Ambiguity and Information Processing in a Model of Intermediary Asset Pricing

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Introduction

- **Heterogeneity in information processing capacity**
  - Financial intermediaries (specialists) are assumed to possess greater channel capacity (Rational Inattention (Sims, 2003)) .
  - Households purchase this capacity by delegating investments to intermediaries.
    - Although households could manage their portfolios themselves, most choose not to do so.
  - Two frictions in financial contract:
    - **Incentive constraint** arises from a moral hazard problem, requires a minimum capital for risk-sharing (He-Krishnamurthy, 2012).
    - **Participation constraint** depends on the heterogeneity in channel capacity.

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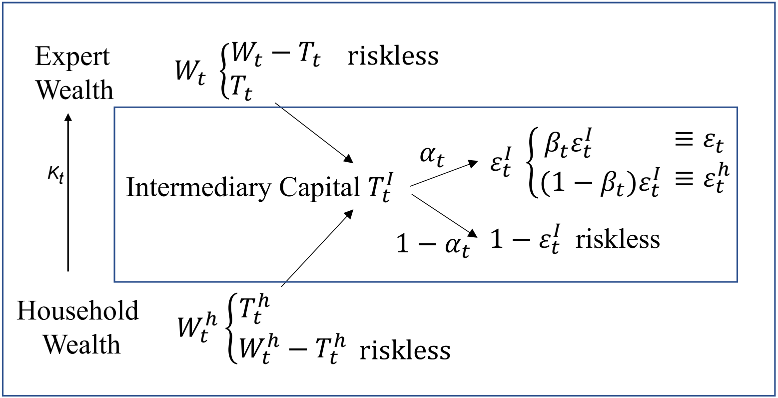
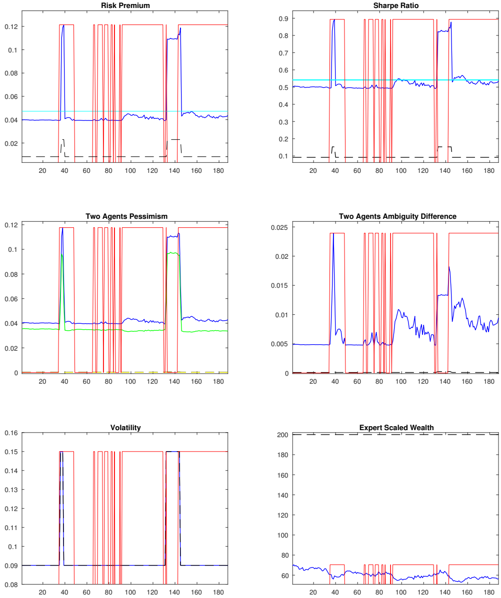
    - **Participation constraint** depends on the heterogeneity in channel capacity.
- **Heterogeneity in beliefs**
  - Knightian uncertainty/Ambiguity/Robustness (Hansen-Sargent, 2008))
  - When volatility increases, so does ambiguity, the drift distortions produce endogenous heterogeneous beliefs.
  - When volatility is high specialists become relatively pessimistic, and this tightens the capital constraint and accelerates the onset of a financial crisis.

Market Structure



- **Effective risk sharing constraint:** $\varepsilon_t^h \leq \tilde{m}\varepsilon_t$.
  - $\tilde{m}$ reflects the financial constraint due to agency friction and ambiguity.
- **Participation constraint:** $k_t \leq a_3(\Sigma - \Sigma^h)$.
  - $a_3 < 0$, $\kappa > \kappa^h \to \Sigma < \Sigma^h$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0.02
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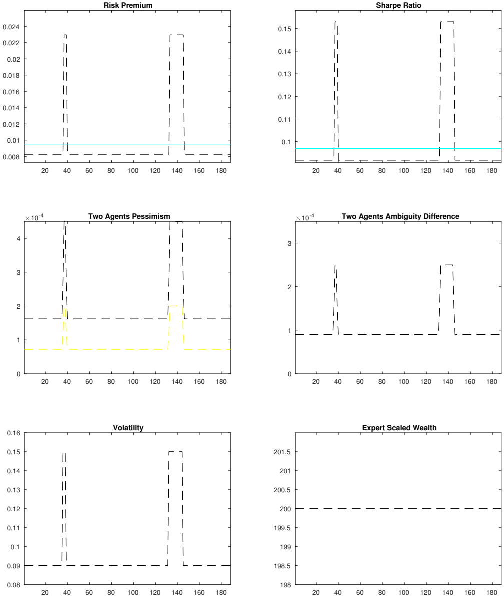
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Model Structure

- Risky asset dividend is governed by stochastic growth rate g_t and volatility σ_t,

$$\frac{dD_t}{D_t} = g_t dt + \sigma_t dZ_t, \tag{1}$$

- Assume the volatility σ_t is a two-state Markov chain with state space $\Sigma_d = \{\sigma_H, \sigma_L\}$, where $\sigma_H > \sigma_L$. The intensity matrix is

$$\begin{bmatrix} -\lambda_H & \lambda_H \\ \lambda_L & -\lambda_L \end{bmatrix}. \tag{2}$$

- Unobservable growth rate follows a (known) mean-reverting process

$$dg_t = \rho_g \left(\bar{g} - g_t\right) dt + \sigma_g dZ_t^u \tag{3}$$

- Agents observe only a noisy signal containing imperfect information

$$ds_t = g_t dt + \sigma_s dZ_t^s \tag{4}$$

Capacity-Constrained Kalman Filter

- The Kalman filter of learning is

$$d\hat{g}_t = \rho_g (\bar{g} - g_t)\, dt + \frac{\Sigma_t}{\sigma_t} d\hat{Z}_t + \frac{\Sigma_t}{\sigma_s} d\hat{Z}_t^s \tag{5}$$

$$d\Sigma_t = \left[\sigma_g^2 - 2\rho_g \Sigma_t - \Sigma_t^2 \left(\frac{1}{\sigma_t^2} + \frac{1}{\sigma_s^2}\right)\right] dt \tag{6}$$

  - Σ_t: signal/noise ratio (estimation variance of the unobserved state).
- Investor has a finite information-processing capacity (Sims, 2003)

$$\mathcal{H}(g_{t+\Delta t} | \mathcal{I}_t) - \mathcal{H}(g_{t+\Delta t} | \mathcal{I}_{t+\Delta t}) \leq \kappa \Delta t, \tag{7}$$

- The Kalman gain is constrained by the agent's channel capacity

$$\frac{1}{2} \frac{\Sigma_t}{\sigma_s^2} \leq \kappa. \tag{8}$$

- Risky asset return

$$dR_t = \frac{D_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} dZ_t. \tag{9}$$

Household Robust Consumption/Portfolio Rules

- Objective

$$V\left(\hat{g}_t^h, \Sigma_t^h, W_t^h; Y_t^h\right) = \sup_{\{C_t^h, \varepsilon_t^h\}} \inf_{\nu_t^h} \mathbb{E} \int_0^\infty e^{-\rho^h t} \left[\ln C_t^h + \frac{1}{2\theta^h}\left(\nu_t^h\right)^2\right] dt \tag{10}$$

$$s.t.\ dW_t^h = \left[\varepsilon_t^h(\pi_{R,t} - k_t) + r_t W_t^h - C_t^h\right] dt + \sigma_{W,t}^h \left(\nu_t^h dt + d\hat{Z}_t\right), \tag{11}$$

$$d\hat{g}_t^h = \rho_g\left(\bar{g} - g_t\right) dt + \frac{\Sigma_t^h}{\sigma_t} d\hat{Z}_t + \frac{\Sigma_t^h}{\sigma_s} d\hat{Z}_t^s \tag{12}$$

$$d\Sigma_t^h = \left[\sigma_g^2 - 2\rho_g \Sigma_t^h - \frac{\left(\Sigma_t^h\right)^2}{\sigma_t^2} - 2\kappa^h\left(\Sigma_t^h\right)^2\right] dt \tag{13}$$

- Optimal rules

$$\nu_t^{h*} = -\frac{\theta^h}{\rho^h} \frac{\varepsilon_t^h \sigma_{R,t}}{W_t^h} \tag{14}$$

$$C_t^{h*} = \rho^h W_t^h \tag{15}$$

$$\varepsilon_t^{h*} = \frac{\pi_{R,t} - k_t}{\gamma^h \sigma_{R,t}^2} W_t^h \tag{16}$$

  - Effective HH risk aversion $\gamma^h = 1 + \frac{\theta^h}{\rho^h}$; θ^h: HH ambiguity aversion degree.

Specialist Robust Consumption/Portfolio Rules

- Objective

$$J\left(\hat{g}_t, \Sigma_t, W_t; Y_t\right) = \sup_{\{C_t, \varepsilon_t\}} \inf_{\nu_t} \mathbb{E} \int_0^\infty e^{-\rho t} \left[\ln C_t + \frac{1}{2\theta} \left(\nu_t\right)^2 \right] dt \qquad (17)$$

$$s.t.\ dW_t = \left[\varepsilon_t \pi_{R,t} + \left(q_t + r_t\right) W_t - C_t\right] dt + \sigma_{W,t} \left(\nu_t dt + d\hat{Z}_t\right) \qquad (18)$$

$$d\hat{g}_t = \rho_g \left(\bar{g} - g_t\right) dt + \frac{\Sigma_t}{\sigma_t} d\hat{Z}_t + \frac{\Sigma_t}{\sigma_s} d\hat{Z}_t^s \qquad (19)$$

$$d\Sigma_t = \left(\sigma_g^2 - 2\rho_g \Sigma_t - \frac{\Sigma_t^2}{\sigma_t^2} - 2\kappa \Sigma_t^2\right) dt \qquad (20)$$

- Optimal rules:

$$\nu_t^* = -\frac{\theta}{\rho} \frac{\varepsilon_t \sigma_{R,t}}{W_t} \qquad (21)$$

$$C_t^* = \rho W_t \qquad (22)$$

$$\varepsilon_t^* = \frac{\pi_{R,t}}{\gamma \sigma_{R,t}^2} W_t. \qquad (23)$$

  - Effective specialist risk aversion $\gamma = 1 + \frac{\theta}{\rho}$; θ: specialist's ambiguity aversion.

Equilibrium

- Intermediation market clears,

$$\varepsilon_t^{h*} = \frac{1-\beta_t^*}{\beta_t^*}\varepsilon_t^*. \tag{24}$$

- Stock market clears,

$$\varepsilon_t^* + \varepsilon_t^{h*} = P_t. \tag{25}$$

- Goods market clears,

$$C_t^* + C_t^{h*} = D_t. \tag{26}$$

Risk Sharing Constraint

- In *unconstrained* region,
  - Slack risk sharing constraint

$$\varepsilon_t^h|_{k_t=0} < m\varepsilon_t \iff \frac{\pi_{R,t}}{\gamma^h \sigma_{R,t}^2} W_t^h < m\frac{\pi_{R,t}}{\gamma \sigma_{R,t}^2} W_t$$
$$\iff T_t^h = W_t^h < \tilde{m} W_t.$$

- In *constrained* region,
  - Binding risk sharing constraint

$$\varepsilon_t^h = m\varepsilon_t \iff W_t^h \geq \tilde{m} W_t = T_t^h.$$

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- **Effective financial constraint:**

$$\tilde{m} \equiv \frac{\gamma^h}{\gamma}m = \frac{1+\theta^h/\rho^h}{1+\theta/\rho}m \tag{27}$$

$$\rho^h \geq \rho, \theta^h = \theta \Rightarrow \gamma^h \leq \gamma \Rightarrow \tilde{m} \leq m \tag{28}$$

- Define scaled specialist wealth as the unique state variable $x_t = W_t/D_t$.
- When the risk sharing constraint just starts to bind, $x^c = \frac{1}{\tilde{m}\rho^h+\rho}$.

Steady State Solution

- In the steady state,

$$\Sigma = \bar{\sigma}^2 \left[-(\kappa + \rho_g) + \sqrt{(\kappa + \rho_g)^2 + (\sigma_g / \bar{\sigma})^2} \right] \tag{29}$$

$$\frac{d\Sigma}{d\kappa} < 0 \tag{30}$$

- Value function

$$J(\hat{g}_t, \Sigma_t, W_t; Y_t) = \frac{1}{\rho} \ln W_t + a_0 + a_1 \hat{g}^2 + a_2 \hat{g} + a_3 \Sigma + Y(x_t), \ a_3 < 0. \tag{31}$$

$$\frac{dJ}{d\kappa} = \frac{dJ}{d\Sigma} \frac{d\Sigma}{d\kappa} = a_3 \frac{d\Sigma}{d\kappa} > 0. \tag{32}$$

  - Agents with higher channel capacity have higher steady state welfare.

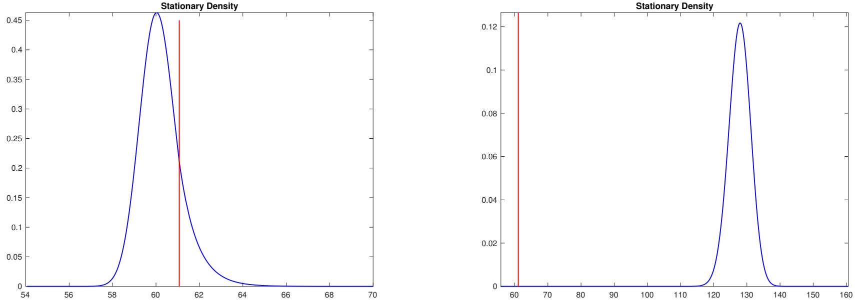
$$\kappa > \kappa^h \rightarrow \bar{k} \equiv J - V = a_3 \left(\Sigma - \Sigma^h \right) > 0. \tag{33}$$

- Participation Constraint: $k_t \leq a_3(\Sigma - \Sigma^h)$.
  - Households will remain in the contract as long as the channel capacity difference is sufficiently greater than the intermediation fee.

Stationary Wealth Distribution (Constant Volatility)

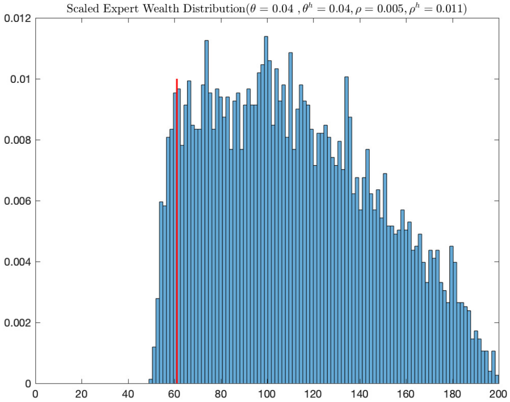
- Endogenous Wealth Evolution

$$\frac{dx_t}{x_t} = \mu_{x,t} dt + \sigma_{x,t} dZ_t.$$

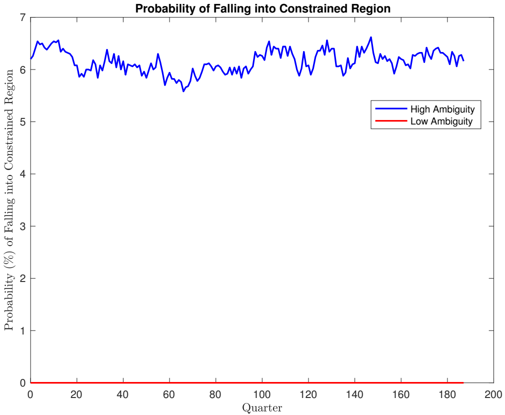


- left: $\sigma_H = 0.15$
- right: $\sigma_L = 0.09$

Simulated Wealth Distribution



Probability of Constraint Binds



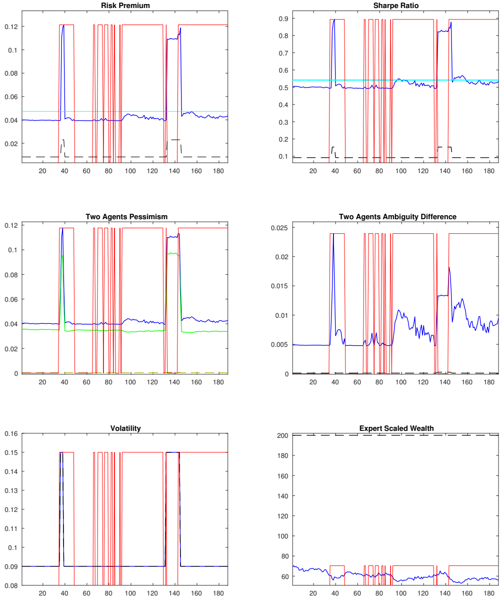
- Probability of Sharpe Ratio Exceed Twice of the Mean: 0.32%

Asset Prices

Table 1.1: Measurements

	Model	
θ	0.0001	0.04
θ^h	0.0001	0.04
γ	1.02	9.26
γ^h	1.01	4.63
Risk Premium (%)	0.92	5.29
Sharpe Ratio (%)	9.59	61.62
Interest Rate (%)	1.59	1.77
Interest Rate Volatility (%)	0.31	0.35
Return Volatility (%)	9.40	8.35
Portfolio Share	1	1.0031
Probability of Sharpe Ratio Exceed Twice of the Mean (%)	0	0.32

This table reports the unconditional simulated results. We simulate 5000 years and 5000 sample paths with quarterly frequency. To match the data from 1970-2017, we report 47 years simulated results in stationary distribution.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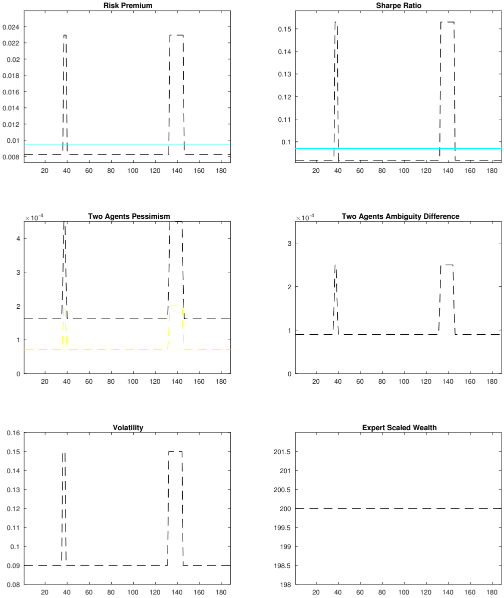
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Conclusion

- **Heterogeneity in information processing capacity**
  - Two frictions in financial contract:
  - Participation constraint depends on the heterogeneity in channel capacity.
  - Incentive constraint requires a minimum capital for risk-sharing, subjected to effective financial constraint.
- **Endogenous heterogeneous beliefs due to ambiguity**
  - When volatility is high specialists become relatively pessimistic, and this tightens the capital constraint and accelerates the onset of a financial crisis.