Welfare Gains from Market Insurance: The Case of Mexican Oil Price Risk [*]

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Abstract

Over the past two decades, Mexico has hedged oil price risk through the purchase of put options. We examine the resulting welfare gains using a standard sovereign default model calibrated to Mexican data. We show that hedging increases welfare by reducing income volatility and reducing risk spreads on sovereign debt. We find welfare gains equivalent to a permanent increase in consumption of 0.44 percent with 90 percent of these gains stemming from lower risk spreads.

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JEL Classification: F3; F4; G1

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1 Introduction

The sharp decline in oil prices that started in late 2014 caught many oil-exporting countries off guard, but not Mexico. Following a long-standing practice, in the fall of 2014, the Ministry of Finance had purchased put options with one year maturity to hedge 228 million barrels of oil, about 28 percent of production, with a strike price of US$ 76.4 per barrel—US$ 31.1 above the actual average oil price in 2015.[1]

Sharp changes in oil prices have coincided with substantial fluctuations in economic activity and inflation (Kilian and Murphy, 2014; Husain et al., 2015). For net oil exporters, the negative consequences of the shock are also often amplified by rising risk spreads on sovereign debt (Baffes et al., 2015). In this context, designing policies to manage risks emerging from the exposure to commodity-price swings remains highly relevant, particularly for commodity exporters (Borensztein et al., 2013). Drawing on Mexico's experience, we assess the benefits and costs of using market insurance to hedge commodity price risk and enhance macroeconomic resilience. To this end, we augment a standard sovereign default model with access to put options —calibrated to Mexican data—to determine the size and main channels of welfare gains relative to a counterfactual scenario without put options. Our main contribution is to analyze how the availability of hedging instruments affects default incentives and welfare.

Our benchmark economy is exposed to price risk of its commodity exports and can borrow through one-period defaultable debt acquired by risk-neutral foreign investors. The default decision and pricing of debt follows a willingness-to-pay framework à la Eaton and Gersovitz (1981). Since the country can default whenever it finds it optimal, bond prices fluctuate with the risk of default. The country can also purchase put options from risk-neutral foreign investors to lock in a minimum price for its commodity exports in the subsequent period. In the absence of put options, consumption smoothing takes place only through defaultable debt. The access to put options allows for additional benefits as they can help smooth income fluctuations arising from oil price volatility. But the upfront cost of put options also reduces consumption in the current period. In a simplified two-period version of the benchmark economy, we establish the aforementioned benefits—net of the cost of hedging—analytically.

After illustrating the main mechanisms in a simple model, we perform quantitative simulations in our full-fledged benchmark economy and compare our results to a version without put options.

[1] The options were in the money in 2009 and 2016 as well.

First, we find that using put options yields welfare gains[2] equivalent to a permanent increase in consumption of 0.44 percent. Second, we decompose these gains between those operating through a reduction of borrowing costs and those from income smoothing. The first channel emerges from the change in default incentives induced by the reduction in downside risks to income through put options. The second channel is similar to Lucas (1987), in which lower income fluctuations translate into a smoother consumption path, which increases welfare for risk averse agents. We conclude that about 90 percent of the welfare gains stem from the borrowing costs channel. Compared to the economy without hedging, risk spreads on debt are 19 basis points lower in the hedging economy.[3]

We also find that the welfare gains decline if the cost of the options includes a premium above the actuarially fair price. However, only a sizable premium would reduce the welfare gains to zero. We also find that welfare gains increase with the strike price of put options, the hedged volume of oil, the volatility of oil prices, and with risk aversion of foreign investors. Finally, we find that selling oil forward can generate larger welfare gains than buying put options because they imply not incurring the upfront cost of insurance. However, political economy considerations cannot be ignored since forwards also imply giving up any revenue windfall if oil prices rise.[4]

Our paper contributes to the literature on welfare gains from market insurance with contributions including Caballero and Panageas (2008), who focus on optimal hedging strategies in countries facing risks of sudden stops in capital flows; and Borensztein et al. (2013) who examine the welfare gains from hedging through options and forwards in the presence of non-defaultable debt. Our paper differs from these studies by exploring synergies between hedging instruments and defaultable debt in increasing welfare. Furthermore, our paper is also related to studies examining the welfare gains from contingent debt, such as Hatchondo and Martinez (2012), who focus on GDP-indexed bonds, and Borensztein et al. (2017), who focus on catastrophic bonds. One closely related paper is Lopez-Martin et al. (forthcoming), calibrated also to Mexican data. However, they model the government and the private sector separately and focus on the income-smoothing aspect of macro hedging. We model the economy as a whole and focus on understanding the sources of welfare gains from macro hedging, by looking at the relative importance of income smoothing and the relaxation

[2] We define welfare gains as the improvement in the present discounted value of the utility derived from consumption.

[3] The welfare gains from income smoothing, at 0.04 percent, are very similar to what comes out of applying Lucas (1987)'s methodology to Mexican consumption series during 1996-2016.

[4] A clear example of political cost is Ecuador, cited in Daniel (2001), whose government conducted several hedging transactions through options and oil swaps in early 1993 that led to significant losses, ultimately triggering heavy criticism and even the appointment of a special committee to investigate allegations of corruption against the monetary authorities.

of borrowing constraints as drivers of those gains. Finally, our paper is also related to the literature on quantitative models of sovereign default such as Aguiar and Gopinath (2006), Arellano (2008), and Hatchondo and Martinez (2009), although our focus is on the welfare gains from relying on hedging instruments as a complement to defaultable debt.

The paper is organized as follows: Section 2 describes Mexico's oil hedging program; Section 4 presents the benchmark model; Section 3 presents a two-period model to understand the benefits and costs of hedging; Section 5 presents quantitative results; Section 6 presents two extensions; and Section 7 concludes.

2 Mexico's Oil Hedging Program

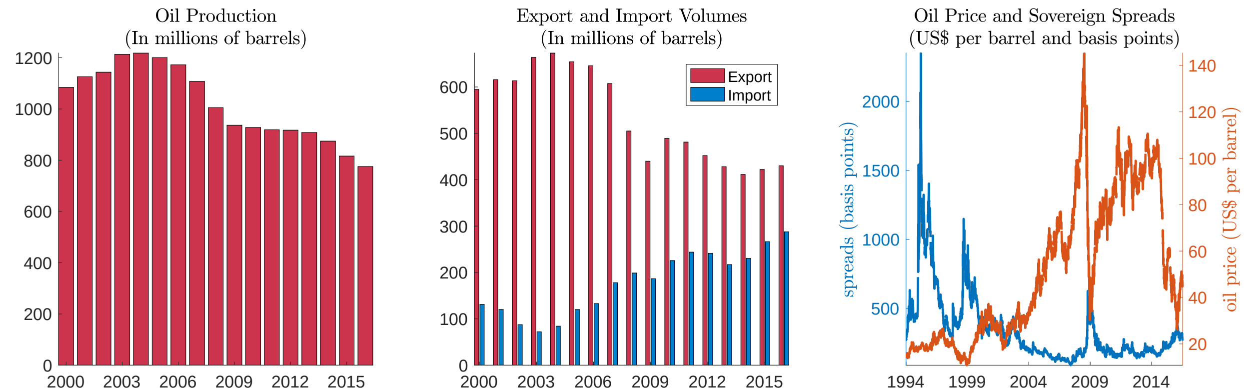
Mexico's government has systematically hedged oil-price risk for at least twenty years through a hedging program that is known to be the largest in the world (Blass, 2017). The program, as it is known today, was set up in 2001, (Duclaud and Garcia, 2012), although Mexico used market hedging instruments as early as 1990 (see Potts and Lippman (1991) and Daniel (2001)); however, details about those earlier operations are scarce. However, Mexico is not the only country that has used these instruments. Ecuador, Ghana, and more recently Uruguay are other examples of countries which have relied on hedging instruments to guard against oil-price volatility.

According to the U.S. International Energy Administration, Mexico is the 12th largest oil producer in the world. The oil sector is controlled by the fully state-owned company, Petroleos Mexicanos (PEMEX). Therefore, oil-related risks directly affect Mexico's public finances. This is the reason why the Mexican treasury conducts the hedging. On average, over 2000-2016, oil-related revenues represented 32 percent of total fiscal revenues, of which, 47 percent corresponded to oil exports, and the remainder to net domestic sales of petroleum products. Over the same period, oil exports averaged 11 percent of total exports. While the importance of oil for the economy and Mexico's public finances has declined since the mid-2000's,[5] oil revenues still represented about 16 percent of total fiscal revenues and close to 5 percent of total exports in 2016. Moreover, there is a high negative correlation between risk spreads on external sovereign debt and oil prices, with a correlation coefficient of -0.59 over the past twenty years (Figure 1). A 2013 constitutional reform opened the oil sector to private investment, but the private oil sector remained in its infancy as of end-2017. However, it is expected to gain importance as private investment picks up and new oil

[5]The decline has been the result of falling oil production due to aging oil fields, lower oil prices since 2014, and higher non-oil tax revenues from a tax reform in place since 2014.

fields are exploited, which would eventually reverse the declining importance of oil in the economy.[6]

Figure 1: Oil Production, Oil Prices, and Sovereign Spreads



Source: INEGI.

The Mexican treasury conducts hedging operations with the main objective of reducing the risk of fiscal revenue shortfalls during any given fiscal year. Specifically, the Mexican treasury includes in its annual budget an assumption on the export price of its oil for the subsequent fiscal year, computed as the weighted average between historical prices and futures. To reduce the risk of a decline in oil-related revenues, the Mexican treasury purchases Asian put options with strike price equal or close to the oil price assumed in the budget. The use of Asian options allows the treasury to lock in a minimum price for the whole fiscal year.[7] The program is executed through several contracts with foreign banks as counterparts. Most of the contracts include Maya oil, a type of Mexican heavy crude oil, as underlying asset, but a small fraction of contracts use the Brent as underlying asset. Maya oil dominates because it represents about 80 percent of Mexico's oil export volumes.

While on average, Mexico produced 1 billion barrels annually over 2000-2016, of which it exported roughly half, Mexico also imported about 178 million barrels of petroleum products annually, over the same period. The domestic sale of imported petroleum products at regulated prices, which did not move one-for-one with international prices, compensated losses (or gains) in crude oil export

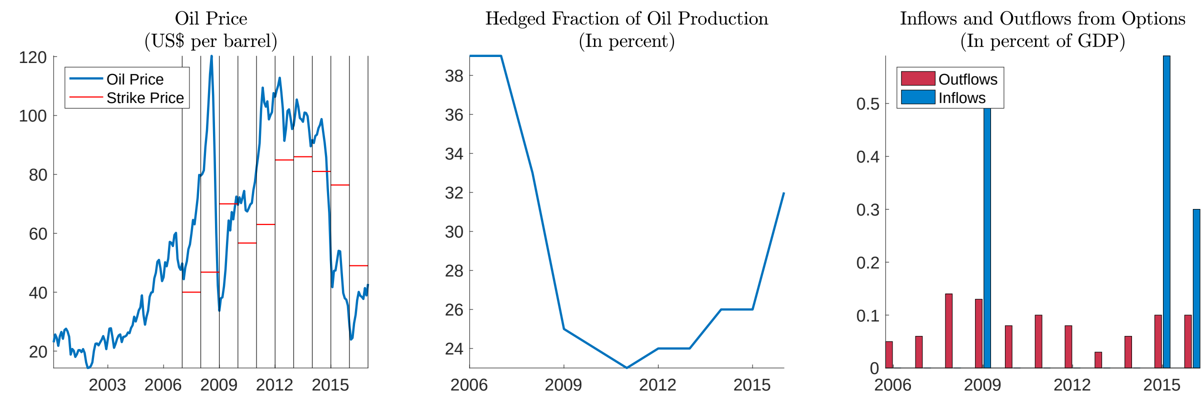
[6] A description of the reform that opened the energy sector to private investment and its potential implications for future oil production can be found in IMF (2014).

[7] An American or European put option is exercised if the spot price on a particular day exceeds the strike price. In contrast, an Asian put option is exercised if the average spot price for a pre-determined period, which in the case of Mexico is one year, exceeds the strike price. In this way, Mexico guarantees a minimum average price of oil for the whole fiscal year.

revenues that resulted from fluctuations in international oil prices.[8] After taking these offsetting factors into account, the Mexican treasury hedged, on average, 29 percent of total production over the past 10 years.

Since 2001, the cost of the options has averaged 0.1 percent of GDP per year and they have been exercised only in three occasions: in 2009, 2015, and 2016, with payoffs reaching 0.5, 0.6, and 0.3 percent of GDP respectively (Figure 2).

Figure 2: Mexico's Oil Hedging Program



Source: INEGI and authors' calculations.

3 Benefits/Costs of Hedging in a Two-period Model

Before proceeding to the quantitative analysis, we use a simple two-period model, $t \in \{0,1\}$ to illustrate analytically the benefits and costs of hedging and its complementarities with defaultable debt. Consumers choose in period 0 how much to issue in bonds d at a price q as to maximize the present discounted value of utility derived from consumption, with discount factor $\beta < 1$. Income is given by y in period 0 while it can take values y^H or $y^L < y^H$ in period 1, with probabilities p and $1-p$, respectively. After income uncertainty is realized in period 1, consumers can default on

[8] A process of liberalization of domestic fuel prices began in 2016 and was completed by end-2017.

their bonds, in which case income equals $y^{def} > 0$. The maximization problem is summarized by

$$\begin{aligned} U_0 &= \max_d \ \log c_0 + \beta E_0 \log c_1 \\ \text{s.t.} \quad & c_0 = qd + y \\ & c_1^i = \max\left\{y^i - d, y^{def}\right\}, i \in \{H, L\} \end{aligned}$$

where for simplicity we assume $u(c) = \log c$. Assuming that the risk-free rate, r^*, equals zero, risk-neutral foreign investors acquire the bonds at a price that satisfies

$$q = \begin{cases} 1, & \text{if } y^L - d \geq y^{def}; \\ p, & \text{if } y^L - d < y^{def} \leq y^H - d; \\ 0, & \text{if } y^H - d < y^{def}, \end{cases}$$

where the first condition implies that risk spreads are zero because in those circumstances default is never optimal. The second condition states that consumers always default under a bad realization of income in period 1, in which case $q = p < 1$. Finally, the third condition implies that the bond is worthless since the consumers would default with probability 1 as it is always optimal to do so. We now introduce hedging in this framework. Suppose that the consumer buys insurance in period 0 that guarantees a level of income of at least $\bar{y}$ in period 1 at a cost ξ that satisfies

$$\xi = \begin{cases} p(\bar{y} - y^H) + (1-p)(\bar{y} - y^L), & \text{if } \bar{y} \geq y^H; \\ (1-p)(\bar{y} - y^L), & \text{if } y^L < \bar{y} < y^H; \\ 0, & \text{if } \bar{y} \leq y^L. \end{cases}$$

Given the structure of put options, the problem for the economy becomes

$$\begin{aligned} U_0^{hedge} &= \max_d \log c_0 + \beta E_0 \log c_1 \\ \text{s.t.} \quad & c_0 = qd + y - \xi \\ & c_1^i = \max\left\{\max\{\bar{y}, y^i\} - d, y^{def}\right\}, i \in \{H, L\} \end{aligned}$$

Role of Hedging. Let us first assume that the insured level of income, $\bar{y}$, equals the unconditional mean of period-1 income, that is $\bar{y} = py^H + (1-p)y^L$. Hedging plays first an income-smoothing role by reducing income fluctuations in period 1 since $y^L < \bar{y} < y^H$ and with hedging, period-1 income is either $\bar{y}$ or y^H. Second, hedging can alter default and borrowing incentives, but not necessarily in an unambiguous way. In the following propositions we demonstrate the various implications of hedging for default incentives and welfare. We also want to stress that this exercise is to compare an economy with hedging and an economy without hedging. In particular, we want to understand whether buying nothing is better or worse than buying a specific quantity of put options with a specific strike price.

Proposition 1. Default Incentives and Hedging

Consider an economy with no hedging in which $\hat{y}^{def}$ *and* $\hat{\hat{y}}^{def}$ *are such that consumers never default if the income loss from default is too large, i.e.* $y^{def} < \hat{y}^{def}$*; they always default if the income loss from default is too small, i.e.* $y^{def} > \hat{\hat{y}}^{def}$*; and they only default after a low realization of income if the income loss from default is neither too large nor too small,* $y^{def} \in (\hat{y}^{def}, \hat{\hat{y}}^{def})$*. Introducing hedging in this economy increases* $\hat{y}^{def}$ *and reduces* $\hat{\hat{y}}^{def}$*.*

Proof. See Appendix B.1. □

Intuitively, Proposition 1 states that hedging can change default thresholds, by either improving or worsening incentives to default. The direction in which those incentives change depends on how costly it is to default. When default is so costly so that it never happens (i.e. the farthest left region in Figure 3, $y^{def} < \hat{y}^{def}$), hedging does not affect default incentives. Reduce default costs a bit and we enter the middle region, i.e. $y^{def} \in (\hat{y}^{def}, \hat{\hat{y}}^{def})$, where changes in the default thresholds can lead to the economy to never default or to always default. In the former case, the result follows from the fact that hedging helps secure a minimum income—above the default level—and therefore reduces incentives to default and the cost of debt. In the latter case, the income under default is higher, and therefore default is less costly. Because hedging requires increasing borrowing to pay for the upfront cost of insurance, it may worsen default incentives given that it is not so costly to default. The farthest right region is in the proposition for completeness only. In this region, the cost of default is so small that the economy would always default. Therefore, it is not an interesting case to analyze since no creditor would lend to consumers who would default with probability 1. In the following propositions, we analyze the implications for welfare under all these cases except for the last one.

Proposition 2. No Default in Equilibrium

When default is too costly, such that the economy does not default in equilibrium, introducing hedging increases social welfare and the country borrows more.

Proof. See Appendix B.2. □

In this case, hedging is clearly beneficial. Income becomes smoother and the economy can afford to borrow more. This insight is similar to the work by Borensztein et al. (2013) who derived welfare implications of hedging in a world with non-defaultable debt.

Proposition 3. Default Only When Income is Low

When the economy defaults only when $y = y^L$, whether hedging increases or decreases welfare depends on its impact on default incentives:

1. *if hedging reduces default incentives, hedging increases welfare, but borrowing might increase or decrease.*
2. *if hedging does not change default incentives, it reduces welfare and increases borrowing.*
3. *if hedging increases default incentives, both social welfare and borrowing decline.*

Proof. See Appendix B.3. □

In case 1, both the income-smoothing and borrowing cost channels imply a welfare gain despite the upfront cost of insurance. However, the impact of hedging on borrowing is ambiguous: On the one hand, more borrowing is desirable to purchase insurance; on the other hand, more borrowing increases the likelihood of default and hence the cost of debt, ultimately reducing incentives to borrow.

In case 2, hedging ensures higher income in the low state of the world than in absence of hedging, only if there is no default; however, if the economy defaults when $y = y^L$, hedging does not change default incentives, nor the level of income since default implies the same level of income under default, y^{def}, as the no-hedging economy. In this case, consumers borrow more in period 0 to purchase insurance, but income in period 1 is the same with or without hedging. As a result, hedging only lowers current disposable income and reduces welfare.

In case 3, if hedging increases default incentives it clearly reduces welfare since it would imply that the economy moves from the region where it only defaults in the bad state of nature to the region where it always defaults.

Figure 3 summarizes key insights from the above propositions. The left regions in the figure correspond to areas where the cost of default is high. In these regions, hedging is always desirable either because both, the borrowing costs channel and the income smoothing channel are at work, which is the case when hedging reduces default risk, or because only the income smoothing channel is at work, which is the case when there is no default in equilibrium. The model also includes regions where hedging reduces welfare because the costs of default are small. However, the fact that defaults are rare events —Mexico has defaulted only 8 times since 1821 —and the empirical literature documents significant output losses following sovereign defaults, the left regions in Figure 3 are likely to be the more empirically relevant cases. We resort now to our quantitative analysis to shed light on the size of welfare gains from hedging.

4 Model Economy

The question of welfare gains from hedging commodity price risk has been explored in models with non-defaultable debt (Borensztein et al., 2013, 2017). Since default risk is in practice not zero, our departure point is a standard sovereign default model as in Aguiar and Gopinath (2006) and Arellano (2008). In this economy, a country can issue one-period bonds in international credit markets on which the country can default when it finds it beneficial to do so. But default is costly. A default implies losing access to international credit markets, although not permanently, and lower income. There is only one source of risk in this economy: oil prices. In addition to issuing defaultable debt, the country can acquire put options to hedge oil price risk in international financial markets. The quantitative assessment of the welfare gains from using put options will be conducted by comparing outcomes with and without put options.

4.1 Benchmark Model with Defaultable Debt and Put Options

The economy is populated by infinitely-living, risk-averse representative agents who make decisions in order to maximize the expected present discounted value—with discount factor β—of the utility derived from consumption:

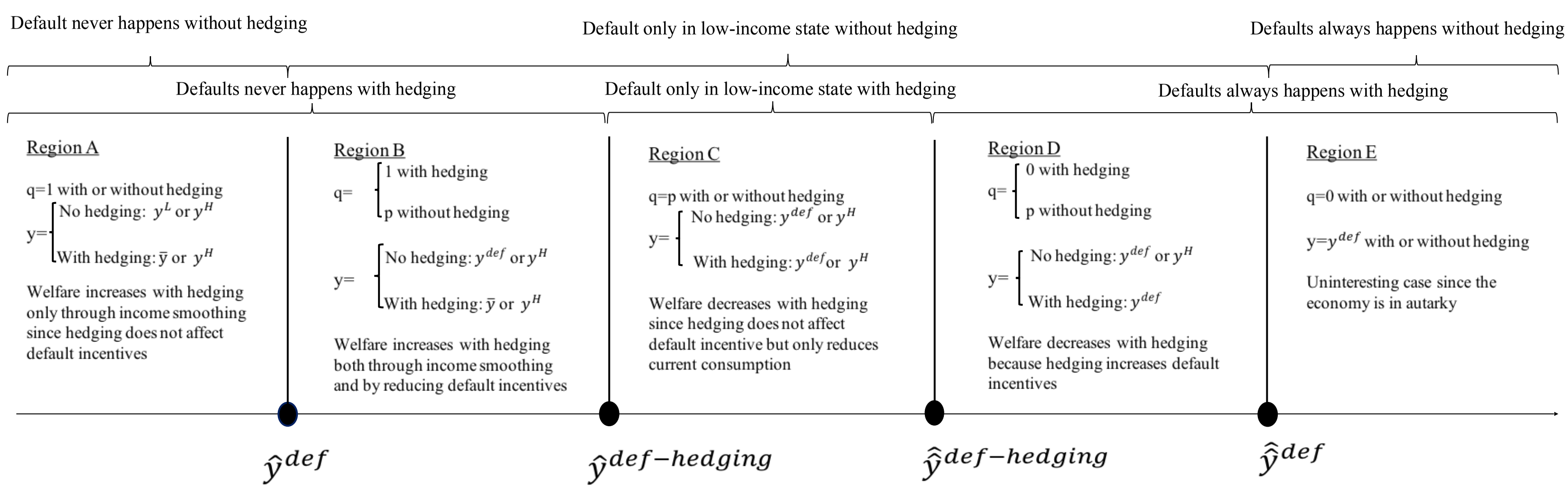
$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right] \tag{1}$$

where risks preferences of the consumer are represented by a standard constant relative risk aversion (CRRA) utility function with coefficient of risk aversion γ.

Total income in this economy, Y_t, has two components: non-oil income (F_t) and oil income (X_t),

$$Y_t = F_t + X_t \equiv F_t + p_t Q_t \tag{2}$$

Figure 3: Two-period Model



Source: Authors' construction.

where p_t and Q_t are the price and quantity of oil production respectively. The only source of risk in this economy is the price of oil, p_t, which is assumed to follow an autoregressive stochastic process, to be defined momentarily. We assume that non-oil income, F_t, grows deterministically at a constant rate G in every period. We normalize all variables by F_t and denote them with lower letters:

$$y_t \equiv \frac{Y_t}{F_t} = 1 + p_t \frac{Q_t}{F_t} \equiv 1 + p_t \mathcal{Q} \tag{3}$$

$$E_0 \left[\sum_{t=0}^{\infty} (\beta G^{1-\gamma})^t \frac{c_t^{1-\gamma}}{1-\gamma} \right] \tag{4}$$

where $\mathcal{Q} = \frac{Q_t}{F_t}$ and $c_t = \frac{C_t}{F_t}$ denote normalized oil production and consumption. To further simplify the exposition, we assume that $\mathcal{Q}$ is constant, which as we will discuss in the calibration section, it is not an inaccurate representation of the data. From now on we will focus on the normalized problem knowing that the original problem can always be recovered by multiplying normalized variables by F_t (See Appendix A for details).

In every period, consumers have an initial level of wealth, w_t, composed by income, y_t, and bonds, b_t, acquired in the previous period: $w_t = y_t + b_t$. Consumers allocate this wealth among consumption, c_t; zero-coupon one-period bonds, b_{t+1}, which they can acquire in international credit markets at a price q_t; and put options acquired in international financial markets at a unit price $\xi(\bar{p}_t)$, which entitles them to sell a fraction $\alpha \mathcal{Q}$ of oil production in period $t+1$ at a pre-determined strike price $\bar{p}_t$.[9] [10]

$$c_t + q_t G b_{t+1} + \alpha \mathcal{Q} G \xi(\bar{p}_t) = w_t \tag{5}$$

where b_{t+1} can take positive or negative values reflecting whether the country lends or borrows in international credit markets. The consumer arrives to the following period, $t+1$, with wealth w_{t+1}, given by

$$w_{t+1} = y_{t+1} + \alpha \mathcal{Q} \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1} \tag{6}$$

where $\alpha \mathcal{Q} \max\{\bar{p}_t - p_{t+1}, 0\}$ reflects the fact that the put options locked in a minimum price, $\bar{p}_t$, for the hedged fraction of oil production. The optimization problem, under no default, is summarized

[9] $\alpha \in (0, 1)$ captures the fact that Mexico hedges only part of production, as discussed in the previous section.

[10] The oil put options in our economy are equivalent to GDP put options since the non-oil income is non-stochastic.

by

$$V^c(w_t, p_t) = \max_{c_t, b_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} E_t \left[V(w_{t+1}, p_{t+1}) \right] \tag{7}$$
$$\text{s.t.} \quad c_t + q_t G b_{t+1} + \alpha \mathcal{Q} G \xi(\bar{p}_t) = w_t$$
$$w_{t+1} = y_{t+1} + \alpha \mathcal{Q} \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1}$$

where $V^c(w_t, p_t)$ denotes the value function under continuation or no default, with the state of the economy summarized by two state variables, $\{w_t, p_t\}$.

Default Decision. In every period, consumers can default on their debt, in which case the economy gets excluded from international financial markets. While in default, consumers cannot borrow nor purchase put options and the economy resorts to financial autarky. Furthermore, consumers cannot exercise the put options and then default. These restrictions on exercising and purchasing put options can be motivated by sanctions in the state of default. Besides the exclusion from international financial markets, default implies an income loss $h(y_t)$ in every period, reflecting the assumption that credit plays an essential role in the economy. This assumption can be justified by the existence of a minimum scale for some investment projects that would not be reached without external financing, preventing those investments from being carried out. Alternatively, the exclusion from international financial markets may obstruct the normal conduct of business of companies operating with non-residents by reducing or eliminating the access to financial services that may be essential to their activity, such as trade finance. The assumption ultimately aims at capturing output losses often linked to sovereign default episodes (e.g. Laeven and Valencia, 2013 and Gornemann, 2014). A default status does not imply permanent exclusion from financial markets. It is assumed that in any given period, there is a probability $\lambda \in (0,1)$ that the economy is "redeemed" and re-enters international credit markets with zero net assets.

Denoting the value function under default as $V_t^d(p_t)$, the problem for an economy that has defaulted becomes

$$V^d(p_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} \left[\lambda E_t V(w_{t+1}, p_{t+1}) + (1-\lambda) E_t V^d(p_{t+1}) \right] \tag{8}$$
$$\text{s.t.} \quad c_t = y_t - h(y_t)$$
$$w_{t+1} = y_{t+1}$$

where $V(w_t, p_t) = \max\left(V^c(w_t, p_t), V^d(p_t)\right)$ reflects that default happens if and only if $V^d(p_t) >$

$V^c(w_t, p_t)$.

Risk-Neutral Foreign Investors. We assume that there is a continuum of risk-neutral foreign investors who can purchase bonds or sell put options to consumers in the benchmark economy. If default happens, foreign investors do not recover any value from the bonds and renege to honor the put options. However, for simplicity, any value foreign investors recover by reneging to honor the options is assumed to be consumed in transaction costs or legal fees. Consequently, recovery values are assumed at zero in the pricing of the bonds. Note that this assumption may ultimately understate the welfare gains from hedging as these recoveries, if not zero, could lead to lower risk spreads on debt.

Denoting r^* the world risk-free rate and $D(w_{t+1}, p_{t+1})$ an indicator default function which equals one if default happens and zero otherwise, no-arbitrage conditions require that

$$\begin{aligned} q(b_{t+1}, p_t) &= \frac{E_t\left[1 - D(\underbrace{y_{t+1} + \alpha Q \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1}}_{w_{t+1}}, p_{t+1})\right]}{1 + r^*} \\ \xi(\bar{p}_t) &= \frac{E_t[\max\{\bar{p}_t - p_{t+1}, 0\}]}{1 + r^*} \end{aligned} \tag{9}$$

The above equations imply that the expected return to the foreign investor from holding bonds or being the counterpart of a put option are equalized and given by the risk-free return. Hedging income appears in the default function, affecting the price of bonds and the risk spreads.

4.2 An Economy without Put Options

The benchmark economy includes the availability of put options because the model will be calibrated to Mexican data over a period where Mexico hedged oil price risk through these instruments. To quantify gains from hedging, we setup a counterfactual economy with no access to put options. To differentiate these two economies, we use a tilde over variables that correspond to the no-hedging economy. State variables $\{w_t, p_t\}$ are defined in the same way as before and value functions are given by

$$\tilde{V}(w_t, p_t) = \max\left(\tilde{V}^c(w_t, p_t), \tilde{V}^d(p_t)\right)$$

where $\tilde{V}^c(w_t, p_t)$ and $\tilde{V}^d(p_t)$ denote the value functions of continuation and default respectively.

As before, the problem under no default or continuation is given by

$$\begin{aligned}\tilde{V}^c(w_t, p_t) &= \max_{c_t, b_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} E_t \left[\tilde{V}(w_{t+1}, p_{t+1})\right] \\ \text{s.t.} \quad & c_t + \tilde{q}_t G b_{t+1} = w_t, \\ & w_{t+1} = y_{t+1} + b_{t+1},\end{aligned} \tag{10}$$

where $\tilde{q}_t$ is the price of the bond in the no-hedging economy. Note also the absence of the terms related to the purchase and exercise of options in the budget constraint equations. Under default, we have

$$\tilde{V}^d(p_t) = \frac{[y_t - h(y_t)]^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} \left[\lambda E_t \tilde{V}(w_{t+1}, p_{t+1}) + (1-\lambda) E_t \tilde{V}^d(p_{t+1})\right]. \tag{11}$$

The pricing of bonds follows the same structure as before:

$$\tilde{q}(b_{t+1}, p_t) = \frac{E_t\left[1 - \tilde{D}(y_{t+1} + b_{t+1}, p_{t+1})\right]}{1 + r^*} \tag{12}$$

4.3 Recursive Equilibrium

As it is standard in the sovereign default literature, we solve the problem from the perspective of a benevolent government, who makes the decision on behalf of private agents in the economy. In what follows we define the recursive equilibrium in this economy.

Definition. *Markov Perfect Equilibrium*

1. *The Markov Perfect Equilibrium of our benchmark model is characterized by a set of value functions* $\{V(w_t, p_t), V^c(w_t, p_t), V^d(w_t, p_t)\}$*, default function* $D(w_t, p_t)$*, consumption function* c_t*, next period bond holding* b_{t+1}*, and bond price* q_t *such that given the state variables* $\{w_t, p_t\}$*, the cost of put options* $\xi(\bar{p}_t)$*, and the strike price* $\bar{p}_t$*, they solve the optimization problems (7) and (8). Furthermore,* $V(w_t, p_t) = \max\left(V^c(w_t, p_t), V^d(p_t)\right)$ *and the price* q_t *satisfies equation (9).*

2. *The Markov Perfect Equilibrium of the economy without put options is characterized by a set of value functions* $\{\tilde{V}(w_t, p_t), \tilde{V}^c(w_t, p_t), \tilde{V}^d(w_t, p_t)\}$*, default function* $\tilde{D}(w_t, p_t)$*, consumption function* $\tilde{c}_t$*, next period bond holding* $\tilde{b}_{t+1}$*, and bond price* $\tilde{q}_t$ *such that given the state variables* $\{w_t, p_t\}$*, they solve the optimization problems (10) and (11). Furthermore,*

$\tilde{V}(w_t, p_t) = \max\left(\tilde{V}^c(w_t, p_t), \tilde{V}^d(p_t)\right)$ *and the price* $\tilde{q}_t$ *satisfies equation (12).*

5 Quantitative Analysis

5.1 Calibration

We calibrate the benchmark model to Mexican data over 1996-2016, a period during which Mexico used put options to hedge oil price risk. The benchmark model has 13 parameters, which we split in three groups before assigning values.

The first group of parameters, comprising $\{r^*, \gamma, \lambda, p, \rho, \sigma, \mathcal{Q}, \alpha, G\}$, are directly taken from the literature or data. The real risk-free interest rate, r^*, equals the average over 1996-2016 of the nominal yield on 1-year U.S. treasury bills, converted to real terms using the U.S. GDP deflator, resulting in a value of 0.64 percent. The risk aversion parameter, γ, is set at 2, the standard value found in the literature. The probability of returning to international financial markets after having defaulted, λ, is calibrated to match the duration of default episodes for Mexico. To get this number, we examine a much longer time period, covering 1821-2016, over which Mexico defaulted 8 times. On average, the duration of default episodes is 9.38 years, which suggests a value of λ equals to 0.11.[11] The parameters of the oil-price process, p, ρ, σ, are obtained from estimating a log AR(1) process of the following form

$$\log p_t = (1-\rho)\left[\log(p) - \frac{1}{2}\frac{\sigma^2}{1-\rho^2}\right] + \rho \log p_{t-1} + \varepsilon_t \tag{13}$$

where the unconditional mean p, the persistence parameter ρ, and volatility σ are estimated using a Maximum Likelihood Estimator (MLE) described in Appendix D over the period 1996-2016. To complete the calibration of the income process, we use actual quantities of Mexican oil production, which over 1996-2016 averaged 1.03 billion barrels per year. Non-oil income, F_t, is approximated by non-oil GDP—measured as total Mexican GDP after subtracting oil and gas extraction. Since the model is written in terms of one tradable good, we convert F_t to U.S. dollars using market exchange rates, and then to real terms using the U.S. GDP deflator. The deterministic annual real growth rate, G, of non-oil income is computed as the average growth of non-oil income over 1996-2016, resulting in a value of 3.13 percent. We calculate the normalized value of oil production, $\mathcal{Q}$, by dividing oil GDP by non-oil income and the oil price. In the model, we are

[11] The default data are taken from Carmen Reinhart's website. See http://www.carmenreinhart.com/data/browse-by-topic/topics/7/.

assuming that this ratio is constant, equal to 0.1 percent, which is not a significant departure from the data. This ratio was fairly stable over 1996-2016, ranging between 0.1 percent and 0.2 percent. The fraction of oil production hedged, α, is set at 29 percent, which corresponds to the average fraction of production hedged by Mexico over 2006-2016. We consider this range only because of lack of publicly available data on the actual fraction hedged prior to 2006.

The second group of parameters, comprising $\{\beta, y^*\}$, is chosen to match relevant moments in the data. In selecting the output loss from default, we follow Arellano (2008) who adopts an asymmetric output cost function which delivers default rates and spreads within the range seen in the data. To this end, the output loss function is given by

$$h(y_t) \quad = \quad \begin{cases} y_t - y^*, & \text{if } y_t \geq y^*, \\ 0, & \text{if } y_t < y^*. \end{cases}$$

We choose β and y^* to match two empirical moments: (1) the Mexican government's gross financing needs—defined as the overall fiscal deficit in any given year plus debt rollover needs—to non-oil fiscal revenue ratio over 2006-2016, of 11.90 percent, a definition of debt that most closely matches the definition of debt in the model; (2) the average risk spreads on sovereign debt over 2000-2016, 1.48 percent. Risk spreads are calculated as the difference between the yield in dollars on Mexico's 1-year government bonds and the yield on U.S. 1-year treasury bills. We compute the average over 2000-2016 to avoid distortions from the sharp increase in spreads around the Tequila crisis of 1995.

The last group of parameters includes the cost and strike price of the options, $\{\xi(\bar{p}_t), \bar{p}_t\}$. As discussed in Section 4 the price is determined by a risk-neutral pricing condition and therefore it emerges endogenously once other parameters in the model have been determined. The exact implementation of the pricing function is given in Appendix E. The strike price is chosen to match the conditional mean of the oil price. While we have data for the actual strike price chosen by Mexico, the sample is too short to estimate a robust empirical relationship between the strike price and the actual oil price. Instead, we proceed as follows. First, we assume that $\bar{p}_t = \mu E_t[p_{t+1}|p_t]$, with the goal of capturing Mexico's actual choice for the strike price, which intends to be close to the oil price assumed in the budget for the subsequent year. This budget oil price is in turn determined by a weighted average between past and future prices implied by forward contracts, which aims at capturing the long-run price of oil, given current market conditions. Second, we choose a value of μ such that the simulated long-run probability of exercising the options is 18.75

percent, consistent with the fact that between 2001 and 2016, the Mexican government exercised the options only 3 times. The approach yields a value of μ equal to 0.77. To cross check that this approach does not result in a number significantly different than the one implied by the data, we compute μ directly from the data by dividing the actual strike price by the average oil price in the year when the options where purchased (Table 1). This alternative approach returns an average value for μ of 0.85, close to the value of 0.77 used in the baseline calibration.

Table 1: Actual Strike Prices from Options

Years	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	Average
Strike price (Nominal US$)	40.00	46.80	70.00	56.69	63.00	84.90	86.00	81.00	76.40	49.00	38.00	62.89
Conditional mean of oil price (Nominal US$)	51.75	53.98	88.86	54.82	69.93	91.86	103.87	99.76	93.80	55.29	51.69	74.15
Ratio to conditional price mean (μ)	0.77	0.87	0.79	1.03	0.90	0.92	0.83	0.81	0.81	0.89	0.74	0.85

Source: Auditoria Superior Federal and authors' calculations.

Finally, it is important to note that a period in this model corresponds to one year, and all values are expressed in 2009 constant U.S. dollar terms. All parameter values are reported in Table 2. The bottom part of the table shows that the model-simulated moments are very close to their data counterparts. It is worth noting also that while our discount factor, at 0.76, appears low for an annual frequency, values in this range are found in the literature, for example Yue (2010) chooses a discount rate at 0.72. It is well-known that sovereign default models with one-period bonds have difficulty in matching both default spreads and debt ratios simultaneously. To achieve both goals, we have to pick a lower value for the discount factor.[12]

We solve the model numerically using value function iteration with the algorithm described in more detail in Appendix C. We use Rouwenhorst method as in Kopecky and Suen (2010) to determine the grid for oil prices. Specifically, we use 21 and 500 grids to approximate oil price and bond holdings respectively.

5.2 Welfare Gains from Hedging

We measure welfare gains from hedging by comparing the utility derived from the stream of consumption under the benchmark economy, and the one in the no-hedging economy. We follow the standard convention in the literature of expressing the welfare gains in terms of a permanent increase in annual consumption. Formally, the definition is given in equation (14).

[12]One rationale from a low β is from a political economy interpretation, which reflects a short-sighted government.

Table 2: Parameters

Parameters	Value	Source
Risk-free rate	$r^* = 0.64\%$	U.S. real interest rate
Risk aversion	$\gamma = 2$	Standard value
Probability of redemption	$\lambda = 0.11$	Average years in default
Growth rate	$G = 1.0313$	Data
Unconditional mean	$p = 54.60$	Data
Persistence	$\rho = 0.71$	Data
Volatility	$\sigma = 0.25$	Data
Oil to non-oil GDP ratio	$p\mathcal{Q} = 6\%$	Data
Strike price	$\bar{p}_t = \mu E_t[p_{t+1}\|p_t] = 0.77E_t[p_{t+1}\|p_t]$	Prob of exercising options
Hedging share	$\alpha = 0.29$	Data
Hedging cost	$\xi(\bar{p}_t)$	Risk-neutral pricing
Discount rate	$\beta = 0.76$	Match debt ratio
Output loss function	$y^* = 0.98E[y] = 1.03$	Match spreads
Target Moments	Data	Model Simulation
Debt-GDP ratio	11.90 %	11.97 %
Sovereign spreads	1.48%	1.40 %

Source: INEGI, Federal Reserve Board of Governors, and authors' calculations.

$$\Delta(w_t, p_t) = 100 * \left[\left(\frac{V(w_t, p_t)}{\tilde{V}(w_t, p_t)} \right)^{\frac{1}{1-\gamma}} - 1 \right] \tag{14}$$

Under this definition, welfare gains are conditional on the values of the state variables, $\{w_t, p_t\}$; therefore, we refer to $\Delta(w_t, p_t)$ as conditional welfare gains.[13] Furthermore, we also define unconditional welfare gains by $E[\Delta(w_t, p_t)]$, where the expectation is taken with respect to the state variables using their ergodic distribution under the benchmark economy.[14] To compute the welfare gains, we first run 100 Monte Carlo simulations of 2,000 periods each for the benchmark economy. We draw oil prices from the estimated stochastic process, given some initial price. This initial condition, together with one for wealth, and the optimal solutions for consumption and borrowing determine the optimal value of these variables for the current period. We then check if default is optimal or not, to then proceed to use the law of motion for wealth and oil prices to determine the value of the state variables for the subsequent period and so on. We repeat this process until we reach 2,000 periods. We throw away the initial 500 periods and approximate welfare—or the value function—by computing the present discounted value of the utility derived from the simulated path for consumption. We construct the counterpart value function for the economy without hedging, using the same procedure and initial conditions for w_t and p_t.

[13]Note that $\Delta(w_t, p_t)$ does not depend on any particular time t. We keep the time script t for consistency of notation.

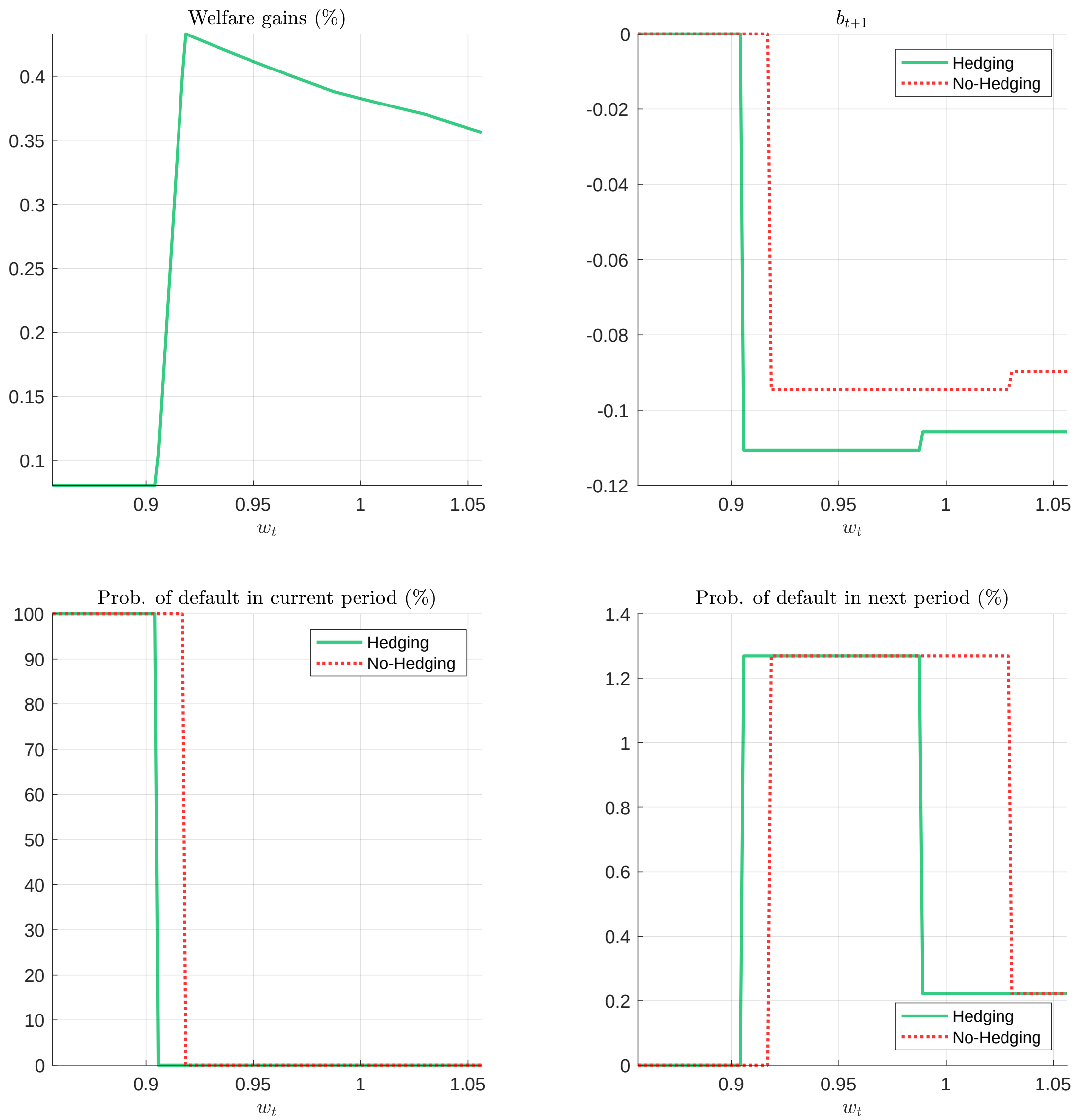
[14]Using instead the ergodic distribution under the economy without hedging yields similar outcomes.

In Figure 4 we show the conditional welfare gains $\Delta(w_t, p_t)$; bond purchase/sale b_{t+1}; the probability of default in the current period t; and the probability of default in the next period $t+1$, given by $E[D(y_{t+1} + b_{t+1}, p_{t+1})|p_t]$; for different values of state variable w_t, and after setting the price of oil, p_t, equal to its unconditional mean. These conditional welfare gains vary from 0 to a 0.45 percent permanent increase in consumption. When the economy has less wealth to start, default incentives are strong, the probability of default in current period t is high, and welfare gains from hedging are small. In this region, hedging does little to improve welfare since default happens regardless, analogous to the result in the farthest right region of Figure 3. When the economy is less indebted, default incentives weaken and the probability of default declines, but more quickly for the economy with hedging than for the one without it. For values of wealth w_t between 0.91 and 0.92, the economy without hedging defaults in the current period, but the economy with hedging does not. This region is analogous to region B in Figure 3. In this region, welfare jumps from close to 0 to 0.43 percent. As wealth increases, welfare gains decline as default becomes less and less relevant. At some point, even the no-hedging economy does not default in the current period and it has the same default probability in the next period as the hedging economy. Welfare declines further since hedging is costly and its benefit through a reduction in borrowing costs is much lower. This result is analogous to the result depicted in the left regions of Figure 3.

We also find unconditional welfare gains equivalent to a permanent increase in annual consumption of 0.44 percent. These gains are within the range found by related studies. Borensztein et al. (2017) finds that the unconditional welfare gains from using catastrophe (CAT) bonds, in the presence of defaultable debt, are typically small: less than 0.12 percent. They rationalize their results by claiming that the CAT bonds do not change the default threshold. Hatchondo and Martinez (2012) explore the welfare gains from issuing GDP-indexed bonds in a model with defaultable debt. They find that GDP-indexed bonds could change the default threshold and find welfare gains equivalent to a permanent increase in consumption of 0.46 percent.

Source of Welfare Gains. As discussed in the context of the 2-period model, we explore two channels, one operating through income smoothing and the second one through default incentives, which ultimately affect borrowing costs. The latter channel can already be appreciated in Table 3, where we compare the stochastic steady state—defined as the average value of the corresponding variables in the long-run simulations—of the benchmark model with the one from the model without hedging. We find that the probability of default is higher in the model without hedging, 1.41 percent versus 1.27 percent, default spreads are also higher, 1.59 percent versus 1.4 percent, and the debt

Figure 4: Welfare Gains, Borrowing, and Probability of Default



Source: Authors' calculations.

Note: Conditional welfare gains, $\Delta(w_t, p_t)$; borrowing, b_{t+1}; the probability of default in the current period; and the probability of default in the next period are plotted against values of wealth, w_t with p_t set equal to its unconditional mean.

level is lower, 10.50 percent versus 11.97 percent. Recall that proposition 3 implied that the impact of hedging on the debt level was ambiguous; however, our quantitative results suggest that debt increases with hedging. It increases due to a lower borrowing cost and also a stronger incentive to borrow because of the additional borrowing needed to purchase the put options.

Table 3: Stochastic Steady State in the Hedging and No-hedging Economies

Economy	debt ratio	default spreads	default probability
Hedging	11.97 %	1.40 %	1.27 %
No-Hedging	10.50 %	1.59 %	1.41 %

Source: Authors' calculations.

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, conditional on no default for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods.

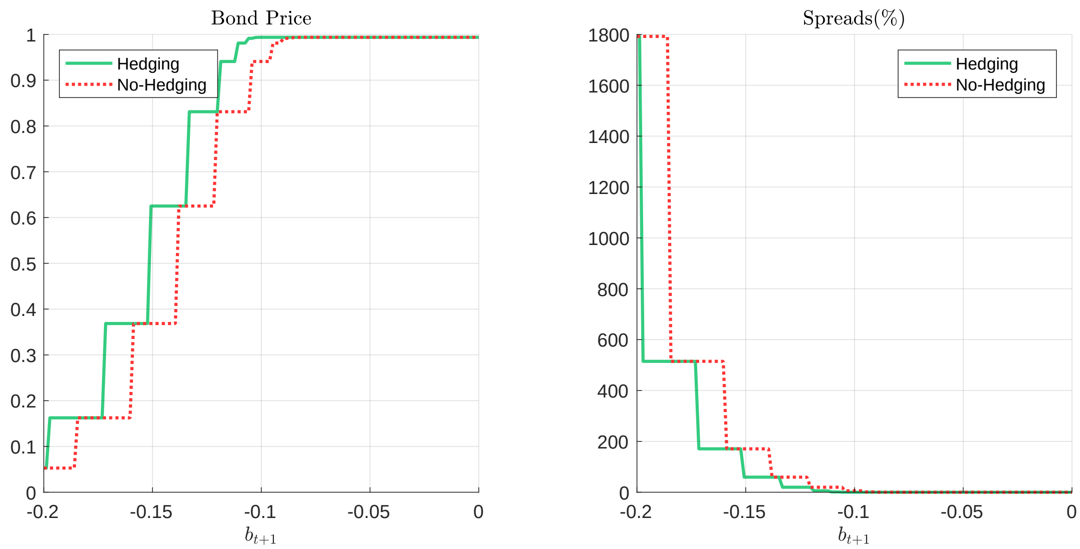
Figure 5 shows bond prices and risk spreads and highlights that the model with hedging has systematically higher bond prices, except in the region where default risk is zero in which case bond prices equal 1 for both the benchmark and the no-hedging model. To decompose more explicitly the borrowing cost and income-smoothing channels of welfare gains, we solve the model without hedging after imposing the same bond price that emerges in the economy with hedging. Note that now we have two versions of the no-hedging economy. One that is solved as if bond prices were the same as if hedging was present, and the standard one where bond prices correspond to the no-hedging world. Since the only difference between these two models is the borrowing cost, the resulting welfare gains stem entirely from the borrowing costs channel.[15] Our simulation suggests that the unconditional welfare gains from the no-hedging economy with hedging bond prices relative to the no-hedging economy with no-hedging bond prices are equivalent to a permanent increase in consumption of 0.40 percent, that is, a 90 percent of the total welfare gains.

To gain further intuition, we examine the dynamic behavior of key variables around default episodes in Figure 6. To this end, we construct a 11,000-period simulation for the hedging and no-hedging economies. After dropping the first 1,000 periods, we identify all default episodes by the no-hedging economy in the remaining 10,000 periods. We look at a 20-period window, centered on the default year, that is, 10 years before and after default, and examine the evolution of key variables within this window. In Figure 6, we plot the average path of the corresponding variable for the hedging and no-hedging economy, keeping in mind that the hedging economy may not have defaulted.

A sharp decline in oil prices, p_t, at time 0, triggers a payoff from the options which compensates the income fall in the hedging economy. The no-hedging economy defaults, which reduces the stock

[15] The remaining channel should include gains from income smoothing, net of the cost of hedging, because the above exercise does not take into account the cost of hedging.

Figure 5: Bond Price and Sovereign Spreads



Source: Authors' calculations.

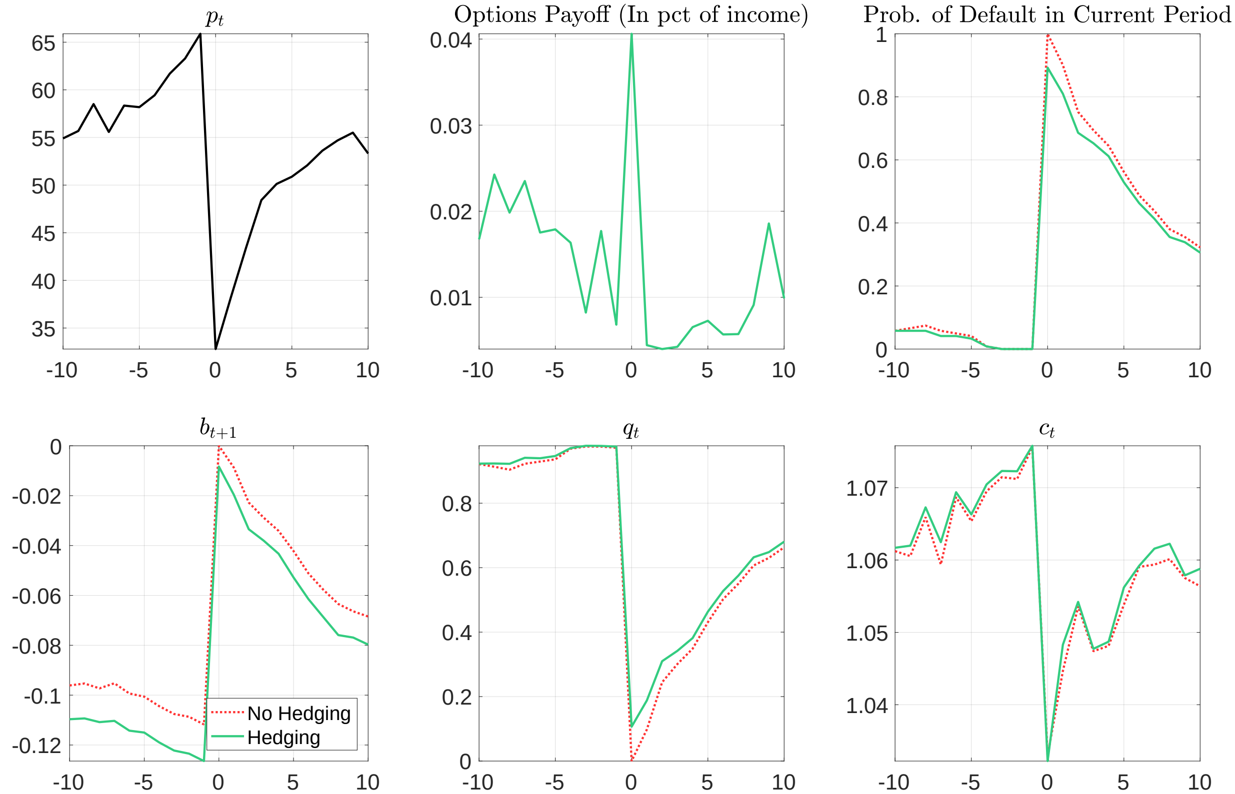
Note: Bond prices, q_t, and spreads are plotted as a function of w_t, with p_t set equal to its unconditional mean. Spreads are computed as the difference between bond yields and the international risk free rate, $100\left(\frac{1}{q_t} - 1 - r^*\right)$.

of debt to zero, but the probability of default rises sharply even in the hedging economy, peaking at 89 percent. The high persistence in the oil price process keeps income prospects weak for some time in both economies, with borrowing being restored gradually, more so in the no-hedging economy than in the hedging economy. This result is the consequence of the temporary exclusion from financial markets and the higher borrowing costs for the no-hedging economy. Finally, the hedging economy is able to sustain higher levels of consumption than in the no-hedging economy, despite the cost of the options, because of lower cost of debt.

5.3 Robustness Check

Cost of Put Options. Our baseline calculation assumes an actuarially fair price for put options. In practice, the actual price can include a premium above the actuarially fair price. This premium may stem from non-competitive behavior, regulatory constraints, risk aversion, and market illiquidity. In the case of Mexico, the use of over-the-counter options with Maya oil as underlying asset could lead to a cost premium given that such instruments are not as liquid as options on the Brent

Figure 6: Event Windows



Source: Authors' calculations.

Note: The event windows are selected by first running an 11,000-period simulation for the hedging and no-hedging economies. After dropping the first 1,000 periods, we identify all default episodes by the no-hedging economy in the remaining 10,000 periods and compute the average evolution of the selected variables depicted in the charts.

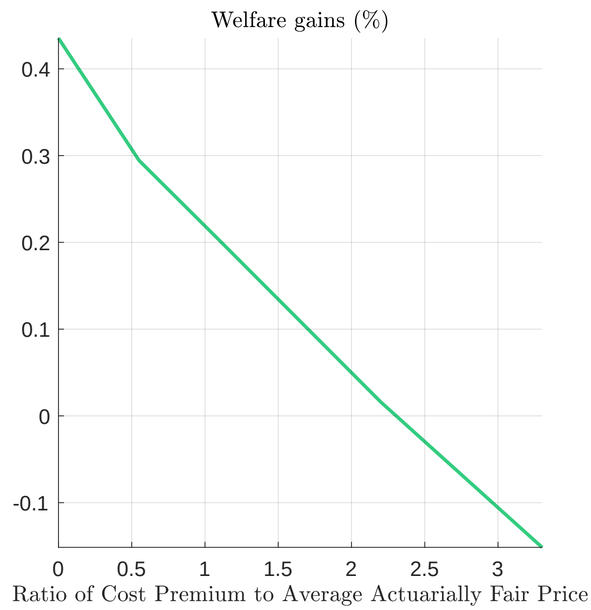
or the West Texas Intermediate (WTI).[16] To examine the implications of such a cost premium, we now assume that there is an additional cost x per barrel of oil, above the actuarially fair price. In Figure 7, we plot the welfare gains from hedging against various levels of the cost premium x, expressed as a ratio to the actuarially fair price. Not surprisingly, the welfare gains decline with x; however, reducing the welfare gains to zero in this model would require a sizable premium, in the order of 2.3 times the actuarially fair price.[17] The reason for the decline in welfare gains is that the options become relatively more expensive than debt, assuming that debt remains fairly priced.

[16] Mexico's decision to use Maya oil as underlying asset is justified on the grounds of avoiding base risk, defined as unexpected movements in Maya oil price not explained by movements in the price of Brent or WTI oil.

[17] The cost premium at which welfare gains are zero, expressed in 2009 constant dollars, is equivalent to US$2.1 per barrel. During 2006-2016, Mexico paid on average US$ 3.5 per barrel to purchase the put options, which is an alternative way to corroborate that the cost premium has to be sizable to reduce welfare gains to zero.

Naturally, if a cost premium also affects the price of debt, then the impact on welfare gains would depend on the relative size of the distortions in debt and option prices.

Figure 7: Welfare Gains under Different Cost Premiums



Source: Authors' calculations.

Note: Unconditional welfare gains as a function of the cost premium x, expressed as a ratio to the actuarially fair price.

Strike Prices. In our benchmark analysis, the strike price is a fraction μ of the expected oil price for next year conditional on the current period's price. We noted in the calibration section that we could compute μ directly in the data, although for only a handful of years for which there was publicly available information. The data suggested a range for μ between 0.72 and 1.14, as shown in Table 1. We arbitrarily chose values for μ of 0.74 and 1.03 and solve the model again to compute the welfare gains. We found that welfare gains increase with the strike price. Moreover, as welfare gains increase, the cost premium computed above also becomes larger, suggesting that the gains becomes less sensitive to the presence of a cost premium in the price of the options. These results, and those described in the remaining of this section, are reported in Table 4.

Oil Price Process. Three parameters govern the oil price process, i.e. persistence ρ, volatility σ and the unconditional mean p. Increasing the unconditional mean for the oil price is inconsequen-

Table 4: Sensitivity Analysis

	Welfare Gains (%)	Debt (%)		Default Spreads (%)		Default Prob. (%)		Cost Premium
	Overall	Hedging	No Hedging	Hedging	No Hedging	Hedging	No Hedging	Hedging
baseline	0.44	11.97	10.50	1.40	1.59	1.27	1.41	2.10
$\mu = 0.74$	0.30	11.71	10.50	1.41	1.59	1.28	1.41	1.74
$\mu = 1.03$	0.75	13.47	10.50	0.94	1.59	0.90	1.41	3.74
$\alpha = 0.23$	0.31	11.52	10.50	1.41	1.59	1.27	1.41	2.10
$\alpha = 0.39$	0.65	12.29	10.50	1.01	1.59	0.96	1.41	2.13
$\rho = 0.8$	0.21	6.85	6.23	3.37	3.75	2.54	2.75	1.32
$\rho = 0.9$	0.03	2.91	3.69	7.29	16.01	3.54	5.81	0.48
$\sigma = 0.1$	-0.00	18.46	18.46	0.80	0.80	0.79	0.79	N.A.
$\sigma = 0.4$	0.37	4.28	2.20	0.12	2.71	0.15	2.07	3.03
$p = 50$	0.44	11.97	10.50	1.40	1.59	1.27	1.41	1.93
$p = 70$	0.44	11.97	10.50	1.40	1.59	1.27	1.41	2.92
$r^* = 2\%$	0.29	9.69	8.81	1.43	1.97	1.28	1.66	2.13
$r^* = 4\%$	0.17	7.92	7.37	1.50	2.26	1.31	1.82	1.75
$\gamma = 3$	0.55	11.64	10.15	1.24	1.71	1.13	1.51	2.24
$\gamma = 4$	0.74	11.70	9.55	1.24	1.75	1.14	1.53	2.42
$\lambda = 0.2$	0.17	5.30	5.36	2.62	4.85	2.29	3.80	2.10
$\lambda = 0.3$	0.04	3.58	4.10	4.54	10.18	3.79	6.93	0.54
$y^* = 0.97E[y]$	0.69	27.97	25.95	0.72	0.95	0.72	0.92	2.21
$y^* = 0.99E[y]$	0.35	5.26	4.05	0.76	2.17	0.73	1.83	2.01
$y^* = 1.05E[y]$	-0.00	0.30	0.00	283.55	4.29	10.53	1.14	N.A.
$G = 1$	0.22	8.22	7.59	1.47	1.94	1.33	1.64	1.93
$G = 1.04$	0.67	14.03	12.20	0.90	1.46	0.87	1.32	2.13
$\beta = 0.9$	0.23	15.56	14.33	0.36	0.48	0.40	0.53	1.70
$\beta = 0.98$	0.08	17.79	16.92	0.13	0.19	0.17	0.23	1.17

Source: Authors' calculations.

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents. Cost premium refers to the cost of options above the actuarially fair price such that the welfare gains from hedging are zero.

tial for our results if volatility and persistence remain the same. This result is intuitive since the exercise leaves risk intact. Increasing the persistence of oil prices reduces welfare gains because, given the one-year horizon of the options, hedging would compensate for a smaller fraction of the cumulative income loss relative to a scenario where oil prices recover more quickly. In turn, the welfare gains increase with the volatility of oil prices, which is associated with higher risk of default, because the borrowing costs channel strengthens. Note that when welfare gains become larger, the cost premium at which gains vanish becomes larger, suggesting as before that the gains become more robust to the presence of a cost premium.

Other Parameters. We also conduct robustness checks with respect to other parameters in our model, with the results also summarized in Table 4. Generally speaking, the benefits from hedging are robust to different parameter values. In particular, the welfare gains are larger when a larger volume of oil production, α, is hedged because a larger fraction of income is protected. Moreover, risk spreads decline as the risk of default is lower. Gains are also larger when consumers are more risk averse, i.e. higher γ, since they dislike income fluctuations more. Welfare gains also increase with G since higher growth in non-oil income increases the desire to borrow, whose cost is reduced by hedging. Welfare gains decline when the international risk-free rate, r^*, increases, which in turns makes borrowing more expensive, reducing the desire to borrow. With lower borrowing, the benefits of hedging through the borrowing cost channel weaken. Welfare gains also decline when the income loss from default is lower. This is represented in the Table by increasing y^*. The result is analogous to what happens in the right regions of Figure 3 depicting the outcomes from our two-period model. Note in Table 4 that for sufficiently low cost of default, i.e. sufficiently high y^*, the welfare gains vanish since hedging in those cases increase default incentives (Proposition 3). In particular, the welfare gains become a negative number in the case of $y^* = 1.05E[y]$ since hedging increases the default probability from 1.14 % in the economy without hedging to 10.53 % in the economy with hedging. Welfare gains decrease with the probability of redemption λ. Since losing access to international financial markets is one component of the cost of default, increasing λ is equivalent to reducing the cost of default; therefore, the result is consistent with what happens when y^* is higher. Intuitively, when default is less costly, the benefits of hedging decline in the presence of defaultable debt, which serves also as a hedging and consumption smoothing instrument. Welfare gains decline with the discount factor, β, since the more patient consumers become, the less they borrow, and the weaker the borrowing costs channel of welfare gains.

6 Extensions

6.1 Selling Oil Forward

In our baseline model, the upfront cost of options generates a tradeoff. On the one hand, hedging helps smooth income, but on the other it implies devoting resources in the current period to the cost of hedging. We contrast the welfare gains with an alternative hedging vehicle: selling oil forward at a predetermined price. There is no upfront cost of insurance, as it is the case for the options, but the country also gives up any revenue windfall if oil prices rise unexpectedly. We maintain the one-year horizon of the hedge. We model this variant of hedging by assuming that the country sells a fraction α of oil production at the conditional mean of the oil price in each period. The new budget constraint and dynamics of beginning-period-of wealth are given by

$$w_t = c_t + q_t G b_{t+1}$$
$$w_{t+1} = 1 + \mathcal{Q}\left\{(1-\alpha)p_{t+1} + \alpha E_t[p_{t+1}]\right\} + b_{t+1}$$

To understand the benefits/costs of forward, we first modify our two-period model to include forwards and derive the following proposition:

Proposition 4. *Forwards and default incentives*

Define y^{def} as the income under default in the no-hedging economy such that the economy does not default when $y^{def} < \hat{y}^{def}$, i.e when the cost of default is sufficiently high. Introducing forwards at a price equal to the conditional mean increases $\hat{y}^{def}$.

Proof. Proof is given in Appendix B.4. □

One implication from Proposition 4 is that the introduction of forwards can reduce default incentives. A similar plot to Figure 3 is presented in Figure 8[18] in the appendix. Similar to options, if default never happens, hedging through forwards increases welfare only through the income smoothing channel (Region A in Figure 8), when the no-hedging economy defaults in the low-income state of the world, introducing forwards reduce the likelihood of default to zero since income is locked in at a level above the income level under default. In in this case, forwards

[18]The figure shows the case where $\hat{y}^{def,forward} < \hat{\hat{y}}^{def}$. However, it is theoretically possible that $\hat{y}^{def,forward} > \hat{\hat{y}}^{def}$.

Table 5: Welfare Gains from Selling Oil Forward

	Welfare Gains (%)	Debt (%)		Default Spreads (%)		Default Prob. (%)	
	Overall	Hedging	No Hedging	Hedging	No Hedging	Hedging	No Hedging
Forwards ($\mu = 1$)	0.89	14.19	10.50	0.96	1.59	0.92	1.41
Put Options ($\mu = 1$)	0.75	13.32	10.50	1.14	1.59	1.06	1.41

Source: Authors' calculations.

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents.

increase welfare through income smoothing and lower borrowing costs (Region B in Figure 8). For completeness, we also describe the implications of the model when default costs are sufficiently low, meaning that $y^H > \hat{y}^{def} > \hat{y}^{def,forward} > \hat{y}^{def} > y^L$. In this case, forwards worsen default incentives. However, it is not an interesting case to examine since it would imply locking in through forwards a level of income below what the economy would get if it defaulted.

Turning now to the quantitative analysis, we find welfare gains from hedging through forwards equivalent to a permanent increase in consumption of 0.89 percent, roughly twice as large as those from our baseline model. However, recall that in our baseline calibration, the strike price for the put options is set at $\bar{p}_t = \mu E_t[p_{t+1}|p_t]$, with $\mu = 0.77$, while in this section, the economy hedges through selling oil forward at a price equal to $E_t[p_{t+1}|p_t]$. Therefore, to conduct a more appropriate comparison between forwards and options, we compute the welfare gains from hedging through put options after setting $\mu = 1$. As shown in Table 5, the resulting welfare gains from options are equivalent to a permanent increase in consumption of 0.75, higher than in the baseline calibration, but still below those from forwards. With forwards, the probability of default and risk spreads are lower than in the model with options, while the economy can afford to borrow more (Table 5).

6.2 Risk Averse Investors

This last extension is intended to understand the benefits of hedging in a world in which global changes in risk appetite affect commodity and other asset prices simultaneously. For simplicity, we model this situation as having risk averse international investors who have a time-variant pricing kernel m_t, i.e. the intertemporal marginal rate of substitution. We follow Arellano (2008) in assuming that m_t is an i.i.d. random variable. The pricing of sovereign bond and options are given by the following formula.

$$q_t(b_{t+1}, p_t) = E_t[m_{t+1}(1 - D(y_{t+1} + b_{t+1}, p_{t+1}))] \tag{15}$$

$$\xi_t(p_t) = E_t[m_{t+1} \max(\bar{p} - p_{t+1}, 0)] \tag{16}$$

with $m_{t+1} = e^{-r^*} e^{-\nu \varepsilon_{t+1}}$ to ensure m_{t+1} is non-negative. After taking logs m_t can be written as

$$\log m_t = -r^* - \nu \varepsilon_t$$

with $E[\log m_t] = -r^*$ and $var(\log m_t) = \nu^2 \sigma^2$. Note that ε_t is the same shock to oil prices, which implies that foreign investors become effectively more or less risk averse when oil prices decrease or increase. We solve the model and compute the welfare gains using the same procedures as before but subject to the above pricing equations. In Table 6 we report the results for various values of ν. Risk aversion implies foreign investors demand a premium above the spread necessary to compensate default risk, making debt more expensive and discouraging borrowing. But foreign investors also demand an extra compensation for risk when selling put options, making the options also more expensive. However, hedging has now an extra channel through which it could increase welfare, through its effect on the risk premium demanded by foreign investors, which depends on the risk of default. Our quantitative analysis in Table 6 suggests that the welfare gains are larger as foreign investors become more risk averse, i.e. ν increases, despite the higher upfront cost of insurance.

7 Conclusion

The sharp unexpected decline in oil prices during 2014-2016 renewed the interest in designing policies to manage such risks in countries highly exposed to swings in commodity prices. Discussions about the various alternatives to countries often start with Mexico, given its longstanding practice of hedging through put options, but analyses of the welfare gains of such policy have been limited. This paper attempts to fill this gap and derives lessons about the benefits and costs for commodity exporters of using market insurance to hedge commodity price risk.

Table 6: Risk Averse Investors: Hedging and No-hedging Economies

	Welfare Gains (%)	Debt (%)		Default Spreads (%)		Default Prob. (%)		Cost Premium
	Overall	Hedging	No Hedging	Hedging	No Hedging	Hedging	No Hedging	Hedging
$\nu = 0$	0.44	11.97	10.50	1.40	1.59	1.27	1.41	2.52
$\nu = 0.25$	0.44	11.77	10.59	1.24	1.89	0.86	1.30	2.10
$\nu = 0.5$	0.59	13.03	11.35	1.45	2.42	0.50	1.10	2.06
$\nu = 0.75$	0.98	17.82	14.15	2.20	2.44	0.32	0.46	2.08

Source: Authors' calculations.

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents.

We have focused our analysis on the role of hedging instruments as a complement to defaultable debt, which in and on itself can be seen as a hedging strategy. Our quantitative assessment concludes that the welfare gains from hedging, in the presence of defaultable debt, can be equivalent to a permanent increase in consumption of about 0.44 percent. We also find that about 90 percent of these gains stem from a reduction in borrowing costs and the difference from income smoothing. The beneficial role of hedging is robust to numerous sensitivity analyses.

In terms of lessons for the design of a program like Mexico's, the welfare gains are lower when option prices exceed their actuarially fair value, a circumstance that may become more likely when using relatively illiquid, over-the-counter options. It may then be worth accepting some base risk to ensure hedging is welfare enhancing. Nevertheless, the model suggests that the premium above the actuarially fair price would have to be very large for the welfare gains to decline to zero.

The model also suggests that selling oil forward generates larger welfare gains than hedging through put options. However, political economy considerations cannot be ignored since selling oil forward implies giving up any potential revenue windfall if oil prices rise. Mexico, through the use of options, seems to have found a good balance between these political economy constraints and the benefits of market instruments to hedge oil price risk.

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A Normalized Economy

In this appendix we show the derivation of the normalized economy starting from the original setup. As described in the main text, the original economy has the following structure:

Preference.

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right]$$

Total Income.

$$Y_t = F_t + p_t Q_t$$

Budget Constraint under No Default.

$$C_t + q_t B_{t+1} + \alpha Q_{t+1}\xi(\bar{p}_t) = Y_t + B_t$$

where agents hedge Q_{t+1} production of oil at period t.

Budget Constraint under Default.

$$C_t = Y_t - H(Y_t)$$

where $H(Y_t) = h(y_t)F_t$.

Given that F_t grows at a constant rate G in every period, C_t and B_{t+1} grow at the same rate as F_t and F_{t+1} respectively. In order to solve a stationary problem, we can normalize the consumers' preferences, total income, and the budget constraints as follows.

Normalized Preference.

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(F_t c_t)^{1-\gamma}}{1-\gamma}\right] = F_0 E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left(\frac{F_t}{F_{t-1}}\frac{F_t-1}{F_{t-2}}\cdots\frac{F_1}{F_0}c_t\right)^{1-\gamma}}{1-\gamma}\right] = F_0 E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(G^t c_t)^{1-\gamma}}{1-\gamma}\right]$$

Normalized Total Income.

$$y_t = \frac{Y_t}{F_t} = \frac{F_t + p_t Q_t}{F_t} = 1 + p_t\frac{Q_t}{F_t} = 1 + p_t\mathcal{Q}$$

Normalized Budget Constraint under No Default.

$$y_t + b_t = \frac{Y_t + B_t}{F_t} = \frac{C_t + q_t B_{t+1} + \alpha Q_{t+1}\xi(\bar{p}_t)}{F_t} = c_t + q_t\frac{F_{t+1}}{F_t}\frac{B_{t+1}}{F_{t+1}} + \alpha\frac{Q_{t+1}}{F_{t+1}}\frac{F_{t+1}}{F_t}\xi(\bar{p}_t) = c_t + q_t G b_{t+1} + \alpha\mathcal{Q}G\xi(\bar{p}_t)$$

Normalized Budget Constraint under Default.

$$c_t = \frac{C_t}{F_t} = \frac{Y_t - H(Y_t)}{F_t} = y_t - h(y_t)$$

Given the normalized preferences, total income, and the budget constraints, we can solve the normalized economy problem knowing that the original problem can always be recovered by multiplying all variables by F_t.

B Proofs

B.1 Proof of Proposition 1

Proof. If the economy defaults in H state, it must default in L state since $y^H > y^L$. Conditional on no default, optimal borrowing d^* satisfies the following condition:

$$\frac{1}{y + d^*} = \beta \left(\frac{p}{y^H - d^*} + \frac{1-p}{y^L - d^*} \right)$$

The economy finds it optimal not to default iff $y^L - d^* > y^{def}$.

Similarly, when the economy defaults only in L state, optimal borrowing d^{**} satisfies

$$\frac{p}{y + pd^{**}} = \beta \frac{p}{y^H - d^{**}}$$

which is consistent iff $y^H - d^{**} > y^{def}$. Define $\hat{y}^{def} = y^L - d^*$ and $\hat{\hat{y}}^{def} = y^H - d^{**}$ and we establish the first part of the proposition.

In an economy with hedging, optimal borrowing $d^{*,hedge}$ with no default satisfies the following condition:

$$\frac{1}{y + d^{*,hedge} - \xi} = \beta \left(\frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}} \right)$$

It is not hard to find $d^{*,hedge} > d^*$ since the marginal benefit of borrowing increases and the marginal cost of borrowing declines. Using the same logic, we find that $d^{**,hedge} > d^{**}$. This implies that $\bar{\bar{y}}^{def,hedge} < \hat{\hat{y}}^{def}$. However, $\bar{y}^{def,hedge} > \hat{y}^{def}$ since $c_1^{L,hedge} > c_1^L$ due to the presence of hedging. $\square$

B.2 Proof of Proposition 2

Proof. If the economy does not default in equilibrium, the interest rate on debt is 1 and the optimal allocation is given by

$$\frac{1}{y+d^*} = \beta\left(\frac{p}{y^H - d^*} + \frac{1-p}{y^L - d^*}\right).$$

With the introduction of hedging, the optimality condition becomes

$$\frac{1}{y+d^{*,hedge} - \xi} = \beta\left(\frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}}\right).$$

It is easy to see that $d^{*,hedge} > d^*$ since the income in first period is reduced by ξ and income in L state has increased by $\bar{y} - y^L$.

We also needs to establish the results that social welfare has been increased. Intuitively, hedging does not change the PDV of income stream but reduces the variance of income. This is beneficial since it increases the welfare in the second period. Denote the social welfare without and with hedging by $U_0(d^*)$ and $U_0^{hedge}(d^{*,hedge})$ respectively. We want to show $U_0^{hedge}(d^{*,hedge}) > U_0(d^*)$ by proving $U_0^{hedge}(d^* + \xi(\bar{y})) > U_0(d^*)$. To see it, we have

$$\begin{aligned}
U_0^{hedge}(d^* + \xi(\bar{y})) - U_0(d^*) &= \beta\left[p\log\left(y^H - \xi(\bar{y}) - d^*\right) + (1-p)\log\left(\bar{y} - \xi(\bar{y}) - d^*\right)\right] \\
&\quad -\beta\left[p\log\left(y^H - d^*\right) + (1-p)\log\left(y^L - d^*\right)\right] \\
&= \beta\left[p\log\left(y^H - (1-p)(\bar{y} - y^L) - d^*\right) + (1-p)\log\left(y^L + p(\bar{y} - y^L) - d^*\right)\right] \\
&\quad -\beta\left[p\log\left(y^H - d^*\right) + (1-p)\log\left(y^L - d^*\right)\right] > 0
\end{aligned}$$

where the last inequality holds since the function

$$f(x) = \beta\left[p\log\left(y^H - (1-p)x - d^*\right) + (1-p)\log\left(y^L + px - d^*\right)\right]$$

increases in $x \in [0, y^H - y^L]$. □

B.3 Proof of Proposition 3

Proof. When the economy defaults only in the low-income state of nature, hedging is beneficial if it reduces default incentives. If the economy does not default in equilibrium, social welfare increases with hedging (See Proposition 2). However, debt might increase or decrease. One can see that from the first order conditions with debt d^{**} and $d^{*,hedge}$ satisfying

$$\frac{p}{y+pd^{**}} = \beta \frac{p}{y^H - d^{**}}$$
$$\frac{1}{y+d^{*,hedge}} = \beta \left(\frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}} \right)$$

It is hard to sign d^{**} and $d^{*,hedge}$ since both the marginal benefit and the marginal cost of borrowing increase with hedging.

If hedging does not change default incentives, social welfare is lower since the economy borrows more (See the proof of Proposition 2 in Appendix B.2) and consumption streams are unambiguously lower. Clearly, social welfare is further reduced if hedging increases default incentives. There is no borrowing in this case. □

B.4 Proof of Proposition 4

Proof. The two-period model with forwards changes into the following form

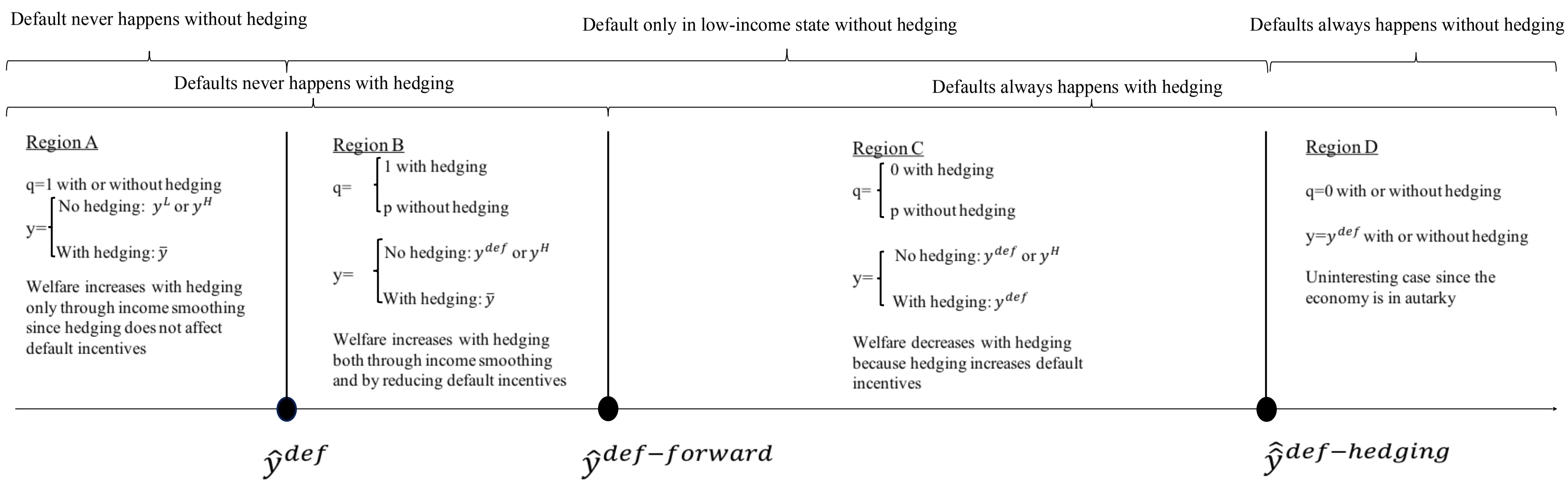
$$\begin{aligned} U_0^{forwards} &= \max_d \log c_0 + \beta \log c_1 \\ \text{s.t.} \quad & c_0 = y + d, \\ & c_1 = \max\{\bar{y} - d, y^{def}\} \end{aligned}$$

where $\bar{y} = py^H + (1-p)y^L$.

The optimality condition when the economy does not default implies that

$$\frac{1}{y+d^{*,forwards}} = \beta \frac{1}{\bar{y} - d^{*,forwards}}$$

Figure 8: Two-period Model with Forwards



Source: Authors' construction.

It is easy to show that $d^* < d^{*,forwards}$ since the marginal cost of borrowing decreases with forwards.[19] Therefore, we have

$$\beta \frac{1}{\bar{y} - d^{*,forwards}} = \frac{1}{y + d^{*,forwards}} < \frac{1}{y + d^*} = \beta \left(\frac{p}{y^H - d^*} + \frac{1-p}{y^L - d^*} \right) < \beta \frac{1}{y^L - d^*},$$

which implies that $\bar{y} - d^{*,forwards} > y^L - d^*$. It follows that $\hat{y}^{def,forwards} > \hat{y}^{def}$. However, it is hard to sign $\hat{y}^{def,forwards}$ and $\hat{\hat{y}}^{def}$. As to welfare, if both economies default in both states, economy with forwards has larger utility due to concavity of log function. If economy defaults and forwards avoid default, welfare is larger in the forwards economy. If economy defaults in low state and forwards economy defaults in both states, welfare is lower in the forwards economy. $\square$

C Algorithm

We solve the model using value function iteration. We create grid spaces for both b_t and p_t and denote them by $\mathscr{B}$ and $\mathscr{P}$ respectively. Starting from an initial guess for the bond price function $q_i(b', p)$ for each $b \in \mathscr{B}$ and $p \in \mathscr{P}$ for iteration $i = 0$, we implement the following algorithm:

1. Staring from an initial guess of $\{V_i(b,p), V_i^d(p), V_i^c(b,p)\}$ for each $b \in \mathscr{B}$ and $p \in \mathscr{P}$ for iteration $i = 0$.

2. Update $V_{i+1}^d(p)$ using equation (8),

3. Update $V_{i+1}^c(b,p)$ according to equation (7).

4. Update $V_{i+1}(b,p)$ using $V_{i+1}(w,p) = \max\left\{V_{i+1}^c(b,p), V_{i+1}^d(p)\right\}$.

5. Calculate the implied bond price as follows

$$q_{i+1}(b', p) = \frac{E_{p'|p}\left[V_{i+1}^c(b', p') \geq V_{i+1}^d(p')\right]}{1 + r^*}$$

6. Iterate until the endogenous objectives $q_j(b', p), V_j(b,p), V_j^c(b', p')$ and $V_j^d(p)$ are close enough for $j = i$ and $j = i + 1$.

[19] One can see that from the inequality

$$f(x) = \frac{p}{y^H - d} + \frac{1-p}{y^L - d} - \frac{1}{\bar{y} - d} = \frac{p(1-p)\left(y^H - y^L\right)^2}{(y^H - d)(y^L - d)(\bar{y} - d)} > 0$$

D Estimation of Oil Price Process

We estimate an AR(1) process for the oil price in logs, with the unconditional oil price given by $\widehat{p} = \frac{1}{T}\sum_{t=1}^{T} p_t$. We then impose the following functional form to get an estimator for the AR(1) coefficient:

$$\log p_t = \underbrace{(1-\rho)\left[\log(\hat{p}) - \frac{1}{2}\frac{\sigma^2}{1-\rho^2}\right] + \rho \log p_{t-1}}_{\mu_{t-1}} + \varepsilon_t,$$

with conditional density

$$f\left(\log p_t | p_{t-1}, p, \rho, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\log p_t - \mu_{t-1})^2}{2\sigma^2}}$$

The likelihood can be written as:

$$L = \Pi_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\log p_t - \mu_{t-1})^2}{2\sigma^2}}$$
$$\log L = -\frac{T-1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=2}^{T}(\log p_t - \mu_{t-1})^2$$

The first order conditions require:

$$\frac{\partial \log L}{\partial \rho} = -\frac{1}{2\sigma^2}\sum_{t=2}^{T} 2(\log p_t - \mu_{t-1})(-\log p_{t-1}) = 0$$
$$\frac{\partial \log L}{\partial \sigma^2} = -\frac{T-1}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{t=2}^{T}(\log p_t - \mu_{t-1})^2 = 0$$

E Option Pricing

The payoff of the put options is given by $\max\{\bar{p} - p_{t+1}, 0\}$ for strike price $\bar{p}$ and current price p_t at time t. For a risk neutral investor, the put option is priced according to the following formula:

$$\xi(p_t) = \frac{E_t\left[\max\{\bar{p} - p_{t+1}, 0\}\right]}{1 + r^*}$$

Since we assume that $\log p_t$ follows an AR(1) process, i.e.

$$\log p_{t+1} \sim N\left(\underbrace{(1-\rho)\left[\log(p) - \frac{1}{2}\frac{\sigma^2}{1-\rho^2}\right] + \rho \log p_t}_{\mu_t}, \sigma^2\right)$$

Hence, we have

$$
\begin{aligned}
\xi(p_t) &= \frac{E_t\left[\max\{\bar{p}-p_{t+1},0\}\right]}{1+r^*} \\
&= \frac{\int_{\log p_{t+1}\leq \log \bar{p}}(\bar{p}-p_{t+1})}{1+r^*} \\
&= \frac{1}{1+r^*}\int_{-\infty}^{\log \bar{p}}(\bar{p}-p_{t+1})\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(\log p_{t+1}-\mu_t)^2}{2\sigma^2}}d\log p_{t+1} \\
&= \frac{\bar{p}}{1+r^*}\Phi\left(\frac{\log \bar{p}-\mu_t}{\sigma}\right)-\frac{1}{1+r^*}\int_{-\infty}^{\log \bar{p}}p_{t+1}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(\log p_{t+1}-\mu_t)^2}{2\sigma^2}}d\log p_{t+1} \\
&= \frac{\bar{p}}{1+r^*}\Phi\left(\frac{\log \bar{p}-\mu_t}{\sigma}\right)-\frac{1}{1+r^*}e^{\mu_t+\frac{\sigma^2}{2}}\Phi\left(\frac{\log \bar{p}-\mu_t-\sigma^2}{\sigma}\right)
\end{aligned}
$$