Protest Puzzles: Tullock's Paradox, Hong Kong Experiment, and the Strength of Weak States

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Two Standing Puzzles

1. **Tullock's "Paradox of Revolution" (1971):** revolution is a public good, revolting is costly, and an individual effect on the likelihood of success is negligible. Thus, revolutions should not happen. Olsonian logic (1965).

2. **Cantoni, Yang, Yuchtman, and Zhang's Hong Kong Experiment (2018):** in the context of the Hong Kong Democracy Movement, potential protesters who were presented with the information that others were more likely to protest, became less likely to protest—**actions are strategic substitutes**.

   - However, almost all current models of protest predict the opposite—**actions are strategic complements**.

Standard Approach: Morris and Shin (2003)

		outcome	
		$n > q$	$n \leq q$
citizen i	*revolt*	$b - c_i$	$-c_i$
	no revolt	0	0

Figure 1: Regime Change Game with Selective Benefits $b > 0$.

- A continuum of citizens indexed by $i \in [0, 1]$.
- $c_i = \theta + \sigma\ \epsilon_i$, and θ and ϵ_is are independent.
- Citizens share an improper prior that θ is distributed uniformly on $\mathbb{R}$, and $\epsilon_i \sim F$ with full support on $\mathbb{R}$.
- The regime collapses whenever the fraction of revolters, n, exceeds a threshold $q \in (0, 1)$.

Standard Approach: Morris and Shin (2003)

		outcome	
		$n > q$	$n \leq q$
citizen i	*revolt*	$b - c_i$	$-c_i$
	no revolt	0	0

- Suppose each citizen revolts whenever $c_i < c^*$.
- Given θ, the measure of revolters is $Pr(c_i < c^*|\theta)$.
- The revolution succeeds whenever, $\theta < \theta^*$, with $Pr(c_i < c^*|\theta^*) = q$.
- The marginal citizen is indifferent: $Pr(\theta < \theta^*|c_i = c^*) \cdot b = c^*$.

A Statistical Property : $Pr(c_i < c^*|\theta^*) = Pr(\theta < \theta^*|c_i = c^*)$.

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$c^* = b\ (1 - q)$ and $\theta^* = c^* - \sigma\ F^{-1}(q) = b\ (1 - q) - \sigma\ F^{-1}(q)$.

A Model of Pivotal Revolutionaries

		outcome	
		$n > qN$	$n \leq qN$
citizen i	*revolt*	$u(N) - c_i$	$-c_i$
	no revolt	$u(N)$	0

Figure 2: Regime Change Game with Public Benefits $u(N) > 0$, with $u'(N) \geq 0$.

- Suppose each citizen revolts whenever $c_j < c^*$.

$$u(N) \int_{\theta=-\infty}^{\infty} \binom{N}{qN} \left(F\left(\frac{c^* - \theta}{\sigma}\right)\right)^{qN} \left(1 - F\left(\frac{c^* - \theta}{\sigma}\right)\right)^{(1-q)N} pdf(\theta|c_i)\, d\theta > c_i.$$

A Model of Pivotal Revolutionaries

		outcome	
		$n > qN$	$n \leq qN$
citizen i	*revolt*	$u(N) - c_i$	$-c_i$
	no revolt	$u(N)$	0

Figure 3: Regime Change Game with Public Benefits $u(N) > 0$, with $u'(N) \geq 0$.

- The marginal citizen with signal $c_i = c^*$ must be indifferent:

$$c^* = u(N) \int_{\theta=-\infty}^{\infty} \binom{N}{qN} \left(F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{qN} \left(1 - F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{(1-q)N} pdf(\theta|c^*)d\theta$$

$$= u(N) \int_{u=0}^{1} \binom{N}{qN} u^{qN} (1-u)^{(1-q)N} du$$

$$= \frac{u(N)}{1+N}. \quad (1)$$

Statistical Property I: Morris and Shin 2003

$$c^* = u(N) \int_{\theta=-\infty}^{\infty} \binom{N}{qN} \left(F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{qN} \left(1 - F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{(1-q)N} pdf(\theta|c^*)d\theta$$

$$= u(N) \int_{u=0}^{1} \binom{N}{qN} u^{qN} (1-u)^{(1-q)N} du$$

$$= \frac{u(N)}{1+N}.$$

- **Economics:** the marginal citizen believes that the probability that a random citizen revolts, conditional on θ, is uniformly distributed on $[0, 1]$:

$$Pr\left(F\left(\frac{c^*-\theta}{\sigma}\right) < A \middle| c_i = c^*\right) = Pr(c^* - \sigma\ F^{-1}(A) < \theta | c_i = c^*)$$

$$= F\left(\frac{c^* - c^* + \sigma\ F^{-1}(A)}{\sigma}\right)$$

$$= A.$$

Statistical Property II: Bayes's Notebook

$$c^* = u(N) \int_{\theta=-\infty}^{\infty} \binom{N}{qN} \left(F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{qN} \left(1-F\left(\frac{c^*-\theta}{\sigma}\right)\right)^{(1-q)N} pdf(\theta|c^*)d\theta$$

$$= u(N) \int_{u=0}^{1} \binom{N}{qN} u^{qN} (1-u)^{(1-q)N} du$$

$$= \frac{u(N)}{1+N}.$$

Intuition (Chamberlain and Rothschild 1981): Consider $N+1$ random variables $\{X_0, X_1, \cdots, X_N\}$ with $X_i \sim iid\ U[0,1]$.

- Consider a random draw for each and rank them in the usual order.
- Because these random variables are identical, the probability that the realization x_0 is the $qN+1$st smallest is $\frac{1}{1+N}$.
- If we knew $x_0 = u$, then the probability that x_0 was the $qN+1$st smallest one would be $\binom{N}{qN} u^{qN} (1-u)^{(1-q)N}$.
- We have a uniform prior that X_0 is uniformly distributed between 0 and 1. Thus, we integrate.

Tullock's Paradox

$$\lim_{N\to\infty} c^*(N) = \lim_{N\to\infty} \frac{u(N)}{1+N} > 0?$$

If $u(N) = b_0 + b_1 N$, with $b_1 > 0$, then in the limit as $N \to \infty$, $c^* = b_1 > 0$. That is, some citizens with positive costs of revolt participate in the revolution.

Abraham Keteltas's 1777 sermon, **God Pleads His Cause**, in the context of the American Revolution is an example (Sandoz 1998, p. 579-605):

> *America will be a glorious land of freedom, knowledge, and religion, an asylum for distressed, oppressed, and persecuted virtue. Let this exhilarating thought, fire your souls, and give new ardor and encouragement to your hopes*—**you contend not only for your own happiness, for your dear relations; for the happiness of the present inhabitants of America; but you contend for the happiness of millions yet unborn.** *Exert therefore, your utmost efforts, strain every nerve, do all you can to promote this cause.*

Hong Kong Experiment

To address Cantoni et al.'s (2018) Experiment, we want to know whether citizen i's incentives to revolt increase or decrease if all other citizens marginally raise their cutoff from c^* in equilibrium.

- Let $B(c_i; c^*)$ be the incremental benefit from revolting versus not revolting: $B(c_i; c^*) - c_i = 0$ at $c_i = c^*$.
- We need the sign of $\frac{\partial B(c_i;c^*)}{\partial c^*}\Big|_{c_i=c^*}$.
- **A real analysis result** (Good and Mayer 1975; Chamberlain and Rothschild 1981):

$$\lim_{N\to\infty} \frac{\partial B(c_i; c^*, \sigma)}{\partial c_i} = \frac{1}{\sigma} \frac{f'\left(\frac{c_i - c^*}{\sigma} + F^{-1}(q)\right)}{f(F^{-1}(q))} \lim_{N\to\infty} \frac{u(N)}{1+N}.$$

Hong Kong Experiment

To address Cantoni, Yang, Yutchman, and Zhang's (2018) Experiment, we want to know whether citizen i's incentives to revolt increase or decrease if all other citizens marginally raise their cutoff from c^* in equilibrium.

$$\lim_{N\to\infty} \frac{\partial B(c_i; c^*)}{\partial c^*}\bigg|_{c_i=c^*} = -\frac{1}{\sigma} \frac{f'\left(F^{-1}(q)\right)}{f(F^{-1}(q))} \lim_{N\to\infty} \frac{u(N)}{1+N}.$$

Suppose $f(\cdot)$ is strictly unimodal, with $q_m = F(mode)$. At equilibrium, actions are strategic substitutes if $q < q_m$ and strategic complements if $q > q_m$.

Hong Kong Experiment

- When the necessary fraction of protesters for a successful protest is below a threshold (q_m), actions are strategic substitutes in equilibrium.
- When success is "easy," free-riding dominates coordination considerations, and when a citizens believes that others are more likely to protest, he has less incentives to protest.
- In contrast, when the regime is strong and goals are grand, so that q is high (e.g., during the months preceding the 1979 Iranian Revolution), actions become strategic complements, and we fall into the realm of standard protest models.

The Strength of Weak States

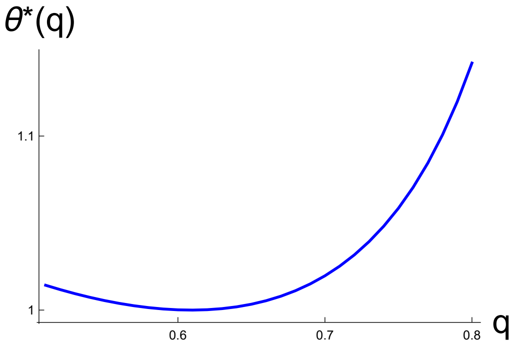

Figure 4: Parameters: $\sigma = 0.6$, $\sigma_0 = 1$, $u(N) = N$, and $N = 1000$. Thus, $c^* = \frac{u(N)}{1+N} \approx 1$.