

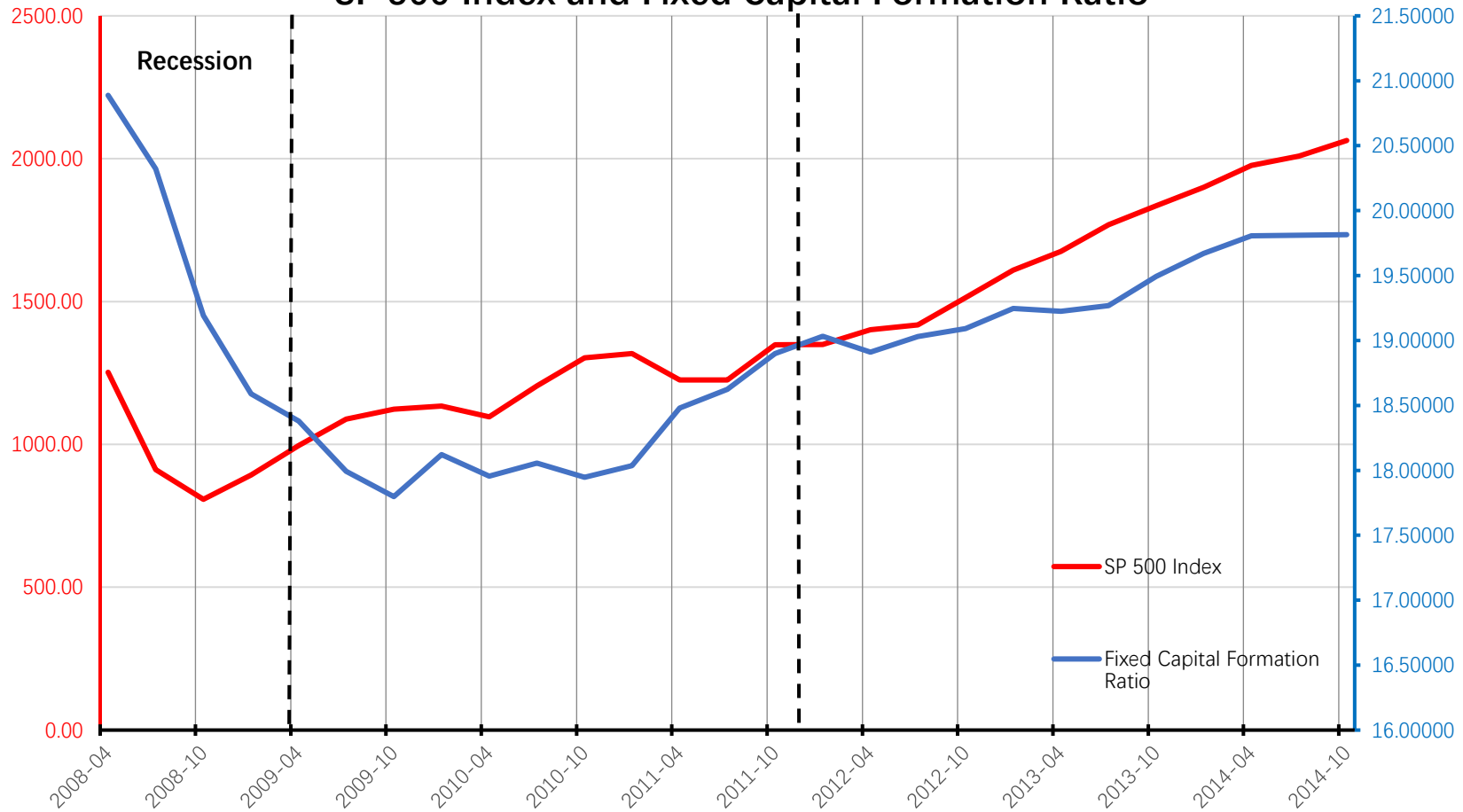
Uncertainty, Liquidity and Financial Cycles

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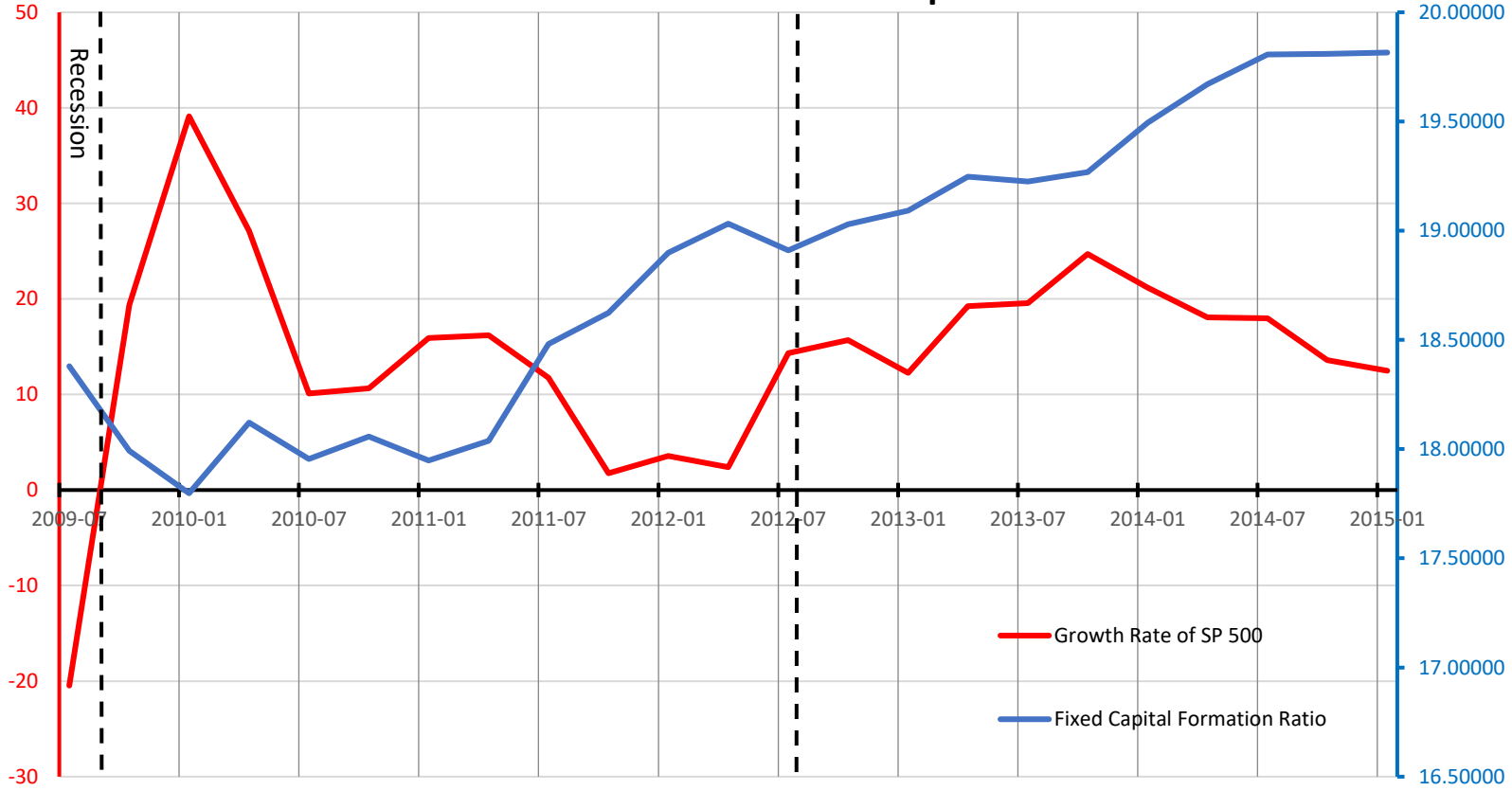
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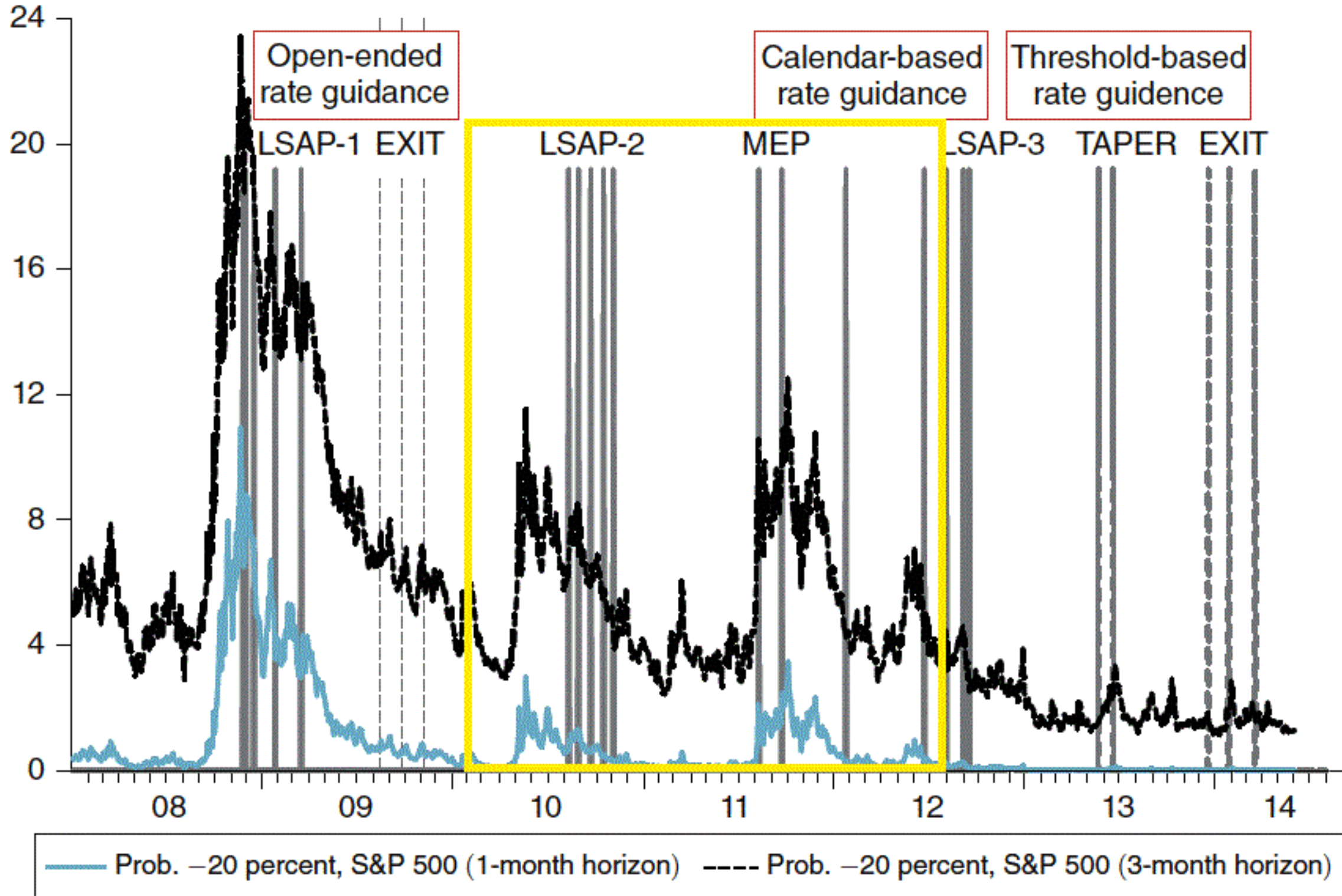
SP 500 Index and Fixed Capital Formation Ratio



Annual Growth Rate of SP 500 and Fixed Capital Formation Ratio



- Obvious Phenomenon: Inconsistence between macroeconomic fluctuations and financial cycles
 - After the Great Recession, a slow economic recovery together with financial booms
 - Macroeconomy and financial markets often blossom together.
- Very FEW Theoretical Explanation



- Main Question: What are the roles of **Uncertainty**?
- My Answer: An endogenous **Liquidity Allocation Mechanism** between real economy and financial system
- A simple theory based on a tractable continuous-time DSGE model of heterogenous agents
- Numerical Analysis of Global Dynamics

Key Assumptions

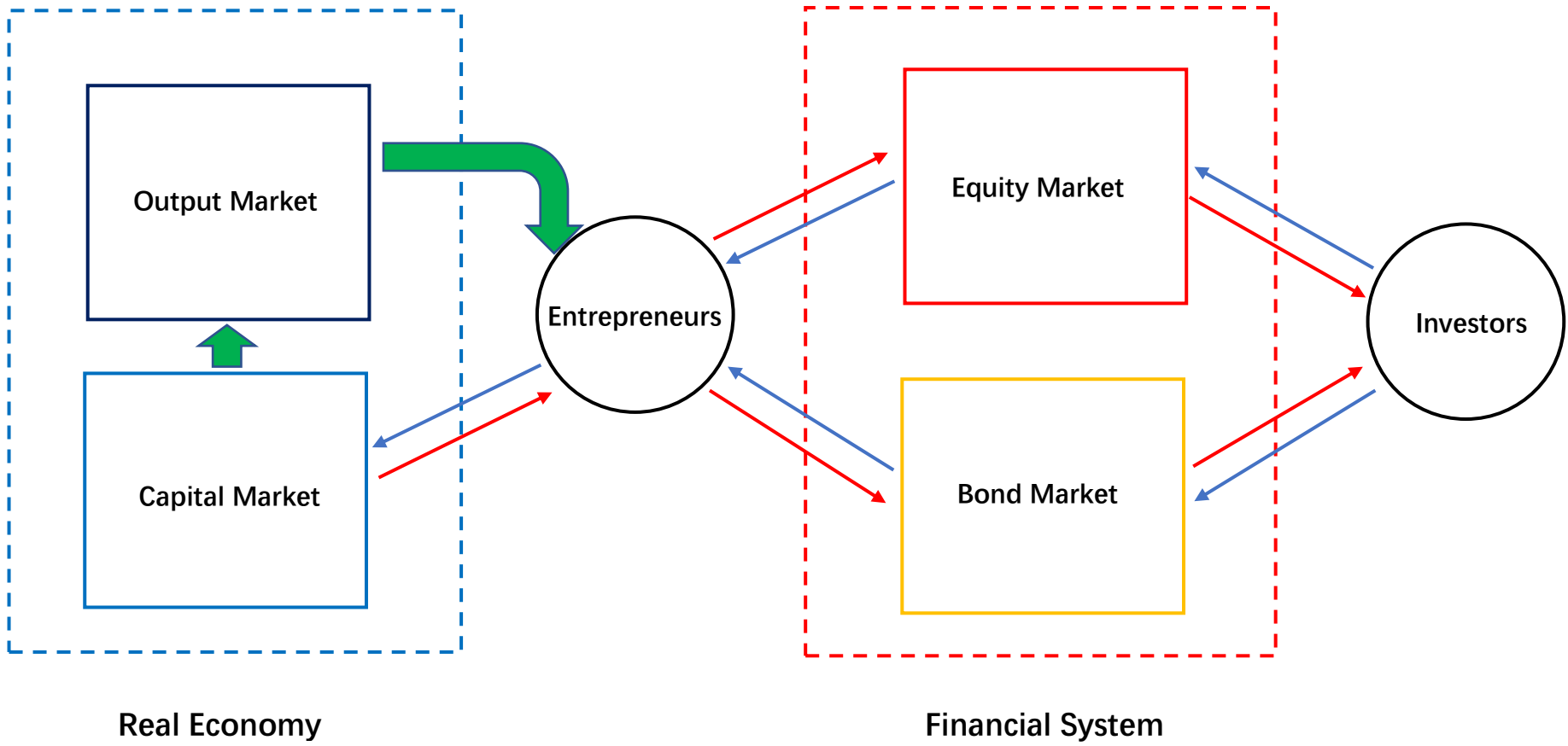
- Slower Adjustment of Macroeconomy than Financial Market.
 - Physical capital has less liquidity than its corresponding equity.
- Financial Frictions: “Skin in the Game”
 - Entrepreneurs have to take partial risk of their own investment.
- This leads to
 - entrepreneur faces a leverage constraint that depends on his capital structure
 - investment depends on not only equity price but also the capital structure

- Net Worth of an entrepreneur
 - determines his ability of risk tolerance
 - affects corporate investment
- Recession Stage:
 - Risk-averse entrepreneurs who have low net worth, will disinvest and hold more financial assets.
 - More funds flows into financial system from real economy.
 - There is a financial booms but a slow economic recovery.
- Booms Stage:
 - Entrepreneurs with high net worth have high investment demand.
 - This leads to high equity prices.

- Bolton, Chen, Wang (JF, 2011)
- Bolton, Wang, Yang (Forthcoming in JF)
- Brunnermeier, Eisenbach and Sannikov (2013)
- Brunnermeier and Sannikov (AER 2014, 2017)
- Di Tella (JPE, 2017)
- He and Kondor (Econometrica, 2016)
- Kiyotaky and Moore (2012)

The Model: Environment

- Infinite identical risk-averse entrepreneurs that totally measured by 1.
- Infinite identical risk-neutral investors that totally measured by 1.
- ONLY entrepreneurs can run physical capital.
- Entrepreneurs raise funds by issuing equity and debt.
- Investors can buy equity and risk-free bonds.



The Model: Entrepreneur's Balance Sheet

Assets	Liabilities
Capital: qK	(Outside) Equity: $(1 - \chi) qK$
(Others') Equity: qH	Bond: $(\nu - 1)W$
	Net Worth: W

The Model: Key Assumption

- Less liquidity of physical capital because of transaction cost:

$$\Psi(\kappa) < \kappa$$

- This implies that capital and equity are imperfect substitutive.
- The market price of physical capital is denoted by p .
- The evolving process of capital is given by

$$dK = [\Phi(l) + \Psi(\kappa) - \delta] K dt + \sigma K dZ$$

- Entrepreneurs face equity issuance constraint: $\chi \geq \bar{\chi}$.
- This leads to
 - a leverage constraint for entrepreneurs from the equity issuance constraint
 - the optimal investment choice depends on equity price, q , and the capital structure

The Model: Entrepreneurs

$$\max_{\{C, \chi, l, \kappa, v, \zeta\}} E \int e^{-\rho t} \frac{C^{1-\gamma}}{1-\gamma} dt$$

s.t.

$$\chi \geq \bar{\chi}$$

$$qH \geq 0$$

$$vW \equiv \chi qK + qH$$

$$dW = vWdR + (1-v)Wr dt - Cdt$$

$$\frac{d(qH)}{qH} = (\zeta - \delta + \mu^q + \sigma\sigma^q) dt + (\sigma + \sigma^q) dZ$$

$$\frac{d(qK)}{qK} = [\Phi(l) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q] dt + (\sigma + \sigma^q) dZ$$

where

$$dR = \left(\frac{A - l - p\kappa}{q} \right) dt + [\Phi(l) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q] dt + (\sigma + \sigma^q) dZ$$

- Optimal Investment Ratio:

$$\left[\frac{(1 - \gamma) \varphi(w) - w\varphi'(w)}{vw\varphi'(w)} + 1 \right] \Phi'(l) = \frac{1}{q}$$

- $w \equiv \frac{W}{qK}$: the capital ratio; $\varphi(w) \equiv J(W, qK) / (qK)^{1-\gamma}$
- Relative Price between capital and equity valued by entrepreneurs:

$$\frac{(1 - \gamma) \varphi(w) - w\varphi'(w)}{vw\varphi'(w)} + 1 = \frac{qKJ'_{qK} + vWJ'_W}{vWJ'_W}$$

The Model: Entrepreneurs

- Trade-Off between Producing Capital and Purchasing Capital:

$$\underbrace{\Phi'(\iota) K}_{\text{Capital Formation by Producing}} = \underbrace{\Psi'(\kappa) K/p}_{\text{Capital Formation by Purchasing}}$$

- Asset Pricing of Inside Equity:

$$\underbrace{\frac{A - \iota - p\kappa}{q} + \Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q - r}_{E(dR)/dt} + \underbrace{\frac{\lambda_2}{V - \gamma\varphi'}}_{\text{Liquidity Premium}}$$
$$= \left[\gamma + (1 - \nu) \frac{w\varphi''}{\varphi'} \right] (\sigma + \sigma^q)^2$$

- λ_2 : the Lagrangian multiplier of equity issuance constraint $\chi \geq \bar{\chi}$

The Model: Differences with BS (2014, 2017)

- BS(2014, 2017): physical capital \Leftrightarrow equity: **relative price is 1**

$$\underbrace{\Phi'(\iota) K}_{\text{Capital Formation by Producing}} = \underbrace{K/q}_{\text{New Capital by Purchasing}}$$

- Non-Arbitrage Condition: one more unit of investment ratio

$$\underbrace{\Phi'(\iota) K}_{\text{Capital Formation by Producing}} = \underbrace{\Psi'(\kappa) K/p}_{\text{Capital Formation by Purchasing}}$$



$$\max_{\{c^t \geq 0, \underline{v}\}} E_0 \left[\int_0^\infty e^{-rt} \underline{C} dt \right]$$

s. t.

$$d\underline{W} = \underline{v} \underline{W} dR + r(1 - \underline{v}) \underline{W} dt - \underline{C} dt$$

- Asset Pricing of (Outside) Equity:

$$\underbrace{\frac{A - \iota}{q} + \Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q}_{E(dR/dt)} = r$$

$$\mu^q q = q'(\eta) \mu^\eta \eta + \frac{1}{2} q''(\eta) (\sigma^\eta \eta)^2$$

$$\begin{aligned} & \left\{ \rho - (1 - \gamma) \left[\Phi(\iota) - \delta + \mu^q + \sigma^q \sigma - \frac{\gamma}{2} (\sigma + \sigma^q)^2 \right] \right\} \varphi \\ = & \max_{\{c, \iota, v\}} \frac{(c\eta)^{1-\gamma}}{1-\gamma} + \frac{1}{2} (v-1)^2 (\sigma + \sigma^q)^2 \eta^2 \varphi'' \\ & + \left\{ v \frac{A-\iota}{q} + (v-1) [\Phi(\iota) - \delta + \mu^q + \sigma^q \sigma] \right. \\ & \left. + (1-v)r - c + (1-v)\gamma(\sigma + \sigma^q)^2 \right\} \eta \varphi' \end{aligned}$$

- $\eta \equiv \frac{\int_0^1 W^i di}{\int_0^1 qK^i di}$: entrepreneurs' wealth ratio over the aggregate wealth

Markov Equilibrium: Parameterization

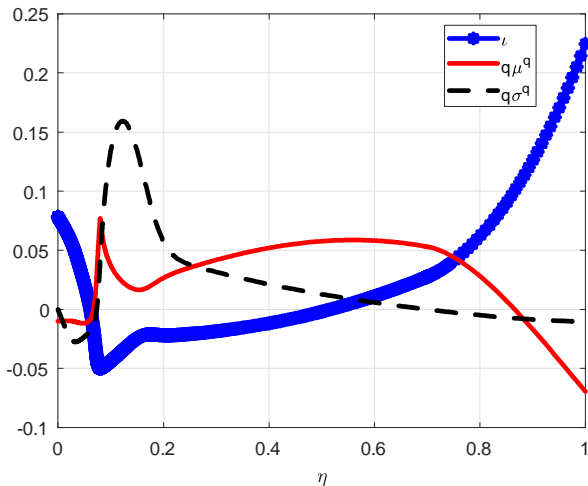
- Set Similar Parameters Values as Brunnermeier and Sannikov (2014):

Parameter	Meaning	Value
ρ	time discount rate of entrepreneurs	6%
r	time discount rate of investors	5%
γ	entrepreneur's risk aversion	2
A	productivity level	12%
δ	depreciation rate	3%
σ	capital quality shock	2%
ϕ	investment function	10
$\bar{\chi}$	equity issuance constraint	70%

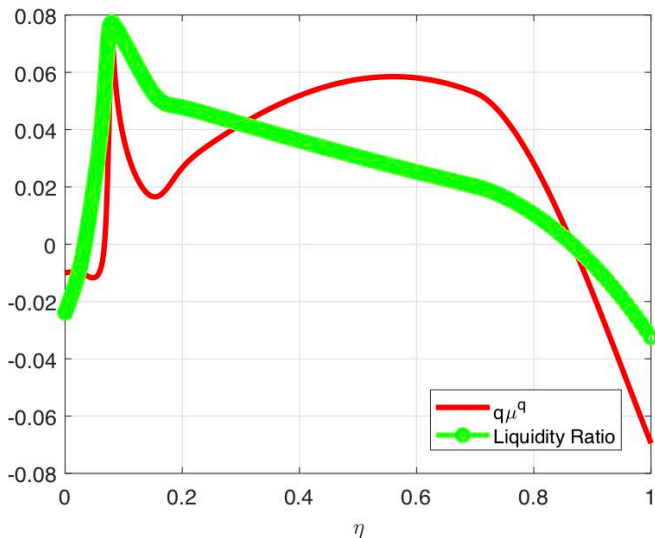
- Investment function:

$$\Phi(l) \equiv \frac{1}{\phi} \left(\sqrt{1 + 2\phi l} - 1 \right)$$

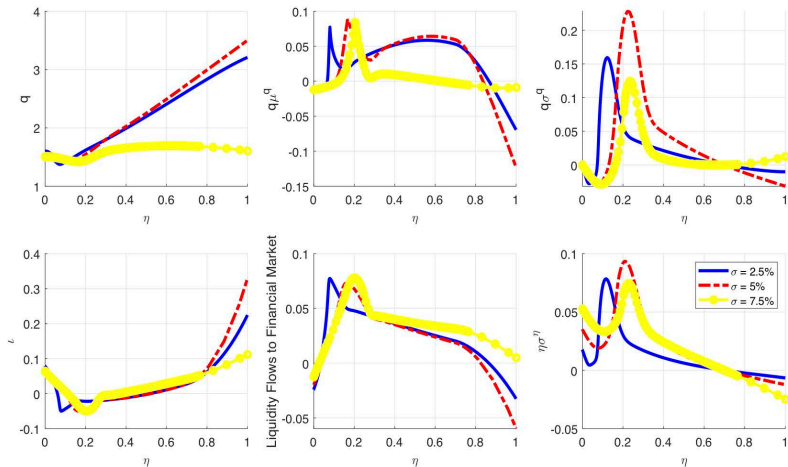
Investment Ratio, Equity Price Growth and Equity Market Risk



Equity Price Growth and Net Liquidity to Financial System



Global Dynamics with Different Extents of Equity Issuance Constraint



- A tractable DSGE model of an endogenous liquidity allocation mechanism between real economy and financial markets.
- The endogenous risks and liquidity allocation are helpful to understand the inconsistency between macroeconomic fluctuations and financial cycles.