Will Fund Managers Survive to the Advent of Robots? An Optimal Contracting Approach

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Abstract

This paper investigates the microfundation of the automation of fund managers in a continuous-time principal-agent framework. In this model, a representative investor delegates the management of a fund either to an agent or to a forthcoming and unsupervised robot. To capture the trade-off between the delegation to an agent and automation, we assume that while the fund manager is inherently subject to an agency friction, he may perform better than the robot by adopting an active management strategy. We derive an optimal long-term contract that distorts the provision of incentives over time. At first, it boosts the fund manager's value and secures the contractual relationship, and then at the advent of robots it lets the principal reassess the agent's value to account for the presence of this new alternative. In line with empirical evidence, we predict that the advent of robots (i) has a skimming effect as it leads to the instantaneous automation of the less successful fund managers, and (ii) it mitigates the agency friction as the fund managers that remain active extract a smaller agency rent. Combined, these factors make the delegation to fund managers more attractive from the perspective of the representative investor after the advent of robots.

As funds are more and more prone to be driven by robots, will fund managers disappear? Indeed, when McKinsey claims that in the banking industry "technology-enabled process transformations are driving efficiency, consistency, speed, and better outcomes"[1], one may expect such jobs to cease to exist.

In this paper, we examine in a principal-agent framework the impact of the advent of robots on the fund manager's optimal contract and on the fund value. We build our dynamic contracting model upon DeMarzo and Sannikov (2006) where a principal delegates to an agent the long-term management of a project, that refers here to a fund. The optimal contract provides incentives to the agent both (i) by postponing payments given after enough success, and (ii) by the threat of being laid off after too many bad outcomes. The novelty of our model is that at the contracting date, the advent of a robot able to automate the fund management is foreseen[2]. It generates a trade-off as the delegation to a fund manager may be more productive than the automation but is inherently subject to a costly agency friction. We interpret the manager's better productivity as his unique ability to actively manage funds, in contrast with the robot that does not seek to *beat the market*.

We derive an optimal contract that distorts the provision of incentives over time. Before the advent of robots, it boosts the continuation value to secure the contractual relationship. Then, at the advent of robots, it lets the principal reassess the fund manager's value[3] to account for the presence of the valuable alternative. While the manager's continuation value drops, its drift is no longer boosted so the threat of contract termination is strengthened. We show that the advent of robots (i) has a skimming effect as a fund manager performing poorly would be instantaneously automated, and (ii) reduces the agency rent given to the

[1] *The transformative power of automation in banking*, McKinsey 2017

[2] The advent of the robot is out of the scope of both parties, as it seems that banking institutions do not internalize the development of robots. For instance, ANZ Group has implemented IBM's Watson in wealth management.

[3] it is standard in the dynamic contracting literature that investigates the impact of a persistent and exogenous shock to compensate at early-stage the agent for the forthcoming reassessment of his value. See among others Hoffmann and Pfeil (2010), Demarzo, Fishman, He, and Wang (2012).

fund manager if he remains active. Combined, these factors make the delegation to fund managers more attractive from the perspective of the representative investor after the advent of robots.

To solve our problem, we adopt the method introduced by Sannikov (2008) in the context of principal-agent models and that is based on the martingale optimality principle. The principal controls the sensitivity of the manager's continuation value to (i) the fund performance through a martingale diffusion term and to (ii) the advent of robots through a martingale jump term. As a consequence, making the continuation value more sensitive to the advent of robots means that the agent accumulates value faster up to the advent of robots before being subject to a larger drop in his value. Due to the zero mean condition of the martingale jump term, such specification solely increases the efficiency of the contract but does not alter the fund manager's decisions.

Unlike standard dynamic contracting model without an exogenous shock where the agent's value is tied to the fund value during all his tenure, it is optimal to let the agent's value drop at the advent of robots, while it increases the fund value. As a consequence, the implementation of the optimal contract cannot be achieved trough standard securities, but rather through a point-based incentive program. It is a compensation scheme where the fund manager's number of points trace his continuation value prior the advent of robots and fluctuates with the fund's performance. After sufficiently good performance, some points can be redeemed and thus converted into cash payments. This particular implementation allows for the expiration of a fraction of the points owned by the fund manager at the advent of robots, and as specified in the optimal contract. When the fund manager holds no more points, he is laid off by the representative investor.

A striking illustration of the skimming effect of automation has been offered in 2017 when BlackRock decided to automate 13 % of their stock pickers[4]. While its CEO Laurence D.

[4] https://www.nytimes.com/2017/03/28/business/dealbook/blackrock-actively-managed-funds-computer-models.html

Fink has justified such action as a necessary “change [of] ecosystem”, it is not clear how the decision whether to lay off or to keep the stock pickers was taken. We show that compared to a baseline model à la DeMarzo Sannikov without robots, the optimal contract that we design postpones the termination of agents that have not performed well enough. Indeed, it is better from the perspective of the representative investor to wait for the advent of a valuable alternative that makes the termination of the contractual relationship more efficient. Therefore, it suggests that BlackRock have strategically postponed the termination of some of the stock pickers because the advent of robots has been foreseen. In addition, we predict that BlackRock dismissed the stock pickers with the worst continuation value which is a certain measure of performance.

The 2018 *asset management compensation study* by Greenwich Associates suggests a decrease in the fund manager's bonuses due to the advent of robots. They claim that the large investment cost in technologies of automation reduces the incentive compensation pool of fund managers. Our optimal contracting approach offers an alternative explanation. We show that even in absence of cash constraints, it is the advent of a valuable alternative able to substitute to the agent that mitigates the agency friction and thus decreases the agency rent of the fund managers that are not instantaneously laid off.

Our work is closely related to Hoffmann and Pfeil (2010), Demarzo et al. (2012) and Li (2017) that theoretically investigate how an exogenous shock on the agent's profitability impacts the optimal compensation. They show that the agent's continuation value reacts instantaneously to such lucky event, and that *rewards for luck* is part of the optimal compensation scheme. It is in line with Garvey and Milbourn (2004), Bertrand and Mullainathan (2013), and Francis, Hasan, John, and Sharma (2013) that show that exogenous events significantly impact the compensation of CEOs and VPs. We rather study a exogenous shock on the contract's termination value, with the specificity that the technology of automation is a real option held by the principal. As the consequence, we assess the optimal time of implementation of the option. It turns out that it is optimal to wait for the manager's continuation value to drop down to the fixed termination boundary before implementing the option. Nevertheless, the principal controls the sensitivity of the agent's continuation value to the advent of robots and he lead an agent that has not performed well enough to be driven down to the termination boundary at the advent of robots.

He (2009) also builds on DeMarzo and Sannikov (2006) but specifies that the cash-flow process follows a geometrical Brownian motion, so changes in firm size generate incentives. Gryglewicz, Hartman-Glaser, and Zheng (2017) present a model where a firm is managed by a risk-averse agent and where the principal holds a growth option. The agent's risk aversion implies that he must be compensated for bearing risks. The study of a growth option is relevant in their framework as the cash flow process follows a geometric Brownian motion so the firm size matters. Grenadier and Wang (2005) study a model with an investment option that encompasses both moral hazard and adverse selection. They show that the agency issue postpones the investment decision taken by the agent. Other papers discuss the interaction between agency issue and real options in signaling games – Grenadier and Malenko (2011)) – or communication – Grenadier, Malenko, and Malenko (2016)).

Finally, our work tries to bridge the gap between the literature on contracting theory and labor economics, where the impact of automation on jobs is broadly investigated at a macro level (see Acemoglu and Autor (2011) for an overview of this literature). Our approach is in the spirit of the study of the *displacement effect* of automation – the irreversible substitution of workers by machines – which according to Acemoglu and Restrepo (2018b) is "both descriptively realistic and leads to distinct and empirically plausible predictions". Autor and Salomons (2018) show that the displacement effect of automation is becoming substantial since the 2000s. Alternatively, automation is considered as a human-augmenting technology in Bessen (2018) or as a capital-augmenting technology in Graetz and Michaels (2015).

I. The Model

A. The Continuous-Time Model

We consider a continuous-time framework of an everlasting fund that builds on a moral-hazard model *à la* DeMarzo and Sannikov (2006), and where a forthcoming technology of automation (hereafter the robot) arises stochastically to substitute to the manager. The fund is owned by a representative investor (hereafter the principal) and he decides whether to hire a fund manager or to implement the robot, so the agent is no longer needed. Here, the robot can be seen as a real option as its implementation at a sunk cost is irreversible. The law of the random advent of robots is foreseen by both parties and exogenous. As a consequence, the presence of robots in the model does not create a new source of asymmetry of information. The dynamics of the fund's value depends on the representative investor's choice of delegation – either to the fund manager or automated – and also to a parameter μ that is interpreted as the quality of the basket of securities included in the fund. The representative investor has access to unlimited fund and both the representative investor and the fund manager are risk-neutral. The fund manager is protected by limited liability and while the representative investor discounts at a rate $r > 0$, the fund manager is more impatient and discounts at $\gamma > r$.

Now, let us characterize the dynamics of the fund value under the two possible alternatives. On the one hand, when the fund manager is in charge, a moral-hazard problem arises because his effort $a_t \in \{0; \bar{a}\}$ that drives the dynamics of the fund value is unobservable. Specifically, as long as the agent manages the fund, the fund value evolves with the dynamics

$$dX_t = a_t \mu dt + \sigma dZ_t \tag{1}$$

where $(Z_t)_t$ is a standard Brownian motion interpreted as the uncertainty of the basket of securities included in the fund and that is formally defined in the probabilistic background of the model in the appendix – section (B) and σ is a positive constant that accounts for the market volatility. The effort process $(a_t)_{t\geq 0}$ is assumed to be progressively measurable with respect to $\mathcal{F}_t$, the information set available at date t that concerns the past realization of the Brownian motion up to t. Whenever the fund manager shirks ($a_t = 0$), he derives a private benefit Bdt, where B is a positive constant. Following perpetually the *no-effort* strategy from date t, i.e. $a_s = 0, \forall\, s \geq t$, gives to him $\frac{B}{\gamma}$. We assume that $\bar{a} > 1$, and it is interpreted as the fund manager's unique ability to actively manage the fund and *beat the market* value of the same basket of securities. The fund has a maximum expected value under active management of $\frac{\bar{a}\mu}{r} > \frac{\mu}{r}$, the expected market value of the same basket of securities. The representative investor can decide to terminate[5] the contract at any time as long as he fulfills his payment promises to the fund manager and we note $\tau \geq 0$ the date of termination of the contract.

On the other hand and at any date after the advent of robots noted T, the representative investor can exert the real option that irreversibly implements the robot to automate the fund. The advent of robots follows an exponential law of parameter λ. We note $\mathcal{H}_t$ the information set at date t that concerns the availability of the technology of automation at t. $\mathcal{H}_t = 0$ as long as the robot is unavailable, and then jumps forever to 1 as soon as it becomes implementable. We note τ_M the date of implementation that requires the payment of a sunk cost $I > 0$, and we assume in addition that the fund manager must have been laid off before the implementation of the robot, so we impose $\tau_M \geq \tau \vee T$. Then, the dynamics of the robot-driven fund with of the same basket of securities follows

$$dX_t = \mu dt + \sigma dZ_t \tag{2}$$

[5]The literature remains vague concerning the definition of contract's termination and agent's lay off. Thus, we use these terms indifferently here, and assume that it means that the agent shirk forever and does not receive payments from the principal anymore.

We note M the value of the automated process at τ_M and it is given by

$$M = \frac{\mu}{r} - I \tag{3}$$

Prior to the advent of robots, the best alternative to the delegation to the fund manager is noted M_0. In the rest of the paper, we assume that the technology of automation is sufficiently valuable to be of interest, and so $M > M_0$. Otherwise the value of the contract termination would always remain at M_0, and the optimal contract would be the one derived in DeMarzo and Sannikov (2006).

B. Formulation of the problem

Following the literature, we assume that both parties fully commit to a long-term contract describing the term of their relationship. Such contract $\Pi = ((U_t)_t; \tau)$ consists of a stream of positive payments $(U_t)_t$ measured in the same unit as the fund manager's private benefit, that depends on both the history of the fund value and the availability of robots that are public signals. Payments are given up to the date of termination of the contract τ. We note $\mathcal{G}_t$ the information set available at date t. Thus, the process $U = (U_t)$ is $\mathcal{G}_t$-adapted and non-decreasing, and τ is a $\mathcal{G}_t$-stopping time that can be infinite. We assume the square integrability of the payments for any effort process $(a_t)_t$, so

$$\mathbb{E}^a \left[\int_0^\tau e^{-\gamma t} dU_t \right]^2 < +\infty \tag{4}$$

Fix an arbitrary contract $\Pi = ((U_t)_t; \tau)$ and assume that the fund manager exerts an effort strategy $a = (a_t)_t$. Then, the fund manager's expected utility at date t is

$$\mathbb{E}^a \left[\int_t^\tau e^{-\gamma(s-t)} (dU_s + \frac{B}{\bar{a}}(\bar{a} - a_s)ds) + e^{-\gamma\tau} \frac{B}{\gamma} \right] \tag{5}$$

where $\frac{B}{\bar{a}}$ reflects the severity of the agency issue in our model. The term $\frac{B}{\gamma}$ refers to the private benefit that the fund manager will obtain once the contract is terminated from shirking perpetually. The fund value at date 0 for a fixed contract Π and an effort strategy $a = (a_t)_t$ is given by

$$max \left(M_0; \mathbb{E}^a \left[\int_0^\tau e^{-rt} (a_t \mu dt - dU_t) + e^{-r\tau} \tilde{M} \right] \right) \tag{6}$$

where $\tilde{M} = M_0 1_{t<T} + M 1_{t \leq T}$ is the value that gets the representative investor at the termination of the fund manager's contract.

We say that the effort process $a^*(\Pi) = (a_t^*(\Pi))_t$ is *incentive-compatible* if it is the fund manager's weakly-preferred response to Π. That is, for any effort process a, the incentive-compatible effort process $a^*(\Pi)$ satisfies

$$\mathbb{E}^{a^*(\Pi)} \left[\int_0^\tau e^{-\gamma t} (dU_t + \frac{B}{\bar{a}}(\bar{a} - a_t^*(\Pi)dt) + e^{-\gamma\tau} \frac{B}{\gamma} \right] \geq \mathbb{E}^a \left[\int_0^\tau e^{-\gamma t} (dU_t + \frac{B}{\bar{a}}(\bar{a} - a_t)dt) + e^{-\gamma\tau} \frac{B}{\gamma} \right] \tag{7}$$

Then, Π belongs to the class of incentive-compatible contracts if it induces the fund manager to follow $a^*(\Pi)$. An optimal contract is an incentive-compatible contract that maximizes the fund value at date 0 and satisfies the participation constraint of the fund manager, so he expects to receive at least his reservation utility w_0. This latter condition can be formulated as

$$\mathbb{E}^{a}\left[\int_{0}^{\tau} e^{-\gamma t}(dU_t + \frac{B}{\bar{a}}(\bar{a} - a_t)dt) + e^{-\gamma\tau}\frac{B}{\gamma}\right] \geq w_0 \tag{8}$$

Thus, the fund value associated to the long-term commitment of a representative investor and a fund manager to a contract Π is given by

$$V = \sup_{\Pi} \mathbb{E}^{a^*(\Pi)}\left[\int_{0}^{\tau} e^{-rt}(a^*(\Pi)\mu dt - dU_t) + e^{-r\tau}\tilde{M}\right] \tag{9}$$

s.t.

- Π is incentive compatible
- Π satisfies the participation constraint (8)

Ex-ante, the representative investor's decision to hire a fund manager at date 0^-, i.e. just before the model starts, is taken by comparing the value he extracts from the two alternatives.

Observation 1: *Assume $a^*(\Pi)$ exists, i.e. Π is incentive compatible. Then,*

$$\mathbb{E}^{a^*(\Pi)}\left[\int_{0}^{\tau} e^{-\gamma t}(dU_t + \frac{B}{\bar{a}}(\bar{a} - a^*_t(\Pi)dt) + e^{-\gamma\tau}\frac{B}{\gamma}\right] \geq \underbrace{\mathbb{E}^{0}\left[\int_{0}^{\tau} e^{-\gamma t}(dU_t + Bdt) + e^{-\gamma\tau}\frac{B}{\gamma}\right]}_{:=\frac{B}{\gamma}} \tag{10}$$

This yields to the natural assumption that the fund manager's reservation utility satisfies $w_0 \geq \frac{B}{\gamma}$.

Observation 2: *Here, we depict some important cases where the representative investor does not offer a contract to the fund manager:*

- *Case 1 - The forthcoming robots will be so valuable that no contract is offered:*

If $\frac{\lambda}{\lambda+r}M \geq \frac{\bar{a}\mu}{r}$, then $V \leq \frac{\lambda}{\lambda+r}M$. As a consequence, the representative investor does not offer a contract to a fund manager as he would be better off waiting for the technology to arise in order to automate.

- *Case 2 - Active fund management is too expensive:*

  If $M \geq \frac{\bar{a}\mu}{r}$, then $V \leq M - w_0$. As a consequence, the principal does not offer a contract to a fund manager that has a reservation utility $w_0 \geq \frac{r}{\lambda+r}M$.

II. Incentive-Compatible Effort and Markov Formulation

In this section, we follow Sannikov (2008) in order to characterize the incentive-compatible effort and give the Markov formulation of the representative investor's problem (7)-(9). Let's take a contract $\Pi = ((U_t)_t; \tau)$ as given and assume for now that the effort strategy chosen by the fund manager is incentive compatible. We define the process $W^{\Pi} = (W_t^{\Pi})_t$ as

$$W_t^{\Pi} = \mathbb{E}^{a(\Pi)}\left[\int_t^{\tau} e^{-\gamma(s-t)}(dU_s + \frac{B}{\bar{a}}(\bar{a} - a_s(\Pi))ds) + e^{-\gamma\tau}\frac{B}{\gamma} \mid \mathcal{G}_t\right] \tag{11}$$

W^{Π} corresponds to the agent's continuation value associated to a contract Π. It is defined as the total value that the fund manager expects to extract from the contract and afterwards starting date t. As the stream of payments $(U_t)_t$ is composed of non-negative terms and the private benefit is positive, by construction $W_t^{\Pi} \geq \frac{B}{\gamma}$ for all $t \leq \tau$ and $W_t^{\Pi} = \frac{B}{\gamma}$ afterwards. Therefore the termination of the contract is necessary by limited liability as soon as the continuation value reaches $\frac{B}{\gamma}$. For this reason, we introduce

$$\tau^{w}_{\frac{B}{\gamma}} = inf\{t \geq 0 \mid W_t^{\Pi} = \frac{B}{\gamma}\} \tag{12}$$

and impose the contract termination to always satisfies $\tau \leq \tau^w_{\frac{B}{\gamma}}$. In the following lemma, we apply the martingale representation theorem to find the dynamics of the continuation value.

Lemma 1: ***Representation of the fund manager's value as a diffusion process*** *Applying the Martingale Representation Theorem, there exists a unique pair of processes $\mathcal{G}_t$-predictable and square-integrable $((\beta_t)_t, (\delta_t)_t) = ((\beta_t)_{t\leq\tau}, (\delta_t)_{t\leq\tau})$ associated to an incentive-compatible contract Π, such that the continuation value W^Π of the fund manager evolves under $\mathbb{P}^a$ as*

$$dW_t^\Pi = (\gamma(W_t^\Pi - \frac{B}{\gamma}) - \frac{B}{\bar{a}}(\bar{a} - a_t))dt + \sigma\beta_t dZ_t^a + \delta_t(dH_t - \lambda dt) - dU_t \quad \textit{for } t \leq \tau \tag{13}$$

Proof. The fund manager's total expected value from entering into a contract Π and seen at date t is

$$\Upsilon_t^{Agent}(\Pi, a(\Pi)) = \int_0^t e^{-\gamma s}(dU_s + \frac{B}{\bar{a}}(\bar{a} - a_s(\Pi))ds) + e^{-\gamma t}(W_t^\Pi - \frac{B}{\gamma}) \qquad \text{for } t \leq \tau \tag{14}$$

$$= \mathbb{E}^{a(\Pi)}\left[\int_0^\tau e^{-\gamma s}(dU_s + \frac{B}{\bar{a}}(\bar{a} - a_s(\Pi))ds) + e^{-\gamma\tau}\frac{B}{\gamma}\right] \qquad \text{for } t \leq \tau \tag{15}$$

It is an uniformly-integrable $\mathbb{P}^a$-martingale. Thus, we can apply the martingale representation theorem (see Karatzas and Shreve (1991)) and we get that there exists a unique $\mathcal{G}_t$-predictable and square-integrable pair of processes (β_t, δ_t) such that, for all $t \leq \tau$

$$\Upsilon_t^{Agent}(\Pi, a(\Pi)) = \Upsilon_0 + \int_0^t e^{-\gamma s}\beta_s\sigma dZ_s^a + \int_0^t e^{-\gamma s}\delta_s(dH_s - \lambda ds) \tag{16}$$

where the observation of the advent of robots is modeled as a jump process $(H_t)_t$ that indicates whether the jump has already occurred ($(H_t)_t = 1 \ \forall t > T$) or not ($(H_t)_t = 0$, up to time T). Differentiating $\Upsilon_t^{Agent}(\Pi, a(\Pi))$ with respect to t yields to (13). □

$\delta_t(dH_t - \lambda dt)$ is a martingale jump term that makes the fund manager's value contingent to the advent of robots. As a consequence, increasing $(\delta)_t$ lets the agent accumulate value faster up to the advent of robots but also makes him being subject to a larger drop in his value. $(\beta_t)_t$ is the sensitivity of the fund manager's continuation value to changes in the market value of the basket of securities that constitutes the fund. As a consequence, the limited-liability condition imposes that at each instant,

$$\delta_t \geq \underline{\delta} := \frac{B}{\gamma} - W_t \tag{17}$$

Otherwise, the agent's continuation value may fall below $\frac{B}{\gamma}$. Whenever (17) is binding at the advent of robots occurs, the contract is instantaneously terminated, and the fund automated.

Now, let us move to the characterization of the incentive-compatible contract. The idea is to apply the martingale optimality principle as introduced by Sannikov (2008) in the context of a principal-agent model. It shows that the representative investor enforces implicitly an effort strategy by controlling at each instant how the fund manager's continuation value is sensitive to (i) the market and (ii) the advent of robots. The latter dynamics depends on whether the robot is available or not. To this end, let us define the controlled process $W^\alpha = (W_t^\alpha)_t$, where $\alpha = (\beta, \delta)$ is any feasible pair of sensitivity processes. It satisfies under $\mathbb{P}^a$

$$\begin{cases} dW_t^\alpha = (\gamma(W_t^\alpha - \frac{B}{\gamma}) + f(\beta_t))dt + \sigma\beta_t dZ_t + \delta_t(dH_t - \lambda dt) - dU_t \\ W_0^\alpha \geq w_0 \end{cases} \tag{18}$$

Thanks to the martingale optimality principle, the following lemma characterizes the incentive compatible contract and the associated optimal effort strategy.

Lemma 2: *For any stream of payments $(U_t)_t$ satisfying (4), for any pair of processes $((\beta_t)_t, (\delta_t)_t)$ $\mathcal{G}_t$-predictable and square-integrable and for the stopping time $\tau \leq \tau^W_{\frac{B}{\gamma}}$, the contract $\Pi =$*

$((U_t)_t, (\beta_t)_t, (\delta_t)_t, \tau \le \tau^W_{\frac{B}{\gamma}})$ *is incentive compatible and the associated optimal effort is* $a^*_t = (\bar{a}1_{\{\beta \ge \underline{\beta}\}})_t$*, where* $\underline{\beta} := \frac{B}{\bar{a}}$*, and* $\delta_t \ge \underline{\delta}$*.*

Proof. Let us consider the stochastic process R^a_t accounting for the fund manager's total value seen from date t when associated to the controlled process W^α.

$$R^a_t = \int_0^t e^{-\gamma s}(dU_s + \frac{B}{\bar{a}}(\bar{a} - a_s(\Pi))ds) + e^{-\gamma t}(W^\alpha_t - \frac{B}{\gamma}) \tag{19}$$

We determine the function f such that $(R^a_t)_t$ is a supermartingale under $\mathbb{P}^a$, so

$$dR^a_t = e^{-\gamma t}\left(dU_t + \frac{B}{\bar{a}}(\bar{a} - a_t)dt + dW^\alpha - \gamma(W^\alpha_t - \frac{B}{\gamma})dt\right) \tag{20}$$

$$= e^{-\gamma t}\left(\frac{B}{\bar{a}}(\bar{a} - a_t)dt + f(\beta_t)dt + \sigma\beta_t dZ^{\bar{a}}_t + \delta_t(dH_t - \lambda dt)\right) \tag{21}$$

$$= e^{-\gamma t}\left(\frac{B}{\bar{a}}(\bar{a} - a_t)dt + f(\beta_t)dt + \sigma\beta_t(dZ^a_t - \frac{a_t - \bar{a}}{\sigma}) + \delta_t(dH_t - \lambda dt)\right) \tag{22}$$

$$= e^{-\gamma t}\left((\frac{B}{\bar{a}} - \beta_t)(\bar{a} - a_t)dt + f(\beta_t)dt + \sigma\beta_t dZ^a_t + \delta_t(dH_t - \lambda dt)\right) \tag{23}$$

Thus, R^a_t is a supermartingale under $\mathbb{P}^a$ if and only if

$$f(\beta) := \underset{a\in\{0,\bar{a}\}}{inf}\left((\bar{a} - a_t)(\beta - \frac{B}{\bar{a}})\right) \tag{24}$$

$$= \begin{cases} \bar{a} & \text{if } \beta \ge \frac{B}{\bar{a}} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

□

As a consequence, R^a_t is a supermartingale under $\mathbb{P}^a$ and $R^{a^*}_t$ is a martingale under $\mathbb{P}^{a^*}$.

It describes the fund manager's total value under the optimal effort strategy. So,

$$W_0^{\alpha} = \mathbb{E}^{a^*}\left[R_\tau^{a^*}\right] \tag{26}$$

$$W_0^{\alpha} \geq \mathbb{E}^{a}\left[R_\tau^{a}\right] \qquad \forall a \in \{0; \bar{a}\} \tag{27}$$

Therefore, we have $W_0^{\alpha} = \sup\limits_{a\in\{0;\bar{a}\}} W_0^{\Pi}$.

This leads to the following Markov formulation of the representative investor's problem (7)-(9):

$$V^p(w_0) = \max(\max_{w\geq w_0} V(w,0); M_0) \tag{28}$$

where

$$V(w,0) = \sup_{\beta\geq\underline{\beta};\delta\geq\underline{\delta}}\left(\sup_{U;\tau\leq\tau^w_{\frac{B}{\gamma}}} \mathbb{E}^{a^*}\left[\int_0^\tau e^{-rt}(a^*\mu dt - dU_t) + e^{-r\tau}\tilde{M}\right]\right) \tag{29}$$

together with

$$a^* = (\bar{a}1_{\{\beta\geq\underline{\beta}\}})_{t\leq\tau},$$

and such that

$$dW_t^{\Pi} = \gamma(W_t^{\Pi} - \frac{B}{\gamma}) + \sigma\beta_t dZ_t^{a^*} + \delta(dH_t - \lambda dt) - dU_t \text{ with } W_0^{\Pi} \geq w_0 \tag{30}$$

III. Optimal Contracting

In the preceding section, we have expressed the representative investor's problem at date 0 as a Markov stochastic control problem. Now, we use the dynamic programming approach to derive the optimal contract. As value functions are forward-looking processes, the value of the fund before the advent of robots depends on its value once they become available. Thus, we have to solve the model using backward induction. We restrict our analysis to the situation where both the fund manager and the representative investor fully commits to a long-term contract, without allowing for renegotiation.

A. *Optimal contracting after the advent of robots*

In this subsection, we consider that automation with value M is already available, i.e. that $H_t = 1$, $\forall t$. As a consequence, the fund can be automated as soon as the fund manager's contract is terminated, so $\tau_M = \tau$. Moreover, his continuation value is here the only remaining and relevant state variable. Its dynamics under $\mathbb{P}^{a^*}$ together with $\beta_t = \underline{\beta}$ $\forall t$ is given by

$$dW_t = \gamma(W_t - \frac{B}{\gamma})dt + \sigma\underline{\beta}dZ_t^{\bar{a}} - dU_t \tag{31}$$

Now, we characterize the fund value after the advent of robots in the following proposition[6].

Proposition 1: *Suppose that automation is currently available, so M is the value of the contract termination. Assume in addition that the fund manager's continuation value evolves according to the dynamics given in (31). Then, under the optimal contract, the high-effort strategy is implemented so $\beta_t = \underline{\beta}$ $\forall t < \tau$. The fund value is concave and solves:*

$$V(w,1) = \frac{\bar{a}\mu}{r} + \frac{\gamma}{r}(w - \frac{B}{\gamma})V'(w,1) + \frac{\underline{\beta}^2\sigma^2}{2r}V''(w,1) \qquad \text{if } w \in [\frac{B}{\gamma};\bar{W}_1]; \tag{32}$$

together with $V(\frac{B}{\gamma},1) = M$ (value-matching condition); $V'(\overline{W}_1,1) = -1$ (smooth-pasting condition) ; and $V''(\overline{W}_1,1) = 0$ (super-contact condition). The value function extends linearly with slope -1 when it attaches the payment frontier $\chi(w,1)$ at the upper boundary of the employment interval $\overline{W_1}$ where payment are given. $\chi(w,1)$ satisfies:

$$\chi(w,1) = \frac{\bar{a}\mu}{r} - \frac{\gamma}{r}(w - \frac{B}{\gamma}) \tag{33}$$

[6] We leave it to the reader that is interested in the proof of this proposition to read DeMarzo and Sannikov (2006) – section III .

It is linear and decreasing in w. In this case, the optimal termination of the contract τ and the optimal implementation of the robot arise at the same instant, so $\tau_M = \tau = \tau^w_{\frac{B}{\gamma}}$.

We keep the discussion brief as the optimal contract after the advent of robots is the one derived in DeMarzo and Sannikov (2006). At first, we note that deferring compensation up to $\bar{W}_1$ is useful to bring the fund manager away from the termination boundary. Payments are made at the upper boundary of the employment interval where the representative investor does not benefit anymore from postponing the payments as the marginal firm's value is constant. Termination of the contract serves as a punishment after the fund has suffered from too many bad outcomes.

B. *Optimal contracting before the advent of robots*

In this subsection, the optimal contract is designed foreseeing the advent of robots, so both W_t and H_t are relevant state variables. As a consequence, automation cannot occur before the robot becomes available and the contract of the fund manager is terminated, so we impose $\tau_M = T \vee \tau$. Here, the dynamics of the fund manager's continuation value taken with $\beta_t = \underline{\beta}\ \forall t$ and $\delta_t \geq \underline{\delta}\ \forall t$ satisfies

$$dW_t = \gamma(W_t - \frac{B}{\gamma})dt + \sigma\underline{\beta}dZ^{\bar{a}}_t + \delta_t(dH_t - \lambda dt) - dU_t \tag{34}$$

Now, we characterize the value function before the advent of robots in the following proposition.

Proposition 2: *Suppose that the automation is not currently available. The principal has to decide between offering a contract to the fund manager and its best alternative that is evaluated M_0. Assume in addition that fund manager's continuation value evolves according to the dynamics given in (34). Then, under the optimal contract, the high-effort strategy is implemented so $\beta_t = \underline{\beta}\ \forall t < \tau \wedge T$ and $\delta_t \geq \underline{\delta}\ \forall t < \tau \wedge T$. The fund value function is concave* *and solves :*

$$\forall w \in [\frac{B}{\gamma}; \overline{W}_0], \quad (\lambda + r)V(w, 0) =$$

$$\bar{a}\mu + \left(\gamma(w - \frac{B}{\gamma}) - \lambda\delta^*(w)\right)V'(w, 0) + \frac{1}{2}\underline{\beta}^2\sigma^2 V''(w, 0) + \lambda V\left(w + \delta^*(w), 1\right) \tag{35}$$

taken together with $V(\frac{B}{\gamma}) = M_0$ *(value-matching condition);* $V'(\overline{W}_0) = -1$ *(smooth-pasting condition) ; and* $V''(\overline{W}_0) = 0$ *(super-contact condition).*

$V(w, 1)$ *is characterized in proposition (1) and the optimal sensitivity to the advent of robots* δ^* *is given by:*

$$\delta^*(w) = = \begin{cases} -(w - \frac{B}{\gamma}) & \textit{if } V'(w, 0) > V'(\frac{B}{\gamma}, 1) \\ \tilde{\delta}(w) & \textit{otherwise} \end{cases} \tag{36}$$

where $\tilde{\delta}(w)$ *is such that* $V'(w + \tilde{\delta}(w), 0) = V'(w, 1)$.

The value function extends linearly with slope -1 when it attaches the barrier frontier $\chi(w, 0)$ *at the upper boundary of the employment interval* $\overline{W}_0$ *where payment are given.* $\chi(w, 0)$ *satisfies*

$$\chi(w, 0) = \frac{1}{\lambda + r}\left[\bar{a}\mu - (\gamma(w - \frac{B}{\gamma}) - \lambda\delta^*(w)) + \lambda V(w + \delta^*(w), 1)\right] \tag{37}$$

In addition, the optimal termination of the contract τ *and the optimal implementation of the* *robot τ_M are given by:*

$$\tau = \begin{cases} \tau^w_{\frac{B}{\gamma}} \wedge T & \text{if } \delta^* = -(w - \frac{B}{\gamma}); \\ \tau^w_{\frac{B}{\gamma}} & \text{otherwise.} \end{cases} \tag{38}$$

and

$$\tau_M = \begin{cases} T & \text{if } \delta^* = -(w - \frac{B}{\gamma}); \\ \tau^w_{\frac{B}{\gamma}} \vee T & \text{otherwise.} \end{cases} \tag{39}$$

In the following section, we discuss such optimal contract.

IV. The Optimal Response to Foreseeing Robots

In this section, we discuss what are the main impact of foreseeing robots. It has effect on both the fund value and the contract of the fund manager, so it is crucial that the representative investor anticipates their future availability from the contracting stage.

At first, let us investigate the fund manager's continuation value. In line with DeMarzo and Sannikov (2006) and Hoffmann and Pfeil (2010), we have shown in the preceding section that the sensitivity of the agent to the production output β remains constant during the delegation to the fund manager. Indeed, it continuously incentivize the manager to actively manage the fund. Its optimal value only depends on the fund manager's private benefit to shirk, which is independent of the changes in the contractual environment. Then, the representative investor sets β^* at the minimum value that induces the agent to exert effort because incentives are costly to provide, so $\beta^* = \underline{\beta}$.

Furthermore, the presence in the model of the forthcoming technology of automation brings forth an additional martingale jump term in the fund manager's continuation value: $\delta_t(dH_t - \lambda dt)$. By construction, its value is zero in expectation, and we refer to δ_t as the fund manager's sensitivity to the advent of robots which is controlled by the principal. At

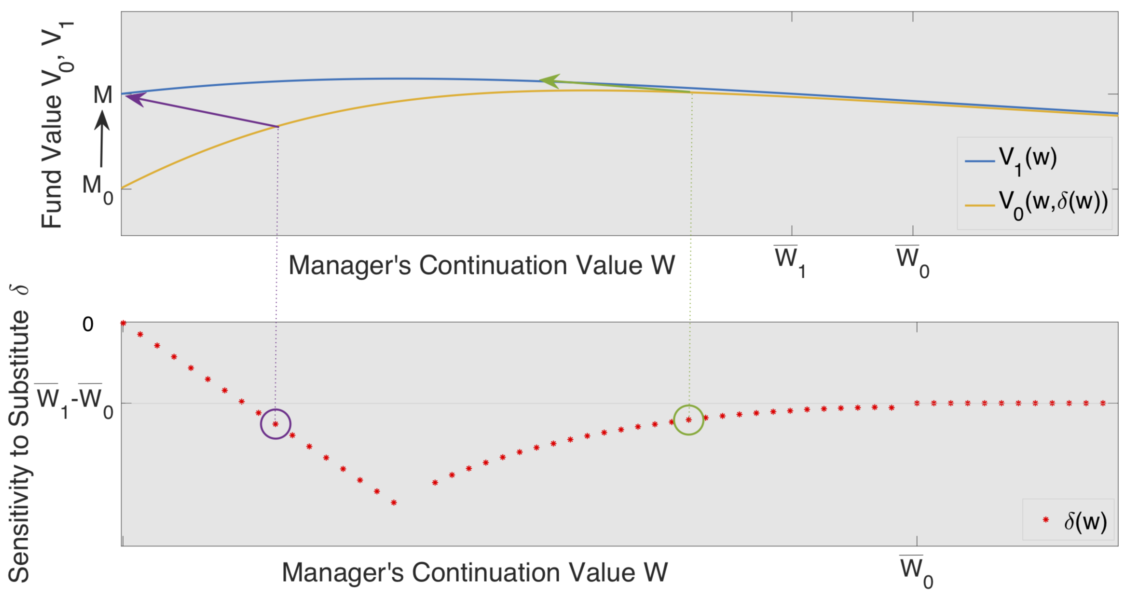

Figure 1. The fund value function before and after the advent of the robots, and the sensitivity $\delta(W)$ of the fund manager's continuation value to the advent of robots.
Parameters are $r = 10\%, \gamma = 15\%, \lambda = 5\%, \bar{a} = 10, \mu = 1, B = 10; \sigma = 5, M = 70$. $\bar{W}_0$ (resp. $\bar{W}_1$) is the payment barrier – a reflecting boundary – before (resp. after) the advent of robots. The value function attaches the payment frontier with slope -1, and then extends linearly. The optimal controlled sensitivity δ keeps if possible the marginal value of delegating to the manager. For small level of the manager's continuation value, it is not possible so instantaneous termination of the contract is triggered by the advent of robots.

the advent of robots, H_T jumps from 0 to 1, and the fund manager's continuation value instantaneously falls by $\delta_t \leq 0$ to reflect the emergence of a valuable alternative to the contractual relationship. Our interpretation is that the advent of robots mitigates the agency friction : it decrease the agency rent that the fund manager is able to extract from the unobservability of his actions by the representative investor.

We note that it contrasts with standard results in contracting theory, where the optimal contract does not rely on exogenous changes in the contractual environment, as in the seminal paper of Holmstrom and Milgrom (1987). Such jump could also be interpreted as a *punishment for luck* to emphasize on the worsening of the agent's value due to an exogenous *lucky* and valuable event from the perspective of the representative investor. A similar response, yet in opposite direction, is exhibited in Hoffmann and Pfeil (2010) or Demarzo et al. (2012). In order to compensate the fund manager for being exposed to such reassessment, the drift of his continuation value is boosted by $-\lambda\delta_t \geq 0$ at each instant before the advent of robots, and it brings the fund manager away from the termination boundary. Therefore, we have derived an optimal contract that secures the delegation to the fund manager prior the advent of robots and then eases the termination of the contract for the purpose of automation when the technology becomes available.

In addition, such optimal contract skims the agents that have performed poorly at the advent of robots. Indeed, for small continuation value the limited liability condition posed on δ is binding. Thus, it is optimal for the representative investor to instantaneously automate any fund if its manager is in such situation at the advent of robots. It is noteworthy that we derive an optimal contract that makes the delegation to a fund manager safer at early stage. Indeed, compared to a baseline model à la DeMarzo Sannikov (2006), the continuation value is boosted until the advent of robots. As a consequence, a fund manager that would have been laid-off with a contract à la DeMarzo Sannikov may have remained active under our optimal contract. For fund managers that are performing well enough and that are not at risk of being skimmed, δ decreases with the fund manager's value. Indeed, making an agent that performs very well too sensitive to such exogenous shock is costly, as it would consist in boosting payments promises close to the payment boundary.

Combined, these two factors make the delegation to a fund manager more attractive from the perspective of the representative investor at the advent of robots.

V. Implementation and Empirical Implications

A. Implementation

In order to implement the optimal contract presented in Proposition (2), we design a *point-based incentive program* where the number of points coincides with the fund manager's continuation value. It rewards the fund manager with points that have the following dynamics

$$dW_t = \left(\gamma(W_t - \frac{B}{\gamma}) - \delta^*(w_t)\lambda\right) dt + \sigma\underline{\beta} dZ_t^{\bar{a}}; \quad \forall t < \left(T \wedge \tau^w_{\frac{B}{\gamma}}\right) \tag{40}$$

$$\Delta W_T = W_T - W_{T^-} = \delta^*(W_{T^-}) \tag{41}$$

$$dW_t = \gamma(W_t - \frac{B}{\gamma})dt + \sigma\underline{\beta} dZ_t^{\bar{a}}; \quad \forall T \leq t \leq \tau^w_{\frac{B}{\gamma}} \tag{42}$$

where T is the date of the technological advent, and where δ^* follows (36). The contract is terminated as soon as W_t hits $\frac{B}{\gamma}$, which is a stopping-time noted $\tau^w_{\frac{B}{\gamma}}$. Whenever the balance W_t would go through the payment boundary, the excess is converted in a lump sum of cash paid to the fund manager. At the date of the advent of robots, $\delta^*(W_T^-)$ points expire and they are removed from the program. This feature may trigger the termination of the contract if it leads W_t to hit the lower boundary. Then and as long as the agent manages the fund, the points accumulate with the dynamics given by (42), at a slower rate than before as there is no martingale jump term included anymore.

As in *He* (2009), the implementation of the optimal contract cannot solely use a cash balance à la DeMarzo and Sannikov (2006) that would mimic the dynamics of the cash-flows to trace the fund manager's continuation value. Otherwise it would be impossible to make it jump at the technological advent T while the drift of the cash-flow process remains constant.

B. *Empirical Implications*

Several theoretical-grounded implications follow from our model. Our model exhibit the skimming of fund managers that have poorly performed at the advent of robots. This is in line with the empirical evidence that has been offered in 2017 when BlackRock decided to automate of 7 out of 50 of their stock pickers[7]. Such decision, justified as a necessary "change [of] ecosystem" by its CEO Laurence D. Fink, is puzzling. Indeed, it is not clear on which criterion it was taken, and which fund managers were dismissed. To explain our empirical implication on that case, let us compare our optimal contract to the one derived in a baseline model à la DeMarzo Sannikov (2006) and that would be optimal if the advent of technologies of automation were not foreseen. In our setting, the termination of agents that have not performed well enough before the advent of robots may be postponed, because the representative investor foresees such decision will be less inefficient in the future. Therefore, we suggest that BlackRock may have strategically postponed the termination of some of these 7 stock pickers. In any case, our model implies that the ones that have be dismissed were those with the poorest performance at the advent of robots.

Second, our model implies that it is necessary to boost fund manager's value prior the advent of robots. The observation of such large promise of payment could be interpreted as a *golden age* of fund management, and it is not abnormal as it serves the purpose of increasing the efficiency to fund management. Then, it allows the representative investor to reassess the value of delegating to a fund manager when technology of automation becomes available. This is in line with the findings of the 2018 *asset management compensation study* by Greenwich Associates that suggests a decrease in the fund manager's bonuses due to the advent of robots. They claim that the large investment cost in technologies of automation reduces the incentive compensation pool of fund managers. Our optimal contracting approach predicts an alternative explanation. We show that the advent of a

[7] https://www.nytimes.com/2017/03/28/business/dealbook/blackrock-actively-managed-funds-computer-models.html

valuable competitor to fund managers mitigates the agency friction and thus decreases the agency rent of the fund managers that are not instantaneously laid off.

More broadly, our results corroborate the *depressing effect* of automation on wages – the reduction of equilibrium wages due to the ability to automate – shown in Acemoglu and Restrepo (2018a) while investigating the irreversible substitution of workers by machines, and which refers in our setting to the jump in value at the advent of robots. It is also in line with He and Zhu (2017) that shows that agents' compensation[8] is negatively correlated with value of a firm's alternative to the manager, here characterized by the forthcoming technology of automation.

Combined, these factors make the delegation to fund managers more attractive from the perspective of the representative investor after the advent of robots, due to its capacity to mitigate the agency friction and its ability to make the termination of the fund manager's contract less inefficient.

VI. Concluding Remarks

In this paper, we show how the contract's characteristics have to adapt to the advent of a technology of automation in the context of fund management. We build a continuous-time principal-agent model à la DeMarzo and Sannikov (2006) with effort, where a real-option to automate the management of a fund emerges stochastically. As in the baseline model without robots, it is optimal to provide incentives both (i) by postponing payments after success, and (ii) by posing the threat of termination after bad outcomes. Nevertheless, foreseeing the advent of robots distorts the provision of incentives over time. Before the advent of robots, the continuation value is boosted to secure the contractual relationship. Thus, termination for incentive reasons is postponed to the advent of a valuable alternative the fund manager. Then, the principal reassess the fund manager's value at the advent of robots. We show

[8] They focus the investigation on signing bonuses

that the advent of robots (i) has a skimming effect as the less successful fund managers are instantaneously automated, and (ii) reduces the agency rent given to the fund managers that remain active. Combined, these factors make the delegation to fund managers more attractive from the perspective of the representative investor after the advent of robots.

The implementation of such contract cannot be achieved trough standard securities, but rather through a point-based incentive program, so it would allow for the expiration of a fraction of the points owned by the fund manager at the advent of robots.

In our model, we show that the advent of robots makes fund management more attractive. In further research looking at the investment and payout policy of the representative investor, it may worth investigating whether it would imply a distortion of such policies over time.

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Appendix A. Optimality of the High-Effort Strategy

In this section, we provide necessary and sufficient conditions for the optimality of the high-effort strategy. It is in line with the proposition 8 in section III in DeMarzo and Sannikov (2006) or with the Appendix A in Hoffmann and Pfeil (2010). Thus, we keep this section brief.

Assume that the representative investor lets the fund manager follow the no-effort strategy on a small period from t up to $t + dt$ without terminating the contract. It provides to the agent a private benefit of $\frac{B}{\gamma}dt$ and is associated to a fund value of 0 during this period. Then, the dynamics of the fund manager's continuation value are given on $[t, t + dt]$ before T by

$$dW_t = \gamma W_t dt - Bdt + \delta_t(dH_t - \lambda dt) \tag{A1}$$

and are given on $[t, t + dt]$ after T by

$$dW_t = \gamma W_t dt - Bdt \tag{A2}$$

Therefore, allowing for no effort is never profitable if, for all W

$$V(W_t, i) \geq e^{-rdt}V(W_t + dW_t, i) \quad \text{where } i = \{0; 1\} \tag{A3}$$

We note that $r\tilde{V}(W, i) = \bar{a}\mu + \gamma(W - \frac{B}{\gamma}V'(w, i) + \frac{\beta^2\sigma^2}{2}V''(w, i)$ and we find similarly to DeMarzo and Sannikov (2006) and to Hoffmann and Pfeil (2010), that sufficient and necessary conditions for the no-effort strategy to never be optimal are given by

$$\begin{cases} \min_{w\in[\frac{B}{\gamma},\bar{W}_0]} \left\{V(w,0)+\frac{\gamma}{r}(\frac{B}{\gamma}-w)V'(w,0)\right\} & \geq 0; \\ \min_{w\in[\frac{B}{\gamma},\bar{W}_1]} \left\{V(w,1)+\frac{\gamma}{r}(\frac{B}{\gamma}-w)V'(w,1)\right\} & \geq 0 \end{cases} \tag{A4}$$

Appendix B. Probabilistic background of the model

Here, we define formally the probability measure induced by any effort process $(a_t)_t$, coming from both the observation of the fund value process and from the observation of the availability of the technology of automation up to date t. We show its equivalence to the standard Weiner measure $\mathbb{P}^0$ on the classical Weiner Space $\Omega = C([0,+\infty),\mathbb{R})$, the set of all continuous real functions that takes their values in $[0,+\infty)$. Let (Z_t^0) be a $\mathcal{F}_t$-Brownian motion under $\mathbb{P}^0$, where $(\mathcal{F}_t)_t$ is the completion of the natural filtration generated by (Z_t^0). Under $\mathbb{P}^0$, we assume that the dynamics of the cash-flow process evolves as

$$dX_t = \sigma dZ_t^0$$

Thus, $\mathbb{P}^0$ corresponds to the probability distribution of the cash flows when no active management or effort is exerted. It can be the case when either the agent shirks or when he is laid off. In addition, one may consider $(\mathcal{H}_t)_t$, the completion of the natural filtration generated by the advent of robots, such that

$$\mathcal{H}_t = \begin{cases} 0 \text{ if } t < T; \\ 1 \text{ otherwise.} \end{cases}$$

We call $(\mathcal{G}_t)_t = (\mathcal{F}_t \vee \mathcal{H}_t)_t$ the information set at date t. For any effort strategy $a = (a_t)_{t\geq 0}$, which is assumed to be $\mathcal{G}_t$-adapted and that takes its values in $\{0, \bar{a}\}$, we define a G_t-predictable process

$$\eta_t(a) = exp(-\frac{1}{\sigma}\int_0^t (a_s\mu)dZ_s^0 - \frac{1}{2\sigma^2}\int_0^t (a_s\mu)^2 ds)$$

$(\eta_t(a))_{t\geq 0}$ is a G_t-martingale as the effort process takes its values in a bounded interval. Its expectation equals 1 when no effort is exerted. A probability measure $\mathbb{P}^a$ on Ω can then be defined as

$$\frac{d\mathbb{P}^a}{d\mathbb{P}^0} \mid G_t = \eta_t(a) \tag{B1}$$

Assuming enough integrability conditions, the process (Z_t^a) defined as

$$Z_t^a = Z_t^0 + \frac{1}{\sigma}\int_0^t a_s\mu ds \tag{B2}$$

is a Brownian motion under $\mathbb{P}^a$. Then, any effort strategy $a = (a_t)_{t\leq\tau}$ induces a probability measure $\mathbb{P}^a$ on Ω for which the dynamics of the cash flows is given by (1).

As a consequence, we have that

- The fund manager's expected value of shirking forever from date t with respect to the filtration G_t is given by:

$$\mathbb{E}^0\left[\int_t^{+\infty} e^{-\gamma(s-t)}Bds \mid G_t\right] = \frac{B}{\gamma} \tag{B3}$$

- The expected value of the fund if $\bar{a}$ is enforced forever from date t is given with respect to the filtration G_t by:

$$\mathbb{E}^{\bar{a}}\left[\int_t^{+\infty} e^{-r(s-t)}(\bar{a}\mu ds + \sigma dZ_s^{\bar{a}}) \mid G_t\right] = \frac{\bar{a}\mu}{r}, \tag{B4}$$

- The expected fund value from automating irreversibly at a sunk cost I at date t is given with respect to the filtration G_t by:

$$M := \mathbb{E}^{0}\left[\int_t^{+\infty} e^{-r(s-t)}(k\mu ds + \sigma dZ_s^{0}) \mid G_t\right] - I = \frac{k\mu}{r} - I, \tag{B5}$$

- The expected value of the forthcoming automation, seen from t before the technological advent is given with respect to the filtration G_t by

$$\mathbb{E}\left[e^{-r(T-t)}M \mid G_t\right] = \frac{\lambda}{\lambda + r}M. \tag{B6}$$

Appendix C. Omitted Proofs

Appendix A. Proof of Proposition 2

We have derived an optimal contract where the optimal sensitivity parameter $\delta^*(w)$ is not differentiable, as it is shown in figure 1. Therefore, we cannot apply directly the proof provided in Hoffmann and Pfeil (2010) or Demarzo et al. (2012) to our proposition,and we provide here an alternative proof that the value function $V(w, 0)$ is concave. Then, we verify that it corresponds indeed to the principal's value function before the advent of robots.

- **Step 1 – Concavity :**

From the boundary condition, there exists $\epsilon > 0$ such that:

$$V''(\bar{W}_0 - \epsilon, 0) > V''(\bar{W}_0, 0) \tag{C1}$$

If we assume that $V(., 0)$ is concave close to the boundary $\bar{W}_0$, then in addition

$$0 > V''(\bar{W}_0 - \epsilon, 0) \tag{C2}$$

Now, let us show that it implies that the function is concave over the whole interval $[\frac{B}{\gamma}; \bar{W}_0]$. In order to do so, let us assume that there exists $\tilde{W} := \sup_{w \in [\frac{B}{\gamma}; \bar{W}_0]} \{V''(W, 0) \geq 0\}$. We have by continuity that $V''(\tilde{W}, 0) = 0$ while $V''(\tilde{W} + h, 0) < 0$, for a small $h > 0$ taken such that $(\tilde{w}+h)V'(\tilde{w}+h, 0) = \tilde{w}V'(\tilde{w}, 0)$. From (35) we have that $V'(\tilde{w}, 0) > 0$.

We can also write the following expression for the difference quotient:

$$\begin{aligned}(r + \lambda) \left[\frac{V(\tilde{w} + h, 0) - V(\tilde{w}, 0)}{h}\right] = \\ \frac{1}{h}[\gamma(\tilde{w} + h + \frac{B}{\gamma}) - \lambda\delta^*(\tilde{w} + h))V'(\tilde{w} + h, 0) - (\gamma(\tilde{w} + \frac{B}{\gamma}) - \lambda\delta^*(\tilde{w})))V'(w, 0) \\ + \underbrace{\frac{\beta^2\sigma^2}{2}V''(\tilde{w} + h, 0)}_{<0} \\ +\lambda \underbrace{(V(w + h + \delta^*(w + h), 1) - V(w + \delta^*(w), 1))}_{\text{roughly zero}}]\end{aligned} \tag{C3}$$

So,

$$(r+\lambda)\left[\frac{V(\tilde{w}+h,0)-V(\tilde{w},0)}{h}\right] <$$
$$\frac{1}{h}\left[\gamma(\tilde{w}+h+\frac{B}{\gamma})-\lambda\delta^*(\tilde{w}+h))V'(\tilde{w}+h,0)-(\gamma(\tilde{w}+\frac{B}{\gamma})-\lambda\delta^*(\tilde{w})))V'(w,0)\right. \tag{C4}$$

Now, we use that $(\tilde{w}+h)V'(\tilde{w}+h,0)=\tilde{w}V'(\tilde{w},0)$, then

$$(r+\lambda)\left[\frac{V(\tilde{w}+h,0)-V(\tilde{w},0)}{h}\right] <$$
$$\frac{1}{h}[-(B+\lambda\delta^*(\tilde{w}+h))V'(\tilde{w}+h,0)+(B+\lambda\delta^*(\tilde{w}))V'(w,0)] \tag{C5}$$

Which translates, if $\lambda\delta^*(w)$ is sufficiently small, to

$$(r+\lambda)\left[\frac{V(\tilde{w}+h,0)-V(\tilde{w},0)}{h}\right] < -B\left[\frac{V'(\tilde{w}+h,0)-V'(\tilde{w},0)}{h}\right] \tag{C6}$$

$$\text{Therefore,}\quad \left[\frac{V(\tilde{w}+h,0)-V(\tilde{w},0)}{h}\right] < 0 \tag{C7}$$

As it contradicts with the assumption that $V'(w)>0$, therefore we have that $V''(W,0)<0\ \forall w \in [\frac{B}{\gamma};\bar{W}_0]$. We conclude that $V(.,0)$ is concave over the whole employment interval as long as it is concave close to $\bar{W}_0$. The complete proof of the concavity of the value function in this case is not provided yet.

- **Step 2 – Verification :** As usual in dynamic contracting theory, our last step is to verify that we have indeed derived an optimal contract. For any incentive-compatible contract, let us define
$\mathrm{F}_t = \int_0^t e^{-rt}(a^*\mu dt - dU_t) + e^{-rt}V(W_t,1)1_{\{t\geq T\}} + e^{-rt}V(W_t,0)1_{\{T>t>\tau\}}$

$+ e^{-rt}\left((\frac{\lambda}{\lambda+r}M) \vee \nu\right) 1_{\{t\leq\tau\}}$

By Itô's lemma, its drift is

$$e^{-rt}\left(\frac{a^*\mu}{r} - M\right) \tag{C8}$$

which is always negative as here $M \geq \frac{a^*\mu}{r}$. Therefore, F_t is a supermartingale and

$$V(W_0, 0) = F_0 \geq \mathbb{E}^{a^*}[F_t \mid \mathcal{G}_t] \tag{C9}$$

with an equality for the contract derived in the proposition (**??**). Then, the optimal choice of sensitivity δ^* satisfies:

$$V'(w, 0) = V'(w + \delta^*, 1) \tag{C10}$$

as long as it remains larger than $-(w - \frac{B}{\gamma})$ to fulfill the limited liability condition.