Common Ownership, Institutional Investors, and Welfare*

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Abstract

This study evaluates the effects of institutional investors' common ownership of firms competing in the same market. Overall, common ownership has two opposing effects: (a) it serves as a device for weakening market competition, and (b) it induces diversification, thereby reducing portfolio risk. We conduct a detailed welfare analysis within which the competition-softening effects of an increased degree of common ownership is weighted against the associated diversification benefits.

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1. Introduction

Recent studies have attracted attention to the increasingly important phenomenon of common ownership of publicly traded firms (for example, Azar, Schmalz, and Tecu (2018), Seldeslachts, Newham, and Banal-Estanol (2017), Gilje, Gormley, and Levit (2017), Schmalz (2018) and Backus, Conlon, and Sinkinson (2019b)). In particular, institutional investors often hold ownership stakes in competing firms belonging to the same industry. He and Huang (2017) present evidence that the proportion of US public firms with common institutional ownership has increased from below 10 percent in 1980 to about 60 percent in 2014. These public firms include institutional owners that simultaneously hold at least 5 percent of the common equity of rival firms in the same industry. Azar (2017) reports a similar trend by reference to a finding that the share of S&P 500 firms with overlapping owners holding at least 3 percent ownership stakes in firms belonging to the same industry has increased from 25 percent to 90 percent during the period from 2000 to 2010.

From a theoretical perspective, common ownership can be expected to soften competition because managers, who maximize the returns to their shareholders, internalize the effects their product market decisions have on rivals. This is the central mechanism developed in the model of overlapping intra-industry ownership by O'Brien and Salop (2000). Several empirical studies have recently estimated the effects on competition of common ownership in different industries. Azar, Schmalz, and Tecu (2018) present evidence to support the conclusion that common ownership raised airfares by relating these prices to measures of concentration which are adjusted to take common ownership into account. Further, Azar, Raina, and Schmalz (2016) show similar effects of common ownership on spreads and fees for banking products.¹ Newham, Seldeslachts, and Banal-Estañol (2018) explore the effects of common ownership on entry. They employ data from the pharmaceutical industry to empirically establish that common ownership with the incumbent brand firm reduces the probability of generic entry.

In this analysis we focus on a configuration where consumers can allocate their savings into one of two competing institutional investors. The institutional investors channel their funds to ac-

¹Gramlich and Grundl (2017) apply different methods for estimating the effects of common ownership on competition in banking, finding mixed and very small effects.

quire ownership in product market firms operating in a duopolistic industry. We can think of these institutional investors as pension funds. We initially show that an increased degree of common ownership relaxes the intensity of product market competition. However, an increased degree of common ownership also reduces the risks in the *intra-industry* portfolios of the institutional owners. Therefore, an increased degree of common ownership defines an interesting tradeoff between relaxed competition in the product market and improved risk diversification in the asset market for risk-averse savers. A detailed welfare analysis associated with this tradeoff is the main contribution of our study.

Our welfare analysis reveals that the socially optimal degree of common ownership is importantly influenced by two factors: (i) the degree of risk aversion and (ii) the relative weight society assigns to consumption of the final product versus that assigned to returns on savings via institutional investors. A low relative weight assigned to consumption of the final product can be interpreted as a society that encourages savings for retirement and discourages excessive consumption. We characterize in detail how the socially optimal degree of common ownership with risk neutrality depends on the relative weight placed on consumption of the final good. In particular, we find that under risk neutrality complete ownership specialization, i.e. no common ownership at all, is socially optimal as long as the relative weight on consumption of the final good is sufficiently high. Further, we show analytically that with risk aversion, and for the class of utility functions with constant relative risk aversion (CRRA), an increase in the degree of risk aversion increases the socially optimal degree of common ownership. The intuition is that the institutional investors offer more diversified investment portfolios to their savers if there is a higher degree of common ownership, and the value savers attach to diversification is increasing as a function of the degree of risk aversion.

Our analysis is linked to a category of theoretical models which have characterized the effects of common ownership or overlapping ownership on market performance by applying industrial economics approaches. This category of models includes O'Brien and Salop (2000), López and Vives (2019), and Shy and Stenbacka (2019), and it is broadly surveyed in Vives (2019) as well as Section 2 of Schmalz (2018). Our present analysis extends this approach to a welfare analysis within which the competition-softening effects of an increased degree of common ownership can be weighted against the associated diversification benefits.

The recent advances in the analysis of the effects of common ownership have initiated an intense debate among economists and legal scholars regarding policy implications. Elhauge (2016) and Posner, Scott Morton, and Weyl (2017) have proposed the introduction of rules to restrict the ability of institutional owners to hold ownership stakes in several firms operating in the same industry. Other researchers, such as Lambert and Sykuta (2018) and Ginsburg and Klovers (2018), have forcefully raised arguments against such restrictions. The debate has also entered the policy arena. For example, in its resolution, dated 19 April 2018, in response to the European Commission's annual report on competition policy, the European Parliament calls on the Commission to "take all necessary measures to deal with the possible anti-competitive effects of common ownership" and to "investigate...the effects of common ownership on European markets, particularly on prices and innovation."² Our welfare analysis could be viewed as a central component in arguments required to derive effects-based policy implications because it highlights the tradeoff between competition-relaxing effects and diversification benefits associated with an increased degree of common ownership.

It should be pointed out that intra-industry common ownership is by no means the only way in which institutional investors in general, and pension funds in particular, can diversity their portfolios. Of course, diversification can also be accomplished by mixing equity from different industries as well as fixed income obligations without owning multiple competing firms within the same industry. However, as frequently observed, investors can further diversify their portfolios by acquiring stocks of competing firms within the same industry, and it is the focus of the present study to analyze this particular aspect. To achieve this goal, we characterize the welfare tradeoff induced by an increased degree of common ownership as we balance the competition-relaxing effects against the associated diversification benefits. In fact, our study matches the priorities suggested in Backus, Conlon, and Sinkinson (2019a) who argue that the "most fruitful direction for future research on competitive effects of common ownership would focus on attempts to mea-

²See, http://www.europarl.europa.eu/sides/getDoc.do?pubRef=-//EP//NONSGML+TA+P8-TA-2018-0187+0+DOC+PDF+V0//EN.

sure the impact of within a single industry, with a focus on pairwise profit weights rather than market-level concentration measures as the variable of interest" (p. 25).

This study is organized as follows. Section 2 designs a static duopoly model in order to measure how the share value of institutional investors varies with the degree of their common ownership in firms competing in the same product market. Section 3 solves for the equilibrium profits of the firms and investors as functions of the degree of common ownership. Section 4 analyzes how common ownership affects investors' portfolio risk. Section 5 conducts the welfare analysis of common ownership. Section 6 explores several extensions of the model. Section 7 presents concluding comments. Appendices provide algebraic derivations.

2. Duopoly competition, institutional investors, and common ownership

Following Shy and Stenbacka (2019) we introduce institutional investors into a modified duopoly model with two firms competing based on production decisions in the product market. The two producing firms are owned by two institutional investors with ownership in both. This section investigates how the investment value of institutional investors is influenced by the degree of their common ownership of the producing firms.

The producing firm 1 and firm 2 are engaged in Cournot quantity competition in an industry facing an aggregate inverse demand function

$$p = \alpha - \beta(q_1 + q_2), \quad \text{where } \alpha > 0, \ \beta > 0.$$
 (1)

The variable p denotes the price of a homogeneous product (or service) sold in this market and q_1 and q_2 are the quantities produced and sold by firms 1 and 2, respectively. Let π_1 and π_2 denote the profits earned by firms 1 and 2, respectively. Then, assuming zero marginal costs, the producing firms' profits as functions of quantities produced are given by

$$\pi_1(q_1, q_2) = pq_1 = [\alpha - \beta(q_1 + q_2)] q_1 \quad \text{and} \quad \pi_2(q_1, q_2) = pq_2 = [\alpha - \beta(q_1 + q_2)] q_2.$$
(2)

Finally, (net) consumer surplus derived from the product market will be evaluated according

to

$$CS(Q) = \int_{0}^{Q} (\alpha - \beta x) dx - pQ = \frac{\beta Q^2}{2},$$
(3)

where $Q = q_1 + q_2$ is aggregate industry output and *p* is substituted from (1).

2.1 Common ownership

Firms 1 and 2 (the producers) are co-owned by two institutional investors labeled *A* and *B*, as illustrated in Figure 1 and formalized in Assumption 1 below.³ Let μ denote investor *A*'s share of ownership in firm 1, and also investor *B*'s share of ownership in firm 2.



Figure 1: The shares of common ownership in firms 1 and 2 by institutional investors A and B.

ASSUMPTION 1. Institutional investor A owns a (weak) majority share μ in firm 1, whereas institutional investor B owns a (weak) majority share μ in firm 2. Formally, $\mu \in [\frac{1}{2}, 1]$.

In an industry with imperfect product market competition, common ownership induces owners to internalize the strategic externalities between the firms. Hansen and Lott (1996) and O'Brien and Salop (2000) have developed formal models to capture such effects and Backus, Conlon, and Sinkinson (2019b) provide an extensive discussion of how firms can apply profit weights in this respect. However, these studies have not explored the welfare implications of common ownership in order to balance diversification benefits against competition-softening effects.

For reasons of transparency we assume that institutional investors A and B are the sole owners

³According to Zingales (2012) (p.233), the share of individuals' ownership of publicly traded equity has decreased dramatically since the 1920s. Azar, Schmalz, and Tecu (2018) cite evidence that the ownership share of institutional investors (such as mutual funds and pension funds) of US publicly traded firms is presently in the 70–80 percent range.

of firms 1 and 2.⁴ Therefore, Assumption 1 implies that institutional investor *A* owns a minority share $(1 - \mu < 50\%)$ in firm 2, whereas institutional investor *B* owns a minority share $(1 - \mu < 50\%)$ in firm 1. In view of Figure 1 and Assumption 1, the profits earned by the institutional investors, as functions of quantity produced by firms 1 and 2, are

$$\pi_A(q_1, q_2) = \mu \pi_1(q_1, q_2) + (1 - \mu) \pi_2(q_1, q_2),$$
(4a)

$$\pi_B(q_1, q_2) = (1 - \mu)\pi_1(q_1, q_2) + \mu\pi_2(q_1, q_2),$$
(4b)

where π_1 and π_2 are defined in (2).

2.2 Introducing risks

We introduce risks by modeling firms that can fail. A failure of a firm is an extreme manifestation of production cost uncertainty, but to focus exclusively on the central underlying economic mechanism, we only model an extreme case where a significant cost increase forces a firm to exit the industry.

Formally, we introduce three probabilities: Let ϕ^{II} be the probability that both firms fail (the letter "phi" stands for "failure"). Also, let ϕ^{I} be the probability that one firm fails while the other does not (two possible events). Finally, ϕ^{0} denotes the probability that neither firm fails. Therefore,

$$\phi^{II} + 2\phi^{I} + \phi^{0} = 1. \tag{5}$$

The probability structure assumed in equation (5) is general in the sense that it can capture both dependent and independent failure events. For example, the case of independence is captured by a binomial distribution with a failure probability $f \in (0, 1)$. In the binomial case, (5) is simplified to $\phi^{II} = f^2$, $\phi^I = f(1 - f)$ (two events), and $\phi^0 = (1 - f)^2$. However, equation (5) assumes a more general probability structure to capture possible correlations between failures of the two firms. Such correlations may stem from possible aggregate industry declines or temporary industry downturns, and could become an important factor in determining investment portfolios.

⁴Section 6.3 shows how this model could be extended to cases where investors A and B are not the sole owners and therefore own smaller shares in each firm.

2.3 Firms' decision process and sequence of events

The literature does not provide a consistent method or a consensus regarding the modeling of how ownership structure actually translates into control of firms' decisions. OECD (2017) presents an extensive discussion of this issue. Because institutional investor A is the majority shareholder in firm 1, investor A can determine firm 1's output level either through the exercise of direct influence or through the control of underlying managerial incentives. This means that investor A controls the production of firm 1, while taking into consideration the profit derived from its minority ownership share in firm 2. Similarly, institutional investor B determines the output level produced by firm 2 taking into account its minority ownership share in firm 1.⁵

The sequence of events is as follows:

- **First Stage:** The fate of each producing firm is realized according to the probabilities defined in the discussion preceding equation (5).
- **Second Stage:** Investor *A* (maintaining a majority share in firm 1) determines the output of firm 1 (if firm 1 does not fail), and investor *B* (maintaining a majority share in firm 2) determines the output of firm 2 (if firm 2 does not fail).

3. Equilibrium in the product market

There are four possible events that could be realized in the First Stage. The simplest case, with probability ϕ^{II} , both firms fail and exit the market. In this case, profits and consumer surplus are

$$\pi_A^{II} = \pi_B^{II} = \pi_1^{II} = \pi_2^{II} = q_1^{II} = q_2^{II} = 0 \text{ and } CS^{II} = 0.$$
 (6)

Next, with probability ϕ^I firm 1 survives and firm 2 exits. Also, with probability ϕ^I firm 1 exits

⁵An alternative modeling method would be to assume that a firm's production decision is made in order to maximize the total portfolio value of its investors, weighted by the proportion of ownership held by these investors. Such an approach has been applied by Hansen and Lott (1996) as well as O'Brien and Salop (2000). The associated distinction between profit maximization and shareholder value maximization and its role for the analysis of strategic competition is discussed in Antón et al. (2018).

and firm 2 remains. This is the monopoly case which is derived in Appendix A. In this case,

Either:
$$q_1^I = \frac{\alpha}{2\beta}, q_{2f}^I = 0, p^I = \frac{\alpha}{2}, \pi_1^I = \frac{\gamma}{4}, \pi_{2f}^I = 0, \pi_A^I = \frac{\mu\gamma}{4}, \pi_B^I = \frac{(1-\mu)\gamma}{4}, \text{ and } CS^I = \frac{\gamma}{8}$$
 (7)
Or: $q_{1f}^I = 0, q_2^I = \frac{\alpha}{2\beta}, p = \frac{\alpha}{2}, \pi_{1f}^I = 0, \pi_2^I = \frac{\gamma}{4}, \pi_A^I = \frac{(1-\mu)\gamma}{4}, \pi_B^I = \frac{\mu\gamma}{4}, \text{ and } CS^I = \frac{\gamma}{8},$

where $\gamma = \alpha^2/\beta$. Note that (7) displays the payoffs of two separate events, each occurs with probability ϕ^I . The subscripts "1*f*" indicates the event when firm 1 fails and "2*f*" the event when firm 2 fails.

Finally, with probability ϕ^0 , neither firm fails, so that the product market is characterized by two competing firms. For given investors' ownership shares μ and $1 - \mu$, investor A determines the output q_1 of firm 1 to maximize (4a) and investor B determines the output q_2 of firm 2 to maximize (4b).

As shown in Appendix A, the Cournot-Nash equilibrium production levels, the corresponding price, profits, and consumer surplus are

$$q_1^0 = q_2^0 = \frac{\alpha\mu}{\beta(2\mu+1)}, p^0 = \frac{\alpha}{2\mu+1}, \pi_A^0 = \pi_B^0 = \pi_1^0 = \pi_2^0 = \frac{\gamma\mu}{(2\mu+1)^2}, \text{ and } CS^0 = \frac{2\gamma\mu^2}{(2\mu+1)^2}, \quad (8)$$

where $\gamma = \alpha^2 / \beta$.

Recall that the parameter μ measures the degree of common ownership. Specifically, $\mu = 0.5$ implies that investors *A* and *B* have equal ownership shares in both firms 1 and 2. In contrast, $\mu = 1$ implies that each producing firm is owned by a single investor (firm 1 is owned by investor *A* and firm 2 is owned by investor *B*). Appendix A derives the following conclusions.

Result 1. Suppose neither firm fails (probability ϕ^0), so the product market operates as a duopoly controlled by investors A and B.

(a) Moving towards more equal co-ownership (μ decreases towards $\frac{1}{2}$) increases price, reduces aggregate industry output, and increases all profits. Formally,

$$\frac{\partial p^0}{\partial \mu} < 0, \quad \frac{\partial Q^0}{\partial \mu} > 0, \quad \frac{\partial \pi_1^0}{\partial \mu} = \frac{\partial \pi_2^0}{\partial \mu} < 0, \quad \frac{\partial \pi_A^0}{\partial \mu} = \frac{\partial \pi_B^0}{\partial \mu} < 0.$$
(9)

(b) The maximum degree of common ownership ($\mu = \frac{1}{2}$) implements the monopoly solution where aggregate investors' profit equals the monopoly profit level $\pi_A^0 + \pi_B^0 = \frac{\gamma}{4}$. (c) The highest degree of market competition is achieved with specialization such that each investor fully owns only one firm ($\mu = 1$). In this case, the market performance is that of the standard Cournot duopoly competition.

Result 1 is illustrated in Figure 2. Sliding in the upward and leftward direction on the curve



Figure 2: Equilibrium price p^0 and aggregate industry output Q^0 under varying degrees of common ownership μ , for the case that neither firm fails (probability ϕ^0).

corresponds to a reduction in μ from $\mu = 1$ towards $\mu = \frac{1}{2}$ which lowers aggregate industry output and increases price and profits. In the limit, equal ownership $\mu = \frac{1}{2}$ generates the highest aggregate industry profit corresponding to a single-firm monopoly operation given in (7).⁶

The effect of common ownership on consumer surplus CS is part of a comprehensive welfare analysis to be conducted in the next section.

4. The effect of common ownership on portfolio risks

Recall that the parameter μ measures the share of investor *A* in firm 1 and the share of investor *B* in firm 2. We can view each investor as a manager of a portfolio containing two assets. This section investigates the effects of varying μ on various statistics that characterize the portfolio of

⁶Similar results were obtained in Rubinstein and Yaari (1983) in a game-theoretic formulation with a perfectlycompetitive stock market, and in Rotemberg (1984) in a model where managers maximize a weighted sum of investors' utilities under circumstances where these investors hold shares in multiple firms within the same industry.

investor *A*. By symmetry, the conclusions of this analysis equally apply to the portfolio managed by investor *B*.

The portfolio of investor *A* consists of two assets which we denote by *A*1 and *A*2 (ownership shares in firms 1 and 2, respectively). In view of the equilibrium profit returns from these ownerships corresponding to the three possible uncertain events given in (6), (7), and (8), we write

$$\pi_{A1}^{II} = \pi_{A2}^{II} = 0, \quad \text{either:} \quad \left[\pi_{A1}^{I} = \frac{\mu\gamma}{4} \text{ and } \pi_{A2f}^{I} = 0\right] \text{ or:} \quad \left[\pi_{A1f}^{I} = 0 \text{ and } \pi_{A2}^{I} = \frac{(1-\mu)\gamma}{4}\right],$$
$$\pi_{A1}^{0} = \frac{\mu\gamma\mu}{(2\mu+1)^{2}}, \text{ and } \pi_{A2}^{0} = \frac{(1-\mu)\gamma\mu}{(2\mu+1)^{2}}. \quad (10)$$

Taking into consideration that investor A owns a share μ in firm 1 and $1 - \mu$ in firm 2, the first expression in (10) is the profit return on assets 1 and 2 in the event that both firms (assets) fail, hence zero. π_{A1}^{I} is the return on asset 1 when only asset 2 fails (probability ϕ^{I}). Similarly, π_{A2}^{I} is the return on asset 2 when only asset 1 fails (also probability ϕ^{I}). Subscripts "A1f" and "A2f" denote the component in investor A's portfolio that yields no return in the case that firm 1 or firm 2 fails, respectively. π_{A1}^{0} and π_{A2}^{0} are the profits made from assets 1 and 2, respectively, when neither asset fails.

The expected returns of the two assets in investor A's portfolio (10) are

$$\mathbf{E}[\pi_{A1}] = \phi^{II} \pi^{II}_{A1} + \phi^{I} \pi^{I}_{A1} + \phi^{I} \pi^{I}_{A1f} + \phi^{0} \pi^{0}_{A1} \quad \text{and} \quad \mathbf{E}[\pi_{A2}] = \phi^{II} \pi^{II}_{A2} + \phi^{I} \pi^{I}_{A2} + \phi^{I} \pi^{I}_{A2f} + \phi^{0} \pi^{0}_{A2}.$$
(11)

Further, the variances of each asset in investor A's portfolio are

$$\operatorname{Var}[\pi_{A1}] = \phi^{II}(\pi_{A1}^{II} - \operatorname{E}[\pi_{A1}])^{2} + \phi^{I}(\pi_{A1}^{I} - \operatorname{E}[\pi_{A1}])^{2} + \phi^{I}(\pi_{A1f}^{I} - \operatorname{E}[\pi_{A1}])^{2} + \phi^{0}(\pi_{A1}^{0} - \operatorname{E}[\pi_{A1}])^{2},$$
(12)

$$\operatorname{Var}[\pi_{A2}] = \phi^{II}(\pi_{A2}^{II} - \operatorname{E}[\pi_{A2}])^{2} + \phi^{I}(\pi_{A2}^{I} - \operatorname{E}[\pi_{A2}])^{2} + \phi^{I}(\pi_{A2f}^{I} - \operatorname{E}[\pi_{A2}])^{2} + \phi^{0}(\pi_{A2}^{0} - \operatorname{E}[\pi_{A2}])^{2}.$$

The variances given in (12) are plotted in Figure 3. Note that $Var[\pi_{A1}]$ increases with μ , whereas $Var[\pi_{A2}]$ declines with μ because higher values of μ correspond to a larger ownership share in firm 1 and a smaller share in firm 2.

Using the formula Cov[X, Y] = E[XY] - E[X]E[Y], the covariance between the two assets in



Figure 3: Variances of each asset in investor *A*'s portfolio given in (12) and the portfolio's variance (15) as functions of the majority shares μ . *Note*: The figure is drawn according to $\gamma = 64$ and probabilities $\phi^{II} = 0.05$, $\phi^{I} = 0.1$, and $\phi^{0} = 0.75$.

the portfolio of investor A is

$$\operatorname{Cov}\left[\pi_{A1},\pi_{A2}\right] = \phi^{II}\pi^{II}_{A1}\pi^{II}_{A2} + \phi^{I}\pi^{I}_{A1}\pi^{I}_{A2f} + \phi^{I}\pi^{I}_{A1f}\pi^{I}_{A2} + \phi^{0}\pi^{0}_{A1}\pi^{0}_{A2} - \operatorname{E}[\pi_{A1}]\operatorname{E}[\pi_{A2}], \quad (13)$$

where the returns are given in (10) and $E[\pi_{A1}]$ and $E[\pi_{A2}]$ are computed in (11).

In order to compute the variance of the entire portfolio managed by investor *A*, we define the (value-based) *portfolio's weights* of the two assets as

$$s_{A1} = \frac{\mathrm{E}[\pi_{A1}]}{\mathrm{E}[\pi_{A1}] + \mathrm{E}[\pi_{A2}]} \quad \text{and} \quad s_{A2} = 1 - s_{A1} = \frac{\mathrm{E}[\pi_{A2}]}{\mathrm{E}[\pi_{A1}] + \mathrm{E}[\pi_{A2}]}.$$
 (14)

Intuitively, (14) defines the weight on asset A1 as the ratio of its expected return divided by the expected return of the entire portfolio managed by investor A. The weight on asset A2 is similarly defined.

We are now ready to specify the variance of the entire portfolio managed by investor A as

$$\operatorname{Var}[\pi_{A}] = s_{A1}^{2} \operatorname{Var}[\pi_{A1}] + (1 - s_{A1})^{2} \operatorname{Var}[\pi_{A2}] + 2s_{A1}(1 - s_{A1}) \operatorname{Cov}[\pi_{A1}, \pi_{A2}].$$
(15)

The solid curve in Figure 3 exhibits the portfolio variance as a function the majority ownership parameter μ . According to Figure 3, as μ increases towards 1 so that investor *A*'s portfolio consists mainly (only) of asset 1, the variance of *A*'s entire portfolio increases, and the return on asset 2

does not play any role. In contrast, as μ declines towards $\frac{1}{2}$, investor *A*'s portfolio become more diversified with equal expected returns on each asset and hence equal portfolio weights (14).

Formally, substituting (10) into (11) and then into (14) yields $s_{A1} = \mu$ and $s_{A2} = 1 - \mu$. Define,

$$m = \begin{bmatrix} \mu \\ 1-\mu \end{bmatrix} \text{ and } \Sigma = \begin{pmatrix} \operatorname{Var}[\pi_{A1}] & \operatorname{Cov}[\pi_{A1}, \pi_{A2}] \\ \operatorname{Cov}[\pi_{A1}, \pi_{A2}] & \operatorname{Var}[\pi_{A2}] \end{pmatrix},$$
(16)

and note that the variance-covariance matrix Σ is positive definite. Therefore, (15) can be written as

$$\operatorname{Var}[\pi_A] = m^T \Sigma \ m. \tag{17}$$

Based on our simulations we can draw the following conclusion. Increasing an investor's majority share μ in one producing firm while reducing the minority share in the other firm increases the investor's portfolio variance. Portfolio variance is minimized when each investor maintains an equal share in each of the product market rivals.⁷

Overall, Result 1 means that an increased degree of common ownership (lower μ) relaxes competition in the product market, whereas more ownership specialization intensifies competition. Our conclusion regarding the portfolio variance is that an increased degree of common ownership reduces portfolio risks. Thus, based on the combination of Result 1 and this conclusion, an increased degree of common ownership defines an interesting tradeoff between the competitionrelaxing effects in the product market and risk diversification in the asset market for risk-averse savers. In the next Section we will conduct a welfare analysis to assess this tradeoff.

5. Welfare evaluations of common ownership

The economy analyzed in this paper consists of two separate groups of agents: A group of buyers whose aggregate welfare is summarized by a function of the aggregate consumer surplus *CS*, and a group of individuals who save (say, for retirement) in institutional investors *A* or *B*. The welfare of the latter group is summarized by functions of the earnings of the investment funds, π_A and π_B .

⁷All the elements in the variance-covariance matrix Σ , defined in (16), depend in fairly complicated ways on μ . For that reason it is not tractable to present a closed-form analytical proof of this conclusion.

To connect the two groups of consumers we need to define a social welfare function with weights assigned to each group. Let U be an increasing weakly concave and differentiable utility function, and let ω ($0 < \omega < 1$) be the weight in social welfare assigned to consumers in the product market. Then, the expected total welfare function EW is defined by

$$EW = \omega \underbrace{EU(CS)}_{\text{consumers' utility } EW_c} + (1 - \omega) \underbrace{[EU(\pi_A) + EU(\pi_B)]}_{\text{savers' utility } EW_s},$$
(18)

where E is the expectation operator and *CS*, π_A as well as π_B are random payoffs distributed according to the three event probabilities (5) and the corresponding three event realizations derived in (6), (7), and (8).

The parameter ω determines the weight society assigns to consumer surplus relative to earnings from savings via institutional investors. Lobbying activities by financial institutions directed at political decision makers (for lower ω) can typically be expected to affect the parameter ω . We can alternatively make the interpretation that the parameter ω captures the tension created by the dual role faced by a representative individual as a consumer as well as an investor in a pension fund.

We next investigate the expected welfare $EW(\mu)$ as a function of the degree of common ownership μ . In light of (5)–(8), the expected consumer surplus is $EU(CS) = 2\phi^I U(CS^I) + \phi^0 U(CS^0)$. Equation (7) implies that $\partial CS^I / \partial \mu = 0$. Hence, the effect of an increase in common ownership μ on expected consumer utility is given by

$$\frac{\partial EU(CS)}{\partial \mu} = \phi^0 U'(CS^0) \frac{4\gamma\mu}{(2\mu+1)^3} > 0.$$
(19)

This means that increased ownership concentration (higher μ) generates expected benefits to consumers because it reduces market power of the producing firms.

From (5)–(8) we can calculate that the expected utility associated with the earnings of institutional investors to be $EU(\pi_A) + EU(\pi_B) = 2\phi^I [U(\pi_A^I) + U(\pi_B^I)] + \phi^0 [U(\pi_A^0) + U(\pi_B^0)]$. Differentiation with respect to μ shows that

$$\frac{\partial \left[\mathrm{E}U(\pi_A) + \mathrm{E}U(\pi_B)\right]}{\partial \mu} = 2\phi^I \frac{\gamma}{4} \left[U'(\frac{\gamma\mu}{4}) - U'(\frac{\gamma(1-\mu)}{4}) \right] + 2\phi^0 U'(\frac{\gamma\mu}{(2\mu+1)^2}) \frac{\gamma(1-2\mu)}{(2\mu+1)^3} < 0.$$
(20)

The first term in (20) is zero with risk neutrality, whereas it is strictly negative with risk aversion. The second term is strictly negative for $\mu > 1/2$. Overall, (20) means that the expected return from ownership of institutional investors decreases with more specialized ownership for the institutional investors (higher μ).

The welfare changes computed in (19) and (20) establish a tradeoff between expected consumer utility and expected utility associated with ownership of institutional investors in response to changes in the degree of common ownership μ . This yields the following general conclusion.

Result 2. An increased degree of common ownership by institutional investors of product market firms (lower μ) decreases expected consumer utility in the product market and increases expected utility associated with earnings generated by institutional investors.

Overall, the tradeoff defined in Result 2 is influenced by two factors: the relative weights placed on the two groups of consumers (ω) and the degree of risk aversion. For reasons of tractability we first analyze this tradeoff by focusing on risk neutrality. Subsequently, we will explore the role played by risk aversion by focusing on the class of utility functions with a constant relative risk aversion (CRRA).

5.1 Welfare evaluation with risk neutrality

In this subsection we focus on risk neutrality, which means a constant marginal utility associated with consumption and investor returns. In light of (18), (19) and (20) the effect of μ on expected welfare is formally captured by

$$\frac{\partial EW}{\partial \mu} = \text{constant} \times \frac{2\gamma\phi^0}{(2\mu+1)^3} \left[2\mu(2\omega-1) - \omega + 1\right],\tag{21}$$

where we have exploited that the marginal utility is constant under risk neutrality. Consequently, we can formulate the following result.⁸

Result 3. Suppose that consumers as well as savers are risk neutral. The institutional investors' degree of common ownership that maximizes total welfare (18) is given by

$$\mu^{*}(\omega) = \begin{cases} \frac{1-\omega}{2(1-2\omega)} & \text{if } 0 \le \omega < \frac{1}{3} \\ 1 & \text{if } \frac{1}{3} \le \omega \le 1. \end{cases}$$
(22)

Result 3 is displayed in Figure 4.



Figure 4: The welfare-maximizing degree of common ownership under risk neutrality.

Figure 4 shows that the welfare-maximizing degree of common ownership by institutional investors, μ^* , declines towards $\frac{1}{2}$ as the weight assigned to consumers (ω in (18)) declines towards 0. This case formally captures the intuition that maximum common ownership ($\mu^* = \frac{1}{2}$) is optimal with a welfare function that disregards buyers' welfare. The socially optimal μ^* is a strictly increasing function of the relative weight placed on consumers (ω) until it reaches the level $\omega = \frac{1}{3}$. Finally, for ω exceeding $\frac{1}{3}$, it is socially optimal that the institutional owners have no common ownership by institutional investors in the sense that the each of the two institutional owners concentrates its investment in one of the producing firms with no overlap between ownerships. In Figure 4 this

⁸Azar and Vives (2018) show the result that the product market markup increases with common ownership does not necessarily generalize to the case in which there are multiple industries in the economy, and in which there is common ownership not just within industry, but across industries.

feature is captured by the fact that $\mu^* = 1$ for $\omega > 1/3$. In particular, this implies that any degree of common ownership is socially inefficient with the equally-weighted sum of consumer surplus and investment returns as the welfare criterion ($\omega = 1/2$). The reason for this is that common ownership relaxes competition. Such a relaxation of competition cannot be consistent with social optimum unless the welfare weights exhibit a sufficiently strong priority for investment returns ($\omega < \frac{1}{3}$).

5.2 Welfare evaluation with risk aversion

We now shift the attention to a configuration with risk averse consumers and savers. All our computations in this subsection will focus on the class of utility functions with constant relative risk aversion (CRRA). More precisely, we focus on utility functions $U(y) = y^{\theta}$ for $0 < \theta \leq 1$. For this utility function, the Arrow-Pratt measure of relative risk aversion is $(1-\theta)$.⁹ A higher value of θ means that the agents have a lower coefficient of relative risk aversion, where $\theta = 1$ corresponds to risk neutrality. A lower value of θ captures more risk averse agents.

Using the three payoff realizations (6), (7), and (8), and substituting the utility function $U(y) = y^{\theta}$ into the general welfare function (18) yields

$$EW = \omega \underbrace{\left[2\phi^{I} \left(\frac{\gamma}{8}\right)^{\theta} + \phi^{0} \left(\frac{2\gamma\mu^{2}}{(2\mu+1)^{2}}\right)^{\theta} \right]}_{\text{consumers' utility } EW_{c}} + (1-\omega) \underbrace{\left\{ 2\phi^{I} \left[\left(\frac{\mu\gamma}{4}\right)^{\theta} + \left(\frac{(1-\mu)\gamma}{4}\right)^{\theta} \right] + \phi^{0} \left[\left(\frac{\gamma\mu}{(2\mu+1)^{2}}\right)^{\theta} + \left(\frac{\gamma\mu}{(2\mu+1)^{2}}\right)^{\theta} \right] \right\}}_{\text{savers' utility } EW_{s}}$$
(23)

Differentiating (23) with respect to μ yields a first-order condition consisting of a high-order polynomial. Therefore, obtaining an algebraic closed-form solution as a basis for an explicit characterization of the socially optimal level of common ownership (μ^*) is not feasible. For that reason we initially apply numerical simulations to illustrate how increased risk aversion (decrease in θ) affects the welfare-maximizing level of μ^* . Figure 5 displays three graphs corresponding to different degrees of risk aversion: $\theta \in \{0.4, 0.6, 0.8\}$.

⁹The index of relative risk aversion for this utility function is $-yu''(y)/u'(y) = 1 - \theta$.



Figure 5: Expected total welfare EW defined in (23) as functions of the magnitude of majority shares μ and the risk aversion parameter $\theta \in \{0.4, 0.6, 0.8\}$. *Note*: The figure is drawn according to $\gamma = 64$, $\omega = 0.5$, and probabilities $\phi^{II} = \phi^{I} = \phi^{0} = 0.25$.

The top graph in Figure 5 corresponds to consumers and savers with low risk aversion where $\theta = 0.8$. Under equal weights ($\omega = \frac{1}{2}$), the welfare-maximizing majority ownership share is $\mu^* = 0.75$, meaning that some degree of investor portfolio diversification is socially optimal. The middle curve in Figure 5 depicts total welfare under higher risk aversion $\theta = 0.6$, where more portfolio diversification (more equal ownership with $\mu^* \approx 0.66$) is socially optimal. Finally, the bottom curve in Figure 5 corresponds to the highest degree of risk aversion ($\theta = 0.4$). For this case the simulation shows that the socially optimal ownership configuration is characterized by a higher degree of diversification as $\mu^* \approx 0.62$. Overall, these simulations suggest that a higher degree of risk aversion (lower θ) tends to induce a higher degree of common ownership (lower μ^*) in the social optimum. We will next theoretically verify this hypothesis.

The analysis of risk neutral consumers and savers in subsection 5.1, and in particular Figure 4, demonstrates the role played by the weight parameter ω in the social welfare function (18). In order to highlight the particular effects of risk aversion in a transparent way we will focus on the case with equal weights $\omega = 1/2$. In Appendix B we prove the following result for $\omega = 1/2$ and a sufficiently large γ ($\gamma > 9$).

Result 4. Suppose $\omega = \frac{1}{2}$ and $\gamma > 9$. Then, an increase in risk aversion (lower θ) increases the socially optimal degree of common ownership (lower μ^*).

The intuition behind Result 4 can be formulated as follows. With a higher degree of common ownership the institutional investors offer more diversified investment portfolios to their savers. The value savers derive from diversification is increasing as a function of the degree of risk aversion. This is the mechanism for why the socially optimal degree of common ownership increases with risk aversion. It should be emphasized that the socially optimal degree of common ownership balances the gains from diversification against the offsetting effects on consumer surplus, and that this tradeoff is importantly determined also by the parameter ω . This tradeoff explains why the simulations illustrated in Figure 5 yield interior solutions for the socially optimal degree of common ownership. It should also be remembered that Result 4 is formulated for $\omega = 1/2$ precisely like the simulations illustrated in Figure 5. In contrast to risk averse consumers, recall that for $\omega = 1/2$ welfare maximization excludes any degree of common ownership under risk neutrality, as we demonstrated in Result 3.

6. Extensions

This section explores a few extensions of the model and provides discussions of various issues related to the assumptions made in the model.

6.1 Investors as consumers

The analysis conducted in the main part of the paper followed the convention that investor and producer decisions are separated from consumption decisions. Following, Nielsen (1979), Farrell (1985), Chapter 3 in Azar (2012), and Azar and Vives (2019), this subsection reformulates the model assuming that investors' production decisions take into consideration the consequences on their own consumption. Formally, suppose that investor *A* and investor *B* each consumes a fraction λ of the total output produced in the market, where $0 \leq \lambda < \frac{1}{2}$. That is, investor *A* (similarly, investor *B*) consumers $\lambda Q = \lambda(q_1 + q_2)$ units of the product. The remaining production $(1 - 2\lambda)Q$ is consumed by non-investors. The analysis in the previous sections was based on a special case where $\lambda = 0$.

Let CS_A , CS_B , and CS_N denote the consumer surplus associated with the product analyzed in Section 2 for investor A, investor B, and non-investors, respectively. Substituting λQ for the consumption of each investor and $(1 - 2\lambda)Q$ for the consumption of non-investors into (3) yields

$$CS_A = CS_B = \frac{\beta \lambda^2 (q_1 + q_2)^2}{2}$$
 and $CS_N = \frac{\beta (1 - 2\lambda)^2 (q_1 + q_2)^2}{2}$. (24)

Define the utility of investor A and investor B as the sum of consumer surplus and profits from partial ownership in the two producing firms. Formally, when producing firms do not fail (event 0), investor A chooses q_1 and investor B chooses q_2 to solve

$$\max_{q_1} U_A(q_1, q_2) = CS_A(q_1, q_2) + \pi_A(q_1, q_2) \quad \text{and} \quad \max_{q_2} U_B(q_1, q_2) = CS_B(q_1, q_2) + \pi_B(q_1, q_2),$$
(25)

where $CS_A(q_1, q_2)$ and $CS_B(q_1, q_2)$ are defined in (24) and π_A and π_B in (4a) and (4b), respectively.

Appendix C derives the equilibrium production, price, and profit associated with (25)

$$q_1 = q_2 = \frac{\alpha\mu}{\beta(1+2\mu-2\lambda^2)}, \quad p = \frac{\alpha(1-2\lambda^2)}{2\mu+1-2\lambda^2}, \quad \pi_A = \pi_B = \frac{\gamma\mu(1-2\lambda^2)}{(2\mu+1-2\lambda^2)^2}.$$
 (26)

Substituting the equilibrium production levels from (26) into (24) and (25) yields

$$CS_{A} = CS_{B} = \frac{2\gamma\mu^{2}\lambda^{2}}{(2\mu + 1 - 2\lambda^{2})^{2}}, \quad CS_{N} = \frac{2\gamma\mu^{2}(1-\lambda)^{2}}{(2\mu + 1 - 2\lambda^{2})^{2}}$$

and
$$U_{A} = U_{B} = \frac{\gamma\mu[1 - 2\lambda^{2}(1-\mu)]}{(2\mu + 1 - 2\lambda^{2})^{2}}.$$
 (27)

Note that this extension of the model is continuous in λ in the sense that (26) and (27) converge to (8) as $\lambda \to 0$.

We now approach the main goal of this extension, which is the investigation of whether and to what extent Result 1 still holds when investors consume a fraction λ of total output produced by firms 1 and 2. Appendix C generalizes Result 1 as follows:

Result 5. Suppose each investor buys a fraction λ from producers 1 and 2, where $0 \le \lambda \le \frac{1}{2}$. Then, moving towards more equal co-ownership (μ decreases towards $\frac{1}{2}$) enhances investors' utility. Formally, $\partial U_A/\partial \mu = \partial U_B/\partial \mu < 0$ for all $\lambda \in [0, \frac{1}{2})$. However, the gains in utility diminish when investors consume a larger proportion λ of the total production. Formally, $\partial^2 U_A/\partial \mu \partial \lambda = \partial^2 U_B/\partial \mu \partial \lambda > 0$.

Result 5 shows that the increase in profit induced by a higher degree of common ownership dominates relative to the associated reduction in consumer surplus also under circumstances when investors operate with dual roles as owners and consumers. The intuition behind Result 5 is as follows. Consider the most extreme case where each investor consumes half of the total output produced by each firm ($\lambda = \frac{1}{2}$), and non-investors consume nothing. Even in this extreme case, the externality on consumption is not fully internalized, because each investor consumes only half of the output produced by the firm in which the investor has a majority share. Note that the only allocation in which this externality could be internalized is when investor *A* buys the entire output q_1 produced by firm 1 and investor *B* consumes the entire output q_2 produced by firm 2. Otherwise, as long as each investor consumes less than the full amount produced by the firm in which the investor has a majority share, the profit-enhancing effect would dominate the consumption effect, thereby inducing the maximum degree of common ownership to be optimal from the perspective of investors' utilities, U_A and U_B . In this respect, the monopoly results obtained in Nielsen (1979) and Farrell (1985) do not carry over to a market structure with oligopoly.¹⁰

With risk neutrality the socially efficient degree of common ownership is invariant to the introduction of the dual role of the investor as owner and consumer. Actually, as the fraction λ of the total output consumed by each institutional investor increases towards $\frac{1}{2}$, the objective function of the institutional owners converges towards (18). With risk aversion it is more complicated to characterize the implications of introducing this dual role of the investor, because the number of factors affecting the socially optimal degree of common ownership increases as the consumer surplus associated with owners has to be separated from that associated with consumers without ownership.

6.2 Multiple producing firms

Section 2 focused on two competing producing firms. Suppose now that investor A owns a majority share $\mu > \frac{1}{2}$ in $N_A \ge 1$ producing firms and investor B owns a majority share $\mu > \frac{1}{2}$ in $N_B \ge 1$ firms. Let q_a denote the production level of each of the N_A firms and q_b the production level of each of the N_B firms. Then, in view of (2), (4a), and (4b), the N_A output choice problems

¹⁰Azar (2017) constructs new indices that capture the internalization effects from consumer/worker control of firms. Brito et al. (2018) develop similar indices in the context of partial acquisitions.

of investor A and N_B output choice problems of investor B are

$$\max_{q_a} \pi_A(q_a, q_b) = \mu N_A q_a \left\{ \alpha - \beta \left[N_A q_A + N_B q_b \right] \right\} + (1 - \mu) N_B q_b \left\{ \alpha - \beta \left[N_A q_A + N_B q_b \right] \right\}, \quad (28a)$$

$$\max_{q_b} \pi_B(q_a, q_b) = (1 - \mu) N_A q_a \left\{ \alpha - \beta \left[N_A q_A + N_B q_b \right] \right\} + \mu N_B q_b \left\{ \alpha - \beta \left[N_A q_A + N_B q_b \right] \right\}.$$
 (28b)

Therefore, the profit of investor A given in (28a) is the sum of profits earned by N_A producing firms in which investor A has a majority ownership μ (each producing q_a), and the sum of N_B firms in which investor A is a minority ownership $1 - \mu$ (each producing q_b units of output). The profit of investor B (28a) is similarly defined, except that the majority and minority shares are reversed.

Solving the investors' profit-maximization problems (28a) and (28b), Appendix C shows that output levels of each firm are $q_a = \frac{\alpha\mu}{N_A\beta(2\mu+1)}$ and $q_b = \frac{\alpha\mu}{N_B\beta(2\mu+1)}$. Comparing these output levels with the output levels when there are only 2 producing firms (8) reveals that $N_Aq_a = N_Bq_b = q_1^0 = q_2^0$. Hence, aggregate production levels are not affected by increasing the number of firms in the product market as long as these are owned by the two institutional owners *A* and *B*. This means that consumer surplus given in (8) remains the same, and as shown in Appendix C, investors' profit also remains the same.

It should be noted that so far we solved only for the case where firms do not fail. A complete extension along this dimension would require a re-specification of the failure probabilities to apply to multiple producing firms. But, such an undertaking is beyond the scope of this study.

6.3 Small ownership shares

Throughout the paper it was assumed that the two investors combined own 100-percent of each firm and that the dominant investor in each firm owns more than 50-percent ($\mu > \frac{1}{2}$). However, such high shares of ownership are rarely empirically observed in industries with significant common ownership. As exemplified by the US airline industry or the German banking industry (see, OECD (2017)), the main investors in significant industries with considerable common ownership seldom have ownership shares exceeding 10 percent. These investors could still significantly influence the producing firms if all other investors have lower ownership shares, and therefore remain

passive investors. We next sketch an extension of our model to capture such a configuration.

Formally, assume that investor A owns a share μ_H in producing firm 1 and a share μ_L in firm 2. Similarly, investor B owns the shares μ_L and μ_H in firms 1 and 2, respectively. This allows for $\mu_H + \mu_L < 1$, meaning that passive investors own the shares $1 - \mu_H - \mu_L$ in each firm. With $\mu_H > \mu_L$, the firm that owns μ_H of the shares is assumed to be the dominant investor, able to influence the firm's production decisions. In view of (2), (4a), and (4b), the investors' output choice problems become

$$\max_{q_1} \pi_A(q_1, q_2) = \mu_H q_1 \left\{ \alpha - \beta \left[q_1 + q_2 \right] \right\} + \mu_L q_2 \left\{ \alpha - \beta \left[q_1 + q_2 \right] \right\},$$
(29a)

$$\max_{q_2} \pi_B(q_1, q_2) = \mu_L q_1 \left\{ \alpha - \beta \left[q_1 + q_2 \right] \right\} + \mu_H q_2 \left\{ \alpha - \beta \left[q_1 + q_2 \right] \right\}.$$
 (29b)

Solving the investors' profit-maximization problems (29a) and (29b), Appendix C derives the equilibrium production $q_1 = q_2 = \frac{\alpha \mu_H}{\beta(3\mu_H + \mu_L)}$. Substituting into (3), (29a), and (29b) yields the equilibrium consumer surplus and investors' profits (when neither firm fails)

$$CS = \frac{2\gamma(\mu_H)^2}{(3\mu_H + \mu_L)^2} \quad \text{and} \quad \pi_A = \pi_B = \frac{\gamma\mu_H(\mu_H + \mu_L)^2}{(3\mu_H + \mu_L)^2}.$$
 (30)

Note that the monopoly outcome when one firm fails given in (7) could also be computed with the revised ownership shares, but we will not repeat this derivation here.

Differentiating (30) with respect to μ_H and μ_L yields $\partial CS/\partial \mu_H = 4\gamma \mu_H \mu_L/[(3\mu_H + \mu_L)^3] > 0$ and $\partial CS/\partial \mu_L = -4\gamma (\mu_H)^2/[(3\mu_H + \mu_L)^3] < 0$. These two opposing effects are consistent with (19) and show that consumers are better off with more unequal ownership shares and worse off with more equal ownership shares. This means that a shift towards a higher degree of overlapping ownership is harmful for consumers.

Finally, (30) also implies that $\partial \pi_A / \partial \mu_H = \partial \pi_B / \partial \mu_H = \gamma (\mu_H + \mu_L) [3(\mu_H)^2 + (\mu_L)^2] / (3\mu_H + \mu_L)^3 > 0$ and $\partial \pi_A / \partial \mu_L = \partial \pi_B / \partial \mu_L = 4\gamma (\mu_H)^2 (\mu_H + \mu_L) / (3\mu_H + \mu_L)^3 > 0$. Thus, in this configuration with small ownership shares, a shift towards a higher degree of common ownership reduces profits for both types of ownership. Note that the change in investors' profit induced by an increase in the majority share μ_H is different from (9), because, under low share ownership, the investor with a higher ownership share grabs more shares from passive investors than from the

other non-passive investor.

6.4 Relaxation of other assumptions

The reported comparative statics and welfare analysis were based on a model with exogenouslygiven ownership shares μ and $1 - \mu$. Based this feature, Section 5 analyzes the welfare-maximizing ownership shares. However, a more complete exploration of investors' behavior could be built on model in which ownership shares are determined as an equilibrium outcome. Such a model would require the definition of "budget constraints", which would reflect the total investment volume collected by the institutional investors. An even more ambitious model could also attempt to endogenize these budget constraints to depend on the market performance of the institutional investors.

Further, the analysis of the product market in Section 2 relies on Cournot competition with linear demand for homogeneous products. An alternative specification would be a Hotelling model in which firms produce differentiated products. In such a model, the degree of competition would be affected also by the degree of product differentiation (often referred to as "transportation costs") in addition to the degree of common ownership μ .

7. Conclusion

We show that the socially optimal degree of common ownership is determined by two factors: (i) the degree of risk aversion and (ii) the relative weight society assigns to consumer surplus associated with the consumption of the final good compared with the returns on savings via institutional investors. We demonstrate that, under risk neutrality, complete ownership specialization with no common ownership at all is socially optimal if the relative weight on consumption of the final good is sufficiently high. Further, we establish analytically that with risk aversion, and for the class of utility functions with constant relative risk aversion (CRRA), an increase in the degree of risk aversion increases the socially optimal degree of common ownership.

Our model characterizes the effects of common ownership through the production decisions under oligopoly competition within the framework of exogenously given default probabilities. It remains an interesting challenge for future research to investigate alternative channels regarding the effects of common ownership on industry performance and welfare. It could be a fruitful approach to separately explore the effects of common ownership within the framework of an oligopoly model where the competing firms explicitly make decisions regarding risks or survival probabilities. Through such an approach, the risks would be an endogenous feature, making it possible to characterize the effects of common ownership on investments in innovation. The outcome of such an extension could be compared with the empirical findings of Aghion, Van Reenen, and Zingales (2013) who identify a positive relationship between innovation and institutional ownership.

Appendix A Algebraic derivations for Section 3

Derivation of the equilibrium values (7). With no loss of generality we focus on the case in which firm 2 fails, so firm 1 becomes a monopoly seller. Setting $q_2 = 0$ into (4a) and maximizing with respect to q_1 yields the first-order condition

$$0 = \frac{\partial \pi_A}{\partial q_1} = \alpha \mu - 2q_1 \beta \mu \quad \text{hence} \quad q_1 = \frac{\alpha}{2\beta} \tag{A.1}$$

which is the monopoly output level. The second-order conditions is $\partial^2 \pi_A / \partial (q_1)^2 = -2\beta \mu < 0$. Substituting into the demand function (1) yields the monopoly price $p = \alpha/2$. Substituting into (4a) and (4b) obtains the profit of investors *A* and *B*, respectively. Substituting into (3) obtains the consumer surplus under monopoly, all are given in (7).

Derivation of the equilibrium values (8). Differentiating (4a) with respect to q_1 and (4b) with respect to q_2 yields

$$0 = \frac{\partial \pi_A}{\partial q_1} = -2q_1\beta\mu - q_2\beta + \alpha\mu, \quad \text{and} \quad 0 = \frac{\partial \pi_B}{\partial q_2} = -2q_2\beta\mu - q_1\beta + \alpha\mu.$$
(A.2)

The second-order conditions are $\partial^2 \pi_A / \partial (q_1)^2 = \partial^2 \pi_B / \partial (q_2)^2 = -2\beta \mu < 0$. The equilibrium output levels and price in (8) are obtained by solving the system of two first-order conditions (A.2) for q_1 and q_2 , and then substituting into (1) to obtain p. Substituting back into (4a) and (4b) yields the profits (8). Finally, substituting the equilibrium price and output levels into (3) obtains

the equilibrium net consumer surplus CS.

Derivation of Result 1. Using (8) and $\frac{1}{2} < \mu < 1$, for every i = 1, 2 and $j = A, B, \partial q_i / \partial \mu = \alpha / [\beta (2\mu + 1)^2] > 0, \partial p / \partial \mu = -2\alpha / (2\mu + 1)^2 < 0$, and $\partial \pi_i / \partial \mu = \partial \pi_j / \partial \mu = \gamma (1 - 2\mu) / (2\mu + 1)^3 < 0$. That proves part (a).

To prove part (b), substituting $\mu = \frac{1}{2}$ into (8) yields $\pi_A = \pi_B = \gamma/8$, and hence $\pi_A + \pi_B = \gamma/4$. To prove part (c), substituting $\mu = 1$ into (8) yields $q_1 = q_2 = \alpha/(3\beta)$, $p = \alpha/3$, and $\pi_1 = \pi_2 = \pi_A = \pi_B = \alpha^2/(9\beta) = \gamma/9$, which are the standard Cournot duopoly competition equilibrium values.

Appendix B Proof of Result 4

We consider the utility function $U(y) = y^{\theta}$ with $0 < \theta \le 1$. Let $\omega = 1/2$. Using (18), (19), and (20), the necessary first-order condition characterizing the socially optimal degree of common ownership is given by

$$\begin{aligned} \frac{\partial EW}{\partial \mu} &= \phi^0 \frac{2\gamma\mu}{(2\mu+1)^3} U'(\frac{2\gamma\mu^2}{(2\mu+1)^2}) + \phi^I \frac{\gamma}{4} \left[U'(\frac{\gamma\mu}{4}) - U'(\frac{\gamma(1-\mu)}{4}) \right] \\ &+ \phi^0 \frac{\gamma(1-2\mu)}{(2\mu+1)^3} U'(\frac{\gamma\mu}{(2\mu+1)^2}) = 0. \end{aligned} \tag{B.1}$$

We assume the second-order condition $\frac{\partial^2 EW}{\partial \mu^2} < 0$ to be satisfied, meaning that the first-order condition (B.1) is also a sufficient condition for the socially optimal degree of common ownership μ^* . By differentiating (B.1) with respect to θ we find the effect θ on the socially optimal degree of common ownership from

$$\frac{\partial^2 EW}{\partial \mu^2} + \frac{\partial^2 EW}{\partial \mu \partial \theta} \frac{\partial \mu^*}{\partial \theta} = 0.$$
 (B.2)

In order to apply (B.2) we need to characterize the second-order mixed derivative $\frac{\partial^2 EW}{\partial \mu \partial \theta}$. We can express the second-order mixed derivative as

$$\frac{\partial^2 EW}{\partial \mu \partial \theta} = \phi^I \frac{\gamma}{4} A(\mu, \theta) + \phi^0 \frac{2\gamma\mu}{(2\mu+1)^3} B(\mu, \theta) + \phi^0 \frac{\gamma}{(2\mu+1)^3} C(\mu, \theta), \tag{B.3}$$

where we have introduced the notation that $\frac{\partial U'(\mu,\theta)}{\partial \theta} = U'_{\theta}(\mu,\theta)$ and where

$$A(\mu, \theta) = U'_{\theta}(\frac{\gamma\mu}{4}) - U'_{\theta}(\frac{\gamma(1-\mu)}{4}),$$
(B.4)

$$B(\mu,\theta) = U'_{\theta}(\frac{2\gamma\mu^2}{(2\mu+1)^2}) - U'_{\theta}(\frac{\gamma\mu}{(2\mu+1)^2}),$$
(B.5)

and

$$C(\mu, \theta) = U'_{\theta}(\frac{\gamma\mu}{(2\mu+1)^2}).$$
 (B.6)

The utility function $U(y) = y^{\theta}$ has the feature that $U'_{\theta}(y) = y^{\theta-1}(1+\theta \ln y)$. Therefore, with $y = \frac{\gamma \mu}{(2\mu+1)^2}$ it can be seen that $\gamma > 9$ is a sufficient condition for $U'_{\theta}(y) > 0$. This is also a sufficient condition to guarantee that U'(y) is strictly convex, because $U'_{\theta\theta}(y) = y^{\theta-1} \ln y(2+\theta \ln y) > 0$. These properties imply that $A(\mu, \theta) > 0$, $B(\mu, \theta) > 0$ and $C(\mu, \theta) > 0$. In light of (B.3) we can therefore draw the conclusion that $\frac{\partial^2 EW}{\partial \mu \partial \theta} > 0$. Consequently, from (B.2) we conclude that

$$\frac{\partial \mu^*}{\partial \theta} = -\frac{\frac{\partial^2 EW}{\partial \mu \partial \theta}}{\frac{\partial^2 EW}{\partial \mu^2}} > 0.$$
(B.7)

This means that stronger risk aversion (lower θ) induces a higher degree of socially optimal common ownership (lower μ^*).

Appendix C Algebraic derivations for Section 6

Derivations of the equilibrium values (26) Differentiating (25) yields

$$0 = \frac{\partial U_A}{\partial q_1} = q_1 \beta (\lambda^2 - 2\mu) + q_2 \beta (\lambda^2 - 1) + \alpha \mu$$

$$0 = \frac{\partial U_B}{\partial q_2} = q_1 \beta (\lambda^2 - 1) + q_2 \beta (\lambda^2 - 2\mu) + \alpha \mu.$$
(C.1)

The second-order conditions are $\partial^2 U_A / \partial (q_1)^2 = \partial^2 U_B / \partial (q_2)^2 = \beta (\lambda^2 - 2\mu) < 0$. Substituting into (1) and (25) yields the remaining expressions in (26).

Derivation of Result 5 Differentiating (27) yields

$$\frac{\partial U_A}{\partial \mu} = \frac{\partial U_B}{\partial \mu} = -\frac{\gamma (2\mu - 1)(1 - 2\lambda^2)^2}{(2\mu + 1 - 2\lambda^2)^3} < 0$$

$$\frac{\partial^2 U_A}{\partial \mu \partial \lambda} = \frac{\partial^2 U_B}{\partial \mu \partial \lambda} = \frac{4\gamma \lambda (2\mu - 1)(1 - 2\lambda^2)(2\lambda^2 + 4\mu - 1)}{(2\mu + 1 - 2\lambda^2)^4} > 0.$$
(C.2)

Derivations for Subsection 6.2 The first-order conditions for the investors' profit-maximization problems are

$$0 = \frac{\partial \pi_A}{\partial q_a} = N_A \left(\alpha \mu - 2N_A q_a \beta \mu - N_B q_b \beta \right)$$

$$0 = \frac{\partial \pi_B}{\partial q_b} = N_B \left(\alpha \mu - 2N_B q_b \beta \mu - N_A q_a \beta \right).$$
(C.3)

The second-order conditions are $\partial^2 \pi_A / \partial (q_a)^2 = -2(N_A)^2 \beta \mu < 0$ and $\partial^2 \pi_B / \partial (q_b)^2 = -2(N_B)^2 \beta \mu < 0$. Solving the system of two equations with two variables (C.3) yields

$$q_a = \frac{\alpha\mu}{N_A\beta(2\mu+1)}, \quad q_b = \frac{\alpha\mu}{N_B\beta(2\mu+1)}, \quad \text{and} \quad \pi_A = \pi_B = \frac{\alpha^2\mu}{\beta(2\mu+1)^2},$$
(C.4)

where the equilibrium profits of investors *A* and *B* are obtained by substituting the equilibrium output levels back into (28a) and (28b).

Derivations for Subsection 6.3 The first-order conditions for the investors' profit-maximization problems are

$$0 = \frac{\partial \pi_A}{\partial q_1} = \alpha \mu_H - 2q_1 \beta \mu_H - q_2 \beta (\mu_H + \mu_L)$$

$$0 = \frac{\partial \pi_B}{\partial q_2} = \alpha \mu_H - q_1 \beta (\mu_H + \mu_L) - 2q_2 \beta \mu_H.$$
(C.5)

The second-order conditions are $\partial^2 \pi_A / \partial (q_1)^2 = \partial^2 \pi_B / \partial (q_2)^2 = -2\beta \mu_H < 0$. Solving the system of two equations with two variables (C.5) yields (30).

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