# An Instrumental Variable Approach to Dynamic Models

work in progress, comments welcome

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# Introduction

We propose an IV approach to the estimation of dynamic models, both single and multiple agent, allowing for an economically meaningful treatment of endogenous market structure. We can handle models that are incomplete and/or set identified because of some combination of initial conditions, multiple equilibria and discrete data.

The approach has close connections to "two-step" methods that build on Hotz-Miller as well as to more computationally based methods that lead to MLE estimation.

# I.I.D. Shocks

Much of the existing empirical dynamic literature models unobserved shocks as

- independently distributed over time
- private information

This greatly simplifies estimation methods, as outlined in papers including Rust (1987), Hotz and Miller (1993) Bajari, Benkard and Levin (2007) , Pakes, Ostrovsky, and Berry (2007) and Pesendorfer and Schmidt-Dengler (2008).

# Independent Private Shocks in Dynamic Models

Given independent private information, current states do not reflect any (persistently) unobserved factors, only accumulated past luck. So, the current state is econometrically exogenous.

This leads naturally to the two-step Hotz-Miller style approaches:

1. first identify the dynamic policy function, "directly from the data" and then
2. use this plus Bellman's equation to identify structural parameters of the single-period return function.

For formal identification, see Magnac and Thesmar (2002) and related literature.

# Problems with Exogenous States

Econometrically exogenous states are greatly at odds with the "static entry" literature, which emphasizes the econometric endogeneity of market structure.

Possible states in an IO model: past entry, number of firms or outlets, capital, quality .... Each is associated with a dynamic decision: entry, store opening, investment in capital or quality. In each case, it's likely that persistent unobservables are correlated with both the current decision and the current observed state.

# Serial Correlation in Optimizing Dynamic Models

- Keane and Wolpin (1997) (single agent discrete-type, no strict identification proofs, moderately computational intensive)
- Arcidiacono and Miller (2011) (oligopoly with mixture models and possible multiple equilibria, computationally intensive, no strict identification proofs, but see Kasahara and Shimotsu (2009).)
- Pakes and Ericson (1996) (oligopoly; similar to us, but "fit to ergodic distribution" for us becomes "fit to IV restrictions").
- Kalouptsidi, Scott, and Souza-Rodrigues (2018) (single-agent special case, same intuition for IV with simple linear implementation)

Literatures on "less structural" dynamic panel models include important work on persistent heterogeneity vs. state dependence and on initial conditions problems

# An IV Alternative

Our approach is similar to the 2-step CCP but we identify (or at least restrict) the policy function via generalized instrumental variable ("GIV") methods following the exposition in Chesher and Rosen (2017).

Chesher and Rosen (2017) builds on a very large prior & on-going literature, including many early papers on incomplete models by Manski (*e.g.* 2003) and/or Tamer, work on discrete entry models by Tamer and also work on sharply identified sets such as by Beresteanu, Molinari, and Molchanov (2011) & Galichon and Henry (2011).

Chesher and Rosen (2017) is useful to us both because of the generality of the approach but also largely for the exposition that focuses on IV intuition in a broad class of models with discrete outcomes and incomplete models.

# IV Intuition

The IV intuition is that past exogenous variation will be correlated with current states. E.g. if Detroit was large and rich 50 years ago, it may have many Sears stores today. Past macro shocks may effect today's market structure, and these may interact with market-level characteristics. Past regulatory regimes may be correlated with market structure (consider hospitals).

In general, natural IVs include *past exogenous cost and demand shifters* that (in the presence of sunk costs, etc) have led to the present endogenous state.

# Outline

Today

- Dynamic Model (notation & examples)
- GIV restrictions
- Identified Set of Policies
- Example: Single Firm Entry
  - Identified sets as serial correlation, IV strength & $T$ vary
  - Inference example
- Oligopoly
  - Broad Idea
  - Worked example with both serially correlated public and i.i.d private info (skip today)
- A simple data example, using Collard-Wexler data

# Model

We begin with a single firm model. We see a large set of markets, each with a single firm, for a small fixed $T$.

Then we consider the oligopoly case.

# Single Period Return

## Monopoly Model

Panel data for a (fixed) $T$ periods, with the periods denoted $t = 1, \ldots, T$. The full set of variables are not necessarily available for any prior history of the firm, $t < 1$, although there may be some partial history.

Single Period Profits for market $i$: $\pi\left(a_{it}, x_{it}, w_{it}, u_{it}; \theta_\pi\right)$

- $x_{it}$ endogenously chosen state(s)
- $a_{it}$ choice (policy) variable(s) (action)
- $w_{it}$ exogenous profit shifter(s)
- $u_{it} \in \mathbb{R}$, unobserved (to us) serially correlated unobservable

Any additional exogenous variables correlated with $x_{it}$ (i.e. policy prior to $t = 1$) are denoted $r_i$. Could extend (at cost) to multiple unobservables (esp. with multiple actions).

# Further notation

$$
\begin{aligned}
x_i &= (x_{i1}, \ldots, x_{iT}) \in \mathbb{X} \\
a_i &= (a_{i1}, \ldots, a_{iT}) \in \mathbb{A} \\
w_i &= (w_{i1}, \ldots, w_{iT}) \in \mathbb{W} \\
u_i &= (u_{i1}, \ldots, u_{iT}) \in \mathbb{U}
\end{aligned}
$$

# Transitions

## Classic setup with added serial correlation

Endogenous States:

$$\Gamma(x_{it+1}|a_{it}, x_{it})$$

Could be deterministic (degenerate) or stochastic state transitions.

Exogenous Observed States

$$\Lambda(w_{it+1}|w_{it})$$

Unobserved (by us) States

$$\bar{\Phi}(u_i; \theta_u)$$

our leading case is first order Markov

$$\Phi(u_{it+1}|u_{it}; \theta_u)$$

Assume $\Gamma$, $\Lambda$ are known and/or identified from data, but $\theta_u$ is unknown.

# Examples

Table: Some Single Agent IO Examples

| State, $x_{it}$ | Action, $a_{it}$ | $\mathcal{A}(x_{it})$ | Transition |
|---|---|---|---|
| Capital | Investment | $\mathbb{R}^+$ | $x_{it+1} = \lambda x_{it} + a_{it}$ |
| Out/In | Entry/Exit | $\{0, 1\}$ | $x_{it+1} = a_{it}$ |
| Retail | # of Stores | $\mathcal{I}^+$ | $x_{it+1} = a_{it}$ |
| Quality | R&D | $\mathbb{R}^+$ | $x_{it+1} \sim f(x_{it}, a_{it})$ |

# IV Assumption

$$z_i = (r_i, w_{i1}, w_{i2}, \ldots, w_{iT}).$$

$$u_i = (u_{i1}, u_{i2}, \ldots, u_{iT}).$$

Independence of the instrument and the unobservables:

$$z_i \perp u_i.$$

# Possible Instruments?

| State | Example IVs? |
|---|---|
| Capital | Past investment cost, past productivity shock |
| Out/In of Market | past market population, past zoning |
| # of Stores | distance from headquarters, interacted with time? |
| Quality | Past R&D shocks, age of firm |

# Dynamic Problem

Bellman Equation:

$$V\left(x_{it}, w_{it}, u_{it}\right) =$$

$$\max_{a_{it}\in\mathcal{A}(x_{it})} \left(\pi\left(a_{it}, x_{it}, w_{it}, u_{it}, \theta_\pi\right) + \delta E\left[V\left(x_{it+1}, w_{it+1}, u_{it+1}\right)|a_{it}, x_{it}, w_{it}, u_{it}\right]\right.$$

where

$$E\left[V\left(x', w', u'\right)|a, x, w, u\right] =$$

$$\int\int\int V\left(x', w', u'\right) d\Gamma(x'|a, x) d\Lambda(w'|w) d\Phi(u'|u; \theta_u).$$

# Policy function

In the true model (and therefore the data), the policy function takes the form

$$a_{it} = \sigma\left(x_{it}, w_{it}, u_{it}\right) \in \mathcal{F}.$$

There may be qualitative restrictions (monotonicity) embodied in the set of functions $\mathcal{F}$.

For a particular $\theta = (\theta_\pi, \theta_u)$, the policy function derived from known transitions and Bellman's equation is:

$$a_{it} = \sigma_\theta(x_{it}, w_{it}, u_{it}).$$

# Identification

For identification, say we observe the true data generating process, across firms or agents, denoted

$$P(a_i, x_i, w_i, r_i).$$

This is equivalent to seeing $T$ period panel on a very large (in fact, infinite) cross-section of firms or agents.

The unknowns are the parameters of profits $\theta_\pi$ and $\theta_u$. Nothing in our general discussion of identification requires these to be finite dimensional, but in practice we consider only finite-dimensional parametric models (including fully flexible profit functions with discrete $(a_i, x_i, w_i)$).

# Incomplete Model

$$
\begin{aligned}
a_{iT} &= \sigma\left(x_{iT}, w_{iT}, u_{iT}\right) \\
x_{it} &\sim \Gamma\left(x_{iT-1}, a_{iT-1}\right), \\
a_{T-1} &= \sigma\left(x_{iT-1}, w_{iT-1}, u_{T-1}\right) \\
x_{T-1} &\sim \Gamma\left(x_{iT-2}, a_{T-2}\right), \\
&\vdots \\
a_1 &= \sigma\left(x_{i1}, w_{i1}, u_{i1}\right).
\end{aligned}
$$

But: there is no model of the endogenous $x_{i1}$, which is inherited from unobserved prior history. If period 1 was the "birth" of the agent, we might have a model for $x_{i1}$, which would complete the single agent model. (Of course, this still doesn't guarantee identification.) Oligopoly will introduce a second source of incompleteness: multiple equilibria.

## Chesher and Rosen GIV

Idea: (set) identify the policy function from classic instrumental variables conditions, extended to "Generalized IV" (GIV) to deal with

1. incomplete model (we only have necessary or sufficient conditions on the unobservables, not necessary and sufficient) and
2. discrete variables, as in entry/exit models
3. lack of point identification of the parameters, even in the absence of problems 1 and 2.

We may have all or none of these problems. Discrete actions naturally lead to conditions on sets of unobservables that give a particular policy $a_{it}$.

# Necessary Conditions

## a familiar step

If the sequence $(x_i, w_i, a_i)$ occurs, then $u_i$ must be in the inverse image set

$$\mathcal{U}(a_i, w_i, x_i, \sigma) = \{u_i : \sigma(x_{it}, w_{it}, u_{it}) = a_{it}, \forall t\}$$

The condition $\{u_i \in \mathcal{U}(a_i, w_i, x_i, \sigma)\}$ is then a necessary condition for event $(x_i, a_i)$.

# Policy Functions consistent with IV conditions

Using Cheshire-Rosen style notation, a pair $(\sigma(x_{it}, w_{it}, u_{it}), \theta_u)$ is then in the identified set iff for all closed sets $\mathcal{S} \in \mathbb{U}$ and $\forall z$

$$\Pr\big(\mathcal{U}(a_i, x_i, w_i, \sigma) \subseteq \mathcal{S} \; | z\big) \leq \Phi(\mathcal{S}; \theta_u) \tag{1}$$

The LHS is the conditional probability of the outcomes $y_i = (a_i, x_i)$ that, according to $\sigma$, have $\{u_i : u_i \in \mathcal{S}\}$ as a necessary condition. For a given $\sigma$ and $z$, this probability is observed in the data. The RHS is the probability of that necessary condition wrt the distribution of $u_i$. The RHS is determined by the distribution of $u_i$, which by assumption does not depend on $z$.

But what "test sets" $\mathcal{S}$ to use?

# Chesher and Rosen Sharply Identified Policy Functions

CR show that to obtain the sharply identified $\theta$ set we only need to check certain sets $\mathcal{S} \in Q(\sigma, w_i)$.

The collection of sets $Q(\sigma, w_i)$ is the "core determining set" as defined in CR (2017) and earlier work, the minimal collection of closed sets $\mathcal{S} \in \mathbb{U}$ that yields the sharp identified set for $\theta$. These include the overlapping sets of $\mathcal{U}(a_i, x_i, w_i, \sigma)$, excluding cases of strict subsets. For simple low dimensional discrete problems, $Q(\sigma, z_i)$ can be easy to compute and not "too big", but it can otherwise grow very (indeed infinitely) large.

This result tells us the necessary conditions we need to check to get sharp identification. Perhaps most useful for smaller problems and to build intuition in examples.

# Equalities and Complete Models

In some cases, some of the inequalities in (1) are equalities, because the necessary conditions are *necessary and sufficient* for particular (sets of) actions.

In a complete model, all of the conditions would be equalities. However, as usual, this does not guarantee that the parameters are point identified.

# Policies identified by GIV Alone

For a given $\theta_u$, denote by

$$\Sigma^{IV}(\theta_u) \subseteq \mathcal{F}.$$

the set of $\sigma$ functions identified from the data and the IV restrictions—*i.e.* those that satisfy condition (1) $\forall \mathcal{S} \in Q(\sigma, z)$ and $\forall z$.

The sets $\Sigma^{IV}$ are determined exclusively by the IV conditions and the data, with no use of the dynamic model.

# Identified Set of Structural Parameters

For any $\theta = (\theta_\pi, \theta_u)$, we can use the Bellman equation to compute the implied policy $\sigma_\theta(x_{it}, w_{it}, u_{it})$. For $\theta = (\theta_\pi, \theta_u)$, this policy is

$$\sigma_\theta(x_{it}, w_{it}, u_{it}) \equiv$$

$$\underset{a_{it} \in \mathcal{A}(x_{it})}{\operatorname{argmax}} \Big( \pi(a_{it}, x_{it}, w_{it}, u_{it}) + \delta E\big[ V(x_{it+1}, w_{it+1}, u_{it+1}) | a_{it}, x_{it}, w_{it}, u_{it} \big] \Big).$$

The sharply identified set of parameters is then

$$\Theta_{ID} \equiv \{\theta = (\theta_\pi, \theta_u) \colon \sigma_\theta(x_{it}, w_{it}, u_{it}) \in \Sigma^{IV}(\theta_u)\} \tag{2}$$

This imposes both the dynamic model and the GIV restrictions. This is the sharply identified set because any $\theta$ in this set generates a policy function that cannot be rejected by the data plus the IV condition.

# Connection to Two Step Models

We could follow a “classic” 2-step approach:

1. identify a (set of) policy function(s) that are consistent with the data and the IV restrictions and then
2. see which structural parameters are consistent with the identified policy function(s). These are the identified $\theta$.

Complications include the presence of parameters for the unobservables (necessary at least to model the degree of serial correlation) and the IV methods in the first step, as opposed to directly “fitting the policy to data.”

# Connection to Two Step Models

## Continued

It's possible that one might point identify the policy function in step 1. In this case, our procedure is just like a 2-step CCP method, except for running a GIV first stage.

For example, when the choice variable is continuous, point identification of the policy is attained under the assumptions in Chernozhukov and Hansen (2005), even using only one transition.

With discrete choice variables, often we'll get only partial identification of the policy (Chesher (2010); Chesher, Rosen and Smolinski (2013)).

# Connection to Two Step Models

## Continued

As a computational alternative, one could search over the space of $\theta$'s, for each possible $\theta$

1. computing $\sigma_\theta(x_{it}, w_{it}, u_{it})$ via the contraction mapping and then
2. testing whether $\sigma_\theta$ survives the IV restrictions applied to the data. If so, that particular $\theta$ is in the identified set, otherwise not.

# Example: Single Firm "Entry / Exit"

A minimal model to think about dynamics.

$$a_{it} \in (0,1)$$

State is In/Out in the prior period

$$x_{it} \in (0,1)$$

Single period payoffs, $\pi(a_{it}, x_{it})$, have a random shock, $\epsilon_{it}$, plus a scalar sunk entry cost, $\gamma$.

$$\begin{aligned}
\pi(0,1) &\equiv \pi(0,1) = 0, \\
\pi(1,1) &= \bar{\pi} - \epsilon_{it}, \text{ with} \\
\pi(1,0) &= \pi(1,1) - \gamma, \text{ and}
\end{aligned}$$

$$\epsilon_{it} = \rho\epsilon_{i,t-1} + \nu_{it},$$

where $\nu_{it} \sim \mathcal{N}(0,1)$. Think of $u_{it}$ as the quantile of $\epsilon_{it}$.

3 structural parameters: $\bar{\pi}, \gamma, \rho$.

# Policy Function in the Example.

Policy function:

$$a_{it} = \sigma(x_{it}, u_{it})$$

where $u_{it} \sim \text{unif}(0,1)$.

The dynamic model generates a monotonicity result that $\sigma$ is weakly increasing in $x$ and weakly decreasing in $u$.

# Policies Identified by GIV from only One Transition

## single firm entry/exit example

This is a binary choice style model *a la* Chesher (2010), using only the marginal distribution of $u$, which is independent of some instrument.

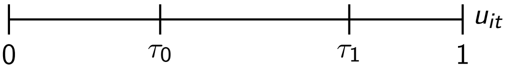



Figure: Policy Cut-offs in the Example

As an example of identification, we look at restrictions only involving the marginal distribution of $u$, next consider IV restrictions on the joint distribution across periods.

# Restrictions Using Only One Choice

Table: Inverse image sets for the Binary-Binary Marginal Example

| $a$ | $x$ | $\mathcal{U}(a,x)$ |
|---|---|---|
| 1 | 1 | $(0,\tau_1)$ |
| 1 | 0 | $(0,\tau_0)$ |
| 0 | 1 | $(\tau_1,1)$ |
| 0 | 0 | $(\tau_0,1)$ |

Table: Restrictions via Elemental Sets for the Binary-Binary Marginal Example

| $\mathcal{S}$ | $\Pr\big(\mathcal{U}(a_i,x_i,\sigma)\subseteq\mathcal{S}\ \lvert z\big)$ | $\leq \Phi(\mathcal{S};\theta_u)$ |
|---|---|---|
| $\mathcal{U}(1,1)$ | $\Pr((1,1)\lvert z)+\Pr((1,0)\lvert z)$ | $\leq \tau_1$ |
| $\mathcal{U}(1,0)$ | $\Pr((1,0)\lvert z)$ | $\leq \tau_0$ |
| $\mathcal{U}(0,1)$ | $\Pr((0,1)\lvert z)$ | $\leq 1-\tau_1$ |
| $\mathcal{U}(0,0)$ | $\Pr((0,0)\lvert z)+\Pr((0,1)\lvert z)$ | $\leq 1-\tau_0$ |

# GIV Restrictions From the Binary-Binary Marginal Example

From the restrictions in the table, get upper & lower bounds for each $z$. Let $P_{11}(z) \equiv \Pr((1,1)|z)$.

$$P_{11}(z) + P_{10}(z) \leq \tau_1 \leq (1 - P_{01}(z)) \quad \Rightarrow$$

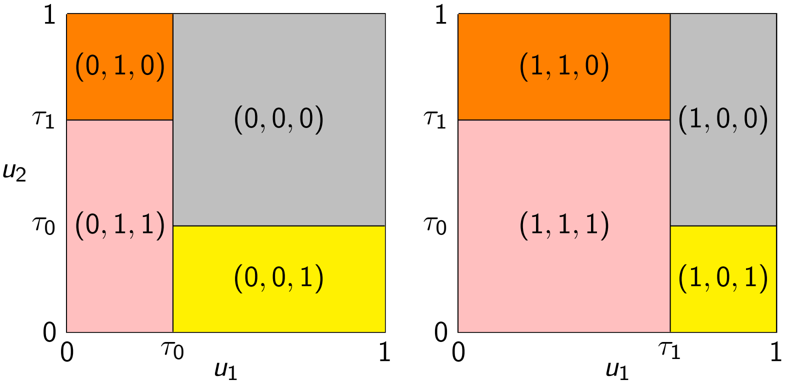
$$\max_z [P_{11}(z) + P_{10}(z)] \leq \tau_1 \leq \min_z [(1 - P_{01}(z))] .$$

Similar result for $\tau_0$.

But from one transition there is limited information on the structural parameters, and these bounds say nothing about serial correlation.

# Elemental Inverse Image Sets, $T = 2$

8 elemental sets $\mathcal{U}(a_i, x_i, \sigma)$, labeled $(x_{i1}, a_{i1}, a_{i2})$

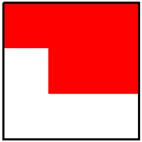



For example, from the necessary condition for $(0, 0, 0)$

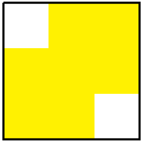
$$\Pr(u_1 > \tau_1, u_2 > \tau_1 | \rho) \geq P_{000}(z) + P_{100}(z) = P_{\cdot 00}$$

# Further Core Determining Sets

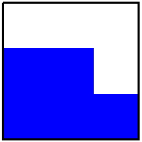
The core determining set includes unions of the “elemental” sets $\mathcal{U}(a_i, x_i, w_i, \sigma)$. The relevant unions are of “partially overlapping” sets.



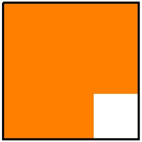
(a) (0,0,0)+(1,1,0)



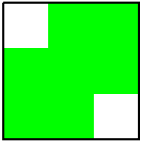
(b) (0,0,0)+(1,1,1)



(c) (1,1,1)+(0,10,1)



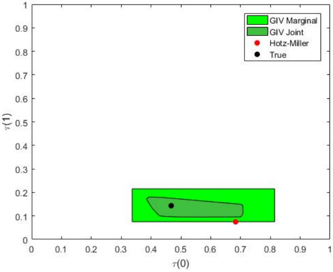
(d) (a)+(b)



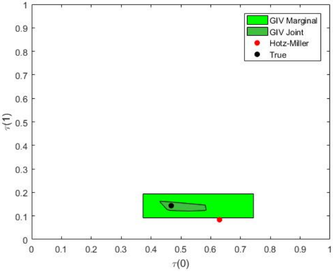
(e) (b)+(c)

# Computing the Identified Set of Policy Function Parameters

## True $\rho = 0.75$, varying IV "strength", Both 1 and 2 periods of data




(a) IV strength = 0.25

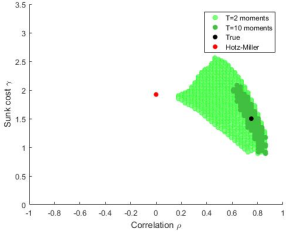



(b) IV strength = 0.56

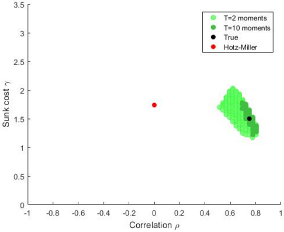
(Example has $\bar{\pi} = 0.5$, $\gamma = 1.5$. IV "strength" is $R^2$ from a regression of $x_{i1}$ on $z_i$. $\rho$ is also a "policy" parameter, see next slide. Policy cutoffs $\tau(a_{it})$ are in the quantiles of $\epsilon$.)

# Computing the Identified Set of Structural Parameters

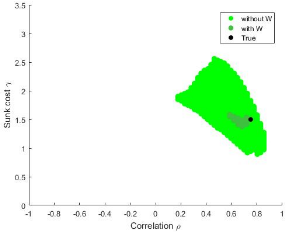
True $\rho = 0.75$, varying IV strength, T=2 or T=10, $w_{it}$ varying or not




(a) IV strength = 0.25



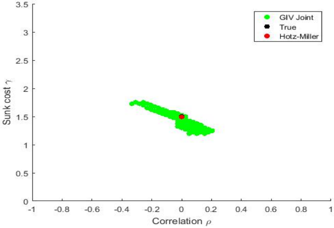

(b) IV strength = 0.56



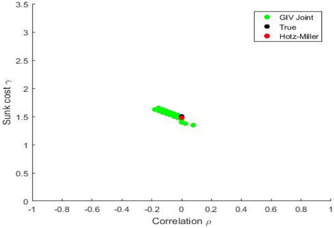

(c) IV strength = 0.25, varying $w_{it}$

# Computing the Identified Set of Structural Parameters

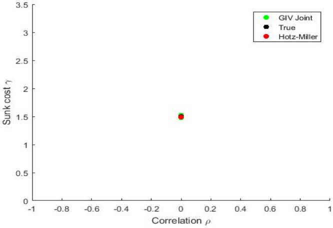
True $\rho = 0$, varying IV strength, 2 transitions in the data




(a) IV strength = 0.25




(b) IV strength = 0.56




(c) IV strength = 1.0

# Inference

We can apply inference procedures from the moment inequalities literature, simply following on the literature from Chernozhukov, Hong, and Tamer (2007).

With discrete IVs, we could work with unconditional moment inequalities (e.g. Andrews and Soares (2010) and Bugni, Canay, and Shi (2017)).

With continuous IVs, we could use conditional moment inequalities (e.g. Andrews and Shi (2013)).

With many moment inequalities (likely in this case) we use Chernozkukov, Chetverikov, and Kato (2018)

# 95% Confidence Sets

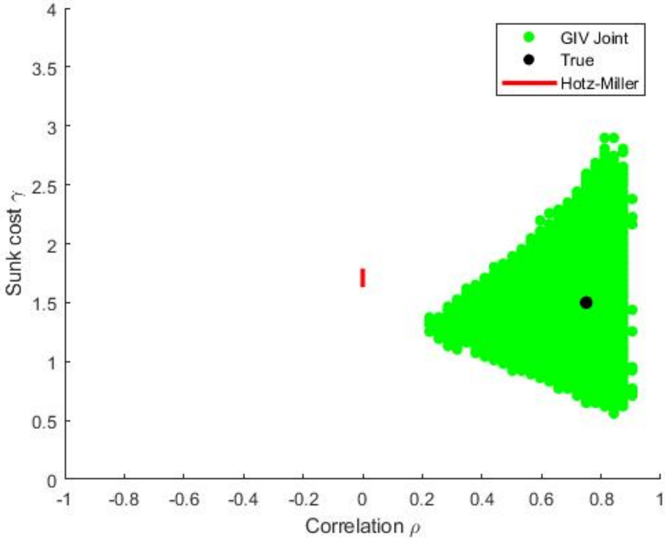



Figure: $\rho$=0.75, IV strength=0.56

# Inference on a Counterfactual 50% Increase in Sunk Costs

**Fraction of Markets with an Increase in $a_{it}$ in at Least 1 Period out of the Next 10 Periods**

| $\rho$ | IV strength | True | GIV Joint | Hotz-Miller |
|---|---|---|---|---|
| 0 | 0.56 | 0.44 | (0.26,0.58) | (0.41,0.51) |
| 0.50 | 0.56 | 0.30 | (0.13,0.65) | (0.49,0.70) |
| 0.75 | 0.56 | 0.18 | (0.05,0.43) | (0.41,0.54) |

# Dynamic Games

more preliminary

Our approach can be extended to models of dynamic strategic interaction. Specifically, consider dynamic games of complete information and allow for serially correlated unobservables. Let $J$ be the number of players. Then agent $j'$s policy in market $i$ at time $t$ is given by

$$a_{jit} = \sigma_j \left( w_{it}, x_{it}, u_{it} \right) \tag{3}$$

with $w_{it} = (w_{1it}, \ldots, w_{Jit})$, etc.

Policies now depend on the unobservables of all players.

With serial correlation, complete information is much easier than private information, but we can add iid private information on top of serially correlated public info, which makes computation (and existence) easier.

# Broad Idea of the Games Approach

As in the single agent case, identification comes from combining the GIV restrictions with the Bellman equation that defines the "best response" of each firm to the actions of the other.

Computationally, it is often very difficult to solve for the set of equilibrium strategies. In this case, we can restrict that computation to those strategies that survive the GIV conditions. In a favorable case, this would be a small set.

Additional assumptions may simplify the task, e.g. ensuring the game is symmetric, that strategies are monotonic in some arguments, etc.

# Computation in the Oligopoly Case

In some case, it may pay to retain the order of the Hotz-Miller two-step, giving a method that is also close to Ericson and Pakes (1995) but which rules out many possible policy functions via the GIV conditions.

1. Find the set of policies that survive the GIV conditions.
2. For each possible structural parameter vector, see if any of those policies map back into themselves in the dynamic equilibrium. (by holding the rival's policy fixed at a GIV-consistent function and solving the single-firm Bellman equation for the own-firm policy).

# Computation in the Oligopoly Case

continued

- Various kinds of monotonicity & symmetry can help a lot.
- We have a worked out computational example with private i.i.d. information plus a discrete serially correlated shock.
- Other good example (not yet computed): continuous oligopoly investment where one might point-identify the policy.

# An Application

Based on Collard-Wexler (2014), who studies the number of firms in small-town ready-mix concrete

- data for the years 1994 to 2006, publicly available.
- the $N$ of concrete firms shifted by local construction employment.
- serially correlated errors but otherwise, like Bresnahan & Riess, an ordered entry model of $N$
- parametric policy function, but no estimates of structural parameters.
- initial conditions set by directly modeling the initial unobservables
- is no multiple equilibria problem under the Abbring-Campbell "last in-first out" conditions.

# Our Extension of Collard-Wexler

- variable profit estimated as a function of $N$ from (limited) public data, outside of the dynamic model.
- a deterministic sunk entry cost & a log-normal per-period fixed cost
- four unknown structural dynamic parameters: sunk cost & mean fixed cost, plus the standard deviation & serial correlation of the unobservable
- We also use the Abbring and Campbell (2010) model of the number of firms in dynamic oligopoly, with their "refinement" to unique equilibrium.
- We have an initial conditions problem

# Our Approach

- $N \in \{0, 1, 2\}$
- For set estimation, we use a large number of inequalities and employ the inference method of Chernozkukov, Chetverikov, and Kato (2018)
- moments include all the 2-period transition core sets + over 12 periods "any entry," "never entry," "$N$ always $= 0$", etc., plus unions of these
- For now, we use a coarse grid search, recent computational methods could be much better.

# IVs in the Data Example

Our instruments for the initial conditions is long-term past income growth, which would have encouraged a local construction industry. We assume that current cement profits demand on current construction profits, which are shifted by construction employment and the unobservable.

Note also that (discretized) within sample variation in the demand shifter also aids identification, as does the long times series in the panel and the multiple levels of $N$ (not just binary entry).

# A Simple Dataset

A balanced panel of 428 markets over the period 1994-2004.

| Variable | Mean | St.Dev. | Min | Max |
|---|---|---|---|---|
| Number of plants | 0.79 | 0.69 | 0.00 | 2.00 |
| Construction Employment | 454 | 524 | 3 | 5,857 |
| Household Income Growth 1969-1989 | 0.15 | 0.11 | -0.18 | 0.69 |

Table: Summary statistics.

# Structural Estimation: Probabilities of Test Sets

## simulation

For some test sets, the probabilities are hard to compute, so use a simulation procedure to obtain test sets. For a given observable event and a given candidate policy function, we

- take many draws for the unobservables
- obtain the equilibrium number of firms over time implied by the candidate policy function
- compute the fraction of markets where the event $\mathcal{E}$ occurs

This gives us the probability on the r.h.s. of the Chesher-Rosen inequality. On the l.h.s. we simply have the probability of the event $\mathcal{E}$, conditional on values of the instruments.

# A Simple Dataset

A balanced panel of 428 markets (smallish towns) over the period 1994-2004.

| Variable | Mean | St.Dev. | Min | Max |
|---|---|---|---|---|
| $N$ | 0.79 | 0.69 | 0.00 | 2.00 |
| Construction Employment | 454 | 524 | 3 | 5,857 |
| Household Income Growth 1969-1989 | 0.15 | 0.11 | -0.18 | 0.69 |

Table: Summary statistics.

# A Descriptive Ordered Probit

| | | | | |
|---|---|---|---|---|
| Log Construction Employment | 0.30** | 0.31** | 0.36** | 0.36** |
| Income Growth 1969-1989 | 0.43** | 0.43** | 0.55** | 0.55** |
| Year Fixed Effects | | × | | × |
| State Fixed Effects | | | × | × |
| Likelihood-Ratio Test p-value | 0.00 | 0.00 | 0.00 | 0.00 |

Table: Ordered probit results. Dependent variable is number of plants. ** denotes significance at the 95% level.

# Confidence Set for 2 of the Parameters

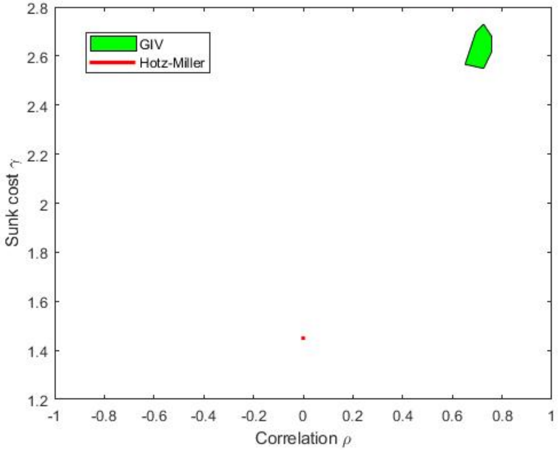



Figure: Autocorrelation and sunk cost: Projection of 95% confidence sets

# Policy Counterfactual Using the Concrete Data

To mimic a technological improvement, we simulate a decrease in the sunk cost by 0.25, corresponding to 17% of its initial level and summarize changes after 5 years.

| | GIV | Hotz-Miller |
|---|---|---|
| Change in # firms | (0.04,0.07) | (0.33,0.36) |
| Old firms exiting | (0.08,0.13) | (0.29,0.32) |
| New firms entering | (0.12,0.19) | (0.64 0.66) |

Table: Subsidy to entry (95% confidence intervals)

# Conclusion

- Market structure in dynamic IO models should not be assumed to be exogenous.
- General IV models are one natural way to handle the econometric endogeneity of states, while preserving much of the intuition of existing two-step methods.
- Intuition of IV: past exogenous shifters (and regulation, etc) are correlated with today's state.
- May have set-identified policies and/or structural parameters. The data, the IV restriction and Bellman's equation together restrict the identified set.
- Empirical policy applications are feasible using current methods

Needed: more applications!

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