Search Friction and Procyclical Mark-ups in New Keynesian Models with Sticky Prices

Zhesheng Qiu[*], José-Víctor Ríos-Rull[†]

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Abstract

New Keynesian models with sticky prices built on the Dixit-Stiglitz framework must have countercyclical mark-ups conditional on monetary shocks, which is inconsistent with empirical evidence based on labor share data. We pose a directed search style shopping friction in goods market to model firms' price setting, on top of Dixit-Stiglitz. Our theory allows for procyclical mark-ups conditional on monetary shocks, without sacrificing the performance of the model along other dimensions. We prove this in a static model, and test it in an estimated medium scale DSGE model. Unlike the literature that criticizes the use of inverse labor shares to get procyclical mark-ups, we provide a complementary view that New Keynesian models can in fact be compatible with procyclical mark-ups conditional on monetary shocks, when there is goods market shopping friction. These results come from the assumption that firms compete via prices for higher production capacity realizations which is bounded above by 100%.

Keywords: Directed Search, Nominal Rigidities, Mark-up Cyclicality

JEL Codes: E12, E32, E52

[*]Department of Economics and Finance, City University of Hong Kong: mailto:zheshqiu@cityu.edu.hk.
[†]Department of Economics, University of Pennsylvania: vr0j@econ.upenn.edu.

1 Introduction

Markup cyclicality in New Keynesian models is puzzling. In the model, higher aggregate demand[1] induces higher real wages, lower price markups, and higher inflation[2]. In the data, markup cyclicality is likely to move in the opposite direction, according to a vast literature including Nekarda and Ramey (2019); Stroebel and Vavra (2019); Anderson, Rebelo, and Wong (2018); Cantore, Ferroni, and León-Ledesma (2019). The inconsistency between model and data on this mark-up channel brings about a challenge to the micro-foundation of New Keynesian models.

We propose a theory of procyclical mark-ups conditional on monetary shocks via directed search in goods market (shopping friction) to address this puzzle. When nested in an estimated medium scale DSGE model, our shopping friction improves the quantitative performance on mark-ups, with all other moments matched with data at least equally well.

Our theory addresses two issues in the mark-up puzzle. First, how can firms pay more to each unit of labor (real waged) but still get higher profit margins (mark-ups)? In our theory, goods need to be found before sold. Higher aggregate demand induces more shopping effort, and hence higher matching probability that looks like higher productivity. This breaks the link between the cost of inputs and profit margins. Second, why do firms still set higher prices when they have lower cost pushing pressure? In our theory, firms care also about how much of their goods can be sold, which is less sensitive to prices when matching probability gets closer to 100%. This generates variable desired mark-ups naturally from matching frictions.

There are a few alternative ways to address this puzzle in literature. Alternative measures may suggest countercyclical mark-ups (Bils, Klenow, and Malin, 2018). Labor market search may break the link between inversed labor share and mark-ups (Cantore, Ferroni, and León-Ledesma, 2019), Composition effects may induce procyclical mark-ups in aggregate (Anderson, Rebelo, and Wong, 2018). Countercyclical shopping efforts on price comparison may lead to procyclical mark-ups for each firm (Kaplan and Menzio, 2016). However, none of them completely address the two issues.

[1] It can be induced by either monetary or fiscal shocks.

[2] There are three commonly asked questions: (1) what if there is no price rigidity, (2) what if there is real rigidity via Kimball aggregator, (3) what if capacity utilization is variable. If there is only nominal wage rigidity, then price mark-ups is constant, but higher inflation is still driven by the pressure of lower price mark-ups. Kimball aggregator is observationally equivalent to additional price rigidity under first order perturbation. Capacity utilization chosen by firms looks like increasing returns to scale, and will induce countercyclical inflation when mark-ups are procyclical.

2 Puzzling Mark-ups

This section starts with a simple model showing under what assumptions can mark-up be measured by the inverse of labor share. Then, a standard medium scale New Keynesian model satisfying these assumptions and estimated by matching a structural VAR is used to demonstrate why the response of labor share to a monetary shock is puzzling. We conclude by a discussion on the current debate in literature for this puzzle, and briefly explain our contribution to this debate.

2.1 How are Markups Measured?

Mark-up, which is defined as the ratio between price and marginal cost of production subtracted by 1, has no direct measures due to the .

the profit margin over marginal cost, neither of which can be directly measured. In literature, it is usually indirectly measured, when some additional assumptions are imposed. Here introduces the basic one that is widely used.

2.2 What is Wrong with Theory?

2.3 Why Was There No Quick Answer?

2.4 A Simple Model of Mark-ups

Consider a firm that chooses the vector of inputs $\mathbf{x} = \{x_i\}_i^n$ to produce a given output y. The production function is $F(\mathbf{x})$, while the nominal cost function is $G(\mathbf{x})$. The firm minimizes the cost of producing y, and solves

$$max_{\mathbf{x}}\{-G(\mathbf{x})\}, \quad s.t. \quad F(\mathbf{x}) \geq y.$$

Make Assumption 1 to further characterize firms' optimal decision.

Assumption 1. *Functions $F, G : \mathbb{R}^n_{\geq 0} \to \mathbb{R}_{\geq 0}$ are twice continuously differentiable and strictly increasing. F is and weakly concave, while G is weakly convex. G is additively separable across all its inputs, and satisfies $G(\mathbf{x}) = \sum_{i=1}^n G^i(x_i)$.*

Assumption 1 allows us to take first order conditions for optimality. The additive separability of the cost function captures the situation in which inputs are purchased from the factor markets separately without a bundle. Note that firms do not have to be price takers in those markets. Given Assumption 1, we can use Λ to denote the Lagrange multiplier, and obtain a first order condition for $\forall i \in \{1, 2, \cdots, n\}$:

$$0 = -G_{x_i}(\mathbf{x}) + \Lambda F_{x_i}(\mathbf{x}).$$

Here Λ can be interpreted as the marginal cost of production that is equalized across all inputs.

Denote $\varepsilon_i^F \equiv \frac{x_i F_{x_i}}{F}$ as the elasticity of production function F with respect to the i'th input x_i, and $\varepsilon_i^G \equiv \frac{x_i G^{i\prime}}{G^i}$ as the elasticity of input i's separate cost function G^i. The first order conditions of the firm yields Proposition 1.

Proposition 1. *Denote p as the price of goods sold in the goods market, then mark-up τ that is defined as profit margin over marginal cost must satisfy*

$$\tau = \frac{\varepsilon_i^F}{\varepsilon_i^G}(\textit{input } i\textit{'s share})^{-1} - 1 = \frac{\varepsilon_i^F}{\varepsilon_i^G}\frac{F/x_i}{G^i/(px_i)} - 1.$$

Proof. See the appendix. □

Proposition 1 provides two insights for mark-up cyclicality, when ε_i^F and ε_i^G are both constant. First, mark-up can be measured by the cyclicality of inverse input shares in data. Second, mark-up is driven by the ratio between input i's productivity and its real average unit cost, and a model that has procyclical endogenous productivity or countercyclical real average cost of inputs may have the potential to generate procyclical mark-ups.[3] In most New Keynesian models, production function is Cobb-Douglas, and cost function is linear, then we have Example 1.

Example 1. $F(\mathbf{x}) = \textit{capital}^{\alpha}\textit{hours}^{1-\alpha}$ *with* $\alpha \in [0, 1]$, $G^{labor} = \textit{wage} \times \textit{hours}$, *and then*

$$1 + \tau = (1-\alpha)(\textit{labor share})^{-1} = (1-\alpha)(\textit{labor productivity})/(\textit{real wage}).$$

[3]Note that constant ε_i^G still allows for convex cost of inputs, so that average cost does not have to be equal to marginal cost, and the firm still does not have to be a price taker.

2.5 Inconsistency between Model and Data

Example 1 implies that the inverse of labor share can be used as a measure of mark-up under the assumptions that we commonly use. In Figure 1, we compare the performance of DSGE to SVAR following Christiano, Eichenbaum, and Trabandt (2016)[4], highlighting the response of labor share. For further reference, we also provide the responses of real wage, inflation, and federal funds rate.

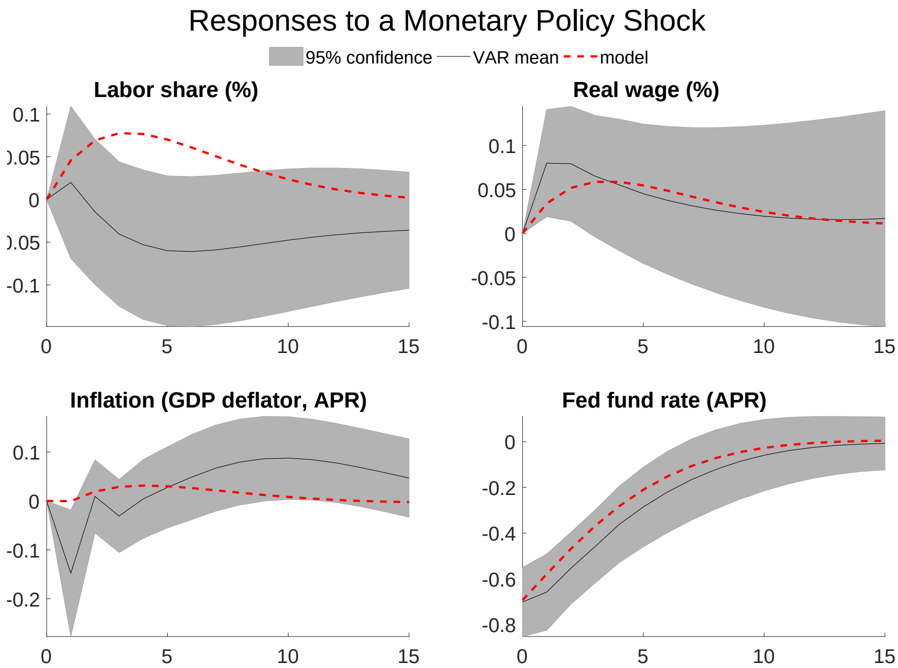

Figure 1: Anomalies in Labor Share Cyclicality

As is in Figure 1, the model implied labor share cyclicality is opposite to data[5]. The right-top panel indicates that the decline in labor share cannot be explained by real wage, because it moves in the opposite direction. The left-bottom panel indicates that the decline in labor share does not induce lower inflation as in standard New Keynesian models. The main challenge here is to fix the labor share response without sacrificing the fits of real wage and inflation.

[4]We take the "Calvo Sticky Wage" version of their model, and make some non-essential changes for tractability. For consistency, we exclude all variables regarding labor search frictions in their SVAR. They need these variables because most versions of their model has them. Both the SVAR and DSGE are estimated in the same way as theirs. More technical details can be found in later sections.

[5]More robust empirical work can be found in Cantore, Ferroni, and León-Ledesma (2019), which also supports our finds.

2.6 Potential Solutions to the Puzzle

Measurement issues A long-standing view in the literature is that the inverse of labor share is not a proper measure of mark-up for cyclicality related issues, because the average cost of labor may be very different from the marginal one. Still, there is no consensus that procyclical mark-ups should be rejected, especially when conditional on monetary shocks. For instance, Rotemberg and Woodford (1999) argues that overhead labor and overtime premium would make the marginal cost of labor more procyclical than the average, while Nekarda and Ramey (2019) still finds procyclical mark-up both unconditionally and conditional on monetary shocks when examining related issues in more details. A more recent example is Kudlyak (2014), which provides evidence from survey data to show that wages for newly hired workers are much more cyclical than those who do not change jobs. Basu and House (2016) confirms their findings, but also shows that a New Keynesian model that features such heterogeneity would require very high level of price rigidity to keep the response of inflation to monetary shocks comparable to data. Bils, Klenow, and Malin (2018) finds procyclical intermediate input shares from industry level data, which indicates that price mark-ups might be countercyclical. On the contrary, Anderson, Rebelo, and Wong (2018) and Stroebel and Vavra (2019) use disaggregate level data from the retail sector in which the cost of goods sold dominates other costs, and find that higher demand does cause higher mark-up, due to the change of households' shopping behavior.

Modeling issues Despite the unsettled debate, we provide a complementary view that procyclical mark-ups conditional on monetary shocks are actually compatible with New Keynesian models at least in the aggregate level. In Example 1, mark-ups are procyclical if labor productivities move up (down) more than real wages during the boomings (recessions). Accordingly, the model must have features such as endogenous productivity or increasing returns to scale. In addition, in order to have higher (lower) inflation in the booms (recessions) when mark-ups are higher (lower), the model must feature some endogenous desired mark-up (mark-up under flexible prices). Directed search in goods market is such a friction that can naturally achieve both of these goals. Compared with other papers that try to rationalize procyclical mark-up such at Anderson, Rebelo, and Wong (2018), our model fully explores the macroeconomic implications.

3 A Static Model

The essence of our theory can be captured by a static model with directed search and price rigidity on top of the Dixit-Stiglitz model. We use the model to demonstrate why standard New Keynesian models with price rigidity have countercyclical mark-ups conditional on monetary shocks, and why directed search can help solve the problem. All results will be analytically proved.

3.1 Price Rigidity under Directed Search

Goods Market There is a measure one of varieties of goods $j \in [0, 1]$, and each one is produced by a monopoly that posts a price and has to deliver the amount of goods demanded at that price. Each one of these firms or varieties has a measure one continuum of locations, each with its own pre-installed labor n_j (and capital later on) and produces y_j. y_j fully depreciates if not sold.

There is a measure one of identical households. They value both the number of varieties and the quantity consumed of each variety. To obtain the varieties, households must search for them, incurring a shopping disutility while doing so. Households that find a variety are randomly allocated to only one of its locations. Each location can be filled with at most one household.

A directed search protocol determines the coordination of firms and households via submarkets indexed by price and tightness $\{p, q\}$. Each household can send shoppers to multiple submarkets while each firm can only go to one of them. Price change by a firm is implemented via switching to a different submarket that has a different price and market tightness. When a firm goes to one submarket, it moves all locations to it.

In submarket $\{p, q\}$ with total shopping effect $D(p, q)$ and measure of firms $J(p, q)$[6], the total number of matches is given by a CRS matching function $\psi(D(p, q), J(p, q))$. The corresponding market tightness q is defined as $q \equiv \frac{D(p,q)}{J(p,q)}$. The number of matches per unit of shopping effort is $\psi^h(q) = \frac{\psi(D(p,q),J(p,q))}{D(p,q)}$ and that per firm is $\psi^f(q) = \frac{\psi(D(p,q),J(p,q))}{J(p,q)}$. $\psi^f(q)$ can also be interpreted as the matching probability of a firm. By definition, these matching functions satisfy $\psi^f(q) = q\psi^h(q)$. The matching function satisfies Assumption 2.

[6] These measure J of firms are producing different varieties. They are pooled into one single submarket because households do not care about which variety to choose, but only how many varieties they find.

Assumption 2. *$\psi^f : \mathbb{R}_{>0} \to (0,1)$ is differentiable, satisfies $\frac{q\psi^{f\prime}(q)}{\psi^f(q)} \in (0,1)$ for $\forall q \in \mathbb{R}_{>0}$, and*

$\lim_{q\to 0} \psi^f(q) = 0$, and $\lim_{q\to+\infty} \psi^f(q) = 1$.

Assumption 2 is quite natural. $\frac{q\psi^{f\prime}(q)}{\psi^f(q)} \in (0,1)$ captures the congestion effect, under which more shopping effort in a submarket increases the total number of matches but reduces the average matches per unit of shopping effort. $\lim_{q\to+\infty} \psi^f(q) = 1$ ensures that the matching probability is bounded above by 1. Note that this limit also rules out the Cobb-Douglas functional form for ψ, and implies that $\lim_{q\to+\infty} \frac{q\psi^{f\prime}(q)}{\psi^f(q)} = 0$. Only with this limit, can we have the standard Dixit-Stiglitz model nested as a special case (more details later on).

Households' problem Given the nominal expenditure e and the set of active submarkets Φ, the representative household chooses the purchase of each variety $c(p,q)$ and its total shopping effort $d(p,q)$ in each submarket $\{p,q\}$. Their decision rules $\{c(e,\Phi,p,q), d(e,\Phi,p,q)\}$ solve

$$\begin{aligned}
V(e,\Phi) = & \max_{\{c(p,q),d(p,q)\}_{\{p,q\}\in\Phi}} u(c^A, d^A), \\
\text{s.t. } e \geq & \int_\Phi d(p,q)\psi^h(q)p\, c(p,q)dpdq, \\
c^A \equiv & \left(\int_\Phi d(p,q)\psi^h(q)c(p,q)^{\frac{\varepsilon-1}{\varepsilon}}dpdq\right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
d^A \equiv & \int_\Phi d(p,q)dpdq,
\end{aligned}$$

where u is a twice differentiable, strictly increasing and strictly concave utility function, and $\varepsilon > 1$.

Consider a situation in which a firm chooses which submarket $\{p,q\}$ to enter, given all other firms in submarket $\{\overline{p},\overline{q}\}$. If a submarket with higher price is potentially active, it must have lower market tightness so that households would be willing to enter, and also lower level of demand due to the substitution between varieties. The corresponding market tightness and demand are denoted as $q^h(e,\{\overline{p},\overline{q}\},p)$ and $c(e,\{\overline{p},\overline{q}\},p,q)$ respectively. Firms will take these functions as given when choosing which submarket to enter. Unlike the directed search models in Moen (1997), we do not explicitly have free entry to pin down the indifference condition for households when deriving these two conditions, but instead rely on the first order conditions of households' problem directly.

Lemma 1. *$c(e, \{\overline{p}, \overline{q}\}, p, q)$ and $q^h(e, \{\overline{p}, \overline{q}\}, p)$ satisfy*

$$\frac{p\ c_p(e, \{\overline{p}, \overline{q}\}, p, q)}{c(e, \{\overline{p}, \overline{q}\}, p, q)} = -\varepsilon, \quad \frac{q\ c_q(e, \{\overline{p}, \overline{q}\}, p, q)}{c(e, \{\overline{p}, \overline{q}\}, p, q)} = 0, \quad \frac{p\ q^h_p(e, \{\overline{p}, \overline{q}\}, p, q)}{c(e, \{\overline{p}, \overline{q}\}, p, q)} = -\frac{\varepsilon - 1}{1 - \mathcal{E}(q)},$$

where $\mathcal{E}(q) \equiv \frac{q\psi^{f\prime}(q)}{\psi^f(q)}$ denotes the elasticity of matching function $\psi^f(\cdot)$.

Proof. See Appendix. □

Lemma 1 characterizes the conditions that the firm needs to take into account when choosing which submarket to go. These elasticities will be sufficient for the firm to consider, and their values do not depend on the functional form of $u(\cdot, \cdot)$. The price elasticity of demand is the same as in Dixit-Stiglitz, while the tightness elasticity of demand is zero. The price elasticity of tightness is a function of $\mathcal{E}(q)$, which has the potential to generate endogenous desired mark-ups. For short, we denote $c^h(e, \{\overline{p}, \overline{q}\}, p) \equiv c(e, \{\overline{p}, \overline{q}\}, p, q)$ for later use.

Lemma 1 helps characterize firms' pricing decisions later on, but is insufficient for households' decisions, unless we impose a specific functional form on the utility function $u(\cdot, \cdot)$. The functional form that we use is GHH, as in Assumption 3.

Assumption 3. *The utility function $u(\cdot, \cdot)$ has a GHH functional form*

$$u(c^A, d^A) = \frac{1}{1-\omega}\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}\right)^{1-\omega}, \text{ with } \nu \geq 0 \text{ and } \omega > 0.$$

Under GHH utility, we can obtain the closed form solutions of shopping effort and consumption demand decisions for the firms choosing to stay in submarket $\{\overline{p}, \overline{q}\}$ when all other firms do so.

Lemma 2. *Under Assumption 3,*

$$d(e, \{\overline{p}, \overline{q}\}, \overline{p}, \overline{q}) = \left[\left(\frac{\zeta^{-1}}{\varepsilon - 1}\frac{e}{\overline{p}}\right)^{\varepsilon-1}\psi^h(\overline{q})\right]^{\frac{1}{(1+\nu)(\varepsilon-1)-1}},$$

$$c(e, \{\overline{p}, \overline{q}\}, \overline{p}, \overline{q}) = \frac{1}{d(e, \{\overline{p}, \overline{q}\}, \overline{p}, \overline{q})\psi^h(\overline{q})}\frac{e}{\overline{p}}.$$

Proof. See Appendix. □

Firms' problem Firm j produces variety j using linear technology $y_j = n_j$ in which labor n_j has unlimited supply at a fixed nominal wage rate W. Now suppose that all firms previously had price p_-. A firm sets price p to produce $y = c^h(e, \{\overline{p}, \overline{q}\}, p)$ in $\psi^f \left[q^h(e, \{\overline{p}, \overline{q}\}, p)\right]$ of its locations, at the expense of price adjustment cost $\chi\left(p/p_-\right) e$. Function $\chi(\cdot)$ is differentiable, strictly increasing and strictly convex in $\mathbb{R}_{>0}$. The firm's decision rule $p(e, W, \{\overline{p}, \overline{q}\}, p_-)$ solves

$$\Omega(e, W, \{\overline{p}, \overline{q}\}, p_-) = \max_p \left(p\psi^f \left[q^h(e, \{\overline{p}, \overline{q}\}, p)\right] - W\right) c^h(e, \{\overline{p}, \overline{q}\}, p) - \chi\left(\frac{p}{p_-}\right) e.$$

Note that the firm choose submarket $\{p, q\}$ to enter only has the degree of freedom to choose p, because for each p, there is only one market tightness $q = q^h(e, \{\overline{p}, \overline{q}\}, p)$ such that the existence of submarket $\{p, q\}$ is justified. The price adjustment cost is normalized by households' nominal expenditure for algebra simplicity. Taking first order condition w.r.t. p and applying Lemma 1 yield Lemma 3, this it further implies Proposition 2.

Lemma 3. *Firms' pricing decision rule $p(e, W, \{\overline{p}, \overline{q}\}, p_-)$ solves*

$$0 = \varepsilon\left(\frac{W}{p} - \frac{\varepsilon - 1}{\varepsilon}\frac{\psi^f(\overline{q})}{1 - \mathcal{E}(\overline{q})}\right) c^h(e, \{\overline{p}, \overline{q}\}, p) - \chi'\left(\frac{p}{p_-}\right)\frac{e}{p_-}.$$

Proof. See Appendix. □

Proposition 2. *The mark-up under flexible prices that is consistent with $p(e, W, \{\overline{p}, \overline{q}\}, p_-)$ is*

$$\frac{\varepsilon}{\varepsilon - 1}[1 - \mathcal{E}(\overline{q})] - 1.$$

Proof. See Appendix. □

Proposition 2 indicates time varying desired mark-up. We expect $\mathcal{E}(\bar{q})$ to be decreasing in $\bar{q}$, as it converges to 0 from above. Then, the desired mark-up will be increasing in $\overline{q}$. The economic implication is that when the market tightness is higher, the goods market is more congested, and it becomes more difficult to increase matching probability further by cutting prices. In another word, price competition along the dimension of directed search is less fierce when the market tightness is higher. This result also indicates that Cobb-Douglas matching function is no long innocuous under directed search when used to study mark-up related issues.

Directed search equilibrium All other firms in the same submarket $\{\overline{p}, \overline{q}\}$ that we have taken as given should be consistent with firms' pricing decisions, and households' shopping decisions. We focus on *symmetric equilibria*, define it in Definition 1, and characterize it in Proposition 3 after combing Definition 1 with Lemma 2 and Lemma 3.

Definition 1. *The directed search equilibrium is defined as a pair of $\{\overline{p}, \overline{q}\}$ satisfying the following consistency conditions:*

$$d(e, \{\overline{p}, \overline{q}\}, \overline{p}, \overline{q}) = \overline{q}, \quad p(e, W, \{\overline{p}, \overline{q}\}, p_{-}) = \overline{p}.$$

Proposition 3. *In the directed search equilibrium, $\{\overline{p}, \overline{q}\}$ must satisfy the following two conditions:*

$$\begin{aligned}
0 &= \frac{\overline{q}^{1+\nu}}{\psi^f(\overline{q})^{\frac{1}{\varepsilon-1}}} - \frac{\zeta^{-1}}{\varepsilon - 1}\frac{e}{\overline{p}}, \\
0 &= \varepsilon\left(\frac{W}{\overline{p}\psi^f(\overline{q})} - \frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\mathcal{E}(\overline{q})}\right) - \chi'\left(\frac{\overline{p}}{p_{-}}\right)\frac{\overline{p}}{p_{-}}.
\end{aligned}$$

Proof. See Appendix. □

These two equations characterize the directed search equilibrium. The first equation captures households' optimal shopping decisions, while the second equation captures firms' optimal pricing decisions. The consistency conditions in Definition 1 have been imposed on both of these equations. We denote the solution for $\{\overline{p}, \overline{q}\}$ as $\{\overline{p}(e, W, p_{-}|\zeta), \overline{q}(e, W, p_{-}|\zeta)\}$. Combined with the definition of mark-up, we have

$$\overline{\tau}(e, W, p_{-}|\zeta) = \frac{\overline{p}(e, W, p_{-}|\zeta)\psi^f[\overline{q}(e, W, p_{-}|\zeta)]}{W} - 1,$$

and then Proposition 3 implies Corollary 1. Corollary 1 is a mirror of Proposition 3.

Corollary 1. $\{\overline{q}(e, W, p_{-}|\zeta), \overline{\tau}(e, W, p_{-}|\zeta)\}$ *solve*

$$\begin{aligned}
0 &= \frac{\overline{q}^{1+\nu}}{\psi^f(\overline{q})^{\frac{\varepsilon}{\varepsilon-1}}} - \frac{\zeta^{-1}}{\varepsilon - 1}\frac{e}{(1+\overline{\tau})W}, \\
0 &= \varepsilon\left(\frac{1}{1+\overline{\tau}} - \frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\mathcal{E}(\overline{q})}\right) - \chi'\left(\frac{(1+\overline{\tau})W}{p_{-}\psi^f(\overline{q})}\right)\frac{(1+\overline{\tau})W}{p_{-}\psi^f(\overline{q})}.
\end{aligned}$$

3.2 What Makes the Mark-up Procyclical?

No search The case without search as in standard New Keynesian models is nested in our model. When the utility cost of shopping goes to 0, households will make infinite shopping effort, allowing firms to sell all of their products for sure. Combining Assumption 2 with Proposition 3 and Corollary 1 yields Corollary 2.

Corollary 2. *$\overline{q}(e, W, p_-|0) = +\infty$ and $\{\overline{p}(e, W, p_-|0), \overline{\tau}(e, W, p_-|0)\}$ must satisfy*

$$0 = \varepsilon \left(\frac{W}{\overline{p}} - \frac{\varepsilon - 1}{\varepsilon} \right) - \chi' \left(\frac{\overline{p}}{p_-} \right) \frac{\overline{p}}{p_-},$$
$$0 = \varepsilon \left(\frac{1}{1+\overline{\tau}} - \frac{\varepsilon - 1}{\varepsilon} \right) - \chi' \left(\frac{(1+\overline{\tau})W}{p_-} \right) \frac{(1+\overline{\tau})W}{p_-}.$$

Proof. See Appendix. □

Assumption 4. *$\chi'(p)p$ is strictly increasing in p, and satisfies $\chi'(1) = \chi(1) = 0$.*

As monetary expansions lead to higher nominal expenditure and nominal wage, Corollary 2 and Assumption 4 allows us to unambiguously show that mark-up is lower during monetary expansions because nominal expenditure does not affect either price or mark-up, while nominal wage partially transmits to price, leading to countercyclical mark-ups. As a result, we have Proposition 4.

Proposition 4. *In the equilibrium with no search, $\{\overline{p}, \overline{\tau}\}$ must satisfy the following conditions:*

$$\overline{p}_e(e, W, p_-|0) = 0, \quad \overline{p}_W(e, W, p_-|0) > 0;$$
$$\overline{\tau}_e(e, W, p_-|0) = 0, \quad \overline{\tau}_W(e, W, p_-|0) < 0.$$

Proof. See Appendix. □

Undirected search Undirected search is insufficient to solve the problem. Suppose firms have to commit to a price ex ante, then households' problem reduces to the case in which $\Phi = \{\overline{p}, \overline{q}\}$, and firms no long take into account $q^h(e, \{\overline{p}, \overline{q}\}, p)$ when setting prices. This is equivalent to impose $\mathcal{E}(q) = 0$ in Corollary 1. As a result, $\overline{\tau}$ and $\overline{p}$ must move in opposite directions, which implies that we must have either countercyclical mark-up or countercyclical inflation under undirected search.

Directed search Comparing Proposition 3 and Corollary 1 with Corollary 2, we find that directed search introduces two additional terms $\psi^f(\overline{q})$ and $\mathcal{E}(\overline{q})$. $\psi^f(\overline{q})$ captures the effect of endogenous productivity, while $\mathcal{E}(\overline{q})$ captures that of endogenous desired mark-up. $\psi^f(\overline{q})$ breaks the positive connection between real wage and labor share, and $\mathcal{E}(\overline{q})$ breaks the positive connection between labor share and inflation. For further characterization, we need the following assumptions.

Assumption 5. $\frac{q^{1+\nu}}{\psi^f(q)^{\frac{\varepsilon}{\varepsilon-1}}}$ *is strictly increasing in* q.

Assumption 6. $\mathcal{E}(q)$ *is weakly decreasing in* q.

Assumption 7. *The matching function* $\psi^f(\cdot)$ *has functional form*

$$\psi^f(q) = \left(1 + q^{-\gamma}\right)^{-\frac{1}{\gamma}}, \text{ with } \gamma \geq 1.$$

Proposition 5. *Under Assumption 2, 3, 4, 5, 6, Proposition 3 and Corollary 1 yield*

$$\overline{q}_e(e, W, p_-|\zeta) > 0, \quad \overline{q}_W(e, W, p_-|\zeta) < 0;$$
$$\overline{\tau}_e(e, W, p_-|\zeta) > 0, \quad \overline{\tau}_W(e, W, p_-|\zeta) < 0;$$
$$\overline{p}_e(e, W, p_-|\zeta) \text{ ?? } 0, \quad \overline{p}_W(e, W, p_-|\zeta) > 0.$$

Assumption 7 is a sufficient condition for Assumption 2, 6. Under Assumption 3, 4, 5, 7, and when p_- *is the directed search equilibrium price for* $(e, W|\zeta)$ *under flexible prices,*

$$\overline{p}_e(e, W, p_-|\zeta) \geq 0, \quad \frac{\partial}{\partial e}\left[\frac{W}{\overline{p}(e, W, p_-|\zeta)}\right] \leq 0, \quad \frac{\partial}{\partial W}\left[\frac{W}{\overline{p}(e, W, p_-|\zeta)}\right] \geq 0.$$

Proof. See Appendix. □

Proposition 5 helps us understand the cyclicality of mark-up, real wage, and inflation conditional on monetary shocks, in partial equilibrium. In general equilibrium, monetary expansion will increase both nominal expenditure e and nominal wage rate W. The impact from W can induce procyclical real wage rates, yet it can not be too strong compared with the impact of e, such that mark-ups can also be procyclical. Whether it is plausible is left for the quantitative work in an estimated New Keynesian model following Christiano, Eichenbaum, and Trabandt (2016) in the next section.

4 Medium Scale DSGE

Our quantitative work is based on the "Calvo Sticky Wage" model in Christiano, Eichenbaum, and Trabandt (2016) with a few variations. Like them, we also model consumption habit, investment adjustment cost, capital utilization, Calvo wage, and Taylor Rule. Unlike them, our baseline model uses Rotemberg pricing instead of Calvo pricing, and removes the government sector, such that the directed search friction can be introduced in a more tractable way. We embed the directed search style shopping friction as is shown in the previous section into the baseline model. It preserves the analytic tractability of Euler equations and the Rotemberg-style New Keynesian Phillips Curve.

4.1 The Model Economy

Time is discrete: $t = 0, 1, 2, 3, \cdots$. The economy is populated by a measure one of representative labor contractors, infinitely lived households indexed by $i \in [0, 1]$, and infinitely lived firms indexed by $j \in [0, 1]$. Each firm j is a monopoly in variety j. The shopping friction in goods market between households and firms is the same as in the static model.

Contractors The representative contractor produces the homogeneous labor input by combining differentiated labor inputs, $\ell_{i,t}$, $i \in [0, 1]$, using the technology

$$L_t = \left(\int_0^1 \ell_{i,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \, di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \quad \varepsilon_w > 1.$$

Labor contractors are perfectly competitive and take as given the nominal wage rate W_t of L_t and wage $W_{i,t}$ of the ith labor type as given. Profit maximization on the part of contractors implies

$$\ell_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\varepsilon_w} L_t. \tag{1}$$

Households Each household $i \in [0, 1]$ is the monopoly supplier of $\ell_{i,t}$ and chooses $W_{i,t}$ subject to (1) and Calvo wage frictions. That is, the household optimizes the wage $W_{i,t}$, with probability $1 - \theta_w$. With probability θ_w, the wage rate is given by

$$W_{i,t} = W_{i,t-1}.$$

With discount factor $\beta \in [0, 1)$, each household $i \in [0, 1]$ has preference on consumption aggregator $c_{i,t}^A$ and shopping effort aggregator $d_{i,t}^A$ defined as in the static model, as well as labor supply $\ell_{i,t}$:

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \left\{ u(c_{i,t}^A, d_{i,t}^A, c_{i,t-1}^A, d_{i,t-1}^A) - v(\ell_{i,t}) \right\},$$

with the following functional forms:

$$u(c_{i,t}^A, d_{i,t}^A, c_{i,t-1}^A, d_{i,t-1}^A) = \frac{1}{1-\omega} \left[\left(c_{i,t}^A - z_t^y \zeta \frac{\left(d_{i,t}^A\right)^{1+\nu}}{1+\nu} \right) - h \left(c_{i,t-1}^A - z_{t-1}^y \zeta \frac{\left(d_{i,t-1}^A\right)^{1+\nu}}{1+\nu} \right) \right]^{1-\omega},$$
$$v(\ell_{i,t}) = (z_t^y)^{1-\omega} \eta \frac{\ell_{i,t}^{1+\xi}}{1+\xi}.$$

Parameter h captures the degree of habit persistence, parameter ξ the inverse of Frisch elasticity of labor supply, parameter η controls the size of working disutility, and variable z_t^y is a composite technology level that measures the trend of output level. Both the shopping disutility and working disutility need to be normalized by this z_t^y such that there exists a balanced growth path.

At time t, household i chooses the shopping effort aggregator $d_{i,t}^A$, with total shopping effort $d_{i,t}(p, q)$, and the purchase of each variety $y_{i,t}(p, q)$ in each active submarket $\{p, q\} \in \Phi_t$ to make goods aggregator $y_{i,t}^A$, using the technology

$$y_{i,t}^A = \left(\int_{\Phi_t} d_{i,t}(p, q) \psi^h(q) y_{i,t}(p, q)^{\frac{\varepsilon-1}{\varepsilon}} dpdq \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1,$$
$$d_{i,t}^A = \int_{\Phi_t} d_{i,t}(p, q) dpdq,$$

and nominal expenditure

$$e_{i,t} = \int_{\Phi_t} d_{i,t}(p, q) \psi^h(q) p \, y_{i,t}(p, q) dpdq.$$

$y_{i,t}^A$ is used to make either consumption aggregator $c_{i,t}^A$ or investment related goods via

$$i_{i,t}^A + a(u_{i,t}) k_{i,t-1} = z_t^i (y_{i,t}^A - c_{i,t}^A).$$

Here $k_{i,t}$ denotes the capital holding, $u_{i,t}$ denotes the capability utilization of capital, $a(\cdot)$ denotes an differentiable, increasing and convex function of maintenance cost incurred by $u_{i,t}$, $i^A_{i,t}$ denotes the investment aggregator for new capital, and z^i_t denotes level of investment specific technology. The inverse of z^i_t can be interpreted as the relative price of investment goods over consumption goods. Capital $k_{i,t}$ is accumulated according to the following law of motion

$$k_{i,t} = (1-\delta_k)k_{i,t-1} + \left[1 - S\left(\frac{i^A_{i,t}}{i^A_{i,t-1}}\right)\right] i^A_{i,t},$$

where $S(\cdot)$ is an differentiable, increasing and convex adjustment cost function. Note that $S(\cdot)$ in our model prevents households from slightly increasing shopping effort to obtain a wedge between the price of variety goods and the rent value of capital made from these variety goods.

Household i's budget constraint is

$$e_{i,t} \leq W_{i,t}\ell_{i,t} + A_{i,t} + R^k_t u^k_{i,t} k_{i,t-1} + R_{t-1}b_{i,t-1} - b_{i,t} + T_t.$$

Here $A_{i,t}$ denotes a state contingent insurance that makes sure that household i's wage income is $W_{i,t}\ell_{i,t} + A_{i,t} = W_t L^s_t$ when optimal decisions are made, R^k_t denotes the rental rate of capital, R_t denote the nominal interest rate from period t to $t+1$, $b_{i,t}$ denotes the holding of risk-free bond carried from period t to $t+1$, and T_t denotes the net lump sum transfer of profit from firms.

Due to the insurance on wage incomes, households' optimal decisions are homogeneous except for labor supply $\{\ell_{i,t}\}$. Hence, we can still derive the counterpart of Lemma 1 for variety demand $y^h_t(\{\overline{p}_t, \overline{q}_t\}, p)$ and market tightness $q^h_t(\{\overline{p}_t, \overline{q}_t\}, p)$ in the potentially active submarket with price p, when all other firms enter a submarket $\{\overline{p}_t, \overline{q}_t\}$. Since $e_{i,t}$ is endogenous, we are no longer able to fully characterize $\{y^h_t(\{\overline{p}_t, \overline{q}_t\}, p), q^h_t(\{\overline{p}_t, \overline{q}_t\}, p)\}$ in analytical forms. Yet, it does not prevents us from characterizing firms' optimal decisions because only the price elasticities of these functions, which still have analytical expressions, are needed. We use $\{y^h_t(p), q^h_t(p)\}$ for short notations.

Lemma 4. *The two functions $\{y^h_t(p), q^h_t(p)\}$ satisfy*

$$\frac{p\, y^{h\prime}_t(p)}{y^h_t(p)} = -\varepsilon, \quad \frac{p\, q^{h\prime}_t(p)}{q^h_t(p)} = -\frac{\varepsilon - 1}{1 - \mathcal{E}(q)}.$$

Proof. See Appendix. □

Firms Each firm j produces goods variety j, with production function

$$y_t = k_t^{\alpha}(z_t^n \ell_t)^{1-\alpha}.$$

Since firms are symmetric, the subscript j is omitted. k_t denotes the capital service rented by the firm, ℓ denotes the homogeneous labor input hired by the firm, and z_t^n denotes the level of neutral technology. Firms take rental rate R_t^k and nominal wage rate W_t as given. Wage has to be paid before production in each period t, so that firms need to borrow from households at gross nominal interest rate R_{t-1} for the cost of labor[7]. Under the optimal allocation of capital and labor inputs, the marginal nominal cost of producing one unit of goods variety becomes

$$MC_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{R_{t-1}W_t/z_t^n}{1-\alpha}\right)^{1-\alpha}.$$

The corresponding demands for capital and labor inputs are

$$k_t = \frac{\alpha MC_t}{R_t^k}y_t, \quad \ell_t = \frac{(1-\alpha)MC_t}{R_{t-1}W_t/z_t^n}y_t.$$

The firm is selling goods variety it produces in two types of markets. The first market is subject to the directed search style shopping friction, in which goods are sold only to households as in the static model. The second one is centralized, in which goods are sold only to firms to alleviate price adjustment costs. The prices for both markets are identical and equal to p_t. The demand function in the first market is denoted by $y_t^h(p_t)$, and the demand function in the second market is denoted by $x_t^h(p_t)$, which has elasticity $\varepsilon > 1$. Use $\chi_t(\cdot)$ to denote the function of price adjustment costs at time t, and λ_t^e for the marginal value of one dollar for households. When all firms are owned evenly by the households, the present value of a firm to be maximized becomes

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \lambda_t^e \left\{ \left[p_t \psi^f(q_t^h(p_t)) - MC_t\right] y_t^h(p_t) + (p_t - MC_t)x_t^h(p_t) - \chi_t\left(\frac{p_t}{p_{t-1}}\right)\right\}.$$

In equilibrium, this yields a New Keynesian Phillips Curve in which market tightness plays a role.

[7]Note that our timing of the loan is different from Christiano, Eichenbaum, and Trabandt (2016), in which firms take within period loan at each period t, with gross nominal interest rate R_t. We change the timing to make it logically consistent with the timing of monetary shocks that requires R_t to be unknown when firms hire labor input.

Consistency and market clearing The consistency conditions for directed search are

$$\int_0^1 d_{i,t}(\overline{p}_t, \overline{q}_t)di = \overline{q}_t \equiv Q_t, \quad p_t = \overline{p}_t \equiv P_t.$$

Market clearing in rental and labor markets requires that

$$k_t = \int_0^1 u_{i,t}^k k_{i,t-1} di, \quad \ell_t = L_t.$$

Market clearing in variety goods markets requires that

$$\int_0^1 y_{i,t}(\overline{p}_t, \overline{q}_t)di = y_t^h(\overline{p}_t) \equiv Y_t^h, \quad \chi_t\left(\frac{\overline{p}_t}{\overline{p}_{t-1}}\right) = \overline{p}_t x_t^h(\overline{p}_t) \equiv P_t X_t^h, \quad Y_t^h + X_t^h = y_t \equiv \mathcal{Y}_t.$$

Market clearing in loanable fund market requires that

$$W_t \ell_t = \int_0^1 b_{i,t-1} di.$$

National accounts The insurance on wage income allows us to omit subscript i for the variables

$$(y_{i,t}^A, c_{i,t}^A, i_{i,t}^A, u_{i,t}^k, k_{i,t-1}, b_{i,t-1}) = (Y_t^A, C_t^A, I_t^A, u_t^k, K_{t-1}, B_{t-1}).$$

Real GDP, real aggregate consumption, and real aggregate investment are defined as

$$Y_t \equiv \psi^f(Q_t)Y_t^h, \quad C_t \equiv \frac{C_t^A}{Y_t^A} Y_t, \quad I_t \equiv \frac{I_t^A + a(u_t^k)K_{t-1}}{Y_t^A} Y_t.$$

Gross inflation rate and real wage rate are defined as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}}, \quad w_t \equiv \frac{W_t}{P_t}.$$

Labor productivity and labor share are defined as

$$\ell p_t \equiv \frac{Y_t}{L_t}, \quad \ell s_t \equiv \frac{w_t L_t}{Y_t}.$$

Aggregate shocks We adopt the following specification of monetary policy:

$$\ln\left(\frac{R_t}{R_{SS}}\right) = \rho_R \ln\left(\frac{R_{t-1}}{R_{SS}}\right) + (1-\rho_R)\left[\phi_\pi \ln\left(\frac{\Pi_t}{\Pi_{SS}}\right) + \phi_y \ln\left(\frac{Y_t}{Y_{t,SS}}\right)\right] + \sigma_R \epsilon_{R,t+1}$$

Here, Π_{SS} denotes the steady state gross inflation rate targeted by the monetary authority, R_{SS} denotes the corresponding steady state federal funds rate, $Y_{t,SS}$ denotes the value of Y_t along the non-stochastic steady state growth path, $\epsilon_{R,t+1}$ denotes the monetary (federal funds rate) shock that is consistent with short-run restriction, and σ_R denotes the standard deviation of it.

We assume that $\ln \mu_t^n \equiv \ln(z_t^n / z_{t-1}^n)$ is i.i.d. We also assume that $\ln \mu_t^i \equiv \ln(z_t^i / z_{t-1}^i)$ follows a first order autoregressive process. The standard deviations of the innovations in both processes are denoted by σ_n and σ_i, respectively. The autocorrelation of $\ln \mu_t^i$ is denoted by ρ_i. The sources of growth in our model are neutral and investment-specific technological progress. Let

$$z_t^y \equiv (z_t^i)^{\frac{\alpha}{1-\alpha}} z_t^n, \quad z_t^k \equiv z_t^i z_t^y, \quad \mu_t^y \equiv z_t^y / z_{t-1}^y, \quad \mu_t^k \equiv z_t^k / z_{t-1}^k.$$

The variables $\{Y_t/z_t^y, C_t/z_t^y, w_t/z_t^y, I_t/z_t^k, K_t/z_t^k\}$ are constants in the non-stochastic steady state. The means of $\{\mu_t^n, \mu_t^i, \mu_t^y, \mu_t^k\}$ in the non-stochastic steady state are denoted by $\{\mu_{SS}^n, \mu_{SS}^i, \mu_{SS}^y, \mu_{SS}^k\}$.

Functional forms We assume that the cost of adjusting investment takes the form:

$$S\left(\frac{I_t^A}{I_{t-1}^A}\right) \equiv \frac{1}{2}\left\{\exp\left[\sqrt{S''}\left(\frac{I_t^A}{I_{t-1}^A} - \mu_{SS}^k\right)\right] + \exp\left[-\sqrt{S''}\left(\frac{I_t^A}{I_{t-1}^A} - \mu_{SS}^k\right)\right]\right\} - 1.$$

The object, S'', is a parameter to be estimated. It is easy to verify that $S(\mu_{SS}^k) = S'(\mu_{SS}^k) = 0$. The cost associated with setting capacity utilization is given by

$$a(u_t^k) = \frac{\sigma_a \sigma_b}{2}(u_t^k)^2 + \sigma_b(1-\sigma_a)u_t^k + \sigma_b\left(\frac{\sigma_a}{2} - 1\right),$$

where σ_a and σ_b are positive scalars. For a given value of σ_a, we select σ_b so that the steady state value of u_t^k is unity. The object, σ_a, is a parameter to be estimated. The matching function $\psi(Q)$ has the following functional form suggested by den Haan, Ramey, and Watson (2000)

$$\psi(D, J) = (D^{-\gamma} + J^{-\gamma})^{-\frac{1}{\gamma}}.$$

The price adjustment cost function $\chi_t(\cdot)$ satisfies

$$\chi_t\left(\frac{p_t}{p_{t-1}}\right) = \frac{\kappa}{2} P_t \mathcal{Y}_t \left(\frac{p_t}{p_{t-1}} - \Pi_{SS}\right)^2.$$

Here, the adjustment cost has been normalized by the equilibrium nominal value of goods varieties produced by all firms $P_t\mathcal{Y}_t$. We use notation $\widetilde{\chi}_t = \frac{\kappa}{2}(\Pi_t - \Pi_{SS})^2$ for short.

New Keynesian Phillips Curve The main mechanism of directed search can be captured by a New Keynesian Phillips Curve in which market tightness plays a role as in Proposition 6.

Proposition 6. *In the equilibrium,*

$$(\Pi_t - \Pi_{SS})\Pi_t = \frac{\varepsilon - 1}{\kappa}\left\{\frac{\varepsilon}{\varepsilon - 1} mc_t - \left[\widetilde{\chi}_t + (1 - \widetilde{\chi}_t)\mathcal{I}_t^{1-\gamma}\right]\right\} + \beta\mathbb{E}_t\left[\mathcal{M}_{t+1}(\Pi_{t+1} - \Pi_{SS})\Pi_{t+1}\right],$$

in which mc_t denotes the real marginal cost, $\mathcal{I}_t$ denotes the measure of matched locations for each firm, and $\mathcal{M}_t$ denotes the stochastic discount factor. These three objects are defined as

$$mc_t \equiv \frac{MC_t}{P_t}, \quad \mathcal{I}_t \equiv \psi^f(Q_t), \quad \mathcal{M}_t \equiv \frac{\lambda_t^e}{\lambda_{t-1}^e}\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}}.$$

Proof. See Appendix. □

An interesting feature is that when $\gamma = 1$, such that $\psi^f(Q) = \frac{Q}{1+Q}$, the New Keynesian Phillips Curve under directed search is observationally equivalent to the standard one with no search, up to first order approximation around the steady state. As a result, the model can have endogenous productivity $\mathcal{I}_t$ as a wedge between labor share and real wage rate, without affecting inflation. The mechanism is more clear before the functional form of match function is imposed.

$$\mathcal{I}_t^{1-\gamma} = \frac{\psi^f(Q_t)}{1 - \mathcal{E}(Q_t)}.$$

The numerator is the effect of endogenous productivity, while the denominator is that of endogenous desired mark-up. When $\gamma = 1$, these two effects exactly cancel out, and the New Keynesian Phillips Curve converges to the standard model with no search.

4.2 Mapping Model to Data

Our model is mapped to data via Baynesian Impulse Reponse Matching estimator as in Christiano, Eichenbaum, and Trabandt (2016). Our procedures to obtain the structural VAR, non-estimated parameters, calibrated parameters, estimated parameters, and mpulse response matchingn results are all compared to theirs. We use "**CET**" for short to denote their Calvo wage model, "no search" for our counterpart, and "directed search" for our shopping model, in tables and figures.

Structural VAR The structural VAR we estimate covers the same period $1951Q1-2008Q4$ as Christiano, Eichenbaum, and Trabandt (2016), with the same data source. For model consistency, we choose a subset of 9 from their 14 variables. The variables we choose are Δ ln(relative price of investment), Δ ln(real GDP/hours), Δ ln(GDP deflator), ln(capacity utilization), ln(hours), ln(real GDP/hours)$-$ ln(real wage), ln(nominal C/nominal GDP), ln(nominal I/nominal GDP), Federal Funds rate. The variables not chosen include unemployment rate, ln(vacancies), job separation rate, job finding rate, ln(hours/labor force). We choose two lags, and linearly detrend all variables before the regression.

Like Christiano, Eichenbaum, and Trabandt (2016), monetary shocks are identified via short-run restrictions, while neutral and investment specific technology shocks are identified via long-run restrictions. The impulse responses of these 9 stationary variables are transformed into those of 9 variables in levels. We get the impulse responses of labor productivity and labor share in addition. These two variables are not targeted by model, but used for validating model performance.

Non-estimated parameters We choose a set of non-estimated parameters outside the model in Table 1, following Christiano, Eichenbaum, and Trabandt (2016) exactly. There are two additional parameters in our models with shopping friction ν and γ. $1+\nu$ captures the curvature of shopping disutility, and γ captures the curvature of matching function. We are agnostic about the values of these two parameters, and hence choose $1+\nu=\gamma=1$ for simplicity.

Calibrated parameters The second set of parameters are chosen to target on their corresponding steady state levels as in Table 2. Note that we do not estimate ε because it is not identifiable with κ under Rotemberg pricing. Under Calvo pricing, the identification of ε is at best weak. We calibrate it to target on 5% steady state profit margin ($\tau_{SS}=0.05$). The profit margin in the real

Table 1: Non-Estimated Parameters

Parameter	Value	Description
		Common in All Models
δ_k	0.025	Depreciation rate of physical capital
θ_w	0.75	Quarterly frequency of not adjusting nominal wage
ε_w	$\frac{1.2}{1.2-1}$	Labor demand elasticity by contractors
ω	1.0	Inverse of intertemporal elasticity of substitution
$400 \ln \mu^y_{SS}$	1.7	Annual output per capita growth rate
$400 \ln \mu^k_{SS}$	2.9	Annual investment per capita growth rate
$400(\Pi_{SS} - 1)$	2.5	Annual net inflation rate
$400(R_{SS}/\Pi_{SS} - 1)$	3.0	Annual net real interest rate
		Only in the Model with Directed Search
$1 + \nu$	1.0	Curvature of shopping disutility
γ	1.0	Curvature of matching function

world may be higher, but our model with directed search has difficulty in generating much higher profit margins due to the extra competition incurred by directed search. The impact of $\mathcal{E}(Q)$ can range from 0 to 1. We are agnostic about the size of this effect, and let the steady state matching probability be 0.70 ($\mathcal{I}_{SS} = 0.70$). Correspondingly, $\mathcal{E}(Q_{SS}) = 1 - \mathcal{I}^{\gamma}_{SS} = 0.30$.

Table 2: Calibrated Parameters

Parameter	**CET**	no search	directed search	Target
β	0.9968 (error)	0.9925	0.9925	$400(R_{SS}/\Pi_{SS} - 1) = 3.0$
σ_b	0.036	0.040	0.040	$u^k_{SS} = 1$
η	cannot find	0.842	2.118	$L_{SS} = 0.945$
ε	$\frac{1.24}{1.24-1} = 5.17$	-	-	estimated
ε	-	21	3	$\tau_{SS} = 0.05$, $\mathcal{I}_{SS} = 0.70$
ζ	-	0.0000	0.1942	$\tau_{SS} = 0.05$, $\mathcal{I}_{SS} = 0.70$

Estimated parameters Table 3 summarizes the results of Bayesian estimation. All parameters but κ are taking the same prior distribution as in Christiano, Eichenbaum, and Trabandt (2016). For κ, we adjust the prior accordingly such that its relatively size to the slop of Phillips Curve $\frac{\varepsilon-1}{\kappa}$ is unchanged. The estimated parameters in our "no search" model are generally aligned with those in **CET**. The "directed search" model fits data much better than the "no search" model.

Table 3: Estimated Parameters

		CET	No Search	Directed Search
	Prior Dist. $\mathcal{D}$,**Mode**,[2.5-97.5%]		Posterior Dist. **Mode**,[2.5-97.5%]	
	Preference and Technology Parameters			
Capital Share, α	$\mathcal{B}$,**0.33**,[0.28-0.38]	**0.33**,[0.27-0.34]	**0.24**,[0.21-0.28]	**0.26**,[0.24-0.29]
Inverse Labor Supply Elasticity, ξ	$\mathcal{G}$,**0.94**,[0.57-1.55]	**0.92**,[0.33-1.01]	**0.38**,[0.26-0.53]	**0.48**,[0.35-0.58]
Consumption Habit, h	$\mathcal{B}$,**0.50**,[0.21-0.79]	**0.68**,[0.65-0.74]	**0.76**,[0.72-0.79]	**0.78**,[0.71-0.83]
Capacity Utilization Ajd. Cost, σ_a	$\mathcal{G}$,**0.32**,[0.09-1.23]	**0.03**,[0.01-0.16]	**1.43**,[0.92-2.19]	**1.58**,[1.16-2.23]
Investment Adjustment Cost, S''	$\mathcal{G}$,**7.50**,[4.57-12.4]	**5.03**,[4.15-7.95]	**13.6**,[9.71-18.2]	**13.2**,[9.58-17.7]
	Price Stickiness Parameters			
Rotemberg Adjustment Cost, κ	$\mathcal{G}$,**139**,[5.06-778]		**371**,[216-585]	
	$\mathcal{G}$,**13.9**,[0.51-77.8]			**24.1**,[11.0-53.7]
Calvo Price Stickiness, θ	$\mathcal{G}$,**0.68**,[0.45-0.84]	**0.74**,[0.67-0.77]		
Implied $\frac{(1-\theta)(1-\beta\theta)}{\theta}$ or $\frac{\varepsilon-1}{\kappa}$		**0.09**,[0.07-0.16]	**0.05**,[0.03-0.08]	**0.08**,[0.04-0.18]
	Monetary Authority Parameters			
Taylor Rule: Inflation, ϕ_π	$\mathcal{G}$,**1.69** [1.42-2.00]	**2.02** [1.82-2.39]	**2.00** [1.75-2.26]	**1.95** [1.68-2.23]
Taylor Rule: GDP, ϕ_y	$\mathcal{G}$,**0.08** [0.03-0.22]	**0.01** [0.00-0.02]	**0.20** [0.15-0.28]	**0.15** [0.11-0.21]
Taylor Rule: Smoothing, ρ_R	$\mathcal{B}$,**0.76** [0.37-0.94]	**0.77** [0.75-0.81]	**0.85** [0.83-0.88]	**0.85** [0.82-0.87]
	Exogenous Processes Parameters			
Std. Dev. Monetary Policy, $400\sigma_R$	$\mathcal{G}$,**0.65** [0.56-0.75]	**0.64** [0.57-0.71]	**0.67** [0.60-0.74]	**0.70** [0.63-0.77]
Std. Dev. Neutral Tech., $100\sigma_n$	$\mathcal{G}$,**0.08** [0.03-0.22]	**0.32** [0.28-0.35]	**0.33** [0.26-0.41]	**0.35** [0.31-0.39]
Std. Dev. Invest. Tech., $100\sigma_i$	$\mathcal{G}$,**0.08** [0.03-0.22]	**0.15** [0.12-0.19]	**0.33** [0.26-0.41]	**0.32** [0.25-0.40]
AR(1) Invest. Technology, ρ_i	$\mathcal{B}$,**0.75** [0.53-0.92]	**0.57** [0.44-0.66]	**0.51** [0.39-0.62]	**0.47** [0.36-0.59]
	Overall Goodness of Fit			
Log Marginal Likelihood (9 Observables):		-	107.2	158.7

Impulse Response Matching results The quantitative results are summarized in Figure 2, 3, 4, and 5. Figure 2 demonstrates that our "directed search" model fixes the labor share cyclicality, without sacrificing the fit of real wage and inflation responses. Moreover, Figure 3, 4, and 5 also demonstrate the 9 impulse responses conditional on 3 structural shocks that we target on in our Bayesian estimation. The last two impulse responses of labor productivity and labor share are not directly targeted, but linear combination of other responses. Even though our "no search" can be qualitatively aligned with the SVAR evidence in Figure 3 for the 9 targeted impulse responses, it is not for the 2 untargeted ones. This can be fixed by the "directed search" model. Figure 4 and 5 indicate that the "directed search" model performs at least as well as the "no search" model under neutral technology and investment specific technology shocks.

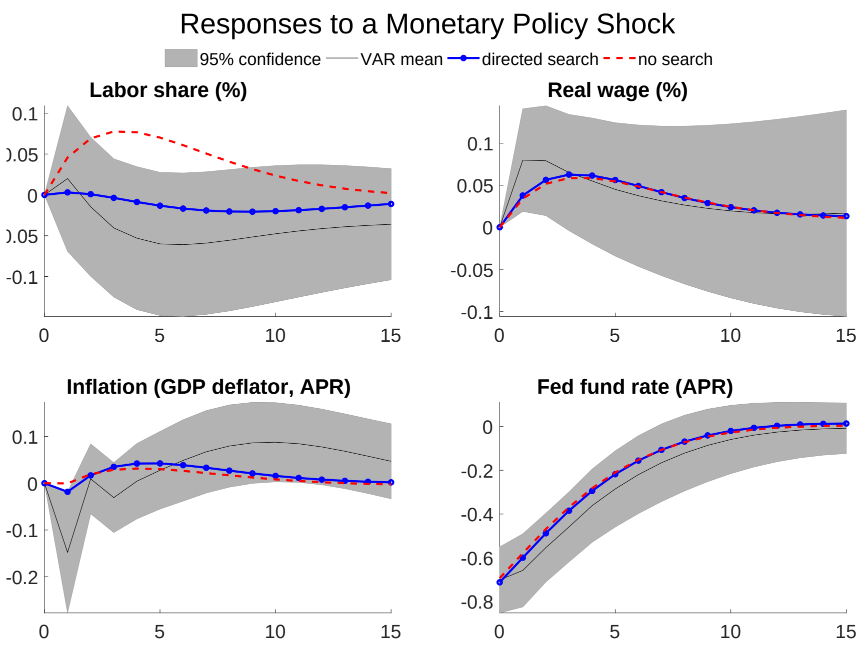

Figure 2: The Solve Labor Share Puzzle

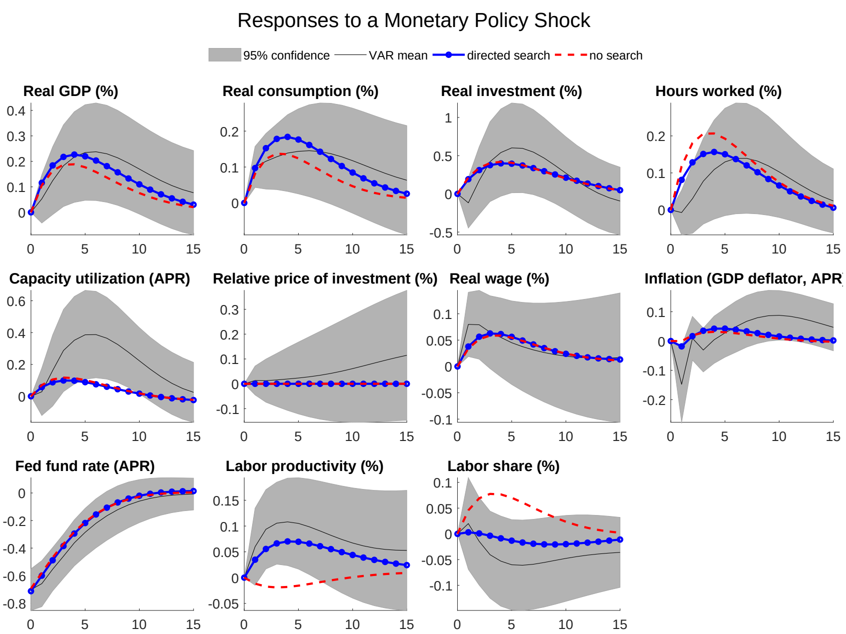

Figure 3: Other Impulse Responses: Monetary Shocks

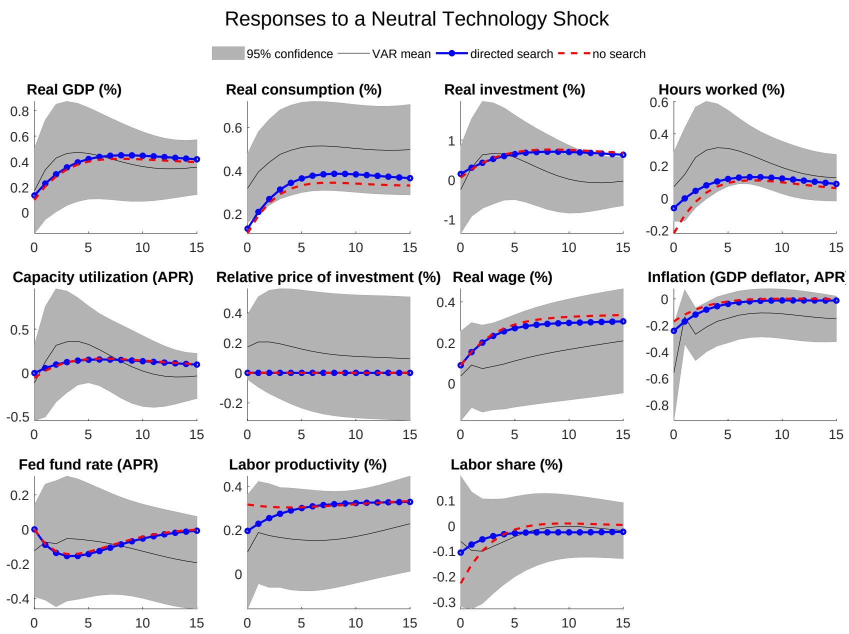

Figure 4: Other Impulse Responses: Neutral Technology Shocks

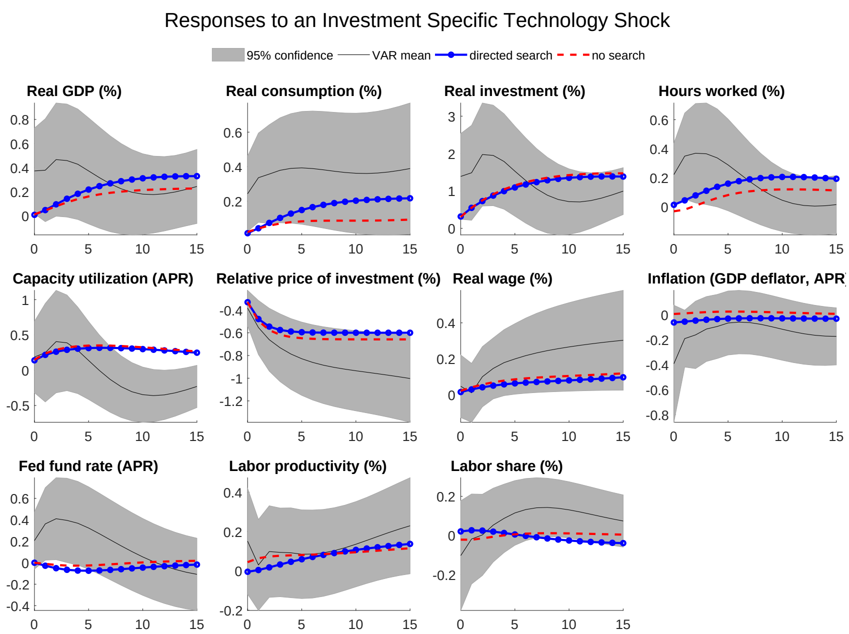

Figure 5: Other Impulse Responses: Investment Specific Technology Shocks

5 Conclusion

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A Proofs

A.1 Proof of Proposition 1

The complete derivation is

$$1+\tau \equiv \frac{p}{\Lambda} = \frac{pF_{x_i}}{G_{x_i}} = \frac{x_i F_{x_i}/F}{x_i G^i_{x_i}/G^i}\left(\frac{G^i}{pF}\right)^{-1} \equiv \frac{\varepsilon^F_i}{\varepsilon^G_i}(input\ i's\ share)^{-1} = \frac{\varepsilon^F_i}{\varepsilon^G_i}\frac{F/x_i}{G^i/(px_i)}.$$

A.2 Proof to Lemma 1

Use λ to denote the Lagrange multiplier on budget, the F.O.C.s to households' problem are

$$0 = \left(\frac{c(p,q)}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} - \lambda p,$$
$$0 = \frac{1}{\varepsilon-1}\left(\frac{c(p,q)}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} + \frac{u_{d^A}}{\psi^h(q)c(p,q)}.$$

When one firm goes to $\{p,q\}$, while all others stay in $\{\overline{p},\overline{q}\}$, the F.O.C.s imply that

$$0 = \left(\frac{c(e,\{\overline{p},\overline{q}\},p,q)}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} - \lambda p, \text{ and } 0 = (\varepsilon-1)\zeta\frac{(d^A)^\nu}{c^A} - \psi^h(q)\left(\frac{\lambda p}{u_{c^A}}\right)^{1-\varepsilon},$$
$$0 = \left(\frac{c(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q})}{c^A}\right)^{-\frac{1}{\varepsilon}} u_{c^A} - \lambda \overline{p}, \text{ and } 0 = (\varepsilon-1)\zeta\frac{(d^A)^\nu}{c^A} - \psi^h(\overline{q})\left(\frac{\lambda \overline{p}}{u_{c^A}}\right)^{1-\varepsilon}.$$

Comparing these two sets of conditions yields

$$c(e,\{\overline{p},\overline{q}\},p,q) = \left(\frac{p}{\overline{p}}\right)^{-\varepsilon} c(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q}),$$
$$q^h(e,\{\overline{p},\overline{q}\},p) = (\psi^h)^{-1}\left[\left(\frac{p}{\overline{p}}\right)^{\varepsilon-1}\psi^h(\overline{q})\right].$$

Given $\psi^f(q) = q\psi^h(q)$, it is easy to prove $\frac{q\psi^{f\prime}(q)}{\psi^f(q)} = \frac{q\psi^{h\prime}(q)}{\psi^f(q)} + 1$. Substitute this back to the previous two equations, and we can obtain all the expressions for all the elasticities $\{\frac{pc_p}{c}, \frac{qc_q}{c}, \frac{pq^h_p}{p}\}$.

A.3 Proof to Lemma 2

Rearranging the F.O.C.s to households' problem yields

$$d(p,q)\psi^h(\overline{q})\left(\frac{c(p,q)}{c^A}\right)^{\frac{\varepsilon-1}{\varepsilon}}c^A = -(\varepsilon-1)\frac{u_{d^A}}{u_{c^A}}d(p,q).$$

Taking integrals on both sides and imposing Assumption 3 yield

$$c^A = (\varepsilon-1)\zeta(d^A)^{1+\nu}.$$

When all households and firms will go to the same submarket $\{\overline{p},\overline{q}\}$, this becomes

$$\psi^h(\overline{q})^{\frac{\varepsilon}{\varepsilon-1}}c(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q}) = (\varepsilon-1)\zeta\ d(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q})^{1+\nu-\frac{\varepsilon}{\varepsilon-1}},$$

and the budget (binding) becomes

$$e = d(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q})\psi^h(\overline{q})\overline{p}\ c(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q}).$$

Combining these two equations and we can solve for $c(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q})$ and $d(e,\{\overline{p},\overline{q}\},\overline{p},\overline{q})$.

A.4 Proof to Lemma 3

The F.O.C. to firms' problem is

$$0 = \left(\psi^f + \frac{q^h\psi^{f\prime}}{\psi^f}\frac{pq_p^h}{q^h}\psi^f + \frac{pc_p^h}{c^h}\psi^f - \frac{W}{p}\frac{pc_p^h}{c^h}\right)c^h - \chi'\left(\frac{p}{p_-}\right)\frac{e}{p_-}.$$

Substituting in the expresions of elasticities in Lemma 1 directly yields Lemma 3.

A.5 Proof to Proposition 2 and 3

Let $\chi'(\cdot)$ goes to 0 in Lemma 3, and use definition $\overline{\tau} \equiv \frac{\overline{p}\psi^f(\overline{q})}{W} - 1$ to prove Proposition 2. Combine Lemma 2, Lemma 3 and Definition 1 to prove Proposition 3.

A.6 Proof to Corollary 2

We only need to prove that $\lim_{\zeta\to 0}\overline{q}(e, W, p_-|\zeta) = +\infty$.
Suppose not, then according to the first condition in Proposition 3, we must have $\lim_{\zeta\to 0}\overline{p}(e, W, p_-|\zeta) = +\infty$, but then the second condition in Proposition 3 cannot hold.

A.7 Proof to Proposition 4

Define auxiliary functions

$$F^p(e, W, p_-, \overline{p}|0) \equiv \varepsilon\left(\frac{W}{\overline{p}} - \frac{\varepsilon-1}{\varepsilon}\right) - \chi'\left(\frac{\overline{p}}{p_-}\right)\frac{\overline{p}}{p_-},$$
$$F^\tau(e, W, p_-, \overline{\tau}|0) \equiv \varepsilon\left(\frac{1}{1+\overline{\tau}} - \frac{\varepsilon-1}{\varepsilon}\right) - \chi'\left(\frac{(1+\overline{\tau})W}{p_-}\right)\frac{(1+\overline{\tau})W}{p_-}.$$

With Assumption 4, it is easy to verify that

$$F^p_e(e, W, p_-, \overline{p}|0) = 0, \quad F^p_W(e, W, p_-, \overline{p}|0) < 0, \quad F^p_{\overline{p}}(e, W, p_-, \overline{p}|0) < 0;$$
$$F^\tau_e(e, W, p_-, \overline{\tau}|0) = 0, \quad F^\tau_W(e, W, p_-, \overline{\tau}|0) > 0, \quad F^\tau_{\overline{\tau}}(e, W, p_-, \overline{\tau}|0) < 0.$$

Since $F^p(e, W, p_-, \overline{p}|0) = F^\tau(e, W, p_-, \overline{\tau}|0) = 0$, we must have

$$\overline{p}_e(e, W, p_-|0) = -\frac{F^p_e}{F^p_{\overline{p}}} = 0, \quad \overline{p}_W(e, W, p_-|0) = -\frac{F^p_W}{F^p_{\overline{p}}} > 0;$$
$$\overline{\tau}_e(e, W, p_-|0) = -\frac{F^\tau_e}{F^\tau_{\overline{\tau}}} = 0, \quad \overline{\tau}_W(e, W, p_-|0) = -\frac{F^\tau_W}{F^\tau_{\overline{\tau}}} < 0.$$

A.8 Proof to Proposition 5

Part 1: Eliminate $\overline{p}$ from Proposition 3, and define an auxiliary function $F^q(e, W, p_-, \overline{q}|\zeta)$.

$$0 = (\varepsilon-1)\left(\frac{\varepsilon\zeta W}{e}\frac{\overline{q}^{1+\nu}}{\psi^f(\overline{q})^{\frac{\varepsilon}{\varepsilon-1}}} - \frac{1}{1-\mathcal{E}(\overline{q})}\right) - \chi'\left(\frac{e}{\zeta p_-}\frac{\psi^f(\overline{q})^{\frac{1}{\varepsilon-1}}}{(\varepsilon-1)\overline{q}^{1+\nu}}\right)\frac{e}{\zeta p_-}\frac{\psi^f(\overline{q})^{\frac{1}{\varepsilon-1}}}{(\varepsilon-1)\overline{q}^{1+\nu}},$$
$$\equiv F^q(e, W, p_-, \overline{q}|\zeta)$$

With Assumption 4, 5, 6, it is easy to verify that

$$F^q_e(e, W, p_-, \overline{q}|\zeta) < 0, \quad F^q_W(e, W, p_-, \overline{q}|\zeta) > 0, \quad F^q_{\overline{q}}(e, W, p_-, \overline{q}|\zeta) > 0.$$

For $F^q(e, W, p_-, \overline{q}|\zeta) = 0$, we must have

$$\overline{q}_e(e, W, p_-|\zeta) = -\frac{F^q_e}{F^q_{\overline{q}}} > 0, \quad \overline{q}_W(e, W, p_-|\zeta) = -\frac{F^q_W}{F^q_{\overline{q}}} < 0.$$

Part 2: Define Auxiliary function

$$F^\tau(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) \equiv \varepsilon\left(\frac{1}{1+\overline{\tau}} - \frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\mathcal{E}(\overline{q})}\right) - \chi'\left(\frac{(1+\overline{\tau})W}{p_-\psi^f(\overline{q})}\right)\frac{(1+\overline{\tau})W}{p_-\psi^f(\overline{q})}.$$

With Assumption 4, 6, it is easy to verify that

$$\begin{aligned}
&F^\tau_e(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) = 0, \quad F^\tau_W(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) < 0, \\
&F^\tau_{\overline{q}}(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) > 0, \quad F^\tau_{\overline{\tau}}(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) < 0.
\end{aligned}$$

For $F^\tau(e, W, p_-, \overline{q}, \overline{\tau}|\zeta) = 0$, we must have

$$\overline{\tau}_e(e, W, p_-|\zeta) = -\frac{F^\tau_e + F^\tau_{\overline{q}}\overline{q}_e}{F^\tau_{\overline{\tau}}} > 0, \quad \overline{\tau}_W(e, W, p_-|\zeta) = -\frac{F^\tau_W + F^\tau_{\overline{q}}\overline{q}_W}{F^\tau_{\overline{\tau}}} < 0.$$

Part 3: Proposition 3 implies that

$$\overline{p} = \frac{e\zeta^{-1}}{\varepsilon-1}\frac{\psi^f(\overline{q})^{\frac{1}{\varepsilon-1}}}{\overline{q}^{1+\nu}} \equiv G^q(e, W, \overline{q}|\zeta).$$

Since $G^q_W(e, W, \overline{q}|\zeta) = 0$ and $G^q_{\overline{q}}(e, W, \overline{q}|\zeta) < 0$,

$$\overline{p}_W(e, W, p_-|\zeta) = G^q_{\overline{q}}(e, W, \overline{q}|\zeta) \cdot \overline{q}_W(e, W, p_-|\zeta) > 0.$$

Part 4: Under Assumption 7, the elasticity $\mathcal{E}(q)$ in the following satisfies Assumption 2, 6.

$$\mathcal{E}(q) = \frac{q\psi^{f\prime}(q)}{\psi^f(q)} = \frac{q^{-\gamma}}{1+q^{-\gamma}} = 1 - \psi^f(q)^\gamma.$$

Part 5: Under Assumption 7, Proposition 3 implies that

$$0 = \varepsilon \frac{W}{\overline{p}} - (\varepsilon - 1)\psi^f(\overline{q})^{1-\gamma} - \psi^f(\overline{q})\chi'\left(\frac{\overline{p}}{p_-}\right)\frac{\overline{p}}{p_-}.$$

For $F^q_{\overline{q}}(e, W, p_-, \overline{q}|\zeta) > 0$ under Assumption 4, 5, 7, it is easy to verify that the solution for $\{\overline{p}, \overline{q}\}$ is unique. As a result, when p_- is the directed search equilibrium price for $(e, W|\zeta)$ under flexible prices, $\overline{p} = p_-$, and

$$\varepsilon \frac{W}{\overline{p}} = (\varepsilon - 1)\psi^f(\overline{q})^{1-\gamma}.$$

Given $\overline{q}_e(e, W, p_-|\zeta) > 0$ and $\overline{q}_W(e, W, p_-|\zeta) < 0$, it is easy to verify that when $\gamma \geq 1$,

$$\overline{p}_e(e, W, p_-|\zeta) \geq 0, \quad \frac{\partial}{\partial e}\left[\frac{W}{\overline{p}(e, W, p_-|\zeta)}\right] \leq 0, \quad \frac{\partial}{\partial W}\left[\frac{W}{\overline{p}(e, W, p_-|\zeta)}\right] \geq 0.$$

under such a p_-.

A.9 Proof to Lemma 4

Household $i \in [0, 1]$ solves

$$\max_{\{y_t(p,q), d_t(p,q), c^A_t, i^A_t, k_t, u^k_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t u(c^A_t, d^A_t, c^A_{t-1}, d^A_{t-1}),$$

$$\begin{aligned}
\text{s.t. } e_t &\equiv \int_{S_t} d_{p,q,t}\psi^h(q)p\, y_{p,q,t}dpdq, \\
y^A_t &\equiv \left(\int_{S_t} d_{p,q,t}\psi^h(q)y_{p,q,t}^{\frac{\varepsilon-1}{\varepsilon}}dpdq\right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
d^A_t &\equiv \int_{S_t} d_{p,q,t}dpdq, \\
(\lambda^e_t) \quad & e_t \leq W_t\ell_t + R^k_t u^k_t k_{t-1} + R_{t-1}b_{t-1} - b_t + T_t, \\
(\lambda^i_t) \quad & i^A_t \leq z^i_t(y^A_t - c^A_t) - a(u^k_t)k_{t-1}, \\
(\lambda^k_t) \quad & k_t \leq (1 - \delta_k)k_{t-1} + \left[1 - S\left(\frac{i^A_t}{i^A_{t-1}}\right)\right] i^A_t.
\end{aligned}$$

The first order conditions w.r.t. $\{y_t(p,q), d_t(p,q)\}$ are

$$
\begin{aligned}
0 &= -\lambda_t^e d_t(p,q)\psi^h(q)p + \lambda_t^i z_t^i \left(\frac{y_t(p,q)}{y_t^A}\right)^{-\frac{1}{\varepsilon}} d_t(p,q)\psi^h(q), \\
0 &= (u_{d^A,t} + \beta \mathbb{E}_t u_{d_-^A,t+1}) - \lambda_t^e \psi^h(q)p\, y_t(p,q) + \frac{\varepsilon}{\varepsilon-1}\lambda_t^i z_t^i \left(\frac{y_t(p,q)}{y_t^A}\right)^{-\frac{1}{\varepsilon}} \psi^h(q) y_t(p,q).
\end{aligned}
$$

For submarket $\{\overline{p}_t, \overline{q}_t\}$ and $\{p,q\}$, these two conditions imply that

$$
\begin{aligned}
\frac{y_t(p,q)}{y_t(\overline{p}_t,\overline{q}_t)} &= \left(\frac{p}{\overline{p}_t}\right)^{-\varepsilon}, \\
\frac{\psi^h(q_t)}{\psi^h(\overline{q}_t)} &= \left(\frac{p_t}{\overline{p}_t}\right)^{\varepsilon-1}.
\end{aligned}
$$

These conditions are identical to those in the static model.

A.10 Proof to Proposition 6

Given marginal cost of production $\{MC_t\}$, each firm solves

$$
\max_{\{p_t\}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \lambda_t^e \left\{ \left[p_t \psi^f(q_t^h(p_t)) - MC_t\right] y_t^h(p_t) + (p_t - MC_t) x_t^h(p_t) - \chi_t\left(\frac{p_t}{p_{t-1}}\right) \right\}.
$$

The first order condition is

$$
\begin{aligned}
0 &= \lambda_t^e \left\{ \psi_t^f y_t^h + p_t \psi_t^{f\prime} q_t^{h\prime} y_t^h + \left(p_t \psi_t^f - MC_t\right) y_t^{h\prime} + x_t^h + (p_t - MC_t) x_t^{h\prime} \right\} \\
&\quad - \lambda_t^e \frac{\chi_t'}{p_{t-1}} + \beta \mathbb{E}_t \lambda_{t+1}^e \frac{p_{t+1}\chi_{t+1}'}{p_t^2}, \\
&= \left\{ 1 + \frac{q_t^h \psi_t^{f\prime}}{\psi_t^f} \frac{p_t q_t^{h\prime}}{q_t^h} + \left(1 - \frac{MC_t}{p_t \psi_t^f}\right) \frac{p_t y_t^{h\prime}}{y_t^h} \right\} \psi_t^f y_t^h + \left(1 + \frac{p_t x_t^{h\prime}}{x_t^h} - \frac{MC_t x_t^{h\prime}}{x_t^h}\right) x_t^h \\
&\quad - \frac{\chi_t'}{p_{t-1}} + \beta \mathbb{E}_t \frac{\lambda_{t+1}^e}{\lambda_t^e} \frac{p_{t+1}\chi_{t+1}'}{p_t^2}, \\
&= (\varepsilon - 1) \left\{ \frac{\varepsilon}{\varepsilon - 1} \frac{MC_t}{p_t} (x_t^h + y_t^h) - x_t^h - \frac{\psi_t^f}{1 - \mathcal{E}(q_t^h)} y_t^h \right\} p_t - \frac{p_t}{p_{t-1}} \chi_t' + \beta \mathbb{E}_t \frac{\lambda_{t+1}^e}{\lambda_t^e} \frac{p_{t+1}}{p_t} \chi_{t+1}'.
\end{aligned}
$$

Imposing all functional forms, consistency and market clearing conditions yields Proposition 6.

B Computation

B.1 Detrended Equilibrium Conditions

Variables $\{\lambda_t^i, C_t^A, Q_t, \lambda_t^k, r_t^k, u_t^k, \lambda_t^e, h_{1,t}, h_{2,t}, w_t^{\#}, w_t, I_t^A, Y_t, \chi_t, \mathcal{I}_t, \mathcal{Y}_t, K_t, L_t, mc_t, \Pi_t, R_{t+1}, \mu_t^n, \mu_t^i, \mu_t^y, \mu_t^k, \tau_t\}$ solves

$$0 = -\lambda_t^e + \lambda_t^i \mathcal{I}_t^{\frac{1}{\varepsilon-1}}, \tag{2}$$

$$0 = -\lambda_t^e + \mathcal{I}_t^{\frac{1}{\varepsilon-1}} \left[\left(U_t - h\, U_{t-1} \left(\mu_t^y\right)^{-1} \right)^{-\omega} - \beta h \left(U_{t+1}\mu_{t+1}^y - h\, U_t \right)^{-\omega} \right], \tag{3}$$

$$0 = -\mathcal{I}_t^{\frac{1}{\varepsilon-1}} Y_t - (\varepsilon - 1)\zeta Q_t^{1+\nu}, \tag{4}$$

$$0 = -\lambda_t^i + \lambda_t^k (1 - S_t - S_t' G_t^i) + \beta \mathbb{E}_t \left[\lambda_{t+1}^k (\mu_{t+1}^u / \mu_{t+1}^k) S_{t+1}' (G_{t+1}^i)^2 \right], \tag{5}$$

$$0 = -\lambda_t^k + \beta \mathbb{E}_t \left[\left\{ \lambda_{t+1}^e r_{t+1}^k u_{t+1}^k - \lambda_{t+1}^i a_{t+1} + \lambda_{t+1}^k (1 - \delta_k) \right\} (\mu_{t+1}^u / \mu_{t+1}^k) \right], \tag{6}$$

$$0 = -\lambda_t^e r_t^k + \lambda_t^i a_t', \tag{7}$$

$$0 = -\lambda_t^e + \beta \mathbb{E}_t \left[\frac{R_{t+1}}{\Pi_{t+1}} \lambda_{t+1}^e \right], \tag{8}$$

$$0 = -h_{1,t} + \eta \left(w_t^{\varepsilon_w} L_t \right)^{1+\xi} + (\beta\theta_w) \mathbb{E}_t \left[(\Pi_{t+1}\mu_{t+1}^y)^{\varepsilon_w(1+\xi)} \mu_{t+1}^u h_{1,t+1} \right], \tag{9}$$

$$0 = -h_{2,t} + \lambda_t^e w_t^{\varepsilon_w} L_t + (\beta\theta_w) \mathbb{E}_t \left[(\Pi_{t+1}\mu_{t+1}^y)^{\varepsilon_w - 1} \mu_{t+1}^u h_{2,t+1} \right], \tag{10}$$

$$0 = -(w_t^{\#})^{1+\varepsilon_w \xi} + \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{h_{1,t}}{h_{2,t}}, \tag{11}$$

$$0 = -w_t^{1-\varepsilon_w} + (1 - \theta_w)(w_t^{\#})^{1-\varepsilon_w} + \theta_w \left(\frac{w_{t-1}}{\Pi_t \mu_t^y} \right)^{1-\varepsilon_w}, \tag{12}$$

$$0 = -I_t^A + \mathcal{I}_t^{\frac{1}{\varepsilon-1}} Y_t - C_t^A - a_t \frac{K_{t-1}}{\mu_t^k}, \tag{13}$$

$$0 = -Y_t + (1 - \chi_t)\mathcal{I}_t \mathcal{Y}_t, \tag{14}$$

$$0 = -\chi_t + \frac{\kappa}{2} \left(\frac{\Pi_t}{\Pi_{SS}} - 1 \right)^2, \tag{15}$$

$$0 = -\mathcal{I}_t + (1 + Q_t^{-\gamma})^{-\frac{1}{\gamma}}, \tag{16}$$

$$0 = -\mathcal{Y}_t + \left(\frac{u_t^k K_{t-1}}{\mu_t^k} \right)^{\alpha} L_t^{1-\alpha}, \tag{17}$$

$$0 = -K_t + (1 - \delta_k) \frac{K_{t-1}}{\mu_t^k} + (1 - S_t) I_t^A, \tag{18}$$

$$0 = -\alpha w_t L_t + (1 - \alpha) r_t^k \frac{u_t^k K_{t-1}}{\mu_t^k}, \tag{19}$$

$$0 = -mc_t + \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{R_t w_t}{1 - \alpha} \right)^{1-\alpha}, \tag{20}$$

$$0 = -(\Pi_t - \Pi_{SS})\Pi_t + \frac{\varepsilon - 1}{\kappa} \left\{ \frac{\varepsilon}{\varepsilon - 1} mc_t - \left[\chi_t + (1 - \chi_t)\mathcal{I}_t^{1-\gamma} \right] \right\} + \beta \mathbb{E}_t \left[\mathcal{M}_{t+1} (\Pi_{t+1} - \Pi_{SS})\Pi_{t+1} \right], \tag{21}$$

$$0 = -\ln \frac{R_t}{R_{SS}} + \rho_R \ln \frac{R_{t-1}}{R_{SS}} + (1 - \rho_R) \left(\phi_\pi \ln \frac{\Pi_{t-1}}{\Pi_{SS}} + \phi_y \ln \frac{Y_{t-1}}{Y_{SS}} \right) + \sigma_R \epsilon_t^R, \tag{22}$$

$$0 = -(\ln \mu_t^n - \ln \mu_{SS}^n) + \sigma_n \epsilon_t^n, \tag{23}$$

$$0 = -(\ln \mu_t^i - \ln \mu_{SS}^i) + \rho_i (\ln \mu_{t-1}^i - \ln \mu^i) + \sigma_i \epsilon_t^i, \tag{24}$$

$$0 = -\ln \mu_t^y + \frac{\alpha}{1 - \alpha} \ln \mu_t^i + \ln \mu_t^n, \tag{25}$$

$$0 = -\ln \mu_t^k + \ln \mu_t^i + \ln \mu_t^y, \tag{26}$$

$$0 = -\tau_t + \frac{\mathcal{I}_t}{mc_t} - 1, \tag{27}$$

with abbreviation for $\{\mu_t^u, U_t, \mathcal{I}_t, G_t^i, S_t, S_t', a_t, a_t', \mathcal{M}_t\}$

$$\mu_t^u \equiv (\mu_t^y)^{1-\omega}, \tag{28}$$

$$U_t \equiv C_t^A - \zeta \frac{Q_t^{1+\nu}}{1+\nu}, \tag{29}$$

$$G_t^i \equiv \frac{I_t^A}{I_{t-1}^A}\mu_t^k, \tag{30}$$

$$S_t \equiv \frac{\exp\left[\sqrt{S''}\left(G_t^i - \mu^k\right)\right] + \exp\left[-\sqrt{S''}\left(G_t^i - \mu^k\right)\right]}{2} - 1, \tag{31}$$

$$S_t' \equiv \sqrt{S''}\frac{\exp\left[\sqrt{S''}\left(G_t^i - \mu^k\right)\right] - \exp\left[-\sqrt{S''}\left(G_t^i - \mu^k\right)\right]}{2}, \tag{32}$$

$$a_t \equiv \frac{\sigma_a\sigma_b}{2}(u_t^k)^2 + \sigma_b(1-\sigma_a)u_t^k + \sigma_b\left(\frac{\sigma_a}{2} - 1\right), \tag{33}$$

$$a_t' \equiv \sigma_a\sigma_b u_t^k + \sigma_b(1-\sigma_a), \tag{34}$$

$$\mathcal{M}_t \equiv \frac{\lambda_t^e}{\lambda_{t-1}^e}\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}}\mu_t^u, \tag{35}$$

and measured consumption and investment $\{C_t, I_t\}$

$$C_t = \mathcal{I}_t^{-\frac{1}{\varepsilon-1}} C_t^A,$$
$$I_t = \mathcal{I}_t^{-\frac{1}{\varepsilon-1}}\left(I_t^A + a_t\frac{K_{t-1}}{\mu_t^k}\right).$$

In the steady state, $(S_{SS}, S_{SS}', S_{SS}'', a_{SS}, a_{SS}', a_{SS}'') = (0, 0, S'', 0, \sigma_b, \sigma_a\sigma_b)$.
Observed variables: $\left(Y_t z_t^y, C_t z_t^y, I_t z_t^k, L_t, u_t^k, (z_t^i)^{-1}, w_t z_t^y, \ln\Pi_t, R_{t+1}, Y_t z_t^y/L_t, w_t L_t/Y_t\right)$.

B.2 Steady State Solver

Initialized parameters:

$$(\mathcal{I}_{SS}, \tau_{SS}, L_{SS}, u_{SS}^k, \chi_{SS}) = (0.70, 0.10, 0.945, 1, 0),$$
$$(400\ln\Pi_{SS}, 400\ln R_{SS}) = (2.5, 3.0),$$
$$(\beta, \omega, \nu, \xi, \gamma, \varepsilon_w, \theta_w, \alpha, \delta_k) = (\Pi_{SS}/R_{SS}, 1, 0, 1, 1, 6, 0.75, 0.33, 0.025),$$
$$(400\ln\mu_{SS}^y, 400\ln\mu_{SS}^k, \ln\mu_{SS}^i, \ln\mu_{SS}^n) = \left(1.7, 2.9, \ln\mu_{SS}^k - \ln\mu_{SS}^y, \ln\mu_{SS}^y - \frac{\alpha}{1-\alpha}\ln\mu_{SS}^i\right).$$

Solved Steady States:

$$
\begin{aligned}
Q_{SS} &= \left(\mathcal{I}_{SS}^{-\gamma} - 1\right)^{-\frac{1}{\gamma}} \\
mc_{SS} &= \frac{\mathcal{I}_{SS}}{1+\tau_{SS}}, \\
\varepsilon &= (1 - mc_{SS}\mathcal{I}_{SS}^{\gamma_{PC}-1})^{-1}, \\
\sigma_b &= \beta^{-1}(\mu_{SS}^y)^{\omega}\mu_{SS}^i - (1-\delta_k), \\
r_{SS}^k &= \mathcal{I}_{SS}^{-\frac{1}{\varepsilon-1}}\sigma_b, \\
w_{SS} &= (1-\alpha)R_{SS}^{-1} mc_{SS}^{\frac{1}{1-\alpha}}\left(\frac{r_{SS}^k}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}, \\
w_{SS}^{\#} &= \left[\frac{1-\theta_w}{1-\theta_w(\Pi_{SS}\mu_{SS}^y)^{\varepsilon_w-1}}\right]^{\frac{1}{\varepsilon_w-1}} w_{SS}, \\
K_{SS} &= \frac{\alpha}{1-\alpha}\frac{w_{SS}}{r_{SS}^k}L_{SS}\mu_{SS}^k, \\
\mathcal{Y}_{SS} &= \left(\frac{K_{SS}}{\mu_{SS}^k}\right)^{\alpha} L_{SS}^{1-\alpha}, \\
I_{SS}^A &= \left(1 - \frac{1-\delta_k}{\mu_{SS}^k}\right)K_{SS}, \\
C_{SS}^A &= \mathcal{I}_{SS}^{\frac{\varepsilon}{\varepsilon-1}}\mathcal{Y}_{SS} - I_{SS}^A, \\
\zeta &= \frac{C_{SS}^A + I_{SS}^A}{(\varepsilon-1)Q_{SS}^{1+\nu}}, \\
U_{SS}^A &= C_{SS}^A - \zeta\frac{Q_{SS}^{1+\nu}}{1+\nu}, \\
\lambda_{SS}^e &= \mathcal{I}_t^{\frac{1}{\varepsilon-1}}\left[1-\beta h(\mu_{SS}^y)^{-\omega}\right]\left[(1-h/\mu_{SS}^y)\,U_{SS}^A\right]^{-\omega}, \\
h_{2,SS} &= \frac{\lambda_{SS}^e w_{SS}^{\varepsilon_w}L_{SS}}{1-\beta\theta_w(\Pi_{SS}\mu_{SS}^y)^{\varepsilon_w-1}(\mu_{SS}^y)^{1-\omega}}, \\
h_{1,SS} &= \frac{\varepsilon_w-1}{\varepsilon_w}(w_{SS}^{\#})^{1+\varepsilon_w\xi}h_{2,SS}, \\
\eta &= \frac{\left[1-\beta\theta_w(\Pi_{SS}\mu_{SS}^y)^{\varepsilon_w(1+\xi)}(\mu_{SS}^y)^{1-\omega}\right]h_{1,SS}}{(w_{SS}^{\varepsilon_w}L_{SS})^{1+\xi}}, \\
\lambda_{SS}^i &= \mathcal{I}_{SS}^{-\frac{1}{\varepsilon-1}}\lambda_{SS}^e, \\
\lambda_{SS}^k &= \lambda_{SS}^i, \\
Y_{SS} &= \mathcal{I}_{SS}\mathcal{Y}_{SS}, \\
C_{SS} &= \mathcal{I}_{SS}^{-\frac{1}{\varepsilon-1}}C_{SS}^A, \\
I_{SS} &= \mathcal{I}_{SS}^{-\frac{1}{\varepsilon-1}}I_{SS}^A.
\end{aligned}
$$

B.3 Bayesian Impulse Response Matching

The model is estimated following Christiano, Eichenbaum, and Trabandt (2016):

- Denote $\psi(\theta_0)$ as the true model implied impulse response functions.
- In finite sample T, the VAR impulse response functions from simulated data satisfy

$$\hat{\psi} \to \mathcal{N}(\psi(\theta_0), V(\theta_0, \zeta_0, T)).$$

- The asymptotically valid approximation of this likelihood is

$$f(\hat{\psi}|\theta, V) = (2\pi)^{-\frac{N}{2}}|V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi} - \psi(\theta)\right)' V^{-1}\left(\hat{\psi} - \psi(\theta)\right)\right].$$

- The Bayesian posterior is

$$f(\theta|\hat{\psi}, V) = \frac{f(\hat{\psi}|\theta, V)p(\theta)}{\int f(\hat{\psi}|\theta, V)p(\theta)d\theta}.$$

- A consistent estimate of V can be obtained through bootstrap:

$$\overline{V} = \frac{1}{M}\sum_{i=1}^{M}(\psi_i - \overline{\psi})(\psi_i - \overline{\psi})'.$$

  In order to improve the small sample efficiency, the off-diagonal elements of $\overline{V}$ are all imposed to be zeros as in Christiano, Eichenbaum, and Trabandt (2016).

- Given the dynare solution of the model, the likelihood function has analytical solution, and we only need to figure out how to perform MCMC Bayesian approach to obtain the posterior.
- We have 100,000 draws for the MCMC chain, and use the last 8,000 for posterior distribution.

B.4 Implement Estimation

$\Theta = \{\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6\}$.

1. Predetermined parameters: $\Theta_1 = \{\omega, \nu, \gamma, \gamma^{PC}, \varepsilon_w, \theta_w, \delta_k\}$.
2. Predetermined steady state: $\Theta_2 = \{\chi_{SS}, \Pi_{SS}, \mu^y_{SS}, \mu^k_{SS}, \mu^i_{SS}\}$.
3. The steady state to be targeted: $\Theta_3 = \{\mathcal{I}_{SS}, \tau_{SS}, L_{SS}, u^k_{SS}, R_{SS}\}$.
4. Estimated parameters: $\Theta_4 = \{\alpha, \xi, h, \sigma_a, S'', \kappa, \phi_\pi, \phi_y, \rho_R, \sigma_R, \sigma_n, \sigma_i, \rho_i\}$.
5. Calibrated parameters to target the steady state: $\Theta_5 = \{\beta, \varepsilon, \sigma_b, \zeta, \eta\}$.
6. The derived: $\Theta_6 = \{Q_{SS}, mc_{SS}, r^k_{SS}, w_{SS}, w^{\#}_{SS}, K_{SS}, \mathcal{Y}_{SS}, I^A_{SS}, C^A_{SS}, \lambda^e_{SS}, h_{2,SS}, h_{1,SS}, \lambda^i_{SS}, \lambda^k_{SS}, Y_{SS}, \mu^n_{SS}\}$.

In the estimation:

1. Read $\{\Theta_1, \Theta_2, \Theta_3, \Theta_4\}$ from *params.m*.
2. If *M_.params* is empty, use *params.m* to solve for $\{\Theta_5, \Theta_6\}$ in *model_steadystate.m*.
3. If *M_.params* is not empty, replace *params.m* with *M_.params* for Θ_4 to solve for $\{\Theta_5, \Theta_6\}$.
4. $\{\Theta_1, \Theta_4, \Theta_5\}$ are assigned to *M_.params*, while $\{\Theta_2, \Theta_3, \Theta_6\}$ are assigned to *model_steadystate.m*.

B.5 Code Adjustment from Christiano, Eichenbaum, and Trabandt (2016)

Our estimation is based on Christiano, Eichenbaum, and Trabandt (2016) with adjustments:

1. The original code for SVAR is never provided, so that I did it by myself following Christiano's lecture note. It has to be recoded because we need the bootstrap confidence intervals for labor productivity and labor share that are not provided in all their published papers.
2. In order to deal with timing issue of monetary shocks, Christiano, Eichenbaum, and Trabandt (2016) define a model with only monetary shocks, and another model with only other shocks, which is unnecessarily complex. We rewrite the code to get around this issue.
3. We have parameters that targets on steady state moments, and have to change values during each loop of estimation. Hence we need to rewrite the structure or loops on parameters.
4. In order to do Bayesian Impulse Response Matching, Christiano, Eichenbaum, and Trabandt (2016) customized the built function in dynare for likelihood evaluation in a specific way that fits their model. We have made it specific for our model.
5. The code provided by Christiano, Eichenbaum, and Trabandt (2016) can run in dynare 4.4.3, but not in dynare 4.5.6 (the newest version is 4.5.7 now). We have not yet solved the problem, but use dynare 4.4.3 at this moment.
6. Codes are rewritten in a more readable way.

B.6 Mistakes in Christiano, Eichenbaum, and Trabandt (2016)

Our paper has identified and corrected a few mistakes in Christiano, Eichenbaum, and Trabandt (2016) when trying to replicate their results in our model with no search.

1. The discount factor β is set to match 3% real annual interest rate, and the correct number should be 0.9925 instead of 0.9968. The original code made this mistake because it did not truly target on 3% real annual interest rate as it stated in the paper.
2. The timing of working capital loans is inconsistent with the timing of monetary shocks.