Mechanism Design with Ambiguous Transfers

Huiyi Guo

Texas A&M University

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Motivation

- In practice, some mechanism designers introduce uncertain rules to the mechanisms, e.g.,
  - Priceline Express Deals,
  - auctions with secret reserve price,
  - "scratch-and-save" promotions.
- No probability over uncertainty, subjective expected utility, ambiguity aversion.
- Can a mechanism designer achieve her first-best outcome by introducing ambiguity?

Preview

Introduces ambiguous transfers to a first-best mechanism design problem.

- Any efficient allocation rule is implementable via an individually rational and budget-balanced mechanism with ambiguous transfers if and only if the Belief Determine Preferences property holds for all agents.
- Strictly weaker than Bayesian implementation conditions (Kosenok & Severinov, 2008).
- By engineering ambiguity, the mechanism designer can obtain first-best outcomes that are impossible under the Bayesian approach.

Literature review

Mechanism design with ambiguity-averse agents

- Exogenous ambiguity, e.g., Bose et al. (2006), Bose and Daripa (2009), Bodoh-Creed (2012), De Castro et al. (2009, 2017), Wolitzky (2016), Song (2016).
- Endogenous ambiguity, e.g., Bose & Renou (2014), Di Tillio et al. (2017).

First-best Bayesian mechanism design

- Full surplus extraction
  - Crémer & McLean (1985, 1988), McAfee & Reny (1992).
- Partial implementation
  - Independent beliefs, e.g., Myerson & Satterthwaite (1983), Dasgupta & Maskin (2000), Jehiel & Moldovanu (2001).
  - Correlated beliefs: McLean & Postlewaite (2004, 2015), Matsushima (1991, 2007), d'Aspremont et al. (2004), Kosenok & Severinov (2008).

Asymmetric information environment

We study the asymmetric information environment given by $\mathcal{E} = \{I, A, (\Theta_i, u_i)_{i=1}^N, p\}$, where

- $I = \{1, ..., N\}$ is the finite set of agents; assume $N \geq 2$;
- A is the compact set of **feasible outcomes** and a is a generic element;
- let $\theta_i \in \Theta_i$ be agent i's **type**; assume $2 \leq |\Theta_i| < \infty$;
- i has a quasi-linear **utility function** $u_i(a, \theta) + b$, where $b \in \mathbb{R}$ is the transfer;
- $p \in \Delta(\Theta)$ is the **common prior**; the conditional probability $p_i(\theta_{-i}|\theta_i)$ represents i's belief.

An allocation rule $q : \Theta \to A$ is ex-post **efficient**, if

$$\sum_{i \in I} u_i\big(q(\theta), \theta\big) \geq \sum_{i \in I} u_i\big(a, \theta\big), \forall a \in A, \theta \in \Theta.$$

Mechanism with ambiguous transfers

Definition

A **mechanism with ambiguous transfers** *is a pair* $\mathcal{M} = (q, \Phi)$*, where* $q : \Theta \to A$ *is a feasible allocation rule, and* Φ *is a set of transfer rules with a generic element* $\phi : \Theta \to \mathbb{R}^N$*. We call the set* Φ **ambiguous transfers**.

The mechanism designer

- announces allocation rule q and tells agents that Φ is the set of potential transfers;
- secretly commits to some $\phi = (\phi_1, ..., \phi_N) \in \Phi$;
- lets agents report their types;
- reveals ϕ;
- realizes transfers and allocations according to reports, q, and ϕ.

Mechanism with ambiguous transfers

Agent faces both risk and ambiguity.

She merely knows the distribution of others' type, which is risk.

She does not know the distribution of the transfer rule adopted by the mechanism designer, which is interpreted as ambiguity.

Assume that agents are ambiguity-averse and use maxmin expected utility. If all agents report truthfully, the interim payoff of type-θ_i is

$$\inf_{\phi \in \Phi} \{ \sum_{\theta_{-i} \in \Theta_{-i}} u_i(q(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i})) p_i(\theta_{-i}|\theta_i) + \sum_{\theta_{-i} \in \Theta_{-i}} \phi_i(\theta_i, \theta_{-i}) p_i(\theta_{-i}|\theta_i) \}.$$

Mechanism with ambiguous transfers

A mechanism with ambiguous transfers (q, Φ) satisfies

- **incentive compatibility** if

$$\inf_{\phi\in\Phi} \sum_{\theta_{-i}\in\Theta_{-i}} [u_i\big(q(\theta_i,\theta_{-i}),(\theta_i,\theta_{-i})\big) + \phi_i(\theta_i,\theta_{-i})]p_i(\theta_{-i}|\theta_i) \geq$$

$$\inf_{\phi\in\Phi} \sum_{\theta_{-i}\in\Theta_{-i}} [u_i\big(q(\theta_i',\theta_{-i}),(\theta_i,\theta_{-i})\big) + \phi_i(\theta_i',\theta_{-i})]p_i(\theta_{-i}|\theta_i)\forall i \in I, \theta_i, \theta_i' \in \Theta_i;$$

- interim **individual rationality** if

$$\inf_{\phi\in\Phi} \sum_{\theta_{-i}\in\Theta_{-i}} [u_i\big(q(\theta_i,\theta_{-i}),(\theta_i,\theta_{-i})\big) + \phi_i(\theta_i,\theta_{-i})]p_i(\theta_{-i}|\theta_i) \geq 0, \forall i \in I, \theta_i \in \Theta_i;$$

- ex-post **budget balance** if $\sum_{i\in I} \phi_i(\theta) = 0, \forall \phi \in \Phi, \theta \in \Theta.$

Key condition

Definition

The **Beliefs Determine Preferences** *(BDP) property holds for agent i if there does not exist $\bar{\theta}_i, \hat{\theta}_i \in \Theta_i$ with $\bar{\theta}_i \neq \hat{\theta}_i$ such that*

$$p_i(\theta_{-i}|\bar{\theta}_i) = p_i(\theta_{-i}|\hat{\theta}_i), \forall \theta_{-i} \in \Theta_{-i}.$$

Beliefs are correlated! Any form of correlation, e.g., positively/negatively correlated.

Why information can be correlated? E.g., common source of information.

Generic in a finite type space with $N \geq 2$ and $|\Theta_i| \geq 2$ for all agents.

An example

The common prior

Consider a two-by-two model with a common prior p given below.

p	θ_2^1	θ_2^2
θ_1^1	0.2	0.3
θ_1^2	0.3	0.2

Observation: both agents satisfy BDP.

The feasible set of alternatives is $A = \{x_0, x_1, x_2\}$, where x_0 gives both agents zero at all states, x_1 and x_2's payoffs are given below (assume $0 < a < B$):

x_1	θ_2^1	θ_2^2
θ_1^1	$a, 0$	a, a
θ_1^2	$a, 0$	a, a

x_2	θ_2^1	θ_2^2
θ_1^1	a, a	$a - 2B, a + B$
θ_1^2	a, a	$a - 2B, a + B$

The efficient allocation rule is $q(\cdot, \theta_2^2) = x_1$ and $q(\cdot, \theta_2^1) = x_2$.

An example

Sufficiency

Let the set of ambiguous transfers be $\Phi = \{\phi, \phi'\}$. Transfers $\phi = (\phi_1, \phi_2)$ and $\phi' = (\phi'_1, \phi'_2)$ are defined as follows.

$$\phi_i(\theta_1, \theta_2) = \begin{cases} c\psi(\theta_1, \theta_2), & \text{if } i = 1, \\ -c\psi(\theta_1, \theta_2), & \text{if } i = 2, \end{cases} \qquad \phi'_i(\theta_1, \theta_2) = \begin{cases} -c\psi(\theta_1, \theta_2), & \text{if } i = 1, \\ c\psi(\theta_1, \theta_2), & \text{if } i = 2, \end{cases}$$

where $c \geq B$, and $\psi : \Theta \to \mathbb{R}$ is given below.

ψ	θ_2^1	θ_2^2
θ_1^1	-3	2
θ_1^2	2	-3

An example
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θ_1^1	0.2	0.3
θ_1^2	0.3	0.2

- When both agents truthfully report, for each agent i and type $\bar{\theta}_i$, $\psi(\bar{\theta}_i, \cdot)$ has zero expected value under belief $p_i(\cdot|\bar{\theta}_i)$.
- When i unilaterally misreports $\hat{\theta}_i \neq \bar{\theta}_i$, $\psi(\hat{\theta}_i, \cdot)$ has a non-zero expected value.

An example
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- When both agents truthfully report, for each agent i and type $\bar{\theta}_i$, $\psi(\bar{\theta}_i, \cdot)$ has zero expected value under belief $p_i(\cdot|\bar{\theta}_i)$. E.g., $\psi(\cdot|\theta_2^2)p_2(\cdot|\theta_2^2) = (2,-3)\cdot(0.6,0.4) = 0$
- When i unilaterally misreports $\hat{\theta}_i \neq \bar{\theta}_i$, $\psi(\hat{\theta}_i, \cdot)$ has a non-zero expected value.

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- When both agents truthfully report, for each agent i and type $\bar{\theta}_i$, $\psi(\bar{\theta}_i, \cdot)$ has zero expected value under belief $p_i(\cdot|\bar{\theta}_i)$. E.g., $\psi(\cdot|\theta_2^2)p_2(\cdot|\theta_2^2) = (2, -3) \cdot (0.6, 0.4) = 0$
- When i unilaterally misreports $\hat{\theta}_i \neq \bar{\theta}_i$, $\psi(\hat{\theta}_i, \cdot)$ has a non-zero expected value. E.g., $\psi(\cdot|\theta_2^1)p_2(\cdot|\theta_2^2) = (-3, 2) \cdot (0.6, 0.4) = -1$

An example

Sufficiency

- (BB) $\phi_1(\theta) + \phi_2(\theta) = 0, \phi_1'(\theta) + \phi_2'(\theta) = 0$ for all $\theta \in \Theta$.
- (IR) By truthfully reporting, both agents get expected payoffs of a. E.g. $IR(\theta_2^2)$: $\min\{a - c[0.6 \times (2) + 0.4 \times (-3)], a + c[0.6 \times (2) + 0.4 \times (-3)]\} = a$.
- (IC) All eight IC constraints hold. E.g. $IC(\theta_2^2\theta_2^1)$.
  By misreporting θ_2^1, agent 2's worst-case expected payoff is $\min\{a+B-c[0.6\times(-3)+0.4\times(2)], a+B+c[0.6\times(-3)+0.4\times(2)]\} = a+B-c \leq a$.

Hence, q is implementable via an interim IR and ex-post BB mechanism with ambiguous transfers.

x_1	θ_2^1	θ_2^2
θ_1^1	$a, 0$	a, a
θ_1^2	$a, 0$	a, a

x_2	θ_2^1	θ_2^2
θ_1^1	a, a	$a - 2B, a + B$
θ_1^2	a, a	$a - 2B, a + B$

An example

Necessity

What if BDP fails for someone, e.g., $\tilde{p}_2(\cdot|\theta_2^1) = \tilde{p}_2(\cdot|\theta_2^2)$?

Suppose implementation can be guaranteed by ambiguous transfers Φ.

From $IC(\theta_2^1\theta_2^2)$ and $IC(\theta_2^2\theta_2^1)$,

$$\inf_{\phi\in\Phi}\{a+\sum_{\theta_{-2}\in\Theta_{-2}}\phi_2(\theta_2^1,\theta_{-2})\tilde{p}_2(\theta_{-2}|\theta_2^1)\} \geq \inf_{\phi\in\Phi}\{0+\sum_{\theta_{-2}\in\Theta_{-2}}\phi_2(\theta_2^2,\theta_{-2})\tilde{p}_2(\theta_{-2}|\theta_2^1)\},$$

$$\inf_{\phi\in\Phi}\{a+\sum_{\theta_{-2}\in\Theta_{-2}}\phi_2(\theta_2^2,\theta_{-2})\tilde{p}_2(\theta_{-2}|\theta_2^2)\} \geq \inf_{\phi\in\Phi}\{a+B+\sum_{\theta_{-2}\in\Theta_{-2}}\phi_2(\theta_2^1,\theta_{-2})\tilde{p}_2(\theta_{-2}|\theta_2^2)\}.$$

Adding gives $2a \geq a + B$, a contradiction.

BDP is necessary.

An example

Compared to Bayesian mechanisms

Kosenok & Severinov (2008): Any efficient allocation is implementable via an IR and BB *Bayesian* mechanism if and only if (i) Convex Independence holds for all agents and (ii) p satisfies Identifiability.

The **Convex Independence** condition holds for agent $i \in I$ if for any type $\bar{\theta}_i \in \Theta_i$ and coefficients $(c_{\hat{\theta}_i})_{\hat{\theta}_i \in \Theta_i} \geq \mathbf{0}$, $p_i(\cdot|\bar{\theta}_i) \neq \sum_{\hat{\theta}_i \in \Theta_i \setminus \{\bar{\theta}_i\}} c_{\hat{\theta}_i} p_i(\cdot|\hat{\theta}_i)$.

The common prior $p(\cdot)$ satisfies the **Identifiability** condition if for any $\tilde{p}(\cdot) \neq p(\cdot)$, there exists an agent $i \in I$ and her type $\bar{\theta}_i \in \Theta_i$, with $\tilde{p}_i(\bar{\theta}_i) > 0$, such that for any $(d_{\hat{\theta}_i})_{\hat{\theta}_i \in \Theta_i} \geq \mathbf{0}$, $\tilde{p}_i(\cdot|\bar{\theta}_i) \neq \sum_{\hat{\theta}_i \in \Theta_i} d_{\hat{\theta}_i} p_i(\cdot|\hat{\theta}_i)$.

An example

Compared to Bayesian mechanisms

The identifiability condition fails in this example. q is not Bayesian implementable.

If some agent i misreports, i will be punished, which by BB makes j better-off. When the identifiability condition is violated, intuitively, some agent j can benefit from misreporting in a way that makes i's truthful report appear untruthful.

With ambiguous transfers, if some agent i misreports, i will not necessarily be punished by the realized transfer rule ϕ. It is possible that under ϕ, misreporting makes i strictly better-off and under $-\phi$ agent i is worse-off. Hence, agent j will not have the above-described incentive.

Main result

Necessary and sufficient condition

Theorem

Given a common prior p, any ex-post efficient allocation rule under any profile of utility functions is implementable via an interim IR and ex-post BB mechanism with ambiguous transfers if and only if the BDP property holds for all agents.

Main result

Two lemmas

Fix any constraint $IC(\bar{\theta}_i\hat{\theta}_i)$. Construct an ex-post budget balanced transfer rule that can be used to guarantee $IC(\bar{\theta}_i\hat{\theta}_i)$ and gives all agents zero expected payoff.

Lemma 1

If the BDP property holds for agent i, then for all $\bar{\theta}_i, \hat{\theta}_i \in \Theta_i$ with $\bar{\theta}_i \neq \hat{\theta}_i$, there exists $\psi^{\bar{\theta}_i\hat{\theta}_i} : \Theta \to \mathbb{R}^n$ such that

1. $\sum_{j \in I} \psi_j^{\bar{\theta}_i\hat{\theta}_i}(\theta) = 0$ for all $\theta \in \Theta$;
2. $\sum_{\theta_{-j} \in \Theta_{-j}} \psi_j^{\bar{\theta}_i\hat{\theta}_i}(\theta_j, \theta_{-j}) p_j(\theta_{-j}|\theta_j) = 0$ for all $j \in I$ and $\theta_j \in \Theta_j$;
3. $\sum_{\theta_{-i} \in \Theta_{-i}} \psi_i^{\bar{\theta}_i\hat{\theta}_i}(\hat{\theta}_i, \theta_{-i}) p_i(\theta_{-i}|\bar{\theta}_i) < 0.$

Proved via Fredholm's theorem of the alternative.

Main result

Two lemmas

Construct a linear combination of all $(\psi^{\bar{\theta}_i \hat{\theta}_i})_{i \in I, \bar{\theta}_i \neq \hat{\theta}_i}$, denoted by ψ, such that the ex-post budget balanced ψ gives all agents zero expected transfers on path and any unilaterally misreporting agents non-zero transfers.

Lemma 2

If the BDP property holds for all agents, then there exists $\psi : \Theta \to \mathbb{R}^n$ such that

1. $\sum_{i \in I} \psi_i(\theta) = 0$ for all $\theta \in \Theta$;
2. $\sum_{\theta_{-i} \in \Theta_{-i}} \psi_i(\theta_i, \theta_{-i}) p_i(\theta_{-i} | \theta_i) = 0$ for all $i \in I$ and $\theta_i \in \Theta_i$;
3. $\sum_{\theta_{-i} \in \Theta_{-i}} \psi_i(\hat{\theta}_i, \theta_{-i}) p_i(\theta_{-i} | \bar{\theta}_i) \neq 0$ for all $i \in I$ and $\bar{\theta}_i, \hat{\theta}_i \in \Theta_i$ with $\bar{\theta}_i \neq \hat{\theta}_i$.

Main result
Proof

Sufficiency.

- Pick any ex-post BB and interim IR transfer rule η.
- Let $\Phi = \{\eta + c\psi, \eta - c\psi\}$ with c sufficiently large.
- IR and BB are trivial. IC can be achieved with c large enough.

Necessity is proved by constructing a counterexample.

Main result

Compared to Bayesian mechanisms

Convex Independence for all agents and Identifiability are necessary and sufficient for Bayesian implementation.

BDP is weaker than Convex Independence.

Identifiability is relaxed under ambiguous transfers.

- e.g., $N = 2$, Convex Independence and Identifiability can never hold simultanously. Impossibility for bilateral trades under Bayesian framework.
- e.g., $N = 3$, $|\Theta_1| = 5$, $|\Theta_2| = 2$, $|\Theta_3| = 2$, Convex Independence fails for 1 with positive probability.
- e.g., $N = 3$ and $|\Theta_i| = 2$ for all agents, Identifiability fails with positive probability.

Extension

Full surplus extraction

If a mechanism can be designed so that under each transfer rule $\phi \in \Phi$, the designer's revenue at each state is equal to ex-post social surplus, i.e.,

$$-\sum_{i \in I} \phi_i(\theta) = \max_{a \in A} \sum_{i \in I} u_i(a, \theta) \, \forall \theta \in \Theta.$$

Theorem

Given a common prior p, full surplus extraction under any profile of utility functions can be achieved via an interim IR mechanism with ambiguous transfers if and only if the BDP property holds for all agents.

- Convex Independence for all agents is necessary and sufficient for Bayesian FSE.
- Hence, ambiguous transfers perform better than Bayesian mechanisms.

Extension

Other ambiguity aversion preferences

The sufficiency parts of the main results hold for alternative models of ambiguity aversion.

- α-maxmin expected utility of Ghirardato (2002)

$$\alpha \inf_{\phi\in\Phi} \sum_{\theta_{-i}\in\Theta_{-i}} [u_i\big(q(\theta_i,\theta_{-i}),\theta\big)p_i(\theta_{-i}|\theta_i) + \sum_{\theta_{-i}\in\Theta_{-i}} \phi_i(\theta_i,\theta_{-i})p_i(\theta_{-i}|\theta_i)]$$

$$+ (1-\alpha)\sup_{\phi\in\Phi}[u_i\big(q(\theta_i,\theta_{-i}),\theta\big)p_i(\theta_{-i}|\theta_i) + \sum_{\theta_{-i}\in\Theta_{-i}} \phi_i(\theta_i,\theta_{-i})p_i(\theta_{-i}|\theta_i)],$$

where $\alpha \in (0.5, 1]$ represents ambiguity-aversion.

Extension

Other ambiguity aversion preferences

- smooth ambiguity aversion preferences of Klibanoff (2005)

$$\int_{\pi\in\Delta(\Phi)} v\Big(\int_{\phi\in\Phi}\Big(\sum_{\theta_{-i}\in\Theta_{-i}}[u_i(q(\theta),\theta)+\phi_i(\theta)]p_i(\theta_{-i}|\theta_i)\Big)d\pi\Big)d\mu,$$

where

- $\forall\pi\in\Delta(\Phi)$, $\pi(\phi)$ is the density that ϕ is the rule drawn by the mechanism designer;
- $\forall\mu\in\Delta(\Delta(\Phi))$, $\mu(\pi)$ is the density that $\pi\in\Delta(\Phi)$ is the right lottery to draw the transfer rule;
- $v:R\to R$ is a strictly increasing function characterizing ambiguity attitude, where a strictly concave v implies ambiguity aversion.

Conclusion

- This paper introduces ambiguous transfers to study first-best mechanism design problems.
- The BDP property is necessary and sufficient for efficient, IR, and BB implementation. It is also necessary and sufficient for FSE.
- As our condition is weaker than those under the Bayesian mechanism design approach, ambiguous transfers can obtain first-best result that cannot be obtained otherwise.
- The BDP property holds generically in any finite type space. Under two-agent settings, ambiguous transfers offer a solution to overcome the negative results on bilateral trading problems generically.

Thank you!