Estimating Policy Functions Implicit In Asset Prices

Jeroen Dalderop
University of Notre Dame

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Motivation

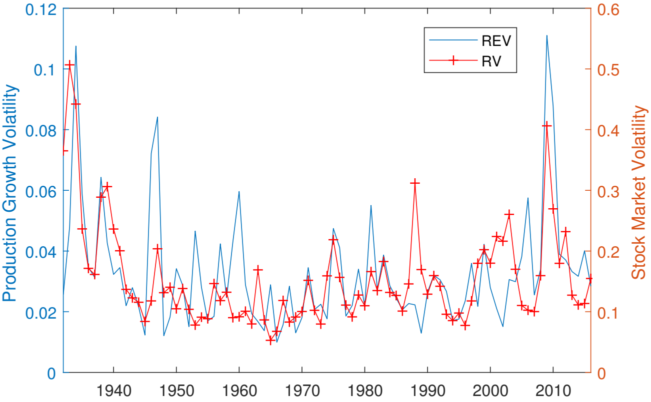


Figure 1: Annual standard deviation of monthly Industrial Production Growth (REV) and daily S&P 500 Index Returns (RV) from 1930-2016, detrended.

Stock market volatility & economic uncertainty: strongest +ve correlation in recessions

- $\hat{\rho}|_{\text{low REV}} = 0.22 \quad \text{vs.} \quad \hat{\rho}|_{\text{high REV}} = 0.53$

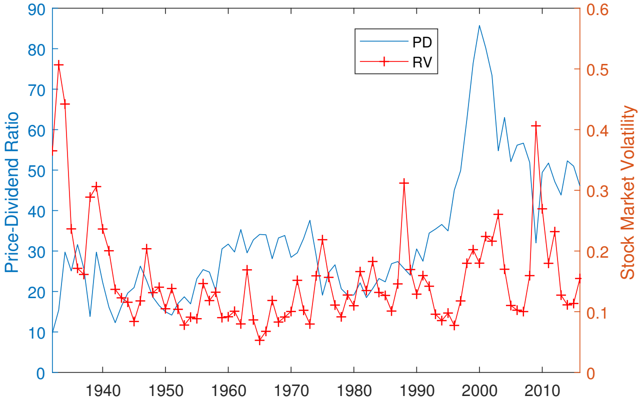


Figure 2: Annual S&P 500 Price-Dividend ratio (PD) versus Realized Volatility (RV) from 1930-2016.

Stock market volatility & stock returns: strongest −ve correlation in recessions

- $\hat{\rho}|_{\text{low REV}} = -0.09$ vs. $\hat{\rho}|_{\text{high REV}} = -0.42$

Benchmark Model

$$P_t = E_t\left(\int_t^\infty \frac{M_s}{M_t} D_s ds\right)$$

$$\text{where } M_t = U_C(t, C_t, V_t)$$

- **Linear models**: consumption (C_t) and dividend (D_t) growth + stochastic discount factor (M_t) log-linear in the state-variables (V_t):

$$\log M_t = -\delta t - \gamma \log C_t - \eta V_t$$

$$\log \frac{D_t}{C_t} = \alpha + \beta V_t + \sqrt{V_t} W_t$$

- Implications:
  1. log-linear price-dividend ratios: $\log \frac{P_t}{D_t} = \alpha_0 + \alpha_1 V_t$
  2. proportional variance of returns: $Var_t(d \log P_t) \propto Var_t(V_t) \propto V_t$

  $\Longrightarrow$ **At odds with the data**

This paper

- Nonlinear consumption and dividend policy and SDF:

$$\log M_t = -\delta t - \gamma \log C_t - H(V_t)$$
$$\log \frac{D_t}{C_t} = \psi(V_t) + \sqrt{V_t} W_t$$

  - V_t: exogenous, Markovian state variables
  - $H(V_t)$: state-dependent discount rate
  - $\psi(V_t)$: consumption and dividend policy functions

- Application: generalizing affine model with time-varying growth volatility and habit
  - Allow for interaction and higher order terms
  - State-dependent impact of volatility shocks (Alfaro, Bloom, Lin, 2016)

Contributions

1. **Cross-sectional** recovery of state variables
   - Robust again functional form assumptions
2. **Estimation**: semiparametric profile maximum likelihood
   - Consistent for long time series and large panel of asset prices
3. **Computational:** policy functions and SDF approximated by polynomials
   - Closed-form price-dividend ratios and volatilities

Outline

Framework

- s_t: Unobserved Markovian state vector
  - Transition density parameterized by α:

$$f(s_{t+1} \mid \mathcal{F}_t) = f\left(s_{t+1} \mid s_t; \alpha\right)$$

- M_t: Choice variables
  - Consumption, investment, dividends, ...
  - Aim: measuring their response to unobserved state $\psi(s_t) = E(M_t \mid s_t)$
- P_t: Prices that depend on the state s_t, the policy functions $\psi(s_t)$, and/or α
  - Form of dependence determined by present value of future payoffs

Estimation

- Measurement equations: for $i = 1, ..., N$

$$M_{it} = \psi_i(s_t) + Z_{it}^M$$

$$P_{it} = \mathbf{g}(s_t, \psi_i(s_t), \alpha) + Z_{it}^P$$

with $Z_t = (Z_t^M, Z_t^P) \perp s_t$

  - Also include state proxies, i.e. realized volatilities
  - "Weak" cross-sectional dependence of $Z_t = (Z_{it})_{i=1}^N$

Profile likelihood

- Parameters of interest: $\theta = (\psi(\cdot), \alpha)$
- Observation vector: $\mathcal{Y}_t = (M_t, P_t)$
- Time-t unconditional likelihood contribution:

$$
\begin{aligned}
L_t(\theta) &= \int f_\theta \left(\mathcal{Y}_t \mid s_t \right) f_\alpha \left(s_t \right) ds_t \\
&= \int \exp \left(N \ell_t(\theta \mid s_t) \right) f_\alpha \left(s_t \right) ds_t
\end{aligned}
$$

- Let $N \to \infty$, and use a Laplace approximation:

$$
\ell_t(\theta) = \frac{1}{N} \log L_t(\theta) \xrightarrow{p} \ell_t(\theta \mid \tilde{s}_t(\theta))
$$

provided the uniqueness of the maximizer

$$
\tilde{s}_t(\theta) = \arg\max_s \; \ell_t(\theta \mid s)
$$

Pairwise profiling

Quasi maximum likelihood

- Let the N transitory deviations $Z_t = (Z_t^M, Z_t^P)$ follow the (Ornstein-Uhlenbeck) process

$$dZ_t = -AZ_t dt + \Sigma dW_t.$$

  - increments of Z_t over any horizon τ are normally distributed

- The measurement density conditional on the current and future states is

$$\log f_\theta \left(\mathcal{Y}_{t+1} \mid s_{t+1}, \mathcal{Y}_t, s_t \right) \propto \left\| Z_{t,pred}(\theta) \right\|^2_{\Omega_\tau}$$

in terms of the generalized residuals

$$Z_{t,pred}(\theta) = Z_t(\theta) - e^{-A\tau} Z_{t-1}(\theta),$$
$$Z_t(\theta) = (M_t - \psi(s_t), P_t - \mathbf{g}(s_t, \psi(s_t), \alpha))^T$$

Identification

The population quasi-maximum likelihood criterion is

$$\bar{Q}(\theta) = \text{plim}_{N\to\infty} \frac{-1}{2N} E\left(\left\| Z_{t,pred}(\theta) \right\|_{\Sigma}^{2} + \log f_{s'|s;\alpha}\left(\bar{s}_{t+1}(\theta) \mid \bar{s}_t(\theta)\right)\right),$$

Global identification condition: for every pair $\theta \neq \theta'$,

$$\psi(\bar{s}(\theta, s_t)) \neq \psi(\bar{s}(\theta', s_t)) \ \text{ or } \ g(\bar{s}(\theta, s_t), \theta) \neq g(\bar{s}(\theta', s_t), \theta')$$

- Without price vector $\mathbf{g}$, could arbitrarily transform state variables in the same parametric class
  - exploit restrictions on how s_t affects prices, or use direct proxies
- Given state dynamics, asset price i identifies product term $\psi_i(s_t)H(s_t)$
  - $\psi_i(s_t)$ identified from observed dividends
  - $H(s_t)$ common across assets

Inference

- Obtain the feasible profile likelihood using consistent estimators $\hat{\Sigma}$ and $\hat{A}$
- Construct finite-dimensional series approximators ψ_L and H_L using basis functions $\underline{p}_L = (p_1(w), ..., p_L(w))$
- If the approximation order is correct, can perform parametric inference on the sieve coefficients:

Theorem

Let regularity conditions and consistency hold. When $N, T \to \infty$, and $\frac{T}{N} \to \kappa$ for $0 < \kappa < \infty$,

$$\sqrt{NT}(\hat{\vartheta} - \vartheta_0) \xrightarrow{d} \mathcal{H}_0^{-1} \times \mathcal{N}\left(\kappa E(\bar{B}_t), \mathcal{V}_0\right)$$

Outline

Setting

- Let $\mathcal{S}_t = (\log Y_t, s_t) \subseteq \mathbb{R}^{D+1}$ be a Markovian state vector consisting of the log output or productivity process $\log Y_t$ and the state variables s_t
- Baseline model for output growth:

$$d \log Y_t = (\mu - \lambda V_t)\, dt + \sqrt{V}_t dW_t$$

$$dV_t = \kappa (\theta - V_t)\, dt + \omega \sqrt{V}_t dB_t$$

$$Cov(dW_t, dB_t) = \rho dt$$

  - $\rho < 0$ captures leverage effect: uncertainty shocks negatively correlate with output shocks
  - $\lambda > \frac{1}{2}$ captures endogenous growth: uncertainty reduces expected growth

- Suppose there is an infinitely-lived representative agent with period utility

$$u(C_t, s_t) = \log(C_t - X_t)H(V_t).$$

    - X_t is a consumption reference level, in line with habit formation (Campbell and Cochrane, 1999)
    - $H(\cdot)$ describes preferences over uncertainty V_t

- Model reference level via the inverse consumption surplus ratio $\frac{C_t}{C_t - X_t} \equiv Q_t$:

$$dQ_t = \kappa^q(\theta^q - Q_t)dt + \eta\sqrt{Q_t}dB_t^q + r_y\sqrt{V}_t dW_t$$
$$Cov(dB_t^q, dW_t) = 0.$$

    - Relative risk aversion Q_t driven by shocks to output growth dW_t and discount rate dB_t^q
    - $r_y < 0$: negative output/income shocks reduce risk bearing capacity

Asset Prices

- Dividend-consumption ratio:

$$\frac{D_t}{C_t} = \psi^d(V_t, Q_t) + Z_t^d, \qquad E(Z_t^d \mid V_t, Q_t) = 0$$

- Price-dividend ratios:

$$\phi(V_t, Q_t, Z_t^d) = E_t\left(\int_0^\infty e^{-\delta\tau}\,\frac{H(V_{t+\tau})}{H(V_t)}\,\frac{Q_{t+\tau}}{Q_t}\,\frac{\psi^d(V_{t+\tau}, Q_{t+\tau}) + Z_{t+\tau}^d}{\psi^d(V_t, Q_t) + Z_t^d}\,d\tau\right)$$

- Under polynomial policy functions ψ_L and H_K,

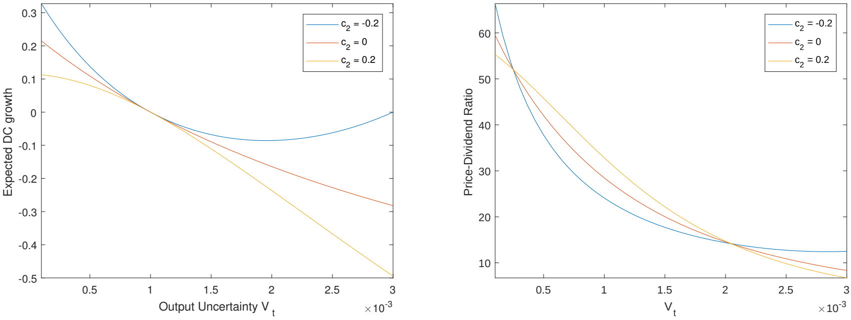
$$\phi_M(s_t) = \frac{\underline{g}^T Q_M \underline{s_t}^M}{\underline{g}^T \underline{s_t}^M},$$

where $M = K + L$, and $\underline{g}$ and Q_M determined by α

- A **convex** dividend-consumption ratio...

$$\frac{D_t}{C_t} = 1 + 0.1 V_t + c_2 V_t^2,$$

...generates **concave** decline in



(a) Expected Dividend growth

(b) Price-Dividend ratio

Figure 3: Expected Dividend minus Consumption growth and Price-Dividend Ratio versus Output Growth Uncertainty.

- Amplification via higher risk aversion generates steeper decline in asset prices...

$$\frac{D_t}{C_t} = 1 + 0.1V_t + 0.2V_t^2 + 0.1Q_tV_t,$$

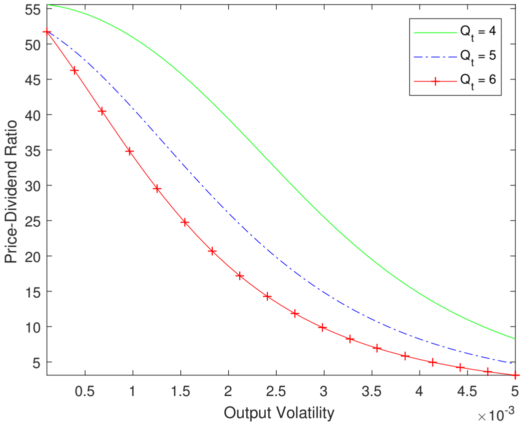


Figure 4: Theoretical Price-Dividend Ratio versus Output Growth Volatility for varying levels of risk aversion.

Empirical Results

- Macro data: U.S. real aggregate output, and consumption from 1926-2016
- Financial data, from 1926-2016:
  - S&P 500 index plus dividends
  - Size-sorted portfolios and dividends from Kenneth French Data Library
- Volatility proxies:
  - realized variation of industrial production growth

$$REV_t = \sum_{m=1}^{12} (\Delta ip_{t+1-m} - \overline{\Delta ip}_t)^2,$$

  - realized variation of stock market returns

$$RV_t = \sum_{d=1}^{252} (\Delta R_{t+1-d} - \overline{R}_t)^2,$$

Table 1 shows the heterogeneous impact of increases in uncertainty on the dividend share of small and large firms based on the regression

$$\frac{D_{it}}{D_t} = \alpha_{0i} + {\beta_i}^T REV_t + z_{it}^d, \qquad E(z_{it}^d \mid REV_t) = 0.$$

Table 1: Parameter estimates and standard errors of regression using annual data from 1926-2016 using one-lag feasible generalized least squares.

Size decile	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}_i$	-2.67	-2.24	-1.35	-1.58	0.77	0.47	2.04	1.13	1.58	1.85
	(4.15)	(1.18)	(0.56)	(0.73)	(0.85)	(1.08)	(0.81)	(0.94)	(0.71)	(0.70)

$\Rightarrow$ consistent with large firms having more liquid debt instruments, which enables them to pay out more dividends in uncertain times - as is priced by investors

Table 2: Estimates and standard errors (in brackets) of the discount rate and transition density parameters.

Estimates based on mixed frequency data, with annual observations from 1926 to 1946 and quarterly observations from 1947 to 2016.

δ	κ	θ	ω	ρ	μ	λ	κ^Q	θ^Q	σ^Q	r_{yq}
0.02	0.38	$3 * 10^{-3}$	0.08	-0.60	0.04	-1.98	0.55	5.45	0.21	-2.86
(0.00)	(0.19)	$(1.7 * 10^{-3})$	(0.10)	(0.12)	(0.01)	(0.73)	(0.67)	(0.30)	(0.84)	(2.25)

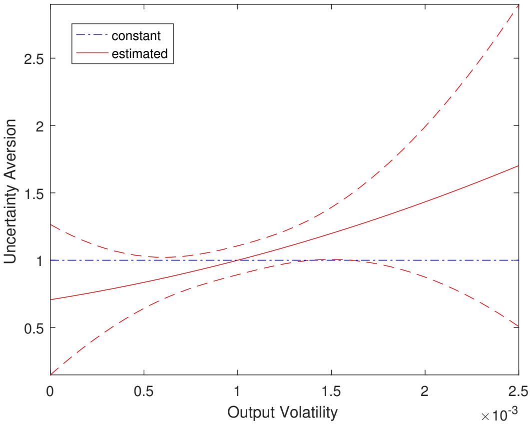
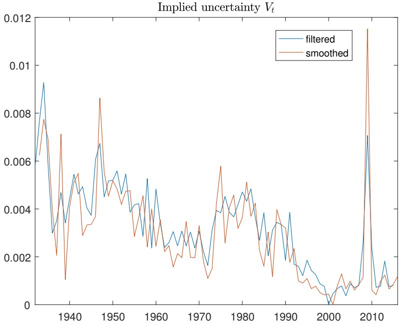
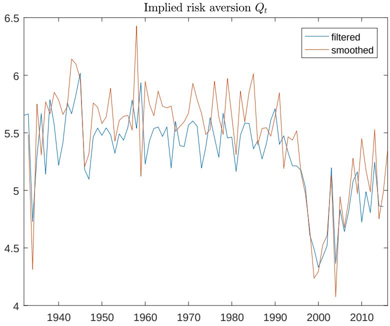


Figure 5: Estimated uncertainty aversion index $\hat{H}_L$ for $L = 2$ against a constant value of one, together with 95% pointwise confidence intervals. Estimates are based on annual observations from 1926 to 1946 and quarterly observations from 1947 to 2016.



(a) Implied uncertainty $\hat{V}_t$.



(b) Implied risk aversion $\hat{Q}_t$.

Figure 6: Comparison of the pairwise concentrated 'filtered' states $\hat{s}'_t$ and 'smoothed' states $\hat{s}_t$ over the period 1930-2016. Filtered time t states use observations from dates $(t, t+1)$; smoothed time t states use observations from dates $(t-1, t)$.

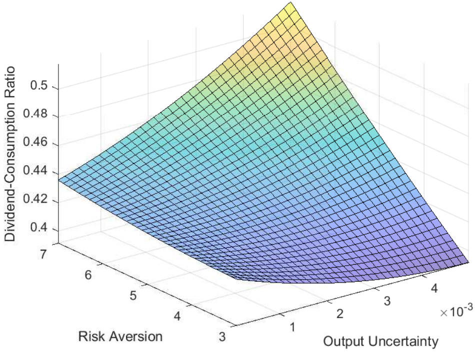


Figure 7: Fitted S&P 500 dividend-consumption ratio as a function of the estimated states using a second-order bivariate expansion.

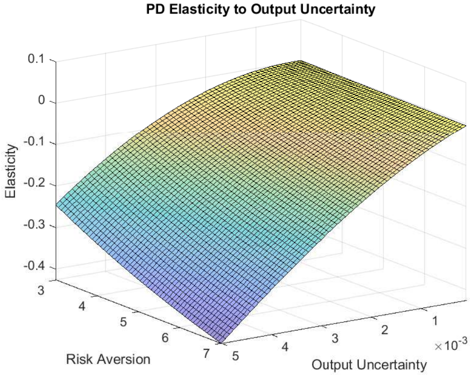


Figure 8: Estimated elasticity of the S&P 500 price-dividend ratio with respect to changes in the variance of economic growth.

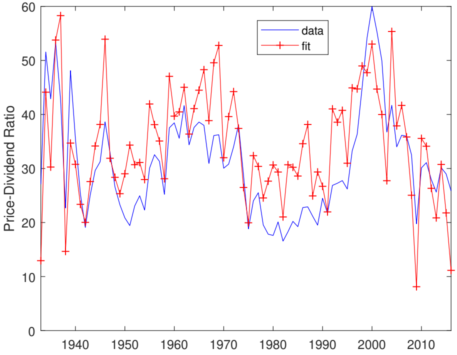


Figure 9: Time-series fitted values of the annual S&P 500 price-dividend ratio as a function of the estimated states using a second-order bivariate expansion.

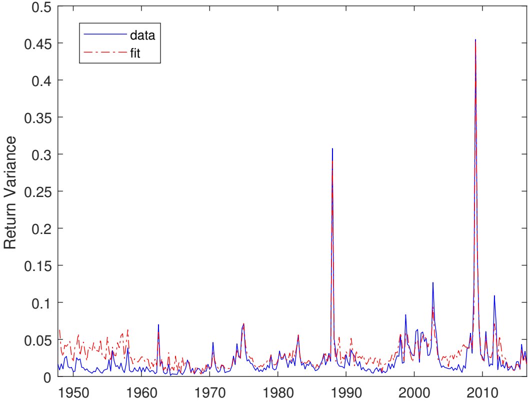


Figure 10: Time-series fitted values of the quarterly realized variance as a function of the estimated states using a second-order bivariate expansion.

Conclusion

- Develop **semiparametric framework** to analyze response of consumption and dividends towards uncertainty and risk aversion shocks
- Exploit **cross-sectional heterogeneity** in dividend policy response to aggregate uncertainty shocks
- Explain **state-dependent** impact of uncertainty shocks on prices and price volatility