Why the Political World is Flat: An Endogenous "Left" and "Right" in Multidimensional Elections

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Abstract

This paper analyzes a multidimensional model of candidate positioning in common interest elections. In an essentially unique equilibrium, polarized candidates bundle issue positions that are logically related, effectively reducing the election to a single summary dimension. The model gives structure to prominent philosophical perspectives, and explains otherwise puzzling empirical features of elections such as why even loosely connected policies are bundled so consistently.

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1 Introduction

The world of public policy is complex and multifaceted. Elected officials must decide tax policy, foreign policy, health policy, education policy, immigration policy, social policy, and many others, each of which encompasses numerous more narrow issues that are themselves complex and multifaceted; indeed, every line of legislation could be viewed as a separate dimension, along which policy could be adjusted. Voters must consider all of the same issues in order to properly evaluate candidates, in addition to personal characteristics such as honesty and management skill. The number of dimensions required to properly model such an environment is enormous. In contrast, existing political economic models are almost exclusively one dimensional.

Efforts to understand multidimensional politics have repeatedly been stymied by one of three challenges. The first is that in many models, such as Plott's (1967) straightforward extension of the classic one-dimensional model of Hotelling (1929) and Downs (1957), equilibrium does not exist, at least in pure strategies.[1] The second is the indeterminacy associated with multiple—often, many—equilibria. In a "citizen-candidate" framework, for example, Besley and Coate (1997) write that "basically *any* pair of candidates who split the voters evenly can be an equilibrium." The third challenge is that in models where a unique equilibrium exists, such as the probabilistic voting models of Hinich (1977, 1978) and Lindbeck and Weibull (1987, 1993), candidates adopt identical policy platforms.[2] Like convergence in one dimension, this is at odds with the substantial polarization that is observed empirically.[3]

One interpretation of the literature above is as a prediction of political "chaos": challengers should always be able to unseat incumbents, contests between symmetric challengers should be unpredictable, and successive majority votes may cycle indefinitely through the same policies, or lead to *any* eventual policy outcome (McKelvey, 1979). As Tullock (1981) points out, however, such instability is not evident empirically.[4] The more common response to this literature is simply to continue using one-dimensional models—in essence, treating the world as if it were flat (in fact, one-dimensional), even though it is known not to be. Actually, this approach has some merit: empirically, voters' preferences across issues turn out to be correlated to a surprising degree, and can thus be effectively summarized by a single-dimensional measure of *ideology*, ranging from liberal to moderate to conservative. In the words of Converse (1964, p. 207), "...if a person is opposed to the expansion of social security, he is probably a conservative and is probably opposed as well to any nationalization of private industries, federal aid to education, sharply progressive income taxation, and so forth." Poole and Rosenthal (1985, 1997, 2001) formalize this statistically, showing that a one-dimensional spatial model correctly predicts almost 90% of the individual roll call votes cast by members of the U.S. House and Senate between 1789 and 1998. They cite similar findings for the European Parliament and the U.N. General Assembly, as well as the British, French, Czech, and Polish parliaments, and Grofman and Brazill (2002) and Shor and McCarty (2011) find the same for the U.S. supreme court and U.S. state legislatures, respectively.[5] Shor (2014) finds the same for ordinary citizens.[6]

The *private interest* literature above treats voter ideology as a taste parameter. In McMurray (2017a) I propose an alternative perspective, based on the *common interest* paradigm of Condorcet (1785). Voters in that model behave like social planners, favoring policies that they believe will best serve society, not their own narrow interests.[7] Ideologies then reflect voters' *opinions* of what is socially optimal, based on their information about the consequences different policies will have. The one-dimensional spatial version of that

[1] See also Duggan and Fey (2005). Duggan and Jackson (2005) prove the existence of mixed-strategy equilibria, but do not provide a characterization. As Austen-Smith and Banks (2005) discuss, the empirical relevance of mixed strategies is unclear in the context of political campaigns.

[2] See also Xefteris (2017).

[3] For example, see Bafumi and Herron (2010) and Shor (2011).

[4] Mueller (2003, ch. 11) reports, for example, that incumbent U.S. governors have historically won reelection by an average margin of 23%.

[5] See also Fowler and Hall (2013).

[6] Tausanovitch and Warshaw (2014) find that voter preferences are also correlated on local and national issues.

[7] As I explain in that paper, this need not assume heroic levels of altruism: large elections can amplify even tiny levels of altruism, so that even voters who are almost purely selfish put almost all decision weight on (their perception of) the public interest. Among other examples, this can explain why wealthy voters often favor redistribution to the poor.

paradigm does well at explaining empirical patterns of voter information, ideology, and participation.[8] In McMurray (2018) I show that this also has a strong polarizing effect, even when political candidates are highly motivated to win, because a candidate who believes that truth is on her side expects voter support even when she is more extreme than her opponent.[9] The contribution of this paper is to extend that work to multiple dimensions. The main analysis treats only two dimensions, but Section 5 explains that similar logic can apply in higher dimensions, as well.

In multiple dimensions, an immediate benefit of an information paradigm is a plausible rationale for why voter attitudes should be correlated across issues. Quite simply, the same logical considerations that favor one policy also favor another. For the purposes of ending an economic recession, for example, it might turn out to be the case that fiscal stimulus is effective while monetary stimulus is not, or vice versa, but ex ante it is more likely either that both forms of stimulus are beneficial (because the economy functions more or less as Keynesian models predict) or that both are wasteful (as in more classical models), so support for one form of stimulus is likely to be correlated with support for the other. Similarly, a belief (or disbelief) in market efficiency or in the competence and integrity of government regulators may jointly determine a voter's support (or lack thereof) for a host of regulations. Beliefs about the relative importance of luck and effort in determining individual fortunes could shape a voter's support for a host of redistributive policies.

The equilibrium analysis below makes clear that the polarizing forces in a one-dimensional setting apply in higher dimensions, as well. With two or more dimensions, however, there are infinitely many directions in which candidates could polarize, and any combination of issues could be bundled together. In a perfectly symmetric specification of the model, any of these bundlings could persist in equilibrium, so indeterminacy is a severe problem, just as in existing literature. On the other hand, if voter opinions are correlated across issues then the number of equilibria falls precipitously. In two dimensions, for example, only two equilibria remain, oriented along the major and minor diagonals of the policy space. The minor equilibrium aggregates private information less efficiently, and therefore serves as a plausible formalization of the concern, often expressed in public discourse, that policy issues have been bundled sub-optimally. That equilibrium seems unlikely to prevail, however, both because it is inferior (and therefore unlikely to be *focal* in the sense of Schelling, 1960) and because it is not stable. Thus, the major equilibrium emerges as the essentially unique behavioral prediction of the model.

Empirically, issue positions that are bundled together as "liberal" or "conservative" in a particular election, place, and time, tend to be consistently bundled together elsewhere, as well. McDonald, Mendes, and Kim (2007) find, for example, that a single ideological dimension categorizes nearly 90% of the individual policy positions of over eighty political parties in seventeen countries, over two and a half decades. Pan and Xu (2017) find that ideology exhibits a similar structure even in authoritarian China, suggesting that the source of attitude correlation also has little to do with political institutions. As Section 2 explains, existing literature offers a few explanations of unidimensionality, but none explain this consistency across elections. Such stability is perfectly consistent with the information model below, however, as logic that links two issues in one setting should link the same issues in other settings, as well.

In many cases, the logical relationships between issues may seem too weak to be a likely source of consistency in how issues are bundled across elections. However, an important feature of the equilibrium predictions below is that *any* non-zero correlation between truth variables is sufficient to orient the equilibrium in the direction of correlation, leading candidates to behave just as they would if that correlation were perfect. The equilibrium predictions of the model also match more detailed empirical features of elections, such as the finding that political candidates typically exhibit greater ideological consistency than voters do.

Information models are challenging to analyze, because to optimize behavior, a voter or candidate must forecast the private information of the many other members of society. Multidimensional models are notoriously complex, as well. At a desirable level of generality, therefore, a multidimensional information model is not tractable. To make headway, the model below imposes a large number of symmetry, monotonicity, or functional form assumptions on the policy space, the utility functions, the joint distributions of truth variables and private signals, the candidate characteristics, the timing of events, and the voting strategies of interest. Following the formal analysis, however, I then explain informally why none of these restrictions is

[8] Among several empirical applications in that paper, a common interest paradigm explains why voters try to persuade political opponents to join their side.

[9] Throughout this paper, feminine pronouns refer to political candidates and masculine pronouns refer to voters.

likely essential for the intuition behind the paper's central results.

2 Literature

One-dimensional private-interest election models typically specify ideology as an exogenous parameter, with no justification beyond its empirical appeal. Exceptions attribute ideology to differences in wealth, which determines demand both for redistribution (Romer, 1975; Meltzer and Richard, 1981) and for public goods (Bergstrom and Goodman, 1973). With multiple public goods or multiple forms of redistribution, this might implicitly provide a justification for unidimensionality, as well. However, this is problematic empirically, as I discuss at length in McMurray (2017a): the wealthy favor redistribution almost as frequently as the poor, for example, and textbook examples of public goods such as defense and environmental protection draw support from opposite ends of the political spectrum. Of course, private interest models also face the challenges outlined in Section 1.

Communication literature treats the subject of unidimensionality explicitly.[10] First, Spector (2000) considers two homogeneous groups of agents with different prior beliefs about a commonly-valued, multi-dimensional state variable. As these groups learn and communicate over time, their beliefs converge in every dimension except the direction of prior disagreement, because communication in this direction lacks credibility. While interesting, that result seems to depend crucially on the exogenous restriction to two homogeneous groups: the logic of Battaglini (2002) suggests that additional groups or heterogeneity within groups would enable individuals to infer full information from the cross-section of others' messages, thereby restoring credibility and facilitating convergence.

DeMarzo, Vayanos, and Zwiebel (2003) consider a model (extended recently by Louis, Troumpounis, and Tsakas, 2016) in which individuals learn by circulating their private information through a social network but, in line with evidence from psychology literature, fail to rationally discount repeated information. Consensus is eventually reached on all dimensions, but the last dimension to converge can be interpreted as the left and right of politics. The orientation of this dimension has nothing to do with the underlying signals or truth variable, but instead depends on the structure of the network. This assumes that a social network obtains no information beyond the initial signals, however; whether unidimensionality would still emerge with a periodic influx of new information, and to what extent that would depend on the communication structure, remain open questions.[11] Importantly, this explanation of unidimensionality gives no account as to why issues should be bundled together similarly from one place and time to the next, where communication networks vary substantially.

In addition to the small number of papers addressing unidimensionality, there is a small number of papers that study information aggregation in multidimensional common-interest settings. However, neither of these address candidate positioning or unidimensionality. Instead, both focus on the ability of elections to aggregate voters' private information efficiently. Feddersen and Pesendorfer (1997) show that voting aggregates information effectively in one dimension in spite of conflicts of interest, but cannot do so in higher dimensions. Barelli, Bhattacharya, and Siga (2015) identify conditions on the information structure that are necessary and sufficient for efficient information aggregation in the absence of conflict. Those conditions are satisfied in the model below.

3 The Model

A society consists of N voters where, as in Myerson (1998, 2000), N is drawn from a Poisson distribution with mean n. Together, these voters must choose a pair $x = (x_1, x_2)$ of policies from the set X. If $x_1 \in [-1, 1]$ and $x_2 \in [-1, 1]$, the most intuitive specification of X would be the Cartesian product $[-1, 1]^2$. Section 5 discusses this possibility, and also the possibility of $K > 2$ dimensions, but for now let X be the unit

[10] In addition to the models listed, Duggan and Martinelli (2011) and Egorov (2014) show how the orientation of political conflict can be influenced by a monolithic media or by candidate messaging, respectively. However, those models take unidimensionality as an exogenous constraint on communication.

[11] With repeated infusions of new information, Jadbabaie et al. (2012) show that such boundedly rational learning converges eventually to Bayesian beliefs.

disk, instead, which provides additional symmetry that makes the analysis more completely tractable. One interpretation of this could be that the origin $(0,0)$ represents a pair of status quo policies, for example, and that the electorate can depart from the status quo in any direction, but only up to some maximal distance, normalized to one.

It is often convenient to represent policies using polar coordinates (r_x, θ_x), where $r_x = \|x\| = \sqrt{x_1^2 + x_2^2}$ is the distance of a policy pair from the origin, and θ_x is the angle formed between x and the horizontal axis. In terms of its polar coordinates, the Cartesian coordinates of x are given by $x_1 = r_x \cos(\theta_x)$ and $x_2 = r_x \sin(\theta_x)$. A policy pair can also be represented as a column vector $x = \binom{x_1}{x_2}$. Multiplying x by the matrix $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ then produces a *rotation* $R_\theta x$ which has the same magnitude as x, but polar angle $\theta_x + \theta$.[12] Multiplying x by $M_\theta = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$ instead produces the *mirror image* through angle θ. That is, $M_\theta x$ has the same magnitude as x, but polar angle $2\theta - \theta_x$, so that x and $M_\theta x$ are equidistant from a vector with angle θ.

Within the set of feasible policy bundles, one is ultimately socially optimal. Denote this as $z = (z_1, z_2)$ (or $z = (r_z, \theta_z)$), and assume that every voter prefers policy pairs that are as close as possible to z.[13] For now, the set of policies that might be optimal is simply Z is identical to X; later, Section 5 considers the possibility that certain feasible policies are known *not* to be optimal, so that Z is a strict subset of X, instead. Away from the optimum, voter utility $u(x,z) = -\|x - z\|^2$ decreases quadratically in the distance $\|x - z\| = \sqrt{(x_1 - z_1)^2 + (x_2 - z_2)^2}$ between x and z. Conditional on information Ω (and dropping terms that do not depend on the policy outcome), expected utility

$$E_z[u(x,z)|\Omega] = -\|x - E(z|\Omega)\|^2 \tag{1}$$

then decreases quadratically in the distance between the policy vector implemented and the updated expectation $E(z|\Omega)$ of the optimum.[14]

As Section 1 explains, the optimal policy positions z_1 and z_2 should be correlated, because of logical connections across issues. To allow this possibility, let the prior density $f(z;\rho)$ depend on a parameter ρ related to the correlation between z_1 and z_2 (where $\rho \geq 0$ without loss of generality). In fact, to make the analysis unambiguous, let f satisfy Condition 1, or *correlative monotonicity*. For positive ρ, this means that f increases in the direction of the *major diagonal* (i.e. the line defined by $z_1 = z_2$) and decreases in the direction of the *minor diagonal* (i.e. defined by $z_1 = -z_2$), and that this pattern becomes more pronounced as ρ increases. The following density is an example that satisfies this condition; for $\rho > 0$, it is illustrated in Figure 1.

$$f(z;\rho) = \frac{1}{\pi}\left(1 + \rho\frac{z_1 z_2}{\|z\|}\right) = \frac{1}{\pi}\left[1 + \rho r_z \cos(\theta_z)\sin(\theta_z)\right] \tag{2}$$

In this example, $\rho \in \left[-\frac{1}{4}, \frac{1}{4}\right]$ and the correlation coefficient between z_1 and z_2 equals $\frac{\rho}{4}$.

Condition 1 (Correlative monotonicity) *$f(z_1, z_2; \rho)$ is differentiable in z_1, z_2, and ρ. Moreover, $\frac{\partial f(z)}{\partial z_1}$ has the same sign as ρz_2 and $\frac{\partial^2 f(z)}{\partial z_1 \partial \rho}$ has the same sign as z_2 and, symmetrically, $\frac{\partial f(z)}{\partial z_2}$ has the same sign as ρz_1 and $\frac{\partial^2 f(z)}{\partial z_2 \partial \rho}$ has the same sign as z_1, implying that $\frac{\partial f(z)}{\partial \theta_z}$ has the same sign as $\rho\cos(2\theta_z)$. Also, $\frac{\partial^2 f(z)}{\partial z_1 \partial z_2}$ has the same sign as ρ, $\frac{\partial f(z)}{\partial \rho}$ has the same sign as $z_1 z_2$ and $\sin(2\theta_z)$, and $\frac{\partial^2 f(z)}{\partial \theta_z \partial \rho}$ has the same sign as $|z_1| - |z_2|$ and $\cos(2\theta_z)$.*

[12] The inverse of R_θ is simply $R_\theta^{-1} = R_{-\theta}$.

[13] The assumption that voters share a common interest may seem inappropriate, given that policies inevitably affect different voters differently. As I explain in McMurray (2017a), however, abundant evidence suggests that voters actually look past their own narrow interests, as if social planners, favoring policies that they believe will be good for society, even if these policies do not favor their own narrow interests. For example, many wealthy voters favor redistribution to the poor.

[14] The quadratic specification here is essential for tractability, but the equilibrium logic below seems only to require that preferences be single-peaked, with a conditional optimum that depends monotonically on a voter's belief about z. With linear utility loss, for example, a voter would favor the median realizations of z_1 and z_2 (conditional on Ω) instead of the mean, but this should have similar implications for behavior.

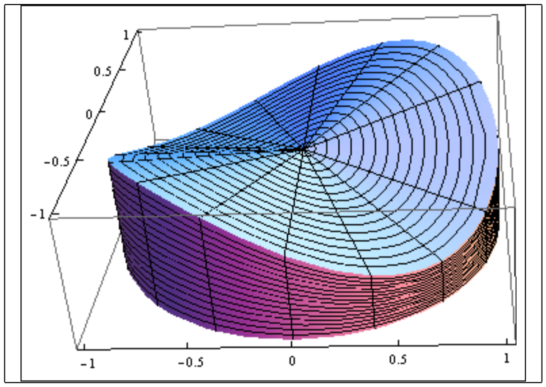

Figure 1: The joint density $f(z)$, for the case of $\rho > 0$.

To make the analysis tractable, assume further that f satisfies Condition 2, or *dimensional symmetry*. This means that f is symmetric around the origin, and that reversing the orientation of one dimension is equivalent simply to reversing the sign of ρ. The density in (2) satisfies this condition, as well. When $\rho = 0$, Condition 1 implies Condition 3, or *radial symmetry*, meaning that the optimal policy pair is equally likely to lie in any direction from the origin.

Condition 2 (Dimensional symmetry) *$f(z_1, z_2) = f(z_2, z_1) = f(-z_1, -z_2) = f(-z_2, -z_1)$ and $f(-z_1, z_2) = f(z_1, -z_2) = f(z_2, -z_1) = f(-z_2, z_1) = f(z_1, z_2; -\rho)$. Equivalently, $f(z) = f\left(M_{\frac{\pi}{4}} z\right) = f(R_\pi z) = f\left(M_{-\frac{\pi}{4}} z\right)$ and $f\left(M_{\frac{\pi}{2}} z\right) = f(M_0 z) = f\left(R_{-\frac{\pi}{2}} z\right) = f\left(R_{\frac{\pi}{2}} z\right) = f(z; -\rho)$.*

Condition 3 (Radial symmetry) *If $\rho = 0$ then $f(R_\theta z) = f(M_\theta z) = f(z)$ for any $\theta \in \mathbb{R}$ and for any $z \in Z$.*

Voters' private opinions regarding the location of the optimal policy pair are determined by pairs $s_i = (s_{i1}, s_{i2})$ of informative private signals, drawn independently (conditional on z) from the set $S = Z$ of possibly optimal policy pairs. Intuitively, s_{i1} should be informative of z_1 and s_{i2} should be informative of z_2. Both to accomplish this and so that posterior beliefs can be tractably characterized, assume further that the conditional density $g(s|z)$ of private signals satisfies Condition 4, or *linear informativeness*. This means that $g(s|z)$ is linear, sloping upward in the direction of z. An example of such a density is the following,

$$g(s|z) = \frac{1}{\pi}(1 + s_1 z_1 + s_2 z_2) = \frac{1}{\pi}\left[1 + r_s r_z \cos(\theta_s - \theta_z)\right] \tag{3}$$

which Figure 2 illustrates for z on the horizontal axis. Condition 4 also implies Conditions 5 and 6. *Rotational symmetry* implies that rotating z merely rotates the entire distribution of signals by the same amount; *error symmetry* implies that a signal s is equally likely to be a clockwise or counter-clockwise deviation from the true state z. Linearity and symmetry do not seem important for the fundamental logic of any of the results below, but without these simplifying assumptions, the analysis becomes intractable when voters combine their private information with inferences based on pivotal voting considerations.[15]

Condition 4 (Linear informativeness) *$g(s|z)$ is an increasing linear function of $s \cdot z$. That is, $g(s|z) = g_0 + g_1(s \cdot z)$ for some $g_0, g_1 > 0$.*

Condition 5 (Rotational symmetry) *For any θ, $g(R_\theta s | R_\theta z) = g(s|z)$.*

Condition 6 (Error symmetry) *If $r_s = r_{s'}$ and $|\theta_s - \theta_z| = |\theta_{s'} - \theta_z|$ then $g(s|z) = g(s'|z)$. If $r_z = r_{z'}$ and $|\theta_s - \theta_z| = |\theta_s - \theta_{z'}|$ then $g(s|z) = g(s|z')$.*

[15] If voters fail to extract information from the event of a pivotal vote, the various restrictions on f and g become unnecessary, as Section 5 discusses.

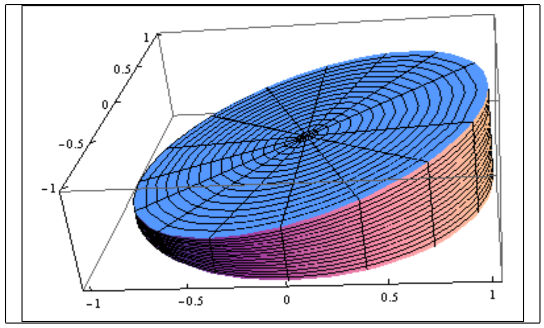

Figure 2: Conditional density $g(s|z)$, for z on the horizontal axis.

The dimensional symmetry of $f(z)$ implies that $E(z_1) = E(z_2) = 0$. Given that, linear informativeness further implies that the marginal distribution of s_i is uniform: $g(s) = E_z g(s|z) = g_0 + g_1 s \cdot E_z(z) = g_0$. Applying Bayes' rule, a voter's posterior expectation of z is then linear in s, as follows.

$$\begin{aligned} E_z(z_1|s) &= E_z\left[z_1 \frac{g(s|z)}{g(s)}\right] \\ &= E_z\left[z_1 + \frac{g_1}{g_0}\left(s_1 z_1^2 + s_2 z_1 z_2\right)\right] \\ &= \frac{g_1}{g_0} s_1 E\left(z_1^2\right) + \frac{g_1}{g_0} s_2 E(z_1 z_2) \\ &= \frac{g_1}{g_0} V(z_1)(s_1 + \rho s_2) \qquad (4) \\ E_z(z_2|s) &= \frac{g_1}{g_0} V(z_2)(\rho s_1 + s_2) \qquad (5) \end{aligned}$$

For the densities (2) and (3), for example, $E(z|s) = \frac{1}{4}\binom{s_1+\rho s_2}{\rho s_1 + s_2}$. Naturally, $E(z_1|s)$ increases in s_1 and $E(z_2|s)$ increases in s_2; if ρ is positive then $E(z_1|s)$ also increases in s_2 and $E(z_2|s)$ also increases in s_1.[16] The distribution of signals is continuous, so despite their common objective, voters develop a myriad of different opinions about which policy combination is optimal.

Voters do not vote directly for policies. Instead, there are two candidates, A and B, who choose platform policy pairs $x_A = (x_{A1}, x_{A2})$ and $x_B = (x_{B1}, x_{B2})$ in X, and voters each vote for one of these.[17] The candidate $w \in \{A, B\}$ who receives the most votes (breaking ties, if necessary, by a fair coin toss) wins the election, takes office, and implements her platform policies. In choosing policies, candidates are assumed to be *truth motivated*, meaning that, like voters, they maximize (1), desiring the final policy outcome to be as close as possible to whatever is truly optimal. This is parsimonious in that candidates are fundamentally no different from other citizens (like the "citizen candidates" of Osborne and Slivinski, 1996, and Besley and Coate, 1997). The one-dimensional analysis in McMurray (2018) considers other possible motivations, as well, but those introduce asymmetries here that make the present analysis intractable.[18]

The behavior of truth motivated candidates depends on their beliefs about the location of the optimal policy. The most natural assumption would be that candidates start from the same prior beliefs as voters,

[16] The assumption that signals on one issue are informative of another issue is consistent with recent evidence from Brunner, Ross, and Washington (2011) that economic conditions have a causal impact on both economic and non-economic vote choices.

[17] The assumption of full participation is to keep the analysis tractable, and seems unimportant for the logic of the results below.

[18] Truth motivation undermines the assumption of binding platform commitments, in that a truth-motivated candidate should only deviate from her platform if she receives additional information that warrants the deviation, to which voters should have no objection. On the other hand, even if the typical candidate shares voters' interests, a culture among voters that enforces platform commitments seems sensible as a safeguard against candidates with ulterior motives (not modeled here). Binding commitments are also crucial for tractability, as they associate each candidate with a unique policy outcome; otherwise, voters can only evaluate candidates by forecasting the myriad of policy adjustments that each might later make, as in the one-dimensional model of McMurray (2017b).

but then update in response to private signals of their own. As I point out in McMurray (2018) and explain further below, however, a type of "pivotal" calculus leads candidates to infer additional information from equilibrium voting behavior. In fact, this equilibrium inference turns out to be so strong as to overwhelm the informational content of a candidate's own signal. To keep the analysis tractable, therefore, candidates' private signals are not modeled at all, and the pivotal inference is candidates' *only* source of information.[19]

An intuitive structure for the electoral game would be for candidates to move first, so that voters could decide how to vote after observing candidates' platform positions. Section 5 considers this possibility for a variant of the model, but sequential structure introduces asymmetry that makes the baseline model intractable. For now, therefore, candidates and voters are instead assumed to move simultaneously. A voting strategy $v : S \to \{A, B\}$ (from the set V) therefore specifies a vote choice $v(s)$ for every possible vector of signals $s \in S$, but does not depend explicitly on candidate positions. This actually does not change the equilibrium analysis of voter behavior, which still must best-respond to candidates' positions. The benefit of a simultaneous structure is in the analysis of candidates, whose beliefs about the optimal policy depend on what they infer from voters' behavior. With a simultaneous game, a candidate can take voting behavior as given, and optimize her response; a sequential game is much more complicated from a candidate's perspective because she must anticipate not only the informational implications of voters' equilibrium response to her current policy position, but also the implications of their responses to the various policy positions to which she could deviate.

With simultaneous structure, the appropriate solution concept is Bayesian Nash equilibrium (BNE). The assumption of Poisson population uncertainty implies that any such equilibrium is necessarily symmetric (Myerson, 1998), in that voters respond identically to identical signals. Such an equilibrium is therefore denoted by a triple (v^*, x_A^*, x_B^*), where the single voting strategy v^* is an optimal response to platforms x_A^* and x_B^* for a voter whose peers all follow v^* as well, and x_A^* and x_B^* both respond optimally to v^* and to each other. The election winner and ultimate policy outcome depend both on these strategies and on the realizations of N and z and of the private information s_i of each voter.

4 Analysis

4.1 Voters

The analysis of voting behavior in response to candidate platforms $x_A, x_B \in X$ closely parallels the one-dimensional treatment of McMurray (2017a). With quadratic utility, a voter prefers the candidate whose policy platform is closest to his expectation of the optimal policy vector. This of course depends on the realization s of his private signal. Lemma 1 now states that, as Austen-Smith and Banks (1996) point out, a voter should also optimally update his beliefs to account for the fact that his vote will only influence his utility in the rare event that it is *pivotal* (event P), meaning that it changes the identity of the election winner, by making or breaking a tie.[20] Proofs of all analytical results are presented in the appendix.

Lemma 1 *The voting strategy v^{br} is a best response to $v \in V$ and $x_A, x_B \in X$ if and only if $v^{br}(s) \in \arg\min_{j \in \{A,B\}} \|x_j - E(z|P, s)\|$ for all $s \in S$.*

It seems intuitive that voting behavior should be monotonic in s, meaning that a citizen whose signal lies closer to the platform of candidate B should be more inclined, not less inclined, to vote for B. If this is the case, then the space of signals can be partitioned into two regions, such that signal realizations in one region

[19] Given the model's other assumptions, adding candidate signals here would have little impact on equilibrium behavior, as Section 4.2 explains.

[20] The pivotal voting calculus is somewhat controversial in that comparing the policy outcome with and without a citizen's vote seems clearly the rational thing to do, but empirically, voters seem unaccustomed to—and perhaps incapable of—computing pivot probabilities (Esponda and Vespa, 2014). This is less troublesome here than in other settings, however, for two reasons. First, in common interest settings such as this, the behavior that is socially optimal constitutes an equilibrium, as McLennan (1998) points out, so a citizen could behave *as if* he were strategic without ever thinking about pivot probabilities, simply by determining the socially optimal voting strategy and then following it. Second, the symmetry of the half-space equilibria described below is such that pivotal considerations have no impact; that is, the voters who support candidates A and B in equilibrium are precisely those with signals closer to x_A and x_B, respectively. Nevertheless, Section 5 comments on an alternative specification of the model, in which voters are unsophisticated, conditioning on s but not on P.

lead a voter to vote A, while signal realizations in the other lead him to vote B. The functional forms in Section 3 are such that $E(z|s)$ is indeed monotonic in s, and the likelihood of a pivotal vote varies little with z, so it seems plausible that $E(z|P,s)$ should be monotonic in s as well. Even with all of the symmetry, monotonicity, and linearity assumed above, however, the intricate relationship between P and z makes this conjecture impossible to verify. Moreover, even if it were true that S can be partitioned into regions of A voters and B voters, the boundary between these regions may be an intricate function of signal realizations, making it difficult to characterize explicitly. To make progress, the analysis below restricts attention to *half-space strategies*, defined in Definition 1.[21]

Definition 1 *$v_h \in V$ is a half-space strategy if h is a unit vector with polar angle $\theta_h \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $v(s) = \begin{cases} A & \text{if } h \cdot s < 0 \\ B & \text{if } h \cdot s \geq 0 \end{cases}$.[22] A Bayesian Nash equilibrium (v_h^*, x_A^*, x_B^*) is a half-space equilibrium if v_h^* is a half-space strategy.*

As its name suggests, a half-space strategy merely divides the electorate in half, in the direction of some unit vector h. That is, voters whose signals lie in the general direction of h vote for candidate B, while those with signals in the opposite direction vote A. (Definition 1 imposes the restriction that $\theta_h \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so that A voters are on the left and B voters are on the right in the horizontal dimension, but this is of course without loss of generality.) As Lemma 2 now states, half-space strategies exhibit symmetry and monotonicity that transfer to electoral outcomes, which is what makes such strategies useful in characterizing equilibrium.

Lemma 2 *If $v_h \in V$ is a half-space strategy then the following hold.*

1. Candidate symmetry: $\Pr(w = A|-z) = \Pr(w = B|z)$ *for all $z \in Z$, and* $\Pr(w = A) = \Pr(w = B) = \frac{1}{2}$.

2. Monotonic voting: for any $z \in Z$, $\Pr(w = B|z)$ *is an increasing function of $h \cdot z$.* $\Pr(P|z)$ *is a decreasing function of $|h \cdot z|$.*

3. Half-space response: if $x_A = -x_B$ then the unique best response to (v_h, x_A, x_B) is a half-space strategy $v_{h^{br}}$.

Part 1 of Lemma 2 simply notes that, when voters follow a half-space strategy, opposite states of the world produce opposite candidate fortunes; averaging across states (and given the symmetry of f), this makes the candidates equally likely to win. Part 2 states that, in the direction of h, $\Pr(w = B|z)$ increases in z. Pivot probabilities $\Pr(P|z)$ first increase then decrease in z, and so are highest when z turns out to be moderate. Part 3 states that if candidate platforms are symmetric around the origin then, given the symmetry of Parts 1 and 2, best-response voting is monotonic. In fact, the boundary between regions of S that lead to A votes and B votes is linear, and passes through the origin. In other words, the best response to a half-space strategy is another half-space strategy.

4.2 Candidates

Like voters, a truth motivated candidate seeks to implement her expectation of the optimal policy. Since candidates are ex ante identical, their basic inclinations would both be to adopt policy platforms at the political center. A candidate's platform only matters if she wins the election, however, so as I explain in McMurray (2018), she optimally restricts her attention to this event, updating her beliefs accordingly, in the same way that a voter conditions on the rare event of a pivotal vote. The consequence of this, as Lemma 3 now states, is that platforms represent candidates' expectations of z, conditional on the event $w = j$ of winning. Note that this depends on the strategy that a candidate expects voters to follow, but not on the platform choice of her opponent.

[21] Alternatively, Section 5 comments briefly on a variant of the model in which voters are unsophisticated, and thus fail to condition on the event of a pivotal vote. In that case, much of the symmetry assumed throughout can be relaxed. In addition to making the analysis tractable, restricting attention to half-space strategies eliminates uninformative equilibria, in which citizens ignore their private signals and in turn are ignored by candidates.

[22] The behavior of voters for whom $h \cdot s = 0$ exactly is inconsequential, as this occurs with zero probability.

Lemma 3 *For any voting strategy $v \in V$, the unique best response for candidate j is given by $x_j^{br} = E(z|w = j)$.*

When voters follow a half-space strategy, candidate A tends to win the election in certain states of the world and candidate B tends to win the election in opposite states. From the event of winning the election, therefore, the two candidates infer opposite information. By the logic of Lemma 3, and given the symmetry identified in Lemma 2, Lemma 4 now states that this leads candidates to adopt symmetric policy platforms, which will be optimal in states of the world where they respectively win.

Lemma 4 *If $v_h \in V$ is a half-space strategy then $x_A^{br} = -x_B^{br} \neq 0$.*

In stating that candidates adopt opposite platforms, Lemma 3 says nothing about the extent of polarization. As I show for a single dimension in McMurray (2018), however, polarization can be substantial, especially when the number of citizens is large. This is because there is a line in Z such that, for realizations of z on one side of the line, candidate A almost surely wins, and for z on the other side, B almost surely wins. Conditional on winning, therefore, candidate A is sure that z lies in one half-space, but B is sure that it lies in the other.[23] That this inference is so strong is what motivates the assertion in Section 3 that adding candidate signals to the model would have little impact on equilibrium behavior: even if candidates observed signals that are much more informative than the typical voter's, the inference from $N+1$ signals would be similar to the inference from N signals, when N is large.[24] In that sense, the assumption that candidates have no private information of their own is unrealistic, but innocuous, given the other assumptions of the model.[25]

4.3 Equilibrium

Lemmas 1 through 4 characterize best response behavior for voters and candidates. Putting those results together, this section now analyzes equilibrium, first for the case in which z_1 and z_2 are uncorrelated, and then for the case of positive correlation. In the first case, $f(z)$ reduces to a uniform density, as Section 3 notes, which in particular exhibits radial symmetry, meaning that the optimal policy pair is equally likely to lie in any direction from the origin. The consequence of this, as Proposition 1 now states, is that *any* unit vector h defines a half-space strategy v_h that, together with candidates' best response policies, constitutes an equilibrium. In such an equilibrium, candidates simply take policy positions in the directions of $-h$ and h, symmetric around the origin. Voter behavior takes the event of a pivotal vote into account, but coincides exactly with the behavior that would prevail if it did not: voters with signals closer to x_A simply vote A, while voters with signals closer to x_B vote B.

Proposition 1 *Let $\rho = 0$. For any unit vector h there exists a half-space equilibrium (v_h^*, x_A^*, x_B^*), with $x_A^* = -x_B^* \neq 0$.*

The logic underlying Proposition 1 is straightforward: when voters follow a half-space voting strategy with normal vector h, candidates A and B deduce that electoral victory will be most likely when the optimal policy lies in the direction of $-h$ and h, respectively, and accordingly adopt platforms in these directions. A voter perceives that his own vote is most likely to be pivotal when z is roughly equidistant from $-h$ and h,

[23] Note that this pivotal logic does not depend at all on the specific functional form of quadratic utility. With linear utility loss functions, for example, a candidate prefers the median realization of z instead of the mean, but her posterior $f(z|w = j)$ would still condition on the event of winning the election.

[24] As I acknowledge in McMurray (2018), it is not clear empirically whether candidates in the real world actually perform the pivotal calculus prescribed in Lemma 3 any more than it is clear that voters actually perform the pivotal calculus prescribed in Lemma 1. It does seem reasonable for public support to bolster a politician's confidence, even if this is subconscious. In any case, it would be irrational for voters or candidates who care about z to ignore any information that is available. Nevertheless, Section 5 comments on an alternative specification of candidate beliefs, for which no such pivotal inference is made.

[25] In a model with candidate signals, each voter would have to infer candidates' private information from their equilibrium policy positions, in addition to inferring the signal realizations of other voters. Similarly, each candidate would have to infer her opponent's signal, in addition to inferring the signal realizations of each voter. These inferences would be complicated by the fact that others' behavior would no longer reflect private information alone, but also guesses of the information that a voter or candidate already possesses, which must be discounted to avoid duplication.

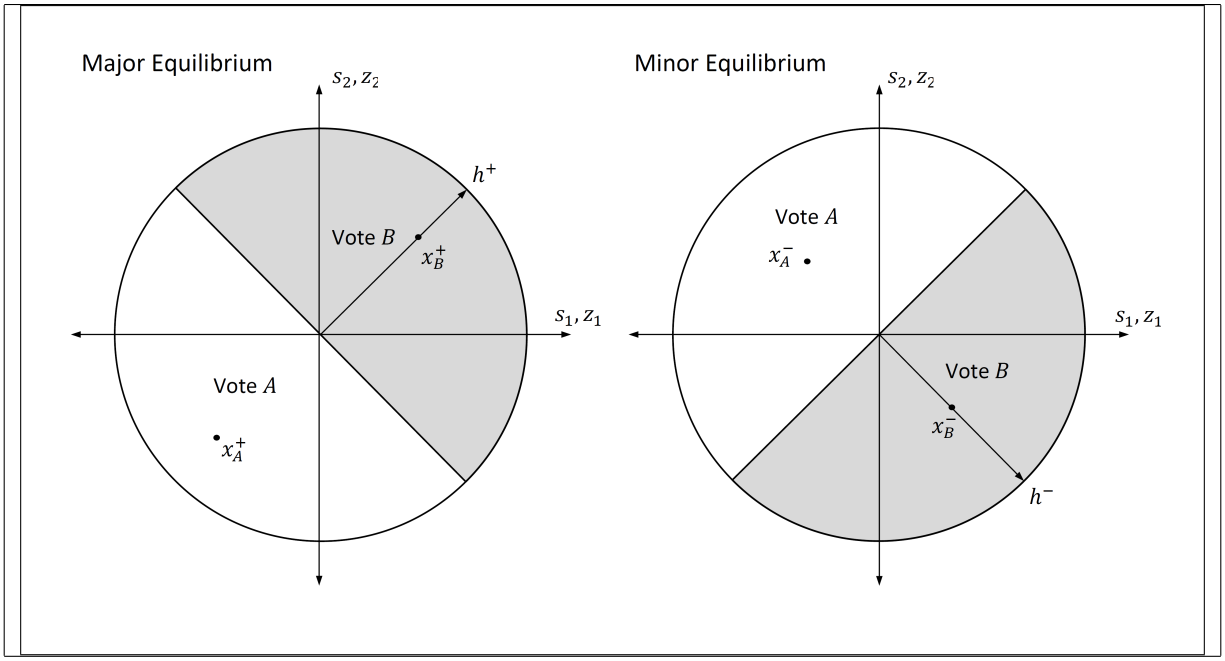

Figure 3: Major and minor equilibria.

and therefore roughly equidistant from x_A and x_B. This conveys nothing about which of the two platforms is superior, so he simply votes for A if his signal is closer to x_A and votes for B if his signal is closer to x_B, just as he would have done if he had not conditioned on the event of a pivotal vote.

Proposition 1 shows how a multidimensional environment reduces to a single dimension in equilibrium, with x_A and x_B endogenously defining "left" and "right" positions on the line between them, and voters dividing according to the projections of their opinions onto this line. This has nothing to do with the specific structure of information, or even with common interests; it follows simply from having two candidates: *any* two positions in a multidimensional space define a line, and if each voter supports the candidate closest to himself (whether to his private interest or to his private opinion of the common interest) then voters will split into two groups, in the direction of that line.[26] However, showing how a *single* election reduces to one dimension does nothing to resolve the puzzle highlighted in Section 1, which is that issues are bundled together consistently so that *different* elections reduce to the *same* line. Finding a unique equilibrium would have resolved the puzzle, by identifying a single orientation that must prevail in every election, but a unit vector h can point in any of an infinite number of directions, and Proposition 1 states that *any* of these can sustain a half-space equilibrium. Thus, the problem of equilibrium indeterminacy in a single election is severe, and so far, there seems to be no reason why different electorates could not settle on different equilibria, therefore bundling issues differently from one another.

Proposition 1 identifies infinitely many equilibria, but with only two policy dimensions, there are really only two ways to bundle the issues; any θ_h between 0 and $\frac{\pi}{2}$ produces a *major* equilibrium, meaning that one candidate is more conservative on both issues than her opponent; any θ_h between $-\frac{\pi}{2}$ and 0 produces a *minor* equilibrium, meaning that each candidate is more liberal on one issue and more conservative on the other. Within these categories, different θ_h correspond to different levels of polarization: for θ_h close to zero, candidates polarize more on issue 1 than issue 2; for θ_h close to $\pm\frac{\pi}{2}$, they polarize more on issue 2 than issue 1. In stating that any θ_h can sustain a half-space equilibrium, then, Proposition 1 implies first that either bundling of issues is possible in equilibrium, and second that either issue can be more polarizing, with a continuum of possible polarization levels.

[26] With more than two candidates, the logic of Duverger's (1954) law suggests that voters should ignore all but the two front runners, which would split the electorate in a similar way.

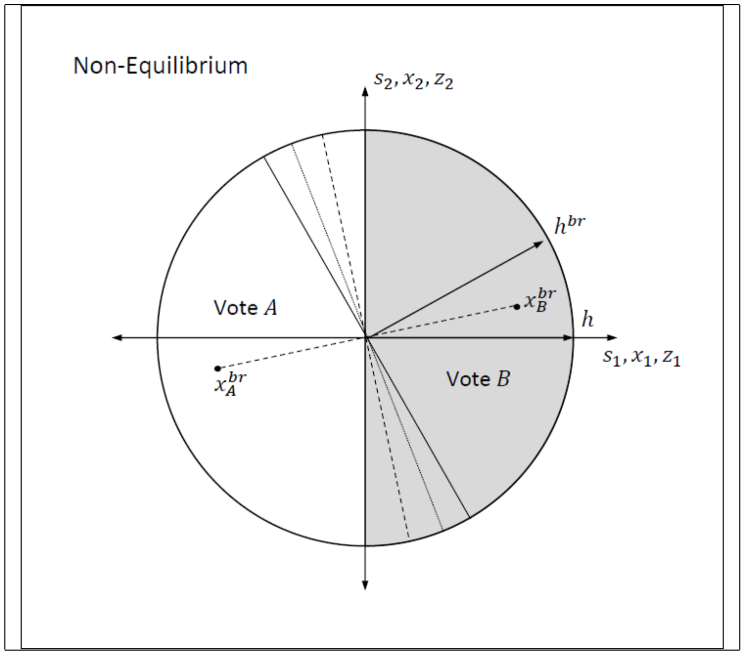

Figure 4: Non-equilibrium half-space strategy.

When $\rho = 0$, the distinction between major and minor equilibria is immaterial. When $\rho > 0$, however, major equilibria bundle issues in the direction of correlation while minor equilibria bundles issues oppositely. In that case, the number of equilibria falls precipitously, as Proposition 2 now states: there is a unique major equilibrium oriented exactly in the direction of the major diagonal, and a unique minor equilibrium oriented exactly in the direction of the minor diagonal. These are illustrated in Figure 3.

Proposition 2 *If $\rho > 0$ then there exists a major half-space equilibrium $\left(v_{h^+}, x_A^+, x_B^+\right)$ with $\theta_{h^+} = \theta_{x_B^+} = \frac{\pi}{4}$ and a minor half-space equilibrium $\left(v_{h^-,} x_A^-, x_B^-\right)$ with $\theta_{h^+} = \theta_{x_B^+} = -\frac{\pi}{4}$. No other half-space equilibrium exists.*

To give some intuition for why equilibrium half-space strategies can only be oriented in directions h^+ and h^-, Figure 4 illustrates the case of a half-space strategy oriented along the horizontal axis—that is, with polar angle $\theta_h = 0$. When following this strategy, voters ignore s_2 completely: those in the unshaded region of the figure observe negative realizations of s_1 and vote A, while those in the shaded region observe positive s_1 and vote B. From this voting behavior, candidate B infers that, if she wins the election, it will likely be because z_1 is positive. For $\rho = 0$, she would learn nothing about z_2, and would adopt a policy position exactly on the horizontal axis. For $\rho > 0$, however, a positive z_1 suggests that z_2 is likely positive as well, so if she wins, candidate B infers that z_1 and z_2 are both positive. Anticipating this, she adopts a platform x_B^{br} strictly above the horizontal axis—that is, with polar angle $\theta_{x_B^{br}} > 0$. Candidate A behaves symmetrically.

If candidates respond to v_h with platforms x_A^{br} and x_B^{br} as illustrated in Figure 4 then the dashed line between them partitions Z such that voters with expectations $E\left(z|P, s\right)$ in the southwest and northeast regions of Z prefer to vote A and B, respectively. Rotated counter-clockwise from the dashed line in Figure 4 is a dotted line. Voters whose expectations $E\left(z|s\right)$ lie southwest of the dotted line on the basis of private information alone form updated expectations $E\left(z|P, s\right)$ southwest of the dashed line after updating in response to the event of a pivotal vote, and therefore prefer to vote A; voters northeast of the dotted line update to be northeast of the dashed line, and prefer to vote B. The reason why these lines do not coincide is that, when a voter's peers vote on the basis of s_1 alone, they are most likely to tie (making a voter's own vote pivotal) when z_1 is close to zero. Thus, for any s, $E\left(z|P, s\right)$ lies closer to the vertical axis than $E\left(z|s\right)$ does. In particular, for a citizen whose signal is such that $E\left(z|s\right)$ lies exactly on the dotted line, $E\left(z|s\right)$ lies exactly on the dashed line, making him indifferent between voting A and voting B.[27]

Corresponding to the dotted and dashed lines in Z is a solid line in S, also depicted in Figure 4. Voters with signal realizations s southeast of this line form expectations $E(z|s)$ and $E(z|P,s)$ southeast of the dotted and dashed lines, respectively, and so prefer to vote A; symmetrically, voters northeast of this line form expectations northeast of the dotted and dash lines, and prefer to vote B. In other words, if his peers follow the voting strategy v_h and candidates adopt the associated best-response platforms x_A^{br} and x_B^{br} then a voter prefers to vote B in response, if and only if s is northeast of the solid line. That is, his best response is the half-space strategy in the direction of h^{br}, where $\theta_{h^{br}} > \theta_{x_B^{br}} > \theta_h$. Since v_h is not the best response to itself (and to candidates' best response platforms), it cannot be sustained in equilibrium.

The logic above is easiest to see for the case of $\theta_h = 0$, but holds more generally. As long as z_1 and z_2 are correlated, information that voters communicate about either of the two dimensions informs candidates about both dimensions. For any half-space strategy with polar angle $\theta_h \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$, voting communicates more about issue 1 than about issue 2; for θ_h below $-\frac{\pi}{4}$ or above $\frac{\pi}{4}$, voting communicates more about issue 2 than about issue 1. Taking the correlation across issues into account lessens the distinction between issues, so that candidates' beliefs and therefore platforms are less disproportionate, and thus closer to the main diagonal than h is. The best-response half-space strategy vector h^{br} must be closer still to the main diagonal, so that the voting behaviors assigned to each signal match the candidate positions even after pivotal considerations are taken into account, which push voters' beliefs back in the direction of the original voting strategy.

If $\theta_h = \pm\frac{\pi}{4}$ then voting is equally informative about the two issues. Even after taking ρ into account, therefore, candidates infer equal information about the two issues. Accordingly, they adopt policy platforms exactly on the diagonal, and since this aligns perfectly with the voting strategy, a voter who conditions on the event of a pivotal vote learns nothing about which candidate is superior. h^{br} then coincides with h, and the voters who favor one candidate after taking pivotal considerations into account are simply those whose signals favor that candidate, as if they had ignored the pivotal voting calculus.

In the major equilibrium, votes for candidate B tend to reflect positive realizations of s_1, suggesting that z_1 is likely positive. They also tend to reflect positive realizations of s_2, suggesting that z_2 is likely positive. Given the positive correlation between issues, the inference that z_2 is positive makes candidate B more confident that z_1 is positive as well, and vice versa. Thus, $E(z_1|w=B)$ and $E(z_2|w=B)$ are both more extreme than they would be if the truth variables were uncorrelated, for the same voting strategy. In the minor equilibrium, votes for candidate B tend to reflect positive realizations of s_1 but negative realizations of s_2, suggesting that z_1 is positive but z_2 is negative. Since the issues are positively correlated, however, the inference that z_2 is positive makes candidate B less confident that z_1 is negative, and vice versa. Thus, $E(z_1|w=B)$ and $E(z_2|w=B)$ are both less extreme than they would be if the truth variables were uncorrelated, for the same voting strategy. Proposition 3 states this formally, and notes further that the degree of polarization is monotonic in ρ.

Proposition 3 $\left\|x_j^+\right\|$ *and* $\left\|x_j^-\right\|$ *increase and decrease with* ρ*, respectively, and are equal if and only if* $\rho = 0$*.*

Standard private interest models highlight the utilitarian value of moderate policies, which compromise between the competing interests at either extreme to minimize the total disutility that voters suffer from policies far from their ideal points. From that perspective, Proposition 3 might seem to indicate that the minor equilibrium promotes greater social welfare. Centrist policies need not hold the same utilitarian appeal in common interest settings, however, as I point out in McMurray (2018), because a voter benefits not from a policy that is close to his current opinion, but from a policy that is close to whatever is truly optimal. In any case, the result that there are two equilibria with differing levels of polarization raises the question of what is best for society in this setting. Defining social welfare $W(v, x_A, x_B)$ is uncontroversial here, unlike many settings, because voters and candidates share the same objective function, which can be

[27] A voter whose expectation $E(z|s)$ is northeast of the dotted line but southwest of the dashed line has a slightly negative signal of z_1 but a strongly positive signal of z_2. Since the two candidates are polarized largely only in the horizontal dimension, his basic inclination would be to vote for candidate A. If his vote is pivotal, however, it is likely that $z_1 \approx 0$. After conditinoning on event P, therefore, he puts relatively higher weight on his signal of z_2 than before, and votes for candidate B instead.

written as follows.

$$W(v, x_A, x_B) = E_{w,z}[u(x_w, z)] = \int_Z \left[\sum_{j=A,B} u(x_j, z) \Pr(w = j|z) \right] f(z)\, dz \tag{6}$$

Proposition 4 now states that, in fact, (6) is higher in the major equilibrium, even though policy outcomes are more extreme. Like polarization, the welfare difference between equilibria is monotonic in ρ.

Proposition 4 *The difference $W\left(v_{h^+}, x_A^+, x_B^+\right) - W\left(v_{h^-}, x_A^-, x_B^-\right)$ in social welfare between the major and minor equilibria has the same sign as ρ and increases in ρ.*

Proposition 4 is simple to state, but its proof is quite involved. This is because the major and minor equilibria are oriented in different directions, and also have different degrees of polarization, making them difficult to compare directly. To establish the result, both equilibria are compared to a third, non-equilibrium strategy combination, which exhibits the polarization of one equilibrium but the orientation of the other. A simple intuition for the result that the major equilibrium is superior to the minor equilibrium is that v_{h^+} and v_{h^-} specify the same voter behavior, but in different states of the world. When z happens to be in quadrant 1 or quadrant 3, v_{h^+} does well at identifying the right quadrant, but v_{h^-} does not; similarly, v_{h^-} is effective at distinguishing between states of the world in quadrants 2 and 4, but v_{h^+} is not. Since quadrants 1 and 3 occur more frequently, v_{h^+} is the more informative voting strategy.[28]

The result that $\left(v_{h^-}, x_A^-, x_B^-\right)$ generates lower welfare than $\left(v_{h^+}, x_A^+, x_B^+\right)$ but both are equilibria implies that an inefficient bundling of political issues could be self-perpetuating: even if it were known that issues had somehow come to be bundled together poorly, the best response for voters and candidates alike would be to go along with the inefficient bundling. If issues are only loosely correlated then Proposition 4 indicates that little welfare is lost, but if issues are strongly correlated then the loss is more severe. In large elections, the familiar logic of Condorcet's (1785) jury theorem implies for either equilibrium that the candidate whose platform is truly closer to the optimal policy vector will win with probability close to one. However, this is not sufficient to eliminate the welfare loss: in the limit as n grows large, the major equilibrium perfectly reveals the true sign of $z \cdot h^+$ while the minor equilibrium perfectly reveals the true sign of $z \cdot h^-$; no matter how large the electorate grows, the first of these remains inherently more valuable.

Proposition 2 shows that a positive correlation reduces the number of equilibria from infinity to two. This is a useful step, but since the two surviving equilibria entail opposite bundlings of the policy issues, it still gives no explanation as to why a major equilibrium should not prevail in one election while a minor equilibrium prevails in another. Proposition 4 is useful in that regard: since the major equilibrium is better than the minor equilibrium for voters and candidates alike, it is likely to be *focal* in the sense of Schelling (1960). However, the proof of Proposition 2 gives an even more compelling reason why the major equilibrium should be the one that prevails, which is that the minor equilibrium is *unstable.* That is, if voters follow a half-space voting strategy oriented in some direction h other than the major and minor diagonals and candidate platforms $x_A^{br}(v_h)$ and $x_B^{br}(v_h)$ best-respond to this voting behavior, an individual voter's best response is also a half-space strategy $v_{h^{br}}$, but oriented in a direction h^{br} that is closer than h to the major diagonal. Rotating slightly away from the major equilibrium therefore produces best-response incentives that rotate back again, but rotating slightly away from the minor equilibrium triggers a series of best responses that converges to the major equilibrium. Both for its efficiency and its stability, then, the major equilibrium emerges as the unique behavioral prediction of the model above. Importantly, this does not require ρ to be large: *any* positive correlation, no matter how small, uniquely pins down both the bundling of political issues and the extent of polarization, in fact prompting the same behavior that would prevail if ρ were equal to one.

[28] McLennan (1998) points out that, in common interest environments such as this, whatever strategy combination is socially optimal is also individually optimal, and therefore constitutes an equilibrium. It is difficult to characterize optimal voting generally for such a complex environment, but it seems reasonable to conjecture that the optimal strategy should exhibit the monotonicity and symmetry of a half-space strategy. If so, Proposition 4 implies that $\left(v_{h^+}, x_A^+, x_A^+\right)$ is the socially optimal strategy vector.

5 Extensions and Alternate Specifications

The analysis above relies on a large number of simplifying assumptions. This section now argues that the main results of the formal analysis are likely to be robust, or extend in natural ways, for various alternative specifications of the model. Section 5.1 considers higher dimensions. Section 5.2 relaxes symmetry. Section 5.3 considers variations on the shape of the underlying policy space.

5.1 Higher Dimensions

The analysis above takes the crucial first step of extending a one dimensional political model to more than one dimension, but the eventual goal is to accommodate not just two, but a large number K of political issues. A thorough treatment of higher dimensions is beyond the scope of this paper, but this section discusses how, as long as the symmetry above is preserved, the results of the two-dimensional analysis extend in a natural way to arbitrary K. To see this, let X be a K-dimensional unit hyperball, denoting the optimal positions on each issue as z_1, $z_2, ..., z_K$, and suppose that the pairwise correlation between any two of these variables is ρ. Then assume further that reversing the positions or the signs of any two z_k and $z_{k'}$ leaves the density $f(z)$ unchanged, which is the multidimensional analog of Condition 2, and that the derivatives $\frac{\partial f(z)}{\partial z_k}$, $\frac{\partial^2 f(z)}{\partial z_k \partial \rho}$, and $\frac{\partial^2 f(z)}{\partial z_k \partial z_{k'}}$ have the same signs as $\rho \prod_{k' \neq k} z_{k'}$, $\prod_{k' \neq k} z_{k'}$, and ρ, respectively, which is the multidimensional analog of Condition 1. An example of a density that satisfies these conditions is $\frac{1}{V_K}\left(1 + \rho \prod_{k=1}^{K} z_k\right)$, where V_K denotes the hypervolume of a K-dimensional unit hyperball. In three dimensions, for example, X is a unit ball and $f(z_1, z_2, z_3) = \frac{3}{4\pi}(1 + \rho z_1 z_2 z_3)$. Then let the conditional density $g(s|z)$ of signal realizations satisfy linear informativeness, as already formulated in Condition 4. An example of such a density is $g(s|z) = \frac{1}{V_K}(1 + s \cdot z)$.

Formal extensions of the results of Section 4 would require the cumbersome notation of hyperspherical coordinates, but it should be clear from the analysis above that, extending the model this way, all of the results above have multidimensional analogs, based on identical reasoning. As in Lemma 1, each voter should still vote for the candidate whose policy platform is closest to his expectation $E(z|P, s)$ of the optimal policy, conditional on the event of a pivotal vote. Half-space strategies can still be defined by a single normal vector, and still imply the symmetry properties of Lemma 2. A candidate's optimal platform choice is still her expectation $E(z|w = j)$ of the optimal policy, conditional on winning, and with a half-space voting strategy, this still implies that candidates will adopt substantially polarized platforms, opposite one another.

When $\rho = 0$, correlative monotonicity still implies that $f(z)$ is uniform. Together with the rotational symmetry of $g(s|z)$ and the symmetry of half-space voting, this guarantees that candidate B's expectation $E(z|w = B)$ lies in the direction of the half-space strategy normal vector h while A's expectation $E(z|w = A)$ lies in the opposite direction. If they adopt these expectations as platforms, it also implies that a voter can thus infer nothing about the magnitude of z in the direction of x_A and x_B, so $E(z|P, s)$ and $E(z|s)$ lie in the same direction, and voters with signals in the direction of h prefer to vote B, while those with signals in the opposite direction prefer to vote A. In other words, v_h constitutes its own best response, for *any* normal vector h. When $\rho > 0$, however, the number of equilibria reduces dramatically, as before. The half-space strategy oriented in the direction of the first Euclidean basis vector e_1 cannot sustain an equilibrium, for example, for the same reason illustrated in Figure 4: if voters voted on the basis of s_1 alone, candidate B would infer upon winning that z_1 is positive, and from this would then infer that z_2 through z_K are positive, as well, and would respond with positive positions on every issue. Relative to h, this reflects a rotation toward the major diagonal (i.e., where $z_k = z_{k'}$ for all k, k'). In response to that, a voter's best response would be a half-space strategy rotated even further toward the major diagonal.

Clearly, a major equilibrium still exists, with the half-space voting strategy and candidate platforms all oriented along the major diagonal. In that equilibrium, a voter votes A if his average signal is negative and B if his average signal is positive. When candidate B wins the election, her updated expectations of z_k are all (equally) positive. Given the positive correlation across issues, the inference that one z_k is positive reinforces the inferences about the other issue dimensions, so that, as in Proposition 3, she adopts a more extreme position than she would have adopted if the issues had been uncorrelated. Candidate A takes an opposite position, and with the voting strategy and both candidate platforms exactly on the major diagonal,

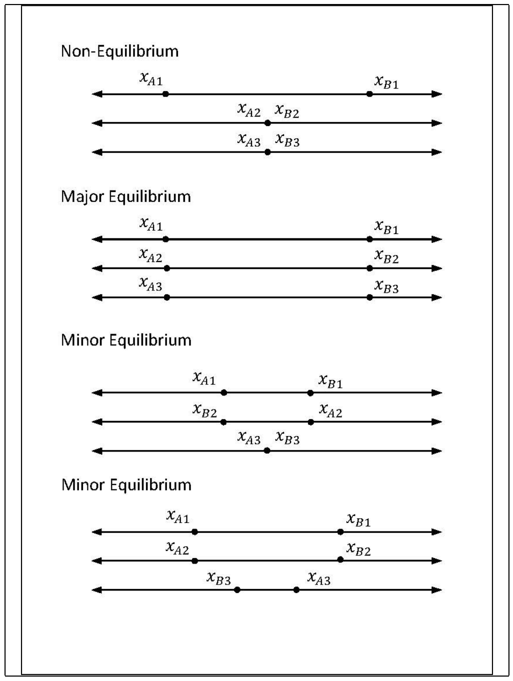

Figure 5: Equilibrium (and non-equilibrium) candidate positions in three dimensions with symmetric prior distribution.

a pivotal vote conveys no information about which candidate is superior, thus sustaining the same voting strategy in response.

For three dimensions, Figure 5 illustrates the candidate platforms that best respond to the non-equilibrium half-space strategy oriented in the direction of e_1, along with major and minor equilibrium platforms. Whereas two dimensions give rise only to a single minor equilibrium, three dimensions produce two distinct types of minor equilibria, with four equilibria of one type and three of the other, for a total of seven. In the first type of minor equilibrium, candidates polarize in opposite directions on two issues, but do not polarize at all in the third dimension. Maintaining the assumption that $x_{A1} \leq x_{B1}$, there are four such equilibria, because there are three issues on which candidates could converge, and if they converge on issue 1, there are two ways to polarize on issues 2 and 3. In the first minor equilibrium of Figure 5, for example, $x_{A3} = x_{B3} = 0$. In response, voters ignore s_3 completely, voting A if $s_1 < s_2$ and voting B if $s_1 > s_2$. Upon winning, candidate B then develops a positive expectation of z_1 and a negative expectation of z_2. Given the positive correlation across issues, these inferences undermine one another, so she is less extreme than she would have been if the issues were uncorrelated, as in Proposition 3. By themselves, the inference that z_1 is positive would lead her to expect z_3 to be positive, and the inference that z_2 is negative would lead her to expect z_3 to be negative. In equilibrium, however, these opposite forces negate one another, so that her expectation of z_3 remains neutral.

In the second type of minor equilibrium, one candidate takes a negative position on two issues and a positive position on one issue, while the other does the opposite. There are three such equilibria, with any of the three issues oriented opposite the other two. In the final example of Figure 5, for example, $x_{A1} < x_{B1}$ and $x_{A2} < x_{B2}$ but $x_{A3} > x_{B3}$. In that case, candidate B infers upon winning that z_1 and z_2 are positive but z_3 is negative. Given the positive correlation across issues, the inference about z_1 and z_2 undermines the inference about z_3, so she takes a position on the third issue that is less extreme than it would be if issues were uncorrelated. The inference that z_1 is positive is similarly undermined by the inference that z_3 is negative, but is bolstered by the inference that z_2 is positive. Thus, she adopts a more extreme position on issue 1 than on issue 3. Issues 1 and 2 are symmetric, so she infers the same amount on both issues, and takes equally extreme positions.

In higher dimensions, the number of minor equilibria grows quickly. In four dimensions, for example, there are sixteen minor equilibria: three with both candidates adopting leftist positions on two issues and rightist positions on two issues; four with one candidate taking leftist positions on three issues and a rightist position on the fourth issue or vice versa; and nine with each candidate taking a leftist position on one issue, a rightist position on one issue, and centrist positions on the remaining two issues.[29] For an arbitrary number of dimensions K, there are enough minor equilibria for a candidate to take leftist or rightist positions on *any* strict subset of the issues.

With so many minor equilibria to choose from when the number of dimensions K is large, it would seem difficult for an electorate to coordinate on any one of them. In contrast, there is only ever a single major equilibrium, no matter how large K is, making this equilibrium quite naturally focal, and therefore likely to prevail. The logic of Proposition 4 suggests that the major equilibrium should also Pareto dominates all of the minor equilibria, as it divides the electorate in a way that is inherently more informative. Furthermore, the many minor equilibria are all unstable in ways similar to that described in Section 4.2. In the first three-dimensional minor equilibrium illustrated in Figure 5, for example, candidates infer nothing about z_3 because they are equally polarized (in opposite directions) on the first two issues, and voters respond with a voting strategy that conveys equal information about s_1 and s_2 and therefore equal information about z_1 and z_2, from which equal but opposite information can be inferred about z_3. Perturbing this equilibrium slightly so that the candidates are more polarized on issue 1 than issue 2, voters would respond with a voting strategy that is rotated so as to place greater weight on s_1 than s_2, leading candidates to infer more information about z_1 than about z_2, and therefore to polarize even more on issue 1 and even less on issue 2 in response. Information about z_3 then no longer cancels out, so that $E\left(z_3|w=A\right) < 0 < E\left(z_3|w=B\right)$ and therefore $x_A^{br} < 0 < x_B^{br}$. If candidates adopt platforms that are more polarized than before on issues 1 and 3 and less polarized than before on issue 2, however, then voters' best response rotates further. As A and B votes increasingly convey information that z_1 and z_3 are positive, platforms on issue 2 become less and less polarized, until they are not polarized at all, and then are polarized in the opposite direction, consistent with issues 1 and 3. In the end, perturbing the minor equilibrium triggers a chain of best responses that converges to the major equilibrium, just as in the case of two dimensions.

5.2 Asymmetries

To keep things tractable, the model of Section 3 adopts a linear g, and assumes a great deal of symmetry: the policy disks X and Z exhibit symmetry in every direction, and so does f when $\rho = 0$ (by Condition 3); even when $\rho > 0$, f exhibits symmetry along both diagonals (Condition 2); g exhibits both rotational symmetry and error symmetry (Conditions 5 and 6); utility u treats the two issues symmetrically; attention is restricted to half-space equilibria, where voting is symmetric in the direction of the normal vector h; and, unlike a sequential game, the simultaneous timing preserves symmetric voting even when candidates consider deviations to platforms that are asymmetric. These many restrictions are worrisome in that linearity and symmetry are knife-edge conditions, and nonlinearity and asymmetry seem entirely plausible. The result that only two equilibria survive when $\rho > 0$, for example, might merely reflect the fact that f is no longer symmetric in every direction, but is still symmetric in two directions. If so, other sources of asymmetry might similarly reduce the set of equilibria, even when issues are uncorrelated. Moreover, if breaking all but two directions of symmetry reduces the number of equilibria to two, one worries that breaking symmetry further might reduce the number of equilibria further, potentially to the point that no pure strategy equilibrium exists at all.

The challenge for tractability stems largely from the intricate way in which the event of a pivotal vote depends on model fundamentals, which makes it difficult to sign the derivative of expected utility, and therefore difficult to characterize behavior for voters who condition on this event. Intuitively, however, it seems that slight deviations from perfect symmetry in the voting behavior or the distribution of signals should produce only slight differences in best response voting, and presumably equilibria that are quite similar to the half-space equilibria above, the main difference being that the boundary partitioning S into A

[29] This accounting of equilibria retains the convention above that the candidate who is weakly to the right on issue 1 is labeled as candidate B. Dropping this convention, there are two minor equilibria in two dimensions, twelve in three dimensions, and twenty-six in four dimensions.

voters and B voters is no longer linear. Without specific functional forms, however, there seems to be little hope of confirming these conjectures.

This section makes marginal progress on this issue by relaxing one form of symmetry, which preserves the half-space structure of equilibrium voting. Consider the following generalized utility function,

$$u(x,z) = -(1+\lambda)(x_1 - z_1)^2 - (1-\lambda)(x_2 - z_2)^2$$

where $\lambda \in [0,1)$. The model of Section 3 corresponds to the case of $\lambda = 0$, but a positive λ captures the plausible possibility that issue 1 is more important to voters than issue 2. Dropping terms that don't depend on the policy outcome, expected utility then generalizes from (1) to the following.

$$E_z[u(x,z)|\Omega] = -(1+\lambda)[x_1 - E(z_1|\Omega)]^2 - (1-\lambda)[x_2 - E(z_2|\Omega)]^2 \tag{7}$$

Proposition 5 now characterizes equilibrium for $\rho = 0$ and $\lambda > 0$, confirming the conjecture that sources of asymmetry other than ρ can reduce the set of equilibria: even if $\rho = 0$, only two half-space equilibria exist when $\lambda > 0$. In that case, there is one equilibrium focused entirely on issue 1 ($\theta_{h^+} = 0$) and another focused entirely on issue 2 ($\theta_{h^-} = -\frac{\pi}{2}$). Clearly, the first of these provides higher welfare, since issue 1 is more important.

Proposition 5 *If $\rho = 0$ and $\lambda > 0$ then there exists one half-space equilibrium with $\theta_{h^*} = 0$ and another with $\theta_{h^*} = \frac{\pi}{2}$. No other half-space equilibria exist.*

The logic of the proof of Proposition 5 begins with the observation that utility weights do not affect candidate strategies; if voters followed a half-space strategy with $\theta_h = \frac{\pi}{4}$ then the logic of Lemma 1 still applies to guarantee that candidates would respond with platforms on the major diagonal. However, λ does affect voting behavior: if candidates adopted platforms on the major diagonal, for example, then a voter with a signal on the minor diagonal was indifferent between the candidate with the superior horizontal position and the candidate with the superior vertical position when $\lambda = 0$, but prefers the former when $\lambda > 0$. As in the case of $\rho > 0$, this drives a wedge between a voting strategy and voters' best response to candidates' best responses to that voting strategy, except when voters and candidates exactly align with one of the major axes, so that $x_B = \begin{pmatrix} x_{B1} \\ x_{B2} \end{pmatrix}$ and $\tilde{x}_B = \begin{pmatrix} (1+\lambda)x_{B1} \\ (1-\lambda)x_{B2} \end{pmatrix}$ lie in the same direction.

When λ and ρ are both zero, the model above is symmetric in every direction, and there are half-space equilibria oriented in every direction. When either parameter is positive, the model is symmetric only in two directions, and only two half-space equilibria remain. If λ and ρ are both positive then the model is no longer symmetric in any direction, which raises the question of whether pure strategy equilibria might cease to exist at all. As Proposition 5 now states, however, this is not the case: a major and a minor equilibrium still remain. These are no longer located exactly on the major and minor diagonals; instead, they have polar angles $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ and $\theta_{h^-} \in \left(-\frac{\pi}{4}, -\frac{\pi}{2}\right)$, respectively, as Figure 6 illustrates.

Proposition 6 *If $\lambda > 0$ and $\rho > 0$ then there exists a major equilibrium $\left(v_{h^+}, x_A^+, x_B^+\right)$ with $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ and a minor equilibrium $\left(v_{h^-}, x_A^-, x_B^-\right)$ with $\theta_{h^-} \in \left(-\frac{\pi}{4}, -\frac{\pi}{2}\right)$.*

The proof of Proposition 6 notes that if voters follow a half-space strategy oriented along the horizontal axis, so that $\theta_h = 0$, then a pivotal vote conveys no information about the vertical location of z. If candidate positions were also located on the horizontal axis, therefore, voters with positive s_1 would prefer to vote B. Voters with signals on the vertical axis would be indifferent between voting A and B, thus sustaining v_h as an equilibrium voting strategy. Because z_1 and z_2 are positively correlated, however, candidate B infers upon winning that both variables are positive, and thus establishes a platform $x_B^{br} = E(z|w = B)$ with $\theta_{x_B^{br}} > 0$. Since issue 1 is more important than issue 2, voters evaluate such a platform as they would evaluate a different platform $\tilde{x}_B^{br} = \begin{pmatrix} (1+\lambda)x_{B1} \\ (1-\lambda)x_{B2} \end{pmatrix}$ under equal weighting. The result that $\theta_{x_B^{br}} > 0$ implies that $\theta_{\tilde{x}_B^{br}} > 0$, as well, so a citizen whose signal lies exactly on the vertical axis prefers to vote B. Thus, v_h is no longer its own best response; instead, the best response to v_h is a half-space strategy with polar angle $\theta_{h^{br}} > 0$.

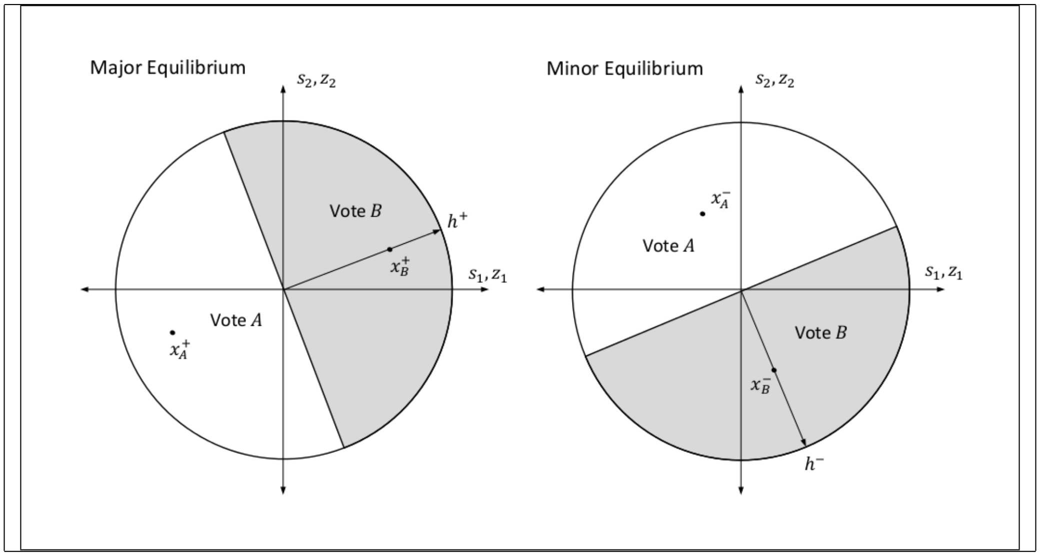

Figure 6: Major and minor equilibria when issue 1 is more important than issue 2

By arguments analogous to these, if voters follow a half-space strategy with polar angle $\theta_h = \frac{\pi}{4}$ and candidates play best response platforms (also on the major diagonal) then the best response for voters is a half-space strategy with polar angle $\theta_{h^{br}} < \frac{\pi}{4}$. By continuity, there exists a polar angle $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ such that the corresponding half-space strategy v_{h^+} is its own best response, and therefore (together with candidates' best-response platforms) constitutes a major equilibrium. Similarly, half-space strategies with polar angles $\theta_h = -\frac{\pi}{4}$ and $\theta_h = -\frac{\pi}{2}$ generate best-response half space strategies with polar angles $\theta_{h^{br}} > -\frac{\pi}{4}$ and $\theta_{h^{br}} < -\frac{\pi}{2}$, respectively, so continuity implies the existence of a polar angle $\theta_{h^-} \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ that (together with candidates' best-response platforms) constitutes a minor equilibrium.

Unlike the equilibria of Proposition 2, the major and minor equilibria of Proposition 6 do not lie exactly on the major and minor diagonals. They still lie in quadrants 1 and 3 of the policy space, however, and therefore represent the same bundling of issues as before. That they are off the diagonals simply means that candidates are now more polarized on the more important issue than on the less important issue. In other words, λ and ρ together determine the relative levels of polarization on the two issues, but ρ alone determines how the issues are bundled.

A formal treatment of asymmetries other than issue importance is beyond the scope of this paper. Some additional intuition can be gained, however, by considering *unsophisticated* voters, who fail to account for pivotality, voting for the candidate whose platform is closest to $E(z|s)$ rather than to $E(z|P,s)$.[30] In that case, equilibrium existence holds quite generally, even relaxing much of the symmetry above. To see this, note that any pair (x_A, x_B) of policy platforms divides Z, and therefore S, into two regions, where voters with signals in one region of S develop expectations $E(z|s)$ closer to x_A than x_B, and thus vote for candidate A, while voters with signals in the other region prefer to vote B. Lemma 3 still implies that a candidate's best response is then given by $x_j^{br} = E(z|w=j)$. Imposing this, $\left(x_A^{br}, x_B^{br}\right)$ can then be interpreted as a function from X^2 into itself. As long as f and g are continuous, X is compact and convex, and Z is compact, a standard fixed point argument guarantees the existence of a pair (x_A^*, x_B^*) of policy platforms that (together with voters' best-response voting strategy) constitute an equilibrium. Simultaneous timing is not important for this logic: if voters move after candidates announce their policy positions then taking unsophisticated voting as given and maximizing expected utility with respect to both x_A and x_B simultaneously as in McLennan (1998) identifies policy positions x_A^* and x_B^* that neither a social planner nor the candidates themselves can improve upon. Such optima exist by the Weierstrass extreme value theorem,

[30] This may actually be realistic; in the experiments of Esponda and Vespa (2014), many voters demonstrated an inability to make inference from the event of a pivotal vote.

as long as f and g are atomless (so that expected utility is continuous) and X is compact.[31]

As long as signals are generally informative of the truth, partitioning the electorate into A voters and B voters will lead the two candidates to infer very different information from the event of winning the election. Quite generally, therefore, the pivotal logic of Lemma 4 should substantially polarize candidates' equilibrium policy platforms. Without all of the symmetry and linearity above, of course, platforms are unlikely to fall exactly on the diagonal of the policy space, but this is unimportant: as emphasized above, a consistent bundling of issues only requires that platforms lie in quadrants 1 and 3. If $\rho > 0$ then, relative to the voting strategy, candidates' best response platforms should still be rotated toward the main diagonal. With the same forces operating, it seems reasonable that equilibrium platforms should remain in the same vicinity as before. Moreover, since the main diagonal can be approached either from a clockwise or a counterclockwise direction, a second (less stable) equilibrium is likely to exist as well, that narrowly avoids swinging in either direction. All of this suggests that the many strong assumptions of Section 3 are necessary only for tractability, and not for the substantive results of Section 4.

5.3 Cartesian Products and Discrete Truth

Defining X as the unit disk implies that certain feasible values of policy x_1 become infeasible when paired with particular values of x_2. Perhaps a more realistic specification is a Cartesian product of intervals $X = [-1,1]^2$—a square and its interior (or a hypercube, in higher dimensions), rather than a disk. In that case, $f(z) = \frac{1}{4}(1 + \rho z_1 z_2)$ is an example of a density that satisfies Conditions 2 and 1 and $g(s|z) = \frac{1}{4}(1 + s_1 z_1 + s_2 z_2)$ is an example of a density that satisfies Condition 4. This f can be defined on $Z = [-1,1]^2$, as in the model above, or alternatively can be interpreted as a mass function on $Z = \{-1,1\}^2$, meaning that the policy that is optimal on either issue ultimately lies at one of the two extremes of the policy interval.[32] This is useful because, as I discuss in McMurray (2017a, 2018), binary truth may be more appropriate for certain applications, and possibly even for the most fundamental ideological questions.[33]

Most of the analysis above does not rely on the policy space being circular, and so proceeds as before, for either of the alternative specifications proposed in this section. In particular, the characterization of best responses for voters and candidates is just as in Lemmas 1 and 3, and still produces the symmetry highlighted in Lemma 2. Without the symmetry of the disk, it seems impossible to determine whether half-space strategies in arbitrary directions can be sustained in equilibrium, but symmetry arguments analogous to those of Section 4 make clear that half-space strategies in the directions of the horizontal and vertical axes only sustain equilibria when $\rho = 0$, while half-space strategies oriented toward the major and minor diagonals sustain equilibria for any ρ.[34] Thus, as before, the major and minor equilibria are robust, while other equilibria are not. Moreover, by the same logic as before, the minor equilibrium can be shown to be inferior and unstable, implying that the major equilibrium (which is more polarized) is uniquely likely to prevail.

[31] The same conditions would guarantee equilibrium existence if the electorate were modeled as a single voter, who perfectly learns which platform (x_A or x_B) is superior, which is another useful approximation to the model of Section 3.

[32] Either way, $g(s|z)$ can be defined on $S = [-1,1]^2$.

[33] To exit a recession, for example, different theories recommend macroeconomic stimulus policies that are as large or as small as possible; intermediate levels of stimulus are also feasible, but are never promoted as optimal. As Harrington (1993) points out, fundamental worldviews may also be binary, such as on the question of whether government action is more likely in general to hamper efficient markets or to correct market inefficiencies.

[34] If voters ignore pivotal considerations, as supposed in Section 5.2, then it can be shown that half-space strategies oriented in directions other than the axes and diagonals can *not* be sustained in equilibrium. This strengthens the conclusion of that section, that the set of equilibria can be reduced by sources of asymmetry other than positive ρ (in this case, a policy square instead of a policy disk). It seems reasonable to conjecture that half-space strategies oriented in other directions would be similarly incompatible with equilibrium when voters take pivotal considerations into account, although candidate polarization in these directions might still be sustainable in equilibrium if, instead of exactly following a half-space strategy, voters split along some non-linear threshold in S, such that after taking pivotal considerations into account, signal realizations exactly on this non-linear threshold develop expectations $E(z|P,s)$ on the line that is equidistant from x_A or to x_B.

6 Applications

6.1 Issue Bundling

From a private interest perspective, it is puzzling that political attitudes should be so unidimensional—that is, that voters who agree on one issue should be so likely to agree on another. As Shor (2014) expresses, for example, "it is not clear why environmentalism necessarily hangs together with a desire for more union prerogatives, but it does." As Section 2 explains, existing explanations for this are few, and focus on factors such as the structure of communication that are election-specific, and therefore seem poorly equipped to explain unidimensionality across elections. However, the analysis above shows that, in a common interest setting, such correlation arises naturally from the logical connections between issues, which should be consistent across space and time. Environmentalism and union support, for example, might both reflect the view that business leaders are overly selfish, and thus willing to abuse employees or environmental resources in pursuit of profits. In fact, such a view could also engender support for minimum wage laws and a host of other pro-labor policies.

Comparing voter and candidate responses to identical survey questions, Shor (2014) finds that political candidates tend empirically to be more ideologically consistent than voters. This pattern, too, finds explanation in the analysis above. With two issues, for example, voter opinions that fall in quadrants 2 and 4 are less common than those in quadrants 1 and 3, but still occur. Candidates, in contrast, only ever position themselves in quadrants 1 and 3. In higher dimensions, candidates adopt consistent positions on every issue. A typical voter adopts consistent positions on many issues, but disagrees with one candidate about one subset of the issues and disagrees with the other candidate on another subset of the issues. As the number of dimensions grows large, the fraction of voters who agree on every issue with one of the candidates shrinks to zero.

In many cases, of course, the logical connection between two issues may seem too weak to explain their being bundled together. Indeed, this seems a reasonable interpretation of the quote above. The analysis here is useful in that regard, because it shows that a little correlation goes a long way. That is, it doesn't matter how small ρ is: *any* non-zero correlation is sufficient to orient the political equilibrium in the direction of the major diagonal, so that candidates bundle issues just as they would if z_1 and z_2 were *perfectly* correlated.

Historically, U.S. political parties have been most polarized on whatever issues are of greatest current interest, while polarization on other issues is less pronounced (see Layman, Carsey, and Horowitz, 2006). This exactly matches the prediction of Section 5.2. Across elections, the relative importance of each issue (captured by λ) is likely to fluctuate substantially, but underlying logical connections (captured by ρ) should remain constant. Even as different elections emphasize different issues, then, the basic bundling of issues tends to remain the same.

Policy realignments do take place occasionally. One possibility is that this reflects learning about ρ. With new insights about the relationship between issues, for example, a correlation that had long been presumed positive might prove to be negative. In that case, what had seemed to be a stable, major equilibrium would suddenly be revealed as a minor equilibrium, and its inherent instability could easily give way to a rebundling of issues.

6.2 Libertarianism

In addition to illuminating ideological consistency, the model above provides a natural framework for formalizing modern political debates. Consider, for example, the U.S. Libertarian party, which is known for taking liberal positions on social issues such as immigration, abortion, and marriage, but conservative positions on economic issues such as taxes and regulation. On its website, the party emphasizes the logical consistency of these positions, arguing that Democrats favor personal liberty and Republicans favor economic liberty, but only the Libertarian party favors liberty of both types.[35] To illustrate this, David Nolan, one of the party founders, created a diagram, reproduced here as Figure 7. This figure bears a striking resemblance to Figure 3 (adapted to a square policy space, as described in Section 5.3), with "left-wing" Democrats and "right-wing" Republicans along the minor diagonal. The model above therefore formalizes the Libertarian

[35] See www.lp.org/platform/, accessed 1/19/18.

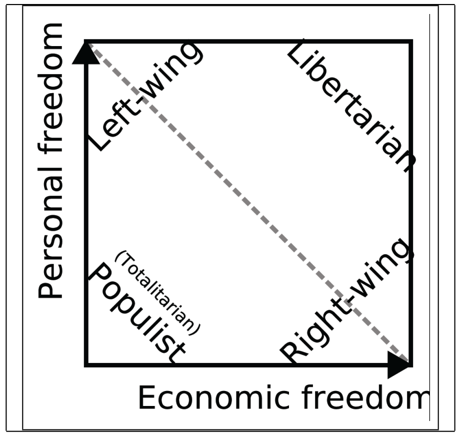

Figure 7: Nolan chart (11/26/2006 version) created by U.S. Libertarian party founder David Nolan, reproduced from upload.Wikimedia.org/wikipedia/commons/3/3e/Nolan-chart.svg (accessed 6/22/17).

perspective as a claim that the U.S. electorate is stuck in a minor equilibrium: if politics could somehow be reoriented so that issues are bundled more sensibly, welfare would improve.

Of course, rotating Figure 7 ninety degrees would place Democrats and Republicans on the major diagonal and Libertarians on the minor diagonal. It may be, therefore, that the traditional bundling of issues is optimal after all. As the names suggest, for example, perhaps "conservative" policies are logically unified by a commitment to preserve both the social and economic institutions of the nation's founding (e.g. limited government, traditional families, etc.), while "liberal" or "progressive" positions seek to modernize on both fronts. As presently constituted, the model above offers little support for the Libertarian perspective, as the inferior bundling of a minor equilibrium is unstable, and therefore unlikely to have prevailed for so long. On the other hand, it may be that appropriately enriching the model would vindicate the Libertarian narrative. Like z, for example, it may be that ρ is imperfectly observed, with different segments of the electorate believing it to be positive or negative. The main point here is not to settle any philosophical debate, but merely to show that the analysis above helps formalize claims that are already prominent in public discourse, thereby clarifying the assumptions that are implicitly being made.

7 Conclusion

Multidimensional models of elections are plagued by counterfactual convergence or equilibrium nonexistence or multiplicity. Empirical unidimensionality may seem to justify ignoring this deficiency, but leaves open the question of why voter tastes should correlate so consistently across elections. This paper has shown that, if voting is driven by opinions rather than tastes, then correlation arises quite naturally from the logical connections between issues. An inferior bundling of issues could persist in equilibrium, mirroring concerns that have been expressed prominently, but stability and efficiency considerations favor a consistent "left" and "right" that may effectively summarize even highly multidimensional decisions.

That a common interest paradigm make progress where private interest literature has not highlights the utility of this general approach to elections (see also McMurray 2017a,b, 2018). Nevertheless, some deviations from perfect common interest seem likely, so adding voter heterogeneity and exploring alternative candidate motivations or beliefs are important directions for future work.[36] The impact of such extensions may only be clear in a sequential game, where candidates expect voters to react to their platform choices, and may therefore necessitate finding variations of the model above, that remain tractable even when symmetry is relaxed. Accommodating asymmetries is also essential for enriching the correlation structure in higher dimensions beyond that described in Section 6.

[36]The logic of McMurray (2018) suggests that equilibrium predictions may be robust to the addition of office motivation, at least in large elections.

It would also be useful to enrich the information structure above. With common priors and independent signals, for example, pairs of voters should easily reach a consensus, simply by sharing and combining their private signals. In large groups, public opinion should be so nearly infallible that a voter who discovers that he belongs to the minority should immediately reverse his opinion. Empirically, of course, individuals routinely disagree with one another, and maintain opinions that they know are unpopular. In McMurray (2018) I discuss informational limitations that might explain these features, and the same discussion applies here, too.[37] A dynamic model of how opinions update between elections would be useful as well: Krasa and Polborn (2014) present evidence, for example, that the correlation of voter attitudes across political issues has increased over time.

Condorcet's (1785) jury theorem emphasizes how elections can reliably identify the better of two alternatives. The logic of that theorem applies here, although with more than two policy quadrants, the ability to ensure that the policy outcome is close to what is optimal is limited. In higher dimensions, candidate platforms may be located in the orthants most likely to contain the optimal policy vector, but the probability of the realized optimum lying exactly in either of these orthants may be vanishingly small as the number of dimensions grows large. Especially if ρ is close to zero, then, the policy outcome may be far from what is optimal—though likely in the right half-space, by the standard logic. An election with more than two candidates does not avoid this problem, because voters should naturally ignore all but two candidates, by the familiar logic of Duverger's (1954) law. If voters voted sincerely for the candidate who seems closest to the truth, adding candidates may improve the election outcome, by placing policy alternatives close to more realizations of the truth variable. Clearly, however, it is infeasible to have as many candidates as issues.

The one-dimensional model of McMurray (2017b) greatly strengthens the jury theorem, showing that the realized margin of victory can act as an informative "mandate" from voters, steering a responsive candidate ex post to the precise optimum from an entire continuum of alternatives. That logic, too, extends naturally to multiple dimensions, but again, since the margin of victory between two candidates is one dimensional, it only reveals the optimal position along the diagonal, not the optimum more globally. In one dimension, votes for minor candidates improve welfare by shaping the winning candidate's beliefs and fine-tuning her mandate. Such votes could have even greater benefit in multiple dimensions, revealing how far off the diagonal the optimum lies, and in which direction. Regardless of whether Democrat and Republican positions reflect a major or a minor equilibrium, for example, Libertarian votes constitute evidence that the truly optimal policy vector lies in an orthogonal direction. Similarly, Green party votes signal strong opinions on environmental issues that votes for major parties cannot convey. As long as policy issues far outnumber political parties, however, the benefit of this is limited. This underscores the importance of letters to legislators, rallies, petitions, public opinion polls, and other political activities, that allow voters to communicate policy-specific opinions that a coarse voting mechanism cannot.[38]

A Appendix

Proof of Lemma 1. When the policy vector that is truly optimal is $z \in Z$, any voter who follows the voting strategy $v \in V$ will vote for candidate $j \in \{A, B\}$ with the following probability,

$$\phi(j|z) = \int_S 1_{v(s)=j}\, g(s|z)\, ds \tag{8}$$

where $1_{v(s)=j}$ is an indicator function that equals one if $v(s) = j$ and zero otherwise. If all voters follow this voting strategy then, by the decomposition property of Poisson random variables, the numbers N_A and N_B of A and B votes are independent Poisson random variables with means $n\phi(A|z)$ and $n\phi(B|z)$, respectively, and the joint probability of vote totals $N_A = a$ and $N_B = b$ is given by the following.

$$\psi(a, b|z) = \frac{e^{-n}}{a!b!} [n\phi(A|z)]^a [n\phi(B|z)]^b \tag{9}$$

[37] Conflicts of interest are a more obvious barrier to consensus, but as that paper discusses, disagreements are just as prevalent on purely speculative questions, where interests should be irrelevant.

[38] Making a similar observation, Besley and Coate (2008) advocate un-bundling complex legislation, allowing separate dimensions to be decided separately.

By the environmental equivalence property of Poisson games, an individual from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by his peers (Myerson, 1998); by voting himself, he can add one to either total. His own vote will be *pivotal* in the election (event P_j) if candidates otherwise tie but j loses the tie-breaking coin toss, or if j wins the coin toss but loses the election by exactly one vote. This occurs with the following probability.

$$\Pr(P_j|z) = \frac{1}{2}\Pr(N_j = N_{-j}) + \frac{1}{2}\Pr(N_j = N_j - 1) \tag{10}$$

The total probability of a vote for either candidate being pivotal (event P) is then given by the following.

$$\Pr(P|z) = \Pr(P_A|z) + \Pr(P_B|z) \tag{11}$$

In terms of (10) and (11), the difference in expected benefit between voting B and voting A is given by the following,

$$\begin{aligned}
\Delta E_{w,z}\left[u(x_w)|s\right] &= E_z\left\{\left[u(x_B|z) - u(x_A|z)\right]\Pr(P_B|z)|s\right\} \\
&\quad - E_z\left\{\left[u(x_A|z) - u(x_B|z)\right]\Pr(P_A|z)|s\right\} \\
&= E_z\left\{\left[u(x_B|z) - u(x_A|z)\right]\Pr(P|z)|s\right\} \\
&= E_z\left\{\sum_{k=1,2}\left[-(x_{Bk} - z_k)^2 + (x_{Ak} - z_k)^2\right]\Pr(P|z)|s\right\} \\
&= E_z\left[\sum_{k=1,2} 2(x_{Bk} - x_{Ak})(z_k - \bar{x}_k)\Pr(P|z)|s\right] \\
&= \Pr(P|s)\sum_{k=1,2} 2(x_{Bk} - x_{Ak})\left[E(z_k|P,s) - \bar{x}_k\right] \\
&= \Pr(P|s)\sum_{k=1,2}\left\{-\left[x_{Ak} - E(z_k|P,s)\right]^2 + \left[x_{Bk} - E(z_k|P,s)\right]^2\right\} \\
&= \Pr(P|s)\left\{\left\|x_A - E(z|P,s)\right\|^2 - \left\|x_B - E(z|P,s)\right\|^2\right\}
\end{aligned} \tag{12}$$

where $\bar{x}_k = \frac{x_{Ak}+x_{Bk}}{2}$ is the average policy position of the two candidates in dimension $k \in \{1,2\}$. This benefit is positive if and only if x_B is closer than x_A to $E(z|P,s)$. ■

Proof of Lemma 2. 1. From Definition 1 it is clear that $v(-s) = A$ if and only if $v(s) = B$. From (8), therefore, it is straightforward to show (utilizing Condition 5) that $\phi(A|-z) = \phi(B|z)$. From (9), it is then clear that $\psi(a,b|-z) = \psi(b,a|z)$ for any $a, b \in Z_+$, implying that

$$\begin{aligned}
\Pr(w = A|-z) &= \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\psi(k+m,k|-z) + \frac{1}{2}\Pr(N_A = N_B) \\
&= \sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\psi(k,k+m|z) + \frac{1}{2}\Pr(N_A = N_B) = \Pr(w = B|z)
\end{aligned}$$

and, integrating over z, that $\Pr(w = A) = \Pr(w = B) = \frac{1}{2}$.

2. The proof of monotonic voting proceeds in several steps. Step 2a shows that $\phi(B|z)$ is constant with respect to changes in z in the direction orthogonal to h. Step 2b shows that, holding the polar angle of z fixed, $\phi(B|z)$ increases in the magnitude r_z, if and only if $h \cdot z > 0$. Step 2c then uses steps 2a and 2b to show that, in general, if $h \cdot z' > h \cdot z$ then there exists a vector z'' that differs from z only in a direction orthogonal to h, and has the same polar angle as z', and that for this vector $\phi(B|z') > \phi(B|z'') = \phi(B|z)$. Step 2d then concludes by showing that $\Pr(w = B|z)$ and $\Pr(P|z)$ are monotonic in $h \cdot z$ and $|h \cdot z|$, respectively.

2a. The derivative of $\phi(B|z)$ in the direction orthogonal to h (i.e. in the direction of $R_{\frac{\pi}{2}}h$) can be written as follows, by rotating the basis vectors,

$$\nabla_z \phi(B|z; h = e_1) \cdot R_{\frac{\pi}{2}}h \quad = \quad \nabla_z\left[\int_{\theta_h - \frac{\pi}{2}}^{\theta_h + \frac{\pi}{2}}\int_0^1 r_s g(s|z;\theta_h)\, dr_s d\theta_s\right] \cdot R_{\frac{\pi}{2}}h$$

$$
\begin{aligned}
&= \nabla_z \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 r_s g\left(R_{\frac{\pi}{2}} s | R_{\frac{\pi}{2}} z; \theta_h = 0\right) dr_s d\theta_s \cdot e_2 \\
&= \nabla_z \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 r_s g\left(s | z; \theta_h = 0\right) dr_s d\theta_s \cdot e_2
\end{aligned}
$$

where $e_1 = \binom{1}{0}$ and $e_2 = \binom{0}{1}$ are the standard Euclidean basis vectors. In this derivation, the final equality follows from Condition 5. Rewriting in Cartesian coordinates, this becomes the following.

$$
\begin{aligned}
&= \frac{\partial}{\partial z_2} \int_{-1}^{1} \int_{-\sqrt{1-s_2^2}}^{\sqrt{1+s_2^2}} g\left(s | z; \theta_h = 0\right) ds_1 ds_2 \\
&= \frac{\partial}{\partial z_2} \int_{-1}^{1} \int_{-\sqrt{1-s_2^2}}^{\sqrt{1+s_2^2}} \left[g_0 + g_1\left(s_1 z_1 + s_2 z_2\right)\right] ds_1 ds_2 = 0
\end{aligned}
$$

Thus, $\phi\left(B|z\right)$ is constant with respect to changes in z in the direction orthogonal to h.

2b. By Condition 4, $g\left(s|z\right)$ is an increasing linear function of $s \cdot z$, say with intercept g_0 and positive slope g_1. Thus, $\phi\left(B|z\right)$ can be written as follows.

$$
\phi\left(B|z\right) = \int_{\{s \in S: v(s) = B\}} \left[g_0 + g_1 r_s r_z \cos\left(\theta_s - \theta_z\right)\right] ds
$$

Differentiating with respect to r_z therefore yields the following.

$$
\begin{aligned}
\frac{\partial}{\partial r_z} \phi\left(B|z\right) &= g_1 \int_{\{s \in S: v(s) = B\}} r_s \cos\left(\theta_s - \theta_z\right) ds \\
&= \frac{g_1}{r_z} \int_{\{s \in S: v(s) = B\}} \left(s \cdot z\right) ds \\
&= \frac{\pi g_1}{r_z} E\left[s | v\left(s\right) = B\right] \cdot z
\end{aligned}
$$

For a half-space strategy, $E\left[s|v\left(s\right) = B\right]$ lies in the direction of h, so $\frac{\partial}{\partial r_z}\phi\left(B|z\right)$ has the same sign as $h \cdot z$.

2c. If $h \cdot z' > h \cdot z$ then let z'' denote the vector with the same polar angle as z' but $h \cdot z'' = h \cdot z$. Steps 2a and 2b imply that $\phi\left(B|z'\right) > \phi\left(B|z''\right) = \phi\left(B|z\right)$.

2d. Conditional on the total number k of voters (and on the state variable z), the number of B votes follows a binomial distribution, with probability parameter $\phi\left(B|z\right)$. The probability that B votes exceed $\frac{k}{2}$ (i.e. $\Pr\left(w = B|k, z\right)$) therefore increases with this parameter. Summing over all k, this implies that the total probability $\Pr\left(w = B|z\right)$ of a B victory is also increasing in $\phi\left(B|z\right)$, so $\nabla_z \Pr\left(w = B|z\right) \cdot h$ has the same sign as $\nabla_z \phi\left(B|z\right) \cdot h$. In other words, $\phi\left(B|z\right)$ and $\Pr\left(w = B|z\right)$ both increase in $h \cdot z$. $\Pr\left(P|z\right)$ is increasing in $\phi\left(B|z\right)$ only if $\phi\left(B|z\right) < \frac{1}{2}$. To see this, rewrite (11) in terms of (8) (using (9) and (10)), as follows,

$$
\begin{aligned}
\Pr\left(P|z\right) &= \sum_{k=m}^{\infty} \left[\psi\left(k, k|z\right) + \frac{1}{2}\psi\left(k+1, k|z\right) + \frac{1}{2}\psi\left(k, k+1|z\right)\right] \\
&= \sum_{k=m}^{\infty} \frac{n^{2k} e^{-n}}{k!k!} \left[\phi\left(A|z\right)\phi\left(B|z\right)\right]^k \left[1 + \frac{n\phi\left(B|z\right)}{2\left(k+1\right)} + \frac{n\phi\left(A|z\right)}{2\left(k+1\right)}\right] \\
&= \sum_{k=m}^{\infty} \frac{n^{2k} e^{-n}}{k!k!} \left\{\left[1 - \phi\left(B|z\right)\right]\phi\left(B|z\right)\right\}^k \left[1 + \frac{n}{2\left(k+1\right)}\right]
\end{aligned}
$$

which is increasing in $\phi\left(B|z\right)$ if and only if $\phi\left(B|z\right) < \frac{1}{2}$. By Part 1 of this lemma, $\phi\left(B|z\right) = \frac{1}{2}$ when $z = 0$, and by step 2a, this implies that $\phi\left(B|z\right) = \frac{1}{2}$ whenever $z \cdot h = 0$. Thus, $\Pr\left(P|z\right)$ increases in $h \cdot z$ when $h \cdot z < 0$ and decreases in $h \cdot z$ when $h \cdot z > 0$. Moreover, the symmetry of the signal density (i.e. $g\left(-s| -z\right) = g\left(s|z\right)$), which follows from Condition 5, induces a symmetry in vote shares (i.e. $\phi(A|-z) = \phi(B|z)$) and therefore pivot probabilities (i.e. $\Pr(P|z) = \Pr(P|-z)$), so that $\Pr(P|z)$ is simply a decreasing function of $|h \cdot z|$, which is the final claim of the lemma.

3. If candidate platforms $x_A = -x_B$ are symmetric then the difference (12) in expected utility between voting B and voting A (conditional both on private information s and on the event P of a pivotal vote) reduces as follows,

$$\begin{aligned}
\Delta E_{w,z}[u(x_w)|s] &= \int_Z \left\{\left[-(x_{B1}-z_1)^2-(x_{B2}-z_2)^2\right]-\left[-(-x_{B1}-z_1)^2-(-x_{B2}-z_2)^2\right]\right\}\Pr(P|z)f(z|s)\,dz \\
&= 4\int_Z (x_{B1}z_1+x_{B2}z_2)\Pr(P|z)f(z|s)\,dz \\
&= 4\int_Z (x_{B1}z_1+x_{B2}z_2)\Pr(P|z)\left[1+\frac{g_1}{g_0}(s_1z_1+s_2z_2)\right]f(z)\,dz \\
&= 4\frac{g_1}{g_0}\int_Z (x_{B1}z_1+x_{B2}z_2)\Pr(P|z)(s_1z_1+s_2z_2)f(z)\,dz
\end{aligned}$$

where the fourth equality follows because Condition 2 and Part 1 of this lemma together imply the following,

$$\begin{aligned}
\int_Z (x_{B1}z_1+x_{B2}z_2)\Pr(P|z)f(z)\,dz_1dz_2 &= \int_{Z_{1,2,3,4}} (x_{B1}z_1+x_{B2}z_2)\Pr(P|z)f(z)\,dz_1dz_2 \\
&\quad+\int_{Z_{1,2,3,4}} (-x_{B1}z_1-x_{B2}z_2)\Pr(P|z)f(z)\,dz_1dz_2 \\
&= 0
\end{aligned}$$

where $Z_{1,2,3,4}$ denotes the half-space of Z with polar angles $\theta_z \in [0,\pi]$ (i.e. the union of octants 1 through 4). This derivation makes clear that $\Delta E_{w,z}[u(x_w)|s]$ is linear in s and equals zero for $(s_1,s_2) = (0,0)$, implying that the best response voting strategy is a half-space strategy. ■

Proof of Lemma 3. Since a candidate is assumed to have no private information of her own, her expected utility can be written as follows.

$$E_{w,z}[u(x,z)] = E_z\left[\sum_{j=A,B} u(x_j,z)\Pr(w=j|z)\right] \tag{13}$$

This depends on both her own policy position and her opponent's position, and on the likelihood that she will win or lose, which in turn depends on the voting strategy v used by voters. Differentiating (13) with respect to the policy choice x_{jk} that candidate $j \in \{A,B\}$ adopts in dimension $k \in \{1,2\}$ yields the following,

$$\frac{\partial}{\partial x_{jk}}E_{w,z}[u(x,z)] = E_z[-2(x_{jk}-z_k)\Pr(w=j|z)]$$

which equals zero if and only if $x_{jk} = E(z_k|w=j)$, which Bayes' rule defines as follows.[39]

$$E(z_k|w=j) = \int_Z z_k\frac{\Pr(w=j|z)f(z)}{\Pr(w=j)}dz \tag{14}$$

The second derivative of (13) is negative, so $x_j^{br} = E(z|w=j)$ is the unique best response to v, no matter what the policy position of a candidate's opponent. ■

[39] Throughout this paper, Lebesgue intergration is with respect to the standard measure. Equivalently,

$$\int_Z dz = \int_{-1}^{1}\int_{-\sqrt{1-z_2^2}}^{\sqrt{1-z_2^2}} dz_1dz_2 = \int_0^{2\pi}\int_0^1 r_z dr_z d\theta_z.$$

Proof of Lemma 4. Lemma 3 states that $x_j^{br} = E(z|w=j)$ for $j = A, B$, and symmetry is a straightforward consequence of the directional symmetry of $f(z)$ (Condition 2) and the candidate symmetry of $g(s|z)$ (Part 1 of Lemma 2).

$$\begin{aligned} x_{Ak}^{br} = \int_Z z_k \frac{\Pr(w=A|z) f(z)}{\Pr(w=A)} dz &= \int_Z (-z_k) \frac{\Pr(w=A|-z) f(-z)}{\Pr(w=A)} dz \\ &= -\int_Z z_k \frac{\Pr(w=B|z) f(z)}{\Pr(w=B)} dz = -x_{Bk}^{br} \end{aligned}$$

Using $h = \binom{h_1}{h_2}$ and its orthogonal rotation $h' = \binom{-h_2}{h_1}$ as basis vectors instead of the standard Euclidean basis vectors, $E(z|w=B) \cdot h$ can be written simply as $E(z_1|w=B)$. This reduces as follows,

$$\begin{aligned} E(z_1|w=B) &= \int_Z z_1 \frac{\Pr(w=B|z_1,z_2) f(z_1,z_2)}{\Pr(w=B)} dz_2 dz_1 \\ &= 2\int_{Z_{1,2}} \left[\begin{array}{c} z_1 \Pr(w=B|z_1,z_2) f(z_1,z_2) - z_1 \Pr(w=B|-z_1,z_2) f(-z_1,z_2) \\ +z_1 \Pr(w=B|z_1,-z_2) f(z_1,-z_2) - z_1 \Pr(w=B|-z_1,-z_2) f(-z_1,-z_2) \end{array} \right] dz_2 dz_1 \\ &= 2\int_{Z_{1,2}} z_1 \left[\Pr(w=B|z_1,z_2) - \Pr(w=B|-z_1,z_2)\right] \left[f(z_1,z_2) + f(z_1,-z_2)\right] dz_2 dz_1 \qquad (15) \end{aligned}$$

where $Z_{1,2}$ denotes the union of the first and second octants (i.e., the first quadrant) and the second equality follows from Lemma 2, since $\Pr(w=B) = \frac{1}{2}$ and since $\Pr(w=B|z)$ is constant in the direction orthogonal to h. That same lemma guarantees that $\Pr(w=B|z_1,z_2) > \Pr(w=B|-z_1,z_2)$ for any $(z_1,z_2) \in Z_{1,2}$, implying that the entire expression is positive. ■

Proof of Proposition 1. Lemma 3 gives candidates' unique best response behavior to any voting strategy as $x_j^{br} = E(z|w=j)$, and by Lemma 4, these platforms are symmetric and distinct when voting follows a half-space strategy. When $\rho = 0$, candidates' expectations lie exactly in the directions of h and $-h$. That is, $x_B^{br} \cdot h$ is positive but, for a vector $h' = \binom{-h_2}{h_1}$ that is orthogonal to h, $x_B^{br} \cdot h' = 0$. The easiest way to demonstrate this is to rewrite x_B^{br} using h and h' as basis vectors in place of the standard Euclidean basis, as in the proof of Lemma 4, so that $x_B^{br} \cdot h$ and $x_B^{br} \cdot h'$ reduce simply to $E(z_1|w=B)$ and $E(z_2|w=B)$, respectively. The first of these is given by (15) in the proof of Lemma 4, and following a similar derivation yields the second as follows,

$$E(z_2|w=B) = 2\int_{Z_{1,2}} z_2 \left[\Pr(w=B|z_1,z_2) - \Pr(w=B|-z_1,z_2)\right] \left[f(z_1,z_2) - f(z_1,-z_2)\right] dz_2 dz_1 \qquad (16)$$

where $Z_{1,2}$ again denotes the union of the first and second octants (i.e., the first quadrant). When $\rho = 0$, the radial symmetry of $f(z)$ (Condition 3) implies that $f(z_1,-z_2) = f(z_1,z_2)$, so (16). Thus, x_B^{br}—and, by symmetry, x_A^{br}—are orthogonal to h', and therefore colinear with h. As shown in the proof of Lemma 4, however, $x_B^{br} \cdot h > 0$. Thus, x_B^{br} lies exactly in the direction of h, and x_A^{br} lies in the opposite direction.

With symmetric candidate platforms, Part 3 of Lemma 2 states that the best voter response is another half-space strategy, $v_{h^{br}}$. By Proposition 1, a voter prefers to vote B if and only if $E(z|P,s) \cdot x_B > 0$, and since x_B is in the direction of h in equilibrium, this is equivalent to the condition that $E(z|P,s) \cdot h > 0$, where the pivotal event P depends implicitly on v_h. With basis vectors h and h' instead of e_1 and e_2, this dot product reduces simply to $E(z_1|P,s)$, which is proportional to the following,

$$\begin{aligned} &\int_{Z_{1,2}} \left[\begin{array}{c} z_1 \Pr(P|z_1,z_2)\, g(s|z_1,z_2) f(z_1,z_2) - z_1 \Pr(P|-z_1,z_2)\, g(s|-z_1,z_2) f(-z_1,z_2) \\ +z_1 \Pr(P|z_1,-z_2)\, g(s|z_1,-z_2) f(z_1,-z_2) - z_1 \Pr(P|-z_1,-z_2)\, g(s|-z_1,-z_2) f(-z_1,-z_2) \end{array} \right] dz_2 dz_1 \\ = &\int_{Z_{1,2}} z_1 \Pr(P|z_1,z_2) \left[\begin{array}{c} \left[g(s|z_1,z_2) - g(s|-z_1,-z_2)\right] f(z_1,z_2) \\ +\left[g(s|z_1,-z_2) - g(s|-z_1,z_2)\right] f(z_1,-z_2) \end{array} \right] dz_2 dz_1 \qquad (17) \end{aligned}$$

where the equality follows from Lemma 2, since $\Pr(P|z_1,z_2)$ is constant with respect to z_2, and depends on the magnitude of z_1 but not the sign. Radial symmetry (Condition 3) implies that $f(-z_1,z_2) =$

$f(z_1,z_2)$ and linear informativeness (Condition 4) implies that $g(s|z_1,z_2)-g(s|-z_1,z_2)$ has the same sign as $\binom{s_1}{s_2}\cdot\binom{z_1}{z_2}-\binom{s_1}{s_2}\cdot\binom{z_1}{z_2}=2s_1z_1$. Since z_1 is positive on $Z_{1,2}$, this has the same sign as s_1, which is equivalent to $s\cdot h$ for the original Euclidean basis vectors. In other words, v_h is the best response to itself, and together with the best-response candidate platforms constitutes a half-space equilibrium. ■

Lemma A1 is useful in establishing Propositions 2 and 3.

Lemma A1 *If* $\theta_h\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ *and* $\theta_z\in\left(\theta_h,\theta_h+\frac{\pi}{2}\right)$ *then* $f(z)-f(M_{\theta_h}z)$ *has the same sign as* $\frac{\pi}{4}-|\theta_h|$.

Proof. $M_{\theta_h}z$ and z are equidistant from h. If $|\theta_h|<\frac{\pi}{4}$ then it is possible for both of these angles to lie in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$; in that case, $f(z)>f(M_{\theta_h}z)$ as claimed, because $f(z)$ increases in θ_z over that entire interval, by correlative monotonicity (Condition 1). If $\theta_{M_{\theta_h}z}$ lies in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ but θ_z exceeds $\frac{\pi}{4}$ then consider the mirror image of z through $\frac{\pi}{4}$, which dimensional symmetry (Condition 2) implies has the same density as z: the assumption that $\theta_h<\frac{\pi}{4}$ implies that $\theta_{M_{\frac{\pi}{4}}z}>\theta_{M_{\theta_h}z}$, and therefore that $f(z)=f\left(\theta_{M_{\frac{\pi}{4}}z}\right)>f\left(\theta_{M_{\theta_h}z}\right)$ (utilizing correlative monotonicity, as before). Similarly, if θ_z lies in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ but $\theta_{M_{\theta_h}z}$ is less than $-\frac{\pi}{4}$ then consider the mirror image of $M_{\theta_z}z$ through $-\frac{\pi}{4}$, which has the same density as $M_{\theta_h}z$: the assumption that $\theta_h>-\frac{\pi}{4}$ implies that $\theta_{M_{\frac{\pi}{4}}M_{\theta_h}z}<\theta_z$, so $f(z)>f\left(\theta_{M_{\frac{\pi}{4}}M_{\theta_h}z}\right)=f\left(\theta_{M_{\theta_h}z}\right)$. If both $\theta_{M_{\theta_h}z}<-\frac{\pi}{4}$ and $\theta_z>\frac{\pi}{4}$ then Conditions 2 and 1 imply for $|\theta_h|<\frac{\pi}{4}$ that $f(z)=f\left(\theta_{M_{\frac{\pi}{4}}z}\right)>f\left(\theta_{M_{\frac{\pi}{4}}M_{\theta_h}z}\right)=f\left(\theta_{M_{\theta_h}z}\right)$. This exhausts all cases, establishing that $f(z)<f(M_{\theta_h}z)$ if $|\theta_h|<\frac{\pi}{4}$. Analogous reasoning implies that $f(z)>f(M_{\theta_h}z)$ if $|\theta_h|>\frac{\pi}{4}$. If $\theta_h>\frac{\pi}{4}$, for example, then the mirror image of z through $\frac{\pi}{4}$ has density equal to that of z, but smaller polar angle and therefore lower density than $M_{\theta_h}z$. ■

Proof of Proposition 2. If his peers vote according to strategy v_h then, according to Lemma 1, a voter's best response is to vote for candidate j if and only if $E(z|P,s)\cdot x_j>0$. For v_h to be its own best response, voters with signals satisfying $s\cdot h>0$ should have a best response to vote B while the rest vote A. A voter with a signal orthogonal to h should be exactly indifferent between voting A and voting B. According to Lemma 3, the best response to v_h for candidate $j=A,B$ is given by the policy $x_j^{br}=E(z|w=j)$ that is optimal in expectation, conditional on winning. Taking these conditions together, a half-space equilibrium requires that $E(z|P,s)\cdot E(z|w=j)$ have the same sign as $s\cdot h$. The logic of this proof is to show that this is possible if and only if $\theta_h\in\left\{-\frac{1}{4},\frac{1}{4}\right\}$. In particular, other values of θ_h produce $E(z|P,s)\cdot E(z|w=j)\neq 0$ for s orthogonal to h.

The cleanest way to compare $E(z|P,s)$ and $E(z|w=j)$ with each other is to compare both vectors with h, and with the orthogonal vector $h'=R_{\frac{\pi}{2}}h=\binom{-h_2}{h_1}$. This is accomplished most simply by using h and h' as basis vectors instead of the standard Euclidean basis, as in the proofs of Lemma 4 and Proposition 1. The dot products $x_j^{br}\cdot h$ and $x_j^{br}\cdot h'$ then reduce to $E(z_1|w=B)$ and $E(z_2|w=B)$, respectively, which are given by (15) and (16) in the proofs of Lemma 4 and Proposition 1. Similarly, $E(z|P,s)\cdot h$ and $E(z|P,s)\cdot h'$ reduce to $E(z_1|P,s)$ and $E(z_2|P,s)$. For generic s, the first of these is given in the proof of Proposition 1 as proportional to (17), and an analogous derivation obtains the second as follows.

$$E(z_2|P,s)\propto\int_{Z_{1,2}}z_2\Pr(P|z_1,z_2)\left[\begin{array}{c}[g(s|z_1,z_2)-g(s|-z_1,-z_2)]\,f(z_1,z_2)\\+[g(s|-z_1,z_2)-g(s|z_1,-z_2)]\,f(z_1,-z_2)\end{array}\right]dz_2dz_1 \tag{18}$$

However, a signal that is orthogonal to h has $s_1=0$; in that case, error symmetry (Condition 6) implies that $g(s|-z_1,z_2)=g(s|z_1,z_2)$ for any z_1, so (17) and (18) reduce to the following.

$$E(z_1|P,s=h')\propto\int_{Z_{1,2}}z_1\Pr(P|z_1,z_2)[g(s|z_1,z_2)-g(s|z_1,-z_2)][f(z_1,z_2)-f(z_1,-z_2)]\,dz_2dz_1 \tag{19}$$

$$E(z_2|P,s=h')\propto\int_{Z_{1,2}}z_2\Pr(P|z_1,z_2)[g(s|z_1,z_2)-g(s|z_1,-z_2)][f(z_1,z_2)+f(z_1,-z_2)]\,dz_2dz_1 \tag{20}$$

Together, $s=h'$ and $z\in Z_{1,2}$ also imply that $g(s|z_1,z_2)>g(s|z_1,-z_2)$.

If $|\theta_h| < \frac{\pi}{4}$ then Lemma A1 implies that z has greater density than $M_{\theta_h} z$. In terms of basis vectors h and h', this means that $f(z_1, z_2) > f(z_1, -z_2)$, implying that (15), (16), (19), and (20) are all positive. That $x_B^{br} \cdot h > 0$ and $x_B^{br} \cdot h' > 0$ implies that x_B^{br} has polar angle strictly between those of h and h'.[40] That $E(z|P, s) \cdot h > 0$ and $E(z|P, s) \cdot h' > 0$, together with the result that x_B^{br} lies between h and h', imply in turn that $E(z|P, s) \cdot x_B^{br} > 0$, as well.[41] In short, $|\theta_h| < \frac{\pi}{4}$ is not compatible with equilibrium: when his peers follow v_h and candidates respond accordingly, equilibrium would require that a voter with orthogonal signal $s = h'$ be indifferent between voting A and voting B, but instead such a voter strictly prefers to vote B.

If $|\theta_h| > \frac{\pi}{4}$ then Lemma A1 implies that z has lower density than $M_{\theta_h} z$, which in terms of basis vectors h and h' means that $f(z_1, z_2) > f(z_1, -z_2)$. In that case, (15) is still positive but (16) is now negative, meaning that $x_B^{br} \cdot h' < 0 < x_B^{br} \cdot h$, so that x_B^{br} has polar angle strictly between those of $-h'$ and h. Similarly, for a voter with orthogonal signal $s = h'$, (20) is still positive but (19) is negative, meaning that $E(z|P, s) \cdot x_B^{br} < 0$.[42] When his peers follow v_h and candidates respond accordingly, therefore, a voter with signal $s = h'$ strictly prefers to vote A in response. Thus, as in the case of $|\theta_h| < \frac{\pi}{4}$, v_h is incompatible with equilibrium.

If $|\theta_h| = \frac{\pi}{4}$ then Lemma A1 implies that z and $M_{\theta_h} z$ have equal density, which in terms of basis vectors h and h' means that $f(z_1, z_2) = f(z_1, -z_2)$. In that case, (15) and (19) are still positive but (16) and (20) are zero. That $x_B^{br} \cdot h > 0$ and $x_B^{br} \cdot h' = 0$ implies that x_B^{br} is colinear with h. That $E(z|P, s) \cdot h > 0$ and $E(z|P, s) \cdot h' = 0$ for a voter with orthogonal signal $s = h'$ then implies that the best response to v_h (and $x_A^{br}(v_h)$ and $x_B^{br}(v_h)$) is v_h, implying that $\left(v_h, x_A^{br}, x_B^{br}\right)$ constitutes a half-space equilibrium. $\theta_h = \frac{\pi}{4}$ produces the major equilibrium, and $\theta_h = -\frac{\pi}{4}$ produces the minor equilibrium. ■

Proof of Proposition 3. Since x_B^+ lies in the direction of h^+ and x_B^- lies in the direction of h^-, their magnitudes can be written as the projection of x_B^+ on h^+ and the projection of x_B^- on h^-, respectively. Generically, (15) gives the projection of x_B^{br} on h, in terms of the basis vectors h and h'. In the major and minor equilibria, candidate platforms are symmetric, so $\Pr(w = B| -z) = \Pr(w = A|z)$, and $|\theta_h| = \frac{\pi}{4}$, so $f(M_{\theta_h} z) = f(z)$ by Lemma A1. Thus, (15) reduces to the following,

$$\|x_B\| = 8 \int_{Z_{1,2}} z_1 \left[\Pr(w = B|z_1, z_2) - \frac{1}{2}\right] f(z_1, z_2)\, dz$$

and can be differentiated as follows.

$$\frac{\partial \|x_B\|}{\partial \rho} = 8 \int_{Z_{1,2}} z_1 \left[\Pr(w = B|z_1, z_2) - \frac{1}{2}\right] \frac{\partial}{\partial \rho} f(z_1, z_2)\, dz$$

A policy vector z under these rotated basis vectors corresponds to a rotated vector $R_{\theta_h} z$ under the original basis vectors. In the cases of the major and minor equilibria, this corresponds to $R_{\frac{\pi}{4}} z = \frac{1}{\sqrt{2}} \binom{z_1 - z_2}{z_1 + z_2}$ and $R_{-\frac{\pi}{4}} z = \frac{1}{\sqrt{2}} \binom{z_1 + z_2}{z_2 - z_1}$, respectively. Reversing z_1 and z_2 therefore corresponds to reversing the sign of either the first or the second component. Either way, because of the dimensional symmetry of f (Condition 2), this is equivalent to reversing the sign of ρ. Thus, $\frac{\partial \|x_B\|}{\partial \rho}$ reduces further, as follows,

$$\begin{aligned}
\frac{\partial \|x_B\|}{\partial \rho} &= 8 \int_{Z_1} \left\{ \begin{array}{c} z_1 \left[\Pr(w = B|z_1, z_2) - \frac{1}{2}\right] \frac{\partial}{\partial \rho} f(z_1, z_2) \\ +z_2 \left[\Pr(w = B|z_2, z_1) - \frac{1}{2}\right] \frac{\partial}{\partial \rho} f(z_2, z_1) \end{array} \right\} dz \\
&= 8 \int_{Z_1} \left\{ \begin{array}{c} z_1 \left[\Pr(w = B|z_1, z_2) - \frac{1}{2}\right] \\ -z_2 \left[\Pr(w = B|z_2, z_1) - \frac{1}{2}\right] \end{array} \right\} \frac{\partial}{\partial \rho} f(z_1, z_2)\, dz
\end{aligned}$$

[40] Formally, $x_B^{br} \cdot h = r_{x_B^{br}} \cos\left(\theta_{x_B^{br}} - \theta_h\right) > 0$ and $x_B^{br} \cdot h' = r_{x_B^{br}} \cos\left(\theta_{x_B^{br}} - \theta_{h'}\right) > 0$ together imply that $\theta_{x_B^{br}} - \theta_h < \frac{\pi}{4}$ and $\theta_{h'} - \theta_{x_B^{br}} < \frac{\pi}{4}$.

[41] Formally, $x_B^{br} = r_{x_B^{br}} [\alpha h + (1 - \alpha) h']$ for some $\alpha \in (0, 1)$ implies that $E(z|\mathcal{P}, s) \cdot x_B^{br} = r_{x_B^{br}} \alpha E(z|\mathcal{P}, s) \cdot h + r_{x_B^{br}} (1 - \alpha) E(z|\mathcal{P}, s) \cdot h' > 0$.

[42] Formally, $x_B^{br} = r_{x_B^{br}} [\alpha h + (1 - \alpha)(-h')]$ for some $\alpha \in (0, 1)$ and $E(z|\mathcal{P}, s) \cdot h < 0 < E(z|\mathcal{P}, s) \cdot h'$ together imply that $E(z|\mathcal{P}, s) \cdot x_B^{br} = r_{x_B^{br}} \alpha E(z|\mathcal{P}, s) \cdot h - r_{x_B^{br}} (1 - \alpha) E(z|\mathcal{P}, s) \cdot h' < 0$.

where $z_1 > z_2 > 0$ for policy pairs in Z_1, implying that $\Pr(w = B|z_1, z_2) > \Pr(w = B|z_2, z_1)$, and therefore that the bracketed difference is positive. With the original basis vectors, correlative monotonicity (Condition 1) states that $\frac{\partial}{\partial\rho} f(z)$ has the same sign as $z_1 z_2$; with the rotated basis vectors, this means that $\frac{\partial}{\partial\rho} f(z)$ has the same sign as $(z_1 - z_2)(z_1 + z_2)$ for the major equilibrium and $(z_1 + z_2)(z_2 - z_1)$ for the minor equilibrium. Since $z_1 > z_2 > 0$ for all policy pairs in Z_1, this implies that $\frac{\partial\|x_B^+\|}{\partial\rho}$ is positive and $\frac{\partial\|x_B^-\|}{\partial\rho}$ is negative, as claimed. ■

Proof of Proposition 4. As a first preliminary step, consider the welfare $W_+(x;\rho)$ associated with the major equilibrium voting strategy v_{h^+}, together with candidate platforms $x_A = (-x,-x)$ and $x_B = (x,x)$ on the major diagonal and symmetric around the origin. Divide Z into octants $Z_1, Z_2, ..., Z_8$ (i.e., with polar angles ranging from 0 to $\frac{\pi}{4}$, $\frac{\pi}{4}$ to $\frac{\pi}{2}$, etc.), and note that octants 4 through 7 can be written in terms of their relationship with octants 1 through 3 and 8, as follows.

$$\begin{aligned} W_+(x;\rho) &= \int_{Z_{1,2,3,8}} \sum_{j=A,B} \left[u(x_j,z)\Pr(w=j|z) f(z) + u(x_j,-z)\Pr(w=j|-z) f(-z)\right] dz \\ &= 2\int_{Z_{1,2,3,8}} \sum_{j=A,B} u(x_j,z)\Pr(w=j|z) f(z)\, dz \end{aligned}$$

The second equality here shows that both half-spaces produce the same utility, and follows because $x_A = -x_B$ implies that $u(x_A,-z) = u(x_B,z)$ and $\Pr(w=A|-z) = \Pr(w=B|z)$. In a similar way, octants 1 and 8 can then be related to octants 2 and 3, and octant 3 can be related to octant 2, so that welfare reduces further to the following,

$$\begin{aligned} &= 2\int_{Z_{2,3}} \sum_{j=A,B} \left[u(x_j,z)\Pr(w=j|z) f(z) + u\left(x_j, M_{\frac{\pi}{4}} z\right)\Pr\left(w=j|M_{\frac{\pi}{4}} z\right) f\left(M_{\frac{\pi}{4}} z\right)\right] dz \\ &= 4\int_{Z_{2,3}} \sum_{j=A,B} u(x_j,z)\Pr(w=j|z) f(z)\, dz \\ &= 4\int_{Z_2} \sum_{j=A,B} \left[u(x_j,z)\Pr(w=j|z) f(z) + u\left(x_j, M_{\frac{\pi}{2}} z\right)\Pr\left(w=j|M_{\frac{\pi}{2}} z\right) f\left(M_{\frac{\pi}{2}} z\right)\right] dz \qquad (21) \end{aligned}$$

with the second equality following because platforms are on the major diagonal, so $u(x_j,z) = u\left(x_j, M_{\frac{\pi}{4}} z\right)$ and $\Pr\left(w=j|M_{\frac{\pi}{4}} z; v_{h^+}\right) = \Pr(w=j|z;v_{h^+})$, and because $f\left(M_{\frac{\pi}{4}} z\right) = f(z)$ by Condition (2).

As a second preliminary step, write the difference in welfare for positive and negative correlation as follows,

$$\begin{aligned} & W_+(x;\rho) - W_+(x;-\rho) \\ =\ & 4\int_{Z_2} \sum_{j=A,B} \left\{ \begin{array}{c} u(x_j,z)\Pr(w=j|z)\left[f(z;\rho) - f(z;-\rho)\right] \\ +u\left(x_j,M_{\frac{\pi}{2}} z\right)\Pr\left(w=j|M_{\frac{\pi}{2}} z\right)\left[f\left(M_{\frac{\pi}{2}} z;\rho\right) - f\left(M_{\frac{\pi}{2}} z;-\rho\right)\right] \end{array} \right\} dz \\ =\ & 4\int_{Z_2} \left[f(z;\rho) - f\left(M_{\frac{\pi}{2}} z;\rho\right)\right] \sum_{j=A,B} \left[\begin{array}{c} u(x_j,z)\Pr(w=j|z) \\ -u\left(x_j,M_{\frac{\pi}{2}} z\right)\Pr\left(w=j|M_{\frac{\pi}{2}} z\right) \end{array} \right] dz \\ =\ & 4\int_{Z_2} \left[f(z;\rho) - f\left(M_{\frac{\pi}{2}} z;\rho\right)\right] \sum_{j=A,B} \left\{ \begin{array}{c} \left[u(x_j,z) - u\left(x_j,M_{\frac{\pi}{2}} z\right)\right]\Pr(w=j|z) \\ +u\left(x_j,M_{\frac{\pi}{2}} z\right)\left[\Pr(w=j|z) - \Pr\left(w=j|M_{\frac{\pi}{2}} z\right)\right] \end{array} \right\} dz \quad (22) \end{aligned}$$

where the second equality follows because $f\left(M_{\frac{\pi}{2}} z;-\rho\right) = f(z;\rho)$. The second portion of this summation reduces to the following,

$$\begin{aligned} & u\left(x_A,M_{\frac{\pi}{2}} z\right)\left[\begin{array}{c} \Pr(w=A|z) \\ -\Pr\left(w=A|M_{\frac{\pi}{2}} z\right) \end{array} \right] + u\left(x_B,M_{\frac{\pi}{2}} z\right)\left[\begin{array}{c} \Pr(w=B|z) \\ -\Pr\left(w=B|M_{\frac{\pi}{2}} z\right) \end{array} \right] \\ =\ & u\left(x_A,M_{\frac{\pi}{2}} z\right)\left[\begin{array}{c} \Pr\left(w=B|M_{\frac{\pi}{2}} z\right) \\ -\Pr(w=B|z) \end{array} \right] + u\left(x_B,M_{\frac{\pi}{2}} z\right)\left[\begin{array}{c} \Pr(w=B|z) \\ -\Pr\left(w=B|M_{\frac{\pi}{2}} z\right) \end{array} \right] \end{aligned}$$

$$= \left[u\left(x_B, M_{\frac{\pi}{2}}z\right) - u\left(x_A, M_{\frac{\pi}{2}}z\right)\right]\left[\Pr(w=B|z) - \Pr\left(w=B|M_{\frac{\pi}{2}}z\right)\right]$$

which is positive for $z \in Z_2$ because $u\left(x_B, M_{\frac{\pi}{2}}z\right) > u\left(x_A, M_{\frac{\pi}{2}}z\right)$ and $\Pr(w=B|z) > \Pr\left(w=B|M_{\frac{\pi}{2}}z\right)$. Since platforms lie on the major diagonal, the utility difference in (22) can be rewritten in Cartesian coordinates as follows,

$$\begin{aligned} u(x_j, z) - u\left(x_j, M_{\frac{\pi}{2}}z\right) &= u\left[(x_{j2}, x_{j2}), (z_1, z_2)\right] - u\left[(x_{j2}, x_{j2}), (-z_1, z_2)\right] \\ &= -(z_1 - x_{j2})^2 - (z_2 - x_{j2})^2 + (-z_1 - x_{j2})^2 + (z_2 - x_{j2})^2 \end{aligned}$$

and reduces to $4x_{j2}z_1$. Therefore, (22) reduces to

$$\begin{aligned} W_+(x;\rho) - W_+(x;-\rho) &= 4\int_{Z_2}\left[f(z;\rho) - f(M_\pi z;\rho)\right]\sum_{j=A,B} 4x_{j2}z_1 \Pr(w=j|z)\,dz \\ &= 16\int_{Z_2} z_1\left[f(z;\rho) - f\left(M_{\frac{\pi}{2}}z;\rho\right)\right]\left\{\begin{array}{c} x_{A2}\left[1 - \Pr(w=B|z)\right] \\ +x_{B2}\Pr(w=B|z) \end{array}\right\}dz \\ &= 32x_{B2}\int_{Z_2} z_1\left[f(z;\rho) - f\left(M_{\frac{\pi}{2}}z;\rho\right)\right]\left[\Pr(w=B|z) - \frac{1}{2}\right]dz \end{aligned}$$

where the third equality follows because platforms are symmetric around the origin, implying that $x_{A2} = -x_{B2}$. By the symmetry and monotonicity of $\Pr(w=B|z)$ (i.e. Parts 1 and 2 of Lemma 2), $\Pr(w=B|z) > \frac{1}{2}$ for any $z \in Z_2$. By Condition 1, the first bracketed difference has the same sign as ρ and increases in ρ. Thus, the entire expression has the same sign as ρ and increases in ρ, as well.

The voting strategy v_{h^+} is merely a rotation of v_{h^-} by the amount $\frac{\pi}{2}$ (i.e. $h^+ = R_{\frac{\pi}{2}}h_-$); accordingly, consider the (non-equilibrium) strategy combination $\left(v_{h^+}, R_{\frac{\pi}{2}}x_A^-, R_{\frac{\pi}{2}}x_B^-\right)$ in which candidate behavior is rotated by the same amount. This strategy combination specifies the same behavior for voters and candidates as $\left(v_{h^-}, x_A^-, x_B^-\right)$, but in different states of the world; recognizing this, the latter can be seen to generate the same welfare as the former, but with a negative correlation coefficient.

$$\begin{aligned} W\left(x_A^-, x_B^-, v_{h^-};\rho\right) &= \int_Z \sum_{j=A,B} u\left(x_j^-, z\right)\Pr\left(w=j|z; v_{h^-}\right) f(z;\rho)\,dz \\ &= \int_Z \sum_{j=A,B} u\left(R_{\frac{\pi}{2}}x_j^-, R_{\frac{\pi}{2}}z\right)\Pr\left(w=j|R_{\frac{\pi}{2}}z; v_{R_{\frac{\pi}{2}}h^-}\right) f\left(R_{\frac{\pi}{2}}z;\rho\right)dz \\ &= \int_Z \sum_{j=A,B} u\left(R_{\frac{\pi}{2}}x_j^-, z\right)\Pr\left(w=j|z;h^+\right) f(z;-\rho)\,dz \\ &= W\left(R_{\frac{\pi}{2}}x_A^-, R_{\frac{\pi}{2}}x_B^-, v_{h^+};-\rho\right) \end{aligned}$$

Since the platforms $R_{\frac{\pi}{2}}x_A^-$ and $R_{\frac{\pi}{2}}x_B^-$ are symmetric and lie on the major diagonal, the preliminary arguments above apply here, guaranteeing that the welfare $W\left(R_{\frac{\pi}{2}}x_A^-, R_{\frac{\pi}{2}}x_B^-, v_{h^+};\rho\right)$ associated with the major equilibrium voting strategy exceeds the welfare $W\left(x_A^-, x_B^-, v_{h^-};\rho\right)$ associated with the minor equilibrium voting strategy.

Moving from $\left(R_{\frac{\pi}{2}}x_A^-, R_{\frac{\pi}{2}}x_B^-, v_{h^+}\right)$ to $\left(x_A^+, R_{\frac{\pi}{2}}x_B^-, v_{h^+}\right)$ and then to $\left(x_A^+, x_B^+, v_{h^+}\right)$ improves welfare a second and third time, as the two candidates optimize in turn. In fact, since it is optimal for candidates to position themselves as close as possible to $E(z|w=j)$ (by Lemma 3), $W_+\left[(x,x);\rho\right]$ increases continuously in x between $(x,x) = R_{\frac{\pi}{2}}x_B^-$ and $(x,x) = x_B^+$. Moreover, the welfare gain increases in ρ. To see this, rewrite (21) in terms of x (and in Cartesian coordinates), as follows.

$$W_+(x;\rho) = 4\int_{(z_1,z_2)\in Z_2}\left\{\begin{array}{c} \left[-((-x)-z_1)^2 - ((-x)-z_2)^2\right]\Pr(w=A|z_1,z_2)\,f(z_1,z_2;\rho) \\ +\left[-((-x)-(-z_1))^2 - ((-x)-z_2)^2\right]\Pr(w=A|-z_1,z_2)\,f(-z_1,z_2;\rho) \\ +\left[-(x-z_1)^2 - (x-z_2)^2\right]\Pr(w=B|z_1,z_2)\,f(z_1,z_2;\rho) \\ +\left[-(x-(-z_1))^2 - (x-z_2)^2\right]\Pr(w=B|-z_1,z_2)\,f(-z_1,z_2;\rho) \end{array}\right\}dz_1dz_2$$

Substituting $f(-z_1, z_2) = f(z_1, z_2; -\rho)$ and differentiating with respect to x and ρ then yields the following, taking into account that $\Pr(w = A|z_1, z_2) = 1 - \Pr(w = B|z_1, z_2)$.

$$\begin{aligned}\frac{\partial^2 W_+(x;\rho)}{\partial x \partial \rho} &= 4\int_{(z_1,z_2)\in Z_2} \left\{ \begin{array}{c} [2((-x)-z_1) + 2((-x)-z_2)]\Pr(w=A|z_1,z_2) \\ -[2((-x)-(-z_1)) + 2((-x)-z_2)]\Pr(w=A|-z_1,z_2) \\ +[-2(x-z_1) - 2(x-z_2)]\Pr(w=B|z_1,z_2) \\ -[-2(x-(-z_1)) - 2(x-z_2)]\Pr(w=B|-z_1,z_2) \end{array} \right\} \frac{\partial}{\partial\rho} f(z_1,z_2)\, dz_1 dz_2 \\ &= 16\int_{(z_1,z_2)\in Z_2} \left\{ \begin{array}{c} -z_1 + (z_1+z_2)\Pr(w=B|z_1,z_2) \\ +(z_1-z_2)\Pr(w=B|-z_1,z_2) \end{array} \right\} \frac{\partial}{\partial\rho} f(z_1,z_2)\, dz_1 dz_2\end{aligned}$$

This is unambiguously positive, since z_1 and z_2 are positive in Z_2, and this implies that $\frac{\partial}{\partial\rho} f(z_1, z_2) > 0$, by Condition 1, and that $\Pr(w = B|z_1, z_2) > \Pr(w = B|-z_1, z_2) > \frac{1}{2}$, by Lemma 2. Increasing (x, x) from $R_{\frac{\pi}{2}} x_B^-$ to x_B^+ therefore increases welfare by a greater amount when ρ is higher. The analysis above also implies that a higher ρ translates into a lower $R_{\frac{\pi}{2}} x_B^-$ and higher x_B^+, so the welfare improvement of moving from $R_{\frac{\pi}{2}} x_B^-$ to x_B^+ is greater still. ■

Proof of Proposition 5. Regardless of λ, Lemma 3 remains valid: the best response for each candidate is to implement her expectation $x_j^{br} = E(z|w = j)$ of the optimal policy, conditional on winning. Lemma 2 remains valid as well: if voters follow a half-space voting strategy then expected vote shares are monotonic in the direction of h, candidates' best response platforms $x_A^{br} = -x_B^{br}$ are symmetric around the origin, and the best voting response is another half-space strategy. The last of these claims can be shown by logic analogous to the proof of Lemma 2: by the same logic as before, the difference in expected utility between voting B and voting A now becomes the following,

$$\begin{aligned}\Delta E_{w,z}[u(x_w)|s] &= \int_Z \left\{ \begin{array}{c} \left[-(1+\lambda)(x_{B1}-z_1)^2 - (1-\lambda)(x_{B2}-z_2)^2\right] \\ -\left[-(1+\lambda)(-x_{B1}-z_1)^2 - (1-\lambda)(-x_{B2}-z_2)^2\right] \end{array} \right\} \Pr(P|z) f(z|s)\, dz \\ &= 4\int_Z [(1+\lambda)x_{B1}z_1 + (1-\lambda)x_{B2}z_2] \left[1 + \frac{g_1}{g_0}(s_1 z_1 + s_2 z_2)\right] f(z)\, dz\end{aligned}$$

which is still linear in s and still equals zero for $(s_1, s_2) = (0, 0)$, implying that the best response voting strategy is a half-space strategy.

From (7), the benefit of voting B has the same sign as $(1+\lambda)x_{B1}E(z_1|P,s) + (1-\lambda)x_{B2}E(z_2|P,s) = E(z|P,s) \cdot \begin{pmatrix} (1+\lambda)x_{B1} \\ (1-\lambda)x_{B2} \end{pmatrix}$. The proof of Proposition 1 shows that if $\rho = 0$ then a voter with $\theta_s = \theta_h + \frac{\pi}{2}$ is indifferent between voting A and B. That is, $E(z|P,s) \cdot \begin{pmatrix} x_{B1} \\ x_{B2} \end{pmatrix} = 0$. Such a voter is no longer indifferent when $\lambda = 0$, unless $x_{B1} = 0$ or $x_{B2} = 0$. That proof shows further that x_B^{br} has the same polar angle as h, however, so equilibrium requires $h_1 = 0$ or $h_2 = 0$, meaning that $\theta_h \in \left\{0, \frac{\pi}{2}\right\}$. ■

Lemma A2 is a technical result that is useful in establishing Proposition 6.

Lemma A2 *Let voters follow a half-space strategy with normal vector h and let x_B^{br} denote the best response platform for candidate B. If $\theta_h \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ then $\theta_{x_B^{br}} \in \left[\theta_h, \frac{\pi}{4}\right]$. If $\theta_h \in \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]$ then $\theta_{x_B^{br}} \in \left[-\frac{3\pi}{4}, \theta_h\right]$. If $\theta_h \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ then $\theta_{x_B^{br}} \in \left[\frac{\pi}{4}, \theta_h\right]$.*

Proof. Write the difference between x_{B1}^{br} and x_{B2}^{br} as follows.

$$\begin{aligned}x_{B1}^{br} - x_{B2}^{br} &= E(z_1|w=B) - E(z_2|w=B) \\ &= \int_Z (z_1 - z_2)\frac{\Pr(w=B|z)}{\Pr(w=B)} f(z)\, dz\end{aligned}$$

Noting that $\Pr(w=B)=\frac{1}{2}$ and expressing all eight octants in terms of the first octant Z_1, this reduces to the following,

$$\begin{aligned}
& 2\int_{Z_1}\left[\begin{array}{c}
(z_1-z_2)\Pr(w=B|z_1,z_2)\,f(z_1,z_2)+(-z_1-z_2)\Pr(w=B|-z_1,z_2)\,f(-z_1,z_2)\\
+(z_1+z_2)\Pr(w=B|z_1,-z_2)\,f(z_1,-z_2)+(-z_1+z_2)\Pr(w=B|-z_1,-z_2)\,f(-z_1,-z_2)\\
+(z_2-z_1)\Pr(w=B|z_2,z_1)\,f(z_2,z_1)+(-z_2-z_1)\Pr(w=B|-z_2,z_1)\,f(-z_2,z_1)\\
+(z_2+z_1)\Pr(w=B|z_2,-z_1)\,f(z_2,-z_1)+(-z_2+z_1)\Pr(w=B|-z_2,-z_1)\,f(-z_2,-z_1)
\end{array}\right]dz\\
= \;& 2\int_{Z_1}\left[\begin{array}{c}
(z_1-z_2)\Pr(w=B|z_1,z_2)\,f(z_1,z_2)-(z_1+z_2)\Pr(w=B|-z_1,z_2)\,f(z_1,z_2;-\rho)\\
+(z_1+z_2)\Pr(w=B|z_1,-z_2)\,f(z_1,z_2;-\rho)-(z_1-z_2)\Pr(w=B|-z_1,-z_2)\,f(z_1,z_2)\\
-(z_1-z_2)\Pr(w=B|z_2,z_1)\,f(z_1,z_2)-(z_1+z_2)\Pr(w=B|-z_2,z_1)\,f(z_1,z_2;-\rho)\\
+(z_1+z_2)\Pr(w=B|z_2,-z_1)\,f(z_1,z_2;-\rho)+(z_1-z_2)\Pr(w=B|-z_2,-z_1)\,f(z_1,z_2)
\end{array}\right]dz\\
= \;& 2\int_{Z_1}\left\{\begin{array}{c}
(z_1-z_2)\left[\begin{array}{c}\Pr(w=B|z_1,z_2)-\Pr(w=B|-z_1,-z_2)\\ -\Pr(w=B|z_2,z_1)+\Pr(w=B|-z_2,-z_1)\end{array}\right]f(z_1,z_2)\\
+(z_1+z_2)\left[\begin{array}{c}-\Pr(w=B|-z_1,z_2)+\Pr(w=B|z_1,-z_2)\\ -\Pr(w=B|-z_2,z_1)+\Pr(w=B|z_2,-z_1)\end{array}\right]f(z_1,z_2;-\rho)
\end{array}\right\}dz\\
= \;& 4\int_{Z_1}\left\{\begin{array}{c}
(z_1-z_2)\left[\Pr(w=B|z_1,z_2)-\Pr(w=B|z_2,z_1)\right]f(z_1,z_2)\\
+(z_1+z_2)\left[\Pr(w=B|z_1,-z_2)-\Pr(w=B|-z_2,z_1)\right]f(z_1,z_2;-\rho)
\end{array}\right\}dz
\end{aligned}$$

where the first equality utilizes Condition 2 and the final equality acknowledges that, by Lemma 2, $\Pr(w=B|-z)=\Pr(w=A|z)=1-\Pr(w=B|z)$. Invoking Lemma 2 a second time, the two differences in brackets have the same signs as $h\cdot(z_1,z_2)-h\cdot(z_2,z_1)=(h_1-h_2)(z_1-z_2)$ and $h\cdot(z_1,-z_2)-h\cdot(-z_2,z_1)=(h_1-h_2)(z_1+z_2)$, respectively, which means that the entire expression has the same sign as h_1-h_2 (since z_1-z_2 and z_1+z_2 are both positive in Z_1). In other words, $x_B^{br}\cdot\binom{1}{-1}$ and $x_B^{br}\cdot\binom{1}{1}$ have the same signs as $h\cdot\binom{1}{-1}$ and $h\cdot\binom{1}{1}$, respectively, so θ_h and $\theta_{x_B^{br}}$ both belong to the same quadrant: either $\left[-\frac{3\pi}{4},-\frac{\pi}{4}\right]$, $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$, $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$, or $\left[\frac{3\pi}{4},\frac{5\pi}{4}\right]$. However, the proof of Theorem 2 shows that x_B^{br} differs from h in the direction of the major diagonal. That is, $\theta_{x_B^{br}}<\theta_h$ if and only if $\frac{\pi}{4}<|\theta_h|<\frac{3\pi}{4}$. ■

Proof of Proposition 6. As in the proof of Proposition 5, the best response for each candidate is to implement her expectation $x_j^{br}=E(z|w=j)$ of the optimal policy, conditional on winning, and if voters follow a half-space voting strategy then expected vote shares are monotonic in z in the direction of h, candidates' best response platforms $x_A^{br}=-x_B^{br}$ are symmetric around the origin, and the best-response voting strategy is another half-space strategy, in the direction h^{br}. Also, the benefit of voting B instead of voting A has the same sign as $E(z|P,s)\cdot\tilde{x}_B$, where $\tilde{x}_B=\begin{pmatrix}(1+\lambda)\,x_{B1}\\(1-\lambda)\,x_{B2}\end{pmatrix}$.

If h has polar angle $\theta_h=0$ then the proof of Theorem 2 shows that $x_B^{br}(v_h)$ has polar angle $\theta_{x_B^{br}}\in\left(0,\frac{\pi}{4}\right)$, which implies that $x_{B1}^{br}>x_{B2}^{br}>0$, and therefore that $(1+\lambda)\,x_{B1}^{br}>(1-\lambda)\,x_{B2}^{br}>0$, or $\tilde{x}_{B1}^{br}>\tilde{x}_{B2}^{br}>0$. Thus, $\tilde{x}_B^{br}(v_h)$ has polar angle $\theta_{\tilde{x}_B^{br}}\in\left(0,\frac{\pi}{4}\right)$. For the same voting strategy, the proof of Theorem 1 shows that if x has polar angle $\theta_x=0$ and a citizen's signal realization has polar angle $\theta_s=\frac{\pi}{2}$ orthogonal to h then $E(z|P,s)\cdot x=0$. For the same signal realization, then,$\theta_{\tilde{x}_B^{br}}>0$ implies that $E(z|P,s)\cdot\tilde{x}_B^{br}>0$. In other words, a voter whose signal is orthogonal to h prefers voting B, and a voter who is indifferent between voting A and B has a signal with polar angle $\theta_s>\frac{\pi}{2}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}}>0$.

If h has polar angle $\theta_h=\frac{\pi}{4}$ then the proof of Theorem 2 shows that $x_B^{br}(v_h)$ has polar angle $\theta_{x_B^{br}}=\frac{\pi}{4}$, which implies that $x_{B1}^{br}=x_{B2}^{br}>0$, and therefore that $(1+\lambda)\,x_{B1}^{br}>(1-\lambda)\,x_{B2}^{br}>0$, or $\tilde{x}_{B1}^{br}>\tilde{x}_{B2}^{br}>0$. Thus, $\tilde{x}_B^{br}(v_h)$ again has polar angle $\theta_{\tilde{x}_B^{br}}\in\left(0,\frac{\pi}{4}\right)$. For the same voting strategy, the proof of Theorem 1 shows that if x has polar angle $\theta_x=\frac{\pi}{4}$ and a citizen's signal realization has polar angle $\theta_s=\frac{3\pi}{4}$ orthogonal to h then $E(z|P,s)\cdot x=0$. For the same signal realization, then, $\theta_{\tilde{x}_B^{br}}<\frac{\pi}{4}$ implies that $E(z|P,s)\cdot\tilde{x}_B^{br}<0$. In other words, a voter whose signal is orthogonal to h prefers voting A, and a voter who is indifferent between voting A and B has a signal with polar angle $\theta_s<\frac{3\pi}{4}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}}<\frac{\pi}{4}$. Since $\theta_{h^{br}(v_h)}$ is a continuous function of θ_h, the results that $\theta_{h^{br}(v_h)}>\theta_h$ for $\theta_h=0$ and $\theta_{h^{br}(v_h)}<\theta_h$ for $\theta_h=\frac{\pi}{4}$ together imply (by the intermediate value theorem) the existence of $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ such that $\theta_{h^{br}\left(v_{h^+}\right)} = \theta_{h^+}$, implying that v_{h^+} (together with the best response candidate platforms) constitutes its own best response, and therefore a half-space equilibrium of the voting game.

If h has polar angle $\theta_h = -\frac{\pi}{4}$ then the proof of Theorem 2 shows that $x_B^{br}(v_h)$ has polar angle $\theta_{x_B^{br}} = -\frac{\pi}{4}$, which implies that $x_{B1}^{br} > 0 > x_{B2}^{br}$ and $\left|x_{B1}^{br}\right| = \left|x_{B2}^{br}\right|$, and therefore that $(1+\lambda)\, x_{B1}^{br} > 0 > (1-\lambda)\, x_{B2}^{br}$ and $\left|(1+\lambda)\, x_{B1}^{br}\right| = \left|(1-\lambda)\, x_{B2}^{br}\right|$, or $\tilde{x}_{B1}^{br} > 0 > \tilde{x}_{B2}^{br}$ with $\left|\tilde{x}_{B1}^{br}\right| > \left|\tilde{x}_{B2}^{br}\right|$. Thus, $\tilde{x}_B^{br}(v_h)$ has polar angle $\theta_{\tilde{x}_B^{br}} \in \left(-\frac{\pi}{4}, 0\right)$. For the same voting strategy, the proof of Theorem 1 shows that if x has polar angle $\theta_x = -\frac{\pi}{4}$ and a citizen's signal realization has polar angle $\theta_s = \frac{\pi}{4}$ orthogonal to h then $E(z|P,s) \cdot x = 0$. For the same signal realization, then, $\theta_{\tilde{x}_{br}} > -\frac{\pi}{4}$ implies that $E(z|P,s) \cdot \tilde{x}_B^{br} > 0$. In other words, a voter whose signal is orthogonal to h prefers voting B, and a voter who is indifferent between voting A and B has a signal with polar angle $\theta_s > \frac{\pi}{4}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}} > -\frac{\pi}{4}$.

If h has polar angle $\theta_h = -\frac{\pi}{2}$ then the proof of Theorem 2 shows that $x_B^{br}(v_h)$ has polar angle $\theta_{x_B^{br}} \in \left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right)$, which implies that $x_{B1}^{br} < 0$ and $x_{B2}^{br} < 0$, and therefore that $(1+\lambda)\, x_{B1}^{br} < 0$ and $(1-\lambda)\, x_{B2}^{br} < 0$, or $\tilde{x}_{B1}^{br} < 0$ and $\tilde{x}_{B2}^{br} < 0$. Thus, $\tilde{x}_B^{br}(v_h)$ has polar angle $\theta_{\tilde{x}_B^{br}} < -\frac{\pi}{2}$. For the same voting strategy, the proof of Theorem 1 shows that if x has polar angle $\theta_x = -\frac{\pi}{4}$ and a citizen's signal realization has polar angle $\theta_s = 0$ orthogonal to h then $E(z|P,s) \cdot x = 0$. For the same signal realization, then, $\theta_{\tilde{x}_{br}} < -\frac{\pi}{4}$ implies that $E(z|P,s) \cdot \tilde{x}_B^{br} < 0$. In other words, a voter whose signal is orthogonal to h prefers voting A, and a voter who is indifferent between voting A and B has a signal with polar angle $\theta_s < 0$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}} < -\frac{\pi}{2}$. Since $\theta_{h^{br}(v_h)}$ is a continuous function of θ_h, the results that $\theta_{h^{br}(v_h)} > \theta_h$ for $\theta_h = 0$ and $\theta_{h^{br}(v_h)} < \theta_h$ for $\theta_h = \frac{\pi}{4}$ together imply (by the intermediate value theorem) the existence of $\theta_{h^-} \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ such that $\theta_{h^{br}\left(v_{h^-}\right)} = \theta_{h^-}$, implying that v_{h^-} (together with the best response candidate platforms) constitutes its own best response, and therefore a half-space equilibrium of the voting game. ■

References

[1] Bafumi, Joseph, and Michael C. Herron. 2010. "Leapfrog Representation and Extremism: A Study of American Voters and Their Members in Congress," *American Political Science Review*, 104(3): 519-542.

[2] Barelli, Paulo, Sourav Bhattacharya, and Lucas Siga. 2017. "On the Possibility of Information Aggregation in Large Elections." Working paper, University of Rochester, Royal Holloway University of London, and New York University.

[3] Bergstrom, Theodore C. and Robert P. Goodman. 1973. "Private Demands for Public Goods." *American Economic Review*, 63(3): 280-296.

[4] Besley, Timothy, and Stephen Coate. 1997. "An Economic Model of Representative Democracy", *Quarterly Journal of Economics*, 112(1): 85-114.

[5] Besley, Timothy, and Stephen Coate. 2008. "Issue Unbundling via Citizens' Initiatives", *Quarterly Journal of Political Science*, 3: 379-397.

[6] Bhattacharya, Sourav. 2012. "Preference Monotonicity and Information Aggregation in Elections." *Econometrica*, forthcoming.

[7] Brunner, Eric, Stephen L. Ross, and Ebony Washington. 2011. "Economics and Policy Preferences: Causal Evidence of the Impact of Economic Conditions on Support for Redistribution and Other Ballot Proposals." *The Review of Economics and Statistics*, 93(3): 888-906.

[8] Condorcet, Marquis de. 1785. *Essay on the Application of Analysis to the Probability of Majority Decisions.* Paris: De l'imprimerie royale. Trans. Iain McLean and Fiona Hewitt. 1994.

[9] Converse, Philip E. 1964. "The Nature of Belief Systems in Mass Publics," in *Ideology and Discontent*, ed. David E. Apter. New York: Free Press.

[10] DeMarzo, Peter M., Dimitri Vayanos, and Jeffrey Zwiebel. 2003. "Persuasion Bias, Social Influence, and Unidimensional Opinions." *Quarterly Journal of Economics*, 118(3): 909-968.

[11] Downs, Anthony. 1957. *An Economic Theory of Democracy.* New York: Harper and Row.

[12] Duggan, John and Mark Fey. 2005. "Electoral Competition with Policy-motivated Candidates." *Games and Economic Behavior*, 51: 490-522.

[13] Duggan, John and Matthew O. Jackson. 2005. "Mixed Strategy Equilibrium and Deep Covering in Multidimensional Electoral Competition." Working paper, University of Rochester.

[14] Duggan, John and Cesar Martinelli. 2011. "A Spatial Theory of Media Slant and Voter Choice." *Review of Economic Studies*, 78: 640-666.

[15] Duverger, Maurice. 1954. *Political Parties: Their Organization and Activity in the Modern State*. New York: Wiley. Translated by Barbara and Robert North.

[16] Egorov, Georgy. 2014. "Single-Issue Campaigns and Multidimensional Politics." Working paper, Northwestern University.

[17] Esponda, Ignacio and Emanuel Vespa. 2014. "Hypothetical Thinking and Information Extraction in the Laboratory." *American Economic Journal: Microeconomics*, 6(4): 180-202.

[18] Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *The American Economic Review*, 86(3): 408-424.

[19] Feddersen, Timothy J. and Wolfgang Pesendorfer. 1997. "Voting Behavior and Information Aggregation in Elections with Private Information." *Econometrica*, 65(5): 1029-1058.

[20] Fowler, Anthony and Andrew B. Hall. 2013. "Conservative Vote Probabilities: An Easier Method for Summarizing Roll Call Data." Working paper, University of Chicago and Harvard University.

[21] Grofman, Bernard and Timothy J. Brazill. 2002. "Identifying the Median Justice on the Supreme Court through Multidimensional Scaling: Analysis of 'Natural Courts'." *Public Choice*, 112: 55-79.

[22] Harrington, Joseph E., Jr. 1993. "Economic Policy, Economic Performance, and Elections." *The American Economic Review*, 83(1): 27-42.

[23] Hinich, Melvin J. 1977. "Equilibrium in Spatial Voting: The Median Voter Result is an Artifact", *Journal of Economic Theory*, 16(2): 208-219.

[24] Hinich, Melvin J. 1978. "The Mean Versus the Median in Spatial Voting Games," in P. Ordeshook, ed., *Game Theory and Political Science*, New York: NYU Press.

[25] Hotelling, Harold. 1929. "Stability in Competition." *Economic Journal*, 39(153): 41-57.

[26] Jadbabaie, Ali, Pooy Molavi, Alvaro Sandroni, and Alireza Tahbaz-Salehi. 2012. "Non-Bayesian Social Learning." *Games and Economic Behavior*, 76: 210-225.

[27] Krasa, Stefan, and Mattias Polborn. 2014. "Policy Divergence and Voter Polarization in a Structural Model of Elections." *Journal of Law and Economics*, 57: 31-76.

[28] Layman, Geoffrey C., Thomas M. Carsey, and Juliana Menasce Horowitz. 2006. "Party Polarization in American Politics: Characteristics, Causes, and Consequences." *Annual Review of Political Science*, 9: 83-110.

[29] Lindbeck, Assar and Jörgen W. Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice*, 52(3): 273-297.

[30] Lindbeck, Assar and Jörgen W. Weibull. 1993. "A Model of Political Equilibrium in a Representative Democracy." *Journal of Public Economics*, 51: 195-209.

[31] Louis, Philippos, Orestis Troumpounis, and Nikolas Tsakas. 2016. "Communication and the Emergence of a Unidimensional World." Working paper, University of New South Wales, Lancaster University, and University of Cyprus.

[32] McDonald, Michael D., Silvia M. Mendes, and Myunghee Kim. 2007. "Cross-temporal and Cross-national Comparisons of Party Left-right Positions." *Electoral Studies*, 26: 62-75.

[33] McKelvey, Richard D. 1979. "General Conditions for Global Intransitivities in Formal Voting Models." *Econometrica*, 47(5): 1085-1112.

[34] McLennan, Andrew. 1998. "Consequences of the Condorcet Jury theorem for Beneficial Information Aggregation by Rational Agents." *American Political Science Review*, 92(2): 413-418.

[35] McMurray, Joseph C. 2013. "Aggregating Information by Voting: The Wisdom of the Experts versus the Wisdom of the Masses." *The Review of Economic Studies*, 80(1): 277-312.

[36] McMurray, Joseph C. 2017a. "Ideology as Opinion: A Spatial Model of Common-value Elections." *American Economic Journal: Microeconomics*, 9(4): 108-140.

[37] McMurray, Joseph C. 2017b. "Voting as Communicating: Mandates, Minor Parties, and the Signaling Voter's Curse." *Games and Economic Behavior*, 102: 199-223.

[38] McMurray, Joseph C. 2018. "Polarization and Pandering in Common Interest Elections." Working paper, Brigham Young University.

[39] Meltzer, Allan H. and Scott F. Richard. "A Rational Theory of the Size of Government." *Journal of Political Economy*, 89(5): 914-927.

[40] Mueller, Dennis C. *Public Choice III*, New York: Cambridge University Press.

[41] Myerson, Roger. 1998. "Population Uncertainty and Poisson Games." *International Journal of Game Theory*, 27: 375-392.

[42] Myerson, Roger. 2000. "Large Poisson Games." *Journal of Economic Theory*, 94: 7-45.

[43] Osborne, Martin J., and Al Slivinski. 1996. "A Model of Political Competition with voter-Candidates." *Quarterly Journal of Economics*, 111(1): 65-96.

[44] Pan, Jennifer and Yiqing Xu. 2017. "China's Ideological Spectrum." *Journal of Politics*, 80(1): 254-273.

[45] Plott, Charles R. 1967. "A Notion of Equilibrium and its Possibility Under Majority Rule." *American Economic Review*, 57(4): 787-806.

[46] Poole, Keith T. and Howard Rosenthal. 1997. *Congress: A Political-Economic History of Roll-Call Voting.* New York: Oxford University Press.

[47] Poole, Keith T. and Howard Rosenthal. 2001. "D-Nominate after 10 Years: A Comparative Update to Congress: A Political-Economic History of Roll-Call Voting." *Legislative Studies Quarterly*, 26(1): 5-29.

[48] Romer, Thomas. 1975. "Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax." *Journal of Public Economics*, 4: 163-185.

[49] Schelling, Thomas C. 1960. *The Strategy of Conflict*. Cambridge: Harvard University Press.

[50] Shor, Boris. 2014. "Congruence, Responsiveness, and Representation in American State Legislatures." Working paper, Georgetown University.

[51] Shor, Boris and Nolan McCarty. 2011. "The Ideological Mapping of American Legislatures," *American Political Science Review*, 105(3): 530-551.

[52] Spector, David. 2000. "Rational Debate and One-Dimensional Conflict," *Quarterly Journal of Economics*, 115(1): 181-200.

[53] Tausanovitch and Warshaw. 2014. "Representation in Municipal Government." *American Political Science Review*, 108(3): 605-641.

[54] Tullock, Gordon. 1981. "Why So Much Stability." *Public Choice*, 37(2): 189-204.

[55] Wittman, Donald. 1983. "Candidate Motivation: A Synthesis of Alternative Theories." *American Political Science Review*, 77(1): 142-157.

[56] Xefteris, Dimitrios. 2017. "Multidimensional Electoral Competition between Differentiated Candidates." *Games and Economic Behavior*, 105: 112-121.