# Tactical Target Date Funds*

Francisco Gomes[†] Alexander Michaelides[‡] Yuxin Zhang[§]

October 2019

**Abstract**

We propose target date funds modified to exploit stock return predictability driven by the variance risk premium. The portfolio rule of these tactical target date funds (TTDFs) is extremely simplified relative to the optimal one, making it easy to implement and communicate to investors. We show that saving for retirement in TTDFs generates economically large welfare gains, even after we introduce turnover restrictions and transaction costs, and after taking into account parameter uncertainty. Crucially, we show that this predictability is uncorrelated with individual household risk, confirming that households are in a prime position to exploit it.

JEL Classification: G11, G50

Key Words: Target date funds, life cycle portfolio choice, retirement savings, variance risk premium, strategic asset allocation, tactical asset allocation, market timing.

*We thank Jinhui Bai, Sylvain Catherine, Joao Cocco, Michael Haliassos, Anthony Lynch, Giovanna Nicodano, Lubos Pastor, Tarun Ramadorai, Grigory Vilkov, Hao Zhou and seminar participants at the CEPR Household Finance Conference, Annual Conference on Macroeconomic Analysis and International Finance at the University of Crete, Cheung Kong Graduate School of Business, Durham University, FEP, Frankfurt School of Finance & Management, IAAE 2019, PBC School of Finance (Tsinghua University), SFS Cavalcade, University of Lugano, and University of Luxembourg for comments and suggestions.

†Department of Finance, London Business School, London NW1 4SA, UK. E-mail: fgomes@london.edu.

‡Department of Finance, Imperial College London, South Kensington Campus, London SW7 2AZ, UK. E-mail: a.michaelides@imperial.ac.uk.

§Hanqing Advanced Institute of Economics and Finance, RenMin University, Beijing, China. E-mail: yx.zhang@ruc.edu.cn.

# 1 Introduction

Conventional financial advice prompts households to invest a larger proportion of their financial wealth in the stock market when young and gradually reduce this exposure as they grow older. This advice is given by several financial planning consultants (for instance, Vanguard[1]) who recommend target-date funds (TDFs) that reduce equity exposure as retirement approaches. The long term investment horizon of these funds, and the slow decumulation of risky assets from the portfolio as retirement approaches, can be thought of as strategic asset allocation (see Campbell and Viceira, 2002), where a long term objective (financing retirement) is optimally satisfied through the TDF. This investment approach arises naturally in the context of life-cycle models with undiversifiable labor income risk (for example, Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007), and Dahlquist, Setty and Vestman (2018)).[2] Moreover, the most recent empirical evidence shows that, even outside of these pension funds, households follow this life-cycle investment pattern (Fagereng, Gottlieb and Guiso (2017)).

In this paper we investigate whether simple portfolio rules designed to capture time variation in expected returns can improve welfare of an investor saving for retirement.[3,4] More precisely, we consider predictability through the variance risk premium (hereafter VRP), as introduced by Bollerslev, Tauchen and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014)). Crucially, we do not implement optimal strategies as implied by the model, instead we explore simple strategies that can be easily implemented by improved target date funds, in the same spirit as the optimal life-cycle strategies are only approximated by the current

[1]See Donaldson, Kinniry, Aliaga-Diaz, Patterson and DiJoseph (2013).

[2]Benzoni, Collin-Duffresne, and Goldstein (2007), Lynch and Tan (2011) and Pastor and Stambaugh (2012) show that this conclusion can be reversed under certain conditions.

[3]In models without labor income Kim and Omberg (1996), Brennan, Schwartz and Lagnado (1997), Brandt (1999), Campbell and Viceira (1999), Balduzzi and Lynch (1999), Barberis (2000), Campbell et. al. (2001 and 2003), Wachter (2002), Liu (2007), Lettau, and Van Nieuwerburgh (2008), and Johannes, Korteweg and Polson (2014) among others, show that optimal stock market exposure varies substantially as a response to time variation in the equity risk premium.

[4]The portfolio choice literature is not limited to the papers studying time variation in the equity risk premium. For example, Munk and Sorensen (2010) and Koijen, Nijman, and Werker (2010) focus on time variation in interest rates and bond risk premia, while Brennan and Xia (2002) study the role of inflation. Chacko and Viceira (2005), Fleming, Kerby and Ostdiek (2001 and 2003) and Moreira and Muir (2017 and 2019) consider time variation in volatility, while Buraschi, Porchia and Trojani (2010) incorporate time-varying correlations.

TDFs.

Existing target date funds do not use the exact policy functions of individual households, they instead offer an approximation that can be implemented at low cost. For example, the exact policy functions imply different portfolio allocations for investors with different levels of wealth (relative to future labor income).[5] Furthermore, the optimal life-cycle asset allocation is actually a convex function of age as the investor approaches retirement, not a linear one. However, the approximate rule is easier to understand for investors that might have limited financial literacy, and they are the ones who decide where to allocate their retirement savings. Therefore, in the same spirit as current TDFs, we approximate the optimal asset allocations with simple linear rules that can be followed by a Tactical Target Date Fund (TTDF). We estimate the best linear rule from regressions on our simulated data, where we include as explanatory factors not only age, but also the predictive factor (i.e. the variance risk premium).[6] We further truncate the fitted linear rule by imposing short-sale constraints. We do this because it might be hard for funds taking short positions to be allowed in some pension plans, and even if that is not a concern, they might be a tough sell among investors saving for retirement that have (on average) limited financial education. Building on our initial discussion, we refer to those modified funds as Tactical Target Date Funds (hereafter TTDFs).

Our focus on the predictability driven by the VRP is motivated not only by its empirical success as a predictive factor but also by the high-frequency nature of this time variation in expected returns. More traditional predictive variables, such as CAY (Lettau and Ludvigson (2001)) or the dividend-yield, capture lower frequency movements (both are more persistent than the VRP) and thus tend to be associated more closely with bad economic conditions and/or discount rate shocks, both of which might affect households directly.[7] The existing literature is well aware of this and therefore carefully mentions that the results should not

[5]In a similar spirit, Dahlquist, Setty and Vestman (2018) study simple adjustments to the portfolio rules of TDFs to take this into account.

[6]We also explore more sophisticated rules which naturally deliver higher wealth accumulation and utility gains but, for reasons just discussed relating to clarity and simplicity in communication, our baseline case remains the simple TTDF.

[7]Bad economic conditions will tend to be associated with negative labor income shocks, while discount rate shocks might reflect increased household risk aversion.

be interpreted as applying to a representative investor, but rather to an investor that is not exposed (or is less exposed than the average) to such risks.

In this paper we propose funds to be used by all investors so it is crucial that the average household is not exposed to these risks. Our underlying hypothesis is that VRP predictability is not related to business cycle fluctuations, and is instead likely driven by constraints on banks, pension funds and mutual funds (e.g. capital constraints or tracking error constraints). Such high frequency predictability is unlikely to be significantly correlated with household-level risks. We make this argument empirically by presenting evidence from the Consumer Expenditure Survey (CEX). Specifically, we document that states of the world with high realizations of the VRP do not predict decreases in household consumption growth, either in the near or in the distant future.

Furthermore, the VRP does not predict increases in cross-sectional consumption risk as captured by the cross-sectional standard deviation, skewness or kurtosis. Importantly, these empirical findings hold regardless of conditioning on stockholder or non-stockholder status, thereby showing that the results do not arise from not conditioning on the stock market participation status (Vissing-Jorgensen (2002)). Importantly, we also show that this holds even though stockholders are shown to bear a disproportionate amount of long run consumption risk as in Malloy, Moskowitz and Vissing-Jorgensen (2009). As a result of this evidence we can conclude that households are in a prime position to "take the other side" and exploit this premium. Furthermore, in general equilibrium, the fact that households own the financial intermediaries adds a further motivation to take the other side of this trade. If those institutional investors are forced to scale down their risky positions when VRP is high because of exogenous constraints, then households should be keen to offset this by increasing the risk exposure in their individual portfolios.

We first show that relative to an investor who assumes i.i.d. expected returns, the investor exploiting VRP predictability (the VRP investor) earns a significantly higher expected return. This result holds even in the presence of short-selling constraints which limit the ability of the VRP investor to exploit the time variation in the risk premium. Her expected return in such a model is still between 2.5% to 4% higher at each age (annually). However, given our goal of designing improved funds for retirement, our focus is not on the welfare gains from following an optimal policy.[8] Instead, we use the output of the model to design an approximate portfolio rule that can be easily implemented by an improved target date fund and thus be transparently communicated to investors. This is an important consideration since individual investors are increasingly expected to be the ones to decide where to allocate their retirement savings, and several of them have limited financial literacy and might be skeptical about complex financial products.[9]

We find that, when compared with the TDF, the TTDF generates substantial increases in age-65 wealth accumulation and certainty equivalent welfare gains. In our analysis we take into account a potential increase in transaction costs implied by the additional trading associated with the VRP strategy. Even with 1% decrease in annual expected returns due to increased portfolio turnover, the certainty equivalent gain from the TTDF versus the standard TDF is still 26% for our baseline calibration. The expected age-65 wealth accumulation is 131% higher. We find that the gains are particular higher for investors with moderate or high risk aversion, essentially households with more wealth. From this analysis we can conclude that if the TTDFs are introduced, then these investors would benefit the most from switching from standard TDFs into these new products.

Given that one drawback of the TTDF is that it implies significant trading, we next consider versions of the fund where we explicitly restrict quarterly turnover to a maximum threshold. It is particularly interesting to discuss the case where we set this threshold so that the average turnover of the constrained TTDF is comparable to the average turnover of the typical mutual fund (78% from Sialms, Starks and Zhang (2013)). Even though the increases in expected wealth accumulation are now smaller, the turnover constraint also decreases the volatility of wealth/consumption. Therefore, even when we impose this constraint the certainty equivalent gains, although smaller, remain economically meaningful. For the baseline parameter values the certainty equivalent gain from the TTDF is still 4%.

One concern with the previous calculations is that the welfare gains were computed in-

[8] Michaelides and Zhang (2017) incorporate stock market predictability through the dividend-yield and compute the welfare gains in the context of a life-cycle model of consumption and portfolio choice.

[9] There is a growing literature documenting the low levels of financial literacy in the population at large. Lusardi and Mitchell (2014) provide an excellent survey. Guiso, Sapienza and Zingales (2008) show that trust is an important determinant of stock market participation decisions.

sample. We address this concern in two ways. First, we estimate the predictive model in an initial sample (1990-1999) and using only that information to design the TTDF, more precisely, the TTDF with a tight turnover restriction. We then compare the real-time performance of this fund relative to the standard TDF over the subsequent period (2000-2016). This period was chosen even though the coefficients of the predictive regression are less stable exactly in the years immediately following our estimation window, before "recovering" in the final part of the sample. Nevertheless, we find that the TTDF with restricted turnover would have outperformed the TDF, and that this improved performance is largely obtained by decreasing the magnitude of the losses in bad years (e.g. 2001, 2002 and 2008). These results highlight that the higher performance of the TTDF is not driven by excessive risk taking, on the contrary, it is often the result of a reduction in risk-taking in anticipation of lower expected returns.

Second, we explicitly model parameter uncertainty using a Bayesian approach. We find that the welfare gains from the new TTDF are almost unchanged relative to the original model. In the presence of parameter uncertainty the optimal portfolio rule is more conservative with the investor more careful about exploiting potential predictability. Nevertheless, quantitatively, with quarterly rebalancing and short selling constraints, the optimal policy functions with parameter uncertainty are not substantially different from the behavior implied without parameter uncertainty. Moreover, the tight turnover restrictions further restrict portfolio changes. As a result, the impact of parameter uncertainty in designing the TTDF is small, and consequently our conclusions when comparing TTDF welfare gains relative to the TDF remain unchanged.

We further show that different natural extensions to the proposed TTDF can lead to even larger welfare gains. Those extensions include relaxing the short-sale constraints, considering a portfolio rule where we allow the age effects to interact with the predictive factor, and extending the TTDF beyond age 65 by adding a linear portfolio rule for the retirement period as well. Despite the improved results we believe that all of the above face non-trivial implementation problems relative to the simpler TTDF, and therefore we only present them as extensions to our baseline case.[10]

[10]The welfare gains would likely be even higher if we considered more recent predictors that have been

Naturally these tactical target date funds could be replicated by a combination of a standard target date fund and a predictability fund that uses the VRP strategy. But this would require an investor who not only has access to that second fund, but is also able to solve for the optimal weights across the two for each state-of-the world. In fact, the same argument can be made even more cleanly for the simple target date fund itself: it can be replicated by combining a pure index fund and cash. Moreover, in this simpler case the weights are only age dependent and therefore the strategy requires very limited financial knowledge to implement. To the extent that limited financial literacy and/or transaction costs (both financial and opportunity cost of time) have created such a large market for the simple TDF, the same forces should be even stronger for introducing the TTDF.

The paper is organized as follows. Section II discusses the VRP measurement and the VAR model for stock returns. In Section III we show that high realizations of the VRP are not associated with increased household risk. Section IV outlines the life-cycle model and discusses the optimal policy functions. Section V discusses the design of the proposed TTDFs and the associated welfare gains, while Section VI incorporates parameter uncertainty and reports the corresponding out-of-sample performance. Section VII explores different extensions, and Section VIII provides concluding remarks.

# 2 Variance Risk Premium and Stock Returns

## 2.1 VAR model for stock returns

The time variation in expected returns is captured by a predictive factor ($f_t$) and following Campbell and Viceira (1999) and Pastor and Stambaugh (2012) we construct the following VAR,

$$r_{t+1} - r_f = \alpha + \beta f_t + z_{t+1}, \tag{1}$$

$$f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1}, \tag{2}$$

shown to outperform the variance risk premium, such as the implied correlation or the correlation risk premium (see Buss, Schonleber and Vilkov (2018)).

where $r_f$ and $r_t$ denote the net risk free rate and the net stock market return, respectively. The two innovations $\{z_{t+1}, \varepsilon_{t+1}\}$ are i.i.d. Normal variables with mean equal to zero and variances $\sigma_z^2$ and $\sigma_\varepsilon^2$, respectively. Following Bollerslev, Tauchen and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014) we consider the variance risk premium (VRP) as the predictive factor, i.e. $f_t \equiv VRP_t$. The formulation allows for contemporaneous correlations between $z_{t+1}$ and $\varepsilon_{t+1}$.[11]

For comparison we will also be reporting results from a model with i.i.d. excess returns, in which case

$$r_{t+1} - r_f = \mu + z_{t+1}. \tag{3}$$

In order for the i.i.d. model to be comparable to the factor model, the first two unconditional moments of returns are set to be equal in both cases. We will also consider cases where additional transaction costs from more active trading negatively impact the expected return earned by the fund that exploits predictability. This will be implemented by adjusting appropriately the value of $\alpha$ in equation (1).

## 2.2 Variance Risk Premium

As in Bollerslev, Tauchen and Zhou (2009) we define the variance risk premium ($VRP_t$) as the difference between the option-implied variance of the stock market ($IV_t$) and its realized variance ($RV_t$),

$$VRP_t \equiv IV_t - RV_t. \tag{4}$$

The data for the quarterly implied variance index ($IV_t$) are taken from the Federal Reserve Bank of St. Louis (FRED), while the data for the monthly realized variance ($RV_t$) from Zhou (2017).[12] We convert the monthly realized variance to quarterly by simply adding the monthly terms. Figure 1 shows the time series variation in implied variance ($IV_t$), realized variance ($RV_t$) and the variance risk premium ($VRP_t$), replicating and extending essentially

[11] Unlike most commonly used predictors of expected returns, the factor that we consider in this paper (the variance risk premium) is not very persistent. Nonetheless, for generality sake, in the numerical solution of the model we approximate this VAR using Floden (2008)'s variation of the Tauchen and Hussey (1991) procedure, designed to better handle the case of a very persistent AR(1) process.

[12] Available here https://sites.google.com/site/haozhouspersonalhomepage/.

the original Bollerslev, Tauchen and Zhou (2009) measure.

### 2.3 VAR Estimation

Table 1 contains the descriptive statistics from the data set. The stock market return has a quarterly mean of 1.98% with a standard deviation equal to 7.9%. Following the life-cycle portfolio choice literature we assume an unconditional equity premium below the historical average, namely 4% at an annual frequency. The net constant real interest rate, $r_f$, equals 0.37% corresponding to 1.5% at an annual frequency.

Table 2 reports the estimation results for the VAR model (equations (1) and (2)). Our quantitative estimates are largely consistent with the ones in Bollerslev et al. (2009). The factor innovation is very smooth with a standard deviation ($\sigma_\varepsilon$) of 0.007. Given these estimates, we can infer the unconditional variance of unexpected stock market returns from

$$\sigma_z^2 = Var(r_t) - \beta^2 \sigma_f^2 \tag{5}$$

The correlation between the factor and the return innovation ($\rho_{z,\varepsilon}$) is a potentially important parameter in determining hedging demands. For most common predictors in the literature (e.g. dividend yield and CAY) this is a large negative number (see, for example, Campbell and Viceira (1999) and Pastor and Stambaugh (2012)). By contrast, when the predictive factor is the VRP, this correlation is estimated as slightly negative, suggesting that hedging demands are not particularly important in this context.[13]

## 3 VRP and Household Consumption Risk

### 3.1 Discussion

The empirical results in the previous section document that a high value of the VRP forecasts high expected stock returns next quarter, consistent with the findings in Bollerslev et al.

[13]Indeed, if we set $\rho_{z,\varepsilon}$ equal to zero in our model the results are not significantly different from the baseline. For that reason we do not explore the role of hedging demands in the paper, but those results are available upon request.

(2009). However, the optimality of increasing the allocation to stocks when the VRP is high will be over-stated if the high expected returns next quarter are accompanied by an increase in risk for households. Therefore, it becomes important for our analysis that this is not the case, and in this section we provide the corresponding supporting evidence.

It is important to clarify that we are not arguing that the changes in expected returns forecasted by the VRP do not reflect risk, as such a discussion is beyond the scope of our paper. We are merely stating that, if it is indeed risk, this risk appears to be faced primarily by other agents in the economy and not by individual households directly. For example, institutional investors such as mutual funds or banks face constraints that might lead them to reduce their risk bearing capacity in these periods.[14] If households are not directly exposed to this risk, it is therefore natural for them to increase their allocation to stocks in these periods and thus earn the additional premium by effectively taking the other side of this trade.[15] Furthermore, from a general equilibrium perspective, and to the extent that it is the same households that own the banks and therefore their own wealth that is invested in pension/mutual funds, a further motivation arises for taking the other side of the VRP. As institutional investors are forced to scale down their risky positions, then households should be keen to offset this by increasing the risk in their individual portfolios.

## 3.2 Data

We use non-durable consumption and services from the Consumer Expenditure Survey (CEX).[16] We exclude durables, implicitly assuming that utility is separable between durables and non-durables and services. This also allows comparison with earlier literature, particularly Malloy et. al. (2009). The service categories relating to durables are also excluded (housing expenses but not costs of household operations), medical care costs, and education costs as they have substantial durable components.

Our CEX sample choice follows Malloy et. al. (2009). We drop household-quarters in which a household reports nonzero consumption for more than 3 or less than 3 months or

[14] For example, tracking error constraints for mutual funds or VAR constraints for banks.

[15] Naturally if we take the view that a high value of the VRP does not represent an increase in risk at all, then the same conclusion applies: households should exploit this predictable variation in the risk premium.

[16] An internet appendix provides further details on data construction.

where consumption is negative. We also drop extreme consumption outliers for which consumption growth is less than 0.2 and greater than 5.0 because these may reflect reporting or coding errors. Furthermore, we drop non-urban households and households residing in student housing, and households with incomplete income responses. To determine stockholders we use the financial information provided in interview five and we also drop any households for which any of the interviews in the second to fifth quarter are missing. To determine stockholder status we use the response to the category "stock, bonds, mutual funds and other such securities". In our data the stock market participation rate is around 19 percent, which is similar to the rate reported by Malloy et. al. (2009) for the earlier version of this sample.

We construct quarterly consumption growth rates for stockholders and non-stockholders from January 1996 to December 2015. The CEX is a repeated cross section with households interviewed monthly over five quarters, enabling us to compute quarterly growth rates at a monthly frequency. Nevertheless, we cannot follow the same household for more than five quarters, and therefore membership in a group is used to create a pseudo-panel to track household risk over longer time periods. Following the literature, we regress the change in log consumption on drivers not in the model (log family size and seasonal dummies) and use the residual as our quarterly consumption growth measure.

Our model applies primarily to stockholders and we know from prior theoretical and empirical work that stockholders face different risks from non-stockholders. We therefore estimate separate measures of risk for the two groups. To determine stockholders we use the financial information provided in interview five and we also drop any households for which any of the interviews in the second to fifth quarter are missing. To determine stockholder status we use the response to the question of owning "stock, bonds, mutual funds and other such securities".

We compute the average consumption growth rate for a particular group of households (for instance, stockholders) for different horizons s=1, 2, 12, 24, by averaging the log consumption growth rates as

$$\frac{1}{N}\sum_{i=1}^{N}[c_{i,t+s} - c_{i,t}] \tag{6}$$

where N can vary depending on group and time period and $c_{i,t}$ is the quarterly log consumption of household $i$ at time $t$.

### 3.3 Empirical Evidence

How can we determine whether consumption growth risks in the short run and long run are affected by the VRP? We address this question by reporting results from regressing different moments of cross-sectional consumption growth on the VRP.

We start by considering regressions of mean consumption growth at different horizons and for different groups on the current VRP. More precisely, we estimate the following regressions

$$\frac{1}{N}\sum_{i=1}^{N}[c_{i,t+s} - c_{i,t}] = \alpha_c^s + \beta_c^s * VRP_t + \varepsilon_t^c,\ s = 1, 2, 12, 24 \tag{7}$$

As discussed above, given the nature of the CEX, we can only compute consumption growth rates for the same agent for up to $s = 2$. However, motivated by the long-run risk literature and by the evidence in Malloy et. al. (2009), we also consider the possibility that a high variance risk premium might signal an increase in long-run consumption risk by investigating the statistical significance of $(\beta_c^s)$ in equation (7) as the horizon $s$ increases.

The estimates of $\beta_c^s$ are shown in Panel A of Table 3. The standard errors are computed using a Newey-West estimator that allows for autocorrelation of up to $s-1$ lags when $s > 1$. For both stockholders and non-stockholders, $\beta_c^1$ is non-significant, indicating that a high value of VRP is not associated with lower expected future consumption growth rate in the next quarter. The same conclusion is obtained for $s = 2$ and the same conclusion arises as we consider consumption growth rates over multiple years ($s = 12$ and $s = 24$). We conclude that there is no significant relationship between VRP and individual short run or long run household consumption risk for either stockholders or non-stockholders.

We can repeat the same analysis for higher cross sectional moments of consumption growth rates. Nevertheless, because higher moments (the standard deviation, skewness and kurtosis) are not additive like the mean consumption growth rate, we can only report the regressions for consumption growth rates for $s = 1$ and $s = 2$, for which measures of these moments can be constructed directly for the same group of households. The results from these regressions are also reported in Table 3 (Panels B, C, and D) to explore the possibility that the VRP might be associated with a future increase in cross-sectional consumption risk. We find that high VRP states are not associated with an increase in either the cross-sectional standard deviation of consumption growth or its kurtosis, or with a decrease in its skewness. We conclude that, given the lack of any statistical significance in these regressions, high VRP states are not associated with an increase in future cross-sectional household consumption risk.

Overall, our results confirm that high VRP states, while predicting high future expected returns, are on average not followed by periods of lower household consumption growth or high cross-sectional dispersion in consumption growth rates.

# 4 Life-Cycle Asset Allocation Model

Time is discrete, but contrary to most of the life-cycle asset allocation literature we solve the model at a quarterly rather than an annual frequency. This is crucial to capture the higher-frequency predictability in expected returns documented by Bollerslev et al. (2009). Households start working life at age 20, retire at age 65, and live (potentially) up to age 100, for a total of 324 quarters. In the notation below we will use $t$ to denote calendar time and $a$ to denote age.

## 4.1 Preferences and Budget Constraint

In the model there are two financial assets available to the investor. The first one is a riskless asset representing a savings account. The second is a risky asset which corresponds to a diversified stock market index. The riskless asset yields a constant gross after tax real return, $R_f$, while the gross real return on the risky asset is potentially time varying as captured by the VAR model described in Section 2 (equations (1) and (2)).

The household has recursive preferences defined over consumption of a single non-durable good ($C_a$), as in Epstein and Zin (1989) and Weil (1990),

$$V_a = \max \left\{ (1-\beta) C_a^{1-1/\psi} + \beta \left( p_a E_a (V_{a+1}^{1-\gamma}) \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{8}$$

where $\beta$ is the time discount factor, $\psi$ is the elasticity of intertemporal substitution (EIS) and $\gamma$ is the coefficient of relative risk aversion. The probability of surviving from age $a$ to age $a+1$, conditional on having survived until age $a$ is given by $p_{a+1}$.

At age $a$, the agent enters the period with invested wealth $W_a$ and receives labor income, $Y_a$. Following Gomes and Michaelides (2005) we assume that an exogenous (age-dependent) fraction $h_a$ of labor income is spent on (un-modelled) housing expenditures.

Letting $\alpha_a$ denote the fraction of wealth invested in stock at age $a$, the dynamic budget constraint is

$$W_{a+1} = [\alpha_a R_{t+1} + (1-\alpha_a) R_f](W_a - C_a) + (1 - h_{a+1}) Y_{a+1} \tag{9}$$

where $R_t$ is the return realized that period (so when $t = a$).In the baseline specification we assume binding short sales constraints on both assets, more precisely

$$\alpha_a \in [0, 1] \tag{10}$$

In practice it is expensive for households to short financial assets and relaxing these assumptions would require introducing a bankruptcy procedure in the model. In the context of the life cycle fund shorting will be cheaper, but still not costless, and this will still require making assumptions about the liquidation process in case of default. For these reasons the baseline model assumes fully binding short-selling constraints but we will also discuss results where we relax these.

## 4.2 Labor Income Process

The labor income follows the standard specification in the literature (e.g. Cocco et al. (2005)), such that the labor income process before retirement is given by[17]

$$Y_a = \exp(g(a)) Y_a^p U_a, \tag{11}$$

$$Y_a^p = Y_{a-1}^p N_a \tag{12}$$

where $g(a)$ is a deterministic function of age and exogenous household characteristics (education and family size), $Y_a^p$ is a permanent component with innovation $N_a$, and $U_a$ a transitory component of labor income. The two shocks, $\ln U_a$ and $\ln N_a$, are independent and identically distributed with mean $\{-0.5 \times \sigma_u^2, -0.5 \times \sigma_n^2\}$, and variances $\sigma_u^2$ and $\sigma_n^2$, respectively. We allow for correlation between the permanent earnings innovation ($\ln N_a$) and the shocks to the expected and unexpected returns ($\varepsilon_{a+1}$ and $z_{a+1}$, respectively).

The unit root process for labor income is convenient because it allows the normalization of the problem by the permanent component of labor income ($Y_a^p$). Letting lower case letters denote the normalized variables the dynamic budget constraint becomes

$$w_{a+1} = \frac{1}{N_{a+1}} [r_{t+1}\alpha_{ia} + r_f(1-\alpha_{ia})](w_a - c_a) + (1 - h_{a+1}) \exp(g(a+1)) U_{ia+1}. \tag{13}$$

As common in the literature the retirement date is exogenous ($a = K$, corresponding to age 65) and income is modelled as a deterministic function of working-time permanent income

$$Y_a = \lambda Y_K^p \text{ for } a > K \tag{14}$$

where $\lambda$ is the replacement ratio of the last working period permanent component of labor income.

[17] We are assuming that the quarterly data generating process for labor income is the same as the one at the annual frequency. The calibration section discusses this in more detail.

## 4.3 Estimation and Calibration

We take the deterministic component of labor income ($g(a)$) from the estimates in Cocco et al. (2005) and linearly interpolate in between years to derive the quarterly counterpart. Likewise we use their replacement ratio for retirement income ($\lambda = 0.68$). Cocco et al. (2005) estimate the variances of the idiosyncratic shocks around 0.1 for both $\sigma_u$ and $\sigma_n$ at an annual frequency. Since we assume that the quarterly frequency model is identical to the annual frequency model it can then be shown that the transitory variance ($\sigma_u^2$) remains the same as in the annual model, while the permanent variance ($\sigma_n^2$) should be divided by four.

Angerer and Lam (2009) note that the transitory correlation between stock returns and labor income shocks does not empirically affect portfolios and this is consistent with simulation results in life cycle models (Cocco, Gomes, and Maenhout (2005)). We therefore set the correlation between transitory labor income shocks and stock returns equal to zero. The baseline correlation between permanent labor income shocks and unexpected stock returns ($\rho_{n,z}$) is set equal to 0.15, consistent with the mean estimates in most empirical work (Campbell et. al. (2001), Davis, Kubler, and Willen (2006), Angerer and Lam (2009) and Bonaparte, Korniotis, and Kumar (2014)). We set the correlation between the innovation in the factor predicting stock returns and the permanent idiosyncratic earnings shocks ($\rho_{n,\varepsilon}$) to zero as there is no available empirical guidance on this parameter.

Finally, we take the fraction of yearly labor income allocated to housing from Gomes and Michaelides (2005). This process is estimated from Panel Study Income Dynamics (PSID) and includes both rental and mortgage expenditures. As before, to obtain an equivalent quarterly process we linearly interpolate across years.

We use preference parameters previously used (Gomes and Michaelides (2005)) or estimated (Cooper and Zhu (2016)) in the literature using U.S. data. The discount factor is 0.9875 (annual equivalent around five percent), the elasticity of intertemporal substitution equal to 0.5 and relative risk aversion coefficient equal to 5.0. Below we report comparative statics with respect to the risk aversion parameter, additional comparative statics are available upon request.

## 4.4 Optimal portfolio allocation

We first document the optimal life-cycle portfolio allocations in the model with time-varying expected returns (henceforth VRP model) for a baseline value of preference parameters for the investor (henceforth VRP investor). These results will form the basis for the next section, where we propose the tactical target date funds (TTDFs). In the VRP model the optimal asset allocation is determined by age, wealth and the realization of the predictive factor (the variance risk premium). In Figure 2 we plot the average share invested in stocks for the VRP investor when the factor is at its unconditional mean ($\alpha_a[E(f)]$), the mean share across all realizations of the factor ($E[\alpha_a(f)]$), and the one obtained under the i.i.d. model ($E[\alpha_a^{iid}]$). In all cases wealth accumulation is being computed optimally using the appropriate policy functions.

The portfolio share from the i.i.d. model follows the classical hump-shape pattern (e.g. Cocco, Gomes and Maenhout (2005)).[18] The optimal allocation of the VRP investor, for the average realization of the predictive factor ($\alpha_a[E(f)]$), shares a very similar pattern and, except for the period in which both are constrained at one, we have

$$\alpha_a[E(f)] < E[\alpha_a^{iid}] \tag{15}$$

Even though under the two scenarios the expected return on stocks is the same, Figure 2 shows that $\alpha_a[E(f)]$ is below one already before age 35 and from then onwards it is always below $E[\alpha_a^{iid}]$. The main driving force behind this result is the difference in wealth accumulation of the two investors. As we show below, the VRP investor is richer and therefore allocates a smaller fraction of her portfolio to risky assets.[19]

We next compare the optimal risky share for the average realization of the factor ($\alpha_a[E(f)]$) with the optimal average risky share across all factor realizations ($E[\alpha_a(f)]$). If the portfolio rule were a linear function of the factor the two curves should overlap exactly. However, Figure 2 shows that there is a substantial difference between the two, particularly early in

[18]The increasing pattern early in life is barely noticeable because under our calibration the average optimal share at young ages is (already) close to one.

[19]The two policy allocations also differ because the policy rules from the VRP model take into account the hedging demands, but that effect is quantitatively much less important.

life. At this early stage of the life-cycle (age below 45) we have

$$E[\alpha_a(f)] < \alpha_a[E(f)] \text{ for } a < 45 \tag{16}$$

This result arises from a combination of the short-selling constraints and the fact that $\alpha_a[E(f)]$ is (much) closer to one than to zero. Given the high average allocation to stocks early in life, for realizations of the factor above its unconditional mean the portfolio rules are almost always constrained at one. On the other hand, for lower realizations of the predictive factor the optimal allocation is "free" to decrease, hence it is lower than $\alpha_a[E(f)]$. In some cases, that depend on the volatility of the predictive factor, the expected next period stock return becomes negative, and the optimal share of wealth in stocks jumps to zero. As a result, the optimal average allocation of the VRP investor is sometimes far below $\alpha_a[E(f)]$.

Building on the previous intuition, it is not surprising to find that the inequality sign flips once the portfolio allocation at the mean factor realization ($\alpha_a[E(f)]$) falls below 50%, which takes place around age 45. Now the more binding constraint is the short-selling constraint on stocks so we have:

$$E[\alpha_a(f)] > \alpha_a[E(f)] \text{ for } a > 45 \tag{17}$$

This comparison suggests that the welfare gains from the VRP model are likely to be much higher if we relax the short-selling constraints, which motivates our discussion of this particular extension in Section VII.

Combining inequalities (15) and (16) it is easy to see that, until age 45, we have:

$$E[\alpha_a(f)] < E[\alpha_a^{iid}] \tag{18}$$

namely that the average portfolio allocation in the VRP model ($E[\alpha_a(f)]$) will be much lower than the one in the i.i.d. model ($E[\alpha_a^{iid}]$), and the intuition follows from the previous discussions. In fact, even after age 45, when (16) is replaced by (17), we see that, although the difference between the optimal allocation of the VRP and i.i.d. investors decreases, equation (18) still holds: the i.i.d. investor has a higher average allocation than the VRP investor.

## 4.5 Portfolio returns

In this section we study the differences in expected returns between the VRP and i.i.d. investors, assuming they start with zero initial financial wealth and they face the same labor income realizations. To avoid repetition we ignore transaction costs in these calculations, since we will naturally consider them in the next section when we discuss the implementation of these portfolio rules in the context of the improved target-date funds. Moreover, the unconditional expected stock returns are the same in both models: the only difference between the average portfolio return calculation is the ability to perform tactical trading in the VRP world, and the implications this ability has on optimal portfolio allocation, saving and wealth accumulation.

In Figure 3 we plot the (annualized) average expected portfolio returns at each age

$$E(R_{t+1}^P) = \alpha_a E_t[R_{t+1}] + (1 - \alpha_a) R_f,\ a = 1, ..., T \tag{19}$$

which are computed by averaging (at each age) across all simulations. Since we are averaging across all possible realizations of the factor, for a constant portfolio allocation ($\overline{\alpha}$), this would be a flat line. For example, if $\overline{\alpha} = 1$, this would be equal to the average equity portfolio return, regardless of age. In the i.i.d. model this line essentially inherits the properties of the optimal $\{\alpha_a\}_{a=1}^T$. The (annualized) expected portfolio return is around 5% early in life, increases slightly in the first years and then decays gradually as the investor approaches retirement and thus shifts towards a more conservative portfolio. In the VRP model the same average life-cycle pattern is present but now, since the household increases (decreases) $\alpha_a$ when the expected risk premium is high (low), the line is shifted upwards. As a result, even though as shown in Figure 2 the VRP investor has on average a lower exposure to stocks than the i.i.d. investor, her expected return is actually higher.

The vertical difference between the two lines gives us a graphical representation of the additional expected excess return that is actually earned by the VRP investor, and to facilitate the exposition we also plot it as a separate line in the figure. From age 37 onwards this difference increases monotonically, as the lower average equity share makes the short-selling constraint less binding and thus the VRP investor is more able to exploit time-variation in the risk premium. As the two agents reach retirement, the difference in expected returns is almost 4%. This difference is therefore at its maximum exactly when these investors have the highest wealth accumulation.

# 5 Tactical Target-Date Funds

In the previous section we derived the optimal life-cycle policy functions from the model. However, these are not feasible options for a mutual fund. For example, current target date funds do not use the exact policy functions of individual households. They instead offer an approximation that can be implemented at low cost, using a roughly linear or piece-wise linear function of age. This is an approximation to the typical optimal solution for the i.i.d. model which follows a hump shape pattern early in life, even though not very pronounced for low levels of risk aversion, and has a convex shape later on as the investor approaches retirement. However, as the exact patterns of optimal policy will vary across individuals based on their preferences and other important factors (e.g. labor income profile and wealth accumulation), the linear function has the dual advantage of being simple to explain and a reasonable approximation to an heterogeneous set of optimal life-cycle profiles. This approach benefits from the further advantage that such a simpler strategy can be more easily communicated to investors that might have limited financial literacy, and are the ones who decide where to allocate their retirement savings.

In the same spirit, and in our baseline specification, we derive a relatively straightforward portfolio rule that can be implemented by an improved target date fund (the TTDF) and which will aim to capture a large fraction of the welfare gains previously described. More precisely, we derive optimal policy rules that consist of linear functions of age and of the predictive factor. If we design more complicated rules we could potentially increase the certainty equivalent gains, and in fact we explore some alternative portfolio rules along these lines. On the other hand, the more complicated rules are more likely to suffer from over-fitting or model mis-specification. Finally, in this section, we impose short-selling constraints on both the TDF and the TTDF. Later on we discuss the results obtained when we relax these constraints.

## 5.1 Designing Tactical Target-Date Funds

The simplest extension of the traditional TDF portfolio that incorporates the predictability channel is obtained by adding the predictive factor as an additional explanatory variable in a linear regression. More precisely, we use the simulated output from the model to estimate

$$\alpha_{iat} = \theta_0 + \theta_1 * a + \theta_2 * f_t + \varepsilon_{iat}. \tag{20}$$

Relative to the optimal simulated profiles this regression is quite restrictive as, in addition to linearity, it implies that both the regression coefficient on age ($\theta_1$) and the intercept ($\theta_0$) are the same regardless of the realization of the factor state. However, as previously argued, this is simple to implement and easier to explain to investors.

Table 4 reports the regression results from these rules for the baseline case of relative risk aversion equal to 5 and, for comparison, the results for the i.i.d. model.[20] Table 4 also reports the fitted linear rules for other values of risk aversion (2 and 10). These would correspond to three different TTDFs, each targeted to investors with different levels of risk aversion.

The life-cycle asset allocations for both the i.i.d. and the VRP baseline model are reasonably well captured by a linear regression rule. Despite the higher complexity of the optimal portfolio rules in the VRP case, the R-squared of the fitted linear regression is actually higher: 74% versus 45%. This is due to the lower implied average allocation to stocks, as already documented in Figure 2, which makes the short-selling constraints less binding. In the regression specification, age is expressed in quarters starting for quarter 1, as in the model. Therefore, the rule age pattern for the i.i.d. case is slightly steeper than the popular "100-age" rule followed by several existing target-date funds, but not far away from it. Similarly, the average age pattern of the VRP rule is slightly flatter than the 100-age rule but not very different from it. Of course under the VRP rule (equation (20)) the allocation also changes with the predictive factor. For example, for sufficiently high (or sufficiently low) values of this factor, the short-selling constraints can become binding. Later on, when

[20]These are regressions on data simulated from the model so the t-statistics are all extremely high almost by definition, and therefore are omitted from the table.

evaluating these strategies, we discuss their implied turnover.

In the last two columns of Table 4 we report the regression results for different values of relative risk aversion. For a risk aversion of 10, the coefficient on the predictive factor is lower, as the more risk averse investor is less willing to take advantage of time variation in expected returns. On the other hand, the less risk averse investor ($\gamma = 2$) should be more willing to exploit the variation in expected returns. However, she also has an average portfolio allocation that is much closer to 1 (the binding portfolio constraint). Therefore, her ability to actually follow the optimal market timing strategy is more limited by the presence of the short-selling constraints. As a result, the coefficient on the predictive factor is actually smaller than for $\gamma = 5$, and this is reflected in the significantly lower regression $R^2$: 58 percent versus 74 percent.

These results are also shown graphically in Figure 4 for the average share of wealth in stocks over the life cycle for the baseline cases. As previously explained, in the VRP world the investor moves more aggressively from positive to sometimes zero investment positions and this explains the lower average share of wealth in stocks relative to the i.i.d. model. This behavior is reflected in the design of the mutual fund associated with each model. The TTDF (TDF) associated with the VRP (i.i.d.) model is drawn based on a linear regression of all simulated portfolios on the factor and age. For simplicity, and for comparison purposes, we show the linear rule by averaging over all factors for the TTDF: this predicted share of wealth in stocks is a straight line across the average share of wealth in stocks generated by the VRP model. On the other hand, the effect of the factor is irrelevant for the TDF because the TDF is based on the i.i.d. model. Figure 4 therefore shows in a parsimonious way the average differences between the TTDF and TDF design.

## 5.2 Utility gains from Tactical Target Date Funds

### 5.2.1 Welfare Metric

Having identified a feasible portfolio rule for the TTDF we now proceed to compute the corresponding certainty-equivalent utility gains. Consistent with the focus of our paper to design improved target date funds, the baseline welfare calculations are computed by keeping pre-retirement consumption constant and comparing age-65 certainty equivalents, following Dahlquist, Setty and Vestman (2018). The differences in certainty equivalents therefore represent the increase or decrease in *risk-adjusted* consumption levels that the agent will register during the retirement period. This procedure guarantees that the pre-retirement utility is the same across cases (TDF and TTDF) and therefore the certainty equivalent gain at retirement captures the full welfare change.

In making the welfare calculations, we use the consumption policy functions associated with the i.i.d. model, when simulating decisions according to the TDF and TTDF models. We simulate returns over working life by the VRP specification and use the same realizations of labor income and initial wealth (zero) to simulate life cycle histories. We also recompute the value functions associated with the i.i.d. consumption rule for both the TDF and TTDF investment rules for every age. We can then compare the differences in outcomes across the two models, and those differences must arise from the two different investment rules. At age 65, we invert the average value function based on the simulations to compare consumption certainty equivalents across models.

In comparing different rules we assume the same asset allocation rules after retirement, that is, we assume that the investor ignores predictability from age 65 onwards. In other words, we are measuring the gains from changing the portfolio rule in the TDF only (that is, during working life). The gains would naturally be larger if we also allowed the investor to exploit time-variation in the risk premium during retirement as well, and we present results for this case in one of our extensions below. Finally, we assume that each investor is able to identify the fund that matches her level of risk aversion, both for the TTDFs and the standard TDF.

### 5.2.2 Results

We now study the welfare gains for the TTDF rule (equation (20)). They are shown in Table 5, for different values of risk aversion ($\gamma$). As previously mentioned, we also take into account a potential increase in transaction costs implied by the market timing strategy. More specifically, we take into account that the TTDF might face an effectively lower expected equity return as a result of these costs. Therefore the results are also shown for different values of the additional transaction costs ($tc$), namely 10 and 25 basis points on a quarterly basis.[21]

When considering the case with $tc = 0.0$ the increases in age-65 wealth accumulation are 103%, 182% and 312%, for risk aversion of 2, 5 and 10, respectively. The associated CE gains are 20.3%, 40.5% and 80.3% showing that the simple rule proposed by equation (20) delivers extremely large welfare gains. This is particularly remarkable if we recall that, in this analysis, we are assuming that the investor does not exploit the predictability in expected returns after retirement, when financial wealth is very high.

Importantly, the welfare gains remain economically large even as we introduce the additional transaction costs. For the baseline calibration of risk aversion ($\gamma = 5$), even with a 1% annual increase in costs, relative to those of the standard TDF, age-65 wealth accumulation is still 131% higher under the TTDF and the certainty equivalent consumption gain is 26.2%. As before, these values are even higher for the less risk-tolerant investor (64.4%) and lower for the more risk-tolerant one (10.1%). One implication of these results is that it would be particularly beneficial to introduce the TTDFs in pension plans with investors with moderate or high risk aversion (5 or 10). The important point is that households with the tendency to be net savers will benefit more from such funds than households with lower saving rates. Equivalently, if such funds are offered in parallel with standard target date funds, investors who save more are the ones who would benefit the most from switching away from the conventional product.

## 5.3 Introducing Turnover Restrictions

### 5.3.1 Approach

One potential concern with the TTDFs, as presented in the previous section, is that their implementation might imply a very high portfolio turnover. The average (annualized) portfolio turnover of the standard TDF (i.e. the one that replicates the optimal allocation of

[21]The standard TDF will also face transaction costs but in our simulations we only explicitly introduce them for the enhanced fund, which is why we view them as additional costs, over and above those already faced by the standard TDF.

the i.i.d. investor) is 23%. For the TTDF investor average turnover rises to 213% indicating that tactical asset allocation implies a more volatile asset allocation behavior over the life cycle. By comparison, the average turnover of the typical mutual fund is 78% (see Sialms, Starks and Zhang (2013)).

In the previous section we included in our analysis additional transaction costs that this high turnover might generate. In this section we follow a more direct approach where we explicitly restrict the fund's turnover. The restriction limits the optimal rebalancing of the portfolio share to a maximum threshold ($k$). More precisely, the portfolio rule is subject to the additional constraint

$$\alpha_a = \begin{cases} \alpha_{a-1} + k & if \ \alpha_a^* > \alpha_{a-1} + k \\ \alpha_a^* & if \ |\alpha_a^* - \alpha_{a-1}| < k \\ \alpha_{a-1} - k & if \ \alpha_a^* < \alpha_{a-1} - k \end{cases} \tag{21}$$

where $\alpha_a^*$ is the optimal allocation in the absence of the constraint.

In our analysis we consider two thresholds, $k = 25\%$ and $k = 15\%$. We impose equation (21) ex-post on the previously estimated policy rules, instead of solving the corresponding dynamic programming problem for two reasons.[22] First, even though the optimal policy function would by definition satisfy constraint (21), that does not guarantee that the corresponding fitted linear rule estimated from the simulated data would as well.[23] Second, from an implementation perspective this again makes the rule more transparent and easy to follow and explain to an investor. The asset allocation of the fund is given by the previous regression specification, which yields $\alpha_a^*$, subject to this constraint.

Figure 5 illustrates the impact of these turnover restrictions. It shows the life-cycle portfolio allocation of both the unconstrained TTDF and the TTDF with the $k = 15\%$ turnover restriction. The figure plots the allocation for different realizations of the predictive factor ($f$), namely its mean (0.494%) and 1.14 standard deviations above and below the mean, re-

[22] Any further mis-specification of the optimal policy functions implied by this approach will only lead us to under-estimate the utility gains since the constraint is more binding for the TTDF than the standard TDF.

[23] This is the same issue we already had before with the short-selling constraints and these also had to be imposed ex-post.

spectively.[24] As we can see, in the absence of any restrictions the TTDF allocation changes by almost 40% for an approximate one standard deviation movement in the predictive factor. Even in the presence of short-selling constraints this creates the very large turnover numbers that we have reported. In contrast, when we impose the $k = 15\%$ constraint the share invested in equities is much less volatile. For a 1 standard deviation movement in the VRP the average change in the risky share is now about 10%.

### 5.3.2 Results

In Table 6 we show the results when we impose constraint (21) for the baseline case of an investor with risk aversion of 5. With a maximum rebalancing limit of 25% the average turnover of the fund falls almost by half to 107%. When the limit is even stricter (15%), the average turnover is now only 69%, which is now even below that of the typical mutual fund (78% as mentioned above). High fund turnover was the motivation for including the additional transaction costs in the previous subsection. Therefore, since we are now limiting fund turnover directly, in these results we only consider the cases with $tc = 0.0$ and $tc = 40$ basis points (annualized).

The constraints naturally limit the fund's ability to exploit time-variation in the risk premium and this is reflected in lower expected wealth accumulation. For example, for $tc = 0$ the expected increase in age-65 wealth accumulation for the baseline case (risk aversion of 5) was 269% in the absence of the turnover constraints, but falls to 45% and 14% for $k = 25\%$ and $k = 15\%$, respectively. However, this is accompanied by an equally significant reduction in the impact on the standard deviation of (age-65) wealth. In the absence of turnover constraints this standard deviation had increased by 462%, whereas now the percentage change is limited to 48% and 5%, respectively.

As we introduce these additional restrictions the extremely large welfare gains that we previously documented are reduced, but we still obtain values that are economically quite meaningful and, we would argue, much more reasonable. With $tc = 0.0$ the certainty equivalent gains for the baseline case (risk aversion of 5) are 11.1% and 3.7%, for $k = 25\%$ and

[24] Those apparently arbitrary values correspond to actual points on the grid for the state space, while plus and minus 1 standard deviations do not.

$k = 15\%$, respectively. Even with $tc = 40$ basis points (annualized) both of these still remain positive: 7.2% and 0.4%, respectively.

As we consider investors with either higher or lower risk aversion we again find that the certainty equivalent gains are particularly larger for the former. Even with the tighter turnover restriction ($k = 15\%$ and $tc = 40$ basis points annualized), the investor with risk aversion of 10 still accumulates 83% more wealth at age 65, on average, by using the TTDF. This corresponds to a certainty equivalent gain of 22%. Across all cases, the investment in the TTDF only leads to certainty equivalent loss for one them: the combination of the tighter turnover restriction *and* additional transaction costs for the investor with risk aversion 2. But as just discussed, even under this combination the investor with risk aversion of 10 still has a certainty equivalent gain of 22%.

Two of the $tc = 0.0$ cases are particularly interesting: the one for the investor with risk aversion of 2 and $k = 25\%$ , and the one for the investor with risk aversion of 5 and $k = 15\%$. In both of these the change in the standard deviation of age-65 wealth is very small, $-2\%$ and 5% respectively, yet there are meaningful differences in wealth accumulation: 23% in the first case and 14% in the second. So for a very similar level of ex-ante risk the investor is obtaining a noticeable difference in average expected wealth. This is reflected in certainty equivalent gains of 4.9% and 3.7%.

Overall, the results in Table 6 confirm that it is possible to design a relatively simple target date fund rule that exploits the risk premium predictability obtained from the VRP, while only requiring standard levels of turnover, and being able to generate economically large welfare gains for a wide range of investors, especially the ones that are net savers over the working life cycle.

# 6 Parameter Uncertainty

One concern with the previous calculations might arise from the welfare gains being computed ignoring parameter uncertainty. In this section we address this concern in two ways. First, we incorporate parameter uncertainty in a Bayesian framework (e.g. Barberis (2000)). Second, we estimate the predictive model in an initial sample (1990-1999) and evaluate the performance of the TTDF over a subsequent period (2000-2016).

## 6.1 Bayesian approach

In this section we take into account for parameter uncertainty using a standard Bayesian approach. For computational reasons we only consider parameter uncertainty over the two more important parameters: $\beta$, the predictive coefficient in the expected return equation, and $\phi$, the persistence of the factor. We assume that the posteriors over the two parameters are independent and, under the assumption of diffuse priors, are given by

$$\beta \sim N(\widehat{\beta}, \widehat{\sigma}_\beta) \text{ and } \phi \sim N(\widehat{\phi}, \widehat{\sigma}_\phi),$$

where $\widehat{\beta}$ and $\widehat{\phi}$ are the corresponding point estimates (3.50 and $-0.18$, respectively), and $\widehat{\sigma}_\beta$ and $\widehat{\sigma}_\phi$ are the standard errors from the estimation (0.78 and 0.09, respectively). We approximate both posterior distributions using standard Gaussian quadrature methods, just as for the other random variables in the model. Using the optimal solution from the model with parameter uncertainty we repeat the previous process. We first fit a new TTDF rule and then evaluate the corresponding welfare gains relative to the TDF rule.

The results are reported in Table 7 for the baseline risk aversion coefficient of 5. The numbers are almost exactly identical to their counterparts in Tables 5 and 6, when we did not consider parameter uncertainty. One explanation for this is the presence of the constraints, namely the short-selling constraints and the turnover restrictions. In the absence of any constraints parameter uncertainty will make the investor follow a more conservative portfolio rule, i.e. a portfolio rule closer to the one implied by the i.i.d. model. However, the presence of constraints, and in particular the tight turnover restrictions, have exactly the same effect. Remember that we are imposing those restriction ex-post when the portfolio rule is implemented, not when solving the model. Therefore, if the investor is already more constrained by the turnover limit than what parameter uncertainty would imply, then adding parameter uncertainty will not change the results. This is indeed what we find.[25] However,

[25]Consider a hypothetical simplified example where, given the current realization of the factor, the unconstrained investor would like to increase her allocation to 80% in the absence of parameter uncertainty and

the results in Table 7 reveal that there is also a second channel at work.

In columns 2 and 3 of Table 7 we report results for the case without turnover restrictions, which can therefore be directly compared to the results in Table 5. The age-65 consumption equivalent gains without parameter uncertainty are now smaller but the difference is very modest. Although here we still have the short-selling constraints, so that the first channel is not fully eliminated, this is also evidence that the actual portfolio rule is not significantly affected by explicitly incorporating parameter uncertainty. We can further confirm this by comparing the coefficients from the actual portfolio rule itself for both cases. The coefficient on the predictive factor is now 45.2, which is only slightly less than the 45.6 obtained for the case without parameter uncertainty (Table 4).[26] Our conclusions are therefore robust to parameter uncertainty concerns.

## 6.2 Predictive model in different sub-samples

In Table 8 we report the estimates of the VAR model for three different sub-samples: 1990-1999, 2000-2009 and 2010-2016. We can see that the 3 different periods yield estimates that are broadly similar but also with some non-trivial differences. For example, even though the coefficient on the predictive regression is always positive, it falls to 2.0 in the middle period, compared with 5.9 and 5.2 in the other two periods. In the full sample estimation (Table 2), the correlation between realized returns and expected returns was only marginally negative. Here, we can see that this is the result of combining a positive correlation in the middle period with negative correlations in the other two. The results in Table 8 suggest that the TTDF's performance is particularly at risk right after our estimation window, and this further motivates our choice of this window for the out-of-sample exercise in the next sub-section.

---

to 75% in the presence of parameter uncertainty. If the maximum turnover constraint already implies that she cannot increase her allocation above 70% then both portfolio rules would give the same allocation.

[26]The constant is now 0.51, same as in Table 4 and the age coefficient is -0.00189, compared with -0.00191 in Table 4.

## 6.3 Out-of-sample returns and wealth accumulation

We now repeat the design of the TTDF but based on the VAR estimated for the period 1990 to 1999. More precisely, we solve and simulate the model again using the data generating process from this VAR, and then use the simulated policy functions to estimate equation (20) again. For computational reasons we keep the TTDF rule constant throughout the exercise and do not update the model every quarter. This lowers the out-of-sample performance of the TTDF. Moreover, motivated by the discussion in subsection 5.3, we further restrict the turnover of the fund with the tightest value of this constraint, i.e. $k = 15\%$.

Figures 6.1 and 6.2 show the results for the 20-year-old investor. We report annual returns to facilitate the exposition but, as before, they are based on an underlying quarterly model and quarterly simulation. Figure 6.1 reports the cumulative returns over the period 2000-2016 for both the TTDF and the TDF. We can see that the TTDF out-performs from early on, and the gap between the two funds increases over time. By the end of the period the investor choosing the TTDF has accumulated 24% more wealth than the other investor. Figure 6.2 shows the (annualized) period-by-period returns and provides a better understanding of this superior performance. In good periods, the TTDF actually tends to under-perform (e.g. 2005 and 2012-2014). It is in bad times that the TTDF does consistently better, namely in the years 2001, 2002 and 2008. In these years the market timing investor is able to mitigate her losses relative to the TDF investor. Returning to figure 6.1 we can indeed confirm that the performance of two funds starts to diverge around 2001 and this difference increases again around 2008.

We can understand the superior performance of the TTDF in bad times from Figure 4, where we see that the average allocations of these funds are very high for young investors. As a result, in the presence of short-selling constraints, these investors can benefit more from decreasing their equity exposure in bad times than from increasing it in good times. It is easier for them to hit the 100% constraint when trying to exploit high expected returns than to hit the 0% constraint when facing low expected returns. To further illustrate this intuition, Figures 7.1 and 7.2 show the results for a 50-year-old investor. As we saw in Figure 4, while the average risky share of the 20-year old investor is close to 70%, for the 50-year old investor this number is almost exactly 50%. As a result, the benefits from the TTDF are now more evenly distributed across booms and busts.

Overall, these results are particularly encouraging given the previous discussion highlighting the period right after our chosen estimation window as the one during which the implied predictive VAR might be very mis-specified. Given that even under this scenario we find that the TTDF outperforms the TDF, this suggests that the gains would be even larger for other potential out-of-sample experiments. Furthermore, they confirm that the higher performance of the TTDF does not arise because of excessive risk taking; on the contrary it often results from lower risk-taking in anticipation of bad states of the world.

# 7 Extensions

## 7.1 Extended Tactical TDF

As previously discussed, the portfolio rule based on equation (20) is very straightforward but quite restrictive. Therefore, in this section we consider an alternative formulation where we fit the simulated shares of wealth in stocks on age using separate regressions conditional on the different realizations of the predictive factor. That is, we run the following series of regressions for each $f_j$ in our discretization grid

$$\alpha_{iat} = I_{f_t=f_j}\theta_0^j + I_{f_t=f_j} * \theta_1^j * a + \varepsilon_{iat}^j, \text{ for each } f_j \tag{22}$$

where $I_{f_t=f_j}$ equals to 1 if $f_t = f_j$ and equals to 0 otherwise. To distinguish it from the fund consider in the previous sections we refer to this one as TTDF2.

The welfare gains from the TTDF2 are reported in Table 9. We first consider the case with no transaction costs ($tc = 0.0$). For all three values of risk aversion the increases in wealth accumulation at age 65 are extremely high: 201%, 260% and 377%. Likewise, the corresponding age-65 certainty equivalent gains are also very large: 37.8%, 55.3% and 95%, respectively. As we introduce differential transaction costs for the TTDF2 these values naturally fall. However, even for a annual transaction cost of 1%, age-65 wealth is higher by more than 100% for all investors. As a result, the utility gains remain quite high: 38.6% for the baseline risk aversion of 5, increasing (decreasing) to 78.9% (23.5%) for the risk aversion of 10 (2). For the reasons that we previously discussed we do not view this rule as a very practical proposition for a TDF. However, these results suggest that individuals with high financial literacy who would potentially be willing to invest in such funds if they were introduced, could obtain very large CE gains from doing so.

## 7.2 Relaxing the short-selling constraints

As shown in Figure 5, the optimal portfolio allocation of the TTDF is sometimes constrained at either 100% or 0%. These results suggest that the utility gains from the VRP strategy are likely to be higher if we relax the short-selling or borrowing constraints. In the life-cycle asset allocation literature it is common to impose fully binding short-selling and borrowing constraints since it is particularly hard or expensive for retail investors to engage in unsecured borrowing or short-selling. Moreover, a mutual fund that takes leveraged positions might not be regarded as an acceptable choice by some pension plan providers. Nevertheless, the proposed TTDF strategy will be implemented by a mutual fund and hence it should be much cheaper and feasible to take both borrowing and short-selling positions.

In this section we therefore investigate the case in which the TTDF can increase its allocation to stocks as far as 200% through borrowing at the same riskless rate, that is:

$$\alpha_a \in [0, 2] \tag{23}$$

For the range of parameter values that we consider the upper bound on this constraint becomes less binding. We could potentially also relax the short-selling constraint on the risky asset and the welfare gains would be even higher, but that particular constraint is less binding given that the average allocation to stocks is above 50%. Furthermore, short-selling the aggregate stock market is typically harder and more expensive to implement than borrowing to invest in stocks.

In the i.i.d. model the household borrows to invest in the stock market early in life and then the pronounced life cycle effect of lowering the share of wealth in stocks takes over.

We use this rule to construct the TDF for the i.i.d. model (the strategic asset allocation benchmark). In this model stock market turnover now rises to 113% relative to 23% in the benchmark analyzed earlier. We follow a similar strategy for the TTDF. Table 10 reports the differences in wealth accumulation and CE gains from taking advantage of the TTDF when we relax the short-selling constraint on the riskless asset for both funds.[27]

Comparing these results with those in Tables 5 and 6, where short-selling was completely ruled out, we find significant increases in certainty equivalent gains. Without any turnover restrictions (columns II to IV) the welfare gains more than double in size, increasing from 40.5% (26.2%) to 91.8% (67.3%) for $tc = 0.0$ (100) basis points (comparison with Table 5). A less tight short-selling constraint (equation (23)), significantly increases the TTDF's ability to exploit the time-variation in the expected risk premium.

One potential concern here is that this strategy implies significantly higher portfolio turnover. In fact, we see that average fund turnover is now 360% as opposed to 213% for the case with fully binding short-selling constraints. To address this concern Table 10 also reports results with the exogenous constraint on trading (equation (21)). As we introduce the tighter version of constraint ($k = 15\%$) portfolio turnover drops significantly, to around 73%. The welfare gains naturally decrease substantially but, as before, remain economically significant. As we compare them with the ones in Table 6 we find that they are very similar but still higher. For example, for $tc = 0.0$, the certainty equivalents are now 13.5% and 5.2% for $k = 25\%$ and $k = 15\%$, respectively, compared with 11.1% and 3.7% in Table 6.

We conclude that relaxing the short-selling constraint on the riskless asset can increase the welfare gains from investing in the TTDF, even if we restrict the fund's turnover to reasonable levels.

## 7.3 Adding VRP strategies during retirement

In the previous section the investor only exploited time variation in expected returns before retirement through the TTDF. The goal was to isolate the role of the TTDF and thus show how introducing these market timing strategies in a target date fund alone could improve

[27] We maintain all other assumptions as in the baseline case, namely relative risk aversion of 5. Results for other values of risk aversion are available upon request.

welfare. In this section we consider the benefits of trying to capture the VRP strategy throughout the life-cycle. For this purpose we consider a combination of the simple TTDF with an otherwise equally designed fund for the retirement period. More precisely, we run a second regression given by equation (20) for ages greater than 65. From this we obtain a linear portfolio rule for the retirement period which complements the TTDF, that is a TTDF in retirement.

The results are shown in Table 11 for the baseline case of risk aversion 5 and with turnover restrictions to keep trading volume consistent with that of typical mutual funds. As expected, the welfare gains are now even larger. For the tighter turnover restrictions ($k = 15\%$) the certainty equivalent gains are between 12.3% and 20.1%, substantially larger than the comparable ones from Table 6 (0.35% to 3.7%, respectively).

# 8 Conclusion

We analyze how target date funds can combine the long term strategic asset allocation perspective of a life cycle investor with the short term market information that gives rise to tactical asset allocation. We rely on the variance risk premium (VRP) as the main factor producing variation in the expected risk premium in quarterly frequency and embed this in a life cycle model to derive optimal saving and asset allocation. We then show how enhanced funds, which we call Tactical Target Date Funds (TTDFs), can be designed in a parsimonious way and can deliver substantial welfare gains. These gains are substantial and remain economically large even after we include transaction costs and further explicitly restrict the turnover of the TTDF. These gains could be potentially increased by considering different extensions to the simplified rule or by considering predictive variables with even higher forecasting power, such as the implied correlation or the correlation risk premium. In unreported experiments we extend the analysis to a wider set of preference parameter configurations and different models of investor behavior during retirement. Further research into the design and commercialization of the proposed TTDFs, and the potential complications that may arise in such implementations, is an interesting topic for future research.

# References


[1] Angerer, X., and P. Lam. "Income Risk and Portfolio Choice: An Empirical Study." *Journal of Finance*, LXIV (2009), 1037–1055.

[2] Balduzzi, P., and A. W. Lynch. "Transaction Costs and Predictability: Some Utility Cost Calculations." *Journal of Financial Economics,* 52 (1999), 47–78.

[3] Barberis, N. "Investing for the Long Run when Returns are Predictable." *Journal of Finance*, 55 (2000), 225–264.

[4] Benzoni, L.; P. Collin-Dufresne; and R. Goldstein. "Portfolio choice over the life-cycle when the stock and labor markets are cointegrated." *Journal of Finance*, 62 (2007), 2123–2167.

[5] Bollerslev, T., G. Tauchen and H. Zhou, "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22 (2009), 4463–4492.

[6] Bollerslev, T., J. Marrone, L. Xu, and H. Zhou, "Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence," *Journal of Financial and Quantitative Analysis*, 49-3 (2014), 633-661.

[7] Bonaparte, Y.; G. Korniotis; and A. Kumar. "Income Hedging and Portfolio Decisions," *Journal of Financial Economics*, 113 (2014), 300–324.

[8] Brandt, M. "Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach," *Journal of Finance*, 54 (1999), 1609–1646.

[9] Brandt, M.W.; A. Goyal; P. Santa-Clara; and J.R. Stroud. "A Simulation Approach to Dynamic Portfolio Choice with an Application to Learning about Return Predictability," *Review of Financial Studies*, 18 (2005), 831–873.

[10] Brennan, M.; E. Schwartz; and R. Lagnado. "Strategic Asset Allocation," *Journal of Economic Dynamics and Control*, 21 (1997), 1377–1403.

[11] Brennan, M., and Y. Xia. “Dynamic Asset Allocation under Inflation,” *Journal of Finance,* 57 (2002), 1201–38.

[12] Buraschi, A., P. Porchia, and F. Trojani, “Correlation risk and optimal portfolio choice,” *The Journal of Finance,* 65, (2010), 393–420.

[13] Buss, A., L. Schonleber and G. Vilkov, “Expected Corrrelation and Future Market Returns,” working paper, INSEAD and Frankfurt School of Finance & Management, 2018.

[14] Campbell, J. Y., and L. Viceira. “Consumption and Portfolio Decisions When Expected Returns are Time Varying,” *Quarterly Journal of Economics*, 114 (1999), 433–495.

[15] Campbell, J. Y., and L. Viceira. Strategic Asset Allocation. 2002. Oxford University Press.

[16] Campbell, J. Y.; J. Cocco; F. Gomes; P. Maenhout; and L. Viceira. “Stock Market Mean Reversion and the Optimal Allocation of a Long Lived Investor,” *European Finance Review*, 5 (2001), 269–292.

[17] Campbell, J.; Y. L. Chan; and L. Viceira. “A Multivariate Model of Strategic Asset Allocation,” *Journal of Financial Economics*, 67 (2003), 41–80.

[18] Chacko, G., and L. Viceira, “Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets,” *Review of Financial Studies*, 18 (2005), 1369–1402.

[19] Cocco, J.; F. Gomes; and P. Maenhout. “Consumption and Portfolio Choice over the Life–Cycle,” *The Review of Financial Studies*, 18 (2005), 491–533.

[20] Cooper, R., and G. Zhu. “Household Finance over the Life-Cycle: What does Education Contribute?” *Review of Economic Dynamics*, 20 (2016), 63-89.

[21] Dahlquist, Magnus, Offer Setty and Roine Vestman. “On the Asset Allocation of a Default Pension Fund,” *Journal of Finance*, 73, (2018), 1893-1936.

[22] Davis, S.; F. Kubler; and P. Willen. "Borrowing Costs and the Demand for Equity over the Life Cycle," *The Review of Economics and Statistics,* 88 (2006), 348–362.

[23] Donaldson, S.; F. Kinniry; R. Aliaga-Diaz; A. Patterson; and M. DiJoseph. "Vanguard's Approach to Target-Date Funds," (2013).

[24] Epstein, L., and S. Zin. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57 (1989), 937–969.

[25] Fagereng, A., C. Gottlieb and L. Guiso, "Asset Market Participation and Portfolio Choice over the Life-Cycle," *Journal of Finance,* 72, (2017), 705-750.

[26] Fleming, J., C. Kirby, and B. Ostdiek, "The economic value of volatility timing," *Journal of Finance*, 56 (2001), 329–352.

[27] Fleming, J., C. Kirby, and B. Ostdiek, "The economic value of volatility timing using 'realized' volatility," *Journal of Financial Economics*, 67 (2003), 473–509.

[28] Floden, M. "A Note on the Accuracy of Markov-chain approximations to highly persistent AR(1) processes," *Economics Letters*, 99 (2008), 516–520.

[29] Gomes, F., and A. Michaelides, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, 60 (2005), 869–904.

[30] — and V. Polkovnichenko. "Optimal Savings with Taxable and Tax-Deferred Accounts," *Review of Economic Dynamics*, 12 (2009), 718–735.

[31] Guiso, L. Sapienza, P. and L. Zingales, "Trusting the Stock Market," *Journal of Finance* 63 (2008), 2557-2600.

[32] Johannes, M,, A. Korteweg, and N. Polson, "Sequential learning, predictability, and optimal portfolio returns," *The Journal of Finance* 69 (2014), 611–644.

[33] Kim, T. S., and E. Omberg. "Dynamic Nonmyopic Portfolio Behavior," *Review of Financial Studies*, 9 (1996), 141–161.

[34] Koijen, R.; T. Nijman; and B. Werker. "When Can Life Cycle Investors Benefit from Time-Varying Bond Risk Premia?" *The Review of Financial Studies*, 23 (2010), 741–780.

[35] Lettau, M., and S. Ludvigson, "Consumption, Aggregate Wealth and Expected Stock Returns," *Journal of Finance*, 56, 3, (2001), 815–849.

[36] Lettau, Martin, and Stijn Van Nieuwerburgh, "Reconciling the return predictability evidence," *Review of Financial Studies*, 21 (2008), 1607–1652.

[37] Liu, J., "Portfolio selection in stochastic environments," *Review of Financial Studies*, 20 (2007), 1–39.

[38] Lusardi, A. and O. Mitchell. "The Economic Importance of Financial Literacy: Theory and Evidence," *Journal of Economic Literature*, 52 (2014), 5-44.

[39] Lynch, A., and S. Tan. "Labor Income Dynamics at Business-Cycle Frequencies: Implications for Portfolio Choice," *Journal of Financial Economics*, 101 (2011), 333–359.

[40] Malloy, C.J., T.J. Moskowitz, and A. Vissing-Jorgensen, "Long-run stockholder consumption risk and asset returns," *The Journal of Finance*, 64(6), (2009), 2427-2479.

[41] Michaelides, A. and Y. Zhang, "Stock Market Mean Reversion and Portfolio Choice over the Life Cycle," *Journal of Financial and Quantitative Analysis*, 52-3 (2017), 1183-1209.

[42] Moreira, A., and T. Muir, "Volatility-managed portfolios," *Journal of Finance* 72 (2017), 1611-1644.

[43] Moreira, A., and T. Muir, "Should Long-Term Investors Time Volatility?" *Journal of Financial Economics* 131-3 (2019), 507-27.

[44] Munk, C., and C. Sorensen. "Dynamic asset allocation with stochastic income and interest rates," *Journal of Financial Economics*, 96 (2010), 433–462.

[45] Pastor, L., and R. F. Stambaugh. "Are Stocks Really Less Volatile in the Long Run?" *The Journal of Finance*, 67 (2012), 431–477.

[46] Polkovnichenko, V.; "Life-Cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk," *The Review of Financial Studies*, 20, 1, (2007), 83-124.

[47] Tauchen, G., and R. Hussey. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59 (1991), 371–396.

[48] Sialms, C., L. Starks and H. Zhang, "Defined Contribution Pension Plans: Sticky or Discerning Money?", Working paper, U. Texas at Austin 2013.

[49] Vissing-Jorgensen, A. "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110 (2002), 825–853.

[50] Wachter, J. "Optimal Consumption and Portfolio Allocation with Mean-Reverting returns: An exact solution for complete markets," *Journal of Financial and Quantitative Analysis*, 37 (2002), 63–91.

[51] Weil, P. "Non-Expected Utility in Macroeconomics," *Quarterly Journal of Economics*, 2 (1990), 29–42.

[52] Zhou, H. "Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty," *Annual Review of Financial Economics*, (2017), forthcoming.

Table 1: Descriptive Statistics for Returns and Variance Risk Premium

Table 1 presents descriptive statistics of quarterly data from 1990Q1 to 2016Q3: $r$ denotes the real return on the S&P 500 index (deflating using the consumer price index (CPI)), IV denotes the quarterly "model free" implied variance or VIX index, and RV is the quarterly "model free" realized variance. Inflation ($\pi$) is derived from CPI. This series and the S&P 500 index are from the Center for Research in Security Prices (CRSP).

| Panel A: Summary Statistics | | | | | |
|---|---|---|---|---|---|
| 1990Q1 –2016Q3 | $r$ | $IV$ | $RV$ | $IV - RV$ | $\pi$ |
| Mean (%) | 1.98 | 1.11 | 0.62 | 0.49 | 0.60 |
| SD (%) | 7.84 | 0.94 | 0.98 | 0.75 | 0.80 |
| Kurtosis | 3.24 | 8.16 | 54.23 | 31.83 | 9.64 |
| Skewness | -0.40 | 2.25 | 6.45 | -3.24 | -1.39 |
| AR(1) | 0.00 | 0.41 | 0.47 | -0.17 | 0.001 |

| Panel B: Correlation Matrix | | | | | |
|---|---|---|---|---|---|
| 1990Q1 –2016Q3 | $r$ | $IV$ | $RV$ | $IV - RV$ | $\pi$ |
| $r$ | 1.00 | -0.52 | -0.42 | -0.10 | -0.11 |
| $IV$ | – | 1.00 | 0.70 | 0.34 | -0.18 |
| $RV$ | – | – | 1.00 | -0.43 | -0.46 |
| $IV - RV$ | – | – | – | 1.00 | 0.38 |
| $\pi$ | – | – | – | – | 1.00 |

Table 2: Predictive Regressions

Table 2 presents predictive regressions based on quarterly data from the first quarter of 1990 to the third quarter of 2016. The parameters related to the predictive regression using VRP as a predictor are estimated from the following restricted VAR:

$$\begin{bmatrix} VRP_{t+1} \\ r_{t+1} - r_f \end{bmatrix} = \begin{bmatrix} Const \\ \alpha \end{bmatrix} + \begin{bmatrix} \phi & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} VRP_t \\ r_t - r_f \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ z_{t+1} \end{bmatrix}$$

Newey-West t-statistics are reported in parentheses ($\alpha$ is set to zero).

| 1990Q1 –2016Q3 | $VRP$ | |
|---|---|---|
| Constant | 0.0058 | (6.72) |
| $\alpha$ | 0.0 | |
| $\beta$ | 3.6 | (4.48) |
| $\phi$ | -0.18 | (-1.84) |
| $\rho_{z,\varepsilon}$ | -0.04 | |
| $\sigma_\varepsilon$ | 0.0074 | |
| $\sigma_z$ | 0.0746 | |
| $\sigma_r$ | 0.079 | |
| Adj. $R^2$ (%) | 15 | |

Table 3: Sensitivity of Household Consumption Growth to VRP Across Horizons

Table 3 presents the sensitivity of different moments of stockholder and non-stockholder consumption growth to the variance risk premium (VRP) over horizons of S = 1, 2, 12, and 24 quarters. Panel A reports the sensitivity of mean consumption growth, while Panels B, C and D report the results for Standard Deviation, Skewness and Kurtosis, respectively. The sensitivity is computed as the regression coefficient from regressing a group's consumption growth over horizon S on current VRP. Below each entry we include the t-stat. Standard errors are computed using a Newey-West estimator that allows for autocorrelation of up to $S-1$ lags when $S > 1$.

| Panel A: Mean consumption Growth | | | | |
|---|---|---|---|---|
| S | 1 | 2 | 12 | 24 |
| Stockholders | 1.15 | 0.58 | -0.24 | 0.90 |
| (t-stat) | (1.59) | (0.98) | (0.29) | (0.99) |
| Non-Stockholders | 0.43 | 0.19 | 0.50 | 0.88 |
| (t-stat) | (1.34) | (0.50) | (0.86) | (1.57) |

| | Panel B: Std. Dev. | | Panel C: Skewness | | Panel D: Kurtosis | |
|---|---|---|---|---|---|---|
| S | 1 | 2 | 1 | 2 | 1 | 2 |
| Stockholders | 0.39 | -0.89 | 0.96 | 3.39 | -8.37 | 14.08 |
| (t-stat) | (0.85) | (1.78) | (0.31) | (0.64) | (1.21) | (0.93) |
| Non-Stockholders | 0.18 | 0.04 | 2.81 | 2.27 | -4.73 | -4.90 |
| (t-stat) | (0.58) | (0.18) | (1.60) | (0.91) | (1.03) | (1.08) |

Table 4: TTDF portfolio rule for different values of risk aversion

Table 4 presents the regression of simulated portfolios on age and factor realizations across different relative risk aversion coefficients (2, 5, 10). More precisely, we use the simulated output from the model to estimate: $\alpha_{iat} = \theta_0 + \theta_1 * a + \theta_2 * f_t + \varepsilon_{iat}$.

| | $VRP$ | $i.i.d.$ | $\gamma = 2$ | $\gamma = 10$ |
|---|---|---|---|---|
| $\theta_0$ | 0.51 | 1.06 | 0.46 | 0.26 |
| $\theta_1$ | $-0.00191$ | $-0.00308$ | $-0.000312$ | $-0.00128$ |
| $\theta_2$ | 45.6 | | 45.1 | 43.0 |
| $R^2$ | 74% | 45% | 58% | 73% |

Table 5: Welfare gains from the TTDF

Table 5 presents results from comparing the TTDF with the standard TDF for different relative risk aversion coefficients and additional transaction costs from trading the TTDF. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. Results are shown for different values of risk aversion ($\gamma$) and different magnitudes of the additional transaction costs faced by the TTDF relative to the TDF ($tc$), expressed in annualized basis points. The results are reported in percentages.

| $\gamma$ | | 2 | | | 5 | | | 10 | |
|---|---|---|---|---|---|---|---|---|---|
| $tc$ (inc.) | 0 | 40 | 100 | 0 | 40 | 100 | 0 | 40 | 100 |
| $W_{65}$ (% inc.) | 103 | 83 | 57 | 182 | 161 | 131 | 312 | 293 | 265 |
| $Std(W_{65})$ (% inc.) | 97 | 77 | 50 | 248 | 223 | 187 | 506 | 491 | 466 |
| Age-65 CE Gain | 20.3 | 15.9 | 10.1 | 40.5 | 34.4 | 26.2 | 80.3 | 73.7 | 64.4 |

Table 6: Welfare gains from the TTDF with turnover restrictions

Table 6 presents results from comparing the TTDF with the standard TDF for different rebalancing restrictions and transaction costs. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. Results are shown for different values of risk aversion ($\gamma$), different magnitudes of the additional transaction costs ($tc$) faced by the TTDF relative to the TDF, expressed in annualized basis points, and different maximum rebalancing constraints. The results are reported in percentages.

| Risk Aversion | 2 | | | | 5 | | | | 10 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Max Rebalancing | 25 | 25 | 15 | 15 | 25 | 25 | 15 | 15 | 25 | 25 | 15 | 15 |
| $tc$ (inc.) | 0 | 40 | 0 | 40 | 0 | 40 | 0 | 40 | 0 | 40 | 0 | 40 |
| Mean Turnover | 108 | 108 | 72 | 72 | 106 | 106 | 69 | 69 | 100 | 100 | 66 | 66 |
| $W_{65}$ (% inc.) | 23 | 10 | 0.6 | $-11$ | 45 | 32 | 14 | 3.4 | 141 | 128 | 94 | 83 |
| $Sd(W_{65})$ (% inc.) | $-2$ | $-14$ | $-27$ | $-36$ | 48 | 33 | 5.0 | $-5.5$ | 275 | 255 | 190 | 173 |
| Age-65 CE Gain | 4.9 | 1.8 | 0.24 | $-2.5$ | 11.1 | 7.2 | 3.7 | 0.35 | 38.5 | 34.0 | 26.2 | 22.3 |

Table 7: Welfare gains from the TTDF with Parameter Uncertainty

Table 7 presents results from comparing the TTDF with the standard TDF when the TTDF rules incorporate parameter uncertainty as described in the main text. Results are shown for different rebalancing restrictions and transaction costs. These results are for the investor with risk aversion equal to 5, and for different magnitudes of the additional transaction costs faced by the TTDF relative to the TDF ($tc$), expressed in annualized basis points, and for different maximum rebalancing constraints (15%, 25% and no rebalancing constraints, denoted by (No Con)). The results are reported in percentages.

| Maximum Rebalancing | No Con | | 25 | | 15 | |
|---|---|---|---|---|---|---|
| $tc$ (inc.) | 0 | 40 | 0 | 40 | 0 | 40 |
| Average Turnover | 212 | 212 | 106 | 106 | 69 | 69 |
| $W_{65}$ (% inc.) | 178 | 157 | 44 | 32 | 14 | 3.4 |
| $Std(W_{65})$ (% inc.) | 245 | 220 | 48 | 33 | 5.4 | $-5.5$ |
| Age-65 CE Gain | 40.1 | 34.0 | 11.0 | 7.2 | 3.7 | 0.35 |

Table 8: Predictive Regressions for Different Sub-Periods

Table 8 presents restricted VAR based on quarterly data for different sub-periods of the sample: 1990:1 to 1999:4, 2000:1 to 2009:4 and 2010:1 to 2016:3. The restricted VAR is given by:

$$\begin{bmatrix} VRP_{t+1} \\ r_{t+1} - r_f \end{bmatrix} = \begin{bmatrix} Const \\ \alpha \end{bmatrix} + \begin{bmatrix} \phi & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} VRP_t \\ r_t - r_f \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ z_{t+1} \end{bmatrix}$$

Newey-West t-statistics are reported in parentheses ($\alpha$ is set to zero).

| | 1990Q1 –1999Q4 | | 2000Q1 –2009Q4 | | 2010Q1 –2016Q3 | |
|---|---|---|---|---|---|---|
| Constant | 0.0051 | (4.21) | 0.0055 | (3.42) | 0.0047 | (3.47) |
| $\alpha$ | 0.0 | | 0.0 | | 0.0 | |
| $\beta$ | 5.85 | (4.88) | 2.01 | (1.62) | 5.23 | (2.85) |
| $\phi$ | 0.18 | (1.15) | -0.30 | (-2.01) | -0.12 | (-0.60) |
| $\rho_{z,\varepsilon}$ | -0.41 | | 0.12 | | -0.61 | |
| $\sigma_\varepsilon$ | 0.0050 | | 0.0098 | | 0.0055 | |
| $\sigma_z$ | 0.0637 | | 0.0893 | | 0.0649 | |
| $\sigma_r$ | 0.0702 | | 0.0917 | | 0.0710 | |
| Adj. $R^2$ (%) | 34.6 | | 3.6 | | 21.4 | |

Table 9: Welfare gains from the Extended Tactical TDF (TTDF2)

Table 9 presents results from comparing the TTDF2 with the standard TDF for different relative risk aversion coefficients and additional transaction costs from trading the TTDF2. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF2 the portfolio allocation also depends on the variance risk premium (VRP), by considering different linear functions of the age for each realization of the VRP. Results are shown for different values of risk aversion ($\gamma$) and different magnitudes of the additional transaction costs ($tc$) faced by the TTDF2 relative to the TDF, expressed in annualized basis points. The results are reported in percentages.

| $\gamma$ | | 2 | | | 5 | | | 10 | |
|---|---|---|---|---|---|---|---|---|---|
| $tc$ (inc.) | 0 | 40 | 100 | 0 | 40 | 100 | 0 | 40 | 100 |
| $W_{65}$ (% inc.) | 201 | 172 | 134 | 260 | 234 | 198 | 377 | 358 | 331 |
| $Std(W_{65})$ (% inc.) | 289 | 254 | 205 | 363 | 336 | 297 | 570 | 561 | 546 |
| Age-65 CE Gain | 37.8 | 31.6 | 23.5 | 55.3 | 48.2 | 38.6 | 95.0 | 88.4 | 78.9 |

Table 10: Welfare gains from the TTDF with less-tight short selling constraints

Table 10 presents results from comparing the TTDF with the standard TDF when both funds are allowed to invest up to 200% in the risky asset. Results are shown for different rebalancing restrictions and transaction costs. In the standard TDF the portfolio allocation rule is a linear function of age only. Under the TTDF the portfolio allocation also depends on the variance risk premium (VRP), which enters as an additional variable in the linear regression. These results are for the case of the investor with risk aversion of 5 (for both funds), different magnitudes of the additional transaction costs ($tc$) faced by the TTDF relative to the TDF, expressed in annualized basis points, and different maximum rebalancing constraints (15% and 25%). The results are reported in percentages.

| Maximum Rebalancing | 100 | | | 25 | | | 15 | | |
|---|---|---|---|---|---|---|---|---|---|
| $tc$ (inc.) | 0 | 40 | 100 | 0 | 40 | 100 | 0 | 40 | 100 |
| Average Turnover | 360 | 360 | 360 | 114 | 114 | 116 | 73 | 73 | 74 |
| $W_{65}$ (% inc.) | 572 | 523 | 452 | 65 | 44 | 16 | 24 | 7 | $-15$ |
| $Std(W_{65})$ (% inc.) | 615 | 599 | 569 | 108 | 82 | 49 | 41 | 21 | $-2.5$ |
| Age-65 CE Gain | 91.8 | 81.5 | 67.3 | 13.5 | 8.1 | 1.1 | 5.2 | 0.6 | $-5.0$ |

Table 11: Welfare gains from using a TTDF both during working life and retirement

Table 11 presents summary statistics comparing results between the VRP model and the i.i.d. model for the baseline specification for different rebalancing restrictions and transaction costs. The portfolio allocations of both the i.i.d. and the VRP investors are given by the corresponding funds both during working life, TDF and TTDF respectively, and during retirement. The asset allocations of the retirement funds are constructed following the same procedure as for the pre-retirement funds. These results are for the case of the investor with risk aversion of 5 (for both funds), different magnitudes of the additional transaction costs ($tc$) faced by the TTDF relative to the TDF, expressed in annualized basis points, and different maximum rebalancing constraints (15% and 25%). Percentage changes reported.

| Maximum Rebalancing | 25 | | 15 | |
|---|---|---|---|---|
| $tc$ (inc.) | 0 | 40 | 0 | 40 |
| Average Turnover | 108 | 108 | 69 | 69 |
| $W_{65}$ (% inc.) | 63 | 41 | 34 | 13 |
| $Std(W_{65})$ (% inc.) | 107 | 71 | 55 | 22 |
| Age-65 CE Gain | 30.0 | 21.6 | 20.1 | 12.3 |

**Figure 1: The Variance Risk Premium**

Figure 1 shows the time series of implied volatility (IV), realized volatility (RV) and variance risk premium (VRP). The series for realized volatility is taken from Zhou (2017) and based on daily US stock market returns from CRSP, while the series for implied volatility is taken from the Federal Reserve Bank of St. Louis. The variance risk premium is the difference between the other two. All data are quarterly from 1990 to 2016.

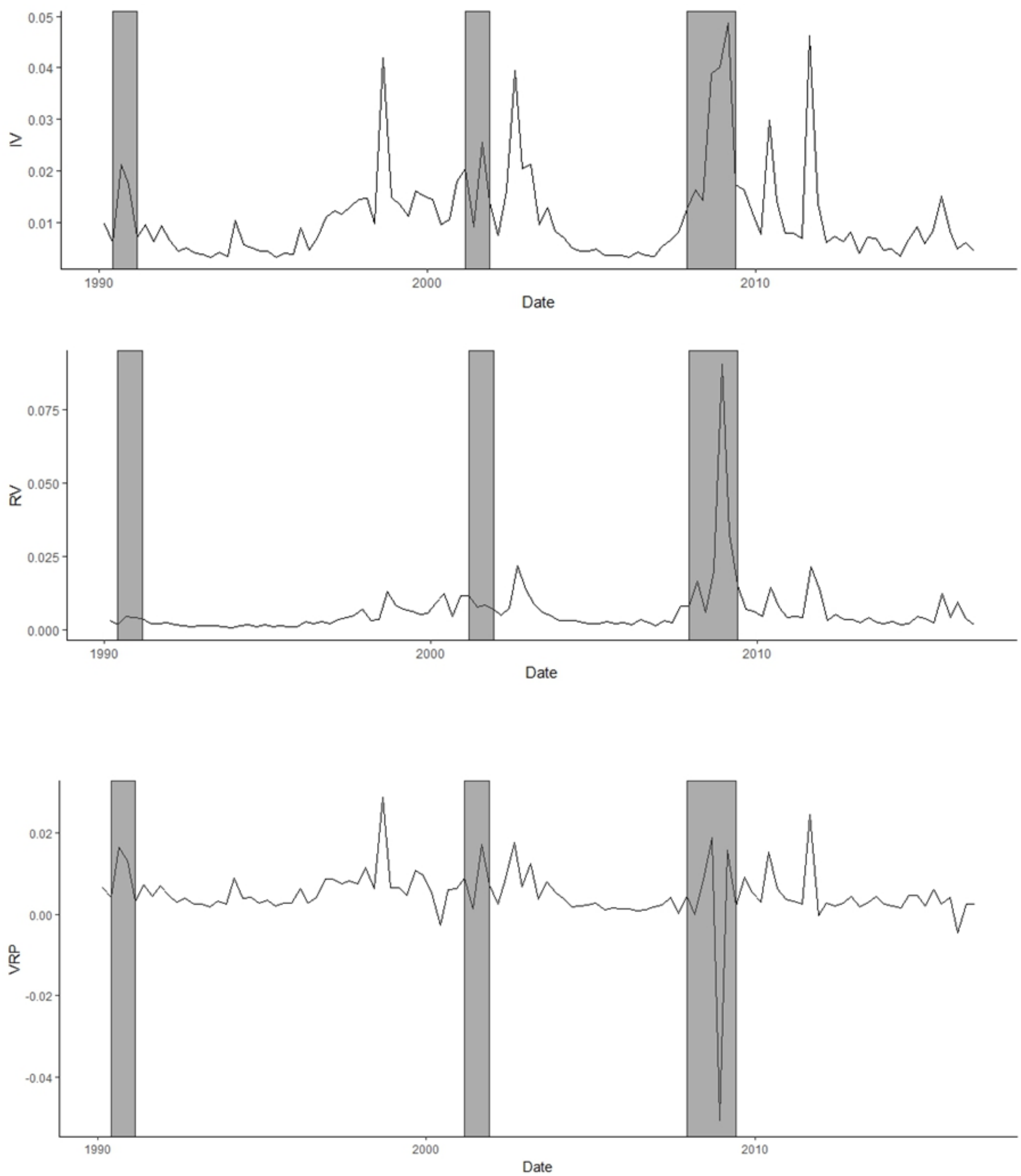


**Figure 2: Average portfolio allocations over the life cycle**

Figure 2 shows the optimal pre-retirement portfolio allocations both for the investor using the i.i.d. model for returns and for the investor using the VRP predictor. For the VRP investor we report both the average allocation and the allocation for the average realization of the predictive factor. In both cases, risk aversion equals 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

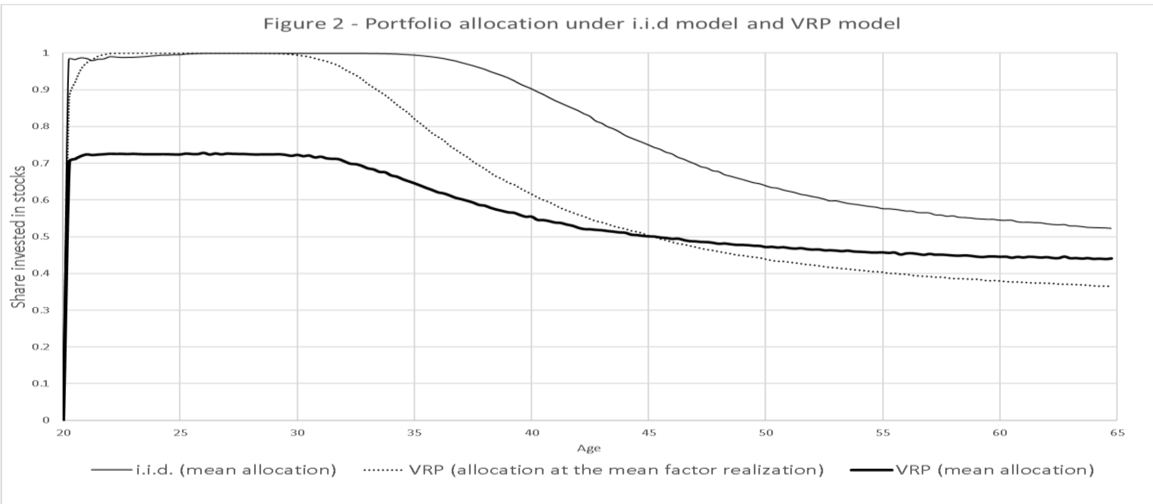



**Figure 3: Expected stock returns for i.i.d. and VRP models**

Figure 3 shows the expected portfolio return both for the investor using the i.i.d. model for returns and for the investor using the VRP predictor. To facilitate comparisons we also plot the difference between the two. In both cases, risk aversion equals 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

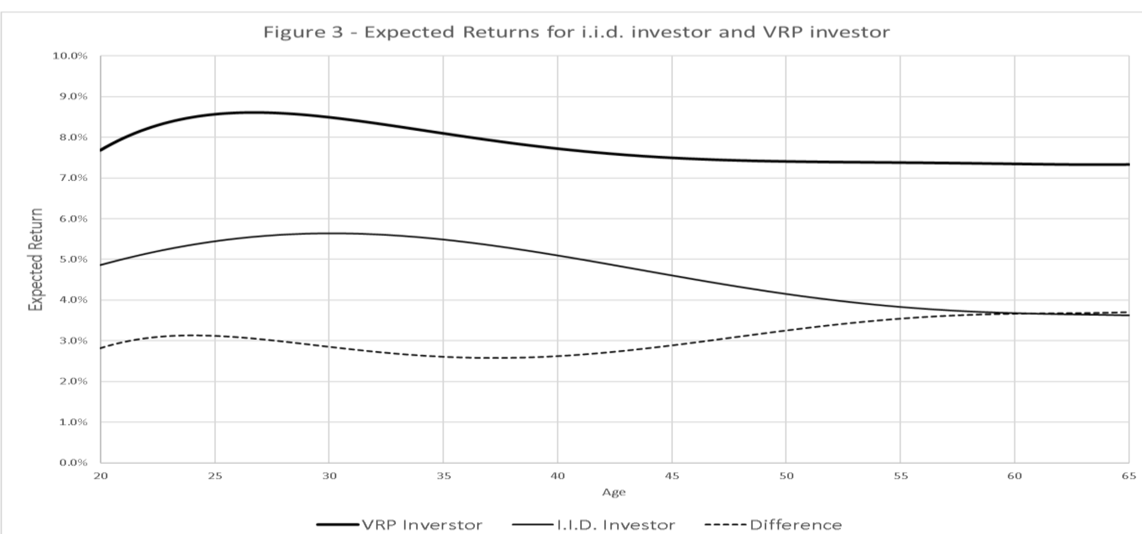


**Figure 4: Portfolio allocation of TDF and TTDF**

Figure 4 shows the mean life-cycle portfolio allocation of both the Target-Date Fund (TDF) and the Tactical Target-Date Fund (TTDF). For comparison we also report the average optimal asset allocation of the investor using the i.i.d. model for returns ("i.i.d. investor") and for the investor using the VRP predictor ("VRP investor"). In both cases, risk aversion equals 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

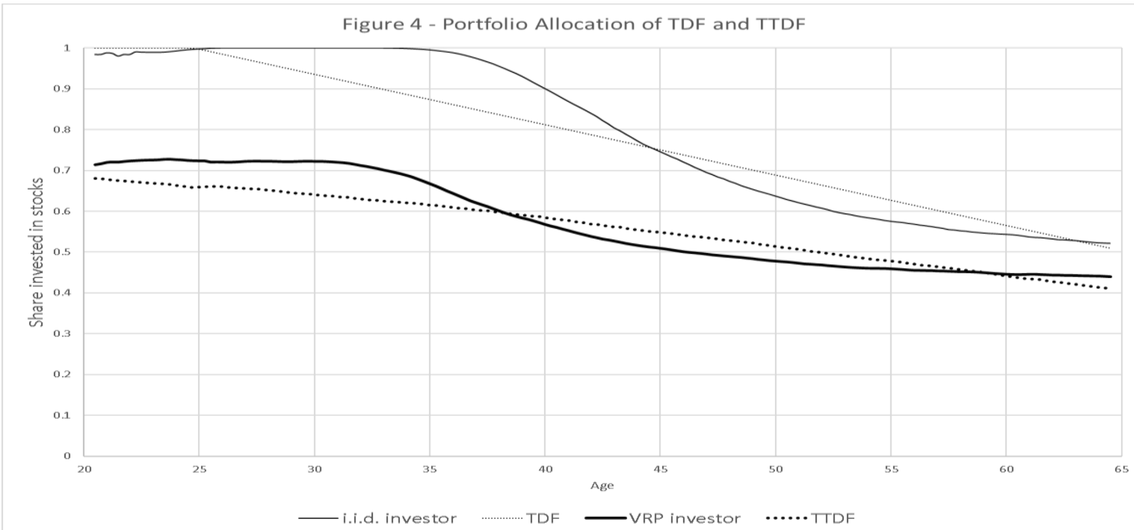



**Figure 5: Portfolio allocation of TTDF with turnover restrictions**

Figure 5 shows the life-cycle portfolio allocation of the Tactical Target-Date Fund (TTDF), both with and without turnover restrictions for different predictive factor (f) realizations. The value of f=0.494% corresponds to its unconditional mean. The values of 1.35% and -0.36% correspond to 1.14 standard deviations above and below the mean, respectively. The case with turnover restrictions considers a maximum turnover limit of 15% per quarter. In both cases, risk aversion is 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

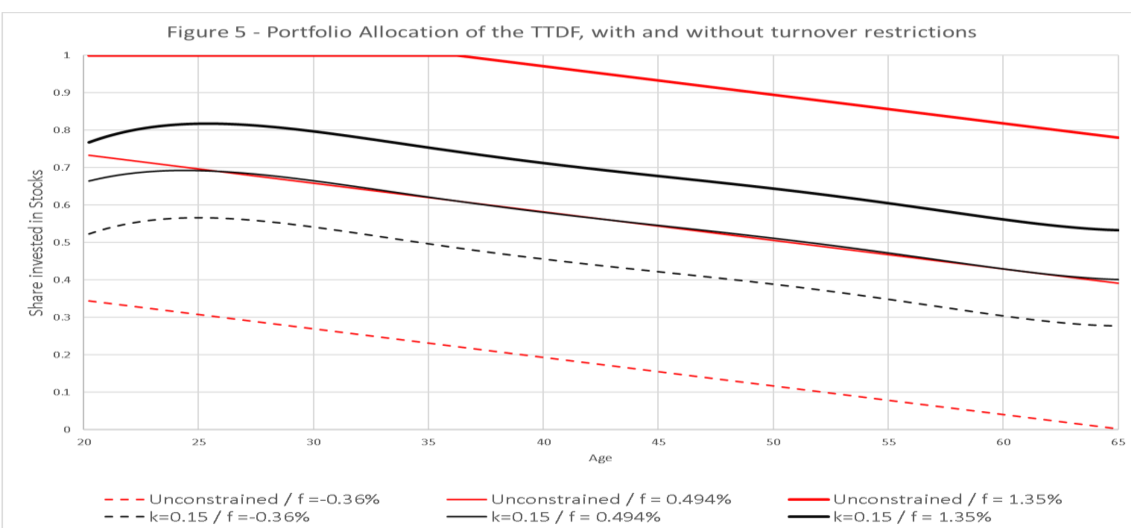


**Figure 6.1: Cumulative out-of-sample returns for Age 20 Investor**

Figure 6.1 shows the results of an out-of-sample comparison between the TTDF (with a 15% turnover restriction) and the TDF. The figure reports the cumulative return to TTDF and TDF funds for a 20-year investor from 2000 onwards, when the TTDF portfolio allocation is based on an estimation of the predictive model that only uses data until 1999. In both cases, risk aversion is 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

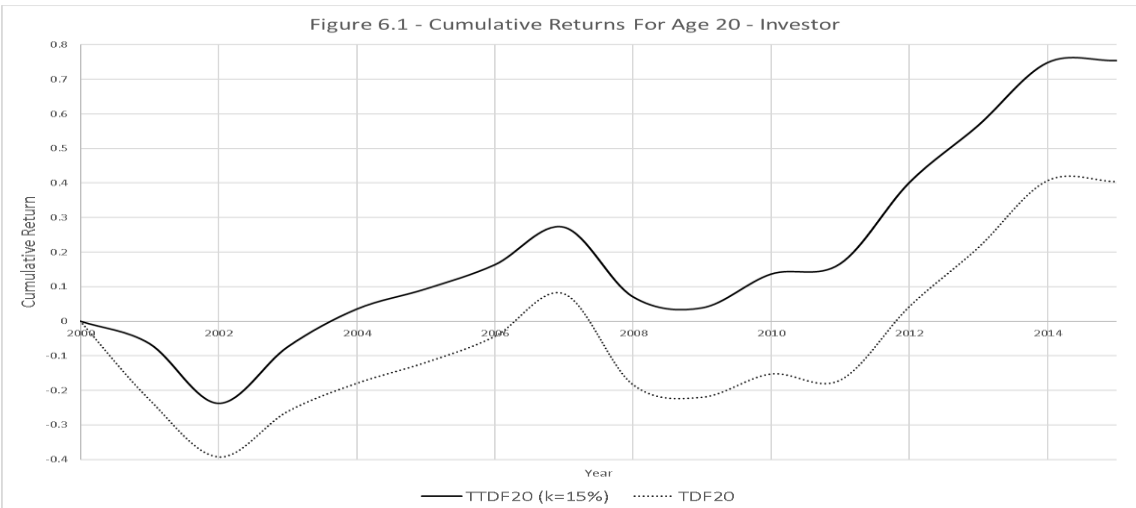



**Figure 6.2: Period-by-period out-of-sample returns for Age 20 Investor**

Figure 6.2 shows the results of an out-of-sample comparison between the TTDF (with a 15% turnover restriction) and the TDF. The figure reports the annualized period-by-period return to the TTDF and TDF funds for the 20-year investor from 2000 onwards, when the TTDF portfolio allocation is based on an estimation using data until 1999. In both cases, risk aversion is 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

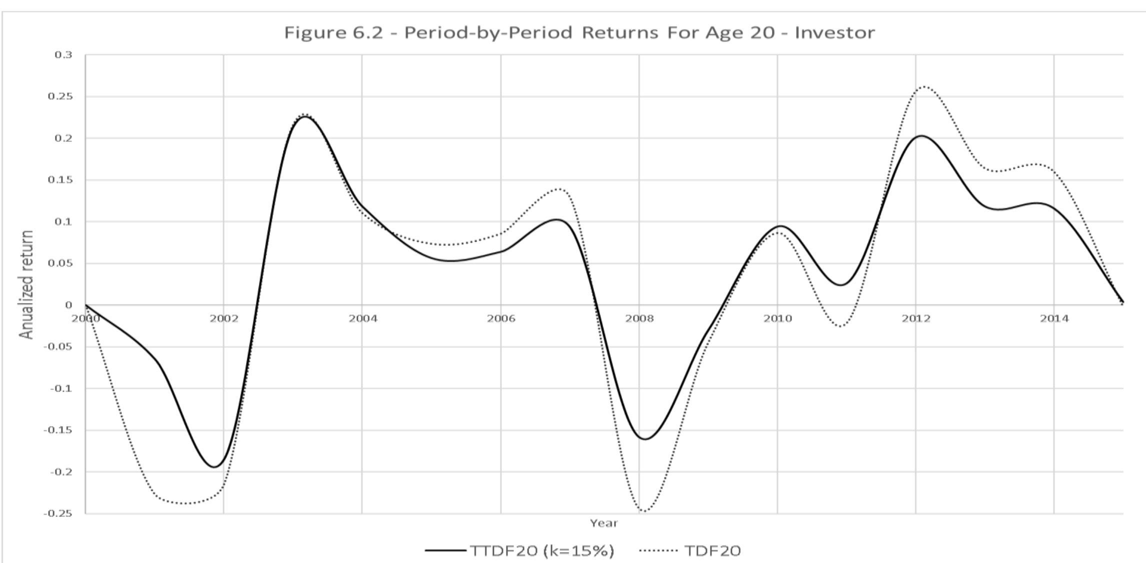


**Figure 7.1: Cumulative out-of-sample returns for Age 50 Investor**

Figure 7.1 shows the results of an out-of-sample comparison between the TTDF (with a 15% turnover restriction) and the TDF. The figure reports the cumulative return to the TTDF and TDF funds for the 50-year investor from 2000 onwards, when the TTDF portfolio allocation is based on an estimation using data until 1999. In both cases, risk aversion is 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.

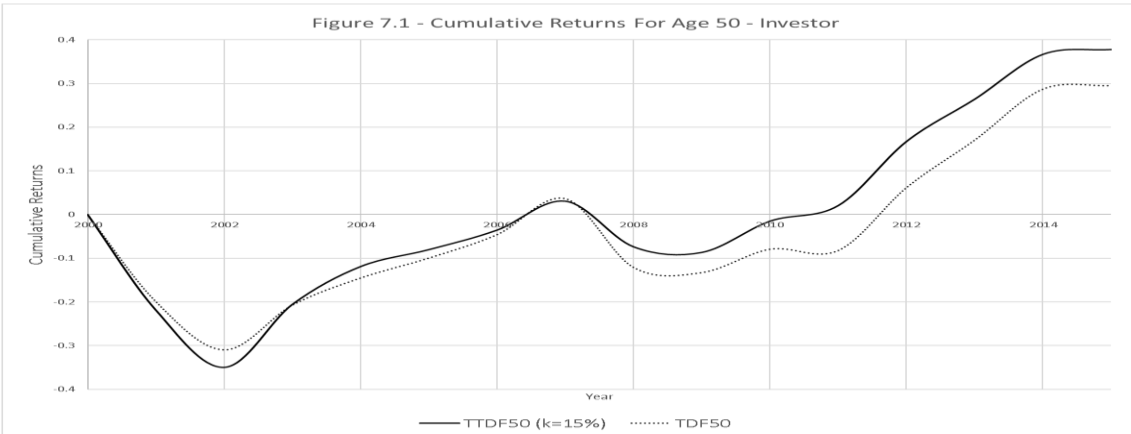
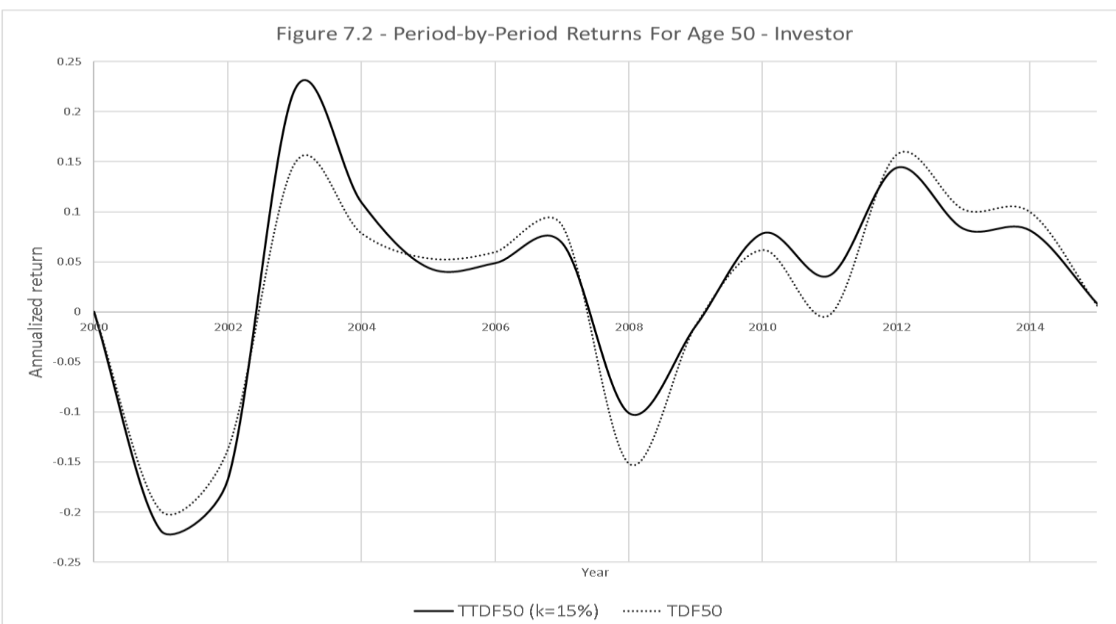



**Figure 7.2: Period-by-period out-of-sample returns for Age 50 Investor**

Figure 7.2 shows the results of an out-of-sample comparison between the TTDF (with a 15% turnover restriction) and the TDF. The figure reports the annualized period-by-period return to the TTDF and TDF funds for the 20-year investor from 2000 onwards, when the TTDF portfolio allocation is based on an estimation using data until 1999. In both cases, risk aversion is 5, the elasticity of intertemporal substitution is 0.5 and the quarterly discount factor is 0.9875.



**Tactical Target Date Funds**

**Internet Appendix**

# 1 Consumer Expenditure Survey

We use data on non-durable consumption and services from the Consumer Expenditure Survey. In this appendix we describe in more detail how we construct the variables used in the regressions in the paper.

## 1.1 Data Description

The Consumer Expenditure Survey (CEX, thereafter) provides data on expenditures, income, and demographic characteristics of consumers in the United States. CEX data are collected by the Census for Bureau of Labor Statistics (BLS, thereafter) in two surveys: the Quarterly Interview Survey (Interview) and the Diary Survey (Diary). The Interview generally tracks consumer units' large expenditures, such as major appliances and cars, while the Diary tracks smaller, everyday expenditures that might be easily forgotten even after a few days, such as a cup of coffee.[1] The CEX data is available from 1996 to 2015 on the website of BLS. In the CEX survey, each household is interviewed every quarter for five times, consecutively. The first interview is not in the data files; the subsequent four interviews are numbered from 2 to 5 (from 2016, the number of the interview is renumbered from 1 to 4). While each household is interviewed at 3-month intervals, interviewers are spread out over the quarter. This means that the CEX data is a rotating panel as shown in Table 1. The NewID is the unique identification number of a household.[2]

The CEX survey consists of six types of data files:

- FMLI - a file with characteristics, income, weights, and summary level expenditures

[1]The CEX data can be downloaded from $https://www.bls.gov/cex/pumd_data.htm\#csv$ or from ICPSR of University of Michigan (ICPSR misses the 2012 data).

[2]For simplicity, we use 1,2 and 3 to represent the household instead of their real NewID. In CEX data, the newid uses 7 digits to represent a household, for example, 2814295. The first 6 digits are also called cuid and the last digit is the interview order which is normally from 2 to 5. But, from 2016, the interview order (the last digit) is renumbered to range from 1 to 4.

for the CU.

- MEMI - a file with characteristics and income for each member in the CU.
- MTBI - a detailed monthly expenditure file categorized by Universal Classification Code (UCC).
- ITBI - a Consumer Unit monthly income file categorized by UCC.
- ITII - a Consumer Unit monthly imputed income file categorized by UCC.4
- NTAXI - a file with federal and state tax information for each tax unit in the CU.5

The files we use in the paper are called the "FMLI" file in the Interview Survey, also refered to as the "Consumer Unit Characteristics and Income" file. It is the summary of the quarterly expenditure of a household. The CEX files' numbering rule is in the form of YYQ, where YY is the 2-digit year and Q is the order of the quarter in the year. For example, the quarterly interview file FMLI in the second quarter of 2010 is FLMI102.[3]

Based on table 1, for each household, we can calculate quarterly consumption growth rates based on reported monthly consumption values. For any household interview, every expenditure item consists of two variables: cq and pq, which stand for consumption in the current quarter and for the previous quarter, respectively. For example, alcbevcq is the alcohol beverages consumed in the current quarter, and alcbevpq refers to the alcohol beverages consumed in the previous quarter.

It is important to remember that the expenditures in the previous and current quarter have different definitions across the different interviews. For instance, in Table 1, the expenditure items for NewID=1 in this quarter are all 0 (the items ended with cq or c, cq items

[3]There is an "X" in the name of quarter 1 files for the current calendar year. This is because files for the first quarter of any calendar year will appear in two CE releases:

- As the "fifth" quarter file in the previous calendar year's release,
- And as the "first" quarter file in the current calendar year's release.

The "X" signifies that the first quarter file of the current calendar year release is not identical to the fifth quarter file of the previous calendar year release. One reason the files are not identical arises from data production processes using the five quarters of data within the release to calculate certain values, such as the critical and topcoded values of income and expenditures.

thereafter) because the interview occurs in the beginning of the quarter. But, his/her expenditure items for the previous quarter (the items ended with pq or p, pq items thereafter) are complete, representing the total expenditure for the previous calendar quarter. So, his/her consumption growth rate in April (the 3rd interview) is defined as the pq items of the 4th interview (the interview reported in July) divided by the pq items of the current interview (the 3rd interview which is done in April).

The cq items for NewID=2 refer to the consumption in the first month of the current quarter, and the pq items are the expenditures in the last two months of the last calendar quarter. For example, the cq items of the 3rd interview in May are the expenditures in April, and the pq items of the current interview (the 3rd interview) are the expenditures in February and March. So, his/her consumption growth rate in May (the 3rd interview) is defined as the sum of pq and cq items of the 4th interview divided by the sum of pq and cq items of the current interview (the 3rd interview).

Similarly, the cq items for NewID=3 refer to the consumption in the first two months of the current quarter, and the pq items are the expenditures in the last month of the previous calendar quarter. For example, the cq items of the 3rd interview in June are the expenditures in April and May, and the pq items of the current interview (the 3rd interview) are the expenditures in March. So, his/her consumption growth rate in June (the 3rd interview) is defined as the sum of pq and cq items of 4th interview divided by the sum of pq and cq items of current interview (the 3rd interview).

This procedure gives us a time series of consumption growth rates for each household. The pq and cq items used in our calculation are as follows:

- RELECTR: Expenditures on electricity for rented vacation homes (1999Q2-2015).
- RFUELOI: Expenditures on fuel oil for rented vacation homes (1999Q2-2015).
- RNATLGA: Expenditures on natural gas for rented vacation homes (1999Q2-2015).
- ROTHRFL: Expenditures on other fuels for rented vacation homes (1999Q1-2015).
- RWATERP: Expenditures on water and public services for rented vacation homes (1999Q1-2015).

- TAIRFAR: Trip expenditures on airfare (1999Q1-2015).
- TALCBEV: Total trip expenditures on alcoholic beverages at restaurants, cafes, and bars (1999Q1-2015).
- TCARTRK: Trip expenditures on car or truck rental (1999Q1-2015).
- TFEESAD: Trip expenditures on miscellaneous entertainment including recreation expenses, participation sport fees, and admission fees to sporting events and movies (1999Q1-2015).
- TFOODAW: Food and non-alcoholic beverages at restaurants, cafes, and fast food places during out-of-town trips (1999Q1-2015).
- TFOODHO: Food and beverages purchased and prepared by CU during out-of-town trips (1999Q1-2015).
- TGASMOT: Trip expenditures on gas and oil (1999Q1-2015).
- TLOCALT: Trip expenditures on local transportation including taxis, buses etc (1999Q1-2015).
- TOTHENT: Trip expenditures on recreational vehicle rentals including campers, boats, and other vehicles (1999Q1-2015).
- TOTHFAR: Tip expenditures on other transportation fares including intercity bus and train fare, and ship fare (1999Q1-2015).
- TOTHRLO: Total trip expenditures on lodging including rent for vacation home, and motels (1999Q1-2015).
- TOTHTRE: Trip expenditures for other transportation expenses including parking fees, and tolls (1999Q1-2015).
- VELECTR: Expenditures on electricity for owned vacation homes (1999Q1-2015).
- VFUELOI: Expenditures on fuel oil for owned vacation homes (1999Q1-2015).

- VMISCHE: Expenditures on miscellaneous household equipment for owned vacation homes (1999Q1-2015).
- VNATLGA: Expenditures on natural gas for owned vacation homes (1999Q1-2015).
- VOTHRFL: Expenditures on other fuels for owned vacation homes (1999Q1-2015).
- VOTHRLO: Expenditures on owned vacation homes including mortgage interest, insurance, taxes, and maintenance (1999Q1-2015).
- VWATERP: Expenditures on water and public services for owned vacation homes (1999Q1-2015).
- TOTHVHR: Trip expenditures on other vehicle rentals (1999Q1-2015, but changed in 2006).
- ECARTKN: Outlays for new vehicle purchases including down payment, principal and interest paid on loans, or if not financed, purchase amount (2000Q1-2015).
- ECARTKU: Outlays for used vehicle purchases including down payment, principal and interest paid on loans, or if not financed, purchase amount (2000Q1-2015).
- EENTMSC: Miscellaneous entertainment outlays including photographic and sports equipment and boat and RV rentals (2000Q1-2015, and changed in 2005Q2).
- EMISCMT: Mortgage principal outlays for other property (2000Q1-2015).
- EMOTRVH: Outlays for motored recreational vehicles (2000Q1-2015).
- EMRTPNV: Mortgage principal outlays for owned vacation home (2000Q1-2015).
- ENOMOTR: Outlays for non-motored recreational vehicles (2000Q1-2015).
- EOTHVEH: Outlays for other vehicle purchases including down payment, principal and interest paid on loans, or if not financed, purchase amount (2000Q1-2015).
- ALCBEV: Alcoholic beverages (1996Q1-2015, and changed in 2007Q2).

- bbyday: Babysitting and child day care (1996Q1-2015, and changed in 2013Q2).
- BOYFIF: Clothing for boys, 2 to 15 (1996Q1-2015, and changed in 2007Q2).
- CHLDRN: Clothing for children under 2 (1996Q1-2015).
- ELCTR: Electricity (1996Q1-2015).
- FDHOME: Food at home (1996Q1-2015, and changed in 2007Q2).
- FDMAP: Meals as pay (1996Q1-2015).
- FDXMAP: Food away excluding meals as pay (1996Q1-2015).
- FEEADM: Fees and admissions (1996Q1-2015, and changed in 2013Q2).
- FLRCVR: Floor coverings (1996Q1-2015, and changed in 1999, 2011, 2013 respectively).
- FOOTWR: Footwear (1996Q1-2015).
- FULOIL: Fuel oil (1996Q1-2015).
- FURNTR: Furniture (1996Q1-2015).
- GASMO: Gasoline and motor oil (1996Q1-2015).
- GRLFIF: Clothing for girls, 2 to 15 (1996Q1-2015, changed in 2017Q2).
- MAINRP: Maintenance and repairs (1996Q1-2015, and changed in 2005, 2013).
- MAJAPP: Major appliances (1996Q1-2015, and changed in 2013).
- MENSIX: Clothing for men, 16 and over (1996Q1-2015, and changed in 2007).
- MISC1: Miscellaneous expenditures (1996Q1-2015, and changed in 2001, 2005).
- MISC2: Miscellaneous expenditures (1996Q1-2015).
- MISCEQ: Miscellaneous household equipment (1996Q1-2015, and changed in 2005, 2007,2011 and 2013).

- MRPINS: Maintenance, repairs, insurance, and other expenses (1996Q1-2015).
- NTLGAS: Natural gas (1996Q1-2015).
- OTHENT: Other entertainment (1996Q1-2015, and changed in 1999,2005,2007).
- OTHFLS: Other fuels (1996Q1-2015, and changed in 2005).
- OTHHEX: Other household expenses (1996Q1-2015, and changed in 1999, 2001, 2005, 2007).
- OTHLOD: Other lodging (1996Q1-2015, and changed in 2007Q2).
- PERSCA: Personal care (1996Q1-2015, and changed in 1999Q2). It is not medical care.
- PETTOY: Pets, toys, and playground equipment (1996Q1-2015, and changed in 2004, 2011).
- READ: Reading (1996Q1-2015, and changed in 2005, 2011, 2013).
- RNTXRP: Rent excluding rent as pay. For example painting wallpaper (1996Q1-2015).
- SMLAPP: Small appliances, miscellaneous housewares (1996Q1-2015).
- TELEPH: Telephone services (1996Q1-2015, and changed in 2001, 2006, 2007,2013).
- TEXTIL: Household textiles (1996Q1-2015).
- TOBACC: Tobacco and smoking supplies (1996Q1-2015).
- TRNOTH: Local public transportation, excluding on trips (1996Q1-2015).
- TRNTRP: Public and other transportation on trips (1996Q1-2015).
- VEHFIN: Vehicle finance charges (1996Q1-2015).
- VEHINS: Vehicle insurance (1996Q1-2015)

- VRNTLO: Vehicle rental, leases, licenses, and other charges (1996Q1-2015, and changed in 2001, 2005, 2006, 2013).
- WATRPS: Water and other public services (1996Q1-2015).
- WOMSIX: Clothing for women, 16 and over (1996Q1-2015, and changed in 2007).
- DMSXCC: Domestic services excluding child care (1996Q1-2015).

In our model, the total expenditure is the sum of variables stated above.[4]

## 1.2 Clean Data

The following data are excluded from our paper.

- Drop household-quarters in which a household reports nonzero consumption for more than 3 or less than 3 months or where consumption is negative.
- Drop extreme outliers because these may reflect reporting or coding errors. Specifically, we drop observations for which the consumption growth ratio $(C^h_{t+1}/C^h_t)$ is less than 0.2 or above 5.0.
- Drop non-urban households (missing for part of the sample) and households residing in student housing, and households with incomplete income responses.
- Drop households reporting a change in age of household head between any two interviews different from 0 or 1 year.

## 1.3 Stockholder Status

The CEX contains information about four categories of financial assets, which is only reported in the 4th interview. We use the following variables to determine the stockholder status of a household.

[4]Note that all variables have two parts: cq and pq. For example, WOMSIXCQ is the clothing for women, 16 and over in the current quarter and WOMSIXPQ is the clothing for women, 16 and over in the previous quarter.

- COMPSECX: The difference in the estimated market value of all stocks, bonds, mutual funds, and other such securities held by the CU last month compared with the value of all securities held a year ago last month (valid from 1996Q1 to 2013Q1).

- PURSSECX: Purchase price of stocks, bonds, or mutual funds including broker fees bought by CU in past 12 months (valid from 1996Q1 to 2013Q1).

- SECESTX: Estimated market value of all stocks, bonds, mutual funds, and other such securities held by the CU on the last day of the previous month (valid from 1996Q1 to 2013Q1).

- SELLSECX: Net amount the CU received from sales of stocks, bonds, or mutual funds after subtracting broker fees (valid from 1996Q1 to 2013Q1).

- STOCKB: Range which best reflects the total value of all directly-held stocks, bonds, and mutual funds (valid from 2013Q2).
  - $0 - $1999
  - $2,000 - $9,999
  - $10,000 - $49,999
  - $50,000 - $199,999
  - $200,000 - $449,999
  - $450,000 and over

- STOCKX: As of today, what is the total value of all directly-held stocks, bonds, and mutual funds (valid from 2013Q2)?

- STOCKYRB: Range which best reflects the total value of all directly-held stocks, bonds, and mutual funds one year ago today (valid from 2013Q2)
  - $0 - $1999
  - $2,000 - $9,999

- $10,000 - $49,999
- $50,000 - $199,999
- $200,000 - $449,999
- $450,000 and over

- STOCKYRX: What was the total value of all directly-held stocks, bonds, and mutual funds one year ago today (valid from 2013Q2)?

Following Malloy et. al. (2009), households with non-missing values for any variable listed above are identified as stockholders. In our data, the stock market participation rate is about 19%.[5]

## 1.4 Consumption Growth Measure

To control for consumption changes driven by changes in family size and for seasonal consumption changes, we regress the change in log consumption on the change in log family size at the household level over the same period plus a set of seasonal dummies. We use the residual as our quarterly consumption growth measure.

## 1.5 Aggregation of Household Consumption Growth Rates

The panel dimension for each household in the CEX allows us to calculate consumption growth rates at the household level. However, because households do not appear in the CEX for more than four quarters, we cannot calculate a long-run consumption growth rate for a particular household. Instead, following Malloy et. al. (2009), we construct a time series of average consumption growth for a particular group of households (e.g., stockholders), by averaging the (log) consumption growth rates for households in that group in each period. We compute the average growth rate ($\bar{gc_t}$) from t to t + 1 for a group as follows:

[5] Malloy et. al. (2009) report a 22.7% of stockholders using CEX data from 1980 to the first quarter of 2005.

$$\frac{1}{H_t^g}\sum_{h=1}^{H_t^g}\left(c_{t+1}^{h,g} - c_t^{h,g}\right) \tag{1}$$

where $c_t^{h,g}$ is the quarterly log consumption of household h in group g for quarter t and $H_t^g$ is the number of households in group g in quarter t.

The consumption growth rate over the horizons of S=1,2,12 is:

$$\sum_{s=0}^{\infty}\beta^s\left[\frac{1}{H_{t+s}^g}\sum_{h=1}^{H_t^g}\left(c_{t+1+s}^{h,g} - c_{t+s}^{h,g}\right)\right] \tag{2}$$

where $\beta$ is the discount factor.

Table 3 is calculated based on this time series average.

### 1.6 Aggregate per capita Nondurable and Service Consumption Growth Rates

We also calculate aggregate per capita nondurable and service consumption growth rates from the National Income and Product Accounts (NIPA, thereafter) from 1996 to the third quarter of 2015. The aggregate consumption growth rates uses real seasonally adjusted monthly aggregate consumption of nondurables from NIPA Table 2.8.3 available starting in January 1959. We use data up to September of 2015 and calculate quarterly consumption growth rates at the monthly frequency to coincide with the CEX data.

The results are consistent with Malloy et. al. (2009).

## 2 VAR

In the paper, we estimate parameters via a restricted VAR imposing some prior information on the parameters. For instance, we force the coefficient of lagged returns to be zero so that the return and predictor do not depend on past returns. Similarly, we restrict the intercept term in the regression of returns on the past factor to be zero. In this appendix, we show that this intercept is statistically and economically insignificant from zero in Table 2. A comparison of the results in this appendix with the results of the baseline used in the paper over the whole sample illustrates that the adjusted R squared slightly rises from 14.4 to 15 percent when this intercept is set to zero, while the rest of the parameter estimates remain essentially unchanged.

# References

[1] Malloy, C.J., T.J. Moskowitz, and A. Vissing-Jorgensen, "Long-run stockholder consumption risk and asset returns," *The Journal of Finance*, 64(6), (2009), 2427-2479.

Table 1: Structure of CEX Data

| | 2010Q1 | | | 2010Q2 | | | 2010Q3 | | | 2010Q4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NewID | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| 1 | 2 | | | 3 | | | 4 | | | 5 | | |
| 2 | | 2 | | | 3 | | | 4 | | | 5 | |
| 3 | | | 2 | | | 3 | | | 4 | | | 5 |

Table 2: VAR Estimation

Table 2 presents predictive regressions based on quarterly data from the first quarter of 1990 to the third quarter of 2016. We relax the restriction that the intercept of regressing return on predictor is zero:

$$\begin{bmatrix} VRP_{t+1} \\ r_{t+1} - r_f \end{bmatrix} = \begin{bmatrix} Const \\ \alpha \end{bmatrix} + \begin{bmatrix} \phi & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} VRP_t \\ r_t - r_f \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ z_{t+1} \end{bmatrix}$$

Newey-West t-statistics are reported in parentheses ($\alpha$ is set to zero).

| 1990Q1 –2016Q3 | $VRP$ | |
|---|---|---|
| Constant | 0.0058 | (6.72) |
| $\alpha$ | 0.0015 | (0.17) |
| $\beta$ | 3.5 | (3.61) |
| $\phi$ | -0.18 | (-1.84) |
| $\rho_{z,\varepsilon}$ | -0.04 | |
| $\sigma_\varepsilon$ | 0.0074 | |
| $\sigma_z$ | 0.0746 | |
| $\sigma_r$ | 0.079 | |
| Adj. $R^2$ (%) | 14.4 | |