Global Risk Aversion and International Return Comovements

Nancy R. Xu*

September 9, 2019

Abstract

I establish three stylized facts about global equity and bond return comovements: Equity return correlations are higher, asymmetric, and countercyclical, whereas bond return correlations are lower, symmetric, and weakly procyclical. To interpret these stylized facts, I formulate a dynamic no-arbitrage asset pricing model that consistently prices international equities and bonds; the model features various time-varying global macroeconomic uncertainties and risk aversion of a global investor. I find that different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the global risk aversion shock can explain the observed comovement differences. Global risk aversion explains 90% (40%) of the fitted global equity (bond) comovement dynamics.

JEL Classification: C1, E3, G12, G15.
Keywords: risk aversion, international return comovements, dynamic correlation models, asset pricing, dynamic factor model.

---

*Xu: nancy.xu@bc.edu, Boston College, Carroll School of Management, 140 Commonwealth Avenue, Chestnut Hill, MA, 02467, USA. I would like to thank Geert Bekaert for many insightful discussions, and Hengjie Ai, Nick Bloom, Eric Engstrom, Bob Hodrick, Ian Martin, and Xiaoyan Zhang for helpful comments. I also greatly benefited from discussions with Simona Abis, Gara Afonso, Ozge Akinci, Nina Boyarchenko, Michael Brennan, Charles Calomiris, David Childers, Richard Crump, Marco Del Negro, Fernando Duarte, Thomas Eisenbach, Marc Giannoi, Thomas Gilbert, Linda Goldberg, Ralph Koijen, Anna Kovner, Hugues Langlois (discussant), Ye Li, Anton Lines, Yang Liu (discussant), David Lucca, Harry Mamaysky, Mark Seasholes (discussant), Or Shachar, Andrea Tambalotti, Stijn Van Nieuwerburgh, Desi Volker, Shang-jin Wei, Kairong Xiao, and seminar/conference participants at 2019 SITE Session 8 "The Macroeconomics of Uncertainty and Volatility", 2019 UBC summer conference, 2018 European Finance Association annual meeting, 2018 China International Conference in Finance, Boston College (Carroll), Emory (Goizueta), Georgetown (McDonough), Johns Hopkins University (Carey), University of California (Riverside), University of Minnesota (Carlson), University of Notre Dame (Mendoza), University of Oklahoma (Price), University of Southern California (Marshall), University of Wisconsin Madison, the Federal Reserve Bank of New York, Columbia University (Economics, Econometrics) and Columbia Business School (Finance). Most of the work was finalized during my internship at the Federal Reserve Bank of New York in the summer of 2017; I appreciate the Dissertation Award from the Bank. First version: November 17, 2017. An Internet Appendix and replicating data are available at `https://www.nancyxu.net/research`. All errors are my own.

1 Introduction

Since the 2007-2008 global financial crisis, there has been renewed interest in understanding how asset returns comove across countries. Global asset return comovements are important inputs when evaluating the benefits of international diversification. From an economic perspective, understanding not only why asset comovements evolve over time this way, but why comovement behaviors are different for different asset classes (equities versus government bonds in particular), is important because it could prove to be highly informative about the property of a "global" stochastic discount factor (SDF) and the relative importance of global risks that transmit internationally. Despite these potentially important empirical and economic implications, we know little about how government bond returns comove across countries (see Cappiello, Engle, and Sheppard (2006) for an exception), while a large empirical literature has focused on quantifying the evolution of international equity return comovements (see e.g. Bekaert, Hodrick, and Zhang (2009) and Christoffersen, Errunza, Jacobs, and Langlois (2012), among many others).

In this paper, I first formally compare global equity return comovement with global bond return comovement using various methods and establish several new stylized facts. Then, I interpret the stylized facts in the context of a dynamic no-arbitrage asset pricing model with time-varying global macroeconomic uncertainties (of output growth, inflation, and the real interest rate) and risk aversion of a global investor. One main advantage of using such an asset pricing model is to motivate economic determinants of global comovements in a consistent pricing framework.

To study the dynamics of global equity and bond comovements, I formulate and estimate a new econometric model of multi-dimensional dynamic dependence of 8 developed countries. My model builds on the Dynamic Equicorrelation model by Engle and Kelly (2012), with improvements in correlation asymmetry and simultaneous fit of domestic equity-bond comovement. It also conducts three tests on the differences between global equity and bond comovements: magnitude, tail behavior, and cyclicality. Using monthly return data from March 1987 to December 2016, I establish the following three stylized facts:

1. Equity return correlations are larger in magnitude than bond return correlations;
2. Equity returns have downside correlations that are significantly higher than upside correlations, while bond return correlations are symmetric;
3. Equity return correlations are countercyclical, while bond return correlations are weakly procyclical.

These stylized facts are then confirmed by non-parametric tests. Dynamic comovement estimates are considered empirical benchmarks throughout the paper.

Why do global equity and bond comovements exhibit such distinct behaviors? A number of papers have attempted to suggest economic determinants of global comovements within individual asset class. For instance, Jotikasthira, Le, and Lundblad (2015) find that around 70 percent of the long-term government bond *yield* comovement is due to the commonality of term premia, or risk compensation state variables. On comoving risky assets, Miranda-Agrippino and Rey (2015) propose that an accommodating U.S. monetary policy lowers the credit constraint and motivates investors to build up leverage using cross-border cash flows, thus driving what they term the "global financial cycle". In contrast to these studies, the present research aims to interpret global equity and bond return comovements in a unified asset pricing framework, potentially offering more information to better assess the relative importance of global risk determinants.

In the second part of the paper, I propose and solve a dynamic no-arbitrage asset pricing model from the perspective of a U.S. (global) investor. The model features time-varying risk aversion of the global investor, or "global risk aversion", and various global macroeconomic uncertainties as key state variables-of-interest. In particular, risk aversion in the present research is motivated as the relative risk aversion in an endowment economy, and its dynamics is driven by both fundamental and non-fundamental shocks. For instance, the fundamental sources of time-varying risk aversion are consistent the structural asset pricing literature (e.g., consumption shocks in Campbell and Cochrane (1999); inflation shocks in Brant and Wang (2003)); the non-fundamental shock, or a *pure* risk aversion shock, is likely linked to mood and anxiety according to Cohn, Engelmann, Fehr, and Maréchal (2015). Next, as commonly assumed in the literature, economic uncertainties are proxied by second moments of macroeconomic variable innovations in the economy, which are important determinants of higher-order asset moments, including correlation and covariance. Hence, to increase the chance of explaining the three stylized facts (established earlier), I allow for a relatively sophisticated shock structure of these state variables to realistically capture heteroskedasticity and conditional non-Gaussianity. In particular, I model dynamic behaviors of downside and upside uncertainties of each macro variable separately, given the recent literature highlighting their different asset pricing implications (e.g., Segal, Shaliastovich, and Yaron (2015) using a long-run risk framework and Bekaert and Engstrom (2017) using a habit-formation framework).

Despite the complex shock structure and the broad set of fundamental and non-fundamental state variables, the asset pricing model obtains a closed-form solution in the affine class. International asset returns can be expressed in a *dynamic factor model* where the global contemporaneous factors are *shocks* of global risk premium determinants (i.e., global risk aversion and economic uncertainties). I highlight two key theoretical implications: First, the dynamics of global equity and bond return comovements are then driven by the second moments of these global shocks, and, second, the difference between the two global comovements in my model is explained by the different sensitivities of international equity and bond returns to these shocks.

The final part of the paper interprets the three stylized facts using this theory-motivated factor model. The core finding is that different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the pure global risk aversion shock drive all three stylized facts. Regarding the first stylized fact, bond comovements are smaller because bond return sensitivities to the risk aversion shock not only are smaller in magnitude but have different signs. Some country government bonds are identified as safe bonds as their bond prices increase with risk aversion, while others are priced as risky bonds. Second, bond comovements are symmetric because the different signs of return sensitivities to the risk aversion shock dampen the role of the positively-skewed risk aversion state variable. Third, during normal periods, all bond returns appear to have positive exposures to the risk aversion shock, whereas during global economic turmoil only a few bonds remain safe (USA and Japan in this sample) while other bond prices start to decrease with risk aversion. This finding squares a procyclical global bond comovement. On the other hand, all international equity return sensitivities to the global risk aversion shock become more negative during global economic turmoil, implying a countercyclical global equity comovement.

There are several other important implications from this analysis. Factor models with time-varying betas spanned by, in particular, economic uncertainty state variables outperform those with constant betas on fitting the empirical benchmark. For instance, global equity comovement varies endogenously through a "global" kernel, spiking during bad and highly uncertain periods, which is exactly consistent with the theoretical predictions in Martin (2013). Furthermore, according to the comovement decomposition results, global risk aversion accounts, on average, for 90% of the fitted global equity comovement, followed by real output growth uncertainties with a 7% share. On the other hand, time-varying economic uncertainties are important determinants of the fitted global bond comovement; inflation and real short rate upside uncertainties account for 49% and 22%, respectively; global risk aversion has a moderate 40% share on average, which is due to the state-dependent sign switches of bond return sensitivities as mentioned above.

1.1 Contributions and Literature

The paper contributes to the finance and economics literature in several ways. First, the core economic finding stresses the importance of a price-of-risk channel, risk aversion of the global investor, in explaining international return comovements. Risk aversion has been suggested as an important source of international financial and risk variables, for instance, capital flow waves (Forbes and Warnock (2012)), monetary policy shock transmission to foreign stock markets (Miranda-Agrippino and Rey (2015)), and interest rate correlations (Jotikasthira, Le, and Lundblad (2015)). My main contribution is to exploit information from both global equity and bond return comovements and quantify the importance of risk aversion (price-of-risk state variable) in the presence of various economic uncertainties (amount-of-risk state variables).

Second, while past work has documented and analyzed the dynamic behaviors of global equity return comovement,[1] the present research is the first to *systematically* examine global bond return comovement in conjunction with the equity counterpart. To be more specific, the asymmetric and countercyclical behaviors of global equity return comovement have been tested and documented using various econometric approaches.[2] Moreover, the first stylized fact is particularly striking that equity return correlations are larger, given that Jotikasthira, Le, and Lundblad (2015) document a very high correlation (>0.9) among international government bond yields.

Third, the present research also speaks to the ongoing debate regarding the estimation of "global" risk factors (see e.g. Bollerslev, Marrone, Xu, and Zhou (2014); Ahir, Bloom, and Furceri (2018)). My evidence suggests that U.S. risk factors demonstrate strong explanatory power of global comovements.

Finally, the econometric model contains two modeling innovations. It allows for testing asymmetric correlations in a high dimension, and proposes a parsimonious way to build in time-varying domestic equity-bond comovement. Both innovations can be shown to improve the statistical fit.

The remainder of the paper is organized as follows. Section 2 establishes the three stylized facts. Section 3 motivates and estimates the relevant global economic determinants of return comovements implied from a dynamic asset pricing model. Section 4 interprets the three stylized facts. Section 5 provides additional evidence including robustness checks. Concluding remarks are presented in Section 6.

2 Stylized Facts of Global Comovements

In this section, I establish new stylized facts of global return comovements using both parametric and non-parametric tests. I focus on three perspectives: magnitude, (a)symmetry, and cyclicality. Sections 2.1–2.4 introduce the parametric model, describe the estimation methodology and data, and discuss the estimation results; Section 2.5 discusses the non-parametric tests.

[1] See, for example, Lin, Engle, and Ito (1994); Longin and Solnik (1995), Karolyi and Stulz (1996); Campbell, Koedijk, and Kofman (2002); Forbes and Rigobon (2002); Karolyi (2003); Bae, Karolyi, and Stulz (2003); Dungey, Fry, González-Hermosillo, and Martin (2005); Campbell, Forbes, Koedijk, and Kofman (2008); Bekaert et al. (2009); Christoffersen et al. (2012); Li (2014); Solnik and Watewai (2016); and the references therein.

[2] On the asymmetry: see e.g. exceedance correlation as in Longin and Solnik (2001); bivariate GARCH models as in Cappiello, Engle and Sheppard (2006); asymmetric copula models as in Christoffersen et al. (2012). On the countercyclicality: see e.g. Longin and Solnik (1995), De Santis and Gerard (1997), Ribeiro and Veronesi (2002).

2.1 An Econometric Model for Global Dynamic Comovements

The present research focuses on global return correlation as the main measure of global comovement. The estimation of a pairwise dynamic correlation system becomes increasingly cumbersome as the size of the system grows. To summarize correlation information at the aggregate level, Engle and Kelly (2012) propose a multivariate dynamic correlation model named Dynamic Equicorrelation (DECO), in which the dynamic equicorrelation is defined as the average of all pairwise dynamic correlations and is determined by maximizing the joint likelihoods of all pairs.

Building on the original DECO framework for its attractive dimension reduction technique, I introduce a general model for global equity return comovement and global bond return comovement with two new features, given the unique economic context. The first feature is intended to accommodate potential asymmetric responses of global comovement to synchronized negative return innovations, which can be motivated by theoretical predictions such as Martin (2013). The second feature aims to simultaneously capture the time-varying economic relation between domestic equity and bond prices through, for instance, a flight-to-safety channel; the original DECO model, by contrast, is suitable only for estimating global comovement within one asset class (if used in this context). The model is detailed as follows.

Consider a world economy of N countries. Denote $\boldsymbol{\varepsilon^E_{t+1}}$ ($\boldsymbol{\varepsilon^B_{t+1}}$) as a $N \times 1$ vector of the residuals of log country equity (bond) return from t to $t+1$.[3] The conditional variance-covariance matrices of the residuals with information set t are denoted as $\boldsymbol{H^E_t} \equiv E_t\left[\boldsymbol{\varepsilon^E_{t+1}\varepsilon^{E\prime}_{t+1}}\right]$ $(N \times N)$ for equities and $\boldsymbol{H^B_t} \equiv E_t\left[\boldsymbol{\varepsilon^B_{t+1}\varepsilon^{B\prime}_{t+1}}\right]$ $(N \times N)$ for bonds. I follow the dynamic conditional correlation literature to express the variance-covariance matrices in a quadratic form and estimate the univariate conditional variances and the conditional correlation matrices in separate steps,

$$\boldsymbol{H^E_t} = \boldsymbol{\Lambda^E_t Corr^E_t \Lambda^E_t}, \tag{1}$$

$$\boldsymbol{H^B_t} = \boldsymbol{\Lambda^B_t Corr^B_t \Lambda^B_t}, \tag{2}$$

where $\boldsymbol{\Lambda^E_t}$ ($\boldsymbol{\Lambda^B_t}$) contains the equity (bond) return conditional volatilites on the diagonal and zeros elsewhere; $\boldsymbol{Corr^E_t}$ ($\boldsymbol{Corr^B_t}$) is the equity (bond) return conditional correlation matrix; and they are all $N \times N$ symmetric matrices. The first step estimates and chooses the conditional variance estimates for each country-asset; the second step estimates the conditional correlation matrices, $\boldsymbol{Corr^E_t}$ and $\boldsymbol{Corr^B_t}$, using the standardized residuals.

[3]Country asset return residuals are obtained by de-centering the return with a constant mean. A constant mean is considered for two reasons. First, it provides empirical convenience as in Cappiello, Engle, and Sheppard (2006). In addition, in terms of economic magnitudes, the amount of return variation driven by expected returns is significantly smaller than that driven by the innovation. As a result, the empirical assumption of a constant mean is unlikely to affect comovement statistics.

2.1.A Global Dynamic Comovement Model: Bonds

Denote $\boldsymbol{z^B_{t+1}}$ $(N \times 1)$ as the standardized residuals of country bond returns. The conditional equicorrelation matrix of $\boldsymbol{z^B_{t+1}}$ is defined by

$$\boldsymbol{Corr^B_t} \equiv E_t[\boldsymbol{z^B_{t+1}} \boldsymbol{z^{B\prime}_{t+1}}] = (1-\rho^B_t)\boldsymbol{I_N} + \rho^B_t \boldsymbol{J_{N\times N}}, \tag{3}$$

where $\boldsymbol{I_N}$ is an identity matrix and $\boldsymbol{J_{N\times N}}$ is a matrix of ones as in Engle and Kelly (2012). The equicorrelation by definition is an equally-weighted average of correlations of all country pairs at information set t:

$$\rho^B_t = \frac{1}{N(N-1)}\left[\boldsymbol{\iota}' \left(\boldsymbol{\widetilde{Q}^B_t}\right)^{-1/2} \boldsymbol{Q^B_t} \left(\boldsymbol{\widetilde{Q}^B_t}\right)^{-1/2} \boldsymbol{\iota} - N\right], \tag{4}$$

where $\boldsymbol{\widetilde{Q}^B_t}$ is $\boldsymbol{Q^B_t}$ with off-diagonal terms equal to zeros (following the Aielli (2013) correction); $\boldsymbol{\iota}$ is a $N \times 1$ vector of ones. Hence, $\boldsymbol{Q^B_t}$ $(N \times N)$ is the key latent high-dimensional variable determining the time variation in $\boldsymbol{Corr^B_t}$.

The original DECO framework models the dynamic process of $\boldsymbol{Q_t}$ (omitting "B" below for simplicity) with a generalized autoregressive heteroskedastic process. In this paper, I use a more general dynamic process to capture the potential (1) slow-moving cyclical dynamics and (2) asymmetric responses to joint negative shocks:

$$\begin{aligned} \boldsymbol{Q_t} &= \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_t} + \beta_1 \left(\boldsymbol{\widetilde{Q}^{\frac{1}{2}}_{t-1} z_t z'_t \widetilde{Q}^{\frac{1}{2}}_{t-1}} - \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right) + \beta_2 \left(\boldsymbol{Q_{t-1}} - \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right) \\ &+ \gamma \left(\boldsymbol{\widetilde{Q}^{\frac{1}{2}}_{t-1} n_t n'_t \widetilde{Q}^{\frac{1}{2}}_{t-1}} - \boldsymbol{\Xi} \circ \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right), \end{aligned} \tag{5}$$

where $\boldsymbol{\overline{Q}}$ $(N \times N)$ is the pre-determined sample bond return correlation matrix; "$\circ$" denotes the element-by-element product operator; β_1, β_2 and γ are unknown parameters capturing the relative importance of the cross products of shock realizations, persistence and asymmetry terms.

The first term "$\boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_t}$" represents the time-varying long-run conditional mean of the conditional covariance matrix, where $\boldsymbol{\Phi_t}$ $(N \times N)$ evolves with the world business condition observable at time t:

$$\boldsymbol{\Phi_t} = \begin{bmatrix} 1 & 1+\phi_t & \cdots \\ 1+\phi_t & 1 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \ \phi_t = \phi \ \widetilde{\theta}^{world}_t, \tag{6}$$

where $\widetilde{\theta}^{world}_t$ is a standardized world recession indicator.[4] By construction, the uncondi-

[4]The recession indicator is assigned 1 during recession periods, and 0 during non-recession periods; then, I standardize the indicator so that the unconditional mean (sample mean) of ϕ_t is 0 for interpretation purposes.

tional sample mean of $\boldsymbol{\Phi_t}$ is a symmetric matrix of ones, and ϕ is an unknown parameter. As in the original DECO model, the second term captures the effect of news (scaled contemporaneous shock products) on the $\boldsymbol{Q_t}$ process, and the third term is an autoregressive term, modeling persistence. To capture potential excess comovement in synchronized downside events, I introduce a new asymmetric term. In the fourth term of Equation (5),

$$\boldsymbol{n_t} = I_{\boldsymbol{z_t}<0} \circ \boldsymbol{z_t}, \tag{7}$$

where $I_{\boldsymbol{z_t}<0}$ $(N \times 1)$ is an indicator of negative returns. The constant symmetric matrix $\Xi \equiv E\left[I_{\boldsymbol{z_t}<0} I'_{\boldsymbol{z_t}<0}\right]$ $(N \times N)$ represents the probability of joint negative shocks during the sample period. The sign of the new coefficient γ is not constrained. The sufficient conditions for this new $\boldsymbol{Q_t}$ to be stationary are $\beta_1 \boldsymbol{J_{N\times N}} + \beta_2 \boldsymbol{J_{N\times N}} + \gamma\Xi < \boldsymbol{J_{N\times N}}$ and $\beta_1, \beta_2 > 0$; see proofs in Appendix C.

The new parameters have important economic interpretations. A positive γ indicates a higher downside comovement when below-average returns co-occur across countries, whereas a negative γ indicates a lower downside comovement. The paper that comes closest to introducing asymmetry to dynamic dependence models in the GARCH class is Cappiello, Engle, and Sheppard (2006); they estimate a bivariate system, whereas the present model works with a multivariate system. In addition, a positive ϕ indicates that the long-run conditional mean of dynamic comovement behaves countercyclically given that the world recession indicator is countercyclical, whereas a negative ϕ indicates procyclical behaviors.

It is worth noting the difference between cyclical and asymmetric behaviors of comovements. In principle, business cycle *news* can affect returns at the high frequency, hence entering $\boldsymbol{z_t z_t'}$ and $\boldsymbol{n_t n_t'}$. However, the actual business cycle *regimes* are rather slow-moving, as captured in these recession indicators (e.g., $\widetilde{\theta}_t^{world}$). This model attempts to differentiate the slow-moving cyclical pattern from the fast-moving return fluctuations through the first and fourth terms as mentioned above, respectively. As a result, new parameters ϕ and γ potentially capture different perspectives. In fact, several papers in the literature have used a similar instrument approach to identify cyclicality in estimations.[5]

2.1.B Global Dynamic Comovement Model: Equities

Next, I propose a "Duo-DECO" framework to potentially accommodate domestic equity-bond return comovement while estimating global equity return comovement across countries. The dynamics of the U.S. equity-bond comovements are difficult to explain (see e.g. Baele et al. (2010); Ermolov (2018)); however, the literature suggests that the time

[5]For instance Bekaert and Harvey (1995) on estimating the world price of risk, Duffee (2005) on testing the cyclicality of the amount of consumption risk, and more recently Xu (2019) on uncovering the procyclical comovement between dividend growth and consumption growth.

variation in domestic correlation is often associated with Flight-to-Safety channels (FTS; see Connolly, Stivers, and Sun (2005)). On a global scale, Baele et al. (2019) identify and characterize FTS episodes for 23 countries individually, finding that the majority of FTS events are country-specific rather than global and that such domestic correlation is generally procyclical.

Each country's standardized equity returns $z^E_{i,t+1}$ is expressed as a linear combination of a part comoving with its standardized bond returns $z^B_{i,t+1}$ with a country-specific conditional sensitivity $b_{i,t}$ and a "bond-purified" equity part denoted as $\breve{z}^E_{i,t+1}$:

$$z^E_{i,t+1} = b_{i,t} z^B_{i,t+1} + \sqrt{1 - b^2_{i,t}}\,\breve{z}^E_{i,t+1}, \tag{8}$$

$$b_{i,t} = 2\frac{\exp(\delta_1 + \delta_2 x_{i,t})}{1 + \exp(\delta_1 + \delta_2 x_{i,t})} - 1, \tag{9}$$

where δ_1 and δ_2 are unknown constant parameters; $x_{i,t}$ is a country recession indicator (1 during recession months and 0 otherwise); $z^B_{i,t+1}$ and $\breve{z}^E_{i,t+1}$ are mutually independent.

There are three immediate implications. First, given the mutual independence of the shocks, the conditional variance has a linear expression as follows:

$$Var_t\left(z^E_{i,t+1}\right) = b^2_{i,t} Var_t\left(z^B_{i,t+1}\right) + (1 - b^2_{i,t}) Var_t\left(\breve{z}^E_{i,t+1}\right). \tag{10}$$

Given that the conditional variances of standardized residuals are 1 at all times, Equation (8) restricts the mean and conditional variance of $\breve{z}^E_{i,t+1}$ to be 0 and 1, respectively. Second, the sensitivity variable $b_{i,t}$ ranges from -1 to 1 (exclusively). To reduce the estimation dimension, δ_1 and δ_2 are assumed to be the same for all countries; but because recession periods indicated by $x_{i,t}$ can be different for different countries, the domestic equity-bond comovements can also be different across countries at each point of time. Third, this return decomposition implies a correlation decomposition, providing estimation convenience. For countries i and j, the conditional correlation between $z^E_{i,t+1}$ and $z^E_{j,t+1}$ is $E_t[z^E_{i,t+1} z^E_{j,t+1}] = b_{i,t} b_{j,t} E_t[z^B_{i,t+1} z^B_{j,t+1}] + \sqrt{1 - b^2_{i,t}}\sqrt{1 - b^2_{j,t}} E_t[\breve{z}^E_{i,t+1} \breve{z}^E_{j,t+1}]$. The conditional correlation matrix of total equity returns, $\boldsymbol{Corr^E_t}$, can be easily expressed as a linear combination of the conditional correlation matrix of bond returns, $\boldsymbol{Corr^B_t}$, and the conditional correlation matrix of the bond-purified equity returns denoted as $\widetilde{\boldsymbol{Corr}}^{\boldsymbol{E}}_{\boldsymbol{t}} \equiv E_t[\breve{\boldsymbol{z}}^{\boldsymbol{E}}_{\boldsymbol{t+1}} \breve{\boldsymbol{z}}^{\boldsymbol{E\prime}}_{\boldsymbol{t+1}}]$.

Similarly, $\widetilde{\boldsymbol{Corr}}^{\boldsymbol{E}}_{\boldsymbol{t}}$ is modeled as a conditional equicorrelation matrix, $(1 - \breve{\rho}^E_t)\boldsymbol{I_N} + \breve{\rho}^E_t \boldsymbol{J_{N\times N}}$, where $\breve{\rho}^E_t$ summarizes a latent conditional covariance matrix $\breve{\boldsymbol{Q}}^{\boldsymbol{E}}_{\boldsymbol{t}}$ $(N \times N)$ as in Equation (4). The dynamics follow a similar process (omitting "E" for simplicity):

$$\breve{\boldsymbol{Q}}_{\boldsymbol{t}} = \overline{\breve{\boldsymbol{Q}}} \circ \breve{\boldsymbol{\Phi}}_{\boldsymbol{t}} + \breve{\beta}_1 \left(\widetilde{\breve{\boldsymbol{Q}}}^{\frac{1}{2}}_{\boldsymbol{t-1}} \breve{\boldsymbol{z}}_{\boldsymbol{t}} \breve{\boldsymbol{z}}'_{\boldsymbol{t}} \widetilde{\breve{\boldsymbol{Q}}}^{\frac{1}{2}}_{\boldsymbol{t-1}} - \overline{\breve{\boldsymbol{Q}}} \circ \breve{\boldsymbol{\Phi}}_{\boldsymbol{t-1}} \right) + \breve{\beta}_2 \left(\breve{\boldsymbol{Q}}_{\boldsymbol{t-1}} - \overline{\breve{\boldsymbol{Q}}} \circ \breve{\boldsymbol{\Phi}}_{\boldsymbol{t-1}} \right)$$

$$+\breve{\gamma}\left(\widetilde{\breve{Q}}^{\frac{1}{2}}_{t-1}\breve{n}_t\breve{n}'_t\widetilde{\breve{Q}}^{\frac{1}{2}}_{t-1}-\breve{\Xi}\circ\overline{\breve{Q}}\circ\breve{\Phi}_{t-1}\right), \tag{11}$$

where $\overline{\breve{\boldsymbol{Q}}}$ is the unconditional correlation matrix of $\breve{\boldsymbol{z}}_t$; $\breve{\boldsymbol{\Phi}}_t$ is the cyclical component of the long-run conditional mean, as similarly modeled in Equation (6); $\widetilde{\breve{\boldsymbol{Q}}}^{\frac{1}{2}}_t$ is $\breve{\boldsymbol{Q}}_t$ with off-diagonal terms equal to zeros, following the Aielli (2013) correction; $\breve{\boldsymbol{n}}_t = I_{\breve{z}_t<0}\circ\breve{\boldsymbol{z}}_t$; $\breve{\boldsymbol{\Xi}} = E\left[I_{\breve{z}_t<0}I'_{\breve{z}_t<0}\right]$; $\breve{\beta}_1$, $\breve{\beta}_2$ and $\breve{\gamma}$ are unknown parameters from this process.

2.2 Estimation Procedure

I pre-determine the return conditional variances of each country-asset return series using four univariate GARCH-class models (Appendix A) and four distributional assumptions (Appendix B). The motivation for considering a wide range of volatility models and distributional assumptions is to carefully control for asymmetry and time variation in the conditional *covariance* that is due to conditional volatilities. The variance models are estimated using the Maximum Likelihood Estimation (MLE) methodology, and conditional volatility estimates are chosen given the goodness of fit criteria (AIC and BIC).

I then estimate the global bond return comovement model in Section 2.1.A using the standardized bond returns obtained from the previous step. There are four unknown parameters from the model, $\{\beta_1, \beta_2, \gamma, \phi\}$, and the estimation is conducted using the MLE methodology. I allow for two multivariate distributional assumptions: a multivariate Gaussian distribution and a multivariate t distribution with an unknown degree of freedom as another free parameter (see Kotz and Nadarajah (2004); Genz and Bretz (2009)).[6] The best estimates of $\boldsymbol{Corr}^{\boldsymbol{B}}_{\boldsymbol{t}}$ are chosen using the AIC and BIC criteria.

The equity correlation model has six unknown parameters, $\{\delta_1, \delta_2, \beta_1, \beta_2, \gamma, \phi\}$, and is estimated using the MLE methodology to maximize over the sum of log likelihoods of the bond-purified equity residuals. Same distributional assumptions and model selection criteria apply. As illustrated in Section 2.1.B, the total equity correlation is a "weighted" average of the estimated bond correlation and bond-purified equity correlation.

Importantly, DECO is a special case of Duo-DECO when $\delta_1 = \delta_2 = 0$ (b^i_t is 0 for all countries during all periods), and thus can be viewed as the null hypothesis in the estimation. In addition, the main motivation of modeling equity return correlations with Duo-DECO instead of bond return correlations in this particular context is that, in theory, the property of government bond returns is only a property of SDF while the property of equity returns is also subject to other factors such as cash flows (or even intermediary

[6] A multivariate Gaussian log likelihood is the sum of a constant and $-\frac{1}{2}\sum_{t=1}^{T}\left(\log\left|\boldsymbol{Corr}^{\boldsymbol{B}}_{\boldsymbol{t}}\right| + \boldsymbol{z}^{\boldsymbol{B}\prime}_{\boldsymbol{t+1}}\left(\boldsymbol{Corr}^{\boldsymbol{B}}_{\boldsymbol{t}}\right)^{-1}\boldsymbol{z}^{\boldsymbol{B}}_{\boldsymbol{t+1}}\right)$; a multivariate t log likelihood is the sum of a constant and $-\frac{1}{2}\sum_{t=1}^{T}\left[\log\left|\boldsymbol{Corr}^{\boldsymbol{B}}_{\boldsymbol{t}}\right| + (df+N)\log\left(1+\frac{1}{df}\boldsymbol{z}^{\boldsymbol{B}\prime}_{\boldsymbol{t+1}}\left(\boldsymbol{Corr}^{\boldsymbol{B}}_{\boldsymbol{t}}\right)^{-1}\boldsymbol{z}^{\boldsymbol{B}}_{\boldsymbol{t+1}}\right)\right]$.

and behavioral elements). The other modeling choice, however, can be explored as well.

2.3 Data

I use monthly log returns denominated in U.S. dollars of eight developed countries: the United States, USA; Canada, CAN; Germany, DEU; France, FRA; United Kingdom, GBR; Switzerland, CHE; Japan, JPN; Australia, AUS. Log equity returns refer to changes in the log total return index of the domestic stock market (United States: S&P500; Canada: S&P/TSX 60; Germany: DAX 30; France: CAC 40; United Kingdom: FTSE 100; Switzerland: SMI; Japan: NIKKEI 225; Australia: S&P/ASX 200). U.S. equity returns use the CRSP value-weighted returns including dividends; for other countries, the total return indices are obtained from DataStream. Log bond returns use changes in the log 10-year government bond index constructed by DataStream. The sample is from March 1987 to December 2016 (a total of 358 months). For the purpose of data source and methodology consistency, country- and world-level recession indicators are obtained from the OECD website; I use the "OECD Major Seven Countries" recession indicator as the proxy for the world business condition.

Table 1 provides the summary statistics of the raw log returns. Panel A shows that the average of pairwise unconditional correlations of the raw log equity (bond) returns is 0.639 (0.465); in fact, using standardized returns, the average equity (bond) return correlation is 0.627 (0.461). From Panel B, the U.S. equity and bond return volatilities are the lowest among the eight countries.[7]

2.4 Estimation Results

2.4.A Global Bond Comovement

In Table 2, Model B(1) denotes the original DECO model. The original DECO model is considered the null model to test whether the new terms (cyclical slow-moving conditional mean and asymmetric responses of joint negative shocks) significantly improve the fit. According to the standard model selection criteria, fitting standardized bond returns with a multivariate t distribution (right panel) exhibits consistently lower AIC and BIC values for all models, suggesting non-Gaussian properties of bond return innovations. Among the models, Model B(1) already exhibits the lowest BIC, and is thus the model chosen for global bond return comovement.

Global bond return conditional correlation implied from the chosen model is highly persistent with $\beta_2 = 0.9164$, and the high persistence coefficient is robust across other

[7]When expressed in local currencies, the U.S. equity volatility remains the lowest. I discuss comovements in local currencies in robustness checks (Section 5) with more details in the Internet Appendix. Local currency returns are also obtained from DataStream.

models in Table 2. The asymmetry parameter γ is insignificant in Model B(2), and remains insignificant after controlling for the time-varying long-run conditional mean as in Model B(4). Therefore, the evidence fails to reject the null that bond correlations are symmetric. The long-run conditional mean of the bond correlation process is estimated to be weakly procyclical (ϕ=-0.0420, t=-1.76) in Model B(4). In terms of economic magnitude, the long-run conditional mean during recession periods is lower than that during non-recession periods by around 0.086 (i.e., $\frac{-0.0420}{0.491}$ where 0.491 is the standard deviation of the OECD world recession indicator). The regression coefficient of the global bond correlation on the world recession indicator is -0.0259 (SE=0.0131), suggesting an overall procyclical behavior.

Model B(5) conducts an equality test, where the pre-determined bond return unconditional correlation matrix $\overline{\boldsymbol{Q}}^{\boldsymbol{B}}$ is modeled as $\overline{\boldsymbol{Q}}^{\boldsymbol{E}} + \nu\,(\boldsymbol{J_{N\times N}} - \boldsymbol{I_N})$ such that the value of ν captures the average difference between the off-diagonal terms of equity and bond return correlation matrices. Under both distributional assumptions, the ν estimate is significant and negative (-0.275 and -0.167), suggesting that the bond correlations are on average smaller than equity correlations. This model is estimated for testing purpose only.

Figure 1 depicts the global bond comovement estimates (dotted black line) along with the OECD world recessions (shaded regions). The first observation is that there might be several structural breaks in the history of international bond market. Global bond comovement experienced a three-year drop starting from 1992, kept increasing over the next 8 years or so until the 2007-08 global financial crisis, and has since then stayed below the pre-crisis level. The biggest monthly increase occurred during January 1999, from 0.215 to 0.414. The formation of the monetary union and the introduction of Euro in Europe plausibly had a positive structural effect on global comovements among national assets, which is consistent with pairwise evidence and discussions in Cappiello, Engle, and Sheppard (2006); Viceira and Wang (2018) also find consistent results using rolling correlations. Another major increase is around the early 2013 after successful fiscal consolidation and implementation of structural reforms among European countries with most government default risk; global bond return correlation increased from 0.398 to 0.552. On the other hand, the biggest *monthly* drop occurred during January 2015 likely due to two major events in the global economy: first, the Greek government-debt crisis kept unfolding, which likely triggered a higher level of fear and anxiety globally and widened country bond risk characteristics; second, the cap-lifting of Swiss Franc against the Euro caused major foreign exchange market turmoil and increases in volatilities.

The second observation is about cyclical behaviors. The global bond comovement has a correlation of -0.1276 (p-value = 0.0157) with the OECD world recession indicator and a correlation of -0.1044 (p-value = 0.0483) with the OECD United States recession indicator, which is consistent with the (weak) procyclicality finding above. After controlling for a HP-filtered trend given the concern of structural breaks, the cyclical part still exhibits a significant and negative correlation of -0.1782 (p-value = 0.0007) with the world recession indicator. Global bond correlation often decreases consecutively in a bad economic environment. For instance, during the 2007-08 global recession, within six months, the estimated global bond correlation dropped from 0.7 to 0.5. The high international bond return variance during recessions (as pictured in the Internet Appendix, Figure A1) is the other force dampening the correlation, but not the primary force as the global bond volatility is in fact uncorrelated with the global bond correlation ($\rho = -0.0546$, p-value = 0.3020).

2.4.B Global Equity Comovement

Table 3 reports the estimation results of global equity comovement models. The multivariate t distributional assumption improves the fit in terms of likelihood, AIC and BIC across all models. Model E(1) is the null model for other models incorporating more flexible assumptions. For instance, the estimates of the asymmetry parameter γ are significant and positive whether controlling for the time-varying long-run conditional mean or not, which rejects the null that global equity comovement is symmetric. The positive sign of γ supports excessive equity return downside comovement. This finding is consistent with the empirical literature (see e.g. Longin and Solnik (2001), Ang and Chen (2002), Cappiello, Engle, and Sheppard (2006), Brunnermeier, Nagel, and Pedersen(2008), among many others) and some recent consumption-based theories on endogenous dependence (see e.g. Martin (2013)). Next, the cyclicality parameter ϕ is found to be significant and positive (around 0.04) in all models, for example, ϕ=0.0426 (p-value=0.0246) in Model E(3) and ϕ=0.0357 (p-value=0.0199) in Model E(4). The long-run conditional mean during recession periods is significantly higher than that during non-recession periods by an average of 0.073 (i.e., $\frac{0.0357}{0.491}$). The sensitivity to the world recession indicator, 0.0185 (SE=0.0097), indicates an overall countercyclical behavior.

The "Duo" part aims to simultaneously accommodate domestic equity-bond return comovement $b_{i,t}$ given each country's local business cycle. As shown in Table 3, δ_2 is estimated to be significant and negative in all models, suggesting a procyclical $b_{i,t}$ given Equation (9). The domestic comovement is estimated to be between 0.2 and 0.3 using this simple two-state model, indicating that the global bond correlation has a small weight ($< 10\%$) in the total global equity correlation. I also re-estimate the equity correlation models using an alternative business cycle variable, standardized country industrial production growth (which, unlike country OECD recession indicator, is a *procyclical* continuous process), and continue to find procyclical behaviors of domestic comovement given significant and positive δ_2 estimates. Finally, I demonstrate that models with the Duo part perform better than those without. Additional evidence is provided in the Internet

Appendix, Tables A3–A5.

The chosen global equity comovement estimates are depicted in Figure 1. I discuss three interesting observations. First, at 0.438, the global equity correlation has a moderately positive correlation with the global bond correlation; their relationship becomes negative (-0.131) using HP-filtered global comovements. From the plot, their cyclical movements tend to diverge significantly during recession periods, for instance during the early 1990's recessions (Gulf war), the 1998 Asian crisis, the recent 2007-08 global financial crisis and the 2012 European debt crisis. This observation is consistent with the findings of countercyclical global equity comovement and procyclical global bond comovement above. Second, given the current estimation, global bond correlation exceeds global equity correlation briefly during July and August of 1992, the second half of 2004, and the last quarter of 2014. Finally, a simple trend regression supports a significant and slightly positive upward trend in both global equity comovement and global bond comovement over the past 30 years; the former evidence is consistent with Christoffersen et al. (2012) who use a different methodology, while the latter evidence is relatively new and likely related to several structure breaks as mentioned before.

2.5 Non-Parametric Tests

In this section, I confirm the stylized facts using unconditional, non-parametric data moments of standardized returns. Table 4 shows that Jennrich (1970)'s χ^2 tests reject the null of equal equity and bond unconditional correlations (see test details in Appendix D). The average pairwise data correlation is 0.6271 for equities and 0.4606 for bonds; the difference is significant ($\chi^2 = 227.087$). The average equity and bond conditional correlation estimates[8] are 0.6583 and 0.4655 using the full sample, respectively, which are statistically close to the data point estimates. Panels B–D demonstrate the fit of conditional model estimates using three 10-year subsamples. The comovement difference in data was the smallest during the second 10 years (1997/03 - 2007/02) and widened up again during and after 2007; both observations are consistent with the parametric evidence in Figure 1. To make the exact comparison, I also simulate 1,000 finite samples (T=358) of country equity and bond returns using the parameter estimates. The simulated moment estimates shown in row "Simulated Model (t)" are statistically close to the data point estimates given a 5% significance test.

To test correlation (a)symmetry in data, I follow Longin and Solnik (2001) and Ang and Chen (2002) and obtain exceedance correlations. Exceedance correlations capture the comovement of returns that fall within the same lower or upper percentiles of their own historical distributions, and are typically found to be smaller than time-series correla-

[8]They are referred to as the empirical benchmarks in the rest of the paper.

tions.[9] The global exceedance correlations using actual data reveal significant asymmetry in equity return correlations. Table 5 show that the exceedance correlation jumps from 0.2619 at the 51st percentile to 0.3292 at the 49th percentile, and the gap is statistically significant. In contrast, symmetry in bond correlations cannot be rejected. To evaluate the model fit, I simulate 100,000 periods of standardized returns using parameter estimates of the best models using both distributional assumptions.[10] The simulated global bond exceedance correlations in row "Simulated Model (t)" are within 95% confidence intervals of the data exceedance correlations across a spectrum of threshold percentiles (25%, 49%, 51%, 75%). On the other hand, asymmetry in the global equity comovement is more difficult to match, as widely acknowledged in the literature;[11] nevertheless, row "Simulated Model (t)" is able to fit the lower percentile exceedance correlations and capture the general pattern around 49%-51%. The model assuming a multivariate Gaussian distribution is rejected, which is consistent with Campbell, Forbes, Koedijk, and Kofman (2008) on the importance of "fat" tails in matching equity return correlation.

Finally, I compare the average pairwise data correlations during the world non-recession and recession periods for each asset. Table 6 shows that the pairwise non-recession sample correlation of standardized equity returns averages at 0.5952, which is significantly lower than the recession counterpart, 0.6571, given two test results.[12] On the other hand, the bond return correlation is slightly higher during non-recession periods, indicating a procyclical behavior. Hence, these non-parametric tests can replicate the cyclical results established using the parametric model.

Given both parametric and non-parametric tests, the present research formally establishes the following three stylized facts:

- Stylized Fact 1: *Equity return correlations are larger in magnitude than bond return correlations.*
- Stylized Fact 2: *Equity return correlations are higher following joint negative shocks, hence asymmetric, while bond return correlations are symmetric.*
- Stylized Fact 3: *Equity return correlations are strongly countercyclical, while bond correlations are weakly procyclical.*

[9] Daily data is typically used to ensure enough observations within each percentile range-of-interest. The exceedance correlation of standardized daily returns ($\widetilde{x}$ and $\widetilde{y}$) of a certain threshold percentile τ is $\rho(\widetilde{x},\widetilde{y}|\widetilde{x} < \Phi_x^{-1}(\tau), \widetilde{y} < \Phi_y^{-1}(\tau))$ if $\tau < 0.5$ or $\rho(\widetilde{x},\widetilde{y}|\widetilde{x} > \Phi_x^{-1}(\tau), \widetilde{y} > \Phi_y^{-1}(\tau))$ if $\tau >= 0.5$, where $\Phi_x^{-1}(\tau)$ denotes the value of a given percentile τ for variable x. Global exceedance correlations are calculated as the equally-weighted average of all pairwise exceedance correlations across 28 unique country pairs.

[10] The previous finite-sample simulation in the equality test (Table 4) is no longer suitable in calculating exceedance correlation because it is easily the case that there might be no data points *jointly* exceeding certain percentiles.

[11] I would like to thank Hugues Langlois (my 2018 EFA discussant) for helpful discussions.

[12] Test 1 is a Jennrich test on the equality between two correlation matrices of differnet sample sizes; Test 2 reports the t statistics of whether the difference between non-recession and recession correlations is zero, given 3000 bootstrapped samples.

3 Economic Determinants

Under standard rational assumptions, asset return innovations can be explained by cash flow and discount rate shocks. Accordingly, the dynamics of global return comovements should be determined by second moments of some common shocks that transmit internationally, while the comovement differences should reflect different asset return sensitivities to these common shocks. Section 3.1 formalizes this intuition using a parsimonious dynamic no-arbitrage asset pricing model that consistently prices international equities and government bonds. The main goal is to identify the economic determinants and common shocks at the equilibrium. Then, I discuss the estimation strategy of these economic determinants and shocks in Section 3.2.

3.1 An Asset Pricing Model

This asset pricing model is defined by a global pricing kernel of a U.S. (global) investor and various state variables (real fundamental, inflation, short rate, and a price-of-risk state variable). The kernel is motivated from both consumption-based and term structure literature. The asset moment of interest in the present research is a higher-order moment, exhibiting rich dynamics and asymmetric behaviors as shown in Section 2. As a result, I assume a flexible shock structure of state variables while the model solution still falls within the closed-form affine class. It is worth noting that, under market completeness, a reduced-form international asset pricing model assuming partial integration (correlated country pricing kernels) can be shown to imply the same economic determinants of *global* comovements. A detailed solution is derived in the Internet Appendix.

3.1.A The Global Real Pricing Kernel

The log real pricing kernel m_{t+1} is assumed with the following process:

$$-m_{t+1} = x_t + \mathcal{J}_t + \boldsymbol{\delta}_{\boldsymbol{m}}' \begin{bmatrix} \omega_{\theta u,t+1} & \omega_{\theta d,t+1} & \omega_{\pi u,t+1} & \omega_{\pi d,t+1} & \omega_{q,t+1} \end{bmatrix}', \tag{12}$$

where x_t is the real short rate, $\mathcal{J}_t$ Jensen's inequality term (see Internet Appendix for a full expression), and $\boldsymbol{\delta}_{\boldsymbol{m}}$ a 5-by-1 constant vector $\begin{bmatrix} \delta_{m\theta u} & \delta_{m\theta d} & \delta_{m\pi u} & \delta_{m\pi d} & \delta_{mq} \end{bmatrix}'$. The five kernel shocks include upside and downside economic real growth shocks, $\omega_{\theta u,t+1}$ and $\omega_{\theta d,t+1}$, upside and downside inflation shocks, $\omega_{\pi u,t+1}$ and $\omega_{\pi d,t+1}$, and a pure risk aversion shock, $\omega_{q,t+1}$. All shocks, defined next, are non-Gaussian, heteroskedastic and mutually independent.

3.1.B The Macro Environment

I denote the log real economic growth from t to $t+1$ as θ_{t+1}. The dynamics allows for a persistent and cyclical conditional mean and a flexible disturbance similar to Bekaert, Engstrom, and Xu (2019; BEX). One advantage of using this disturbance is to jointly incorporate realistic higher-order moment properties such as non-Gaussianity, asymmetry and heteroskedasiticity as documented in the literature (see e.g. Hamilton (1990), Fagiolo, Napoletano, and Roventini (2008), Gambetti, Pappa, and Canova (2008), Adrian, Boyarchenko, and Giannone (2019)).

Specifically, the growth disturbance consists of two independent centered gamma shocks, $\omega_{\theta u,t+1}$ and $\omega_{\theta d,t+1}$, governing the upside and downside tail behaviors separately:

$$\begin{aligned}
\theta_{t+1} &= m_{\theta,t} + \delta_{\theta\theta u}\omega_{\theta u,t+1} - \delta_{\theta\theta d}\omega_{\theta d,t+1}, && (13)\\
\omega_{\theta u,t+1} &\sim \widetilde{\Gamma}\left(\theta u_t, 1\right), && (14)\\
\omega_{\theta d,t+1} &\sim \widetilde{\Gamma}\left(\theta d_t, 1\right), && (15)\\
\theta u_{t+1} &= \overline{\theta u} + \rho_{\theta u}(\theta u_t - \overline{\theta u}) + \delta_{\theta u}\omega_{\theta u,t+1}, && (16)\\
\theta d_{t+1} &= \overline{\theta d} + \rho_{\theta d}(\theta d_t - \overline{\theta d}) + \delta_{\theta d}\omega_{\theta d,t+1}, && (17)
\end{aligned}$$

where $m_{\theta,t}$ is the conditional mean;[13] $\delta_{\theta u}$ and $\delta_{\theta d}$ are positive scale parameters; $\widetilde{\Gamma}\left(x, 1\right)$ denotes a centered gamma distribution with shape parameter x and a unit scale parameter. The two shape parameters θu_t and θd_t govern the higher conditional moments of the real upside and downside shocks, respectively (see Appendix E for statistical properties of a gamma-distributed shock). Increases in θu_t monotonically increase both the volatility and skewness, and thus this shape parameter can be dubbed as "good" uncertainty in economic sense; on the other hand, increases in θd_t lower the skewness, indicating a "bad" economic environment.[14] Finally, as the last two equations show, the shape parameters follow autoregressive processes and have positive exposures to the upside and downside shocks, respectively. This serves to capture the plausible relation between the first and second moments of macro variables: When there is a large positive realization of output growth, one might expect more good uncertainty in the future; similarly, more bad uncertainty is expected when a large negative output shock arrives.

Next, on modeling the inflation shocks, I allow inflation to have exposures to the real growth shocks, which is potentially in line with a standard New Keynesian AS curve relating inflation to the output gap. The inflation-specific innovation is similarly decomposed into two shocks governing the behaviors of the left- and right-tails of the distribution; this

[13] The conditional mean is analogous to a GARCH-in-mean structure, $m_{\theta,t} = \overline{\theta} + \rho_\theta(\theta_t - \overline{\theta}) + m_{\theta u}(\theta u_t - \overline{\theta u}) + m_{\theta d}(\theta d_t - \overline{\theta d})$.

[14] In mathematical expressions, given the moment generating function (MGF) of gamma-distributed shocks, the conditional variance of θ_{t+1} can be derived as $\delta^2_{\theta\theta u}\theta u_t + \delta^2_{\theta\theta d}\theta d_t$, and the conditional unscaled skewness as $2\delta^3_{\theta\theta u}\theta u_t - 2\delta^3_{\theta\theta d}\theta d_t$.

is to intuitively capture the potential asymmetry and different economic explanations for upside and downside inflation uncertainties. Denote π_{t+1} as the inflation rate from t to $t+1$. The inflation process is assumed with the following reduced-form dynamics:

$$\pi_{t+1} = m_{\pi,t} + (\delta_{\pi\theta u}\omega_{\theta u,t+1} + \delta_{\pi\theta d}\omega_{\theta d,t+1}) + (\delta_{\pi\pi u}\omega_{\pi u,t+1} - \delta_{\pi\pi d}\omega_{\pi d,t+1}), \tag{18}$$

$$\omega_{\pi u,t+1} \sim \widetilde{\Gamma}(\pi u_t, 1), \tag{19}$$

$$\omega_{\pi d,t+1} \sim \widetilde{\Gamma}(\pi d_t, 1), \tag{20}$$

$$\pi u_{t+1} = \overline{\pi u} + \rho_{\pi u}(\pi u_t - \overline{\pi u}) + \delta_{\pi u}\omega_{\pi u,t+1} \tag{21}$$

$$\pi d_{t+1} = \overline{\pi d} + \rho_{\pi d}(\pi d_t - \overline{\pi d}) + \delta_{\pi d}\omega_{\pi d,t+1}, \tag{22}$$

where the conditional mean $m_{\pi,t}$ is a persistent process,[15] and the two inflation shocks follow independent centered gamma distributions with time-varying shape parameters. Similarly, πu_t and πd_t are upside and downside inflation uncertainties, respectively. This inflation process is new to the literature.

Together, the four macro shocks are mutually independent and their shape parameters determine time-varying fundamental uncertainties.[16]

3.1.C Risk Aversion

Risk aversion, although conventionally perceived as a slow-moving cyclical variable or simply a constant, has been shown *empirically* to fluctuate significantly given high-frequency cognitive changes (such as fear and anxiety) in a lab experiment by Cohn et al. (2015). Moreover, Martin (2017) implies that risk premium and risk aversion need be extremely volatile in order to reconcile with option prices.

Hence, given the recent micro and macro evidence, I model risk aversion to receive both fundamental and non-fundamental shocks. In particular, from the perspective of equilibrium models, a non-fundamental risk aversion shock entering the pricing kernel can be motivated from a HARA-type utility function. This is because such utility functions allow for an exogenous time-varying reference point to determine the relative risk aversion (RRA), and the reference point can is not necessarily driven by fundamentals, as pointed out by BEX. I hereby follow the literature to denote the risk aversion state variable as q_t and assume a parsimonious linear process:

$$q_{t+1} = m_{q,t} + \underbrace{\delta_{q\theta u}\omega_{\theta u,t+1} + \delta_{q\theta d}\omega_{\theta d,t+1} + \delta_{q\pi u}\omega_{\pi u,t+1} + \delta_{q\pi d}\omega_{\pi d,t+1}}_{\text{fundamental shock exposure}} + \delta_{qq}\omega_{q,t+1}, \tag{23}$$

$$\omega_{q,t+1} \sim \widetilde{\Gamma}(q_t, 1), \tag{24}$$

[15] $m_{\pi,t} = \overline{\pi} + \rho_{\pi\theta}(\theta_t - \overline{\theta}) + \rho_{\pi\theta u}(\theta u_t - \overline{\theta u}) + \rho_{\pi\theta d}(\theta d_t - \overline{\theta d}) + \rho_{\pi\pi}(\pi_t - \overline{\pi}) + \rho_{\pi\pi u}(\pi u_t - \overline{\pi u}) + \rho_{\pi\pi d}(\pi d_t - \overline{\pi d})$.

[16] Specifically, the conditional variance of inflation can be derived as $\left(\delta^2_{\pi\theta u}\theta u_t + \delta^2_{\pi\theta d}\theta d_t\right) + \left(\delta^2_{\pi\pi u}\pi u_t + \delta^2_{\pi\pi d}\pi d_t\right)$, and its conditional unscaled skewness is $\left(2\delta^3_{\pi\theta u}\theta u_t + 2\delta^3_{\pi\theta d}\theta d_t\right) + \left(2\delta^3_{\pi\pi u}\pi u_t - 2\delta^3_{\pi\pi d}\pi d_t\right)$.

where the conditional mean $m_{q,t}$ is a linear function of $q_t, \theta_t, \theta u_t, \theta d_t, \pi_t, \pi u_t$ and πd_t, and the risk aversion disturbance is sensitive to the four macro shocks and an orthogonal, non-fundamental shock ("pure risk aversion shock"). Similarly, the conditional higher-order moments of risk aversion contain both fundamental and non-fundamental parts.[17]

I share two insights next. First, the modeling of such a non-fundamental shock is rather exploratory given the lack of empirical evidence. However, it is plausible that there is a positive relation between the level and volatility of risk aversion: When the current aggregate risk aversion is high (low), we expect future risk aversion to fluctuate more (less). My model is agnostic about the exact non-fundamental source. Second, while conventional wisdom considers risk aversion and fundamental shocks to enter the kernel in a multiplicative way, as in Campbell and Cochrane (1999; CC), the present kernel and the CC kernel are still consistent. It is because, in both models, risk aversion contributes asymmetry to the kernel innovation, and thus to the asset prices at the equilibrium. One advantage of the present kernel is the possibility of a closed-form solution given the linear shock structure.

3.1.D Real Short Rate

I assume an isomorphic process for the real short rate,

$$x_{t+1} = m_{x,t} + f_x\left(\omega_{\theta u,t+1}, \omega_{\theta d,t+1}, \omega_{\pi u,t+1}, \omega_{\pi d,t+1}, \omega_{q,t+1}\right) + \delta_{xu}\omega_{xu,t+1} - \delta_{xd}\omega_{xd,t+1}, \quad (25)$$

where the conditional mean $m_{x,t}$ is a linear function of x_t, q_t, θ_t, θu_t, θd_t, π_t, πu_t, πd_t and two new variances of short rate shocks, xu_t and xd_t. The real short rate disturbance has a "systematic" component $f_x()$ which is sensitive to the common fundamental and risk aversion shocks. While its exposure to the real and nominal macro shocks are intuitive, its exposure to the risk aversion shock can also be motivated by the Campbell and Cochrane model through, for instance, a precautionary savings channel.

The residual is decomposed into two centered gamma shocks with autoregressive shape parameters:

$$\omega_{xu,t+1} \sim \widetilde{\Gamma}\left(xu_t, 1\right); xu_{t+1} = \overline{xu} + \rho_{xu}(xu_t - \overline{xu}) + \delta_{xu}\omega_{xu,t+1}, \quad (26)$$

$$\omega_{xd,t+1} \sim \widetilde{\Gamma}\left(xd_t, 1\right); xd_{t+1} = \overline{xd} + \rho_{xd}(xd_t - \overline{xd}) + \delta_{xd}\omega_{xd,t+1}. \quad (27)$$

The two real short rate shocks, $\omega_{xu,t+1}$ and $\omega_{xd,t+1}$, can be interpreted as discretionary monetary policy shocks in this framework because they are "cleansed" from various sys-

[17]Specifically, the model-implied conditional variance of risk aversion is $\left(\delta^2_{q\theta u}\theta u_t + \delta^2_{q\theta d}\theta d_t\right) + \left(\delta^2_{q\pi u}\pi u_t + \delta^2_{q\pi d}\pi d_t\right) + \delta^2_{qq}q_t$, and the conditional unscaled skewness $\left(2\delta^3_{q\theta u}\theta u_t + 2\delta^3_{q\theta d}\theta d_t\right) + \left(2\delta^3_{q\pi u}\pi u_t + 2\delta^3_{q\pi d}\pi d_t\right) + 2\delta^3_{qq}q_t$.

tematic monetary policy determinants. I assume no feedback from short rate shocks to the risk aversion process for simplicity.

3.1.E Closed-Form Solution

To price U.S. equities, the real dividend growth process is assumed to receive global real shocks ($\omega_{\theta u}$ and $\omega_{\theta d}$), and its residual is the global dividend shock ω_g. The global dividend shock is assumed to be homoskedastic. To price individual country equities, country dividend growth processes are sensitive to global real and dividend shocks ($\omega_{\theta u}$, $\omega_{\theta d}$ and ω_g); similarly, to price international nominal assets, country inflation processes are sensitive to global real and nominal shocks ($\omega_{\theta u}$, $\omega_{\theta d}$, $\omega_{\pi u}$ and $\omega_{\pi d}$). Country-specific dividend growth and inflation shocks are then assumed to be homoskedastic and mutually independent across countries.

In this global model, assets are priced from the perspective of a U.S. (global) investor. Given the MGF of a gamma-distributed variable and the Euler equation, it can be shown that the model has a closed-form solution in an (quasi) exponential affine class; Internet Appendix V provides detailed derivations. As a result, the log asset return for country-asset i from t to $t+1$, r^i_{t+1}, can be expressed with the following general process similar to a *dynamic factor model*:

$$\begin{aligned} r^i_{t+1} &= \text{Conditional Mean} + \text{Global Shock Exposure} + \text{Country-Specific Residual}, \\ &\quad (+ \text{Approximation Error}), \end{aligned} \tag{28}$$

where the approximation error, assumed to be homoskedastic and Gaussian, comes from linearizing $\ln\left(\frac{PD_{t+1}+1}{PD_t}\right)$ where the price dividend ratio PD follows an exact exponential affine function of the state variables; this error is absent from the log bond return solution. The global shock exposure is key to determining the return comovement dynamics. The model implies seven heteroskedastic, mutually independent global shocks,

$$\{\omega_q, \omega_{\theta u}, \omega_{\theta d}, \omega_{\pi u}, \omega_{\pi d}, \omega_{xu}, \omega_{xd}\},$$

and a single homoskedastic Gaussian shock that summarizes the global dividend shock, country-specific shocks, and a potential approximation error.[18]

Hence, the present asset pricing model sheds light on the stylized facts established in Section 2 in two ways. (1) The time variation in global return comovements is driven by second moments of these seven heteroskedastic global shocks. These second moment state variables are the relevant *economic determinants* of global comovements in the present model. (2) Different asset return sensitivities to these global shocks determine the comovement differences across different asset classes.

[18]The sum of Gaussian shocks also follows a Gaussian distribution.

3.2 Identification of Economic Determinants and Shocks

In this section, I describe the estimation results of the seven economic determinants and shocks that are motivated from the model solution. I first filter the real uncertainties, θu_t and θd_t, from the monthly changes in log industrial production index (source: FRED) as the proxy for real economic growth; the estimation adopts Bates (2006)'s Approximate Maximum Likelihood (AML) estimation methodology, which conveniently allows filtering non-Gaussian shocks and exploits exponential affine characteristic functions. Then, given the shock orthogonality assumption, I project the month changes in log consumer price index (source: FRED) as the proxy for inflation onto real uncertainties and shocks, and then filter the inflation-*specific* uncertainties, πu_t and πd_t, and shocks from the residual using AML. I use the longest sample available (from January 1947 to December 2016) to estimate the real and inflation processes.

Figure 2 presents the dynamics of the seven economic determinants of global co-movements in a balanced sample (1987.03–2015.02).[19] As depicted in the second row, the real upside uncertainty (left plot) can be tested to be acyclical using the long sample but weakly procyclical using the paper sample (1987.03–2015.02) with a NBER correlation coefficient of -0.38. The real downside uncertainty (right plot) is strongly countercyclical with a positive NBER recession correlation coefficient of 0.71 using the long sample.

It is noteworthy that, to correctly interpret their economic magnitudes, these uncertainty state variables in Figure 2 need scaling. For instance, the total real growth conditional variance is $\delta^2_{\theta\theta u}\theta u_t + \delta^2_{\theta\theta d}\theta d_t$. My estimation shows that $\delta_{\theta\theta u} = 0.0001$ and $\delta_{\theta\theta d} = 0.0019$ (see Table A6 of the Internet Appendix). Although θu_t centers at 500 and θd_t is on average around 13, the downside variability in the real economic growth accounts for 90% of the total variability (e.g. $\frac{0.0019^2 \times 13}{0.0019^2 \times 13 + 0.0001^2 \times 500}$).

As depicted in the third row of Figure 2, the inflation-specific upside uncertainty state variable πu_t is weakly countercyclical with a NBER recession correlation of 0.16 using the long sample (starting 1947) and of 0.39 using the balanced sample (starting 1987). The inflation-specific upside and downside uncertainties are uncorrelated at 0.07. In the long sample, high inflation upside uncertainty seems to appear in clusters, for example in the 1973 recession, the 1980s recession, and the recent financial crisis.[20] The countercyclicality in the inflation variability is potentially consistent with Ball (1992). On the one hand, when the actual and expected inflation is low, there is a consensus that the monetary authority will try to keep them low. On the other hand, when inflation is high, the public does not know whether the policymaker will disinflate or keep inflation high due to the fear that disinflation could result in a recession; this dispersion in beliefs potentially results in increases in the inflation upside uncertainty. This "high inflation-

[19] Long sample plots of economic uncertainties are available in the Internet Appendix, Figures A3 and A4.

[20] Figure A4 of the Internet Appendix provides the long-sample plot.

high upside uncertainty" theory is also consistent with the modeling of inflation shocks in the inflation uncertainty processes in Section 3.1.B. The inflation-specific downside uncertainty πd_t exhibits a significant upward trend in the past 30 years.

Given that real and nominal macro shocks are orthogonal, this framework is able to quantify the relative importance of the four shocks in explaining the total inflation variability. I find that inflation-specific upside and downside uncertainties on average account for the majority of the total inflation variance (47.63% and 50.25%, respectively), while the real upside and downside uncertainties together explain less than 3% during the sample period. While the persistent inflationary pressure from 2005 till the last quarter of 2008 explains the high inflation upside uncertainty, there is a significant and sharp increase in the inflation downside uncertainty during the Lehman Brothers aftermath; between 2008 and the end of the sample, the inflation downside uncertainty *dominates* the total inflation uncertainty with an average share of 71%, suggesting the potentially more important role of aggregate demand in the recent decades. This result sheds light on the ongoing debate on the relative importance of aggregate demand and supply.

Investor risk aversion is difficult to measure. Bekaert, Engstrom, and Xu (2019) filter a utility-based risk aversion index from a wide set of macro and financial risk variables, given asset pricing implications derived at the equilibrium. I use their risk aversion estimate as the empirical proxy for q_t in my paper because of the consistent discount rate part of both models; specifically, both pricing kernels can be motivated from a representative agent, endowment economy with a HARA utility, allowing both fundamental and non-fundamental shocks to span time-varying RRA. It is true that my model accommodates short rate and inflation processes (in order to price nominal assets) and inflation uncertainties and shocks feed back on risk aversion, while their model focuses on pricing risky assets. This feedback turns out to have small economic significance, as the filtered inflation-specific shocks in my estimation are tested to be empirically uncorrelated with the risk aversion shock in their estimates using the overlapping sample ($\rho(\omega_q^{BEX}, \omega_{\pi u})$=0.038, $\rho(\omega_q^{BEX}, \omega_{\pi d})$=-0.037). The first plot of Figure 2 depicts the RRA state variable, q_t, from Bekaert, Engstrom, and Xu (2019), and RRA can be obtained as $\gamma \exp(q_t)$ with $\gamma = 2$ being the utility curvature. Risk aversion is weakly countercyclical and much more transitory than that implied from the Campbell and Cochrane model. It can also spike up during non-NBER recession periods, for instance during the October 1987 crash, the LTCM collapse in 1998, the end of the TMT bull in August 2002, and the Euro area debt crisis in late 2011. Finally, this risk aversion measure is highly correlated with VIX, a finding that Martin (2017) also implies.

Lastly, the estimation procedure of the latent real short rate uncertainties exploits the no-arbitrage condition and the assumed reduced-form pricing kernel shock structure. That is, given the closed-form solution (see Table A8 and Internet Appendix V), the latent real 30-day T-bill rate is a part of the observed nominal 30-day T-bill rate (source: CRSP);

with the gamma shock assumptions, uncertainties and shocks of the real short rate and unknown kernel loadings ($\boldsymbol{\delta_m}$) can be simultaneously estimated using Bates (2006)'s AML methodology.[21]Given the estimation results, exposures to inflation shocks explain around 64% of the total real short rate variability, while risk aversion explains 17% with a negative coefficient which is consistent with the precautionary savings effect. Between the two short rate-specific uncertainties, the short rate-specific downside uncertainty explains a share of 73% on average, i.e. $E\left[\frac{\delta_{xd}^2 xd}{\delta_{xu}^2 xu+\delta_{xd}^2 xd}\right]$. From the bottom two plots of Figure 2, the real short rate upside uncertainty between 1986 and 1989 is high; in the early 1990s, however, the downside uncertainty starts to become relatively more relevant. Interestingly, from the period covering the last 10 years, the nominal short rate is close to the zero boundary; given the positive expected inflation, the real short rate is effectively negative, and the share of xd has grown to 88%.

4 A Theory-Motivated Dynamic Factor Model

In this section, I evaluate the ability of the international dynamic factor model implied from Section 3 to fit (Section 4.1) and interpret (Sections 4.2–4.3) the three stylized facts established in Section 2. Suppose there are N (8) log asset return series for each asset class and P (7) global factors with time-varying second moments. To summarize, log equity and bond returns, $\boldsymbol{r_{t+1}}$, can be expressed with the following general process:

$$\underbrace{\boldsymbol{r_{t+1}}}_{2N\times 1} = \underbrace{\boldsymbol{E_t}\left[\boldsymbol{r_{t+1}}\right]}_{2N\times 1} + \underbrace{\begin{bmatrix} \boldsymbol{\Omega_{t+1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega_{t+1}} & \cdots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Omega_{t+1}} \end{bmatrix}}_{2N\times 2NP} \underbrace{\begin{bmatrix} \boldsymbol{\beta_{1,t}} \\ \boldsymbol{\beta_{2,t}} \\ \vdots \\ \boldsymbol{\beta_{2N,t}} \end{bmatrix}}_{2NP\times 1} + \underbrace{\boldsymbol{\varepsilon_{t+1}}}_{2N\times 1}, \tag{29}$$

where $\boldsymbol{E_t}\left[\boldsymbol{r_{t+1}}\right]$ denotes a column vector of expected returns which, in my model, is a linear function of state variables; $\boldsymbol{\Omega_{t+1}}$ denotes a row vector of the global heteroskedastic shocks (factors) as motivated and defined in Section 3,

$$\boldsymbol{\Omega_{t+1}} = \begin{bmatrix} \omega_{q,t+1} & \omega_{\theta u,t+1} & \omega_{\theta d,t+1} & \omega_{\pi u,t+1} & \omega_{\pi d,t+1} & \omega_{xu,t+1} & \omega_{xd,t+1} \end{bmatrix}.$$

Given the empirical focus of the paper, I also allow the possibility that betas of global factors are time-varying. For each country-asset class, the return sensitivity to each shock

[21]This methodology of filtering the real short rate using the no-arbitrage assumption and appropriate shock assumptions is commonly used in the bond pricing literature (e.g., Chen and Scott (1993), Ang, Bekaert, and Wei (2008)); here, the shock assumption is non-Gaussian.

is defined as a linear function of a standardized instrument s_t:

$$\beta_t = \beta_0 + \beta_1 s_t. \tag{30}$$

The residuals are assumed with a parsimonious correlated structure as motivated in Section 3.1.E:

$$E\left[\boldsymbol{\varepsilon_{t+1}}|\boldsymbol{\Omega_{t+1}}\right] = \mathbf{0}, \tag{31}$$

$$E\left[\boldsymbol{\varepsilon_{t+1}}\boldsymbol{\varepsilon'_{t+1}}|\boldsymbol{\Omega_{t+1}}\right] = \boldsymbol{\Sigma}. \tag{32}$$

This dynamic factor model conveniently implies a pairwise conditional covariance between Countries i and j as follows:

$$\boldsymbol{\beta'_{i,t}} Var_t\left(\boldsymbol{\Omega_{t+1}}\right)\boldsymbol{\beta_{j,t}} + Cov(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}), \tag{33}$$

where $\boldsymbol{\beta_{i,t}}$ $(P \times 1)$ and $\boldsymbol{\beta_{j,t}}$ $(P \times 1)$ are return sensitivities to $\boldsymbol{\Omega_{t+1}}$, and $Var_t\left(\boldsymbol{\Omega_{t+1}}\right)$ $(P \times P)$ a conditional covariance-variance matrix of common shocks with conditional variances $\{q_t, \theta u_t, \theta d_t, \pi u_t, \pi d_t, xu_t, xd_t\}$ on the diagonal and zeros on the off-diagonal. The model-implied pairwise conditional correlation is

$$\frac{\boldsymbol{\beta'_{i,t}} Var_t\left(\boldsymbol{\Omega_{t+1}}\right)\boldsymbol{\beta_{j,t}} + Cov(\varepsilon_{i,t+1}, \varepsilon_{j,t+1})}{\sqrt{\boldsymbol{\beta'_{i,t}} Var_t\left(\boldsymbol{\Omega_{t+1}}\right)\boldsymbol{\beta_{i,t}} + Var(\varepsilon_{i,t+1})}\sqrt{\boldsymbol{\beta'_{j,t}} Var_t\left(\boldsymbol{\Omega_{t+1}}\right)\boldsymbol{\beta_{j,t}} + Var(\varepsilon_{j,t+1})}}. \tag{34}$$

The *global* correlation is then the equally-weighted average of all unique pairwise correlations, an assumption consistent with the DECO-class empirical models presented in Section 2.

4.1 Estimation and Model Fit

This dynamic factor model is a system of regression equations with correlated residuals, which is in the class of Zellner (1962)'s Seemingly Unrelated Regression (SUR). I use feasible Generalized Least Squares (FGLS) estimators for betas and the residual covariance matrix (see Zellner (1962) and Zellner and Huang (1962)), and estimate them jointly with MLE.[22] On the choice of (standardized) instrument s_t, I use inflation upside uncertainty πu_t as the instrument to span equity betas, given Ball (1992)'s argument

[22]Here is some more reasoning. With correlated residuals, the Ordinary Least Squares (OLS) estimators are no longer Best Linear Unbiased Estimators (BLUE), whereas Generalized Least Squares (GLS) estimators are, by construction (Greene, 2003). Both OLS and GLS estimators are unbiased and consistent; however, the variance of the OLS estimator is biased and inefficient. Then again, GLS assumes a known residual covariance matrix, which is an unrealistic assumption. The FGLS estimator is preferred because it assumes Σ is unknown.

that this uncertainty is very informative about dispersion in beliefs.[23] On the other hand, there is little theoretical or empirical research examining time-varying bond betas. Therefore, the beta instruments for bond returns are selected based on the empirical fit, and real upside uncertainty θu_t filtered from industrial production growth is chosen.[24] In sum, the equity beta instrument (bond beta instrument) is countercyclical (procyclical) with a NBER recession correlation of -0.39 (0.30) during the paper sample period. Equation-level parameter estimates are available in the Internet Appendix.

Figures 3 and 4 compare the factor model-implied global equity and bond return conditional correlations with the empirical benchmarks from Section 2, respectively. Re-

[23]Time-varying betas, or conditional betas, in *equity* returns can be motivated from existing empirical and theoretical findings. Empirical studies have found that responses of stock return volatility to macroeconomic news depends on the conditional states of the business cycle (see e.g. Andersen, Bollerslev, Diebold, and Vega (2007)). In addition, time-varying betas can also arise in economic models exhibiting various departures from rational expectations, and I briefly discuss two plausible mechanisms below. First, according to the Bayesian Learning theory by David and Veronesi (2013), investors take time to learn about shifts in economic states by observing signals from fundamental and non-fundamental shocks, during high economic uncertainty periods. In times of precise prior beliefs, large news is not necessary to move posterior probabilities (i.e., small betas); but when there is large uncertainty, which may be correlated with economic uncertainty measures, even small news moves posterior distributions (i.e., large betas). As a second theoretical motivation, Bansal and Shaliastovich (2010) propose a Confidence Risk Theory which suggests that a widening cross-section of variance in economic signals lowers investors' confidence placed in future growth forecasts, leading to larger declines in asset prices, although there are no large moves in consumption. These mechanisms suggest that the time variation in equity betas might be related to dispersion in beliefs and economic uncertainty in their theoretical predictions. In fact, inflation upside uncertainty as the instrument indeed improves the empirical fit the most, among the six economic uncertainties in this paper.

[24]Here is the exact procedure. 1. I discuss several potential structure breaks identified in the empirical estimates in Section 2.4; in particular, the early 1990s and 1999 sudden increases in the global bond correlation closely relate to the massive economic and monetary union restructuring (or "structural breaks") in the European sovereign debt market, which is also documented in separate studies (see e.g. Codogno, Favero, and Missale (2003), Cappiello, Engle, and Sheppard (2006), Bernoth, Von Hagen, and Schuknecht (2012)). Because my model does not involve *central banks*, the focus of the present research is to explain the cyclical behaviors rather than trends due to structural breaks. 2. To find the best instruments for bond betas, I examine three aspects to evaluate the closeness:

(1) Evidence from the correlation fit. After filtering out a Hodrick-Prescott trend (using a coefficient of 14400 as suggested when using monthly data; Hodrick and Prescott (1997)), I correlate the cyclical part with the model-implied global correlations using 6 time-varying betas that are spanned by 6 economic uncertainty state variables in the paper. The closeness improves from only 0.02 (insignificant from 0) with constant betas to a positive value of 0.0967 when the instrument is the real upside uncertainty state variable, θu. That is from the perspective of the fit of correlation.

(2) Evidence from the variance fit. An economically meaningful factor model to explain the covariance between country returns should also perform well in explaining conditional variances of country returns. I calculate the average explanatory power of the bond model using each of the economic state variables. The time-varying beta model with θu as the instrument exhibits the highest R^2 of 0.145; for other instruments, 0.132 for θd, 0.123 for for πu, 0.126 for πd, 0.128 for xu, 0.126 for xd.

(3) Evidence from the covariance fit. In analogy, I also compare the average pure covariances explained by the model out of the data covaraince (both excluding the diagonal terms) across all country bond equations; that is, $1-\frac{\text{sum of model-implied residual covariances}}{\text{sum of data covariances}}$. The time-varying beta model with θu as the instrument exhibits the highest explanatory of the pure covariance terms of 0.156; for other instruments, 0.129 for θd, 0.120 for for πu, 0.117 for πd, 0.114 for xu, 0.128 for xd.

garding the fit of global equity return correlation, the constant beta model already generates a reasonably high correlation (0.525) with the empirical benchmark. From the top plot of Figure 3, the constant-beta global equity comovement measure is able to match the October 1987 spike and the drops during the early 1990s and the long expansion between 2001 and 2007. However, the constant beta model underestimates the global equity return comovement during the peak of the 2007-2008 financial crisis by 0.1, and overestimates by as much as 0.2 during the 1990s. From the bottom plot, the time-varying beta model significantly improves the constant beta model with a higher correlation with the empirical benchmark (0.623). Graphically, the time-varying beta model is able to match the global equity comovement during the 2007-2008 financial crisis and generate a wider gap between recession and non-recession global equity correlation levels.

It is important to mention that the present asset pricing theory does not model central banks, which precludes the model from generating several potential structural breaks in international sovereign bond markets (as discussed in Section 2.4 and again in Footnote[24]); hence, a fair comparison is between the factor model-implied bond correlation and the cyclical component of the empirical benchmark. From the right plots of Figure 4, the dynamic fit of global bond return correlation thus improves from a correlation of 0.02 (top) to 0.1 (bottom) after allowing for time-varying betas, despite the parsimony of a linear factor structure. One obvious improvement appears around October 1987 where international bond prices were highly correlated but then started to diverge immediately after the Black Monday. The other improvement appears around 1995-97. The time-varying beta instrument, real upside uncertainty θu, plays the role of switching economic environment to capture the possibility that bond risk characteristics might change.

Table 7 formally evaluates the ability of this theory-motivated dynamic factor model to fit the three stylized facts established in Section 2. I express each stylized fact by two numerical moments. The main result is that time-varying beta models with a full set of global shocks are able to match *all* three stylized facts. On the first stylized fact, both constant and time-varying beta models are able to match the magnitudes given the insignificant t statistics. Next, constant beta models fail to match the second stylized fact given that the upside comovement difference (0.2307) is higher than the downside comovement difference (0.2269), whereas the time-varying beta model is able to generate a slightly higher downside comovement difference. Regarding the third stylized fact, both models generate cyclicality coefficients that are statistically close to the data point estimates calculated using the original global bond correlation benchmark. After controlling for the potential structural breaks in global bond correlation mentioned above, the (pro)cyclicality coefficient implied from the time-varying beta model (-0.034) is much closer to the data coefficient (-0.029) than that implied from the constant beta model (-0.012), which further supports the dominant performance of time-varying beta models.

4.2 Global Comovement Decomposition

One advantage of this dynamic factor model is that it provides a parsimonious framework for cleanly evaluating the contribution of each factor to the fit. To achieve this, I conduct two global comovement decomposition exercises.

4.2.A Conditional Covariance

The first decomposition exercise aims to understand the sources of time-varying global *covariance* of an asset class. Because the "$Cov(\varepsilon_{i,t+1}, \varepsilon_{j,t+1})$" part in Equation (33) only accounts for (on average) < 10% of the total conditional covariance and this section focuses on the dynamics decomposition, I construct and evaluate the share explained by factor ω_κ out of the total time-varying part:

$$\frac{\beta_{i,t,\kappa} Var_t(\omega_{\kappa,t+1}) \beta_{j,t,\kappa}}{\boldsymbol{\beta'_{i,t}} Var_t(\boldsymbol{\Omega_{t+1}}) \boldsymbol{\beta_{j,t}}}, \tag{35}$$

where $\beta_{i,t,\kappa} = \beta_{i,0,\kappa} + \beta_{i,1,\kappa} s_t$ ($\beta_{j,t,\kappa} = \beta_{j,0,\kappa} + \beta_{j,1,\kappa} s_t$) denotes the sensitivities of country asset return i (j) on $\omega_{\kappa,t+1}$; the values of βs are given by the full estimation results; $Var_t(\omega_{\kappa,t+1})$ denotes the conditional variance of that factor (e.g., $Var_t(\omega_{q,t+1}) = q_t$, $Var_t(\omega_{\theta u,t+1}) = \theta u_t$).

Then, I obtain the average of the shares (across all months and all 28 unique country pairs) and report the global covariance decomposition results in Table 8. Three observations are worth highlighting:

First, the risk aversion factor explains around 90% of the fitted equity return covariance, in both the constant beta and time-varying beta models. This quantitative result immediately contributes to the ongoing debate about the relative importance of U.S. risk factors that transmit into international risky asset markets and drive a global financial cycle (Miranda-Agrippino and Rey (2015)). The present independent finding supports the potentially stronger role of a pure risk compensation channel as opposed to other fundamental variables modeled in this paper (e.g., production comovement). My finding also adds to the recent stock return predictability literature given the close relation between variance risk premium and risk aversion.[25] The extant literature typically finds robust and positive predictive power of variance risk premium for equity excess returns, but weak predictive power of fundamental macro uncertainties — which often appear with wrong signs.[26] Besides the dominant role of risk aversion in explaining international

[25]Bekaert, Engstrom, and Xu (2019) show that risk aversion filtered from equity and corporate bond markets appears to be highly correlated with variance risk premium, which confirms their close relation as suggested in the literature.

[26]Domestic evidence includes, but not limited to, Bollerslev, Tauchen, and Zhou (2009), Bekaert and Hoerova (2014); international evidence includes Bollerslev, Marrone, Xu, and Zhou (2014), Londono and Xu (2019), among many others.

equity return covariances, real economic uncertainties also explain a share of around 5% to 7%.

Second, with respect to the bond covariance decomposition, inflation upside uncertainties has the highest share (48.6%) in the factor model with time-varying betas, as shown in the last column of Table 8; the constant part "$[\beta_0]$", $\frac{\beta_{i,0} Var_t(\omega_{\pi u,t+1})\beta_{j,0}}{\boldsymbol{\beta}'_{i,t} Var_t(\boldsymbol{\Omega}_{t+1})\boldsymbol{\beta}_{j,t}}$, already admits 43.1%. Risk aversion is the second statistically important factor with a share of 40%, followed by real short rate upside uncertainty with a share of 22%. While it is economically intuitive to think that global factors of inflation risk, risk aversion and short rate risk influence international bond prices, the main goal in the present research is to propose their relative importance in a consistent framework. Given that inflation and short rate have immediate "cash flow" effects on the bond/yield pricing, my evidence shows that the global risk aversion factor highlights an equally important "risk compensation" channel.

Based on my findings, this global risk aversion factor captures the fact that international bonds are not always priced as safe assets. Equation-level parameter estimates (Internet Appendix, Table A10) show that the U.S. and Japan government bond returns exhibit significant and positive β_0 estimates with the global risk aversion shock while other countries start with negative or insignificant β_0 loadings on risk aversion. When real upside uncertainty (or good uncertainty) increases substantively during good times ($s_b > 0$), the sensitivities of almost all foreign bond returns to the global risk aversion shock turn positive given that β_1 estimates are mostly significant and positive; or $\beta_0 + \beta_1 s_b > 0$. Country bonds are all priced as safe assets when worldwide economic growth is anticipated in the future, thus driving up the global bond return comovement. On the other hand, during bad times with low expectations of good volatility ($s_b < 0$), risk characteristics of international bond prices seem to diverge as some risk aversion sensitivities turn negative while others remain positive, thus driving down the global bond return comovement. As a result, we observe a procyclical global bond return comovement.

To be exact, a one standard deviation (SD) increase in real upside uncertainty ($s_b = 1$) is enough to switch the signs of bond return sensitivities to the global risk aversion shock from negative to positive for all four European countries (Germany, France, United Kingdom, Switzerland); with a 2.79 SD increase, all bonds are "safe". The October 1987 event corresponds to a 1.67 SD in the global real upside uncertainty (above its average). In a further improvement, the time-varying beta model now is able to capture the increase in global bond comovement during the expansion between 1995 and 1997. This improvement is again due to the beta instrument reflecting good and bad states of the global economy; the peaks correspond to around a 2.8 SD increase in the real upside economic uncertainty.

It is important to also mention that other factor loadings do not diverge the way risk aversion loadings do during bad times. For the inflation upside shock, *all* bond prices with significant loadings decrease with $\omega_{\pi u}$ during bad times ($s_b < 0$; Internet Appendix, Table A10); however, during good times ($s_b > 0$), *all* bond prices now increase with $\omega_{\pi u}$. Therefore, global bond comovement increases with a πu shock during all periods, rendering the effect of risk aversion a unique explanation of the cyclical behaviors of global bond return comovement.

Third, a global factor model with constant betas already explains 49.4% of the total equity return *covariance*, as shown in the penultimate row in Table 8. With time-varying betas, the factor model has better explanatory power at 54.6%.[27] On the other hand, fitting bond return covariances with a constant-beta factor model is admittedly difficult with 0.9% explained. However, a time-varying beta model demonstrates a significant improvement, 15.6%.

Figure 5 shows pairwise conditional covariances implied from the time-varying beta models for equities (left panel) and bonds (right panel); the share is calculated across all country pairs in a given month, and its time-series average is presenteds in Table 8. The main observation is that all pairwise equity covariances are positive during all times; on the other hand, some bond covariances can turn significantly negative during certain periods, which can be interpreted as divergence in international bond risk characteristics. Moreover, while risk aversion on average explains 90% of the total equity covariance, macroeconomic uncertainties (real and inflation) explain around 40% during the immediate aftermath of the Lehman Brothers collapse. It is interesting to find that risk aversion and economic uncertainties – both being strongly countercyclical premium state variables – exhibit different contributions to asset price comovements: global risk aversion consistently exhibits greater explanatory power than a wide range of uncertainties (real, inflation, monetary policy) considered in this paper during *normal* periods. Such pattern has not been suggested by extant theories.

4.2.B Conditional Correlation

While covariances can be easily linearly decomposed, a correlation decomposition is not as straightforward. I devise a correlation decomposition test next. Denote $CORR_{0,t}$ as the factor model-implied global conditional correlation using all the seven heteroskedastic factors, $CORR_{\backslash\kappa,t}$ as the factor model-implied global conditional correlation using all factors except for factor ω_κ given the re-estimation,[28] and BM_t as the empirical benchmark.[29] Under the null that a particular factor ω_κ has zero contribution,

[27] The share of total explained covariance is calculated by dividing the time-series average of pairwise conditional covariance by the unconditional pairwise covariance matrix, and then taking the equal-weight cross-sectional average.

[28] Whether I re-estimate the system or not does not make major difference between all shocks are assumed and estimated to be mutually independent.

[29] As motivated above, the empirical benchmark for global bond correlation uses its HP-filtered measure for a fair comparison.

$\rho(CORR_{0,t}, BM_t) - \rho(CORR_{\setminus\kappa,t}, BM_t)$ should be indifferent from zero; the larger (more positive) the difference is, the higher the marginal contribution this factor has.

The first row of Table 9 reports the sample correlations between the empirical benchmark and the full-shock factor model estimates implied from the time-varying beta models. The second row shows that the marginal contribution of the risk aversion shock in the fit of global equity correlation (global bond correlation) is 0.866 (0.138); to interpret these numbers, it means that dropping the risk aversion shock could result in zero or even *negative* correlations with the empirical benchmarks. Furthermore, the rest of the table reports the marginal contributions of economic uncertainties in the dynamic fit. The results consistently show that dropping any of the six uncertainty shocks from the equity factor model does not weaken the dynamic fit as much as dropping risk aversion. However, uncertainty shocks matter more for the bond factor model; in particular, eliminating the inflation upside uncertainty shock could drop the total dynamic fit by half (i.e., comparing 0.044 with 0.097 from the first row).

4.3 Economic Significance of Risk Aversion

The previous section presents evidence on the crucial role of risk aversion in determining the dynamics of the fitted global comovement, using covariance and correlation decomposition methodologies. In this section, I formally evaluate the economic significance of risk aversion in terms of the three stylized facts.

In the last two columns of Table 7, I report the fit of a factor model omitting the risk aversion shock ω_q. Without ω_q, both constant and time-varying beta models (after re-estimation) fail to fit the facts. In particular, the model-implied comovement differences become inconspicuous and rejected by the data; for instance, global equity correlation becomes significantly smaller (by 0.15), cannot be rejected from symmetry tests, and exhibit no cyclical behaviors, while global bond correlation has essentially a zero sensitivity to the world recession indicator. To evaluate the economic significance of other factors, Table 10 extends Table 7 and reports the fit of factor models omitting economic uncertainty shocks one at a time. It is important to recognize that these models are still able to generate reasonable comovement differences, although some statistics might exceed the 95% confidence interval.

Figure 6 plots the difference between model-implied global equity and bond correlations against the empirical benchmark. The model-implied comovement difference has a correlation of 0.61 using all factors but a correlation of 0.20 after omitting only the risk aversion shock. I discuss two observations that are in line with Tables 7 and 10. First, omitting uncertainty shocks mainly result in a higher global equity correlation, thus the solid lines in the bottom six plots of Figure 6 are overall higher than the empirical benchmark. This observation in turn suggests that equity and bond returns across countries have more or less similar signs of sensitivities to these macroeconomic risk factors.

Second, omitting risk aversion shock mainly weakens the cyclical behaviors of global equity and bond correlations, as observed in Table 7, with the biggest misfit during major recessions in the sample period. I hereby review and rationalize why global risk aversion could reconcile all three stylized facts through the lens of the model:

■ Stylized Fact 1: *International equity return sensitivities to the global risk aversion shock are significant and negative (i.e. risky), whereas bond return sensitivities are not only much smaller in magnitude but have different signs in different countries during different periods. Therefore, global equity comovement is higher in magnitude than global bond comovement.*

■ Stylized Fact 2: *The second moment of the risk aversion shock is positively-skewed,* $skew(q_t) > 0$*. With bond returns displaying relatively weaker return sensitivities with mixed signs to the risk aversion shock, asymmetry in global bond return comovement is thus less clear than that in global equity return comovement.*

■ Stylized Fact 3: *The global investor demands higher risk compensations from investing in international equities when her risk aversion is high. Moreover, equity prices decrease with the global risk aversion shock even further during periods of economic turmoil, resulting in a robust countercyclical global equity comovement. On the other hand, while all bonds are priced as safe assets during normal times, there is increasing divergence in the risk characteristics of international bonds during bad times. Some bond prices still increase with global risk aversion (e.g., USA, Japan) while others decrease, resulting in a procyclical global bond comovement.*

5 Robustness and Extensions

In the final section, I consider three additional exercises on the three stylized facts. In particular, I examine the role of one particular country in determining the dynamics of global correlations, estimate an alternative model using average pairwise DCC estimates as the empirical proxy for global correlations, and evaluate the three stylized facts using local currency returns. The latter two exercises can be considered as robustness tests.

Extension (I). The first exercise is a "Jackknife" exercise: The global correlation models (as introduced in Section 2) are re-estimated after omitting one country at a time. From the previous analysis, country bond return sensitivities to the global risk aversion shock depend on different states of the world economy, and some country bonds seem to remain "safe" during all times, such as those of U.S. and Japan, while others appear to turn "risky" during economic turmoil. On the other hand, all country equities exhibit risky-asset behaviors during all times. As a result, this jackknife exercise aims to confirm the factor model results on different international bond risk characteristics, and does not serve as a robustness check (as global bond correlation omitting a safe country bond must be different from that omitting a risky country bond).

From Table 11, the omission of United States does not change the dynamics and level of global equity return comovement significantly; the correlation with the empirical benchmark is 0.99 (statistically close to 1), and the t test statistics is -0.23. Similar results are found for other country equities except for Japan. According to Column "JPN", the t statistics is positive and significant, 10.02, and the global equity correlation without Japan would be higher by around 0.061, which is likely due to the distant economic and financial market environment between Japan and the rest of the world. On the other hand, the omission of United States significantly increased the global bond correlation (t statistics=3.64), which is consistent with the aforementioned safe asset theory. Interestingly, global bond correlation without any European countries is significantly lower than the empirical benchmark, with Switzerland having the smallest negative impact. Figure 7 provides relevant graphical illustrations.

Extension (II). While a DECO framework is convenient in estimating an aggregate-level dynamic dependence and conducting tests within the system, there might be concerns that the structural assumption is overly simplified. To address it, in the second exercise, I calculate alternative global dynamic correlation measures using the average of all pairwise correlations (28 unique pairs) where each pairwise correlation is individually estimated using DCC-class models (Engle (2002)).[30]

Table 12 first demonstrates that the average pairwise correlation is highly statistically close to the paper DECO estimates from Section 2. The time-series correlation is 0.953 for equity comovement (Panel A) and 0.964 for bond comovement (Panel B). Panel C tests the equality between the two global comovement estimates (paper and pairwise) for each asset class; I fail to reject the null. The left plot of Figure 8 compares the DECO estimates drawn from Figure 1 and the average pairwise correlation estimates constructed here. During most periods, the two measures reach similar magnitudes; simple regression tests show that this alternative global equity comovement measure is countercyclical (i.e., coefficient on the world output growth=-0.737, SE=0.331; coefficient on the world recession indicator=0.0074, SE=0.0067), whereas this alternative global bond comovement measure is procyclical (i.e., coefficient on the world output growth=0.259, SE=0.138; coefficient on the world recession indicator=-0.0229, SE=0.0085). Hence, there is strong evidence on the closeness between the paper and alternative estimates.

Extension (III). Finally, while the present research focuses on the perspective of a U.S. (global) investor, following Bekaert et al. (2009) and Christoffersen et al. (2012),

[30] Similarly, the DCC models allow for asymmetry, cyclicality in the long-run component and potential procyclical equity-bond comovement within a country; the estimates are chosen based on the standard goodness of fit criteria.

it is admittedly important to acknowledge the potential role of foreign exchange risk in the dynamics of global asset return comovement. There are mainly two effects. On the one hand, dollar (USD) returns can be decomposed into local currency (LC) returns and changes in the log exchange rates, and it is a well-known fact that dollar volatility – in particular that of bond returns – could double.[31] This effect of exchange rates *through* volatility potentially dampens the global correlation estimates. On the other hand, exchange rates also influence the numerator of return correlations. The covariance of two dollar asset returns linearly includes both the covariance of currency-hedged returns and three *covariances with/of exchange rate changes*; the overall economic magnitude of the latter three covariances in the total dollar return covariance might be difficult to predict: (1) this paper analyzes 8 countries with 7 different currencies; (2) these currencies carry different levels of riskiness; for instance, it is likely to see that the Japanese Yen appreciates (the Euro depreciates) while the U.S. bond price increases, indicating a positive (negative) covariance between U.S. bond returns and JPY/USD (EUR/USD).[32]

In the third exercise, I first re-estimate the DECO system of equities and bonds using local currency returns (or currency-hedged returns) and same model variants as discussed in Section 2. I continue to find strong evidence of the three stylized facts. Table 12 shows that the global equity comovement estimated using local currency returns exhibits a high correlation (0.886) with the empirical benchmark using the dollar returns. The global bond comovement in local currencies has a moderate correlation (0.672) with the empirical benchmark — which is expected because exchange rates matter more in the total dynamics of bond returns than equity returns. In addition, the global conditional correlation estimates exhibiting an average of 0.647 (Equity, LC) and 0.564 (Bond, LC) are statistically close to the data. Moreover, the global equity correlation comoves positively with the OECD world recession indicator (coefficient=0.0183, SE=0.0076), while the global bond correlation comoves negatively with the indicator (coefficient=-0.0248, SE=0.0153).

Importantly, Table 13 presents *non-parametric* evidence of the three stylized facts in currency-hedged global comovement measures. First, the Jennrich (1970) test still rejects the null that local currency equity and bond return correlations are the same. The next test examines the asymmetry of correlations in response joint left-tail events using exceedance correlations using daily local currency returns. I continue to find excessive left-tail comovement in equity returns but not in bond returns. With respect to the

[31] For instance, the monthly dollar return volatility of the 10-year Japanese (German) government bond is 13.24% (11.54%) during the paper sample period, while its local currency return volatility is 5.53% (5.50%).

[32] Suppose the local currency return of Country 1 bond is $r^{LC}_{1,t+1}$ and its dollar return is $r^{USD}_{1,t+1} = r^{LC}_{1,t+1} + s_{t+1}$ where s_{t+1} denotes the log changes in the exchange rate (>0: appreciation; <0: depreciation); the U.S. bond return is $r_{US,t+1}$. Then, the covariance between dollar returns is $cov(r_{US,t+1}, r^{USD}_{1,t+1}) = cov(r_{US,t+1}, r^{LC}_{1,t+1}) + cov(r_{US,t+1}, s_{t+1})$. In particular, $cov(r_{US,t+1}, s_{t+1})$ would be positive (negative) if the local currency appreciates (depreciates) as the U.S. bond price increases.

third stylized fact, both test results show that the average pairwise sample correlation of local currency equity (bond) returns during recession periods is significantly higher (lower) than that during non-recession periods, rendering a countercyclical (procyclical) comovement.

To address the similar modeling concern as in the second exercise, I also construct the average pairwise correlations using local currency returns and compare them with the DECO estimates. Table 12 shows that the correlation between the two measures of global comovement is as high as 0.972 (0.977) for equities (bonds). It is worth noting that a similar pattern of the currency-hedged global bond comovement (the right plot of Figure 8) can be found in Viceira and Wang (2018) who use the average rolling correlations as the proxy.

Given the close relation between the DECO and average pairwise DCC estimates, I extend the analysis and compute each country's average comovement with the rest of the world. Panel D of Table 12 shows that Australasian countries (Japan and Australia) exhibit consistently lower (than average) return comovement than other countries over the past 30 years, which is robust considering different currency domains and different asset classes. In addition, for all countries except for Japan, bond comovement denominated in U.S. dollars is *lower* than that denominated in local currencies. As discussed above, this observation can be rationalized by the possibility that local bond prices have a negative relation with exchange rates; for instance, during recessions, the U.S. bond price increases while EUR/USD decreases, or the Euro depreciates. The impact of exchange rates in global equity comovement is less obvious, from the first two columns in Panel D. The Germany government bond market comoves the strongest with the rest of the world, regardless of the currency domain (0.581 using dollar returns and 0.647 using local currency returns), indicating the likely smallest international diversification benefit in this market.

6 Conclusion

In this paper, I formally establish three new stylized facts contrasting global equity and bond comovements using parametric and non-parametric methodologies: (1) equity return correlations are larger than bond return correlations; (2) equity returns have higher downside than upside correlations, while bond return correlations are rather symmetric; and (3) equity return correlations are countercyclical while bond return correlations are (weakly) procyclical. The global dynamic comovement model designed to accommodate asymmetry and domestic comovement demonstrates a potential methodological contribution. Next, I motivate and identify economic determinants of global comovements in a dynamic no-arbitrage asset pricing model with time-varying global economic uncertainties (of output growth, inflation, and the real short rate) and risk aversion (of the global investor). Finally, I bring the model solution to the data and interpret the three stylized facts in a theory-motivated dynamic factor framework. I find that different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the global risk aversion shock are crucial in driving all three stylized facts.

While Miranda-Agrippino and Rey (2015) suggest that global risk aversion drives the global risky-asset cycle, my paper establishes that global risk aversion is a major source of not only the dynamics but also the differences between global equity comovement and global bond comovement, a finding which is new to the literature. In addition, the present research has the potential to offer updated quantitative evidence for the relative importance of fundamental global factors in jointly explaining these global comovement facts.

One interesting byproduct of this research is the suggestion that conditional betas are likely to be spanned by economic uncertainties, which confirms some previous theories that have not been formally empirically tested (e.g., Ball (1992), Bansal and Shaliastovich (2010), David and Veronesi (2013)). Finally, while equity returns can be explained reasonably well by a dynamic factor model, my evidence suggests that bond returns seem to have a weaker factor structure, a finding that merits further scrutiny.

References

[1] Adrian, T., Boyarchenko, N., and Giannone, D. Vulnerable growth. *American Economic Review 109*, 4 (2019), 1263–89.

[2] Ahir, H., Bloom, N., and Furceri, D. The world uncertainty index. *Working paper* (2018).

[3] Aielli, G. P. Dynamic conditional correlation: on properties and estimation. *Journal of Business & Economic Statistics 31*, 3 (2013), 282–299.

[4] Andersen, T. G., Bollerslev, T., Diebold, F. X., and Vega, C. Real-time price discovery in global stock, bond and foreign exchange markets. *Journal of International Economics 73*, 2 (2007), 251–277.

[5] Ang, A., Bekaert, G., and Wei, M. The term structure of real rates and expected inflation. *The Journal of Finance 63*, 2 (2008), 797–849.

[6] Ang, A., and Chen, J. Asymmetric correlations of equity portfolios. *Journal of financial Economics 63*, 3 (2002), 443–494.

[7] Backus, D. K., Foresi, S., and Telmer, C. I. Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*, 1 (2001), 279–304.

[8] Bae, K.-H., Karolyi, G. A., and Stulz, R. M. A new approach to measuring financial contagion. *The Review of Financial Studies 16*, 3 (2003), 717–763.

[9] Baele, L., Bekaert, G., and Inghelbrecht, K. The determinants of stock and bond return comovements. *The Review of Financial Studies 23*, 6 (2010), 2374–2428.

[10] Baele, L., Bekaert, G., Inghelbrecht, K., and Wei, M. Flights to safety. *Review of Financial Studies* (2019).

[11] Ball, L. Why does high inflation raise inflation uncertainty? *Journal of Monetary Economics 29*, 3 (1992), 371–388.

[12] Bansal, R., Shaliastovich, I., et al. Confidence risk and asset prices. *American Economic Review 100*, 2 (2010), 537–541.

[13] Bates, D. S. Maximum likelihood estimation of latent affine processes. *The Review of Financial Studies 19*, 3 (2006), 909–965.

[14] Bekaert, G., and Engstrom, E. Asset return dynamics under habits and bad environment–good environment fundamentals. *Journal of Political Economy 125*, 3 (2017), 713–760.

[15] Bekaert, G., Engstrom, E., and Xu, N. R. The time variation in risk appetite and uncertainty. *working paper* (2019).

[16] Bekaert, G., and Harvey, C. R. Time-varying world market integration. *The Journal of Finance 50*, 2 (1995), 403–444.

[17] Bekaert, G., Hodrick, R. J., and Zhang, X. International stock return comovements. *The Journal of Finance 64*, 6 (2009), 2591–2626.

[18] Bekaert, G., and Hoerova, M. The vix, the variance premium and stock market volatility. *Journal of Econometrics 183*, 2 (2014), 181–192.

[19] Bernoth, K., Von Hagen, J., and Schuknecht, L. Sovereign risk premiums in the european government bond market. *Journal of International Money and Finance 31*, 5 (2012), 975–995.

[20] Bollerslev, T. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*, 3 (1986), 307–327.

[21] Bollerslev, T., Marrone, J., Xu, L., and Zhou, H. Stock return predictability and variance risk premia: statistical inference and international evidence. *Journal of Financial and Quantitative Analysis 49*, 3 (2014), 633–661.

[22] Bollerslev, T., Tauchen, G., and Zhou, H. Expected stock returns and variance risk premia. *The Review of Financial Studies 22*, 11 (2009), 4463–4492.

[23] Brandt, M. W., and Wang, K. Q. Time-varying risk aversion and unexpected inflation. *Journal of Monetary Economics 50*, 7 (2003), 1457–1498.

[24] Brunnermeier, M. K., Nagel, S., and Pedersen, L. H. Carry trades and currency crashes. *NBER Macroeconomics Annual 23*, 1 (2008), 313–348.

[25] Campbell, J. Y., and Cochrane, J. H. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*, 2 (1999), 205–251.

[26] Campbell, R., Koedijk, K., and Kofman, P. Increased correlation in bear markets. *Financial Analysts Journal 58*, 1 (2002), 87–94.

[27] Campbell, R. A., Forbes, C. S., Koedijk, K. G., and Kofman, P. Increasing correlations or just fat tails? *Journal of Empirical Finance 15*, 2 (2008), 287–309.

[28] Cappiello, L., Engle, R. F., and Sheppard, K. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial econometrics 4*, 4 (2006), 537–572.

[29] Chen, R.-R., and Scott, L. Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates. *The Journal of Fixed Income 3*, 3 (1993), 14–31.

[30] Christoffersen, P., Errunza, V., Jacobs, K., and Langlois, H. Is the potential for international diversification disappearing? a dynamic copula approach. *The Review of Financial Studies 25*, 12 (2012), 3711–3751.

[31] Codogno, L., Favero, C., and Missale, A. Yield spreads on emu government bonds. *Economic Policy 18*, 37 (2003), 503–532.

[32] Cohn, A., Engelmann, J., Fehr, E., and Maréchal, M. A. Evidence for countercyclical risk aversion: An experiment with financial professionals. *American Economic Review 105*, 2 (2015), 860–85.

[33] Connolly, R., Stivers, C., and Sun, L. Stock market uncertainty and the stock-bond return relation. *Journal of Financial and Quantitative Analysis 40*, 1 (2005), 161–194.

[34] David, A., and Veronesi, P. What ties return volatilities to price valuations and fundamentals? *Journal of Political Economy 121*, 4 (2013), 682–746.

[35] De Santis, G., and Gerard, B. International asset pricing and portfolio diversification with time-varying risk. *The Journal of Finance 52*, 5 (1997), 1881–1912.

[36] Duffee, G. R. Time variation in the covariance between stock returns and consumption growth. *The Journal of Finance 60*, 4 (2005), 1673–1712.

[37] Dungey, M., Fry, R., González-Hermosillo, B., and Martin, V. L. Empirical modelling of contagion: a review of methodologies. *Quantitative Finance 5*, 1 (2005), 9–24.

[38] Engle, R. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics 20*, 3 (2002), 339–350.

[39] Engle, R., and Kelly, B. Dynamic equicorrelation. *Journal of Business & Economic Statistics 30*, 2 (2012), 212–228.

[40] Ermolov, A. Time-varying risk of nominal bonds: How important are macroeconomic shocks? *working paper* (2018).

[41] Fagiolo, G., Napoletano, M., and Roventini, A. Are output growth-rate distributions fat-tailed? some evidence from oecd countries. *Journal of Applied Econometrics 23*, 5 (2008), 639–669.

[42] Forbes, K. J., and Rigobon, R. No contagion, only interdependence: measuring stock market comovements. *The Journal of Finance 57*, 5 (2002), 2223–2261.

[43] Forbes, K. J., and Warnock, F. E. Capital flow waves: Surges, stops, flight, and retrenchment. *Journal of International Economics 88*, 2 (2012), 235–251.

[44] French, K. R., Schwert, G. W., and Stambaugh, R. F. Expected stock returns and volatility. *Journal of Financial Economics 19*, 1 (1987), 3–29.

[45] Gambetti, L., Pappa, E., and Canova, F. The structural dynamics of us output and inflation: what explains the changes? *Journal of Money, Credit and Banking 40*, 2-3 (2008), 369–388.

[46] Genz, A., and Bretz, F. *Computation of multivariate normal and t probabilities*, vol. 195. Springer Science & Business Media, 2009.

[47] Glosten, L. R., Jagannathan, R., and Runkle, D. E. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance 48*, 5 (1993), 1779–1801.

[48] Greene, W. H. *Econometric analysis.* Pearson Education India, 2003.

[49] Hamilton, J. D. Analysis of time series subject to changes in regime. *Journal of Econometrics 45*, 1-2 (1990), 39–70.

[50] Hentschel, L. All in the family nesting symmetric and asymmetric garch models. *Journal of Financial Economics 39*, 1 (1995), 71–104.

[51] Hodrick, R. J., and Prescott, E. C. Postwar us business cycles: an empirical investigation. *Journal of Money, credit, and Banking* (1997), 1–16.

[52] Jennrich, R. I. An asymptotic $\chi 2$ test for the equality of two correlation matrices. *Journal of the American Statistical Association 65*, 330 (1970), 904–912.

[53] Jotikasthira, C., Le, A., and Lundblad, C. Why do term structures in different currencies co-move? *Journal of Financial Economics 115*, 1 (2015), 58–83.

[54] Karolyi, G. A. Does international financial contagion really exist? *International Finance 6*, 2 (2003), 179–199.

[55] Karolyi, G. A., and Stulz, R. M. Why do markets move together? an investigation of us-japan stock return comovements. *The Journal of Finance 51*, 3 (1996), 951–986.

[56] Kenourgios, D., Samitas, A., and Paltalidis, N. Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. *Journal of International Financial Markets, Institutions and Money 21*, 1 (2011), 92–106.

[57] Kotz, S., and Nadarajah, S. *Multivariate t-distributions and their applications.* Cambridge University Press, 2004.

[58] Li, F. Identifying asymmetric comovements of international stock market returns. *Journal of Financial Econometrics 12*, 3 (2013), 507–543.

[59] Li, Q., Yang, J., Hsiao, C., and Chang, Y.-J. The relationship between stock returns and volatility in international stock markets. *Journal of Empirical Finance 12*, 5 (2005), 650–665.

[60] Lin, W.-L., Engle, R. F., and Ito, T. Do bulls and bears move across borders? international transmission of stock returns and volatility. *Review of Financial Studies 7*, 3 (1994), 507–538.

[61] Londono, J. M., and Xu, N. R. Variance risk premium components and international stock return predictability. *working paper* (2019).

[62] Longin, F., and Solnik, B. Is the correlation in international equity returns constant: 1960–1990? *Journal of international money and finance 14*, 1 (1995), 3–26.

[63] Longin, F., and Solnik, B. Extreme correlation of international equity markets. *The Journal of Finance 56*, 2 (2001), 649–676.

[64] Martin, I. The lucas orchard. *Econometrica 81*, 1 (2013), 55–111.

[65] Martin, I. What is the expected return on the market? *The Quarterly Journal of Economics 132*, 1 (2017), 367–433.

[66] Miranda-Agrippino, S., and Rey, H. World asset markets and the global financial cycle. Tech. rep., 2015.

[67] Nelson, D. B. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica* (1991), 347–370.

[68] Ribeiro, R., and Veronesi, P. Excess comovement of international stock markets in bad times: a rational expectations equilibrium model. In *II Encontro Brasileiro de Finanças* (2002).

[69] Segal, G., Shaliastovich, I., and Yaron, A. Good and bad uncertainty: Macroeconomic and financial market implications. *Journal of Financial Economics 117*, 2 (2015), 369–397.

[70] Solnik, B., and Watewai, T. International correlation asymmetries: Frequent-but-small and infrequent-but-large equity returns. *The Review of Asset Pricing Studies 6*, 2 (2016), 221–260.

[71] Viceira, L. M., and Wang, Z. K. Global portfolio diversification for long-horizon investors. *working paper* (2018).

[72] Wu, G. The determinants of asymmetric volatility. *The Review of Financial Studies 14*, 3 (2001), 837–859.

[73] Xu, N. R. Procyclicality of the comovement between dividend growth and consumption growth. *Working Paper* (2019).

[74] Zakoian, J.-M. Threshold heteroskedastic models. *Journal of Economic Dynamics and Control 18*, 5 (1994), 931–955.

[75] Zellner, A. An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. *Journal of the American statistical Association 57*, 298 (1962), 348–368.

[76] Zellner, A., and Huang, D. S. Further properties of efficient estimators for seemingly unrelated regression equations. *International Economic Review 3*, 3 (1962), 300–313.

Appendices

A Univariate Conditional Variance Models

The univariate variance model for each return series is selected using the Bayesian information criterion (BIC) from a class of models capable of capturing the common features of financial asset return variance: persistent, clustering, and (sometimes) asymmetric. Although commonly-acknowledged, these features do not appear in conditional variances of all asset returns. For example, as asymmetry in both domestic stock returns and international stock returns is widely documented (see, e.g., French, Schwert, and Stambaugh (1987), Hentschel (1995), Wu (2001), Li et al. (2005), Kenourgios, Samitas, and Paltalidis (2011) among many others), little evidence of asymmetry is found in bond returns, both domestically or internationally (see a thorough examination in Cappiello, Engle, and Sheppard (2006) for instance). As a result, in this paper, I consider four conditional variance models in the GARCH class with four residual distributional assumptions; thus, 16 models are included in the model selection.

Suppose the residual follows a conditional distribution, $\varepsilon_{t+1} \sim D(0, h_t)$ where h_t denotes the conditional variance. The first conditional variance model follows an autoregressive conditional heteroskedastic process with one lag of the innovation and one lag of volatility, or "GARCH" as in Bollerslev (1986):

$$h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1} \tag{A1}$$

where α_1 denotes the sensitivity of conditional variance to news and α_2 the autoregressive coefficient. Then, I use three widely-used asymmetric GARCH models that introduce non-linearity to different transformations of the conditional variance h_t. The second model is the exponential GARCH, or "EGARCH" as in Nelson (1991), which includes a signed standardized residual term to capture the (potential) higher downside risk variance. The third model is the threshold GARCH, or "TARCH" as in Zakoian (1994), which introduces asymmetry to conditional volatility, whereas the fourth model, Glosten, Jagannathan, and Runkle (1993)'s "GJR-GARCH", introduces asymmetry to conditional variance. The model specifications are shown below:

$$\ln(h_t) = \alpha_0 + \alpha_1 \frac{|\varepsilon_t|}{\sqrt{h_{t-1}}} + \alpha_2 \ln(h_{t-1}) + \alpha_3 \frac{\varepsilon_t}{\sqrt{h_{t-1}}}, \tag{A2}$$

$$\sqrt{h_t} = \alpha_0 + \alpha_1 |\varepsilon_t| + \alpha_2 \sqrt{h_{t-1}} + \alpha_3 I_{\varepsilon_t<0} |\varepsilon_t|, \tag{A3}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1} + \alpha_3 I_{\varepsilon_t<0} \varepsilon_t^2, \tag{A4}$$

where α_3 is the asymmetry term. If the downside uncertainty is higher than the upside uncertainty, then α_3 in Equation (A2) is expected to be negative because downside risk in these models is identified when residuals are negative, whereas α_3 in Equations (A3) and (A4) are expected to be positive because the asymmetry terms in last two models are sign-independent.

The standardized residuals, z_{t+1}, are defined to be $\frac{\varepsilon_{t+1}}{\sqrt{h_t}}$.

B Four distributional assumptions in estimating the conditional variances of return series in Section A.

I consider four distributions. First, Gaussian distribution; $\varepsilon_{t+1} \sim N(0, h_t)$ with conditional probability density function equal to $\frac{1}{\sqrt{2\pi h_t}} \exp^{-\frac{\varepsilon_{t+1}}{2h_t}}$. Second, Student's t distribution; $\varepsilon_{t+1} \sim STD(0, h_t, \zeta_1)$ with conditional probability density function equal to
$\frac{\Gamma\left[\frac{1}{2}(\zeta_1+1)\right]}{\sqrt{\pi}\Gamma\left(\frac{1}{2}\zeta_1\right)} \left[(\zeta_1 - 2)h_t\right]^{-\frac{1}{2}} \left[1 + \frac{\varepsilon_{t+1}^2}{(\zeta_1-2)h_t}\right]^{-\frac{1}{2}(\zeta_1+1)}$ where $\zeta_1 > 2$ denotes the degree of freedom capturing the thickness of both tails and Γ denotes the gamma distribution. A higher ζ_1 indicates a thinner tail. Third, Generalized error distribution; $\varepsilon_{t+1} \sim GED(0, h_t, \zeta_1)$ with conditional probability density function equal to $\frac{\zeta_1}{2\sqrt{h_t}\Gamma\left(\frac{1}{\zeta_1}\right)} \exp^{-\left(\frac{\varepsilon_{t+1}}{\sqrt{h_t}}\right)^{\zeta_1}}$. Platykurtic densities (with tails lighter than Gaussian) are defined if $\zeta_1 > 2$; on the other hand, leptokurtic densities (with tails heavier than Gaussian) are defined if $1 < \zeta_1 < 2$. Fourth, Skewed student t distribution; $\varepsilon_{t+1} \sim SKEWT(0, h_t, \zeta_1, \zeta_2)$ where conditional probability density function (according to Hansen, 1994) equals

$$f(\varepsilon_{t+1}|h_t, \zeta_1, \zeta_2) = \begin{cases} bc\left[1 + \frac{1}{\zeta_1-2}\left(\frac{b\frac{\varepsilon_{t+1}}{\sqrt{h_t}}+a}{1-\zeta_2}\right)^2\right]^{-\frac{\zeta_1+1}{2}} & \frac{\varepsilon_{t+1}}{\sqrt{h_t}} < -\frac{a}{b}, \\ bc\left[1 + \frac{1}{\zeta_1-2}\left(\frac{b\frac{\varepsilon_{t+1}}{\sqrt{h_t}}+a}{1+\zeta_2}\right)^2\right]^{-\frac{\zeta_1+1}{2}} & \frac{\varepsilon_{t+1}}{\sqrt{h_t}} \geqslant -\frac{a}{b}, \end{cases} \tag{A5}$$

where $2 < \zeta_1 < \infty$, $-1 < \zeta_2 < 1$, constants $a = 4\zeta_2 c\left(\frac{\zeta_1-2}{\zeta_1-1}\right)$, $b^2 = 1+3\zeta_2^2-a^2$, and $c = \frac{\Gamma\left[\frac{1}{2}(\zeta_1+1)\right]}{\sqrt{\pi}\Gamma\left(\frac{1}{2}\zeta_1\right)} \left[(\zeta_1 - 2)h_t\right]^{-\frac{1}{2}}$. The density function is continuous, and has a single mode at $-\frac{a}{b}$, which is of opposite sign with the parameter ζ_2. Thus if $\zeta_2 > 0$, the mode of the density is to the left of zero and the distribution is right-skewed, and vice-versa when $\zeta_2 < 0$. To summarize, all distributions except for the first distribution allow for thick tails; in addition, the last distribution also captures the skewness.

C Prove Covariance Stationarity of the Global Dynamic Comovement Model in Equation (5).

In this section, I prove that $\boldsymbol{Q_t}$ $(N \times N)$ is a stationary process. As introduced in Section 2.1.A, $\boldsymbol{Q_t}$ follows a generalized autoregressive heteroskedastic process,

$$\begin{aligned}\boldsymbol{Q_t} &= \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_t} + \beta_1 \left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{z_t z_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) + \beta_2 \left(\boldsymbol{Q_{t-1}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) \\ &+ \gamma \left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{n_t n_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \boldsymbol{\Xi} \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right), \end{aligned} \tag{A6}$$

where "$\circ$" denotes the Hadamard product operator (element-by-element); $\overline{\boldsymbol{Q}}$ is the unconditional component of the long-run conditional mean; $\widetilde{\boldsymbol{Q}}_{\boldsymbol{t}}$ is $\boldsymbol{Q_t}$ with off-diagonal terms being zeros, which is a modification to Engle (2002) proposed by Aielli (2013); $\boldsymbol{n_t}(N \times 1) = \boldsymbol{I_{z_t<0}} \circ \boldsymbol{z_t}$, where $\boldsymbol{I_{z_t<0}}$ $(N \times 1)$ is assigned 1 if the residual is less than 0, and assigned 0 otherwise; $\boldsymbol{\Xi} = E[\boldsymbol{I_{z_t<0} I'_{z_t<0}}]$; $\boldsymbol{\Phi_t}(N \times N) = \begin{bmatrix} 1 & 1+\phi_t & 1+\phi_t & \cdots \\ 1+\phi_t & 1 & 1+\phi_t & \cdots \\ 1+\phi_t & 1+\phi_t & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$ where $\phi_t = \phi\ \widetilde{\theta}_t^{world}$, $\widetilde{\theta}_t^{world}$ is the standardized world recession indicator and ϕ is an unknown constant parameter.

C.1 Time-Invariant Mean

First, given that $E_{t-1}\left(\boldsymbol{z_t z_t'}\right) = \boldsymbol{Corr_{t-1}} = \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{-\frac{1}{2}} \boldsymbol{Q_{t-1}} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{-\frac{1}{2}}$, one-period conditional mean has the following process,

$$\begin{aligned} E_{t-1}\left(\boldsymbol{Q_t}\right) &= \overline{\boldsymbol{Q}} \circ E_{t-1}\left(\boldsymbol{\Phi_t}\right) + \beta_1 \left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} E_{t-1}\left(\boldsymbol{z_t z_t'}\right) \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) + \beta_2 \left(\boldsymbol{Q_{t-1}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) \\ &+ \gamma \left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} E_{t-1}\left(\boldsymbol{n_t n_t'}\right) \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \boldsymbol{\Xi} \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right), \end{aligned} \tag{A7}$$

$$\begin{aligned} &= \overline{\boldsymbol{Q}} \circ E_{t-1}\left(\boldsymbol{\Phi_t}\right) + \beta_1 \left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{Corr_{t-1}} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) + \beta_2 \left(\boldsymbol{Q_{t-1}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) \\ &+ \gamma \left(E_{t-1}\left(\boldsymbol{I_{z_t<0} I'_{z_t<0}}\right) \circ \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{Corr_{t-1}} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} - \boldsymbol{\Xi} \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right), \end{aligned} \tag{A8}$$

$$\begin{aligned} &= \overline{\boldsymbol{Q}} \circ E_{t-1}\left(\boldsymbol{\Phi_t}\right) + \beta_1 \left(\boldsymbol{Q_{t-1}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) + \beta_2 \left(\boldsymbol{Q_{t-1}} - \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right) \\ &+ \gamma \left(E_{t-1}\left(\boldsymbol{I_{z_t<0} I'_{z_t<0}}\right) \circ \boldsymbol{Q_{t-1}} - \boldsymbol{\Xi} \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{\Phi_{t-1}} \right). \end{aligned} \tag{A9}$$

Given the law of iterated expectation and $E\left[E_{t-1}\left(\boldsymbol{I_{z_t<0} I'_{z_t<0}}\right)\right] = \boldsymbol{\Xi}$, the unconditional mean of $\boldsymbol{Q_t}$ can be shown to be time-invariant as below,

$$E\left[E_{t-1}\left(\boldsymbol{Q_t}\right)\right] = \overline{\boldsymbol{Q}} \circ E\left[\boldsymbol{\Phi_t}\right] + \left(\beta_1 \boldsymbol{\iota} + \beta_2 \boldsymbol{\iota} + \gamma \boldsymbol{\Xi}\right) \circ \left(E\left[\boldsymbol{Q_{t-1}}\right] - \overline{\boldsymbol{Q}} \circ E\left[\boldsymbol{\Phi_{t-1}}\right] \right), \tag{A10}$$

$$= \overline{\boldsymbol{Q}} \circ E\left[\boldsymbol{\Phi_t}\right] + \left(\beta_1 \boldsymbol{\iota} + \beta_2 \boldsymbol{\iota} + \gamma \boldsymbol{\Xi}\right) \circ \left(E\left[\boldsymbol{Q_{t-1}}\right] - \overline{\boldsymbol{Q}} \circ E\left[\boldsymbol{\Phi_{t-1}}\right] \right), \tag{A11}$$

where $\boldsymbol{\iota}$ is a $N \times N$ matrix of 1s. Given that by construction $E\left[\boldsymbol{\Phi_t}\right] = \boldsymbol{\iota}$,

$$E\left[\boldsymbol{Q_t}\right]\left(\boldsymbol{\iota} - \beta_1 \boldsymbol{\iota} - \beta_2 \boldsymbol{\iota} - \gamma \boldsymbol{\Xi}\right) = \overline{\boldsymbol{Q}} \circ E\left[\boldsymbol{\Phi_t}\right]\left(\boldsymbol{\iota} - \beta_1 \boldsymbol{\iota} - \beta_2 \boldsymbol{\iota} - \gamma \boldsymbol{\Xi}\right), \tag{A12}$$

$$E\left[\boldsymbol{Q_t}\right] = \overline{\boldsymbol{Q}}. \tag{A13}$$

C.2 Time-Invariant Variance

$$\begin{aligned} Var\left(\boldsymbol{Q_t}\right) &= \overline{\boldsymbol{Q}} \circ Var\left(\boldsymbol{\Phi_t}\right) \circ \overline{\boldsymbol{Q}} + \beta_1 \left(\underbrace{Var\left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{z_t z_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \right)}_{\equiv[\boldsymbol{A}]} - \overline{\boldsymbol{Q}} \circ Var\left(\boldsymbol{\Phi_{t-1}}\right) \circ \overline{\boldsymbol{Q}} \right) \\ &+ \beta_2 \left(Var\left(\boldsymbol{Q_{t-1}}\right) - \overline{\boldsymbol{Q}} \circ Var\left(\boldsymbol{\Phi_{t-1}}\right) \circ \overline{\boldsymbol{Q}} \right) \\ &+ \gamma \left(\underbrace{Var\left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{n_t n_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \right)}_{\equiv[\boldsymbol{B}]} - \boldsymbol{\Xi} \circ \overline{\boldsymbol{Q}} \circ Var\left(\boldsymbol{\Phi_t}\right) \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{\Xi} \right), \end{aligned} \tag{A14}$$

where

$$[\boldsymbol{A}] = Var\left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{z_t z_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \right) = E\left[Var_{t-1}\left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{z_t z_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \right) \right] + Var\left[E_{t-1}\left(\widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \boldsymbol{z_t z_t'} \widetilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}^{\frac{1}{2}} \right) \right], \tag{A15}$$

$$= E\left[\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}} Var_{t-1}\left(\boldsymbol{z_t z_t'}\right)\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\right] + Var\left[\boldsymbol{Q_{t-1}}\right], \tag{A16}$$

$$= E\left[\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\left[\underbrace{E_{t-1}\left[\left(\boldsymbol{z_t z_t'}\right)\left(\boldsymbol{z_t z_t'}\right)'\right]}_{(N\times N),\equiv[\boldsymbol{C1}]} - \underbrace{E_{t-1}\left(\boldsymbol{z_t z_t'}\right)E'_{t-1}\left(\boldsymbol{z_t z_t'}\right)}_{(N\times N),\equiv[\boldsymbol{C2}]}\right]\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\right] + Var\left[\boldsymbol{Q_{t-1}}\right], \tag{A17}$$

$$[\boldsymbol{B}] = Var\left(\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\boldsymbol{n_t n_t'}\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\right) = E\left[Var_{t-1}\left(\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\boldsymbol{n_t n_t'}\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\right)\right] + Var\left[E_{t-1}\left(\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\boldsymbol{n_t n_t'}\tilde{\boldsymbol{Q}}^{\frac{1}{2}}_{\boldsymbol{t-1}}\right)\right], \tag{A18}$$

$$= E\left[\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}} Var_{t-1}\left(\boldsymbol{n_t n_t'}\right)\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\right] + Var\left[E_{t-1}\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)\right]\circ Var\left[\boldsymbol{Q_{t-1}}\right], \tag{A19}$$

$$= E\left[\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\left[\underbrace{E_{t-1}\left[\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)'\right]\circ[\boldsymbol{C1}]}_{(N\times N),\equiv[\boldsymbol{D1}]} - \underbrace{E_{t-1}\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)E'_{t-1}\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)\circ[\boldsymbol{C2}]}_{(N\times N),\equiv[\boldsymbol{D2}]}\right]\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}\right]$$

$$+ Var\left[E_{t-1}\left(I_{\boldsymbol{z_t}<0}I'_{\boldsymbol{z_t}<0}\right)\right]\circ Var\left[\boldsymbol{Q_{t-1}}\right]. \tag{A20}$$

Given that $\boldsymbol{z_t}$ is assumed to be a stationary vector, higher moments of $\boldsymbol{z_t}$ is time-invariant; it immediately suggests that the unconditional means of Components $[\boldsymbol{C1}]$ and $[\boldsymbol{C2}]$ in the equation above are time-invariant. Given the stationary $\tilde{\boldsymbol{Q}}_{\boldsymbol{t-1}}$ process as shown earlier, the unconditional mean of products of stationary processes in $[\boldsymbol{A}]$ are time-invariant. Similar arguments can be applied to $[\boldsymbol{B}]$.

D The Jennrich (1970) Correlation Test

Suppose two N-variate sample correlation matrices, $\boldsymbol{R_1}$ $(N\times N)$ and $\boldsymbol{R_2}$ $(N\times N)$ with sample sizes t_1 and t_2 (per variate), the test statistics is, $\chi^2 = \frac{1}{2}tr(\boldsymbol{ZZ}) - diag(\boldsymbol{Z})'\boldsymbol{S^{-1}}diag(\boldsymbol{Z})$ where "tr" calculates the matrix trace and "diag" the diagonal terms; $\boldsymbol{Z}(N\times N) = c^{1/2}\bar{\boldsymbol{R}}^{\boldsymbol{-1}}(\boldsymbol{R_1} - \boldsymbol{R_2})$ where $c = \frac{t_1 t_2}{t_1+t_2}$ and $\bar{\boldsymbol{R}} = (t_1\boldsymbol{R_1} + t_2\boldsymbol{R_2})/(t_1+t_2)$; $\boldsymbol{S}(N\times N) = \boldsymbol{I_N} + \bar{\boldsymbol{R}}\circ\bar{\boldsymbol{R}}^{\boldsymbol{-1}}$ where $\boldsymbol{I_N}$ is the identity matrix and "$\circ$" denotes the Hadamard product operator (element-by-element). The test statistics (see further details in Jennrich, 1970) has an asymptotic χ^2 distribution with degrees of freedom $N(N-1)/2$.

E Review on the Statistical Properties of a Gamma Distribution

For a Gamma random variable, $y\sim\Gamma(s,\theta)$ where s denotes the shape parameter and θ the scale parameter, it has the following PDF,

$$f_Y^{Gamma}(y;s,\theta) = \frac{1}{\Gamma(s)\theta^s}y^{s-1}\exp\left(-\frac{y}{\theta}\right), \tag{A21}$$

where $\Gamma(v)$ is a complete Gamma function.

The moment generating function is,

$$M_Y^{Gamma}(t;s,\theta) = (1-\theta t)^{-s}, \forall t < \frac{1}{\theta}. \tag{A22}$$

The mean is θs; the variance is $\theta^2 s$; the unscaled skewness is $2\theta^3 s$.

F Conditional Covariance Decomposition

In this part, I demonstrate the decomposition of conditional covariance that is explained by a specific factor $\omega_{\kappa,t+1}$. The instrument to span the time-varying betas s_t is standardized, and all factors are zero mean. Then, the conditional covariance related to $\omega_{\kappa,t+1}$ is,

$$\begin{aligned}\beta_{i,t,\kappa}Var_t(\omega_{\kappa,t+1})\beta_{j,t,\kappa} &= (\beta_{i,0,\kappa}+\beta_{i,1,\kappa}s_t)Var_t(\omega_{\kappa,t+1})(\beta_{j,0,\kappa}+\beta_{j,1,\kappa}s_t)\\ &= \underbrace{\beta_{i,0,\kappa}Var_t(\omega_{\kappa,t+1})\beta_{j,0,\kappa}}_{\text{pure constant part}} + \left[\beta_{i,0,\kappa}\beta_{j,1,\kappa}+\beta_{i,1,\kappa}\beta_{j,0,\kappa}\right]s_t Var_t(\omega_{\kappa,t+1})\\ &+ \underbrace{\beta_{i,1,\kappa}Var_t(\omega_{\kappa,t+1})s_t^2\beta_{j,1,\kappa}}_{\text{pure time-varying part}}. \end{aligned} \tag{A23}$$

Table 1: Summary Statistics.

This table presents the unconditional correlation matrices of USD-denominated log returns of 8 developed countries (United States, USA; Canada, CAN; Germany, DEU; France, FRA; United Kingdom, GBR; Switzerland, CHE; Japan, JPN; Australia, AUS) in Panel A and unconditional univariate moments (with bootstrapped standard errors in parentheses) in Panel B. Mean and standard deviations are presented in annualized percentages. "Equity" return refers to the change in log total return index of domestic country stock market (United States: S&P500; Canada: S&P/TSX 60; Germany: DAX 30; France: CAC 40; United Kingdom: FTSE 100; Switzerland: SMI; Japan: NIKKEI 225; Australia: S&P/ASX 200); CRSP value-weighted return is used to obtain the USA equity return; other return series are obtained from DataStream. "Bond" return refers to the change in log 10-year government bond index constructed by DataStream. Data is at monthly frequency. The sample is from March 1987 to December 2016 (T=358). Bold (italics) values indicate <5% (10%) significance level.

Panel A. Unconditional Correlation Matrices, 8 countries, 1987/03 - 2016/12								
	North America		*Europe*				*Australasia*	
	USA	CAN	DEU	FRA	GBR	CHE	JPN	AUS
(A.1) Equity								
USA	1	**0.782**	**0.725**	**0.720**	**0.759**	**0.671**	**0.434**	**0.672**
CAN		1	**0.649**	**0.649**	**0.696**	**0.578**	**0.442**	**0.723**
DEU			1	**0.872**	**0.743**	**0.726**	**0.436**	**0.606**
FRA				1	**0.763**	**0.740**	**0.477**	**0.625**
GBR					1	**0.741**	**0.509**	**0.720**
CHE						1	**0.472**	**0.594**
JPN							1	**0.473**
AUS								1
(A.2) Bond								
USA	1	**0.457**	**0.436**	**0.436**	**0.343**	**0.344**	**0.324**	**0.282**
CAN		1	**0.415**	**0.439**	**0.396**	**0.267**	**0.201**	**0.599**
DEU			1	**0.958**	**0.685**	**0.812**	**0.503**	**0.440**
FRA				1	**0.666**	**0.768**	**0.464**	**0.460**
GBR					1	**0.573**	**0.402**	**0.385**
CHE						1	**0.540**	**0.360**
JPN							1	**0.239**
AUS								1

Panel B. Unconditioanl Univariate Moments (annualized percentages)								
	North America		*Europe*				*Australasia*	
	USA	CAN	DEU	FRA	GBR	CHE	JPN	AUS
(B.1) Equity								
Mean	**9.321**	**8.331**	*7.476*	*7.138*	**7.347**	**8.820**	*2.299*	**9.284**
	(2.741)	(3.552)	(4.220)	(3.907)	(3.120)	(3.195)	(3.853)	(4.198)
S.D.	**15.017**	**19.551**	**23.143**	**21.340**	**17.177**	**17.497**	**21.169**	**23.174**
	(0.950)	(1.317)	(1.224)	(1.018)	(0.948)	(0.890)	(0.907)	(2.264)
Skewness	**-1.149**	**-1.374**	**-0.961**	**-0.566**	**-1.244**	**-1.265**	**-0.504**	*-3.175*
	(0.364)	(0.386)	(0.252)	(0.255)	(0.596)	(0.441)	(0.244)	(1.661)
(B.2) Bond								
Mean	**5.863**	**7.348**	**6.287**	**7.333**	**7.178**	**6.224**	**5.112**	**9.781**
	(0.984)	(1.652)	(1.879)	(1.861)	(1.823)	(2.001)	(2.180)	(2.173)
S.D.	**7.240**	**9.972**	**11.537**	**11.360**	**11.135**	**12.307**	**13.238**	**13.170**
	(0.330)	(0.500)	(0.520)	(0.509)	(0.526)	(0.539)	(0.672)	(0.616)
Skewness	0.016	-0.416	-0.006	-0.014	0.049	0.230	0.278	**-0.550**
	(0.230)	(0.261)	(0.186)	(0.188)	(0.197)	(0.181)	(0.244)	(0.195)

Table 2: Estimation Results of Global Bond Comovement.

This table presents the estimation results of the global bond return comovement models as described in Section 2. Denote $\boldsymbol{z_{t+1}^B}$ $(N \times 1)$ as the standardized residuals of country bond returns from t to $t+1$. The conditional equicorrelation matrix of $\boldsymbol{z_{t+1}^B}$ is $\boldsymbol{Corr_t^B} \equiv E_t[\boldsymbol{z_{t+1}^B z_{t+1}^{B\prime}}] = (1-\rho_t^B)\boldsymbol{I_N} + \rho_t^B \boldsymbol{J_{N\times N}}$; $\boldsymbol{I_N}$ is an identity matrix; $\boldsymbol{J_{N\times N}}$ is a matrix of ones; ρ_t^B is the equicorrelation, $\frac{2}{N(N-1)}\sum_{i>j}\frac{q_{i,j,t}^B}{\sqrt{q_{i,i,t}^B q_{j,j,t}^B}}$ where $q_{i,j,t}^B$ is the (i,j)-th element of a symmetric matrix $\boldsymbol{Q_t^B}$ $(N \times N)$ which follows a generalized autoregressive heteroskedastic process (omitting "B" below),

$$\boldsymbol{Q_t} = \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_t} + \beta_1\left(\boldsymbol{\widetilde{Q}_{t-1}^{\frac{1}{2}} z_t z_t' \widetilde{Q}_{t-1}^{\frac{1}{2}}} - \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right) + \beta_2\left(\boldsymbol{Q_{t-1}} - \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right)$$
$$+ \gamma\left(\boldsymbol{\widetilde{Q}_{t-1}^{\frac{1}{2}} n_t n_t' \widetilde{Q}_{t-1}^{\frac{1}{2}}} - \boldsymbol{\Xi} \circ \boldsymbol{\overline{Q}} \circ \boldsymbol{\Phi_{t-1}}\right).$$

The model detail is explained in Section 2.1.A. The unknown parameters are $\{\beta_1, \beta_2, (\nu), \gamma, \phi\}$, where ν is estimated separately for the equality test. The model is estimated using the MLE methodology, considering two distributions: (1) multivariate Gaussian; (2) multivariate t with an unknown degree of freedom parameter *df*. Data is at monthly frequency covering period from March 1987 to December 2016 (T=358). Model specifications such as sum of log likelihood, AIC and BIC are included at the end of the table. Bold (italics) values indicate <5% (10%) significance level.

	Multivariate Gaussian					*Multivariate t*				
	B (1)	B (2)	B (3)	B (4)	B (5)	B (1)	B (2)	B (3)	B (4)	B (5)
β_1	**0.0946**	**0.0799**	**0.0713**	**0.0584**	**0.0858**	**0.0610**	*0.0407*	**0.0894**	*0.0311*	**0.0745**
	(0.0368)	(0.0375)	(0.0213)	(0.0225)	(0.0334)	(0.0060)	(0.0218)	(0.0144)	(0.0159)	(0.0222)
β_2	**0.9050**	**0.8982**	**0.9256**	**0.9234**	**0.9141**	**0.9164**	**0.9017**	**0.9106**	**0.9216**	**0.8776**
	(0.0363)	(0.0410)	(0.0191)	(0.0224)	(0.0335)	(0.0061)	(0.0284)	(0.0144)	(0.0246)	(0.0285)
ν					**-0.2746**					**-0.1665**
					(0.0551)					(0.0621)
γ		0.0203		0.0170			0.0263		0.0214	
		(0.0186)		(0.0146)			(0.0228)		(0.0199)	
ϕ			-0.0375	-0.0572				-0.0434	*-0.0420*	
			(0.0529)	(0.0466)				(0.0284)	(0.0238)	
df						**6.7254**	**6.6772**	**6.6921**	**6.7167**	**6.3292**
						(0.9285)	(0.8372)	(0.9195)	(0.8433)	(0.7835)
LL	-3509.93	-3507.17	-3507.87	-3504.27	-3508.88	-3374.61	-3372.09	-3374.24	-3374.11	-3375.28
AIC	7023.87	7020.34	7021.74	7016.53	7023.75	6755.22	6752.17	6756.47	6758.21	6758.56
BIC	7031.63	7031.99	7033.38	7032.06	7035.40	6766.86	6767.70	6772.00	6777.62	6774.09
						Chosen				

Table 3: Estimation Results of Global Equity Comovement.

This table presents the estimation results of the global equity return comovement models as described in Section 2. Denote $z^E_{i,t+1}$ as the standardized residual of Country i's equity return from t to $t+1$. Define a simple domestic equity-bond comovement process of each country captured by $b_{i,t}$ with the following process:

$$z^E_{i,t+1} = b_{i,t} z^B_{i,t+1} + \sqrt{1-b^2_{i,t}}\,\breve{z}^E_{i,t+1},$$
$$b_{i,t} = 2\frac{\exp(\delta_1+\delta_2 x_{i,t})}{1+\exp(\delta_1+\delta_2 x_{i,t})} - 1,$$

where δ_1 and δ_2 are unknown constant parameters and $x_{i,t}$ is a country recession indicator (1 during recession months; 0 otherwise; source: OECD); by design, the bond-purified equity return residual $\breve{\boldsymbol{z}}^{\boldsymbol{E}}_{\boldsymbol{t+1}}$ $(N \times 1)$ has variance equal to 1. The conditional equicorrelation matrix of $\breve{\boldsymbol{z}}^{\boldsymbol{E}}_{\boldsymbol{t+1}}$ is $\widetilde{\boldsymbol{Corr}}^{\boldsymbol{E}}_{\boldsymbol{t}} \equiv E_t[\breve{\boldsymbol{z}}^{\boldsymbol{E}}_{\boldsymbol{t+1}} \breve{\boldsymbol{z}}^{\boldsymbol{E\prime}}_{\boldsymbol{t+1}}] = (1-\breve{\rho}^E_t)\boldsymbol{I_N} + \breve{\rho}^E_t \boldsymbol{J_{N\times N}}$. The equicorrelation $\breve{\rho}^E_t$ is $\frac{2}{N(N-1)}\sum_{i>j}\frac{\breve{q}^E_{i,j,t}}{\sqrt{\breve{q}^E_{i,i,t}\breve{q}^E_{j,j,t}}}$, where $\breve{q}^E_{i,j,t}$ is the (i,j)-th element of a symmetric matrix $\breve{\boldsymbol{Q}}^{\boldsymbol{E}}_{\boldsymbol{t}}$ $(N \times N)$ which follows an isomorphic generalized autoregressive heteroskedastic process (see Equation (11) in Section 2.1.B). The unknown parameters are $\{\delta_1, \delta_2, \beta_1, \beta_2, \gamma, \phi\}$. The model is estimated using the MLE methodology, considering two distributions: (1) multivariate Gaussian; (2) multivariate t with an unknown df. The log likelihood can be formulated using $\breve{\boldsymbol{z}}^{\boldsymbol{E}}_{\boldsymbol{t+1}}$ and $\widetilde{\boldsymbol{Corr}}^{\boldsymbol{E}}_{\boldsymbol{t}}$. Data is at monthly frequency covering period from March 1987 to December 2016 (T=358). Model specifications such as sum of log likelihood, AIC and BIC are included at the end of the table. Bold (italics) values indicate <5% (10%) significance level.

	Multivariate Gaussian					*Multivariate t*				
	E (1)	E (2)	E (3)	E (4)	E (5)	E (1)	E (2)	E (3)	E (4)	E (5)
β_1	**0.0883**	0.0775	**0.0722**	0.0630	*0.0725*	**0.0745**	**0.0182**	0.0476	0.0173	0.0281
	(0.0296)	(0.0560)	(0.0346)	(0.2528)	(0.0384)	(0.0364)	(0.0087)	(0.0488)	(0.0177)	(0.0249)
β_2	**0.8708**	**0.8803**	**0.8961**	*0.9015*	**0.8942**	**0.8879**	**0.9069**	**0.9341**	**0.9689**	**0.9520**
	(0.0397)	(0.0460)	(0.0496)	(0.4759)	(0.0449)	(0.0568)	(0.0115)	(0.0808)	(0.0171)	(0.0348)
ν										
γ		*0.0254*		*0.0203*	**0.0259**		**0.0221**		*0.0207*	**0.0279**
		(0.0132)		(0.0112)	(0.0106)		(0.0058)		(0.0107)	(0.0135)
ϕ			*0.0381*	*0.0363*				*0.0426*	*0.0357*	
			(0.0208)	(0.0187)				(0.0246)	(0.0200)	
δ_1	**0.5260**	**0.4997**	**0.5114**	**0.5012**		**0.5470**	**0.5303**	**0.5461**	**0.5130**	
	(0.0333)	(0.0376)	(0.0334)	(0.0354)		(0.0306)	(0.0316)	(0.0308)	(0.0354)	
δ_2	**-0.0553**	**-0.0526**	**-0.0555**	**-0.0528**		**-0.0616**	**-0.0549**	**-0.0620**	**-0.0556**	
	(0.0214)	(0.0242)	(0.0213)	(0.0228)		(0.0202)	(0.0213)	(0.0202)	(0.0177)	
df						**11.0408**	**9.5987**	**11.1170**	**9.8033**	**9.8748**
						(2.5590)	(1.6707)	(2.6198)	(0.8433)	(1.8115)
LL	-2991.32	-2985.38	-2988.97	-2983.65	-3117.03	-2916.02	-2887.87	-2914.39	-2886.37	-3025.19
AIC	5990.64	5980.77	5987.94	5979.30	6240.06	5842.05	5787.75	5840.77	5786.74	6058.37
BIC	6006.16	6000.17	6007.34	6002.58	6251.70	5861.45	5811.03	5864.06	5813.91	6073.90
							Chosen			

Table 4: Non-Parametric Tests: Magnitudes of Global Comovements.

This table replicates the Stylized Fact 1 using non-parametric method. "Data" reports the equally-weighted unconditional pairwise correlations (28) of standardized returns. "S.E." reports the bootstrapped standard errors. "Jennrich's χ^2" is a statistical test constructed in Jennrich (1970) to test the equality between two correlation matrices of two samples; ***: 1% significance test; see Appendix D for the test details. "Conditional Model" reports the sample average of the empirical benchmarks obtained from the chosen models according to Tables 2 and 3. "Simulated Model (t)" reports the average of 1000 unconditional average pairwise correlations of finite-sample simulated standardized returns using the chosen model specifications; this statistics is not available for subsamples. Bold values indicate the model point estimates are within 95% confidence intervals of the data point estimates.

	Equity	Bond
	Panel A. Full Sample	
Data	0.6271	0.4606
S.E.	(0.0254)	(0.0233)
Jennrich's χ^2	227.087(***)	
Conditional Model	**0.6583**	**0.4655**
Simulated Model (t)	**0.6712**	**0.4219**
	Panel B. 1987/03 - 1997/02	
Data	0.5923	0.3907
S.E.	(0.0299)	(0.0213)
Jennrich's χ^2	91.701(***)	
Conditional Model	**0.5787**	**0.3707**
	Panel C. 1997/03 - 2007/02	
Data	0.6401	0.5469
S.E.	(0.0270)	(0.0223)
Jennrich's χ^2	116.729(***)	
Conditional Model	**0.6644**	**0.5109**
	Panel D. 2007/03 - 2017/01	
Data	0.7538	0.5021
S.E.	(0.0268)	(0.0225)
Jennrich's χ^2	124.005(***)	
Conditional Model	**0.7329**	**0.5156**

Table 5: Non-Parametric Tests: Asymmetries in Global Comovements.

This table replicates the Stylized Fact 2 using data. The exceedance correlation of standardized daily returns (x and y) at a certain threshold percentile τ is $\rho(x, y|x < \Phi_x^{-1}(\tau), y < \Phi_y^{-1}(\tau))$ if $\tau < 0.5$ or $\rho(x, y|x > \Phi_x^{-1}(\tau), y > \Phi_y^{-1}(\tau))$ if $\tau >= 0.5$ (see Longin and Solnik (2001) and Ang and Chen (2002)). Global exceedance correlations are obtained using the equal-average of 28 unique pairwise exceedance correlations. "Data" reports the global exceedance correlation using daily standardized returns (which are similarly obtained for each country given the best GARCH-class conditional volatility estimates). "S.E." reports the standard errors for global exceedance correlations as in Ang and Chen (2002). Two models are simulated: (1) Chosen models assuming multivariate t ("B (1)", Table 2; "E (2)", Table 3). (2) Models assuming multivariate Gaussian ("B (1)", Table 2; "E (2)", Table 3). Bold values indicate the model point estimates are within 95% confidence intervals of the data point estimates.

	Equity			
	25%	49%	51%	75%
Data	0.3682	0.3292	0.2619	0.2469
S.E.	(0.0199)	(0.0147)	(0.0153)	(0.0216)
Simulated Model (t)	**0.3425**	**0.3468**	0.3279	0.3125
Simulated Model (n)	0.2253	0.2626	**0.2590**	**0.2125**
	Bond			
	25%	49%	51%	75%
Data	0.3029	0.3024	0.3079	0.3245
S.E.	(0.0209)	(0.0149)	(0.0149)	(0.0206)
Simulated Model (t)	**0.3157**	**0.3031**	**0.3047**	**0.3303**
Simulated Model (n)	0.2309	0.2358	0.2338	0.2254

Table 6: Non-Parametric Tests: Cyclicalities of Global Comovements.

This table replicates the Stylized Fact 3 using data. "Non-recession" ("Recession") periods are identified by the OECD world recession indicator. "Data" reports the average pairwise unconditional correlations of standardized returns in each period; "Boot. S.E." reports the bootstrapped standard errors. "Test 1" tests the equality between non-recession and recession period global correlation; "Test 2" obtains 3000 bootstrapped sample differences between non-recession and recession global correlations, and tests whether 3000 bootstrapped differences are indifferent from zero; ***, **: 1%, 5% significance. "Conditional Model" reports the average conditional correlation during each period. Bold values indicate the model point estimates are within 95% confidence intervals of the data point estimates.

	Equity		Bond	
	Non-Recession	Recession	Non-Recession	Recession
Data	0.5952	0.6571	0.4685	0.4520
Boot. S.E.	(0.0410)	(0.0329)	(0.0332)	(0.0265)
Test 1: Jennrich's χ^2	42.386 (**)		47.093 (**)	
Test 2: t(Non-Recession - Recession)	-65.71 (***)		22.20 (***)	
Conditional Model	**0.6537**	**0.6674**	**0.4740**	**0.4484**

Table 7: Dynamic Factor Model Fit & Economic Significance of Risk Aversion.

This table evaluates the fit of dynamic factor models and demonstrates the economic significance of risk aversion in explaining global return correlations. Column "Empirical BM" denotes the chosen conditional models (see Tables 2 and 3). Four dynamic factor models motivated from a dynamic asset pricing model are considered: constant or time-varying betas with a full set of factors or a subset of factors excluding the risk aversion shock ω_q. Equation-level estimation results of the return loadings are relegated to Tables A9 (constant beta) and A10 (time-varying beta) in the Internet Appendix. Each stylized fact is summarized by 2 statistics to be matched; Fact 1: average global conditional correlations; Fact 2: the difference between the average global conditional equity and bond correlations when the world stock return is $\geqslant$ or < 0; Fact 3: the sensitivity of global conditional correlations to the OECD world recession indicator. Bootstrapped standard errors of BM statistics are reported in parentheses; other columns report the absolute t statistics. "Yes" ("No") indicates that the model fits (fails to fit) the stylized facts reasonably well.

	Empirical BM:		*Dynamic Factor Models:*		
Betas		Constant	Time-varying	Constant	Time-varying
Factors		Full	Full	Exclud. ω_q	Exclud. ω_q
Test Fact 1: Equity Correlation > Bond Corrleation					
	{Moment 1: Average Conditional Global Correlations}				
Global Equity Correlation	0.6583	0.6919	0.6762	0.5000	0.5008
(Boot. S.E.) $\|t\|$	(0.0412)	[0.816]	[0.435]	[3.841]	[3.823]
Global Bond Correlation	0.4655	0.4628	0.4523	0.6219	0.4482
	(0.0356)	[0.076]	[0.370]	[4.395]	[0.484]
Fit Fact 1?		Yes	Yes	No	No
Test Fact 2: Excessive Left-Tail Global Correlation in Equities					
	{Moment 2: Global Equity Correlation − Global Bond Correlation}				
$r^{World} \geqslant 0$	0.1864	0.2307	0.2210	-0.1224	0.0517
(Boot. S.E.) $\|t\|$	(0.0074)	[6.017]	[4.703]	[41.947]	[18.298]
$r^{World} < 0$	0.1994	0.2269	0.2279	-0.1212	0.0537
	(0.0083)	[3.316]	[3.433]	[38.564]	[17.517]
Fit Fact 2?		No	Yes	No	No
Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Corrleation					
	{Moment 3: Sensitivity to OECD World Recession Indicator}				
Global Equity Correlation	0.0185	0.0196	0.0337	-0.0002	0.0015
(Boot. S.E.) $\|t\|$	(0.0097)	[0.117]	[1.566]	[1.918]	[1.743]
Global Bond Correlation	-0.0259	-0.0116	-0.0337	-0.0007	-0.0007
	(0.0131)	[1.091]	[0.598]	[1.927]	[1.931]
Fit Fact 3?		Yes	Yes	No	No

Table 8: Global Return Covariance Decomposition.

This table calculates the extent to which each state variable contributes to the global equity conditional covariance and global bond conditional covariance. For Country i and Country j ($i \neq j$), the covariance share explained by factor ω_κ is,

$$\frac{\beta_{i,t,\kappa} Var_t(\omega_{\kappa,t+1})\beta_{j,t,\kappa}}{\boldsymbol{\beta'_{i,t}} Var_t(\boldsymbol{\Omega_{t+1}})\boldsymbol{\beta_{j,t}}},$$

where $\beta_{t,\kappa} = \beta_{0,\kappa} + \beta_{1,\kappa} s_t$ denotes the sensitivity of country asset return on ω_κ, conditional at information set t; $\boldsymbol{\beta_t}$ denotes the vector of betas for the same country; s_t is the standardized business condition instrument. For time-varying beta factor models, $\beta_{i,t,\kappa} Var_t(\omega_{\kappa,t+1})\beta_{j,t,\kappa}$ can be further decomposed into a β_0 part, $\beta_{i,0,\kappa} Var_t(\omega_{\kappa,t+1})\beta_{j,0,\kappa}$ and a β_1 beta part, $\beta_{i,1,\kappa} Var_t(\omega_{\kappa,t+1})\beta_{j,1,\kappa} s_t^2$. The numbers below report the average relative share of each part of each state variable across time and across all unique country pairs; rows in bold summarize the sum. In the second last row, the share of total explained comovement is obtained by dividing the average pairwise total model-implied conditional covariance by the unconditional pairwise average covariance in data.

		○ Constant Beta		○ Time-Varying Beta			
		Equity	Bond	Equity		Bond	
Risk Aversion:	$\boldsymbol{\omega_q}$	**90.3%**	**78.2%**	**90.5%**		**40.0%**	
				[β_0]	88.6%	[β_0]	27.2%
				[β_1]	1.9%	[β_1]	12.9%
Real Uncertainties:	**Total**	**5.2%**	**-1.9%**	**7.4%**		**-3.8%**	
	$\omega_{\theta u}$	5.5%	-0.4%	5.1%		3.2%	
				[β_0]	4.9%	[β_0]	-3.8%
				[β_1]	0.2%	[β_1]	7.0%
	$\omega_{\theta d}$	-0.3%	-1.5%	2.3%		-6.9%	
				[β_0]	0.9%	[β_0]	-5.3%
				[β_1]	1.4%	[β_1]	-1.7%
Inflation Uncertainties:	**Total**	**2.8%**	**33.6%**	**1.1%**		**46.8%**	
	$\omega_{\pi u}$	1.8%	10.8%	1.3%		48.6%	
				[β_0]	0.1%	[β_0]	43.1%
				[β_1]	1.2%	[β_1]	5.5%
	$\omega_{\pi d}$	1.0%	22.8%	-0.2%		-1.8%	
				[β_0]	0.3%	[β_0]	0.7%
				[β_1]	-0.5%	[β_1]	-2.5%
Real Short Rate Uncertainties:	**Total**	**1.7%**	**-9.9%**	**1.0%**		**17.0%**	
	ω_{xu}	-0.2%	-10.1%	-0.1%		21.9%	
				[β_0]	-0.3%	[β_0]	14.7%
				[β_1]	0.2%	[β_1]	7.2%
	ω_{xd}	1.9%	0.1%	1.1%		-4.9%	
				[β_0]	1.1%	[β_0]	-5.5%
				[β_1]	-0.1%	[β_1]	0.5%
Share of Explained Comovement		49.4%	0.9%	54.6%		15.6%	
Excluding Risk Aversion		4.8%	0.2%	5.2%		9.3%	

Table 9: Global Return Correlation Decomposition.

This table presents the extent to which each factor contributes to fitting the factor model-implied global correlations. Row "All Shocks" reports the correlation between the factor model-implied correlation and the empirical benchmark, or $\rho(CORR_{0,t}, BM_t)$; the bootstrapped standard errors are shown in the parentheses. The correlation between factor model-implied correlation using all shocks excluding Shock ω_κ and the empirical benchmarks is denoted as $\rho(CORR_{\backslash\kappa,t}, BM_t)$. The rest of the rows reports $\rho(CORR_{0,t}, BM_t) - \rho(CORR_{\backslash\kappa,t}, BM_t)$ (the higher the number, the more marginal contribution a factor has). Due to the potential structural break in bond markets around January 1999, BM_t is HP-filtered and is also depicted in the right plots in Figure 4 (see discussions in Section 2.4 and Footnote[24]).

	Equity		Bond	
All Shocks	0.623	(0.038)	0.097	(0.055)
Risk Aversion, ω_q	0.866		0.138	
Real Upside Uncertainty, $\omega_{\theta u}$	0.010		0.039	
Real Downside Uncertainty, $\omega_{\theta d}$	0.017		0.035	
Inflation Upside Uncertainty, $\omega_{\pi u}$	-0.003		0.044	
Inflation Downside Uncertainty, $\omega_{\pi d}$	0.015		0.032	
Real Short Rate Upside Uncertainty, ω_{xu}	0.013		0.024	
Real Short Rate Downside Uncertainty, ω_{xd}	0.014		0.034	

Table 10: Dynamic Factor Model Fit & Economic Significance of Other State Variables.

This table evaluates the fit of dynamic factor models excluding one factor shock at a time to demonstrate its economic significance. Panel A (Panel B) considers constant beta models (time-varying beta models). Other details are described in Table 7.

	*Panel A. Dynamic Factor Model with **Constant Betas**:*					
Excluding:	$\omega_{\theta u}$	$\omega_{\theta d}$	$\omega_{\pi u}$	$\omega_{\pi d}$	ω_{xu}	ω_{xd}
Test Fact 1: Equity Correlation > Bond Corrleation						
	{Moment 1: Average Conditional Global Correlations}					
Global Equity Correlation	0.7732	0.7733	0.7752	0.7737	0.7730	0.7747
$\lvert t \rvert$	[2.790]	[2.792]	[2.837]	[2.803]	[2.784]	[2.826]
Global Bond Correlation	0.4713	0.4623	0.4621	0.4655	0.4602	0.4640
	[0.163]	[0.088]	[0.093]	[0.002]	[0.148]	[0.040]
Fit Fact 1?	Yes	Yes	Yes	Yes	Yes	Yes
Test Fact 2: Excessive Left-Tail Global Correlation in Equities						
	{Moment 2: Global Equity Correlation–Global Bond Correlation}					
$r^{World} \geqslant 0$	0.3032	0.3122	0.3142	0.3095	0.3141	0.3120
$\lvert t \rvert$	[15.870]	[17.089]	[17.366]	[16.719]	[17.350]	[17.058]
$r^{World} < 0$	0.3002	0.3093	0.3113	0.3064	0.3109	0.3088
	[12.129]	[13.221]	[13.467]	[12.880]	[13.420]	[13.169]
Fit Fact 2?	No	No	No	No	No	No
Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Corrleation						
	{Moment 3: Sensitivity to OECD World Recession Indicator}					
Global Equity Correlation	0.0099	0.0105	0.0097	0.0097	0.0100	0.0098
$\lvert t \rvert$	[0.884]	[0.823]	[0.902]	[0.903]	[0.867]	[0.896]
Global Bond Correlation	-0.0116	-0.0114	-0.0123	-0.0115	-0.0115	-0.0117
	[1.093]	[1.108]	[1.043]	[1.099]	[1.101]	[1.087]
Fit Fact 3?	Yes	Yes	Yes	Yes	Yes	Yes
	*Panel B. Dynamic Factor Model with **Time-Varying Betas***					
Excluding:	$\omega_{\theta u}$	$\omega_{\theta d}$	$\omega_{\pi u}$	$\omega_{\pi d}$	ω_{xu}	ω_{xd}
Test Fact 1: Equity Correlation > Bond Corrleation						
	{Moment 1: Average Conditional Global Correlations}					
Global Equity Correlation	0.7601	0.7600	0.7645	0.7622	0.7592	0.7619
$\lvert t \rvert$	[2.472]	[2.470]	[2.579]	[2.523]	[2.450]	[2.514]
Global Bond Correlation	0.3528	0.3460	0.3529	0.3502	0.3598	0.3600
	[3.165]	[3.356]	[3.161]	[3.238]	[2.969]	[2.961]
Fit Fact 1?	Yes	Yes	Yes	Yes	Yes	Yes
Test Fact 2: Excessive Left-Tail Global Correlation in Equities						
	{Moment 2: Global Equity Correlation–Global Bond Correlation}					
$r^{World} \geqslant 0$	0.4035	0.4103	0.4075	0.4081	0.3962	0.3978
$\lvert t \rvert$	[29.486]	[30.416]	[30.039]	[30.121]	[28.495]	[28.718]
$r^{World} < 0$	0.4126	0.4192	0.4171	0.4173	0.4039	0.4073
	[25.654]	[26.444]	[26.195]	[26.220]	[24.605]	[25.015]
Fit Fact 2?	Yes	Yes	Yes	Yes	Yes	Yes
Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Corrleation						
	{Moment 3: Sensitivity to OECD World Recession Indicator}					
Global Equity Correlation	0.0202	0.0221	0.0169	0.0198	0.0209	0.0203
$\lvert t \rvert$	[0.180]	[0.370]	[0.163]	[0.134]	[0.243]	[0.188]
Global Bond Correlation	-0.0446	-0.0437	-0.0465	-0.0451	-0.0505	-0.0460
	[1.430]	[1.361]	[1.578]	[1.466]	[1.884]	[1.533]
Fit Fact 3?	Yes	Yes	Yes	Yes	Yes	Yes

Table 11: Extensions: Jackknife Exercise for Global Comovement.

This table presents the closeness between the global comovement estimates using the full country sample (empirical benchmarks) and those omitting one country at a time. Two statistics are reported: 1. correlation with the empirical benchmarks; 2. mean difference (new global correlation minus the empirical benchmarks) with a t statistics reported.

Country omitted:	USA	CAN	DEU	FRA	GBR	CHE	JPN	AUS
Global equity comovement								
1. Correlation w/ BM	0.99	0.99	0.99	0.99	0.99	0.98	0.96	0.89
2. Mean–BM	-0.002	0.007	-0.001	-0.008	-0.014	0.005	0.061	0.010
t stats	-0.23	1.03	-0.12	-1.14	-2.11	0.74	10.02	1.48
Global bond comovement								
1. Correlation w/ BM	0.96	0.94	0.99	0.99	0.98	0.98	0.94	0.86
2. Mean–BM	0.034	0.017	-0.043	-0.041	-0.020	-0.018	0.019	0.019
t stats	3.64	1.83	-4.94	-4.59	-2.29	-2.03	2.29	2.77

Table 12: Extensions: Alternative Global Comovement Measures.

This table presents the closeness between the paper and alternative global comovement measures. "Paper" refers to the multivariate dynamic correlation estimates using the DECO-class model in the paper; "Pairwise" refers to the average of pairwise dynamic correlation estimates using DCC-class models where each pair's dynamic correlation is estimated separately; "USD" and "LC" denote U.S. dollar and local currency denomination (source: DataStream), respectively. Panels A and B present the correlation matrices among measures. Panel C tests whether the average of pairwise DCC estimates and the paper's DECO estimates are equal; the three rows represent average magnitudes and the relevant t statistics in absolute terms. Panel D reports the average comovement magnitudes (between one country and the other seven countries) using the DCC estimates; bold values indicate that this country's comovement is greater than the global pairwise average.

Panel A. Global Equity Comovement Measures				
	Paper,USD	Pairwise,USD	Paper,LC	Pairwise,LC
Paper,USD	1	0.953	0.886	0.904
Pairwise,USD		1	0.845	0.877
Paper,LC			1	0.972
Pairwise,LC				1
Panel B. Global Bond Comovement Measures				
	Paper,USD	Pairwise,USD	Paper,LC	Pairwise,LC
Paper,USD	1	0.963	0.672	0.676
Pairwise,USD		1	0.755	0.767
Paper,LC			1	0.977
Pairwise,LC				1
Panel C. Equality Test				
	Equity,USD	Equity,LC	Bond,USD	Bond,LC
Paper	0.658	0.647	0.465	0.564
Pairwise	0.666	0.657	0.458	0.558
$\|t\|$	[1.108]	[1.315]	[0.561]	[0.493]
Panel D. Average Comovement by Countries (bold: > DECO)				
	Equity,USD	Equity,LC	Bond,USD	Bond,LC
USA	**0.692**	**0.723**	0.345	**0.589**
CAN	**0.674**	**0.666**	0.393	**0.589**
DEU	**0.690**	**0.685**	**0.581**	**0.647**
FRA	**0.712**	**0.705**	**0.580**	**0.623**
GBR	**0.733**	**0.709**	**0.497**	**0.611**
CHE	0.665	0.657	**0.510**	0.513
JPN	0.514	0.474	0.379	0.371
AUS	0.652	0.636	0.381	0.520

Table 13: Extensions: Stylized Facts Using Local Currency Data.

This table presents evidence of the 3 stylized facts using asset returns denominated in local currencies. Non-parametric tests and methodologies can be found in Tables 4, 5 and 6. ***, **, *: <1%, 5%,10% significance.

Test Fact 1: Equity Correlation > Bond Corrleation				
	Equity, LC	Bond, LC		
Data	0.6418	0.5504		
S.E.	(0.0339)	(0.0240)		
Jennrich's χ^2	81.166 (***)			
Test Fact 2: Excessive Left-Tail Global Correlation in Equity Returns				
		Equity, LC		
	25%	49%	51%	75%
Data	0.3101	0.3279	0.2522	0.1922
S.E.	(0.0208)	(0.0147)	(0.0154)	(0.0222)
		Bond, LC		
	25%	49%	51%	75%
Data	0.2295	0.2333	0.2295	0.2227
S.E.	(0.0218)	(0.0156)	(0.0156)	(0.0219)
Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Corrleation				
	Equity, LC		Bond, LC	
	Non-Recession	Recession	Non-Recession	Recession
Data	0.6143	0.6618	0.5503	0.5307
S.E.	(0.0414)	(0.0559)	(0.0297)	(0.0411)
Test 1: Jennrich's χ^2	35.73 (*)		90.44 (***)	
Test 2: t(Non-Recession - Recession)	-42.81 (***)		24.59 (***)	

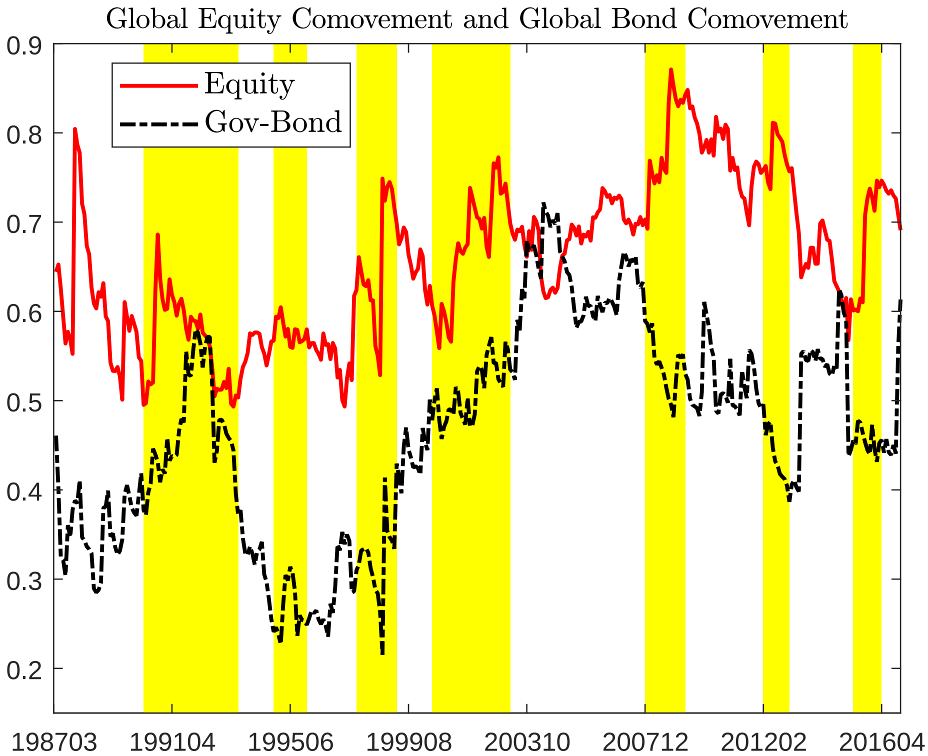

Figure 1: Global Dynamic Comovement Estimates.

This plot presents the dynamics of global comovements estimated from the empirical model in Section 2. The solid (dashed) line depicts the global equity (10-year government bond) correlations using log returns denominated in U.S. dollars. The shaded regions are OECD world recession months from the OECD website. Model estimation details are presented in Tables 2 and 3.

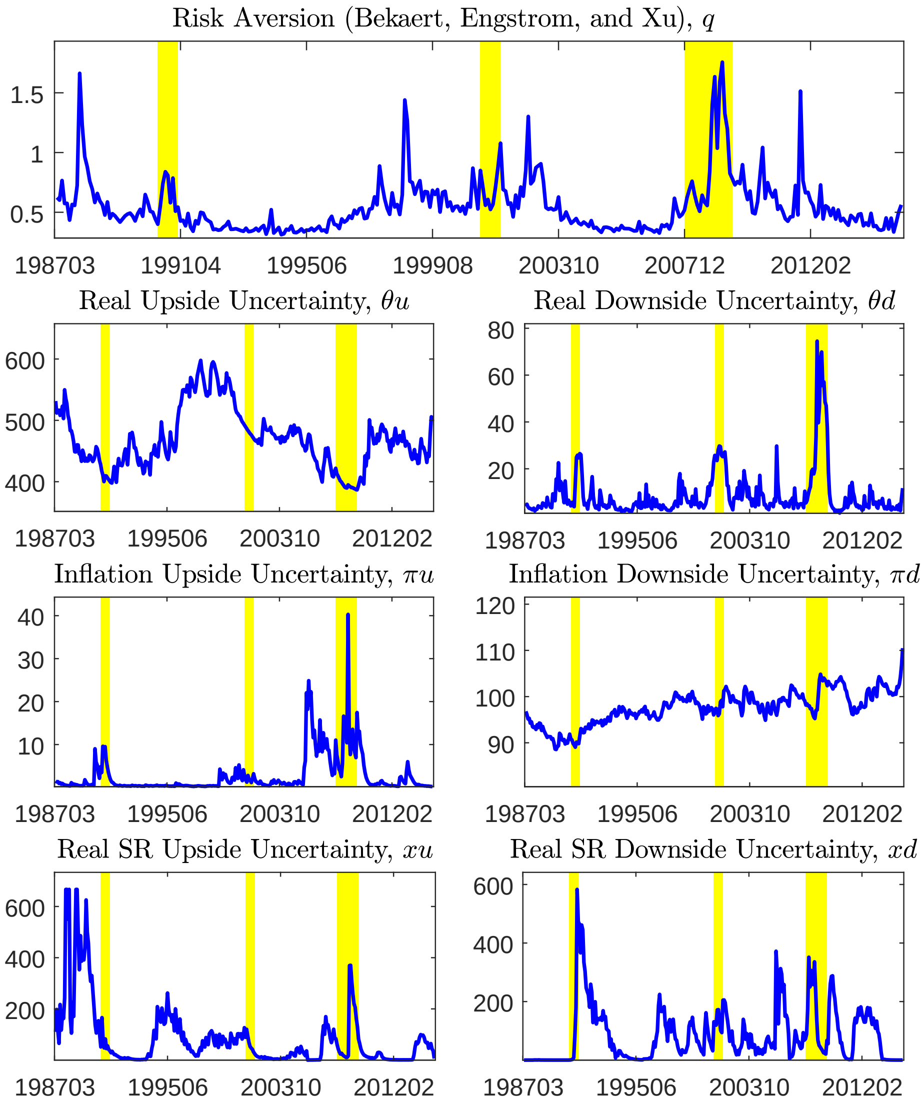

Figure 2: Dynamics of the Seven Economic Determinants.

This plot presents the empirical estimates of the 7 economic determinants of global comovements as implied from the asset pricing model. All state variables are shown in a balanced sample from 1987/03 to 2015/02; the actual estimations are conducted using the longest sample possible of each economic variable (real output growth and inflation, 1947/01–2016/12; real short rate, 1987/03–2015/02). According to the model, these state variables are shape parameters of the following gamma-distributed shocks (from top to bottom, left to right): pure risk aversion shock (ω_q), upside and downside real shocks ($\omega_{\theta u}$ and $\omega_{\theta d}$), upside and downside inflation shocks ($\omega_{\pi u}$ and $\omega_{\pi d}$), upside and downside short rate shocks (ω_{xu} and ω_{xd}). The magnitudes of actual uncertainties are combinations of shape parameters and scale parameters, for instance, real output growth uncertainty is $\sigma^2_{\theta\theta u}\theta u_t + \sigma^2_{\theta\theta d}\theta d_t$ according to Equation (13). The magnitudes of the scale parameters and long-sample plots are available in the Internet Appendix. The shaded regions are NBER U.S. recession month from the NBER website.

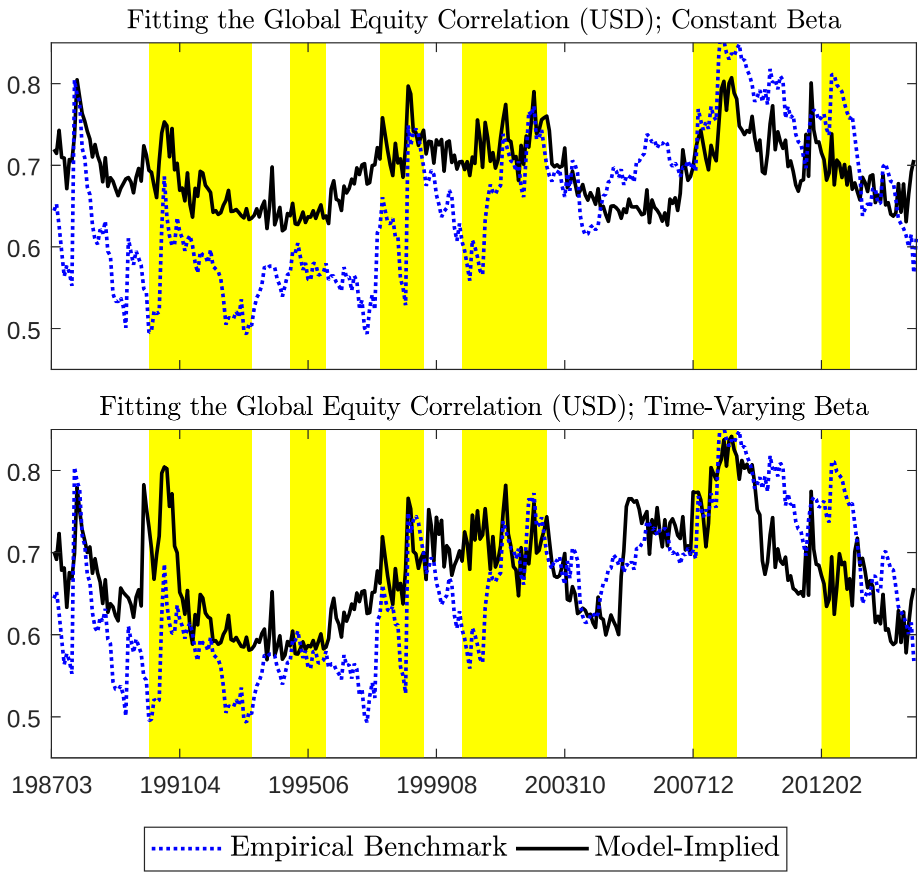

Figure 3: Empirical Benchmark and Model-Implied (Dynamic Factor Model) Global Equity Return Comovements.

The shaded regions are OECD world recession months from the OECD website.

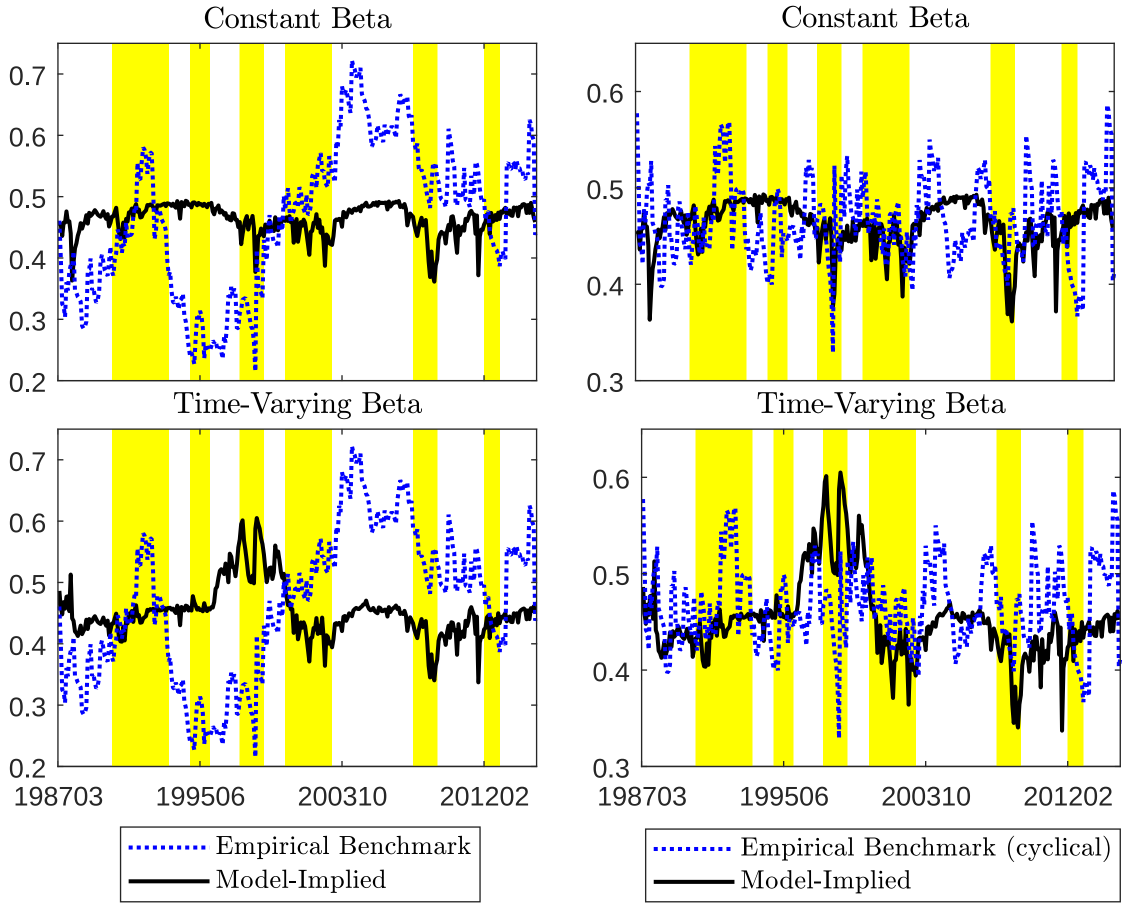

Figure 4: Empirical Benchmark and Model-Implied (Dynamic Factor Model) Global Bond Return Comovements.

The left plots provide comparisons using the empirical benchmark (as in Figure 1); the right plots provide comparisons using HP-filtered global bond comovement ($\lambda = 14400$ as suggested in Hodrick and Prescott (1997)). The shaded regions are OECD world recession months from the OECD website.

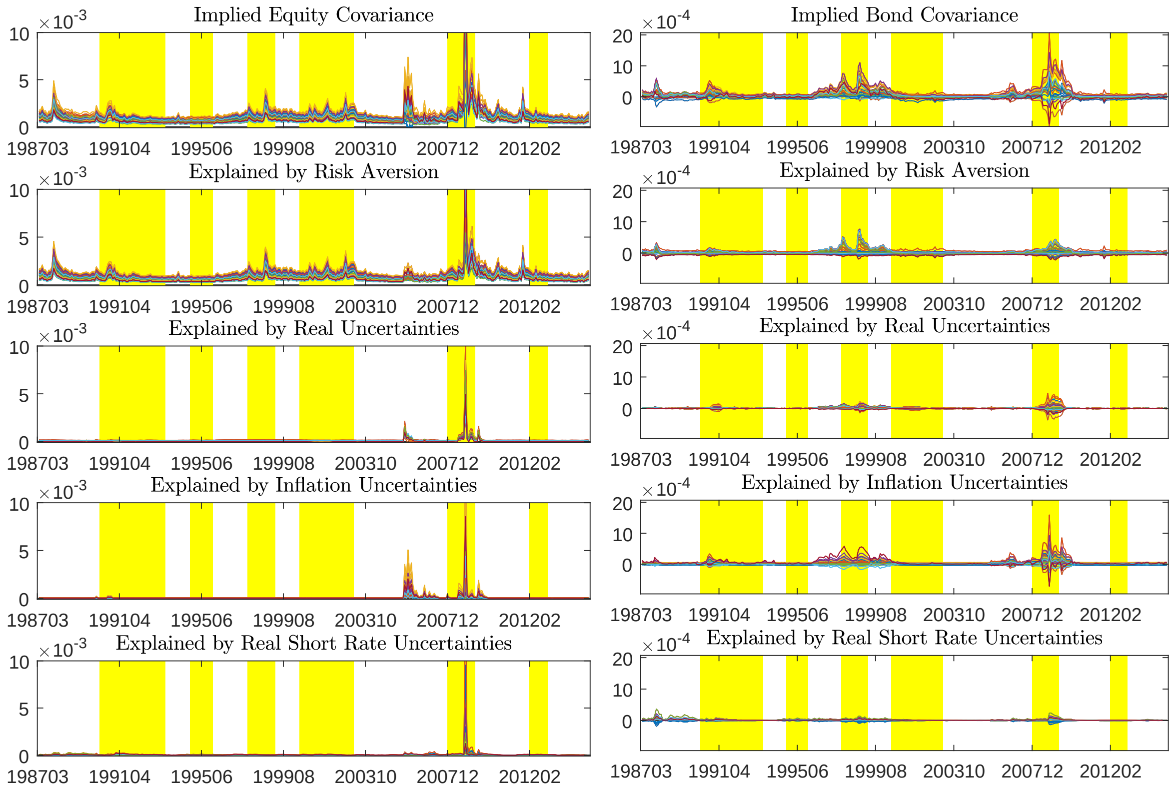

Figure 5: Model-Implied (Dynamic Factor Model) Conditional Covariances Explained by Economic Determinants.

The plot depicts the time variation in conditional covariances explained by each of the four determinant categories (risk aversion, real uncertainties, inflation uncertainties, short rate uncertainties) for all 28 unique country pairs. This plot can be thought of as the dynamic version of Table 8. The shaded regions are OECD world recession months from the OECD website.

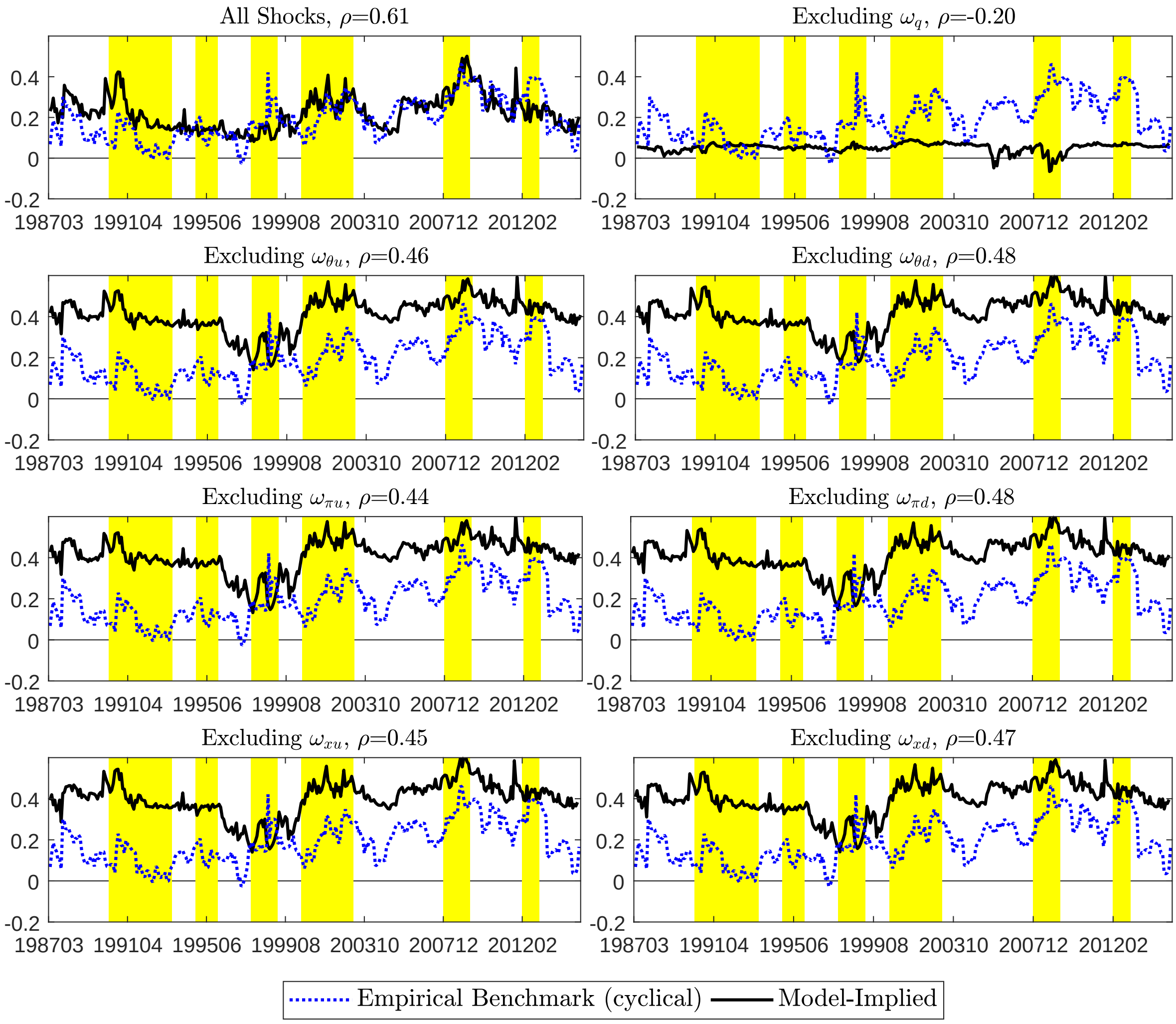

Figure 6: The Difference between Model-Implied Global Equity Return Correlation and Global Bond Return Correlation; Full Shocks and Omitting One Shock.

The benchmark (model-implied) differences are depicted in dashed blue (solid black) lines. Due to the structural break concern, the global bond correlation benchmark value is HP-filtered and is as depicted in the right plots in Figure 4. The shaded regions are OECD world recession months from the OECD website.

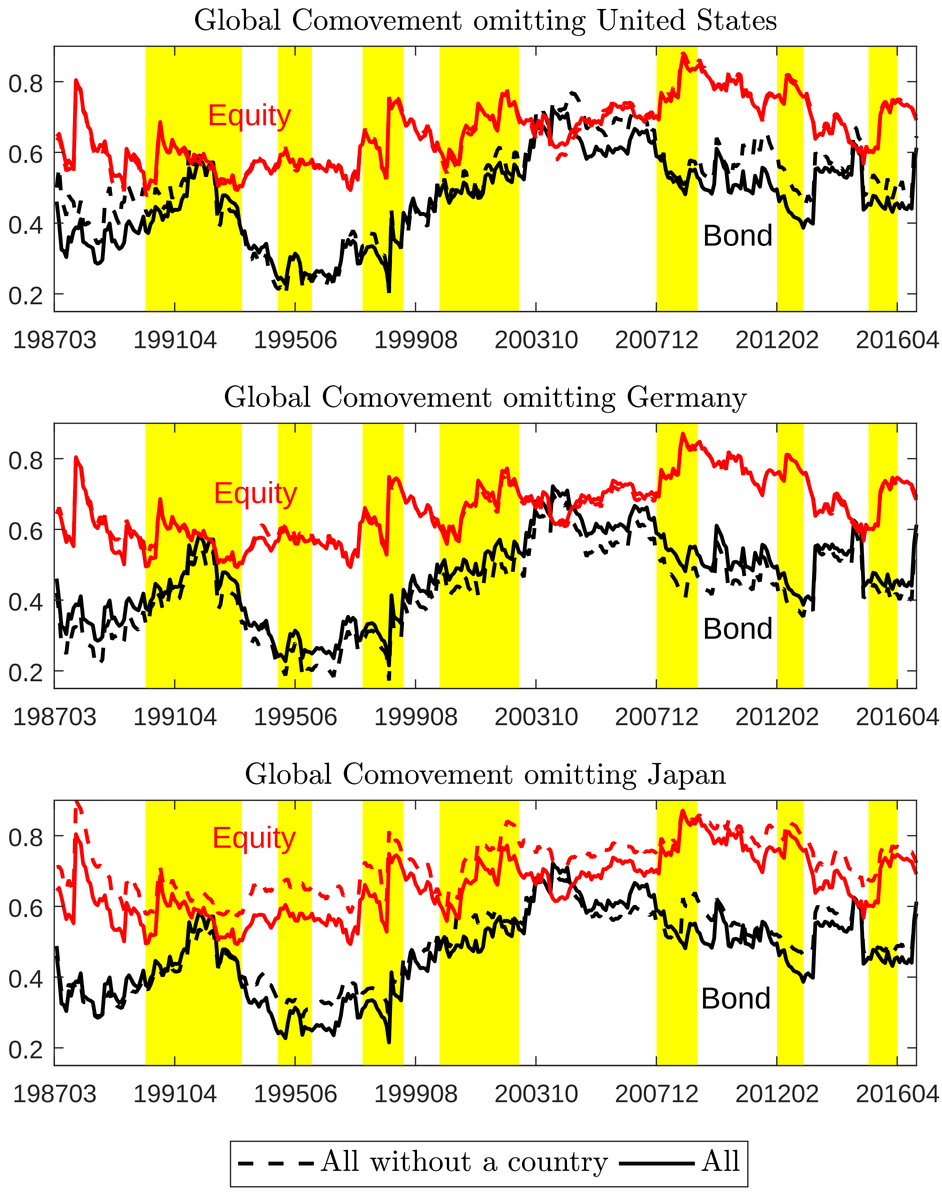

Figure 7: Jackknife Exercise for Global Comovement: Omitting United States, Germany, Japan.

The figure compares the global comovement estimates using the full country sample (empirical benchmarks) and those omitting certain countries (Exercise (I) in Section 5). The thick (dashed) lines in all three plots are the empirical benchmarks as depicted in Figure 1 (new estimates). Table 11 offers more information for other countries. The shaded regions are OECD world recession months from the OECD website.

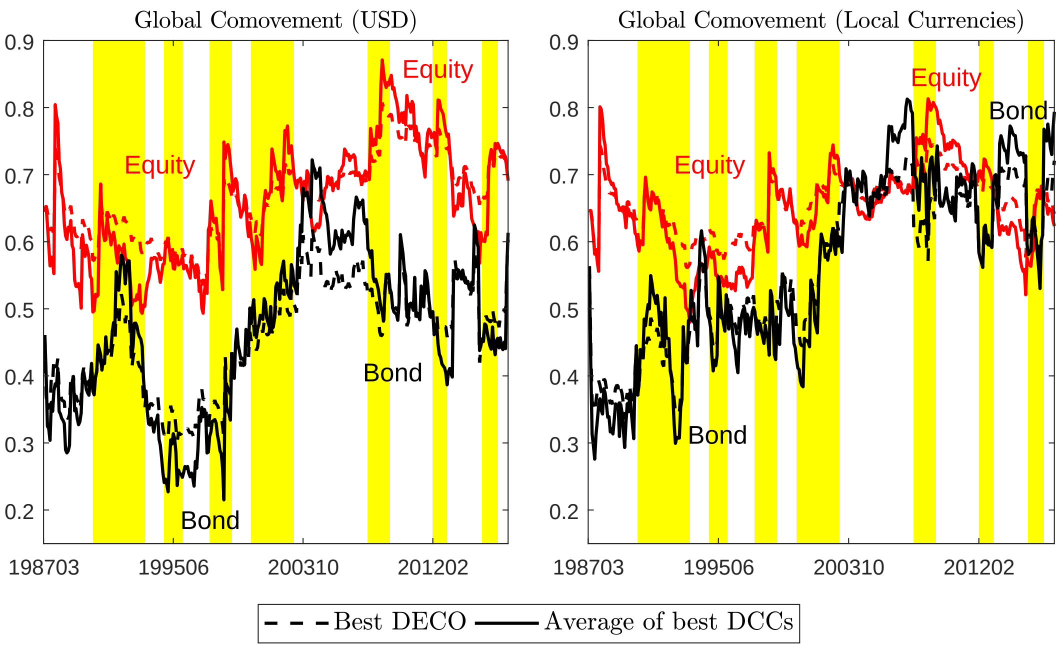

Figure 8: Alternative Global Dynamic Comovement Measures.

This figure presents alternative global return correlation estimates using the average of pairwise DCC models (Exercise (II) in Section 5) and using local currency returns (Exercise (II) in Section 5). The solid lines in the left plots are empirical benchmarks as depicted in Figure 1. The shaded regions are OECD world recession months from the OECD website.

Internet Appendices for
"Global Risk Aversion and International Return Comovements"

Nancy R. Xu
September 9, 2019

In this file, I provide additional tables and figures to supplement the main results.

I. Additional tables for Section 2, the empirical model:
Table A1: Best conditional volatility models for each country asset
Table A3: Flight-to-safety estimates
Table A2: Values of constant matrices "Ξ" in the Global Dynamic Comovement Model that captures the percentage of joint negative events between two countries
Table A4: Robustness check for the empirical model estimation, using alternative country-level recession indicators
Table A5: Robustness check for the empirical model estimation, without the Duo structure

II. Additional tables for Section 3, the economic determinants:
Table A6: Estimation results of the U.S. real output growth upside and downside uncertainties and shocks
Table A7: Estimation results of the U.S. inflation upside and downside uncertainties and shocks
Table A8: Estimation results of the U.S. real short rate upside and downside uncertainties and shocks

III. Additional tables for Section 4, the dynamic factor model:
Table A9: Factor exposures in a SUR framework; constant betas
Table A10: Factor exposures in a SUR framework; time-varying betas; return in U.S. dollars
Table A11: Factor exposures in a SUR framework; time-varying betas; return in local currencies
Table A12: Model-implied conditional variance decomposition

IV. Additional figures:
Figure A1: Time variations in the global bond return volatility
Figure A2: Time variations in the global equity return volatility
Figure A3: time variations in the U.S. real output growth upside and downside uncertainties
Figure A4: Time variation in the U.S. inflation upside and downside uncertainties
Figure A5: Time variation in the U.S. real short rate upside and downside uncertainties
Figure A6: Time variation in regional dynamic comovements.

V Solving an International Asset Pricing Model

In the main text (Section 3), for simplicity, I assume that there exists a global investor who prices both U.S. and foreign country assets (equities and Treasury bonds), and thus the asset prices are solved from the perspective of this global investor. The advantage of that parsimonious framework is to motivate a global dynamic factor model examined in Section 4.

In this appendix section, I acknowledge the exchange rates dynamics and different real pricing kernel of each country. For each country, its domestic investor prices domestic assets where (1) the domestic macro environment and investor risk aversion receive global state variable exposures, and (2) the domestic investor's pricing kernel reflects partial integration. Section V.1 introduces the U.S. state variables and real pricing kernel and solves the U.S. asset prices; Section V.2 discusses the individual country real pricing kernels and state variables as well as model solutions.

The main take-away is that a global dynamic factor model still holds.

V.1 The U.S. Asset Market

V.1.1 U.S. State Variable Dynamics

V.1.1.a Matrix representation In a matrix representation, the U.S. state vector at time t is denoted as $\boldsymbol{X_{t+1}}$ (11 $\times$ 1),

$$\begin{bmatrix} \theta_{t+1} & \text{Industrial production growth} \\ \theta u_{t+1} & \text{Real upside uncertainty} \\ \theta d_{t+1} & \text{Real downside uncertainty} \\ \pi_{t+1} & \text{Inflation} \\ \pi u_{t+1} & \text{Nominal upside uncertainty} \\ \pi d_{t+1} & \text{Nominal downside uncertainty} \\ x_{t+1} & \text{Real short rate} \\ xu_{t+1} & \text{Real short rate upside uncertainty} \\ xd_{t+1} & \text{Real short rate downside uncertainty} \\ g_{t+1} & \text{Dividend growth} \\ q_{t+1} & \text{Global risk aversion} \end{bmatrix},$$

which follows this general dynamics:

$$\boldsymbol{X_{t+1}} = \boldsymbol{\xi_{X,t}} + Jensen's\,(\boldsymbol{\delta_X}, \boldsymbol{S_t}) + \boldsymbol{\delta_X \omega_{t+1}}, \tag{A1}$$

$$\boldsymbol{\omega_{t+1}} \sim \Gamma(\boldsymbol{S_t}, \boldsymbol{1}) - \boldsymbol{S_t}, \tag{A2}$$

where $\boldsymbol{\xi_{X,t}}$ (11 $\times$ 1) denotes the conditional mean vector; $\boldsymbol{\omega_{t+1}}$ (8 $\times$ 1) denotes the global state variable shock matrix $\begin{bmatrix}\omega_{\theta u,t+1} & \omega_{\theta d,t+1} & \omega_{\pi u,t+1} & \omega_{\pi d,t+1} & \omega_{xu,t+1} & \omega_{xd,t+1} & \omega_{g,t+1} & \omega_{q,t+1}\end{bmatrix}'$ where the shocks are mutually independent; $\boldsymbol{\delta_X}$ (11 $\times$ 8) denotes the constant coefficient matrix to the state variable shocks $\boldsymbol{\omega_{t+1}}$; $\boldsymbol{S_t}$ (8 $\times$ 1) is the vector of the shock shape parameters $\begin{bmatrix}\theta u_t & \theta d_t & \pi u_t & \pi d_t & xu_t & xd_t & v & q_t\end{bmatrix}'$; $Jensen's\,(\boldsymbol{\delta_X}, \boldsymbol{S_t})$ denotes the Jensen's inequality term from Gamma distributions; $\Gamma(s, 1)$ denotes the Gamma random variable with a shape parameter s and a scale parameter 1.

The six uncertainty state variables and their shocks are denoted as,

$$\boldsymbol{U_t} = \begin{bmatrix}\theta u_t & \theta d_t & \pi u_t & \pi d_t & xu_t & xd_t\end{bmatrix}',$$

$$\boldsymbol{\omega_{U,t+1}} = \begin{bmatrix}\omega_{\theta u,t+1} & \omega_{\theta d,t+1} & \omega_{\pi u,t+1} & \omega_{\pi d,t+1} & \omega_{xu,t+1} & \omega_{xd,t+1}\end{bmatrix}'.$$

V.1.1.b Output growth and uncertainties I follow Bekaert, Engstrom, and Xu (2019) to model industrial production growth innovation with two centered independent gamma shocks where each shock has a time-varying shape parameter that governs the higher moments of the shock. I name the shape parameter that governs the right-tail (left-tail) skewness the real upside (downside) uncertainty, θu (θd).[33] Formally, θ_{t+1} has the following process,

$$\theta_{t+1} = \overline{\theta} + \rho_{\theta\theta}(\theta_t - \overline{\theta}) + \rho_{\theta\theta u}(\theta u_t - \overline{\theta u}) + \rho_{\theta\theta d}(\theta d_t - \overline{\theta d}) + u^{\theta}_{t+1}, \tag{A3}$$

where the growth shock is decomposed into two independent shocks,

$$u^{\theta}_{t+1} \quad = \quad \delta_{\theta\theta u}\omega_{\theta u,t+1} - \delta_{\theta\theta d}\omega_{\theta d,t+1}. \tag{A4}$$

[33] Note that Bekaert, Engstrom, and Xu (2019) name them "good" and "bad" uncertainties to assign economic meanings of real uncertainties, whereas my notation here is more general and consistent as, for example, inflation upside uncertainty (later) is not typically considered as "good" uncertainty.

The shocks follow centered Gamma distributions with time-varying shape parameters,

$$\omega_{\theta u,t+1} \sim \widetilde{\Gamma}(\theta u_t, 1) \tag{A5}$$
$$\omega_{\theta d,t+1} \sim \widetilde{\Gamma}(\theta d_t, 1), \tag{A6}$$

where $\widetilde{\Gamma}(y, 1)$ denotes a centered Gamma-distributed random variable with shape parameter y and a unit scale parameter. The shape factors, θu_t and θd_t, follow autoregressive processes,

$$\theta u_{t+1} = \overline{\theta u} + \rho_{\theta u}(\theta u_t - \overline{\theta u}) + \delta_{\theta u}\omega_{\theta u,t+1} \tag{A7}$$
$$\theta d_{t+1} = \overline{\theta d} + \rho_{\theta d}(\theta d_t - \overline{\theta d}) + \delta_{\theta d}\omega_{\theta d,t+1}, \tag{A8}$$

where ρ_y denotes the autoregressive term of process y_{t+1}, δ_y the sensitivity to $\omega_{y,t+1}$, and $\overline{y}$ the constant long-run mean. Given that Gamma distributions are right-skewed by design, the growth shock with a negative loading on $\omega_{\theta d,t+1}$ models the left-tail events; hence, $\omega_{\theta d,t+1}$ is interpreted as the downside uncertainty shocks, and θd_t the real downside uncertainty.
State variables: $\{\theta, \theta u, \theta d\}$.
State variable shocks: $\{\omega_{\theta u}, \omega_{\theta d}\}$.

V.1.1.c Inflation and uncertainties Inflation process receives contemporaneous shocks from the real side. Denote π_{t+1} as the change in the log consumer price index for all urban consumers, πu_t the nominal upside uncertainty and πd_t the nominal downside uncertainty. The inflation system follows this reduced-form dynamics,

$$\begin{aligned}\pi_{t+1} = \overline{\pi} + \rho_{\pi\theta}(\theta_t - \overline{\theta}) + \rho_{\pi\theta u}(\theta u_t - \overline{\theta u}) + \rho_{\pi\theta d}(\theta d_t - \overline{\theta d})\\ + \rho_{\pi\pi}(\pi_t - \overline{\pi}) + \rho_{\pi\pi u}(\pi u_t - \overline{\pi u}) + \rho_{\pi\pi d}(\pi d_t - \overline{\pi d}) + u^{\pi}_{t+1},\end{aligned} \tag{A9}$$

where the inflation disturbance is sensitive to the two real uncertainty shocks, and the residual is decomposed into two nominal uncertainty shocks that are mutually independent of one another,

$$u^{\pi}_{t+1} = (\delta_{\pi\theta u}\omega_{\theta u,t+1} + \delta_{\pi\theta d}\omega_{\theta d,t+1}) + (\delta_{\pi\pi u}\omega_{\pi u,t+1} - \delta_{\pi\pi d}\omega_{\pi d,t+1}). \tag{A10}$$

The shocks follow centered Gamma distributions with time-varying shape parameters,

$$\omega_{\pi u,t+1} \sim \widetilde{\Gamma}(\pi u_t, 1) \tag{A11}$$
$$\omega_{\pi d,t+1} \sim \widetilde{\Gamma}(\pi d_t, 1), \tag{A12}$$
$$\pi u_{t+1} = \overline{\pi u} + \rho_{\pi u}(\pi u_t - \overline{\pi u}) + \delta_{\pi u}\omega_{\pi u,t+1} \tag{A13}$$
$$\pi d_{t+1} = \overline{\pi d} + \rho_{\pi d}(\pi d_t - \overline{\pi d}) + \delta_{\pi d}\omega_{\pi d,t+1}. \tag{A14}$$

Importantly, the theoretical structural representation of the inflation dynamics above is,

$$\pi_{t+1} = \xi_{\pi,t} + [\boldsymbol{\delta_\pi} - \ln(\mathbf{1} + \boldsymbol{\delta_\pi})]\boldsymbol{S_t} + \boldsymbol{\delta_\pi}\boldsymbol{\omega_{t+1}}, \tag{A15}$$

where $\boldsymbol{\delta_\pi} = \begin{bmatrix}\delta_{\pi\theta u} & \delta_{\pi\theta d} & \delta_{\pi\pi u} & -\delta_{\pi\pi d} & 0 & 0 & 0 & 0\end{bmatrix}$ so that the relevant shocks are $\omega_{\theta u,t+1}$, $\omega_{\theta d,t+1}$, $\omega_{\pi u,t+1}$, and $\omega_{\pi d,t+1}$. The signs of the the innovation loadings on the two real uncertainty shocks, $\omega_{\theta u,t+1}$ and $\omega_{\theta d,t+1}$, are not restricted in the model, whereas $\delta_{\pi\pi u}$ and $\delta_{\pi\pi d}$ are assumed to be positive.
State variables: $\{\pi, \pi u, \pi d\}$.
State variable shocks: $\{\omega_{\pi u}, \omega_{\pi d}\}$.

V.1.1.d Risk aversion

Denote q_t as the time-varying risk aversion variable,[34]

$$\begin{aligned} q_{t+1} = {} & \overline{q} + \rho_{q\theta}(\theta_t - \overline{\theta}) + \rho_{q\theta u}(\theta u_t - \overline{\theta u}) + \rho_{q\theta d}(\theta d_t - \overline{\theta d}) \\ & + \rho_{q\pi}(\pi_t - \overline{\pi}) + \rho_{q\pi u}(\pi u_t - \overline{\pi u}) + \rho_{q\pi d}(\pi d_t - \overline{\pi d}) + \rho_{qq}(q_t - \overline{q}) + u^q_{t+1}, \end{aligned} \tag{A16}$$

where the risk aversion shock is sensitive to the real and nominal uncertainty shocks, the short rate shock and a risk aversion-specific heteroskedastic shock,

$$u^q_{t+1} = (\delta_{q\theta u}\omega_{\theta u,t+1} + \delta_{q\theta d}\omega_{\theta d,t+1}) + (\delta_{q\pi u}\omega_{\pi u,t+1} + \delta_{q\pi d}\omega_{\pi d,t+1}) + \delta_{qq}\omega_{q,t+1}, \tag{A17}$$

where the risk aversion-specific shock follows a centered heteroskedastic Gamma distribution,

$$\omega_{q,t+1} \sim \widetilde{\Gamma}\left(q_t, 1\right). \tag{A18}$$

State variables: $\{q\}$.
State variable shocks: $\{\omega_q\}$.

V.1.1.e Real short rate and uncertainties

Denote x_t as the latent real short rate,

$$\begin{aligned} x_{t+1} = {} & \overline{x} + \rho_{x\theta}(\theta_t - \overline{\theta}) + \rho_{x\theta u}(\theta u_t - \overline{\theta u}) + \rho_{x\theta d}(\theta d_t - \overline{\theta d}) \\ & + \rho_{x\pi}(\pi_t - \overline{\pi}) + \rho_{x\pi u}(\pi u_t - \overline{\pi u}) + \rho_{x\pi d}(\pi d_t - \overline{\pi d}) \\ & + \rho_{xx}(x_t - \overline{x}) + \rho_{xxu}(xu_t - \overline{xu}) + \rho_{xxd}(xd_t - \overline{xd}) + \rho_{xq}(q_t - \overline{q}) + u^x_{t+1}, \end{aligned} \tag{A19}$$

where the short rate shock is sensitive to the real and nominal uncertainty shocks as well as a short rate-specific homoskedastic shock,

$$u^x_{t+1} = (\delta_{x\theta u}\omega_{\theta u,t+1} + \delta_{x\theta d}\omega_{\theta d,t+1}) + (\delta_{x\pi u}\omega_{\pi u,t+1} + \delta_{x\pi d}\omega_{\pi d,t+1}) + \delta_{xq}\omega_{q,t+1} + \delta_{xxu}\omega_{xu,t+1} - \delta_{xxd}\omega_{xd,t+1}, \tag{A20}$$

where the (exogenous) short rate shocks follow centered Gamma distributions with time-varying shape parameters,

$$\omega_{xu,t+1} \sim \widetilde{\Gamma}\left(xu_t, 1\right), xu_{t+1} = \overline{xu} + \rho_{xu}(xu_t - \overline{xu}) + \delta_{xu}\omega_{xu,t+1}, \tag{A21}$$

$$\omega_{xd,t+1} \sim \widetilde{\Gamma}\left(xd_t, 1\right), xd_{t+1} = \overline{xd} + \rho_{xd}(xd_t - \overline{xd}) + \delta_{xd}\omega_{xd,t+1}. \tag{A22}$$

State variables: $\{x, xu, xd\}$.
State variable shocks: $\{\omega_{xu}, \omega_{xd}\}$.

V.1.1.f Real dividend growth

Denote g_t as the change in log real dividend per share,

$$g_{t+1} = \overline{g} + \rho_{g\theta}(\theta_t - \overline{\theta}) + \rho_{g\theta u}(\theta u_t - \overline{\theta u}) + \rho_{g\theta d}(\theta d_t - \overline{\theta d}) + \rho_{gg}(g_t - \overline{g}) + u^g_{t+1}, \tag{A23}$$

where the dividend growth shock is sensitive to the real and nominal uncertainty shocks as well as a dividend-specific homoskedastic shock,

$$u^g_{t+1} = (\delta_{g\theta u}\omega_{\theta u,t+1} + \delta_{g\theta d}\omega_{\theta d,t+1}) + \delta_{gg}\omega_{g,t+1}, \tag{A24}$$

where the sign of δ_{gg} is not restricted, and the dividend-specific shock is assumed to follow a homoskedastic Gamma distribution,

$$\omega_{g,t+1} \sim \widetilde{\Gamma}\left(v, 1\right). \tag{A25}$$

[34] It is a risk aversion variable, because the exact definition is risk aversion (motivated form a HARA utility is $\gamma \exp(q_t)$).

Importantly, the theoretical structural representation of the real growth dynamics above is,

$$g_{t+1} = \xi_{g,t} + [\boldsymbol{\delta_g} + \ln(\mathbf{1} - \boldsymbol{\delta_g})]\boldsymbol{S_t} + \boldsymbol{\delta_g}\boldsymbol{\omega_{t+1}}, \tag{A26}$$

where $\boldsymbol{\delta_g} = \begin{bmatrix}\delta_{g\theta u} & \delta_{g\theta d} & 0 & 0 & 0 & 0 & \delta_{gg} & 0\end{bmatrix}$ so that the relevant shocks are $\omega_{\theta u,t+1}$, $\omega_{\theta d,t+1}$, and $\omega_{g,t+1}$.
State variables: $\{g\}$.
State variable shocks: $\{\omega_g\}$.

V.1.2 U.S. Real Pricing Kernel

I specify the (minus) logarithm of the real global pricing kernel to be affine to the global state variable levels and shocks,

$$-m_{t+1} \quad = \quad x_t + [\boldsymbol{\delta_m} - \ln(\mathbf{1} + \boldsymbol{\delta_m})]\,\boldsymbol{S_t} + \boldsymbol{\delta_m}\boldsymbol{\omega_{t+1}}, \tag{A27}$$

where the drift x_t is the real short rate, $\boldsymbol{\delta_m}$ (1×8) prices of risks, $\boldsymbol{\omega_{t+1}}$ (8×1) the state variable shock matrix defined earlier, and $[\boldsymbol{\delta_m} - \ln(1 + \boldsymbol{\delta_m})]\,\boldsymbol{S_t}$ the Jensen's inequality term given the Gamma distributional assumptions.

The real global pricing kernel is spanned by five global shocks: the real upside and downside uncertainty shocks ($\omega_{\theta u}$ and $\omega_{\theta d}$), the inflation upside and downside uncertainty shocks ($\omega_{\pi u}$ and $\omega_{\pi d}$), and the risk aversion shock (ω_q). First, the two real-side uncertainty shock and the risk aversion shock span the pricing kernel, which can be motivated in Campbell and Cochrane (1999) and Bekaert, Engstrom, and Xu (2019). Second, the two inflation uncertainty shocks span the real pricing kernel, which is to induce the inflation risk premium.

V.1.3 U.S. Asset Prices and Risk Premiums

V.1.3.a Nominal Treasury Bonds The real global short rate ($y_{t,1} = -\ln\{[E_t[\exp(m_{t+1})]\}$) and the nominal global short rate ($\widetilde{y}_{t,1} = -\ln\{[E_t[\exp(m_{t+1} - \pi_{t+1})]\}$) are solved in closed forms,

$$y_{t,1} = x_t, \tag{A28}$$

$$\widetilde{y}_{t,1} = x_t + \underbrace{\xi_{\pi,t} + \ln\left[(\mathbf{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi}) \circ (\mathbf{1} + \boldsymbol{\delta_m})^{\circ -1} \circ (\mathbf{1} + \boldsymbol{\delta_\pi})^{\circ -1}\right]\boldsymbol{S_t}}_{\text{inflation compensation}}, \tag{A29}$$

[35] where "ln(.)" is the element-wise logarithm operator, "$\circ$" the Hadamard product of two identically sized matrices (or element-by-element matrix multiplication), and "$(.)^{\circ -1}$" the Hadamard inverse. The three components in nominal short rate are the real short rate (x_t), the expected inflation rate ($\xi_{\pi,t}$), and the inflation risk premium to compensate investors for bearing the inflation risk associated with the nominal bonds. It is noteworthy that the linear approximation of the inflation risk premium, $\ln\left[(\mathbf{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi}) \circ (\mathbf{1} + \boldsymbol{\delta_m})^{\circ -1} \circ (\mathbf{1} + \boldsymbol{\delta_\pi})^{\circ -1}\right]$, is $-(\boldsymbol{\delta_m} \circ \boldsymbol{\delta_\pi})\,\boldsymbol{S_t}$, or $Cov_t(m_{t+1}, \pi_{t+1})$ as derived in the Gaussian-augmented nominal term structure literature (see, e.g., Campbell, Sunderam, and Viceira, 2017).

The price of the n-period zero-coupon nominal bond ($\widetilde{P}^b_{t,n}$) can be then solved recursively in exact closed forms, and is an exponential affine function of a set of time-varying state variables.

$$\widetilde{P}^b_{t,n} = E_t\left[\exp\left(\widetilde{p}^b_{t+1,n-1} + m_{t+1} - \pi_{t+1}\right)\right] \tag{A30}$$

$$= E_t\left[\exp\left(x_{t+1} + \xi_{\pi,t+1} + \ln\left[(\mathbf{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi}) \circ (\mathbf{1} + \boldsymbol{\delta_m})^{\circ -1} \circ (\mathbf{1} + \boldsymbol{\delta_\pi})^{\circ -1}\right]\boldsymbol{S_{t+1}} + m_{t+1} - \pi_{t+1}\right)\right] \tag{A31}$$

[35] In this paper, $\widetilde{(.)}$ denotes nominal variables.

$$= \exp\left(\boldsymbol{A_{0,n}} + \boldsymbol{A_{1,n}}\boldsymbol{X_t}\right), \tag{A32}$$

where $A_{0,n}, \boldsymbol{A_{1,n}}$ are constant scalars or matrices.

The log return of the global nominal n-period zero-coupon bonds from t to $t+1$ can be expressed as follows,

$$\begin{aligned}\widetilde{r}^b_{t+1,n} &\equiv \ln\left(\frac{\widetilde{P}^b_{t+1,n-1}}{\widetilde{P}^b_{t,n}}\right),\\ &= \Omega^b_{0,n} + \boldsymbol{\Omega^b_{1,n}}\boldsymbol{X_t} + \boldsymbol{\Omega^b_{2,n}}\boldsymbol{\omega_{t+1}} + \left[\boldsymbol{\Omega^b_{2,n}} + \ln\left(1-\boldsymbol{\Omega^b_{2,n}}\right)\boldsymbol{S_t}\right] + \epsilon^b_{t+1,n}, \qquad \text{(A33)}\end{aligned}$$

where $\epsilon^b_{t+1,n} \sim N(0,\sigma^2_b)$ is a homoskedastic Gaussian shock to potentially capture approximation error.

V.1.3.b Bond Risk Premium Given the no-arbitrage condition, $E_t[\exp(\widetilde{m}_{t+1}+\widetilde{r}^b_{t+1,n})] = 1$, the global bond risk premium (ignoring the Jensen's inequality terms) has a closed-form solution,

$$E_t[\widetilde{r}^b_{t+1,n}] - \widetilde{y}_{t,1} + \frac{1}{2}\sigma^2_b = \ln\left[(1+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi}-\boldsymbol{\Omega^b_{2,n}})\circ(1+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\circ(1-\boldsymbol{\Omega^b_{2,n}})^{\circ-1}\right]\boldsymbol{S_t}, \tag{A34}$$

[36] which in a quadratic Gaussian approximation has the following expression,

$$\approx \left[(\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})\circ\boldsymbol{\Omega^b_{2,n}}\right]\boldsymbol{S_t} = -Cov_t(\widetilde{m}_{t+1},\widetilde{r}^b_{t+1,n}). \tag{A35}$$

[37] where $\boldsymbol{\delta_m}$ is the SDF loading on the four global uncertainty shocks subject to the time-varying global risk aversion as discussed in Section V.1.2, and $\boldsymbol{\delta_\pi}$ is the inflation rate loading on the four global uncertainty shocks as discussed in Section V.1.1.

V.1.3.c Equities Bekaert, Engstrom, and Xu (2019) show that log equity returns is quasi-affine to the state variable levels and shocks as below,

$$\widetilde{r}^e_{t+1} \equiv \ln\left(\frac{PD_{t+1}+1}{PD_t}\frac{\widetilde{D}_{t+1}}{\widetilde{D}_t}\right), \tag{A36}$$

$$= \Omega^e_0 + \boldsymbol{\Omega^e_1}\boldsymbol{X_t} + \boldsymbol{\Omega^e_2}\boldsymbol{\omega_{t+1}} + \left[\boldsymbol{\Omega^e_2} + \ln\left(1-\boldsymbol{\Omega^e_2}\right)\boldsymbol{S_t}\right] + \epsilon^e_{t+1}, \tag{A37}$$

where $\epsilon^e_{t+1} \sim N(0,\sigma^2_e)$ is a homoskedastic Gaussian shock to potentially capture approximation error.

V.1.3.d Equity Risk Premium Given the no-arbitrage condition, $E_t[\exp(\widetilde{m}_{t+1}+\widetilde{r}^e_{t+1})] = 1$, the global equity risk premium has a closed-form solution using the return process,

$$E_t[\widetilde{r}^e_{t+1}] - \widetilde{y}_{t,1} + \frac{1}{2}\sigma^2_e = \ln\left[(1+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi}-\boldsymbol{\Omega^e_2})\circ(1+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\circ(1-\boldsymbol{\Omega^e_2})^{\circ-1}\right]\boldsymbol{S_t}, \tag{A38}$$

$$\approx \left[(\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})\circ\boldsymbol{\Omega^e_2}\right]\boldsymbol{S_t} = -Cov_t(\widetilde{m}_{t+1},\widetilde{r}^e_{t+1}). \tag{A39}$$

[36] Note that, the non-linearity is due to the non-linearities in the moment generating function of gamma shocks.

[37] The quadratic Taylor approximation for "$y - \ln(1+y)$" is $\frac{1}{2}y^2$.

V.1.3.e Variances

The physical variance for Asset $a \in \{b, e\}$,

$$V_t^{a,P} \equiv E_t\left[\left(\tilde{r}_{t+1}^a - E_t(\tilde{r}_{t+1}^a)\right)^2\right], \tag{A40}$$

$$= \boldsymbol{\Omega_2^a S_t \Omega_2^{a\prime}} + \sigma_a^2, \tag{A41}$$

where the parameter matrices are discussed in Equations (A33) and (A36).

The risk-neutral variance for Asset $a \in \{b, e\}$,

$$V_t^{a,Q} \equiv E_t^Q\left[\left(\tilde{r}_{t+1}^a - E_t(\tilde{r}_{t+1}^a)\right)^2\right] \tag{A42}$$

$$= \frac{\partial^2 mgf_t^Q(\tilde{r}_{t+1}^a;\nu)}{\partial \nu^2}|_{\nu=0} - \left(\frac{\partial mgf_t^Q(\tilde{r}_{t+1}^a;\nu)}{\partial \nu}|_{\nu=0}\right)^2 \tag{A43}$$

$$= \left[\boldsymbol{\Omega_2^a} \circ (\boldsymbol{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})^{\circ -1}\right] \boldsymbol{S_t} \left[\boldsymbol{\Omega_2^a} \circ (\boldsymbol{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})^{\circ -1}\right]' + \sigma_a^2, \tag{A44}$$

where the moment generating function is $mgf_t^Q(\tilde{r}_{t+1}^a;\nu) = \frac{E_t\left[\exp\left(\tilde{m}_{t+1}+\nu\tilde{r}_{t+1}^a\right)\right]}{E_t\left[\exp(\tilde{m}_{t+1})\right]}$. "$\circ$" is the Hadamard product of two identically sized matrices (or element-by-element matrix multiplication), and "$(.)^{\circ -1}$" is the Hadamard inverse. $\boldsymbol{\Omega_2^a}$ is the asset return loading vector on the common shocks, or an "amount-of-risk" loading vector; $(\boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})$ represents the nominal pricing kernel loading vector on the common shocks, or a "price-of-risk" loading vector. Intuitively, a positive downside uncertainty shock is perceived as bad news, driving up the intertemporal marginal rates of substitution; the sensitivity of the pricing kernel on the downside uncertainty shock is expected to be higher (positive) than that on the upside uncertainty shock, or $\delta_{m\theta d,t}$ in the minus m_{t+1} expression is smaller than 0 and less than $\delta_{m\theta u,t}$.

V.1.3.f Variances as Assets: Variance Risk Premium

Hence, the solutions of variances in closed form imply a premium of $V_t^{a,Q}$ over $V_t^{a,P}$. For asset $a \in \{b, e\}$,

$$\begin{aligned} VRP_t^a &= V_t^{a,Q} - V_t^{a,P} \\ &= \left[\boldsymbol{\Omega_2^a} \circ (\boldsymbol{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})^{\circ -1}\right] \boldsymbol{S_t} \left[\boldsymbol{\Omega_2^a} \circ (\boldsymbol{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})^{\circ -1}\right]' - \boldsymbol{\Omega_2^a S_t \Omega_2^{a\prime}}. \end{aligned} \tag{A45}$$

V.2 Other Asset Markets

This world economy is partially integrated. Each market is complete. Each country-level state variable has a global component with constant exposures to the global levels and shocks and an idiosyncratic component. Idiosyncratic shocks are uncorrelated across countries. Under the no-arbitrage assumption, there exists closed-form solutions for country equity and bond prices.

V.2.1 Local State Variables: Matrix representation

In a matrix representation, the regional state vector denoted as $\boldsymbol{X_{t+1}^i}$ (11×1),

$$\begin{bmatrix}\theta_{t+1}^i & \theta u_{t+1}^i & \theta d_{t+1}^i & \pi_{t+1}^i & \pi u_{t+1}^i & \pi d_{t+1}^i & x_{t+1}^i & xu_{t+1}^i & xd_{t+1}^i & g_{t+1}^i & q_{t+1}^i\end{bmatrix}',$$

follows this general dynamics:

$$\boldsymbol{X_{t+1}^i} = \boldsymbol{\alpha_X^i} \circ \boldsymbol{\xi_{X,t}} + (\boldsymbol{1} - \boldsymbol{\alpha_X^i}) \circ \boldsymbol{\xi_{X,t}^i} + \underbrace{Jensen's\left(\boldsymbol{\alpha_X^i} \circ \boldsymbol{\delta_X^i}, \boldsymbol{S_t}\right) + Jensen's\left((\boldsymbol{1} - \boldsymbol{\alpha_X^i}) \circ \boldsymbol{X_\omega^i}, \boldsymbol{S_t^i}\right)}_{\text{Jensen's inequality terms}}$$

$$+ \left(\boldsymbol{\alpha^i_X} \circ \boldsymbol{\delta^i_X}\right) \boldsymbol{\omega_{t+1}} + \left((\boldsymbol{1} - \boldsymbol{\alpha^i_X}) \circ \boldsymbol{X^i_\omega}\right) \boldsymbol{\omega^i_{t+1}}, \tag{A46}$$

$$\boldsymbol{\omega^i_{t+1}} \sim \Gamma(\boldsymbol{S^i_t}, \boldsymbol{1}) - \boldsymbol{S^i_t}, \tag{A47}$$

where $\boldsymbol{\xi_{X,t}}$ (11×1) denotes the conditional mean vector of the global state variables $\boldsymbol{X_{t+1}}$ in Section V.1.1, $\boldsymbol{\omega_{t+1}}$ (9×1) the global state variable shock matrix, $\boldsymbol{\delta^i_X}$ (11×9) the constant local coefficient vector to the global state variable shocks $\boldsymbol{\omega_{t+1}}$ (which are not constraint to be the same with global state variable loadings on global shocks $\boldsymbol{\delta_X}$), $\boldsymbol{S_t}$ (9×1) the time-varying shape parameters of global shocks, and $\Upsilon\left(\boldsymbol{\alpha^i_X} \circ \boldsymbol{\delta^i_X}, \boldsymbol{S_t}\right)$ is the Jensen's inequality term from Gamma distributions. The local counterparts are defined as follows. $\boldsymbol{\xi^i_{X,t}}$ (11×1) denotes the local component of the conditional mean vector of the regional state variables, $\boldsymbol{\omega^i_{t+1}}$ (11×1) the local state variable shock matrix,

$$\begin{bmatrix}\omega^i_{\theta u,t+1} & \omega^i_{\theta d,t+1} & \omega^i_{\pi u,t+1} & \omega^i_{\pi d,t+1} & \omega^i_{x,t+1} & \omega^i_{xu,t+1} & \omega^i_{xd,t+1} & \omega^i_{g,t+1} & \omega^i_{q,t+1}\end{bmatrix}',$$

$\boldsymbol{X^i_\omega}$ (11×9) the constant coefficient vector to the local state variable shocks $\boldsymbol{\omega^i_{t+1}}$, $\boldsymbol{S^i_t}$ (9×1) the time-varying shape parameters of local shocks,

$$\begin{bmatrix}\theta u^i_t & \theta d^i_t & \pi u^i_t & \pi d^i_t & xu^i_t & xd^i_t & v^i & q^i_t\end{bmatrix}'.$$

Most important, $\boldsymbol{\alpha^i_X}$ (11×1) captures the constant global exposures.

The shock structures of each local state variables follow the global counterparts to ensure local shocks are also mutually independent from each other.

V.2.2 Local Real Pricing Kernel

I specify the logarithm of the local real local pricing kernel to be affine to the global and local state variable levels and shocks,

$$\begin{aligned}-m^i_{t+1} = {} & \alpha^i_m \left(x_t + \boldsymbol{\delta^i_m}\boldsymbol{\omega_{t+1}}\right) + (1-\alpha^i_m)\left(x^i_t + \boldsymbol{m^i_\omega}\boldsymbol{\omega^i_{t+1}}\right) \\ & \underbrace{+ \left[\alpha^i_m\boldsymbol{\delta^i_m} - \ln\left(\boldsymbol{1} + \alpha^i_m\boldsymbol{\delta^i_m}\right)\right]\boldsymbol{S_t} + \left[(1-\alpha^i_m)\boldsymbol{m^i_\omega} - \ln\left(\boldsymbol{1} + (1-\alpha^i_m)\boldsymbol{m^i_\omega}\right)\right]\boldsymbol{S^i_t}}_{\text{Jensen's Inequality Terms}},\end{aligned} \tag{A48}$$

where $\boldsymbol{\omega_{t+1}}$ (9×1) and $\boldsymbol{\omega^i_{t+1}}$ (9×1) are the global and local state variable shock matrix defined earlier. $\boldsymbol{\delta^i_m}$ (1×9) denotes a vector of constant sensitivities to global shocks. Similarly, $\boldsymbol{m^i_\omega}$ (1×7) denotes a vector of constant sensitivities to local shocks.

The drift term, $\alpha^i_m x_t + (1-\alpha^i_m)x^i_t$, is the real regional short rate.

V.2.3 Local Asset Prices and Risk Premiums

V.2.3.a Nominal Treasury Bonds The real local short rate ($y^i_{t,1} = -\ln\{[E_t[\exp(m^i_{t+1})]]\}$) and the nominal regional short rate ($\widetilde{y}^i_{t,1} = -\ln\{[E_t[\exp(m^i_{t+1} - \pi^i_{t+1})]]\}$) are solved in closed forms,

$$y^i_{t,1} = \alpha^i_m x_t + (1-\alpha^i_m)x^i_t, \tag{A49}$$

$$\begin{aligned}\widetilde{y}^i_{t,1} = {} & \alpha^i_m x_t + \alpha^i_\pi \xi_{\pi,t} + (1-\alpha^i_m)x^i_t + (1-\alpha^i_\pi)\xi^i_{\pi,t} \\ & + \ln\left[(\boldsymbol{1} + \alpha^i_m\boldsymbol{\delta_m} + \alpha^i_\pi\boldsymbol{\delta^i_\pi}) \circ (\boldsymbol{1} + \alpha^i_m\boldsymbol{\delta_m})^{\circ -1} \circ (\boldsymbol{1} + \alpha^i_\pi\boldsymbol{\delta^i_\pi})^{\circ -1}\right]\boldsymbol{S_t} \\ & + \ln\left[(\boldsymbol{1} + (1-\alpha^i_m)\boldsymbol{m^i_\omega} + (1-\alpha^i_\pi)\boldsymbol{\pi^i_\omega}) \circ (\boldsymbol{1} + (1-\alpha^i_m)\boldsymbol{m^i_\omega})^{\circ -1} \circ (\boldsymbol{1} + (1-\alpha^i_\pi)\boldsymbol{\pi^i_\omega})^{\circ -1}\right]\boldsymbol{S^i_t}.\end{aligned} \tag{A50}$$

where "ln(.)" is the element-wise logarithm operator, "$\circ$" is the Hadamard product of two identically sized matrices (or element-by-element matrix multiplication), and "$(.)^{\circ -1}$" is the Hadamard inverse. The three components in nominal short rate represent the real short rate $(\alpha_m^i x_t + (1-\alpha_m^i)x_t^i)$, the expected inflation rate $(\alpha_\pi^i \xi_{\pi,t} + (1-\alpha_\pi^i)\xi_{\pi,t}^i)$, and the inflation risk premium (+ Jensen's inequality term).

The price of n-period zero-coupon nominal bond $(\widetilde{P}_{t,n}^{b,i})$ can be then solved recursively in exact closed forms, given the shock specifications. The nominal local bond return from t to $t+1$ can be approximately expressed as follows,

$$\widetilde{r}_{t+1,n}^{b,i} \equiv \ln\left(\frac{\widetilde{P}_{t+1,n-1}^{b,i}}{\widetilde{P}_{t,n}^{b,i}}\right), \tag{A51}$$

$$\begin{aligned} &= \Omega_{0,n}^{b,i} + \boldsymbol{\Omega_{1,n}^{b,i} X_t} + \boldsymbol{\Omega_{2,n}^{b,i} \omega_{t+1}} + \left[\boldsymbol{\Omega_{2,n}^{b,i}} + \ln\left(1-\boldsymbol{\Omega_{2,n}^{b,i}}\right)\boldsymbol{S_t}\right] \\ &+ \boldsymbol{\Omega_{3,n}^{b,i} X_t^i} + \boldsymbol{\Omega_{4,n}^{b,i} \omega_{t+1}^i} + \left[\boldsymbol{\Omega_{4,n}^{b,i}} + \ln\left(1-\boldsymbol{\Omega_{4,n}^{b,i}}\right)\boldsymbol{S_t^i}\right] + \epsilon_{t+1}^{b,i}, \end{aligned} \tag{A52}$$

where $\epsilon_{t+1}^{b,i}$ is a homoskedastic Gaussian shock with volatility σ_b^i to capture approximation error.

V.2.3.b Bond Risk Premium Given the no-arbitrage condition, $E_t[\exp(\widetilde{m}_{t+1}^i + \widetilde{r}_{t+1,n}^{b,i})] = 1$ where $\widetilde{r}_{t+1,n}^{b,i}$ is the nominal bond return, the regional bond risk premium has a closed-form solution,

$$\begin{aligned} E_t[\widetilde{r}_{t+1,n}^{b,i}] - \widetilde{y}_{t,1}^i + \frac{1}{2}{\sigma_b^i}^2 &= \underbrace{\ln\left[\left(1+\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}-\boldsymbol{\Omega_{2,n}^{b,i}}\right)\circ\left(1+\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}\right)^{\circ-1}\circ(1-\boldsymbol{\Omega_{2,n}^{b,i}})^{\circ-1}\right]\boldsymbol{S_t}}_{\text{(1) compensation for global risk exposure}} \\ &+ \underbrace{\ln\left[\left(1+(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}-\boldsymbol{\Omega_{4,n}^{b,i}}\right)\circ\left(1+(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\circ(1-\boldsymbol{\Omega_{4,n}^{b,i}})^{\circ-1}\right]\boldsymbol{S_t^i}}_{\text{(2) compensation for regional risk exposure}}, \end{aligned} \tag{A53}$$

[38] which in a quadratic Gaussian approximation has the following expression,

$$\approx \underbrace{\left(\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}\right)\circ\boldsymbol{\Omega_{2,n}^{b,i}S_t}}_{\approx(1)} + \underbrace{\left[(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}\right]\circ\boldsymbol{\Omega_{4,n}^{b,i}S_t^i}}_{\approx(2)} = -Cov_t(\widetilde{m}_{t+1}^i, \widetilde{r}_{t+1,n}^{b,i}). \tag{A54}$$

[39]

V.2.3.c Equities The nominal local equity return from t to $t+1$ can be approximately expressed as follows,

$$\widetilde{r}_{t+1}^{e,i} \equiv \ln\left(\frac{\widetilde{P}_{t+1,n-1}^{e,i}}{\widetilde{P}_{t}^{e,i}}\right), \tag{A55}$$

$$\begin{aligned} &= \Omega_{0}^{e,i} + \boldsymbol{\Omega_{1}^{e,i} X_t} + \boldsymbol{\Omega_{2}^{e,i} \omega_{t+1}} + \left[\boldsymbol{\Omega_{2}^{b,i}} + \ln\left(1-\boldsymbol{\Omega_{2}^{b,i}}\right)\boldsymbol{S_t}\right] \\ &+ \boldsymbol{\Omega_{3}^{e,i} X_t^i} + \boldsymbol{\Omega_{4}^{e,i} \omega_{t+1}^i} + \left[\boldsymbol{\Omega_{4}^{b,i}} + \ln\left(1-\boldsymbol{\Omega_{4}^{b,i}}\right)\boldsymbol{S_t^i}\right] + \epsilon_{t+1}^{e,i}. \end{aligned} \tag{A56}$$

[38]Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.

[39]The quadratic Taylor approximation for "$y - \ln(1+y)$" is $\frac{1}{2}y^2$.

where $\epsilon_{t+1}^{e,i}$ is a homoskedastic Gaussian shock with volatility σ_e^i to capture approximation error.

V.2.3.d Equity Risk Premium Given the no-arbitrage condition, $E_t[\exp(\widetilde{m}_{t+1}^i+\widetilde{r}_{t+1}^{e,i})] = 1$ where $\widetilde{r}_{t+1}^{e,i}$ is the nominal equity return, the regional equity risk premium has a closed-form solution,

$$E_t[\widetilde{r}_{t+1}^{e,i}] - \widetilde{y}_{t,1}^i + \frac{1}{2}\sigma_b^{i2} = \underbrace{\ln\left[\left(1+\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}-\boldsymbol{\Omega_2^{e,i}}\right)\circ\left(1+\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}\right)^{\circ-1}\circ(1-\boldsymbol{\Omega_2^{e,i}})^{\circ-1}\right]\boldsymbol{S_t}}_{\text{(1) compensation for global risk exposure}}$$
$$+\underbrace{\ln\left[\left(1+(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}-\boldsymbol{\Omega_4^{e,i}}\right)\circ\left(1+(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\circ(1-\boldsymbol{\Omega_4^{e,i}})^{\circ-1}\right]\boldsymbol{S_t^i}}_{\text{(2) compensation for regional risk exposure}}, \tag{A57}$$

[40] which in a quadratic Gaussian approximation has the following expression,

$$\approx\underbrace{\left(\alpha_m^i\boldsymbol{\delta_m}+\alpha_\pi^i\boldsymbol{\delta_\pi^i}\right)\circ\boldsymbol{\Omega_2^{e,i}S_t}}_{\approx(1)}+\underbrace{\left[(1-\alpha_m^i)\boldsymbol{m_\omega^i}+(1-\alpha_\pi^i)\boldsymbol{\pi_\omega^i}\right]\circ\boldsymbol{\Omega_4^{e,i}S_t^i}}_{\approx(2)} = -Cov_t(\widetilde{m}_{t+1}^i,\widetilde{r}_{t+1}^{e,i}). \tag{A58}$$

V.2.3.e Variances The physical variance for Asset $a\in\{b,e\}$,

$$V_t^{a,i,P} \equiv E_t\left[\left(\widetilde{r}_{t+1}^{a,i}-E_t(\widetilde{r}_{t+1}^{a,i})\right)^2\right], \tag{A59}$$
$$= \boldsymbol{\Omega_2^{a,i}S_t\Omega_2^{a,i\prime}}+\boldsymbol{\Omega_4^{a,i}S_t^i\Omega_4^{a,i\prime}}+\sigma_a^{i2}, \tag{A60}$$

where the parameter matrices are discussed in Equations (A51) and (A55).

The risk-neutral variance for Asset $a\in\{b,e\}$,

$$V_t^{a,i,Q} \equiv E_t^Q\left[\left(\widetilde{r}_{t+1}^{a,i}-E_t(\widetilde{r}_{t+1}^{a,i})\right)^2\right], \tag{A61}$$
$$= \left[\boldsymbol{\Omega_2^{a,i}}\circ(\boldsymbol{1}+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\right]\boldsymbol{S_t}\left[\boldsymbol{\Omega_2^{a,i}}\circ(\boldsymbol{1}+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\right]'$$
$$+\left[\boldsymbol{\Omega_4^{a,i}}\circ\left(\boldsymbol{1}+\boldsymbol{m_\omega^i}+\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\right]\boldsymbol{S_t^i}\left[\boldsymbol{\Omega_4^{a,i}}\circ\left(\boldsymbol{1}+\boldsymbol{m_\omega^i}+\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\right]'+\sigma_a^{i2}. \tag{A62}$$

V.2.3.f Variances as Assets: Variance Risk Premium The present tripartite model derives closed-form solutions for VRP which show potentials to capture its empirical time variation characteristics. For asset $a\in\{b,e\}$,

$$VRP_t^a = V_t^{a,Q}-V_t^{a,P}$$
$$= \left[\boldsymbol{\Omega_2^a}\circ(\boldsymbol{1}+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\right]\boldsymbol{S_t}\left[\boldsymbol{\Omega_2^a}\circ(\boldsymbol{1}+\boldsymbol{\delta_m}+\boldsymbol{\delta_\pi})^{\circ-1}\right]'-\boldsymbol{\Omega_2^aS_t\Omega_2^{a\prime}}$$
$$+\left[\boldsymbol{\Omega_4^{a,i}}\circ\left(\boldsymbol{1}+\boldsymbol{m_\omega^i}+\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\right]\boldsymbol{S_t^i}\left[\boldsymbol{\Omega_4^{a,i}}\circ\left(\boldsymbol{1}+\boldsymbol{m_\omega^i}+\boldsymbol{\pi_\omega^i}\right)^{\circ-1}\right]'-\boldsymbol{\Omega_4^{a,i}S_t^i\Omega_4^{a,i\prime}}, \tag{A63}$$

[40] Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.

where $\boldsymbol{\Omega_2^a}$ and $\boldsymbol{\Omega_4^{a,i}}$ are the "amount-of-risk" coefficients, and $\boldsymbol{\delta_m}$ and $\boldsymbol{m_\omega^i}$ are the "price-of-risk" coefficients that are linear to the global and regional risk aversions respectively. In the tripartite framework, the variance risk premium can be decomposed into a global component and a regional component.

V.2.3.g Foreign Exchange Returns Denote $s^{\$/i}$ as the log of the spot exchange rate in units of dollars per foreign currency i at region i. As stated in the Proposition 1 of Backus, Foresi, and Telmer (2011), the change in the nominal exchange rate, $\Delta s_{t+1}^{\$/i} = s_{t+1}^{\$/i} - s_t^{\$/i}$, in a frictionless world is equivalent to the nominal pricing kernel difference,

$$\Delta s_{t+1}^{\$/i} = m_{t+1}^i - m_{t+1} + \pi_{t+1} - \pi_{t+1}^i. \tag{A64}$$

An increase in $s^{\$/i}$ means a depreciation in dollars (and an appreciation in region i currency). In this model, a hypothetical world with perfect integration (i.e, $\alpha_m^i = 1 \forall^i$) still obtains a time-varying spot rate to address the inflation risk amid the real macroeconomic risks. The regional currency excess return is the log return to U.S. investors of borrowing in dollars to hold foreign investment currency i can be expressed as an exact dynamic factor model,

$$\tilde{r}_{t+1}^{fx,i} \equiv \Delta s_{t+1}^{\$/i} + \tilde{y}_{t,1}^i, \tag{A65}$$

$$= \Omega_0^{fx,i} + \boldsymbol{\Omega_1^{fx,i} X_t} + \boldsymbol{\Omega_2^{fx,i} \omega_{t+1}} + \boldsymbol{\Omega_3^{fx,i} X_t^i} + \boldsymbol{\Omega_4^{fx,i} \omega_{t+1}^i} + \varepsilon_{t+1}^{fx,i} + Jensen's + \epsilon_{t+1}^{fx,i}, \tag{A66}$$

where $\Omega_0^{fx,i}$, $\boldsymbol{\Omega_1^{fx,i}}$,$\boldsymbol{\Omega_2^{fx,i}}$,$\boldsymbol{\Omega_3^{fx,i}}$ and $\boldsymbol{\Omega_4^{fx,i}}$ are constant matrices; $\epsilon_{t+1}^{fx,i}$ is the approximation error term that follows a homoskedastic Gaussian distribution with volatility σ_{fx}^i.

V.2.3.h Foreign Exchange Risk Premium Given the no-arbitrage condition, $E_t[\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1}^{fx,i})] = 1$ where $\tilde{r}_{t+1}^{fx,i}$ is the nominal foreign exchange return (from the U.S. investor's view point), the foreign exchange risk premium has a closed-form solution,

$$E_t[\tilde{r}_{t+1}^{fx,i}] - \tilde{y}_{t,1} + \frac{1}{2}{\sigma_{fx}^i}^2 = \ln\left[\left(1 + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi} - \boldsymbol{\Omega_2^{fx,i}}\right) \circ (1 + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi})^{\circ -1} \circ (1 - \boldsymbol{\Omega_2^{fx,i}})^{\circ -1}\right] \boldsymbol{S_t}, \tag{A67}$$

[41] which in a quadratic Gaussian approximation has the following expression,

$$\approx (\boldsymbol{\delta_m} + \boldsymbol{\delta_\pi}) \circ \boldsymbol{\Omega_2^{fx,i} S_t} = -Cov_t(\tilde{m}_{t+1}, \tilde{r}_{t+1}^{fx,i}). \tag{A68}$$

[41]Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.

Table A1: Conditional Volatility Models for Asset Returns.

This table presents best GARCH-class models and distributional assumptions for asset return conditional volatility. The four GARCH-class models are GARCH ("GARCH"), exponential GARCH ("EGARCH"), Threshold GARCH ("TARCH"), and Glosten-Jagannathan-Runkle GARCH ("GJRGARCH"). The four distributions-of-interest are Gaussian (" "), Student t ("t" characterized by a tail parameter ζ_1), GED ("GED" characterized by a tail parameter ζ_1), and Skewed t ("Skewt" characterized by a tail parameter ζ_1 and an asymmetry parameter ζ_2) distributions. Suppose $r_{t+1} = \mu + \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim D(0, h_t)$.

(1) GARCH, Bollerslev (1986) : $h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1}$

(2) EGARCH, Nelson (1991) : $\ln(h_t) = \alpha_0 + \alpha_1 \frac{|\varepsilon_t|}{\sqrt{h_{t-1}}} + \alpha_2 \ln(h_{t-1}) + \alpha_3 \frac{\varepsilon_t}{\sqrt{h_{t-1}}}$

(3) TARCH, Zakoian (1994) : $\sqrt{h_t} = \alpha_0 + \alpha_1 |\varepsilon_t| + \alpha_2 \sqrt{h_{t-1}} + \alpha_3 I_{\varepsilon_t<0} |\varepsilon_t|$

(4) GJRGARCH, Glosten, Jagannathan, and Runkle (1993) : $h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1} + \alpha_3 I_{\varepsilon_t<0} \varepsilon_t^2$.

Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold values indicate <5% significance level.

Asset	Best Model	Variance Equation Parameters			Distribution Parameters	
		α_1	α_2	α_3	Thick Tail (ζ_1)	Skew (ζ_2)
USA Equity	EGARCH-Skewt	**0.2652**	**0.8694**	**-0.1635**	**7.9925**	**-0.3664**
CAN Equity	GARCH-Skewt	**0.1111**	**0.8079**		**7.5999**	**-0.2775**
DEU Equity	EGARCH-Skewt	**0.2178**	**0.8603**	**-0.1164**	**7.3323**	**-0.2923**
FRA Equity	EGARCH-Skewt	**0.1749**	**0.8325**	**-0.2215**	21.9996	**-0.2914**
GBR Equity	EGARCH-Skewt	**0.1515**	**0.8269**	**-0.1881**	**11.3674**	**-0.1898**
CHE Equity	GJRGARCH-Skewt	0.0345	**0.2317**	**0.2989**	**6.5014**	**-0.1673**
JPN Equity	EGARCH	0.2339	**0.9369**	**-0.1193**		
AUS Equity	EGARCH-Skewt	**0.1257**	**0.9192**	-0.0685	**6.4191**	**-0.2395**
USA Gov-Bond	TARCH	**0.3669**	**0.6959**	-0.1259		
CAN Gov-Bond	GARCH-t	0.0702	**0.6549**		**9.4227**	
DEU Gov-Bond	TARCH	**0.2506**	**0.7814**	-0.0644		
FRA Gov-Bond	GARCH	**0.0774**	**0.8484**			
GBR Gov-Bond	GARCH-GED	**0.0463**	**0.9278**		**1.3353**	
CHE Gov-Bond	GARCH	**0.1284**	**0.4380**			
JPN Gov-Bond	GARCH-GED	0.1093	**0.7756**		**1.3036**	
AUS Gov-Bond	GARCH-Skewt	**0.1330**	**0.5543**		13.7548	**-0.2537**

Table A2: Values of Constant Matrices "Ξ" in the Global Dynamic Comovement Model (Equations (7) and (17) of the main paper).

In the econometric model, constant symmetric matrix $\Xi = E\left[I_{\check{z}_t<0} I'_{\check{z}_t<0}\right]$ $(N \times N)$ is crucial to maintain a stationary $\check{\boldsymbol{Q}}_{\boldsymbol{t}}$ process, where $I_{\check{z}_t<0}$ $(N \times 1)$ is assigned 1 if the residual is less than 0, and assigned 0 otherwise. This table presents pre-determined empirical estimates of Ξ of each estimation (by asset-currency) using the sample in the econometric part of the paper (March 1987 – December 2016).

Equity, USD	USA	CAN	DEU	FRA	GBR	CHE	JPN	AUS
USA	0.43	0.34	0.31	0.34	0.34	0.31	0.30	0.30
CAN		0.46	0.31	0.34	0.35	0.31	0.30	0.34
DEU			0.46	0.37	0.36	0.34	0.31	0.29
FRA				0.47	0.36	0.34	0.33	0.32
GBR					0.50	0.36	0.35	0.36
CHE						0.47	0.33	0.30
JPN							0.50	0.33
AUS								0.47
Bond, USD	USA	CAN	DEU	FRA	GBR	CHE	JPN	AUS
USA	0.50	0.31	0.33	0.32	0.31	0.32	0.31	0.30
CAN		0.47	0.30	0.29	0.31	0.29	0.28	0.32
DEU			0.50	0.46	0.38	0.41	0.34	0.30
FRA				0.49	0.36	0.42	0.32	0.29
GBR					0.51	0.37	0.31	0.31
CHE						0.51	0.35	0.30
JPN							0.50	0.28
AUS								0.47

Table A3: Model Fit: Flight-to-Safety Channel, Given the Chosen Model in Table 3

The model implicitly include a FTS channel. To provide the right empirical moments to be compared with, "Empirical" reports the average of time-varying correlation (estimated using a parsimonious dynamic conditional correlation model as in Engle (2002)) between standardized monthly equity returns and bond returns—both denominated in USD as consistently used in this paper. Then, "Conditional Model" reports the time-series averages of the model-implied equity beta (=correlation given the standardization) Table 3. **States:** Good (Bad) states, when country recession indicator = 0 (1). Bold (italics) values indicate the model point estimates are within 95% (99%) confidence intervals of the corresponding data moments.

	Full States	Good	Bad
Empirical	0.3360	0.3799	0.2946
S.E.	(0.0481)	(0.0468)	(0.0494)
Conditional Model	**0.2501**	*0.2591*	**0.2333**

Table A4: Estimation Results of Global Equity Comovement: $x_{i,t}$ =Standardized Country Output Growth.

This table provides one of the robustness checks of the global equity correlation estimates involving the FTS channel (as in Table 3). Here, I use standardized country output growth (industrial production growth) as $x_{i,t}$ in the FTS process. Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold (italics) values indicate <5% (10%) significance level.

	Multivariate Gaussian				*Multivariate t*			
	E (6)	E (7)	E (8)	E (9)	E (6)	E (7)	E (8)	E (9)
β_1	**0.0884**	**0.0777**	**0.0724**	0.0634	**0.0753**	**0.0176**	0.0490	0.0172
	(0.0291)	(0.0355)	(0.0341)	(0.0539)	(0.0353)	(0.0082)	(0.0566)	(0.0200)
β_2	**0.8708**	**0.8801**	**0.8958**	**0.9008**	**0.8864**	**0.9676**	**0.9315**	**0.9692**
	(0.0390)	(0.0450)	(0.0487)	(0.0828)	(0.0547)	(0.0111)	(0.0935)	(0.1289)
ν								
γ		*0.0255*		*0.0305*		**0.0218**		**0.0205**
		(0.0134)		(0.0183)		(0.0055)		(0.0081)
ϕ			*0.0382*	0.0264			0.0421	0.0257
			(0.0208)	(0.0215)			(0.0257)	(0.0223)
δ_1	**0.4958**	**0.4856**	**0.4943**	**0.4855**	**0.4238**	**0.4130**	**0.4221**	**0.4123**
	(0.0265)	(0.0270)	(0.0263)	(0.0263)	(0.0243)	(0.0185)	(0.0242)	(0.0656)
δ_2	**0.0686**	**0.0623**	**0.0678**	**0.0605**	**0.0387**	*0.0328*	**0.0377**	**0.0331**
	(0.0214)	(0.0219)	(0.0212)	(0.0214)	(0.0175)	(0.0185)	(0.0176)	(0.0166)
df					**11.0510**	**9.6308**	**11.1301**	**9.8784**
					(2.5636)	(1.6799)	(2.6175)	(1.9001)
LL	-2991.66	-2985.61	-2989.32	-2983.89	-2916.31	-2888.03	-2914.75	-2886.50
AIC	5991.32	5981.21	5988.64	5979.77	5842.62	5788.06	5841.49	5787.01
BIC	6006.84	6000.62	6008.04	6003.05	5862.02	5811.34	5864.78	5814.17

Table A5: Estimation Results of Global Equity Comovement: DECO Estimates, No Domestic Comovement Part.

This table provides one of the robustness checks of the global equity correlation estimates involving the FTS channel (as in Table 3). Here, I directly estimate the DECO model with tests. Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold (italics) values indicate <5% (10%) significance level.

	Multivariate Gaussian					*Multivariate t*				
	E (10)	E (11)	E (12)	E (13)	E (14)	E (10)	E (11)	E (12)	E (13)	E (14)
β_1	**0.0883**	*0.0725*	**0.0497**	0.0515	**0.0985**	**0.0612**	0.0281	**0.0599**	**0.0225**	**0.1041**
	(0.0313)	(0.0384)	(0.0219)	(0.1016)	(0.0329)	(0.0138)	(0.0249)	(0.0117)	(0.0104)	(0.0287)
β_2	**0.8905**	**0.8942**	**0.9485**	**0.9058**	**0.8693**	**0.9388**	**0.9520**	**0.9401**	**0.9617**	**0.8482**
	(0.0431)	(0.0449)	(0.0281)	(0.1418)	(0.0444)	(0.0143)	(0.0348)	(0.0123)	(0.0142)	(0.0349)
ν					**0.2072**					**0.2800**
					(0.0539)					(0.0358)
γ		**0.0259**		0.0188			**0.0279**		**0.0233**	
		(0.0106)		(0.0136)			(0.0135)		(0.0065)	
ϕ			*0.0432*	**0.0394**				*0.0259*	*0.0386*	
			(0.0252)	(0.0201)				(0.0141)	(0.0207)	
δ_1										
δ_2										
df						**11.3522**	**9.8748**	**11.6657**	**10.1025**	**10.6052**
						(2.5334)	(1.8115)	(2.6764)	(1.9130)	(2.1257)
LL	-3120.24	-3117.03	-3116.13	-3114.15	-3120.47	-3042.07	-3025.19	-3038.64	-3023.17	-3042.15
AIC	6244.48	6240.06	6238.25	6236.31	6246.94	6090.13	6058.37	6085.28	6056.33	6092.29
BIC	6252.24	6251.70	6249.89	6251.83	6258.58	6101.77	6073.90	6100.80	6075.74	6107.81

Table A6: Estimation Results of the Latent U.S. (Global) Output Growth Upside and Downside Uncertainties and Shocks.

I follow Bekaert, Engstrom, and Xu (2019) to model both the real upside uncertainty (θu_t) and real downside uncertainty (θd_t) from decomposing the industrial production growth shocks, and use their estimation results in the present research. The (log) global real growth rate of technology (or output growth), θ_t, has time-varying conditional moments governed by two state variables: θu_t (upside uncertainty) and θd_t (downside uncertainty). Formally, θ_t has the following process,

$$\theta_{t+1} = \overline{\theta} + \rho_{\theta\theta}(\theta_t - \overline{\theta}) + \rho_{\theta\theta u}(\theta u_t - \overline{\theta u}) + \rho_{\theta\theta d}(\theta d_t - \overline{\theta d}) + u^{\theta}_{t+1}, \tag{A69}$$

where the growth shock is decomposed into two independent shocks,

$$u^{\theta}_{t+1} = \delta_{\theta\theta u}\omega_{\theta u,t+1} - \delta_{\theta\theta d}\omega_{\theta d,t+1}. \tag{A70}$$

The shocks follow de-meaned Gamma distributions with time-varying shape parameters,

$$\omega_{\theta u,t+1} \sim \widetilde{\Gamma}(\theta u_t, 1) \tag{A71}$$

$$\omega_{\theta d,t+1} \sim \widetilde{\Gamma}(\theta d_t, 1), \tag{A72}$$

where $\widetilde{\Gamma}(y, 1)$ denotes a de-meaned Gamma-distributed random variable with shape parameter y and a unit scale parameter. The shape factors, θu_t and θd_t, follow autoregressive processes,

$$\theta u_{t+1} = \overline{\theta u} + \rho_{\theta u}(\theta u_t - \overline{\theta u}) + \delta_{\theta u}\omega_{\theta u,t+1} \tag{A73}$$

$$\theta d_{t+1} = \overline{\theta d} + \rho_{\theta d}(\theta d_t - \overline{\theta d}) + \delta_{\theta d}\omega_{\theta d,t+1}, \tag{A74}$$

where ρ_y denotes the autoregressive term of process y_{t+1}, δ_y the sensitivity to $\omega_{y,t+1}$, and $\overline{y}$ the constant long-run mean. Given that Gamma distributions are right-skewed by design, the growth shock with a negative loading on $\omega_{\theta d,t+1}$ influences the negative skewness, and θd_t positive skewness.

	θ_t	VARC	θu_t	θd_t
	Conditional Mean			
mean	-1.01E-04		500	**12.9194**
	(4.39E-04)		(fix)	(1.7274)
AR	**0.1395**	79.40%	**0.9993**	**0.9040**
	(0.0328)		(0.0002)	(0.0152)
$\rho_{\theta\theta u}$	1.31E-05	7.66%		
	(4.72E-04)			
$\rho_{\theta\theta d}$	**-2.18E-04**	12.94%		
	(2.39E-05)			
	Shock Structure			
$\omega_{\theta u,t}$ loading	**1.12E-04**	25.04%	**0.7860**	
	(1.10E-05)		(0.0843)	
$\omega_{\theta d,t}$ loading	**-0.0019**	74.96%		**2.0579**
	(0.0002)			(0.1485)

Table A7: Estimation Results of the Latent U.S. (Global) Inflation Upside and Downside Uncertainties and Shocks.

The U.S. inflation rate has the following process,

$$\pi_{t+1} = \overline{\pi} + \rho_{\pi\theta}(\theta_t - \overline{\theta}) + \rho_{\pi\theta u}(\theta u_t - \overline{\theta u}) + \rho_{\pi\theta d}(\theta d_t - \overline{\theta d}) + \rho_{\pi\pi}(\pi_t - \overline{\pi}) + \rho_{\pi\pi u}(\pi u_t - \overline{\pi u}) + \rho_{\pi\pi d}(\pi d_t - \overline{\pi d}) + u^{\pi}_{t+1}, \tag{A75}$$

where θ_t denotes the change in log industrial production index (real) from $t-1$ to t, θu_t the real upside uncertainty and θd_t the real downside uncertainty. The estimates of θu_t and θd_t are obtained from Table A6. π_t denotes the inflation rate, πu_t the nominal upside uncertainty and πd_t the nominal downside uncertainty. The nominal upside and downside uncertainties are latent variables in this system. $\overline{x}$ denotes the unconditional mean of Variable x; $\rho_{\pi x}$ denotes the sensitivity of inflation to Variable x in the conditional mean process. The inflation disturbance, u^{π}_{t+1}, is sensitive to the two real uncertainty shocks and the two nominal uncertainty shocks that are mutually independent of one another,

$$u^{\pi}_{t+1} = (\delta_{\pi\theta u}\omega_{\theta u,t+1} + \delta_{\pi\theta d}\omega_{\theta d,t+1}) + (\delta_{\pi\pi u}\omega_{\pi u,t+1} - \delta_{\pi\pi d}\omega_{\pi d,t+1}). \tag{A76}$$

The shocks follow de-meaned Gamma distributions with time-varying shape parameters,

$$\omega_{\pi u,t+1} \sim \widetilde{\Gamma}(\pi u_t, 1) \tag{A77}$$

$$\omega_{\pi d,t+1} \sim \widetilde{\Gamma}(\pi d_t, 1), \tag{A78}$$

$$\pi u_{t+1} = \overline{\pi u} + \rho_{\pi u}(\pi u_t - \overline{\pi u}) + \delta_{\pi u}\omega_{\pi u,t+1} \tag{A79}$$

$$\pi d_{t+1} = \overline{\pi d} + \rho_{\pi d}(\pi d_t - \overline{\pi d}) + \delta_{\pi d}\omega_{\pi d,t+1}. \tag{A80}$$

The estimation of the inflation system uses Bates (2006)'s filtration-based AML estimation. Sample period ranges from January 1947 to December 2016. Bold (italic) values indicate <5% (10%) significance level.

A. π_t Shock Structure			
$\omega_{\theta u,t}$	$\omega_{\theta d,t}$	$\omega_{\pi u,t}$	$\omega_{\pi d,t}$
-8.49E-06	8.58E-06	**4.22E-04**	**-3.56E-04**
(1.39E-06)	(1.57E-05)	(1.90E-05)	(8.78E-06)
B. πu_t, Upside Uncertainty			
$\overline{\pi u}$	AR	$\omega_{\pi u,t}$	
3.9091	**0.9730**	**1.4593**	
(1.1494)	(0.0082)	(0.1963)	
C. πd_t, Downside Uncertainty			
$\overline{\pi d}$	AR	$\omega_{\pi d,t}$	
100	**0.9881**	**0.1915**	
(fix)	(0.0219)	(0.0055)	

Table A8: Estimation Results of the Latent U.S. (Global) Real Short Rate Upside and Downside Uncertainties and Shocks.

Assume a reduced-form (minus) real pricing kernel,

$$-m_{t+1} = x_t + [\boldsymbol{\delta_m} - \ln(1 + \boldsymbol{\delta_m})]\boldsymbol{S_t} + \boldsymbol{\delta_m}\boldsymbol{\omega_{t+1}}, \tag{A81}$$

where the shock structure is determined by a linear combination of shocks in the U.S. economy: risk aversion shock denoted as ω_q (Bekaert, Engstrom, and Xu, 2019), real upside uncertainty shock $\omega_{\theta u}$ (This Paper, Table A6), real downside uncertainty shock $\omega_{\theta d}$ (This Paper, Table A6), nominal upside and downside uncertainty shocks $\omega_{\pi u}$ and $\omega_{\pi d}$ (This Paper, Table A7). The shocks are assumed to follow Gamma distributions with time-varying shape parameters,

$$\boldsymbol{\omega_{t+1}} \sim \Gamma(\boldsymbol{S_t}, \boldsymbol{1}) - \boldsymbol{S_t}. \tag{A82}$$

Given the no-arbitrage assumption and the inflation process as in Table A7, one-period real interest rate can be shown to be x_t, and one-period nominal interest rate is,

$$\begin{aligned}\widetilde{y}_{t,1} &= -\ln\{[E_t[\exp(m_{t+1} - \pi_{t+1})]\} \\ &= x_t + \underbrace{\xi_{\pi,t} + \ln\left[(\boldsymbol{1} + \boldsymbol{\delta_m} + \boldsymbol{\delta_\pi}) \circ (\boldsymbol{1} + \boldsymbol{\delta_m})^{\circ -1} \circ (\boldsymbol{1} + \boldsymbol{\delta_\pi})^{\circ -1}\right] \boldsymbol{S_t}}_{\text{inflation compensation}},\end{aligned} \tag{A83}$$

where inflation shock loadings $\boldsymbol{\delta_\pi} = [\delta_{\pi\theta u}, \delta_{\pi\theta d}, \delta_{\pi\pi u}, -\delta_{\pi\pi d}]$ and expected inflation rate $\xi_{\pi,t}$ are presented in Table A7. The real short rate is assumed with the following reduced-form expression,

$$\begin{aligned}x_{t+1} &= \overline{x} + \rho_{x\theta}(\theta_t - \overline{\theta}) + \rho_{x\theta u}(\theta u_t - \overline{\theta u}) + \rho_{x\theta d}(\theta d_t - \overline{\theta d}) + \rho_{x\pi}(\pi_t - \overline{\pi}) + \rho_{x\pi u}(\pi u_t - \overline{\pi u}) + \rho_{x\pi d}(\pi d_t - \overline{\pi d}) \\ &\quad + \rho_{xx}(x_t - \overline{x}) + \rho_{xxu}(xu_t - \overline{xu}) + \rho_{xxd}(xd_t - \overline{xd}) + \rho_{xq}(q_t - \overline{q}) + u^x_{t+1},\end{aligned} \tag{A84}$$

where the short rate shock is sensitive to the real and nominal uncertainty shocks as well as a short rate-specific homoskedastic shock,

$$u^x_{t+1} = \delta_{xq}\omega_{q,t+1} + (\delta_{x\theta u}\omega_{\theta u,t+1} + \delta_{x\theta d}\omega_{\theta d,t+1}) + (\delta_{x\pi u}\omega_{\pi u,t+1} + \delta_{x\pi d}\omega_{\pi d,t+1}) + \delta_{xu}\omega_{xu,t+1} - \delta_{xd}\omega_{xd,t+1}, \tag{A85}$$

where the short rate-specific shocks are assumed to follow de-meaned Gamma distributions with time-varying shape parameters,

$$\omega_{xu,t+1} \sim \widetilde{\Gamma}(xu_t, 1), xu_{t+1} = \overline{xu} + \rho_{xu}(xu_t - \overline{xu}) + \delta_{xu}\omega_{xu,t+1}, \tag{A86}$$

$$\omega_{xd,t+1} \sim \widetilde{\Gamma}(xd_t, 1), xd_{t+1} = \overline{xd} + \rho_{xd}(xd_t - \overline{xd}) + \delta_{xd}\omega_{xd,t+1}. \tag{A87}$$

The estimation of the inflation system uses Bates (2006)'s filtration-based AML estimation; the unknown parameters are $\boldsymbol{\delta_m}, \delta_\pi, \delta_{xu}, \delta_{xd}, \rho_{xu}, \rho_{xd}$, and other parameters can be derived using linear projection within the system; the estimation outputs are ω_{xu}, ω_{xd}, xu, xd and x. Sample period begins when first risk aversion estimate is available, 1986/06-2015/02). Bold (italic) values indicate <5% (10%) significance level.

A. x_t Shock Structure						
$\omega_{q,t}$	$\omega_{\theta u,t}$	$\omega_{\theta d,t}$	$\omega_{\pi u,t}$	$\omega_{\pi d,t}$	$\omega_{xu,t}$	$\omega_{xd,t}$
-0.7579	**-0.0039**	*-0.0322*	**-0.2428**	**0.0959**	**0.0379**	**-0.0500**
(0.7662)	(0.0058)	(0.0520)	(0.0568)	(0.0212)	(0.0008)	(0.0012)
B. xu_t, Upside Uncertainty						
$\overline{xu}$	AR	$\omega_{xu,t}$				
22.9586	**0.8759**	**5.9808**				
(0.9786)	(0.0408)	(0.3801)				
C. xd_t, Downside Uncertainty						
$\overline{xd}$	AR	$\omega_{xd,t}$				
8.9025	**0.8536**	**4.9358**				
(2.5225)	(0.0419)	(0.2301)				

Table A9: Factor Exposures of Global Asset Returns in a Seemingly Unrelated Regression (SUR) Frameworks; Constant Beta.

In this table, I jointly estimate the constant exposures of global equity and bond returns to global factor shocks in a SUR framework. The error terms may have cross-equation contemporaneous correlations. SUR models are estimated with MLE. The sample period covers from March 1987 to February 2015; February 2015 is the last month given the availability of the risk aversion estimate from Bekaert, Engstrom, and Xu (2019). Standard errors are shown in the parantenses. Bold (italics) values indicate <5% (10%) significance level.

	ω_q		$\omega_{\theta u}$		$\omega_{\theta d}$		$\omega_{\pi u}$		$\omega_{\pi d}$		ω_{xu}		ω_{xd}	
Panel A. Returns in USD														
USA Equity	**-0.1734**	(0.0128)	**-0.0003**	(0.0001)	-0.0001	(0.0008)	**-0.0022**	(0.0010)	0.0001	(0.0004)	0.0002	(0.0002)	0.0003	(0.0002)
CAN Equity	**-0.1811**	(0.0178)	**-0.0005**	(0.0001)	-0.0006	(0.0011)	**-0.0038**	(0.0015)	**-0.0016**	(0.0005)	-0.0001	(0.0003)	*0.0006*	(0.0003)
DEU Equity	**-0.2245**	(0.0218)	-0.0001	(0.0001)	0.0004	(0.0014)	*-0.0032*	(0.0018)	-0.0004	(0.0006)	0.0004	(0.0003)	0.0003	(0.0004)
FRA Equity	**-0.1942**	(0.0202)	**-0.0003**	(0.0001)	-0.0005	(0.0013)	**-0.0040**	(0.0016)	-0.0001	(0.0006)	**0.0007**	(0.0003)	0.0005	(0.0003)
GBR Equity	**-0.1452**	(0.0163)	**-0.0004**	(0.0001)	-0.0014	(0.0010)	-0.0017	(0.0013)	-0.0002	(0.0005)	*-0.0005*	(0.0003)	0.0002	(0.0003)
CHE Equity	**-0.1524**	(0.0170)	**-0.0003**	(0.0001)	-0.0009	(0.0011)	-0.0010	(0.0014)	0.0002	(0.0005)	0.0001	(0.0003)	0.0004	(0.0003)
JPN Equity	**-0.0833**	(0.0225)	-0.0001	(0.0002)	0.0018	(0.0014)	-0.0021	(0.0018)	-0.0003	(0.0006)	-0.0001	(0.0004)	0.0005	(0.0004)
AUS Equity	**-0.1829**	(0.0220)	**-0.0007**	(0.0001)	-0.0020	(0.0014)	*-0.0031*	(0.0018)	-0.0007	(0.0006)	-0.0005	(0.0004)	**0.0008**	(0.0004)
USA Gov-Bond	**0.0280**	(0.0076)	0.0000	(0.0001)	0.0007	(0.0005)	**-0.0016**	(0.0006)	**0.0005**	(0.0002)	0.0000	(0.0001)	-0.0001	(0.0001)
CAN Gov-Bond	**-0.0345**	(0.0103)	0.0000	(0.0001)	0.0009	(0.0007)	**-0.0027**	(0.0008)	-0.0003	(0.0003)	-0.0001	(0.0002)	0.0002	(0.0002)
DEU Gov-Bond	0.0095	(0.0123)	0.0000	(0.0001)	0.0005	(0.0008)	**-0.0024**	(0.0010)	**-0.0009**	(0.0004)	0.0000	(0.0002)	0.0000	(0.0002)
FRA Gov-Bond	0.0043	(0.0121)	0.0000	(0.0001)	0.0004	(0.0008)	**-0.0027**	(0.0010)	**-0.0008**	(0.0003)	0.0000	(0.0002)	0.0000	(0.0002)
GBR Gov-Bond	**0.0241**	(0.0119)	0.0001	(0.0001)	0.0002	(0.0008)	-0.0014	(0.0010)	-0.0004	(0.0003)	*-0.0003*	(0.0002)	0.0000	(0.0002)
CHE Gov-Bond	0.0125	(0.0134)	0.0001	(0.0001)	0.0003	(0.0009)	-0.0014	(0.0011)	-0.0003	(0.0004)	-0.0002	(0.0002)	0.0000	(0.0002)
JPN Gov-Bond	**0.0334**	(0.0142)	0.0000	(0.0001)	0.0002	(0.0009)	0.0010	(0.0012)	-0.0002	(0.0004)	0.0000	(0.0002)	0.0001	(0.0002)
AUS Gov-Bond	**-0.0467**	(0.0132)	**-0.0003**	(0.0001)	*-0.0014*	(0.0008)	**-0.0030**	(0.0011)	*-0.0006*	(0.0004)	-0.0003	(0.0002)	*0.0004*	(0.0002)
Panel B. Returns in LC														
USA Equity	**-0.1734**	(0.0128)	**-0.0003**	(0.0001)	-0.0001	(0.0008)	**-0.0022**	(0.0010)	0.0001	(0.0004)	0.0002	(0.0002)	0.0003	(0.0002)
CAN Equity	**-0.1397**	(0.0142)	**-0.0004**	(0.0001)	-0.0008	(0.0009)	-0.0017	(0.0012)	*-0.0008*	(0.0004)	0.0001	(0.0002)	*0.0004*	(0.0002)
DEU Equity	**-0.2058**	(0.0206)	-0.0002	(0.0001)	0.0004	(0.0013)	-0.0018	(0.0017)	0.0009	(0.0006)	0.0005	(0.0003)	0.0005	(0.0003)
FRA Equity	**-0.1747**	(0.0187)	**-0.0003**	(0.0001)	-0.0005	(0.0012)	*-0.0027*	(0.0015)	**0.0011**	(0.0005)	**0.0008**	(0.0003)	**0.0007**	(0.0003)
GBR Equity	**-0.1478**	(0.0143)	**-0.0004**	(0.0001)	-0.0014	(0.0009)	-0.0015	(0.0012)	*0.0008*	(0.0004)	-0.0003	(0.0002)	0.0003	(0.0002)
CHE Equity	**-0.1467**	(0.0164)	**-0.0004**	(0.0001)	-0.0012	(0.0010)	-0.0011	(0.0013)	**0.0010**	(0.0005)	0.0002	(0.0003)	**0.0006**	(0.0003)
JPN Equity	**-0.1035**	(0.0207)	-0.0001	(0.0001)	0.0016	(0.0013)	**-0.0034**	(0.0017)	0.0002	(0.0006)	0.0000	(0.0003)	*0.0006*	(0.0003)
AUS Equity	**-0.1182**	(0.0163)	**-0.0005**	(0.0001)	-0.0001	(0.0010)	-0.0005	(0.0013)	0.0002	(0.0005)	-0.0003	(0.0003)	0.0004	(0.0003)
USA Gov-Bond	**0.0280**	(0.0076)	0.0000	(0.0001)	0.0007	(0.0005)	**-0.0016**	(0.0006)	**0.0005**	(0.0002)	0.0000	(0.0001)	-0.0001	(0.0001)
CAN Gov-Bond	0.0070	(0.0074)	0.0000	(0.0000)	0.0007	(0.0005)	-0.0006	(0.0006)	**0.0006**	(0.0002)	0.0000	(0.0001)	0.0000	(0.0001)
DEU Gov-Bond	**0.0282**	(0.0056)	0.0000	(0.0000)	0.0005	(0.0004)	**-0.0010**	(0.0005)	**0.0004**	(0.0002)	0.0000	(0.0001)	0.0001	(0.0001)
FRA Gov-Bond	**0.0238**	(0.0061)	0.0000	(0.0000)	0.0004	(0.0004)	**-0.0014**	(0.0005)	**0.0005**	(0.0002)	0.0000	(0.0001)	0.0001	(0.0001)
GBR Gov-Bond	**0.0215**	(0.0070)	0.0000	(0.0000)	0.0002	(0.0004)	**-0.0012**	(0.0006)	**0.0006**	(0.0002)	-0.0001	(0.0001)	0.0001	(0.0001)
CHE Gov-Bond	**0.0181**	(0.0050)	0.0000	(0.0000)	0.0000	(0.0003)	**-0.0015**	(0.0004)	**0.0004**	(0.0001)	-0.0001	(0.0001)	**0.0002**	(0.0001)
JPN Gov-Bond	**0.0133**	(0.0061)	0.0000	(0.0000)	0.0000	(0.0004)	-0.0003	(0.0005)	0.0003	(0.0002)	0.0000	(0.0001)	0.0001	(0.0001)
AUS Gov-Bond	**0.0180**	(0.0078)	-0.0001	(0.0001)	0.0005	(0.0005)	-0.0004	(0.0006)	0.0003	(0.0002)	-0.0001	(0.0001)	-0.0001	(0.0001)

Table A10: Factor Exposures of Global Asset Returns in Seemingly Unrelated Regression (SUR) Framework; USD; Time-Varying Beta.

In this table, I jointly estimate the time-varying exposures of global equity and bond returns (in USD) to global factor shocks in a SUR framework. The error terms may have cross-equation contemporaneous correlations. SUR models are estimated with MLE. The sample period covers from March 1987 to February 2015. Standard errors are shown in the parantenses. Bold (italics) values indicate <5% (10%) significance level.

β_0	ω_q		$\omega_{\theta u}$		$\omega_{\theta d}$		$\omega_{\pi u}$		$\omega_{\pi d}$		ω_{xu}		ω_{xd}	
USA Equity	**-0.1687**	(0.0129)	**-0.0003**	(0.0001)	0.0011	(0.0010)	*-0.0034*	(0.0018)	0.0002	(0.0004)	0.0001	(0.0002)	0.0003	(0.0002)
CAN Equity	**-0.1790**	(0.0179)	**-0.0005**	(0.0001)	0.0011	(0.0015)	-0.0031	(0.0024)	**-0.0015**	(0.0005)	-0.0003	(0.0003)	*0.0007*	(0.0003)
DEU Equity	**-0.2254**	(0.0220)	-0.0001	(0.0002)	0.0020	(0.0018)	0.0001	(0.0030)	-0.0005	(0.0007)	0.0006	(0.0004)	0.0002	(0.0004)
FRA Equity	**-0.1932**	(0.0205)	**-0.0003**	(0.0001)	0.0007	(0.0017)	-0.0016	(0.0028)	-0.0001	(0.0006)	**0.0008**	(0.0004)	0.0003	(0.0004)
GBR Equity	**-0.1432**	(0.0163)	**-0.0004**	(0.0001)	0.0007	(0.0013)	-0.0006	(0.0022)	-0.0002	(0.0005)	*-0.0005*	(0.0003)	-0.0001	(0.0003)
CHE Equity	**-0.1493**	(0.0172)	**-0.0003**	(0.0001)	0.0001	(0.0014)	-0.0003	(0.0023)	0.0002	(0.0005)	0.0001	(0.0003)	0.0003	(0.0003)
JPN Equity	**-0.0858**	(0.0225)	-0.0001	(0.0002)	*0.0035*	(0.0018)	0.0005	(0.0031)	-0.0001	(0.0007)	-0.0001	(0.0004)	0.0005	(0.0004)
AUS Equity	**-0.1846**	(0.0222)	**-0.0007**	(0.0002)	-0.0008	(0.0018)	0.0004	(0.0030)	-0.0004	(0.0007)	*-0.0006*	(0.0004)	*0.0007*	(0.0004)
USA Gov-Bond	**0.0303**	(0.0078)	0.0000	(0.0001)	*0.0010*	(0.0006)	0.0001	(0.0013)	**0.0005**	(0.0002)	0.0001	(0.0001)	-0.0002	(0.0002)
CAN Gov-Bond	**-0.0386**	(0.0105)	0.0000	(0.0001)	*0.0016*	(0.0008)	0.0013	(0.0017)	-0.0002	(0.0003)	-0.0002	(0.0002)	0.0001	(0.0002)
DEU Gov-Bond	-0.0024	(0.0125)	0.0001	(0.0001)	0.0010	(0.0010)	0.0003	(0.0020)	**-0.0008**	(0.0004)	-0.0002	(0.0002)	-0.0002	(0.0002)
FRA Gov-Bond	-0.0083	(0.0122)	0.0000	(0.0001)	0.0009	(0.0010)	0.0002	(0.0020)	**-0.0007**	(0.0003)	-0.0002	(0.0002)	-0.0002	(0.0002)
GBR Gov-Bond	0.0134	(0.0122)	0.0001	(0.0001)	0.0002	(0.0010)	0.0006	(0.0020)	-0.0003	(0.0003)	**-0.0004**	(0.0002)	-0.0003	(0.0002)
CHE Gov-Bond	-0.0012	(0.0137)	0.0001	(0.0001)	-0.0001	(0.0011)	0.0005	(0.0022)	-0.0003	(0.0004)	-0.0003	(0.0002)	-0.0002	(0.0003)
JPN Gov-Bond	**0.0314**	(0.0146)	0.0000	(0.0001)	-0.0003	(0.0012)	-0.0008	(0.0024)	-0.0002	(0.0004)	-0.0001	(0.0002)	0.0002	(0.0003)
AUS Gov-Bond	**-0.0614**	(0.0132)	**-0.0002**	(0.0001)	-0.0013	(0.0010)	0.0027	(0.0022)	-0.0004	(0.0004)	**-0.0005**	(0.0002)	0.0002	(0.0003)
β_1 for Equities	$\omega_q * s_e$		$\omega_{\theta u} * s_e$		$\omega_{\theta d} * s_e$		$\omega_{\pi u} * s_e$		$\omega_{\pi d} * s_e$		$\omega_{xu} * s_e$		$\omega_{xd} * s_e$	
USA Equity	-0.0068	(0.0142)	0.0001	(0.0001)	*-0.0011*	(0.0006)	0.0006	(0.0008)	0.0006	(0.0006)	-0.0004	(0.0003)	0.0000	(0.0002)
CAN Equity	-0.0144	(0.0198)	-0.0001	(0.0002)	**-0.0020**	(0.0008)	-0.0004	(0.0011)	0.0002	(0.0008)	*-0.0009*	(0.0005)	-0.0003	(0.0003)
DEU Equity	**-0.0527**	(0.0243)	0.0000	(0.0002)	-0.0016	(0.0010)	**-0.0029**	(0.0014)	-0.0011	(0.0010)	0.0003	(0.0006)	-0.0001	(0.0004)
FRA Equity	-0.0299	(0.0226)	0.0002	(0.0002)	-0.0013	(0.0010)	-0.0018	(0.0013)	-0.0009	(0.0009)	-0.0003	(0.0005)	-0.0001	(0.0003)
GBR Equity	-0.0109	(0.0181)	0.0001	(0.0002)	**-0.0020**	(0.0008)	-0.0013	(0.0010)	-0.0007	(0.0007)	-0.0001	(0.0004)	0.0001	(0.0003)
CHE Equity	*-0.0349*	(0.0190)	0.0003	(0.0002)	-0.0007	(0.0008)	-0.0010	(0.0011)	-0.0002	(0.0008)	0.0001	(0.0005)	0.0001	(0.0003)
JPN Equity	**-0.0537**	(0.0249)	0.0002	(0.0002)	-0.0013	(0.0011)	-0.0015	(0.0014)	0.0011	(0.0010)	-0.0001	(0.0006)	0.0003	(0.0004)
AUS Equity	**-0.0500**	(0.0245)	0.0000	(0.0002)	-0.0014	(0.0010)	-0.0017	(0.0014)	0.0002	(0.0010)	-0.0008	(0.0006)	0.0000	(0.0004)
β_1 for Bonds	$\omega_q * s_b$		$\omega_{\theta u} * s_b$		$\omega_{\theta d} * s_b$		$\omega_{\pi u} * s_b$		$\omega_{\pi d} * s_b$		$\omega_{xu} * s_b$		$\omega_{xd} * s_b$	
USA Gov-Bond	0.0035	(0.0068)	0.0000	(0.0001)	0.0006	(0.0005)	*0.0018*	(0.0010)	-0.0003	(0.0003)	**-0.0003**	(0.0001)	-0.0002	(0.0001)
CAN Gov-Bond	0.0138	(0.0092)	0.0000	(0.0001)	**0.0017**	(0.0007)	**0.0036**	(0.0013)	-0.0001	(0.0003)	0.0001	(0.0002)	0.0000	(0.0002)
DEU Gov-Bond	**0.0372**	(0.0109)	*0.0002*	(0.0001)	0.0008	(0.0008)	0.0018	(0.0015)	-0.0003	(0.0004)	0.0001	(0.0002)	-0.0002	(0.0002)
FRA Gov-Bond	**0.0370**	(0.0106)	**0.0002**	(0.0001)	0.0011	(0.0008)	0.0020	(0.0015)	-0.0004	(0.0004)	0.0002	(0.0002)	-0.0001	(0.0002)
GBR Gov-Bond	**0.0276**	(0.0106)	**0.0002**	(0.0001)	0.0003	(0.0008)	0.0010	(0.0015)	0.0001	(0.0004)	0.0003	(0.0002)	-0.0003	(0.0002)
CHE Gov-Bond	**0.0411**	(0.0120)	0.0002	(0.0001)	-0.0003	(0.0009)	0.0010	(0.0017)	0.0002	(0.0004)	0.0001	(0.0002)	0.0000	(0.0002)
JPN Gov-Bond	-0.0011	(0.0128)	0.0001	(0.0001)	-0.0010	(0.0009)	-0.0021	(0.0018)	**-0.0012**	(0.0005)	0.0001	(0.0002)	0.0000	(0.0003)
AUS Gov-Bond	**0.0350**	(0.0116)	0.0000	(0.0001)	0.0010	(0.0008)	**0.0042**	(0.0016)	*-0.0007*	(0.0004)	**0.0005**	(0.0002)	0.0000	(0.0002)

Table A11: Factor Exposures of Global Asset Returns in a SUR Framework; Local Currencies; Time-Varying Beta.

In this table, I jointly estimate the time-varying exposures of global equity and bond returns (in local currencies) to global factor shocks in a SUR framework. The error terms may have cross-equation contemporaneous correlations. SUR models are estimated with MLE. The sample period covers from March 1987 to February 2015. Standard errors are shown in the parantenses. Bold (italics) values indicate <5% (10%) significance level.

β_0	ω_q		$\omega_{\theta u}$		$\omega_{\theta d}$		$\omega_{\pi u}$		$\omega_{\pi d}$		ω_{xu}		ω_{xd}	
USA Equity	**-0.1687**	(0.0129)	**-0.0003**	(0.0001)	0.0011	(0.0010)	*-0.0034*	(0.0018)	0.0002	(0.0004)	0.0001	(0.0002)	0.0003	(0.0002)
CAN Equity	**-0.1401**	(0.0144)	**-0.0004**	(0.0001)	0.0005	(0.0012)	-0.0007	(0.0020)	-0.0007	(0.0004)	0.0000	(0.0002)	*0.0005*	(0.0003)
DEU Equity	**-0.2066**	(0.0209)	-0.0001	(0.0001)	0.0016	(0.0017)	-0.0002	(0.0029)	0.0008	(0.0006)	0.0006	(0.0004)	0.0006	(0.0004)
FRA Equity	**-0.1734**	(0.0190)	**-0.0003**	(0.0001)	0.0004	(0.0015)	-0.0021	(0.0026)	*0.0011*	(0.0006)	**0.0008**	(0.0003)	*0.0007*	(0.0004)
GBR Equity	**-0.1472**	(0.0145)	**-0.0004**	(0.0001)	-0.0004	(0.0012)	-0.0016	(0.0020)	**0.0010**	(0.0004)	-0.0004	(0.0003)	0.0002	(0.0003)
CHE Equity	**-0.1452**	(0.0167)	**-0.0003**	(0.0001)	-0.0004	(0.0014)	-0.0014	(0.0023)	**0.0010**	(0.0005)	0.0001	(0.0003)	**0.0007**	(0.0003)
JPN Equity	**-0.1048**	(0.0206)	-0.0001	(0.0001)	**0.0033**	(0.0017)	-0.0006	(0.0028)	0.0006	(0.0006)	-0.0003	(0.0004)	0.0006	(0.0004)
AUS Equity	**-0.1179**	(0.0165)	**-0.0005**	(0.0001)	0.0014	(0.0013)	0.0005	(0.0023)	0.0003	(0.0005)	-0.0003	(0.0003)	0.0003	(0.0003)
USA Gov-Bond	**0.0256**	(0.0080)	0.0000	(0.0001)	0.0007	(0.0005)	**-0.0015**	(0.0006)	0.0003	(0.0002)	0.0001	(0.0002)	-0.0001	(0.0001)
CAN Gov-Bond	0.0065	(0.0078)	0.0000	(0.0001)	0.0007	(0.0005)	-0.0006	(0.0006)	**0.0005**	(0.0002)	0.0002	(0.0002)	0.0000	(0.0001)
DEU Gov-Bond	**0.0272**	(0.0060)	0.0000	(0.0000)	0.0005	(0.0004)	**-0.0010**	(0.0005)	**0.0003**	(0.0002)	0.0001	(0.0002)	0.0001	(0.0001)
FRA Gov-Bond	**0.0221**	(0.0065)	0.0000	(0.0000)	0.0004	(0.0004)	**-0.0014**	(0.0005)	**0.0005**	(0.0002)	0.0001	(0.0002)	0.0001	(0.0001)
GBR Gov-Bond	**0.0224**	(0.0074)	0.0000	(0.0000)	0.0002	(0.0004)	*-0.0012*	(0.0006)	**0.0005**	(0.0002)	0.0001	(0.0002)	0.0000	(0.0001)
CHE Gov-Bond	**0.0192**	(0.0053)	0.0000	(0.0000)	0.0000	(0.0003)	**-0.0015**	(0.0004)	**0.0004**	(0.0001)	0.0000	(0.0001)	0.0001	(0.0001)
JPN Gov-Bond	**0.0168**	(0.0065)	0.0000	(0.0000)	0.0000	(0.0004)	-0.0005	(0.0005)	*0.0003*	(0.0002)	0.0000	(0.0002)	0.0000	(0.0001)
AUS Gov-Bond	**0.0182**	(0.0081)	**-0.0001**	(0.0001)	0.0006	(0.0005)	-0.0002	(0.0006)	0.0001	(0.0002)	0.0001	(0.0002)	-0.0002	(0.0001)
β_1 for Equities	$\omega_q * s_e$		$\omega_{\theta u} * s_e$		$\omega_{\theta d} * s_e$		$\omega_{\pi u} * s_e$		$\omega_{\pi d} * s_e$		$\omega_{xu} * s_e$		$\omega_{xd} * s_e$	
USA Equity	-0.0068	(0.0142)	0.0001	(0.0001)	*-0.0011*	(0.0006)	0.0006	(0.0008)	0.0006	(0.0006)	-0.0004	(0.0003)	0.0000	(0.0002)
CAN Equity	-0.0105	(0.0159)	-0.0001	(0.0001)	**-0.0014**	(0.0007)	-0.0006	(0.0009)	0.0001	(0.0006)	-0.0004	(0.0004)	-0.0002	(0.0002)
DEU Equity	-0.0186	(0.0231)	0.0001	(0.0002)	-0.0012	(0.0010)	-0.0016	(0.0013)	-0.0007	(0.0009)	0.0002	(0.0006)	-0.0002	(0.0003)
FRA Equity	0.0048	(0.0210)	0.0003	(0.0002)	-0.0008	(0.0009)	-0.0004	(0.0012)	-0.0004	(0.0008)	-0.0003	(0.0005)	-0.0002	(0.0003)
GBR Equity	0.0005	(0.0160)	0.0001	(0.0001)	-0.0008	(0.0007)	0.0002	(0.0009)	0.0007	(0.0006)	-0.0003	(0.0004)	0.0002	(0.0002)
CHE Equity	0.0043	(0.0184)	**0.0004**	(0.0002)	-0.0005	(0.0008)	0.0002	(0.0010)	0.0004	(0.0007)	-0.0002	(0.0004)	0.0000	(0.0003)
JPN Equity	**-0.0537**	(0.0228)	0.0002	(0.0002)	*-0.0016*	(0.0010)	-0.0011	(0.0013)	0.0014	(0.0009)	-0.0009	(0.0006)	0.0002	(0.0003)
AUS Equity	-0.0217	(0.0182)	0.0001	(0.0002)	*-0.0013*	(0.0008)	-0.0008	(0.0010)	0.0003	(0.0007)	-0.0002	(0.0004)	0.0001	(0.0003)
β_1 for Bonds	$\omega_q * s_b$		$\omega_{\theta u} * s_b$		$\omega_{\theta d} * s_b$		$\omega_{\pi u} * s_b$		$\omega_{\pi d} * s_b$		$\omega_{xu} * s_b$		$\omega_{xd} * s_b$	
USA Gov-Bond	*0.0150*	(0.0082)	0.0000	(0.0000)	0.0004	(0.0005)	0.0005	(0.0009)	**0.0007**	(0.0002)	0.0001	(0.0002)	0.0000	(0.0001)
CAN Gov-Bond	0.0123	(0.0081)	0.0000	(0.0000)	-0.0001	(0.0005)	0.0005	(0.0009)	**0.0005**	(0.0002)	0.0001	(0.0002)	0.0000	(0.0001)
DEU Gov-Bond	0.0097	(0.0062)	0.0000	(0.0000)	-0.0002	(0.0004)	0.0002	(0.0007)	*0.0003*	(0.0002)	0.0001	(0.0001)	0.0000	(0.0001)
FRA Gov-Bond	0.0042	(0.0067)	0.0001	(0.0000)	-0.0005	(0.0004)	0.0003	(0.0007)	0.0003	(0.0002)	0.0000	(0.0001)	-0.0001	(0.0001)
GBR Gov-Bond	0.0055	(0.0077)	0.0001	(0.0000)	-0.0001	(0.0005)	0.0007	(0.0008)	**0.0004**	(0.0002)	0.0002	(0.0002)	-0.0002	(0.0001)
CHE Gov-Bond	0.0059	(0.0055)	0.0000	(0.0000)	0.0001	(0.0003)	0.0008	(0.0006)	0.0002	(0.0002)	0.0001	(0.0001)	-0.0001	(0.0001)
JPN Gov-Bond	-0.0047	(0.0067)	0.0000	(0.0000)	-0.0002	(0.0004)	**0.0021**	(0.0007)	0.0001	(0.0002)	0.0001	(0.0001)	-0.0001	(0.0001)
AUS Gov-Bond	*0.0148*	(0.0084)	**0.0002**	(0.0001)	0.0007	(0.0005)	-0.0001	(0.0009)	0.0003	(0.0002)	0.0002	(0.0002)	-0.0001	(0.0001)

Table A12: Conditional Variance Decomposition.

Panel A. Constant Beta

	ω_q	$\omega_{\theta u}$	$\omega_{\theta d}$	$\omega_{\pi u}$	$\omega_{\pi d}$	ω_{xu}	ω_{xd}	Explained
USA Equity	93.6%	4.0%	0.0%	1.2%	0.2%	0.3%	0.6%	56.7%
CAN Equity	71.3%	7.2%	0.2%	2.4%	17.1%	0.0%	1.7%	47.4%
DEU Equity	95.4%	0.5%	0.1%	1.6%	0.9%	1.0%	0.6%	39.6%
FRA Equity	88.1%	3.9%	0.2%	2.9%	0.2%	3.2%	1.5%	37.3%
GBR Equity	84.6%	8.8%	2.0%	1.0%	0.6%	2.6%	0.4%	33.9%
CHE Equity	89.8%	7.0%	0.8%	0.4%	0.4%	0.1%	1.5%	33.7%
JPN Equity	76.2%	1.8%	7.8%	3.4%	3.7%	0.1%	7.1%	8.2%
AUS Equity	72.9%	15.0%	2.2%	1.7%	3.1%	1.3%	3.9%	33.8%
USA Gov-Bond	43.4%	0.2%	5.6%	9.0%	40.5%	0.1%	1.3%	13.9%
CAN Gov-Bond	57.0%	0.6%	7.7%	18.1%	10.6%	1.9%	4.0%	9.4%
DEU Gov-Bond	3.2%	1.2%	2.4%	12.2%	80.7%	0.2%	0.1%	8.7%
FRA Gov-Bond	0.9%	0.3%	1.3%	17.5%	79.8%	0.1%	0.1%	7.4%
GBR Gov-Bond	38.7%	6.4%	0.5%	8.6%	30.7%	15.1%	0.0%	5.1%
CHE Gov-Bond	20.7%	7.6%	2.0%	15.5%	43.8%	10.0%	0.4%	2.1%
JPN Gov-Bond	78.8%	1.7%	0.5%	5.2%	13.0%	0.3%	0.6%	3.3%
AUS Gov-Bond	36.3%	16.2%	7.3%	9.4%	21.1%	4.4%	5.3%	14.3%

Panel B. Time-Varying Beta

	ω_q		$\omega_{\theta u}$		$\omega_{\theta d}$		$\omega_{\pi u}$		$\omega_{\pi d}$		ω_{xu}		ω_{xd}		Explained
	β_0	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0	β_1	
USA Equity	87.6%	0.1%	3.6%	0.2%	0.9%	0.8%	2.4%	0.2%	0.6%	2.0%	0.0%	0.7%	0.9%	0.0%	60.2%
CAN Equity	68.1%	0.2%	8.2%	0.3%	0.6%	1.8%	1.3%	0.1%	14.2%	0.1%	0.4%	1.8%	2.4%	0.6%	52.7%
DEU Equity	87.2%	2.2%	0.4%	0.0%	1.7%	0.7%	0.0%	1.9%	1.3%	2.9%	1.5%	0.1%	0.2%	0.1%	50.3%
FRA Equity	86.0%	1.1%	3.0%	0.4%	0.2%	0.7%	0.4%	1.2%	0.1%	2.8%	3.2%	0.2%	0.7%	0.0%	41.3%
GBR Equity	82.1%	0.2%	7.6%	0.1%	0.5%	2.6%	0.1%	1.0%	0.3%	2.7%	2.7%	0.1%	0.0%	0.1%	38.3%
CHE Equity	87.4%	2.6%	5.1%	1.9%	0.0%	0.3%	0.0%	0.8%	0.5%	0.3%	0.2%	0.1%	0.7%	0.1%	35.8%
JPN Equity	55.0%	6.7%	0.7%	2.0%	19.0%	1.1%	0.1%	1.0%	0.3%	9.1%	0.4%	0.0%	4.1%	0.4%	19.5%
AUS Equity	72.3%	2.8%	15.2%	0.0%	0.3%	0.8%	0.0%	1.1%	0.9%	0.2%	2.2%	1.3%	2.7%	0.0%	38.6%
USA Gov-Bond	33.2%	0.3%	0.2%	0.5%	8.8%	1.9%	0.1%	5.4%	31.2%	7.0%	0.3%	4.1%	4.4%	2.7%	24.1%
CAN Gov-Bond	46.9%	3.7%	0.2%	0.3%	17.1%	10.1%	2.8%	12.0%	2.4%	1.0%	2.4%	0.8%	0.3%	0.0%	20.1%
DEU Gov-Bond	0.2%	18.6%	1.6%	4.7%	5.8%	2.3%	0.1%	3.5%	52.8%	3.3%	1.9%	0.7%	3.8%	0.8%	15.8%
FRA Gov-Bond	2.2%	17.8%	0.8%	6.6%	5.7%	3.5%	0.1%	4.0%	44.6%	6.9%	2.2%	1.9%	3.3%	0.4%	16.9%
GBR Gov-Bond	10.6%	16.9%	7.8%	10.9%	0.5%	0.4%	1.5%	1.7%	11.6%	0.2%	18.2%	5.1%	10.1%	4.4%	10.6%
CHE Gov-Bond	0.1%	38.6%	6.6%	7.8%	0.3%	0.5%	1.7%	1.7%	20.7%	3.2%	10.7%	1.0%	7.1%	0.2%	8.5%
JPN Gov-Bond	33.8%	0.0%	1.0%	2.6%	1.0%	2.4%	1.6%	3.5%	5.4%	43.9%	1.4%	0.6%	2.8%	0.0%	15.8%
AUS Gov-Bond	40.3%	7.6%	9.8%	0.0%	4.0%	1.5%	4.8%	7.0%	5.6%	8.8%	6.2%	3.5%	0.9%	0.0%	29.0%

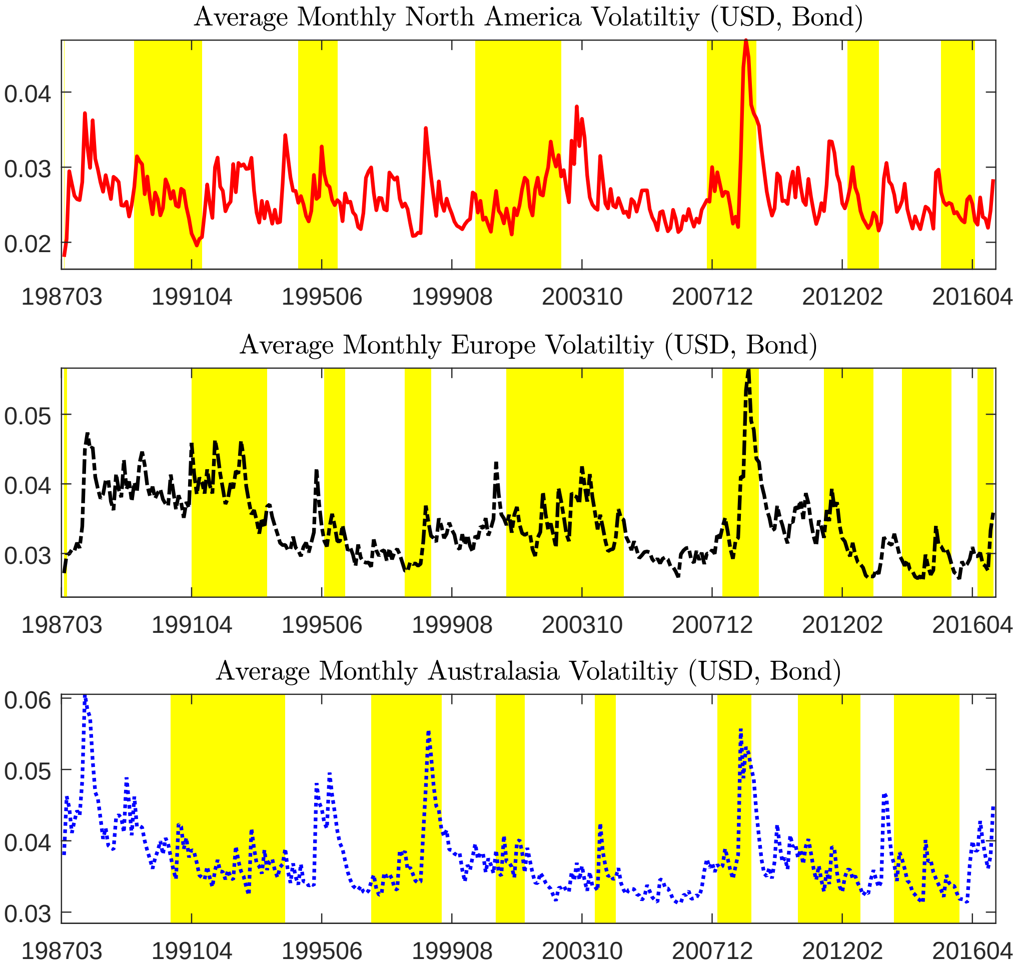

Figure A1: Average (equal-weight) monthly USD-denominated bond return conditional volatility for North America, Europe, and Australasia.

The shaded regions are OECD recession indicators (from peak to trough) for United States (top plot), Germany (middle plot), and Japan (bottom plot) obtained from Federal Reserve Bank of St. Louis. More details on obtaining the conditional volatilities are shown in Table A1 in the main manuscript.

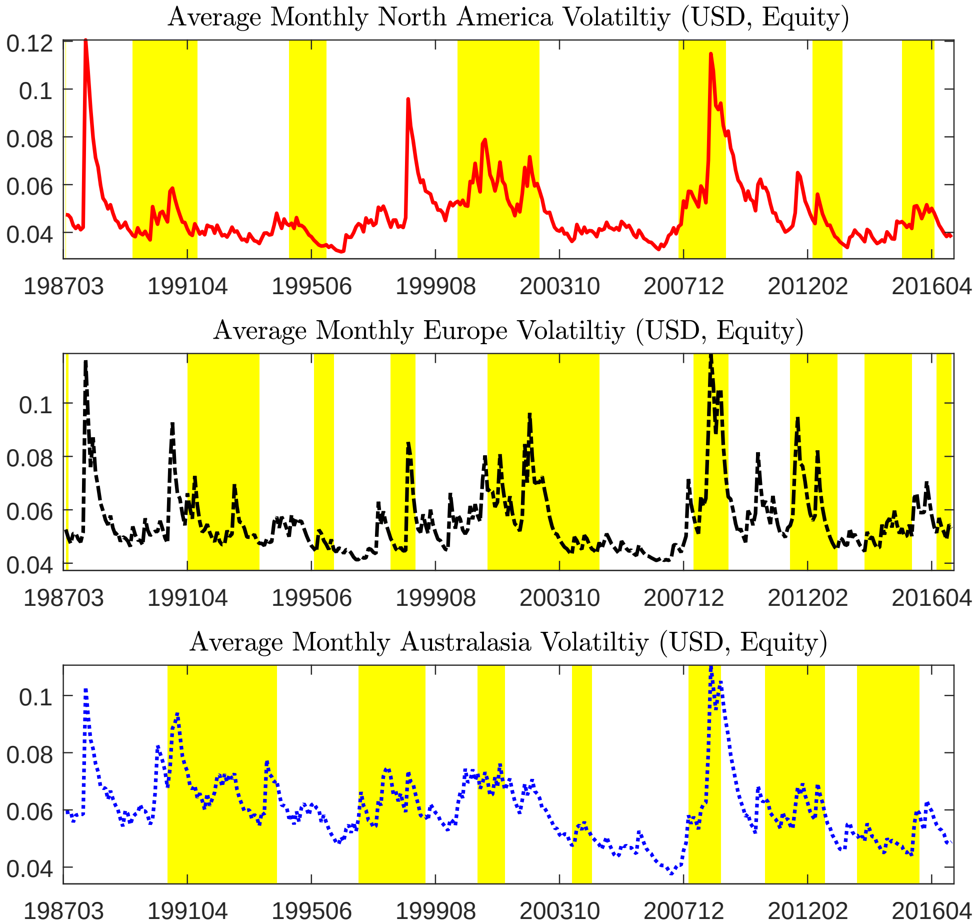

Figure A2: Average (equal-weight) monthly USD-denominated equity return conditional volatility for North America, Europe, and Australasia.

The shaded regions are OECD recession indicators (from peak to trough) for United States (top plot), Germany (middle plot), and Japan (bottom plot) obtained from Federal Reserve Bank of St. Louis. More details on obtaining the conditional volatilities are shown in Table 2 of the main paper.

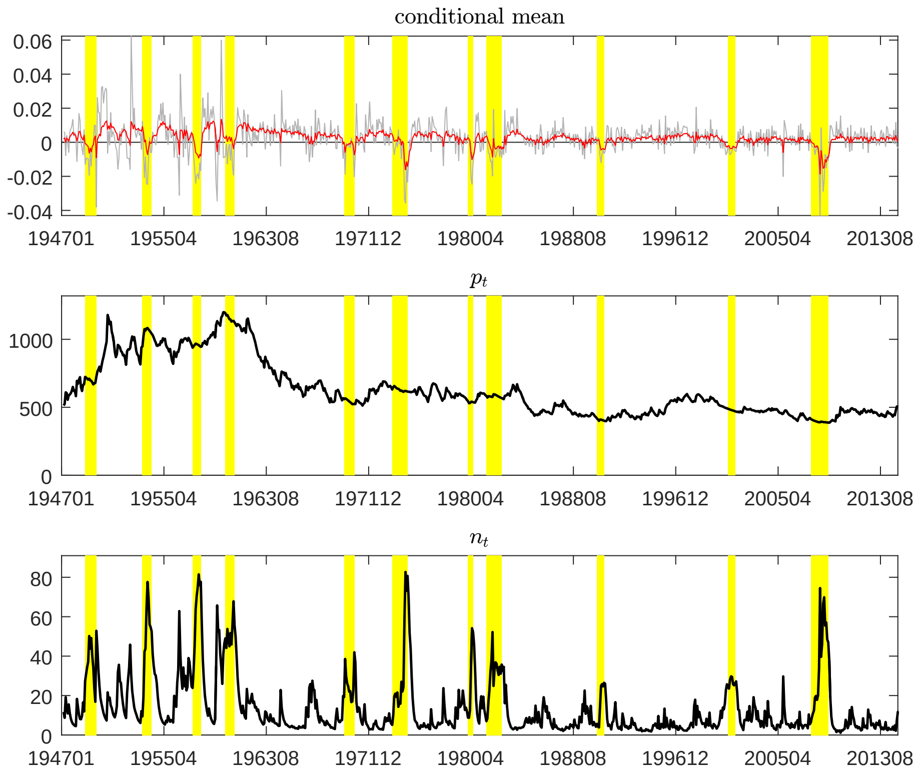

Figure A3: Estimation Results of the U.S. Real Upside (θu) and Downside (θd) Uncertainties; sample period begins from 1947/01 to 2016/12.

The model is detailed in Table A6. The shaded regions are U.S. NBER recession indicators.

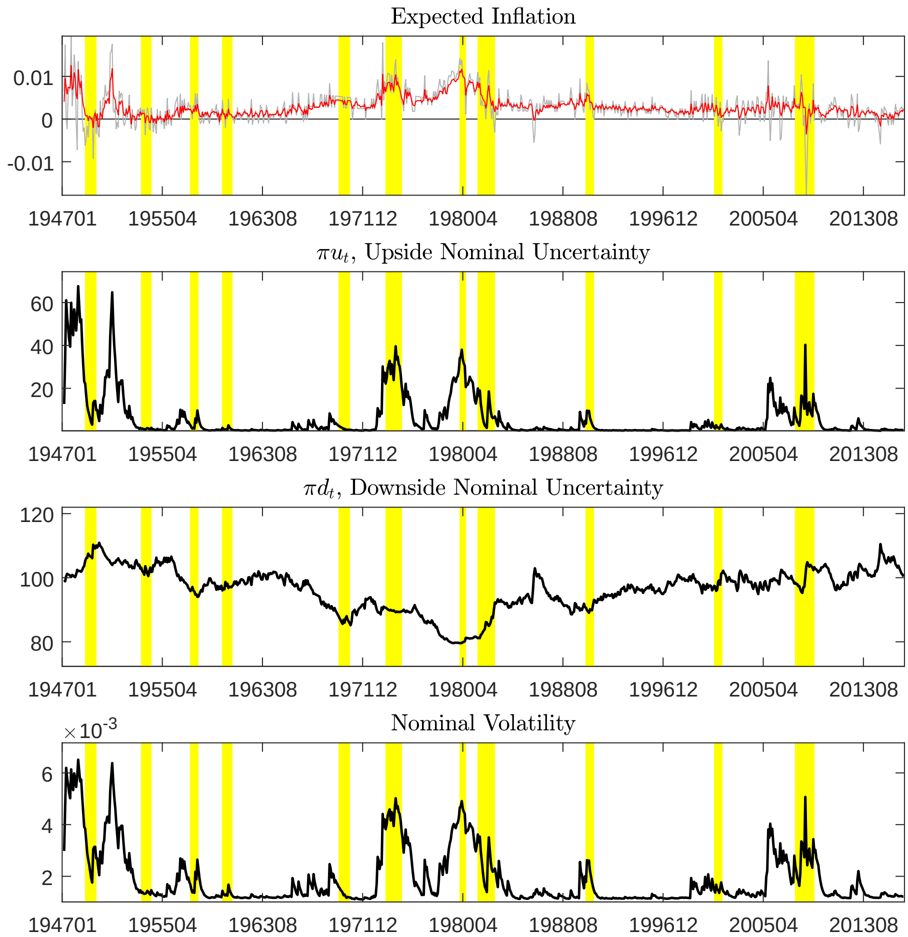

Figure A4: Estimation Results of the U.S. Inflation Upside (πu) and Downside (πd) Uncertainties (Long Sample, 1947/01-2016/12).

The model is detailed in Table A7. The shaded regions are U.S. NBER recession indicators.

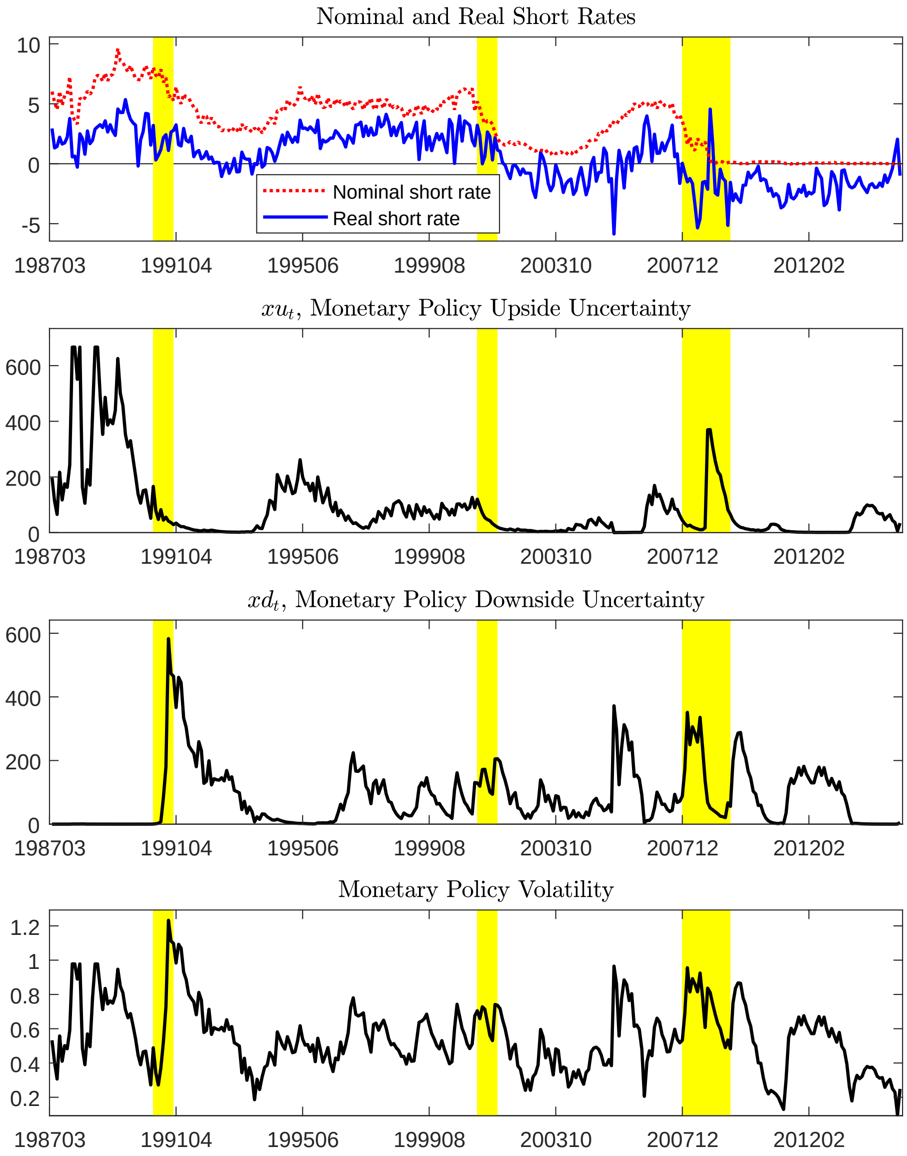

Figure A5: Estimation Results of the U.S. Real Short Rate Upside (xu) and Downside (xd) Uncertainties; sample period begins when first risk aversion estimate is available, 1986/06-2015/02).

The model is detailed in Table A8. The shaded regions are U.S. NBER recession indicators.

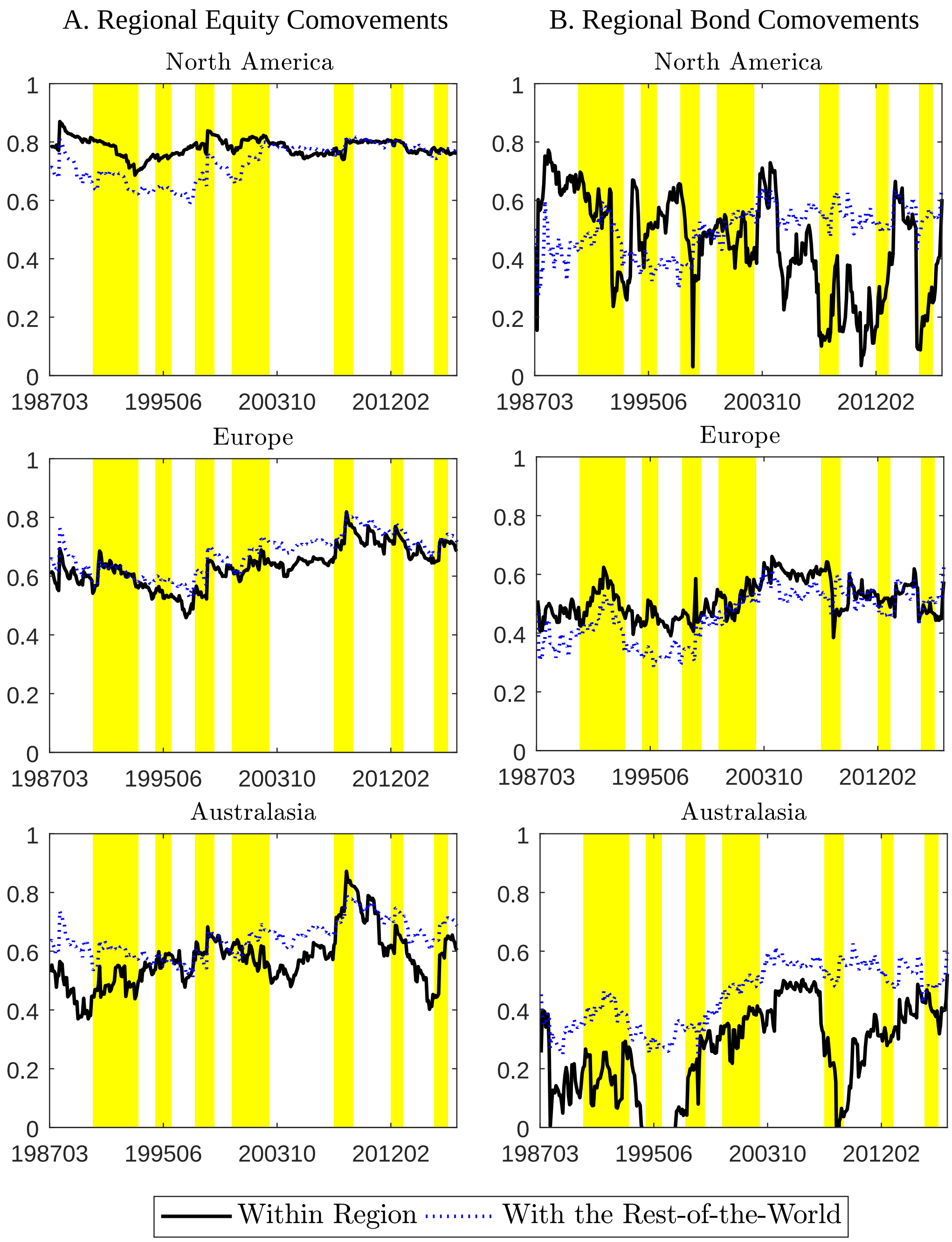

Figure A6: Regional Dynamic Comovements.

This figure presents the dynamics of a region's internal correlation (solid lines) and its correlation with the rest-of-the-world (dotted lines). Both correlations are constructed using equal averages and across unique country pairs. alternative global return correlation estimates using the average of pairwise DCC models (dashed lines) and using local currency returns (right plot). The shaded regions are OECD world recession months from the OECD website.