d}t = L \cdot \mathbb{E}u'(w - c(x^*) + \widetilde{t}) \tag{58}$$

and

$$v'(w; x^*) = \int_{-L}^{0} u''(w - c(x^*) + t)\mathrm{d}t = L \cdot \mathbb{E}u''(w - c(x^*) + \widetilde{t}), \tag{59}$$

where $\widetilde{t}$ is uniformly distributed on $[-L, 0]$ with density $\frac{1}{L} \cdot \mathbf{1}_{[-L,0]}$. Its mean is given by $L/2$, and risk vulnerability of u implies

$$-\frac{v'(w; x^*)}{v(w; x^*)} = -\frac{\mathbb{E}u''(w - c(x^*) + \widetilde{t})}{\mathbb{E}u'(w - c(x^*) + \widetilde{t})} \geq -\frac{u''(w - c(x^*) - L/2)}{u'(w - c(x^*) - L/2)} = \mathcal{A}(w - c(x^*) - L/2). \tag{60}$$

So if $c'(x^*)\mathcal{A}(w - c(x^*) - L/2)$ exceeds $\rho(x^*, \kappa)$, then $-c'(x^*)v'(w; x^*)/v(x; x^*)$ does *a fortiori*.