## SHIFT-SHARE DESIGNS: THEORY AND INFERENCE\*

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#### **Abstract**

We study inference in shift-share regression designs, such as when a regional outcome is regressed on a weighted average of sectoral shocks, using regional sector shares as weights. We conduct a placebo exercise in which we estimate the effect of a shift-share regressor constructed with randomly generated sectoral shocks on actual labor market outcomes across U.S. Commuting Zones. Tests based on commonly used standard errors with 5% nominal significance level reject the null of no effect in up to 55% of the placebo samples. We use a stylized economic model to show that this overrejection problem arises because regression residuals are correlated across regions with similar sectoral shares, independently of their geographic location. We derive novel inference methods that are valid under arbitrary cross-regional correlation in the regression residuals. We show using popular applications of shift-share designs that our methods may lead to substantially wider confidence intervals in practice.

JEL codes: C12, C21, C26, F16, F22

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## 1 Introduction

We study how to perform inference in shift-share designs: regression specifications in which one studies the impact of a set of shocks, or "shifters", on units differentially exposed to them, with the exposure measured by a set of weights, or "shares". Specifically, shift-share regressions have the form

$$Y_i = \beta X_i + Z_i' \delta + \epsilon_i$$
, where  $X_i = \sum_{s=1}^S w_{is} \mathcal{X}_s$ ,  $w_{is} \ge 0$  for all  $s$ , and  $\sum_{s=1}^S w_{is} \le 1$ . (1)

For example, in an investigation of the impact of sectoral demand shifters on regional employment changes,  $Y_i$  is the change in employment in region i, the shifter  $\mathcal{X}_s$  is a measure of the change in demand for the good produced by sector s, and the share  $w_{is}$  may be measured as the initial share of region i's employment in sector s. Other observed characteristics of region i are captured by the vector  $Z_i$ , which includes the intercept, and  $\varepsilon_i$  is the regression residual. Shift-share specifications are increasingly common in many contexts (see, e.g., Bartik (1991), Blanchard and Katz (1992), Card (2001), or Autor, Dorn and Hanson (2013)). However, their formal properties are relatively understudied.

Our starting point is the observation that usual standard error formulas may substantially understate the true variability of OLS estimators of  $\beta$  in eq. (1). We illustrate the importance of this issue through a placebo exercise. As outcomes, we use 2000–2007 changes in employment rates and average wages for 722 Commuting Zones in the United States. We build a shift-share regressor by combining actual sectoral employment shares in 1990 with randomly drawn sector-level shifters for 396 4-digit SIC manufacturing sectors. The placebo samples thus differ exclusively in the randomly drawn sectoral shifters. For each sample, we compute the OLS estimate of  $\beta$  in eq. (1) and test if its true value is zero. Since the shifters are randomly generated, their true effect is indeed zero. Valid 5% significance level tests should therefore reject the null of no effect in at most 5% of the placebo samples. We find, however, that usual standard errors—clustering on state as well as heteroskedasticity-robust errors—are much smaller than the standard deviation of the OLS estimator and, as a result, lead to severe overrejection. Depending on the labor market outcome used, the rejection rate for 5% level tests can be as high as 55% for heteroskedasticity-robust standard errors and 45% for standard errors clustered on state, and it is never below 16%.

To explain the source of this overrejection problem, we introduce a stylized economic model featuring multiple regions, each of which produces output in multiple sectors. The key ingredients of our model are a sector- and region-specific labor demand and a regional labor supply. We assume that labor demand in each sector-region pair has a sector-specific elasticity with respect to wages and an intercept that aggregates several sector-specific components (e.g. sectoral productivities and demand shifters for the corresponding sectoral good). Labor supply in each region is upward-sloping and has a region-specific intercept that may aggregate group-specific labor supply shifters (e.g. push factors that raise immigration from different countries of origin). Up to a first-order approximation, the impact of sector-level shocks on labor market outcomes takes the form of a shift-share specification similar to that in eq. (1).

A key insight of our model is that the regression residual  $\epsilon_i$  in eq. (1) will generally account for shift-share components that aggregate all unobserved sector-level shocks using the same shares

 $w_{is}$  that enter the construction of the regressor  $X_i$ , as well as shift-share components that aggregate unobserved group-specific labor supply shifters using exposures  $\tilde{w}_{ig}$  of region i to group-g specific shocks. Thus, the residual may incorporate multiple shift-share terms with shares correlated with those defining the shift-share regressor  $X_i$ . Consequently, whenever two regions have similar shares, they will not only have similar exposure to the shifters  $\mathcal{X}_s$ , but will also tend to have similar values of the residuals  $\epsilon_i$ . While traditional inference methods allow for some forms of dependence between the residuals, such as spatial dependence within a state, they do not directly address the possible dependence between residuals generated by unobserved shift-share components. This is why, in our placebo exercise, traditional inference methods underestimate the variance of the OLS estimator of  $\beta$ , creating the overrejection problem.

We then establish the large-sample properties of the OLS estimator of  $\beta$  in eq. (1) under repeated sampling of the shifters  $\mathcal{X}_s$ , conditioning on the realized shares  $w_{is}$ , controls  $Z_i$ , and residuals  $\epsilon_i$ . This sampling approach is motivated by our economic model: we are interested in what would have happened to outcomes if the sector-level shocks  $\mathcal{X}_s$  had taken different values, holding everything else constant. Our framework allows for heterogeneous effects of the shifters: one unit increase in  $\mathcal{X}_s$  causes the outcome in region i to increase by  $w_{is}\beta_{is}$ , where  $\beta_{is}$  is an unknown parameter.

Our key assumption is that, conditional on the controls and the shares, the shifters are as good as randomly assigned and independent across sectors. An advantage of this assumption is that it allows us to do inference conditionally on  $e_i$ ; as a result, we can allow for *any* correlation structure of the regression residuals across regions.<sup>1</sup> In contrast, if, instead of assuming independence of the shifters across sectors, we modeled the correlation structure in the residual, as in the spatial econometrics literature (e.g. Conley, 1999) or in the interactive fixed effects literature (e.g. Bai, 2009; Gobillon and Magnac, 2016), the resulting inference would be sensitive to the validity of the modeling assumptions. We show that the regression estimand  $\beta$  in eq. (1) corresponds to a weighted average of the heterogeneous parameters  $\beta_{is}$  and derive novel confidence intervals that are valid in samples with many regions and sectors. We also derive an analogous formula when  $X_i$  is used as an instrument in an instrumental variables regression, which follows directly from the fact that the associated first-stage and reduced-form regressions take the form in eq. (1).

To gain intuition for our formula, it is useful to consider the special case in which each region is fully specialized in one sector (i.e. for every i,  $w_{is} = 1$  for some sector s). In this case, our procedure is identical to using the usual clustered standard error formula, but with clusters defined as groups of regions specialized in the same sector. This is in line with the rule of thumb that one should "cluster" at the level of variation of the regressor of interest. In the general case, our standard error formula essentially forms sectoral clusters, the variance of which depends on the variance of a weighted sum of the regression residuals  $\epsilon_i$ , with weights that correspond to the shares  $w_{is}$ .

We extend our baseline results in three ways. We provide versions of our standard errors that only require the shifters to be independent across "clusters" of sectors, allowing for arbitrary correlation

<sup>&</sup>lt;sup>1</sup>This is similar to the insight in Barrios et al. (2012), who consider cross-section regressions estimated at an individual level when the variable of interest varies only across groups of individuals. They show that, as long as the regressor of interest is as good as randomly assigned and independent across the groups, standard errors clustered on groups are valid under any correlation structure of the residuals.

among sectors belonging to the same "cluster." We also show how to apply our framework to panel data settings in which we have multiple observations of each region over time. Finally, we cover applications in which the shifter is unobserved, but can be estimated using observable local shocks.

We illustrate the finite-sample properties of our novel inference procedure in the same placebo exercise that we use to show the bias of the usual standard error formulas. Our new formulas give a good approximation to the variability of the OLS estimator across the placebo samples; consequently, they yield rejection rates that are close to the nominal significance level. As predicted by the theory, our standard error formula remains accurate under alternative distributions of both the shifters and the regression residuals. When the number of sectors is small or there is a sector that is significantly larger than the rest, our method overrejects, although the overrejection is milder in comparison with the usual standard error formulas. If the shifters are not independent across sectors, we show that it is important to properly account for their correlation structure.

In the final part of the paper, we illustrate the implications of our new inference procedure for two popular applications of shift-share regressions. First, we study the effect of changes in sector-level Chinese import competition on labor market outcomes across U.S. Commuting Zones, as in Autor, Dorn and Hanson (2013). Second, we use changes in sector-level national employment to estimate the regional inverse labor supply elasticity, as in Bartik (1991).<sup>2</sup> Our new confidence intervals for the effects of Chinese competition on local labor markets increase by 23%–66% relative to those implied by state-clustered or heteroskedasticity-robust standard errors, although these effects remain statistically significant. In contrast, our confidence intervals for the inverse labor supply elasticity estimated using the procedure in Bartik (1991) are very similar to those constructed using standard approaches.

Shift-share designs have been applied to estimate the effect of a wide range of shocks. For example, in seminal papers, Bartik (1991) and Blanchard and Katz (1992) use shift-share designs to analyze the impact on local labor markets of shifters measured as changes in national sectoral employment. More recently, shift-share strategies have been applied to investigate the local labor market impact of various shocks, including international trade competition (Topalova, 2007, 2010; Kovak, 2013; Autor, Dorn and Hanson, 2013; Dix-Carneiro and Kovak, 2017; Pierce and Schott, 2018), credit supply (Greenstone, Mas and Nguyen, 2015), technological change (Acemoglu and Restrepo, 2019, 2018), and industry reallocation (Chodorow-Reich and Wieland, 2018). Shift-share regressors have been used as well to estimate the impact of immigration on labor markets, as in Card (2001) and many other papers following his approach; see reviews in Lewis and Peri (2015) and Dustmann, Schönberg and Stuhler (2016). Furthermore, recent papers use shift-share strategies to estimate how firms respond to changes in outsourcing costs and foreign demand (Hummels et al., 2014; Aghion et al., 2018).<sup>3</sup>

Our paper is related to two other papers studying the statistical properties of shift-share instrumental variables. First, Goldsmith-Pinkham, Sorkin and Swift (2018) consider using the full vector of

<sup>&</sup>lt;sup>2</sup>Additionally, in Online Appendix F, we use changes in the stock of immigrants from various origin countries to investigate the impact of immigration on employment and wages, following Altonji and Card (1991) and Card (2001).

<sup>&</sup>lt;sup>3</sup>Shift-share regressors have also been used to study the impact of sectoral shocks on political preferences (Autor et al., 2017; Che et al., 2017; Che et al., 2017; Colantone and Stanig, 2018), marriage patterns (Autor, Dorn and Hanson, 2018), crime levels (Dix-Carneiro, Soares and Ulyssea, 2018), and innovation (Acemoglu and Linn, 2004; Autor et al., 2019). In addition to using shift-share designs to estimate the overall impact of a shifter of interest, other work has used them as part of a more general structural estimation approach; see Diamond (2016), Adão (2016), Galle, Rodríguez-Clare and Yi (2018), Burstein et al. (2018), Bartelme (2018). Baum-Snow and Ferreira (2015) review additional applications in the context of urban economics.

shares  $(w_{i1},...,w_{iS})$  as an instrument for endogenous treatment. They conclude that this approach requires the entire vector of shares to be as good as randomly assigned conditional on the shifters. Second, Borusyak, Hull and Jaravel (2018), focusing on the use of a shift-share regressor as an instrument, show it is a valid instrument if the set of shifters is as good as randomly assigned conditional on the shares, and discuss consistency of the instrumental variables estimator in this context. We follow Borusyak, Hull and Jaravel (2018) by modeling the shifters as randomly assigned, since this approach follows naturally from our economic model. Using this assumption, we point out the potential bias of standard inference procedures when applied to shift-share designs, and provide a novel inference procedure that is valid in this context.

While our paper focuses on the statistical properties of the OLS estimator of  $\beta$  in eq. (1), there exists a prior literature that has focused on studying the validity of different economic interpretations that one may attach to the estimand  $\beta$ . For example, this prior literature has studied how this interpretation may be affected by the presence of cross-regional general equilibrium effects (Beraja, Hurst and Ospina, 2019; Adão, Arkolakis and Esposito, 2019), slow adjustment of labor market outcomes to the shifters  $\mathcal{X}_s$  (Jaeger, Ruist and Stuhler, 2018), and heterogeneous effects of the shifters across sectors and regions (Monte, Redding and Rossi-Hansberg, 2018).

The rest of this paper is organized as follows. Section 2 presents a placebo exercise illustrating the properties of the usual inference procedures. Section 3 introduces a stylized economic model and maps its implications into a potential outcome framework. Section 4 establishes the asymptotic properties of the OLS estimator of  $\beta$  in eq. (1), as well as the properties of an instrumental variables estimator that uses a shift-share variable as an instrument. Section 5 discusses extensions of our baseline framework. Section 6 examines the performance of our novel inference procedures in a series of placebo exercises. Section 7 revisits two prior applications of shift-share designs, and Section 8 concludes. Proofs and additional results are collected in an Online Appendix.

# 2 Overrejection of usual standard errors: placebo evidence

In this section, we implement a placebo exercise to evaluate the finite-sample performance of the two inference methods most commonly applied in shift-share regression designs: (a) Eicker-Hubert-White—or heteroskedasticity-robust—standard errors, and (b) standard errors clustered on groups of regions geographically close to each other. In our placebo, we regress observed changes in U.S. regional labor market outcomes on a shift-share regressor that is constructed by combining actual data on initial sectoral employment shares for each region with randomly generated sector-level shocks. We describe the setup in Section 2.1 and discuss the results in Section 2.2.

## 2.1 Setup and Data

We generate 30,000 placebo samples indexed by m. Each of them contains N = 722 regions and S = 396 sectors. We identify each region i with a U.S. Commuting Zone (CZ) and each sector s with a 4-digit SIC manufacturing industry.

Using the notation from eq. (1), the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$ , and the outcomes  $\{Y_i\}_{i=1}^N$  are identical

in each placebo sample. The shares correspond to employment shares in 1990, and the outcomes correspond to changes in employment rates and average wages for different subsets of the population between 2000 and 2007. Our source of data on employment shares is the County Business Patterns, and our measures of changes in employment rates and average wages are based on data from the Census Integrated Public Use Micro Samples in 2000 and the American Community Survey for 2006 through 2008. Given these data sources, we construct our variables following the procedure described in the Online Appendix of Autor, Dorn and Hanson (2013).

The placebo samples differ exclusively in the shifters  $\{\mathcal{X}_s^m\}_{s=1}^N$ , which are drawn i.i.d. from a normal distribution with zero mean and variance equal to five in each placebo sample m. Since the shifters are independent of both the outcomes and the shares, the parameter  $\beta$  is zero; this is true irrespective of the dependence structure between the outcomes and the shares.

For each placebo sample m, given the observed outcome  $Y_i$ , the generated shift-share regressor  $X_i^m$  and a vector of controls  $Z_i$  including only an intercept, we compute the OLS estimate of  $\beta$ , the heteroskedasticity-robust standard error (which we label Robust), and the standard error that clusters CZs in the same state (labeled Cluster).

#### 2.2 Results

Table 1 presents the median and standard deviation of the empirical distribution of the OLS estimates of  $\beta$  across the 30,000 placebo samples, along with the median standard error estimates, and rejection rates for 5% significance level tests of the null hypothesis  $H_0$ :  $\beta = 0$ . We present these statistics for several outcome variables, which are listed in the leftmost column.

Column (1) of Table 1 shows that, up to simulation error, the average of the OLS estimates is zero for all outcomes. Column (2) reports the standard deviation of the estimated coefficients. This dispersion is the target of the estimators of the standard error of the OLS estimator.<sup>4</sup> Columns (3) and (4) report the median standard error estimates for the *Robust* and *Cluster* procedures, respectively, and show that both standard error estimators are downward biased. On average across all outcomes, the median magnitudes of the heteroskedasticity-robust and state-clustered standard errors are, respectively, 55% and 46% lower than the standard deviation.

The downward bias in the *Robust* and *Cluster* standard errors translates into a severe overrejection of the null hypothesis  $H_0$ :  $\beta = 0$ . Since the true value of  $\beta$  equals 0 by construction, a correctly behaved test with significance level 5% should have a 5% rejection rate. Columns (5) and (6) in Table 1 show that traditional standard error estimators yield much higher rejection rates. For example, when the outcome variable is the CZ's employment rate, the rejection rate is 48.5% and 38.1% when *Robust* and *Cluster* standard errors are used, respectively. These rejection rates are very similar when the dependent variable is instead the change in the average log weekly wage.

These results are quantitatively important. To see this, consider the following thought-experiment. Suppose we were to provide the 30,000 simulated samples to 30,000 researchers without disclosing the origin of the data to them. Instead, we would tell them that the shifters correspond to changes in a

<sup>&</sup>lt;sup>4</sup>Figure D.1 in Online Appendix D.1 reports the empirical distribution of the OLS estimates when the dependent variable is the change in each CZ's employment rate. Its distribution resembles a normal distribution centered around  $\beta = 0$ .

Table 1: Standard errors and rejection rate of the hypothesis  $H_0$ :  $\beta = 0$  at 5% significance level.

	Est	imate	Median	std. error	Rejection rate				
	Mean (1)	Std. dev. (2)	Robust (3)	Cluster (4)	Robust (5)	Cluster (6)			
Panel A: Change in the share of working-age population									
Employed	-0.01	2.00	0.73	0.92	48.5%	38.1%			
Employed in manufacturing	-0.01	1.88	0.60	0.76	55.7%	44.8%			
Employed in non-manufacturing	0.00	0.94	0.58	0.67	23.2%	17.6%			
Panel B: Change in average log v	veekly w	vage							
Employed	-0.03	2.66	1.01	1.33	47.3%	34.2%			
Employed in manufacturing	-0.03	2.92	1.68	2.11	26.7%	16.8%			
Employed in non-manufacturing	-0.02	2.64	1.05	1.33	45.4%	33.7%			

Notes: For the outcome variable indicated in the leftmost column, this table indicates the mean and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median standard error estimates (columns (3) and (4)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (5) and (6)). *Robust* is the Eicker-Huber-White standard error, and *Cluster* is the standard error that clusters CZs in the same state. Results are based on 30,000 placebo samples.

sectoral shock of interest—for instance, trade flows, tariffs, or national employment. If the researchers set out to test the null that the impact of this shock is zero using standard inference procedures at a 5% significance level, then over a third of them would conclude that our computer generated shocks had a statistically significant effect on the evolution of employment rates between 2000 and 2007.

The following remark summarizes the results of our placebo exercise.

**Remark 1.** In shift-share regressions, traditional inference methods may suffer from a severe overrejection problem, and yield confidence intervals that are too short.

To understand the source of this overrejection problem, note that the standard error estimators reported in Table 1 assume that the regression residuals are either independent across all regions (for *Robust*), or between geographically defined groups of regions (for *Cluster*). Given that shift-share regressors are correlated across regions with similar employment shares  $\{w_{is}\}_{s=1}^{S}$ , these methods generally lead to a downward bias in the standard error estimate whenever regions with similar employment shares  $\{w_{is}\}_{s=1}^{S}$  also have similar regression residuals. In the next section, we show how such correlations between regression residuals may arise.

# 3 Stylized economic model

This section presents a stylized economic model mapping labor demand and labor supply shocks to labor market outcomes for a set of regional economies. The aim of the model is twofold. First, it illustrates the economic mechanisms behind the overrejection problem documented in Section 2.2. Second, it provides guidance on how to estimate: (i) the impact of sector-specific labor demand shifters on regional labor market outcomes; and (ii) the regional inverse labor supply elasticity. We

describe the model fundamentals in Section 3.1, discuss its main implications in Section 3.2, and map these implications to a potential outcome framework in Section 3.3.

#### 3.1 Environment

We consider an economy with multiple sectors s = 1,...,S and multiple regions i = 1,...,N. We assume that the labor demand in sector s and region i,  $L_{is}$ , is given by

$$\log L_{is} = -\sigma_s \log \omega_i + \log D_{is}, \qquad \sigma_s > 0, \tag{2}$$

where  $\omega_i$  is the wage rate in region i,  $\sigma_s$  is the labor demand elasticity in sector s, and  $D_{is}$  is a regionand sector-specific labor demand shifter. This shifter may account for multiple sectoral components. Specifically, we decompose  $D_{is}$  into a sectoral shifter of interest  $\chi_s$ , other shifters that vary by sector  $\mu_s$ , and a residual region- and sector-specific shifter  $\eta_{is}$ :

$$\log D_{is} = \rho_s \log \chi_s + \log \mu_s + \log \eta_{is}. \tag{3}$$

We assume that the labor supply in region i is given by

$$\log L_i = \phi \log \omega_i + \log v_i, \qquad \phi > 0, \tag{4}$$

where  $\phi$  is the labor supply elasticity, and  $v_i$  is a region-specific labor supply shifter. We allow this shifter to have a shift-share structure that yields region-specific aggregates of group-specific labor supply shocks. In particular, indexing labor groups by g = 1, ..., G, we decompose

$$\log v_i = \sum_{g=1}^G \tilde{w}_{ig} \log v_g + \log v_i, \tag{5}$$

where  $v_g$  is a group-specific labor supply shifter,  $\tilde{w}_{ig}$  measures the exposure of region i to group g labor supply shifter, and  $v_i$  captures region-specific factors affecting labor supply. The variable  $v_g$  captures factors that affect the supply of labor of group g in all regions in the population of interest. Workers may be classified into groups according to their education level, gender, or country of origin.

We assume that workers cannot move across regions but are freely mobile across sectors. Thus, labor markets clear if

$$L_i = \sum_{s=1}^{S} L_{is}, \qquad i = 1, \dots, N.$$
 (6)

## 3.2 Labor market equilibrium

We assume that, in each period, the model described by eqs. (2) to (6) characterizes the labor market equilibrium in every region, and that, across periods, changes in the labor market outcomes  $\{\omega_i, L_i\}_{i=1}^N$  are due to changes in either the labor demand shifters,  $\{\chi_s, \mu_s\}_{s=1}^S$  and  $\{\eta_{is}\}_{i=1,s=1}^{N,S}$ , or the labor supply shifters,  $\{\nu_g\}_{g=1}^G$  and  $\{\nu_i\}_{i=1}^N$ .

We use  $\hat{z} = \log(z^t/z^0)$  to denote log-changes in a variable z between a period t = 0 and some other period t. We assume that the realized changes between any two periods in all labor demand and supply shifters are draws from a joint distribution  $F(\cdot)$ :

$$\left( \{ \hat{\chi}_s, \hat{\mu}_s \}_{s=1}^S, \{ \hat{\eta}_{is} \}_{i=1,s=1}^{N,S}, \{ \hat{v}_g \}_{g=1}^G \{ \hat{v}_i \}_{i=1}^N \right) \sim F(\cdot). \tag{7}$$

Up to a first-order approximation around the initial equilibrium, eqs. (2) to (6) imply that the changes in employment and wages in region i are given by

$$\hat{L}_i = \sum_{s=1}^S l_{is}^0 (\theta_{is} \hat{\chi}_s + \lambda_i \hat{\mu}_s + \lambda_i \hat{\eta}_{is}) + (1 - \lambda_i) \left( \sum_{g=1}^G \tilde{w}_{ig} \hat{v}_g + \hat{v}_i \right), \tag{8}$$

$$\hat{\omega}_i = \phi^{-1} \sum_{s=1}^S l_{is}^0 (\theta_{is} \hat{\chi}_s + \lambda_i \hat{\mu}_s + \lambda_i \hat{\eta}_{is}) - \phi^{-1} \lambda_i (\sum_{g=1}^G \tilde{w}_{ig} \hat{\nu}_g + \hat{\nu}_i), \tag{9}$$

where  $l_{is}^0 = L_{is}^0/L_i^0$  is the initial employment share of sector s in region i,  $\lambda_i = \phi \left[\phi + \sum_{s=1}^S l_{is}^0 \sigma_s\right]^{-1}$ , and  $\theta_{is} = \rho_s \lambda_i$ .

Consider first the model's implications for the impact on regional labor market outcomes of changes in sector-specific labor demand. We focus here on the impact of the demand shocks  $\{\hat{\chi}_s, \hat{\mu}_s\}_{s=1}^S$  on the change in the employment rate  $\hat{L}_i$ ; however, given the symmetry between eqs. (8) and (9), the model's implications for the impact of these shocks on the change in the wage level  $\hat{\omega}_i$  are analogous.

According to eq. (8), the change in the employment rate in region i depends on two shift-share components that aggregate the impact of the sector-specific labor demand shocks. In both components, the "share" term is the initial employment share  $l_{is}^0$ ; the "shift" term corresponds in each of them to one of the two sector-specific labor demand shocks,  $\hat{\chi}_s$  or  $\hat{\mu}_s$ . Furthermore,  $\hat{L}_i$  also depends on additional shift-share terms that aggregate the impact of group-specific labor supply shocks. In this case, the "share" term is the region's exposure to each group-specific shock,  $\tilde{w}_{ig}$ . Conditional on a sector s and a labor group s, the shares  $\{l_{is}^0\}_{i=1}^N$  and  $\{\tilde{w}_{ig}\}_{i=1}^N$  may be correlated. Settings in which the outcome of interest depends on multiple shift-share terms with potentially correlated shares is central to understanding the placebo results presented in Section 2.

Another implication of eq. (8) is that, even conditional on the initial employment share  $l_{is}^0$ , the impact of sectoral labor demand shocks on regional employment may be heterogeneous across sectors and regions; e.g., the impact of  $\hat{\chi}_s$  on  $\hat{L}_i$  depends not only on  $l_{is}^0$  but also on  $\theta_{is}$ , which may vary across i and s. While datasets usually contain information on the initial employment shares for every sector and region  $\{l_{is}^0\}_{i=1,s=1}^{N,S}$ , the parameters  $\{\theta_{is}\}_{i=1,s=1}^{N,S}$  are not generally known.

We summarize the discussion in the last two paragraphs in the following remark:

**Remark 2.** In our model, the equilibrium equations for the change in regional labor market outcomes combines multiple shift-share terms, and the shifter effects depend on unknown parameters that may be heterogeneous.

Online Appendices B and C show that there are multiple microfoundations consistent with the insights summarized in Remark 2. Alternative microfoundations may differ in the mapping between the labor demand and supply elasticities,  $\sigma_s$  and  $\phi$ , and structural parameters, or in the interpre-

tation of the different terms entering the labor demand shifter  $D_{is}$  in eq. (3).<sup>5</sup> In addition, Online Appendix C.3 shows that similar insights arise in a model that allows for migration across regions. In this case, the change in regional employment depends not only on the region's own shift-share terms included in eq. (8), but also on a component, common to all regions, that combines the shift-share terms corresponding to all N regions. In this environment,  $l_{is}^0\theta_{is}$  is the partial effect of the shifter  $\hat{\chi}_s$  on  $\hat{L}_i$  conditional on a fixed effect that absorbs cross-regional spillovers created by migration.

Turning to the estimation of the inverse labor supply elasticity, eqs. (4) and (5) imply that

$$\hat{\omega}_i = \tilde{\phi} \hat{\mathcal{L}}_i - \tilde{\phi} (\sum_{g=1}^G \tilde{w}_{ig} \hat{v}_g + \hat{v}_i) \quad \text{with} \quad \tilde{\phi} = \phi^{-1}. \tag{10}$$

It follows from eq. (8) that the change in region i's employment rate,  $\hat{L}_i$ , also depends on the term  $\sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_g + \hat{v}_i$ . Thus, the two terms on the right-hand side of eq. (10) are correlated with each other, creating an endogeneity problem. The instrumental variables solution to this problem relies on the observation that using eqs. (8) and (9), one can write the inverse labor supply elasticity as the ratio of the impact of a sector-specific labor demand shock (e.g.  $\hat{\chi}_s$ ) on wages to that on employment:

$$ilde{\phi} = rac{\partial \hat{\omega}_i}{\partial \hat{\chi}_s} igg/rac{\partial \hat{L}_i}{\partial \hat{\chi}_s}.$$

In Sections 4 and 5, we use the model described here to provide an economic interpretation for the econometric assumptions we impose when discussing identification and estimation in shift-share designs. These assumptions imply restrictions on the distribution of labor supply and demand shocks  $F(\cdot)$  introduced in eq. (7). In Section 7, we return to this economic model when interpreting empirical estimates of the impact of sector-specific labor demand shifters on regional labor market outcomes (Section 7.1); and the regional inverse labor supply elasticity (Section 7.2).

## 3.3 From economic model's equilibrium conditions to a potential outcome framework

We build on the results in Section 3.2 to propose a general framework for the estimation of the impact of shifters on outcomes measured at a different unit of observation. For concreteness, we refer to the level at which shifters vary as sectors and to the level at which the outcome varies as regions.

To make precise what we mean by "the effect of shifters on an outcome", we use the potential outcomes notation, writing  $Y_i(x_1,...,x_S)$  to denote the potential (counterfactual) outcome that would occur in region i if the shocks to the S sectors were exogenously set to  $\{x_s\}_{s=1}^S$ . Consistently with eqs. (8) and (9), we assume that the potential outcomes are linear in the shocks,

$$Y_i(x_1,...,x_S) = Y_i(0) + \sum_{i=1}^{S} w_{is} x_s \beta_{is}, \quad \text{where} \quad w_{is} \ge 0 \text{ for all } s, \quad \sum_{s=1}^{S} w_{is} \le 1,$$
 (11)

<sup>&</sup>lt;sup>5</sup>In Online Appendix B, we derive eqs. (8) and (9) from a multisector gravity model with endogenous labor supply that follows closely that in Adão, Arkolakis and Esposito (2019). In Online Appendix C.1, we show that Remark 2 is consistent with a Jones (1971) model featuring sector-specific production inputs, as in Kovak (2013). In Online Appendix C.2, we show that it is also consistent with a Roy (1951) model featuring workers with heterogeneous preferences for employment across sectors, as in Galle, Rodríguez-Clare and Yi (2018), Lee (2018) and Burstein, Morales and Vogel (2019).

and  $Y_i(0) = Y_i(0,...,0)$  denotes the potential outcome in region i when all shocks  $\{x_s\}_{s=1}^S$  are set to zero. Thus, increasing  $x_s$  by one unit, holding the shocks to the other sectors constant, leads to an increase in region i's outcome of  $w_{is}\beta_{is}$  units. This is the treatment effect of  $x_s$  on  $Y_i(x_1,...,x_S)$ . The actual (observed) outcome is given by  $Y_i = Y_i(X_1,...,X_S)$ , which depends on the realization of the shifters,  $(X_1,...,X_S)$ .

If the shifters of interest are the sectoral labor demand shocks  $\{\hat{\chi}_s\}_{s=1}^S$ , and the outcome of interest is the employment change  $\hat{L}_i$ , we can map eq. (8) into eq. (11) by defining

$$Y_{i} = \hat{L}_{i}, \ w_{is} = l_{is}^{0}, \ x_{s} = \hat{\chi}_{s}, \ \beta_{is} = \theta_{is}, \ Y_{i}(0) = \lambda_{i} \sum_{s=1}^{S} w_{is}(\hat{\mu}_{s} + \hat{\eta}_{is}) + (1 - \lambda_{i})(\sum_{g=1}^{G} \tilde{w}_{ig}\hat{v}_{g} + \hat{v}_{i}).$$
 (12)

Observe that  $Y_i(0)$  aggregates all shifters other than the sectoral shocks of interest  $\{\hat{\chi}_s\}_{s=1}^S$ .

We are interested in the properties of the OLS estimator  $\hat{\beta}$  of the coefficient on the shift-share regressor  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$  in a regression of  $Y_i$  onto  $X_i$ .<sup>7</sup> To focus on the key conceptual issues, we abstract away from any additional covariates or controls for now, and assume that  $\mathcal{X}_s$  and  $Y_i$  have been demeaned, so that we can omit the intercept in a regression of  $Y_i$  on  $X_i$  (see Section 4.2 for the case with controls). In this simplified setting, the OLS estimator of the coefficient on  $X_i$  is given by

$$\hat{\beta} = \frac{\sum_{i=1}^{N} X_i Y_i}{\sum_{i=1}^{N} X_i^2},\tag{13}$$

and we can write the regression equation as

$$Y_i = \beta X_i + \epsilon_i$$
, where  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$ . (14)

The definition of the estimand  $\beta$  in eq. (14) and the properties of the estimator  $\hat{\beta}$  will depend on: (a) what is the population of interest; and (b) how we think about repeated sampling. For (a), we define the population of interest to be the observed set of N regions, as opposed to focusing on a large superpopulation of regions from which the N observed regions are drawn. Consequently, we are interested in the parameters  $\{\beta_{is}\}_{i=1,s=1}^{N,S}$  and the treatment effects  $\{w_{is}\beta_{is}\}_{i=1,s=1}^{N,S}$  themselves, rather than the distributions from which they are drawn, which would be the case if we were interested in a superpopulation of regions.<sup>8</sup> For (b), given our interest on estimating the *ceteris paribus* impact of a specific set of shocks  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$ , we consider repeated sampling of these shocks, while holding the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$ , the parameters  $\{\beta_{is}\}_{i=1,s=1}^{N,S}$ , and the potential outcomes  $\{Y_i(0)\}_{i=1}^{N}$  fixed.

<sup>&</sup>lt;sup>6</sup>Given the mapping in eq. (12), the expression in eq. (11) captures the first-order impact of the labor demand shocks  $\{\hat{\chi}_s\}_{s=1}^S$  on changes in the employment rate. We focus on this first-order impact because it helps connecting our analysis to linear specifications used extensively in the shift-share literature. See Online Appendix D.5 for a discussion of the approximation error arising from the linear specification imposed in eq. (8).

<sup>&</sup>lt;sup>7</sup>We assume for now that the shifters  $\{\mathcal{X}_s\}_{s=1}^S$  are directly observable. In Section 5.3, we consider the case in which we only observe noisy estimates of these shifters.

<sup>&</sup>lt;sup>8</sup>Treating the set of observed regions as the population of interest is common in applications of the shift-share approach. For example, the abstract of Autor, Dorn and Hanson (2013) reads: "We analyze the effect of rising Chinese import competition between 1990 and 2007 on U.S. local labor markets". Similarly, the abstract of Dix-Carneiro and Kovak (2017) reads: "We study the evolution of trade liberalization's effects on Brazilian local labor markets" (emphases added).

Given these assumptions, the estimand  $\beta$  is defined as the population analog of eq. (13) under repeated sampling of the shocks  $\mathcal{X}_s$ ,

$$\beta = \frac{\sum_{i=1}^{N} E[X_i Y_i \mid \mathcal{F}_0]}{\sum_{i=1}^{N} E[X_i^2 \mid \mathcal{F}_0]}, \quad \text{with} \quad \mathcal{F}_0 = \{Y_i(0), \beta_{is}, w_{is}\}_{i=1, s=1}^{N, S},$$
(15)

and, given eqs. (11) and (14), the regression error  $\epsilon_i$  is then defined as the residual

$$\epsilon_i = Y_i - X_i \beta = Y_i(0) + \sum_{i=1}^S w_{is} \mathcal{X}_s(\beta_{is} - \beta). \tag{16}$$

Thus, the statistical properties of the regression residual  $\varepsilon_i$  depend on the properties of the potential outcome  $Y_i(0)$ , the shifters  $\{\mathcal{X}_s\}_{s=1}^S$ , the shares  $\{w_{is}\}_{s=1}^S$ , and the difference between the parameters  $\{\beta_{is}\}_{s=1}^S$  and the estimand  $\beta$ . Importantly, as illustrated in eq. (12), the potential outcome  $Y_i(0)$  will generally incorporate terms that have a shift-share structure with shares that are either identical to (e.g. the term  $\sum_{s=1}^S w_{is}\hat{\mu}_s$ ) or different from but potentially correlated with (e.g. the term  $\sum_{g=1}^G \tilde{w}_{ig}\hat{\nu}_g$ ) the shares  $\{w_{is}\}_{s=1}^S$  that define the shift-share regressor  $X_i$ . It then follows from eq. (16) that the residuals  $\varepsilon_i$  and  $\varepsilon_{i'}$  will generally be correlated for any pair of regions i and i' with similar values of the shift-share regressor.

We summarize this discussion in the following remark.

**Remark 3.** Correct inference for the coefficient on a shift-share regressor requires taking into account potential cross-regional correlation in residuals across observations with similar values of the shift-share covariate of interest. One possible source of such correlation is the presence in these residuals of shift-share components with shares identical to or correlated with those entering the covariate of interest.

Remark 3 has important implications for estimating the sampling variability of  $\hat{\beta}$ . In particular, traditional inference procedures do not account for correlation in  $\epsilon_i$  among regions with similar shares and, therefore, tend to underestimate the variability of  $\hat{\beta}$ . As we formalize in the next section, this is the main reason for the overrejection problem described in Section 2.

# 4 Asymptotic properties of shift-share regressions

In this section, we formulate the statistical assumptions that we impose on the data generating process (DGP), use them to derive asymptotic results, and provide an economic interpretation of these assumptions using the model introduced in Section 3. In Section 4.1, we consider the case in which there is a single shift-share regressor and no controls. We account for controls in Section 4.2. In Section 4.3, we consider using the shift-share variable as an instrument for a regional treatment variable. All proofs and technical details are collected in Online Appendix A.

We follow the notation from eq. (1) by writing sector-level variables (such as the shifter  $\mathcal{X}_s$ ) in script font style and region-level aggregates (such as  $X_i$ ) in normal style. We use standard matrix and vector notation. In particular, for a (column) L-vector  $A_i$  that varies at the regional level, A denotes the  $N \times L$  matrix with the ith row given by  $A'_i$ . For an L-vector  $\mathcal{A}_s$  that varies at the sectoral level,  $\mathcal{A}$ 

denotes the  $S \times L$  matrix with the sth row given by  $\mathcal{A}'_s$ . If L = 1, then A and  $\mathcal{A}$  are an N-vector and an S-vector, respectively. Let W denote the  $N \times S$  matrix of shares, so that its (i,s) element is given by  $w_{is}$ , and let B denote the  $N \times S$  matrix with (i,s) element given by  $\beta_{is}$ .

## 4.1 Simple case without controls

We focus here on the statistical properties of the OLS estimator  $\hat{\beta}$  defined in eq. (13).

#### **Assumptions**

We consider large-sample properties of  $\hat{\beta}$  as the number sectors goes to infinity,  $S \to \infty$ . The assumptions below imply that  $N \to \infty$  as  $S \to \infty$ . To assess how large S needs to be in order that these asymptotics provide a good approximation to the finite sample distribution of  $\hat{\beta}$ , we conduct a series of placebo simulations in Section 6. We describe here the main substantive assumptions, and collect technical regularity conditions in Online Appendix A.1.1. As in eq. (15), let  $\mathcal{F}_0 = (Y(0), B, W)$ .

**Assumption 1** (Identification). (i) The observed outcome is given by  $Y_i = Y_i(\mathcal{X}_1, \dots, \mathcal{X}_S)$ , such that eq. (11) holds; (ii) The shifters are as good as randomly assigned conditional on  $\mathcal{F}_0$  in the sense that, for all  $s = 1, \dots, S$ ,

$$E[\mathcal{X}_s \mid \mathcal{F}_0] = 0. \tag{17}$$

Assumption 1(i) requires that the potential outcomes are linear in the shifters  $\{\mathcal{X}_s\}_{s=1}^S$ . As discussed in Section 3.3, one can generate such linear specification from a first-order approximation of the impact of the shifters  $(\mathcal{X}_1, \dots, \mathcal{X}_S)$  on the outcome  $Y_i$ . This approximation may be subject to error. In Online Appendix A.1.1, we generalize eq. (11) to allow for a linearization error and derive restrictions on this error under which our inference procedures remain valid.

Assumption 1(ii) imposes that the sectoral shifters  $\mathcal{X}$  are mean independent of the shares W, potential outcomes Y(0), and parameters B; the assumption that the shifters are mean zero is a normalization to allow us to drop the intercept; we relax it in Section 4.2. This random assignment assumption is a key assumption for identifying the causal impact of a shift-share covariate; a version of this assumption has been previously proposed by Borusyak, Hull and Jaravel (2018).

If we are interested in studying the effect of labor demand shifters in the context of the model in Section 3 (i.e.  $\mathcal{X}_s = \hat{\chi}_s$ ), Assumption 1(ii) will hold if the shifters  $\{\hat{\chi}_s\}_{s=1}^S$  are mean independent of the other labor demand shifters,  $\{\hat{\mu}_s\}_{s=1}^S$  and  $\{\hat{\eta}_{is}\}_{i=1,s=1}^{N,S}$ , and of the labor supply shifters,  $\{\hat{\nu}_g\}_{g=1}^G$  and  $\{\hat{\nu}_i\}_{i=1}^N$ . The plausibility of this restriction depends on the specific empirical application. For example, if all N regions in the sample are regions within a small open economy,  $\hat{\chi}_s$  denotes changes in international prices in sector s, and  $\hat{\mu}_s$  denotes changes in the tariffs that this small open economy charges on its sector s imports; then, Assumption 1(ii) requires these changes in tariffs to be independent of the changes in tariffs in any country that is large enough for their tariff changes to affect international prices (see Online Appendix B.4 for additional details).

**Assumption 2** (Consistency and Inference). (i) The shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$  are independent conditional on  $\mathcal{G}_0$ ; (ii)  $\max_s n_s / \sum_{t=1}^S n_t \to 0$ , where  $n_s = \sum_{s=1}^S w_{is}$  denotes the total share of sector s; (iii)  $\max_s n_s^2 / \sum_{t=1}^S n_t^2 \to 0$ .

Assumption 2(i) requires the shifters to be independent. It adapts to our setting the assumption underlying randomization-style inference in randomized controlled trials that the treatment assignment is independent across entities (see Imbens and Rubin, 2015, for a review). An independence or a weak dependence assumption of this type is generally necessary in order to do inference. One could alternatively impose assumptions on the correlation structure of the regression residuals, either by imposing a particular structure on them, as in the literature on interactive fixed effects (e.g. Gobillon and Magnac, 2016), or by imposing a distance metric on the observations, as in the spatial econometrics literature (e.g. Conley, 1999). However, as the economic model in Section 3 shows, the structure of the residuals may be very complex. The residuals may include potentially correlated region-specific terms as well as several shift-share terms, which may or may not use the same shares as the covariate of interest  $X_i$ . It is thus difficult to conceptualize which exact restriction on their joint distribution one should impose.

By instead imposing restrictions on the distribution of the vector of shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$  conditional on  $\mathcal{F}_0 = (Y(0), B, W)$ , Assumption 2(i) ensures that the standard errors we derive remain valid under *any* dependence structure between the shares  $w_{is}$  across sectors and regions, and under *any* correlation structure of the potential outcomes  $Y_i(0)$  or, equivalently, of the regression errors  $\epsilon_i$ , across regions.<sup>10</sup> We thus do not have to worry about correctly specifying this correlation structure, as one would under the alternative approaches mentioned above. Our approach allows (but does not require) the residual to have a shift-share structure; it similarly allows all  $\{w_{is}\}_{i=1,s=1}^{N,S}$  to be equilibrium objects responding to the same economic shocks, and thus be correlated across regions and sectors.<sup>11</sup> In Section 5.1, we relax Assumption 2(i) and allow for a non-zero correlation in the shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$  within clusters of sectors; we only require that the shifters are independent across the clusters. Additionally, in the context of the empirical application in Section 7.1, we discuss how to perform inference in a setting in which all shifters of interest are generated by a common shock that has heterogeneous effects across sectors.

In the economic model in Section 3, if  $\mathcal{X}_s = \hat{\chi}_s$  and we interpret these shocks as, for example, sector-specific productivity shocks, Assumption 2(i) requires that there is no common component driving the changes in sectoral productivities. Our approach does not require the shifters  $\{\mathcal{X}_s\}_{s=1}^S$  to be identically distributed; we allow, for example, the variance of the shock to differ across sectors.

Assumptions 2(ii) and 2(iii) are our main regularity conditions. 12 Assumption 2(ii) is needed for

<sup>&</sup>lt;sup>9</sup>For example, for inference on average treatment effects, which is commonly the goal when running a regression, one typically assumes that the sample is a random sample from the population of interest and, thus, that the treatment variable is independent across the individuals in the sample.

<sup>&</sup>lt;sup>10</sup>Since our inference is valid conditional on  $\{\epsilon_i\}_{i=1}^N$ , it accounts for any correlation structure they may have, including spatial, or, in applications with multiple periods, temporal correlations. See Section 5.2 for settings with multiple periods.

<sup>&</sup>lt;sup>11</sup>This conceptualization of all the shares  $w_{is}$  as equilibrium objects that respond (at least partly) to the same set of shocks is consistent with the model in Section 3. As shown in eq. (12), each share  $w_{is}$  corresponds to the share of workers in region i employed in sector s in an initial equilibrium,  $l_{is}^0$ . Furthermore, each of these initial employment shares will be a function of the same sector-specific demand shocks and group-specific labor supply shocks; consequently  $l_{is}^0$  will generally be correlated with  $l_{i's'}^0$  even for  $i \neq i'$  and  $s \neq s'$ .

 $<sup>^{12}</sup>$ In the context of a shift-share instrumental variables regression, Goldsmith-Pinkham, Sorkin and Swift (2018) discuss similar conditions stated in terms of Rotemberg weights. This is convenient under the baseline assumption considered in Goldsmith-Pinkham, Sorkin and Swift (2018) that the vector of shares  $(w_{i1}, \ldots, w_{iS})$  is exogenous, because the Rotemberg weights determine the asymptotic bias of the estimator under local failures of this exogeneity condition. Since we do not assume exogeneity of the shares, this interpretation is not available under our setup.

consistency: it requires that the size of each sector,  $n_s$ , is asymptotically negligible. This assumption is analogous to the standard consistency condition in the clustering literature that the largest cluster be asymptotically negligible. To see the connection, consider the special case with "concentrated sectors", in which each region i specializes in one sector s(i); i.e.  $w_{is}=1$  if s=s(i) and  $w_{is}=0$  otherwise, and  $n_s$  is thus the number of regions that specialize in sector s. In this case,  $X_i=\mathcal{X}_{s(i)}$ , so that, if eq. (17) holds,  $\hat{\beta}$  is equivalent to an OLS estimator in a randomized controlled trial in which the treatment varies at a cluster level; here the sth cluster consists of regions that specialize in sector s. The condition  $\max_s n_s / \sum_{t=1}^S n_t \to 0$  then reduces to the assumption that the largest cluster be asymptotically negligible. Assumption 2(iii) is needed for asymptotic normality—it ensures that the Lindeberg condition holds. It strengthens Assumption 2(ii) slightly by requiring that the contribution of each sector to the asymptotic variance is asymptotically negligible; otherwise the estimator will not generally be asymptotically normal, even if it is consistent.

In terms of the economic model introduced in Section 3, Assumptions 2(ii) and 2(iii) require that no sector dominates the rest in terms of initial employment at the national level; i.e.  $\sum_{i=1}^{N} l_{is}^{0}$  is not too large for any sector. Section 6.1 shows that this assumption is reasonable for the U.S. if the S sectors used to construct the treatment of interest  $X_i$  correspond to the 396 4-digit manufacturing sectors (see Section 2.1). In Section 6.2, we illustrate the consequences of the failure of this assumption due to the inclusion of a large aggregate sector, the non-manufacturing sector, in  $X_i$ .

#### Asymptotic theory

We now establish that the OLS estimator in eq. (13) is consistent and asymptotically normal.

**Proposition 1.** Suppose Assumption 1, Assumptions 2(i) and 2(ii), and Assumptions A.1(i) to A.1(iii) in Online Appendix A.1.1 hold. Then

$$\beta = \frac{\sum_{i=1}^{N} \sum_{s=1}^{S} \pi_{is} \beta_{is}}{\sum_{i=1}^{N} \sum_{s=1}^{S} \pi_{is}}, \quad and \quad \hat{\beta} = \beta + o_p(1),$$
(18)

where  $\pi_{is} = w_{is}^2 \operatorname{var}(\mathcal{X}_s \mid \mathcal{F}_0)$ .

This proposition gives two results. First, it shows that the estimand  $\beta$  in eq. (15) can be expressed as a weighted average of the region- and sector-specific parameters  $\{\beta_{is}\}_{i=1,s=1}^{N,S}$ , with the weight  $\pi_{is}$  increasing in the share  $w_{is}$  and in the conditional variance of the shifter  $\text{var}(\mathcal{X}_s \mid \mathcal{F}_0)$ . Second, it states that the OLS estimator  $\hat{\beta}$  converges to this estimand as  $S \to \infty$ . The special case with concentrated sectors is again useful in interpreting Proposition 1. In this case,  $\sum_{s=1}^{S} \pi_{is}\beta_{is} = \text{var}(\mathcal{X}_{s(i)} \mid \mathcal{F}_0)\beta_{is(i)}$  and, therefore, the first result in Proposition 1 reduces to the standard result from the randomized controlled trials literature with cluster-level randomization (with each "cluster" defined as all regions specialized in the same sector) that the weights are proportional to the variance of the shock.

The estimand  $\beta$  does not in general equal a weighted average of the heterogeneous treatment effects. As discussed in Section 3.3, the effect on the outcome in region i of increasing the value of the sector s shock in one unit is equal to  $w_{is}\beta_{is}$ ; weighting this effect using a set of region- and

sector-specific weights  $\{\xi_{is}\}_{i=1,s=1}^{N,S}$ , yields the weighted average treatment effect

$$au_{\xi} = rac{\sum_{i=1}^{N} \sum_{s=1}^{S} \xi_{is} w_{is} eta_{is}}{\sum_{i=1}^{N} \sum_{s=1}^{S} \xi_{is}}.$$

Alternatively, the total effect of increasing the shifters simultaneously in every sector by one unit is  $\sum_{s=1}^{S} w_{is}\beta_{is}$ ; weighting it using a set of region-specific weights  $\{\zeta_i\}_{i=1}^{N}$  yields the weighted total treatment effect  $\tau_{\zeta}^{T} = \sum_{i=1}^{N} \zeta_i \sum_{s=1}^{S} w_{is}\beta_{is} / \sum_{i=1}^{N} \zeta_i$ . If  $\beta_{is}$  is constant across i and s, then  $\beta = \tau_{\zeta}^{T}$ , provided  $\sum_{s=1}^{S} w_{is} = 1$  in every region i; otherwise, we can consistently estimate  $\tau_{\zeta}^{T}$  by  $\hat{\beta} \cdot \sum_{i=1}^{N} \zeta_i \sum_{s=1}^{S} w_{is} / \sum_{i=1}^{N} \zeta_i$ . Similarly, if  $\beta_{is}$  is constant across i and s,  $\tau_{\zeta}$  is consistently estimated by  $\hat{\beta} \cdot \sum_{i=1}^{N} \sum_{s=1}^{S} \zeta_{is} w_{is} / \sum_{i=1}^{N} \sum_{s=1}^{S} \zeta_{is}$ . On the other hand, if  $\beta_{is}$  varies across regions and sectors, then it is not clear in general how to exploit knowledge of the estimand  $\beta$  defined in eq. (18) to learn something about  $\tau_{\zeta}$  or  $\tau_{\zeta}^{T}$ . A special case in which it is possible to consistently estimate  $\tau_{\zeta}$  even if  $\beta_{is}$  varies across i or s arises when  $\mathcal{X}_s$  is homoskedastic,  $var(\mathcal{X}_s \mid \mathcal{F}_0) = \sigma^2$ , and  $\zeta_{is} = w_{is}$ ; in this case, a consistent estimate of  $\tau_{\zeta}$  is given by  $\hat{\beta} \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}^2 / \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}$ .

**Proposition 2.** Suppose Assumptions 1 and 2, and Assumption A.1 in Online Appendix A.1.1 hold. Suppose also that

$$V_N = \frac{1}{\sum_{s=1}^{S} n_s^2} \operatorname{var} \left( \sum_{i=1}^{N} X_i \epsilon_i \mid \mathcal{F}_0 \right)$$

converges in probability to a non-random limit. Then

$$\frac{N}{\sqrt{\sum_{s=1}^{S} n_s^2}} (\hat{\beta} - \beta) = \mathcal{N}\left(0, \frac{\mathcal{V}_N}{\left(\frac{1}{N} \sum_{i=1}^{N} X_i^2\right)^2}\right) + o_p(1).$$

This proposition shows that  $\hat{\beta}$  is asymptotically normal, with a rate of convergence equal to  $N(\sum_{s=1}^{S} n_s^2)^{-1/2}$ . If all sector sizes  $n_s$  are of the order N/S, the rate of convergence equals  $\sqrt{S}$ . However, if the sizes are unequal, the rate may be slower.

According to Proposition 2, the asymptotic variance formula has the usual "sandwich" form. Since  $X_i$  is observed, to construct a consistent standard error estimate, it suffices to construct a consistent estimate of  $V_N$ , the middle part of the sandwich. To motivate our standard error formula, suppose that  $\beta_{is}$  is constant across i and s,  $\beta_{is} = \beta$ . Then it follows from eq. (17) and Assumption 2(i) that

$$V_N = \frac{\sum_{s=1}^S \operatorname{var}(\mathcal{X}_s \mid \mathcal{G}_0) R_s^2}{\sum_{s=1}^S n_s^2}, \qquad R_s = \sum_{i=1}^N w_{is} \epsilon_i.$$
(19)

<sup>&</sup>lt;sup>13</sup>In general, one can consistently estimate  $\tau_{\xi}$  or  $\tau_{\xi}^{T}$  by imposing a mapping between  $\beta_{is}$  and structural parameters, and obtaining consistent estimates of these structural parameters. However, since this mapping will vary across models, the consistency of such estimator will not be robust to alternative modeling assumptions, even if all these assumptions predict an equilibrium relationship like that in eq. (8); e.g. see Online Appendix B and Online Appendices C.1 and C.2 for examples of this mapping in different models.

Replacing  $var(\mathcal{X}_s \mid \mathcal{F}_0)$  by  $\mathcal{X}_s^2$ , and  $\epsilon_i$  by the regression residual  $\hat{\epsilon}_i = Y_i - X_i \hat{\beta}$ , we obtain the estimate

$$\hat{V}_{AKM}(\hat{\beta}) = \frac{\hat{V}_{AKM}(\hat{\beta})}{\left(\sum_{i=1}^{N} X_i^2\right)^2}, \qquad \hat{V}_{AKM}(\hat{\beta}) = \sum_{s=1}^{S} \mathcal{X}_s^2 \hat{R}_s^2, \qquad \hat{R}_s = \sum_{i=1}^{N} w_{is} \hat{\epsilon}_i.$$
 (20)

When  $\beta_{is} = \beta$ , we show formally that this variance estimate leads to valid inference under regularity conditions in Section 4.2. In Online Appendix A.1.6 we show that this variance estimate remains valid under heterogeneous  $\beta_{is}$  under further regularity conditions.

To gain intuition for the variance estimate in eq. (20), consider the case with concentrated sectors. Then the numerator in eq. (20) becomes  $\sum_{s=1}^{S} \mathcal{X}_{s}^{2} \hat{R}_{s}^{2} = \sum_{s=1}^{S} (\sum_{i=1}^{N} \mathbb{I}\{s(i) = s\} X_{i} \hat{\epsilon}_{i})^{2}$ , so that eq. (20) reduces to the cluster-robust variance estimate that clusters on the sector that each region is specialized. This is consistent with the rule of thumb that one should "cluster" at the level of variation of the regressor of interest. More generally, the variance estimate essentially forms sectoral clusters with variance that depends on the variance of  $\hat{R}_{s}$ , a weighted sum of the regression residuals  $\{\hat{\epsilon}_{i}\}_{i=1}^{N}$ , with weights that correspond to the shares  $\{w_{is}\}_{i=1}^{N}$ . An important advantage of  $\hat{V}_{AKM}(\hat{\beta})$  is that it allows for an arbitrary structure of cross-regional correlation in residuals:

**Remark 4.** In the expression for  $V_N$  in eq. (19), the expectation is only taken over  $\{X_s\}_{s=1}^S$ —we do not take any expectation over the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$  or the residuals  $\{\epsilon_i\}_{i=1}^N$ . This is because our inference is conditional on the realized values of the shares and on the potential outcomes  $\{Y_i(0)\}_{i=1}^N$ . In terms of the regression in eq. (14), this means that we consider properties of  $\hat{\beta}$  under repeated sampling of  $X_i = \sum_{s=1}^S w_{is} X_s$  conditional on the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$  and on the residuals  $\{\epsilon_i\}_{i=1}^N$  (as opposed to, say, considering properties of  $\hat{\beta}$  under repeated sampling of the residuals conditional on  $\{X_i\}_{i=1}^N$ ). As a result, our inference method allows for arbitrary dependence between the residuals  $\{\epsilon_i\}_{i=1}^N$ .

To understand the source of the overrejection problem discussed in Section 2, let us compare the variance estimate  $\hat{V}_{AKM}(\beta)$  with the cluster-robust variance estimate when the residuals  $\hat{e}_i$  are computed at the true  $\beta$  (so that  $\hat{e}_i = e_i$ ). These variance estimates differ in the middle sandwich, with the cluster-robust estimate replacing  $\hat{V}_{AKM}(\beta)$  in eq. (20) with  $\hat{V}_{CL}(\beta) = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{I}\{c(i) = c(j)\}X_iX_je_ie_j$ , where c(i) denotes the cluster that region i belongs to (the comparison with heteroskedasticity-robust standard errors obtains as a special case if c(i) = i, so that each region belongs to its own cluster). Assuming for simplicity that the conditional variance of  $\mathcal{X}_s$  does not depend on Y(0), it follows by simple algebra that the expectation of the difference between these terms is given by

$$E[\hat{\mathcal{V}}_{AKM}(\beta) - \hat{\mathcal{V}}_{CL}(\beta) \mid W] = \sum_{s=1}^{S} \operatorname{var}(\mathcal{X}_{s} \mid W) \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{I}\{c(i) \neq c(j)\} w_{is} w_{js} E[\epsilon_{i} \epsilon_{j} \mid W]. \tag{21}$$

This expression is non-negative so long as the correlation between the residuals is non-negative. The magnitude of the difference will be large if regions located in different clusters (so that  $c(i) \neq c(j)$ ) that have similar shares (i.e. large values of  $\sum_{s=1}^{S} w_{is}w_{js}$ ) also tend to have similar residuals (i.e. large values of  $E[\epsilon_i\epsilon_j \mid W]$ ). For illustration, consider a simplified version of the model described in Section 3 in which: (a)  $\sigma_s \geq 0$  for all s and  $\phi \geq 0$ , so that  $0 \leq \lambda_i \leq 1$ ; (b) region-specific labor demand and supply shocks  $\{\hat{\eta}_{is}\}_{s=1}^{S}$  and  $\hat{v}_i$  are independent across regions; and (c) all labor demand and supply shocks

are independent of each other. Then, it follows from eqs. (12) and (16) that, for any  $i \neq j$ ,

$$E[\epsilon_i \epsilon_j \mid W, \tilde{W}] = \lambda_i \lambda_j \sum_{s=1}^S w_{is} w_{js} E[\hat{\mu}_s^2 \mid W, \tilde{W}] + (1 - \lambda_i)(1 - \lambda_j) \sum_{g=1}^G \tilde{w}_{jg} \tilde{w}_{ig} E[\hat{v}_g^2 \mid W, \tilde{W}] \ge 0, \quad (22)$$

which by the law of iterated expectations implies that  $E[\hat{V}_{AKM}(\beta) - \hat{V}_{CL}(\beta) \mid W] \geq 0$ . This expression illustrates that regions with similar shares will tend to have similar residuals in two cases. First, if the variance of the unobserved shifter  $\hat{\mu}_s$  is large, so that  $E[\hat{\mu}_s^2 \mid W, \tilde{W}]$  is large. In other words, standard inference methods lead to overrejection if the residual contains important shift-share terms that affect the outcome of interest through the same shares  $\{w_{is}\}_{s=1}^S$  as those defining the covariate of interest  $X_i$ . Second, if the variance of the unobserved shifter  $\hat{v}_g$  is large, so that  $E[\hat{v}_g^2 \mid W, \tilde{W}]$  is large, and the shares  $\tilde{w}_{ig}$  through which these shifters affect the outcome variable have a correlation structure that is similar to that of  $w_{is}$  (so that  $\sum_{g=1}^G \tilde{w}_{ig}\tilde{w}_{jg}$  is large whenever  $\sum_{s=1}^S w_{is}w_{js}$  is large). Thus, standard inference methods may overreject even when the unobserved shifters contained in the residual vary along a different dimension than the shift-share covariate of interest.

#### 4.2 General case with controls

We now study the properties of the OLS estimator  $\hat{\beta}$  of the coefficient on  $X_i$  in a regression of  $Y_i$  onto  $X_i$  and a K-vector of controls  $Z_i$ . To this end, let Z denote the  $N \times K$  matrix with i-th row given by  $Z'_i = (Z_{i1}, \ldots, Z_{iK})$ , and let  $\ddot{X} = X - Z(Z'Z)^{-1}Z'X$  denote an N-vector with i-th element equal to the regressor  $X_i$  with the controls  $Z_i$  partialled out (i.e. the residual from regressing  $X_i$  onto  $Z_i$ ). Then, by the Frisch–Waugh–Lovell theorem,  $\hat{\beta}$  can be written as

$$\hat{\beta} = \frac{\sum_{i=1}^{N} \ddot{X}_{i} Y_{i}}{\sum_{i=1}^{N} \ddot{X}_{i}^{2}} = \frac{\ddot{X}' Y}{\ddot{X}' \ddot{X}}.$$
(23)

The controls may play two roles. First, they may be included to increase the precision of  $\hat{\beta}$ . Second, and more importantly, they may be included because one may worry that the shifters  $\{X_s\}_{s=1}^S$  are correlated with the potential outcomes  $\{Y_i(0)\}_{i=1}^N$ , violating Assumption 1(ii). To formalize how  $Z_i$ , a regional variable, may be a control variable for the shifters, which vary at a sectoral level, we project  $Z_i$  onto the sectoral space using the same shares as those defining the shift-share regressor  $X_i$ ,

$$Z_i = \sum_{s=1}^S w_{is} \mathcal{Z}_s + U_i. \tag{24}$$

We think of  $\{\mathcal{Z}_s\}_{s=1}^S$  as latent sector-level shocks that may have an independent effect on the outcome Y and may also be correlated with the shifters  $\{\mathcal{X}_s\}_{s=1}^S$ , with  $U_i$ , the residual in this projection, mean-independent of the shifters. If the kth control  $Z_{ik}$  is included for precision, then the sector-level shocks  $\{\mathcal{Z}_{sk}\}_{s=1}^S$  and, thus,  $Z_{ik}$ , are uncorrelated with  $X_i$ . If  $Z_{ik}$  is included because one worries that otherwise  $X_i$  may not be as good as randomly assigned, we interpret  $Z_{ik}$  as a proxy for the confounding sector-level shocks  $\{\mathcal{Z}_{sk}\}_{s=1}^S$ , and think of  $U_{ik}$  as a measurement error in this proxy.

To make this concrete, consider the model in Section 3, with the equivalences in eq. (12). Then

we may include  $Z_{ik} = \sum_{s=1}^{S} l_{is}^0 \hat{\mu}_s$  as a control. Here the measurement error in eq. (24) is zero, and  $\mathcal{Z}_{sk} = \hat{\mu}_s$ . If the shifters  $\{\hat{\chi}_s\}_{s=1}^{S}$  are correlated with the demand shocks  $\{\hat{\mu}_s\}_{s=1}^{S}$ , then not including this control will generate omitted variable bias. Alternatively, we may include  $Z_{ik} = \sum_{s=1}^{S} w_{is} \hat{\eta}_{is}$  as a control. Here  $\mathcal{Z}_{sk} = 0$ , and  $U_{ik} = Z_{ik}$  is a regional aggregation of idiosyncratic region- and sector-specific labor-demand shocks that are independent of  $\mathcal{X}_s$ . In this case, if the shifters  $\{\hat{\chi}_s\}_{s=1}^{S}$  are independent of the demand shocks  $\{\eta_{is}\}_{i=1,s=1}^{N,S}$ , then including the control will help increase the precision of  $\hat{\beta}$ , but it is not necessary for consistency.

## Assumptions

For clarity of exposition, we focus here on the main substantive assumptions and relegate technical regularity conditions to Online Appendix A.1.1. Let  $\mathcal{F}_0 = (Y(0), W, B, \mathcal{Z}, U)$ ; without controls, this set of variables reduces to (Y(0), B, W), as in Section 4.1. Here,  $\mathcal{Z}$  denotes the  $S \times K$  matrix with sth row given by  $\mathcal{Z}'_s$ , and U denotes the  $N \times K$  matrix with i-th element given by  $U'_i$ .

We maintain Assumption 2 with  $\mathcal{F}_0 = (Y(0), W, B, \mathcal{Z}, U)$ . The inclusion of controls allows us to weaken Assumption 1 and instead impose the following identification assumption:

**Assumption 3** (Identification with controls). (i) The observed outcome satisfies  $Y_i = Y_i(\mathcal{X}_1, \dots, \mathcal{X}_S)$ , such that eq. (11) holds, and the controls  $Z_i$  satisfy eq. (24); (ii) The shifters are as good as randomly assigned in the sense that, for every s,

$$E[\mathcal{X}_s \mid \mathcal{F}_0] = E[\mathcal{X}_s \mid \mathcal{Z}_s], \tag{25}$$

and the right-hand side is linear in  $\mathcal{Z}_s$ ,

$$E[\mathcal{X}_s \mid \mathcal{Z}_s] = \mathcal{Z}_s' \gamma; \tag{26}$$

(iii) For elements k such that  $\gamma_k \neq 0$ ,  $N^{-1} \sum_{i=1}^N E[U_{ik}^2] \rightarrow 0$ ; (iv) For elements k such that  $\gamma_k \neq 0$ ,  $(\sum_{s=1}^S n_s^2)^{-1/2} \sum_{i=1}^N E[U_{ik}^2] \rightarrow 0$ .

Assumption 3(ii) weakens Assumption 1(ii) by only requiring the shifters to be as good as randomly assigned conditional on  $\mathcal{Z}$ , in the sense that eq. (25) holds. To interpret this restriction, consider a projection of the regional potential outcomes onto the sectoral space. For simplicity, consider the case with constant effects,  $\beta_{is} = \beta$  for all i and s, and project  $Y_i(0)$  onto the shares  $(w_{i1}, \ldots, w_{iS})$ , so that we may write  $Y_i(0) = \sum_{s=1}^{S} w_{is} \mathcal{Y}_s(0) + \kappa_i$ . Then, eq. (25) holds if (i)  $\mathcal{Y}_s(0)$  is spanned by the vector of controls  $\mathcal{Z}_s$ ; and (ii)  $\{\mathcal{X}_s\}_{s=1}^{S}$  is mean-independent of the projection residuals  $\{\kappa_i\}_{i=1}^{N}$ .

As an example, consider again the model in Section 3, with the outcomes  $Y_i$  generated by eq. (12). Then eq. (25) holds, for example, if we set  $\mathcal{Z}_s = \mathcal{Y}_s(0) = \hat{\mu}_s$  and if, conditional on the sector-specific labor demand shocks  $\{\hat{\mu}_s\}_{s=1}^S$ , the shifters of interest  $\{\hat{\chi}_s\}_{s=1}^S$  are mean independent of the sector- and region-specific labor demand shocks  $\{\hat{\eta}_{is}\}_{i=1,s=1}^{N,S}$  and of the labor supply shocks  $\{\hat{\nu}_g\}_{g=1}^G$  and  $\{\hat{\nu}_i\}_{i=1}^N$ . Suppose, for instance, the shocks of interest  $\{\hat{\chi}_s\}_{s=1}^S$  are changes in tariffs (e.g. Kovak, 2013) and that other potential labor demand shocks are those induced by automation and robots (e.g Acemoglu and Restrepo, 2019). Splitting the impact of automation into nationwide sector-specific effects, as captured

by  $\{\hat{\mu}_s\}_{s=1}^S$ , and sector- and region-specific deviations from the nationwide effects, as captured by  $\{\hat{\eta}_{is}\}_{i=1,s=1}^{N,S}$ , eq. (25) allows the political entity responsible for setting the tariffs to do so influenced by the nationwide sector-specific effects of automation, but not by any region-specific deviation from those national effects. In contrast, Assumption 1(ii) would require that the tariffs are also independent of the nationwide effects of automation.

Under eq. (25), one generally needs to include the controls non-parametrically; by imposing eq. (26), we ensure that it suffices to include the controls as additional covariates in a linear regression. If the shifters  $\mathcal{X}_s$  are not mean zero (in the sense that the regression intercept on the right-hand side of eq. (26) is non-zero), eq. (26) requires that we include a constant  $\mathcal{Z}_{sk} = 1$  as one of the controls. If the shares sum to one,  $\sum_{s=1}^{S} w_{is} = 1$ , this amounts to including an intercept  $Z_{ik} = 1$  as a control in the regression. Importantly, if the shares do not sum to one, this amounts to including  $\sum_{s=1}^{S} w_{is}$  as a control (see Borusyak, Hull and Jaravel, 2018, for a more extensive discussion of this point). For instance, if the shares  $w_{is}$  correspond to labor shares in different manufacturing sectors, one needs to include the size of the manufacturing sector  $\sum_{s=1}^{S} w_{is}$  in each region as a control.

Given Assumption 3(ii), if we observed  $\{\mathcal{Z}_s\}_{s=1}^S$  directly, we could include the vector  $Z_i^* = \sum_{s=1}^S w_{is}\mathcal{Z}_s$  directly as control. However, the definition of each regional control  $Z_i$  in eq. (24) allows for  $Z_i^*$  to be observed with measurement error  $U_i$ . If  $\gamma_k = 0$ , such as when  $Z_{ik}$  is included for precision, then this measurement error in  $Z_{ik}^*$  does not matter; if  $\gamma_k \neq 0$ , this measurement error will in general induce a bias in  $\hat{\beta}$ . This is analogous to the classic linear regression result that measurement error in a control variable generally leads to a bias in the estimate of the coefficient on the variable of interest. Assumption 3(iii) ensures that any such bias disappears in large samples by imposing that the variance of the measurement error for controls that matter (i.e. those with  $\gamma_k \neq 0$ ) converges to zero as  $S \to \infty$ . This ensures consistency of  $\hat{\beta}$ . For asymptotic normality, we need to strengthen this condition in Assumption 3(iv) by requiring that the variance of the measurement error converges to zero sufficiently fast. Assumption 3(iv) holds, for instance, if  $U_i = S^{-1} \sum_{s=1}^S \psi_{is}$ , where  $\psi_{is}$  is an idiosyncratic measurement error that is independent across s. In intuitive terms, this condition guarantees that  $Z_i$  is a sufficiently good proxy for the confounding latent shocks  $\{\mathcal{Z}_s\}_{s=1}^S$ .

#### Asymptotic theory

The following result generalizes Proposition 1:

**Proposition 3.** Suppose Assumptions 2(i) and 2(ii) and Assumptions A.1(i) to A.1(iii) in Online Appendix A.1.1 hold with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y(0), B, W)$ . Suppose also that Assumptions 3(i) to 3(iii) and Assumptions A.2(i) and A.2(ii) in Online Appendix A.1.1 hold. Then

$$\beta = \frac{\sum_{i=1}^{N} \sum_{s=1}^{S} \pi_{is} \beta_{is}}{\sum_{i=1}^{N} \sum_{s=1}^{S} \pi_{is}}, \quad \text{and} \quad \hat{\beta} = \beta + o_p(1),$$
 (27)

where  $\pi_{is} = w_{is}^2 \operatorname{var}(\mathcal{X}_s \mid \mathcal{F}_0)$ .

The only difference in the characterization of the probability limit relative to Proposition 1 is that the weights  $\pi_{is}$  now reflect the variance of  $\mathcal{X}_s$  that also conditions on the controls.

To state the asymptotic normality result, define  $\delta = E[Z'Z]^{-1}E[Z'(Y-X\beta)]$ , so that we can define the regression residual in eq. (1) as  $\epsilon_i = Y_i - X_i\beta - Z_i'\delta$ .

**Proposition 4.** Suppose Assumptions 2 and 3 and Assumptions A.1 and A.2 in Online Appendix A.1.1 hold with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y(0), B, W)$ . Suppose, in addition, that

$$V_N = \frac{1}{\sum_{s=1}^{S} n_s^2} \operatorname{var} \left( \sum_{i=1}^{N} (X_i - Z_i' \gamma) \epsilon_i \mid \mathcal{F}_0 \right)$$

converges in probability to a non-random limit. Then

$$\frac{N}{\sqrt{\sum_{s=1}^{S} n_s^2}} (\hat{\beta} - \beta) = \mathcal{N}\left(0, \frac{\mathcal{V}_N}{\left(\frac{1}{N} \sum_{i=1}^{N} \ddot{X}_i^2\right)^2}\right) + o_p(1).$$

Relative to Proposition 2, the main difference is that  $X_i$  in the definition of  $\mathcal{V}_N$  is replaced by  $X_i - Z_i'\gamma$ , and that  $X_i$  is replaced by  $\ddot{X}_i$  in the outer part of the "sandwich." To motivate our standard error formula, suppose that  $\beta_{is} = \beta$  for all i and s. Under  $\beta_{is} = \beta$ , it follows from eq. (25) and Assumption 2(i) that

$$\mathcal{V}_N = rac{\sum_{s=1}^S ext{var}( ilde{\mathcal{X}}_s \mid \mathcal{Y}_0) R_s^2}{\sum_{s=1}^S n_s^2}, \qquad R_s = \sum_{i=1}^N w_{is} \epsilon_i, \qquad ilde{\mathcal{X}}_s = \mathcal{X}_s - \mathcal{Z}_s' \gamma.$$

A plug-in estimate of  $R_s$  can be constructed by replacing  $\epsilon_i$  with the estimated regression residuals  $\hat{\epsilon}_i = Y_i - X_i \hat{\beta} - Z_i \hat{\delta}$ , where  $\hat{\delta} = (Z'Z)^{-1}Z'(Y - X\hat{\beta})$  is an OLS estimate of  $\delta$ . We can estimate the variance  $\text{var}(\tilde{\mathcal{X}}_s \mid \mathcal{F}_0)$  by  $\hat{\mathcal{X}}^2$ , where

$$\widehat{\mathcal{X}} = (W'W)^{-1}W'\ddot{X} \tag{28}$$

projects the estimate  $\ddot{X}$  of  $X - Z'\gamma$  onto the sectoral space by regressing it onto the shares W. To carry out the regression in eq. (28), W must be full rank; this requires that there are more regions than sectors,  $N \geq S$ . These steps lead to the standard error estimate

$$\widehat{se}(\hat{\beta}) = \frac{\sqrt{\sum_{s=1}^{S} \widehat{X}_{s}^{2} \widehat{R}_{s}^{2}}}{\sum_{i=1}^{N} \ddot{X}_{i}^{2}}, \qquad \widehat{R}_{s} = \sum_{i=1}^{N} w_{is} \widehat{\epsilon}_{i}.$$
(29)

The next remark summarizes the steps needed for the construction of the standard error  $\widehat{se}(\hat{\beta})$ :

**Remark 5.** To construct the standard error estimate in eq. (29):

- 1. Obtain the estimates  $\hat{\beta}$  and  $\hat{\delta}$  by regressing  $Y_i$  onto  $X_i = \sum_{s=1}^S w_{is} \mathcal{X}_s$  and the controls  $Z_i$ . The estimate  $\hat{\epsilon}_i$  corresponds to the estimated regression residuals.
- 2. Construct  $\ddot{X}_i$ , the residuals from regressing  $X_i$  onto  $Z_i$ . Compute  $\widehat{X}_s$ , the regression coefficients from regressing  $\ddot{X}$  onto W.
- 3. Plug the estimates  $\hat{\epsilon}_i$ ,  $\ddot{X}_i$ , and  $\hat{X}_s$  into the standard error formula in eq. (29).

To gain intuition for the procedure in Remark 5, it is useful to consider again the case with concentrated sectors. Suppose that  $U_i = 0$  for all i, so that the regression of  $Y_i$  onto  $X_i$  and  $Z_i$  is identical to the regression of  $Y_i$  onto  $X_{s(i)}$  and  $Z_{s(i)}$ . Then the standard error formula in eq. (29) reduces to the usual cluster-robust standard error, with clustering on s(i).

The cluster-robust standard error is generally biased due to estimation noise in estimating  $\epsilon_i$ , which can lead to undercoverage, especially in cases with few clusters (see Cameron and Miller, 2014 for a survey). Since the standard error in eq. (29) can be viewed as generalizing the cluster-robust formula, similar concerns arise in our setting. We thus consider a modification  $\hat{se}_{\beta_0}(\hat{\beta})$  of  $\hat{se}(\hat{\beta})$  that imposes the null hypothesis when estimating the regression residuals to reduce the estimation noise in estimating  $\epsilon_i$ .<sup>14</sup> To calculate the standard error  $\hat{se}_{\beta_0}(\hat{\beta})$  for testing the hypothesis  $H_0$ :  $\beta = \beta_0$  against a two-sided alternative at significance level  $\alpha$ , one replaces  $\hat{e}_i$  with  $\hat{e}_{\beta_0,i}$ , the residual from regressing  $Y_i - X_i\beta_0$  onto  $Z_i$  ( $\hat{e}_{\beta_0,i}$  is an estimate of the residuals with the null imposed). The null is rejected if the absolute value of the t-statistic ( $\hat{\beta} - \beta_0$ )/ $\hat{se}_{\beta_0}(\hat{\beta})$  exceeds  $z_{1-\alpha/2}$ , the  $1-\alpha/2$  quantile of a standard normal distribution (1.96 for  $\alpha = 0.05$ ). To construct a confidence interval (CI) with coverage  $1-\alpha$ , one collects all hypotheses  $\beta_0$  that are not rejected. The endpoints of this CI are a solution to a quadratic equation, and are thus available in closed form—one does not have to numerically search for all the hypotheses that are not rejected. The next remark summarizes this procedure.

**Remark 6** (Confidence interval with null imposed). To test the hypothesis  $H_0$ :  $\beta = \beta_0$  with significance level  $\alpha$  or, equivalently, to check whether  $\beta_0$  lies in the confidence interval with confidence level  $1 - \alpha$ :

- 1. Obtain the estimate  $\hat{\beta}$  by regressing  $Y_i$  onto  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$  and the controls  $Z_i$ . Obtain the restricted regression residuals  $\hat{\epsilon}_{\beta_0,i}$  as the residuals from regressing  $Y_i X_i\beta_0$  onto  $Z_i$ .
- 2. Construct  $\ddot{X}_i$ , the residuals from regressing  $X_i$  onto  $Z_i$ . Compute  $\hat{X}_s$ , the regression coefficients from regressing  $\ddot{X}$  onto W (this step is identical to step 2 in Remark 5).
- 3. Compute the standard error as

$$\widehat{se}_{\beta_0}(\hat{\beta}) = \frac{\sqrt{\sum_{s=1}^{S} \widehat{\mathcal{X}}_s^2 \hat{R}_{\beta_0,s}^2}}{\sum_{i=1}^{N} \ddot{X}_i^2}, \qquad \widehat{R}_{\beta_0,s} = \sum_{i=1}^{N} w_{is} \hat{\epsilon}_{\beta_0,i}.$$
(30)

4. Reject the null if  $|(\hat{\beta} - \beta_0)/\widehat{se}_{\beta_0}(\hat{\beta})| > z_{1-\alpha/2}$ . A confidence set with coverage  $1 - \alpha$  is given by all nulls that are not rejected,  $CI_{1-\alpha} = \{\beta_0 : |(\hat{\beta} - \beta_0)/\widehat{se}_{\beta_0}(\hat{\beta})| < z_{1-\alpha/2}\}$ . This set is an interval with endpoints given by

$$\hat{\beta} - A \pm \sqrt{A^2 + \frac{\widehat{se}(\hat{\beta})^2}{Q/(\ddot{X}'\ddot{X})^2}}, \qquad A = \frac{\sum_{s=1}^{S} \widehat{\mathcal{X}}_s^2 \hat{R}_s \sum_{i=1}^{N} w_{is} \ddot{X}_i}{Q}, \tag{31}$$

where  $Q = (\ddot{X}'\ddot{X})^2/z_{1-\alpha/2}^2 - \sum_{s=1}^S \widehat{\mathcal{X}}_s^2(\sum_i w_{is}\ddot{X}_i)^2$  and  $\widehat{se}(\hat{\beta})$  and  $\hat{R}_s$  are given in eq. (29).

<sup>&</sup>lt;sup>14</sup>Alternatively, one could construct a bias-corrected variance estimate; see, for example, Bell and McCaffrey (2002) for an example of this approach in the context of cluster-robust inference.

**Proposition 5.** Suppose that the assumptions of Proposition 4 hold, and that  $\beta_{is} = \beta$ . Suppose also that  $N \ge S$ , W is full rank, and that either  $\max_s \sum_{i=1}^N |((W'W)^{-1}W')_{si}|$  is bounded and  $\max_i E[(U'_i\gamma)^4 \mid W] \to 0$ , or else that  $U_i = 0$  for i = 1, ..., N. Define  $\widehat{\mathcal{X}}$  as in eq. (28), and let  $\widehat{R}_s = \sum_{i=1}^N w_{is}\widetilde{\epsilon}_i$ , where  $\widetilde{\epsilon}_i = Y_i - X_i\widetilde{\beta} - Z'_i\widetilde{\delta}$ , and  $\widetilde{\beta}$  are consistent estimators of  $\delta$  and  $\beta$ . Then

$$\frac{\sum_{s=1}^{S} \widehat{\mathcal{X}}_{s}^{2} \widehat{R}_{s}^{2}}{\sum_{s=1}^{S} n_{s}^{2}} = \mathcal{V}_{N} + o_{p}(1).$$
(32)

Since in both  $\hat{e}_i$  and  $\hat{e}_{\beta_0,i}$  are consistent estimates of the residuals, this proposition shows that the procedures in Remarks 5 and 6 both yield asymptotically valid confidence intervals. The additional assumptions of Proposition 5 ensure that the estimation error in  $\hat{\mathcal{X}}_s$  that arises from having to back out the sector-level shocks  $\mathcal{Z}_s$  from the controls  $Z_i$  is not too large. If the sectors are concentrated, then  $((W'W)^{-1}W')_{si} = \mathbb{I}\{s(i) = s\}/n_s$ , so that  $\max_s \sum_{i=1}^N |((W'W)^{-1}W')_{si}| = 1$ , and the assumption always holds. We show in Online Appendix A.1.6 that the procedures in Remarks 5 and 6 continue to yield valid inference if  $\beta_{is}$  is heterogeneous across regions and sectors, as long as further regularity conditions hold.

Although both standard errors  $\widehat{se}_{\beta_0}(\hat{\beta})$  and  $\widehat{se}(\hat{\beta})$  are consistent (and one could further show that the resulting confidence intervals are asymptotically equivalent), they will in general differ in finite samples. In particular, it can be seen from eq. (31) that the confidence interval with the null imposed is not symmetric around  $\hat{\beta}$ , but its center is shifted by  $A.^{15}$  As we show in Section 6, this recentering tends to improve the finite-sample coverage properties of the confidence interval. On the other hand, the confidence interval described in Remark 6 tends to be longer on average than that in Remark 5.

### 4.3 Instrumental variables regression

We now turn to the problem of estimating the effect of a regional treatment variable  $Y_{2i}$  on a regional outcome  $Y_{1i}$  using the shift-share variable  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$  as an instrumental variable (IV). To set up the problem precisely, we again use the potential outcome framework. In particular, we assume that

$$Y_{1i}(y_2) = Y_{1i}(0) + y_2\alpha, (33)$$

where  $\alpha$ , our parameter of interest, measures the causal effect of  $Y_{2i}$  onto  $Y_{1i}$ . We assume for simplicity that this causal effect is linear and constant across regions. In analogy with eq. (11), we denote the region-i treatment level that would occur if the region received shocks  $(x_1, \ldots, x_S)$  as

$$Y_{2i}(x_1,\ldots,x_S) = Y_{2i}(0) + \sum_{s=1}^{S} w_{is} x_s \beta_{is}.$$
 (34)

<sup>&</sup>lt;sup>15</sup>This is analogous to the differences in likelihood models between confidence intervals based on the Lagrange multiplier test (which imposes the null and is not symmetric around the maximum likelihood estimate) and the Wald test (which does not impose the null and yields the usual confidence interval).

<sup>&</sup>lt;sup>16</sup>If we weaken the assumption of constant treatment effects and instead assume  $Y_{1i}(y_2) = Y_{1i}(0) + y_2\alpha_i$ , then it follows by a mild extension of the results in Online Appendix A.2 that our methods would deliver inference on the estimand  $\sum_{i=1}^{N} \pi_i \alpha_i / \sum_{i=1}^{N} \pi_i$ , with  $\pi_i = \sum_{s=1}^{S} w_{is}^2 \operatorname{var}(\mathcal{X}_s \mid \mathcal{F}_0) \beta_{is}$ , where  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, \alpha, W)$ , and  $\beta_{is}$  is defined in eq. (34).

The observed outcome and treatment variables are given by  $Y_{1i} = Y_{1i}(Y_{2i})$  and  $Y_{2i} = Y_{2i}(\mathcal{X}_1, \dots, \mathcal{X}_S)$ , respectively.

The framework in eqs. (33) and (34) maps directly to the problem of estimating the regional inverse labor supply elasticity. In particular, in the context of the model in Section 3, eqs. (8) and (10) map directly into eqs. (33) and (34) if we define

$$Y_{1i} = \hat{\omega}_i, \ Y_{2i} = \hat{L}_i, \ \alpha = \tilde{\phi}, \ Y_{1i}(0) = -\tilde{\phi}(\sum_{g=1}^G \tilde{w}_{ig}\hat{v}_g + \hat{v}_i), \ w_{is} = l_{is}^0, \ \mathcal{X}_s = \hat{\chi}_s, \ \beta_{is} = \theta_{is},$$
 (35)

and  $Y_{2i}(0)$  is given by the expression for  $Y_i(0)$  in eq. (12).<sup>17</sup> As this mapping illustrates, the potential outcome  $Y_{1i}(0)$  will generally have a shift-share structure, with the shifters being group-specific labor supply shocks (e.g. growth in the number of workers by education group). Consequently, the regression residual in the structural equation will generally have a shift-share structure. Similarly, as eq. (12) illustrates, the potential outcome  $Y_{2i}(0)$  will also generally include several shift-share components, with the shifters being either sector-specific labor demand shocks or the same group-specific labor supply shocks appearing in  $Y_{1i}(0)$ . Thus, the regression residual in the first-stage regression of  $Y_{2i}$  onto  $X_i$  will also generally have a shift-share structure.

Our estimate of  $\alpha$  is given by an IV regression of  $Y_{1i}$  onto  $Y_{2i}$  and a K-vector of controls  $Z_i$ , with  $X_i$  used as an instrument for  $Y_{2i}$ . This IV estimate can be written as

$$\hat{\alpha} = \frac{\sum_{i=1}^{N} \ddot{X}_{i} Y_{1i}}{\sum_{i=1}^{N} \ddot{X}_{i} Y_{2i}},$$
(36)

where, as in Section 4.2,  $\ddot{X}_i$  denotes the residual from regressing  $X_i$  onto  $Z_i$ .

#### **Assumptions**

Assumption 4 is a generalization of Assumption 3. Let  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W)$ .

**Assumption 4** (IV Identification). (i) The observed outcome and treatment variables satisfy  $Y_{1i} = Y_{1i}(Y_{2i})$  and  $Y_{2i} = Y_{2i}(X_1, ..., X_s)$  such that eqs. (33) and (34) hold, and the controls  $Z_i$  satisfy eq. (24); (ii) The shifters are exogenous in the sense that, for every s,

$$E[\mathcal{X}_s \mid \mathcal{G}_0] = E[\mathcal{X}_s \mid \mathcal{Z}_s],\tag{37}$$

and the right-hand side satisfies eq. (26); (iii) Assumptions 3(iii) and 3(iv) hold; (iv)  $\sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}^2 \cdot \text{var}(\mathcal{X}_s \mid \mathcal{F}_0) \beta_{is} \neq 0$ .

Assumption 4(ii) adapts the standard instrument exogeneity condition (see, e.g., Condition 1 in Imbens and Angrist, 1994) to our setting. Our approach follows Borusyak, Hull and Jaravel (2018), who impose a similar identification condition. To illustrate the restrictions that Assumption 4(ii) may impose, consider again the problem of estimating the inverse labor supply elasticity within the

<sup>&</sup>lt;sup>17</sup>In some applications of shift-share IVs, the shifters  $\{\mathcal{X}_s\}_{s=1}^S$  are unobserved and have to be estimated. We assume here that  $\mathcal{X}_s$  is directly measurable for every sector s, and study the case with estimated shifters in Section 5.3.

context of the model in Section 3, with the mapping between this model and the potential outcomes in eqs. (33) and (34) given in eqs. (12) and (35). If the controls  $\{\mathcal{Z}_s\}_{s=1}^S$  correspond to the shocks  $\{\hat{\mu}_s\}_{s=1}^S$ , then eq. (37) requires that, conditional on  $\{\hat{\mu}_s\}_{s=1}^S$ , the labor demand shocks  $\{\hat{\chi}_s\}_{s=1}^S$  used to construct our IV are mean-independent of the idiosyncratic labor demand shocks  $\{\hat{\eta}_{is}\}_{i=1,s=1}^{N,S}$  and of the labor supply shifters  $\{\hat{v}_i\}_{i=1}^N$  and  $\{\hat{v}_g\}_{g=1}^G$ . For example, if  $\{\hat{\chi}_s\}_{s=1}^S$  are sectoral productivity shocks, then these productivity shocks need to be independent of shocks to individuals' willingness to work in different groups and regions. Assumption 4(iv) requires that the coefficient on the instrument in the first-stage equation, which can be written as  $\beta = \sum_{i=1}^N \sum_{s=1}^S w_{is}^2 \operatorname{var}(\mathcal{X}_s \mid \mathcal{F}_0) \beta_{is} / \sum_{i=1}^N \sum_{s=1}^S w_{is}^2 \operatorname{var}(\mathcal{X}_s \mid \mathcal{F}_0)$ , is non-zero—this is the standard IV relevance assumption. For consistency and inference, in an analogy to the OLS case, we assume that Assumption 2 holds with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W)$ .

In a recent paper, Goldsmith-Pinkham, Sorkin and Swift (2018) explore a different approach to identification and inference on the treatment effect  $\alpha$ . Focusing here for simplicity on the case without controls, in place of Assumption 4(ii), they assume that the shares  $\{w_{is}\}_{s=1}^{S}$  are as good as randomly assigned conditional on the shifters  $\{\mathcal{X}_s\}_{s=1}^{S}$ ; so that they are mean-independent of the potential outcomes  $Y_1(0)$  and  $Y_2(0)$  conditional on  $\mathcal{X}$ . As Goldsmith-Pinkham, Sorkin and Swift (2018) show, under this alternative assumption, one can replace the shift-share instrument  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$  by the full vector of shares  $(w_{i1}, \ldots, w_{iS})$  in the first-stage equation. For estimation and inference, this alternative approach requires that, conditionally on the shifters, either the shares  $(w_{i1}, \ldots, w_{iS})$  or else the structural residuals be independent across regions or clusters of regions.

For estimating the inverse labor supply elasticity in the context of the model in Section 3, eq. (35) illustrates that this alternative identification assumption requires that, conditional on  $\{\hat{\chi}_s\}_{s=1}^S$ , the region-specific employment shares in the initial equilibrium  $\{l_{is}^0\}_{s=1}^S$  are mean-independent of both the region-specific exposure shares  $\{\tilde{w}_{ig}\}_{g=1}^G$ , and the region-specific labor supply shock  $v_i$ . This assumption is violated if regions more exposed to labor demand shocks in a sector s (e.g. to changes in tariffs in the food sector) are also more exposed to labor supply shocks affecting workers of a group g (e.g. currency crisis in Mexico affecting the number of Mexican migrants; see Monras, 2018). <sup>19</sup>

In terms of inference, since the structural residuals will not be independent across regions unless they contain no shift-share component (which, according to the economic model in Section 3, is unlikely), the approach in Goldsmith-Pinkham, Sorkin and Swift (2018) generally requires that the shares are independent across (clusters of) regions. This assumption is, from the perspective of the model in Section 3, conceptually very different from assuming independence of the shifters  $\mathcal{X}_s$  across

 $<sup>^{18}</sup>$ If, instead of eq. (34), we defined the first stage as simply the projection of  $Y_{2i}$  onto the shift-share instrument, we could further relax this condition and only require  $\{\hat{\chi}_s\}_{s=1}^S$  to be mean-independent of the labor supply shifters. An advantage of the current setup is that it allows us to derive primitive conditions for the consistency of the estimates of the first-stage regression and, thus, of the IV estimator.

<sup>&</sup>lt;sup>19</sup>To allow for a shift-share component in the structural residual, Goldsmith-Pinkham, Sorkin and Swift (2018) view the shares  $(w_{i1}, \ldots, w_{iS})$  as "invalid" instruments, since, in this case,  $E[\epsilon_i w_{is} \mid \mathcal{X}] \neq 0$ , where  $\epsilon_i$  denotes the structural error. Goldsmith-Pinkham, Sorkin and Swift (2018) show that if these shares are used to construct a single shift-share instrument  $X_i$ , the bias in the IV estimator coming from the correlation between any  $w_{is}$  and the structural residual averages out under certain conditions as  $S \to \infty$ , as in the many invalid instrument setting studied in Kolesár et al. (2015). Under the current setup, in contrast, eq. (37) implies that  $X_i$  is a valid instrument for any fixed S. Leveraging exogeneity of  $\mathcal{X}_s$  is a key difference between our approach and that in Kolesár et al. (2015) and Goldsmith-Pinkham, Sorkin and Swift (2018). It allows us to do inference without imposing a particular correlation structure on the residuals  $\epsilon_i$ , and it allows us to achieve identification without requiring  $S \to \infty$ ; the latter is only needed for consistency and inference.

sectors. Since the shifters  $\mathcal{X}_s = \hat{\chi}_s$  are exogenous, the latter only involves assumptions on model fundamentals by restricting the distribution in eq. (7). In contrast, each share  $w_{is} = l_{is}^0$  corresponds to the employment allocation across sectors in a region i in an initial equilibrium, so that the former involves imposing restrictions on an endogenous outcome of the model. Furthermore, since all the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$  depend on the same set of sector-specific labor demand shifters  $\{(\chi_s, \mu_s)\}_{s=1}^S$ , they will generally be correlated across regions.<sup>20</sup>

Which identification and inference approach is more attractive depends on the context of each particular empirical application. While the economic model in Section 3 motivates the approach we pursue here, this does not mean that our approach is generally more attractive. In other empirical applications (e.g. when the shares are exogenous variables from the perspective of an economic framework), the approach of Goldsmith-Pinkham, Sorkin and Swift (2018) may be more appropriate.

#### Asymptotic theory

It follows by adapting the arguments in the proof of Proposition 4 that, if Assumption 4 holds, and Assumption 2 holds with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W)$ , then, under mild technical regularity conditions (see Online Appendix A.2 for details and proof),

$$\frac{N}{\sqrt{\sum_{s=1}^{S} n_s^2}} (\hat{\alpha} - \alpha) = N \left( 0, \frac{V_N}{(\frac{1}{N} \sum_{i=1}^{N} \ddot{X}_i Y_{2i})^2} \right) + o_p(1), \ V_N = \frac{\sum_{s=1}^{S} \text{var}(\tilde{X}_s \mid \mathcal{G}_0) R_s^2}{\sum_{s=1}^{S} n_s^2}, \ R_s = \sum_{i=1}^{N} w_{is} \epsilon_i,$$
(38)

where  $\epsilon_i = Y_{1i} - Y_{2i}\alpha - Z_i'\delta$  is the residual in the structural equation, with  $\delta = E[Z'Z]^{-1}E[Z'(Y_1 - Y_2\alpha)]$ . This suggests the standard error estimate

$$\widehat{se}(\hat{\alpha}) = \frac{\sqrt{\sum_{s=1}^{S} \widehat{\mathcal{X}}_{s}^{2} \hat{R}_{s}^{2}}}{|\sum_{i=1}^{N} \ddot{X}_{i} Y_{2i}|} = \frac{\sqrt{\sum_{s=1}^{S} \widehat{\mathcal{X}}_{s}^{2} \hat{R}_{s}^{2}}}{\sum_{i=1}^{N} \ddot{X}_{i}^{2} |\hat{\beta}|}, \qquad \widehat{R}_{s} = \sum_{i=1}^{N} w_{is} \hat{\epsilon}_{i},$$
(39)

where  $\widehat{\mathcal{X}}_s$  is constructed as in Remark 5,  $\widehat{\epsilon} = Y_1 - Y_2 \widehat{\alpha} - Z'(Z'Z)^{-1} Z'(Y_1 - Y_2 \widehat{\alpha})$  is the estimated residual of the structural equation, and  $\widehat{\beta} = \sum_{i=1}^N \ddot{X}_i Y_{2i} / \sum_{i=1}^N \ddot{X}_i^2$  is the first-stage coefficient.

The difference between the IV standard error formula in eq. (39) and the OLS version in eq. (29) is analogous to the difference between IV standard errors and OLS heteroskedasticity-robust standard errors for the corresponding reduced-form specification: the residual  $\hat{\epsilon}_i$  corresponds to the residual in the structural equation, and the denominator is scaled by the first-stage coefficient. To obtain the IV analog of the standard error estimator under the null  $H_0$ :  $\alpha = \alpha_0$ , we use the formula in eq. (39) except that, instead of  $\hat{\epsilon}_i$ , we use the structural residual computed under the null,  $\hat{\epsilon}_{\alpha_0} = (I - Z'(Z'Z)^{-1}Z')(Y_1 - Y_2\alpha_0)$ . The resulting confidence interval is a generalization of the Anderson and Rubin (1949) confidence interval (which assumes that the structural errors are independent). For this reason, this confidence interval will remain valid even if the shift-share instrument is weak.

The specific properties  $T_{is}^{20}$  For instance, if  $\sigma_s = \sigma$  for all s, then  $I_{is}^0 = D_{is}^0/(\sum_{t=1}^S D_{it}^0)$ , where  $D_{is}^0$  is the labor demand shifter of sector s in region i in the initial equilibrium. According to eq. (3), for any s, all shifters  $\{D_{is}^0\}_{i=1}^N$  depend on the same sector-level demand shocks,  $\{(\chi_s, \mu_s)\}_{s=1}^S$  and, thus, the labor shares  $I_{is}^0$  will generally be correlated across all regions for any given sector.

## 5 Extensions

We now discuss three extensions to the basic setup. In Section 5.1, we relax the assumption that the shifters  $\{X_s\}_{s=1}^S$  are independent, allowing them to be correlated within clusters of sectors. Section 5.2 generalizes our results to settings in which we have multiple observations for each region. Section 5.3 considers the case in which the shifters are not directly observed, and have to be estimated.

#### 5.1 Clusters of sectors

Suppose that the sectors can be grouped into larger units, which we refer to as "clusters", with  $c(s) \in \{1, ..., C\}$  denoting the cluster that sector s belongs to; e.g., if each s corresponds to a four-digit industry code, c(s) may correspond to a three-digit code. With this structure, we replace Assumption 2(i) with the weaker assumption that, conditional on  $\mathcal{F}_0$ , the shocks  $\mathcal{X}_s$  and  $\mathcal{X}_k$  are independent if  $c(s) \neq c(k)$ , and we replace Assumption 2(iii) with the assumption that, as  $C \to \infty$ , the largest cluster makes an asymptotically negligible contribution to the asymptotic variance; i.e.  $\max_c \tilde{n}_c^2 / \sum_{d=1}^C \tilde{n}_d^2 \to 0$ , where  $\tilde{n}_c = \sum_{s=1}^S \mathbb{I}\{c(s) = c\}n_s$  is the total share of cluster c.

Under this setup, by generalizing the arguments in Section 4.2, one can show that, as  $C \to \infty$ ,

$$\frac{N}{\sqrt{\sum_{c=1}^{C} \tilde{n}_c^2}} (\hat{\beta} - \beta) = n \left( 0, \frac{\mathcal{V}_N}{\left( \frac{1}{N} \sum_{i=1}^{N} \ddot{X}_i^2 \right)^2} \right) + o_p(1),$$

and, assuming that  $\beta_{is} = \beta$  for every region and sector, the term  $V_N$  is now given by

$$V_N = \frac{\sum_{c=1}^C \sum_{s=1,t=1}^{S,S} \mathbb{I}\{c(s) = c(t) = c\} E[\tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t \mid W, \mathcal{Z}] R_s R_t}{\sum_{c=1}^C \tilde{n}_c^2}, \qquad R_s = \sum_{i=1}^N w_{is} \epsilon_i, \quad \tilde{\mathcal{X}}_s = \mathcal{X}_s - \mathcal{Z}_s' \gamma.$$

As a result, we replace the standard error estimate in eq. (29) with a version that clusters  $\widehat{\mathcal{X}}_s \hat{R}_s$ ,

$$\widehat{se}(\hat{\beta}) = \frac{\sqrt{\sum_{c=1}^{C} \sum_{s,t} \mathbb{I}\{c(s) = c(t) = c\} \widehat{\mathcal{X}}_s \widehat{R}_s \widehat{\mathcal{X}}_t \widehat{R}_t}}{\sum_{i=1}^{N} \ddot{X}_i^2}, \qquad \widehat{R}_s = \sum_{i=1}^{N} w_{is} \widehat{e}_i, \tag{40}$$

where  $\widehat{\mathcal{X}}_s$  is defined as in Remark 5. Confidence intervals with the null imposed can be constructed as in Remark 6, replacing  $\hat{e}_i$  with  $\hat{e}_{\beta_0,i}$  in eq. (40). In the IV setting considered in Section 4.3, the standard error for  $\hat{a}$  is analogous to that in eq. (40), except that  $\hat{e}_i$  denotes the residual in the structural equation, and we divide the expression by the absolute value of the first-stage coefficient,  $\sum_{i=1}^N \ddot{X}_i Y_{2i} / \sum_{i=1}^N \ddot{X}_i^2$ .

#### 5.2 Panel data

Consider a setting with j = 1, ..., J regions, k = 1, ..., K sectors, and t = 1, ..., T periods. For each period t, we have data on shifters  $\{X_{kt}\}_{k=1}^{K}$ , outcomes  $\{Y_{jt}\}_{j=1}^{J}$ , and shares  $\{w_{jkt}\}_{j=1,k=1}^{J,K}$ . This setup maps into the potential outcome framework in eq. (11) if we identify a "sector" with a sector-period pair s = (k, t), and a "region" with a region-period pair i = (j, t), so that we can index outcomes and

shifters as  $Y_i = Y_{jt}$  and  $\mathcal{X}_s = \mathcal{X}_{kt}$ , with the shares given by

$$w_{is} = \begin{cases} w_{jkt} & \text{if } i = (j, t) \text{ and } s = (k, t), \\ 0 & \text{if } i = (j, t), s = (k, t'), \text{ and } t \neq t'. \end{cases}$$
(41)

If the shifters  $\mathcal{X}_{kt}$  are independent across time and sectors, Propositions 3 and 4 immediately give the large-sample distribution of the OLS estimator. In general, however, it will be important to allow the shifters  $\mathcal{X}_{kt}$  to be correlated across time within each sector k. In this case, one can use the clustered standard error derived in Section 5.1 by grouping observations over time for each sector k into a common cluster, so that c(k,t) = c(k',t') if k = k'. We can then apply the formula in eq. (40) to allow for any arbitrary time-series correlation in the sector-level shocks  $\mathcal{X}_{kt}$  for any given sector k. Regardless of whether the sector-period pairs (k,t) are clustered, as discussed in Remark 4, our standard error formulas allow for arbitrary dependence patterns in the regression residuals—in particular, they account for potential serial dependence in the regression residuals.

If the shift-share regressor is used as an IV in a regression of an outcome  $Y_{1jt}$  onto a treatment  $Y_{2jt}$ , the mapping to eqs. (33) and (34) is analogous, and one can use an IV version of the formula in eq. (40) for inference.

#### 5.3 IV with estimated shifters

We now consider a setting in which the sectoral shifters  $\{\mathcal{X}_s\}_{s=1}^S$  that define the shift-share IV studied in Section 4.3 are not directly observed. We follow the setup in Section 4.3 but assume that, instead of observing  $\mathcal{X}_s$  directly, we only observe a noisy measure of it,

$$X_{is} = \mathcal{X}_s + \psi_{is} \tag{42}$$

for each sector-region pair. We consider IV regressions that use two different estimates of  $X_i = \sum_{s=1}^{S} w_{is} \mathcal{X}_s$ . First, an estimate that replaces  $\mathcal{X}_s$  with an estimate  $\hat{\mathcal{X}}_s = \sum_{i=1}^{N} \check{w}_{is} X_{is} / \check{n}_s$ , where  $\check{n}_s = \sum_{i=1}^{N} \check{w}_{is}$  and the weights  $\check{w}_{is}$  are not necessarily related to  $w_{is}$ . The resulting estimate of  $X_i$  is

$$\hat{X}_{i} = \sum_{s=1}^{S} w_{is} \hat{\mathcal{X}}_{s} = \sum_{s=1}^{S} w_{is} \frac{1}{\check{n}_{s}} \sum_{i=1}^{N} \check{w}_{js} X_{js}, \tag{43}$$

and it yields the IV estimate  $\tilde{\alpha} = \hat{X}'Y_1/\hat{X}'Y_2$ , where  $\hat{X} = \hat{X} - Z(Z'Z)^{-1}Z'\hat{X}$  is the residual from regressing  $\hat{X}_i$  onto  $Z_i$ . Second, we consider the leave-one-out estimator

$$\hat{X}_{i,-} = \sum_{s=1}^{S} w_{is} \hat{\mathcal{X}}_{s,-i} = \sum_{s=1}^{S} w_{is} \frac{1}{\check{n}_{s,-i}} \sum_{j=1}^{N} \mathbb{I}\{j \neq i\} \check{w}_{js} X_{js}, \qquad \check{n}_{s,-i} = \sum_{j=1}^{N} \mathbb{I}\{j \neq i\} \check{w}_{js}, \tag{44}$$

where  $\hat{\mathcal{X}}_{s,-i} = \sum_{j=1}^{N} \mathbb{I}\{j \neq i\} \check{w}_{js} X_{js} / \check{n}_{s,-i}$  is an estimate of  $\mathcal{X}_s$  that excludes region i. A version of this estimator has been used in Autor and Duggan (2003). This leave-one-out estimator of the shift-share instrument  $X_i$  yields the IV estimate  $\hat{a}_- = \ddot{X}'_- Y_1 / \ddot{X}'_- Y_2$ , where  $\ddot{X}_- = \hat{X}_- - Z'(Z'Z)^{-1} Z' \hat{X}_-$ .

While we assume that  $\mathcal{X}_s$  satisfies the exogeneity restriction in Assumption 4(ii) for every s, we

allow the measurement errors  $\psi_i = (\psi_{i1}, \dots, \psi_{iS})'$  to be potentially correlated with the potential outcomes  $Y_{1i}(0)$  and  $Y_{2i}(0)$  in the same region i. We assume, however, that  $\psi_i$  is independent of the errors  $\psi_j$  and of the potential outcomes  $Y_{1j}(0)$  and  $Y_{2j}(0)$  for any region  $j \neq i$  (see Online Appendix A.2 for a formal statement). In Online Appendix E.2.3, we use the model in Section 3 to discuss these assumptions in the context of estimating the inverse labor supply elasticity.<sup>21</sup>

The potential correlation between  $\psi_i$  and the potential outcomes in region i implies that the estimation error in  $\hat{X}_i$ , which is a function on  $\psi_i$ , may be correlated with the residual in the structural equation. Thus, including the ith observation in the construction of  $\hat{X}_i$  induces an own-observation bias in the IV estimator  $\tilde{\alpha}$  of  $\alpha$ . See Goldsmith-Pinkham, Sorkin and Swift (2018) and Borusyak, Hull and Jaravel (2018) for a discussion. This bias is analogous to the bias of the two-stage least squares estimator in settings with many instruments (e.g. Bekker, 1994; Angrist, Imbens and Krueger, 1999), such as when one uses group indicators as instruments.<sup>22</sup> We show in Online Appendix A.2 that the magnitude of the bias is of the order  $\frac{1}{N}\sum_{i=1}^{N}\sum_{s=1}^{S}\frac{w_{is}\hat{w}_{is}}{\hat{n}_s} \leq S/N$ , so that consistency of  $\tilde{\alpha}$  generally requires the number of sectors to grow more slowly than the number of regions. Furthermore, to ensure that the asymptotic bias in  $\tilde{\alpha}$  does not induce undercoverage of the resulting confidence intervals, one generally requires  $S^{3/2}/N \to 0$ .

The estimator  $\hat{\alpha}_{-}$ , which can be thought of as a shift-share analog of the jackknife IV estimator studied in Angrist, Imbens and Krueger (1999), remains consistent, as shown in Borusyak, Hull and Jaravel (2018) and in Online Appendix A.2. We also show in this appendix that, under regularity conditions, its asymptotic distribution is given by

$$\frac{N}{\sqrt{\sum_{s=1}^{S} n_s^2}} (\hat{\alpha}_- - \alpha) = N \left( 0, \frac{V_N + W_N}{\left( \frac{1}{N} \sum_{i=1}^{N} \ddot{X}_i Y_{2i} \right)^2} \right) + o_p(1), \tag{45}$$

with  $V_N$  defined as in eq. (38), and

$$W_{N} = \frac{1}{\sum_{s=1}^{S} n_{s}^{2}} (\sum_{j=1}^{N} (\sum_{i=1}^{N} S_{ij})^{2} + \sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij} S_{ji}), \qquad S_{ij} = \sum_{s=1}^{S} \mathbb{I}\{i \neq j\} \frac{w_{is} \check{w}_{js} \psi_{js} \epsilon_{i}}{\check{n}_{s,-i}}.$$

The term  $W_N$  accounts for the additional uncertainty stemming from the fact that the shift-share IV is estimated. It is analogous to the many-instrument term in the jackknife IV estimator under many instrument asymptotics (see Chao et al., 2012). Using simulations, we show in Online Appendix E.2.4 several designs in which, while correcting for the own-observation bias by using  $\hat{\alpha}_-$  instead of  $\tilde{\alpha}$  is quantitatively important, accounting for the additional variance term  $W_N$  is less important.

<sup>&</sup>lt;sup>21</sup>Specifically, we show in Online Appendix E.2.3 that, if  $X_{is}$  corresponds to employment growth rates, then  $\psi_i$  will generally not be independent of  $(\psi_j, Y_{1j}(0), Y_{2j}(0))$  in others regions  $j \neq i$ , unless one makes restrictive assumptions about the demand elasticities  $\sigma_s$ , such as  $\sigma_s = 0$ . We also construct alternative shift-share IVs that satisfy this independence assumption under weaker restrictions on  $\sigma_s$ , but require adjusting the shifter used in estimation.

<sup>&</sup>lt;sup>22</sup>See, e.g., Maestas, Mullen and Strand (2013); Dobbie and Song (2015); Aizer and Doyle (2015), or Silver (2016).

## 6 Performance of new methods: placebo evidence

In Section 6.1, we revisit the placebo exercise in Section 2 to examine the finite-sample properties of the inference procedures described in Remarks 5 and 6. In Section 6.2, we show that our baseline placebo results are robust to several changes in the placebo design.

## 6.1 Baseline specification

We first consider the performance of the standard error estimator in eq. (29) (which we label AKM), and the standard error and confidence interval in eqs. (30) and (31) (with label AKM0) in the baseline placebo design described in Section 2.<sup>23,24</sup>

For the *AKM* and *AKM0* inference procedures, Table 2 presents median standard error estimates and rejection rates for 5% significance level tests of the null hypothesis  $H_0$ :  $\beta = 0$ . In the case of *AKM0*, since the standard error depends on the null being tested, the table reports the median "effective standard error", defined as the length of the 95% confidence interval divided by  $2 \times 1.96$ .

The results in Table 2 show that the inference procedures introduced in Section 4 perform well. The median AKM standard error is slightly lower than the standard deviation of  $\hat{\beta}$ , by about 5% on average across all outcomes. The median AKM0 effective standard error is slightly larger than the standard deviation of  $\hat{\beta}$ , by about 11% on average. The implied rejection rates are close to the 5% nominal rate: the AKM procedure has rejection rates between 7.5% and 9.1% and the AKM0 rejection rates are always between 4.3% and 4.5%. As discussed in Section 4.2, the AKM and AKM0 confidence intervals are asymptotically equivalent. The differences in rejection rates between the AKM and AKM0 inference procedures are thus due to differences in finite-sample performance. As noted in other contexts (see, e.g., Lazarus et al., 2018), imposing the null can lead to improved finite-sample size control. The better size control of the AKM0 procedure is consistent with these results.

## 6.2 Alternative placebo specifications

In Section 4, we show theoretically that the *AKM* and *AKM0* inference procedures are valid in large samples only if: (a) the number of sectors goes to infinity; (b) all sectors are asymptotically "small"; (c) the sectoral shocks are independent across sectors. Given these conditions, these inference procedures remain valid under (d) any distribution of the sectoral shifters; and (e) arbitrary correlation structure of the regression residuals. In this section, we evaluate the sensitivity of these inference procedures to requirements (a) to (c) above, and illustrate points (d) and (e) by documenting the robustness of these procedures to alternative distributions of the shifters and the residuals. In all cases, we also report *Robust* and *Cluster* standard errors estimates and rejection rates. We focus on the change in the share of working-age population employed as the outcome variable of interest.

We first evaluate how the performance of different inference procedures depends on the number of sectors. Panel A of Table 3 shows that the overrejection problem affecting standard inference

<sup>&</sup>lt;sup>23</sup>We fix the matrix Z to be a column of ones when implementing the formulas in eqs. (29) and (31).

<sup>&</sup>lt;sup>24</sup>In Online Appendix D.8, we explore the sensitivity of our results to using counties (instead of CZs) as the regional unit of analysis, and occupations (instead of sectors) as the unit at which the shifter is defined.

Table 2: Median standard errors and rejection rates for  $H_0$ :  $\beta = 0$  at 5% significance level.

	Est	imate	Media	n eff. s.e.	Rejection rate				
	Mean (1)	Std. dev (2)	AKM (3)	AKM0 (4)	AKM (5)	AKM0 (6)			
Panel A: Change in the share of working-age population									
Employed	-0.01	2.00	1.90	2.21	7.8%	4.5%			
Employed in manufacturing	-0.01	1.88	1.77	2.06	8.0%	4.3%			
Employed in non-manufacturing	0.00	0.94	0.89	1.04	8.2%	4.5%			
Panel B: Change in average log w	veekly w	vage							
Employed	-0.03	2.66	2.57	2.99	7.5%	4.3%			
Employed in manufacturing	-0.03	2.92	2.74	3.18	9.1%	4.5%			
Employed in non-manufacturing	-0.02	2.64	2.55	2.96	7.8%	4.5%			

Notes: For the outcome variable indicated in the leftmost column, this table indicates the mean and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) and (4)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (5) and (6)). *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. The median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples.

procedures worsens when the number of sectors decreases: the rejection rates of 5% significance level tests based on *Robust* and *Cluster* standard errors reach 70.6% and 56.1%, respectively, when we construct the shift-share covariate using 20 2-digit SIC sectors (instead of the 396 4-digit SIC sectors we use in the baseline placebo). In line with the findings of the literature on clustered standard errors with few clusters, the rejection rates of hypothesis tests that rely on *AKM* standard errors also increase to 12%, but rejection rates for hypothesis tests that apply the *AKM0* inference procedure remain very close to the nominal 5% significance level.

Panels B to D of Table 3 examine the robustness of the results in Tables 1 and 2 to alternative distributions of the shifters. In Panel B, as in our baseline placebo exercise, the shifters are drawn i.i.d. from a normal distribution, but we change the variance to both a lower ( $\sigma^2 = 0.5$ ) and a higher value ( $\sigma^2 = 10$ ) than in the baseline ( $\sigma^2 = 5$ ). In Panel C, we draw the shifters from a log-normal distribution re-centered to have mean zero and scaled to have the same variance as in the baseline. Panel D investigates the robustness of our results to heteroskedasticity in the sector-level shocks. We set variance of the shock in each sector s, to  $\sigma_s^2 = 5 + \lambda(n_s - S/N)$ . Thus, the cross-sectional average of the variance of the sector-level shocks is the same as in the baseline (which corresponds to setting  $\lambda = 0$ ), but this variance now varies across sectors. Comparison of the results in Panels B to D of Table 3 to those in Tables 1 and 2 suggests that our baseline results are not sensitive to specific details of the distribution of sector-level shifters. This is consistent with the claim (d) above.

Panels E and F of Table 3 explore the robustness of our baseline results to different patterns of correlation in the regression residuals. In the baseline placebo, since  $\beta = 0$ , the regression residuals inherit the correlation patterns in the outcome variable. Here, we modify these patterns by adding a random shock  $\eta_i^m$  in each placebo sample m to the outcome  $Y_i$ . Panel E explores the impact of increasing the correlation between the regression residuals of CZs that belong to the same state.

Table 3: Alternative number of sectors, shifter distributions and residuals' correlation patterns

	Est	imate		Median	eff. s.e.		Rejection rate					
	Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)	AKM (5)	AKM0 (6)	Robust (7)	Cluster (8)	AKM (9)	AKM0 (10)		
Panel A: Sensitiv	vity to t	he numbe	er of sect	ors								
2-digit ( $S = 20$ )	-0.01	3.19	0.65	0.96	2.84	6.06	70.6%	56.1%	12.0%	5.8%		
3-digit ( $S = 136$ )	0.00	2.25	0.73	0.94	2.18	2.72	54.2%	42.5%	7.5%	4.5%		
Panel B: Sensitivity to the variance of the shifters												
$\sigma^2 = 0.5$	-0.04	6.33	2.33	2.91	6.04	7.02	48.5%	38.0%	7.9%	4.5%		
$\sigma^2 = 10$	0.00	1.41	0.52	0.65	1.35	1.57	48.1%	37.8%	7.5%	4.5%		
Panel C: Log-normal shifters												
$\sigma^2 = 5$	0.27	2.26	0.86	1.05	2.17	3.7	44.6%	35.3%	7.7%	5.2%		
Panel D: Heteros	kedast	ic shifters										
$\lambda = 3$	-0.01	1.63	0.55	0.72	1.51	2.14	52.1%	40.1%	8.7%	4.0%		
$\lambda = 7$	0.01	1.38	0.44	0.58	1.23	2.01	53.7%	41.1%	9.5%	4.2%		
Panel E: Simulat	ed state	e-level sho	ocks in r	egressior	ı residu	ıal						
	0.00	2.11	0.86	1.11	1.99	2.32	42.8%	30.4%	7.9%	4.6%		
Panel F: Simulated 'large' sector shifter in regression residual												
	-0.01	2.01	0.74	0.92	1.90	2.21	48.4%	37.8%	7.9%	4.6%		
Panel G: Including a 'large' sector in shift-share regressor												
	-0.02	4.25	0.59	0.76	1.18	1.34	92.0%	89.6%	77.2%	76.3%		

Notes: All estimates in this table use the change in the share of the working-age population employed in each CZ as the outcome variable  $Y_i$  in eq. (1). This table indicates the mean and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples. This table presents results for placebo simulations that depart from the baseline; the results should thus be compared to those in Tables 1 and 2. In Panel A, we reduce the number of sectors relative to the baseline. In Panel B, we change the variance of the distribution from which all shifters are drawn. In Panel C, we assume that the distribution from which all shifters are drawn is log-normal (re-centered at zero) with variance equal to five. In Panel D, we allow the variance of the shock in each sector to be heteroskedastic,  $\sigma_s^2 = 5 + \lambda (n_s - S/N)$ . In Panel E, we simulate state-level shocks and include them in our regression residual. In Panels F and G, we simulate a shifter for the non-manufacturing sector and include it in our regression residual and in our shift-share regressor, respectively.

Specifically, we generate a random variable  $\tilde{\eta}_k^m$  for each state k and simulation m such that  $\tilde{\eta}_k^m \sim \mathcal{N}(0,6)$ . We then set  $\eta_i^m = \tilde{\eta}_{k(i)}^m$  where k(i) is the state of CZ i. Since we have now increased the relative importance of the correlation pattern accounted for by *Cluster* standard errors, the resulting overrejection decreases from 38.3% to 30.4%. In line with claim (e) above, the rejection rates of the AKM and AKM0 inference procedures are not affected. In Panel F, we evaluate the robustness of our results to adding a shock to the non-manufacturing sector that is included in the regression residual. Specifically, in each simulation m, we set  $\eta_i^m = (1 - \sum_{s=1}^S w_{is})\hat{\eta}_s^m$  with  $\hat{\eta}_s^m \sim \mathcal{N}(0,5)$ , where  $\sum_{s=1}^S w_{is}$  is the 1990 aggregate employment share of the 396 4-digit SIC manufacturing sectors included in the definition of the shift-share regressor of interest. The results in Panel F of Table 3 show that adding

this component to the regression residual does not affect the rejection rates.

Lastly, Panel G in Table 3 explores the consequences of adding the non-manufacturing sector to the shift-share regressor. In Panel F, the shock to the non-manufacturing sector is part of the regression residual; in Panel G, we use this shock, in combination with the shocks to all manufacturing sectors, to construct the shift-share regressor. Across CZs, the average initial employment share in the non-manufacturing sector is 77.5%; i.e.  $N^{-1}\sum_{i=1}^{N}(1-\sum_{s=1}^{S}w_{is})=77.5\%$ . Including such a large sector in the shift-share regressor violates Assumptions 2(ii) and 2(iii). As a result, the *AKM* and *AKM0* inference procedures overreject severely; standard inference procedures fare even worse, with rejection rates reaching up to 92%. The results in Panels F and G suggest that, provided that the shifters are independent across sectors, it is better to exclude large sectors from the shift-share regressor of interest, and thus let the shocks associated with them enter the regression residual. One should, however, bear in mind that, if  $\beta_{is}$  in eq. (11) varies across sectors, excluding large sectors from the shift-share regressor will change the estimand  $\beta$  (see Proposition 3).

In the placebo simulations described in Tables 1 to 3, we have drawn the shifters independently from a mean-zero distribution. In Table 4, we allow for non-zero correlation in the shifters within "clusters" of sectors.<sup>25</sup> Specifically, we report results from placebo exercises in which the shifters are drawn from the joint distribution  $(\mathcal{X}_1^m,\ldots,\mathcal{X}_S^m)\sim \mathcal{N}\left(0,\Sigma\right)$ , where  $\Sigma$  is an  $S\times S$  covariance matrix with elements  $\Sigma_{sk}=(1-\rho)\sigma\mathbb{I}\{s=k\}+\rho\sigma\mathbb{I}\{c(s)=c(k)\}$  and c(s) indicates the "cluster" that industry s belongs to. In panels A, B, and C, these clusters correspond to the 3-, 2-, and 1-digit SIC sector that the 4-digit SIC sector s belongs to, respectively.

Panel A of Table 4 shows that introducing correlation within 3-digit SIC sectors has a moderate effect on the rejection rates of both the traditional methods and versions of the *AKM* and *AKM0* methods that assume that the sectoral shocks are independent. Rejection rates close to 5% are obtained with versions of the *AKM* and *AKM0* inference procedures that cluster the shifters at a 2-digit SIC level (see Section 5.1). As shown in Panel B, the overrejection problem affecting both traditional inference procedures and versions of the *AKM* and *AKM0* procedures that assume independence of shifters is more severe when the shifters are correlated at the 2-digit level. However, the last two columns show that, in this case, the versions of *AKM* and *AKM0* that cluster the sectoral shocks at the 2-digit level achieve rejection rates close to the nominal level. Finally, Panel C shows that the overrejection problem is much more severe in the presence of high correlation in shifters within the two 1-digit aggregate sectors, and this problem is not solved by clustering at the 2-digit level.

The last panel in Table 4 illustrates the inferential problems that arise in empirical applications of shift-share designs when all shifters are correlated with each other. Such correlations also arise, for example, when all shifters are generated (at least in part) by a common shock with potentially heterogeneous effects across sectors. As simulations presented in Table E.2 in Online Appendix E.1.1 illustrate, if there is a common component affecting all shifters, it is important to first estimate this common component and to control for it in the shift-share regression of interest. Otherwise, hy-

 $<sup>^{25}</sup>$ In Online Appendix D.2, we study the impact of drawing the shifters from a distribution with non-zero mean. We show that, in line with the discussion in Section 4.2, it is important to control for the region-specific sum of shares  $\sum_{s=1}^{S} w_{is}$ .  $^{26}$ There is an extensive empirical literature documenting the importance of common factors driving changes in sector-specific variables such as sectoral industrial production, employment and value added (see, e.g., Altonji and Ham, 1990; Shea, 2002; Foerster, Sarte and Watson, 2011).

Table 4: Correlation in sectoral shocks

	Estimate		Median eff. s.e.						Rejection rate					
					Independent		2-digit SIC				Independent		2-digit SIC	
	Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)	AKM (5)	AKM0 (6)	AKM (7)	AKM0 (8)	Robust (9)	Cluster (10)	AKM (11)	AKM0 (12)	AKM (13)	AKM0 (14)
Panel A: Simulated shifters with correlation within 3-digit SIC sectors														
$\rho = 0.00$	-0.01	2.00	0.74	0.92	1.91	2.22	1.86	2.64	48.6%	37.8%	7.9%	4.6%	8.9%	4.7%
$\rho = 0.50$	0.02	2.14	0.77	1.07	2.03	2.16	2.24	2.87	49.2%	32.3%	6.4%	7.1%	4.4%	4.7%
$\rho = 1.00$	0.01	2.27	0.76	1.08	1.99	2.10	2.38	3.14	52.2%	35.0%	8.6%	9.7%	4.7%	4.8%
Panel B:	Simulat	ed shifters	with cor	relation w	ithin 2-	digit SIC	sectors							
$\rho = 0.00$	-0.01	1.99	0.73	0.92	1.90	2.22	1.86	2.65	48.2%	37.7%	7.6%	4.5%	8.8%	4.5%
$\rho = 0.50$	-0.01	2.73	0.73	1.13	1.82	1.89	2.92	4.07	62.3%	43.2%	20.5%	23.5%	5.8%	5.0%
$\rho = 1.00$	0.01	3.20	0.69	1.18	1.67	1.63	3.38	6.16	68.4%	48.1%	31.2%	35.7%	6.3%	5.2%
Panel C: Simulated shifters with correlation within 1-digit SIC sectors														
$ \rho = 0.00  \rho = 0.50  \rho = 1.00 $	0.01 0.02 0.42	1.98 4.95 59.63	0.73 0.74 0.71	0.92 1.41 1.74	1.90 1.88 1.86	2.21 1.59 1.02	1.85 2.42 2.94	2.65 2.36 1.67	48.5% 84.2% 90.2%	37.7% 72.3% 78.3%	7.3% 58.3% 75.2%	4.4% 64.7% 82.7%	8.5% 42.7% 52.1%	4.4% 53.2% 65.6%

Notes: All estimates in this table use the change in the share of the working-age population employed in each CZ as the outcome variable  $Y_i$  in eq. (1). This table indicates the mean and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (8)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (9) to (14)). Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters CZs in the same state; in columns (5) and (11), AKM is the standard error in Remark 5; in columns (7), and (13), AKM is the standard error in eq. (40) for 2-digit SIC sector clusters; in columns (6) and (12), AKM0 is the confidence interval in Remark 6 as indicated in Section 5.1. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples.

pothesis tests based on standard inference procedures as well as on the *AKM* and *AKM0* inference procedures may suffer from an overrejection problem.

We summarize the conclusions from Tables 3 and 4 in the following remark.

**Remark 7.** In shift-share regressions, overrejection of the usual inference procedures is more severe when there is a small number sectors. In this case, the methods we provide attenuate the overrejection problem, but may still overreject when the number of sectors is very small. Our methods perform well under different distributions of shifters and regression residuals, but they lead to an overrejection problem when the shift-share covariate aggregates over a large sector. Finally, when the shifters are not independent across sectors, it is important to properly account for their correlation structure.

In Online Appendices D.3 to D.7 we present results from additional placebo simulations in which we investigate the consequences of: (a) the violation of the assumption that the shifters of interest are as good as randomly assigned; (b) the presence of serial correlation in both the shifters of interest and the regression residuals, in panel data settings; (c) the true potential outcome function being nonlinear, implying that the linearly additive potential outcome framework in eq. (11) is misspecified; (d) the presence in the regression residuals of shift-share components with shares correlated in different degrees with those entering the shift-share covariate of interest; and, (e) the presence of treatment heterogeneity across regions and sectors.

## 7 Empirical applications

We now apply the *AKM* and *AKM0* inference procedures to two empirical applications. First, the effect of Chinese competition on U.S. local labor markets, as in Autor, Dorn and Hanson (2013). Second, the estimation of the local inverse elasticity of labor supply, as in Bartik (1991). Additionally, in Online Appendix F, we apply the *AKM* and *AKM0* inference procedures to the study of the impact of immigration on labor market outcomes of U.S. natives.

## 7.1 Effect of Chinese exports on U.S. labor market outcomes

Autor, Dorn and Hanson (2013, henceforth ADH), explore the impact of exports from China on labor market outcomes across U.S. CZs. Specifically, ADH present IV estimates for a specification that fits within the panel data setting described in Section 5.2, with each region j = 1, ..., 722 denoting a CZ, each sector k = 1, ..., 396 denoting a 4-digit SIC industry, and each period t = 1, 2 denoting either 1990–2000 changes or 2000–2007 changes. As in Section 5.2, we index here the intersection of a region j and a period t by t, and the intersection of a sector t and a period t by t. In ADH, the outcome t is a ten-year equivalent change in a labor-market outcome, the endogenous treatment is t is a ten-year equivalent change in U.S. imports from China normalized by the start-of-period total U.S. employment in the sector, and t is the start-of-period employment share of a sector in a CZ. ADH use the shift-share IV t is a ten-year-lag of the start-of-period total U.S. employment in the sector, and t is the ten-year-lag of the employment share t in the sector, and t is the ten-year-lag of the employment share t in the sector.

these variables, we use the data sources described in Section 2.1. In all regression specifications, we include a vector of controls  $Z_i$  corresponding to the largest set of controls used in ADH.<sup>27</sup>

Table 5 reports 95% CIs computed using different methodologies for the specifications in Tables 5 to 7 in ADH. Panels A, B, and C present the IV, reduced-form and first-stage estimates, respectively. Following Autor et al. (2014), the *AKM* and *AKM0* CIs cluster the shifters  $\{\mathcal{X}_s\}_{s=1}^S$  by 3-digit SIC industry; thus, the *AKM* and *AKM0* CIs we report are robust to serial correlation in the shifters as well as to cross-sectoral correlation in the shifters within 3-digit SIC industries. Tables E.4 and E.5 in Online Appendix E.1 report *AKM* and *AKM0* CIs for alternative definitions of clusters.

In Online Appendix E.1, we present placebo simulations that depart from our baseline placebo design in ways that explore specific features of the empirical setting studied in this section. In Table E.1, we draw the shifters from the empirical distribution of shifters used to construct the ADH IV (instead of drawing them from a normal distribution); the resulting rejection rates are very similar to those in the baseline simulation. In Table E.2, we draw shifters that have a common component with factor structure; since the resulting correlation structure cannot be captured by clustering, we show that it is important in this case to include an estimate of the common factor component as an additional control.<sup>28</sup>

In Table 5, state-clustered CIs are very similar to the heteroskedasticity-robust ones. In contrast, our proposed CIs are wider than those implied by state-clustered standard errors. For the IV estimates reported in Panel A, the average increase across all outcomes in the length of the 95% CI is 24% with the *AKM* procedure and 65% with the *AKM0* procedure. When the outcome is the change in the manufacturing employment rate, the length of the 95% CI increases by 26% with the *AKM* procedure and by 65% with the *AKM0* procedure. In light of the lack of impact of state-clustering on the 95% CI, the wider intervals implied by our inference procedures indicate that cross-region residual correlation is driven by similarity in sectoral compositions rather than by geographic proximity.

Panel B of Table 5 reports CIs for the reduced-form specification. In this case, the increase in the CI length is slightly larger than for the IV estimates: across outcomes, it increases on average by 54% for *AKM* and 130% for *AKM*0. The smaller relative increase in the CI length for the IV estimate relative to its increase for the reduced-form estimate is a consequence of the fact that all inference procedures yield similar CIs for the first-stage estimate, as reported in Panel C.

As discussed in Section 6, the differences between *AKM* (or *AKM0*) CIs and state-clustered CIs are related to the importance of shift-share components in the regression residual. The results in Panel C suggest that, once we account for changes in sectoral imports from China to other high-income countries, there is not much sectoral variation left in the first-stage regression residual; i.e., there are no other sectoral variables that are important to explain changes in sectoral imports from China to the

 $<sup>^{27}</sup>$ See column (6) of Table 3 in ADH. The vector  $Z_i$  aims to control for labor supply shocks and labor demand shocks other than the changes in imports from China, and it includes the start-of-period percentage of employment in manufacturing. The discussion in Section 4.2 implies that one should instead control for the ten-year-lagged of the start-of-period employment share in manufacturing, to match the shares that enter the definition of the shift-share IV. However, to facilitate the comparison with the original results in ADH, we use their vector of controls. As shown in Borusyak, Hull and Jaravel (2018), controlling for the ten-year-lagged manufacturing employment shares does not substantively affect the estimates.

<sup>&</sup>lt;sup>28</sup>For placebo simulation evidence under our baseline assumption that the shifters are independent across 3-digit clusters, using data for outcomes  $Y_{1i}$  and shares  $w_{is}$  identical to that used in this section, see Online Appendix D.4.

Table 5: Effect of Chinese exports on U.S. commuting zones—Autor, Dorn and Hanson (2013)

	Change in the employment share			Change in avg. log weekly wage			
	All	Manuf.	Non-Manuf.	All	Manuf.	Non-Manuf.	
	(1)	(2)	(3)	(4)	(5)	(6)	
Panel A:	Panel A: 2SLS Regression						
$\hat{eta}$	-0.77	-0.60	-0.18	-0.76	0.15	-0.76	
Robust	[-1.10, -0.45]	[-0.78, -0.41]	[-0.47, 0.12]	[-1.23, -0.29]	[-0.81, 1.11]	[-1.27, -0.25]	
Cluster	[-1.12, -0.42]	[-0.79, -0.40]	[-0.45, 0.10]	[-1.26, -0.26]	[-0.81, 1.11]	[-1.28, -0.24]	
AKM	[-1.25, -0.30]	[-0.84, -0.35]	[-0.54, 0.18]	[-1.37, -0.15]	[-0.81, 1.11]	[-1.42, -0.10]	
AKM0	[-1.69, -0.39]	[-1.01, -0.36]	[-0.84, 0.14]	[-1.77, -0.17]	[-1.49, 1.05]	[-1.97, -0.19]	
Panel B:	Panel B: OLS Reduced-Form Regression						
$\hat{eta}$	-0.49	-0.38	-0.11	-0.48	0.10	-0.48	
Robust	[-0.71, -0.27]	[-0.48, -0.28]	[-0.31, 0.08]	[-0.80, -0.16]	[-0.50, 0.69]	[-0.83, -0.13]	
Cluster	[-0.64, -0.34]	[-0.45, -0.30]	[-0.27, 0.05]	[-0.78, -0.18]	[-0.51, 0.70]	[-0.81, -0.15]	
AKM	[-0.81, -0.17]	[-0.52, -0.23]	[-0.35, 0.12]	[-0.88, -0.07]	[-0.50, 0.69]	[-0.93, -0.03]	
AKM0	[-1.24, -0.24]	[-0.67, -0.25]	[-0.64, 0.08]	[-1.27, -0.10]	[-1.16, 0.61]	[-1.47, -0.11]	
Panel C:	2SLS First-Sta	ge					
$\hat{eta}$			0.63				
Robust			[0.46, 0.80]				
Cluster			[0.45, 0.81]				
AKM			[0.53, 0.73]				
AKM0			[0.54, 0.84]				

Notes: N = 1,444 (722 CZs  $\times$  2 time periods). Observations are weighted by the start of period CZ share of national population. All regressions include the full vector of baseline controls in ADH; i.e. those in column (6) of Table 3 in Autor, Dorn and Hanson (2013). 95% confidence intervals are reported in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; AKM is the standard error in eq. (40) with 3-digit SIC clusters; AKM0 is the confidence interval with 3-digit SIC clusters described in the last sentence of Section 5.1.

U.S.<sup>29</sup> To investigate this claim, Table E.3 in Online Appendix E.1 reports the rejection rates implied by a placebo exercise designed to match the first-stage specification reported in Panel C of Table 5. The placebo results show that, while traditional methods still suffer from severe overrejection when no controls are included, the overrejection is attenuated once we include as controls the shift-share IV and the control vector  $Z_i$  we use in Table 5, indicating that these variables soak up much of the cross-CZ correlation in the treatment variable used in ADH.

Overall, Table 5 shows that, despite the wider confidence intervals obtained with our procedures, the qualitative conclusions in ADH remain valid at usual significance levels. However, the increased width of the 95% CI shows that the uncertainty regarding the magnitude of the impact of Chinese import exposure on U.S. labor markets is greater than that implied by usual inference procedures. In particular, the *AKM0* CI is much wider than that based on state-clustered standard errors; furthermore due to its asymmetry around the point estimate, using the *AKM0* CI, we cannot rule out impacts of

<sup>&</sup>lt;sup>29</sup>This is analogous to what we would observe in a regression in which the regressor of interest varies at the state level, and we control for all state-specific covariates affecting the outcome variable: state-clustered standard errors would be similar to heteroskedasticity-robust standard errors, since there is little within-state correlation left in the residuals.

the China shock that are two to three times larger than the point estimates of these effects.<sup>30</sup>

## 7.2 Estimation of inverse labor supply elasticity

In our second application, we estimate the inverse labor supply elasticity. Specifically, using the notation of Section 3, we estimate the parameter  $\tilde{\phi}$  in the equation

$$\hat{\omega}_i = \tilde{\phi}\hat{L}_i + \delta Z_i + \epsilon_i, \qquad \tilde{\phi} = \phi^{-1}, \tag{46}$$

where  $\hat{L}_i$  denotes the log change in the employment rate in CZ i,  $\hat{\omega}_i$  denotes the log change in wages,  $Z_i$  is a vector of controls, and  $\epsilon_i$  is a regression residual. We use the same sample, data sources, and vector of controls  $Z_i$  as in Section 7.1.<sup>31</sup>

The model in Section 3 has implications for the properties of different strategies for estimating the inverse labor supply elasticity  $\tilde{\phi}$ . By eq. (10), the residual  $\epsilon_i$  in eq. (46) accounts for changes in labor supply shocks,  $\sum_{g=1}^{G} \tilde{w}_{ig}\hat{v}_g + \hat{v}_i$ , not controlled for by the vector  $Z_i$ . Second, it follows from eq. (8) that, up to a first-order approximation around an initial equilibrium, changes in regional employment rates,  $\hat{L}_i$ , can be written as a function of both shift-share aggregators of sectoral labor demand shocks and the same labor supply shocks potentially entering  $\epsilon_i$  in eq. (10),  $\sum_{g=1}^{G} \tilde{w}_{ig}\hat{v}_g + \hat{v}_i$ . Thus,  $\hat{L}_i$  and  $\epsilon_i$  will generally be correlated and the OLS estimator of  $\tilde{\phi}$  in eq. (46) will be biased. However, as discussed in Section 4.3, the model in Section 3 also implies that we can instrument for  $\hat{L}_i$  using shift-share aggregators of sectoral labor demand shocks that are independent of the unobserved labor supply shocks (see Online Appendix E.2 for more details).

In this section, we use three different shift-share IVs to estimate  $\tilde{\phi}$  in eq. (46). For each of them, Table 6 presents the reduced-form, first-stage and 2SLS estimates. First, in Panel A, we use the instrumental variable in Bartik (1991); i.e.  $\hat{X}_i = \sum_{i=1}^N w_{is} \hat{L}_s$ , where  $\hat{L}_s$  denotes the nation-wide employment growth in sector s. Second, in Panel B, we use the leave-one-out version of this instrument; i.e.  $\hat{X}_i = \sum_{i=1}^N w_{is} \hat{L}_{s,-i}$ , where  $\hat{L}_{s,-i}$  denotes the employment growth in sector s over all CZs excluding CZ i.<sup>32</sup> Third, in Panel C, we use the IV used in Autor, Dorn and Hanson (2013), which we denote as ADH IV and describe in detail in Section 7.1.<sup>33</sup> As in Section 7.1, we report versions of the AKM and AKM0 CIs with shifters clustered at the 3-digit SIC industry for all periods.

Column (3) of Table 6 shows that the estimates of the inverse labor supply elasticity are similar no matter which IV we use: 0.80 when using the original Bartik IV, 0.82 when using the leave-one-out

 $<sup>^{30}</sup>$ It follows from Remark 6 (see the expression for the quantity A) that the asymmetry in the AKM0 CI comes from the correlation between the regression residuals  $\hat{R}_s$  and the shifters cubed. In large samples, this correlation is zero and the AKM and AM0 CIs are asymptotically equivalent. The differences between both CIs in Table 5 thus reflect differences in their finite-sample properties. This notwithstanding, the placebo exercise presented in Online Appendix D.4 shows that both inference procedures yield close to correct rejection rates in a sample analogous to that used in ADH.

<sup>&</sup>lt;sup>31</sup>Table E.7 in Online Appendix E.2 investigates the robustness of our results to alternative sets of controls.

<sup>&</sup>lt;sup>32</sup>The leave-one-out version of the instrument in Bartik (1991) was originally proposed by Autor and Duggan (2003). In Online Appendix E.2.3, we clarify the assumptions under which the model in Section 3 is consistent with the validity of the leave-one-version of the Bartik IV. Online Appendix E.2.4 presents placebo exercises attesting that the *AKM* and *AKM0* CIs reported in this section have appropriate coverage in the context of this empirical application.

<sup>&</sup>lt;sup>33</sup>The effect of these IVs on the changes in the employment rate may be heterogeneous across regions and sectors (see eq. (8)). This does not affect the validity of our inference procedures since, as discussed in Section 4.3, we allow for heterogeneous effects in the first-stage regression.

Table 6: Estimation of inverse labor supply elasticity

	First-Stage	Reduced-Form	2SLS			
Dependent variable:	$\hat{L}_i$	$\hat{\omega}_i$	$\hat{\omega}_i$			
•	(1)	(2)	(3)			
Panel A: Bartik IV—Not leave-one-out estimator						
$\hat{eta}$	0.90	0.73	0.80			
Robust	[0.70, 1.10]	[0.54, 0.91]	[0.64, 0.97]			
Cluster	[0.64, 1.16]	[0.47, 0.98]	[0.60, 1.01]			
AKM	[0.65, 1.16]	[0.49, 0.96]	[0.62, 0.98]			
AKM0	[0.61, 1.17]	[0.44, 0.96]	[0.59, 1.02]			
Panel B: Bartik IV—Leave-one-out estimator						
$\hat{eta}$	0.87	0.71	0.82			
Robust	[0.68, 1.06]	[0.53, 0.89]	[0.65, 0.98]			
Cluster	[0.62, 1.12]	[0.46, 0.96]	[0.60, 1.03]			
AKM (leave-one-out)	[0.59, 1.15]	[0.47, 0.94]	[0.61, 1.02]			
AKM0 (leave-one-out)	[0.53, 1.15]	[0.42, 0.94]	[0.59, 1.09]			
Panel C: ADH IV						
$\hat{eta}$	-0.72	-0.48	0.67			
Robust	[-1.04, -0.39]	[-0.80, -0.16]	[0.36, 0.98]			
Cluster	[-0.93, -0.50]	[-0.78, -0.18]	[0.35, 0.99]			
AKM	[-1.19, -0.24]	[-0.88, -0.07]	[0.27, 1.07]			
AKM0	[-1.83, -0.35]	[-1.27, -0.10]	[0.18, 1.14]			

Notes: N=1,444 (722 CZs  $\times$  2 time periods). The variable  $\hat{L}_i$  denotes the log-change in the employment rate in CZ i. The variable  $\hat{\omega}_i$  denotes the log change in mean weekly earnings. Observations are weighted by the start of period CZ share of national population. All regressions include the full vector of baseline controls in ADH; i.e. those in column (6) of Table 3 in Autor, Dorn and Hanson (2013). 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; AKM is the standard error in eq. (40) with 3-digit SIC clusters; AKM0 is the confidence interval with 3-digit SIC clusters described in the last sentence of Section 5.1; AKM (leave-one-out) is the standard error in Section 5.3 with 3-digit SIC clusters; AKM0 (leave-one-out) is the confidence interval with 3-digit SIC clusters described in Section 5.3.

version of this estimator, and 0.67 when using the ADH IV.<sup>34</sup> In both Panel A and Panel B, the *AKM* and *AKM0* CIs are very similar to the state-clustered CI. In Panel C, the *AKM* and *AKM0* CIs are only moderately wider than those obtained with state-clustered standard errors.

Columns (1) and (2) of Table 6 show the first-stage and reduced-form estimates, respectively. In Panel A and Panel B, the *AKM0* CIs are similar to the state-clustered CIs; in contrast, in Panel C, the first-stage and reduced-form *AKM0* CIs more twice as wide, and more than three times as wide as the state-clustered CI, respectively. Thus, the first-stage and reduced-form *AKM* and *AKM0* CIs differ more from the state-clustered CI when the ADH IV is used than when the Bartik IV is used. A possible explanation for this finding is that the shift-share component of the first-stage and reduced-form regression residuals is much smaller in the latter than in the former case. The Bartik IV absorbs

<sup>&</sup>lt;sup>34</sup>One explanation for the similarity between the leave-one-out and the original Bartik IV is that, as discussed in Section 5.3, the bias of the original Bartik IV is of the order  $\frac{1}{N}\sum_{i=1}^{N}\sum_{s=1}^{S}\frac{w_{is}\check{w}_{is}}{\check{n}_{s}}$ . This quantity equals 0.004 in this application, indicating that the own-observation bias is likely to be small.

the bulk of the shift-share covariates that affect the change in the employment rate and wages across CZs. In contrast, the ADH IV is just one of the possibly various shift-share terms affecting the change in the outcome and endogenous treatment of interest. With the remaining shift-share entering the regression residual, it becomes quantitatively important to use our inference procedures to obtain CIs with the right coverage.

## 8 Concluding remarks

This paper studies inference in shift-share designs. We show that standard economic models predict that changes in regional outcomes depend on observed and unobserved sector-level shocks through several shift-share terms. Our model thus implies that the residual in shift-share regressions is likely to be correlated across regions with similar sectoral composition, independently of their geographic location, due to the presence of unobserved shift-share terms. Such correlations are not accounted for by inference procedures typically used in shift-share regressions, such as when standard errors are clustered on geographic units. To illustrate the importance of this shortcoming, we conduct a placebo exercise in which we study the effect of randomly generated sector-level shocks on actual changes in labor market outcomes across CZs in the United States. We find that traditional inference procedures severely overreject the null hypothesis of no effect. We derive two novel inference procedures that yield correct rejection rates.

It has become standard practice to report cluster-robust standard errors in regression analysis whenever the variable of interest varies at a more aggregate level than the unit of observation. This practice guards against potential correlation in the residuals that arises whenever these residuals contain unobserved shocks that also vary at the same level as the variable of interest. In the same way, we recommend that researchers report confidence intervals in shift-share designs that allow for a shift-share structure in the residuals, such as one of the two confidence intervals that we propose.

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# Online supplement to:

"Shift-Share Designs: Theory and Inference"

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## Appendix A Proofs and additional theoretical results

Appendix A.1 gives proofs and additional details for the results in Sections 4.1 and 4.2. Appendix A.2 gives proofs and additional details for the results in Sections 4.3 and 5.3.

#### A.1 Proofs and additional details for OLS regression

Since Propositions 1 and 2 are special cases or Propositions 3 and 4, we only prove Propositions 3, 4 and 5. We give the proofs under a slightly more general setup that allows for a linearization error in the potential outcome equation. We introduce this more general setup in Appendix A.1.1, where we also collect the assumptions that we impose on the DGP. We collect some auxiliary Lemmata used in the proofs in Appendix A.1.2, and we prove these propositions in Appendices A.1.3, A.1.3 and A.1.5. Appendix A.1.6 discusses inference when the effects  $\beta_{is}$  are heterogeneous.

Throughout the Appendix, we assume that  $\sum_{s=1}^{S} w_{is} \leq 1$  for all i. Thus,  $\sum_{s=1}^{S} n_s \leq N$ , where  $n_s = \sum_{i=1}^{N} w_{is}$  denotes the size of sector s. We use the notation  $A_S \leq B_S$  to denote  $A_S = O(B_S)$ , i.e. there exists a constant C independent of S such that  $A_S \leq CB_S$ . Let  $\mathcal{F}_0$  denote the  $\sigma$ -field generated by  $(\mathcal{Z}, U, Y(0), B, W)$  (for the case with no covariates,  $\mathcal{F}_0$  denotes the  $\sigma$ -field generated by (Y(0), B, W)). Define  $\overline{w}_{st} = \sum_{i=1}^{N} w_{is}w_{it}$ ,  $\tilde{\mathcal{X}}_s = \mathcal{X}_s - \mathcal{Z}_s'\gamma$ , and  $\sigma_s^2 = \text{var}(\mathcal{X}_s \mid \mathcal{F}_0)$ . Finally, let  $r_N = (\sum_s n_s^2)^{-1}$ , and let  $E_W$  denote expectation conditional on W.

### A.1.1 General setup and assumptions

We first list and discuss the regularity conditions needed for the results in Section 4.1. We then generalize the setup from Section 4.2 by allowing for a linearization error in the potential outcome equation (11). Unless stated otherwise, all limits are taken as  $S \to \infty$ . We leave the dependence of the number of regions  $N = N_S$  on S implicit.

For the results in Section 4.1, we assume that the observed data (Y, X, W) is generated by the variables  $(Y(0), B, W, \mathcal{X})$ , which we model as a triangular array, so that the distribution of the data may change with the sample size.<sup>1</sup> The additional regularity conditions we impose on these variables, in addition to Assumptions 1 and 2 as follows:

**Assumption A.1.** (i) The support of  $\beta_{is}$  is bounded; (ii)  $\frac{1}{N}\sum_{i=1}^{N}\sum_{s=1}^{S} \text{var}(\mathcal{X}_{s} \mid \mathcal{F}_{0})w_{is}^{2}$  converges in probability to a strictly positive non-random limit; (iii) For some  $\nu > 0$ ,  $E[|\mathcal{X}_{s}|^{2+\nu} \mid \mathcal{F}_{0}]$  exists and is uniformly bounded, and conditional on W, the second moments of  $Y_{i}(0)$  exist, and are bounded uniformly over i; (iv) For some  $\nu > 0$ ,  $E[|\mathcal{X}_{s}|^{4+\nu} \mid \mathcal{F}_{0}]$  is uniformly bounded, and conditional on W, the fourth moments of  $Y_{i}(0)$  exist, and are bounded uniformly over i.

The bounded support condition on  $\beta_{is}$  in Assumption A.1(i) is made to keep the proofs simple and can be relaxed. Assumption A.1(ii) is a standard regularity condition ensuring that the shocks  $\mathcal{X}$  have sufficient variation so that the denominator of  $\hat{\beta}$ , scaled by N, does not converge to zero.

<sup>&</sup>lt;sup>1</sup>In other words, to allow the distribution of the data to change with the sample size S, we implicitly index the data by S. Making this index explicit, for each S, the data is thus given by the array  $\{(Y_{iS}(0), \beta_{isS}, w_{isS}, \mathcal{X}_{sS}): i = 1, ..., N_S, s = 1, ..., S\}$ .

This requires that there is at least one "non-negligible" sector in most regions in the sense that its share  $w_{is}$  is bounded away from zero. This implies that  $\sum_{s=1}^{S} n_s/N$  is also bounded away from zero. Assumption A.1(iii) imposes some mild assumptions on the existence of moments of  $\mathcal{X}$  and  $Y_i(0)$ . Assumption A.1(iv), which is only needed for asymptotic normality, strengthens this condition.

For the results in Section 4.2, we generalize the setup in the main text by allowing for a linearization error in the expression for potential outcomes,

$$Y_i(x_1,...,x_S) = Y_i(0) + \sum_{s=1}^S w_{is} x_s \beta_{is} + L_i(x_1,...,x_S), \qquad \sum_{s=1}^S w_{is} \le 1,$$
 (A.1)

and we weaken Assumption 3(i) by replacing it with the assumption that the observed outcome is given by  $Y_i = Y_i(\mathcal{X}_1, \dots, \mathcal{X}_S)$ , such that eq. (A.1) holds with  $L_i(\mathcal{X}_1, \dots, \mathcal{X}_S) = L_i$ .

We assume that the observed data  $(Y, \mathcal{X}, Z, W)$  is generated by the triangular array of variables  $(Y(0), B, W, U, \mathcal{X}, \mathcal{Z}, L)$ . Let  $\check{\delta} = (Z'Z)^{-1}Z'(Y - X\beta)$  denote the regression coefficient in a regression of  $Y - X\beta$  on Z, that is, the regression coefficient on  $Z_i$  in a regression in which  $\hat{\beta}$  is restricted to equal to the true value  $\beta$ .

**Assumption A.2.** (i)  $N^{-1}\sum_{i=1}^{N}E[L_i^2]^{1/2}\to 0$ , and conditional on W, the second moments of  $U_i$  and  $\mathcal{Z}_s$  exist and are bounded uniformly over i and s; (ii) Z'Z/N converges in probability to a positive definite non-random limit; (iii)  $(\sum_s n_s^2)^{-1/2}\sum_{i=1}^{N}E[L_i^2]^{1/2}\to 0$ ,  $\max_i E[L_i^4\mid W]\to 0$ , and conditional on W, the fourth moments of  $\mathcal{Z}_s$ , and  $U_i$  exist and are bounded uniformly over s and i; (iv)  $\delta - \delta = O_p(q_s)$  for some sequence  $q_S\to 0$ ; (v)  $q_S^2N/\sum_s n_s^2\cdot\sum_i E[(U_i'\gamma)^2]\to 0$  and  $\gamma'U'\epsilon=o_p((\sum_s n_s^2)^{1/2})$ .

Assumption A.2(i) imposes some mild moment restrictions on the controls  $Z_i$ . It also requires that on average, the variance of the linearization error  $L_i$  vanishes with sample size. This ensures that the linearization error does not impact the consistency of  $\hat{\beta}$ . Assumption A.2(ii) ensures that the controls are not collinear.

Assumptions A.2(iii) to A.2(v) are only needed for asymptotic normality. Assumption A.2(iii) strengthens the moment conditions in Assumption A.2(i). It also imposes a stricter condition on the linearization error: it requires that, on average over N, the standard deviation of  $L_i$  is of smaller order than  $(\sum_s n_s^2)^{1/2}/N$ , the rate of convergence of  $\hat{\beta}$ . A sufficient condition is that  $L_i = o_p(S^{-1/2})$ . This ensures that the linearization error is of smaller order than the variance of the estimator, so that the distribution of  $\hat{\beta}$  does not suffer from asymptotic bias. This formalizes the assumption that the linearization error is "small". The condition that  $\max_i E[L_i^4 \mid W] \to 0$  is only needed for showing consistency of the standard error estimator; it is not needed for asymptotic normality. Assumption A.2(iv) requires that  $\check{\delta}$  is consistent, which ensures that the error in estimation of  $\delta$  does not affect the asymptotic distribution of  $\hat{\beta}$ . Finally, Assumption A.2(v) imposes conditions on  $U'_i\gamma$ , the measurement error for controls that matter, which ensure that measurement error in the controls that matter does not impact the asymptotic distribution of  $\hat{\beta}$ . They are stated as high-level conditions to cover a range of different cases, and depend on the rate of convergence  $q_S$  of  $\check{\delta}$ . In typical cases, the rate will be  $q_S = (\sum_s n_s^2)^{1/2}/N$ , the same as that of  $\hat{\beta}$ , and the condition  $q_S^2 N / \sum_s n_s^2 \cdot \sum_i E[(U_i' \gamma)^2] \to 0$ is implied by Assumption 3(iii). Let  $U_{1i}$  denote the subset of elements of  $U_i$  for which  $\gamma_k \neq 0$ , and let  $U_{2i}$  denote the remaining elements. If  $U_{i1}$  is mean zero and independent across i conditional on the

remaining variables ( $(Y(0), W, B, \mathcal{Z}, \mathcal{X}, U_2)$ ), so that these elements are pure measurement error, then the second condition is implied by Assumption 3(iv).

#### A.1.2 Auxiliary results

**Lemma A.1.**  $\{\mathcal{A}_{S1}, \ldots, \mathcal{A}_{SS}\}_{S=1}^{\infty}$  be a triangular array of random variables. Fix  $\eta \geq 1$ , and let  $A_{Si} = \sum_{s=1}^{S} w_{is} \mathcal{A}_{Ss}$ ,  $i = 1 \ldots, N_S$ . Suppose  $E[|\mathcal{A}_{Ss}|^{\eta} \mid W]$  exists and is uniformly bounded. Then  $E[|A_{Si}|^{\eta} \mid W]$  exists and is bounded uniformly over S and i.

*Proof.* The result follows by triangle inequality for  $\eta = 1$ . Suppose therefore that  $\eta > 1$ . By Hölder's inequality,

$$E[|A_{Si}|^{\eta} \mid W] = E\left[\left|\sum_{s=1}^{S} w_{is}^{\frac{\eta-1}{\eta}} w_{is}^{\frac{1}{\eta}} \mathcal{A}_{Ss}\right|^{\eta} \mid W\right] \leq \left(\sum_{s=1}^{S} w_{is}\right)^{\eta-1} \sum_{s=1}^{S} w_{is} E[|\mathcal{A}_{Ss}|^{\eta} \mid W]$$

$$\leq \max_{s} E[|\mathcal{A}_{Ss}|^{\eta} \mid W] \cdot (\sum_{s=1}^{S} w_{is})^{\eta} \leq \max_{s} E[|\mathcal{A}_{Ss}|^{\eta} \mid W],$$

which yields the result.

**Lemma A.2.**  $\{A_{S1}, \ldots, A_{SN_S}\}_{S=1}^{\infty}$  be a triangular array of random variables. Suppose  $E[A_{Si}^2 \mid W]$  exists and is uniformly bounded. Then  $\sum_{s=1}^S E[(\sum_{i=1}^N w_{is} A_{Si})^2 \mid W] \leq \sum_s n_s^2$ .

Proof. By Cauchy-Schwarz inequality,

$$\sum_{s=1}^{S} E\left[\left(\sum_{i=1}^{N} w_{is} A_{Si}\right)^{2} \mid W\right] \leq \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{is} w_{js} E[A_{Si}^{2} \mid W]^{1/2} E[A_{Sj}^{2} \mid W]^{1/2}$$

$$\leq \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{is} w_{js} = \sum_{s=1}^{S} n_{s}^{2}.$$

**Lemma A.3.** Let  $\{A_{S1}, \ldots, A_{SN_S}, B_{S1}, \ldots, B_{SN_S}, \mathcal{A}_{S1}, \ldots, \mathcal{A}_{SS}\}_{S=1}^{\infty}$  be a triangular array of random variables. Suppose  $E[A_{Si}^4 \mid W]$ ,  $E[B_{Si}^4 \mid W]$ , and  $E[\mathcal{A}_{Ss}^2 \mid W]$  exist and are uniformly bounded. Then  $(\sum_s n_s^2)^{-1} \cdot \sum_{i,j,s} w_{is} w_{js} A_{Si} B_{Sj} \mathcal{A}_{Ss} = O_p(1)$ .

*Proof.* Let  $R_S = (\sum_s n_s^2)^{-1} \sum_{i,j,s} w_{is} w_{js} A_{Si} B_{Sj} \mathcal{A}_{Ss}$ . By the triangle and Cauchy-Schwarz inequalities,

$$\begin{split} E[|R_S| \mid W] &\leq \frac{1}{\sum_s n_s^2} \sum_{i,j,s} w_{is} w_{js} E[|A_{Si} B_{Sj} \mathcal{A}_{Ss}| \mid W] \\ &\leq \frac{1}{\sum_s n_s^2} \sum_{i,j,s} w_{is} w_{js} E[|B_{Sj}|^4 \mid W]^{1/4} E[|A_{Si}|^4 \mid W]^{1/4} E[\mathcal{A}_{Ss}^2 \mid W]^{1/2} \preceq \frac{1}{\sum_s n_s^2} \sum_{i,j,s} w_{is} w_{js} = 1. \end{split}$$

The result then follows by Markov inequality.

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#### A.1.3 Proof of Proposition 3

First we show that

$$Z'W\tilde{\mathcal{X}} = O_p(1/\sqrt{r_N}). \tag{A.2}$$

Conditional on *W*, the left-hand side has mean zero by Assumption 3(ii), and by Assumption 2(i), the variance of the *k*th row given by

$$\operatorname{var}\left(\sum_{i,s} w_{is} \tilde{\mathcal{X}}_s Z_{ik} \mid W\right) = \sum_s E_W \sigma_s^2 \left(\sum_i w_{is} Z_{ik}\right)^2 \preceq \sum_s E_W \left(\sum_i w_{is} Z_{ik}\right)^2.$$

By Lemma A.1, Assumption A.2(i), and the  $C_r$ -inequality,  $E_W[Z_{ik}^2] = E_W[(\sum_s w_{is} \mathcal{Z}_{sk} + U_{ik})^2]$  is uniformly bounded. Therefore, by Lemma A.2, the right-hand side is bounded by  $\sum_s n_s^2$ , so the result follows by Markov inequality and dominated convergence theorem.

Since  $X = W\tilde{X} + Z\gamma - U\gamma$ , it follows from eq. (A.2) and Assumption A.2(ii) that

$$\hat{\gamma} - \gamma = (Z'Z/N)^{-1}Z'W\tilde{X}/N - (Z'Z/N)^{-1}Z'U\gamma/N = o_p(1), \tag{A.3}$$

where  $\hat{\gamma} = (Z'Z)^{-1}Z'X$ , and the last equality follows since  $\sum_s n_s^2/N^2 \le \max_s n_s/N \to 0$  by Assumption 2(ii), and since  $Z'U\gamma/N = o_p(1)$  by the Cauchy-Schwarz inequality and Assumption 3(iii).

Next, we will show that

$$\ddot{X}'\ddot{X}/N = \frac{1}{N} \sum_{i,s} w_{is}^2 \sigma_s^2 + o_p(1). \tag{A.4}$$

To this end, we have

$$\begin{split} \ddot{X}'\ddot{X}/N &= (W\tilde{\mathcal{X}} - U\gamma - Z(\hat{\gamma} - \gamma))'(W\tilde{\mathcal{X}} - U\gamma - Z(\hat{\gamma} - \gamma))/N \\ &= (W\tilde{\mathcal{X}})'(W\tilde{\mathcal{X}})/N + o_p(1) \\ &= \frac{1}{N} \sum_s \overline{w}_{ss} \sigma_s^2 + \frac{2}{N} \sum_{s < t} \overline{w}_{st} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t + \frac{1}{N} \sum_s \overline{w}_{ss} (\tilde{\mathcal{X}}_s^2 - \sigma_s^2) + o_p(1). \end{split}$$

where the first line follows from the decomposition

$$\ddot{X} = X - Z(Z'Z)^{-1}Z'X = X - Z\hat{\gamma} = W\tilde{\mathcal{X}} - U\gamma - Z(\hat{\gamma} - \gamma), \tag{A.5}$$

the second line follows by the Cauchy-Schwarz inequality, Assumption 3(iii), and eq. (A.3), and the third line follows by expanding  $(W\tilde{X})'(W\tilde{X})/N$ . Therefore, to show eq. (A.4), it suffices to show that the second and third term in the above expression are  $o_p(1)$ . Since the second term has mean zero conditional on W, it suffices to show that its variance converges to zero. To that end,

$$\begin{aligned} \operatorname{var}\left(\frac{2}{N}\sum_{s< t}\tilde{\mathcal{X}}_{s}\tilde{\mathcal{X}}_{t}\overline{w}_{st}\mid W\right) &= \frac{4}{N^{2}}\sum_{s< t}E_{W}[\sigma_{s}^{2}\sigma_{t}^{2}]\overline{w}_{st}^{2} \leq \frac{1}{N^{2}}\sum_{s, t}\overline{w}_{st}^{2} = \frac{1}{N^{2}}\sum_{i, j, s, t}w_{is}w_{it}w_{js}w_{jt} \\ &\leq \frac{1}{N^{2}}\sum_{i, j, s, t}w_{is}w_{it}w_{js} \leq \frac{1}{N^{2}}\sum_{i, j, s}w_{is}w_{js} = \frac{1}{N^{2}}\sum_{s}n_{s}^{2} \leq \frac{\max_{t}n_{t}\sum_{s}n_{s}}{N^{2}} \to 0. \end{aligned}$$

where the convergence to 0 follows by Assumption 2(ii). By the inequality of von Bahr and Esseen, Assumption A.1(iii), and the inequality  $\overline{w}_{ss} \leq n_s$ ,

$$E[N^{-1}|\sum_{s}(\tilde{\mathcal{X}}_{s}^{2}-\sigma_{s}^{2})\overline{w}_{ss}|^{1+\nu/2}|\mathcal{F}_{0}] \leq \frac{2}{N^{1+\nu/2}}\sum_{s}\overline{w}_{ss}^{1+\nu/2}E[|\tilde{\mathcal{X}}_{s}^{2}-\sigma_{s}^{2}|^{1+\nu/2}|\mathcal{F}_{0}]$$

$$\leq \frac{1}{N^{1+\nu/2}}\sum_{s}\overline{w}_{ss}^{1+\nu/2}\leq (\max_{s}n_{s}/N)^{\nu/2}, \quad (A.6)$$

which converges to zero by Assumption 2(ii). Equation (A.4) then follows by Markov inequality.

Next, we show that

$$\ddot{X}'Y/N = \frac{1}{N} \sum_{i,s} \sigma_s^2 w_{is}^2 \beta_{is} + o_P(1)$$
(A.7)

Using eq. (A.5), we can write the left-hand side as

$$\begin{split} \ddot{X}'Y/N &= \tilde{\mathcal{X}}'W'Y/N - \gamma'U'Y/N - Y'Z/N \cdot (\hat{\gamma} - \gamma) \\ &= \tilde{\mathcal{X}}'W'Y/N + o_p(1) \\ &= \frac{1}{N} \sum_{s,i} w_{is} \tilde{\mathcal{X}}_s L_i + \frac{1}{N} \sum_{s,i} w_{is}^2 (\tilde{\mathcal{X}}_s \mathcal{X}_s - \sigma_s^2) \beta_{is} + \frac{1}{N} \sum_{s,i} w_{is} \tilde{\mathcal{X}}_s Y_i(0) \\ &+ \frac{1}{N} \sum_{s < t} \sum_{i} w_{is} w_{it} \tilde{\mathcal{X}}_s \mathcal{X}_t \beta_{it} + \frac{1}{N} \sum_{s < t} \sum_{i} w_{is} w_{it} \tilde{\mathcal{X}}_t \mathcal{X}_s \beta_{is} + \frac{1}{N} \sum_{s,i} w_{is}^2 \sigma_s^2 \beta_{is} + o_p(1) \end{split}$$

where the second line follows since by the  $C_r$ -inequality, Lemma A.1, Assumptions A.1(i), A.2(i) and A.1(iii),  $N^{-1}\sum_i E[Y_i^2]$  is bounded, so that  $Y'Z/N = O_p(1)$  and  $\gamma'U'Y/N = o_p(1)$  by Cauchy-Schwarz inequality and Assumption 3(iii), and the third line follows by expanding  $\tilde{\mathcal{X}}'W'Y$ . We therefore need to show that the first five terms in the expression above are  $o_p(1)$ . By the Cauchy-Schwarz inequality, the expectation of the absolute value of the first term is bounded by

$$N^{-1} \sum_{i} E[L_{i}^{2}]^{1/2} (E \sum_{s} w_{is}^{2} \sigma_{s}^{2})^{1/2} \leq N^{-1} \sum_{i} E[L_{i}^{2}]^{1/2},$$

which converges to zero by Assumption A.2(i). Thus, the first term is  $o_p(1)$  by Markov inequality and the dominated convergence theorem. The second term is  $o_p(1)$  by an argument analogous to eq. (A.6). The third to fifth terms are mean zero conditional on  $\mathcal{F}_0$ , so it suffices to show that their variances conditional on W converge to zero. The variance of the third summand is bounded by

$$\operatorname{var}\left(\frac{1}{N}\sum_{s}\tilde{\mathcal{X}}_{s}\sum_{i}w_{is}Y_{i}(0)\mid W\right)=\frac{1}{N^{2}}\sum_{s}E_{W}\sigma_{s}^{2}\left(\sum_{i}w_{is}Y_{i}(0)\right)^{2} \leq \frac{1}{N^{2}}\sum_{s}E_{W}\left(\sum_{i}w_{is}Y_{i}(0)\right)^{2},$$

which converges to zero by Lemma A.2. The variance of the fourth term is bounded by

$$\operatorname{var}\left(\frac{1}{N}\sum_{s< t}\sum_{i}w_{is}w_{it}\tilde{\mathcal{X}}_{s}\mathcal{X}_{t}\beta_{it}\mid W\right) = \frac{1}{N^{2}}\sum_{s< t, t'}\sum_{i, i'}w_{is}w_{it}\sigma_{s}^{2}E_{W}[\mathcal{X}_{t}\mathcal{X}_{t'}]\beta_{it}w_{i's}w_{i't'}\beta_{i't'}$$

$$\leq \frac{1}{N^2} \sum_{s,t,t',i,i'} w_{is} w_{it} w_{i's} w_{i't'} \leq \frac{1}{N^2} \sum_{s} n_s^2 \leq \max_{s} n_s / N \to 0.$$

Variance of the fifth term converges to zero by analogous arguments.

Combining eq. (A.4) with eq. (A.7) and Assumption A.1(ii) then yields the result.

#### A.1.4 Proof of Proposition 4

Using eq. (A.5), we have

$$\begin{split} r_N^{1/2}(\ddot{X}'\ddot{X})(\hat{\beta}-\beta) &= r_N^{1/2}X'(I-Z(Z'Z)^{-1}Z')(Z\delta+\epsilon) = r_N^{1/2}X'(I-Z(Z'Z)^{-1}Z')\epsilon \\ &= r_N^{1/2}\tilde{X}'W'\epsilon - r_N^{1/2}\gamma'U'\epsilon - r_N^{1/2}(\hat{\gamma}-\gamma)'Z'\epsilon. \end{split}$$

The third term can be written as

$$\begin{split} r_N^{1/2}(\hat{\gamma} - \gamma)' Z' \epsilon &= r_N^{1/2} \epsilon' Z (Z'Z)^{-1} (Z'W\tilde{\mathcal{X}} - Z'U\gamma) = r_N^{1/2} (\check{\delta} - \delta)' (Z'W\tilde{\mathcal{X}} - Z'U\gamma) \\ &= (\check{\delta} - \delta)' (O_p(1) - r_N^{1/2} Z'U\gamma) \\ &= o_p(1) - O_p(1) \cdot q_S r_N^{1/2} Z'U\gamma = o_p(1), \end{split}$$

where the first line follows from the decomposition in eq. (A.3), the second line follows from eq. (A.2), the third line follows by Assumption A.2(iv), and the last equality follows since by Cauchy-Schwarz inequality and Assumption A.2(v),  $q_S r_N^{1/2} E[|Z_k' U \gamma|] \leq \sqrt{q_S^2 r_N N \sum_i E(U_i' \gamma)^2} \rightarrow 0$ . Since  $r_N^{1/2} \gamma' U' \epsilon = o_p(1)$  by Assumption A.2(v), and since by eq. (A.4) and Assumption A.1(ii),  $(\ddot{X}'\ddot{X}/N)^{-1} = (1 + o_p(1)) \cdot (N^{-1} \sum_{i,s} \pi_{is})^{-1}$ , it follows that

$$\frac{N}{(\sum_{s} n_{s}^{2})^{1/2}}(\hat{\beta} - \beta) = (1 + o_{p}(1)) \frac{1}{N^{-1} \sum_{i,s} \pi_{is}} r_{N}^{1/2} \sum_{s,i} \tilde{\mathcal{X}}_{s} w_{is} \epsilon_{i} + o_{p}(1).$$

Therefore, it suffices to show

$$r_N^{1/2} \sum_{s,i} \tilde{\mathcal{X}}_s w_{is} \epsilon_i = \mathcal{N}(0, \text{plim } \mathcal{V}_N) + o_p(1). \tag{A.8}$$

Define  $V_i = Y_i(0) - Z_i'\delta + \sum_t w_{it} \mathcal{Z}_t' \gamma(\beta_{it} - \beta)$ , and

$$a_s = \sum_i w_{is} V_i, \qquad b_{st} = \sum_i w_{is} w_{it} (\beta_{it} - \beta). \tag{A.9}$$

Then we can write  $\epsilon_i = V_i + \sum_t w_{it} \tilde{\mathcal{X}}_t(\beta_{it} - \beta) + L_i$ . Since

$$E|r_N^{1/2}\sum_{i,s}\tilde{\mathcal{X}}_sw_{is}L_i| \leq r_N^{1/2}\sum_{i}(\sum_s Ew_{is}^2\sigma_s^2)^{1/2}E[L_i^2]^{1/2} \leq r_N^{1/2}\sum_{i}E[L_i^2]^{1/2} \to 0$$

by Assumption A.2(iii), and since  $0 = \sum_{i,s} \pi_{is}(\beta_{is} - \beta) = \sum_{s} \sigma_s^2 b_{ss}$ , we can decompose

$$r_N^{1/2}\sum_{s,i} ilde{\mathcal{X}}_sw_{is}\epsilon_i=r_N^{1/2}\sum_s ilde{\mathcal{X}}_s\sum_iw_{is}\left(V_i+\sum_tw_{it} ilde{\mathcal{X}}_t(eta_{it}-eta)+L_i
ight)=r_N^{1/2}\sum_s\mathcal{Y}_s+o_P(1),$$

where

$$\mathcal{Y}_s = \tilde{\mathcal{X}}_s a_s + (\tilde{\mathcal{X}}_s^2 - \sigma_s^2) b_{ss} + \sum_{t=1}^{s-1} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t (b_{st} + b_{ts}).$$

Observe that  $\mathcal{Y}_s$  is a martingale difference array with respect to the filtration  $\mathcal{F}_s = \sigma(\mathcal{X}_1, \dots, \mathcal{X}_s, \mathcal{F}_0)$ .

By the dominated convergence theorem and the martingale central limit theorem, it suffices to show that  $r_N^{1+\nu/4} \sum_{s=1}^S E_W[\mathcal{Y}_s^{2+\nu/2}] \to 0$  for some  $\nu > 0$  so that the Lindeberg condition holds, and that the conditional variance converges,

$$r_N \sum_{s=1}^{S} E[\mathcal{Y}_s^2 \mid \mathcal{F}_{s-1}] - \mathcal{V}_N = o_p(1).$$

To verify the Lindeberg condition, by the  $C_r$ -inequality, it suffices to show that

$$r_N^2 \sum_s E_W[\tilde{\mathcal{X}}_s^4 a_s^4] \to 0, \qquad \qquad r_N^{1+\nu/4} \sum_s E_W[(\tilde{\mathcal{X}}_s^2 - \sigma_s^2)^{2+\nu/2} b_{ss}^{2+\nu/2}] \to 0, \ r_N^2 \sum_s E_W\left(\sum_{t=1}^{s-1} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t b_{st}\right)^4 \to 0, \qquad \qquad r_N^2 \sum_s E_W\left(\sum_{t=1}^{s-1} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t b_{ts}\right)^4 \to 0.$$

Note that since  $E(\sum_t w_{it} \mathcal{Z}_t' \gamma(\beta_{it} - \beta))^4 \preceq (\sum_t w_{it})^4 \preceq 1$ , it follows from Assumptions A.2(iii) and A.1(iv), and the  $C_r$  inequality that the fourth moment of  $V_i$  exists and is bounded. Therefore, by arguments as in the proof of Lemma A.2,  $\sum_s E_W[a_s^4] \preceq \sum_s n_s^4$ , so that

$$r_N^2 \sum_{s} E_W[\tilde{\mathcal{X}}_s^4 a_s^4] = r_N^2 \sum_{s} E_W[E[\tilde{\mathcal{X}}_s^4 \mid \mathcal{F}_0] a_s^4] \leq r_N^2 \sum_{s} E_W[a_s^4] \leq r_N^2 \sum_{s} n_s^4 \leq \max_{s} n_s^2 r_N \to 0$$
 (A.10)

by Assumption 2(iii). Second, since  $\beta_{is}$  is bounded by Assumption A.1(i), we have  $b_{ss} \leq \sum_i w_{is}^2 \leq n_s$ , so that

$$r_N^{1+\nu/4} \sum_s E_W[(\tilde{\mathcal{X}}_s^2 - \sigma_s^2)^{2+\nu/2} b_{ss}^{2+\nu/2}] \leq r_N^{1+\nu/4} \sum_s n_s^{2+\nu/2} \leq (r_N \max_s n_s^2)^{\nu/4} \to 0.$$

Third, by similar arguments

$$r_N^2 \sum_{s} E_W \left( \sum_{t=1}^{s-1} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t b_{st} \right)^4 = r_N^2 \sum_{s} E_W E[\tilde{\mathcal{X}}_s^4 \mid \mathcal{F}_0] E\left[ \left( \sum_{t=1}^{s-1} \tilde{\mathcal{X}}_t b_{st} \right)^4 \mid \mathcal{F}_0 \right]$$

$$\leq r_N^2 \sum_{s} \left( \sum_{t=1}^{s-1} \sum_{i} w_{is} w_{it} \right)^4 \leq r_N^2 \sum_{s} n_s^4 \to 0.$$

The claim that  $r_N^2 \sum_s E_W \left( \sum_{t=1}^{s-1} \tilde{\mathcal{X}}_s \tilde{\mathcal{X}}_t b_{ts} \right)^4 \to 0$  follows by similar arguments.

It remains to verify that the conditional variance converges. Since  $V_N$  can be written as

$$\begin{aligned} \mathcal{V}_N &= \frac{1}{\sum_{s=1}^S n_s^2} \operatorname{var} \left( \sum_i (X_i - Z_i' \gamma) \epsilon_i \mid \mathcal{F}_0 \right) = r_N \sum_s E[\mathcal{Y}_s^2 \mid \mathcal{F}_0] + o_P(1) \\ &= r_N \sum_s \left[ E\left[ (\tilde{\mathcal{X}}_s a_s + (\tilde{\mathcal{X}}_s^2 - \sigma_s^2) b_{ss})^2 \mid \mathcal{F}_0 \right] + \sum_{t=1}^{s-1} \sigma_s^2 \sigma_t^2 (b_{st} + b_{ts})^2 \right] + o_P(1), \end{aligned}$$

we can decompose

$$r_N \sum_{s} E[\mathcal{Y}_s^2 \mid \mathcal{F}_{s-1}] - \mathcal{V}_N = 2D_1 + D_2 + 2D_3 + o_p(1),$$

where

$$D_{1} = r_{N} \sum_{s} (\sigma_{s}^{2} a_{s} + E[\tilde{\mathcal{X}}_{s}^{3} \mid \mathcal{G}_{0}] b_{ss}) \sum_{t=1}^{s-1} \tilde{\mathcal{X}}_{t} (b_{st} + b_{ts}),$$

$$D_{2} = r_{N} \sum_{s} \sigma_{s}^{2} \sum_{t=1}^{s-1} (\tilde{\mathcal{X}}_{t}^{2} - \sigma_{t}^{2}) (b_{st} + b_{ts})^{2},$$

$$D_{3} = r_{N} \sum_{s} \sigma_{s}^{2} \sum_{t=1}^{s-1} \sum_{u=1}^{t-1} \tilde{\mathcal{X}}_{t} \tilde{\mathcal{X}}_{u} (b_{st} + b_{ts}) (b_{su} + b_{us}).$$

It therefore suffices to show that  $D_j = o_p(1)$  for j = 1, 2, 3. Since  $E[D_j \mid \mathcal{F}_0] = 0$ , it suffices to show that  $var(D_j \mid W) = E_W[var(D_j \mid \mathcal{F}_0)]$  converges to zero. Since  $b_{st} + b_{ts} \preceq \overline{w}_{st}$ , and since  $E_W[|a_s a_t|] \preceq n_s n_t$ , and  $|b_{ss}| \preceq \overline{w}_{ss} \leq n_s$ , it follows that

$$\operatorname{var}(D_{1} \mid W) = r_{N}^{2} \sum_{t} E_{W} \left[ \sigma_{t}^{2} \left( \sum_{s=t+1}^{S} (b_{st} + b_{ts}) (\sigma_{s}^{2} a_{s} + E[\tilde{\mathcal{X}}_{s}^{3} \mid \mathcal{F}_{0}] b_{ss}) \right)^{2} \right]$$

$$\preceq r_{N}^{2} \sum_{t} \left( \sum_{s=t+1}^{S} \overline{w}_{st} n_{s} \right)^{2} \leq r_{N}^{2} \max_{s} n_{s}^{2} \sum_{t} \left( \sum_{s} \overline{w}_{st} \right)^{2} = r_{N} \max_{s} n_{s}^{2} \to 0,$$

where the convergence to zero follows by Assumption 2(iii). By similar arguments, since  $\overline{w}_{st} \leq n_s$ 

$$var(D_2 \mid W) = r_N^2 \sum_{t} E_W (\tilde{\mathcal{X}}_t^2 - \sigma_t^2)^2 \left( \sum_{s=t+1}^{S} \sigma_s^2 (b_{st} + b_{ts})^2 \right)^2 \leq r_N^2 \sum_{t} \left( \sum_{s=t+1}^{S} \overline{w}_{st}^2 \right)^2 \\
\leq r_N^2 \sum_{t} \left( \sum_{s=t+1}^{S} n_s \overline{w}_{st} \right)^2 \leq r_N \max_{s} n_s^2 \to 0.$$

Finally,

$$\operatorname{var}(D_{3} \mid W) = r_{N}^{2} \sum_{t} \sum_{u=t+1}^{S} E_{W} \sigma_{t}^{2} \sigma_{u}^{2} \left( \sum_{s=u+1}^{S} \sigma_{s}^{2} (b_{st} + b_{ts}) (b_{su} + b_{us}) \right)^{2}$$

$$\leq r_{N}^{2} \sum_{t} \sum_{u=t+1}^{S} \left( \sum_{s=u+1}^{S} \overline{w}_{st} \overline{w}_{su} \right)^{2} \leq r_{N}^{2} \sum_{s,t,u,v} \overline{w}_{st} \overline{w}_{su} \overline{w}_{vt} \overline{w}_{vu} \leq r_{N} \max_{s} n_{s}^{2} \to 0,$$

where the last line follows from the fact that since  $\sum_s \overline{w}_{st} = n_t$  and  $\overline{w}_{st} \leq n_s$ ,

$$\sum_{s,t,u,v} \overline{w}_{st} \overline{w}_{su} \overline{w}_{vt} \overline{w}_{vu} \leq \max_{s} n_{s} \sum_{s,t,u,v} \overline{w}_{su} \overline{w}_{vt} \overline{w}_{vu} = \max_{s} n_{s} \sum_{u,v} n_{u} n_{v} \overline{w}_{vu} \\
\leq \max_{s} n_{s}^{2} \sum_{u,v} n_{v} \overline{w}_{vu} = \max_{s} n_{s}^{2} / r_{N}. \quad (A.11)$$

Consequently,  $D_j = o_p(1)$  for j = 1, 2, 3, the conditional variance converges, and the theorem follows.

#### A.1.5 Proof of Proposition 5

We'll prove a more general result that doesn't assume constant treatment effects. In particular, we will show that under the conditions of the proposition when the condition  $\beta_{is} = \beta$  is dropped, the variance estimator  $\hat{V}_N = r_N \sum_s \hat{X}_s \hat{R}_s^2$ , where  $r_N = 1/\sum_{s=1}^s n_s^2$  satisfies

$$\hat{V}_N = r_N \sum_{s=1}^{S} E[\tilde{\mathcal{X}}_s^2 R_s^2 \mid \mathcal{F}_0] + o_p(1), \tag{A.12}$$

where, using the definitions of  $a_s$  and  $b_{st}$  in eq. (A.9),

$$R_s = \sum_{i=1}^{N} w_{is} \epsilon_i = a_s + \sum_{i=1}^{N} w_{is} L_i + \sum_{t=1}^{S} \tilde{\mathcal{X}}_t b_{st}.$$

Since under constant treatment effects,  $V_N = r_N \sum_{s=1}^S E[\tilde{X}_s^2 R_s^2 \mid \mathcal{F}_0]$ , the assertion of the proposition follows from eq. (A.12).

Throughout the proof, we write  $E_{\mathcal{F}_0}[\cdot]$  and  $E_W[\cdot]$  to denote expectations conditional on  $\mathcal{F}_0$ , and W, respectively. Let  $\tilde{\theta} = (\tilde{\beta}, \tilde{\delta}')'$ ,  $\theta = (\beta, \delta)$ ,  $M_i = (X_i, Z_i')'$ . We can decompose the variance estimator as

$$\hat{V}_{N} = r_{N} \sum_{s} (\hat{\mathcal{X}}_{s}^{2} - \tilde{\mathcal{X}}_{s}^{2}) \hat{R}_{s}^{2} + r_{N} \sum_{s} \tilde{\mathcal{X}}_{s}^{2} (\hat{R}_{s}^{2} - R_{s}^{2}) + r_{N} \sum_{s} (\tilde{\mathcal{X}}_{s}^{2} R_{s}^{2} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} R_{s}^{2}]) + r_{N} \sum_{s} E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} R_{s}^{2}].$$
(A 13)

We need to show that the first three terms are  $o_p(1)$ . Since  $\tilde{\epsilon}_i = \epsilon_i + M_i'(\theta - \tilde{\theta})$ , with  $\epsilon_i = V_i + L_i + \sum_t w_{it} \tilde{\mathcal{X}}_t(\beta_{it} - \beta)$ , we can decompose

$$\hat{R}_s^2 = \sum_{i,j} w_{is} w_{js} \tilde{\epsilon}_i \tilde{\epsilon}_j = R_s^2 + 2 \sum_{i,j} w_{js} w_{is} M_i'(\theta - \tilde{\theta}) \epsilon_j + \sum_{i,j} w_{is} w_{js} M_i'(\theta - \tilde{\theta}) M_j'(\theta - \tilde{\theta}). \tag{A.14}$$

Therefore, the second term in eq. (A.13) satisfies

$$r_{N} \sum_{s} \tilde{\mathcal{X}}_{s}^{2} (\hat{R}_{s}^{2} - R_{s}^{2}) = 2(\theta - \tilde{\theta})' \left[ r_{N} \sum_{s,i,j} w_{is} w_{js} \tilde{\mathcal{X}}_{s}^{2} M_{i} \epsilon_{j} \right] + (\theta - \tilde{\theta})' \left[ r_{N} \sum_{s,i,j} \tilde{\mathcal{X}}_{s}^{2} w_{is} w_{js} M_{i} M_{j}' \right] (\theta - \tilde{\theta})$$

$$= (\theta - \tilde{\theta})' O_{p}(1) + (\theta - \tilde{\theta})' O_{p}(1) (\theta - \tilde{\theta}) = o_{p}(1),$$

where the second line follows by applying Lemma A.3 to the terms in square brackets. Next, the third term in (A.13) can be decomposed as

$$r_{N} \sum_{s} (\tilde{\mathcal{X}}_{s}^{2} R_{s}^{2} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} R_{s}^{2}]) =$$

$$+ r_{N} \sum_{s} b_{ss}^{2} (\tilde{\mathcal{X}}_{s}^{4} - E_{\mathcal{I}_{0}}[\mathcal{X}_{s}^{4}]) + r_{N} \sum_{s < t} (b_{st}^{2} + b_{ts}^{2}) (\tilde{\mathcal{X}}_{s}^{2} \tilde{\mathcal{X}}_{t}^{2} - \sigma_{s}^{2} \sigma_{t}^{2}) + 2r_{N} \sum_{s} \sum_{t < u} b_{st} b_{su} \tilde{\mathcal{X}}_{s}^{2} \tilde{\mathcal{X}}_{t} \tilde{\mathcal{X}}_{u}$$

$$+ r_{N} \sum_{s} (\tilde{\mathcal{X}}_{s}^{2} - \sigma_{s}^{2}) a_{s}^{2} + r_{N} \sum_{i,j,s} w_{js} w_{is} (\tilde{\mathcal{X}}_{s}^{2} L_{i} L_{j} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} L_{i} L_{j}]) + 2r_{N} \sum_{i,s} w_{is} a_{s} (\tilde{\mathcal{X}}_{s}^{2} L_{i} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} L_{i}])$$

$$+ 2r_{N} \sum_{s < t} a_{s} b_{st} \tilde{\mathcal{X}}_{s}^{2} \tilde{\mathcal{X}}_{t} + 2r_{N} \sum_{s < t} a_{t} b_{ts} \tilde{\mathcal{X}}_{t}^{2} \tilde{\mathcal{X}}_{s} + 2r_{N} \sum_{s} a_{s} b_{ss} (\tilde{\mathcal{X}}_{s}^{3} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{3}])$$

$$+ r_{N} \sum_{i,s,t} w_{is} b_{st} (\tilde{\mathcal{X}}_{s}^{2} \tilde{\mathcal{X}}_{t} L_{i} - E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{s}^{2} \tilde{\mathcal{X}}_{t} L_{i}]). \quad (A.15)$$

We will show that all terms are of the order  $o_p(1)$ . By the inequality of von Bahr and Esseen, since  $b_{ss}$  is bounded by a constant times  $\overline{w}_{ss} \leq n_s$ ,

$$E_{\mathcal{F}_0}|r_N\sum_s b_{ss}^2(\tilde{\mathcal{X}}_s^4 - E_{\mathcal{F}_0}[\mathcal{X}_s^4])|^{1+\nu/4} \leq r_N^{1+\nu/4}\sum_s n_s^{2+\nu/2}E_{\mathcal{F}_0}|(\tilde{\mathcal{X}}_s^4 - E_{\mathcal{F}_0}[\mathcal{X}_s^4])|^{1+\nu/4} \leq (\max_s n_s^2 r_N)^{\nu/4} \to 0$$

by Assumption 2(iii), so that the first term is  $o_p(1)$ . The second term can be written as

$$r_{N} \sum_{s < t} (b_{st}^{2} + b_{ts}^{2}) (\tilde{\mathcal{X}}_{s}^{2} - \sigma_{s}^{2}) (\tilde{\mathcal{X}}_{t}^{2} - \sigma_{t}^{2}) + r_{N} \sum_{s \neq t} (b_{st}^{2} + b_{ts}^{2}) (\tilde{\mathcal{X}}_{s}^{2} - \sigma_{s}^{2}) \sigma_{t}^{2}$$

The conditional variance of both summands is bounded by a constant times  $r_N^2 \sum_s (\sum_t \overline{w}_{st}^2)^2 \le r_N^2 \cdot \sum_s n_s^4 \to 0$ , so that the second term is also  $o_p(1)$ . The third term admits the decomposition

$$2r_{N}\sum_{s}\sum_{t\leq u}b_{st}b_{su}\tilde{\mathcal{X}}_{s}^{2}\tilde{\mathcal{X}}_{t}\tilde{\mathcal{X}}_{u}=2r_{N}\sum_{s,t}\sum_{s\notin\{t,u\}}b_{st}b_{su}\tilde{\mathcal{X}}_{s}^{2}\tilde{\mathcal{X}}_{t}\tilde{\mathcal{X}}_{u}+2r_{N}\sum_{t\neq u}b_{tt}b_{tu}E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{t}^{3}]\tilde{\mathcal{X}}_{u}$$

$$2r_{N}\sum_{u\leq t}b_{tt}b_{tu}(\tilde{\mathcal{X}}_{t}^{3}-E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{t}^{3}])\tilde{\mathcal{X}}_{u}+2r_{N}\sum_{t\leq u}b_{tt}b_{tu}(\tilde{\mathcal{X}}_{t}^{3}-E_{\mathcal{I}_{0}}[\tilde{\mathcal{X}}_{t}^{3}])\tilde{\mathcal{X}}_{u}.$$

The conditional variance of the first summand is bounded by a constant times  $r_N^2 \sum_{t,u,s,v} \overline{w}_{st} \overline{w}_{su} \overline{w}_{vt} \overline{w}_{vu}$ , which converges to zero by the inequality in eq. (A.11). The conditional variance of the second summand is bounded by a constant times  $r_N^2 \sum_{s,t,u} \overline{w}_{tt} \overline{w}_{tu} \overline{w}_{ss} \overline{w}_{su} \leq r_N^2 \max_s n_s^2 \sum_s n_s^2 \to 0$ . Since  $(\tilde{\mathcal{X}}_t^3 - E_{\mathcal{F}_0}[\tilde{\mathcal{X}}_t^3]) \sum_{u=1}^{t-1} b_{tt} b_{tu} \tilde{\mathcal{X}}_u$  and  $\tilde{\mathcal{X}}_u \sum_{t=1}^{u-1} b_{tt} b_{tu} (\tilde{\mathcal{X}}_t^3 - E_{\mathcal{F}_0}[\tilde{\mathcal{X}}_t^3])$  are both martingale differences, by the inequality of von Bahr and Esseen, the 4/3-th absolute moment of the last two terms is bounded by a constant times  $r_N^{4/3} \sum_{s,t} \overline{w}_{tt}^{4/3} \overline{w}_{ts}^{4/3} \leq (\max_s n_s^2 r_N)^{1/3} r_N \sum_t n_t^2 \to 0$ . Thus, all summands in the above display are of the order  $o_p(1)$ , and the third term in eq. (A.15) is therefore also  $o_p(1)$ . The fourth term is  $o_p(1)$  by arguments in eq. (A.10). By the triangle and Cauchy-Schwarz inequalities, the conditional expectation of the absolute value of the fifth term is bounded by

$$2r_N \sum_{i,j,s} w_{js} w_{is} E_W[\tilde{\mathcal{X}}_s^4]^{1/2} E_W[L_i^4]^{1/4} E_W[L_j^4]^{1/4} \leq \max_i E_W[L_j^4]^{1/2} \to 0.$$

Similarly, conditional expectation of the absolute value of the sixth term is bounded by

$$4r_N \sum_{i,j,s} w_{is} w_{js} E_W[V_j^4]^{1/4} E[\tilde{\mathcal{X}}_s^4]^{1/2} E_W[L_i^4]^{1/4} \preceq \max_i E_W[L_j^4]^{1/4} \to 0.$$

Thus, by the Markov inequality, the fifth and sixth terms are both of the order  $o_p(1)$ . The conditional variance of the seventh and eighth terms is bounded by a constant times  $r_N^2 \sum_{s,t,u} n_s n_u \overline{w}_{st} \overline{w}_{ut} \le r_N \max_s n_s^2 \to 0$ , so that they are both  $o_p(1)$  by Markov inequality. By the inequality of von Bahr and Esseen, the 4/3-th absolute moment of the last ninth term is bounded by a constant times  $r_N^{4/3} \sum_s E_W[|a_s|^{4/3}] n_s^{4/3} \le (\max_s n_s^2 r_N)^{1/3} \to 0$ , since by Jensen's inequality,  $E|a_s|^{4/3} \le (Ea_s^2)^{2/3}$ , which is bounded by a constant times  $n_s^{4/3}$ . Finally, the expectation of the absolute value of the last term in eq. (A.15) is bounded by a constant times

$$r_N \sum_{i,s,t} w_{is} \overline{w}_{st} E_W[\tilde{\mathcal{X}}_s^4]^{1/2} E_W[\tilde{\mathcal{X}}_t^4]^{1/4} E_W[L_i^4]^{1/4} \preceq \max_i E_W[L_i^4]^{1/4} \to 0.$$

It remains to show that the first term in eq. (A.13) is  $o_p(1)$ . It follows from eq. (A.5) and eq. (24) that

$$\widehat{\mathcal{X}} = (W'W)^{-1}W'\ddot{X} = \widetilde{\mathcal{X}} - (W'W)^{-1}W'U(\hat{\gamma} - \gamma) - \mathcal{Z}(\hat{\gamma} - \gamma) - (W'W)^{-1}W'U\gamma,$$

where  $\hat{\gamma} = (Z'Z)^{-1}Z'X$ . Let  $\mathcal{U} = (W'W)^{-1}W'U$ , and denote the sth row by  $\mathcal{U}'_s$ . Since  $\mathcal{U}^4_{sk} = (\sum_i ((W'W)^{-1}W')_{si}U_{ik})^4$ , it follows by the Cauchy-Schwarz inequality that

$$E[\mathcal{U}_{sk}^4 \mid W] \leq \max_{s} E[(\sum_{i} ((W'W)^{-1}W')_{si}\mathcal{U}_{ik})^4 \mid W] \leq \max_{s} (\sum_{i} |((W'W)^{-1}W')_{si}|)^4,$$

which is bounded assumption of the proposition. Therefore, the fourth moments of  $\mathcal{U}_s$  are bounded uniformly over s. Observe also that  $E_W[\epsilon_i^4]$  is bounded uniformly over s by assumptions of the proposition. Therefore, by applying Lemma A.3 after using the expansion in eq. (A.14), we get

$$\begin{split} r_N \sum_s (\hat{\mathcal{X}}_s^2 - \tilde{\mathcal{X}}_s^2) \hat{R}_s^2 &= r_N \sum_s \hat{R}_s^2 (\mathcal{U}_s' \gamma)^2 - 2r_N \sum_s \hat{R}_s^2 \tilde{\mathcal{X}}_s \mathcal{U}_s' \gamma \\ &+ r_N \sum_s \hat{R}_s^2 \left[ 2\mathcal{U}_s' \gamma - 2\tilde{\mathcal{X}}_s + (\mathcal{Z}_s + \mathcal{U}_s)' (\hat{\gamma} - \gamma) \right] (\mathcal{Z}_s + \mathcal{U}_s)' (\hat{\gamma} - \gamma) \\ &= r_N \sum_s R_s^2 (\mathcal{U}_s' \gamma)^2 - 2r_N \sum_s R_s^2 \tilde{\mathcal{X}}_s \mathcal{U}_s' \gamma + O_p(1) (\hat{\gamma} - \gamma) + o_p(1). \end{split}$$

By Cauchy-Schwarz inequality,

$$r_N \sum_{s} E_W |R_s^2 (\mathcal{U}_s' \gamma)^2| \le r_N \sum_{s} (E_W [R_s^4])^{1/2} (E_W (\mathcal{U}_s' \gamma)^4)^{1/2} \le \max_{s} (E_W (\mathcal{U}_s' \gamma)^4)^{1/2} r_N \sum_{s} n_s^2 \to 0,$$

since  $\max_s E_W[(\mathcal{U}_s'\gamma)^4] \leq \max_i E_W(\mathcal{U}_i'\gamma)^4 \max_s (\sum_i |((W'W)^{-1}W')_{si}|)^4$ , which converges to zero by assumption of the proposition. By similar arguments,  $2r_N \sum_s E_W |R_s^2 \tilde{\mathcal{X}}_s \mathcal{U}_s' \gamma| \to 0$  also, so that

$$r_N \sum_{s} (\hat{\mathcal{X}}_s^2 - \tilde{\mathcal{X}}_s^2) \hat{R}_s^2 = o_p(1) + O_p(1) (\hat{\gamma} - \gamma) = o_p(1),$$

where the second equality follows from eq. (A.3).

#### A.1.6 Inference under heterogeneous effects

For valid (but perhaps conservative) inference under heterogeneous effects, we need to ensure that when  $\beta_{is} \neq \beta$ , eq. (32) holds with inequality, that is,

$$\frac{\sum_{s=1}^{S} \widehat{\mathcal{X}}_{s}^{2} \widehat{R}_{s}^{2}}{\sum_{s=1}^{S} n_{s}^{2}} \ge \mathcal{V}_{N} + o_{p}(1). \tag{A.16}$$

To discuss conditions under which this is the case, suppose, for simplicity, that  $L_i = 0$  so that eq. (11) holds, and  $R_s = \sum_s w_{is} \epsilon_i$ , where  $\epsilon_i = Y_i(0) - Z_i' \delta + \sum_s \mathcal{X}_s w_{is} (\beta_{is} - \beta)$  is the regression residual. Then the "middle sandwich" in the asymptotic variance sandwich formula,  $V_N$ , as defined in Proposition 4, can be decomposed into three terms:

$$V_{N} = \frac{\operatorname{var}\left(\sum_{s} \tilde{X}_{s} R_{s} \mid \mathcal{G}_{0}\right)}{\sum_{s=1}^{S} n_{s}^{2}} = \frac{\sum_{s} E[\tilde{X}_{s}^{2} R_{s}^{2} \mid \mathcal{G}_{0}]}{\sum_{s=1}^{S} n_{s}^{2}} - \frac{\sum_{s} E[\tilde{X}_{s} R_{s} \mid \mathcal{G}_{0}]^{2}}{\sum_{s=1}^{S} n_{s}^{2}} + \frac{\sum_{s \neq t} \operatorname{cov}(\tilde{X}_{s} R_{s}, \tilde{X}_{t} R_{t} \mid \mathcal{G}_{0})}{\sum_{s=1}^{S} n_{s}^{2}} = D_{1} + D_{2} + D_{3}, \quad (A.17)$$

where

$$D_{1} = \frac{\sum_{s} E[\tilde{X}_{s}^{2} R_{s}^{2} \mid \mathcal{F}_{0}]}{\sum_{s=1}^{S} n_{s}^{2}}, \qquad D_{2} = -\frac{\sum_{s} \left(\sum_{i} \sigma_{s}^{2} w_{is}^{2} (\beta_{is} - \beta)\right)^{2}}{\sum_{s=1}^{S} n_{s}^{2}}, D_{3} = \frac{\sum_{s \neq t} \sigma_{s}^{2} \sigma_{t}^{2} \sum_{i,j} w_{is} w_{it} (\beta_{it} - \beta) w_{jt} w_{js} (\beta_{js} - \beta)}{\sum_{s=1}^{S} n_{s}^{2}}.$$

As shown in the proof of Proposition 5 (see eq. (A.12)), the standard error estimator consistently estimates  $D_1$ . Under homogeneous effects,  $D_2 = D_3 = 0$ , and it follows that the standard error estimator is consistent. To ensure valid inference under heterogeneous effects, one needs to ensure that  $D_2 + D_3 \le o_p(1)$ . This is the case under several sufficient conditions, and we give two such conditions below.

The term  $D_2$  reflects the variability of the treatment effect and it is always negative. It therefore makes the variance estimate that we propose conservative if  $D_3 = o_p(1)$ . An analogous term, also reflecting the variability of the treatment effect, is present in randomized, and cluster-randomized trials, which is why the robust and cluster-robust standard error estimators yield conservative inference in these settings (see, for example Imbens and Rubin, 2015, Chapter 6). The term  $D_3$  reflects correlation between the treatment effects. It arises due to aggregating the sectoral shocks  $\mathcal{X}_s$  to a regional level to form the shifter  $X_i$ , and it has no analog in cluster-randomized trials. Indeed, in the example with "concentrated sectors", which is analogous to cluster-randomized trials if there are no covariates, the term equals zero, since in that case  $w_{is}w_{it} = 0$  for  $s \neq t$ . Our standard errors are thus valid, although conservative, in this case.

More generally, a sufficient condition for validity of our standard error estimator under treatment effect heterogeneity is that  $T_N = \sum_{s \neq t} (\sum_i w_{is} w_{it})^2 / \sum_s n_s^2 \to 0$ , since  $D_3 = O_p(T_N)$ . The condition  $T_N \to 0$  requires that the shares are sufficiently concentrated so that not too many regions "specialize" in more than one sector (in the sense that the sectoral share  $w_{is}$  is bounded away from zero as  $S \to \infty$ 

for more than one sector). For example,  $T_N \to 0$  if the share of the second-largest sector goes to zero as  $S \to \infty$ , that is  $\max_{i,s \neq s_i} w_{is} \to 0$ , where  $s_i$  denotes the largest sector in region i. This follows from the inequalities

$$\begin{split} \sum_{i,j} \sum_{s \neq t} w_{is} w_{it} w_{js} w_{jt} &= \sum_{i,j,s,t} I(s = s_i, t \neq s_i) w_{is} w_{it} w_{js} w_{jt} + \sum_{i,j} \sum_{s \neq t} I(s \neq s_i) w_{is} w_{it} w_{js} w_{jt} \\ &\leq \sum_{i,j,s,t} I(t \neq s_i) w_{is} w_{it} w_{js} w_{jt} + \sum_{i,j,s,t} I(s \neq s_i) w_{is} w_{it} w_{js} w_{jt} \\ &\leq 2 \max_{i,s \neq s_i} w_{is} \sum_{i,j,s,t} w_{it} w_{js} w_{jt} \leq 2 \max_{i,s \neq s_i} w_{is} \sum_{t} n_t^2 = o(r_N). \end{split}$$

For illustration, in the empirical application in Section 7.1,  $T_N = 0.0014$ .

A second sufficient condition for the asymptotic negligibility of  $D_3$  is that the conditional variance of the shifters  $\mathcal{X}_s$ ,  $\sigma_s^2 = E[(\mathcal{X}_s - \mathcal{Z}_s'\gamma)^2 \mid \mathcal{F}_0]$  and the weighted treatment effects  $\sigma_s^2\beta_{is}$  are meanindependent of the shares W, provided some additional mild regularity conditions are satisfied, as shown in the lemma below. Importantly, this condition still allows the treatment effects to depend on the controls Z, or other aspects of the model, such as  $Y_i(0)$ : the covariance assumptions in the lemma allow the treatment effects  $\beta_{is}$  to be correlated within a region and/or within a sector. The assumption that  $\sum_i \sum_{s\neq t} w_{is}^2 w_{it}^2 / \sum_{s'} n_{s'}^2 \to 0$  holds if either a vanishing fraction of regions "specialize" in more than one sector (in the sense that the sectoral share  $w_{is}$  is bounded away from zero as  $S \to \infty$  for more than one sector). It also holds if  $S/\sum_s n_s \to 0$ , that is, the number of regions grows faster than the number of sectors.<sup>2</sup> For illustration, the quantity equals 0.00022 in the empirical example in Section 7.1. The lemma uses the notation defined at the beginning of Appendix A.1.5.

**Lemma A.4.** Suppose that the assumptions of Proposition 4 hold. Suppose, in addition, that the conditional expectations  $E[\sigma_s^2\beta_{is}\mid W]=E[(\mathcal{X}_s-\mathcal{Z}_s'\gamma)^2\beta_{is}\mid W]$  and  $E[\sigma_s^2\mid W]=E[(\mathcal{X}_s-\mathcal{Z}_s'\gamma)^2\mid W]$  do not depend on W, i, or s. Suppose also that  $\text{cov}(\sigma_s^2\beta_{is},\sigma_t^2\beta_{jt}\mid W)=0$  unless i=j or s=t, that  $\text{cov}((\sigma_s^2\beta_{is},\sigma_s^2),\sigma_t^2\mid W)=0$  unless s=t, and that  $\sum_{s\neq t}\sum_i w_{is}^2w_{it}^2/\sum_s n_s^2\to 0$ . Then  $D_3=o_p(1)$ .

*Proof.* By Assumptions A.1(i) and A.1(iii),

$$r_N \sum_{s \neq t} \sum_i E_W |\sigma_s^2 \sigma_t^2 w_{is}^2 w_{it}^2 (\beta_{it} - \beta) (\beta_{js} - \beta)| \leq r_N \sum_{s \neq t} \sum_i w_{is}^2 w_{it}^2,$$

and the right-hand side converges to zero by assumption of the lemma. Therefore, by Markov inequality,  $D_3 = r_N \sum_{s \neq t} \sum_{i \neq j} w_{is} w_{it} \sigma_t^2(\beta_{it} - \beta) w_{jt} w_{js} \sigma_s^2(\beta_{js} - \beta) + o_p(1)$ . By Assumptions A.1(i), A.2(iii) and A.1(iv), and assumptions of the lemma, the variance of  $\sum_{i,s} w_{is}^2 \sigma_s^2 \beta_{is}/N$  and of  $\sum_{i,s} w_{is}^2 \sigma_s^2/N$  conditional on W is bounded by a constant times  $\sum_{i,j,s} w_{is}^2 w_{js}^2/N^2 + \sum_{i,s,t} w_{is}^2 w_{it}^2/N^2 \leq 2 \max_s n_s/N \to 0$ . Therefore, by Assumption A.1(ii),  $\beta = \mu/\sigma + o_p(1)$ , where  $\mu = E_W[(\mathcal{X}_s - \mathcal{Z}_s'\gamma)^2 \beta_{is}]$  and  $\sigma = E_W[\sigma_s^2]$ . It then follows that

$$D_{3} = r_{N} \sum_{s \neq t} \sum_{i \neq j} w_{is} w_{it} w_{jt} w_{js} (\sigma_{s}^{2} \beta_{js} - \mu) (\sigma_{t}^{2} \beta_{it} - \mu) - 2r_{N} \sum_{s \neq t} \sum_{i \neq j} w_{is} w_{it} w_{jt} w_{js} (\mu - \sigma_{t}^{2} \mu / \sigma) (\sigma_{s}^{2} \beta_{js} - \mu)$$

<sup>&</sup>lt;sup>2</sup>This follows from the inequalities  $\sum_{i,s,t} w_{is}^2 w_{it}^2 \leq \sum_s n_s$ , and  $\sum_s n_s^2 \geq (\sum_s n_s)^2 / S$ .

$$+ r_N \sum_{s \neq t} \sum_{i \neq j} w_{is} w_{it} w_{jt} w_{js} (\mu - \sigma_s^2 \mu / \sigma) (\mu - \sigma_t^2 \mu / \sigma) + o_p(1).$$

Each term in the above display has mean zero, and variance bounded by a constant times

$$\begin{split} r_N^2 \sum_{s \neq t} (\sum_{i \neq j} w_{is} w_{it} w_{jt} w_{js})^2 + r_N^2 \sum_{i \neq j} (\sum_{s \neq t} w_{is} w_{it} w_{jt} w_{js})^2 \\ & \leq r_N^2 \max_s n_s^2 \sum_{i,j,s,t} w_{is} w_{it} w_{js} w_{jt} + r_N^2 \sum_{i,j,s,t} w_{it} w_{jt} w_{is} w_{js} \leq 2r_N \max_s n_s^2 \to 0. \end{split}$$

Therefore,  $D_3 = o_p(1)$  by Markov inequality and dominated convergence theorem.

Although both the condition  $T_N \to 0$  and the conditions in Lemma A.4 may be restrictive in some applications, note that both of these conditions are merely sufficient, but not necessary for  $D_3 + D_2 \le o_p(1)$ .

#### A.2 Proofs and additional details for IV regression

We prove eqs. (38) and (45), and show that the bias of the estimator  $\tilde{\alpha}$  is of the order  $\frac{1}{N}\sum_{i,s}w_{is}\check{w}_{is}/\check{n}_s$ . We also discuss how the case with estimated shifters relates to the literature on many instruments.

#### A.2.1 Assumptions

To compactly state the assumptions, let  $\mathcal{G}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W, \check{W})$ , and put  $\check{W} = W$ , and  $\psi_{is} = 0$  if the shifters  $\mathcal{X}$  are observed.

We impose an instrumental variables version of the regularity conditions Assumptions A.1 and A.2:

**Assumption A.3.** (i) For some  $\nu > 0$ ,  $E[\mathcal{X}_s^{2+\nu} \mid \mathcal{F}_0]$  exists and is uniformly bounded. The support of  $\beta_{is}$  is bounded. Conditional on  $(W, \check{W})$ , the second moments of  $Y_{1i}(0), Y_{2i}(0), U_i$  and  $\mathcal{Z}_s$  exist, and are bounded uniformly over i and s. Z'Z/N converges in probability to a positive definite nonrandom limits; (ii) For some  $\nu > 0$ ,  $E[|\mathcal{X}_s|^{4+\nu} \mid \mathcal{F}_0, \Psi]$  is uniformly bounded, and  $\mathcal{X}_s$  are independent across s conditional on  $(\mathcal{F}_0, \Psi)$ , with  $E[\mathcal{X}_s \mid \mathcal{F}_0, \Psi] = E[\mathcal{X}_s \mid \mathcal{Z}]$ . Conditional on  $(W, \check{W})$ , the fourth moments of  $Y_{1i}(0)$ ,  $U_i$  and  $\mathcal{Z}_s$  exist, and are bounded uniformly over i and s. Assumption A.2(iv) and Assumption A.2(v) hold  $\delta = E[Z'Z]^{-1}E[Z'Y_1(0)], \check{\delta} = (Z'Z)^{-1}Z'Y_1(0), \text{ and } \varepsilon_i = Y_{1i} - Y_{2i}\alpha - Z'_i\delta$ .

Assumption A.3(i) is needed for consistency, and Assumption A.3(ii) is needed for asymptotic normality. When the shifters are observed, these assumptions are natural analogs of the regularity conditions in the OLS case that are needed for consistency (Assumptions A.1(i) and A.1(iii) and Assumptions A.2(i) and A.2(ii) and asymptotic normality (Assumption A.1(iv) and Assumptions A.2(iii) to A.2(v)). When the shifters are not directly observed, Assumption A.3(ii) strengthens Assumption 4(ii) so that it holds conditionally on  $\Psi$  also.

If  $X_i$  is not observed, we need to impose additional conditions on  $\psi_{is}$  and the weights  $\check{w}_{is}$ :

**Assumption A.4.** Let  $A_{-i}$  denote the vector A with the ith element removed. Let  $\mathcal{F}_{-i} = \sigma(Y_{1,-i}(0), Y_{2,-i}(0), U_{-i}, W, \check{W}, \mathcal{Z})$ . (i) For all s and i,  $E[\check{w}_{is}\psi_{is} \mid \mathcal{F}_{-i}] = 0$ , and  $E[\check{w}_{is}^2\psi_{is}^2 \mid \mathcal{F}_{0}]$  is bounded by a universal constant times  $\check{w}_{is}^2$ ; (ii) For all s, t, and all  $i \neq j$ ,  $E[\check{w}_{is}\check{w}_{jt}\psi_{is}\psi_{jt} \mid \mathcal{F}_{-i}] = 0$ ; (iii)  $\max_{i,s}\check{w}_{is}/\sum_{j=1}^{N}\check{w}_{js}$ 

is bounded away from 1; (iv)  $\max_i \sum_s \frac{n_s}{\tilde{n}_s} \check{w}_{is}$  is bounded; (v) There exist variables  $\{C_i, \eta_i\}_{i=1}^N$  such that  $(Y_{i1}(0), U_i) = C_i + \eta_i$ , and conditional on (C, W, Z),  $\{\check{w}_{i1}\psi_{i1}, \ldots, \check{w}_{iS}\psi_{iS}, \eta_i\}$  are independent across i, with uniformly bounded second moments, and  $E[(\check{w}_{is}\psi_{is}, \eta_i) \mid C, W, \check{W}, Z] = 0$ . Conditional on  $(W, \check{W})$ , the fourth moments of  $\eta_i$  and  $C_i$  are uniformly bounded; (vi)  $E_{W,\check{W}}[\check{w}_{is}\psi_{js}]^4$  is bounded by a constant times  $\check{w}_{is}^4$ ; (vii)  $N/(\sum_s n_s^2)^2 \to 0$ .

Assumption A.4(i) requires that the local shock  $\psi_{is}$  in region i is mean zero, and unrelated to the regional variables  $(Y_{1j}(0), Y_{2j}(0), U_j)$  in other regions. Importantly, it allows these local shocks to be correlated with the regional variables in region i. In particular, in some applications, it may be the case that  $Y_{2i} = \sum_s w_{is} X_{is} + \eta_i$ , with the additional term  $\eta_i$  potentially zero. In this case  $\psi_{is}$  is always mechanically correlated with  $Y_{2i}$  (and hence also  $Y_{1i}$  if there is endogeneity). As we will show below, this correlation causes bias in the estimator  $\tilde{\alpha}$  that ignores the estimation error in the shifters.

Assumption A.4(ii) requires that these local shocks are uncorrelated across regions: this ensures consistency of the leave-one-out estimator. One could relax this assumption and instead only require no correlation across clusters of regions, in which case one would have to leave out region i's cluster when constructing an estimate of  $X_i$ . The local shocks are allowed to be correlated across industries in the same region. The scaling by  $\check{w}_{is}$  in the statement of the assumption allows for the possibility that  $X_{is}$  gives an uninformative signal about  $\mathcal{X}_s$  if  $\check{w}_{is} = 0$ . Assumption A.4(iii) imposes two mild regularity conditions on the weights; it ensures that no single weight  $\check{w}_{is}$  is so large that it dominates a particular sector, which is necessary for the leave-one-out estimator to be well-defined.

Assumption A.4(iv) ensures that the weights  $\check{w}_{is}$  are balanced in the sense that no single region i is asymptotically non-negligible. The condition holds under equal weighting,  $\check{w}_{is}=1$ , since in this case  $\sum_s n_s \check{w}_{is}/\check{n}_s = \sum_s n_s/N \le 1$ . Oftentimes, the weights  $\check{w}_{is}$  take the form  $\check{w}_{is}=L_i w_{is}$ , where  $L_i$  is a measure of the size or region i. In this case,  $\sum_s n_s \check{w}_{is}/\check{n}_s = \sum_s \frac{L_i w_{is}}{\bar{L}_s}$ , where  $\bar{L}_s = \check{n}_s/n_s = \sum_i L_i w_{is}/\sum_j w_{js}$  is the sector-weighted average size of a region. Thus, the condition requires that the sector-weighted size of region i,  $w_{is}L_i$ , is non-negligible relative to the national average for at most a fixed number of sectors. Since  $\sum_s n_s \check{w}_{is}/\check{n}_s \le \frac{\max_i L_i}{\min_j L_j}$ , a sufficient condition is that the ratio of the largest to the smallest region is bounded.

Assumptions A.4(v) to A.4(vii) are only needed for asymptotic normality. Assumption A.4(v) effectively imposes that only the part of  $(Y_{i1}(0), U_i)$  that's independent of  $\psi_i$  is allowed to be correlated across i; the part that's related to  $\psi_i$  must be independent across i. Assumption A.4(vii) imposes a very mild condition on the sector sizes, and holds, for example, if  $n_s \ge 1$ .

#### A.2.2 Asymptotic results

When the shifters are observed, we obtain the following result, which implies eq. (38) in the main text:

**Proposition A.1.** Suppose that Assumptions 2(i) and 2(ii) and Assumption 4 hold with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W)$ , and that Assumption A.3(i) holds. Then the estimator  $\hat{\alpha}$  in eq. (36) is consistent. If, in addition, Assumption 2(iii) and Assumption A.3(ii) hold, then  $\hat{\alpha}$  satisfies eq. (38), provided  $V_N$  converges to a non-random limit.

The consistency result follows since by arguments analogous to those in the proof of Proposition 3 (see, in particular, eq. (A.7)),  $N^{-1}\sum_i\ddot{X}_iY_{1i}(0)=o_p(1)$ , and  $N^{-1}\sum_i\ddot{X}_iY_{2i}(0)=N^{-1}\sum_{i,s}\sigma_s^2w_{is}^2\beta_{is}+o_p(1)$ . Furthermore, since  $N^{-1}\sum_{i,s}\sigma_s^2w_{is}^2\beta_{is}\neq 0$  by Assumption 4(iv), it follows by Slutsky's lemma that

$$\hat{\alpha} - \alpha = \frac{N^{-1} \sum_{i} \ddot{X}_{i} Y_{1i}(0)}{N^{-1} \sum_{i} \ddot{X}_{i} Y_{2i}(0)} = o_{p}(1).$$

The asymptotic normality result follows since  $r_N^{1/2} \sum_i \ddot{X}_i Y_{1i}(0) = \mathcal{N}(0, \mathcal{V}_N) + o_p(1)$  by arguments analogous to those in proof of Proposition 4 (see, in particular, eq. (A.8)).

**Proposition A.2.** Suppose that Assumptions 2(i) and 2(ii) and Assumption 4 hold with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W, \check{W})$ , and that Assumption A.3(i) and Assumptions A.4(i) to A.4(iv) hold. Then the estimator  $\hat{\alpha}_-$  is consistent for  $\alpha$ . Furthermore, the estimator  $\tilde{\alpha}$  satisfies  $\tilde{\alpha} = \alpha + O_p\left(\frac{1}{N}\sum_{i,s}\frac{w_{is}\check{w}_{is}}{\check{n}_s}\right)$ , provided that  $(\mathring{X}'Y_2/N)^2$  converges to a strictly positive probability limit.

The asymptotic bias  $\tilde{\alpha}$  is analogous to the own observation bias of the two-stage least squares (2SLS) estimator in settings with many instruments. To see the connection, consider the special case in which  $Y_{2i} = \sum_s w_{is} X_{is} = \sum_s w_{is} \mathcal{X}_s + \sum_s w_{is} \psi_{is}$ , and each region specializes in a single sector,  $w_{is} = \mathbb{I}\{s(i) = s\}$ , with  $\check{w}_{is} = w_{is}$ . Then we can write  $Y_{2i} = \mathcal{X}_{s(i)} + \psi_{is(i)}$ , and  $\hat{X}_i = \frac{1}{n_s} \sum_i \mathbb{I}\{s(i) = s\}Y_{2i}$ . This setting is isomorphic to a many instrument setting, where the instruments are group indicators  $\mathbb{I}\{s(i) = s\}$ , individuals are assigned to groups, and the average treatment intensity depends on group membership (for example, the endogenous variable may be the length of a sentence, the groups are groups of individuals assigned to the same judge, and judges differ in their average sentencing severity  $\mathcal{X}_s$ ). Then the first-stage predictor used by the 2SLS estimator is  $\hat{X}_i$ . Since  $\hat{X}_i$  puts weight  $1/n_s$  on the first-stage regression error  $\psi_{is(i)}$ , this generates a bias in the 2SLS estimate, which persists in large samples unless the weight  $1/n_s$  is negligible. In our setting, Proposition A.2 shows that the bias is of the order  $\frac{1}{N}\sum_{i,s}\frac{w_{is}w_{is}}{n_s} \leq \frac{1}{N}\sum_{i,s}\frac{\hat{w}_{is}}{n_s} = S/N$ . Thus, a sufficient condition for consistency is that the number of sectors grows more slowly than the number of regions. This is analogous to the requirement for 2SLS consistency in the many instruments literature that the number of instruments grows more slowly than the number of observations.

**Proposition A.3.** Suppose that Assumptions 2 and 4 hold with  $\mathcal{F}_0 = (\mathcal{Z}, U, Y_1(0), Y_2(0), B, W, \check{W})$ , and that Assumptions A.3 and A.4 hold. Suppose that  $V_N$  and  $W_N$ , defined in eq. (45), converge in probability to non-random limits. Then

$$\frac{N}{\sqrt{\sum_{s=1}^{S} n_s^2}} (\hat{\alpha}_- - \alpha) = \mathcal{N}\left(0, \frac{\mathcal{V}_N + \mathcal{W}_N}{\left(\frac{1}{N} \sum_i \ddot{X}_i Y_{2i}\right)^2}\right) + o_p(1).$$

The additional term  $W_N$  in the expression for the asymptotic variance of  $\hat{\alpha}_-$ , which is absent if X is observed, is of the order

$$\frac{1}{\sum_{s} n_s^2} \sum_{j} \left( \sum_{s} \frac{n_s \check{w}_{js}}{\check{n}_s} \right)^2 + \frac{1}{\sum_{s} n_s^2} \sum_{i,j,s,t} \frac{w_{is} \check{w}_{js}}{\check{n}_s} \frac{w_{jt} \check{w}_{it}}{\check{n}_t} \preceq \frac{N+S}{\sum_{s} n_s^2} \preceq S/N + (S/N)^2,$$

where the second inequality follows Assumption A.4(iv), and the last inequality follows by  $\ell_1$ - $\ell_2$  norm inequality  $\sqrt{S \sum_s n_s^2} \ge \sum_s n_s$ , and we assume that  $\sum_s w_{is}$  is bounded away from zero so that  $\sum_s n_s$  is of the same order as N. Therefore, if the number of regions grows faster than the number of sectors, the term will be asymptotically negligible. This is similar to the result in the many IV literature that the usual standard error formula for the jackknife IV estimator is valid if the number of instruments grows more slowly than the sample size. The term  $W_N$  also has a similar structure to the many-instrument term in the standard error for jackknife IV (see Chao et al. (2012)).

#### A.2.3 Proof of Proposition A.2

By the arguments in the proof of Proposition 3, for the first part of the proposition, it suffices to show that  $(\ddot{X}_{-} - \ddot{X})'Y_1/N = o_p(1)$  and  $(\ddot{X}_{-} - \ddot{X})'Y_2/N = o_p(1)$ , which in turn follows if we can show that for  $A_i \in \{Y_{1i}, Y_{2i}, Z_i\}$ ,

$$\frac{1}{N} \sum_{i} (\hat{X}_{i,-} - X_i) A_i = \frac{1}{N} \sum_{j,i,s} \mathbb{I}\{j \neq i\} \frac{w_{is} \check{w}_{js}}{\check{n}_{s,-i}} \psi_{js} A_i = o_p(1), \tag{A.18}$$

where  $\check{n}_{s,-i} = \sum_{j=1}^{N} \check{w}_{js} - \check{w}_{is}$ . By Assumption A.4(i), conditional on W, this term has mean zero. Since by Assumption A.4(ii),  $\mathbb{I}\{j \neq j'\}\mathbb{I}\{j \neq i\}\mathbb{I}\{j' \neq i'\}E_{W,\check{W}}[w_{js}\psi_{js}A_i \cdot w_{j't}\psi_{j't}A_{i'}] = 0$  unless j = i' and j' = i, the variance of this term is given by

$$\begin{split} \frac{1}{N^2} \sum_{j,i,i',s,t} \mathbb{I}\{j \neq i,i'\} w_{is} w_{i't} \frac{E_{W,\check{W}}[\check{w}_{js}\psi_{js}A_i\check{w}_{jt}\psi_{jt}A_{i'}]}{\check{n}_{s,-i}\check{n}_{t,-i'}} \\ &+ \frac{1}{N^2} \sum_{j,i,s,t} \mathbb{I}\{j \neq i\} w_{is} w_{jt} \frac{E_{W,\check{W}}[\check{w}_{js}\psi_{js}\check{w}_{it}\psi_{it}A_iA_j]}{\check{n}_{s,-i}\check{n}_{t,-j}}. \end{split}$$

Now, by Assumption A.3(i),  $E_{W,\check{W}}[\check{w}_{js}\psi_{js}A_i\check{w}_{jt}\psi_{jt}A_{i'}] \leq \check{w}_{js}\check{w}_{jt}E_{W,\check{W}}[A_iA_{i'}]$ , which is bounded by a constant times  $\check{w}_{js}\check{w}_{jt}$  since the second moment of  $A_i$  is uniformly bounded by Assumption A.4(i). Similarly,  $E_{W,\check{W}}[\check{w}_{js}\psi_{js}\check{w}_{it}\psi_{it}A_iA_j]$  is bounded by a constant times  $\check{w}_{js}\check{w}_{it}$ . Therefore, the expression in the preceding display is bounded by a constant times

$$\frac{1}{N^{2}} \sum_{j,i,i',s,t} w_{is} w_{i't} \frac{\check{w}_{js} \check{w}_{jt}}{\check{n}_{s,-i} \check{n}_{t,-i'}} + \frac{1}{N^{2}} \sum_{j,i,s,t} w_{is} w_{jt} \frac{\check{w}_{js} \check{w}_{it}}{\check{n}_{s,-i} \check{n}_{t,-j}} \\
\leq \frac{1}{N^{2}} \max_{is} \frac{\check{n}_{s}^{2}}{\check{n}_{s,-i}^{2}} \left[ \sum_{j} \left( \sum_{s} n_{s} \frac{\check{w}_{js}}{\check{n}_{s}} \right)^{2} + N \right] \leq \frac{1}{N'}$$

where the first inequality follows since  $\sum_{j,i,s,t} w_{is} w_{jt} \frac{\psi_{js} \psi_{it}}{\check{n}_s \check{n}_t} \leq \sum_{j,i,s,t} w_{is} w_{jt} \frac{\check{w}_{js}}{\check{n}_s} \leq \sum_{j,s} n_s \frac{\check{w}_{js}}{\check{n}_s} = N$ , and the second inequality follows since Assumption A.4(iii) implies  $\max_{is} \check{n}_s / \check{n}_{s,-i} = 1/(1 - \max_{is} \check{w}_{is} / \check{n}_{is})$  is bounded, and since Assumption A.4(iv) implies that  $\sum_j \left(\sum_s n_s \frac{\check{w}_{js}}{\check{n}_s}\right)^2 \leq \sum_j 1 = N$ . Therefore, eq. (A.18) holds by Markov inequality and the dominated convergence theorem.

To show the second part of the proposition, decompose

$$\frac{1}{N} \sum_{i} A_{i} (\hat{X}_{i} - \hat{X}_{i,-}) = \frac{1}{N} \sum_{i,s} \frac{w_{is} \check{w}_{is}}{\check{n}_{s}} \psi_{is} A_{i} - \frac{1}{N} \sum_{i,j,s} \mathbb{I} \{j \neq i\} \frac{\check{w}_{is}}{\check{n}_{s}} \frac{w_{is} \check{w}_{js}}{\check{n}_{s,-i}} \psi_{js} A_{i}.$$

By arguments similar to those above, conditional on  $(W, \check{W})$ , the second term has mean zero and variance that converges to zero. By Assumption A.4(i) and Jensen's inequality, the mean of the first term is of the order  $\frac{1}{N}\sum_{i,s} \frac{w_{is}\check{w}_{is}}{\check{n}_s}$ . Consequently, provided that  $(\mathring{X}'Y_2/N)^2$  converges to a strictly positive limit, we have

$$\tilde{\alpha} - \alpha = \frac{O_p(\frac{1}{N}\sum_{i,s} \frac{w_{is}\check{w}_{is}}{\check{n}_s})}{\ddot{X}'Y_2/N} = O_p\left(\frac{1}{N}\sum_{i,s} \frac{w_{is}\check{w}_{is}}{\check{n}_s}\right),$$

as required.

#### A.2.4 Proof of Proposition A.3

Since  $Nr_N^{1/2}(\hat{\alpha}_- - \alpha) = r_N^{1/2}\hat{X}'_-Y_1(0)/\hat{X}'_-Y_2/N = r_N^{1/2}\hat{X}'_-Y_1(0)\cdot(\beta_{FS}N^{-1}\sum_{i,s}w_{is}^2\sigma_s^2)^{-1}(1+o_P(1))$ , it suffices to show that

$$r_N^{1/2} \hat{X}'_- Y_1(0) = \mathcal{U}(0, \mathcal{V}_N + \mathcal{W}_N) + o_p(1).$$

By arguments as in the proof of Proposition 4,

$$\begin{split} r_{N}^{1/2}\hat{X}_{-}'Y_{1}(0) &= r_{N}^{1/2}(W\tilde{X} - U\gamma + (\hat{X}_{-} - X))'(Z(\delta - \check{\delta}) + \epsilon_{\Delta}) \\ &= r_{N}^{1/2}(W\tilde{X})'\epsilon_{\Delta} + r_{N}^{1/2}(\hat{X}_{-} - X)'(Z(\delta - \check{\delta}) + \epsilon_{\Delta}) + o_{p}(1) \\ &= r_{N}^{1/2}(W\tilde{X} + (\hat{X}_{-} - X))'\epsilon_{\Delta} + o_{p}(1), \end{split}$$

where the last line follows since  $(\hat{X}_- - X)'Z/N = o_p(1)$  by eq. (A.18). Let  $C_{\Delta,i} = C_{iY(0)} - C'_{iU}\delta - \sum_s w_{is} \mathcal{Z}'_s \delta$  and  $\eta_{\Delta,i} = \eta_{iY(0)} - \eta'_{iU}\delta$ , so that  $\epsilon_{\Delta,i} = Y_{i1}(0) - Z'_i\delta = \eta_{\Delta,i} + C_{\Delta,i}$ . Then we can decompose

$$r_N^{1/2}(W\tilde{\mathcal{X}}+(\hat{X}_--X))'\epsilon_{\Delta}=r_N^{1/2}\sum_{j=1}^{N+S}\mathcal{Y}_j,$$

where

$$\mathcal{Y}_{j} = \begin{cases} \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is} \check{w}_{js} \frac{\mathbb{I}\{j \neq i\} \psi_{js} C_{\Delta,i}}{\check{n}_{s,-i}} + \sum_{i=1}^{j-1} \sum_{s=1}^{S} \left[ \frac{w_{is} \check{w}_{js} \psi_{js} \eta_{\Delta,i}}{\check{n}_{s,-i}} + \frac{\check{w}_{is} w_{js} \eta_{\Delta,j} \psi_{is}}{\check{n}_{s,-j}} \right], & j = 1, \dots, N, \\ \tilde{\mathcal{X}}_{j-N} \sum_{i} w_{i,j-N} \epsilon_{\Delta,i}, & j = N+1, \dots, N+S. \end{cases}$$

Let H denote the matrix with rows  $\eta_i'$ , and define the  $\sigma$ -fields  $\mathcal{G}_i = \sigma(W, \check{W}, \mathcal{Z}, C, \eta_1, \ldots, \eta_i, \psi_1, \ldots, \psi_i)$ ,  $i = 1, \ldots, N$ ,  $\mathcal{G}_i = \sigma(W, \check{W}, \mathcal{Z}, C, H, \Psi, \mathcal{X}_1, \ldots, \mathcal{X}_{j-N})$ ,  $j = N+1, \ldots, N+S$ . Then, under Assumption A.4(v),  $\mathcal{Y}_j$  is a martingale difference array with respect to the filtration  $\mathcal{G}_j$ . Since by the arguments in the proof of Proposition 4,  $r_N^{1+\nu/4} \sum_{j=N+1}^{N+S} E_{W,\check{W}}[\mathcal{Y}_j^{2+\nu/2}] \to 0$ , and  $r_N \sum_{j=N+1}^{N+S} E[\mathcal{Y}_j^2 \mid \mathcal{G}_{j-1}] - \mathcal{V}_N = o_p(1)$ , it suffices to show that  $r_N^2 \sum_{j=1}^N E_{W,\check{W}}[\mathcal{Y}_j^4] \to 0$ , and  $r_N \sum_{j=1}^N E[\mathcal{Y}_j^2 \mid \mathcal{G}_{j-1}] - \mathcal{W}_N = o_p(1)$ . The result then follows by a martingale central limit theorem.

Since  $\check{n}_s/\check{n}_{s,-i}$  is bounded, and  $\sum_s w_{js} \leq 1$ , and since  $\sum_{s=1}^S \frac{n_s \check{w}_{js}}{\check{n}_s}$  is bounded by Assumption A.4(iv), we have the bound

$$r_{N}^{2} \sum_{j=1}^{N} E_{W,\check{W}} \left( \sum_{i=1}^{j-1} \sum_{s=1}^{S} w_{is} \check{w}_{js} \frac{\psi_{js} \eta_{\Delta,i}}{\check{n}_{s,-i}} \right)^{4} \preceq r_{N}^{2} \sum_{j} \left( \sum_{i=1}^{j-1} \sum_{s=1}^{S} \frac{w_{is} \check{w}_{js}}{\check{n}_{s}} \right)^{4} \leq r_{N}^{2} \sum_{j=1}^{N} \left( \sum_{s=1}^{S} \frac{n_{s} \check{w}_{js}}{\check{n}_{s}} \right)^{4} \leq r_{N}^{2} N,$$

which converges to zero by Assumption A.4(vii). By an analogous argument, the conditional expectation of  $r_N^2 \sum_{j=1}^N \left(\sum_{i=1}^N \sum_{s=1}^S w_{is} \check{w}_{js} \frac{\mathbb{I}\{j \neq i\} \psi_{js} C_{\Delta,i}}{\check{n}_{s,-i}}\right)^4$  and of  $r_N^2 \sum_{j=1}^N \left(\sum_{i=1}^{j-1} \sum_{s=1}^S \check{w}_{is} w_{js} \frac{\eta_{\Delta,j} \psi_{is}}{\check{n}_{s,-j}}\right)^4$  is also bounded by  $r_N^2 N$ , so that  $r_N^2 \sum_{j=1}^N E_{W,\check{W}}[\mathcal{Y}_j^4] \to 0$  by  $C_r$ -inequality.

It remains to show that the conditional variance  $r_N \sum_{j=1}^N E[\mathcal{Y}_j^2 \mid \mathcal{G}_{j-1}]$  converges. Expanding the expectation yields

$$\begin{split} r_{N} \sum_{j=1}^{N} E[\mathcal{Y}_{j}^{2} \mid \mathcal{G}_{j-1}] &= 2r_{N} \sum_{i,j,s,t} \sum_{i'}^{j-1} \frac{\mathbb{I}\{j \neq i\} E_{\mathcal{G}_{0}}[\check{w}_{js}\check{w}_{jt}\psi_{js}\psi_{jt}]}{\check{n}_{s,-i}} \frac{w_{is}w_{i't}C_{\Delta,i}\eta_{\Delta,i'}}{\check{n}_{t,-i'}} \\ &+ 2r_{N} \sum_{i,j,s,t} \sum_{i'=1}^{j-1} \frac{\mathbb{I}\{j \neq i\} E_{\mathcal{G}_{0}}[\check{w}_{js}w_{jt}\psi_{js}\eta_{\Delta,j}]}{\check{n}_{s,-i}} \frac{w_{is}\check{w}_{i't}C_{\Delta,i}\psi_{i't}}{\check{n}_{t,-j}} \\ &+ r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \sum_{i'=1}^{j-1} \mathbb{I}\{i \neq i'\} \frac{E_{\mathcal{G}_{0}}[\check{w}_{js}\check{w}_{jt}\psi_{js}\psi_{jt}]}{\check{n}_{s,-i}} \frac{w_{i't}w_{is}\eta_{\Delta,i}\eta_{\Delta,i'}}{\check{n}_{t,-i'}} \\ &+ 2r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \sum_{i'=1}^{j-1} \mathbb{I}\{i \neq i'\} \frac{E_{\mathcal{G}_{0}}[\check{w}_{js}w_{jt}\psi_{js}\eta_{\Delta,j}]}{\check{n}_{s,-i}} \frac{w_{i't}\check{w}_{is}\check{w}_{i't}\eta_{\Delta,i}\psi_{i't}}{\check{n}_{t,-j}} \\ &+ r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \sum_{i'=1}^{j-1} \mathbb{I}\{i \neq i'\} \frac{w_{jt}w_{js}E_{\mathcal{G}_{0}}[\eta_{\Delta,j}]}{\check{n}_{s,-j}} \frac{\check{w}_{i't}\check{w}_{is}\psi_{i't}\psi_{is}}{\check{n}_{s,-j}} \frac{\check{w}_{i't}\check{w}_{is}\psi_{i't}\psi_{is}}{\check{n}_{s,-j}} \\ &+ r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{w_{jt}w_{js}E_{\mathcal{G}_{0}}[\eta_{\Delta,j}\eta_{\Delta,j}]}{\check{n}_{s,-j}} \frac{\check{w}_{is}\check{w}_{it}\psi_{is}\psi_{it}}{\check{n}_{t,-j}} + 2r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{E_{\mathcal{G}_{0}}[\check{w}_{js}w_{jt}\psi_{js}\eta_{\Delta,j}]}{\check{n}_{s,-i}} \frac{w_{is}\check{w}_{it}\eta_{\Delta,i}\psi_{it}}{\check{n}_{t,-j}} \\ &+ r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{E_{\mathcal{G}_{0}}[\check{w}_{js}\check{w}_{jt}\psi_{js}\psi_{jt}]}{\check{n}_{s,-i}} \frac{w_{is}w_{it}\eta_{\Delta,i}}{\check{n}_{t,-j}} + r_{N} \sum_{j=1}^{N} E_{\mathcal{G}_{0}}\left(\sum_{i=1}^{N} \sum_{s=1}^{S} \frac{\mathbb{I}\{j \neq i\}w_{is}\check{w}_{js}\psi_{js}C_{\Delta,i}}{\check{n}_{s,-i}}\right)^{2}. \end{split}$$

Conditional on  $(W, \check{W})$ , the first five terms are mean zero. The variance of the first term is bounded by a constant times

$$r_N^2 \sum_{i'} \left( \sum_{i,j,s,t} \frac{w_{is} w_{i't} \check{w}_{js} \check{w}_{jt}}{\check{n}_s \check{n}_t} \right)^2 = r_N^2 \sum_{i'} \left( \sum_{j,t} \frac{w_{i't} \check{w}_{jt}}{\check{n}_t} \sum_s \frac{n_s \check{w}_{js}}{\check{n}_s} \right)^2 \preceq r_N^2 N.$$

Similarly, the variance of the second, third, fourth, and fifth term can be shown to be bounded by a constant times  $r_N^2N$ . Next, the expectation conditional on  $(W, \check{W})$  of the absolute value of the sixth term is bounded by a constant times

$$r_{N} \sum_{i,j} \left( \sum_{s} \frac{\check{w}_{is} w_{js}}{\check{n}_{s}} \right) \left( \sum_{t} \frac{w_{jt} \check{w}_{it}}{\check{n}_{t}} \right) \leq r_{N} \sum_{i} \max_{i'} \sum_{j} \left( \sum_{s} \frac{\check{w}_{is} w_{js}}{\check{n}_{s}} \right) \left( \sum_{t} \frac{w_{jt} \check{w}_{i't}}{\check{n}_{t}} \right)$$

$$= r_N \sum_{i} \max_{i'} \sum_{j} \left( \sum_{s} \frac{\check{w}_{is} w_{js}}{\check{n}_s} \right) \left( \sum_{t} \frac{w_{jt} \check{w}_{i't}}{\check{n}_t} \right)$$

Consequently, by Markov inequality,

$$r_{N} \sum_{j=1}^{N} E[\mathcal{Y}_{j}^{2} \mid \mathcal{G}_{j-1}] = r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{w_{jt} w_{js} E_{\mathcal{G}_{0}}[\eta_{\Delta,j} \eta_{\Delta,j}]}{\check{n}_{s,-j}} \frac{\check{w}_{is} \check{w}_{it} \psi_{is} \psi_{it}}{\check{n}_{t,-j}} + 2r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{E_{\mathcal{G}_{0}}[\check{w}_{js} w_{jt} \psi_{js} \eta_{\Delta,j}]}{\check{n}_{s,-i}} \frac{w_{is} \check{w}_{it} \eta_{\Delta,i} \psi_{it}}{\check{n}_{t,-j}} + r_{N} \sum_{j=1}^{N} E_{\mathcal{G}_{0}} \left( \sum_{i=1}^{N} \sum_{s=1}^{S} \frac{\mathbb{I}\{j \neq i\} w_{is} \check{w}_{js} \psi_{js} C_{\Delta,i}}{\check{n}_{s,-i}} \right)^{2} + o_{p}(1). \quad (A.19)$$

Similarly, expanding the expression for  $W_N$  yields

$$\begin{split} \mathcal{W}_{N} &= \frac{1}{r_{N}} \sum_{i,i',j,s,t} \mathbb{I}\{j \neq i,i'\} \, \mathbb{I}\{i \neq i'\} \frac{\check{w}_{js}\check{w}_{jt}\psi_{jt}\psi_{js}}{\check{n}_{s,-i}} \frac{w_{is}w_{i't}\eta_{\Delta,i}\eta_{\Delta,i'}}{\check{n}_{t,-i'}} \\ &\quad + \frac{2}{r_{N}} \sum_{i,i',j,s,t} \mathbb{I}\{j \neq i,i'\} \frac{\check{w}_{js}\check{w}_{jt}\psi_{jt}\psi_{js}}{\check{n}_{s,-i}} \frac{w_{is}w_{i't}C_{\Delta,i}\eta_{\Delta,i'}}{\check{n}_{t,-i'}} \\ &\quad + \frac{1}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i \neq j\} \frac{w_{is}\check{w}_{js}\psi_{it}C_{\Delta,i}}{\check{n}_{s,-i}} \frac{w_{jt}\check{w}_{it}\psi_{js}\eta_{\Delta,j}}{\check{n}_{t,-j}} + \frac{1}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i \neq j\} \frac{w_{is}\check{w}_{js}\psi_{it}\eta_{\Delta,i}}{\check{n}_{s,-i}} \frac{w_{jt}\check{w}_{it}\psi_{js}C_{\Delta,j}}{\check{n}_{t,-j}} \\ &\quad + \frac{1}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i \neq j\} \frac{w_{is}\check{w}_{js}\psi_{jt}C_{\Delta,i}}{\check{n}_{s,-i}} \frac{w_{jt}\check{w}_{it}\psi_{js}C_{\Delta,i}}{\check{n}_{s,-i}} + \frac{2}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i < j\} \frac{w_{is}\check{w}_{js}\psi_{it}\eta_{\Delta,i}}{\check{n}_{s,-i}} \frac{w_{jt}\check{w}_{it}\psi_{js}\eta_{\Delta,j}}{\check{n}_{t,-j}} \\ &\quad + \frac{1}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i \neq j\} \frac{w_{is}\check{w}_{js}\psi_{jt}\psi_{jt}}{\check{n}_{s,-i}} \frac{w_{is}w_{it}\eta_{\Delta,i}}{\check{n}_{t,-i}} + \frac{2}{r_{N}} \sum_{i,j,s,t} \mathbb{I}\{i < j\} \frac{w_{is}\check{w}_{js}\psi_{it}\eta_{\Delta,i}}{\check{n}_{s,-i}} \frac{w_{jt}\check{w}_{it}\psi_{js}C_{\Delta,i}}{\check{n}_{t,-j}} \right)^{2}. \end{split}$$

Conditional on  $(W, \check{W})$ , the first five terms are mean zero. The variance of the first term is bounded by a constant times

$$\begin{split} \frac{1}{r_N^2} \sum_{i'} \left( \sum_{i,j,s,t} \frac{\check{w}_{js} \check{w}_{jt}}{\check{n}_s} \frac{w_{is} w_{i't}}{\check{n}_t} \right)^2 + \frac{1}{r_N^2} \sum_{i'} \left( \sum_{i,j,s,t} \frac{\check{w}_{js} \check{w}_{jt}}{\check{n}_s} \frac{w_{is} w_{i't}}{\check{n}_t} \right) \left( \sum_{i_2,j_2,s_2,t_2} \frac{\check{w}_{j_2s_2} \check{w}_{j_2t_2}}{\check{n}_{s_2}} \frac{w_{i's_2} w_{i_2t_2}}{\check{n}_{t_2}} \right) \\ + \frac{1}{r_N^2} \sum_{i'} \left( \sum_{i,j,s,t} \frac{\check{w}_{js} \check{w}_{jt}}{\check{n}_s} \frac{w_{is} w_{i't}}{\check{n}_t} \right) \left( \sum_{i_2,j_2,s_2,t_2} \frac{\check{w}_{i's_2} \check{w}_{i't_2}}{\check{n}_{s_2}} \frac{w_{is_2} w_{j_2t_2}}{\check{n}_{t_2}} \right) \preceq \frac{N}{r_N^2}. \end{split}$$

Similarly, the variance of the second, third, fourth and fifth term can also be shown to be bounded by a constant times  $Nr_N^2$ . Therefore by Markov inequality, in view of eq. (A.19),

$$r_{N} \sum_{j=1}^{N} E[\mathcal{Y}_{j}^{2} \mid \mathcal{G}_{j-1}] - \mathcal{W}_{N} = r_{N} \sum_{j,s,t} \sum_{i=1}^{j-1} \frac{w_{jt} w_{js} E_{\mathcal{G}_{0}}([\eta_{\Delta,j}^{2}] - \eta_{\Delta,j}^{2})}{\check{n}_{s,-j}} \frac{\check{w}_{is} \check{w}_{it} \psi_{is} \psi_{it}}{\check{n}_{t,-j}}$$

$$\begin{split} &+2r_{N}\sum_{j,s,t}\sum_{i=1}^{j-1}\frac{\check{w}_{js}w_{jt}(E_{G_{0}}[\psi_{js}\eta_{\Delta,j}]-\psi_{js}\eta_{\Delta,j})}{\check{n}_{s,-i}}\frac{w_{is}\check{w}_{it}\eta_{\Delta,i}\psi_{it}}{\check{n}_{t,-j}}\\ &+r_{N}\sum_{j,s,t}\sum_{i=1}^{j-1}\frac{\check{w}_{js}\check{w}_{jt}(E_{G_{0}}[\psi_{js}\psi_{jt}]-\psi_{js}\psi_{jt})}{\check{n}_{s,-i}}\frac{w_{is}w_{it}\eta_{\Delta,i}^{2}}{\check{n}_{t,-i}}\\ &+r_{N}\sum_{j=1}^{N}\sum_{i,i',s,t}\mathbb{I}\{j\neq i,i'\}\frac{w_{is}C_{\Delta,i}w_{i't}C_{\Delta,i'}}{\check{n}_{s,-i}}\frac{\check{w}_{jt}\check{w}_{js}(E_{G_{0}}[\psi_{js}\psi_{jt}]-\psi_{js}\psi_{jt})}{\check{n}_{t,-i'}}+o_{p}(1). \end{split}$$

All terms in this expression have mean zero conditional on W, and the variance of each term can be shown to be bounded by a constant times  $r_N N$ , so that  $r_N \sum_{j=1}^N E[\mathcal{Y}_j^2 \mid \mathcal{G}_{j-1}] - \mathcal{W}_N = o_p(1)$  as required.

## Appendix B Stylized economic model: baseline microfoundation

Appendices B.1 and B.2 provide a microfoundation for the stylized economic model presented in Section 3.1. In Appendix B.3, we use this microfoundation to derive expressions analogous to those in eqs. (8) and (9) in Section 3.2. In Appendix B.4, we exploit again our microfoundation and outline a set of restrictions on the model fundamentals such our main identification restriction, Assumption 1(ii) in Section 4.1, holds.

#### **B.1** Environment

We consider a model with multiple sectors s = 1, ..., S and multiple regions i, j = 1, ..., N. Regions are partitioned into countries indexed by c = 1, ..., C, and we denote the set of regions located in a country c by  $N_c$ . Region i has a population of  $M_i$  individuals who cannot move across regions. Each individual belongs to a different group, g = 1, ..., G. The share of group g in the population of region i is  $n_{ig}$ .

**Production.** Each sector s in region i has a representative firm that produces a differentiated good using only local labor. For simplicity, we assume that workers of different groups are perfect substitutes in production. The quantity  $Q_{is}$  produced by sector s in region i is produced using labor with productivity  $A_{is}$ ; i.e.

$$Q_{is} = A_{is}L_{is}, \tag{B.1}$$

where  $L_{is}$  denotes the number of workers (irrespective of their group) employed by the representative firm in this sector-region pair. Regions thus differ in terms of their sector-specific productivity  $A_{is}$ .

**Preferences for consumption goods.** Every individual has identical nested preferences over the sector- and region-specific differentiated goods. Specifically, we assume that individuals have Cobb-Douglas preferences over sectoral composite goods,

$$C_j = \prod_{s=1}^{S} \left( C_{js} \right)^{\gamma_s}, \tag{B.2}$$

where  $C_j$  is the utility level of a worker located in region j that obtains utility  $C_{js}$  from consuming goods in sector s, and  $C_{js}$  is a CES aggregator of the sector s goods produced in different regions:

$$C_{js} = \left[\sum_{i=1}^{N} \left(c_{ijs}\right)^{\frac{\sigma_{s}-1}{\sigma_{s}}}\right]^{\frac{\sigma_{s}}{\sigma_{s}-1}}, \qquad \sigma_{s} \in (1, \infty),$$
(B.3)

where  $c_{ijs}$  denotes the consumption in region j of the sector s good produced in region i. This preference structure has been previously used in Armington (1969), Anderson (1979) and multiple papers since (e.g. Anderson and van Wincoop, 2003; Arkolakis, Costinot and Rodríguez-Clare, 2012).

**Preferences for sectors and non-employment.** Individuals of every group g have the choice of being employed in one of the sectors s = 1, ..., S of the economy or opting for non-employment, which we index as s = 0. Conditional on being employed, all workers of group g have identical homogeneous preferences over their sector of employment, but workers differ in their preferences for non-employment. Specifically, conditional on obtaining utility  $C_j$  from the consumption of goods, the utility of a worker  $\iota$  of group g living in region j is

$$U(\iota \mid C_j) = \begin{cases} u(\iota)C_j & \text{if employed in any sector } s = 1, \dots, S, \\ C_j & \text{if not employed } (s = 0). \end{cases}$$
 (B.4)

We assume that each individual  $\iota$  belonging to group g and living in a region located in country c independently draws  $u(\iota)$  from a Pareto distribution with scale parameter  $\nu_{cg}$  and shape parameter  $\phi$ , so that the cumulative distribution function of  $u(\iota)$  is given by

$$F_{ig}^{u}(u) = 1 - \left(\frac{u}{v_{cg}}\right)^{-\phi}, \qquad u \ge v_{cg}, \qquad \phi > 1.$$
 (B.5)

If a worker living in region j chooses to be employed, she will earn wage  $\omega_j$ . In equilibrium, wages are equalized across sectors and groups because (i) firms are indifferent between workers of different groups, (ii) workers are indifferent about the sector of employment, and (iii) workers are freely mobile across sectors. If a worker chooses to not be employed, she receives a benefit  $b_j$ . We denote the total number of employed workers of group g in region j by  $L_{jg}$ , the total employment in region j as  $L_j = \sum_{g=1}^G L_{jg}$ , and the employment rate in j as  $E_j \equiv L_j/M_j$ .

**Market structure.** Goods and labor markets are perfectly competitive.

**Trade costs.** We assume that there are no trade costs, which implies that the equilibrium price of the good produced in a region is the same in every other region; i.e.  $p_{ijs} = p_{is}$  for j = 1, ..., N. Thus,

<sup>&</sup>lt;sup>3</sup>We assume that benefits are paid by a national government that imposes a flat tax  $\chi_c$  on all income earned in country c. The budget constraint of the government is thus  $\sum_{j\in N_c}\{\chi_c(\omega_jE_j+b_j(1-E_j))M_j\}=\sum_{j\in N_c}\{b_j(1-E_j)M_j\}$ . Alternatively, we could think of the option s=0 as home production and assume that workers that opt for home production in region j obtain  $b_j$  units of the final good, which they consume. This alternative model is isomorphic to that in the main text.

for every sector s there is a composite sectoral good that has identical price  $P_s$  in all regions; i.e.

$$(P_s)^{1-\sigma_s} = \sum_{s=1}^{S} (p_{is})^{1-\sigma_s},$$
 (B.6)

and the final good's price is  $P = \prod_{s=1}^{S} (P_s)^{\gamma_s}$ .

#### **B.2** Equilibrium

We now characterize the equilibrium wage  $\omega_j$  and total employment  $L_j$  of all regions j = 1, ..., N.

**Consumption.** We first solve the expenditure minimization problem of an individual residing in region j. Given the sector-level utility in eq. (B.3) and the condition that  $p_{ijs} = p_{is}$  for j = 1, ..., N, all regions j have identical spending shares  $x_{is}$  on goods from region i, given by

$$x_{is} = \left(\frac{p_{is}}{P_s}\right)^{1-\sigma_s}. (B.7)$$

**Labor supply.** Every worker maximizes the utility function in eq. (B.4) in order to decide whether to be employed. Consequently, conditional on the wage  $\omega_i$  and the non-employment benefit  $b_i$ , the total employment of individuals of group g in region i is  $L_{ig} = n_{ig}M_i \Pr[u_i(\iota)\omega_i > b_i]$ . It therefore follows from eq. (B.5) that  $L_i = \sum_{g=1}^G L_{jg}$  is

$$L_i = \omega_i^{\phi} v_i \tag{B.8}$$

such that

$$v_i = \nu_i \sum_{g=1}^G n_{ig} \nu_{cg} \tag{B.9}$$

with  $v_i \equiv M_i b_i^{-\phi}$ , and  $v_{cg} \equiv v_{cg}^{\phi}$ .

Producer's problem. In perfect competition, firms must earn zero profits and, therefore,

$$p_{is} = \frac{\omega_i}{A_{is}}. ag{B.10}$$

**Goods market clearing.** Given that labor is the only factor of production and firms earn no profits, the income of all individuals living in region i is  $W_i \equiv \sum_s \omega_i L_{is}$ , and world income is  $W \equiv \sum_i W_i$ . We normalize world income to one, W = 1. Given preferences in eq. (B.2), all individuals spend a share  $\gamma_s$  of their income on sector s, so that world demand for the differentiated good s produced in region i is  $x_{is}\gamma_s$ . Goods market clearing requires world demand for good s produced in region i to equal total revenue of the representative firm operating in sector s in region i,  $\omega_i L_{is}$ . Thus, using the

expression in eq. (B.7), we obtain

$$L_{is} = (\omega_i)^{-\sigma_s} (A_{is} P_s)^{\sigma_s - 1} \gamma_s. \tag{B.11}$$

Note that this labor demand equation is analogous to that in eq. (2) of Section 3, with the region- and sector-specific demand shifter  $D_{is}$  defined as  $D_{is} = (A_{is}P_s)^{\sigma_s-1} \gamma_s$ .

If, without loss of generality, we split the region- and sector-specific productivity  $A_{is}$  into a country and sector-specific component  $A_{cs}$  and a residual  $\tilde{A}_{is}$ ,

$$A_{is} = A_{cs}\tilde{A}_{is}. (B.12)$$

**Labor market clearing.** Given the sector- and region-specific labor demand in eq. (B.11), total labor demand in region *i* is

$$L_i = \sum_{s=1}^{S} (\omega_i)^{-\sigma_s} (A_{is} P_s)^{\sigma_s - 1} \gamma_s.$$
(B.13)

Labor market clearing requires labor supply in eq. (B.8) to equal labor demand in eq. (B.13):

$$v_i(\omega_i)^{\phi} = \sum_{s=1}^{S} (\omega_i)^{-\sigma_s} (A_{is} P_s)^{\sigma_s - 1} \gamma_s.$$
(B.14)

**Equilibrium.** Given technology parameters  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , preference parameters  $\{(\sigma_s, \gamma_s)\}_{s=1}^{S}$ , labor supply parameters  $\phi$ ,  $\{v_i\}_{i=1}^{N}$ ,  $\{n_{ig}\}_{i=1,g=1}^{N,G}$  and  $\{v_{cg}\}_{c=1,g=1}^{C,G}$ , and normalizing world income to equal 1, W=1, we can use eqs. (B.6), (B.9), (B.10), (B.12) and (B.14) to solve for the equilibrium wage in every world region,  $\{\omega_i\}_{i=1}^{N}$ , the equilibrium price of every sector-region specific good  $\{p_{is}\}_{i=1,s=1}^{N,S}$ , and the sectoral price indices  $\{P_s\}_{s=1}^{S}$ . Given these equilibrium wages and sectoral price indices, we can use eq. (B.13) to solve for the equilibrium level of employment in every region,  $\{L_i\}_{i=1}^{N}$ .

## B.3 Labor market impact of sectoral shocks: equilibrium relationships

We assume that, in every period, the model described in Appendices B.1 and B.2 characterizes the labor market equilibrium in every region  $i=1,\ldots,N$ . Across periods, we assume that the parameters  $\{\sigma_s\}_{s=1}^S$ ,  $\{n_{ig}\}_{i=1,g=1}^{N,G}$ , and  $\phi$  are fixed, and that all changes in the labor market outcomes  $\{\omega_i,L_i\}_{i=1}^N$  are generated by changes in technology  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , sectoral preferences  $\{\gamma_s\}_{s=1}^S$ , and labor supply parameters  $\{\nu_i\}_{i=1}^N$  and  $\{\nu_{cg}\}_{c=1,g=1}^{C,G}$ .

We focus here on understanding how changes in these exogenous parameters affect the labor market equilibrium in all regions located in a given country c; i.e. all regions belonging to the set  $N_c$ .

In our model, the sectoral prices mediate the impact of all foreign technology and labor supply shocks on the labor market equilibrium of every region in country c; i.e. the changes in  $\{(\omega_i, L_i)\}_{i \in N_c}$  depend on the changes in  $\{\tilde{A}_{is}\}_{s=1,i \notin N_c}^S$ ,  $\{v_i\}_{i \notin N_c}$ , and  $\{v_{c'g}\}_{g=1,c'\neq c}^G$  only through changes in  $\{P_s\}_{s=1}^S$ . Therefore, we can write the changes in wages and employment in every region i of the population of

interest  $N_c$  as a function of the changes in the sectoral prices, and the changes in the productivity and labor supply shocks in region i.

**Isomorphism.** As in Section 3.2, we use  $\hat{z} = \log(z^t/z^0)$  to denote log-changes in any given variable z between some initial period t = 0 and any other period t. Up to a first-order approximation around the initial equilibrium, eqs. (B.13) and (B.14) imply that

$$\hat{L}_{i} = \sum_{s=1}^{S} l_{is}^{0} \left[ \theta_{is} \hat{P}_{s} + \lambda_{i} ((\sigma_{s} - 1) \hat{A}_{cs} + \hat{\gamma}_{s}) + \lambda_{i} ((\sigma_{s} - 1) \hat{A}_{is}) \right] + (1 - \lambda_{i}) (\sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{cg} + \hat{v}_{i}),$$
 (B.15)

with  $l_{is}^0 \equiv L_{is}^0/L_i^0$ ,  $\tilde{w}_{ig} \equiv L_{ig}^0/L_i^0$ ,  $\theta_{is} = (\sigma_s - 1)\lambda_i$  and  $\lambda_i \equiv \phi \left[\phi + \sum_s l_{is}^0 \sigma_s\right]^{-1}$ . Combining eqs. (B.8), (B.9) and (B.15), we can similarly obtain

$$\hat{\omega}_{i} = \sum_{s=1}^{S} l_{is}^{0} \left[ \theta_{is} \hat{P}_{s} + \lambda_{i} ((\sigma_{s} - 1) \hat{A}_{cs} + \hat{\gamma}_{s}) + \lambda_{i} ((\sigma_{s} - 1) \hat{A}_{is}) \right] - \phi^{-1} \lambda_{i} (\sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{cg} + \hat{v}_{i}).$$
 (B.16)

Given our emphasis on understanding the changes in labor market outcomes for regions located in the same country, all regions in the population of interest will share the same value of  $A_{cs}$  for every sector s, and the same value of  $\hat{v}_{cg}$  for every labor group g; thus, we can simplify the notation by writing  $\hat{A}_{cs} = \hat{A}_s$  and  $\hat{v}_{cg} = \hat{v}_g$  for all s and g, respectively. Given this notational simplification and the following equivalences

$$\chi_s = P_s, \tag{B.17}$$

$$\mu_s = (A_s)^{\sigma_s - 1} \gamma_s, \tag{B.18}$$

$$\eta_{is} = (\tilde{A}_{is})^{\sigma_s - 1},\tag{B.19}$$

we can easily see that the expressions in eqs. (B.15) and (B.16) are identical to those in eqs. (8) and (9) in Section 3.2, respectively. Consequently, the environment described in Appendices B.1 and B.2 does indeed provide a microfoundation for the equilibrium relationships in eqs. (8) and (9).

### B.4 Identification of labor market impact of sectoral prices

As the mapping in eq. (B.17) illustrates, we may think of the changes in sectoral prices  $\{\hat{P}_s\}_{s=1}^S$  as our sectoral shocks of interest. Given data on changes in a labor market outcome (e.g. changes in the employment rate  $\hat{L}_i$ ) for all units of a population of interest formed by all regions of a particular country c, and data on the changes in sectoral prices  $\{\hat{P}_s\}_{s=1}^S$ , Assumption 1(ii) in Section 4.1 indicates that identifying the coefficient in front of a shift-share term that aggregates these sectoral price changes requires that these are as good as randomly allocated.

In the context of the equilibrium relationship in eq. (B.15), the sectoral price changes  $\{\hat{P}_s\}_{s=1}^S$  will satisfy Assumption 1(ii) if they are mean independent of: country c-specific sectoral productivity changes  $\{\hat{A}_{cs}\}_{s=1}^S$ ; country c-specific labor-group supply shocks  $\{\hat{v}_{cg}\}_{g=1}^G$ ; region and sector-specific productivity shocks, for all sectors and all regions in country c,  $\{\hat{A}_{is}\}_{s=1,i\in N_c}^S$ ; region-specific labor

supply shocks, for all regions in country c,  $\{\hat{v}_i\}_{i\in N_c}$ . This mean independence restriction will hold if the following two conditions are satisfied.

First, country c is "small"; i.e. all labor demand and labor supply shocks in country c have no impact on the changes in sectoral prices  $\{\hat{P}_s\}_{s=1}^S$ .

Second, labor demand and labor supply shocks affecting any region i in the country or population of interest c are mean independent of any labor demand and labor supply shock affecting any other region of the world economy that is "large" (i.e. any other region whose labor demand and supply shocks have an impact on the changes in sectoral prices).

In summary, if the vector of shifters of interest  $\{\mathcal{X}_s\}_{s=1}^S$  corresponds to the sectoral price changes  $\{\hat{P}_s\}_{s=1}^S$ , the researcher is interested on the impact of these shifters on a collection of "small" regions, and labor market shocks in these "small" regions are independent of the corresponding shocks in any "large" region, then the identification condition in Assumption 1(ii) is satisfied.

### B.4.1 Impact of labor demand and supply shocks on sector-specific price indices

In general equilibrium, the price change in every sector s,  $\hat{P}_s$ , depends on the shocks  $A_{cs}$ ,  $\hat{A}_{is}$ ,  $\hat{\gamma}_s$ ,  $\hat{v}_i$ , and  $v_{cg}$  of all sectors, labor groups, and regions in the world economy. Specifically, the change in the sector-specific price index is

$$\hat{P}_{s} = -\sum_{s'} \alpha_{ss'} \sum_{j=1}^{N} x_{js'}^{0} (\hat{A}_{js'} + \tilde{\lambda}_{j} \hat{v}_{j} - \tilde{\lambda}_{j} \sum_{k} l_{jk}^{0} [\hat{\gamma}_{k} + (\sigma_{k} - 1) \hat{A}_{jk}]), \tag{B.20}$$

where  $\tilde{\lambda}_j \equiv \left[\phi + \sum_s l_{is}^0 \sigma_s\right]^{-1}$ ,  $\left\{\alpha_{ss'}\right\}_{s=1,s'=1}^{S,S}$  are positive constants, and  $x_{js}^0$  is the share of the world production in sector s that corresponds to region j in the initial equilibrium; i.e.  $x_{js}^0 \equiv X_{js}^0 / \sum_{i=1}^N X_{is}^0$ . Imposing that all regions in a country c verify that  $x_{js}^0 \approx 0$  for all  $j \in N_c$  and for  $s = 1, \ldots, S$ , we can rewrite the change in the sector-specific price index as

$$\hat{P}_{s} = -\sum_{s'} \alpha_{ss'} \sum_{j \notin N_{c}} x_{js'}^{0} (\hat{A}_{js'} + \tilde{\lambda}_{j} \hat{\sigma}_{j} - \tilde{\lambda}_{j} \sum_{k} l_{jk}^{0} [\hat{\gamma}_{k} + (\sigma_{k} - 1) \hat{A}_{jk}]). \tag{B.21}$$

In this case,  $\hat{P}_s$  does not depend on the labor supply shocks and technology shocks in any region j included in country c; i.e.  $\hat{P}_s$  depends neither on  $\{\hat{A}_{cs}\}_{s=1}^{S}$ , nor  $\{\hat{v}_{cg}\}_{g=1}^{G}$ , nor  $\{\hat{A}_{is}\}_{s=1,i\in N_c}^{S}$ , nor  $\{\hat{V}_i\}_{i\in N_c}$ .

**Proof of eq. (B.20).** Equations (B.7) and (B.14) imply that

$$\hat{P}_s - \sum_k \tilde{\alpha}_{sk} \hat{P}_k = \sum_j x_{js}^0 (\tilde{\lambda}_j \sum_k l_{jk}^0 [\hat{\gamma}_k + (\sigma_k - 1) \hat{A}_{jk}] - \tilde{\lambda}_j \hat{\sigma}_j - \hat{A}_{js}),$$

where  $\tilde{\alpha}_{sk} \equiv \sum_j x_{js}^0 l_{jk}^0 \tilde{\lambda}_j (\sigma_k - 1)$ . Let us use bold variables to denote vectors,  $\mathbf{y} \equiv [y_s]_s$ , and bar bold variables to denote matrices,  $\bar{\mathbf{a}} \equiv [a_{sk}]_{s,k}$ . Thus, we can rewrite the equation above in matrix form as

$$(I-\bar{\boldsymbol{\alpha}})\,\hat{\boldsymbol{P}}=\hat{\boldsymbol{\eta}},$$

with  $\hat{\eta}_s \equiv \sum_j x_{js}^0 \left( \tilde{\lambda}_j \sum_k l_{jk}^0 \left[ \hat{\gamma}_k + (\sigma_k - 1) \hat{A}_{jk} \right] - \tilde{\lambda}_j \hat{\sigma}_j - \hat{A}_{js} \right)$ . In order to obtain eq. (B.20), it is sufficient to show that  $(I - \bar{\alpha})$  is a nonsingular m-matrix and, therefore, it has a positive inverse matrix. To establish this result, notice first that  $\tilde{\alpha}_{sk} \in (0,1)$  for every s and k; to show this, it is sufficient to show that, for every s, s, and s, it holds that s, it holds that s, and s, it holds that s, ithe holds that s, it holds that s, ith holds that s, it hold

$$0 < l_{jk}^0 \tilde{\lambda}_j(\sigma_k - 1) = \frac{l_{jk}^0(\sigma_k - 1)}{\phi + \sum_k l_{ik}^0 \sigma_k} < \frac{l_{jk}^0 \sigma_k}{\phi + \sum_k l_{ik}^0 \sigma_k} < 1,$$

where the last two inequalities arise from  $\sigma_k > 1$  and  $\phi > 0$ .

Finally, to show that  $(I - \bar{\alpha})$  is nonsingular, it is sufficient to establish that it is diagonal dominant:

$$\begin{aligned} |1 - \tilde{\alpha}_{sk}| - \sum_{k \neq s} |\tilde{\alpha}_{sk}| &= 1 - \sum_{j} x_{js}^{0} \frac{l_{js}^{0}(\sigma_{s} - 1)}{\phi + \sum_{k} l_{jk}^{0} \sigma_{k}} - \sum_{k \neq s} \sum_{j} x_{js}^{0} \frac{l_{jk}^{0}(\sigma_{k} - 1)}{\phi + \sum_{k} l_{jk}^{0} \sigma_{k}}, \\ &= \sum_{j} x_{js}^{0} \left( 1 - \frac{\sum_{k} l_{jk}^{0}(\sigma_{k} - 1)}{\phi + \sum_{k} l_{jk}^{0} \sigma_{k}} \right) \\ &= \sum_{j} x_{js}^{0} \left( \frac{\phi + 1}{\phi + \sum_{k} l_{jk}^{0} \sigma_{k}} \right) > 0. \blacksquare \end{aligned}$$

# Appendix C Stylized economic model: Extensions

In Appendices C.1 and C.2, we provide alternative microfoundations for the equilibrium relationship in eq. (8). Finally, in Appendix C.3, we incorporate migration into the baseline microfoundation described in Appendix B.

# C.1 Sector-specific factors of production

We extend here the model described in Appendix B to incorporate other factors of production. In particular, we introduce a sector-specific factor, as in Jones (1971) and, more recently, Kovak (2013).

#### **C.1.1** Environment

The only difference with respect to the setting described in Appendix B.1 is that the production function in eq. (B.1) is substituted for a Cobb-Douglas production function that combines labor and capital inputs:

$$Q_{is} = A_{is} \left( L_{is} \right)^{1-\theta_s} \left( K_{is} \right)^{\theta_s}.$$

We assume that capital is a sector-specific factor of production (sector-s capital has no use in any other sector) and that, for every sector, each region has an endowment of sector-specific capital  $\bar{K}_{is}$ .

### C.1.2 Equilibrium

**Consumption.** The consumer's problem is identical to that in Appendix B.2.

**Labor supply.** The labor supply decision is identical to that in Appendix B.2.

**Producer's problem.** Conditional on the region-i equilibrium wage  $\omega_i$  and rental rate of sector-s capital  $R_{is}$ , the cost minimization problem of the sector-s region-i representative firm and the market clearing condition for sector-s region-i specific capital imply that

$$\frac{1-\alpha_s}{\alpha_s}\frac{\bar{K}_{is}}{L_{is}}=\frac{\omega_i}{R_{is}}.$$

Conditional on the sector-s region-i final good price  $p_{is}$ , the firm's zero profit condition implies that

$$p_{is}A_{is}\tilde{\alpha}_s=\left(\omega_i\right)^{1- heta_{is}}\left(R_{is}\right)^{ heta_{is}}$$
 ,

where  $\tilde{\alpha}_s \equiv (\alpha_s)^{\alpha_s} (1 - \alpha_s)^{1 - \alpha_s}$ . The combination of these two conditions yields the demand for labor in sector s and region i,

$$L_{is} = \frac{1 - \alpha_s}{\alpha_s} \bar{K}_{is} \left( \frac{p_{is} A_{is} \tilde{\alpha}_s}{\omega_i} \right)^{\frac{1}{\alpha_s}}, \tag{C.1}$$

and the total sales of the sector-s region-i good as a function of the output price  $p_{is}$ ,

$$X_{is} = \frac{1}{1 - \alpha_s} \omega_i L_{is} = \frac{\bar{K}_{is}}{\alpha_s} \left( p_{is} A_{is} \tilde{\alpha}_s \right)^{\frac{1}{\alpha_s}} \left( \omega_i \right)^{1 - \frac{1}{\alpha_s}}. \tag{C.2}$$

**Goods market clearing.** Applying the same normalization as in Appendix B.1, W = 1, the total expenditure in the sector-s region-i good is equal to  $x_{is}\gamma_s$ , with  $x_{is}$  defined in eq. (B.7) as a function of the equilibrium prices  $p_{is}$ . Equating  $x_{is}\gamma_s$  and eq. (C.2), we can solve for the equilibrium value of  $p_{is}$  as a function of the sector-s price index  $P_s$ :

$$p_{is} = \left[ \frac{\bar{K}_{is}}{\alpha_s} \left( A_{is} \tilde{\alpha}_s \right)^{\frac{1}{\alpha_s}} \left( \omega_i \right)^{1 - \frac{1}{\alpha_s}} \frac{(P_s)^{1 - \sigma_s}}{\gamma_s} \right]^{-\theta_{is} \eta_{is}}, \tag{C.3}$$

where  $\delta_s \equiv (1 + \alpha_s(\sigma_s - 1))^{-1} \in (0,1)$ . Additionally, combining eqs. (C.1) and (C.3), we obtain an expression for labor demand in sector-*s* region-*i* as a function of the equilibrium wage  $\omega_i$ , the sector-*s* price  $P_s$  and other exogenous determinants:

$$L_{is} = \kappa_{is} \gamma_s^{\delta_s} \left( A_{is} P_s \right)^{(\sigma_s - 1)\delta_s} \left( \omega_i \right)^{-\sigma_s \delta_s}, \tag{C.4}$$

where  $\kappa_{is} \equiv (1 - \alpha_s)(\bar{K}_{is}\tilde{\alpha}_s^{\frac{1}{\alpha_s}}/\alpha_s)^{1-\delta_s}$ . Note that this labor demand equation is analogous to that in eq. (2), with the region- and sector-specific demand shifter  $D_{is}$  defined as

$$D_{is} = \kappa_{is} (\gamma_s)^{\delta_s} (A_{is} P_s)^{(\sigma_s - 1)\delta_s}$$
,

and with the labor demand elasticity now defined as  $\sigma_s \delta_s$ . Note that the labor demand elasticity in eq. (2) is identical to that in eq. (C.4) in the specific case in which  $\delta_s = 1$ , which will hold when

 $\alpha_s = 0$ . Without loss of generality, we split the region- and sector-specific productivity  $A_{is}$  according to eq. (B.12).

**Labor market clearing.** Given the sector- and region-specific labor demand in eq. (C.4), total labor demand in region i is

$$L_{i} = \sum_{s=1}^{S} \kappa_{is} \gamma_{s}^{\delta_{s}} \left( A_{is} P_{s} \right)^{(\sigma_{s} - 1)\delta_{s}} \left( \omega_{i} \right)^{-\sigma_{s} \delta_{s}}. \tag{C.5}$$

Labor market clearing requires labor supply in eq. (B.8) to equal labor demand in eq. (C.5):

$$v_i(\omega_i)^{\phi} = \sum_{s=1}^{S} \kappa_{is} \gamma_s^{\delta_{is}} \left( A_{is} P_s \right)^{(\sigma_s - 1)\delta_{is}} \left( \omega_i \right)^{-\sigma_s \delta_{is}}, \qquad j = 1, \dots, N.$$
 (C.6)

**Equilibrium.** Given the technology parameters  $\{\alpha_s\}_{s=1}^S$ ,  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , sector- and region-specific capital inputs  $\{\bar{K}_{is}\}_{i=1,s=1}^{N,S}$ , preference parameters  $\{(\sigma_s, \gamma_s)\}_{s=1}^S$ , labor supply parameters  $\phi$ ,  $\{v_i\}_{i=1}^N$ ,  $\{n_{ig}\}_{i=1,g=1}^{N,G}$  and  $\{v_{cg}\}_{c=1,g=1}^{C,G}$ , and normalizing world income to equal 1, W = 1, we can use eqs. (B.6), (B.9), (B.12), (C.3) and (C.6) to solve for the equilibrium wage in every world region,  $\{\omega_i\}_{i=1}^N$ , the equilibrium price of every sector-region specific good  $\{p_{is}\}_{i=1,s=1}^{N,S}$ , and the sectoral price indices  $\{P_s\}_{s=1}^S$ . Given these equilibrium wages and sectoral price indices, we can use eq. (C.5) to solve for the equilibrium level of employment in every region,  $\{L_i\}_{i=1}^N$ .

### C.1.3 Labor market impact of sectoral shocks

We assume that, in every period, the model described in Appendices C.1.1 and C.1.2 characterizes the labor market equilibrium in every region  $i=1,\ldots,N$ . Across periods, we assume that the parameters  $\{(\sigma_s,\alpha_s)\}_{s=1}^S$ ,  $\{n_{ig}\}_{i=1,g=1}^{N,G}$ , and  $\phi$  are fixed, and that all changes in the labor market outcomes  $\{\omega_i,L_i\}_{i=1}^N$  are generated by changes in technology  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , sectoral preferences  $\{\gamma_s\}_{s=1}^S$ , and labor supply parameters  $\{v_i\}_{i=1}^N$  and  $\{v_{cg}\}_{c=1,g=1}^{C,G}$ . We focus here on understanding how changes in these exogenous parameters affect the labor market equilibrium in all regions located in a given country c; i.e. all regions belonging to the set  $N_c$ .

**Isomorphism.** Following steps analogous to those in Appendix B.3, we can show that eqs. (C.5) and (C.6) imply that

$$\hat{L}_{i} = \sum_{s=1}^{S} l_{is}^{0} \left[ \theta_{is} \hat{P}_{s} + \lambda_{i} ((\sigma_{s} - 1)\delta_{s} \hat{A}_{cs} + \delta_{s} \hat{\gamma}_{s}) + \lambda_{i} ((\sigma_{s} - 1)\delta_{s} \hat{A}_{is} + \hat{\kappa}_{is}) \right]$$

$$+ (1 - \lambda_{i}) \left( \sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{cg} + \hat{v}_{i} \right),$$
(C.7)

with  $\theta_{is} = (\sigma_s - 1)\delta_s\lambda_i$  and  $\lambda_i \equiv \phi(\phi + \sum_s l_{is}^0\sigma_s\delta_s)^{-1}$ . As in Appendix B.3, given our emphasis on understanding the changes in labor market outcomes for regions located in the same country, all regions in the population of interest will share the same value of  $A_{cs}$  for every sector s, and the same value of  $\hat{v}_{cg}$  for every labor group g; thus, we can simplify the notation by writing  $\hat{A}_{cs} = \hat{A}_s$ 

and  $\hat{v}_{cg} = \hat{v}_g$  for all s and g, respectively. Given this notational simplification and the following equivalences

$$\chi_s = P_s, \tag{C.8}$$

$$\mu_s = (A_s)^{(\sigma_s - 1)\delta_s} (\gamma_s)^{\delta_s}, \tag{C.9}$$

$$\eta_{is} = \kappa_{is} (\tilde{A}_{is})^{(\sigma_s - 1)\delta_s}. \tag{C.10}$$

we can easily see that the expression in eq. (C.7) is identical to that in eq. (8) in Section 3.2. Consequently, the environment described in Appendices C.1.1 and C.1.2 does indeed provide a microfoundation for the equilibrium relationship in eq. (8).

# C.2 Sector-specific preferences

We extend the model described in Appendix B to allow workers to have idiosyncratic preferences for being employed in the different s = 1, ..., S sectors and for being non-employed s = 0. In order to maintain the analysis simple, we assume here that there is a single worker group G = 1.

#### C.2.1 Environment

The only difference with respect to the setting described in Appendix B.1 is that the utility function in eqs. (B.4) and (B.5) is substituted by an alternative utility function that features workers idiosyncratic preferences for being employed in the different s = 1, ..., S sectors and for being non-employed s = 0. Specifically, we assume here that, conditional on obtaining utility  $C_i$  from the consumption of goods, the utility of a worker  $\iota$  living in region i is

$$U_{is} = u_s(\iota)C_i, \tag{C.11}$$

and, to simplify the analysis, we assume that  $u_s(\iota)$  is i.i.d. across individuals  $\iota$  and sectors s with a Fréchet cumulative distribution function; i.e. for every region i = 1, ..., N and sector s = 0, ..., S,

$$F_u(u) = e^{-v_{is}u^{-\phi}}, \qquad \phi > 1.$$
 (C.12)

This modeling of workers' sorting patterns across sectors is similar to that in Galle, Rodríguez-Clare and Yi (2018) and Burstein, Morales and Vogel (2019). See Adão (2016) for a framework that relaxes the distributional assumption in eq. (C.12). Given that individuals have heterogeneous preferences for employment in different sectors, workers are no longer indifferent across sectors and, thus, equilibrium wages  $\{\omega_{is}\}_{s=1}^{S}$  may vary across sectors within a region i. As in the main text, we assume that workers that choose the non-employment sector s=0 in region i receive non-employment benefits  $b_i$ .

#### C.2.2 Equilibrium

**Consumption.** The consumer's problem is identical to that in Appendix B.2.

**Labor supply.** Conditional on the equilibrium wages  $\{\omega_{is}\}_{s=1}^{S}$ , the labor supply in sector  $s=1,\ldots,S$  of region i is

$$L_{is} = M_i \frac{v_{is}(\omega_{is})^{\phi}}{\Phi_i} \quad \text{with} \quad \Phi_i \equiv v_{i0}b_i^{\phi} + \sum_{s=1}^{S} v_{is}(\omega_{is})^{\phi}, \tag{C.13}$$

and the labor supply in the non-employment sector s = 0 is

$$L_{i0} = M_i \frac{v_{i0}(b_i)^{\phi}}{\Phi_i}.$$
 (C.14)

Producer's problem. In perfect competition, firms must earn zero profits and, therefore,

$$p_{is} = \frac{\omega_{is}}{A_{is}}. (C.15)$$

Goods market clearing. The conditions determining the equilibrium in the good's market and, consequently, the region- and sector-specific labor demand equations are identical to those in Appendix B.2.

**Labor market clearing.** Combining the region- and sector-specific labor supply in eq. (C.13) with the region- and sector-specific labor demand in eq. (B.11), and imposing the normalization W = 1, the labor market clearing condition in every sector s = 1, ..., S and region i = 1, ..., N is

$$M_i \frac{v_{is}(\omega_{is})^{\phi}}{\Phi_i} = (\omega_{is})^{-\sigma_s} (A_{is}P_s)^{\sigma_s - 1} \gamma_s.$$
 (C.16)

**Equilibrium.** Given productivity parameters  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , preference parameters  $\{\sigma_s, \gamma_s\}_{s=1}^{S}$ , labor supply parameters  $\phi$  and  $\{v_{is}\}_{i=1,s=0}^{N,S}$ , and normalizing world income to equal 1, W=1, we can use eqs. (B.6), (B.12), (C.15) and (C.16) to solve for the equilibrium wage in every sector and region,  $\{\omega_{is}\}_{i=1,s=1}^{N,S}$ , the equilibrium price of every sector- and region-specific good  $\{p_{is}\}_{i=1,s=1}^{N,S}$ , and the sectoral price indices  $\{P_s\}_{s=1}^{S}$ . Given these equilibrium wages and sectoral price indices, we can use eqs. (C.13) and (C.14) to solve for the equilibrium level of employment in every sector and region,  $\{L_{is}\}_{i=1,s=0}^{N,S}$ .

### C.2.3 Labor market impact of sectoral shocks

We assume that, in every period, the model described in Appendices C.2.1 and C.2.2 characterizes the labor market equilibrium in every region  $i=1,\ldots,N$ . Across periods, we assume that the parameters  $\{\sigma_s\}_{s=1}^S$ , and  $\phi$  are fixed, and that all changes in the labor market outcomes  $\{\omega_i,L_i\}_{i=1}^N$  are generated by changes in technology  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , sectoral preferences  $\{\gamma_s\}_{s=1}^S$ , and labor supply parameters  $\{v_{is}\}_{i=1,s=1}^{N,S}$ . We focus here on understanding how changes in these exogenous parameters affect the labor market equilibrium in all regions located in a given country c; i.e. all regions belonging to the set  $N_c$ .

**Isomorphism.** Given that the total population of a region,  $M_i$ , is fixed across time periods, it holds that, to a first-order approximation,  $l_{i0}^0 \hat{L}_{i0} + (1 - l_{i0}^0) \hat{L}_i = 0$ , where  $\hat{L}_i$  denotes the log-change in total population in region i. Therefore, the change in total employment in region i may be written as

$$\hat{L}_{i} = -\frac{l_{i0}^{0}}{1 - l_{i0}^{0}} \hat{L}_{i0} 
= \frac{l_{i0}^{0}}{1 - l_{i0}^{0}} (\hat{\Phi}_{i} - \phi \hat{b}_{i} - \hat{v}_{i0}) 
= \frac{l_{i0}^{0}}{1 - l_{i0}^{0}} (\sum_{s=0}^{S} l_{is}^{0} \hat{v}_{is} + \phi l_{i0}^{0} \hat{b}_{i} + \phi \sum_{s=1}^{S} l_{is}^{0} \hat{\omega}_{is} - \phi \hat{b}_{i} - \hat{v}_{i0}).$$
(C.17)

From eq. (C.16), we can express the changes in wages in every sector and every region of country c as

$$\hat{\omega}_{is} = (\phi + \sigma_s)^{-1} \left( \hat{\Phi}_i + \hat{\gamma}_s + (\sigma_s - 1)(\hat{A}_{is} + \hat{P}_s) - \hat{v}_{is} \right). \tag{C.18}$$

Combining eqs. (C.17) and (C.18), we can re-express the change in total employment in region i as

$$\hat{L}_{i} = \sum_{s=1}^{S} l_{is}^{0} [\theta_{is} \hat{P}_{s} + \lambda_{i} (\phi + \sigma_{s})^{-1} ((\sigma_{s} - 1) \hat{A}_{cs} + \hat{\gamma}_{s}) + \lambda_{i} (\phi + \sigma_{s})^{-1} (\sigma_{s} - 1) \hat{\bar{A}}_{is}] + \hat{v}_{i},$$
 (C.19)

where 
$$\hat{v}_i = l_{i0}^0 (1 - l_{i0}^0)^{-1} (\hat{v}_i - \phi \hat{b}_i - \hat{v}_{i0})$$
,  $\hat{v}_i = (1 - \phi \sum_{s=1}^S l_{is}^0 (\phi + \sigma_s)^{-1})^{-1} (\phi l_{i0}^0 \hat{b}_i + l_{i0}^0 \hat{v}_{i0} + \sum_{s=1}^S l_{is}^0 \sigma_s (\phi + \sigma_s)^{-1} \hat{v}_{is})$ ,  $\beta_{is} = (\sigma_s - 1)(\phi + \sigma_s)^{-1} \lambda_i$ , and  $\lambda_i = \phi l_{i0}^0 (1 - l_{i0}^0)^{-1} (1 - \phi \sum_{s=1}^S l_{is}^0 (\phi + \sigma_s)^{-1})^{-1}$ .

As in Appendix B.3, given our emphasis on understanding the changes in labor market outcomes for regions located in the same country, all regions in the population of interest will share the same value of  $A_{cs}$  for every sector s; thus, we can simplify the notation by writing  $\hat{A}_{cs} = \hat{A}_s$  for all s. Given this notational simplification, the following equivalences

$$\chi_s = P_s,$$
 $\mu_s = (A_s)^{(\sigma_s - 1)(\phi + \sigma_s)^{-1}} (\gamma_s)^{(\phi + \sigma_s)^{-1}},$ 
 $\eta_{is} = (\tilde{A}_{is})^{(\sigma_s - 1)(\phi + \sigma_s)^{-1}},$ 

and the adjustment of the expression for  $\lambda_i$  and  $\hat{v}_i$ , the expression in eq. (C.19) is identical to that in eq. (8) in Section 3.2. Consequently, the environment described in Appendices C.2.1 and C.2.2 does indeed provide a microfoundation for the equilibrium relationship in eq. (8).

### C.3 Allowing for regional migration

We extend here the baseline environment described in Appendix B.1 to allow for mobility of individuals across regions within a single country c. As in Appendix C.2, to maintain the analysis simple, we focus on the special case with a single worker group, G = 1.

#### C.3.1 Environment

We still assume that the number of individuals living in each country c is fixed and equal to  $M_c$ . The only difference with respect to the setting described in Appendix B.1 is that the mass of individuals living in a region i,  $M_i$ , is no longer fixed. We assume that, before the realization of the shock  $u(\iota)$  in eq. (B.4), individuals must decide their preferred region of residence taking into account their idiosyncratic preferences for local amenities in each region. Specifically, we assume that the utility to individual  $\iota$  of residing in region i is

$$U(\iota) = \tilde{u}_i(\iota) \left( \bar{U}_i(\omega_i/P, b_i/P) - 1 \right) \tag{C.20}$$

where  $\bar{U}_i(\omega_i/P, b_i/P)$  is the expected utility of residing in region i, as determined by eqs. (B.4) and (B.5), and  $\tilde{u}_i(\iota)$  is the idiosyncratic amenity level of region i for individual  $\iota$ . For simplicity, we assume that individuals draw their idiosyncratic amenity level independently (across individuals and regions) from a Type I extreme value distribution:

$$\tilde{u}_i(\iota) \sim F_{\tilde{u}}(\tilde{u}) = e^{-\tilde{u}^{-\tilde{\phi}}}, \qquad \tilde{\phi} > 0.$$
 (C.21)

A similar modeling of labor mobility has been previously imposed, among others, in Allen and Arkolakis (2016), Redding (2016), Allen, Arkolakis and Takahashi (2018), and Fajgelbaum et al. (2019), among others. See Redding and Rossi-Hansberg (2017) for additional references.

### C.3.2 Equilibrium

**Consumption.** The consumer's problem is identical to that in Appendix B.2.

**Labor supply.** To characterize the labor supply in region *i*, we first compute  $\bar{U}_i(w_i/P, b_i/P)$ :

$$\begin{split} \bar{U}_{i}(\omega_{i}/P,b_{i}/P) &= \frac{\omega_{i}}{P} \int_{b_{i}/\omega_{i}}^{\infty} u dF_{u}(u) + \frac{b_{i}}{P} \int_{\nu_{i}}^{b_{i}/\omega_{i}} dF_{u}(u), \\ &= \phi \frac{\omega_{i}}{P} \int_{b_{i}/\omega_{i}}^{\infty} \left(\frac{u}{\nu_{i}}\right)^{-\phi} du + \frac{b_{i}}{P} \int_{\nu_{i}}^{b_{i}/\omega_{i}} \frac{\phi}{\nu_{i}} \left(\frac{u}{\nu_{i}}\right)^{-\phi-1} du, \\ &= \frac{\phi}{\phi - 1} \frac{\omega_{i}}{P} \nu_{i}^{\phi} \left(\frac{\omega_{i}}{b_{i}}\right)^{\phi-1} + \frac{b_{i}}{P} \left(1 - \nu_{i}^{\phi} \left(\frac{\omega_{i}}{b_{i}}\right)^{\phi}\right), \\ &= \frac{b_{i}}{P} \left(1 + \frac{1}{\phi - 1} \nu_{i}^{\phi} \left(\frac{\omega_{i}}{b_{i}}\right)^{\phi}\right). \end{split}$$

To simplify the analysis, we assume that the unemployment benefit is identical in all regions and equal to the price index P; i.e.  $b_i = P$  for all  $i \in N$ . Defining  $v_i \equiv (v_i/b_i)^{\phi}$  as in eq. (B.8), the assumption that  $b_i = P$  for all  $i \in N$  implies that  $v_i \equiv v_i/P$  and, thus,

$$\bar{U}_i(\omega_i/P,b_i/P) = 1 + \frac{1}{\phi - 1}v_i\left(\frac{\omega_i}{P}\right)^{\phi}$$
,

and the share of national population in region i is

$$M_{i} = \Pr \left[ \tilde{u}_{i}(\iota) \left( \bar{U}_{i}(\omega_{i}/P, b_{i}/P) - 1 \right) > \tilde{u}_{j}(\iota) \left( \bar{U}_{j}(\omega_{j}/P, b_{j}/P) - 1 \right), \ \forall j \in N_{c} \right]$$
$$= \Pr \left[ \tilde{u}_{i}(\iota) v_{i}(\omega_{i})^{\phi} > \tilde{u}_{j}(\iota) v_{j}(\omega_{j})^{\phi}, \ \forall j \in N_{c} \right].$$

Given the distributional assumption in eq. (C.21), it holds that

$$M_i = \frac{v_i(\omega_i)^{\phi_m}}{\Phi_c} M_c$$
 such that  $\Phi_c = \sum_{j \in N_c} v_j(\omega_j)^{\phi_m}$  and  $\phi_m \equiv \tilde{\phi}\phi$ . (C.22)

Given the value of  $M_i$ , total employment in region i is determined as in eq. (B.8). Therefore, the total labor supply in region i is

$$L_i = \frac{v_i(\omega_i)^{\phi_m}}{\sum_{j \in N_c} v_j(\omega_j)^{\phi_m}} M_c v_i(\omega_i)^{\phi}.$$
 (C.23)

**Producer's problem.** The producer's problem is identical to that in Appendix B.2.

Goods market clearing. The conditions determining the equilibrium in the good's market and, consequently, the region- and sector-specific labor demand equations are identical to those in Appendix B.2.

**Labor market clearing.** Combining the region- and sector-specific labor supply in eq. (C.23) with the aggregate labor demand in eq. (B.13), and imposing the normalization W = 1, the labor market clearing condition in every region  $i \in N_c$  is

$$\frac{v_i(\omega_i)^{\phi_m}}{\sum_{j\in N_c} v_j(\omega_j)^{\phi_m}} M_c v_i(\omega_i)^{\phi} = \sum_s (\omega_i)^{-\sigma_s} \left(A_{is} P_s\right)^{\sigma_s - 1} \gamma_s, \tag{C.24}$$

or, equivalently,

$$(\Phi_c)^{-1} M_c v_i(\omega_i)^{\phi + \phi_m} = \sum_s (\omega_i)^{-\sigma_s} (A_{is} P_s)^{\sigma_s - 1} \gamma_s, \tag{C.25}$$

for every region i in every country c.

**Equilibrium.** Given productivity parameters  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , preference parameters  $\{\sigma_s, \gamma_s\}_{s=1}^{S}$ , labor supply parameters,  $\phi$ ,  $\phi_m$ , and  $\{v_i\}_{i=1}^{N}$ , and normalizing world income to equal 1, W=1, we can use eqs. (B.6), (B.10), (B.12) and (C.25) to solve for the equilibrium wage in every region,  $\{\omega_i\}_{j=1}^{N}$ , the equilibrium price of every sector- and region-specific good  $\{p_{is}\}_{i=1,s=1}^{N,S}$ , and the sectoral price indices  $\{P_s\}_{s=1}^{S}$ . Given these equilibrium wages and sectoral price indices, we can use eq. (C.23) to solve for the equilibrium level of employment in every region,  $\{L_i\}_{i=1}^{N}$ .

### C.3.3 Labor market impact of sectoral shocks

We assume that, in every period, the model described in Appendices C.3.1 and C.3.2 characterizes the labor market equilibrium in every region i = 1, ..., N. Across periods, we assume that the param-

eters  $\{\sigma_s\}_{s=1}^S$ ,  $\phi$  and  $\phi_m$  are fixed, and that all changes in the labor market outcomes  $\{\omega_i, L_i\}_{i=1}^N$  are generated by changes in technology  $\{A_{cs}\}_{c=1,s=1}^{C,S}$  and  $\{\tilde{A}_{is}\}_{i=1,s=1}^{N,S}$ , sectoral preferences  $\{\gamma_s\}_{s=1}^S$ , and labor supply parameters  $\{v_i\}_{i=1}^N$ . We focus here on understanding how changes in these exogenous parameters affect the labor market equilibrium in all regions located in a given country c; i.e. all regions belonging to the set  $N_c$ .

**Isomorphism.** According to eq. (C.23), the change in employment in any region i in country c is

$$\hat{L}_i = 2\hat{v}_i + (\phi + \phi_m)\hat{\omega}_i - \hat{\Phi}_c. \tag{C.26}$$

Assuming that  $\{M_c\}_{c=1}^C$ ,  $\{\sigma_s\}_{s=1}^S$ , and  $(\phi, \phi_m)$  are fixed and totally differentiating eq. (C.24) with respect to the remaining determinants of  $\hat{\omega}_i$ , we can express the changes in wages in every region i of country c as

$$\hat{\omega}_i = \lambda_i \hat{\Phi}_c + \lambda_i \sum_{s=1}^S l_{is}^0 \left[ \hat{\gamma}_s + (\sigma_s - 1)(\hat{A}_{is} + \hat{P}_s) \right] - \lambda_i \hat{v}_i, \tag{C.27}$$

where  $\lambda_i \equiv (\phi + \phi_m + \sum_s l_{is}^0 \sigma_s)^{-1}$ . Using the expression in eq. (C.22), we can also express

$$\hat{\Phi}_c = \sum_{i \in N_c} m_i^0 \left( \phi_m \hat{\omega}_i + \hat{v}_i \right), \tag{C.28}$$

where  $m_i^0$  is the share of individuals living in country c that had residence in region i at the initial period 0; i.e.  $m_i^0 \equiv M_i^0/M_c^0$ , with  $M_c^0 \equiv \sum_{i \in N_c} M_i^0$ .

Combining eqs. (C.26) and (C.27), we can express the change in total employment in region i as

$$\hat{L}_{i} = [(\phi + \phi_{m})\lambda_{i} - 1]\hat{\Phi}_{c} + \sum_{s=1}^{S} l_{is}^{0} [\theta_{is}\hat{P}_{s} + \lambda_{i}(\phi + \phi_{m})((\sigma_{s} - 1)\hat{A}_{cs} + \hat{\gamma}_{s}) + \lambda_{i}(\phi + \phi_{m})(\sigma_{s} - 1)\hat{A}_{is}] + [2 - (\phi + \phi_{m})\lambda_{i}]\hat{v}_{i}$$
(C.29)

where  $\theta_{is} = (\sigma_s - 1)(\phi + \phi_m)\lambda_i$ . As in Appendix B.3, given our emphasis on understanding the changes in labor market outcomes for regions located in the same country, all regions in the population of interest will share the same value of  $A_{cs}$  for every sector s; thus, we can simplify the notation used in eq. (C.29) by writing  $\hat{A}_{cs} = \hat{A}_s$  for all s. Given this notational simplification, if it were to be the case that  $\hat{\Phi}_c = 0$ , the expression in eq. (C.29) would be analogous to that in eq. (8) under the following equivalences

$$\chi_s = P_s$$
,  $\mu_s = (A_s)^{(\sigma_s-1)(\phi+\phi_m)}(\gamma_s)^{(\phi+\phi_m)}$ ,  $\eta_{is} = (\tilde{A}_{is})^{(\sigma_s-1)(\phi+\phi_m)}$ ,

and the necessary adjustment of the expression for  $\lambda_i$  and  $\hat{v}_i$ . However, the term  $\hat{\Phi}_c$  will generally not be zero and, as indicated in eq. (C.28), it will generally capture the effect of shocks to all regions

in the same country c as the region of interest i. In the specific case in which  $\sigma_s = \sigma$  for all sectors s, it will be the case that  $\lambda_i = \lambda$  for all regions i, and, consequently, the term  $[(\phi + \phi_m)\lambda_i - 1]\hat{\Phi}_c$  will be common to all regions i belonging to the same country c. In this special case, the parameter  $\theta_{is}$  will no longer capture the total effect of the price shifters  $\hat{P}_s$  but the differential effect of this price shifter on region i relative to all other regions in the same country c.

# Appendix D Additional placebo exercises

This section presents additional placebo exercises that complement the results in Sections 2 and 6. Appendix D.1 reports the empirical distribution of the estimated coefficients and standard errors of the baseline placebo exercise in Sections 2 and 6. Appendix D.2 investigates the importance of controlling for the size of the residual sector in shift-share specifications. In Appendix D.3, we present results illustrating the impact of confounding sector-level shocks on different estimators of the coefficient on the shift-share covariate of interest. Appendix D.4 investigates the consequences of serial correlation in panel data applications of shift-share specifications. Appendix D.5 analyzes the consequences of misspecification of our baseline linearly additive potential outcome framework. Appendix D.6 reports results investigating the performance of inference procedures in the presence of unobserved shift-share components whose shares differ from those of the shift-share variable of interest. Appendix D.7 studies the consequences of treatment heterogeneity. In Appendix D.8, we provide additional results for the placebo exercises described in Sections 2 and 6. <

# D.1 Placebo exercise: empirical distributions

Figure D.1 reports the empirical distribution of the estimated coefficients when: (a) the dependent variable is the 2000–2007 change in each CZ's employment rate; in each simulation draw m, we draw a random vector  $(\mathcal{X}_1^m, \dots, \mathcal{X}_{S-1}^m)$  of i.i.d. normal random variables with zero mean and variance  $\text{var}(\mathcal{X}_s^m) = 5$ , and set  $\mathcal{X}_S^m = 0$ ; and (c) the vector of controls  $Z_i$  only includes a constant. The empirical distribution of the estimated coefficients resembles a normal distribution centered around  $\beta = 0$ . For more details in the placebo exercise that generates this distribution of estimated coefficients, see Section 2.

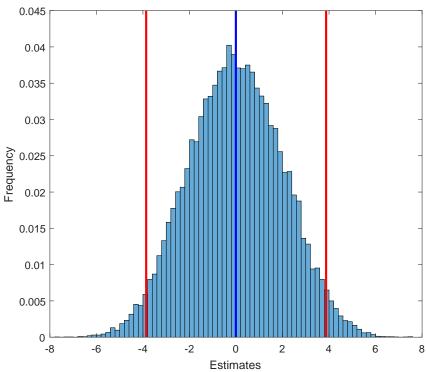


Figure D.1: Empirical distribution of estimated coefficients in the placebo exercise.

Notes: The blue line indicates the average estimated coefficient; the red lines indicate the 2.5% and 97.5% percentiles of distribution of  $\hat{\beta}^m$  across the  $m=1,\ldots,30,000$  simulations. The dependent variable is the 2000–2007 change in the employment rate.

## D.2 Controlling for size of the residual sector

In the placebo simulations described in Tables 1 to 3, we have drawn the shifters from a mean-zero distribution. In Table D.1, we depart from the mean-zero assumption.

As discussed in Section 4.2, controlling for the region-specific sum of shares,  $\sum_{s=1}^{S} w_{is}$ , is important if the shifters have non-zero mean. In our placebo setting, this is equivalent to controlling for the CZ-specific share of employment in the non-manufacturing sector in 1990,  $1 - \sum_{s=1}^{S} w_{is}$ ; we refer to this control here as the "residual sector control". Panel A in Table D.1 shows that, when the shifters are mean zero, the mean of  $\hat{\beta}$  is not affected by whether we include the residual sector control. However, including the residual sector control attenuates the overrejection problem of traditional inference methods. Intuitively, this control soaks part of the correlation in residuals that traditional inference methods do not take into account. Panel B in Table D.1 shows that, if the shifter mean is non-zero, the OLS estimate of  $\beta$  in eq. (1) suffers from substantial bias when the residual sector control is not included in the regression; this bias disappears once it is included. Specifically, in Panel B,  $\mathcal{X}_s^m \sim \mathcal{N}(1,5)$ , and the estimator in the first row of this panel suffers from negative bias because the positive mean of the shifters creates a positive correlation between the shift-share regressor of interest and the control  $\sum_{s=1}^{S} w_{is}$ , which captures the larger secular decline in the employment rate in regions initially specialized in manufacturing production.

Table D.1: Controlling for the size of the residual sector in each CZ

	Est	imate		Median	eff. s.e.		Rejection rate				
	Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)				Cluster (8)	AKM (9)	AKM0 (10)	
Panel A: Shifters wit	h mear	n equal to	zero							_	
No controls			0.74	0.92	1.91	2.23	48.0%	37.7%	7.6%	4.5%	
Control: $1 - \sum_{s=1}^{S} w_{is}$	-0.02	1.43	0.74	0.84	1.31	1.52	33.6%	28.4%	11.2%	4.7%	
Panel B: Shifters with	h mear	differen	t from z	ero							
No controls			0.71	0.94	1.48	1.66	99.1%	97.8%	85.4%	87.6%	
Control: $1 - \sum_{s=1}^{S} w_{is}$	0.00	1.43	0.74	0.84	1.31	1.52	33.3%	27.8%	11.1%	4.6%	

Notes: All estimates in this table use the change in the share of the working-age population employed in each CZ as the outcome variable  $Y_i$  in eq. (1). This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0: \beta = 0$  using a 5% significance level test (columns (7) to (10)). Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters CZs in the same state; AKM is the standard error in Remark 5; AKM0 is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by  $2 \times 1.96$ . Results are based on 30,000 placebo samples. In Panel A,  $(X_1^m, \dots, X_S^m)$  is drawn i.i.d. from a normal distribution with zero mean and variance equal to five in each placebo sample. For each of the two panels, the first row presents results in which no control is accounted for in the estimating equation; the second row presents results in which we control for the size of the residual sector.

### D.3 Confounding sector-level shocks: omitted variable bias and solutions

In this appendix, we illustrate the consequences of violations of the assumption that the shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$  are independent of other shocks affecting the outcome variable of interest. Specifically, we show the impact that the presence of latent sector-specific shocks correlated with the shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$  has on the properties of the OLS estimator of the coefficient on the shift-share regressor of interest  $X_i \equiv \sum_{s=1}^S \mathcal{X}_s$ . We also illustrate the properties of two solutions to this problem: (i) the inclusion of regional controls as a proxy for sector-level unobserved shocks (see Section 4.2), and (ii) the use of a shift-share instrumental variable constructed as a weighted average of exogenous sector-level shocks (see Section 4.3).

To generate the shifters of interest, the confounding sectoral shocks, and the exogenous sectorspecific shocks that will enter the instrumental variable, we extend the baseline placebo exercise and, for each sector s and simulation m, we take a draw of a three-dimensional vector

$$(\mathcal{X}_s^{a,m}, \mathcal{X}_s^{b,m}, \mathcal{X}_s^{c,m}) \sim N(0; \tilde{\Sigma}),$$

where  $\mathcal{X}_s^a$  is the shifter of interest,  $\mathcal{X}_s^b$  is the unobserved confounding shock,  $\mathcal{X}_s^c$  is an exogenous shifter. Specifically, the matrix  $\tilde{\Sigma}$  is such that  $var(\mathcal{X}_s^a) = var(\mathcal{X}_s^b) = var(\mathcal{X}_s^c) = \tilde{\sigma}$ ,  $cov(\mathcal{X}_s^a, \mathcal{X}_s^b) = cov(\mathcal{X}_s^a, \mathcal{X}_s^c) = \tilde{\rho}\tilde{\sigma}$ , and  $cov(\mathcal{X}_s^b, \mathcal{X}_s^c) = 0$ . Thus, we impose that  $\mathcal{X}_s^a$  has a correlation of  $\tilde{\rho}$  with both  $\mathcal{X}_s^b$  and  $\mathcal{X}_s^c$ , but  $\mathcal{X}_s^b$  and  $\mathcal{X}_s^c$  are independent. In our simulations, we set  $\tilde{\rho} = 0.7$  and  $\tilde{\sigma} = 12$ .

To assign the role of a confounding effect to  $\mathcal{X}_s^b$ , we generate an outcome variable as

$$Y_i^m = Y_i^{obs} + \delta \sum_{s=1}^S w_{is} \mathcal{X}_s^{b,m},$$

where  $Y_i^{obs}$  is the observed 2000–2007 change in the employment rate in CZ i, and  $\delta$  is a parameter controlling the impact of the unobserved sectoral shocks  $(\mathcal{X}_1^b,\ldots,\mathcal{X}_S^b)$  on the simulated outcome  $Y_i^m$ . Thus, the parameter  $\delta$  captures the magnitude of the impact that the unobserved shocks  $(\mathcal{X}_1^b,\ldots,\mathcal{X}_S^b)$  have on the outcome variable. We simulate data both with  $\delta=0$  and with  $\delta=6$ .

In addition, we assume that we observe a regional variable that is a noisy measure of CZ i's exposure to the unobserved sectoral shocks  $(\mathcal{X}_1^b, \dots, \mathcal{X}_S^b)$ ,

$$X_i^{b,m} = u_i^m + \sum_s w_{is} \mathcal{X}_s^{b,m}$$
 where  $u_i^m \sim N(0, \sigma_u)$ .

The parameter  $\sigma_u$  thus modulates the measurement error in  $X_i^b$  as a proxy for the impact of the unobserved shocks  $(\mathcal{X}_1^b, \dots, \mathcal{X}_S^b)$  on CZ i. We simulate data both with  $\sigma_u = 0$  and with  $\sigma_u = 6$ .

For each set of parameters  $(\delta, \sigma_u)$  and for each simulation draw, we compute three estimators of the impact of  $X_i^a \equiv \sum_{s=1}^S w_{is} \mathcal{X}_s^a$  on  $Y_i$ . First, we ignore the possible endogeneity problem and compute the OLS estimator without controls; i.e. the estimator in eq. (13). Second, we consider the OLS estimator of the coefficient on  $X_i^a$  in a regression that includes  $X_i^b$  as a proxy for the vector of unobserved confounding sectoral shocks; i.e. the estimator in eq. (23). Third, we consider the IV estimator that uses  $X_i^c \equiv \sum_i w_{is} \mathcal{X}_s^c$  as the instrumental variable; i.e. the estimator in eq. (36). For each of these three estimators, we implement four inference procedures: *Robust*, *Cluster*, *AKM* and *AKM0*. All results are reported in Table D.2.

When there is no confounding sectoral shock ( $\delta = 0$ ), Panel A shows that all three estimators yield an average coefficient close to zero. Panels B and C report results in the presence of confounding sectoral shocks ( $\delta > 0$ ); in this case, the OLS estimator of the coefficient on  $X_i^a$  in a simple regression of  $Y_i$  on  $X_i^a$  without additional covariates is positively biased ( $\hat{\beta} = 4.2$ ). The introduction of the regional control only yields unbiased estimates when it is a good proxy for the latent confounding sectoral shock (i.e. if  $\sigma_u = 0$  as in Panel B). In contrast, the IV estimate always yields an average estimated coefficient close to zero.

As illustrated in Table D.2, traditional inference methods always under-predict the dispersion in the estimated coefficient. As discussed in eq. (21), this is driven by the correlation between the unobservable residuals of regions with similar sector employment compositions. The *AKM* and *AKM0* inference procedures impose no assumption on the cross-regional pattern of correlation in the regression residuals and yield, on average, estimates of the median length of the 95% confidence interval that are equal or higher to the standard deviation of the empirical distribution of estimates. As a result, as Table D.2 reports, while traditional methods overreject the null  $H_0: \beta = 0$  in the context of both OLS and IV estimation procedures, our methods yield the correct test size for both estimators.

<sup>&</sup>lt;sup>4</sup>Using the notation in Section 4.2, the simulated variable  $\mathcal{X}_s^a$  corresponds to  $\mathcal{X}_s$ , the simulated variable  $\mathcal{X}_s^b$  is an element of  $\mathcal{X}_s$ ,  $u_i$  corresponds to  $U_i$ , and  $X_i^b$  to  $Z_i$ . The value of the parameter  $\gamma$  in eq. (26) is thus equal to  $\tilde{\rho}$ .

Table D.2: Magnitude of standard errors and rejection rates—Confounding effects

	Est	imate		Median	eff. s.e.		Reject. $H_0$ : $\beta = 0$ at 5%			
	Mean	Std. dev	Robust	Cluster	AKM	AKM0	Robust	Cluster	AKM	AKM0
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: No confo	unding	g effect ( $\delta$	$\tilde{s}=0$ )							
OLS no controls	0.00	1.28	0.47	0.59	1.23	1.43	48.2%	37.6%	7.7%	4.5%
OLS with controls	0.00	1.80	0.67	0.83	1.72	1.97	47.6%	37.9%	7.9%	4.7%
2SLS	0.00	1.84	0.69	0.85	1.76	2.02	47.7%	37.7%	7.7%	4.6%
Panel B: Confound	ding ef	fect ( $\delta =$	6) and p	erfect re	gional	control	$(\sigma_u=0)$			
OLS no controls	4.19	1.47	0.58	0.70	1.38	1.60	97.9%	96.8%	80.9%	72.2%
OLS with controls	-0.01	1.81	0.67	0.83	1.72	1.97	48.2%	38.3%	8.1%	4.6%
2SLS	-0.01	1.85	0.69	0.85	1.75	2.02	48.1%	38.3%	8.0%	4.7%
Panel C: Confound	ding ef	fect ( $\delta =$	6) and i	mperfect	t region	nal cont	rol ( $\sigma_u =$	= 2)		
OLS no controls	4.20	1.47	0.58	0.70	1.37	1.60	97.9%	96.8%	81.4%	72.6%
OLS with controls	4.10	1.46	0.58	0.70	1.39	1.61	97.7%	96.3%	79.4%	71.3%
2SLS	-0.22	2.46	0.93	1.10	2.12	2.66	41.7%	34.0%	8.1%	4.6%

Notes: All estimates in this table use the change in the share of the working-age population employed in each CZ as the outcome variable  $Y_i$  in eq. (1). This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta=0$  using a 5% significance level test (columns (7) to (10)). The median effective standard error refers to the median length of the 95% confidence interval across the simulated datasets divided by  $2\times1.96$  *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. All results are based on 30,000 simulation draws.

### D.4 Panel data: serial correlation in residuals and shifters

In this appendix, we focus on panel data applications and perform several placebo exercises that illustrate the consequences of serial correlation in either the shifters  $(\mathcal{X}_1,\ldots,\mathcal{X}_S)$  or the regression residuals on the properties of several standard error estimates. For each of our placebo exercises, we generate 30,000 placebo samples indexed by m. Each of them contains 722 regions, 397 sectors, and 2 periods: the first period corresponds to 1990–2000 changes, and the second period corresponds to 2000–2007 changes. As in the baseline placebo, each region corresponds to a U.S. Commuting Zone (CZ), and each sector corresponds to a 4-digit SIC manufacturing industry. We index each region by j and each sector by k. When implementing the AKM and AKM0 in this context, we follow the approach in Section 5.2 by defining "generalized regions" as i = (j,t), "generalized sectors" as s = (k,t), and shares  $w_{is}$  as in eq. (41).

As in our baseline placebo, each simulated sample m has identical values of the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$ . Specifically, the shares in periods 1 and 2 correspond to employment shares in 1990 and 2000, respectively. Depending on the placebo exercise, the placebo samples may differ across simulated samples in terms of the outcomes  $\{Y_i\}_{i=1}^N$ . Finally, all placebo samples always differ in the shifters  $(\mathcal{X}_1, \ldots, \mathcal{X}_S)$ .

For each simulated sample m, we draw the random vector of shifters  $(\mathcal{X}_1^m, \dots, \mathcal{X}_S^m)$  from the joint distribution

$$(\mathcal{X}_1^m, \dots, \mathcal{X}_S^m) \sim \mathcal{N}(0, \Sigma^2),$$
 (D.1)

where  $\Sigma^2$  is a  $S \times S$  covariance matrix with  $\Sigma^2_{sk} = (1 - \rho^2)\sigma^2 \mathbb{I}\{s = k\} + \rho^2\sigma^2 \mathbb{I}\{c(s) = c(k)\}$  and, for every s, c(s) indicates the "cluster" that the generalized sector s belongs to. We incorporate serial correlation in the sector-level shocks by defining clusters of generalized sectors associated with the same underlying sector in different periods. We follow the baseline placebo by setting  $\sigma^2 = 5$ . The value of  $\rho^2$  controls the degree of correlation across shifters of different generalized sectors that correspond to the same underlying industrial sector at different points in time.

For each simulated sample m, we generate the outcome of region i in the placebo sample m as

$$Y_i^m = Y_i + \eta_i^m, \tag{D.2}$$

where  $Y_i$  denotes the change in the employment rate in the generalized region i. By changing the distribution from which the term  $\eta_i^m$  is drawn, we change the distribution of the regression residuals. We implement different placebo exercises in which  $\{\eta_i^m\}_{i=1}^N$  is drawn from different distributions.

In some placebo exercises, we allow for serial correlation in  $\eta_i$  for every region i but impose that  $\eta_i$  is independent of  $\eta_j$  for any two different regions i and j; specifically,

$$(\eta_1^m, \dots, \eta_{1444}^m) \sim \mathcal{N}(0, \Sigma^1),$$
 (D.3)

where  $\Sigma^1$  is a  $1444 \times 1444$  covariance matrix with  $\Sigma^1_{ii'} = (1 - \rho^1)\sigma^1 \mathbb{I}\{i = i'\} + \rho^1\sigma^1 \mathbb{I}\{j(i) = j(i')\}$  and j(i) is the region associated with the generalized observation i. We set  $\sigma^1 = Var(Y_i)/2$  and generate different placebo samples for different values of  $\rho^1$ . The value of  $\rho^1$  controls the degree of correlation

across regression residuals of different generalized regions that correspond to the same geographic region at different points in time.

In some other placebo exercises, we assume that  $\eta_i^m$  has a shift-share structure with shares identical to those entering the shift-share component of interest. Specifically, we assume that

$$\eta_i^m = \sum_{s=1}^S w_{is} \mu_s^m \quad \text{such that} \quad (\mu_1^m, \dots, \mu_S^m) \sim \mathcal{N}\left(0, \Sigma^2\right),$$
(D.4)

where  $\Sigma^2$  is identical to the variance matrix of the shifters  $(\mathcal{X}_1^m, \dots, \mathcal{X}_S^m)$  introduced in eq. (D.1).

We start by evaluating the robustness of our results to the existence of serial correlation in regional outcomes or regression residuals. In Panel A of Table D.3, we implement a placebo exercise in which the shifters are drawn according to eq. (D.1) with  $\rho^2 = 0$  (i.e. no serial correlation in sectoral shifters) and the outcome variables are drawn according to eqs. (D.2) and (D.3) with three different values of  $\rho^1$  (i.e. different degree of serial correlation in the regression residuals). The rejection rates of all six inference procedures we consider (*Robust*, *Cluster*, *AKM* and *AKM0*, the last two both in a version that assumes that the shifters are independent, and in a version that allows them to be serially correlated) are robust to different degrees of serial correlation in the regression residuals. The reason is that, as illustrated in column (4) of Table D.3, the standard deviation of the estimator  $\hat{\beta}$  is invariant to these patterns of serial correlation in the regression residuals.

In Panel B, we implement a placebo exercise in which the shifters are drawn according to eq. (D.1) with  $\rho^2$  equal to either 0, 0.5 or 1 (i.e. different degrees of serial correlation in sectoral shifters) and the distribution of the simulated outcome variables is identical to their empirical distribution (i.e.  $\eta_i^m = 0$  for every region i and placebo sample m). The results indicate that the larger the serial correlation in the sector-level shifters, the larger the rejection rates implied by the *Robust* and *Cluster* standard errors, as well as those implied by an implementation of the *AKM* and *AKM0* inference procedures that wrongly assumes that the shifters are independent across generalized sectors. Conversely, as illustrated in columns (15) and (16) in Panel B of Table D.3, the *AKM* and *AKM0* become very close to the nominal rejection rate of 5% once we cluster across generalized sectors that correspond to the same underlying sector at different points in time.

In Panel C, we depart from the setting described in Panel B in that we draw values of  $\eta_i^m$  according to the distribution described in eq. (D.4). The sector-level shifters entering the shift-share covariate of interest  $X_i^m$  and the term  $\eta_i^m$  are thus drawn from the same distribution. The results are very similar to those in Panel B.

Finally, in Panel D, we draw shifters  $(\mathcal{X}_1^m,\ldots,\mathcal{X}_S^m)$  that are not only serially correlated but also correlated across 4-digit industries belonging to the same 3-digit sector. Columns (11) to (14) show that, when ignored by the corresponding inference procedure, such correlation patterns in the shifters of interest lead to an overrejection problem, the severity of which depends on the correlation in the shifters. Columns (15) and (16) show that this overrejection problem disappears when we implement the *AKM* and *AKM0* inference procedures clustering across all generalized shifters that correspond to pairs of a 4-digit sectors and time period such that the 4-digit sector is associated to the same 3-digit industry.

Table D.3: Magnitude of standard errors and rejection rates: panel data with serially correlated shifters

Seria	l Correl.	Est	imate		Median eff. s.e.						Rejectio	n rate o	of H <sub>0</sub> : β	= 0 at 5%	vo
$\overline{ ho^1}$	$\rho^2$	Mean	Std. dev	Robust	Cluster	AKM	AKM0		AKM0 (cluster)		Cluster	AKM	AKM0		AKM0 (cluster)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Panel	A: Corr	elation	over tim	e in a re	sidual re	gion-l	evel con	nponent							
0	0	-0.01	1.12	0.55	0.67	1.06	1.15	1.05	1.17	34.3%	24.0%	6.7%	5.0%	7.0%	4.8%
0.5	0	-0.01	1.10	0.55	0.66	1.07	0.00	1.06	1.18	32.7%	24.6%	6.6%	5.0%	6.7%	4.8%
1	0	-0.01	1.09	0.55	0.66	1.05	0.00	1.05	1.16	32.7%	24.2%	5.9%	4.7%	6.5%	4.3%
Panel	B: Corre	elation	over time	e in shif	ter of int	erest									
0	0	-0.01	1.05	0.46	0.60	1.02	1.1	1.01	1.12	39.4%	27.0%	6.4%	4.6%	6.7%	4.5%
0	0.5	0.00	1.13	0.47	0.59	1.04	1.12	1.09	1.22	42.9%	31.6%	8.1%	6.1%	6.9%	4.7%
0	1	0.00	1.22	0.47	0.57	1.07	1.14	1.18	1.37	46.2%	36.5%	10.1%	7.9%	7.6%	4.5%
Panel	C: Corre	elation	over time	e in shif	ter of in	terest a	nd in a	residual	shift-sha	re comp	onent				
0	0	0.00	1.06	0.47	0.60	1.03	1.11	1.02	1.14	39.7%	27.5%	6.7%	4.8%	6.9%	4.7%
0	0.5	0.00	1.14	0.47	0.59	1.05	1.12	1.1	1.23	42.6%	31.5%	8.0%	6.2%	6.8%	4.5%
0	1	0.01	1.22	0.48	0.58	1.07	1.14	1.19	1.38	45.8%	36.1%	9.6%	7.5%	7.2%	4.2%
Panel	D: Corr	elation	over tim	e and w	ithin 3-d	igit se	ctors in	shifter of	interest	and in a	residua	l shift-	share co	mponent	
0	0	0.00	1.06	0.46	0.60	1.03	1.11	1.01	1.16	39.6%	27.1%	6.6%	4.9%	7.2%	4.7%
0	0.5	0.02	1.24	0.47	0.61	1.03	1.1	1.19	1.39	47.3%	34.6%	11.8%	9.8%	7.4%	4.5%
0	1	0.00	1.40	0.47	0.61	1.02	1.09	1.35	1.68	53.5%	41.7%	16.9%	14.5%	8.1%	4.2%

Notes: All estimates in this table use the change in the share of the working-age population employed in each CZ as the outcome variable  $Y_i$  in eq. (1). This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (8)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (8) to (15)). The median effective standard error refers to the median length of the 95% confidence interval across the simulated datasets divided by  $2 \times 1.96$  Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters CZs in the same state; AKM is the standard error in Remark 5; AKM0 is the confidence interval in Remark 6. In Panels A, BA, and C, AKM (cluster) and AKM0 (cluster) assume that the shifter corresponding to each 4-digit SIC shifter is distributed independently of those corresponding to other 4-digit shifter, but allow for correlation over time in these 4-digit SIC shifters. In Panel D, AKM (cluster) and AKM0 (cluster) additionally allow for correlation across 4-digit SIC shifters that belong to the same 3-digit SIC sector. All results are based on 30,000 simulation draws.

### D.5 Misspecification in linearly additive potential outcome framework

In this appendix section, we study the consequences of potential misspecification in the linearly additive potential outcome framework introduced in eq. (11) in Section 3.3. The extent to which this linearly additive framework is misspecified obviously depends on what the true potential outcome framework is. Inspired by the economic model described in Section 3, we outline a nonlinear potential outcome framework in Appendix D.5.1. In Appendix D.5.2, we determine theoretically the asymptotic properties of the OLS estimator of the coefficient on the shift-share component in the linearly additive potential outcome framework; specifically, we compare the treatment effects implied by the linear framework to those implied by the nonlinear one. In Appendix D.5.3, we present simulation results that quantify the bias in the estimation of treatment effects that arise from assuming a linearly additive potential outcome framework when the true one corresponds to the nonlinear framework described in Appendix D.5.1.

### D.5.1 Nonlinear potential outcome framework

Consider the special case of the model of Section 3 in which the labor demand elasticity is identical in all sectors, i.e.  $\sigma_s = \sigma$  for all s. We also set  $\rho_s \equiv 1$  for all s. In this case, region i's labor demand in sector s is

$$L_{is} = (\omega_i)^{-\sigma} (\chi_s \mu_s \eta_{is}),$$

which implies that the total labor demand in region i is

$$L_i = (\omega_i)^{-\sigma} \sum_{s=1}^S (\chi_s \mu_s \eta_{is}).$$

By equalizing this expression with the expression for region i's labor supply in eq. (4) in Section 3, we obtain the following relationship between equilibrium wages in region i and both labor supply and labor demand shocks in i:

$$\log \omega_i = \check{\beta} \log \left( \sum_{s=1}^{S} (\chi_s \mu_s \eta_{is}) \right) - \check{\beta} \log \nu_i$$
 (D.5)

where  $\check{\beta} \equiv (\phi + \sigma)^{-1}$ .

We focus here on determining the impact on log-changes in regional wages  $\omega_i$  of log-changes in the sectoral demand shifters  $\{\chi_s\}_{s=1}^S$ ; i.e. using the notation introduced in Section 3.2, we focus on characterizing the impact of  $\{\hat{\chi}_s\}_{s=1}^S$  on  $\hat{\omega}_i$ . Because of the nonlinear nature of the relationship between labor demand shocks and wages in eq. (D.5), the impact of  $\{\hat{\chi}_s\}_{s=1}^S$  on  $\hat{\omega}_i$  depends on the changes in all other labor demand and supply shocks. For simplicity, we focus on the case in which all these other labor demand and supply shocks remain constant at their initial level. From eq. (D.5), the wages in the new and old equilibria are given by

$$\log \omega_i = \check{eta} \log \left( \sum_{s=1}^{\mathcal{S}} \chi_s^0 \mu_s^0 \eta_{is}^0 e^{\hat{\chi}_s} 
ight) - \check{eta} \log 
u_i^0,$$

$$\log \omega_i^0 = \check{\beta} \log \left( \sum_{s=1}^S \chi_s^0 \mu_s^0 \eta_{is}^0 \right) - \check{\beta} \log \nu_i^0,$$

where we use a superscript zero to denote the value of the variables in the initial equilibrium and the absence of superscript denotes the value of the corresponding variable in the new equilibrium. By taking the difference between these two expressions,

$$\hat{\omega}_{i} = \check{\beta} \log \left( \sum_{s=1}^{S} \frac{\chi_{s}^{0} \mu_{s}^{0} \eta_{is}^{0}}{\sum_{k=1}^{S} \chi_{k}^{0} \mu_{k}^{0} \eta_{ik}^{0}} e^{\hat{\chi}_{s}} \right) = \check{\beta} \log \left( \sum_{s=1}^{S} \frac{L_{is}^{0} \left(\omega_{i}^{0}\right)^{\sigma}}{\sum_{k=1}^{S} L_{ik}^{0} \left(\omega_{i}^{0}\right)^{\sigma}} e^{\hat{\chi}_{s}} \right) = \check{\beta} \log \left( \sum_{s=1}^{S} \frac{L_{is}^{0}}{L_{i}^{0}} e^{\hat{\chi}_{s}} \right)$$
(D.6)

where the second equality follows from rearranging the terms in the labor demand expression in eq. (2) in Section 3 to obtain the equality  $\chi_s^0 \mu_s^0 \eta_{is}^0 = L_{is}^0 \left(\omega_i^0\right)^\sigma$  for every region and sector, and the third equality follows from the fact that labor market clearing yields  $L_i^0 = \sum_{s=1}^S L_{is}^0$ .

Note that, by using data on the labor allocation across sectors for every region in some initial equilibrium (i.e.  $L_{is}^0/L_i^0$ , for every i and s), the expression in eq. (D.6) allows to compute the effect of changes in the sector-specific labor demand shifters  $\{\chi_s\}_{s=1}^S$  while calibrating the value of the overall labor demand shifter  $(\chi_s^0)^{\rho_s}\mu_s^0\eta_{is}^0$  at the initial equilibrium. Furthermore, the last expression in eq. (D.6) has the advantage that, conditional on values of  $\{\hat{\chi}_s\}_{s=1}^S$  that are of interest, it depends exclusively on the parameter  $\check{\beta}$ ; specifically, it does not depend on the labor demand parameter  $\sigma$ .

We can map the expression in eq. (D.6) to a nonlinear potential outcome framework by setting  $\mathcal{X}_s = \hat{\chi}_s$ ,  $Y_i = \hat{\omega}_i$ , and  $w_{is} = L_{is}^0 / L_i^0$  for every region and sector; i.e.

$$Y_i(\mathcal{X}_1, \dots, \mathcal{X}_S) = \check{\beta} \log \left( \sum_{s=1}^S w_{is} e^{\mathcal{X}_s} \right).$$
 (D.7)

According to the model in Section 3, this nonlinear potential outcome function yields the exact expression for the change in wages implied by a change in the labor demand shifters  $\{\hat{\chi}_s\}_{s=1}^S$ . Using eq. (D.7) we can also compute the treatment effect on region i of changing the shifters from  $\{\mathcal{X}_s\}_{s=1}^S$  to  $\{\mathcal{X}_s'\}_{s=1}^S$ ,

$$Y_i(\mathcal{X}_1, \dots, \mathcal{X}_S) - Y_i(\mathcal{X}_1', \dots, \mathcal{X}_S') = \check{\beta} \Big[ \log \Big( \sum_{s=1}^S w_{is} e^{\mathcal{X}_s} \Big) - \log \Big( \sum_{s=1}^S w_{is} e^{\mathcal{X}_s} \Big) \Big].$$
 (D.8)

and the average treatment effect

$$\bar{Y}(\mathcal{X}_1, \dots, \mathcal{X}_S) - \bar{Y}(\mathcal{X}_1', \dots, \mathcal{X}_S') = \check{\beta} \frac{1}{N} \sum_{i=1}^N \left[ \log \left( \sum_{s=1}^S w_{is} e^{\mathcal{X}_s} \right) - \log \left( \sum_{s=1}^S w_{is} e^{\mathcal{X}_s} \right) \right]. \tag{D.9}$$

The linearly additive function in eq. (11) in Section 3.3 provides a first-order approximation to the nonlinear function in eq. (D.8). In the next two subsections, we study the extent to which the linear expression in eq. (11) provides an accurate approximation to the nonlinear one in eq. (D.7). Specifically, we explore the extent to which the treatment effects in eqs. (D.8) and (D.9) are well approximated by those computed on the basis of the linear potential outcome framework introduced

in Section 3.3.

The extent to which the linear approximation is accurate will depend on the distribution of  $\{\mathcal{X}_s\}_{s=1}^S$ . Throughout this section, we assume that  $\{\mathcal{X}_s\}_{s=1}^S$  are independently drawn from a normal distribution,

$$\mathcal{X}_s \sim \mathcal{N}(0, \gamma^2),$$
 (D.10)

so that  $e^{x_s}$  is log-normally distributed with  $E[(e^{x_s})^k] = e^{k^2 \gamma^2/2}$ .

### D.5.2 Asymptotic properties of the shift-share linear specification

We consider here the asymptotic properties of the OLS estimator of  $\beta$  in the linear shift-share regression,

$$Y_i = \alpha + \beta \sum_{s=1}^{S} w_{is} \mathcal{X}_s + \epsilon_i, \tag{D.11}$$

when the distribution of  $\mathcal{X}_s$  for every sector s is given by eq. (D.10), and the distribution of  $Y_i$  for every region i is given by the potential outcome framework in eq. (D.7). Since  $\mathcal{X}_s$  has mean zero, the constant does not affect the regression estimand, which is given by

$$\beta = \frac{\sum_{i=1}^{N} E[X_i Y_i]}{\sum_{i=1}^{N} E[X_i^2]},$$
(D.12)

where, under eqs. (D.7) and (D.10),

$$\sum_{i=1}^{N} E[X_i^2] = \gamma^2 \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}^2,$$
(D.13)

and

$$\sum_{i=1}^{N} E[X_i Y_i] = \check{\beta} E \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is} \mathcal{X}_s \log \left( \sum_{k=1}^{S} w_{ik} e^{\mathcal{X}_k} \right). \tag{D.14}$$

Using eqs. (D.12) to (D.14), we can obtain an expression for  $\beta$ , the OLS estimand in a regression of  $Y_i$  on  $\sum_{s=1}^{S} w_{is} \mathcal{X}_s$ ,

$$\beta = \check{\beta} \frac{\sum_{i=1}^{N} \sum_{s=1}^{S} w_{is} E[\gamma Z_s \log(\sum_{k=1}^{S} w_{ik} e^{\gamma Z_k})]}{\gamma^2 \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}^2},$$
(D.15)

as well as for the difference between this value of  $\beta$  and the parameter from the nonlinear model in eq. (D.5):

$$\beta - \check{\beta} = \check{\beta} \frac{\sum_{i=1}^{N} \sum_{s=1}^{S} w_{is} E[\gamma Z_s \log(\sum_{k=1}^{S} w_{ik} e^{\gamma Z_k})]}{\gamma^2 \sum_{i=1}^{N} \sum_{s=1}^{S} w_{is}^2} - \check{\beta}, \tag{D.16}$$

where  $\{Z_s\}_{s=1}^S$  are *i.i.d* standard normal. As it is clear from this expression, the difference between  $\beta$  and  $\check{\beta}$  depends on the shares  $\{w_{is}\}_{i=1,s=1,\prime}^{N,S}$  the value of the  $\gamma$  (i.e. the standard deviation of  $\mathcal{X}_s$  for every s, according to eq. (D.10)), and the value of  $\check{\beta}$  itself.

The expression analogous to that in eq. (D.8) when the linear potential outcome framework in

eq. (11) in Section 3.3 is assumed is the following,

$$Y_i(\mathcal{X}_1,\ldots,\mathcal{X}_S) - Y_i(\mathcal{X}_1',\ldots,\mathcal{X}_S') = \beta \Big(\sum_{s=1}^S w_{is}(\mathcal{X}_s - \mathcal{X}_s')\Big).$$
 (D.17)

and the expression analogous to that in eq. (D.8) is

$$\bar{Y}(\mathcal{X}_1,\ldots,\mathcal{X}_S) - \bar{Y}(\mathcal{X}_1',\ldots,\mathcal{X}_S') = \beta \frac{1}{N} \sum_{i=1}^N \left( \sum_{s=1}^S w_{is}(\mathcal{X}_s - \mathcal{X}_s') \right). \tag{D.18}$$

### D.5.3 Simulation

In this section, we construct a simulation exercise to quantify: (a) the difference between  $\beta$  and  $\check{\beta}$ , using eq. (D.16) to compute such difference; (b) the correlation coefficient between the *i*-specific treatment effects in eq. (D.8) and those in eq. (D.17); and, (c) the difference between the average treatment effect in eq. (D.9) and that in eq. (D.18).

In all simulations, we calibrate the labor supply elasticity to equal 2,  $\sigma = 2$ , and the inverse labor supply elasticity to equal 0.5,  $\phi = 0.5$ , implying that  $\check{\beta} = 0.4$ . To remain close to our baseline placebo exercise, we calibrate the shares  $\{w_{is}\}_{i=1,s=1}^{N,S}$  using 1990 data on sector-region employment shares for 722 US CZs and 396 4-digit manufacturing sectors. Concerning the value of the variance of the sectoral shifters, we present results for five different values of  $var(\mathcal{X}_s) = \gamma^2$  varying between  $\gamma^2 = 0.5$  and  $\gamma^2 = 10$ . For each value of  $\gamma$ , we then generate 30,000 samples indexed by m such that  $\{\mathcal{X}_s^m\}_{s=1}^{396}$  are independently drawn according to eq. (D.10) and  $\{Y_i^m\}_{i=1}^{722}$  are constructed according to eq. (D.7).

For each placebo sample m, we compute the OLS estimator  $\hat{\beta}$  of the parameter  $\beta$  defined in eq. (D.12), confidence intervals for  $\beta$  according to the *Robust*, *Cluster*, *AKM* and *AKM0* inference procedures, the true linear approximation to the i-specific treatment effect and to the average treatment effect (i.e. the expressions in eqs. (D.17) and (D.18) with  $\check{\beta}$  instead of  $\beta$ ), the estimated linear approximation to the i-specific treatment effect and to the average treatment effect (i.e. the expressions in eqs. (D.17) and (D.18) with  $\hat{\beta}$  instead of  $\beta$ ), and the true i-specific treatment effects and their average (i.e. the expressions in eqs. (D.8) and (D.9) with  $\check{\beta}=0.4$ ).

A comparison of columns (2) and (3) in Table D.4 illustrates that the average across the placebo samples generated under the same value of  $\gamma$  of the OLS estimates of  $\beta$ ,  $\overline{\hat{\beta}} \equiv (30,000)^{-1} \sum_{m=1}^{30,000} \hat{\beta}^m$  (reported in column (3)) is very close to the true value of the parameter  $\beta$  (reported in column (2)). We compute this true value of  $\beta$  using the expression in eq. (D.15) and Monte Carlo integration based on 50,000 draws of  $(Z_1,\ldots,Z_S)$  from the distribution in eq. (D.10). Thus, as expected, the average value of  $\hat{\beta}^m$  is very close to its theoretical value.

Columns (4)–(7) of Table D.4 report different measures of the average treatment effect across simulated samples. Specifically, we compute in these three columns, in this order, the average across the 30,000 placebo samples of: (a) the true linear approximation to the average treatment effect (i.e. the expression in eq. (D.18) with the value  $\beta$  set to the expression in eq. (D.15)); the estimated linear approximation to the average treatment effect (i.e. the expression in eq. (D.18) with  $\hat{\beta}^m$  instead of  $\beta$ ); and the true average treatment effect (i.e. the expression in eq. (D.9)). When the variance of sector-

level shocks is low ( $\gamma^2 = 0.1$ ), the first row in Table D.4 shows that all these three averages are very close to each other. As the variance of sector-level shocks grows, the remaining rows in Table D.4 show that the bias in the linear approximations to the average treatment effect grows. Columns (6) and (7) of Table D.4 illustrate that not only the linear approximation to the average treatment effects worsen as  $\gamma^2$  increases, but the average (across the 30,000 placebo samples) correlation coefficient between the *i*-specific linear treatment effects in eq. (D.17) (computed with  $\check{\beta}$  instead of  $\beta$ ) and the nonlinear ones in eq. (D.8) becomes much lower.

In summary, Table D.4 shows that, when the value of the variance of the sector-level shocks is small, the difference between  $\beta$  and  $\check{\beta}$  reported in eq. (D.16) is small, and the linear approximations to the treatment effects in eqs. (D.17) and (D.18) remain very close to their non-linear counterparts in eqs. (D.8) and (D.9). Conversely, these approximations become much worse as the variance of the sector-level shocks increases.

In Table D.5, we study the performance of different inference methods in their capacity to provide information about the value of  $\beta$  in eq. (D.15) or about the parameter  $\check{\beta}$ . Columns (2)–(6) report the standard deviation of the OLS estimated coefficients  $\hat{\beta}^m$  and the average estimated standard errors obtained with different inference procedures. Columns (7)–(10) report the rejection rate of the null hypothesis that  $\beta = \check{\beta}$  and columns (11)–(14) report the rejection rate of the null hypothesis that  $\beta$ coincides with the expression in eq. (D.15). Results are similar for all levels of  $\gamma^2$ : robust and stateclustered standard errors significantly underestimate the standard deviation of the OLS estimator, while the AKM and AKM0 are much closer to this standard deviation. In line with these results, when testing the null that  $\beta$  coincides with the expression in eq. (D.15) at the 5% significance level, columns (13)-(14) show that the rejection rates are close to 5% for AKM and AKM0 inference procedures, but columns (11)-(12) show that the analogous rejection rates are around 50% for the Robust and Cluster inference procedures. Given the difference (reported in Table D.4) between the value of  $\beta$  in eq. (D.15) and the value of  $\mathring{\beta}$ , it is not surprising that, as illustrated in columns (7)–(10) of Table D.5, rejection rates for the null that  $\beta$  equals  $\dot{\beta}$  are larger than for the null that  $\beta$  equals the expression in eq. (D.15), no matter what inference procedure we use. However, it is remarkable that, when the AKM0 inference procedure is used, these rejection rates remain quite close to 5% and always below 10%.

In summary, Table D.5 shows that, no matter what the value of the variance of the sector-level shocks is, the relative performance of the four different inference procedures that we consider in all our placebo simulations is consistent with what we have documented in Sections 2 and 6.1. *Robust* and *Cluster* lead to overrejection of the estimand of the OLS estimator, while *AKM* and *AKM0* maintain their good coverage properties for this estimand. Interestingly, even when the OLS estimated does not coincide with the structural parameter  $\check{\beta}$ , the *AKM0* inference procedure maintains good coverage for this structural parameter; the reason is that, as the variance of the sector-level shocks increases and the OLS estimand becomes more different from  $\check{\beta}$ , the length of the *AKM0* confidence interval also increases, and it does so at a rate such that it contains  $\check{\beta}$  in a fraction of placebo samples that is always between 5% and 10%.

Table D.4: First-order approximation error: bias in  $\hat{\beta}$  and in estimated average treatment effect

$var(\mathcal{X}_s)$	eq. (D.15)	$\overline{\hat{eta}}$	Avg. Ti	reatme	nt Effect	Correlation between
			Linear		Non-linear	linear & non-linear
			Estimated	True	True	avg. treatment effect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.1	0.41	0.41	(4) $(5)$ $0.00$ $0.00$		0.00	0.96
1	0.48	0.48	0.00	0.00	0.05	0.76
2	0.54	0.53	0.00	0.00	0.11	0.64
5	0.63	0.62	0.01 0.0		0.34	0.47
10	0.65	0.62	0.01	0.00	0.81	0.36

Notes: The sectoral shifters  $\mathcal{X}_s$  are i.i.d, drawn from a normal distribution with mean zero and variance  $var(\mathcal{X}_s)$ . Column (1) indicates the different values of  $var(\mathcal{X}_s)$  that we consider in our simulation exercise; for each value of  $var(\mathcal{X}_s)$  listed in column (1), we generate 30,000 simulated samples. Given a set of draws of the shifters  $(\mathcal{X}_1^m,\ldots,\mathcal{X}_s^m,\ldots,\mathcal{X}_s^m)$  for a simulated sample indexed by m, their true impact on the outcome of a region i is  $\beta \log(\sum_s w_{is} \exp(\mathcal{X}_s^m))$  and the first-order approximation to this expression is  $\beta \sum_s w_{is} \mathcal{X}_s^m$ . We set  $\beta = 0.4$  for all our simulation exercises. Given this value of  $\beta$  and the value of  $var(\mathcal{X}_s)$  in column (1), we report in column (2) the value of  $\beta$ , the estimand of the OLS estimator in a regression of  $Y_i$  on  $X_i$  computed according to the expression in eq. (D.15). We report in column (3) the average (across the simulated samples) value of this OLS estimator  $\beta^m$ . Column (4) and (5) reports the average (across the simulated samples) value of the linearly approximated average treatment effect in eq. (D.18), with the only difference being whether the value of  $\beta$  in this expression is set to the value in eq. (D.15) or to the average of the OLS estimator  $\beta^m$ . Column (6) reports the average (across the simulated samples) value of the true average treatment effect in eq. (D.9). Column (7) reports the median (across the simulated samples) value of the correlation coefficient between the true treatment effect in eq. (D.9) and that arising from the first-order approximation in eq. (D.18). See the description in Appendix D.5.3 for additional details.

Table D.5: First-order approximation error: impact on standard errors and rejection rates.

$\overline{var(\mathcal{X}_s)}$	Est	imate		Median	eff. s.e.		Rejection rate of $H_0$ : $\beta = \check{\beta}$				Rejection rate of $H_0$ : $\beta = eq.$ (D.15)			
	Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)	AKM (5)	AKM0 (6)	Robust (7)	Cluster (8)	AKM (9)	AKM0 (10)	Robust (11)	Cluster (12)	AKM (13)	AKM0 (14)
0.1	0.41	0.07	0.03	0.03	0.07	0.08	45.3%	38.7%	9.6%	3.9%	45.1%	38.4%	9.5%	4.0%
1	0.48	0.10	0.03	0.04	0.09	0.10	60.7%	56.3%	15.3%	5.9%	52.8%	47.9%	10.4%	2.8%
2	0.53	0.14	0.04	0.05	0.12	0.14	66.7%	62.6%	17.3%	7.7%	54.1%	49.1%	9.1%	2.5%
5	0.62	0.19	0.06	0.07	0.18	0.21	72.5%	68.4%	18.2%	9.1%	53.9%	48.6%	7.7%	2.4%
10	0.62	0.22	0.07	0.08	0.22	0.25	67.8%	63.0%	14.2%	6.8%	53.5%	47.4%	7.3%	3.0%

Notes: The sectoral shifters  $\mathcal{X}_s$  are i.i.d drawn from a normal distribution with mean zero and variance  $var(\mathcal{X}_s)$ . Column (1) indicates the different values of  $var(\mathcal{X}_s)$  that we consider in our simulation exercise; for each value of  $var(\mathcal{X}_s)$  listed in column (1), we generate 30,000 simulated samples. This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), the percentage of placebo samples for which we reject the null hypothesis  $H_0: \beta = \hat{\beta}$  using a 5% significance level test (columns (7) to (10)), and the percentage of placebo samples for which we reject the null hypothesis that  $\beta$  coincides with the expression in eq. (D.15) using a 5% significance level test (columns (11) to (14)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96.

### D.6 Unobserved shift-share components with different shares

Equation (21) in Section 4.1 characterizes the source of the overrejection problem affecting traditional inference methods in shift-share specifications, showing that heteroskedasticity-robust and cluster-robust standard errors overreject whenever the correlation between residuals is positive. This positive correlation arises when the residual has a shift-share structure in eq. (22), the unobserved shifters may vary at the same level as the shift-share covariate of interest (e.g. sectors) or a different one (e.g. countries of origin of immigrants). In this section, we conduct a placebo simulation to illustrate the bias in both robust and state clustered standard errors that arises when the regression residual has a shift-share component.

We generate 30,000 placebo samples indexed by *m* with 722 US CZs and 396 4-digit SIC manufacturing industries. As in the baseline placebo exercise discussed in Sections 2 and 6.1, we compute the shift-share covariate of interest using the sectoral employment shares of US CZs in 1990 and sectoral shifters that are drawn independently from a normal distribution with mean equal zero and variance equal to five; i.e.

$$X_i^m = \sum_{s=1}^{396} w_{is} \mathcal{X}_s^m$$
 such that  $\mathcal{X}_s^m \sim N(0,5)$ .

The difference between the simulation exercise we consider here and the baseline placebo simulation in Sections 2 and 6.1 is that the outcome variable is no longer taken from the observed data. Instead, this outcome variable varies across placebo samples and it is drawn randomly for each simulated sample m as

$$Y_i^m = \sum_{s=1}^{396} \tilde{w}_{is} \mathcal{A}_s^m$$
 such that  $\mathcal{A}_s^m \sim N(0,5)$ ,

where  $\tilde{w}_{is}$  are shares that may be different from (but possibly correlated with) the baseline sectoral employment shares in each CZ; i.e.  $\tilde{w}_{is}$  may be different from  $w_{is}$ . Specifically, for all placebo samples, we generate a single set of alternative shares as

$$\tilde{w}_{is} = \frac{\exp(u_{is} + \ln(w_{is} + v_{is}))}{\sum_{k=1}^{396} \exp(u_{ik} + \ln(w_{ik} + v_{ik}))} \left(\sum_{k=1}^{396} w_{ik}\right)$$
(D.19)

where  $u_{is}$  and  $v_{is}$  drawn randomly such that  $u_{is} \sim N(0, \sigma_u^2)$  and  $v_{is} \sim U[0, \sigma_v]$ .

Given a pair of values  $(\sigma_u, \sigma_v)$ , for each placebo sample we compute: (a) the OLS estimator of the regression of  $Y_i^m$  on  $X_i^m$  and a constant; (b) effective standard errors according to the robust, state-clustered, AKM and AKM0 inference procedures; (c) for each of these inference procedures, the outcome of a 5% significance level test of hypothesis of the null hypothesis  $H_0$ :  $\beta = 0$ . Each row of Table D.6 reports several summary statistics of the distribution of these quantities across the 30,000 placebo samples. Each row does so for placebo samples generated by different values of  $\sigma_u$  and  $\sigma_v$ .

The first row of Table D.6 considers the case in which  $\sigma_v = \sigma_u = 0$ . In this case,  $w_{is} = \tilde{w}_{is}$  for every i and s and, thus, the correlation coefficient between the shares entering the covariate of interest and those entering the regression residual equal 1 (see column (3) in Table D.6). In this case, as in our baseline placebo, robust and state-cluster standard errors have rejection rates for a 5% significance

Table D.6: Bias in standard errors when regression residual is a shift-share term with shares correlated with those entering the shift-share covariate of interest

			Est	imate	Median eff. s.e.				Reject	tion rate	of <i>H</i> <sub>0</sub> :	$\beta = 0$
$\sigma_u^2$ (1)	$\sigma_v$ (2)	$ ho_{w_{is},\tilde{w}_{is}}$ (3)	Mean (4)	Std. dev (5)	Robust (6)	Cluster (7)	AKM (8)	AKM0 (9)	Robust (10)	Cluster (11)	AKM (12)	AKM0 (13)
0	0	1.00	0.00	0.17	0.08	0.08	0.14	0.16	34.8%	31.0%	10.2%	3.7%
1	0	0.77	0.00	0.16	0.08	0.09	0.14	0.16	31.5%	27.3%	9.9%	4.0%
3	0	0.55	0.00	0.15	0.09	0.09	0.13	0.15	23.8%	22.7%	9.8%	4.1%
5	0	0.44	0.00	0.14	0.10	0.10	0.13	0.15	18.0%	17.4%	9.6%	4.3%
0	0.001	1.00	0.00	0.11	0.06	0.06	0.10	0.11	31.6%	28.7%	10.0%	3.7%
1	0.001	0.70	0.00	0.10	0.05	0.06	0.09	0.10	28.1%	26.5%	9.8%	4.2%
3	0.001	0.41	0.00	0.09	0.06	0.06	0.08	0.09	19.0%	19.3%	8.8%	4.1%
5	0.001	0.28	0.00	0.09	0.06	0.06	0.08	0.09	13.4%	14.5%	8.1%	4.4%
0	0.01	1.00	0.00	0.04	0.02	0.02	0.04	0.04	25.3%	23.4%	9.4%	3.6%
1	0.01	0.38	0.00	0.05	0.03	0.04	0.04	0.05	14.3%	14.2%	7.6%	3.8%
3	0.01	0.18	0.00	0.04	0.03	0.03	0.04	0.05	11.6%	12.3%	7.7%	4.4%
5	0.01	0.10	0.00	0.05	0.05	0.04	0.05	0.06	7.9%	9.0%	7.5%	4.2%

Notes: We impose that, for every simulated sample  $m=1,\ldots,30000$ , the outcome variable is  $Y_i^m=\sum_s \tilde{w}_{is}\mathcal{A}_s^m$ , with  $\mathcal{A}_s^m$  drawn from a normal distribution with mean zero and variance equal to five. The shares  $\{\tilde{w}_{is}\}_{i,s}$  vary across the cases described in each of the rows in the table above but, for each of these rows, are fixed across the 30,000 simulated samples. Specifically, given shares  $\{w_{is}\}_{i,s}$  that capture the employment share in CZ i employed in sector s in 1990, we generate each  $\tilde{w}_{is}$  according to the expression in eq. (D.19), with  $u_{is}$  and  $v_{is}$  drawn randomly according to the distributions  $u_{is} \sim \mathcal{N}(0,\sigma_u^2)$  and  $U[0,\sigma_v]$ . The first two columns in the table above indicate the values of  $\sigma_u$  and  $\sigma_v$  used to generate  $\{\tilde{w}_{is}\}_{i,s}$  in each case. As illustrated in the third column, the larger the value of either  $\sigma_u$  or  $\sigma_v$ , the lower the correlation coefficient  $\rho_{w_{is},\tilde{w}_{is}}$  between  $w_{is}$  and  $\tilde{w}_{is}$  across regions and sectors. Given the generated outcome variables  $\{Y_i^m\}_i$  for each simulated sample m, we compute the OLS estimate of  $\beta$  in the regression  $Y_i^m = \beta X_i^m + \epsilon_i^m$ , with  $X_i^m = \sum_s w_{is} \mathcal{X}_s^m$  and each  $\mathcal{X}_s^m$  drawn randomly from a normal distribution with mean zero and variance equal to 5. We indicate the mean and standard deviation of the OLS estimates of  $\beta$  across the simulated samples (columns (4) and (5)), the median effective standard error estimates (columns (6) to (9)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0: \beta = 0$  using a 5% significance level test (columns (10) to (13)). Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters CZs in the same state; AKM is the standard error in Remark 5; AKM0 is the confidence interval in Remark 6.

level test that are around 30%–35%. In contrast, the *AKM* and *AKM0* inference procedures exhibit rejection rates that are 10% and 4%, respectively. The remaining rows of Table D.6 show that, as we increase the value of  $\sigma_v$  and  $\sigma_u$ , the correlation between  $w_{is}$  and  $\tilde{w}_{is}$  declines, which attenuates the overrejection problem affecting testing procedures that rely on robust and state-clustered standard errors. However, the rejection rates of these two inference methods are still above 10% even when the correlation between  $w_{is}$  and  $\tilde{w}_{is}$  is as low as 0.18. For all cases, the rejection rates of the *AKM* and *AKM0* testing procedures remain stable and close to 5%.

### D.7 Heterogeneous treatment effects

We now present a placebo exercise to evaluate the performance of our inference procedures in the presence of heterogeneous treatment effects. For each placebo sample m, we construct the dependent variable as

$$Y_i^m = Y_i + \sum_s w_{is} \mathcal{X}_s^m \beta_{is}$$
 such that  $\beta_{is} = \lambda w_{is}$ .

In all placebo samples,  $Y_i$  is the change in the share of working-age population employed in CZ i and  $w_{is}$  is the share of sector s in total employment of CZ i. As before, in each placebo sample, we take

Table D.7: Heterogeneous treatment effects

		Est	imate		Median (	eff. s.e.		Rejection rate of $H_0: \beta = \beta_0$				
λ	$eta_0$	Mean	Std. dev	Robust	Cluster	AKM	AKM0	Robust	Cluster	AKM	AKM0	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
0	0.00	0.00	1.98	0.73	0.92	1.91	2.22	0.48	0.38	0.07	0.04	
1	0.14	0.15	1.98	0.73	0.92	1.91	2.22	0.48	0.38	0.07	0.04	
3	0.43	0.45	1.98	0.74	0.92	1.91	2.22	0.48	0.38	0.07	0.04	
5	0.72	0.74	1.98	0.74	0.93	1.91	2.23	0.48	0.37	0.08	0.04	

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (3) and (4)), the median effective standard error estimates (columns (5) to (8)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0: \beta = \beta_0$  using a 5% significance level test (columns (9) to (12)) where the true value of  $\beta_0$  shown in column (2) is given in eq. (D.20). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples.

independent draws of the sector-level shifters from a normal distribution with a mean of zero and a variance of 5.

The parameter  $\lambda$  controls the degree of heterogeneity in the treatment effect of the sector-level shifters. When  $\lambda=0$ , this placebo exercise is identical to our baseline placebo exercise in Section 2. We are interested in inference on the OLS estimand. By Proposition 1, it is given by

$$\beta_0 = \sum_{i,s} w_{is}^2 \beta_{is} / \sum_{i,s} w_{is}^2 = \lambda \sum_{i,s} w_{is}^3 / \sum_{i,s} w_{is}^2,$$
 (D.20)

which is linear in  $\lambda$ .

Table D.7 presents the results of the placebo exercise for different values of  $\lambda$ . For all values of  $\lambda$ , the average OLS estimate in column (3) is similar to  $\beta_0$ . Results indicate that both the standard deviation of the OLS estimator and the performance of the inference procedures are not sensitive to the value of  $\lambda$ .

#### D.8 Other extensions

In Table D.8, we report results analogous to those in Table D.1 for outcome variables  $Y_i$  other than the employment rate in CZ i. The rejection rates that we obtain are very similar to those reported in Table D.1 and discussed in Section 6.2.

In Table D.9, we investigate the sensitivity of our results to an alternative definition of "region". We report results for a placebo exercise that is analogous to the baseline placebo exercise discussed in Sections 2 and 6.1 except for the use of counties instead of CZs as regions. We use the County Business Patterns data to construct employment by county and sector using the imputation procedure in Autor, Dorn and Hanson (2013). Since this procedure does not yield wage bill information at the county level, we only implement the placebo exercise for the outcome variables used in Panel A of Tables 1 and 2: employment rate; employment rate in manufacturing; and, employment rate in non-manufacturing. The results show that the rejection rates of all four inference procedures we consider

are very similar to those obtained in the baseline placebo exercise, which are reported precisely in Panel A of Tables 1 and 2.

In Table D.10, we investigate the sensitivity of our results to an alternative definition of "sector". We report results for a placebo exercise that is analogous to the baseline placebo exercise discussed in Sections 2 and 6.1 except for the use of 331 occupations instead of 396 sectors as the unit of observation at which the shifters vary. The results in Table D.10 show that the overrejection problem affecting tradition inference procedures is even more severe when the shift-share covariate aggregates occupation-specific shifters than when it aggregates sectoral shifters. Actually, only the AKM0 inference procedure yields rejection rates for the null hypothesis  $H_0$ :  $\beta = 0$  that are below the 5% significance level of the test.

Table D.8: Controlling for the size of the residual sector in each CZ

	Est	imate		Median	eff. s.e.		Reject	tion rate	of $H_0$ :	$\beta = 0$
	Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)	AKM (5)	AKM0 (6)	Robust (7)	Cluster (8)	AKM (9)	AKM0 (10)
Panel A: Shifters v	vith ze	ro mean								
Outcome variable: ch	ange in	the share	of workin	ıg-age pop	pulation	in manı	ıfacturin	g		
No controls	-0.02	1.87	0.60	0.76	1.78	2.06	55.5%	44.2%	8.1%	4.2%
Control: $1 - \sum_{s} w_{is}$	0.00	1.03	0.56	0.63	0.97	1.12	30.1%	25.8%	10.0%	4.4%
Change in the share of	of worki	ng-age pop	vulation i	n non-ma	ınufactı	ıring				
No controls	0.00	0.94	0.58	0.67	0.89	1.04	23.0%	17.5%	8.1%	4.5%
Control: $1 - \sum_s w_{is}$	0.00	1.05	0.60	0.68	0.97	1.12	27.5%	22.6%	9.8%	5.4%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees				
No controls	0.05	2.67	1.02	1.34	2.58	3.00	47.0%	33.9%	7.8%	4.4%
Control: $1 - \sum_s w_{is}$	0.00	1.21	0.95	1.07	1.15	1.33	12.9%	8.9%	7.9%	4.8%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees in	manufaci	turing		
No controls	0.02	2.94	1.69	2.11	2.75	3.19	27.0%	17.3%	9.3%	4.5%
Control: $1 - \sum_s w_{is}$	0.01	2.13	1.66	1.92	1.98	2.28	12.5%	8.0%	7.7%	4.5%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees in	non-man	ufacturin	g	
No controls	0.00	2.62	1.05	1.33	2.56	2.98	44.5%	32.8%	7.6%	4.4%
Control: $1 - \sum_{s} w_{is}$	0.00	1.24	0.98	1.08	1.17	1.35	12.8%	9.5%	8.5%	4.7%
Panel B: Shifters w	vith no	n-zero me	ean							
Outcome variable: ch	ange in	the share	of workin	ig-age pop	pulation	in manı	ıfacturin	ς		
No controls	-3.92	1.12	0.57	0.81	1.34	1.51	98.7%	97.6%	80.6%	78.7%
Control: $1 - \sum_{s} w_{is}$	0.00	1.05	0.56	0.63	0.97	1.12	31.1%	26.4%	10.3%	4.6%
Outcome variable: ch	ange in	the share	of workin	ig-age pop	pulation	in non-1	nanufact	uring		
No controls	-0.75	0.71	0.48	0.64	0.76	0.86	37.2%	22.2%	14.2%	13.9%
Control: $1 - \sum_s w_{is}$	0.01	1.05	0.60	0.68	0.97	1.13	27.6%	22.5%	9.7%	5.2%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees				
No controls	-6.52	1.55	0.97	1.58	1.91	2.15	99.6%	98.3%	90.9%	90.5%
Control: $1 - \sum_s w_{is}$	0.01	1.22	0.95	1.08	1.15	1.33	13.4%	9.1%	8.1%	4.9%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees in	manufaci	turing		
No controls	-5.38	1.88	1.54	2.29	1.94	2.17	89.3%	69.8%	75.1%	71.0%
Control: $1 - \sum_{s} w_{is}$	-0.02	2.13	1.66	1.91	1.98	2.28	12.5%	8.1%	7.8%	4.7%
Outcome variable: ch	ange in	average lo	g-weekly	wage of a	all empl	oyees in	non-man	ufacturin	8	
	. 01	1 -1	0.00	1 50	1 00	2.15	00.49/	07.00/	00.00/	00 60/
No controls	-6.31	1.54	0.99	1.58	1.90	2.15	99.4%	97.8%	89.0%	88.6%

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta=0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by  $2 \times 1.96$ . Results are based on 30,000 placebo samples. In Panel A,  $(X_1^m, \dots, X_{S-1}^m)$  is drawn i.i.d. from a normal distribution with zero mean and variance equal to 5 in each placebo sample. In Panel B,  $(X_1^m, \dots, X_{S-1}^m)$  is drawn i.i.d. from a normal distribution with mean equal to one and variance equal to 5 in each placebo sample. For each of the two panels, the first row presents results in which no control is accounted for in the estimating equation; the second row presents results in which we control for the size of the residual sector,  $1 - \sum_s w_{is}$ .

Table D.9: Magnitude of standard errors and rejection rates: county-level analysis

	Est	imate	]	Median (	eff. s.e.		Rejection rate of $H_0$ : $\beta = 0$			
	Mean	Std. dev	Robust	Cluster	AKM	AKM0	Robust	Cluster	AKM	AKM0
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: Change in the	share	of worki	ng-age p	opulatio	on					
employed (all)	0.00	0.65	0.24	0.30	0.61	0.67	47.3%	36.3%	8.0%	4.8%
employed (manuf.)	0.00	0.77	0.18	0.27	0.71	0.78	65.5%	51.4%	8.1%	4.6%
employed (non-manuf.)	0.00	0.37	0.21	0.22	0.35	0.39	27.9%	25.3%	8.8%	4.6%

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters *CZs* in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples.

Table D.10: Magnitude of standard errors and rejection rates: occupation-specific shifters

	Est	imate	1	Median	eff. s.e		Rejection rate of $H_0$ : $\beta = 0$			
	Mean	Std. dev	Robust	Cluster	AKM	AKM0	Robust	Cluster	AKM	AKM0
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: Change in the	share	of worki	ng-age	populat	ion					
employed (all)	0.01	8.59	1.13	2.45	7.46	27.82	83.5%	62.4%	24.9%	4.0%
employed (manuf.)	0.02	8.13	0.80	1.82	6.55	25.50	89.7%	75.3%	32.9%	3.2%
employed (non-manuf.)	-0.01	4.03	0.96	1.76	3.06	9.86	65.1%	38.4%	17.9%	3.8%
Panel B: Change in ave	rage lo	g weekl	y wage							
employed (all)	0.00	12.58	1.74	4.23	10.1	38.10	84.8%	62.6%	30.6%	3.4%
employed (manuf.)	-0.07	11.11	3.24	6.18	9.41	31.96	56.2%	27.2%	11.7%	4.9%
employed (non-manuf.)	0.01	12.60	1.77	4.19	9.96	37.96	84.9%	64.1%	31.8%	3.2%

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples.

# Appendix E Empirical applications: additional results

### E.1 Effect of Chinese exports on U.S. labor market outcomes

This section presents additional results that complement the estimates in Section 7.1 of the effect of Chinese import competition on US local labor markets following the approach in Autor, Dorn and Hanson (2013, ADH hereafter).

### E.1.1 Placebo exercise: alternative distributions of shifters

The reduced-form and the first-stage specifications have a panel data structure discussed Section 5.2. Since the outcome data and the share matrix *W* is the same as in the placebo exercise in Appendix D.4, the results of that placebo exercise are informative about the finite-sample properties of the four inference procedures that we consider (robust standard errors, state-clustered standard errors, and the *AKM* and *AKM0* procedures) in the ADH empirical application. In this section, we investigate the robustness of the results in Appendix D.4 to alternative distributions of the sectoral shifters. In particular, instead of assuming that the shifters are i.i.d. according to a normal distribution, we consider distributions that are arguably closer to the distribution of the actual shifters employed in ADH (the growth in sectoral Chinese exports to high-income countries other than the US).

First, we consider a placebo exercise that differs from that in Appendix D.4 only in that the sectoral shifters are drawn independently from the empirical distribution of the shifters used in ADH. The results are presented in Panel A of Table E.1. As in the analysis in Appendix D.4, although the data generating process for our placebo exercise implies that  $\beta = 0$ , the rejection rates of a 5% significance level test of the null hypothesis  $H_0$ :  $\beta = 0$  are substantially above 5% when robust and state-clustered standard errors are used. The rejection rates implied by the *AKM* and *AKM0* procedures are much closer to 5%, with rejection rates are close to 10%.

Second, to get closer to the specification in ADH, we incorporate into our placebo specification the baseline set of controls that ADH use (see, e.g., column (6) of Table 3 in ADH). In particular, we draw the sectoral shifters from the empirical distribution of shifters used in ADH after partialling out the baseline set of controls used in ADH.<sup>5</sup> Panel B of Table E.1 reports the results. For the Robust, Cluster and AKM testing procedures, the rejection rates in Panel B are very similar to those in Panel A, while the AKM0 rejection rate is much closer to the nominal level.

Next, we consider relaxing the assumption that the sectoral shifters are independent, or independent across clusters. This specification is motivated by the concern that the 1990–2000 and 2000–2007 sector-specific growth rates in Chinese exports to high-income countries other than the US were determined at least partly by a common factor that had possibly heterogeneous effects across sectors. We formalize this by modeling year-t imports from China of goods in sector s by high-income countries

<sup>&</sup>lt;sup>5</sup>To partial out a set of controls (which vary by region) from the shifters (which vary by sector), we implement the following two-step procedure. First, we obtain the residual of a regression of the shift-share instrumental variable  $X_i$  used in ADH on the set of controls listed in column (6) of Table 3 in ADH; let  $\ddot{X}_i$  denote this residual. We then draw the shifters from the empirical distribution of the residualized sectoral shifters  $\mathcal{X}^{res}$ , which correspond to the regression coefficients from regressing  $\ddot{X}_i$  onto the vector of shares  $(w_{i1}, \ldots, w_{iS})$ , i.e.  $\mathcal{X}^{res} = (W'W)^{-1}W'\ddot{X}$ .

Table E.1: Alternative distributions of sectoral shifters: placebo

	Estimate			Median	eff. s.e.		Rejection rate of $H_0$ : $\beta = 0$				
			Robust Cluster Al							AKM0	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
Panel A: Empirica	l distri	bution o	f ADH (	2013) sh	ocks						
period: 1990-2000	0.11	0.49	0.16	0.19	0.38	0.85	48.5%	39.9%	10.7%	9.3%	
period: 2000–2007	0.04	0.16	0.05	0.06	0.13	0.31	47.9%	39.4%	11.0%	9.5%	
Panel B: Empirica	l distri	bution of	residua	lized Al	DH (20	13) shoc	cks				
period: 1990-2000	0.00	0.14	0.05	0.06	0.12	0.21	46.5%	37.9%	10.4%	3.7%	
period: 2000–2007	0.00	0.07	0.03	0.03	0.06	0.11	46.4%	37.7%	10.9%	3.7%	

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta=0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples. In Panel A, each  $\mathcal{X}_s^m$  is drawn from the empirical distribution of shifters  $\mathcal{X}_s$  observed in the data; i.e. from the empirical distribution of changes in sectoral exports from China to high-income countries other than the US. In Panel B, each  $\mathcal{X}_s^m$  is drawn from the empirical distribution of residualized shifters  $\mathcal{X}_s$  observed in the data; i.e. from the empirical distribution of projecting the changes in sectoral exports from China to high-income countries other than the US on the full vector of baseline controls in ADH; i.e. those in column 6 of Table 3 in Autor, Dorn and Hanson (2013).

other than the US,  $IMP_{st}$ , as

$$IMP_{st} = X_{st}^{Ch} + \epsilon_{st}, \tag{E.1}$$

where  $X_{st}^{Ch}$  is a sectoral component of Chinese exports common to all destinations (i.e. it accounts for export supply factors), and  $\epsilon_{st}$  is sector- and destination-specific component (i.e. it accounts for export demand factors). We impose the following factor structure on  $X_{st}^{Ch}$ :

$$X_{st}^{Ch} = \eta_s \bar{X}_t^{Ch} + e_{st}. \tag{E.2}$$

The term  $\bar{X}_t^{Ch}$  captures unobserved factors that may potentially impact Chinese exports across all sectors (e.g. growth in Chinese labor productivity). The row-vector of sector-specific loadings  $\eta_s$  indicates how Chinese exports in each sector s react to changes in the common unobserved factors captured by  $\bar{X}_t^{Ch}$  (e.g. how sensitive each sector s is to growth in Chinese labor productivity). Finally,  $e_{st}$  is a sector- and year-specific idiosyncratic component of Chinese exports. Note that, as long as the distribution of  $\bar{X}_t^{Ch}$  is not degenerate, the shifter  $IMP_{st}$  will be correlated across any two sectors s and s' unless the loadings  $\eta_s$  and  $\eta_{s'}$  are orthogonal. This correlation in shifters violates the independence assumption imposed by Assumption 2(i) in Section 4.1 in a way that is not accounted for by the clustering extension considered in Section 5.1. In the placebo simulations that follow, we explore the consequences of the violation of this assumption, as well as modifications of the AKM and AKM0 procedures that account for the potential factor structure in the shifters.

Combining eqs. (E.1) and (E.2) yields

$$IMP_{st} = \eta_s \bar{X}_t^{Ch} + \varepsilon_{st}, \quad \text{with} \quad \varepsilon_{st} = \varepsilon_{st} + e_{st}.$$
 (E.3)

Figure E.1: Histogram of estimates of  $\{\eta_s\}_{s=1}^S$ 

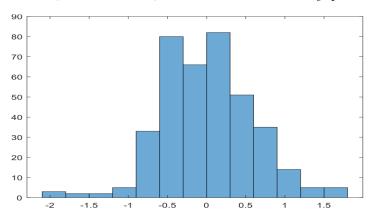
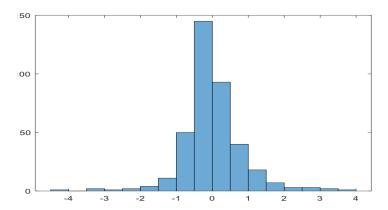


Figure E.2: Histogram of estimates of  $\{u_{s,2007} - u_{s,1991}\}_{s=1}^{S}$ 

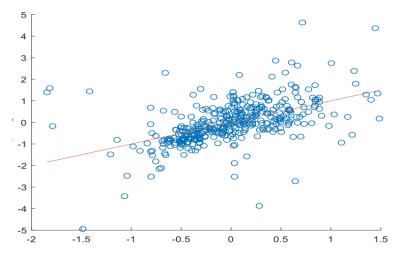


To remain as close as possible to the empirical application in ADH, we use annual data on sector-specific exports from China to other high-income countries between 1991 and 2007 (which corresponds to the variable  $IMP_{st}$  above) to estimate the common factor  $\bar{X}_t^{Ch}$ , the factor loadings  $\{\eta_s\}_{s=1}^S$ , and the residuals  $\{\varepsilon_{st}\}_{s=1}^S$  for every year t and 4-digit SIC manufacturing sectors s using the interactive fixed effects estimator in Bai (2009), as implemented by Gomez (2017).

Figure E.1 reports the histogram of the estimates of  $\{\eta_s\}_{s=1}^S$ . There is considerable dispersion in the factor loadings across sectors. The estimates also reveal substantial variation across sectors and years in the idiosyncratic component of Chinese export growth  $\varepsilon_{st}$ ; this can be seen in Figure E.2, which presents a histogram of the sector-specific changes in  $\varepsilon_{st}$  between 1991 and 2007. To provide a graphical illustration of the relative importance of the two terms entering the right-hand side of eq. (E.3), Figure E.3 provides a scatterplot of the variables  $\{IMP_{s,2007} - IMP_{s,1991}\}_{s=1}^S$  against the estimates of the terms  $\{\eta_s(\bar{X}_{2007}^{Ch} - \bar{X}_{1991}^{Ch})\}_{s=1}^S$ ; these terms explain only 27% of the cross-sectoral variation in export growth from China to high-income countries other than the US between 1991 and 2007.

Table E.2 reports the results of a placebo exercise illustrating the effects of the correlation in sectoral shifters implied by the estimated version of the model in eq. (E.3) on the finite-sample properties of the *AKM* and *AKM0* procedures. Specifically, we modify the baseline placebo exercise described

Figure E.3: Scatterplot of  $\{IMP_{s,2007} - IMP_{s,1991}\}_{s=1}^{S}$  against  $\{\eta_s(\bar{X}_{2007}^{Ch} - \bar{X}_{1991}^{Ch})\}_{s=1}^{S}$ 



Notes: Observed data on sector-specific export flows from China to high-income countries other than the US (i.e.  $IMP_{s,2007} - IMP_{s,1991}$ ) appear in the vertical axis; estimates of  $\eta_s(\bar{X}_{1991}^{Ch})$  appear in the horizontal axis. The  $R^2$  of this regression is 0.273.

in Section 6.1 by instead generating the simulated sectoral shifters as

$$\mathcal{X}_{s}^{m} = \kappa \eta_{s}^{m} \Delta \hat{\bar{X}}_{Ch} + u_{s}^{m}, \quad \text{with} \quad \Delta \hat{\bar{X}}^{Ch} = \hat{\bar{X}}_{2007}^{Ch} - \hat{\bar{X}}_{1991}^{Ch}$$
 (E.4)

where  $\hat{X}^{Ch}_t$  denotes the estimate of  $\bar{X}^{Ch}_t$  for t=1991 and t=2007. The parameter  $\kappa$  controls the relative importance of the factor component in the simulated shifters. For each simulated sample m, the residuals  $u^m_s$  are drawn independently from a distribution that we vary across specifications. The term  $\eta^m_s$  is either fixed across the placebo samples m and set to equal to the estimate  $\hat{\eta}_s$ , or else drawn independently from the empirical distribution of  $\hat{\eta}_s$ . Whether the factor loadings  $\eta_s$  are fixed across the placebo samples or random (and independent across s) is important for the properties of the AKM and AKM0 inference procedures. If the loadings are random and independent, the shifters  $\mathcal{X}_s$  will also be independent across s, so that Assumption 2(i) in Section 4.1 holds, and we expect the AKM and AKM0 inference procedures to have good asymptotic properties even if conditionally on the loadings, the interactive fixed effects structure in eq. (E.3) applies. On the other hand, if the loadings are fixed across simulation samples, the shifters will be correlated, so that the asymptotic results in Section 4 do not apply.

In Panels A and B in Table E.2, we fix  $\eta_s^m = \hat{\eta}_s$  for every sector s and placebo sample m, with  $u_s^m$  drawn i.i.d. from mean-zero normal distribution with variance 5 in Panel A, and from the empirical the distribution of  $\hat{\varepsilon}_{s,2007} - \hat{\varepsilon}_{s,1991}$  in Panel B, where  $\hat{\varepsilon}_{st}$  is the interactive fixed effects estimate of the term  $\varepsilon_{st}$  in eq. (E.3). In the first three rows of each panel, when no controls are included, larger values of  $\kappa$  (which imply a larger weight on the interactive fixed effects component  $\eta_s^m \Delta \hat{X}_{Ch}$  in eq. (E.4)) imply larger rejection rates of the null  $H_0$ :  $\beta = 0$  when we use either the AKM or the AKM0 inference procedures. For  $\kappa = 1$ , which corresponds to the specification in ADH, the rejection rates for AKM0 are close to the nominal rates, and AKM suffers from moderate overrejection. Importantly, this overrejection problem can be fixed by controlling for the term  $\eta_s^m \Delta \hat{X}_{Ch}$  as an additional covariate

in our regression specification (see rows 4 to 6 in Panels A and B in Table E.2). This is in line with our theory, since conditioning on this control restores the independence assumption on the shifters. The takeaway form the results in Panels A and B in Table E.2 is thus that, if one thinks that the true data generating process for the sectoral shifters  $\{X_s\}_{s=1}^S$  corresponds to the model in eq. (E.4), then one should obtain a consistent estimate of  $\eta_s \Delta \hat{X}_{Ch}$  and control for it in the regression specification in order to ensure that the shifters are independent conditional on the controls, so that Assumption 2(i) holds once we condition on the control vector  $Z_i$ .

In Panel C of Table E.2, instead of holding the loadings fixed, we draw both  $\eta_s^m$  and  $v_s^m$  in each placebo sample m from the empirical distribution of the interactive fixed effects estimates, independently across s. This makes the shifters independent across s, so that, as discussed above, Assumption 2(i) in Section 4.1 holds even without conditioning on  $\eta_s \Delta \hat{X}_{Ch}$ . As a result, the rejection rates for the AKM and AKM0 inference procedures reported in Panel C are similar to those reported in Table 2 in Section 6.1 and unaffected by the value of the parameter  $\kappa$  in eq. (E.4). In particular, the AKM0 inference procedure yields always rejection rates that are very close to 5%.

### E.1.2 Placebo exercise: accounting for controls in the first-stage regression

The placebo exercise described in Sections 2.2 and 6 use the outcome variables  $Y_i$  and the shares  $w_{is}$  used in Autor, Dorn and Hanson (2013) for the period 2000–2007. The placebo exercise discussed in Appendix D.4 gets closer to the reduced-form empirical specification in Autor, Dorn and Hanson (2013) by incorporating information on outcome variables and shares both for the period 1990–2000 and for the period 2000–2007. However, these two placebo exercises implement a specification that differs from that in Autor, Dorn and Hanson (2013) in that it includes no controls. As argued in Section 3.3, the overrejection problem affecting robust and state-clustered standard errors that is documented in the simulations is caused by cross-regional correlation in residuals across observations with similar shares. The inclusion of controls may improve the performance these methods, since the controls may soak up some (or even most) of the cross-regional correlation in the residuals.

In Table E.3, we introduce a placebo sample for the first-stage regression in Autor, Dorn and Hanson (2013). In Panel A, when we do not include any controls, both robust and state-clustered standard errors over-reject the null hypothesis  $H_0$ :  $\beta_1 = 0$ . In Panel B, we include as a control the shift-share instrumental variable used in Autor, Dorn and Hanson (2013), and the rejection rate for these procedures decreases to about 20%. Finally, in Panel C, we additionally include all controls used in the baseline specification in Autor, Dorn and Hanson (2013), and the *Robust* and *Cluster* rejection rates get closer to 14%. It can also be seen from Table E.3 that the rejection rates for the *AKM* and *AKM0* procedures are always very close to the 5% nominal level.

#### E.1.3 Additional empirical results

In Tables E.4 and E.5 we extend the results presented in Table 5 in Section 7.1. Specifically, Tables E.4 and E.5 present results not only for all workers (in Panel A), but also two subsets of workers: college graduates (in Panel B) and non-college graduates (in Panel C). Additionally, while the *AKM* and *AKM0* confidence intervals presented in Table 5 cluster observations belonging to the same 3-digit

Table E.2: Simulation for common China shock with heterogeneous sectoral exposure

		Est	imate		Median	eff. s.e.		Rejection rate of $H_0$ : $\beta = 0$			
κ	Control for	Mean	Std. dev	Robust	Cluster	AKM	AKM0	Robust	Cluster	AKM	AKM0
	$\eta_s^m \Delta \hat{\bar{X}}_{Ch}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Pa	$\mathbf{nel} \ \mathbf{A:} \ \eta_s^m =$	$=\hat{\eta}_s$ for	all m an	$ds; u_s^m \sim$	$\sim n(0,5)$						
0	No	0.00	0.17	0.08	0.09	0.14	0.17	35.4%	31.3%	10.3%	3.9%
1	No	0.00	0.15	0.07	0.07	0.12	0.14	38.2%	33.6%	12.4%	5.1%
3	No	0.00	0.09	0.04	0.04	0.06	0.07	42.2%	35.9%	17.5%	8.4%
0	Yes	0.00	0.16	0.08	0.08	0.14	0.16	34.9%	31.6%	10.4%	4.2%
1	Yes	0.00	0.16	0.08	0.08	0.14	0.16	35.1%	31.8%	10.5%	4.3%
3	Yes	0.00	0.16	0.08	0.08	0.14	0.16	34.9%	31.9%	10.5%	4.3%
Pa	$\mathbf{nel} \; \mathbf{B:} \; \eta_s^m =$	$\hat{\eta}_s$ for	all $m$ and	$\mathbf{d} s; u_s^m \sim$	$F_{emp}$						
0	No	0.00	0.43	0.20	0.21	0.35	0.49	36.5%	33.2%	12.1%	3.5%
1	No	0.00	0.26	0.11	0.12	0.18	0.21	42.3%	36.3%	17.3%	8.2%
3	No	0.00	0.10	0.04	0.05	0.07	0.08	43.9%	37.3%	18.8%	9.4%
0	Yes	0.00	0.43	0.19	0.21	0.34	0.46	36.7%	33.7%	12.7%	3.9%
1	Yes	0.00	0.43	0.19	0.21	0.34	0.46	36.0%	33.1%	12.3%	3.7%
3	Yes	0.00	0.43	0.19	0.21	0.34	0.46	36.3%	33.4%	12.3%	3.6%
Pa	nnel C: $(\eta_s^m,$	$u_s^m) \sim$	$F_{emp}$								
0	No	0.00	0.43	0.20	0.21	0.35	0.49	36.7%	33.1%	12.0%	3.5%
1	No	0.00	0.26	0.12	0.13	0.22	0.26	36.0%	32.1%	10.5%	3.8%
3	No	0.00	0.10	0.05	0.05	0.09	0.11	35.3%	31.4%	10.3%	3.7%
0	Yes	0.00	0.43	0.19	0.21	0.34	0.46	36.2%	33.1%	12.1%	3.5%
1	Yes	0.00	0.43	0.19	0.21	0.34	0.46	37.1%	33.5%	12.4%	3.9%
3	Yes	0.00	0.42	0.18	0.20	0.32	0.42	37.8%	34.4%	13.5%	5.2%

Notes: We impose the assumption that the year-specific sectoral shifters  $IMP_{st}$  are generated from the model in eq. (E.3). We compute the estimates of the parameters in this model using Gomez (2017), which implements the estimation approach in Bai (2009). To compute these estimates, we use annual data on exports from China to high-income countries other than the US,  $IMP_{st}$ , between 1991 and 2007 (i.e. the same sectoral exports used to construct the instrumental variable in Autor, Dorn and Hanson (2013)) for all sectors used in our baseline placebo exercise. We use these estimates to construct a treatment variable  $X_i^m \equiv \sum_s w_{is} \mathcal{L}_s^m$ , with each  $\mathcal{L}_s^m$  defined as in eq. (E.4), for every simulated sample  $m = 1, \dots, 30,000$ . The different panels impose different assumptions on the distribution of  $(\eta_s^m, u_s^m)$  across sectors and simulated samples. In Panels A and B in Table E.2, we fix  $\eta_s^m = \hat{\eta}_s$  for every sector s and placebo sample m. The placebo simulations whose results we present in these two panels differ in the distribution from which  $u_s^m$  is drawn. In Panel A, we draw  $u_s^m$  independently across sectors and placebo samples either from a normal distribution with mean zero and variance equal to five. In Panel B, we draw  $u_s^m$  independently from the distribution of  $\hat{\varepsilon}_{s,2007} - \hat{\varepsilon}_{s,1991}$  across sectors, where, for t = 2007 and t = 1991,  $\hat{\varepsilon}_{st}$  is the estimate of the term  $\varepsilon_{st}$  in eq. (E.3) (in Panel B). The placebo exercises in Panel C of Table E.2 differs from that in Panel B in that, in the former, each  $\eta_s^m$  is independently drawn across sectors s and placebo samples m from the distribution of  $\hat{\eta}_s$  across sectors, where  $\hat{\eta}_s$  is our estimate of the term  $\eta_s$  in eq. (E.3). In all three panels, we compute the outcome variable as  $Y_i^m = \sum_s w_{is} \mu_s^m$ , with  $\mu_s^m$  drawn randomly from a normal distribution with mean zero and variance equal to 5. Given the variables  $Y_i^m$  and  $X_i^m$  for each simulated sample m, we compute an estimate of  $\beta$  in the regression  $Y_i = \beta X_i^m + \epsilon_i$  (whenever there is a 'No' in the second column) or in the regression  $Y_i = \beta X_i^m + \gamma \sum_s \eta_s^m \Delta \bar{X}_{Ch} + \epsilon_i$  (whenever there is a 'Yes' in the second column). We indicate the median and standard deviation of the OLS estimates of  $\beta$  across the simulated samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta = 0$  using a 5% significance level test (columns (7) to (10)). Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters CZs in the same state; AKM is the standard error in Remark 5; AKM0 is the confidence interval in Remark 6.

Table E.3: Placebo exercise for the first-stage regression in Autor, Dorn and Hanson (2013)

Est	imate		Median	eff. s.e.		Re	ejection rat	e H <sub>0</sub> : β =	= 0
Mean (1)	Std. dev (2)	Robust (3)	Cluster (4)	AKM (5)	AKM0 (6)	Robust (7)	Cluster (8)	AKM (9)	AKM0 (10)
Panel A:	No controls	5							
0.01	1.73	0.72	0.81	1.63	1.88	41.5%	36.7%	6.5%	4.0%
Panel B: 0	Controls: A	DH IV							
0.01	1.01	0.63	0.63	0.93	1.06	20.6%	21.3%	7.8%	4.3%
Panel C: 0	Controls: A	DH IV ar	nd all cont	rols incl	uded in	Гable 3, co	ol. 6 of in	Autor et	al. (2013)
0.00	0.68	0.51	0.51	0.64	0.72	14.4%	14.1%	5.6%	3.8%

Notes: This table indicates the median and standard deviation of the OLS estimates of  $\beta$  in eq. (1) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (6)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0\colon \beta=0$  using a 5% significance level test (columns (7) to (10)). *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters CZs in the same state; *AKM* is the standard error in Remark 5; *AKM0* is the confidence interval in Remark 6. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by 2 × 1.96. Results are based on 30,000 placebo samples. In all three panels, each  $\mathcal{X}_s^m$  is *i.i.d* drawn from a normal distribution with mean zero and variance equal to 5. In Panel A, we introduce no controls in the regression equation. In Panel B, we control for the instrumental variable used in Autor, Dorn and Hanson (2013); i.e. the shift-share aggregator of changes in sectoral exports from China to high-income countries other than the US. In Panel C, we control for the instrumental variable used in Autor, Dorn and Hanson (2013) and for the broadest set of controls used in that paper; i.e. the set of controls used in column 6 of Table 3 of Autor, Dorn and Hanson (2013).

sector in different periods (which we denote in Tables E.4 and E.5 as AKM (3d cluster) and AKM0 (3d cluster)), Tables E.4 and E.5 also present AKM and AKM0 confidence intervals that only cluster on time (denoted as AKM (4d cluster) and AKM0 (4d cluster)), and AKM and AKM0 that treat shifters as independent both across 4-digit sectors and across time periods (denoted as AKM (indep.) and AKM0 (indep.))

There are several takeaways from the results in Tables E.4 and E.5. First, accounting for the possible correlation in the shifters has only a minimal impact on the *AKM* confidence intervals (i.e. the *AKM* (*indep.*), *AKM* (*4d cluster*), and *AKM* (*3d cluster*) confidence intervals are always very similar); the impact on the *AKM0* confidence intervals is a bit larger but also quite small. Second, while the *AKM* and *AKM0* confidence intervals are quite similar to the *Robust* and *Cluster* ones in the case of college graduates (Panel B), they are much larger for non-college graduates (Panel C). Finally, similarly to what we observed in Table 5 in Section 7.1, the *AKM0* confidence interval is not centered around the point estimate: it includes more values of the parameter to the left of the point estimate than it does to the right.

Table E.4: Effect of Chinese on U.S. Commuting Zones in Autor, Dorn and Hanson (2013): Reduced-Form Regression

	Change is	n the employ	ment share	Change is	n avg. log we	ekly wage
	All	Manuf.	Non-Manuf.	All	Manuf.	Non-Manuf.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: All Work	cers					
$\hat{eta}$	-0.49	-0.38	-0.11	-0.48	0.10	-0.48
Robust	[-0.71,-0.27]	[-0.48,-0.28]	[-0.31,0.08]	[-0.80,-0.16]	[-0.50,0.69]	[-0.83,-0.13]
Cluster	[-0.64, -0.34]	[-0.45,-0.30]	[-0.27,0.05]	[-0.78,-0.18]	[-0.51,0.70]	[-0.81,-0.15]
AKM (indep.)	[-0.79,-0.18]	[-0.52,-0.24]	[-0.33,0.10]	[-0.84,-0.12]	[-0.47,0.66]	[-0.88,-0.08]
AKM0 (indep.)	[-1.08,-0.25]	[-0.63,-0.26]	[-0.51,0.07]	[-1.08,-0.15]	[-0.91,0.58]	[-1.22,-0.15]
AKM (4d cluster)	[-0.79,-0.19]	[-0.52,-0.23]	[-0.33,0.10]	[-0.87,-0.09]	[-0.49,0.68]	[-0.90,-0.07]
AKM0 (4d cluster)	[-1.10,-0.26]	[-0.66,-0.25]	[-0.52,0.07]	[-1.16,-0.13]	[-0.99,0.59]	[-1.28,-0.14]
AKM (3d cluster)	[-0.81,-0.17]	[-0.52,-0.23]	[-0.35,0.12]	[-0.88,-0.07]	_	[-0.93,-0.03]
AKM0 (3d cluster)	[-1.24,-0.24]	[-0.67,-0.25]	[-0.64,0.08]	[-1.27,-0.10]	[-1.16,0.61]	[-1.47,-0.11]
Panel B: College (	Graduates					
$\hat{eta}$	-0.27	-0.37	0.11	-0.48	0.29	-0.47
Robust	[-0.42,-0.12]	[-0.48,-0.26]	[-0.04, 0.25]	[-0.82,-0.13]	[-0.10,0.68]	[-0.83,-0.11]
Cluster	[-0.39,-0.14]	[-0.48,-0.27]	[-0.04,0.26]	[-0.83,-0.13]	[-0.14,0.72]	[-0.81,-0.12]
AKM (indep.)	[-0.45,-0.09]	[-0.50, -0.25]	[-0.03, 0.24]	[-0.82,-0.13]	[-0.11,0.69]	[-0.83,-0.11]
AKM0 (indep.)	[-0.57,-0.11]	[-0.56,-0.24]	[-0.11,0.24]	[-1.00,-0.13]	[-0.35,0.68]	[-1.07,-0.14]
AKM (4d cluster)	[-0.45,-0.09]	[-0.51,-0.23]	[-0.04, 0.25]	[-0.85,-0.10]	[-0.14,0.72]	[-0.85,-0.09]
AKM0 (4d cluster)	[-0.58,-0.11]	[-0.59,-0.23]	[-0.11,0.25]	[-1.08,-0.11]	[-0.41,0.70]	[-1.14,-0.13]
AKM (3d cluster)	[-0.45,-0.08]	[-0.52,-0.23]	[-0.04, 0.25]	[-0.88,-0.08]	[-0.14,0.72]	[-0.89,-0.05]
AKM0 (3d cluster)	[-0.62,-0.09]	[-0.59,-0.20]	[-0.17,0.25]	[-1.20,-0.08]	[-0.46,0.73]	[-1.32,-0.09]
Panel C: Non-Col	lege Graduat	es				
$\hat{eta}$	-0.70	-0.37	-0.34	-0.51	-0.06	-0.52
Robust	[-1.02,-0.38]	[-0.48, -0.25]	[-0.60,-0.07]	[-0.90,-0.13]	[-0.69,0.56]	[-0.94, -0.10]
Cluster	[-0.92,-0.48]	[-0.47,-0.26]	[-0.55,-0.12]	[-0.84,-0.19]	[-0.53,0.40]	[-0.87,-0.17]
AKM (indep.)	[-1.18,-0.22]	[-0.55,-0.19]	[-0.68,0.01]	[-1.08,0.05]	[-0.70,0.57]	[-1.15,0.11]
AKM0 (indep.)	[-1.68,-0.34]	[-0.72,-0.23]	[-1.01,-0.06]	[-1.59,-0.06]	[-1.26,0.45]	[-1.78,-0.04]
AKM (4d cluster)	[-1.17,-0.23]	[-0.55,-0.18]	[-0.67,0.00]	[-1.09,0.06]	[-0.70,0.57]	[-1.14,0.11]
AKM0 (4d cluster)	[-1.69,-0.35]	[-0.74,-0.23]	[-1.01,-0.07]	[-1.64,-0.05]	[-1.30,0.45]	[-1.80,-0.04]
AKM (3d cluster)	[-1.22,-0.18]	[-0.55,-0.18]	[-0.71,0.04]	[-1.10,0.07]	[-0.72,0.60]	[-1.16,0.13]
AKM0 (3d cluster)	[-1.95,-0.32]	[-0.79,-0.23]	[-1.21,-0.04]	[-1.80,-0.04]	[-1.55,0.46]	[-2.02,-0.02]

Notes: N=1,444 (722 CZs  $\times$  two time periods). Models are weighted by start of period CZ share of national population. All regressions include the full vector of baseline controls in ADH; i.e. those in column 6 of Table 3 in Autor, Dorn and Hanson (2013). 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* (*indep.*) is the standard error in Remark 5; *AKM* (*4d cluster*) is the standard error in eq. (40) with 4-digit SIC clusters; *AKM* (*3d cluster*) is the standard error in eq. (40) with 3-digit SIC clusters; *AKM* (*indep.*) is the confidence interval in Remark 6; *AKM*0 (*4d cluster*) is the confidence interval with 4-digit SIC clusters described in the last sentence of Section 5.1; and *AKM*0 (*3d cluster*) is the confidence interval with 3-digit SIC clusters described in the last sentence of Section 5.1.

Table E.5: Effect of Chinese on U.S. Commuting Zones in Autor, Dorn and Hanson (2013): 2SLS Regression

	Change is	n the employ:	ment share	Change is	n avg. log we	ekly wage
	All	Manuf.	Non-Manuf.	All	Manuf.	Non-Manuf.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: All Work	cers					
$\hat{eta}$	-0.77	-0.60	-0.18	-0.76	0.15	-0.76
Robust	[-1.10,-0.45]	[-0.78,-0.41]	[-0.47,0.12]	[-1.23,-0.29]	[-0.81,1.11]	[-1.27,-0.25]
Cluster	[-1.12,-0.42]	[-0.79,-0.40]	[-0.45,0.10]	[-1.26,-0.26]	[-0.81,1.11]	[-1.28,-0.24]
AKM (indep.)	[-1.19,-0.36]	[-0.81,-0.38]	[-0.50,0.15]	[-1.30,-0.22]	[-0.76,1.06]	[-1.32,-0.20]
AKM0 (indep.)	[-1.40,-0.42]	[-0.89,-0.39]	[-0.65,0.11]	[-1.48,-0.23]	[-1.14,0.99]	[-1.58,-0.25]
AKM (4d cluster)	[-1.19,-0.36]	[-0.84,-0.36]	[-0.50,0.15]	[-1.35,-0.17]	[-0.80,1.10]	[-1.36,-0.17]
AKM0 (4d cluster)	[-1.46, -0.43]	[-0.96,-0.38]	[-0.66,0.12]	[-1.61,-0.21]	[-1.24,1.03]	[-1.69,-0.24]
AKM (3d cluster)	[-1.25,-0.30]	[-0.84, -0.35]	[-0.54, 0.18]	[-1.37,-0.15]	[-0.81,1.11]	[-1.42, -0.10]
AKM0 (3d cluster)	[-1.69,-0.39]	[-1.01,-0.36]	[-0.84,0.14]	[-1.77,-0.17]	[-1.49,1.05]	[-1.97,-0.19]
Panel B: College (	Graduates					
$\hat{eta}$	-0.42	-0.59	0.17	-0.76	0.46	-0.74
Robust	[-0.64,-0.20]	[-0.81,-0.37]	[-0.08, 0.41]	[-1.29,-0.22]	[-0.19,1.11]	[-1.29,-0.20]
Cluster	[-0.67,-0.18]	[-0.84,-0.34]	[-0.07,0.41]	[-1.37,-0.14]	[-0.22,1.14]	[-1.34,-0.15]
AKM (indep.)	[-0.69,-0.16]	[-0.83,-0.36]	[-0.07, 0.40]	[-1.30,-0.22]	[-0.22, 1.14]	[-1.28,-0.20]
AKM0 (indep.)	[-0.78,-0.16]	[-0.87,-0.33]	[-0.14, 0.40]	[-1.44,-0.19]	[-0.45,1.13]	[-1.47,-0.21]
AKM (4d cluster)	[-0.70,-0.15]	[-0.85,-0.33]	[-0.07,0.41]	[-1.34,-0.17]	[-0.27,1.18]	[-1.31,-0.18]
AKM0 (4d cluster)	[-0.82,-0.17]	[-0.93,-0.32]	[-0.15, 0.42]	[-1.56,-0.17]	[-0.53,1.18]	[-1.57,-0.21]
AKM (3d cluster)	[-0.71,-0.13]	[-0.86,-0.32]	[-0.08,0.42]	[-1.37,-0.14]	[-0.25,1.17]	[-1.37,-0.11]
AKM0 (3d cluster)	[-0.90,-0.14]	[-0.96,-0.27]	[-0.23,0.42]	[-1.71,-0.13]	[-0.61,1.21]	[-1.82,-0.15]
Panel C: Non-Col	lege Graduat	es				
$\hat{eta}$	-1.11	-0.58	-0.53	-0.81	-0.10	-0.82
Robust	[-1.58,-0.64]	[-0.76,-0.40]	[-0.93,-0.13]	[-1.35,-0.28]	[-1.07,0.87]	[-1.41,-0.23]
Cluster	[-1.61,-0.61]	[-0.77,-0.39]	[-0.94,-0.13]	[-1.28,-0.34]	[-0.84,0.63]	[-1.31,-0.33]
AKM (indep.)	[-1.76,-0.47]	[-0.83,-0.33]	[-1.02,-0.04]	[-1.62,0.00]	[-1.09,0.89]	[-1.71,0.07]
AKM0 (indep.)	[-2.12,-0.58]	[-0.95,-0.37]	[-1.27,-0.11]	[-2.01,-0.10]	[-1.55,0.79]	[-2.20,-0.07]
AKM (4d cluster)	[-1.75,-0.47]	[-0.85,-0.32]	[-1.01,-0.05]	[-1.64,0.02]	[-1.10,0.90]	[-1.72,0.07]
AKM0 (4d cluster)	[-2.19,-0.59]	[-1.02,-0.36]	[-1.29,-0.12]	[-2.12,-0.09]	[-1.63,0.79]	[-2.28,-0.07]
AKM (3d cluster)	[-1.86,-0.36]	[-0.86,-0.30]	[-1.09,0.03]	[-1.68,0.06]	[-1.14,0.93]	[-1.78,0.14]
AKM0 (3d cluster)	[-2.62,-0.52]	[-1.13,-0.35]	[-1.59,-0.07]	[-2.43,-0.07]	[-2.00,0.79]	[-2.69,-0.04]

Notes: N=1,444 (722 CZs  $\times$  two time periods). Models are weighted by start of period CZ share of national population. All regressions include the full vector of baseline controls in ADH; i.e. those in column 6 of Table 3 in Autor, Dorn and Hanson (2013). 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* (*indep.*) is the standard error in eq. (39); *AKM* (*4d cluster*) is the standard error in eq. (39) with an adjustment analogous to that in eq. (40) with 4-digit SIC clusters; *AKM* (*indep.*) is the standard error in eq. (39) with an adjustment analogous to that in eq. (40) with d-digit SIC clusters; *AKM0* (*indep.*) is the confidence interval built using the standard error in eq. (39) with the residual  $(I-Z'(Z'Z)^{-1}Z')(Y_1-Y_2\alpha_0)$  instead of the estimate  $\hat{\epsilon}_{\Delta}=(I-Z'(Z'Z)^{-1}Z')(Y_1-Y_2\hat{\alpha})$ ; *AKM0* (4d cluster) and *AKM0* (3d cluster) impose the same adjustment to the procedure in *AKM* (4d cluster) and *AKM* (3d cluster), respectively.

# E.2 Estimation of inverse labor supply elasticity

Shift-share IV regressions have been used extensively to estimate inverse local labor supply elasticities. Using the notation in Section 3, we can write the inverse labor supply in each region *i* as

$$\log \omega_i = \tilde{\phi} \log L_i - \tilde{\phi} \log v_i, \quad \text{with} \quad \tilde{\phi} \equiv \phi^{-1}, \tag{E.5}$$

and, consequently, we can relate log changes in wages and log changes employment rates (or number of employees) for each region i between any two time periods as

$$\hat{\omega}_i = \tilde{\phi} \hat{L}_i - \tilde{\phi} \left( \sum_{g=1}^G \tilde{w}_{ig} \hat{v}_g + \hat{v}_i \right). \tag{E.6}$$

### E.2.1 Bias in OLS estimate of inverse labor supply elasticity

Using data on log changes in wages and employment rates for a set of regions,  $\{(\hat{\omega}_i, \hat{L}_i)\}_i$ , one may consider using OLS to compute an estimate of  $\tilde{\phi}$ . However, such estimator will be inconsistent. To show this formally, note that, up to a first-order approximation around the initial equilibrium, we can write the change in employment in any given region i as

$$\hat{L}_{i} = \sum_{s=1}^{S} l_{is}^{0} \left[ \theta_{is} \hat{\chi}_{s} + \lambda_{i} \hat{\mu}_{s} + \lambda_{i} \hat{\eta}_{is} \right] + (1 - \lambda_{i}) \left( \sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{g} + \hat{v}_{i} \right), \tag{E.7}$$

and the change in wages as

$$\hat{\omega}_i = \tilde{\phi} \sum_{s=1}^{S} l_{is}^0 (\theta_{is} \hat{\chi}_s + \lambda_i \hat{\mu}_s + \lambda_i \hat{\eta}_{is}) - \tilde{\phi} \lambda_i (\sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_g + \hat{v}_i).$$
 (E.8)

Using eq. (E.6), the probability limit of the OLS estimator of  $\tilde{\phi}$ ,  $\hat{\phi}_{OLS}$ , can be written as

$$plim(\hat{\phi}_{OLS}) = \frac{cov(\hat{\omega}_i, \hat{L}_i)}{var(\hat{L}_i)} = \tilde{\phi} + \frac{cov(-\tilde{\phi}(\sum_{g=1}^G \tilde{w}_{ig}\hat{v}_g + \hat{v}_i), \hat{L}_i)}{var(\hat{L}_i)}, \tag{E.9}$$

where  $cov(-\tilde{\phi}(\sum_{g=1}^{G} \tilde{w}_{ig}\hat{v}_g + \hat{v}_i), \hat{L}_i)/var(\hat{L}_i)$  captures the asymptotic bias in  $\hat{\phi}_{OLS}$  as an estimator of  $\tilde{\phi}$ . To characterize this term, we assume here that the of labor supply shocks  $\{\hat{v}_g\}_g$  and  $\{\hat{v}_i\}_i$  are independent of the vector of all labor demand shocks  $(\{\hat{\chi}_s\}_s, \{\hat{\mu}_s\}_{i,s}, \{\hat{\eta}_{is}\}_{i,s})$  conditional on the matrix of weights  $W \equiv \{l_{is}^0\}_{i,s}$  and the matrix of parameters  $B \equiv (\{\beta_{is}\}_{i,s}, \{\lambda_i\}_i)$ 

$$(\{\hat{\chi}_s\}_s, \{\hat{\mu}_s\}_s, \{\hat{\eta}_{is}\}_{i,s}) \perp (\{\hat{\nu}_g\}_g, \{\hat{\nu}_i\}_i) \mid (W, B).$$
(E.10)

Given this assumption and eq. (E.7), we can rewrite  $plim(\hat{\phi}_{OLS})$  in eq. (E.9) as

$$plim(\hat{\tilde{\phi}}_{OLS}) = \tilde{\phi} + \frac{cov(-\tilde{\phi}(\sum_{g=1}^{G}\tilde{w}_{ig}\hat{v}_g + \hat{v}_i), (1 - \lambda_i)(\sum_{g=1}^{G}\tilde{w}_{ig}\hat{v}_g + \hat{v}_i))}{var(\hat{L}_i)}$$

$$= \tilde{\phi} - \tilde{\phi}(1 - \lambda) \frac{var(\sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_g + \hat{v}_i)}{var(\hat{L}_i)}, \tag{E.11}$$

where the second equality follows if we additionally assume that elasticity of labor demand in eq. (2) does not vary across sectors,  $\sigma_s = \sigma$  for all s, so that  $\lambda_i = \lambda$  for all i. As indicated in Section 3.2, in this case,  $\lambda \equiv \phi [\phi + \sigma \sum_{s=1}^{S} l_{is}^0]^{-1}$ . Thus, if  $\sigma > 0$  (which guarantees that  $\lambda < 1$ ) and  $\tilde{\phi} > 0$ , then the OLS will underestimate the inverse labor supply elasticity in the sense that  $p lim(\hat{\phi}_{OLS}) < \tilde{\phi}$ .

### E.2.2 Consistency of IV estimate of inverse labor supply elasticity

Using data for a set of regions and sectors on log changes in wages and employment rates  $\{(\hat{\omega}_i, \hat{L}_i)\}_i$ , initial employment shares  $\{l_{is}^0\}_{i,s}$ , and sectoral labor demand shifters  $\{\hat{\chi}_s\}_s$ , we can write the probability limit of the IV estimator of  $\tilde{\phi}$  that uses  $X_i \equiv \sum_{s=1}^{S} l_{is}^0 \hat{\chi}_s$  as IV,  $\hat{\phi}_{IV}$ , as

$$plim(\hat{\phi}_{IV}) = \frac{cov(\hat{\omega}_i, X_i)}{cov(\hat{L}_i, X_i)}.$$

Given the expressions for  $\hat{L}_i$  and  $\hat{\omega}_i$  in eqs. (E.7) and (E.8), respectively, and the independence assumption in eq. (E.10), we can rewrite

$$\begin{split} plim(\hat{\phi}_{IV}) &= \frac{cov(\tilde{\phi}\sum_{s=1}^{S}l_{is}^{0}\left[\theta_{is}\hat{\chi}_{s} + \lambda_{i}\hat{\mu}_{s} + \lambda_{i}\hat{\eta}_{is}\right] - \tilde{\phi}\lambda_{i}(\sum_{g=1}^{G}\tilde{w}_{ig}\hat{v}_{g} + \hat{v}_{i}), X_{i})}{cov(\sum_{s=1}^{S}l_{is}^{0}\left[\theta_{is}\hat{\chi}_{s} + \lambda_{i}\hat{\mu}_{s} + \lambda_{i}\hat{\eta}_{is}\right] + (1 - \lambda_{i})\left(\sum_{g=1}^{G}\tilde{w}_{ig}\hat{v}_{g} + \hat{v}_{i}), X_{i}\right)} \\ &= \tilde{\phi}\frac{cov(\sum_{s=1}^{S}l_{is}^{0}\left[\theta_{is}\hat{\chi}_{s} + \lambda_{i}\hat{\mu}_{s} + \lambda_{i}\hat{\eta}_{is}\right], X_{i})}{cov(\sum_{s=1}^{S}l_{is}^{0}\left[\theta_{is}\hat{\chi}_{s} + \lambda_{i}\hat{\mu}_{s} + \lambda_{i}\hat{\eta}_{is}\right], X_{i})} \\ &= \tilde{\phi}. \end{split}$$

Therefore, under the distributional assumptions in eq. (E.10), the IV estimator that uses a shift-share instrument that aggregates sector-specific labor demand shifters is a consistent estimator of the inverse labor supply elasticity. Notice that the heterogeneity in  $\theta_{is}$  does not affect the consistency of  $\hat{\phi}$ . However, the consistency of  $\hat{\phi}$  will depend on the specific labor demand shock being employed by the researcher to construct its shift-share IV being independent of the specific labor supply shocks that have been prevalent in the set of regions belonging the population of interest.

# E.2.3 Evaluation of leave-one-out IV through the lens of the model in Section 3

We describe in this section how one may use the model in Section 3 to frame the approach to the estimation of the inverse labor supply elasticity described in Section 7.2. This approach is described in general terms in Section 5.3.

In Section 7.2, we focus on the estimation of the inverse labor supply elasticity  $\tilde{\phi}$  and we base the estimation of this parameter on the estimating equation

$$\hat{\omega}_i = \tilde{\phi}\hat{L}_i + \delta Z_i + \epsilon_i, \quad \text{with} \quad \tilde{\phi} = \phi^{-1}.$$
 (E.12)

For simplicity, we assume here that we use no controls (i.e.  $\delta = 0$ ) and that, thus, we can rewrite the estimating equation above as

$$\hat{\omega}_i = \tilde{\phi}\hat{L}_i + \epsilon_i, \quad \text{with} \quad \tilde{\phi} = \phi^{-1}.$$
 (E.13)

The advantage of focusing on the version without controls is that, in this case, the model in Section 3 clarifies that  $\epsilon_i = -\tilde{\phi}(\sum_{g=1}^G \tilde{w}_{ig}\hat{v}_g + \hat{v}_i)$ , where  $\{\hat{v}_g\}_g$  and  $\{\hat{v}_i\}_i$  are labor supply shocks. Thus, in the version without controls, there is a clear mapping between the regression residual of the structural equation,  $\epsilon_i$ , and the labor supply shocks in our economic model.

As discussed in Appendix E.2.1, the OLS estimator of  $\tilde{\phi}$  will be biased. However, as discussed in Appendix E.2.2, one may obtain a consistent estimate of  $\tilde{\phi}$  by computing an IV estimator that instruments for the log change in employment in region i,  $\hat{L}_i$ , using as an instrument a shift-share aggregator of labor demand shocks  $\{\mathcal{X}_s\}_s$ . In terms of the model in Section 3,  $\mathcal{X}_s$  is any (possibly sector s-specific) function of the sector s-specific labor demand shocks  $\chi_s$  and  $\mu_s$  (see eqs. (2) and (3)). These sector-specific labor demand shocks are in many cases unobserved to the researcher. In these cases, following Bartik (1991) and the subsequent literature on the estimation of inverse local labor supply elasticities, it has become typical to estimate  $\tilde{\phi}$  using as instruments one of two different IVs: either a shift-share aggregator of the growth in national employment in every sector s,

$$X_{i} = \sum_{s=1}^{S} l_{is}^{0} \hat{L}_{s}, \quad \text{with} \quad \hat{L}_{s} = \sum_{j=1}^{N} \frac{L_{js}^{0}}{\sum_{j'=1}^{N} L_{j's}^{0}} \frac{L_{js}^{t} - L_{js}^{0}}{L_{js}^{0}}, \quad (E.14)$$

or a shift-share aggregator of the leave-one-out measure of the growth in national employment in sector *s*,

$$X_{i,-} = \sum_{s=1}^{S} l_{is}^{0} \hat{L}_{s,-i}, \quad \text{with} \quad \hat{L}_{s,-i} = \sum_{j=1, j \neq i}^{N} \frac{L_{js}^{0}}{\sum_{j'=1, j' \neq i}^{N} L_{j's}^{0}} \frac{L_{js}^{t} - L_{js}^{0}}{L_{js}^{0}}.$$
 (E.15)

We focus here on outlining the restrictions that one should impose on the sector-specific labor demand shifters  $\{(\hat{\chi}_s, \hat{\mu}_s)\}_s$ , region- and sector-specific labor demand shifters  $\{\hat{\eta}_{is}\}_{i,s}$ , group-specific labor supply shifters,  $\{\hat{v}_g\}_g$ , and region-specific labor supply shifters  $\{\hat{v}_i\}_i$  (all of them introduced in the model in Section 3) so that the IV estimator that uses  $X_{i,-}$  as an instrument yields a consistent estimate of  $\tilde{\phi}$ .

The variable  $X_{i,-}$  in eq. (E.15) is a valid instrument as long as we can write

$$\hat{L}_{is} = \mathcal{X}_s + \psi_{is},\tag{E.16}$$

and the following restrictions hold

$$E[\mathcal{X}_s|\hat{\omega}(0), \hat{L}(0), L^0] = E[\mathcal{X}_s], \qquad \text{for all } s, \tag{E.17}$$

$$E[l_{is}^{0}\psi_{is}|\hat{\omega}_{-i}(0),\hat{L}_{-i}(0),L^{0}] = 0,$$
 for all  $i$  and  $s$ , (E.18)

$$E[l_{is}^{0}\psi_{is}l_{is}^{0}\psi_{js}|\hat{\omega}_{-i}(0),\hat{L}_{-i}(0),L^{0}] = 0, mtext{for all } i \neq j \text{ and } s, mtext{(E.19)}$$

where  $\hat{\omega}_{-i}(0)$  ( $\hat{L}_{-i}(0)$ ) denotes the change in wages (employment shares) in every region other than i when the sectoral shock of interest equals 0 for all sectors (i.e.  $\mathcal{X}_s = 0$  for all s), and  $L^0$  is the vector all region- and sector-specific shares in the initial equilibrium (i.e.  $L^0 = \{l_{is}^0\}_{i,s}$ ).

According to the model in Section 3, we can express the changes in employment in sector s in a region i as

$$\hat{L}_{is} = -\sigma_s \hat{\omega}_i + \rho_s \hat{\chi}_s + \hat{\mu}_s + \hat{\eta}_{is}.$$

Combining this expression with the expression for  $\hat{\omega}_i$  in eq. (E.8) in Appendix E.2.1, we can rewrite the change in employment in sector s and region i approximately as

$$\hat{L}_{is} = -\sigma_s \tilde{\phi} \sum_{s'=1}^{S} l_{is'}^0 \left[ \theta_{is'} \hat{\chi}_{s'} + \lambda_i \hat{\mu}_{s'} + \lambda_i \hat{\eta}_{is'} \right] + \sigma_s \tilde{\phi} \lambda_i \left( \sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_g + \hat{v}_i \right) + \rho_s \hat{\chi}_s + \hat{\mu}_s + \hat{\eta}_{is},$$
 (E.20)

with 
$$\lambda_i \equiv \phi \left[ \phi + \sum_{s=1}^S l_{is}^0 \sigma_s \right]^{-1}$$
,  $\theta_{is} \equiv \rho_s \lambda_i$ , and  $\tilde{\phi} = \phi^{-1}$ .

Without imposing any restrictions on the values of the labor demand and supply elasticities, the expression for  $\hat{L}_{is}$  in eq. (E.20) will not satisfy the restrictions in eq. (E.16) to eq. (E.19). To illustrate this point, we can map the different terms in eq. (E.20) into those in eq. (E.16) as

$$\mathcal{X}_s = \rho_s \hat{\chi}_s + \hat{\mu}_s, \tag{E.21}$$

$$\psi_{is} = -\sigma_{s}\tilde{\phi} \sum_{s'=1}^{S} l_{is'}^{0} \left[ \theta_{is'} \hat{\chi}_{s'} + \lambda_{i} \hat{\mu}_{s'} + \lambda_{i} \hat{\eta}_{is'} \right] + \sigma_{s}\tilde{\phi} \lambda_{i} \left( \sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{g} + \hat{v}_{i} \right) + \hat{\eta}_{is}.$$
 (E.22)

Under this definition of the labor demand shock  $\mathcal{X}_s$ , the potential outcomes  $\hat{\omega}_i(0)$  and  $\hat{L}_i(0)$  are

$$\hat{\omega}_i(0) = \tilde{\phi} \sum_{s=1}^S l_{is}^0 \lambda_i \hat{\eta}_{is} - \tilde{\phi} \lambda_i \left( \sum_{g=1}^G \tilde{w}_{ig} \hat{\nu}_g + \hat{\nu}_i \right), \tag{E.23}$$

$$\hat{L}_{i}(0) = \sum_{s=1}^{S} l_{is}^{0} \lambda_{i} \hat{\eta}_{is} + (1 - \lambda_{i}) \left( \sum_{g=1}^{G} \tilde{w}_{ig} \hat{v}_{g} + \hat{v}_{i} \right).$$
 (E.24)

Given the expressions in eqs. (E.22) to (E.24), the restriction on  $\psi_{is}$  in eq. (E.19) will not be satisfied: for any two regions i and i',  $\psi_{is}$  and  $\psi_{i's}$  are a function of the same set of sectoral demand shocks  $\{\hat{\chi}_s\}_s$  and  $\{\hat{\mu}_s\}_s$  and, thus,  $\psi_{is}$  and  $\psi_{i's}$  will generally be correlated with each other. Thus, unless additional restrictions are imposed, the IV estimator that uses the variable described in eq. (E.15) as instrument for  $\hat{L}_i$  in eq. (E.13) will not be a consistent estimator of  $\tilde{\phi}$ .

However, under the restriction that  $\sigma_s = 0$  for every sector s, the expression for  $\hat{L}_{is}$  in eq. (E.20) will satisfy the restrictions in eq. (E.16) to eq. (E.19). In this case,

$$\mathcal{X}_s = \rho_s \hat{\chi}_s + \hat{\mu}_s, \tag{E.25}$$

$$\psi_{is} = \hat{\eta}_{is},\tag{E.26}$$

and  $\hat{\omega}_i(0)$  and  $\hat{L}_i(0)$  correspond to the expressions in eq. (E.23) and eq. (E.24). Thus, if the sector-specific labor demand shocks  $\{(\hat{\chi}_s, \hat{\mu}_s)\}_s$  are mean independent of the region-specific labor supply

shocks  $\{\hat{v}_g\}_g$  and  $\{\hat{v}_i\}_i$  as well as of the region- and sector-specific labor demand shocks  $\{\hat{\eta}_{is}\}_{i,s}$ , the restriction in eq. (E.17) will hold. Additionally, under the additional assumption that  $\eta_{is}$  is mean zero and uncorrelated with  $\eta_{js}$  for every  $i \neq j$  and s, the restrictions in eqs. (E.18) and (E.19) will hold. Thus, if these additional restrictions on the model in Section 3 hold, the IV estimator that uses the variable described in eq. (E.15) as instrument to estimate  $\tilde{\phi}$  in eq. (E.13) will be consistent.

There are two alternative instrumental variables that do not use data on any specific labor demand shock and that lead to consistent estimates of the inverse labor supply elasticity  $\tilde{\phi}$  under weaker restrictions than those needed for the instrument in eq. (E.15) to be valid.

First, conditional on a calibrated value of  $\sigma_s$  for every sector s, one may estimate  $\tilde{\phi}$  using as an instrument for  $\hat{L}_i$  the following leave-one-out estimator:

$$\tilde{X}_{i,-} = \sum_{s=1}^{S} l_{is}^{0} \hat{\bar{L}}_{s,-i}, \quad \text{with} \quad \hat{\bar{L}}_{s,-i} = \sum_{j=1, j \neq i}^{N} \frac{L_{js}^{0}}{\sum_{j'=1, j' \neq i}^{N} L_{j's}^{0}} \hat{\bar{L}}_{js}, \quad \text{and} \quad \hat{\bar{L}}_{is} = \hat{L}_{is} - \sigma_{s} \hat{\omega}_{i}. \quad (E.27)$$

Combining the expression for  $\hat{L}_{is}$  in eq. (E.20) and the expression for  $\hat{\omega}_i$  in eq. (E.8), we can write

$$\hat{\tilde{L}}_{is} = \rho_s \hat{\chi}_s + \hat{\mu}_s + \hat{\eta}_{is}. \tag{E.28}$$

Thus, we can define  $\mathcal{X}_s$  and  $\psi_{is}$  as in eq. (E.25) and eq. (E.26). Consequently, as discussed above, eqs. (E.17) to (E.19) will hold if: (a) the sector-specific labor demand shocks  $\{(\hat{\chi}_s, \hat{\mu}_s)\}_s$  are mean independent of the region-specific labor supply shocks  $\{\hat{v}_g\}_g$  and  $\{\hat{v}_i\}_i$  as well as of the region- and sector-specific labor demand shocks  $\{\hat{\eta}_{is}\}_{i,s}$ ; and (b)  $\eta_{is}$  is mean zero and uncorrelated with  $\eta_{js}$  for every  $i \neq j$  and s. Thus, under these two sets of assumptions, the IV estimator that uses the variable described in eq. (E.27) as instrument for  $\hat{L}_i$  in eq. (E.13) will be a consistent estimator of  $\tilde{\phi}$  no matter what the value of the labor demand elasticities  $\{\sigma_s\}_s$  is.

Second, under the assumption that the labor demand elasticity is constant across sectors (i.e.  $\sigma_s = \sigma$  for every s), the residual from projecting  $\hat{L}_{is}$ , as defined in eq. (E.20), on a set of region-specific fixed effects is equivalent to  $\hat{L}_{is}$ , as defined in eq. (E.28). Therefore, once we define  $\mathcal{X}_s$  and  $\psi_{is}$  as in eq. (E.25) and eq. (E.26), the IV estimator that uses the variable described in eq. (E.27) as an instrument for  $\hat{L}_i$  in eq. (E.13) will be a consistent estimator of  $\tilde{\phi}$  if two assumptions hold: (a) the sector-specific labor demand shocks  $\{(\hat{\chi}_s, \hat{\mu}_s)\}_s$  are mean independent of the region-specific labor supply shocks  $\{\hat{v}_g\}_g$  and  $\{\hat{v}_i\}_i$  as well as of the region- and sector-specific labor demand shocks  $\{\hat{\eta}_{is}\}_{i,s}$ ; and (b)  $\eta_{is}$  is mean zero and uncorrelated with  $\eta_{js}$  for every i,j, and s.

#### E.2.4 Placebo exercise

In this section, we implement a placebo exercise to evaluate the finite-sample properties of our suggested inference procedures when using the shift-share IVs introduced in Section 5.3. For each placebo sample  $m=1,\ldots,30,000$ , we construct sector- and region-specific shocks  $X_{is}^m=\mathcal{X}_s^m+\psi_{is}^m$ , where  $\mathcal{X}_s^m$  and  $\psi_{is}^m$  are independently drawn from normal distributions with variances equal to 5 and 10, respectively. We then use data on employment shares of U.S. CZs by 4-digit manufacturing

sectors,  $\{w_{is}\}_{i=1,s=1}^{N,S}$  to compute

$$Y_{i2} = \sum_{s=1}^{S} w_{is} X_{is}^{m}, \qquad Y_{i1} = \rho \sum_{s=1}^{S} w_{is} \psi_{is}^{m} + \sum_{s=1}^{S} w_{is} A_{s}^{m},$$

where  $A_s^m$  is independently drawn from a normal distribution with variance equal to 20.

Our goal is to estimate the effect  $\alpha$  of  $Y_{i2}^m$  on  $Y_{i1}^m$ ,

$$Y_{i1}^m = Y_{i2}^m \alpha + \epsilon_i^m. \tag{E.29}$$

Note that, by the above construction,  $\alpha=0$ . Therefore, the residual is  $\epsilon_i^m=Y_{i1}^m=\rho\sum_{s=1}^S w_{is}\psi_{is}^m+\sum_{s=1}^S w_{is}A_s^m$ , which indicates that there is a potential endogeneity problem stemming from the fact that  $\psi_{is}^m$  affects both  $Y_{i1}^m$  and  $Y_{i2}^m$  whenever  $\rho\neq 0$ .

We consider three different shift-share IVs. First, we consider the IV constructed directly with the shock  $\mathcal{X}_s^m$ :

$$X_i^m = \sum_{s=1}^S w_{is} \mathcal{X}_s^m.$$

Second, we consider an IV constructed with the aggregate growth in  $X_{is}$ :

$$\hat{X}_i^m = \sum_{s=1}^S w_{is} \hat{\mathcal{X}}_s^m \quad \text{such that} \quad \hat{\mathcal{X}}_s^m \equiv \sum_{i=1}^N \left( \frac{\check{w}_{is}}{\sum_{j=1}^N \check{w}_{js}} \right) X_{is}^m$$

where  $\check{w}_{is} = L_{is}^0 / \sum_{j=1}^N L_{js}^0$  is the share of CZ i in the national employment of sector s in 1990. Third, we consider an IV constructed with leave-one-out aggregate growth in  $X_{is}$ :

$$\hat{X}_{i,-}^m = \sum_{s=1}^S w_{is} \hat{\mathcal{X}}_{s,-i}^m \quad \text{such that} \quad \hat{\mathcal{X}}_{s,-i}^m \equiv \sum_{j=1, j \neq i}^N \left( \frac{\check{w}_{js}}{\sum_{o=1, j \neq i}^N \check{w}_{os}} \right) X_{js}^m.$$

The instruments  $X_i^m$  and  $\hat{X}_{i,-}^m$  are always valid in our setting. However, whenever  $\rho \neq 0$ , the instrument  $\hat{X}_i^m$  is invalid since  $\{\psi_{is}^m\}_{s=1}^S$  affect  $\hat{X}_i^m$  and  $\epsilon_i$ .

Table E.6 reports the results of this placebo exercise for different values of  $\rho$ . In Panel A, we report results using  $X_i^m$  as an instrument; we denote this instrument as the "infeasible" IV, as its construction requires observing the shifters  $\{\mathcal{X}_s\}_s$ . As expected, for all values of  $\rho$ , the median  $\hat{\alpha}^m$  across placebo samples is zero. Because of the shift-share structure of  $\epsilon_i$ , robust and state-clustered standard error estimators underestimate the variability of the estimates, while AKM and AKM0 inference procedures yield good coverage. Panel B presents the results based on the feasible shift-share IV  $\hat{X}_i^m$ . When using this IV, higher levels of  $\rho$  yield higher average estimates of  $\alpha$ . This follows from the endogeneity problem created by the fact that  $\{\psi_{is}^m\}_s$  are part of both the dependent variable,  $Y_{i1}^m$ , and the instrument,  $\hat{X}_i^m$ . Finally, Panel C presents results based on the leave-one-out IV  $\hat{X}_{i,-}^m$ . This instrument is not affected by an endogeneity problem, as it does not use information on region i-specific shocks  $\{\psi_{is}^m\}_s$  when constructing the region i-specific variable  $\hat{X}_{i,-}^m$ . Thus, the average of the IV estimates of  $\alpha$  that use  $\hat{X}_{i,-}^m$  as an instrument is also very close to zero for all values of  $\rho$ . The results in Panel C also show

Table E.6: Mismeasured shifter: impact on standard errors and rejection rates.

ρ	Estin	nate		1	Median	eff. s.e.				Reject	ion rate	of $H_0$ : $\beta$	=0	
_	Median	eff. s.e.	Robust	Cluster	AKM	AKM0	AKM	AKM0	Robust	Cluster	AKM	AKM0	AKM	AKM0
							Leave-	one-out					Leave-	one-out
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Pane	el A: Unfo	easible sl	hift-share	e IV										
0	0.00	0.35	0.16	0.18	0.29	0.37	_	_	0.33	0.29	0.09	0.04	_	_
5	0.01	0.97	0.77	0.76	0.80	1.05	_	_	0.08	0.09	0.08	0.04	_	_
10	-0.02	1.86	1.53	1.50	1.52	2.01	_	_	0.07	0.08	0.08	0.04	_	_
Pane	el B: Shif	t-share I	W with ag	gregate s	ector-le	vel grow	<b>th</b>							
0	0.00	0.23	0.11	0.12	0.20	0.22	_	_	0.33	0.29	0.09	0.04	_	_
5	1.66	0.50	0.39	0.40	0.45	0.51	_	_	0.89	0.89	0.85	0.79	_	_
10	3.30	0.93	0.77	0.77	0.84	0.95	_	_	0.91	0.90	0.88	0.83	_	-
Pane	el C: Shif	t-share I	V with ag	ggregate s	ector-le	vel grow	th (leav	ve-one-o	ut)					
0	0.00	0.36	0.17	0.18	0.30	0.40	0.31	0.32	0.32	0.29	0.09	0.04	0.08	0.03
5	-0.07	1.06	0.82	0.80	0.87	1.24	0.93	0.95	0.07	0.08	0.06	0.05	0.05	0.03
10	-0.16	2.03	1.62	1.59	1.66	2.37	1.78	1.80	0.06	0.07	0.06	0.05	0.05	0.03

Notes: This table reports the median and effective standard error estimates of the IV estimates of  $\alpha$  in eq. (E.29) across the placebo samples (columns (1) and (2)), the median effective standard error estimates (columns (3) to (8)), and the percentage of placebo samples for which we reject the null hypothesis  $H_0$ :  $\beta=0$  using a 5% significance level test (columns (9) to (14)). For each value of  $\rho$ , we generate 30,000 simulated samples with sector-region shocks  $X_{is}^m = \mathcal{X}_s^m + \psi_{is}^m$ , where  $\mathcal{X}_s^m \sim \mathcal{N}(0, 10)$ . We then construct  $Y_{12} = \sum_{s=1}^S w_{is} X_{is}^m$  and  $Y_{11} = \rho \sum_s w_{is} \psi_{is}^m + \sum_s w_{is} X_s^n$ , where  $A_s^m \sim \mathcal{N}(0, 20)$ . Robust is the Eicker-Huber-White standard error is the standard error that clusters CZs in the same state; AKM is the standard error in Remark 5; AKM0 is the confidence interval in Remark 6. AKM (leave-one-out) and AKM0 (leave-one-out) are the versions of AKM and AKM0 in Section 5.3. For each inference procedure, the median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by  $2 \times 1.96$ .

Table E.7: Estimation of inverse labor supply elasticity: robustness with different control sets

	(1)	(2)	(3)	(4)
Panel A: Bartik IV, Not leave-one-out estimate	mator			
$\hat{eta}$	0.75	0.8	0.83	0.8
Robust	[0.48, 1.03]	[0.64, 0.97]	[0.56, 1.1]	[0.64, 0.96]
Cluster	[0.44, 1.07]	[0.60, 1.01]	[0.55, 1.11]	[0.59, 1.02]
AKM	[0.59, 0.92]	[0.62, 0.98]	[0.60, 1.06]	[0.62, 0.98]
AKM0	[0.56, 0.95]	[0.59, 1.02]	[0.61, 1.21]	[0.59, 1.01]
Panel B: Bartik IV, Leave-one-out estimate	or			
$\hat{eta}$	0.76	0.82	0.83	0.81
Robust	[0.48, 1.03]	[0.65, 0.98]	[0.56, 1.10]	[0.65, 0.98]
Cluster	[0.43, 1.08]	[0.60, 1.03]	[0.55, 1.11]	[0.59, 1.04]
AKM	[0.59, 0.92]	[0.62, 1.01]	[0.58, 1.08]	[0.63, 1.00]
AKM0	[0.57, 0.96]	[0.60, 1.07]	[0.59, 1.28]	[0.60, 1.04]
AKM (leave-one-out)	[0.59, 0.92]	[0.61, 1.02]	[0.58, 1.08]	[0.62, 1.01]
AKM0 (leave-one-out)	[0.56, 0.97]	[0.59, 1.09]	[0.59, 1.29]	[0.59, 1.06]
Controls:				
Period dummies	Yes	Yes	Yes	Yes
Controls in Autor et al. (2013)	No	Yes	No	Yes
Controls in Amior and Manning (2018)	No	No	Yes	Yes

Notes: N=1,444 (722 CZs  $\times$  2 time periods). The dependent variable is the log-change in mean weekly earnings in CZ i, and the regressor is the log-change in the employment rate in CZ i. Observations are weighted by the 1980 CZ share of national population. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; AKM is the standard error in eq. (40) with 3-digit SIC clusters; AKM0 is the confidence interval with 3-digit SIC clusters described in Section 5.1; AKM (*leave-one-out*) is the standard error in section 5.3 with 3-digit SIC clusters; AKM0 (*leave-one-out*) is the confidence interval with 3-digit SIC clusters described in section 5.3. Baseline controls in Autor et al. (2013) are the controls in column 6 of Table 3 in ADH. Amenity controls in Amior and Manning (2018): binary indicator for presence of coastline, three temperature indicators, log population density in 1900, log distance to the closest CZ.

that the leave-one-out versions of the *AKM* and *AKM0* inference procedures (see Section 5.3) yield slightly larger median effective standard errors than the baseline versions of the *AKM* and *AKM0* procedures (see Section 4.3). In this particular application, the magnitude of the adjustment is modest: the implied rejection rates for the null hypothesis  $H_0$ :  $\alpha = 0$  differ by less than 2 percentage points.

#### **E.2.5** Additional results

Table E.7 reports estimates of the inverse labor supply elasticity with alternative sets of controls. Column (2) replicates the estimates of Panels A and B in column (3) of Table 6. Table E.7 shows that these results are robust to controlling (a) only for period dummies (column (1)); (b) for period dummies and the proxies for region-specific labor supply shocks included in Amior and Manning (2018) (column (3)); (c) for period dummies, the controls included in Autor, Dorn and Hanson (2013) and the proxies for region-specific labor supply shocks in Amior and Manning (2018) (column (4)).

# Appendix F Effect of immigration on U.S. local labor markets

To complement the empirical applications discussed in Section 7, we present here the results of estimating of the impact of immigration on labor market outcomes in the US. To this end, we estimate the model

$$Y_{it} = \beta \Delta Imm Share_{it} + Z'_{it} \delta + \epsilon_{it}, \tag{F.1}$$

where, for observation or cell i,  $Y_{it}$  is the change in a labor market outcome for native workers between years t and t - 10,  $\Delta ImmShare_{it}$  is the change in the share of immigrants in total employment between years t and t - 10, and  $Z_{it}$  is a control vector that includes fixed effects.

Following Dustmann, Schönberg and Stuhler (2016), one may classify different approaches to the estimation of  $\beta$  in eq. (F.1) on the basis of the definition of the cell i: in the *skill-cell approach*, i corresponds to an education-experience cell defined at the national level (e.g. Borjas, 2003); in the *spatial approach*, i corresponds to a region (e.g. Altonji and Card, 1991); in the *mixed approach*, i corresponds to the intersection of a region and an occupation, or a region and an education group (e.g. Card, 2001).

In the *spatial* and *mixed* approaches, since Altonji and Card (1991) and Card (2001), it has become common to instrument for the change in the immigrant share  $\Delta ImmShare_{it}$  using a shift-share IV:

$$X_{it} = \sum_{g=1}^{G} ImmShare_{igt_0} \frac{\Delta Imm_{gt}}{Imm_{gt_0}},$$
 (F.2)

where g indexes countries (or groups of countries) of origin of immigrants, and  $t_0$  is some pre-sample or beginning-of-the-sample time period. The variable  $ImmShare_{igt_0}$  plays the role of the share  $w_{is}$  in eq. (1) and denotes the share of immigrants from origin g in total immigrant employment in cell i in year  $t_0$ ; the ratio  $\Delta Imm_{gt}/Imm_{gt_0}$  plays the role of the shifter  $\mathcal{X}_s$  in eq. (1), with  $\Delta Imm_{gt}$  denoting the change in the total number of immigrants coming from origin g between years f and f and f and f denoting the total number of immigrants from region g at the national level in year f and f and f and f denoting the total number of immigrants from region g at the national level in year f and f and f and f denoting the total number of immigrants from region g at the national level in year f and f and f are f and f and f are f and f and f are f are f and f are f are f and f are f are f and f are f and f are f are f and f are f and f are f are f and f are f and f are f are f and f are f are f and f are f and f are f are f are f and f are f are f are f and f are f are f and f are f are f and f are f and f are f and f are f are f and f are f and f are f are f are f are f and f are f are f are f are f are f and f are f and f are f a

When estimating the parameter of interest  $\beta$  in eq. (F.1), the researcher must make a choice on the sample period or time frame of the analysis, and on the G countries (or areas) of origin used to construct the shift-share IV. In Appendix F.1, we discuss two different sample periods previously used in the literature, and present a list of areas of origin of immigrants for which information is available in each of the two sample periods. In Appendix F.2, we present placebo evidence that illustrates the finite-sample properties of the different inference procedures when applied to the two sample periods discussed in Appendix F.1 and when using different sets of countries of origin of immigrants to construct the shift-share IV in eq. (F.2). The main conclusion that arises from these placebo simulations is that restricting the set of countries of origin used in the construction of the shift-share IV to those with a relatively small value of  $ImmShare_{igt_0}$  generally improves the finite-sample coverage of all different inference procedures. Consequently, in Appendix F.3, we present estimates of  $\beta$  in eq. (F.1) that use information on a restricted set of countries when building the shift-share IV in eq. (F.2). For the sake of comparison, in Appendix F.4, we present estimates that use information on all countries for which information on immigration flows into the US is available for the relevant sample period.

In Appendices F.3 and F.4, we present estimates of specifications that follow either the *spatial* approach or the *mixed approach*. In all specifications, information on all variables entering eqs. (F.1) and (F.2) comes from the Census Integrated Public Use Micro Samples for 1980–2000 and the American Community Survey for 2008–2012. In all regressions, the vector of controls  $Z_{it}$  includes period dummies and, when implementing the *mixed approach*, we also add occupation- or education-group-specific dummies to the vector  $Z_i$ . All tables referenced in this section are included at the end.

# F.1 Sample periods and list of countries of origin of immigrants

The results we present use one of two time frames. The first one uses information on immigrant shares (i.e. the variable  $ImmShare_{igt_0}$  in eq. (F.2)) measured in 1980, and information on the outcome variables, endogenous treatment, and shifters of interest (i.e. the variables  $Y_{it}$ ,  $\Delta ImmShare_{it}$  and  $\Delta Imm_{gt}$  in eqs. (F.1) and (F.2)) for the periods 1980–1990, 1990–2000, and 2000–2010. Table F.1 lists all countries or areas of origin that we consider for which information on the number of immigrants in the U.S. is available for all periods in this time frame (i.e. 1980, 1990, 2000, and 2010).

The second time frames uses information on immigrant shares measured in 1960, and information on the outcome variables, endogenous treatment, and shifters of interest for the period 1970–1980. Table F.2 lists all countries or areas of origin that we consider for which information on the number of immigrants in the U.S. is available for all periods in this time frame (i.e. 1960, 1970 and 1980).

In both Tables F.1 and F.2, we have marked in italics those countries or areas of origin that account for a relatively large share (larger than 3%) of the overall immigrant U.S. population in the corresponding base year (this base year is 1980 for Table F.1 and 1960 for Table F.2).

# F.2 Placebo simulations

In Tables F.3 and F.4, we present the results of placebo exercises that illustrate the properties of different inference procedures for the parameter on the shift-share covariate in eq. (F.2) in regressions of labor market outcomes for native workers on this shift-share covariate. The only difference between the analysis in Table F.3 and the analysis in Table F.4 is in the set of areas of origin of immigrants used to construct the shift-share covariate in eq. (F.2). While the former uses information only on those countries of origin whose total share of immigrants in the corresponding baseline year  $t_0$  (either 1960 or 1980, depending on the specification) is below 3% (i.e. it uses information only on those countries of origin g that satisfy  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} \leq 0.03$ ), the latter uses information on all areas of origin of immigrants listed in the tables described in Appendix F.1.

We present results for four outcome variables: the change in employment ( $\Delta \log E_i$ ) and average wages ( $\Delta \log w_i$ ) across all native workers, and the change in average wages for high-skill and low-skill workers. For each of the four outcome variables, we consider several regressions in which we vary both the definition of a cell or unit of observation, and the sample period. The first four rows of each panel in Tables F.3 and F.4 implement a purely *spatial approach*, defining each unit of observation as a commuting zone (CZ) or as a metropolitan statistical area (MSA). The last four rows follow a *mixed approach*, defining each unit as the intersection of a CZ and either one of the fifty occupations defined in Burstein et al. (2018) (CZ-50 Occ.), one of seven aggregate occupations defined similarly to

Card (2001) (CZ-7 Occ.), or one of two education groups (CZ-Educ.). In terms of sample periods, we explore two alternatives. We either define the weights in 1980 and measure the outcome variable as the 1980–1990, 1990–2000, and 2000–2010 changes in log employment or log wages or, alternatively, we measure the weights in 1960 and measure the outcome variable as the 1970–1980 change in the variable of interest.

Tables F.3 and F.4 yield three key takeaways. First, robust standard errors are generally biased downward, leading frequently to an overrejection problem.

Second, when we construct the shift-share covariate in eq. (F.2) relying only on countries of origin with relatively small shares of U.S. immigrant population in the baseline year, state-clustered standard errors yield adequate rejection rates when the unit of observation is defined as the intersection of a CZ and fifty detailed occupation groups, shares are measured in 1980, and the outcome is defined as the subsequent three decadal changes. In all other cases, inference procedures based on state-clustered standard errors tend to overreject.

Third, the *AKM* and *AKM0* inference procedures perform much better when the shift-share covariate in eq. (F.2) is constructed using only countries of origin with relatively small shares of U.S. immigrant population in the baseline year, so that Assumptions 2(ii) and 2(iii) more plausibly hold. Furthermore, these inference procedures also tend to perform better in specifications that apply a *mixed approach* than in those that apply a purely *spatial approach*. One possible explanation for this pattern is that our asymptotics require that the number of observations  $N \to \infty$ ; thus, the behavior of the *AKM* and *AKM0* inference procedures is generally better in samples with a larger number of observations, and the *mixed approach*, which intersects each region with several occupations or education groups, yields larger sample sizes. Importantly, while the *AKM* inference procedure may still lead to confidence intervals that are too short in several specifications, the *AKM0* inference procedure generally yields accurate rejection rates. However, confidence intervals based on the *AKM0* inference procedure may be very conservative for certain specifications.

# F.3 Results with a restricted set of origin countries

All results presented in this section exploit information only on those countries of origin whose total share of immigrants in the corresponding baseline year  $t_0$  (either 1960 or 1980, depending on the specification) is below 3%. More precisely, these results presented here are computed using an IV such as that in eq. (F.2) constructed excluding those countries of origin g for which  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} > 0.03$ . We exclude large origin countries so that Assumptions 2(ii) and 2(iii) more plausibly hold. The simulations in Appendix F.2 also suggest that excluding large origin countries should lead to better finite-sample performance of the inference procedures that we propose.<sup>6</sup>

Table F.5 presents results for three different implementations of the *mixed approach*. In all three cases, the data comes from a three-period panel with  $t = \{1990, 2000, 2010\}$  and  $t_0 = 1980$ . The implementations differ in the definition of a cell. In columns (1) to (4) of Table F.5, a cell corresponds

<sup>&</sup>lt;sup>6</sup>Our theory currently does not provide guidance on the particular threshold that one should choose. While we find that the 3% threshold works well in the placebo exercises in this particular application, we leave the question of what threshold one should in general pick to ensure that Assumptions 2(ii) and 2(iii) plausibly hold to future research.

to the intersection of a CZ and one of the 50 occupations defined in Appendix F of Burstein et al. (2018). In columns (5) and (6), we define a cell as the intersection of a CZ and one of two education groups: high school-equivalent or college-equivalent educated workers (see Card, 2009). In columns (7) to (10), a cell corresponds to the intersection of a CZ and one of seven aggregate occupations (see Card, 2001).<sup>7</sup>

Although Table F.5 adopts occupational definitions that build on those in Burstein et al. (2018) and Card (2001), our specifications do not exactly match their definition of shares and shifters. Thus, our estimates should not be viewed as a test of the robustness of the results presented in these studies. Furthermore, no matter which definition of cell we use, when interpreting our estimates, one should bear in mind that, as discussed in Jaeger, Ruist and Stuhler (2018a), these may conflate the short- and the long-run responses to immigration shocks.

The magnitude and statistical significance of the estimates of  $\beta$  in eq. (F.1) is generally consistent across the specifications studied in Table F.5. In terms of the impact of immigration on native employment, we find that a one percentage point increase in the share of immigrants in total employment reduces the number of native workers employed by 1.19–1.49%, with all estimates of  $\beta$  being statistically different from zero at the 5% level for all four inference procedures that we consider. In terms of the impact of immigration on natives' average weekly wages, we find that the estimated impact of an increase in the immigrant share is not statistically different from zero at the 5% significance level according to the *AKM* and *AKM0* CIs; this is true for all three cell definitions and no matter whether we compute average wages for all workers, only for high-skill workers or only for low-skill workers. *Robust* and *Cluster* CIs also indicate that the effect of immigration on natives' average weekly wages is not statistically different from zero at the 5% significance level when each cell corresponds to the intersection of a CZ and an education group, but these standard inference procedures sometimes predict that immigration has a positive effect on the wages of high-skill workers when occupations are used to define the unit of analysis (see columns (3) and (9) in Table F.5).

While all inference procedures broadly agree in the statistical significance (at the 5% significance level) of the impact of immigration on natives' labor market outcomes, there is considerable heterogeneity across specifications in the length of the *AKM* and *AKM0* confidence intervals relative to those based on *Robust* and *Cluster* standard errors. In columns (1) to (4), which use detailed occupations to define cells, *AKM* and *AKM0* CIs tend to be very similar (in some cases, even slightly smaller) to those based on state-clustered standard errors, although they are generally much larger than those based on robust standard errors. In contrast, for the other two cell definitions, the IV *AKM* and *AKM0* CIs are on average, 200% and 356% wider than those based on state-clustered standard errors, and the reduced-form *AKM* and *AKM0* CIs are on average 228% and 358% wider than those based on state-clustered standard errors. Similarly, the CIs for the first-stage coefficient, reported in Panel C, *AKM* and *AKM0* CIs are more than twice as wide as *Robust* and *Cluster* CIs.<sup>8</sup>

<sup>&</sup>lt;sup>7</sup>We group the 50 disaggregated occupations used in Burstein et al. (2018) into seven aggregate occupations: laborers, farm workers and low-skilled service workers; operatives and craft workers; clerical workers; sales workers; managers; professional and technical workers; and others.

<sup>&</sup>lt;sup>8</sup>The results in Table F.5 are consistent with the placebo simulation results in Table F.3, which show that state-clustered standard errors lead to rejection rates that are very close to the nominal level when a cell is defined as the intersection of CZs and 50 occupations, but lead to overrejection for the other two cell definitions.

To understand why standard inference procedures may lead to overrejection of the null hypothesis of no effect in certain cases, recall from the discussion in Section 4.1 that robust and state-clustered standard errors may be biased downward even if there is no shock in the structural residual that varies exactly at the same level as the shifters of interest; a downward bias will arise so long as there is a shift-share component in the residual with shares that have a correlation structure similar to that of the shares used to construct the shift-share instrument. We present simulations that illustrate this point in Appendix D.6.

Tables F.6 and F.7 present results for different versions of the *spatial approach*, using CZs and MSAs as unit of observation, respectively. We present both estimates that measure the immigrant shares  $ImmShare_{igt_0}$  in 1980 and use data on shifters and outcomes for the periods 1980–1990, 1990–2000, and 2000–2010, and estimates that measure the immigrant shares in 1960 and use data on outcomes only for the period 1970–1980. While the first sample definition mimics that in Table F.5, the second one is suggested in Jaeger, Ruist and Stuhler (2018b) as being more robust to potential bias in the estimates of  $\beta$  that arise from the combination of serial correlation in the shifters  $\Delta Imm_{gt}$  and the potentially slow adjustment of labor market outcomes to these immigration shocks.

The placebo simulation results for the different specifications considered in Tables F.6 and F.7 (see Table F.3) reveal that, due to the relatively small number of observations (i.e. small number of MSAs and CZs; small value of N) and, in the case of the specification that relies on immigrants shares measured in 1960, the relatively small number of countries of origin of immigrants (i.e. small value of G), only the AKM0 inference procedure consistently yields rejection rates that are close to the nominal level of 5%. However, the AKM0 procedure yields CIs with an implied median effective standard error that is much larger than the true standard deviation of the estimator. It is thus conservative. Thus, the placebo results suggest that, for most of the specifications considered in Tables F.6 and F.7, the AKM0 CIs may be conservative and the Robust, Cluster and AKM CIs may be too small. It is thus not surprising that, for the different specifications considered in Tables F.6 and F.7, the AKM0 CIs are much larger than than those implied by the other three inference procedures.

Finally, Tables F.8 to F.10 report p-values for the null hypothesis of no effect for all specifications considered in Tables F.5 to F.7, respectively.

# F.4 Results with all origin countries

Tables F.11 to F.16 present results analogous to those in Tables F.5 to F.10, respectively. While the latter set of tables, as described in Appendix F.3, use a shift-share instrument that excludes countries of origin that account for more than 3% of the overall immigrant population in the baseline year, the former uses all areas of origin of immigrants listed in Tables F.1 and F.2.

As the results of placebo simulations presented in Table F.4 show, using all countries of origin to construct the shift-share instrumental variable of interest results in the *Robust*, *Cluster* and *AKM* standard errors underestimating the sampling variability of the estimator of interest. Table F.4 also shows that not excluding any country of origin from the construction of the instrument in eq. (F.2)

<sup>&</sup>lt;sup>9</sup>This is particularly noticeable for the IV results in Panel A; however, to interpret these CIs, one should bear in mind that, as the first-stage results in Panel C show, the shift-share IV is weak in these specifications. In the presence of weak IVs, only the *AKM0* confidence interval remains valid in general (see discussion in Section 4.3).

Table F.1: Origin countries (1980 weights)

Afghanistan	France	Liechtenstein and Lux.	Scandinavia
Africa	Greece	Malaysia	Scotland
Albania	Gulf States	Maldives	Singapore
Andorra and Gibraltar	India	Malta	South America
Austria	Indonesia	Mexico	Spain
Belgium	Iran	Nepal	Switzerland
Brunei	Iraq	Netherlands	Syria
Cambodia	Ireland	Oceania	Thailand
Canada	Israel/Palestine	Other	Turkey
Central America	Italy	Other Europe	Vietnam
China	Japan	Other USSR and Russia	Wales
Cuba and West Indies	Jordan	Philippines	Yemen
Cyprus	Korea	Portugal	
Eastern Europe	Laos	Rest of Asia	
England	Lebanon	Saudi Arabia	

Notes: In italics, countries that are dropped from the sample when considering only countries whose share is below 3%; i.e. those countries in italics are countries of origin g such that  $(\sum_{i=1}^{N} ImmShare_{ig,1980} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig',1980}) > 0.03$ .

Table F.2: Origin countries (1960 weights)

	France	Liechtenstein and Lux.	Scandinavia
Africa	Greece		Scotland
Albania			
	India		South America
Austria		Mexico	Spain
Belgium			Switzerland
		Netherlands	Syria
	Ireland	Oceania	
Canada	Israel/Palestine	Other	Turkey
Central America	Italy	Other Europe	•
China	Japan	Other USSR and Russia	Wales
Cuba and West Indies		Philippines	
	Korea	Portugal	
Eastern Europe		Rest of Asia	
England	Lebanon		

Notes: In italics, countries that are dropped from the sample when considering only countries whose share is below 3%; i.e. those countries in italics are countries of origin g such that  $(\sum_{i=1}^{N} ImmShare_{ig1960} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig'1960}) > 0.03$ .

results in the *AKM0* inference procedure being too conservative: the 95% confidence interval often has an infinite length and the rejection rates are generally much smaller than the 5% nominal rate.

Given the relatively poor performance of all inference procedures in the placebo simulations, one should use caution when extracting conclusions from the estimates presented in Tables F.11 to F.16.

Table F.3: Reduced-form placebo with origin countries below 3% of total immigrant share

			Me	dian eff. s.e.		Rej	ection rate	e for $H_0$ : $\beta =$	0 at 5%
		$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$	
		All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill
Unit obs.	Weights	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Sta	andard dev	iation of p	lacebo	estimate					
CZ	1980	1.14	0.46	0.30	1.04				
CZ	1960	0.96	0.81	0.80	1.51				
MSA	1980	1.94	0.49	0.74	0.83				
MSA	1960	1.99	0.25	0.33	0.71				
CZ-50 Occ.	1980	0.17	0.06	0.07	0.06				
CZ-7 Occ.	1980	1.59	0.40	0.47	0.42				
CZ-Educ.	1980	2.88	0.54	_	_				
Panel B: Ro	bust standa	ard error							
CZ	1980	0.49	0.21	0.20	0.25	37.82%	43.17%	12.45%	73.21%
CZ	1960	0.77	0.31	0.32	0.39	9.24%	47.70%	37.09%	68.84%
MSA	1980	1.82	0.33	0.35	0.37	8.12%	20.42%	43.80%	42.74%
MSA	1960	1.47	0.24	0.27	0.30	20.12%	2.73%	3.62%	48.74%
CZ-50 Occ.	1980	0.11	0.05	0.06	0.06	20.22%	12.46%	8.57%	4.51%
CZ-7 Occ.	1980	0.47	0.16	0.19	0.18	63.04%	44.33%	41.78%	45.89%
CZ-Educ.	1980	0.65	0.15	_	_	67.22%	66.12%	_	_
Panel C: Sta	te-clustere	d standard	l error						
CZ	1980	0.70	0.26	0.27	0.34	23.76%	30.73%	5.44%	62.41%
CZ	1960	1.03	0.46	0.48	0.60	2.01%	26.87%	12.68%	51.41%
MSA	1980	2.11	0.29	0.32	0.43	3.42%	24.52%	49.14%	36.30%
MSA	1960	1.62	0.20	0.23	0.35	17.72%	7.59%	12.52%	41.45%
CZ-50 Occ.	1980	0.14	0.07	0.06	0.06	9.21%	5.24%	5.01%	1.64%
CZ-7 Occ.	1980	0.64	0.23	0.26	0.25	49.70%	27.94%	27.90%	29.90%
CZ-Educ.	1980	0.98	0.23	_	_	50.31%	50.35%		
Panel D: AI	KM standar	d error							
CZ	1980	0.91	0.40	0.24	0.95	12.01%	23.27%	5.40%	30.31%
CZ	1960	0.73	0.67	0.62	1.37	11.27%	15.35%	9.30%	20.06%
MSA	1980	1.60	0.39	0.62	0.73	25.50%	15.31%	22.26%	17.92%
MSA	1960	1.39	0.20	0.26	0.55	22.88%	11.71%	7.66%	23.57%
CZ-50 Occ.	1980	0.15	0.06	0.06	0.05	11.63%	10.77%	8.16%	10.02%
CZ-7 Occ.	1980	1.49	0.35	0.40	0.37	19.94%	14.79%	12.98%	20.63%
CZ-Educ.	1980	2.40	0.49	_	_	17.46%	25.09%	_	_
Panel E: AK	M0 standa	rd error							
CZ	1980	$\infty$	$\infty$	$\infty$	$\infty$	2.74%	1.63%	3.07%	1.00%
CZ	1960	3.04	3.13	2.67	6.31	3.10%	3.16%	5.04%	2.26%
MSA	1980	30.16	7.92	17.82	12.70	1.93%	1.25%	0.80%	1.41%
MSA	1960	$\infty$	$\infty$	$\infty$	$\infty$	2.85%	2.84%	2.30%	0.52%
CZ-50 Occ.	1980	0.25	0.09	0.10	0.08	4.43%	4.13%	3.63%	4.35%
CZ-7 Occ.	1980	2.89	0.66	0.76	0.72	4.67%	3.72%	4.08%	4.08%
CZ-Educ.	1980	8.10	1.75	_	_	4.65%	3.17%	_	

Notes:  $\Delta \log E_i$  denotes  $\log$  change in native employment;  $\Delta \log w_i$  denotes  $\log$  change in average weekly wages of native workers. Whenever we use 1980 weights, we use observations for the time periods 1980–1990, 1990–2000, 2000–2010. Whenever we use 1960 weights, we use observations for the time period 1970–1980. We use information on 722 CZs when combined with both 1960 and 1980 weights, on 257 MSAs when combined with 1980 weights, and 217 MSAs when combined with 1960 weights. Models are weighted by the start-of-period share of national population. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. The median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by  $2 \times 1.96$ . Results are based on 30,000 placebo samples.

Table F.4: Reduced-form placebo with all origin countries

			Me	dian eff. s.e.		Rej	ection rate	e for $H_0: \beta =$	0 at 5%
		$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$	
		All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill
Unit obs.	Weights	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Sta	ndard dev	iation of p	lacebo	estimate					
CZ	1980	0.21	0.18	0.15	0.25				
CZ	1960	0.27	0.20	0.18	0.32				
MSA	1980	0.40	0.06	0.06	0.13				
MSA	1960	0.51	0.12	0.10	0.26				
CZ-50 Occ.	1980	0.11	0.04	0.02	0.04				
CZ-7 Occ.	1980	0.51	0.15	0.2	0.22				
CZ-Educ.	1980	0.60	0.15	_	_				
Panel B: Rol	bust standa	ard error							
CZ	1980	0.12	0.05	0.04	0.06	3.57%	73.01%	81.63%	38.91%
CZ	1960	0.13	0.05	0.05	0.06	15.86%	55.42%	57.59%	60.02%
MSA	1980	0.20	0.03	0.03	0.04	25.90%	18.07%	9.97%	53.39%
MSA	1960	0.34	0.08	0.08	0.09	9.21%	9.52%	2.18%	41.43%
CZ-50 Occ.	1980	0.04	0.02	0.02	0.02	62.52%	48.73%	9.07%	38.04%
CZ-7 Occ.	1980	0.15	0.05	0.04	0.07	75.86%	75.24%	86.16%	80.99%
CZ-Educ.	1980	0.13	0.04	_	_	82.08%	49.75%	_	_
Panel C: Sta						0_10075			
CZ	1980	0.09	0.07	0.05	0.10	3.46%	46.56%	68.91%	13.85%
CZ	1960	0.16	0.09	0.07	0.12	8.50%	20.66%	43.84%	29.18%
MSA	1980	0.28	0.04	0.03	0.07	10.98%	5.77%	7.30%	21.38%
MSA	1960	0.39	0.09	0.09	0.13	2.80%	7.02%	1.29%	33.24%
CZ-50 Occ.	1980	0.06	0.02	0.02	0.02	48.40%	34.88%	5.15%	30.04%
CZ-7 Occ.	1980	0.22	0.06	0.06	0.06	30.90%	60.28%	74.47%	75.69%
CZ-Educ.	1980	0.18	0.08	_	_	73.77%	9.82%	_	_
Panel D: AK			0.00			701770	7.0270		
CZ	1980	0.09	0.06	0.04	0.12	4.44%	56.32%	66.26%	10.19%
CZ	1960	0.14	0.11	0.08	0.19	17.27%	21.85%	37.77%	13.12%
MSA	1980	0.20	0.03	0.04	0.07	40.31%	30.36%	3.86%	31.97%
MSA	1960	0.33	0.08	0.07	0.20	23.68%	11.11%	1.43%	13.19%
CZ-50 Occ.	1980	0.07	0.03	0.02	0.03	32.38%	32.88%	25.60%	28.50%
CZ-7 Occ.	1980	0.21	0.06	0.07	0.08	54.43%	59.39%	61.74%	62.08%
CZ-Educ.	1980	0.23	0.07	<del>-</del>	<del></del>	53.00%	39.53%	O1.7 170	02.0070
Panel E: AK			0.07			00.0070	07.0070		
CZ	1980	∞	$\infty$	$\infty$	$\infty$	2.52%	0.27%	0.33%	0.44%
CZ	1960	∞	∞	∞	∞	2.52%	0.32%	2.03%	0.30%
MSA	1980	∞	∞	∞	∞	0.54%	0.97%	2.47%	0.08%
MSA	1960	∞	∞	∞	∞	1.06%	0.12%	1.58%	0.06%
CZ-50 Occ.	1980	0.42	0.15	0.1	0.14	1.73%	2.49%	3.17%	2.72%
CZ-7 Occ.	1980	∞	∞	∞	∞	1.33%	0.73%	0.83%	0.56%
CZ-Educ.	1980	∞	∞	_	<del>~</del>	1.16%	1.20%	J.05 /0	0.5070

Notes:  $\Delta \log E_i$  denotes  $\log$  change in native employment;  $\Delta \log w_i$  denotes  $\log$  change in average weekly wages of native workers. Whenever we use 1980 weights, we use observations for the time periods 1980–1990, 1990–2000, 2000–2010. Whenever we use 1960 weights, we use observations for the time period 1970–1980. We use information on 722 CZs when combined with both 1960 and 1980 weights, on 257 MSAs when combined with 1980 weights, and 217 MSAs when combined with 1960 weights. Models are weighted by the start-of-period share of national population. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. The median effective standard error is equal to the median length of the corresponding 95% confidence interval divided by  $2 \times 1.96$ . Results are based on 30,000 placebo samples.

Table F.5: Effect of immigration: analysis by CZ-Occupations and CZ-Education groups (excluding large origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$	$\Delta \log w_i$	$\Delta \log E_i$		$\Delta \log w_i$	
Workers:	All	All	High-Skill	Low-Skill	All	All	All	All	High-Skill	Low-Skill
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		CZ-50 Occ. (1	1980 weights)		CZ-Educ. (19	980 weights)		CZ-7 Occ. (1	980 weights)	
Panel A: 2	SLS									
$\hat{eta}$	-1.19	0.05	0.26	-0.14	-1.49	0.18	-1.39	0.08	0.24	-0.14
Robust	[-1.55, -0.83]	[-0.09, 0.20]	[0.11, 0.41]	[-0.32, 0.03]	[-2.09, -0.90]	[-0.20, 0.56]	[-1.89, -0.90]	[-0.17, 0.33]	[0.00, 0.47]	[-0.43, 0.15]
Cluster	[-1.89, -0.49]	[-0.35, 0.46]	[-0.14, 0.67]	[-0.69, 0.40]	[-2.14, -0.85]	[-0.04, 0.39]	[-1.87, -0.92]	[-0.11, 0.27]	[0.10, 0.38]	[-0.27, -0.01]
AKM	[-1.55, -0.83]	[-0.36, 0.47]	[-0.17, 0.69]	[-0.64, 0.35]	[-2.31, -0.68]	[-0.53, 0.88]	[-2.02, -0.76]	[-0.51, 0.67]	[-0.31, 0.79]	[-0.81, 0.53]
AKM0	[-1.66, -0.72]	[-0.53, 0.54]	[-0.32, 0.81]	[-0.92, 0.39]	[-3.00, -0.14]	[-0.90, 1.60]	[-2.35, -0.54]	[-0.75, 0.94]	[-0.51, 1.07]	[-1.12, 0.80]
Panel B: R	Reduced-Form									
$\hat{eta}$	-0.89	0.04	0.2	-0.11	-1.29	0.15	-1.05	0.06	0.18	-0.11
Robust	[-1.17, -0.61]	[-0.07, 0.15]	[0.06, 0.33]	[-0.23, 0.02]	[-1.86, -0.73]	[-0.19, 0.50]	[-1.38, -0.73]	[-0.13, 0.25]	[-0.01, 0.37]	[-0.32, 0.11]
Cluster	[-1.37, -0.41]	[-0.27, 0.35]	[-0.16, 0.55]	[-0.47, 0.25]	[-1.69, -0.90]	[-0.02, 0.33]	[-1.29, -0.82]	[-0.08, 0.21]	[0.07, 0.29]	[-0.20, -0.01]
AKM	[-1.35, -0.43]	[-0.28, 0.36]	[-0.18, 0.57]	[-0.44, 0.23]	[-1.99, -0.60]	[-0.48, 0.79]	[-1.54, -0.57]	[-0.39, 0.52]	[-0.26, 0.62]	[-0.59, 0.38]
AKM0	[-1.55, -0.39]	[-0.27, 0.55]	[-0.16, 0.79]	[-0.44, 0.42]	[-2.25, -0.11]	[-0.53, 1.56]	[-1.62, -0.36]	[-0.42, 0.78]	[-0.29, 0.86]	[-0.62, 0.67]
Panel C: F	irst-Stage									
$\hat{eta}$		0.7	75		0.8	7		0.3	76	
Robust		[0.56,			[0.55,			[0.56,		
Cluster		[0.62,			0.66,			0.60,		
AKM		[0.36,	,		[0.44,			[0.42,		
AKM0		[0.38,	,		[0.34,	,		0.38,		

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native workers. The specifications *CZ-50 Occupations*, *CZ-2 Education Groups*, and *CZ-7 Occupations* differ in the definition of the unit of observation. In all three specifications, we use 1980 weights and three time periods, 1980–1990, 1990–2000, and 2000–2010. Thus, N=108,300 (722 CZs  $\times$  50 occupations  $\times$  3 time periods) for the *CZ-50 Occupations* specification; N=4,332 (722 CZs  $\times$  2 education groups  $\times$  3 time periods) for the *CZ-2 Education Groups* specification; and N=15,162 (722 CZs  $\times$  7 occupations  $\times$  3 time periods) for the *CZ-7 Occupations* specification. Models are weighted by start-of-period occupation-region (or education group-region) share of national population. All regressions include occupation (or education group) and period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in 1980 is larger than 3%; i.e.  $\sum_i ImmShare_{ig^it_0} / \sum_i \sum_{g^i} ImmShare_{ig^it_0} > 0.03$ . See Table F.1 for a list of the origin countries included in the analysis.

Table F.6: Effect of immigration: analysis by CZ (excluding large origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$	
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Commuting Zon	e (1980 weights)			Commuting	Zone (1960 weig	hts)
Panel A: 2	SLS Regression	1						
$\hat{\beta}$	-0.89	0.42	0.56	-0.10	-1.29	-0.71	-0.34	-1.14
Robust	[-1.48, -0.30]	[-0.06, 0.90]	[0.16, 0.95]	[-0.68, 0.47]	[-3.69, 1.10]	[-1.10, -0.33]	[-1.01, 0.33]	[-1.66, -0.61]
Cluster	[-1.33, -0.45]	[0.15, 0.69]	[0.39, 0.72]	[-0.31, 0.11]	[-3.96, 1.37]	[-1.17, -0.26]	[-1.12, 0.44]	[-1.71, -0.56]
AKM	[-1.85, 0.08]	[-0.44, 1.28]	[-0.12, 1.24]	[-1.17, 0.97]	[-3.83, 1.24]	[-1.31, -0.12]	[-1.16, 0.48]	[-1.89, -0.38]
AKM0		$[-\infty, -230.52]$ $\cup [-2.73, \infty]$		$[-\infty, -158.12]$ $\cup [-7.24, \infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$
Panel B: R	Reduced-Form F	Regression						
$\hat{\beta}$	-0.75	0.35	0.47	-0.08	-1.31	-0.72	-0.34	-1.15
Robust	[-1.16, -0.33]	[-0.10, 0.81]	[0.07, 0.86]	[-0.55, 0.38]	[-3.39, 0.78]	[-1.06, -0.38]	[-0.94, 0.25]	[-1.61, -0.68]
Cluster	[-1.02, -0.47]	[0.12, 0.59]	[0.31, 0.62]	[-0.26, 0.09]	[-3.57, 0.96]	[-1.10, -0.35]	[-1.02, 0.33]	[-1.56, -0.73]
AKM	[-1.47, -0.02]	[-0.44, 1.15]	[-0.21, 1.14]	[-0.96, 0.79]	[-3.55, 0.94]	[-1.21, -0.23]	[-1.09, 0.40]	[-1.73, -0.57]
AKM0	[-1.71, 6.41]	[-0.73, 8.03]	[-0.44, 6.99]	[-1.29, 8.26]	$[-\infty,\infty]$		[-3.37, -1.87]	[-4.01, -2.12]
Panel C: 2	SLS First-Stage	!						
Â		0.	84				1.01	
Robust		[0.51]	1.18]				[0.46, 1.56]	
Cluster			1.03				0.65, 1.37	
AKM			1.31				0.70, 1.32	
AKM0		[0.00]					$[-\infty,\infty]$	

Notes:  $\Delta \log E_i$  denotes log change in native employment;  $\Delta \log w_i$  denotes log change in average weekly wages of native workers. In the specification CZ (1980 weights), we use information on 722 CZs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=2, 166 (722  $CZs \times 3$  time periods). In the specification CZ (1960 weights), we use information on 722 CZs, 1960 weights and one time period, 1970–1980; thus, N=722 (722  $CZs \times 1$  time period). Models are weighted by start-of-period CZ share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in year  $t_0$  is larger than 3%; i.e.  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{igt_0} > 0.03$ . See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.7: Effect of immigration: analysis by MSA (excluding large origin countries)

: $\Delta \log E_i$	Outcome:		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$		
All	Workers:	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill	
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	
		MSA (1980	) weights)		MSA (1960 weights)				
2SLS Regress	Panel A: 2	n							
-1.87	β	0.50	0.63	-0.10	-1.55	-0.32	-0.17	-0.57	
[-3.17, -0.58]	Robust	[-0.06, 1.06]	[0.17, 1.09]	[-0.93, 0.73]	[-3.68, 0.57]	[-0.72, 0.08]	[-0.83, 0.49]	[-1.08, -0.06]	
[-2.97, -0.78]	Cluster	[0.23, 0.77]	[0.48, 0.77]	[-0.47, 0.27]	[-3.94, 0.84]	[-0.74, 0.09]	[-0.88, 0.53]	[-1.06, -0.08]	
[-3.77, 0.03]	AKM	[-0.44, 1.44]	[-0.15, 1.40]	[-1.49, 1.29]	[-5.61, 2.51]	[-0.90, 0.25]	[-1.07, 0.73]	[-1.26, 0.12]	
$[-\infty,\infty]$	AKM0	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	
Reduced-Form	Panel B: R	Regression							
-1.47	$\hat{eta}$	0.39	0.49	-0.08	-1.48	-0.31	-0.16	-0.54	
[-2.06, -0.8]	Robust	[-0.17, 0.95]	[0.00, 0.98]	[-0.70, 0.54]	[-3.29, 0.34]	[-0.61, -0.01]	[-0.75, 0.42]	[-0.90, -0.19]	
[-1.94, -0.9]	Cluster	[0.09, 0.69]	[0.27, 0.71]	[-0.35, 0.19]	[-3.45, 0.49]	[-0.63, 0.01]	[-0.79, 0.46]	[-0.86, -0.22]	
[-2.43, -0.50]	AKM	[-0.50, 1.28]			[-4.79, 1.83]		[-0.96, 0.63]	[-0.98, -0.11]	
	AKM0	[-0.79, 9.41]					$[-\infty,\infty]$	$[-\infty,\infty]$	
2SLS First-Sta	Panel C: 2	e							
	Ĝ	0.73	8			0	.95		
		£ .							
					[0.49, 1.42]				
			4.				1		
	ß Robust Cluster AKM AKM0	0.73 [0.29,1 [0.51,1 [0.10,1 [-0.61,	1.28] 1.05] 1.47]			[0.46 [0.64 [0.49	5,1.45] -,1.27]		

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native workers. In the specification MSA (1980 weights), we use information on 257 MSAs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=651 (257 MSAs  $\times$  3 time periods). In the specification MSA (1960 weights), we use information on 217 MSAs, 1960 weights and one time period, 1970–1980; thus, N=217 (217 CZs  $\times$  1 time period). Models are weighted by start-of-period MSA share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error rior that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in year  $t_0$  is larger than 3%; i.e.  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} > 0.03$ . See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.8: Immigration: p-values by CZ-Occ. and CZ-Educ. (excluding large origin countries)

	$\Delta \log E_i$		$\Delta \log w$	'i	$\Delta \log E_i$	$\Delta \log w_i$	$\Delta \log E_i$		$\Delta \log u$	$\gamma_i$	
	All	All	High-Skill	Low-Skill	All	All	All	All	High-Skill	Low-Skill	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
		CZ-5	0 Occupation	ns	CZ-	CZ- Educ. CZ-			7 Occupations		
Panel A	: 2SLS	Regre	ssion								
Robust	0.000	0.459	0.001	0.116	0.000	0.357	0.000	0.519	0.046	0.348	
Cluster	0.001	0.790	0.205	0.611	0.000	0.102	0.000	0.412	0.001	0.042	
AKM	0.000	0.793	0.235	0.576	0.000	0.620	0.000	0.788	0.395	0.684	
AKM0	0.006	0.796	0.273	0.566	0.043	0.621	0.018	0.788	0.410	0.687	
Panel B	: Reduc	ed-Fo	rm Regress	ion							
Robust	0.000	0.473	0.004	0.091	0.000	0.379	0.000	0.528	0.057	0.330	
Cluster	0.000	0.795	0.275	0.565	0.000	0.089	0.000	0.409	0.001	0.036	
AKM	0.000	0.801	0.304	0.538	0.000	0.635	0.000	0.792	0.422	0.674	
AKM0	0.006	0.796	0.273	0.566	0.043	0.621	0.018	0.788	0.410	0.687	
Panel C	: First-S	tage									
Robust			0.000		0.000			0.000			
Cluster			0.000		0.000			0.000			
AKM			0.000		0.000			0.000			
AKM0			0.002		0.0	017			0.006		

Notes:  $\Delta \log E_i$  denotes  $\log$  change in native employment;  $\Delta \log w_i$  denotes  $\log$  change in average weekly wages of native workers. The specifications CZ-50 Occupations, CZ-Educ., and CZ-7 Occupations differ in the definition of the unit of observation. In all three specifications, we use 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010. Thus, N=108,300 (722  $CZs \times 50$  occupations  $\times$  3 time periods) for the CZ-50 Occupations specification; N=4,332 (722  $CZs \times 2$  education groups  $\times$  3 time periods) for the CZ-Educ. specification; and N=15,162 (722  $CZs \times 7$  occupations  $\times$  3 time periods) for the CZ-7 Occupations specification. Models are weighted by start-of-period occupation-region (or education group-region) share of national population. All regressions include occupation (or education group) and period dummies. 95% confidence intervals in square brackets. Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in year 1980 is larger than 3%; i.e.  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} > 0.03$ . See Table F.1 for a list of the origin countries included in the analysis.

Table F.9: Effect of immigration: p-values by CZ (excluding large origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w$	i	$\Delta \log E_i$		$\Delta \log w_i$	į		
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
	Com	muting	Zone (1980 v	weights)	Commuting Zone (1960 weights)					
Panel A: 2	SLS Reg	ression	1							
Robust	0.003	0.083	0.005	0.731	0.291	0.000	0.317	0.000		
Cluster	0.000	0.002	0.000	0.338	0.342	0.002	0.390	0.000		
AKM	0.071	0.335	0.109	0.853	0.318	0.018	0.416	0.003		
AKM0	0.216	0.360	0.174	0.855	0.393	0.179	0.472	0.142		
Panel B: R	educed-l	Form R	Regression							
Robust	0.000	0.129	0.020	0.722	0.220	0.000	0.254	0.000		
Cluster	0.000	0.003	0.000	0.330	0.258	0.000	0.316	0.000		
AKM	0.044	0.384	0.174	0.850	0.254	0.004	0.366	0.000		
AKM0	0.216	0.360	0.174	0.855	0.393	0.179	0.472	0.142		
Panel C: F	irst-Stag	e								
Robust			0.000				0.000			
Cluster			0.000				0.000			
AKM			0.001		0.000					
AKM0			0.050				0.062			

Notes:  $\Delta \log E_i$  denotes  $\log$  change in native employment;  $\Delta \log w_i$  denotes  $\log$  change in average weekly wages of native workers. In the specification CZ (1980 weights), we use information on 722 CZs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=2, 166 (722 CZs  $\times$  3 time periods). In the specification CZ (1960 weights), we use information on 722 CZs, 1960 weights and one time period, 1970–1980; thus, N=722 (722 CZs  $\times$  1 time period). Models are weighted by start-of-period CZ share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in year  $t_0$  is larger than 3%; i.e.  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} > 0.03$ . See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.10: Effect of immigration: p-values by MSA (excluding large origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$	i	$\Delta \log E_i$		$\Delta \log w$	i	
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
		MSA	(1980 weight:	s)	MSA (1960 weights)				
Panel A: 2	SLS Reg	ression							
Robust	0.005	0.080	0.008	0.812	0.152	0.115	0.610	0.028	
Cluster	0.001	0.000	0.000	0.594	0.203	0.128	0.631	0.022	
AKM	0.053	0.300	0.112	0.887	0.453	0.272	0.708	0.106	
AKM0	0.146	0.361	0.203	0.887	0.532	0.376	0.732	0.284	
Panel B: R	educed-I	orm R	egression						
Robust	0.000	0.173	0.048	0.803	0.110	0.043	0.585	0.003	
Cluster	0.000	0.012	0.000	0.564	0.141	0.059	0.607	0.001	
AKM	0.003	0.388	0.213	0.884	0.381	0.150	0.687	0.015	
AKM0	0.146	0.361	0.203	0.887	0.532	0.376	0.732	0.284	
Panel C: F	irst-Stage	2							
Robust			0.002				0.000		
Cluster			0.000		0.000				
AKM			0.025		0.000				
AKM0			0.096				0.126		

Notes:  $\Delta \log E_i$  denotes  $\log$  change in native employment;  $\Delta \log w_i$  denotes  $\log$  change in average weekly wages of native workers. In the specification MSA (1980 weights), we use information on 257 MSAs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=651 (257 MSAs  $\times$  3 time periods). In the specification MSA (1960 weights), we use information on 217 MSAs, 1960 weights and one time period, 1970–1980; thus, N=217 (217 CZs  $\times$  1 time period). Models are weighted by start-of-period MSA share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We exclude from the analysis those countries of origin whose immigrant share in year  $t_0$  is larger than 3%; i.e.  $\sum_{i=1}^{N} ImmShare_{igt_0} / \sum_{i=1}^{N} \sum_{g'=1}^{G} ImmShare_{ig't_0} > 0.03$ . See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.11: Effect of immigration: analysis by CZ-Occupations and CZ-Education groups (including all origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$	$\Delta \log w_i$	$\Delta \log E_i$		$\Delta \log w_i$			
Workers:	All	All	High-Skill	Low-Skill	All	All	All	All	High-Skill	Low-Skill		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
		CZ-50 Oc	ccupations		CZ- 2 Education Groups CZ-7 Occupations							
Panel A: 2	SLS Regression	n										
$\hat{\beta}$	-0.73	-0.07	0.15	-0.24	-0.53	-0.01	-0.79	-0.08	0.08	-0.27		
Robust	[-1.04, -0.42]	[-0.22, 0.09]	[0.01, 0.29]	[-0.42, -0.06]	[-1.03, -0.02]	[-0.45, 0.44]	[-1.25, -0.33]	[-0.39, 0.22]	[-0.17, 0.33]	[-0.64, 0.09]		
Cluster	[-1.15, -0.31]	[-0.49, 0.36]	[-0.23, 0.52]	[-0.80, 0.32]	[-1.03, -0.02]	[-0.24, 0.23]	[-1.34, -0.25]	[-0.33, 0.16]	[-0.09, 0.25]	[-0.47, -0.08]		
AKM	[-1.22, -0.24]	[-0.42, 0.29]	[-0.18, 0.47]	[-0.66, 0.18]	[-1.85, 0.79]	[-0.81, 0.80]	[-1.67, 0.09]	[-0.74, 0.57]	[-0.52, 0.67]	[-0.99, 0.44]		
AKM0	[-1.61, 0.24]	[-0.52, 0.94]	[-0.24, 1.11]	[-0.97, 0.68]	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$		
Panel B: R	educed-Form I	Regression										
$\hat{\beta}$	-0.19	-0.02	0.04	-0.06	-0.19	0.00	-0.25	-0.03	0.02	-0.09		
Robust	[-0.27, -0.11]	[-0.05, 0.02]	[0.00, 0.08]	[-0.10, -0.02]	[-0.41, 0.04]	[-0.16, 0.15]	[-0.41, -0.10]	[-0.12, 0.06]	[-0.06, 0.11]	[-0.18, 0.01]		
Cluster	[-0.32, -0.06]	[-0.12, 0.09]	[-0.08, 0.15]	[-0.17, 0.05]	[-0.38, 0.01]	[-0.08, 0.08]	[-0.42, -0.09]	[-0.10, 0.04]	[-0.04, 0.09]	[-0.13, -0.05]		
AKM	[-0.38, 0.01]	[-0.10, 0.07]	[-0.06, 0.13]	[-0.15, 0.02]	[-0.74, 0.37]	[-0.29, 0.28]	[-0.62, 0.11]	[-0.23, 0.17]	[-0.17, 0.22]	[-0.29, 0.11]		
AKM0	[-1.01, 0.03]	[-0.08, 0.51]	[-0.04, 0.60]	[-0.14, 0.42]	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	[-1.28, -0.26]	[-1.18, -0.20]	[-1.13, -0.37]		
Panel C: F	irst-Stage											
Â		0.	26		0.3	35			0.32			
Robust		[0.19]	, 0.32]		[0.17,	0.53]		]	0.20, 0.44]			
Cluster	[0.17, 0.35]				[0.24, 0.46]			[0.21, 0.43]				
AKM	[0.12, 0.39]					[0.10, 0.60]			[0.12, 0.52]			
AKM0			.0.84]		[-∞			L	[−∞.∞]			

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native workers. The specifications CZ-50 Occupations, CZ-2 Education Groups, and CZ-7 Occupations differ in the definition of the unit of observation. In all three specifications, we use 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010. Thus, N=108,300 (722  $CZs \times 50$  occupations  $\times 3$  time periods) for the CZ-50 Occupations specification; N=4,332 (722  $CZs \times 2$  education groups  $\times 3$  time periods) for the CZ-2 Education Groups specification; and N=15,162 (722  $CZs \times 7$  occupations  $\times 3$  time periods) for the CZ-7 Occupations specification. Models are weighted by start-of-period occupation-region (or education group-region) share of national population. All regressions include occupation (or education group) and period dummies. 95% confidence intervals in square brackets. Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We include all countries of origin in the analysis. See Table F.1 for a list of the origin countries included in the analysis.

Table F.12: Effect of immigration: analysis by CZ (including all origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$		
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
	(	Commuting Zon	e (1980 weight		Commuting Z	lone (1960 weigh	its)		
Panel A: 2	SLS Regression	on							
$\hat{eta}$	-0.49	0.13	0.27	-0.2	0.05	-0.25	0.09	-0.52	
Robust	[-1.12, 0.14]	[-0.37, 0.63]	[-0.09, 0.64]	[-0.85, 0.44]	[-0.96, 1.07]	[-0.52, 0.02]	[-0.16, 0.35]	[-0.86, -0.18]	
Cluster	[-0.98, 0.01]	[-0.15, 0.41]	[0.08, 0.47]	[-0.49, 0.08]	[-0.93, 1.03]	[-0.59, 0.09]	[-0.16, 0.34]	[-0.92, -0.11]	
AKM	[-1.74, 0.77]	[-0.88, 1.14]	[-0.53, 1.08]	[-1.42, 1.01]	[-2.39, 2.50]	[-1.14, 0.64]	[-0.76, 0.95]	[-1.64, 0.61]	
AKM0	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	
Panel B: R	Reduced-Form	Regression							
$\hat{eta}$	-0.19	0.05	0.11	-0.08	$0.04 \qquad -0.17 \qquad 0.06 \qquad -0.36$				
Robust	[-0.39, 0.02]	[-0.16, 0.26]	[-0.07, 0.28]	[-0.30, 0.14]	[-0.66, 0.73]	[-0.40, 0.05]	[-0.12, 0.24]	[-0.67, -0.04]	
Cluster	[-0.37, 0.00]	[-0.07, 0.17]	[0.01, 0.20]	[-0.17, 0.01]	[-0.63, 0.71]	[-0.48, 0.14]	[-0.11, 0.24]	[-0.79, 0.08]	
AKM	[-0.71, 0.33]	[-0.36, 0.46]	[-0.24, 0.45]	[-0.52, 0.37]	[-1.64, 1.72]	[-0.82, 0.47]	[-0.51, 0.64]	[-1.20, 0.49]	
AKM0	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	$[-\infty,\infty]$	
Panel C: 2	SLS First-Stag	ge							
$\hat{eta}$		0.	38				0.69		
Robust		[0.20,	, 0.57]		[0.41, 0.97]				
Cluster		0.27	, 0.49		[0.35, 1.03]				
AKM			, 0.67		[0.53, 0.85]				
AKM0		[-x	o,∞] <sup>¹</sup>			[-	$-\infty,\infty$		

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native workers. In the specification *CZ* (1980 weights), we use information on 722 *CZs*, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=2, 166 (722 *CZs* × 3 time periods). In the specification *CZ* (1960 weights), we use information on 722 *CZs*, 1960 weights and one time period, 1970–1980; thus, N=722 (722 *CZs* × 1 time period). Models are weighted by start-of-period *CZ* share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of *CZs* in the same state; *AKM* is the standard error in Remark 5; and *AKM*0 is the confidence interval in Remark 6. We include all countries of origin in the analysis. See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.13: Effect of immigration: analysis by MSA (including all origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$		$\Delta \log E_i$		$\Delta \log w_i$			
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
		MSA (1980	) weights)		MSA (1960 weights)					
Panel A: 2	2SLS Regression									
$\hat{eta}$	-1.41	0.16	0.28	-0.21	-0.18	-0.14	0.18	-0.35		
Robust	[-2.62, -0.21]	[-0.38, 0.71]	[-0.11, 0.68]	[-1.06, 0.63]	[-1.11, 0.75]	[-0.30, 0.02]	[-0.05, 0.42]	[-0.56, -0.14]		
Cluster	[-2.55, -0.28]	[-0.14, 0.46]	[0.06, 0.51]	[-0.62, 0.20]	[-1.21, 0.86]	[-0.31, 0.04]	[-0.08, 0.44]	[-0.52, -0.17]		
AKM	[-3.24, 0.41]	[-0.91, 1.23]	[-0.61, 1.18]	[-1.64, 1.21]	[-5.04, 4.69]	[-0.92, 0.64]	[-0.90, 1.26]	[-1.26, 0.56]		
AKM0	$[-\infty,\infty]$									
Panel B: F	Reduced-Form R	egression								
$\hat{eta}$	-0.41	0.05	0.08	-0.06	-0.12	-0.10	0.13	-0.24		
Robust	[-0.61, -0.20]	[-0.13, 0.23]	[-0.07, 0.24]	[-0.27, 0.15]	[-0.78, 0.54]	[-0.22, 0.03]	[-0.05, 0.31]	[-0.43, -0.05]		
Cluster	[-0.62, -0.19]	[-0.06, 0.15]	[0.00, 0.17]	[-0.16, 0.03]	[-0.83, 0.59]	[-0.23, 0.04]	[-0.08, 0.33]	[-0.42, -0.06]		
AKM	[-0.91, 0.10]	[-0.28, 0.37]	[-0.21, 0.37]	[-0.45, 0.33]	[-3.47, 3.23]	[-0.64, 0.45]	[-0.60, 0.85]	[-0.89, 0.41]		
AKM0	$[-\infty,\infty]$									
Panel C: I	First-Stage									
$\hat{eta}$	· ·	0.2	9			(	).69			
Robust		[0.10,	0.48]			[0.3	5, 1.02]			
Cluster		0.16,			[0.34, 1.03]					
AKM		[0.03,	•		[0.44, 0.93]					
AKM0		[-∞]	_ 1			L .	∞,∞]			

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native worker. In the specification MSA (1980 weights), we use information on 257 MSAs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=651 (257 MSAs  $\times 3$  time periods). In the specification MSA (1960 weights), we use information on 217 MSAs, 1960 weights and one time period, 1970–1980; thus, N=217 (217  $CZs \times 1$  time period). Models are weighted by start-of-period MSA share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; Cluster is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We include all countries of origin in the analysis. See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.14: Immigration: p-values by CZ-Occ. and CZ-Educ. (including all origin countries)

	$\Delta \log E_i$		$\Delta \log w$	i	$\Delta \log E_i$	$\Delta \log w_i$	$\Delta \log E_i$	$\log E_i$ $\Delta \log w_i$			
	All	All	High-Skill	Low-Skill	All	All	All	All	High-Skill	Low-Skill	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
		CZ-5	0 Occupation	ns	CZ- Educ.			CZ-7	CZ-7 Occupations		
Panel A	: 2SLS	Regre	ssion								
Robust	0.000	0.402	0.042	0.011	0.040	0.978	0.001	0.586	0.543	0.139	
Cluster	0.001	0.764	0.445	0.403	0.040	0.958	0.004	0.496	0.363	0.005	
AKM	0.004	0.721	0.380	0.267	0.434	0.988	0.077	0.801	0.798	0.454	
AKM0	0.074	0.734	0.359	0.330	0.532	0.988	0.274	0.808	0.793	0.504	
Panel E	: Reduc	ed-Fo	rm Regress	ion							
Robust	0.000	0.372	0.072	0.002	0.106	0.978	0.001	0.560	0.567	0.074	
Cluster	0.005	0.752	0.515	0.282	0.068	0.957	0.002	0.447	0.422	0.000	
AKM	0.059	0.704	0.442	0.161	0.513	0.988	0.170	0.793	0.805	0.395	
AKM0	0.074	0.734	0.359	0.330	0.532	0.988	0.274	0.808	0.793	0.504	
Panel C	C: First-S	Stage									
Robust			0.000		0.000			0.000			
Cluster			0.000		0.000			0.000			
AKM			0.000		0.005			0.001			
AKM0			0.021		0.3	134			0.067		

Notes:  $\triangle \log E_i$  denotes log change in native employment;  $\triangle \log w_i$  denotes log change in average weekly wages of native workers. The specifications CZ-50 Occupations, CZ-Educ., and CZ-7 Occupations differ in the definition of the unit of observation. In all three specifications, we use 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010. Thus, N=108,300 (722  $CZs \times 50$  occupations  $\times$  3 time periods) for the CZ-50 Occupations specification; N=4,332 (722  $CZs \times 2$  education groups  $\times$  3 time periods) for the CZ-Educ. specification; and N=15,162 (722  $CZs \times 7$  occupations  $\times$  3 time periods) for the CZ-7 Occupations specification. Models are weighted by start-of-period occupation-region (or education group-region) share of national population. All regressions include occupation (or education group) and period dummies. 95% confidence intervals in square brackets. Robust is the Eicker-Huber-White standard error; Cluster is the standard error that clusters of CZs in the same state; AKM is the standard error in Remark 5; and AKM0 is the confidence interval in Remark 6. We all origin countries in the analysis. See Table F.1 for a list of the origin countries included in the analysis.

Table F.15: Effect of immigration: p-values by CZ (including all origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$	į	$\Delta \log E_i$		$\Delta \log w$	i		
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
	Com	muting	Zone (1980 t	veights)	Commuting Zone (1960 weights)					
Panel A: 2	SLS Reg	ressior	1							
Robust	0.130	0.607	0.142	0.536	0.917	0.072	0.478	0.003		
Cluster	0.055	0.353	0.005	0.154	0.914	0.152	0.464	0.013		
AKM	0.449	0.799	0.504	0.741	0.965	0.581	0.830	0.367		
AKM0	0.578	0.797	0.522	0.759	0.965	0.619	0.830	0.463		
Panel B: R	leduced-l	Form R	Regression							
Robust	0.075	0.640	0.252	0.485	0.916	0.136	0.483	0.025		
Cluster	0.047	0.404	0.032	0.091	0.913	0.274	0.473	0.108		
AKM	0.481	0.808	0.553	0.730	0.965	0.600	0.827	0.408		
AKM0	0.578	0.797	0.522	0.759	0.965	0.619	0.830	0.463		
Panel C: 2	SLS First	t-Stage								
Robust			0.000				0.000			
Cluster			0.000				0.000			
AKM			0.008				0.000			
AKM0			0.179				0.156			

Notes:  $\Delta \log E_i$  denotes log change in native employment;  $\Delta \log w_i$  denotes log change in average weekly wages of native workers. In the specification CZ (1980 weights), we use information on 722 CZs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=2, 166 (722 CZs  $\times$  3 time periods). In the specification CZ (1960 weights), we use information on 722 CZs, 1960 weights and one time period, 1970–1980; thus, N=722 (722 CZs  $\times$  1 time period). Models are weighted by start-of-period CZ share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. We include all origin countries in the analysis. See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

Table F.16: Effect of immigration: p-values by MSA (including all origin countries)

Outcome:	$\Delta \log E_i$		$\Delta \log w_i$	į	$\Delta \log E_i$		$\Delta \log w_i$	i			
Workers:	All	All	High-Skill	Low-Skill	All	All	High-Skill	Low-Skill			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)			
	Com	muting	Zone (1980 v	veights)	Com	Commuting Zone (1960 weights)					
Panel A: 2	SLS Reg	ression	1								
Robust	0.021	0.562	0.159	0.621	0.709	0.096	0.131	0.001			
Cluster	0.015	0.294	0.014	0.306	0.737	0.118	0.168	0.000			
AKM	0.130	0.768	0.534	0.769	0.943	0.727	0.739	0.450			
AKM0	0.343	0.771	0.554	0.778	0.944	0.753	0.730	0.562			
Panel B: R	educed-l	Form R	Regression								
Robust	0.000	0.616	0.302	0.571	0.717	0.127	0.172	0.012			
Cluster	0.000	0.375	0.061	0.203	0.737	0.168	0.237	0.010			
AKM	0.115	0.782	0.584	0.757	0.943	0.731	0.735	0.467			
AKM0	0.343	0.771	0.554	0.778	0.944	0.753	0.730	0.562			
Panel C: F	irst-Stag	e									
Robust			0.003				0.000				
Cluster			0.000		0.000						
AKM			0.029		0.000						
AKM0			0.165				0.200				

Notes:  $\Delta \log E_i$  denotes log change in native employment;  $\Delta \log w_i$  denotes log change in average weekly wages of native workers. In the specification CZ (1980 weights), we use information on 722 CZs, 1980 weights and three time periods, 1980–1990, 1990–2000, 2000–2010; thus, N=2, 166 (722 CZs  $\times$  3 time periods). In the specification CZ (1960 weights), we use information on 722 CZs, 1960 weights and one time period, 1970–1980; thus, N=722 (722 CZs  $\times$  1 time period). Models are weighted by start-of-period CZ share of national population. All regressions include period dummies. 95% confidence intervals in square brackets. *Robust* is the Eicker-Huber-White standard error; *Cluster* is the standard error that clusters of CZs in the same state; *AKM* is the standard error in Remark 5; and *AKM0* is the confidence interval in Remark 6. We include all origin countries in the analysis. See Tables F.1 and F.2 for a list of the origin countries included in the analysis.

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