

Bail-ins and Bail-outs: Incentives, Connectivity, and Systemic Stability

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We develop a framework for analyzing how banks can be incentivized to make contributions to a voluntary bail-in and ascertaining the kinds of inter-bank linkages that are most conducive to a bail-in. A bail-in is possible only when the regulator's threat to not bail out insolvent banks is credible. Incentives to join a rescue consortium are stronger in networks where banks have a high exposure to default contagion, and weaker if banks realize that a large fraction of the benefits resulting from their contributions accrue to others. Our results reverse existing presumptions about the relative merits of different network topologies for moderately large shock sizes: while diversification effects reduce welfare losses in models without intervention, they inhibit the formation of bail-ins by introducing incentives for free-riding. We provide a nuanced understanding of why certain network structures are preferable, identifying the impact of the network structure on the credibility of bail-in proposals.

Financial institutions are linked to each other via bilateral contractual obligations and are thus exposed to counterparty risk of their obligors. If one institution is in distress, it will default on its agreements, thereby affecting the solvency of its creditors. Since the creditors are also borrowers, they may not be able to repay what they owe and default themselves—problems in one financial institution spread to others in what is called financial contagion. Large shocks can trigger a cascade of defaults with potentially devastating effects for the economy. The government is thus forced to intervene in some way and stop the cascade to reduce the

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negative externalities imposed on the economy. The extent of these cascades—the magnitude of the systemic risk—depends on the nature of the linkages, i.e., the topology of the financial system. In the 2008 crisis, it became apparent that the financial system had evolved in a way which enhanced its ability to absorb small shocks but made it more fragile in the face of a large shock. While a few studies called attention to these issues before the crisis, it was only after the crisis that the impact of the network structure on systemic risk became a major object of analysis.¹ Most of the existing studies analyze the systemic risk implications of a default cascade, taking into account the network topology, asset liquidation costs, and different forms of inefficiencies that arise at default. Many of these models, however, do not account for the possibility of intervention to stop the cascade. There is either no rescue of insolvent banks or the regulator (or central bank or other government institution) intervenes by following an exogenously specified protocol. The goal of our paper is to endogenize the intervention mechanism as the outcome of the strategic interaction between regulator and financial institutions.

The most common default resolution procedure is the *bailout*, in which the government injects liquidity to help distressed banks servicing their debt. For example, during the global financial crisis, capital was injected into banks to prevent fire sale losses, including the intervention of the Bank of England and the U.S. Treasury Department’s Asset Relief Program (TARP); see also Duffie (2010) for a related discussion.² Since the East Asia crisis, critics of bailouts have suggested *bail-ins* as an alternative, which are financed through voluntary contributions by the banks within the network.³ Bail-ins allow creditor banks to participate in losses by effectively swapping their non-performing interbank claims for equity in the distressed banks. Creditors save some of their investment and the distressed banks remain solvent, with the burden of losses placed on creditors as opposed to taxpayers. A prominent example of a bail-in is the consortium organized by the Federal Reserve

¹Most notably, Allen and Gale (2000) and Greenwald and Stiglitz (2003). See also Boissay (2006), Castiglionesi (2007), May, Levin and Sugihara (2008), and Nier et al. (2007). One of the reasons for the limited study is the scarce availability of data on interbank linkages. An early construction of Japan’s interbank network, done before the crisis but published afterwards, is De Masi et al. (2011). With the exception of Haldane at the Bank of England, remarkably, central bankers paid little attention to the interplay of systemic risk and network topology; see Haldane (2009).

²The Bush administration bailed out large financial institutions (AIG insurance, Bank of America and Citigroup) and government sponsored entities (Fannie Mae, Freddie Mac) at the heart of the crisis. The European Commission intervened to bail out financial institutions in Greece and Spain. It is widely believed that the AIG bailout was an indirect bailout of Goldman Sachs.

³Several attempted bail-ins failed, simply because the threat of not undertaking a bail-out was not credible. See Stiglitz (2002). In the aftermath of these failures, there have been proposals for bonds that would automatically convert into equity in the event of a crisis.

Bank of New York to rescue the hedge fund Long-Term Capital Management.⁴ As a third default resolution procedure we consider *assisted bail-ins*, where the regulator provides some liquidity assistance to incentivize the formation of bail-in consortia. Such a strategy strikes a balance between the contributions of creditors and taxpayers. Assisted bail-ins have been used in the recent financial crisis.⁵

We investigate the structure of default resolution plans that arise in equilibrium, when the regulator cannot credibly commit to an ex-post suboptimal resolution policy. Moral hazard may prevent constrained efficient bail-ins from being implemented: if creditor banks anticipate a bailout by the government, they have no incentive to participate in a bail-in. Therefore, the government’s ability to engineer a bail-in crucially depends on the credibility of its no-bailout threat. We show how the network structure affects the regulator’s negotiation power and the banks’ incentives to participate in a rescue.

We model the provision of liquidity assistance as a sequential game between regulator and banks that consists of three stages. At the beginning of the game, banks have already observed the realization of asset returns. In the first stage, the regulator proposes an assisted bail-in allocation policy, specifying the contributions by each solvent bank, as well as the additional liquidity injections (subsidies) that he will provide to each bank. In the second stage, each bank decides whether or not to accept the regulator’s proposal. If all banks accept, the game ends with the proposed rescue consortium and financial contagion is stopped; otherwise we move to the third stage where the regulator has three options: (i) contribute the amount that was supposed to be covered by the banks which rejected the proposal, (ii) abandon the bail-in coordination and resort to a public bailout, or (iii) avoid any rescue and let the default cascade occur. After transfers are made, liabilities of banks are cleared simultaneously in the spirit of Eisenberg and Noe (2001). Unlike their paper, however, but as in Rogers and Veraart (2013) and Battiston et al.

⁴Long-Term Capital Portfolio collapsed in the late 1990s. On September 23, 1998, a recapitalization plan of \$3.6 billion was coordinated under the supervision of the Federal Reserve Bank of New York. A total of sixteen banks, including Bankers Trust, Barclays, Chase, Credit Suisse, Deutsche Bank, Goldman Sachs, Merrill Lynch, Morgan Stanley, Salomon Smith Barney, UBS, Société Générale, Paribas, Crédit Agricole, Bear Stearns, and Lehman Brothers originally agreed to participate. However, Bear Stearns and Lehman Brothers later declined to participate and their agreed-upon contributions was instead provided by the remaining fourteen banks.

⁵A noticeable example of an assisted bail-in is Bear Stearns. JPMorgan Chase and the New York Federal Reserve stepped in with an emergency cash bailout in March, 2008. The provision of liquidity by the Federal Reserve was taken to avoid a potential fire sale of nearly U.S. \$210 billion of Bear Stearns’ assets. The Chairman of the Fed, Ben Bernanke, defended the bail-in by stating that Bear Stearns’ bankruptcy would have affected the economy, causing a “chaotic unwinding” of investments across the U.S. markets and a further devaluation of other securities across the banking system.

(2016), bankruptcies are costly. When there are no bankruptcy costs, the system is “conservative” and the clearing of liabilities simply reduces to a redistribution of wealth in the network. In the presence of bankruptcy costs, there are real losses that propagate through the financial system if banks cannot fully honor their liabilities. These losses are amplified through self-reinforcing feedback loops among defaulting banks and the size of the amplification depends crucially on the network structure.

The regulator’s option of standing idly by in the last stage is what we call the regulator’s *threat of no intervention*. The threat, however, may not be credible if walking away from the proposal decreases the regulator’s welfare function. In that case, the regulator cannot incentivize any bank to participate in a bail-in consortium because all banks are aware that without their participation, the regulator will resort to a public bailout. If the threat is credible, a bail-in can be organized. Our first result characterizes the assisted bail-in that arises as the generically unique equilibrium outcome. Our subsequent results explore properties of the equilibrium outcome. We show that the credibility of the regulator’s threat is tightly linked to the topology of the financial system. The threat is credible if and only if the amplification of the shock through the network does not exceed a certain threshold. This means that for moderately large shock sizes, the no-intervention threat may not be credible in networks where interbank liabilities are well diversified because a high degree of interconnectedness between defaulting banks increases the amplification of the shock through the network. High amplification leads to large social losses, which make the threat non-credible and leave a public bailout as the only possible rescue option. If, by contrast, the shock’s amplification is small as in the case of concentrated networks, where financial contagion can be quarantined to a specific area, the regulator’s threat is credible. This permits constrained efficient assisted bail-ins to be coordinated for shock sizes and recovery rates where more dense networks would require a more costly bailout.

We demonstrate that a bank is willing to contribute a larger amount to a bail-in if the losses in absence of intervention are more concentrated. This loosening of the banks’ participation constraints allows the government to shift the burden of losses away from taxpayers towards creditors of failing banks. The intuition underlying this result is that more concentrated networks allow bail-in plans with benefits more targeted to the bail-in contributors. This implies the existence of a *network multiplier*: the total contribution from banks required to stop the initial shortfall caused by fundamentally defaulting banks is lower than the aggregate losses

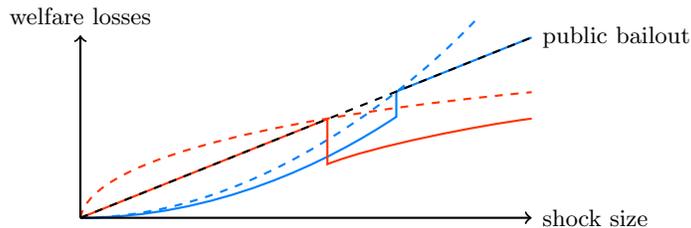


Figure 1: The figure compares equilibrium welfare losses in a diversified (blue) and a concentrated network (red) in the presence (solid lines) and absence (dashed lines) of intervention.⁶ When the no-intervention losses exceed the costs of a public bailout (black dashed line), the government’s threat to not intervene is not credible and a public bailout is the only possible equilibrium rescue. In the absence of intervention, a diversified network is better for small shocks, while a concentrated network is preferred for large shocks. There will be a public bailout for diversified networks if the shock is large, and for concentrated networks if the shock is small. When a public bailout is not credible, then a bail-in becomes feasible, lowering overall welfare costs discontinuously. This increases the range of shocks over which a concentrated network is desirable.

in the system if no intervention arises. These gains from a rescue, however, accrue to the entire financial sector. An individual bank is willing to contribute if and only if the share of these gains accrued to the contributing bank itself is at least one (conditional on every other bank’s contribution decision). Hence, the same exact force that creates an absorption mechanism for losses in a diversified network (see Allen and Gale (2000) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015*a*)) leads to free-riding when the banks are faced with the decision of how much to contribute to a bail-in. The impacts of these forces on equilibrium welfare losses are illustrated in Figure 1. Once the possibility (desirability) of government intervention to prevent systemic collapse is taken into account, the relative desirability of different network typologies is reversed, at least for some shock sizes.

Our analysis indicates that the regulator’s threat of inaction is less credible if the costs of inefficient asset liquidation are higher. This result contributes to explain some of the decisions made by the sovereign authorities during distress scenarios. For instance, a private bail-in was coordinated to rescue the Long-Term Capital Management hedge fund in 1998. In contrast, the government of the United States rescued Citigroup through a public bailout in November 2008. Compared to the period when Long-Term Capital Management was rescued, at the time when Citigroup entered into distress the capitalization of the entire financial system was

⁶The figure displays our results in the stylized case of a continuum of banks to highlight the key differences between welfare losses in a ring and a complete network. In our model, the financial network consists of a finite number of banks, which leads to additional discontinuities in welfare losses. We refer to Section 4.2 for a numerical example consisting of a finite number of banks.

much lower (i.e., its leverage was much higher).⁷ Because the amplification of a shock is high in a lowly capitalized network, this suggests that the no-intervention threat may not have been credible.⁸

The remainder of the paper is organized as follows. In Section 1, we position our work relative to the existing literature. We develop the model in Section 2. We characterize the optimal bail-in plan and the equilibrium outcome for any financial network in Section 3. We analyze the relation between the credibility of the regulator’s threat and welfare losses in Section 4. Section 5 concludes. Appendix A generalizes the results of Section 3. Proofs are delegated to appendices B and C.

1 Literature Review

Our paper is related to a vast branch of literature on financial contagion in interbank networks.⁹ Pioneering works include Allen and Gale (2000) and Eisenberg and Noe (2001), and further developments were made in more recent years by Acemoglu, Ozdaglar and Tahbaz-Salehi (2015*a*), Elliott, Golub and Jackson (2014), Gai, Haldane and Kapadia (2011), Glasserman and Young (2015), and Capponi, Chen and Yao (2016). We refer to Glasserman and Young (2016) for a thorough survey on financial contagion.¹⁰ These works study how an initial shock is amplified through the interbank network based on the network topology, the inefficient liquidation of non-interbank assets, the trade-off between diversification and integration on the level of interbank exposures, and the complexity of the network. Different from our study, in these models agents execute exogenously specified contractual agreements but do not take any strategic action to resolve distress in the network. A different branch of the literature has focused on the coordinating role of the central bank in

⁷Prior to its needing a bail-out, seven major credit events occurred in the month of September 2008 alone, involving Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir and Kaupthing.

⁸This may not be the only explanation: politics may play more than a little role.

⁹There is, in addition, a closely related literature on financial contagion in intercountry and real production networks. See Battiston et al. (2007) and Acemoglu et al. (2012). These literatures have largely grown up independently, with little cross-references to each other. Even within the financial networks literature, there are two different strands which have developed relatively independently, one, as here, exploring contagion through network models using matrix analysis, the other (such as Battiston et al. (2007) and Battiston et al. (2012)) exploring the consequences of interlinkages on the dynamics of default cascades.

¹⁰Other related contributions include Gai and Kapadia (2010), which analyze how knock-on effects of distress can lead to write down the value of institutional assets; Cifuentes, Ferrucci and Shin (2005), which analyze the impact of fire sales on financial network contagion; Cabrales, Gottardi and Vega-Redondo (2017), which study the trade-off between the risk-sharing generated by more dense interconnection and the greater potential for default cascades; Battiston et al. (2012), which demonstrate that systemic risk does not necessarily decrease if the connectivity of the underlying financial network increases; and Elsinger, Lehar and Summer (2006), which study transmission of contagion in the Austrian banking system;

stopping financial contagion through provision of liquidity (Freixas, Parigi and Rochet (2000) and Gorton (2010)). These studies, however, do not consider resolution strategies where the government provides assistance that goes beyond the simple provision of liquidity (to financial institutions that are supposed to be solvent), but at the same time feature the involvement of the private sector, such as the bail-ins considered in our paper. The presence of this default resolution option questions the credibility of the no-intervention commitment by the government, which we show to be tightly linked to the network structure and the size of the initial shock.

Our paper contributes to the debate about bail-ins and bailouts, and the roles played by moral hazard and government credibility. Cooper and Nikolov (2017) analyze the relation between the government choice to bail out banks and the incentives for banks to self-insure through equity buffers. In their analysis, moral hazard arises because the incentive for banks to hold government debt without equity buffers depends on the anticipated bailout choice of the government. While their focus is on fragility of the sovereign debt market, our study targets the design of optimal resolution policies for the interbank debt market. Keister (2016) studies the interaction between a government's bailout policy during a crisis and banks' willingness to bail-in their investors. Similar to our study, banks may not go along with a bail-in if they anticipate being bailed out. Different from their study, our focus is on the interplay between the optimal sustainable resolution policy and the topology of the financial network.

In our model, the regulator determines the optimal rescue plan after the banks have already decided on the amount of risk undertaken, and shocks to outside assets have occurred. We do not consider here the moral hazard problem of banks taking excessive risks, knowing that they will be rescued if the market moves against them. Moral hazard problems arising in this context have been thoroughly investigated in the literature, but our model, like the rest of the literature, does not account for the endogenous structure of the interbank network (the exception is a recent study by Erol (2016), which develops an endogenous network formation model in which the government can intervene to stop contagion). Important contributions include Gale and Freixas (2002), who argue that a bailout is optimal *ex post*, but *ex ante* it should be limited to control moral hazard; Acharya and Yorulmazer (2007), who show that banks may find it optimal to invest in highly correlated assets in anticipation of a bailout triggered by the occurrence of many simultaneous failures; Farhi and Tirole (2012), who support Acharya and Yorulmazer (2007)'s findings

by showing that safety nets can provide perverse incentives and induce correlated behavior that increases systemic risk; Chari and Kehoe (2016), who show that if the regulator cannot commit to avoid bailouts ex post, then banks may overborrow ex ante; and Keister (2016), who finds that prohibiting bailouts may lead intermediaries to invest into too liquid assets which lower aggregate welfare. In contrast with these studies, in our model banks do not strategically decide on interbank or outside asset investments. The channel of contagion comes from the propagation of shocks through the exogenous network of liabilities.¹¹

Related to our model is the study by Rogers and Veraart (2013), who analyze situations in which banks can stop the insolvency from spreading by stabilizing the financial system through mergers. In their paper, however, the question of whether such a merger is incentive compatible for the shareholders of an individual bank is not addressed and the government does not take an active role. By contrast, our model focuses on the credibility of the regulator’s actions and the free-riding problem that arises because the stability of the financial system is to the benefit of every participant.

2 Model

We consider an interbank network with simultaneous clearing in the spirit of Eisenberg and Noe (2001). Banks $i = 1, \dots, n$ are connected through interbank liabilities $L = (L^{ij})_{i,j=1,\dots,n}$, where L^{ij} denotes the liability of bank j to bank i . We denote by $L^j := \sum_{i=1}^n L^{ij}$ the total liability of bank j to other banks in the network. Define the relative liability matrix $\pi = (\pi^{ij})$ by setting $\pi^{ij} = L^{ij}/L^j$ if $L^j \neq 0$ and $\pi^{ij} = 0$ otherwise. Our framework can accommodate lending from the private sector by adding a “sink node” that has only interbank assets but no interbank liabilities. Banks have investments in outside assets with values $e = (e^1, \dots, e^n)$, cash holdings $c_h = (c_h^1, \dots, c_h^n)$ and financial commitments $c_f = (c_f^1, \dots, c_f^n)$ with a higher seniority than the interbank liabilities. These commitments include deposits, wages and other operating expenses. If a bank i is not able to meet its liabilities out of current income, it will liquidate a part $\ell^i \in [0, e^i]$ of its outside investments, but will recover only a fraction $\alpha \in (0, 1]$ of the value of those investments.¹² If a

¹¹In the presence of bail-ins, banks bear the costs of their excessive risk taking in more states of the nature than in a model with bailouts only. This reduces the extent of moral hazard.

¹²In reality, recovery rates are asset-specific and some assets may directly be transferred to the creditors of defaulting institutions without liquidation. The parameter α is to be understood as an average recovery rate across all assets. It is equal to 1 if all assets are transferred to the creditors.

bank i cannot meet its liabilities even after liquidating all of its outside assets, it will default. As in Rogers and Veraart (2013) and Battiston et al. (2016), the default of a bank is costly and only a fraction $\beta \in (0, 1]$ of the defaulting bank's value is paid to creditors. Because the financial commitments c_f have higher seniority than the interbank liabilities, many parts of the model will depend c_f and c_h only through the difference $c = c_h - c_f$, which can be viewed as a net cash balance.

We denote by (L, π, c_h, c_f, e) the financial system after risks have been taken and after an exogenous shock has hit the system. The shock may lower the value of banks' outside assets or the net cash holdings of the banks. This may result in a negative net cash balance if, for example, a bank intended to use the returns from an investment to cover the operating expenses, but the returns turned out to be lower than expected.

We refer to the defaults that occur as an immediate consequence of the shock as *fundamental defaults* and denote their index set by $\mathcal{F} := \{i \mid L^i > c^i + \alpha e^i + (\pi L)^i\}$, where $(\pi L)^i = \sum_{j=1}^n \pi^{ij} L^j$ is the book value of bank i 's interbank assets. Fundamentally defaulting banks cannot meet their obligations even if every other bank repays its liabilities in full. Because fundamentally defaulting banks are able to only partially repay their creditors, their defaults may lead to additional defaults in the system, resulting in a default cascade. If, however, banks in \mathcal{F} receive a liquidity injection so that they can meet their obligations, the financial system is stabilized. In this section, we first characterize the outcome of a default cascade and then elaborate on the different types of rescues.

2.1 Default cascade

A defaulting bank will recall its assets and repay its creditors according to their seniority. Depositors are the most senior creditors, hence they are given priority over lenders from the interbank network and the private sector, to whom we refer as junior creditors. Creditors with the same seniority are repaid proportionally to their claim sizes. How much a bank is able to recall from its interbank assets depends on the solvency of the other banks in the system. A clearing payment vector is a set of repayments, simultaneously executed by all banks, for which every solvent bank repays its liabilities in full and every insolvent bank pays its entire value (net of costs from asset liquidation and bankruptcy) to its creditors.¹³

¹³In practice, liabilities may be cleared sequentially rather than simultaneously and the order of clearing may impact the outcome. This method of simultaneous clearing is standard in the literature and may

Definition 2.1. A clearing payment vector $p = (p^1, \dots, p^n)$ for a financial system (L, π, c_h, c_f, e) is a fixed point of

$$p^i = \begin{cases} L^i & \text{if } c^i + \alpha e^i + \sum_{j=1}^n \pi^{ij} p^j \geq L^i, \\ \left(\beta(c_h^i + \alpha e^i + \sum_{j=1}^n \pi^{ij} p^j) - c_f^i \right)^+ & \text{otherwise,} \end{cases}$$

where we denote by $x^+ = \max(x, 0)$ the positive part of x .

For a clearing payment vector p , we denote by $\mathcal{D}(p) := \{i \mid p^i < L^i\}$ the set of defaulting banks. Each bank i liquidates a nominal amount equal to

$$\ell^i(p) = \min \left(\frac{1}{\alpha} \left(L^i - c^i - \sum_{j=1}^n \pi^{ij} p^j \right)^+, e^i \right),$$

that is, each solvent bank liquidates just enough to remain solvent and each defaulting bank liquidates the outside assets in their entirety. A defaulting bank pays its entire value to its creditors. If the payment p^i is positive, it is divided pro-rata among bank i 's junior creditors and the senior creditors are paid in full. If $p^i = 0$, the junior creditors do not receive anything and the senior creditors suffer a loss of

$$\delta^i(p) := \left(c_f^i - \beta \left(c_h^i + \alpha e^i + \sum_{j=1}^n \pi^{ij} p^j \right) \right)^+.$$

Our notion of clearing payment vectors extends the corresponding notion in Rogers and Veraart (2013), by allowing banks to partially liquidate their outside assets, and the corresponding notion in Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a) by incorporating bankruptcy costs. The value of bank i 's equity after liabilities are cleared with the clearing payment vector p is equal to

$$V^i(p) := (\pi p + c + e - (1 - \alpha)\ell(p) - p)^i \mathbf{1}_{\{p^i = L^i\}}, \quad (1)$$

where $\mathbf{1}_{\{p^i = L^i\}}$ is the indicator function, taking the value 1 if $p^i = L^i$ and 0 otherwise. Welfare losses $w_\lambda(p)$ are defined as weighted sum of losses due to default costs, i.e.,

$$w_\lambda(p) := (1 - \alpha) \sum_{i=1}^n \ell^i(p) + (1 - \beta) \sum_{i \in \mathcal{D}(p)} \left(c_h^i + \alpha e^i + \sum_j \pi^{ij} p^j \right) + \lambda \sum_{i \in \mathcal{D}(p)} \delta^i(p). \quad (2)$$

represent the fact that clearing of liabilities occurs on a smaller time scale than the formation of bail-ins.

The first two terms in (2) are deadweight losses due to inefficient asset liquidation and bankruptcy costs: If bank i liquidates a nominal amount $\ell^i(p)$, only $\alpha\ell^i(p)$ is recovered and $(1 - \alpha)\ell^i(p)$ is lost. A defaulting bank i recovers a fraction β of its assets and a fraction $(1 - \beta)$ is lost. The last term in (2) are losses borne by senior creditors, weighted by a constant $\lambda \geq 0$. The weight λ captures the importance that the regulator assigns to the senior creditors' losses relative to the deadweight losses from asset liquidation and bankruptcy proceedings. The regulator's goal is to minimize welfare losses and the parameter λ captures his priorities in doing so. A regulator with $\lambda = 0$ views the senior creditors' losses merely as transfers of wealth and not as losses to the economy. A higher value of λ indicates a higher priority to the welfare of the economy exclusive of the banking sector. The coefficient λ may also be interpreted as a measure of political pressure on the regulator since a haircut on deposits is likely to result in unhappy voters. We obtain the following existence result for clearing payment vectors in analogy with Rogers and Veraart (2013).

Lemma 2.1. *For any financial system (L, π, c_h, c_f, e) , there exist a greatest and a lowest clearing payment vector \bar{p} and \underline{p} , respectively, with $\bar{p}^i \geq p^i \geq \underline{p}^i$ for any clearing payment vector p and any bank i . Moreover, \bar{p} is Pareto dominant, i.e., $V^i(\bar{p}) \geq V^i(p)$ for every bank i and $w_\lambda(\bar{p}) \leq w_\lambda(p)$ for any $\lambda > 0$.*

As is standard in the literature, liabilities are cleared with the Pareto-dominant clearing payment vector \bar{p} .

2.2 Coordination of rescues

Definition 2.2. An *assisted bail-in* (b, s) specifies for each bank $i = 1, \dots, n$ the transfer b^i of bank i to fundamentally defaulting banks as well as the size s^i of the subsidy bank i receives. The government's contribution to the bail-in is $\sum_{i=1}^n (s^i - b^i)$, which is imposed to be non-negative.

An assisted bail-in is a set of centralized transfers. The government allocates the contributions received by the participating banks, with bank i receiving subsidy s^i .

Remark 2.1. Assisted bail-ins contain public bailouts and privately backed bail-ins as special cases. A public bailout is an assisted bail-in, in which the banks' contributions are equal to 0, that is, $b = 0$. In a private bail-in, the government contributions are 0, i.e., $\sum_{i=1}^n b^i = \sum_{i=1}^n s^i$.

In addition to contributing to a bail-in financially, the regulator also serves to coordinate among different bail-ins. Specifically, the regulator may propose a bail-in allocation, but cannot force banks to participate.¹⁴

Organization of a rescue.

1. The regulator proposes an assisted bail-in (b, s) .
2. Each bank i from $\{j \in \mathcal{F}^c \mid b^j > 0\}$ chooses a binary action $a^i \in \{0, 1\}$, indicating whether or not it agrees to contribute b^i .¹⁵
3. The regulator has the following three options:

- (i) $a^0 = \text{"bail-in"}$: Proceed with the planned subsidies s without the contributions of banks which rejected the proposal and make up for those contributions using taxpayer money. Bank i 's cash holdings and its financial commitments in the rescue are $c_h^i(s) := c_h^i + s^i$ and $c_f^i(b, a) := c_f^i - b^i 1_{\{a^i=1\}}$, respectively. Let $\bar{p}(b, s, a)$ denote the Pareto-dominant clearing payment vector in the financial system $(L, \pi, c_h(s), c_f(b, a), e)$. The value of bank i after the rescue is

$$V^i(b, s, a) := V^i(\bar{p}(b, s, a)).$$

Welfare losses in the presence of intervention are a straightforward modification of (2), where we additionally account for the social costs of government subsidies:

$$w_\lambda(b, s, a) := w_\lambda(\bar{p}(b, s, a)) + \lambda \sum_{i=1}^n (s^i - b^i 1_{\{a^i=1\}}).^{16} \quad (3)$$

¹⁴Duffie and Wang (2017) consider bail-in strategies which are done contractually, rather than by a central planner. In their model, prioritization of bail-ins is based on the type of the instrument and not based on mitigating failure contagion through the network. Under strong axioms and assumptions on bilateral bargaining conditions, they show that the efficient choice of bail-in arrangements is made voluntarily.

¹⁵For a set of banks \mathcal{X} , we denote by $\mathcal{X}^c := \{1, \dots, N\} \setminus \mathcal{X}$ the complement with respect to $\{1, \dots, N\}$. A bank i with $b^i = s^i = 0$, is not expected to make a contribution to the bail-in, nor does it receive a subsidy. We assume that such a bank has no power to reject the proposal because it is simply not involved in the discussion. Similarly, a fundamentally defaulting bank cannot afford to reject a bail-in proposal. For ease of notation, these banks have only the singleton $\{1\}$ available to choose from in stage 2.

¹⁶In reality, the regulator may assign a weight λ_1 to depositor's losses and a different weight λ_2 to taxpayer contributions. In many situations, $\lambda_1 > \lambda_2$ so that the depositors are bailed out. However, the crises in Cyprus and Iceland have shown that this is not always the case. Our results do not crucially depend on the assumption that $\lambda_1 = \lambda_2$ and we make it for notational convenience.

- (ii) $a^0 = \text{"bailout"}$: Resort to a public bailout $(0, \tilde{s})$ with subsidies \tilde{s} decided by the regulator. After accounting for the subsidies, the cash holdings of bank i is equal to $c_h^i(\tilde{s}) = c_h^i + \tilde{s}^i$. We denote by $\bar{p}(\tilde{s})$ the greatest clearing payment vector in $(L, \pi, c_h(\tilde{s}), c_f, e)$. The banks' values and welfare losses are defined analogously to (i).
- (iii) $a^0 = \text{"no intervention"}$: Abandon the rescue, which results in a default cascade as in Section 2.1. We denote the welfare losses in a default cascade by $w_N := w_\lambda(\bar{p}(0, 0, 0))$ for the sake of brevity.

Definition 2.3. A bail-in (b, s) is *feasible* if it does not require any contributions of fundamentally defaulting banks and each non-fundamentally defaulting bank i can afford to contribute b^i , given the vector s of subsidies. More precisely, $b^i = 0$ for any $i \in \mathcal{F}$ and $L^i + c_f^i + b^i \leq c_h^i + s^i + \alpha e^i + (\pi \bar{p}(b, s, 1))^i$ for $i \notin \mathcal{F}$. A bail-in is thus feasible if it can be accepted by all banks.

The goal of the next section is to characterize all subgame perfect equilibria of this game when the regulator is restricted to propose feasible bail-ins. The notion of subgame perfection is closely related to the government's lack of commitment: it eliminates the non-credible threat of the regulator to abandon the rescue in the third stage when, in fact, he prefers a public bailout over a default cascade. The regulator cannot incentivize any bank i to participate in a bail-in if welfare losses are lowest in a bailout that protects bank i . Indeed, knowing that the regulator will inevitably resort to a public bailout after a rejection of his proposal, bank i has no punishments to fear for not cooperating. In reality, the coordination of a bail-in might involve more than one round of negotiation between regulator and banks. Some banks might reject the regulator's initial proposal, after which the regulator revises his proposal to accommodate the banks.¹⁷ In a game-theoretic model with complete information, the regulator can anticipate the banks' responses and need not make a proposal that is not incentive compatible. The negotiation is thus collapsed into a single stage. To maintain the dynamic flavor of a negotiation, we require that the banks' response to a proposal be *renegotiation proof* in the sense of Farrell and Maskin (1989).

¹⁷The original proposal made by the Federal Reserve Bank of New York for the rescue of Long Term Capital Management involved a total of 16 of LTCM's creditors. However, Bear Stearns and Lehman Brothers later declined to participate. Upon the rejection of these two banks, the Fed adjusted its proposal so that the contributions of Bear Stearns and Lehman Brothers was covered by the remaining 14 banks.

Definition 2.4. At any time t , let $\mathcal{A}_t(h_t)$ denote the set of players who are still active after history h_t has occurred.¹⁸ A subgame-perfect equilibrium (SPE) σ is *weakly renegotiation proof* if after every history h_t , there exists no continuation SPE σ_t such that the outcome of σ_t for players in \mathcal{A}_t Pareto-dominates the outcome of the continuation profile $\sigma|_{h_t}$ for those players.

This concept is a suitable equilibrium selection criterion in our setting because the coordination of a bail-in is precisely a negotiation: during the bail-in of Long Term Capital Management, Peter Fisher of the Federal Reserve Bank of New York sat down with representatives of LTCM’s creditors to find an appropriate solution; and it is implausible that they would have ever agreed on a bail-in plan that is Pareto-dominated.

3 Characterization of equilibria

In this section, we characterize the set of equilibria via backward induction. In the main body of the paper, we focus on the case of all-or-nothing interventions, where the regulator considers only rescues in which every bank of the system is saved. The more general case, where the regulator can propose interventions that rescue some banks but not others, is treated in Appendix A. As the intuition is largely the same for complete and general rescues, this allows us to present the economic forces behind the formation of bail-ins without the added mathematical complexity and the notational overhead of general rescues.

In complete rescues, the network’s structure affects the coordination of bail-ins only through the credibility of the regulator’s threat, which stands at the heart of this analysis. The payoffs in a rescue, however, decouple from the network structure because in a complete rescue, every bank recovers the nominal value of its assets. If general rescues are allowed, the regulator performs an additional minimization step in the first stage, optimizing proposals over rescues of all possible subsets of banks. However, for each subset considered, the formation of bail-ins works analogously to a complete bail-in.

3.1 Public bailout

In a complete bailout, the regulator provides a subsidy to every distressed bank. For a fundamentally defaulting bank i , there are two potentially sensible choices for

¹⁸A player is *active* at time t , if he/she faces at least one non-trivial decision after time t .

the size of the subsidy s^i . The regulator can either give a subsidy that is just large enough so that bank i does not have to liquidate any of its investments or he can give a subsidy that is just large enough to prevent bank i from defaulting, but that requires bank i to liquidate its projects. Which of the two choices minimizes the welfare losses depends on the the values of λ and α : if taxpayer money is expensive relative to liquidation costs ($\lambda\alpha \geq 1 - \alpha$), the regulator prefers that banks liquidate their outside assets to contribute a larger amount, whereas if taxpayer money is considered cheap ($\lambda\alpha < 1 - \alpha$), the regulator prefers to contribute a larger amount himself so as to avoid the liquidation of banks' outside assets.

Lemma 3.1. *The welfare-maximizing complete bailout s_P is given by*

$$s_P^i = \begin{cases} (L^i - c^i - (\pi L)^i)^+ & \text{if } \lambda\alpha < 1 - \alpha, \\ (L^i - c^i - \alpha e^i - (\pi L)^i)^+ & \text{if } \lambda\alpha \geq 1 - \alpha. \end{cases}$$

We denote the resulting welfare losses by $w_P = w_\lambda(0, s_P, 1)$.

Corollary 3.2. *Let (b, s) be the proposed bail-in with response vector a of the banks. In any sequential equilibrium,*

$$a^0(b, s, a) = \begin{cases} \text{"bail-in"} & \text{if } w_\lambda(b, s, a) < \min(w_N, w_P), \\ \text{"no intervention"} & \text{if } w_N \leq \min(w_\lambda(b, s, a), w_P), \\ \text{"bailout"} & \text{otherwise.} \end{cases}$$

It is clear that the regulator will choose whichever action minimizes his welfare losses if the minimizer is unique. To see why ties are broken according to "no intervention" \succ "bailout" \succ "bail-in", observe that banks have no incentive to contribute to a bail-in if they anticipate a public bailout. Thus, if the regulator commits to choosing "no intervention" over "bailout" when $w_N = w_P$, he is able to incentivize banks to contribute without violating sequential rationality. Similarly, by giving least priority to "bail-in" the regulator is able to deter unilateral deviations of banks in Stage 2 of the game; see Lemma 3.5 and Footnote 21 below for details.

3.2 Equilibrium response of banks

For any given feasible bail-in proposal, there may be many equilibrium responses by the banks. Suppose that the regulator proposes a bail-in, which requires the

participation of at least 5 banks for it to be preferable over the alternatives of a default cascade or a public bailout. Then any response with at most 3 banks accepting the proposal is trivially an equilibrium because a deviation of a single bank is not going to change the outcome. More interesting is the multiplicity among equilibria where the regulator proceeds with the bail-in because there may be several coalitions of banks, with which the regulator is happy to coordinate an intervention. A crucial separation among continuation equilibria is whether or not the banks' responses trigger the regulator's decision to proceed with the bail-in according to Corollary 3.2.

Definition 3.1. Given a bail-in proposal (b, s) , an equilibrium response a by the banks is called an *accepting equilibrium* if $a^0(b, s, a) = \text{"bail-in"}$ and it is called a *rejecting equilibrium* otherwise. Two equilibria (b, s, a) and $(\tilde{b}, \tilde{s}, \tilde{a})$ are *equivalent* if they demand the same net contribution $b^i - s^i = \tilde{b}^i - \tilde{s}^i$ from each bank i and $a = \tilde{a}$.¹⁹

Remark 3.1. Note that the banks' responses do not have to be unanimous in an accepting/rejecting equilibrium. Some banks may reject the proposal in an accepting equilibrium and vice versa. However, a sufficient proportion of banks accepts/rejects the proposal in an accepting/rejecting for the regulator to proceed with/abandon the bail-in.

Our first result in this section shows that most rejecting equilibria are indeed trivial equilibria that arise from a miscoordination. The requirement that equilibria be weakly renegotiation proof eliminates precisely these equilibria.

Lemma 3.3. *Suppose that a complete bail-in proposal (b, s) admits at least one accepting continuation equilibrium. Then every accepting continuation equilibrium is weakly renegotiation proof. Moreover, a rejecting continuation equilibrium is weakly renegotiation proof only if $(b, s) = (0, s_P)$.*

Lemma 3.3 shows that banks do not reject a bail-in proposal that can also be accepted in equilibrium. However, two accepting equilibria may not be Pareto-comparable, which raises the question of how banks coordinate their responses. The following result shows that the regulator can preempt the coordination problem by altering his proposed bail-in so that it is incentive compatible only for one coalition to accept the proposal.

¹⁹Because net contributions are identical it follows that also $a^0(b, s, a) = a^0(\tilde{b}, \tilde{s}, \tilde{a})$.

Lemma 3.4. *Let (b, s) be a proposed bail-in plan with accepting equilibrium responses $\{a_1, \dots, a_m\}$. For any $a_k, k = 1, \dots, m$, there exists a proposal (\tilde{b}, \tilde{s}) , to which a_k is the unique accepting equilibrium response (up to equivalence). Moreover, a_k is the Pareto-dominant equilibrium response to (\tilde{b}, \tilde{s}) .*

Our final result of this section characterizes the banks' incentive compatible responses in an accepting equilibrium.

Lemma 3.5. *Let (b, s) be a feasible proposal of a complete bail-in. In an accepting equilibrium a , bank i with $b^i > 0$ accepts if and only if:²⁰*

1. $w_\lambda(b, s, (0, a^{-i})) \geq \min(w_N, w_P)$, and
2. $b^i - s^i \leq \begin{cases} \sum_{j=1}^n \pi^{ij} (L^j - p_N^j) & \text{if } w_N \leq w_P, \\ -s_P^i & \text{if } w_P < w_N. \end{cases}$

Let us break down the intuition behind this result. If $w_N \leq w_P$, then the regulator prefers a default cascade over a bailout. The first condition thus states that there is no possibility for free-riding: if bank i were to reject the proposed bail-in, the regulator is not going to make up for i 's contribution and lets a default cascade occur instead.²¹ The second condition states how much bank i is willing to contribute to prevent a default cascade. Bank i is willing to make a net contribution to the bail-in up to the amount the bank would lose in a default cascade. If $w_N > w_P$, the two conditions state that the regulator's threat to not bail out the banks is not credible. Because a rejection of bank i leads to a bailout by Condition 1, bank i accepts only bail-ins that are a greater net subsidy than the bailout by Condition 2. If $w_N > w_P$, the regulator can thus do no better than resorting to a public bailout.

3.3 Optimal proposal of the regulator

If a proposal admits a unique accepting equilibrium, its acceptance is the unique weakly renegotiation proof continuation by Lemma 3.3. For such a proposal, the regulator can anticipate the banks' responses and what the resulting welfare losses will be. It is thus suboptimal for the regulator to make a proposal that admits more than one accepting equilibrium: according to Lemma 3.4, the regulator could have

²⁰We use the standard notation in game theory and denote by $(0, a^{-i})$ the action profile where bank i rejects the proposal and each other player's action (including the regulator's) is the same as in action profile a .

²¹This explains why the regulator gives least preference to continuation $a^0 = \text{"bail-in"}$ in Corollary A.2.

revised his proposal to select his preferred response unambiguously. Among those proposals that admit only one accepting equilibrium, the regulator will thus propose the bail-in that minimizes welfare losses, subject to the conditions of Lemma 3.5.

Consider a feasible bail-in proposal (b, s) with accepting equilibrium a . Because every bank is rescued, each bank recovers its interbank assets in full. The nominal amount that any bank i has to liquidate to retrieve its net contribution $b^i 1_{\{a^i=1\}} - s^i$ thus decouples from the contributions of others and hence the decisions of the other banks. Specifically, bank i has to liquidate an amount $\ell_c^i(b^i 1_{\{a^i=1\}} - s^i)$, where

$$\ell_c^i(x) = \min\left(\frac{1}{\alpha}(L^i + x - c^i - (\pi L)^i)^+, e^i\right) \quad (4)$$

is the nominal amount that has to be liquidated by bank i to recover the cash amount x , conditional on a complete rescue taking place (hence the subscript c). Bank i 's contribution to the bail-in reduces welfare losses by

$$f^i(b, s, a) := \lambda b^i 1_{\{a^i=1\}} - (1 - \alpha)(\ell_c^i(b^i 1_{\{a^i=1\}} - s^i) - \ell_c^i(-s^i)).$$

It lowers taxpayer losses by b^i , but will create deadweight losses if the bank needs to liquidate part of its outside assets to retrieve the contributed amount. The “no free-riding” condition of Lemma 3.5 implies that the welfare losses after the proposal's rejection by any net contributor $i \in \mathcal{R}(b, s, a) := \{i \mid b^i - s^i > 0, a^i = 1\}$ are bounded from below by w_N . Therefore, welfare losses in (b, s, a) admit the lower bound

$$w_\lambda(b, s, a) \geq w_N - \min_{i \in \mathcal{R}(b, s, a)} f^i(b, s, a). \quad (5)$$

The regulator thus strives to include banks in the bail-in which offer a high contribution to the rescue consortium and generate low deadweight losses when they liquidate their outside assets to retrieve the contributed amount. Which choice of b is optimal for the regulator depends on the values of λ and α : if taxpayer money is expensive relative to liquidation costs ($\lambda\alpha \geq 1 - \alpha$), the regulator prefers that banks liquidate their outside assets to contribute a larger amount, whereas if taxpayer money is cheap ($\lambda\alpha < 1 - \alpha$), the regulator prefers to contribute a larger amount himself. We are now ready to state the main result for complete rescues, which characterizes the proposals and the associated welfare losses that arise in any weakly renegotiation proof equilibrium.

Theorem 3.6. Let $i_1, \dots, i_{|\mathcal{F}^c|}$ be a non-increasing ordering of banks according to $\nu^i := \lambda\eta^i - (1 - \alpha)(\ell_c^i(\eta^i - s_P^i) - \ell_c^i(-s_P^i))$, where s_P is defined in Lemma 3.1 and

$$\eta^i := \begin{cases} \min\left(\sum_{j=1}^n \pi^{ij}(L^j - p_N^j), (c^i + \alpha e^i + (\pi L)^i - L^i)^+\right) & \text{if } \lambda\alpha \geq 1 - \alpha, \\ \min\left(\sum_{j=1}^n \pi^{ij}(L^j - p_N^j), (c^i + (\pi L)^i - L^i)^+\right) & \text{if } \lambda\alpha < 1 - \alpha. \end{cases}$$

Let $m := \min\left(k \mid w_P - \sum_{j=1}^k \nu^{i_j} < w_N\right)$.

Welfare losses: If $w_P < w_N$, then the unique equilibrium outcome is the public bailout s_P by the regulator. If $w_N \leq w_P$, then the welfare losses in any weakly renegotiation proof equilibrium are equal to

$$w_* = \min\left(w_P - \sum_{j=1}^m \nu^{i_j}, w_N - \nu^{i_{m+1}}\right).$$

Bail-in proposal: If $w_* = w_P - \sum_{j=1}^m \nu^{i_j}$, there exists a unique weakly renegotiation proof equilibrium with proposal (b_*, s_*) , where banks $j = i_1, \dots, i_m$ each contribute η^j and subsidies are given by $s_* = s_P$. If $w_* = w_N - \nu^{i_{m+1}}$, any proposal (b_*, s_*) made in a weakly renegotiation proof equilibrium satisfies:

1. $s_* \geq s_P$,
2. $\eta^{i_{m+1}} \leq b_*^j - s_*^j \leq \eta^j$ for any $j = i_1, \dots, i_{m+1}$, and
3. $\sum_{i=1}^n (s_*^i - s_P^i) = \frac{1}{\lambda} \left(w_N + \sum_{j=1}^m \nu^{i_j} - w_P \right)$.

As highlighted before, the equilibrium outcome depends crucially on whether or not the regulator's threat is credible. Note here that the credibility of the threat is a function of exogenous variables: the welfare losses w_P in the optimal bailout is the result of a minimization problem solved by the regulator in the last stage of the game to find the optimal subset of bailed out banks, and w_N are the welfare losses in absence of any action. We discuss how the credibility of the regulator's threat is affected by the network topology and the regulator's preference parameter in the next section. If the no-intervention threat of the regulator fails to be credible, then the unique equilibrium outcome is the public bailout characterized in Lemma 3.1. Because banks are aware that a default cascade is too costly for the regulator, they know that without their cooperation the regulator will resort to a public bailout,

which is the preferred outcome by the banks. If the no-intervention threat is credible, a bail-in will be organized in equilibrium.

The quantity η^i is the welfare-maximizing incentive-compatible contribution of bank i . There may be a cost associated with retrieving the amount η^i if taxpayer money is deemed expensive ($\lambda\alpha \geq 1 - \alpha$) and bank i is forced to liquidate its outside assets. The welfare impact of bank i 's contribution is thus equal to ν^i . It captures the trade-off between willingness of bank i to contribute and marginal liquidation losses caused by asset liquidation. A bank's contributions to a rescue consortium benefits other banks in the system because it enhances the stability of the financial network. As such, the coordination of bail-ins has an inherent free-riding problem, where each bank prefers to let others contribute in its stead. By adding banks to the rescue consortium in the decreasing order i_1, i_2, \dots , the regulator first asks for contributions of banks, which get the largest benefit from the rescue. Note that the absolute benefit determines the order—because the regulator cares only about welfare losses—and not the cost-to-benefit ratio, which might be the case in a privately organized bail-in. To avoid incentives for free-riding among contributors, the regulator includes only the m (or $m + 1$) largest benefactors into the rescue. If the regulator decides to include bank i_{m+1} into the bail-in consortium, he must “burn” an amount equal to $\frac{1}{\lambda} \left(w_N + \sum_{j=1}^m \nu^{i_j} - w_P \right)$ by giving it away as subsidies. These subsidies can be distributed arbitrarily among banks as long as the smallest net contribution is equal to $\eta^{i_{m+1}}$. This latter condition ensures that there is no free-riding among the net contributors because the rejection by one contributor would trigger the regulator's decision to let a default cascade occur.

Remark 3.2. If the regulator had the power to commit to playing N in the third stage, the equilibrium outcome would improve to w_* even if $w_P < w_N$. Commitment power would thus improve social welfare by $(w_P - w_*)\mathbf{1}_{\{w_P < w_N\}}$.

Theorem 3.6 characterizes the equilibrium outcome of the game up to the credibility of the no-intervention threat. Next, we show how the credibility of the regulator's threat can be completely characterized in terms of the model primitives. As we demonstrate next, it critically depends on the relation between the amplification of the shock through the network, the asset liquidation costs, and the regulator's trade-off between senior creditors' losses and inefficiencies arising at default. The main component of welfare losses in a public bailout is the aggregate shortfall $\sum_{i=1}^n (L^i - c^i - \alpha e^i - (\pi L)^i)^+$ of fundamentally defaulting banks. This can be understood as a measure of the size of the exogenous shock hitting the financial

system. The welfare losses triggered by a default cascade, on the other hand, are a measure of the shock size after the shock propagates through the financial system. Because senior creditors absorb a part of the losses, the portion of the shortfall that is amplified through the system is

$$\chi_0 := \sum_{i=1}^n (L^i - c^i - \alpha e^i - (\pi L)^i)^+ - \sum_{i \in \mathcal{D}(p_N)} \delta^i(p_N). \quad (6)$$

The losses accruing to the financial system after the shock has spread through the network are equal to

$$\chi_N := \sum_{i \in \mathcal{F}} (1 - \alpha) e^i + \sum_{i \notin \mathcal{F}} (c^i + e^i + (\pi L)^i - L^i - V^i(p_N)). \quad (7)$$

Our result states that the regulator's threat is credible if and only if the amplification $\chi_N - \chi_0$ of losses through the financial system in the absence of intervention is smaller than a certain threshold.

Lemma 3.7. *The regulator's threat is credible if and only if*

$$\chi_N - \chi_0 \leq \lambda \chi_0 + \min(\lambda \alpha, 1 - \alpha) \sum_{i=1}^n \ell_c^i(0). \quad (8)$$

Lemma 3.7 establishes a link between the credibility of the no-intervention threat and the existing literature on financial networks without intervention, which often ranks the social desirability of network topologies according to the welfare loss criterion $\chi_N - \chi_0$. In conjunction with results from this literature (e.g. Allen and Gale (2000) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a)), our result indicates that for large shock sizes and low recovery rates, dense interconnections between defaulting banks are detrimental to the formation of bail-ins because they serve as an amplifier of the shock.

Both the left and right hand sides of (8) depend only on exogenous variables. Notice that the amplification of losses $\chi_N - \chi_0$ depends on the clearing payment vector p_N , that is the fixed point of a function whose arguments are the model primitive parameters. Moreover, the left-hand side of (8) is independent of the regulator's preference parameter λ . An immediate consequence is that the regulator's threat becomes more credible if he assigns a larger weight to taxpayers' and senior creditors' money. This is because a bailout is perceived to be more costly if λ increases, making the threat of no bailout more credible.

4 Credibility of the regulator's threat

In this section, we study how the credibility of the regulator's threat and the network topology affect equilibrium welfare losses. Section 4.1 investigates this relation in detail. Section 4.2 develops a supporting parametric example featuring two topologies, the ring and the complete network, respectively representing a concentrated and a more diversified pattern of interbank liabilities.

4.1 Welfare losses and loss concentration

In this section, we compare equilibrium welfare losses between two arbitrary network topologies. It is clear from our Theorem 3.6 that the credibility of the regulator's no-intervention threat plays a major role. Our first result states that if the threat is credible in one network but not the other, equilibrium welfare losses are lower in the network with the credible threat.

Lemma 4.1. *For fixed $L, c_h, c_f, e, \alpha, \beta$, the equilibrium welfare losses after intervention are smaller in network π_1 than in network π_2 if the regulator's threat is credible in network π_1 but not in network π_2 .*

Lemma 4.1 is intuitive. Because the regulator's threat is credible in network π^1 but not in network π^2 , Theorem 3.6 implies that $w_*(\pi^1) < w_N(\pi^1) \leq w_P = w_*(\pi^2)$. If the threat fails to be credible in both networks, the only available option for the regulator in either network is a public bailout. Therefore, equilibrium welfare losses are identical in both networks.

Lastly, consider the situation where the threat is credible in both networks. Then the socially preferable network crucially depends on the relation between the topologies of the interbank liability matrices. This is because the liability matrix of a network determines how losses in the absence of intervention are concentrated.²² There are two counteracting economic forces that determine the relative merits of one network topology over the other. On the one hand, contributions are larger in a network where no-intervention losses are concentrated on a smaller number of banks, leading to a higher reduction of equilibrium welfare losses. On the other hand, because welfare losses without intervention are larger in the more concentrated

²²If interbank liabilities $(L^{ij})_{ij}$ are more concentrated in network π_1 than in network π_2 , then first-order interbank losses are more concentrated in π_1 than in π_2 . Higher-order losses, however, that arise as a result of the amplification of losses through the network, may not preserve this monotonicity. This is why we state Proposition 4.2 in terms of concentration of interbank losses rather than interbank exposures.

network, the no free-riding condition of Lemma A.3 implies that fewer banks can be included into the bail-in consortium, thereby leading to an increase in welfare losses. Which force dominates the other depends on the precise structure of the network. In the remainder of the section, we study the conditions under which the network topology with more concentrated losses is socially preferable.

Because the dependence of equilibrium welfare losses on the network topology is highly non-linear, we need to impose additional regularity assumptions on the network to obtain more quantitative comparisons between different topologies. Following Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a), we say that a financial system (L, π, c_h, c_f, e) is *regular* if $L^i = L^j$ for every pair of banks i, j and $L = \pi L$. Because each bank's interbank claims are equal to its interbank liabilities, the set of fundamentally defaulting banks in a regular financial system is $\mathcal{F} := \{i \mid c^i + \alpha e^i < 0\}$. Note that for regular networks, the set of fundamentally defaulting banks does not depend on the precise structure of the interbank network other than the relation $L = \pi L$. Thus, for any other network topology π' with $\pi' L = L$, the system (L, π', c_h, c_f, e) is regular and has the same set of fundamentally defaulting banks.

A regular financial system has *homogeneous cash holdings* if $c_h^i = c_h^j$ and $c_f^i = c_f^j$ for every pair of banks $i, j \in \mathcal{F}$ and every pair of banks $i, j \notin \mathcal{F}$. A regular financial system with homogeneous cash holdings would arise, for example, if all banks have homogeneous cash holdings ex ante, but some banks are hit by a shock of identical size causing them to fundamentally default. Considering regular financial systems with homogeneous cash holdings guarantees that differences in equilibrium welfare losses between two networks stem only from differences in their topologies. We do not impose any restriction on the banks' holdings of outside assets.

We next introduce a notion of concentration that will allow us to compare networks, and establish the merit of one topology over the other.

Definition 4.1. Consider two financial systems (L, π, c_h, c_f, e) (L, π', c_h, c_f, e) . For any $k < n$, interbank losses are *strongly k -more concentrated* in network π than in π' if $\eta^{(i)}(\pi) \geq \eta^{(i)}(\pi')$ for every $i \leq k$, where $x^{(i)}$ is the i^{th} -largest entry of vector x .

Observe that the welfare losses in a network π equal the sum of the banks' incentive compatible contributions, i.e., $w_N(\pi) = \sum_{i=1}^n \eta^i(\pi)$. The above notion of strongly k -more concentrated losses implies that the largest k losses in network π are higher than the corresponding losses incurred by banks in the network π' . If $w_N(\pi) = w_N(\pi')$, then the total losses would be identical in both networks but the

losses would be more evenly distributed in network π' than in network π .

Proposition 4.2. *Suppose that (L, π, c_h, c_f, e) and (L, π', c_h, c_f, e) are two regular financial systems with homogeneous cash holdings. If interbank losses are strongly $m(\pi) + 1$ -more concentrated in network π than in π' and*

$$w_N(\pi) \leq w_N(\pi') + \nu^{i_{m(\pi)+1}(\pi)}(\pi) - \nu^{i_{\min(m(\pi), m(\pi'))+1}(\pi')}(\pi') \quad (9)$$

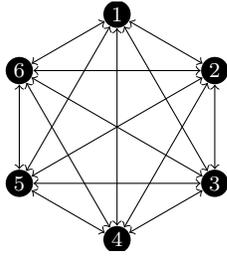
holds, then equilibrium welfare losses are lower in π than in π' .

We show in the proof in Appendix C that $m(\pi) \leq m(\pi')$ if $w_N(\pi) \geq w_N(\pi')$. Because the interbank losses are strongly $m(\pi) + 1$ -more concentrated in network π than in π' , this implies that the right-hand side of (9) is greater than or equal to $w_N(\pi')$. Therefore, Proposition 4.2 implies that even if welfare losses without intervention are larger in the network with strongly more concentrated losses, then – as long as the losses are not too much larger – equilibrium welfare losses will be lower in the more concentrated network. We present a numerical example to illustrate this fact in Section 4.2.

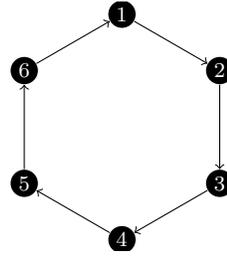
Overall, our analysis shows that the presence of intervention enlarges the range of shock sizes for which networks with a higher concentration of losses are socially preferable. Prior studies have shown that, in the absence of government intervention, dense interconnections between defaulting banks may amplify rather than absorb initial losses (e.g. Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a) and Haldane (2009)). Our results suggest that this phenomenon is strengthened in the presence of intervention for two fundamental reasons. First, in a more concentrated network, bail-ins can be tailored in a way that the benefits accrue more strongly to the contributors, which increases the sizes of a bank’s incentive compatible contribution (Proposition 4.2). Second, because the amplification of a large shock is expected to be smaller in the network with a higher liability concentration, the government can credibly stand by idly in this network but may not be able to do so in the more diversified network (Lemma 3.7). As a result, the coordination of a bail-in plan in the more concentrated network would lead to lower welfare losses (Lemma 4.1).

4.2 Comparison between ring and complete network

In this section we illustrate the general results from the previous sections on stylized network topologies. We consider the ring network as a representative structure of a



(a) The complete network.



(b) The ring network.

Figure 2: We compare equilibrium welfare losses in the highly concentrated ring network (b) to the diversified complete network (a).

concentrated network, and the complete network as a representative structure of a diversified network. We consider a regular financial system of $n = 6$ banks in which we normalize interbank liabilities to 1. The relative liability matrices in the ring and complete network are given by π_R and π_C , respectively, with $\pi_R^{ij} = 1$ if and only if $i = j + 1$ (modulo n) and $\pi_C^{ij} = \frac{1}{n-1} = 0.2$ for every $i \neq j$. We provide a graphical representation of the two networks in Figure 2. We choose $\lambda = 1$ and fix the recovery rates to $\alpha = 0.75$, $\beta = 0.85$. We consider an ex-post scenario where an exogenous shock has rendered bank 1 insolvent. The vector of net cash balances is given by $c = (-1, 0.05, 0.05, 0.05, 0.05, 0.05)$ and the nominal value of outside assets are approximately equal to $e = (0.5, 0.5, 0.4, 0.7, 0.2, 0.1)$ so that $w_N(\pi_R) = 0.652$ and $w_N(\pi_C) = 0.611$. Since $w_P = 0.75$, the regulator's threat is credible in both networks, hence an equilibrium bail-in can be organized.

In the complete network, the shock is spread evenly among banks in absence of intervention, resulting in the contagious defaults of the banks with the lowest level of capitalization, namely banks 5 and 6. In the ring network, banks 2 and 3 default as a result of financial contagion because these are the closest to the fundamentally defaulting bank in the chain of creditors. Clearing payment vectors as well as the vectors of interbank losses and maximal welfare impact are summarized in Table 1. Note that $m(\pi_R) = 1$ as $w_P - \nu^{(1)}(\pi_R) < w_N(\pi_R)$. Since interbank losses of banks 2 and 3 in the ring network are larger than interbank losses of any two banks in the complete network, losses are strongly more concentrated in the ring network, guaranteeing that welfare losses in the equilibrium bail-in plan are lower in the ring network by Proposition 4.2.

We conclude this section by illustrating numerically how equilibrium welfare losses change with sizes of the initial shock c^1 and recovery rate α of the outside assets, keeping other parameters as in the previous example. The equilibrium wel-

Network	π_R	π_C
$p_N(\pi)$	(0.225, 0.616, 0.874, 1, 1, 1)	(0.198, 1, 1, 1, 1, 0.839)
$\eta(\pi)$	(0.425, 0.35, 0.126, 0, 0)	(0.193, 0.193, 0.193, 0.193, 0.125)
$\nu(\pi)$	(0.3, 0.25, 0.1, 0, 0)	(0.145, 0.145, 0.145, 0.145, 0.1)

Table 1: Shown are the clearing payment vectors $p_N(\pi)$, the vector $\eta(\pi)$ of interbank losses to banks 2, \dots , 5, and the vector $\nu(\pi)$ of maximal welfare impact of an incentive-compatible contribution by banks 2, \dots , 5 for the two networks $\pi \in \{\pi_R, \pi_C\}$.

fare losses in the two networks are shown in Figure 3. The vertical steps indicate the welfare impacts that contributions of the banks have when they join/leave the equilibrium bail-in consortium. The continuous changes in welfare losses are due to reduced/increased liquidation losses as well as generally lower/larger shock sizes. At the “top of the stairs”, where no bank makes a contribution, the equilibrium intervention is a public bailout. Thus, the corresponding area in the (c^1, α) -plane is the region where the no-intervention threat fails to be credible. We observe that the threat is not credible in the ring network but credible in the complete network only for a small set of parameters where the shock size is small and recovery rates are large. For low recovery rates or large shocks, the threat is more credible in the ring network. We also observe that welfare losses are lower for almost every pair (c^1, α) , where the threat is credible in the ring network. This is due to the larger size of incentive-compatible contributions in the ring network, which is visually confirmed by the larger size of the vertical steps in Figure 3. Note that equilibrium welfare losses are 0 for some pairs (c^1, α) , where a bail-in can be financed completely by the private sector. Nevertheless, the regulator plays an important role in the coordination of the private rescue.

5 Concluding Remarks

Government support of financial institutions designated to be too big or too important to fail is costly. Various initiatives have been undertaken by central governments and monetary authorities, especially after the global financial crisis, to expand resolution plans and tools, including voluntary bail-in consortia, in which creditors of distressed banks make contributions. The central questions studied in this paper are: do credible bail-in strategies actually exist? What is the structure of optimal bailouts and bail-ins? How do they depend on the network structure and how does the possibility of strategic intervention affect the desirability of different network

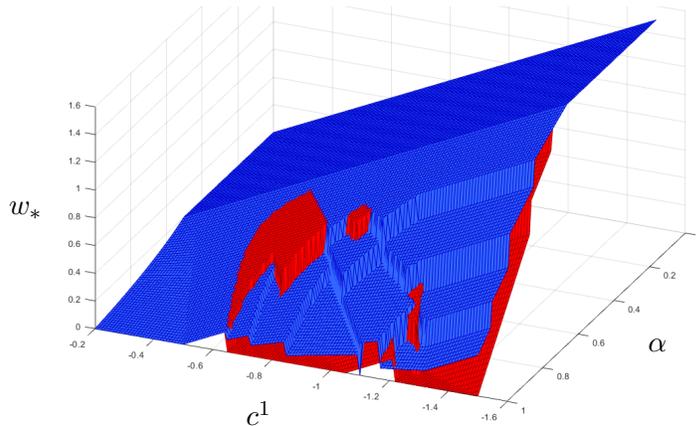


Figure 3: Equilibrium welfare losses are shown in the complete network (blue) and the ring network (red). The network with lower welfare losses is socially preferable. The vertical “steps” illustrate the welfare impacts when banks join/leave the equilibrium bail-in consortium. If, for a pair (c^1, α) , the equilibrium bail-in plan does not involve contributions by any banks (at the “top of the stairs” so to speak), the regulator’s no-intervention threat fails to be credible and equilibrium welfare losses are equal to the welfare costs of a public bailout. If equilibrium welfare losses are 0, a privately backed bail-in is incentive compatible.

structures? We have shown that the existence of credible bail-ins is tightly linked to the amplification of initial shocks through the network. If shocks are strongly amplified by inefficient asset liquidation, bankruptcy costs, and negative feedback effects between interconnected banks in distress, the regulator cannot credibly threaten the banks to not intervene himself. This leaves a public bailout as the only incentive-compatible rescue option. If shocks are only moderately amplified and the threat is credible, the creditors of the defaulting banks can be incentivized to contribute to a rescue in order to avoid a default cascade.

Our analysis shows that the option to implement bail-in strategies strongly affects the socially desirable network structure. While in network models without intervention, dense interconnections create a mechanism for the absorption of a shock and, as a result, enhance welfare, this is no longer desirable if banks and the regulator can strategically coordinate a default resolution plan. Sparser networks enlarge the range of shocks for which a credible bail-in strategy exists; and banks are willing to make larger contributions to a rescue consortium because they recoup a larger fraction of their social contributions.

Our paper makes a first step towards a systematic analysis of the incentives to which alternative resolution plans give rise. In an extension to the model presented here, banks would anticipate which bail-in consortia are credible for which

network structures, and hence choose their counterparties so to take into account their ex-ante expected contributions to the equilibrium bail-in plan. While the current analysis indicates that, regardless of the network structure, the threat of non-intervention is credible if recovery rates on external and interbanking assets are high enough, the network structure may play an important role if recovery rates are low. To prevent banks constructing (either in a coordinated or in a non-coordinated way) on a network configuration under which the regulator’s threat fails to be credible, the regulator may decide to impose structural policies which restrict the class of allowable network configurations. Examples of these policies include limiting the size of exposures toward individual counterparties, such as the “Large exposures” framework put forward by the Basel Committee.²³ Such an endogenous network formation model adds an additional dimension to the moral hazard literature: besides maximizing the value of their bailout option through excessive risk taking as in Farhi and Tirole (2012), banks can control the likelihood of a public bailout when social losses are high through interbank linkages. Accounting for the network formation incentives driven by strategic intervention would also complement and extend the existing literature on endogenous network formation. Existing studies (see, for instance, Farboodi (2014) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015*b*)) highlight the inefficiencies arising from the overexposure to counterparty risk by banks which make risky investment.²⁴ However, they do not account for the influence that bailout strategies plans may have on banks’ contractual decisions.

In this study, we have positioned ourselves in an ex-post scenario, i.e., after the realization of asset value shocks. In a future work, it would be desirable to account for the (ex-ante) risk taking decisions by banks, which choose their outside investments to maximize shareholder value, while accounting for counterparty risk created by the network. Accounting for ex-ante risk taking behavior will lead to a comprehensive framework for the analysis of welfare maximizing rescue policies. Such a framework would extend existing literature (e.g., Acharya and Yorulmazer (2007) and Acharya, Shin and Yorulmazer (2011)), which has primarily focused on endogenous (non-interbank) asset correlation and risks of liquidity arising when banks make their investment decisions in anticipation of a bailout.

²³Under this framework, which is expected to be fully implemented by January 1, 2019, the set of acceptable exposures is computed so as to guarantee that the maximum possible loss incurred by any bank would not be high enough to induce its own default.

²⁴These inefficiencies are, in fact, just another manifestation of the general market inefficiencies which arise in the presence of incomplete markets and imperfect information; see Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986).

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A General interventions

In this appendix, we characterize the optimal intervention plan if the government is not bound to propose complete rescues, i.e., the government may propose bail-ins or conduct bailouts that rescue only a subset of banks. We also refer to these interventions as *partial* interventions.

A.1 Public bailout

When the regulator is not forced to rescue every bank, he will essentially minimize the welfare losses over all possible sets of banks that he could bail out. The first lemma describes this minimization procedure.

Lemma A.1. *Fix a set of banks $\mathcal{B} \subseteq \{1, \dots, N\}$. The welfare-maximizing bailout $s_P(\mathcal{B})$ that ensures solvency of all banks in \mathcal{B} is computed as follows:*

1. Let $p(\mathcal{B})$ be the greatest fixed-point of

$$p^i = \begin{cases} L^i & \text{if } i \in \mathcal{B} \text{ or } c^i + \alpha e^i + \sum_j \pi^{ij} p^j \geq L^i, \\ \left(\beta(c_h^i + \alpha e^i + (\pi p)^i) - c_f^i \right)^+ & \text{otherwise,} \end{cases}$$

and set

$$s^i(\mathcal{B}) := \left(L^i - c^i - \alpha e^i 1_{\{\lambda\alpha > 1 - \alpha\}} - \sum_{j=1}^n \pi^{ij} p^j(\mathcal{B}) \right)^+ 1_{\{i \in \mathcal{B}\}}. \quad (10)$$

2. The welfare-maximizing bailout that guarantees solvency of banks in \mathcal{B} is $s_P(\mathcal{B}) = s(\chi(\mathcal{B}))$, where

$$\chi(\mathcal{B}) := \arg \min_{\mathcal{B}' \supseteq \mathcal{B}} w_\lambda(s(\mathcal{B}')). \quad (11)$$

In words, $s(\mathcal{B})$ is the welfare-maximizing bailout among all bailouts that rescue \mathcal{B} by giving direct subsidies only to banks in \mathcal{B} . The welfare-maximizing bailout $s_P(\mathcal{B})$

among all bailouts that rescue \mathcal{B} may involve indirect subsidies to rescue banks in \mathcal{B}^c so that banks in \mathcal{B} remain solvent because they are sufficiently insulated from losses propagating through \mathcal{B}^c . The welfare-maximizing partial bailout is thus $s_P(\emptyset)$, where the government is not committed to saving any specific bank. This includes the possibility of not saving any banks at all, i.e., $\chi(\emptyset) = \emptyset$, if it is optimal to let the default cascade play out in full.

Proof of Lemma A.1. Let s be any set of subsidies that rescues banks in \mathcal{B} by awarding direct subsidies only to banks in \mathcal{B} . For any such vector of subsidies, the greatest clearing payment vector is given by $\bar{p}(s) = p(\mathcal{B})$. Indeed, for any such vector of subsidies, banks in \mathcal{B} are able to repay their liabilities in full. Contingent on full repayment by banks in \mathcal{B} , banks in \mathcal{B}^c are not affected by the subsidies as they are awarded only to banks in \mathcal{B} , hence $\bar{p}(s) = p(\mathcal{B})$. The welfare-maximizing vector of direct subsidies that banks $i \in \mathcal{B}$ require to afford payment L^i is thus given by (a) the shortfall $(L^i - c^i - (\pi p(\mathcal{B}))^i)^+$ before liquidation if $\lambda\alpha \leq 1 - \alpha$ and (b) the shortfall $(L^i - c^i - \alpha e^i - (\pi p(\mathcal{B}))^i)^+$ after liquidation if $\lambda\alpha > 1 - \alpha$.

To see that it is optimal to give direct subsidies to banks in $\chi(\mathcal{B})$, observe that it is clearly optimal to give direct subsidies to the set

$$\chi'(\mathcal{B}) := \arg \min_{\mathcal{B}': \mathcal{S}(\mathcal{B}') \supseteq \mathcal{B}} w_\lambda(s(\mathcal{B}')), \quad (12)$$

where $\mathcal{S}(\mathcal{B}')$ denotes the set of all banks that are solvent if the regulator provides subsidies $s(\mathcal{B}')$. By definition, $s(\mathcal{S}(\mathcal{B}')) = s(\mathcal{B}')$, hence any set of subsidies that is taken into account in the minimization (12) is also taken into account in (11). In particular, $s(\chi'(\mathcal{B})) = s(\mathcal{S}(\chi'(\mathcal{B}))) = s(\chi(\mathcal{B}))$, which establishes the claim. \square

Corollary A.2. *Let (b, s) be the proposed bail-in with response vector a of the banks. In the last stage, the regulator chooses to implement $s_P(\emptyset)$ if and only if $w_\lambda(s_P(\emptyset)) \leq w_\lambda(b, s, a)$. We denote by $p_* = p(s_P(\emptyset))$ the clearing payment vector in the optimal bailout (which may involve no subsidies at all).*

A.2 Equilibrium response of banks

The following lemma is the analogue to Lemma 3.5, characterizing the banks' maximal incentive-compatible contributions in an accepting equilibrium.

Lemma A.3. *Let (b, s) be a feasible bail-in proposal. In an accepting equilibrium a , bank i with $b^i > 0$ accepts if and only if the following conditions hold:*

1. $w_\lambda(b, s, (0, a^{-i})) \geq w_\lambda(s_P(\emptyset))$,
2. $b^i - s^i \leq \sum_{j=1}^n \pi^{ij} (\bar{p}^j(b, s, (1, a^{-i})) - p_*^j) - s_P^i(\emptyset)$.

The presentation of Lemma A.3 differs slightly from Lemma 3.5 because $s_P(\emptyset)$ may be the default cascade or a non-trivial bailout. Nevertheless, the intuition behind the result is the same: Condition 1 states that no contributing bank has an incentive to free-ride and Condition 2 states that the maximal incentive-compatible net contribution of a bank is equal to the excess losses it experiences in the optimal bailout over the losses in the bail-in (b, s) .

Similarly to the case of complete rescues, the regulator can select among different accepting equilibria by modifying his proposal. The following result is the analogue to Lemma 3.4.

Lemma A.4. *Let (b, s) be a bail-in proposal with accepting equilibrium responses $\{a_1, \dots, a_m\}$. For any $a_k, k = 1, \dots, m$, there exists a proposal (\tilde{b}, \tilde{s}) , to which a_k is the unique accepting equilibrium response (up to equivalence).*

Differently from the case of complete rescues, accepting equilibria do not necessarily Pareto-dominate rejecting equilibria. A bank which is not being rescued in the proposed bail-in might be worse off than under the optimal bailout. In a reasonable proposal, however, every bank that is involved in the bail-in coordination is better off in an accepting equilibrium than in any rejecting equilibrium as we will see shortly.

A.3 Optimal proposal of the regulator

To incentivize banks to contribute to a bail-in, the regulator would like to threaten the banks that he will let a default cascade occur if they reject the proposal. This threat may not be credible, however, if doing so leads to higher welfare losses. For a given proposal (b, s) , the maximal incentive-compatible net contribution of bank i is $t^i(b, s) = \left(\sum_j \pi^{ij} (\bar{p}^j(b, s, 1) - p_*^j) - s_P^i(\emptyset) \right)^+$ by Lemma A.3. We say that the threat towards bank i is *credible* if $t^i(b, s) > 0$.

Similarly to the optimal bailout, the regulator will minimize welfare losses among possible subsets of banks that can be rescued. For a fixed set of banks $\mathcal{B} \subseteq \{1, \dots, n\}$, we introduce the auxiliary game, where the regulator can propose only feasible bail-ins from the set

$$\Xi(\mathcal{B}) := \{(b, s) \mid s^i = 0 \text{ for } i \notin \mathcal{B} \text{ and } \mathcal{B} \subseteq \mathcal{S}(s(\mathcal{B}))\}$$

of bail-ins, whose subsidies guarantee solvency of banks in \mathcal{B} by awarding direct subsidies only to banks in \mathcal{B} . Bailouts, however, can target any set of banks. As argued in the proof of Lemma A.1, the clearing payment vector after any such proposal is equal to $p(\mathcal{B})$ if the proposal is accepted. Thus, in an accepting equilibrium of any proposal from $\Xi(\mathcal{B})$, any bank's maximal incentive-compatible contribution, the associated welfare impact, and the necessary liquidation amount disentangle from the other banks' responses. The maximal incentive-compatible contribution of bank i is equal to

$$t^i(\mathcal{B}) := \left(\sum_j \pi^{ij} (\bar{p}^j(\mathcal{B}) - p_*^j) - s_P^i(\emptyset) \right)^+. \quad (13)$$

In the following definition, we describe the set $\Xi_*(\mathcal{B})$ of bail-ins that will be proposed by the regulator in equilibrium; see Theorem A.6 below. We isolate the definition of $\Xi_*(\mathcal{B})$ from the main result of this appendix because we wish to present the analogue to Lemma 3.3 before our main result of this appendix.

Definition A.1. For a set of banks \mathcal{B} , define

$$\eta^i(\mathcal{B}) := \begin{cases} \min\left(t^i(\mathcal{B}), (c^i + \alpha e^i + (\pi p(\mathcal{B}))^i - L^i)^+\right) & \text{if } \lambda\alpha \geq 1 - \alpha, \\ \min\left(t^i(\mathcal{B}), (c^i + (\pi p(\mathcal{B}))^i - L^i)^+\right) & \text{if } \lambda\alpha < 1 - \alpha, \end{cases} \quad (14)$$

and set $\nu^i(\mathcal{B}) := \lambda\eta^i(\mathcal{B}) + (1 - \alpha)(\ell_{\mathcal{B}}^i(-s^i(\mathcal{B})) - \ell_{\mathcal{B}}^i(\eta^i(\mathcal{B}) - s^i(\mathcal{B})))$, where

$$\ell_{\mathcal{B}}(x) = \min\left(\frac{1}{\alpha}(L + x - c - \pi p(\mathcal{B}))^+, e\right) \quad (15)$$

and the minimum is taken componentwise. Let $i_1, \dots, i_{|\mathcal{F}^c|}$ be a non-increasing order of non-fundamentally defaulting banks according to $\nu^i(\mathcal{B})$ and set

$$m(\mathcal{B}) := \min\left(k \left| w_\lambda(s(\mathcal{B})) - \sum_{j=1}^k \nu^{i_j}(\mathcal{B}) < w_\lambda(s_P(\emptyset)) \right.\right). \quad (16)$$

Let $b(\mathcal{B})$, defined by $b^j(\mathcal{B}) := \eta^j(\mathcal{B})$ for $j = i_1, \dots, i_{m(\mathcal{B})}$ and $b^j(\mathcal{B}) = 0$ otherwise. Let $\Xi_*(\mathcal{B})$ denote the set of proposals (b, s) such that:

1. $b = b(\mathcal{B})$ and $s = s(\mathcal{B})$ if $w_\lambda(s(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B}) < w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$.
2. If $w_\lambda(s(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B}) \geq w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$, then

- (a) $s \geq s(\mathcal{B})$,
- (b) $\eta^j(\mathcal{B}) \geq b^j - s^j \geq \eta^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ for every bank j with $b^j > 0$, and
- (c) $\sum_{i=1}^n (s^i - s^i(\mathcal{B})) = \frac{1}{\lambda} \left(w_\lambda(s_P(\emptyset)) + \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B}) - w_\lambda(s(\mathcal{B})) \right)$.

Observe that $\Xi_*(\mathcal{B}) \subseteq \Xi(\mathcal{B})$ since $s \geq s(\mathcal{B})$ guarantees that all banks in \mathcal{B} remain solvent. The meaning of the variables will be analogous to the main body: $\eta^i(\mathcal{B})$ is the welfare-maximizing contribution of bank i towards a bail-in in $\Xi(\mathcal{B})$, which has a welfare impact of $\nu^i(\mathcal{B})$.

Lemma A.5. *Let $(b, s) \in \Xi_*(\mathcal{B})$ be a partial bail-in proposal that admits at least one accepting equilibrium. Then every accepting equilibrium is weakly renegotiation proof. Moreover, a rejecting equilibrium is weakly renegotiation proof only if $(b, s) = (0, s_P(\emptyset))$.*

In the auxiliary game, where a fixed set of banks \mathcal{B} has to be rescued in any proposed bail-in, all weakly renegotiation proof equilibria involve a bail-in proposal from $\Xi_*(\mathcal{B})$ as stated by the following result.

Theorem A.6. *Using the terminology of Definition A.1, let*

$$w_\lambda(\mathcal{B}) := \min \left(w_\lambda(s(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B}), w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B}) \right). \quad (17)$$

If $w_\lambda(\mathcal{B}) > w_\lambda(s_P(\emptyset))$, then the unique equilibrium outcome in the auxiliary game is the public bailout $s_P(\emptyset)$. If $w_\lambda(\mathcal{B}) \leq w_\lambda(s_P(\emptyset))$, then welfare losses in any weakly renegotiation proof equilibrium of the auxiliary game are equal to $w_\lambda(\mathcal{B})$. Moreover, $(b, s) \in \Xi_(\mathcal{B})$ is proposed and accepted by all banks.*

The intuition behind Theorem A.6 is exactly the same as in Theorem 3.6. The regulator can include banks into the bail-in only if the threat towards that bank is credible. Moreover, the regulator will first include banks that have the largest exposure to contagion, hence the least incentive for free-riding. The regulator can include only the first $m(\mathcal{B})$ or $m(\mathcal{B}) + 1$ banks into the bail-in because of the “no free-riding” condition of Lemma A.3. Observe that Theorem 3.6 is recovered as a special case of Theorem A.6 by setting $\mathcal{B} = \{1, \dots, N\}$. The following corollary characterizes all weakly renegotiation proof equilibria of the original game with partial bail-ins.

Corollary A.7. *Let $\psi(\mathcal{B}) := \arg \min_{\mathcal{B}' \supseteq \mathcal{B}} w_\lambda(\mathcal{B}')$. If $w_\lambda(\psi(\emptyset)) > w_\lambda(s_P(\emptyset))$, then the unique sequential equilibrium outcome is the public bailout $s_P(\emptyset)$. If $w_\lambda(\psi(\emptyset)) \leq w_\lambda(s_P(\emptyset))$, then welfare losses in any weakly renegotiation proof equilibrium are equal to $w_\lambda(\psi(\emptyset))$. Moreover, in any weakly renegotiation proof equilibrium, a bail-in from $\Xi_*(\psi(\emptyset))$ is proposed and accepted by all banks.*

Proof. Corollary A.7 follows from Theorem A.6 with a similar argument as in the proof of Lemma A.1. A bail-in proposal $(b, s) \in \Xi_*(\mathcal{B})$ for any \mathcal{B} admits a unique weakly renegotiation proof continuation, in which the proposal is accepted by all banks. While $(b, s) \in \Xi_*(\mathcal{B})$ minimizes welfare losses among all bail-ins that rescue \mathcal{B} by giving direct subsidies only to banks in \mathcal{B} , a bail-in $(b, s) \in \Xi_*(\psi(\mathcal{B}))$ minimizes welfare losses among all bail-ins that rescue the banks in \mathcal{B} . The welfare-minimizing assisted bail-in proposal is thus $(b, s) \in \Xi_*(\psi(\emptyset))$. Note that it admits a unique weakly renegotiation proof equilibrium continuation by Theorem A.6. \square

B Proof of Theorems 3.6 and A.6

We begin by proving the results in Section 3 and Appendix A building up to Theorems 3.6 and A.6, respectively. Because the results in Appendix A are generalizations of the results in the main body of the paper, we give only the proofs to the results from Appendix A unless the corresponding statements in the main body contain a refinement that does not translate to the more general setting.

Proof of Lemma A.3. Fix a feasible proposal (b, s) with accepting equilibrium response a and fix a bank i with $b^i > 0$. We first show necessity of the stated conditions. To this end, we suppose towards a contradiction that $a^i = 1$. Consider first the case where Condition 1 is violated. Then the regulator proceeds with a bail-in even if bank i rejects the proposal. Therefore, the resulting subsidies are the same under a and $(0, a^{-i})$, the only difference being that b^i is paid for by bank i in the former and by the regulator in the latter case. Since $b^i > 0$, bank i is strictly better off under $(0, a^{-i})$, contradicting the assumption that a is an equilibrium. Consider now the case where Condition 2 is violated. Because (b, s) is feasible, bank i can afford to pay its liabilities L^i and the bail-in contribution b^i by liquidating at most e^i , i.e.,

$$\ell^i(\bar{p}(b, s, a)) = \frac{1}{\alpha} \left(L^i - c^i - s^i + b^i - \sum_j \pi^{ij} \bar{p}^j(b, s, a) \right)^+. \quad (18)$$

Because Condition 2 is violated, the right-hand side of (18) is strictly larger than $\ell^i(p_*)$, that is, bank i has to liquidate a larger amount of assets in the bail-in than in the bailout. Because the negation of Condition 2 also implies that bank i receives a larger sum of repayments and subsidies in $s_P(\emptyset)$ than in (b, s) , we obtain $V^i(\bar{p}(b, s, a)) < V^i(p_*)$. This is again a contradiction.

For sufficiency, observe that Condition 1 implies that a rejection will provoke the regulator to resort to a public bailout and Condition 2 implies that such an outcome makes bank i worse off in the same way as in the previous paragraph. It is thus optimal for bank i to accept the proposal. \square

Observe that Lemma 3.5 is just a special case of Lemma A.3, using that $\bar{p}(b, s, a) = L$ in any accepting equilibrium response a of a complete bail-in proposal (b, s) . Indeed, because s_P is the welfare-maximizing complete bailout, $w_\lambda(s_P(\emptyset))$ is precisely equal to the minimum of the welfare losses w_P in s_P and the welfare losses w_N in a default cascade. Condition 1 of Lemma A.3 thus reduces to Condition 1 of Lemma 3.5. For Condition 2, we distinguish whether the threat is credible or not. If $w_P < w_N$, then $p_* = L$ because s_P is a complete rescue, hence Condition 2 reduces to $b^i - s^i \leq -s_P^i$. If $w_P \geq w_N$, then $s_P(\emptyset) = 0$, hence Condition 2 reduces to $b^i - s^i \leq \sum_{j=1}^n \pi^{ij} (L^j - p_N^j)$.

Proof of Lemma A.4. Fix a_k and set $\mathcal{B} := \{i \mid a_k^i = 1\}$ for the sake of brevity. Let $\tilde{b}^i = b^i 1_{\{i \in \mathcal{B}\}}$ and $\tilde{s}^i = s^i$ for $i = 1, \dots, n$. Because a_k is an accepting equilibrium response to (b, s) , Conditions 1 and 2 of Lemma A.3 are satisfied for any $i \in \mathcal{B}$. It follows again from Lemma A.3 that a_k is also an accepting equilibrium response to (\tilde{b}, \tilde{s}) because each bank makes exactly the same contributions in $(\tilde{b}, \tilde{s}, a_k)$ as in (b, s, a_k) . To show uniqueness, observe that responses by banks in \mathcal{B}^c have no effect on the outcome because they make no contributions to (\tilde{b}, \tilde{s}) . Due to Condition 1 of Lemma A.3, the regulator will not proceed with the bail-in if only a proper subset of \mathcal{B} accepts the proposal. Therefore, a_k is the unique accepting equilibrium (up to equivalence). \square

Proof of Lemma 3.4. Fix an accepting equilibrium a_k and set $\mathcal{B} = \{i \mid a_k^i = 1\}$. The proof of the first statement is analogous to the proof of Lemma A.4: a_k is the unique accepting equilibrium to (\tilde{b}, s) , where $\tilde{b}^i = b^i 1_{\{i \in \mathcal{B}\}}$ for $i = 1, \dots, n$. For Pareto-dominance, note that Condition 2 of Lemma 3.5 states that the contributions of a bank $i \in \mathcal{B}$ to a bail-in are at most as large as the losses in a rejecting equilibrium.

Therefore, any $i \in \mathcal{B}$ weakly prefers a_k over any rejecting equilibrium. Similarly, any $i \notin \mathcal{B}$ prefers a_k weakly over any rejecting equilibrium because i recovers its assets in full without making any contributions. Finally, the regulator prefers an accepting equilibrium over a rejecting equilibrium by definition. \square

Next, we proceed with the proofs of Lemmas 3.3 and A.5, showing that accepting equilibria are weakly renegotiation proof. Because the statement of Lemma 3.3 in the main text is stronger, asserting that accepting equilibria after *every* proposal are weakly renegotiation proof, we present here its proof in full length and show only afterwards how Lemma A.5 is derived.

Proof of Lemma 3.3. Fix a complete bail-in proposal (b, s) with accepting equilibrium a . For a bank i that makes a positive net contribution to the bail-in, Lemma 3.5 implies that $V^i(s_P) \leq V^i(b, s, a)$ as otherwise bank i would have not accepted the proposal. For any other bank i , it follows immediately that $V^i(s_P) \leq V^i(b, s, a)$ since everybody is rescued in a complete rescue (b, s, a) and hence bank i can reclaim the full value of its interbank assets. Finally, the regulator is weakly better off in a than in any rejecting equilibrium by definition of an accepting equilibrium. This shows that no rejecting equilibrium can Pareto-dominate a . It also implies the second statement of the lemma because a rejecting equilibrium is weakly renegotiation proof only if it is equivalent to (b, s, a) . Since there are only two possible outcomes in a rejecting equilibrium (public bailout and no rescue), the complete bail-in proposal (b, s) has to coincide with the public bailout as, by definition, it rescues every bank in the system.

The first statement will follow once we show that any two accepting equilibria are Pareto-incomparable. Let \tilde{a} be an accepting equilibrium that is not equivalent to a . Let $\mathcal{A} := \{i \mid a^i = 1, b^i > 0\}$ and $\tilde{\mathcal{A}} = \{i \mid \tilde{a}^i = 1, b^i > 0\}$ denote the set of banks that make a positive contribution in a and in \tilde{a} , respectively. By Condition 1 of Lemma A.3, the regulator rejects the bail-in if only a strict subset of either \mathcal{A} or $\tilde{\mathcal{A}}$ accepts the proposal. This shows that $\mathcal{A} \setminus \tilde{\mathcal{A}} \neq \emptyset$ and $\tilde{\mathcal{A}} \setminus \mathcal{A} \neq \emptyset$. Since the subsidies and hence the clearing payment vector are identical in any accepting equilibrium, every bank in $\mathcal{A} \setminus \tilde{\mathcal{A}}$ is strictly better off under \tilde{a} and every bank in $\tilde{\mathcal{A}} \setminus \mathcal{A}$ is strictly better off under a , showing that neither Pareto-dominates the other. \square

Proof of Lemma A.5. Fix $(b, s) \in \Xi_*(\mathcal{B})$ with accepting equilibrium a , which is the unique accepting equilibrium by Lemma B.2. It is thus sufficient to show that (b, s, a)

is not dominated by any rejecting equilibrium. Let $\mathcal{A} := \{i \in \mathcal{F}^c \mid s^i \neq 0 \text{ or } b^i \neq 0\}$ denote the set of banks facing a non-trivial decision in Stage 2. If $b^i > 0$ for some bank $i \in \mathcal{A}$, then Lemma A.3 implies $b^i - s^i \leq \sum_{j=1}^n \pi^{ij} (\bar{p}^j(b, s, a) - p_*^j) - s_P^i(\emptyset)$. Therefore, i is better off in a than in any rejecting equilibrium. If $b^i = 0$ for $i \in \mathcal{A}$, then $s^i > 0$ as otherwise bank i would not be included in the bail-in coordination. Because any proposal (b, s) prescribes subsidies only to banks in \mathcal{B} that need rescuing, it follows that

$$V^i(s_P(\emptyset)) \leq e^i 1_{\{\lambda\alpha \leq 1-\alpha\}} = V^i(s(\mathcal{B})) \leq V^i(b, s, a). \quad (19)$$

The central equality in (19) holds since $e^i 1_{\{\lambda\alpha \leq 1-\alpha\}}$ is the value of a bank i that is rescued by the subsidies $s(\mathcal{B}')$ for any $\mathcal{B}' \ni i$. Since $s(\mathcal{B}')$ are the welfare-maximizing direct subsidies that rescue \mathcal{B}' , the regulator covers the shortfall after banks liquidate their outside assets if $\lambda\alpha > 1 - \alpha$ and prior to the assets' liquidation if $\lambda\alpha \leq 1 - \alpha$. In the latter case, bank i is left precisely with its outside assets. The first inequality in (19) holds since

$$V^i(s_P(\emptyset)) = V^i(s(\chi(\emptyset))) = e^i 1_{\{\lambda\alpha \leq 1-\alpha\}} 1_{\{i \in \chi(\emptyset)\}}.$$

The last inequality in (19) holds because $s^i(\mathcal{B})$ is the subsidy provided to bank i under the welfare-maximizing bailout guaranteeing solvency of banks in \mathcal{B} , thus $s^i \geq s^i(\mathcal{B})$ and hence $\bar{p}(b, s, a) \geq \bar{p}(\mathcal{B})$. Finally, the regulator is weakly better off in a than in any rejecting equilibrium by definition of an accepting equilibrium. This shows that no rejecting equilibrium dominates a for the regulator and the banks facing a non-trivial decision. It also implies the second statement of the lemma by showing that a rejecting equilibrium is weakly renegotiation proof only if it is equivalent to (b, s, a) . \square

Next, we proceed with the proof of Theorem A.6. We start by giving four precursory lemmas, which will be invoked in the proof of the theorem.

Lemma B.1. *Let $(b, s) \in \Xi(\mathcal{B})$. For any response a with $a^0(b, s, a) = \text{"bail-in"}$, welfare losses are equal to*

$$w_\lambda(b, s, a) = w_\lambda(s) - \sum_{i=1}^n \left(\lambda b^i + (1 - \alpha) (\ell_{\mathcal{B}}^i(-s) - \ell_{\mathcal{B}}^i(b - s)) \right) 1_{\{a^i=1\}}.$$

Proof. Fix $(b, s) \in \Xi(\mathcal{B})$ and a with $a^0(b, s, a) = \text{"bail-in"}$. It follows in the same way

as in the proof of Lemma A.1 that $\bar{p}(b, s, a) = p(\mathcal{B})$. Thus, the clearing payment vector does not depend on the particular choice of (b, s) and hence each bank i liquidates a nominal amount $\ell_{\mathcal{B}}^i(b1_{\{a=1\}} - s)$, where $\ell_{\mathcal{B}}$ is given in (15). Welfare losses in the public bailout s are equal to

$$\begin{aligned} w_{\lambda}(s) &= \sum_{i=1}^n (1 - \alpha) \ell_{\mathcal{B}}^i(-s) + \lambda s^i + \sum_{i \in \mathcal{D}(\mathcal{B})} (1 - \beta) (c_h^i + \alpha e^i + (\pi p(\mathcal{B}))^i) + \lambda \delta^i(p(\mathcal{B})) \\ &= w_{\lambda}(b, s, a) + \sum_{i=1}^n \left(\lambda b^i + (1 - \alpha) (\ell_{\mathcal{B}}^i(-s) - \ell_{\mathcal{B}}^i(b - s)) \right) 1_{\{a^i=1\}}, \end{aligned}$$

where we have used equations (2) and (3) in the last equality. \square

Lemma B.2. *Any $(b, s) \in \Xi_*(\mathcal{B})$ admits a unique accepting continuation equilibrium a with $a^i = 1$ for every bank i and $w_{\lambda}(b, s, a) = w_{\lambda}(\mathcal{B})$.*

Proof. Let $(b, s) \in \Xi_*(\mathcal{B})$ and let a with $a^i = 1$ for every bank i and $a^0 = \text{"bail-in"}$. If $w_{\lambda}(\mathcal{B}) = w_{\lambda}(s(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B})$, then the definition of $\Xi_*(\mathcal{B})$ in Definition A.1 imposes that $b^i = \eta^i(\mathcal{B})$ for any bank $i \in \{i_1, \dots, i_{m(\mathcal{B})}\}$. The participation of any such bank i in the bail-in thus lowers welfare losses by $\nu^i(\mathcal{B})$ as argued in Definition A.1. This readily implies $w_{\lambda}(b, s, a) = w_{\lambda}(\mathcal{B})$. If $w_{\lambda}(\mathcal{B}) = w_{\lambda}(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ instead, then by Lemma B.1 we obtain

$$\begin{aligned} w_{\lambda}(b, s, a) &= w_{\lambda}(s) - \sum_{j=1}^{m(\mathcal{B})+1} \nu^{i_j}(\mathcal{B}) \\ &= w_{\lambda}(s(\mathcal{B})) + \lambda \sum_{i=1}^n (s^i - s^i(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})+1} \nu^{i_j}(\mathcal{B}) \\ &= w_{\lambda}(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B}), \end{aligned}$$

where we have used property 2.(c) of a bail-in in $\Xi_*(\mathcal{B})$; see Definition A.1.

To see that a is indeed a continuation equilibrium, we verify the necessary and sufficient conditions given in Lemma A.3. By definition of $\Xi_*(\mathcal{B})$, any bank i 's net contribution $b^i - s^i$ is smaller than its maximal incentive-compatible contribution $t^i(\mathcal{B})$ given in (13). Therefore, the second condition in Lemma A.3 is satisfied for every bank. If $w_{\lambda}(\mathcal{B}) = w_{\lambda}(s(\mathcal{B})) - \sum_{j=1}^{m(\mathcal{B})} \nu^{i_j}(\mathcal{B})$, then Lemma B.1 together with the fact that $b^j = \eta^j(\mathcal{B})$ for $(b, s) \in \Xi_*(\mathcal{B})$ and $j \in \{i_1, \dots, i_{m(\mathcal{B})}\}$ implies that a

deviation by bank i_k with $k = 1, \dots, m(\mathcal{B})$ would lead to welfare losses of

$$w_\lambda(b, s, (0, a^{-i_k})) = w_\lambda(b, s, a) + \nu^{i_k}(\mathcal{B}) \geq w_\lambda(b, s, a) + \nu^{i_{m(\mathcal{B})}}(\mathcal{B}) \geq w_\lambda(s_P(\emptyset)),$$

where the first inequality holds because $i_1, \dots, i_{m(\mathcal{B})}$ is a decreasing order of $\nu^j(\mathcal{B})$ and the second inequality holds by definition of $m(\mathcal{B})$ given in (16). This shows that Condition 1 in Lemma A.3 is satisfied and hence a is indeed an equilibrium response. It also implies that the regulator will not agree to proceed with the bail-in if only a subset of banks accepts, thereby showing uniqueness. If $w_\lambda(\mathcal{B}) = w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ holds instead, then the argument works analogously, where now $\nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ is the smallest welfare impact of a contribution by a bank in the consortium. \square

Lemma B.3. *For any proposed bail-in (b, s) with accepting equilibrium response a , there exists a proposal (\tilde{b}, \tilde{s}) with accepting continuation equilibrium \tilde{a} with $\tilde{b}^i \tilde{s}^i = 0$ and $\tilde{a}^i = 1$ for every bank i such that $w(\tilde{b}, \tilde{s}, \tilde{a}) = w(b, s, a)$.*

Proof. Fix a bail-in proposal (b, s) with equilibrium response a . Define the proposal (\tilde{b}, \tilde{s}) by setting $\tilde{b}^i = (b^i 1_{\{a^i=1\}} - s^i)^+$ and $\tilde{s}^i = (s^i - b^i 1_{\{a^i=1\}})^+$, and let \tilde{a} be the response vector of unanimous acceptance, i.e., $\tilde{a}^i = 1$ for every bank i . Then $\tilde{b}^i 1_{\{\tilde{a}^i=1\}} - \tilde{s}^i = b^i 1_{\{a^i=1\}} - s^i$ for any bank i and hence each bank's net contribution remains unchanged. It follows that $\bar{p}(\tilde{b}, \tilde{s}, \tilde{a}) = \bar{p}(b, s, a)$, hence the definition of welfare losses in (3) yields

$$\begin{aligned} w_\lambda(\tilde{b}, \tilde{s}, \tilde{a}) &= w_\lambda(\bar{p}(\tilde{b}, \tilde{s}, \tilde{a})) + \lambda \sum_{i=1}^n (\tilde{s}^i - \tilde{b}^i 1_{\{\tilde{a}^i=1\}}) \\ &= w_\lambda(\bar{p}(b, s, a)) + \lambda \sum_{i=1}^n (s^i - b^i 1_{\{a^i=1\}}) = w_\lambda(b, s, a). \end{aligned}$$

It remains to check that \tilde{a} is an equilibrium response. It follows from the definition of \tilde{b} that only banks in $\mathcal{R} = \{b^i - s^i > 0, a^i = 1\}$ make a positive net contribution to (\tilde{b}, \tilde{s}) . Lemma A.3 implies that for every $i \in \mathcal{R}$,

$$\tilde{b}^i - \tilde{s}^i = b^i - s^i \leq \sum_{j=1}^n \pi^{ij} (\bar{p}^j(b, s, a) - p_*^j) - s_P^i(\emptyset).$$

Because $\bar{p}(\tilde{b}, \tilde{s}, \tilde{a}) = \bar{p}(b, s, a)$, this shows that $(\tilde{b}, \tilde{s}, \tilde{a})$ satisfies Condition 2 of Lemma A.3.

Let \mathcal{B} be the set $\mathcal{S}(\tilde{s})$ of banks that are solvent if banks receive subsidies \tilde{s} . Note that $\mathcal{S}(\tilde{s}) = \mathcal{S}(s)$ and recall that $\ell^i(\tilde{b}, \tilde{s}, \tilde{a}) = \ell_{\mathcal{B}}^i(\tilde{b}1_{\{\tilde{a}=1\}} - \tilde{s})$ and $\ell^i(b, s, a) = \ell_{\mathcal{B}}^i(b1_{\{a=1\}} - s)$, where $\ell_{\mathcal{B}}$ is defined in (15). To verify that Condition 1 of Lemma A.3 is satisfied for any $i \in \mathcal{R}$, observe that $\tilde{s} \leq s$ implies $\ell_{\mathcal{B}}^i(-\tilde{s}) \geq \ell_{\mathcal{B}}^i(-s)$. Equation (3) thus yields the bound

$$\begin{aligned} w(\tilde{b}, \tilde{s}, (0, \tilde{a}^{-i})) &= w(\tilde{b}, \tilde{s}, \tilde{a}) + \lambda(\tilde{b}^i - \tilde{s}^i) - (1 - \alpha)(\ell_{\mathcal{B}}^i(\tilde{b} - \tilde{s}) - \ell_{\mathcal{B}}^i(-\tilde{s})) \\ &\geq w(b, s, a) + \lambda(b^i - s^i) - (1 - \alpha)(\ell_{\mathcal{B}}^i(b - s) - \ell_{\mathcal{B}}^i(-s)) \\ &= w(b, s, (0, a^{-i})). \end{aligned}$$

Since a is an equilibrium response to (b, s) , it follows from Lemma A.3 that $w_{\lambda}(s_P(\emptyset)) \leq w(b, s, (0, a^{-i})) \leq w(\tilde{b}, \tilde{s}, (0, \tilde{a}^{-i}))$, hence $(\tilde{b}, \tilde{s}, \tilde{a})$ satisfies Condition 1 of Lemma A.3. Therefore, \tilde{a} is an equilibrium response to (\tilde{b}, \tilde{s}) by yet another application of Lemma A.3. \square

Lemma B.4. *In any equilibrium response a to $(b, s) \in \Xi(\mathcal{B})$, welfare losses are bounded from below by $w_{\lambda}(\mathcal{B})$. Moreover, if $w_{\lambda}(b, s, a) = w_{\lambda}(\mathcal{B})$, then $(b, s) \in \Xi_*(\mathcal{B})$.*

Proof. Fix a proposal $(b, s) \in \Xi(\mathcal{B})$ with continuation equilibrium a . If a is a rejecting equilibrium, then $w_{\lambda}(b, s, a) = w_{\lambda}(s_P(\emptyset))$. It follows straight from the definitions of $m(\mathcal{B})$ and $w_{\lambda}(\mathcal{B})$ in (16) and (17), respectively, that $w_{\lambda}(b, s, a)$ is bounded from below by $w_{\lambda}(\mathcal{B})$. Suppose, therefore, that a is an accepting continuation equilibrium. Due to Lemma B.3, we may assume that $s^i = 0$ for any bank i that makes a positive net contribution by passing to an equivalent equilibrium. It follows from Lemma B.1 that the contribution of a bank $i \in \mathcal{R} := \{j \mid b^j > 0, a^j = 1\}$ reduces welfare losses by

$$f^i(b, a) = \left(\lambda b^i + (1 - \alpha)(\ell_{\mathcal{B}}^i(0) - \ell_{\mathcal{B}}^i(b)) \right) 1_{\{a^i=1\}}.$$

Lemma A.3 yields that $w_{\lambda}(b, s, a) + f^i(b, a) = w_{\lambda}(b, s, (0, a^{-i})) \geq w_{\lambda}(s_P(\emptyset))$, showing that welfare losses in the continuation equilibrium a are bounded by

$$w_{\lambda}(b, s, a) \geq w_{\lambda}(s_P(\emptyset)) - f^i(b, a) \tag{20}$$

for any bank $i \in \mathcal{R}$. Welfare losses in an accepting equilibrium thus cannot be lowered from $w_{\lambda}(s_P(\emptyset))$ by more than the contribution of any participating bank in the rescue.

Next, we show that the welfare impact of bank i 's contribution is maximized by $\eta^i(\mathcal{B})$ as defined in (14). Since (b, s) is feasible, bank i 's budget constraint implies that i 's contribution of b^i is bounded above by $B^i = (c^i + \alpha e^i + (\pi p(\mathcal{B}))^i - L^i)^+$. Moreover, Lemma A.3 shows that incentive-compatible contributions are bounded above by $t^i(\mathcal{B})$, hence $b^i \leq \min(t^i(\mathcal{B}), B^i)$. For contributions $x^i \leq B^i$ that satisfy the budget constraint, it holds that $\ell_{\mathcal{B}}^i(x) = \frac{1}{\alpha}(L^i + x - c^i - (\pi p(\mathcal{B}))^i)^+$. The welfare impact $f^i(b, a)$ is thus non-decreasing in b^i if $\lambda\alpha \geq 1 - \alpha$ and it attains its maximum where $L^i + b^i - c^i - (\pi p(\mathcal{B}))^i = 0$ if $\lambda\alpha < 1 - \alpha$. Therefore, the welfare impact among incentive-compatible contributions is maximized by a contribution of size $\eta^i(\mathcal{B})$.

We will conclude the proof of the first statement by distinguishing cases according to the number $|\mathcal{R}|$ of contributing banks. Observe first that $|\mathcal{R}| \geq m(\mathcal{B})$ by the definition of $m(\mathcal{B})$ in (16). Indeed, if $|\mathcal{R}| < m(\mathcal{B})$, then

$$w_\lambda(b, s, a) \geq w_\lambda(s(\mathcal{B})) - \sum_{k=1}^{m(\mathcal{B})-1} \nu^{i_k}(\mathcal{B}) > w_\lambda(s_P(\emptyset))$$

because $\eta(\mathcal{B})$ is the vector of welfare maximizing contributions whose welfare impacts $\nu(\mathcal{B})$ are non-increasingly ordered according to $i_1, \dots, i_{|\mathcal{F}^c|}$. This contradicts the fact that a is an accepting equilibrium by Corollary A.2. Suppose next that $|\mathcal{R}| = m(\mathcal{B})$. Because $s(\mathcal{B})$ vector of direct subsidies to \mathcal{B} with the smallest sum among subsidies that guarantee solvency of banks in \mathcal{B} by Lemma A.1, it follows that $w_\lambda(s) \geq w_\lambda(s(\mathcal{B}))$. Because the welfare impacts of the banks' contributions are non-increasingly ordered according to $i_1, \dots, i_{|\mathcal{F}^c|}$, we obtain

$$w_\lambda(b, s, a) \geq w_\lambda(s(\mathcal{B})) - \sum_{i \in \mathcal{R}} f^i(b, a) \geq w_\lambda(s(\mathcal{B})) - \sum_{k=1}^{m(\mathcal{B})} \nu^{i_k}(\mathcal{B}) \geq w_\lambda(\mathcal{B}), \quad (21)$$

where the last inequality follows directly from (17). Finally, suppose that $|\mathcal{R}| \geq m(\mathcal{B}) + 1$. Because $i_1, \dots, i_{|\mathcal{F}^c|}$ is a non-increasing ordering of the welfare impacts of the banks' contributions, it follows that there is at least one bank $j \in \mathcal{R}$ with $f^j(b, a) \leq \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$. It thus follows from (20) that

$$w_\lambda(b, s, a) \geq w_\lambda(s_P(\emptyset)) - \min_{i \in \mathcal{R}} f^i(b, a) \geq w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B}) \geq w_\lambda(\mathcal{B}). \quad (22)$$

The second statement follows along the lines of the previous paragraph. Consider first the case where $w_\lambda(s(\mathcal{B})) - \sum_{k=1}^{m(\mathcal{B})} \nu^{i_k}(\mathcal{B}) < w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$. Then any

(b, s) with $|\mathcal{R}| \geq m(\mathcal{B}) + 1$ does not attain $w_\lambda(\mathcal{B})$ by (22). Thus $|\mathcal{R}| = m(\mathcal{B})$ and the statement follows because (21) holds with equality if and only if $(b, s) = (b(\mathcal{B}), s(\mathcal{B}))$. If $w_\lambda(s(\mathcal{B})) - \sum_{k=1}^{m(\mathcal{B})} \nu^{i_k}(\mathcal{B}) \geq w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$, then the bound $w_\lambda(s_P(\emptyset)) - \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ can be attained. It follows from (22) that the bound is attained by any (b, s) with $\min_{i \in \mathcal{R}} b^i \geq \nu^{i_{m(\mathcal{B})+1}}(\mathcal{B})$ and $w_\lambda(b, s, a) = w_\lambda(s_P(\emptyset)) - \min_{i \in \mathcal{R}} b^i$. The latter condition is equivalent to property 2 of $\Xi_*(\mathcal{B})$ in Definition A.1, proving that $(b, s) \in \Xi_*(\mathcal{B})$. \square

Proof of Theorem A.6. Lemma B.2 shows that any $(b, s) \in \Xi_*(\mathcal{B})$ has a unique accepting equilibrium with welfare losses equal to $w_\lambda(\mathcal{B})$. By Lemma A.5, this accepting equilibrium is the unique weakly renegotiation proof continuation equilibrium. The regulator is thus aware that any bail-in from $\Xi_*(\mathcal{B})$ that he proposes in the first stage will be implemented in equilibrium. Moreover, an accepted bail-in proposal from $\Xi_*(\mathcal{B})$ is strictly preferred by the regulator over any other accepted proposal $(b', s') \notin \Xi_*(\mathcal{B})$ by Lemma B.4. Therefore, if $w_\lambda(\mathcal{B}) \leq w_\lambda(s_P(\emptyset))$, the regulator's only rational choice in stage 1 is to propose a bail-in from $\Xi_*(\mathcal{B})$. If, on the other hand, $w_\lambda(\mathcal{B}) > w_\lambda(s_P(\emptyset))$, then regulator will implement the public bailout $s_P(\emptyset)$ from Lemma A.1 either by proposing it in the first stage or by choosing it in the third stage after an arbitrary proposal in the first stage. \square

Theorem 3.6 follows from Theorem A.6 by setting $\mathcal{B} = \{1, \dots, n\}$ and considering that in the case of all-or-nothing interventions, the subsidies $s_P(\emptyset)$ of Corollary A.2 are either given by the complete bailout from Lemma 3.1 or they are the vector of zero subsidies, which results in a default cascade.

Proof of Theorem 3.6. Let $\mathcal{B} = \{1, \dots, n\}$. If $w_P < w_N$, then $s_P(\emptyset)$ is the complete bailout of Lemma 3.1. Therefore, $t^i(\mathcal{B}) = 0$ for every bank i , hence also $\eta^i(\mathcal{B}) = 0$, showing that no contributions are incentive-compatible. The unique sequential equilibrium outcome is thus the complete bailout s_P . If $w_P \geq w_N$, then $s_P(\emptyset)$ corresponds to the no-intervention outcome, where $s_P^i(\emptyset) = 0$. Therefore, $t^i(\mathcal{B}) = \sum_{j=1}^n \pi^{ij} (L^j - p_N^i)$, where we have used that $p(\mathcal{B}) = L$ in a complete rescue. It follows from Theorem A.6 that welfare losses and proposed bail-ins in an accepting equilibrium are of the desired form, and it follows from Lemma 3.3 that all accepting equilibria are precisely the weakly renegotiation proof equilibria. \square

C Proofs of additional results

Lemma 2.1 follows as a consequence of the following result, which is a straightforward adaptation of Theorem 1 in Rogers and Veraart (2013) to our setting.

Lemma C.1. *Let $\varphi^{(k)}$ denote the k -fold application of the operator*

$$\varphi^i(p) := \begin{cases} L^i & \text{if } c^i + \alpha e^i + (\pi p)^i \geq L^i, \\ \left(\beta(c_h + \alpha e + \pi p)^i - c_f^i \right)^+ & \text{otherwise.} \end{cases}$$

Then $\bar{p} = \lim_{k \rightarrow \infty} \varphi^{(k)}(L)$.

Proofs of Lemma 2.1 and C.1. Any clearing payment vector p is a fixed point of φ because a bank i defaults if and only if it cannot repay its liabilities after liquidating its assets. Since φ is a monotone operator, Tarski's fixed-point theorem implies that there exists a smallest and a greatest fixed point \underline{p} and \bar{p} , respectively. Because any fixed point of φ is bounded above by L , it follows that $\bar{p} = \lim_{k \rightarrow \infty} \varphi^{(k)}(L)$. The final statement of Lemma 2.1 follows from the monotonicity of the banks' value of equity and of the welfare losses in the clearing payment vector p . \square

Proof of Lemma 3.7. We begin by showing the identity

$$w_N = \sum_{i=1}^n (c^i + e^i - V^i(p_N)) + (1 + \lambda) \sum_{i \in \mathcal{D}(p_N)} \delta^i(p_N), \quad (23)$$

where we denote by p_N the greatest clearing vector in absence of intervention. It follows from Definition 2.1 that the clearing payment vector p_N satisfies the relation $\beta(c_h^i + \alpha e^i + (\pi p_N)^i) = p_N^i + c_f^i - \delta^i(p_N)$ for any defaulting bank $i \in \mathcal{D}(p_N)$, where $(\pi x)^i = \sum_{j=1}^n \pi^{ij} x^j$ for any vector x . Using equations (1) and (2), we obtain

$$\begin{aligned} \sum_{i=1}^n V^i(p_N) &= \sum_{i \notin \mathcal{D}(p_N)} (c^i + e^i - (1 - \alpha)\ell^i(p_N) + (\pi p_N)^i - p_N^i) \\ &= \sum_{i=1}^n (c^i + e^i - (1 - \alpha)\ell^i(p_N)) - \sum_{i \in \mathcal{D}(p_N)} ((1 - \beta)(c_h^i + \alpha e^i + (\pi p)^i) - \delta^i(p_N)) \\ &= \sum_{i=1}^n (c^i + e^i) + (1 + \lambda) \sum_{i \in \mathcal{D}(p_N)} \delta^i(p_N) - w_N, \end{aligned}$$

where we have used that $\sum_{i=1}^n (\pi x)^i = \sum_{i=1}^n x^i$ for any vector x because π is a stochastic matrix. This readily implies (23). Together with the definitions of χ_0 and χ_N in (6) and (7), respectively, it follows that

$$\begin{aligned}\chi_N &= \sum_{i \in \mathcal{F}} (-c^i - \alpha e^i - (\pi L)^i + L^i) + \sum_{i=1}^n (c^i + e^i - V^i(p_N)) \\ &= \chi_0 + w_N - \lambda \sum_{i \in \mathcal{D}(p_N)} \delta^i(p_N).\end{aligned}\tag{24}$$

The characterization of the complete bailout in Lemma 3.1 and the definition of χ_0 imply that $w_P = \lambda \chi_0 + \lambda \sum_{i \in \mathcal{D}(p_N)} \delta^i(p_N) + \min(\lambda \alpha, 1 - \alpha) \sum_{i=1}^n \ell_c^i(0)$. Therefore, solving (24) for w_N and subtracting w_P , we obtain

$$w_N - w_P = \chi_N - (1 + \lambda) \chi_0 - \min(\lambda \alpha, 1 - \alpha) \sum_{i=1}^n \ell_c^i(0),$$

which establishes the claim. \square

Proof of Proposition 4.2. We start by showing that in any regular financial system with homogeneous cash holdings, $\nu^i(\pi)$ is a monotonically increasing transformation of $\eta^i(\pi)$ that is independent of the precise form of π or the identity of the bank i , i.e., $\nu^i(\pi_1) = f(\eta^i(\pi_1))$ and $\nu^j(\pi_2) = f(\eta^j(\pi_2))$ for the same function f . First, due to Lemma B.3 we can assume that banks make contributions only if they do not receive a subsidy, that is, $s_P^i = 0$ for any contributing bank i and therefore $\nu^i(\pi) = \lambda \eta^i(\pi) - (1 - \alpha)(\ell_c^i(\eta^i(\pi)) - \ell_c^i(0))$. Because banks have homogeneous cash holdings, there exists c such that $c^i = c$ for every non-fundamentally defaulting bank i . Regularity of a financial system π then implies that

$$\ell_c^i(x) = \min\left(\frac{1}{\alpha}(x - c)^+, e^i\right).$$

We distinguish two cases: If $\lambda \alpha \geq 1 - \alpha$, the definition of $\nu^i(\pi)$ in Theorem 3.6 and the regularity of the financial system imply that $(\eta^i(\pi) - c)^+ \leq \alpha e^i$, hence $\ell_c^i(\eta^i(\pi)) = \frac{1}{\alpha}(\eta^i(\pi) - c)^+$. Therefore, $\nu^i(\pi) = \lambda \eta^i(\pi) - \frac{1 - \alpha}{\alpha}(\eta^i(\pi) - c)^+$ is indeed a monotonically increasing transformation of $\eta^i(\pi)$ that does not depend on the precise form of π and the identity of bank i . If $\lambda \alpha < 1 - \alpha$, note that $c \leq 0$ implies that $\eta^i(\pi) = 0$ for any bank i and hence $\nu^i(\pi) = 0$ is trivially an increasing transformation of $\eta^i(\pi)$. If $c > 0$, then $\ell_c^i = 0$ for any bank i and hence $\nu^i(\pi) = \lambda \eta^i(\pi)$ is an

increasing function of $\eta^i(\pi)$ independent of π and i .

Because ν is a monotonic transformation of η independent of the network, the assumption that interbank losses are strongly $m(\pi) + 1$ -more concentrated in π than in π' implies that $\nu^{i_k(\pi)}(\pi) \geq \nu^{i_k(\pi')}(\pi')$ for every $k \leq m(\pi) + 1$, where $i_k(\pi)$ and $i_k(\pi')$ are non-increasing orders of $\nu^i(\pi)$ and $\nu^i(\pi')$, respectively, as in Theorem 3.6. Observe that the subsidies s_P in the public bailout of Lemma 3.1 do not depend on the particular network structure, given that the network is regular. Therefore, $w_P(\pi) = w_\lambda(0, s_P, 1) = w_P(\pi')$. The assumption that interbank losses are strongly $m(\pi) + 1$ -more concentrated in π than in π' thus implies that

$$w_P - \sum_{j=1}^k \nu^{i_j(\pi)}(\pi) \leq w_P - \sum_{j=1}^k \nu^{i_j(\pi')}(\pi') \quad (25)$$

for any value of $k \leq m(\pi) + 1$.

Consider first the case where $w_*(\pi') = w_P - \sum_{j=1}^{m(\pi')} \nu^{i_j(\pi')}(\pi')$. For the sake of brevity, write $m = m(\pi)$ and $m' = m(\pi')$. If $m' \leq m$, then the statement follows immediately from (25). If $m' \geq m + 1$, then the definition of m' implies that

$$w_*(\pi') \geq w_N(\pi') - \nu^{i_{m'}(\pi')}(\pi') \geq w_N(\pi') - \nu^{i_{m+1}(\pi')}(\pi').$$

It now follows from (9) that $w_*(\pi') \geq w_N(\pi) - \nu^{i_{m(\pi)+1}(\pi)}(\pi) \geq w_*(\pi)$ as desired. If $w_*(\pi') = w_N(\pi') - \nu^{i_{m'+1}(\pi')}(\pi')$, then it follows straight from (9) that $w_*(\pi') \geq w_N(\pi) - \nu^{i_{m(\pi)+1}(\pi)}(\pi) \geq w_*(\pi)$, thereby concluding the proof. \square