The impact of gender and race segregation on labor organization in a social interaction model

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Abstract

This paper presents three versions of a social interaction model that introduces heterogeneity within the working class. The paper departs from the belief that bringing gender and racial perspectives into modeling economic phenomena can help better understand certain results. Specifically, it argues that dynamics of class struggle can only be represented by adding one layer of complexity and introducing different agents within the working class.

Three versions of a social interaction game representing decreasing unionization rates were built. Although emerging from the interaction of their individual pursuits, the falling unionization rate is a harmful result on the working class as a whole.

The presentation of the types of games in normal form lay the groundwork for the discussion. An extended version with utility specification is used to show how the assumption of competition between two different groups of agents within the working class can help explain the decrease in the unionization rate. A parameter is introduced to explain the degree of competition between the groups. The result is that unless there is no competition between workers the Nash equilibrium unionization rate of the game deviates from the Pareto optimal rate.

Finally, a social interaction statistical equilibrium model with entropy is built. When temperature is very low, the model have three equilibria, two stable and one unstable. The value of the competition parameter defines which of the two stable unionization rate equilibria workers reach . When competition is high, unionization rate is zero, when there is no competition all workers unionize. A higher value of the temperature parameter shifts the model from three equilibria to one middle equilibrium whose value also depends on the competition parameter. Overall the three models depicts how competition amongst workers lead to a lower unionization rate.

Keywords: Labor Markets; Social Coordination Problems; Labor Unions; Gender and Racial economics;

JEL Codes: B54, C72, D74, J71

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"-Esto pasa por meter a las mujeres en cosas de hombres -bromeó.
-¡No! ¡Esto pasa por meter a los burgueses en las cosas del pueblo! -replicó la joven indignada."

Isabel Allende - La Casa de los Espirítus (Allende, 2014)

1 Introduction

Many macroeconomic growth models within the heterodox tradition encapsulate class and distribution by focusing on the interaction between capitalists and workers. Some seminal examples include ? (?), Goodwin (1967), and Pasinetti (1983). In those models classes are acknowledged but the working class is assumed to be homogeneous. This traditional view does not convincingly explain the observed lower unionization rate among women nor the historically reported hostility of men to the organization of women. Milkman (2007) thoroughly describes the gender structures of labor unions, from the low participation of women in union leadership to the concentration of their membership into public sectors. The literature on the history of women and labor unions also describes the hostility of men to the organization of women, particularly to women's attempts to join men's unions.[1]

The hostility of labor unions to non-white workers and specifically to African Americans was explored by Du Bois (2017). W. E. B. Du Bois proposed that the segregation of the proletariat along racial lines is a key explanation for the labor movement's lack of strength. Beck (1980) (Beck, 1980) tests the hypothesis that unionization increases inequality between white and nonwhite males. Their results "provide support for the hypothesis that unionization has been a mechanism for protecting the economic interests of white labor" (Beck, 1980).

This paper formalizes the historical evidence of racial and gender conflicts within the working class as a game theory model. The motivation for this work comes from my observation and involvement with the International Women's Strike (IWS, 2019) specifically the attempts to organize a women's movement that does not benefit already privileged groups at the cost of non-white women. Or, in their own words, the launch of a new feminist wave for the 99%. The launch of the book "Feminism for the 99%: A Manifesto" is an expression of the theoretical formalization of this movement that gained momentum after Donald Trump's 2016 U.S. presidential election but has since struggled to grow. The present paper paper builds on this

[1] See Boston (Boston, 2015).

literature and offer a mathematical formalization that dialogues with the economic literature.

The model formalization builds a framework to show how the fall in the unionization rate in the U.S. can be understood as a social coordination problem. To do so, I confront the homogeneity of the working class by representing the fall in the unionization rate as a harmful result to workers as a class but that emerges from the interaction of their individual pursuits.

Three versions of a social interaction model are used to explain the fall in the unionization rate as a result of competition within the workers. The existence of a coordination problem creates an equilibrium at a low unionization rate, which is not the best social outcome for the workers. U.S. data is used to complement the models. I also argue that bringing a feminist and racial perspective helps to better understand the dynamics of class struggle. As argued in the first chapter, adding identity matters as a layer of complexity can drastically change the results of economic models.

The next section motivates the work by analyzing segregation data and identifying certain patterns. Following, the three versions of the model are presented in separate sections. Finally, the conclusions of the models are contrasted to the U.S. data and conclusions are drawn.

2 Occupational Diversity by Sex and Race in the data

I The data

In this section I discuss the occupational diversity of work force and of labor unions using the Current Population Survey data by the U.S. Bureau of Labor Statistics. This data is available from 1983 through 2017. I am interested in the union membership status, occupation, race, and gender values. The average sample size by year with observations that contains complete information in all of those categories is 168,367. The average number of occupations is 500. Gender is categorized as male or female throughout the years. Race starts with three categories in 1983 and ends with 26 categories in 2017. All of my calculations are done by recategorizing race in a binary way as whites and non-whites. The unionization rate within this period falls by approximately 50%, starting at 19.6% of workers unionized in 1983 and ending at 10.% in 2017.

II The Information Theory Theil Index

The Information Theory Index (TI) [2] was used by Theil and Finizza (1971) to study racial segregation in Chicago public schools. Comparing with the Index of Dissimilarity (ID) [3] proposed by Duncan and Duncan (1955), the TI does not distinguish by which group is over represented. For example, the contribution of a sector that is 25% female and 75% male is the same as one that is 75% female and 25% male.

The choice to use the TI instead of the ID in this study is due to the fact that the ID does not behave well when the population of one group is smaller than 100,[4] which is the case for the population of women unionized in certain occupations. The TI also has the advantage of being decomposable. Finally, Roberto (2015) argues that the Information Theory Index (ITI) is a measurement of diversity instead of segregation. For the argument that I will put forward the level of heterogeneity in each sector is what matters.

To calculate the TI, I first need to calculate the entropy of total population and for each group of the population, in our case for each occupation. The entropy can be understood as measurement of surprise that a distribution carries. When measuring the entropy of a population according to the distribution between male and female, if the entropy is equal to 0, you are certain of the gender when you pick one individual at random. In other words, a population of only male or a population of only female would have an entropy of zero. The maximum entropy is associated with an equal distribution. In this case, the probability of guessing the sex of a random individual is as low as possible, therefore the surprise factor and the information content of the gender is higher. Formally, entropy is the weighted average of the probability of an outcome (r) occurring. The entropy of each outcome (E_r) and the overall entropy (E) are respectively:

$$E_r = log(\frac{1}{\pi_r}) \tag{1}$$

$$E = \sum_{r=1}^{R} \pi_r * E_r \tag{2}$$

Where π_r represents the probability that a certain individual identifies with a specific group and is measured as the proportion of such identity within the population. When r

[2] Formalized below.
[3] Formalized in Appendix A.
[4] See Anker (1998)

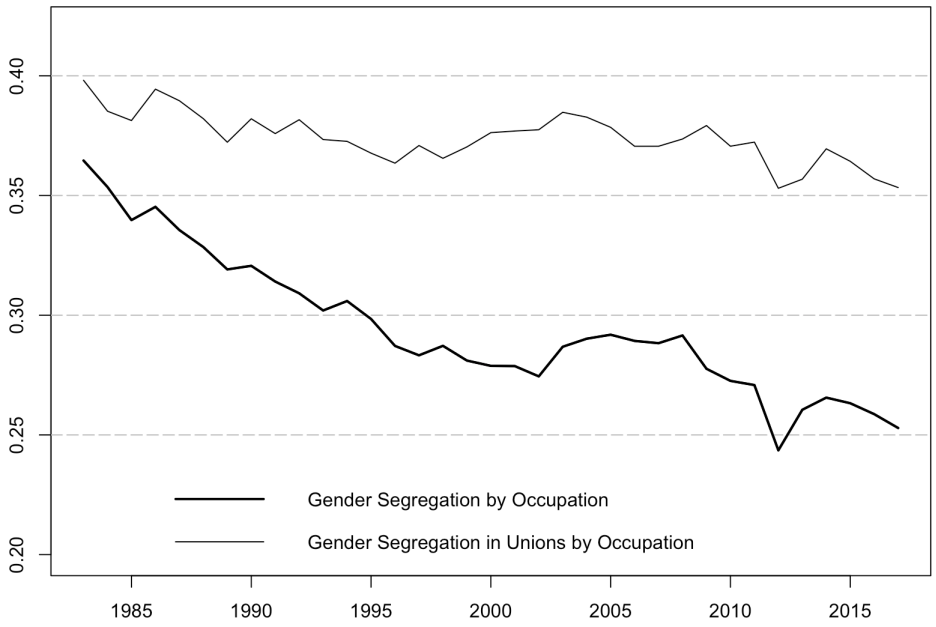

Figure 1: Information Theory Theil Index of Segregation by gender

Source: Author's calculation from Current Population Survey Data

represents gender,π_m and π_w will be the proportion of male and female in the total population respectively.

The TI of occupational segregation measures the divergence between the population entropy and the entropy in each occupation:

$$TI = \sum_{i=1}^{n} \frac{ti}{T} * \frac{E - E_i}{E} \tag{3}$$

The TI ranges between 0 and 1, where a value of 1 is interpreted as high segregation or no diversity, and 0 as complete integration. The occupational TI for the work force and for labor union members is calculated for gender and for race. The results are presented in Figure 1 and Figure 2. In these cases, occupational segregation within the workforce, as measured by the TI for gender and race, decreased between 1983 and 2017. Nonetheless the decrease of segregation within labor unions is much slower. This indicates that although there is more diversity of gender and race in the work force within occupations, this has not been translated at the same level within labor unions.

This result can be read as a reaction in labor unions to a higher degree of competition within the occupations. Labor unions can be used as an instrument to protect the privileges of those already employed in an occupation against the entrance of workers of a discriminated

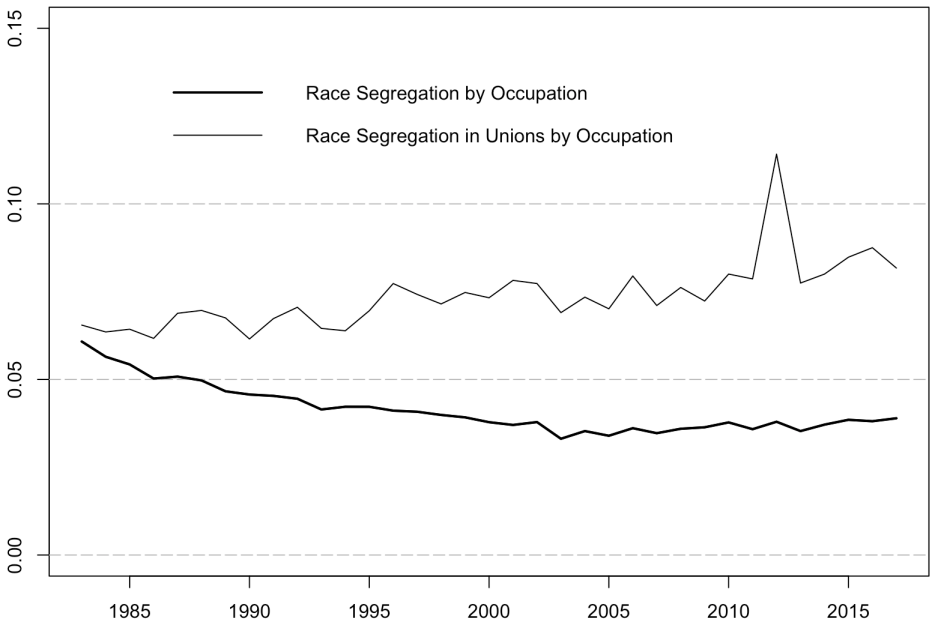

Figure 2: Information Theory Theil Index of Segregation by Race

Source: Author's calculation from Current Population Survey Data

identity, who are frequently paid a lower wage.[5]. Therefore the increase in diversity within the occupations is a trigger, making labor unions a less welcoming environment for individuals who do not identify with the norm. Furthermore, this indicates that in order for labor unions to thrive it is necessary to build solidarity across identities. Racism and sexism within the work force in an obstacle to this objective. Labor unions depend on the cooperation among workers and across identities. If the diverse population is not integrated within the labor unions, bargaining power will fall. This argument will be formalized using game theory framework. Sexism and racism if defined in this case as decreasing the payoff of all workers, where both add a cost in the utility function and decrease the degree of cooperation necessary for the union to achieve class-wide results. The increase in work force diversity needs to be accompanied by integration and the creation of work force solidarity, otherwise there is a fall in the unionization rate.

III Intersectionality as a way to strengthen the labor movement

The term intersectionality has been used within social organizations to express the need to build movements in response to the needs of oppressed groups. The coining of the term is attributed to Kimberle Crenshaw. The main argument in Crenshaw (2018) is that the single-axis framework of analysis used in anti-discrimination law that emerges from treating race

[5]This argument has been developed in Chapter one

and gender as mutually exclusive categories leads to a marginalization of black women in the feminist movement and in antiracist politics. The term has since been more broadly applied, particularly to express discontent with the existence of oppression on the basis of identity within social movements. Examples of this include, but are not limited to, racism within the feminist movement, sexism within the black movement and both racism and sexism within the labor movement.

Intersectionality, like any overly used and misunderstood term, has attracted a lot of criticism. In practice, intersectional politics depends on a recognition of the identities subject to oppression. This leads to an effort to enumerate in a finite and discrete way the identities that should be acknowledged within social movements as well as an exhaustive discussion on who is more oppressed. The main problem with is that such a discrete finite list does not exist since identity is fluid and constantly changing. Therefore it is problematic to center social movements on fixed categories of oppression. Based on the understanding that gender and race, the main categories explored here, are social constructs from which the capitalist system profits, I believe that it is important to generalize my argument on the basis of any discrimination by identity.

Oppression on the basis of identity is viewed here both as an emergent property of the capitalist system as well as a reinforcement of pre-capitalist social constructs, where profit seeking behavior engenders the exploitation and expropriation of oppressed groups. This behavior reproduces existing oppression and creates new forms of oppression as a way of maximizing capitalist profits. For example, the fact that the responsibility of reproductive labor typically falls to one specific sex and that women are identified with certain characteristics is not a natural aspect, it is socially constructed and reinforced by the capitalist system. It is also a source of expropriation since this allows for capitalists not to pay for reproduction costs and therefore to extract even more surplus value from the system.

Nonetheless, problematic outcomes can arise when feminist movements are solely focused on gender discrimination. For instance, the liberal feminists movement has been criticized for improving the situation of white females at the expense of nonwhite women. In cases such as this, instead of creating affinity based on what ties the groups together, problems arise as the interaction of identities leads to prejudice and the exclusion of people on the basis of their non-belonging.

The quote from "House of Spirits" by Isabel Allende (Allende, 2014) presented at the beginning of the paper depicts this quite well. This quote refers to the character Alba, then

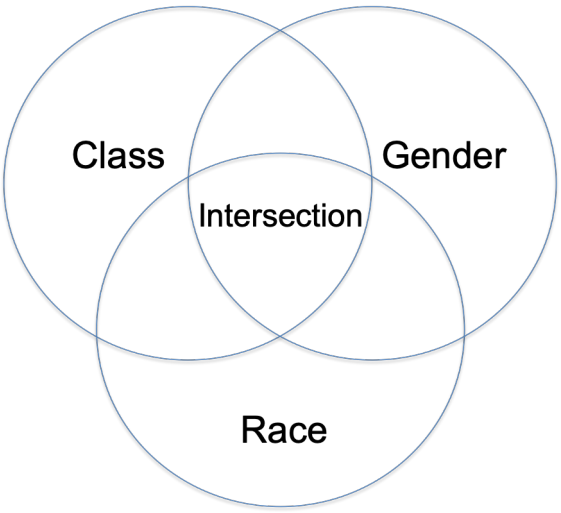

Figure 3: Intersectionality

an 18 year-old girl, who is the granddaughter of a powerful conservative senator. Through the influence of Pedro Garcia Tercero, her biological father and an icon of the proletariat movement, and Miguel, the boy she is in love with, she grows sympathetic to the communist movement. Alba had experienced gender discrimination at an early age during a sexual harassment episode. Nonetheless, she is perceived as a privileged bourgeois. Eventually, she decides to join an occupation at her university in solidarity with the workers. When she suddenly gets a heavy flow period and is sick and unable to stay in the occupation, she is criticized by a man, who claims that this is what happens when women meddle with men's business. A girl then replies saying that this is what happens when a bourgeois tries to meddle with the people's business. Alba is oppressed in terms of her gender and is rejected because of her lack of experience of oppression in terms of class. We could have imagined a different scenario where her experience of oppression based on her gender is enough to create solidarity across oppressed groups.

This vignette illustrates how a social movement aimed at fighting oppression on any specific terms can only be successful if it is focused on the sources of such oppression. For example, a feminist movement that is not centered on the needs of all women is bound to benefit the already privileged group of women at the cost of racialized women, as Fraser, Arruzza, and Batachari argue (Fraser & Arruzza, 2019). Therefore, a successful social movement, be it a movement for women's rights, black people's rights or worker's rights, should focus on the particiapants' common denominator, allowing them to fight oppression on the basis of identity and enabling the creation of solidarity across different identity groups.

This paper aims at formalizing the argument for intersectionality and for the argument

that identity is a fluid category. The two versions of the model described in the following sections explain why it is in the interest of a working class movement to empower minorities. It also shows the limits of the concept of intersectionality and how any oppression on the basis of identity is harmful for the labor movement. The next two sections present a literature review of games that deal with coordination problems and games that discuss labor conflict before proceeding to the formalization of the problem of coordination of a segregated working class.

3 Types of Games

The problem of interaction between men and women can be described as a normal form game with a pay-off matrix. In this section, the types of games are presented according to the properties of their Nash equilibria: an assurance game, a disagreement game, a prisoner's dilemma game, and an invisible hand game. In an assurance game, there are two Nash equilibria but one of them is Pareto superior to the other. The problem of defining which equilibrium we are at depends mostly on how the two players interact. Initially, the group of women and the group of men are the two different players in this game. The highest payoff for each player comes when they both decide to join the union. A player that joins the union does not get as many benefits and bare the cost alone while the other group gets no benefit at all. Therefore, if one decides not to join the union, the other's best response is also to not join the union.

Assurance Game		Men	
		Not Join	Join
Women	Not Join	0,0*	0,-1
	Join	-1,0	2,2*

In a disagreement game there are also two Nash equilibria but they are not Pareto comparable. This would be the case if the cost of men and women interacting together in a union offsets the benefits of having more members. In this case the highest payoff for each group is when either group joins the union alone.

Disagreement Game		Men	
		Not Join	Join
Women	Not Join	0,0	0,3*
	Join	3,0*	1,1

The prisoner's dilemma game is characterized by the fact that there is only one Nash equilibrium that is not Pareto Optimal. When both men and women join the union, each individual group has an incentive to not join the union and collect the benefits of a labor organization without having to incur in the cost of joining the union nor of interacting with the other group. This would lead men and women to not join the union and the game would settle at a situation that is worst for both.

Prisoner's dilemma Game		Men	
		Not Join	Join
Women	Not Join	-2,-2*	0,-3
	Join	-3,0	-1,-1

Finally, the invisible hand game is characterized by the fact that the unique Nash Equilibrium is also Pareto efficient. In this case, the individual pursuits of each group would lead them to cooperate. In this game there is no conflict involved.

Invisible hand Game		Men	
		Not Join	Join
Women	Not Join	0,0	0,1
	Join	1,0	2,2*

Except in the case of an invisible hand game, the achievement of a social optimum depends on an effort of cooperation and demonstrates that even in the case where the equilibrium solution is Pareto efficient there might be a need for institutional effort or else the players might be trapped in a less than ideal situation. The description of the games in terms of pay-off matrices oversimplifies the problem and does not shed light on the specific dynamics between the two groups discussed in the last section. Nonetheless, this helps shed some light on how conflict and the costs of joining a union could play different roles depending on the player's assumptions and can lead to different situations. The next section presents an expanded version of a game with utility functions that will be used to calculate the Nash equilibrium and Pareto Optimal. I will discuss the nature of the game according to the taxonomy presented in this section, and evaluate if competition between the two groups can change the nature of the game with respect to the number of equilibria and the Pareto efficiency.

4 An Expanded Game

The model presented in this section attributes utility functions to each of the two groups of workers, women and men. The constraining assumption made here is that the group of workers per gender act in order to maximize their utility as a group and decide on the optimal level of unionization for their group. This version will have qualitatively different results than the normal form game given its assumptions. The main feature is the fact that a negative interaction between the genders leads to an equilibrium that is Pareto Inferior to the one that would be reached without competition.

Although this model does not realistically represent the way individual workers make choices about whether or not to join the union, it is extremely useful to illustrate how the competition among the workers can explain a fall in the unionization rate across workers. The model also shows that the Nash equilibrium will only be Pareto Optimal if there is no competition between genders. If there is any negative interaction between genders, the equilibrium unionization rate will be below the Pareto optimal level.

The group of male and female workers utility in joining the union, U_m and U_w respectively is represented by the following equations:

$$\begin{aligned} U_m &= f(s_m, s_w) = \alpha((1 - \gamma_m * s_w)s_m - \beta s_m^2) \\ U_w &= f(s_w, s_m) = \alpha((1 - \gamma_w * s_m)s_w - \beta s_w^2) \end{aligned} \tag{4}$$

Where s_m and s_w are respectively the share of male and female workers that join the union, i.e. the unionization rate per gender, and γ_m and γ_w are the interaction parameters. For a given positive value of γm, a higher unionization rate amongst women will have a stronger negative effect on the utility of the group of male workers. For a given positive value of γw, a higher unionization rate amongst men will have a stronger negative effect on the utility of the group of female workers.

It is important to note that the utility functions represented here are assumed to be symmetrical for simplicity purposes. Nonetheless, this imposes a restriction on the kind of problems that the model is able to analyze. The assumption of symmetry means there is competition between two similar groups and that the burden that one group imposes on the other is the same. This is not representative of the observed fact that the two groups do not have the same power. Therefore, this version of the model shows how the existence of competition

influences the interaction between the groups but it does not shed light on how patriarchy and sexism is a burden that affects one group more heavily. An extension of this model would be necessary to show the unequal distribution of power between the two groups.

The model supposes that each group of workers will choose their own unionization rate depending on the unionization rate of the other group in order to maximize their own utilities. The unionization rate is calculated through the best response function, which is the choice of strategy that maximizes a player's utility given the other players choice of strategy. The best response function is calculated differentiating a players own utility with respect to their strategy while taking the other player's strategy as constant. The optimal point is where the derivative is equal to zero as long as the second derivative is smaller than 0, FOC and SOC, respectively. The solution to the first order condition yields the following best response functions, while the solution to the second order condition shows that this is a maximization point:

$$\begin{aligned} s_m = f(s_w) &= \frac{1 - s_w\gamma_m}{2\beta} \\ s_w = f(s_m) &= \frac{1 - s_m\gamma_w}{2\beta} \\ \delta\delta_{s_m}(U_m) = \delta\delta_{s_w}(U_w) &= -2\alpha\beta \end{aligned} \tag{5}$$

The Nash equilibrium unionization rates s_m^* and s_w^* are calculated by simultaneously solving for both best response functions.

$$s_m^* = \frac{\gamma_m - 2\beta}{\gamma_m\gamma_w - 4\beta^2} \tag{6}$$

$$s_w^* = \frac{\gamma_w - 2\beta}{\gamma_m\gamma_w - 4\beta^2} \tag{7}$$

Calculating the Pareto Optimal outcomes requires a social planner that does a utility maximization where the unionization rate for each group to be the same. The constrained utility maximization for the Pareto Optimal solutions as a function of γ is described in Equation 8.

$$\begin{aligned} &Max\ U_m + U_w \\ &s.t.\ s_m = s_w. \end{aligned} \tag{8}$$

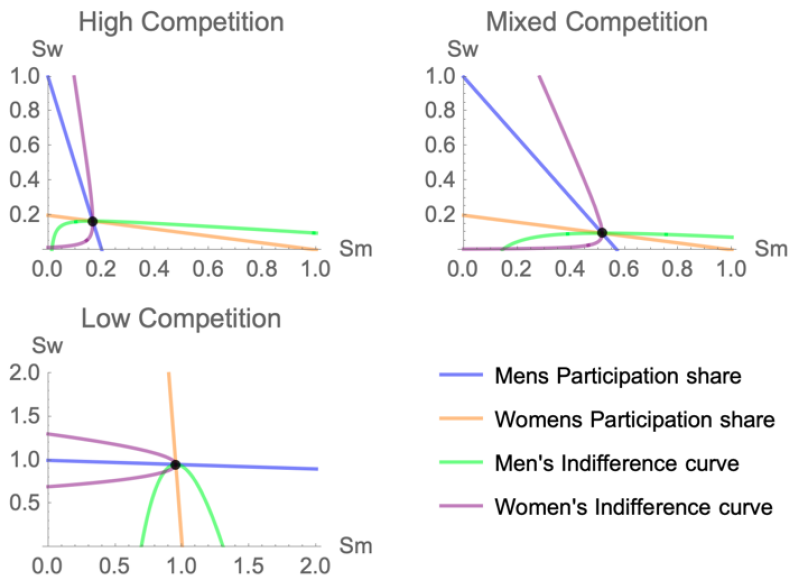

Figure 4: Unionization rate response to the degree of competition between workers

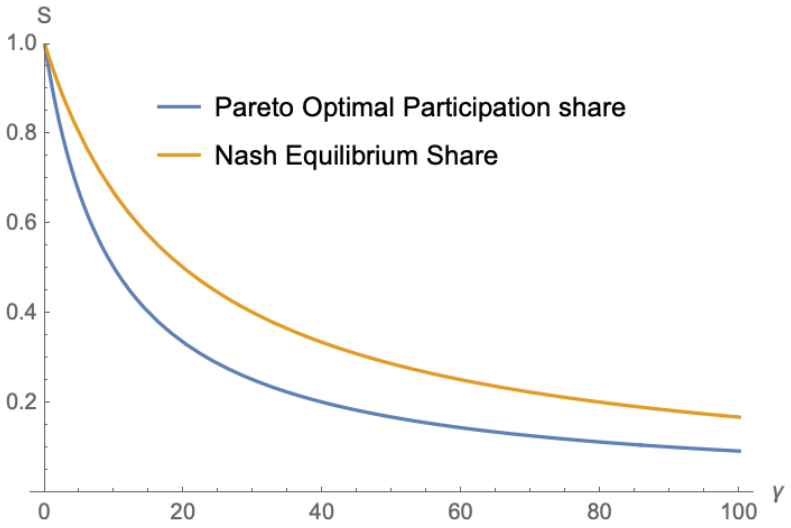

Figure 5: Pareto Optimal Unionization Rate versus Nash Equilibria Unionization Rates

The Pareto Optimal levels of unionization as a function of γ are:

$$s^p = \frac{1}{2\beta + \gamma_m + \gamma_w} \tag{9}$$

Figure 4 shows how changes in the value of the interaction parameters shift the equilibrium. When there is high competition, i.e. interaction, the equilibrium unionization rate is very low for both men and women and the indifference curves are perpendicular to each other. This indicates that there is room for a Pareto improvement. The mixed interaction graph represents a situation where $\gamma_m > \gamma_w$ and shows that when men perceive women as a threat, they act in such a way to decrease the unionization rate of women. Finally, if both γ_m and γ_w are low, the

equilibrium is a high unionization rate for both groups and the indifference curves are tangent to each other, representing a Pareto Optimal situation.

If the unionization rates and the interaction parameters are fixed to be equal among the two groups, we can observe the findings described above. Figure 5 shows three interesting consequences of the interaction between works. First, the higher the negative interaction parameter, i.e. the higher the competition between the two groups of workers, the lower the equilibrium and Pareto optimal unionization rates. Second, only when γ is equal to zero is the Nash equilibrium of the game also a Pareto optimal outcome. Lastly, for all values of γ different from zero, the Nash equilibrium unionization rate is higher than the Pareto optimal one. This means that the competition between workers engenders a social coordination problem. In other words, when each group pursues their own self interest, they end up in a worse situation than if they had coordinated. This happens because each group tries to compensate for the negative effect of the other by putting more effort in unionizing and also dragging the other group unionization rate to a higher than ideal level. For a given value of γ, the utility of each group could increase if both coordinate to try to decrease their level of unionization, therefore decreasing both the cost of unionizing as well as the burden that one group imposes on the other.

A more interesting way of increasing each group's utility is by discussing what factors could contribute to a fall in the value of the γ parameter. Efforts to decrease the competition between the two groups or to build affinity between men and women could lead them to a more unified working class where individual and class objectives are aligned with each other. The fact that we only observe a coincidence between the Nash equilibrium and the Pareto optimal outcome when there is no negative interaction between the two groups shows that a unified working class would mean that the individual pursuits lead to the best social outcome. This would change the nature of the game from a "prisoner's dilemma" to an "invisible hand" game. In the first type of game, the interaction between the two players following their own self-interest lead them to a worse situation. In an "invisible hand" game, as the name suggests, when players follow their own self-interest they jointly lead themselves to their best social and individual results.

I Introducing heterogeneity within groups

The model so far has assumed that the group of men and women act as one individual making an optimal choice. This further depends on the assumption that there is no conflict of

interest within each group. Nonetheless, we have no reason to believe that gender is the only identity that leads to competition. If ethnicity and race are also barriers for the creation of solidarity within the groups of males or females, then we would expect the results of the game to be different. Within the framework of the model presented in this section we could think of this problem as if there is another game happening within each group. The decision of females as a group would emerge from the interaction between the different racial an ethnic groups of women. The intersectionality argument in this framework can then be understood from the perspective of oppression happening on multiple layers. The problem of trying to apply this is that we would need beforehand to identify all social identities according to which oppression exists. Alternatively, we can think of the problem of identity as a general competition amongst individuals. Instead of assigning different utility functions for each social identity group, we can assign a utility function for individuals and then introduce heterogeneity as a parameter that would decrease the utility of the individual. This is presented in the next section.

5 The Social Interaction Game

To better explain how the model presented above works from the individual point of view, I make changes to the utility function to represent the individual utilities. This allows the introduction of a social interaction model to explain the unionization rate through the interaction between workers. The model is based on the work of Bowles (2009) and the idea that microeconomic problems need to be modeled by assessing their social interaction features.

I start by supposing that the body of workers consists of N agents that are similar in terms of the decision problem. The agents face the same utility function when trying to decide if they should or should not join a union. Their utility depends on their own decision of joining the union or not and on the number of workers that decide to unionize.

$$U = f(x, z) \tag{10}$$

The individual decision to join the union is represented as $x = 1$, while not joining sets x to zero. Z ranges between 0 and 1 and represents the unionization rate, or, the proportion of workers that have joined the union.

$$U(x, z) = \alpha x - \beta x + \gamma xz + \eta z \tag{11}$$

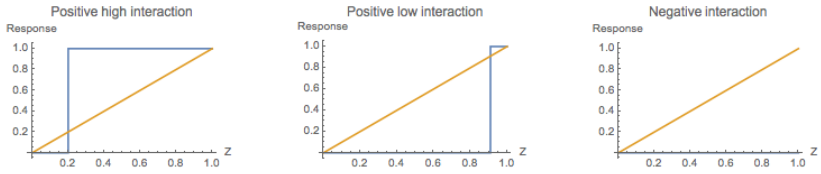

Figure 6: Nash Equlibrium for different levels of interaction

The term αx represents the benefits that members of the union enjoy once they joined it. This can be seen as the benefits that a union offers exclusively for members. The term βx represents the cost of joining the union. The term ηz is the benefits that everyone is able to enjoy because some people join the union, such as a wage increase that a union negotiates for all workers in a certain category regardless of their membership status or efforts. The γxz is the interaction term and can be seen as the change in the benefits that the joint efforts are able to negotiate but are exclusive for those who are putting some effort into the union.

All parameters are assumed to be positive, but β has a negative effect on utility since it represents the cost.

A best response function is derived by first looking into the individual utility of each strategy for a given Z, with joining the union being $U(1|z)$ and not joining the union as $U(0|z)$.

$$\begin{aligned} U(1|z) &= \alpha - \beta + \gamma z + \eta z \\ U(0|z) &= \alpha - \beta + \gamma z + \eta z \end{aligned} \tag{12}$$

The best response function will then depend on the value of the parameters and Z in the following way:

If $\alpha - \beta + \gamma z + \eta z > \eta z$, the individual will join the union.

If $\eta z > \alpha - \beta + \gamma z + \eta z$, the individual will not join the union.

If $\eta z = \alpha - \beta + \gamma z + \eta z$, the individual will be indifferent to joining the union or not.

Rewriting the above, we can see that the individual decision depends on wether the benefits of joining and the strength of the interaction term are bigger than the cost of joining.

If $\alpha + \gamma z > \beta$, the individual will join the union.

If $\beta > \alpha + \gamma z$, the individual will not join the union.

If $\alpha + \gamma z = \beta$, the individual will be indifferent to joining the union or not.

Figure 6 displays the existence of three equilibria. The first two are stable equalibria: in the first case no one joins the union, and in the second everyone joins the union. In the third

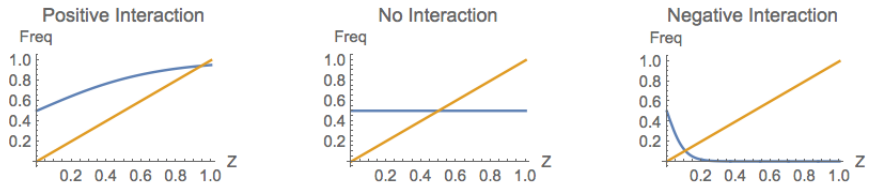

Figure 7: Nash Equlibrium for different levels of interaction in an Statistical Equilibrium Model

case, unstable equilibrium is dependent on the value of the interaction variable and depends on the threshold for the step function. When there is high positive interaction, the threshold for workers to cooperate and join unions is much lower. Therefore, it is much more likely that the high equilibrium will be reached. If the interaction term is weak, one individual's best response would be joining the union only when a very high percentage of the others have already joined, thus making it much more likely that no one will join the union. Lastly, when the interaction between the workers is set to a negative value, there is only one equilibrium.

6 The statistical equilibrium model

This section introduces entropy into the problem presented above, and thus treat the reaction of individuals to the threshold in a more realistic way. The maximum utility solution of the problem is found by maximizing entropy with a constraint on utility. The solution to the maximization problem is:

$$f[x|z] = \frac{e^{\frac{\alpha x - \beta x - \gamma x z + \eta z}{T}}}{e^{\frac{\alpha - \beta - \gamma z + \eta z}{T}} + e^{\frac{\eta z}{T}}} \tag{13}$$

$$\begin{aligned} f[1|z] &= \frac{e^{\frac{\alpha - \beta - \gamma z + \eta z}{T}}}{e^{\frac{\alpha - \beta - \gamma z + \eta z}{T}} + e^{\frac{\eta z}{T}}} \\ f[0|z] &= \frac{e^{\frac{\eta z}{T}}}{e^{\frac{\alpha - \beta - \gamma z + \eta z}{T}} + e^{\frac{\eta z}{T}}} \end{aligned} \tag{14}$$

Figure 7 shows that when $T = 0.01$ and all parameters but β are positive, there are three equilibria. The equilibrium is characterized by the fact that Pbar, which represents the average probability of joining the union, is equal to the frequency of workers that join the union. If all workers are seen to be equal, then only points along the 45^o line (orange line in the graph) are possible.

From the three equilibria, only two are stable. The middle one is unstable and any shock on that point would bring workers to either almost all unionize (i.e. high equilibrium) or almost

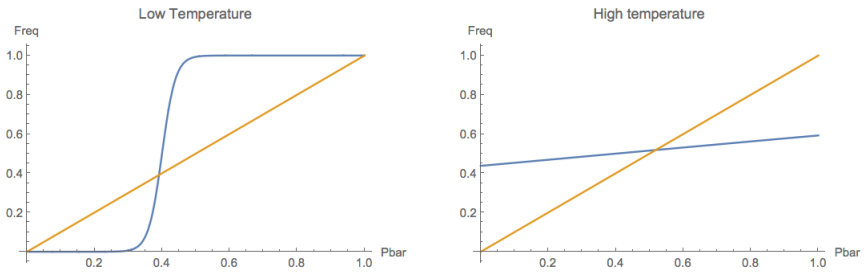

Figure 8: From an Assurance Game towards a Prisoner's Dillema

all decide to not unionize (i.e. low equilibrium).

The decision temperature can be understood as a degree of coherence within the decisions of the individuals. It has been defined that all individuals are subject to the same utility function and therefore the best response for them is always the same given that they have equal parameters. The decision temperature can be understood as the degree to which they are able to answer with the exact best response. A very low temperature means that overall the individuals are very effective at choosing whether or not to unionize according to their utility. A higher temperature would mean that individuals will more frequently decide to unionize even though their utility would be higher by unionizing, and vice-versa.

Figure 8 shows that if workers have a higher decision temperature, the medium level equilibrium becomes stable and unique. In this case we can think of workers as not having enough information to decide on their strategy, and therefore their decision on whether or not to join the union is a somewhat random one.

The level at which the stable medium equilibrium sets nonetheless depends on the value of the three parameters: α, β, and γ. I can then solve to express all the equilibria for a given value of α and β as a function of γ.

One interesting aspect of this is the fact that when temperature rises and the two corner equilibria are eliminated, the nature of the game changes. In the case of low temperature, one of the Nash Equilibria is also Pareto optimum. This is an assurance game, and therefore depending on which equilibrium we are at, there is no need for social coordination attempts. When temperature increases, there is only one equilibrium which is not Pareto Optimum. In this case the game becomes a prisoner's dilemma and therefore there is a need for coordination attempts in order to change the results and to achieve a better social outcome.

7 Reflections on the data

The three versions of the model presented above indicate that there is a trade-off between an increase in the participation of minority identities in the work force and the overall unionization rate.

Figures 9 and 10 shed more light on this correlation. They are scatter plots from 1983 to 2017 of the normalized proportion of union members by the proportion of gender homogeneity and racial homogeneity respectively, with the loess smooth fitting curve[6]. Figure 9 shows that there is a high correlation between gender homogeneity and union membership. In other words, industries that are either dominated by men or dominated by women have higher unionization rates than those that have a balanced mix of both genders. Figure 10 shows that there is also correlation between racial homogeneity and unionization rate. Given that the unionization rate among African Americans is higher than among whites, this correlation is particularly interesting. This indicates that the higher unionization rate within each of those groups are concentrated in sectors where the diversity is smaller, which corroborates the conclusions of the models.

8 Conclusion

This paper proposed three versions of a game to explain how the lack of solidarity within the work force can lead to harmful results for the labor movement. Specifically, I proposed that competition between workers will (1) create a coordination problem, (2) lead the working class to unionize at a rate that is not optimal, and (3) decrease the unionization rate.

These results can be used to argue that economic models that discuss the labor market without considering the existence of competition between workers will fail to explain certain phenomena. Neoclassical models that suppose methodological individualism will not be able to address the possibility of a coordination problem. Marxian models based on bargaining power will fail to explain the difficulties that the labor force faces in attempting to unionize.

From a political perspective, the results presented in this paper are coherent with a point made by many radical scholars[7] concerning the importance of a labor movement that is aimed

[6]The Loess is a non-parametric method for graphical assesment of correlation. For more see Jacoby (2000) (Jacoby, 2000).

[7]Henessy and Igraham argues that "Women's cheap labor (guaranteed through racist and patriarchal gender systems) is fundamental to the accumulation of surplus value - the basis for capitalist profit making and expansion.

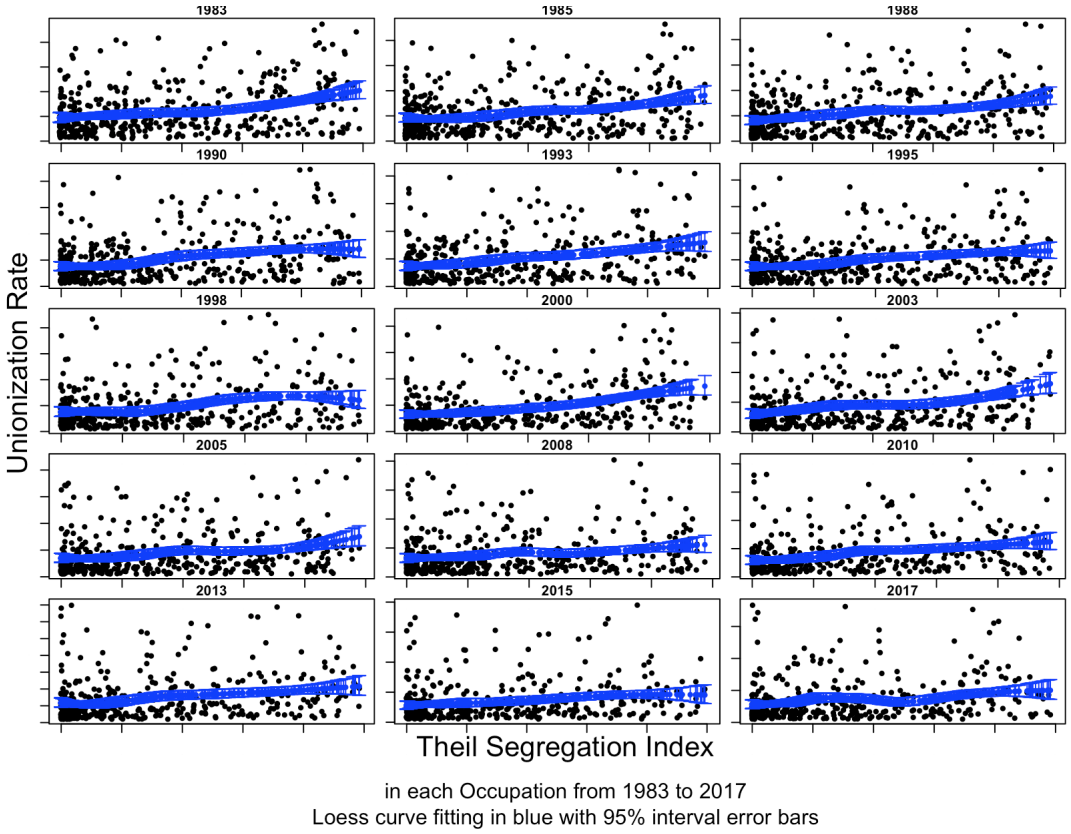

Figure 9: Unionization Rate by Gender Segregation by Occupation

Source: Author's calculation from Current Population Survey Data

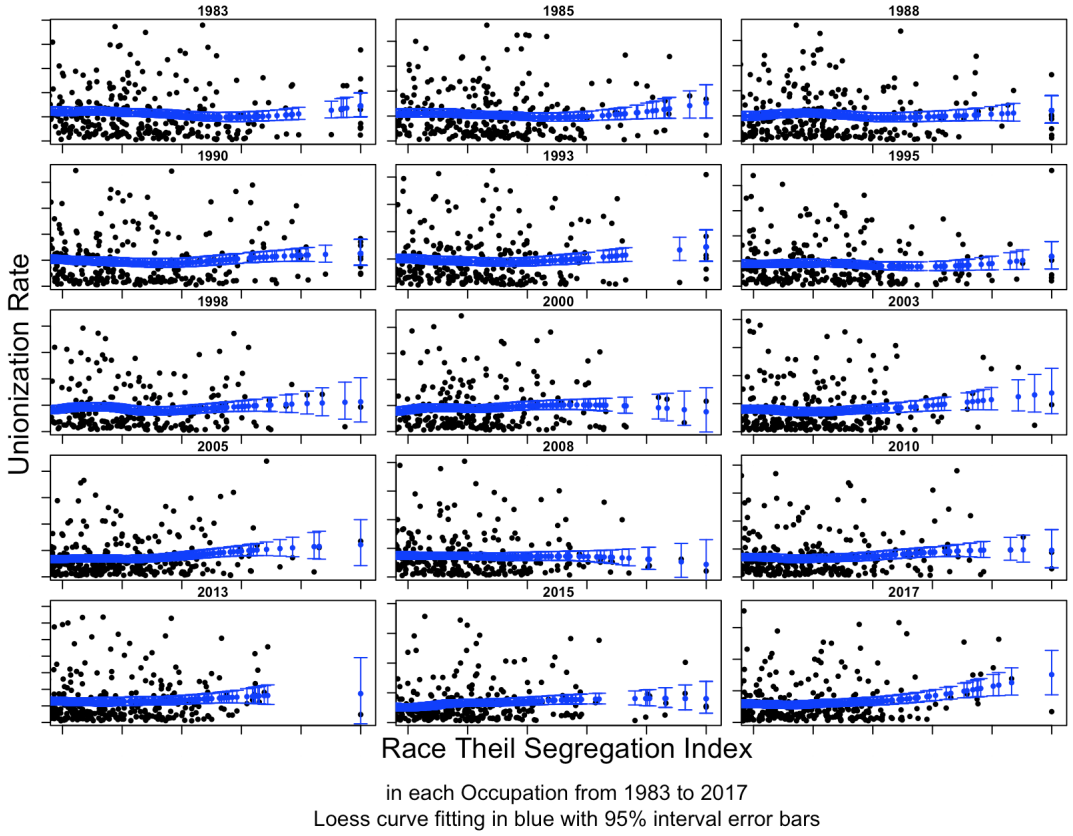

Figure 10: Unionization Rate by Race Homogeneity

Source: Author's calculation from Current Population Survey Data

at providing benefits for all workers, specifically a feminist movement aimed at improving the life of all women and a black movement aimed at improving the life of all African Americans.

Therefore, any movement that seeks to improve the life of all workers cannot ignore the sexist and racist reality of capitalism.

A The Dissimilarity index

The Dissimilarity Index (DI), also frequently referred as the Duncan Index, was formalized by Duncan and Duncan (1955) and has been since broadly used as a measure of segregation. Originally developed to discuss geographical segregation of different population groups, it has been "by far the most commonly used inequality index" (Anker (1988)) applied to economics to measure occupational segregation by sex of the labor force. The DI can be calculated as follows:

$$\sum_i |\frac{f_i}{F} - \frac{m_i}{M}| \tag{15}$$

Where f_i represents the number of female workers in occupation i, m_i represents the number of male workers in occupation i, F represents the number of females in the labor force and M the number of males in the labor force.

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A feminism that aims to improve the life of all women and at the same time recognizes their differential relation to one another cannot ignore the material reality of capitalism's class system in women's lives." (Henessy and Igraham pp. 3 (Hennessey & Ingraham, 1997). Fraser and Arruzza (2019) (Fraser & Arruzza, 2019) present a manifesto claiming for such a movement.

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