Factor Momentum and the Momentum Factor*

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Abstract

Momentum in individual stock returns emanates from momentum in factor returns. Most factors are positively autocorrelated: the average factor earns a monthly return of 1 basis point following a year of losses and 53 basis points following a positive year. Factor momentum explains all forms of individual stock momentum. Stock momentum strategies indirectly time factors: they profit when the factors remain autocorrelated, and crash when these autocorrelations break down. Our key result is that momentum is not a distinct risk factor; it aggregates the autocorrelations found in all other factors.

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1 Introduction

Momentum appears to violate the efficient market hypothesis in its weakest form. Past returns should not predict future returns if asset prices respond to new information immediately and to the right extent—unless past returns correlate with changes in systematic risk. Researchers have sought to explain the profitability of momentum strategies with time-varying risk, behavioral biases, and trading frictions.[1] At the same time, the pervasiveness of momentum over time and across asset classes has given momentum the status of an independent factor: models without momentum cannot explain it and those with momentum cannot explain anything more than just momentum (Fama and French, 2016).[2] In this paper we show that momentum is not a distinct risk factor: it aggregates the autocorrelations found in all other factors. Rather than being unrelated to the other factors, momentum in fact relates to *all* of them.

We first show that factors' prior returns are informative about their future returns. Small stocks, for example, are likely to outperform big stocks when they have done so over the prior year. This effect is economically and statistically large among the 20 factors we study: The average factor earns 53 basis points per month following a year of gains but just 1 basis points following a year of losses. The difference in these average returns is significant with a t-value of 4.67. This result is not specific to the use of obscure asset pricing factors: we work with the major factors that are regularly updated and published by academics and a hedge fund.

A time-series factor momentum strategy is a strategy that bets on this continuation in factor

[1] See, for example, Conrad and Kaul (1998), Berk et al. (1999), Johnson (2002), and Sagi and Seasholes (2007) for risk-based explanations; Daniel et al. (1998), Hong and Stein (1999), Frazzini et al. (2012), Cooper et al. (2004), Griffin et al. (2003), and Asness et al. (2013) for behavioral explanations; and Korajczyk and Sadka (2004), Lesmond et al. (2004), and Avramov et al. (2013) for trading friction-based explanations.

[2] Jegadeesh (1990) and Jegadeesh and Titman (1993) document momentum in the cross section of stocks, Jostova et al. (2013) in corporate bonds, Beyhaghi and Ehsani (2017) in corporate loans, Hendricks et al. (1993), Brown and Goetzmann (1995), Grinblatt et al. (1995), and Carhart (1997) in mutual funds, Baquero et al. (2005), Boyson (2008), and Jagannathan et al. (2010) in hedge funds, Bhojraj and Swaminathan (2006), Asness et al. (2013), and Moskowitz et al. (2012) in major futures contracts, Miffre and Rallis (2007) and Szakmary et al. (2010) in commodity futures, Menkhoff et al. (2012) in currencies, and Lee et al. (2014) in credit default swaps.

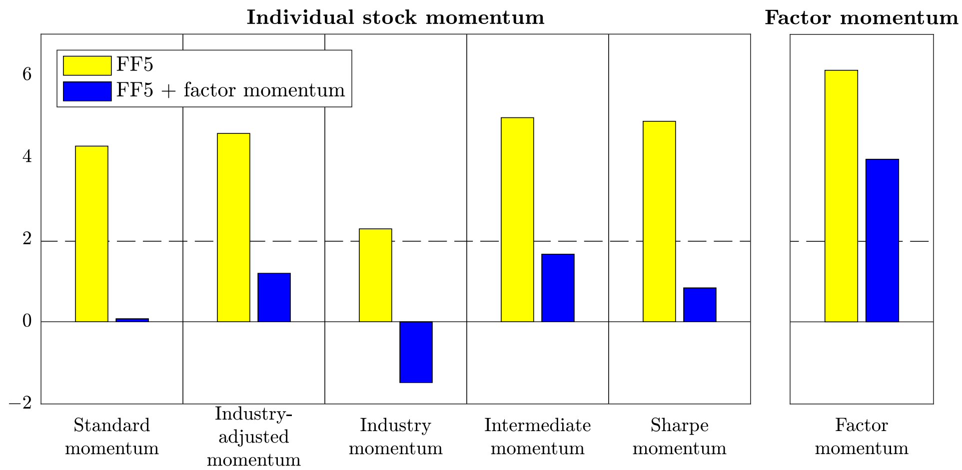

Figure 1: **Individual stock momentum versus factor momentum.** This figure shows t-values associated with alphas for five momentum strategies that trade individual stocks and a factor momentum strategy that trades 20 factors. For individual stock momentum strategies, we report t-values from the five-factor model (yellow bars) and this model augmented with factor momentum (blue bars). For the factor momentum strategy, we report t-values from the five-factor model (yellow bar) and this model augmented with all five individual stock momentum strategies (blue bar). The dashed line denotes a t-value of 1.96.

returns. It is long the factors with positive returns and short those with negative returns. This *time-series* momentum strategy earns an annualized return of 4.2% (t-value = 7.04). We show that this strategy dominates the *cross-sectional* strategy because it is a pure bet on the positive autocorrelations in factor returns. A cross-sectional strategy, by contrast, also bets that a high return on a factor predicts low returns on the other factors (Lo and MacKinlay, 1990); in the data, however, high return on any factor predicts high returns on all factors.[3]

Momentum in factor returns transmits into the cross section of security returns, and the amount that transmits depends on the dispersion in factor loadings. The more these loadings differ across assets, the more of the factor momentum shows up as *cross-sectional* momentum in individual security returns. If stock momentum is about the autocorrelations in factor returns, factor momentum should subsume individual stock momentum. Indeed, we show that a momentum factor

[3] Goyal and Jegadeesh (2017) and Huang et al. (2018) note that time-series momentum strategies that trade individual assets (or futures contracts) are not as profitable as they might seem because they are not zero-net investment long-short strategies. In factor momentum, however, each "asset" that is traded is already a long-short strategy. The cross-sectional and time-series factor momentum strategies are therefore directly comparable.

constructed in the space of factor returns describes average returns of portfolios sorted by prior one-year returns better than Carhart's (1997) UMD, a factor that *directly* targets momentum in stock returns.

Factor momentum also explains other forms of stock momentum: industry momentum, industry-adjusted momentum, intermediate momentum, and Sharpe ratio momentum. The left-hand side of Figure 1 shows that factor momentum renders all individual stock momentum strategies statistically insignificant. We report two pairs of t-values for each version of momentum. The first is that associated with the strategy's Fama and French (2015) five-factor model alpha; the second one is from the model that adds factor momentum. The right-hand side of the same figure shows that a five-factor model augmented with *all five forms of individual stock momentum* leaves factor momentum with an alpha that is significant with a t-value of 3.96.

Our results suggest that equity momentum is not a distinct risk factor; it is an accumulation of the autocorrelations in factor returns. A momentum strategy that trades individual securities indirectly times factors. This strategy profits as long as the factors remain positively autocorrelated. Factors' autocorrelations, however, vary over time, and an investor trading stock momentum loses when they turn negative. We show that a simple measure of the continuation in factor returns determines both when momentum crashes and when it earns outsized profits. A theory of momentum would need to explain, first, why factor returns are typically positively autocorrelated and, second, why most of the autocorrelations sometimes, and abruptly, turn negative at the same time.

Our results suggest that factor momentum may be a reflection of how assets' values first diverge and later converge towards intrinsic values. The strength of factor momentum significantly varies by investor sentiment of Baker and Wurgler (2006). Conditional on low sentiment, factors that earned positive return over the prior year outperform those that lost money by 71 basis points per month (t-value = 4.79). In the high-sentiment environment, this performance gap is just 18 basis points (t-value = 1.32). This connection suggests that factor momentum may stem from asset values *drifting* away from, and later towards, fundamental values, perhaps because of slow-moving capital (Duffie, 2010). Under this interpretation, factors may, at least in part, be about mispricing (Kozak et al., 2018; Stambaugh et al., 2012)

Our results relate to McLean and Pontiff (2016), Avramov et al. (2017), and Zaremba and Shemer (2017) who show that anomaly returns predict the cross section of anomaly returns at the one-month and one-year lags. A companion paper to this study, Arnott et al. (2019), shows that the short-term industry momentum of Moskowitz and Grinblatt (1999) also stems from factor momentum; it is the short-term cross-sectional factor momentum that explains short-term industry momentum. That alternative form of factor momentum, however, explains none of individual stock momentum, consistent with the finding of Grundy and Martin (2001) that industry momentum is largely unrelated to stock momentum.

We show that the profits of cross-sectional momentum strategies derive almost entirely from the autocorrelation in factor returns; that time-series factor momentum fully subsumes momentum in individual stock returns (in all its forms); that the characteristics of stock momentum returns change predictably alongside the changes in the autocorrelation of factor returns; and that momentum is not a distinct risk factor—rather, momentum factor aggregates the autocorrelations found in the other factors. Because almost all factor returns are autocorrelated, for reasons we do not yet understand, momentum is inevitable.

2 Data

We take the factor and portfolio data from three public sources: Kenneth French's, AQR's, and Robert Stambaugh's data libraries.[4] Table 1 lists the factors, start dates, average annualized

[4]These data sets are available at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`, `https://www.aqr.com/insights/datasets`, and `http://finance.wharton.upenn.edu/~stambaug/`.

Table 1: Descriptive statistics

This table reports the start date, the original study, and the average annualized returns, standard deviations, and t-values for 15 U.S. and seven global factors. The universe of stocks for the global factors is the developed markets excluding the U.S. The end date for all factors is December 2015.

Factor	Original study	Start date	Annual return Mean	SD	t-value
U.S. factors					
Size	Banz (1981)	Jul 1963	3.1%	10.6%	2.11
Value	Rosenberg et al. (1985)	Jul 1963	4.0%	9.9%	2.96
Profitability	Novy-Marx (2013)	Jul 1963	3.0%	7.4%	2.92
Investment	Titman et al. (2004)	Jul 1963	3.6%	7.0%	3.77
Momentum	Jegadeesh and Titman (1993)	Jul 1963	8.5%	14.7%	4.18
Accruals	Sloan (1996)	Jul 1963	2.6%	6.7%	2.81
Betting against beta	Frazzini and Pedersen (2014)	Jul 1963	10.1%	11.2%	6.52
Cash-flow to price	Rosenberg et al. (1985)	Jul 1963	3.3%	10.2%	2.36
Earnings to price	Basu (1983)	Jul 1963	4.0%	10.1%	2.89
Liquidity	Pástor and Stambaugh (2003)	Jan 1968	5.0%	12.1%	2.87
Long-term reversals	Bondt and Thaler (1985)	Jul 1963	3.2%	8.7%	2.70
Net share issues	Loughran and Ritter (1995)	Jul 1963	3.1%	8.2%	2.73
Quality minus junk	Asness et al. (2017)	Jul 1963	4.2%	8.2%	3.73
Residual variance	Ang et al. (2006)	Jul 1963	1.5%	17.6%	0.64
Short-term reversals	Jegadeesh (1990)	Jul 1963	5.9%	10.9%	3.92
Global factors					
Size	Banz (1981)	Jul 1990	0.4%	7.5%	0.27
Value	Rosenberg et al. (1985)	Jul 1990	4.5%	7.5%	3.04
Profitability	Novy-Marx (2013)	Jul 1990	4.5%	4.9%	4.63
Investment	Titman et al. (2004)	Jul 1990	2.3%	6.3%	1.87
Momentum	Jegadeesh and Titman (1993)	Nov 1990	8.8%	12.6%	3.49
Betting against beta	Frazzini and Pedersen (2014)	Jul 1990	10.0%	9.9%	5.11
Quality minus junk	Asness et al. (2017)	Jul 1990	5.1%	7.1%	3.57

returns, standard deviations of returns, and t-values associated with the average returns. If the return data on a factor is not provided, we use the portfolio data to compute the factor return. We compute factor return as the average return on the three top deciles minus that on the three bottom deciles, where the top and bottom deciles are defined in the same way as in the original study.

The 15 anomalies that use U.S. data are accruals, betting against beta, cash-flow to price, investment, earnings to price, book-to-market, liquidity, long-term reversals, net share issues, quality minus junk, profitability, residual variance, market value of equity, short-term reversals, and momentum. Except for the liquidity factor of Pástor and Stambaugh (2003), the return data for these factors begin in July 1963; those for the liquidity factor begin in January 1968. The seven global factors are betting against beta, investment, book-to-market, quality minus junk, profitability, market value of equity, and momentum. Except for the momentum factor, the return data for these factors begin in July 1990; those for the momentum factor begin in November 1990. We use monthly factor returns throughout this study.

Table 1 highlights the significant variation in average annualized returns. The global size factor, for example, earns 0.4%, while both the U.S. and global betting against beta factors earn 10.0%. Factors' volatilities also vary significantly. The global profitability factor has an annualized standard deviation of returns of 4.9%; that of the U.S. momentum factor is 14.7%.

3 Factor momentum

3.1 Factor returns conditional on past returns

Table 2 shows that factor returns are significantly predictable by their own prior returns. We estimate time-series regressions in which the dependent variable is a factor's return in month t, and the explanatory variable is an indicator variable for the factor's performance over the prior year from month $t-12$ to $t-1$. This indicator variables takes the value of one if the factor's return is positive, and zero otherwise. We also estimate a pooled regression to measure the average amount of predictability in factor returns.[5]

[5] Table A1 in Appendix A.1 shows estimates from regressions of factor returns on prior one-year factor returns. We present the indicator-variable specification of Table 2 as the main specification because it is analogous to a strategy that signs the positions in factors based on their prior returns.

The intercepts in Table 2 measure the average factor returns earned following a year of underperformance. The slope coefficient represents the difference in returns between the up- and down-years. In these regressions all slope coefficients, except that for the U.S. momentum factor, are positive and nine of the estimates are significant at the 5% level. Although all factors' unconditional means are positive (Table 1), the intercepts show that eight anomalies earn a negative average return following a year of underperformance. The first row shows that the amount of predictability in factor premiums is economically and statistically large. We estimate this regression using data on all 20 non-momentum factors. The average anomaly earns a monthly return of just 1 basis point (t-value = 0.06) following a year of underperformance. When the anomaly's return over the prior year is positive, this return increases by 51 basis points (t-value = 4.67) to 52 basis points.

3.2 Average returns of time-series and cross-sectional factor momentum strategies

We now measure the profitability of strategies that take long and short positions in factors based on their prior returns. A time-series momentum strategy is long factors with positive returns over the prior one-year period (winners) and short factors with negative returns (losers). A cross-sectional momentum strategy is long factors that earned above-median returns relative to the other factors over the prior one-year period (winners) and short factors with below-median returns (losers). We rebalance both strategies monthly.[6] We exclude the two stock momentum factors, U.S. and global UMD, from the set of factors to avoid inducing a mechanical correlation between factor momentum and individual stock momentum. The two factor momentum strategies therefore trade a maximum of 20 factors; the number of factors starts at 13 in July 1964 and increases to 20 by July 1991

[6] In Appendix A.2 we construct alternative strategies in which the formation and holding periods range from one month to two years.

Table 2: Average factor returns conditional on their own past returns

The table reports estimates from univariate regressions in which the dependent variable is a factor's monthly return and the independent variable takes the value of one if the factor's average return over the prior year is positive and zero otherwise. We estimate these regressions using pooled data (first row) and separately for each anomaly (remaining rows). In the pooled regression, we cluster the standard errors by month.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.01	0.06	0.52	4.67
		U.S. factors		
Size	−0.15	−0.77	0.70	2.76
Value	0.15	0.78	0.28	1.16
Profitability	0.01	0.08	0.38	2.14
Investment	0.14	1.06	0.24	1.44
Momentum	0.78	2.03	−0.10	−0.23
Accruals	0.10	0.79	0.14	0.88
Betting against beta	−0.16	−0.58	1.29	4.10
Cash-flow to price	0.10	0.52	0.28	1.14
Earnings to price	0.16	0.81	0.27	1.07
Liquidity	0.14	0.58	0.44	1.43
Long-term reversals	−0.18	−1.10	0.71	3.42
Net share issues	0.19	1.27	0.10	0.53
Quality minus junk	−0.04	−0.23	0.61	3.04
Residual variance	−0.52	−1.73	1.19	2.89
Short-term reversals	0.34	1.28	0.20	0.66
		Global factors		
Size	−0.08	−0.42	0.23	0.95
Value	−0.04	−0.18	0.63	2.31
Profitability	0.11	0.58	0.32	1.54
Investment	−0.13	−0.75	0.51	2.34
Momentum	0.52	1.03	0.30	0.55
Betting against beta	0.03	0.08	1.15	2.95
Quality minus junk	0.28	1.18	0.20	0.72

because of the variation in the factors' start dates (Table 1).

Table 3 shows the average returns for the time-series and cross-sectional factor momentum strategies as well as an equal-weighted portfolio of all 20 factors. The annualized return on the average factor is 4.2% with a t-value of 7.60. In the cross-sectional strategy, both the winner and

Table 3: Average returns of time-series and cross-sectional factor momentum strategies

This table reports annualized average returns, standard deviations, and Sharpe ratios for different combinations of up to 20 factors. The number of factors increases from 13 in July 1964 to 20 by July 1991 (see Table 1). The equal-weighted portfolio invests in all factors with the same weights. The time-series factor momentum strategy is long factors with positive returns over the prior one-year period (winners) and short factors with negative returns (losers). The cross-sectional momentum strategy is long factors that earned above-median returns relative to other factors over the prior one-year period (winners) and short factors with below-median returns (losers). The time-series strategy is on average long 11.0 factors and short 5.8 factors. The cross-sectional strategy is balanced because it selects factors based on their relative performance. We rebalance all strategies monthly.

	Annualized return			
Strategy	Mean	SD	t-value	Sharpe ratio
Equal-weighted portfolio	4.21	3.97	7.60	1.06
Time-series factor momentum	4.19	4.27	7.04	0.98
Winners	6.26	4.70	9.54	1.33
Losers	0.28	6.38	0.31	0.04
Cross-sectional factor momentum	2.78	3.64	5.74	0.76
Winners	6.94	5.54	8.98	1.25
Losers	1.42	5.23	1.95	0.27

loser portfolios have, by definition, the same number of factors. In the time-series strategy, the number of factors in these portfolios varies. For example, if there are five factors with above-zero returns and 15 factors with below-zero returns over the one-year period, then the winner strategy is long five factors and the loser strategy is long the remaining 15 factors. The time-series momentum strategy takes positions in all 20 factors with the sign of the position in each factor determined by the factor's prior return. We report the returns both for the factor momentum strategies as well as for the loser and winner portfolios underneath these strategies. These loser and winner strategies are equal-weighted portfolios..

Consistent with the results on the persistence in factor returns in Table 2, both winner strategies outperform the equal-weighted benchmark, and the loser strategies underperform it. The portfolio of time-series winners earns an average return of 6.3% with a t-value of 9.54, and cross-sectional winners earn an average return of 7.0% with a t-value of 8.98. The two loser portfolios earn average returns of 0.3% and 1.4%, and the t-values associated with these averages are 0.31 and 1.95.

The momentum strategies are about the spreads between the winner and loser portfolios.[7] The time-series factor momentum strategy earns an annualized return of 4.2% (t-value = 7.04); the cross-sectional strategy earns a return of 2.8% (t-value = 5.74). Because time-series losers earn premiums that are close to zero, the choice of being long or short a factor following periods of negative returns is inconsequential from the viewpoint of average returns. However, by diversifying across all factors, the time-series momentum strategy has a lower standard deviation than the winner portfolio alone (4.3% versus 4.7%).

The difference between time-series and cross-sectional factor momentum strategies is statistically significant. In a regression of the time-series strategy on the cross-sectional strategy, the estimated slope is 1.0 and the alpha of 1.4% is significant with a t-value of 4.44. In the reverse regression of the cross-sectional strategy on time-series strategy, the estimated slope is 0.7 and the alpha of −0.2% has a t-value of −1.02. The time-series factor momentum therefore subsumes the cross-sectional strategy, but not vice versa.

An important feature of factor momentum is that, unlike factor investing, it is "model-free." If factors are autocorrelated, an investor can capture the resulting momentum premium without prespecifying which leg of the factor *on average* earns a higher return. Consider, for example, the SMB factor. This factor earns an average return of 27 basis points per month (see Table 2), but its premium is 55 basis points following a positive year and −15 basis points after a negative year. For the momentum investor, this factor's "name" is inconsequential. By choosing the sign of the position based on the factor's prior return, this investor earns an average return of 55 basis points per month by trading the "SMB" factor after small stocks have outperformed big stocks; and a return of 15 basis points per month by trading a "BMS" factor after small stocks have

[7] The mean return of the cross-sectional strategy is half of the difference between its winner and loser legs. The mean for the time-series strategy is closer to the mean of its winner leg because the strategy, on average, includes more long positions than short.

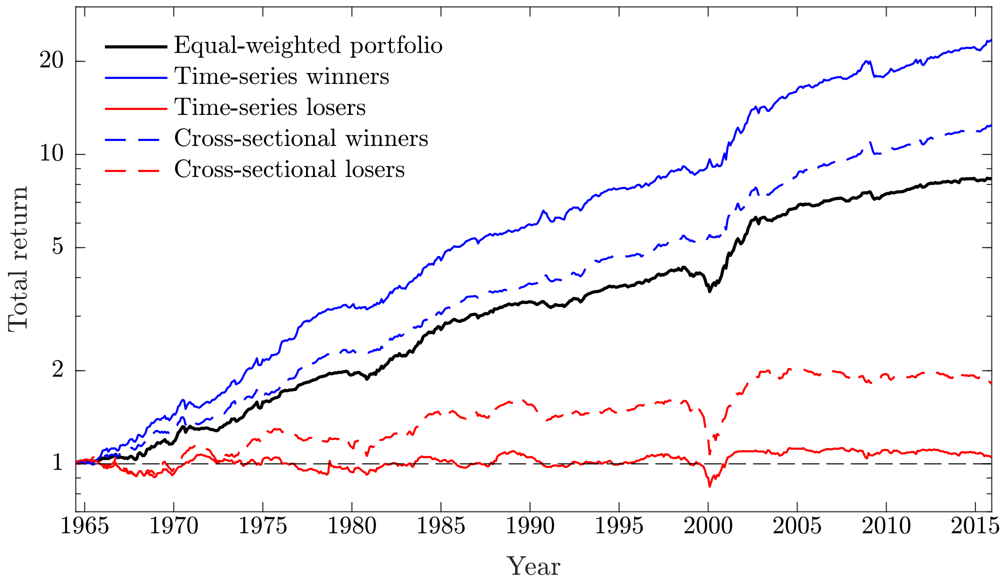

Figure 2: **Profitability of time-series and cross-sectional factor momentum strategies, July 1964–December 2015.** This figure displays total return on an equal-weighted portfolio of all factors and the returns on factors partitioned into winners and losers by their past performance. Time-series winners and losers are factors with above- or below-zero return over the prior one-year period. Cross-sectional winners and losers are factors that have out- or underperformed the median factor over this formation period. Each portfolio is rebalanced monthly and each portfolio's standard deviation is standardized to equal to that of the equal-weighted portfolio.

underperformed big stocks.

Figure 2 plots the cumulative returns associated the equal-weighted portfolio and the winner and loser portfolios of Table 3. We leverage each strategy in this figure so that each strategy's volatility is equal to that of the equal-weighted portfolio. Consistent with its near zero monthly premium, the total return on the time-series loser strategy remains close to zero even at the end of the 52-year sample period. The time-series winner strategy, by contrast, has earned three times as much as the passive strategy by the end of the sample period. Although the cross-sectional winner strategy in Panel A of Table 3 earns the highest average return, it is more volatile, and so it underperforms the time-series winner strategy on a volatility-adjusted basis. The cross-sectional loser strategy earns a higher return than the time-series loser strategy: factors that underperformed other factors but that still earned *positive* returns tend to earn positive returns the next month.

The winner-minus-loser gap is therefore considerably wider for the time-series strategies than what it is for the cross-sectional strategies.

3.3 Decomposing factor momentum profits: Why does the cross-sectional strategy underperform the time-series strategy?

We found a statistically significant difference between the cross-sectional and time-series factor momentum strategies. We redefine these strategies as in Lo and MacKinlay (1990) and Lewellen (2002) to analytically decompose their profits. The decompositions allow us to quantify the sources of profits to each strategy, and to identify the primary cause of their difference.

The cross-sectional decomposition chooses portfolio weights that are proportional to demeaned past returns. The weight on factor f in month t is positive if the factor's past return is above average and negative if it is below average:[8]

$$w_t^f = r_{-t}^f - \bar{r}_{-t}, \tag{1}$$

where r_{-t}^f is factor f's past return over some formation period such as from month $t-12$ to month $t-1$ and $\bar{r}_{-t}$ is the cross-sectional average of all factors' returns over the same formation period. The month-t return that results from the position in factor f is therefore

$$\pi_t^f = (r_{-t}^f - \bar{r}_{-t})\, r_t^f, \tag{2}$$

where r_t^f is factor f's return in month t. We can decompose the profits by averaging the profits in

[8] The key idea of the Lo and MacKinlay (1990) decomposition is the observation that, by creating a strategy with weights proportional to past returns, the strategy's expected return is the expected product of lagged and future returns. This expected product can then be expressed as the product of expectations plus the covariance of returns.

equation (2) across the F factors and taking expectations:

$$\mathrm{E}[\pi_t^{\mathrm{XS}}] = \mathrm{E}\big[\sum_{f=1}^{F} \frac{1}{F}(r_{-t}^f - \bar{r}_{-t})r_t^f\big] = \frac{1}{F}\sum_{f=1}^{F}\mathrm{cov}(r_{-t}^f, r_t^f) - \mathrm{cov}(\bar{r}_{-t}, \bar{r}_t) + \frac{1}{F}\sum_{f=1}^{F}(\mu^f - \bar{\mu})^2, \quad (3)$$

where μ^f is the unconditional expected return of factor f. The three potential sources of profits can be isolated by writing equation (3) in matrix notation (Lo and MacKinlay, 1990):

$$\begin{aligned} \mathrm{E}[\pi_t^{\mathrm{XS}}] &= \frac{1}{F}\mathrm{Tr}(\Omega) - \frac{1}{F^2}1'\Omega 1 + \sigma_\mu^2 \\ &= \frac{F-1}{F^2}\mathrm{Tr}(\Omega) - \frac{1}{F^2}(1'\Omega 1 - \mathrm{Tr}(\Omega)) + \sigma_\mu^2, \end{aligned} \quad (4)$$

where $\Omega = \mathrm{E}\big[(r_{-t}^f - \mu)(r_t^f - \mu)'\big]$ is the autocovariance matrix of factor returns, $\mathrm{Tr}(\Omega)$ is the trace of this matrix, and σ_μ^2 is the cross-sectional variance of mean factor returns.

The representation in equation (4) separates cross-sectional momentum profits to three sources:

1. Autocorrelation in factor returns: a past high factor return signals future high return,
2. Negative cross-covariances: a past high factor return signals low returns on other factors, and
3. Cross-sectional variance of mean returns: some factors earn persistently high or low returns.

The last term is independent of the autocovariance matrix; that is, factor "momentum" can emerge even in the absence of any time-series predictability (Conrad and Kaul, 1998). A cross-sectional strategy is long the factors with the highest past returns and short the factors with the lowest past returns; therefore, if past returns are good estimates of factors' unconditional means, a cross-sectional momentum strategy earns positive returns even in the absence of auto- and cross-serial covariance patterns.

Table 4 shows that the cross-sectional strategy in equation (4) earns an average annualized return of 2.5% with a t-value of 3.49. The autocovariance term contributes an average of 2.9%,

Table 4: Decomposition of factor momentum profits

Panel A reports the amount that each term in equation (4) contributes to the profits of the cross-sectional factor momentum strategy. Panel B reports the contributions of the terms in equation (5) to the profits of the time-series factor momentum strategies. We report the premiums in percentages per year. We multiply the cross-covariance term by -1 so that these terms represent their net contributions to the returns of the cross-sectional and time-series strategies. We compute the standard errors by block bootstrapping the factor return data by month. When month t is sampled, we associate month t with the factors' average returns from month $t-12$ to $t-1$ to compute the terms in the decomposition.

Strategy	Annualized Premium (%)	Standard error
Cross-sectional factor momentum	2.48	0.71
Autocovariance	2.86	0.97
$(-1)\times$ Cross-covariance	-1.00	0.54
Cross-sectional variance of mean returns	0.53	0.16
Time-series factor momentum	4.88	1.05
Autocovariance	3.01	1.02
Mean squared return	1.88	0.43

more than all of the cross-sectional strategy's profits. The cross-covariance term is positive and, therefore, it negatively contributes (-1.0% per year) to this cross-sectional strategy's profits. A positive return on a factor predicts positive returns also on the other factors, and the cross-sectional strategy loses by trading against this cross-predictability. This negative term more than offsets the positive contribution of the cross-sectional variation in means (0.5% per year).

Whereas the cross-sectional strategy's weights are based on the factors' *relative* performance, those of the time-series strategy are based on the factors' *absolute* performance. The time-series strategy is a pure bet on factor autocorrelations; in principle, this strategy could be long or short all factors at the same time whereas the cross-sectional strategy is always a balanced mix of long and short positions. The weight on factor f in month t is now just its return over the formation period, $w_t^f = r_{-t}^f$. Following Moskowitz et al. (2012), the time-series momentum strategy's expected return decomposes as:

$$\mathrm{E}[\pi_t^{\mathrm{TS}}] = \frac{1}{F}\mathrm{E}\big[\sum_{f=1}^{F} r_{-t}^f r_t^f\big] = \frac{1}{F}\sum_{f=1}^{F}\big[\mathrm{cov}(r_{-t}^f, r_t^f) + (\mu^f)^2\big] = \frac{1}{F}\mathrm{Tr}(\Omega) + \frac{1}{F}\sum_{f=1}^{F}(\mu^f)^2, \tag{5}$$

where $\mathrm{Tr}(\Omega)$ is the trace of the autocovariance matrix of factor returns and μ^f is the mean return of factor f. Equation (5) shows that the time-series momentum profits stem either from autocorrelation in factor returns or from mean returns that are either very positive or negative.[9]

Table 4 shows that the monthly premium of the time-series strategy is 4.9% with a t-value of 4.65. The decomposition of these profits into the autocorrelation and mean-squared components shows that this premium largely derives from the autocorrelation in factor returns; the annualized premiums associated with these two components are 3.0% (t-value of 2.61) and 1.9% with (t-value = 4.49). The time-series strategy outperforms the cross-sectional strategy because it does not bet on factors exhibiting negative cross-covariance; it is a pure bet on the autocorrelations in factor returns.

4 Factor momentum and individual stock momentum

4.1 Transmission of factor momentum into the cross section of stock returns: Framework

If stock returns obey a factor structure, then factor momentum transmits into the cross section of stock returns in the form of *cross-sectional* stock momentum of Jegadeesh and Titman (1993). In multifactor models of asset returns, such as the Intertemporal CAPM of Merton (1973) and the Arbitrage Pricing Theory of Ross (1976), multiple sources of risk determine expected returns.

[9] Autocovariane in factor returns appears in decompositions of both cross-sectional and time-series strategies. The scaling factor of the autocovariance term, however, is slightly different. This is because we isolate the diagonal elements of the covariance matrix to express the cross-sectional strategy profits in separate auto- and cross-covariance components.

Consider a factor model in which asset excess returns obey an F-factor structure,

$$R_{s,t} = \sum_{f=1}^{F} \beta_s^f r_t^f + \varepsilon_{s,t}, \tag{6}$$

where R_s is stock s's excess return, r^f is the return on factor f, β_s^f is stock s's beta on factor f, and ε_s is the stock-specific return component that should not command a risk premium in the absence of arbitrage. We assume that the factors do not exhibit any lead-lag relationships with the stock-specific return components, that is, $\mathrm{E}[r_{t'}^f \varepsilon_{s,t}] = 0$.

We now assume that asset prices evolve according to equation (6) and examine the payoffs to a cross-sectional momentum strategy; this strategy, as before, chooses weights that are proportional to stocks' performance relative to the cross-sectional average. The expected payoff to the position in stock s is

$$\mathrm{E}[\pi_{s,t}^{\mathrm{mom}}] = \mathrm{E}\big[(R_{s,-t} - \bar{R}_{-t})(R_{s,t} - \bar{R}_t)\big], \tag{7}$$

where $\bar{R}$ is the return on an equal-weighted index. Under the return process of equation (6), this expected profit becomes

$$\begin{aligned} \mathrm{E}[\pi_{s,t}^{\mathrm{mom}}] \;\; = \;\; & \sum_{f=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^f)\,(\beta_s^f - \bar{\beta}^f)^2\big] + \sum_{\substack{f=1 \\ f \neq g}}^{F} \sum_{g=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^g)\,(\beta_s^g - \bar{\beta}^g)\,(\beta_s^f - \bar{\beta}^f)\big] \\ & + \mathrm{cov}(\varepsilon_{s,-t}, \varepsilon_{s,t}) + (\eta_s - \bar{\eta})^2, \end{aligned} \tag{8}$$

where η_s is stock s's unconditional expected return. The expectation of equation (8) over the cross section of N stocks gives the expected return on the cross-sectional momentum strategy,

$$\begin{aligned} \mathrm{E}[\pi_t^{\mathrm{mom}}] \quad = \quad & \underbrace{\sum_{f=1}^{F} \left[\mathrm{cov}(r_{-t}^f, r_t^f)\, \sigma_{\beta_f}^2 \right]}_{\text{factor autocovariances}} + \underbrace{\sum_{f=1}^{F} \sum_{\substack{g=1 \\ f \neq g}}^{F} \left[\mathrm{cov}(r_{-t}^f, r_t^g)\, \mathrm{cov}(\beta^f, \beta^g) \right]}_{\text{factor cross-covariances}} \\ & + \underbrace{\frac{1}{N} \sum_{s=1}^{N} \left[\mathrm{cov}(\varepsilon_{-t}^s, \varepsilon_t^s) \right]}_{\substack{\text{autocovariances} \\ \text{in residuals}}} + \underbrace{\sigma_\eta^2}_{\substack{\text{variation in} \\ \text{mean returns}}}, \end{aligned} \tag{9}$$

where N is the number of stocks and $\sigma_{\beta_f}^2$ and σ_η^2 are the cross-sectional variances of the portfolio loadings and unconditional expected returns.

Equation (9) shows that the profits of the cross-sectional stock momentum strategy can emanate from four sources:

1. Positive autocorrelation in factor returns induces momentum profits through the first term. Cross-sectional variation in betas amplifies this effect.

2. The lead-lag return relationships between factors could also contribute to stock momentum profits. The strength of this effect depends both on the cross-serial covariance in factor returns and the covariances between factor loadings. This condition is restrictive: the cross-serial correlation of returns and the covariances of betas have to have the same signs. It would need to be, for example, that (1) SMB return in period 1 positively predicts HML returns in period 2 and (2) that SMB and HML loadings also positively correlate. For this channel to matter, this condition would need be satisfied for the average pair of factors.[10]

3. Autocorrelation in stocks' residual returns can also contribute to the profitability of the cross-sectional momentum strategy.

[10] In Appendix A.3 we show that this term is negligible relative to the autocovariance term in the five-factor model because of this joint condition.

4. The cross-sectional variation in mean returns of individual securities can also contribute to momentum profits. If stocks' past returns are good estimates of their unconditional means, a cross-sectional momentum is long stocks with high mean returns and short those with low means (Conrad and Kaul, 1998).

4.2 Explaining the returns on portfolios sorted on equity momentum

Does factor momentum contribute to the returns of cross-sectional momentum strategies? We focus on the role of the first term of equation (9); this is the term through which the autocorrelation in factor returns could add to the profits of cross-sectional momentum strategies. We measure the connection between the profitability of these strategies and time-series factor momentum. The time-series factor momentum strategy is the same as above: it is long factors that have earned positive returns over the prior year and short those that have earned negative returns.

In Table 5 we compare the performance of three asset pricing models in pricing portfolios sorted by prior one-year returns skipping a month; the sorting variable is the same as that used to construct the UMD factor of Carhart (1997).[11] The first model is the Fama-French five-factor model; the second model is this model augmented with the UMD factor; and the third model is the five-factor model augmented with the factor time-series momentum strategy. We report alphas for the deciles and, for the models 2 and 3, the factor loadings against UMD and factor time-series momentum.

Stock momentum is evident in the alphas of the Fama-French five-factor model. The alpha for the loser portfolio is -0.78% per month (t-value $= -4.06$) and for the winner portfolio it is 0.61% (t-value $= 4.89$). The average absolute alpha across the deciles is 27 basis points. We significantly improve the model's ability to price these portfolios by adding Carhart's momentum factor. The average absolute monthly alpha falls to 13 basis points, and the profitability of the

[11] We use the portfolio return data made available by Kenneth French at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html as the test assets.

Table 5: Time-series regressions on momentum sorted portfolios

This table compares the performance of three asset pricing models in explaining the monthly excess returns on ten portfolios sorted by prior one-year returns skipping a month. The three models are: (1) the Fama-French five-factor model with the market, size, value, profitability, and investment factors; (2) the five-factor model augmented with Carhert's UMD factor; and (3) the five-factor model augmented with the time-series factor momentum strategy. The time-series factor momentum strategy is long the factors with positive prior one-year returns and short those with negative prior one-year returns. The 20 factors used in constructing this strategy are listed in Table 1. We report alphas for each of the three models and loadings against the UMD factor and the time-series factor momentum strategy. We compute Gibbons et al. (1989) test statistics using the returns on the decile portfolios. This test statistic is distributed as $F(N, T-N-1)$ under the null hypothesis that the alphas are jointly zero, where $N = 10$ is the number of test assets and $T = 618$ is the number of observations in the time series. The sample period starts in July 1964 and ends in December 2015.

	Asset pricing model				
	FF5	FF5 + UMD		FF5 + TSMOM	
Decile	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{b}_{\text{umd}}$	$\hat{\alpha}$	$\hat{b}_{\text{tsmom}}$
Low	−0.78	−0.10	−0.92	−0.05	−2.44
	(−4.06)	(−0.92)	(−36.32)	(−0.33)	(−19.46)
2	−0.34	0.17	−0.69	0.18	−1.76
	(−2.53)	(2.50)	(−45.18)	(1.70)	(−20.25)
3	−0.22	0.18	−0.54	0.18	−1.32
	(−1.92)	(2.92)	(−37.36)	(1.90)	(−17.37)
4	−0.16	0.08	−0.33	0.13	−0.96
	(−1.89)	(1.24)	(−21.97)	(1.74)	(−16.43)
5	−0.19	−0.06	−0.17	−0.05	−0.47
	(−2.74)	(−0.98)	(−11.88)	(−0.70)	(−8.84)
6	−0.16	−0.12	−0.06	−0.09	−0.23
	(−2.44)	(−1.79)	(−3.76)	(−1.38)	(−4.27)
7	−0.11	−0.16	0.07	−0.14	0.09
	(−1.72)	(−2.47)	(4.46)	(−2.09)	(1.76)
8	0.06	−0.10	0.22	−0.06	0.42
	(0.93)	(−1.71)	(16.57)	(−0.93)	(7.82)
9	0.10	−0.14	0.32	−0.10	0.65
	(1.17)	(−2.33)	(22.99)	(−1.28)	(10.47)
High	0.61	0.19	0.58	0.18	1.43
	(4.89)	(2.43)	(32.30)	(1.72)	(16.61)
High − Low	1.39	0.29	1.50	0.24	3.87
	(4.94)	(2.53)	(56.85)	(1.09)	(22.31)
Avg. $\lvert\hat{\alpha}\rvert$	0.27	0.13		0.12	
GRS F-value	4.43	3.26		2.55	
GRS p-value	0.00%	0.04%		0.50%	

long-short portfolio falls from 1.4% to 0.3%. Yet, the alpha associated with the long-short portfolio is statistically significant with a t-value of 2.53. The UMD slope monotonically increases from -0.92 to 0.58 as we move from the bottom to top decile.

The model augmented with the time-series factor momentum strategy performs just as well as—or even better than—the Carhart (1997) six-factor model in pricing the momentum portfolios. The average absolute alpha falls to 12 basis points per month; the Gibbons et al. (1989) test statistic falls from 3.26 to 2.55; and the alpha of the high-minus low falls from 0.29% to 0.24% (t-value = 1.09). Similar to the Carhart (1997) model, the estimated slopes against the factor momentum strategy increase monotonically from bottom decile's -2.44 to top decile's 1.43.

The fact that the five-factor model augmented with factor momentum performs as well as the model augmented with UMD is surprising. The Carhart six-factor model sets a high standard because both the factor and the test assets sort on the same variable; that is, UMD targets momentum as directly as, say, HML targets portfolios sorted by book-to-market.

4.3 Alternative momentum factors: Spanning tests

In Table 6 we show that, in addition to the "standard" individual stock momentum of Jegadeesh and Titman (1993), factor momentum fully subsumes the informativeness of other cross-sectional momentum strategies.[12] In addition to the UMD factor, which sorts by stocks' prior one-year returns skipping a month, we construct three other momentum factors using the same methodology: Industry-adjusted momentum of Cohen and Polk (1998) sorts stocks' by their industry-adjusted returns; intermediate momentum of Novy-Marx (2012) sorts stocks by their returns from month $t-12$ to $t-7$; and Sharpe ratio momentum of Rachev et al. (2007) sorts stocks by the returns scaled

[12] We use the term "span" to indicate that an unconstrained investor trading the right-hand-side factors would not gain anything by trading the left-hand side strategy. See, for example, Novy-Marx (2013) and Fama and French (2015). Tests of mean-variance spanning also constrain the betas to add up to one (Kan and Zhou, 2012). We could alter the implicit leverage of the factor momentum strategy to set the sum of the beta to one.

by the volatility of returns. We also construct the industry momentum strategy of Moskowitz and Grinblatt (1999). This strategy sorts 20 industries based on their prior six-month returns and takes long and short positions in the top and bottom three industries.

Panel A of Table 6 introduces the alternative momentum factors alongside the time-series factor momentum strategy. Each factor earns statistically significant average returns and Fama-French five-factor model alphas. Although the average return associated with the time-series momentum strategy is the lowest—0.35% per month—it is also the least volatile by a wide margin. Its Sharpe and information ratio, which are proportional to the t-values associated with the average returns and five-factor model alphas, are therefore the highest among all the factors.

The first two columns of Panel B show estimates from spanning regressions in which the dependent variable is one of the momentum factors. The model is the Fama-French five factor model augmented with factor momentum. These regressions can be interpreted both from the investment and asset pricing perspectives. From an investment perspective, a statistically significant alpha implies that an investor would have earned a higher Sharpe ratio by having traded the left-hand side factor in addition to the right-hand side factors (Huberman and Kandel, 1987). From an asset pricing perspective, a statistically significant alpha implies that the asset pricing model that only contains the right-hand side variables is dominated by a model that also contains the left-hand side factor (Barillas and Shanken, 2017).

Although all definitions of momentum earn statistically significant average returns and five-factor model alphas, factor momentum spans all of them. Consistent with Table 5, time-series factor momentum leaves standard momentum (UMD) with an alpha of 0.01% per month (t-value = 0.07). Table 6 shows that factor momentum also spans the other four forms of momentum. The maximum t-value across the five specifications is intermediate momentum's 1.64.

The last two columns of Table 6 show that none of the alternative definitions of momentum

Table 6: Alternative definitions of momentum: Spanning tests

Panel A reports monthly average returns and Fama-French five-factor model alphas for alternative momentum factors. Every factor, except for industry momentum, is similar to the UMD factor of Jegadeesh and Titman (1993) ("standard momentum"). We sort stocks into six portfolios by market values of equity and prior performance. A momentum factor's return is the average return on the two high portfolios minus that on the two low portfolios. Industry momentum uses the Moskowitz and Grinblatt (1999) methodology; it is long the top three industries based on prior six-month returns and short the bottom three industries, where each stock is classified into one of 20 industries following (Moskowitz and Grinblatt, 1999, Table I). Panel A also reports references for the original studies that use these alternative definitions. Panel B reports estimates from spanning regressions in which the dependent variable is the monthly return on either one of the momentum factors or factor momentum. When the dependent variable is one of the momentum factors, we estimate regressions that augment the five-factor model with factor momentum. We report the intercepts and the slopes for factor momentum. When the dependent variable is factor momentum, we estimate regressions that augment the five-factor model with one of the momentum factors or, on the last row, with all five momentum factors, We report the intercepts and the slopes for the momentum factors. The sample begins in July 1964 and ends in December 2015.

Panel A: Factor means and Fama-French five-factor model alphas

		Monthly returns			FF5 model	
Momentum definition	Reference	$\bar{r}$	SD	$t(\bar{r})$	$\hat{\alpha}$	$t(\hat{\alpha})$
	Individual stock momentum					
Standard momentum	Jegadeesh and Titman (1993)	0.70	4.27	4.10	0.74	4.28
Ind.-adjusted momentum	Cohen and Polk (1998)	0.47	2.80	4.18	0.50	4.58
Industry momentum	Moskowitz and Grinblatt (1999)	0.35	4.13	2.09	0.39	2.26
Intermediate momentum	Novy-Marx (2012)	0.58	3.12	4.60	0.63	4.97
Sharpe ratio momentum	Rachev et al. (2007)	0.63	3.43	4.55	0.69	4.88
	Factor momentum					
Factor momentum		0.35	1.23	7.05	0.30	6.13

span time-series factor momentum. Across all six specifications reported in this panel, the lowest t-value for the alpha earned by the factor momentum is 3.73. The last specification augments the Fama-French five-factor model with all five momentum factors. In this specification factor momentum's alpha is significant with a t-value of 3.96. Table 6 indicates that factor momentum contains information not present in any other forms of momentum and yet, at the same time, no other form of momentum is at all informative about the cross section of stock returns when

Panel B: Spanning regressions

Individual stock momentum definition, SMOM	Dependent variable = Individual stock momentum $\hat{\alpha}$	FMOM	Factor momentum $\hat{\alpha}$	SMOM
Standard momentum	0.01 (0.07)	2.43 (23.49)	0.16 (4.32)	0.20 (23.49)
Industry-adjusted momentum	0.11 (1.18)	1.32 (17.95)	0.17 (4.17)	0.26 (17.95)
Industry momentum	−0.22 (−1.48)	2.02 (17.29)	0.24 (5.87)	0.16 (17.29)
Intermediate momentum	0.17 (1.64)	1.52 (17.92)	0.16 (3.89)	0.23 (17.92)
Sharpe ratio momentum	0.09 (0.83)	2.00 (24.02)	0.13 (3.73)	0.24 (24.02)
All of above			0.14 (3.96)	.†

†Note: This regression includes all six individual stock momentum factors as explanatory variables in addition to the five factors of the Fama-French five-factor model: standard momentum, industry-adjusted momentum, industry momentum, intermediate momentum, and Sharpe ratio momentum.

controlling for factor momentum.

Factor momentum's ability to span individual stock momentum, but not vice versa, suggests that individual stock momentum is a manifestation of factor momentum. An investor who trades individual stock momentum indirectly times factors, and an investor who directly times factors performs better. The indirect method loses out because it also takes positions based on noise. The other possible sources of momentum profits do not contribute to these profits, and so their inclusion renders the strategy unnecessarily volatile.

4.4 Individual stock momentum versus factor momentum with alternative sets of factors

The factor momentum strategy takes positions in up to 20 factors. Tables 5 and 6 show that this "full" version of factor momentum explains individual stock momentum. In Figure 3 we measure the extent to which this result is sensitive to the number and identity of the factors included in

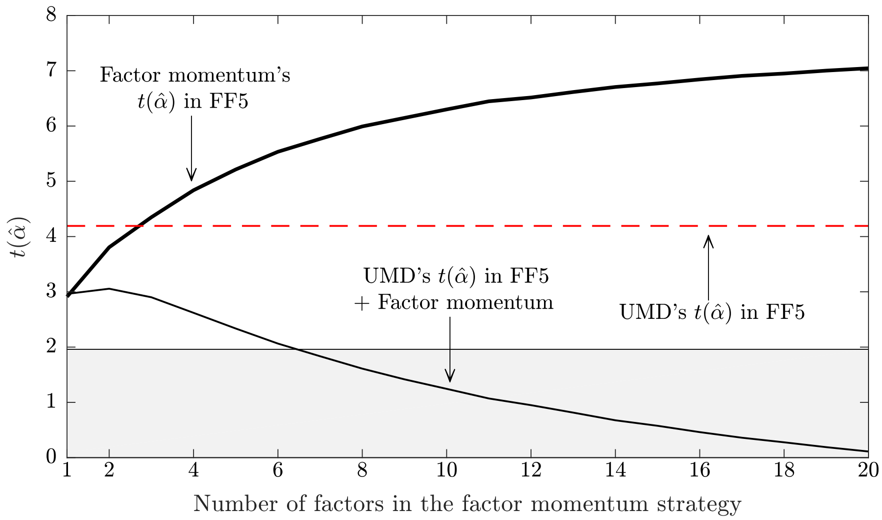

Figure 3: **Individual stock momentum versus factor momentum as a function of the number of factors.** We form random subsets of the 20 non-momentum factors listed in Table 1 and form time-series factor momentum strategies that trade these factors. A time-series factor momentum strategy is long factors with positive returns over the prior year and short those with negative returns. In this figure the number of factors ranges from 1 to 20. The thick line represents the factor momentum strategy's average $t(\hat{\alpha})$ from the Fama-French five-factor model regression; the thin line represents UMD's average $t(\hat{\alpha})$ from a regression that augments the five-factor model with the factor momentum strategy; and the dashed line denotes UMD's $t(\hat{\alpha})$ from the Fama-French five-factor model regression. The shaded region indicates t-values below 1.96.

factor momentum.

In this figure we construct random combinations of factors, ranging from one factor to the full set of 20 factors. We then construct a factor momentum strategy that trades this random set of factors and estimate two regressions. The first regression is the Fama-French five-factor model and the dependent variable is the factor momentum strategy. The dependent variable in the second regression is UMD and the model is the is the Fama-French five-factor model augmented with factor momentum. We draw 20,000 random combinations of factors for each set size, record the t-values associated with the alphas from these models, and then plot averages of these t-values as a function of the number of factors. In Figure 3 we also show, for reference, the t-value associated with UMD's alpha in the five-factor model.

Figure 3 shows that the t-value associated with factor momentum's five-factor model alpha monotonically increases in the number of factors. When the strategy alternates between long and short positions in just one factor, the average t-value is 2.91; when it trades 10 factors, it is 6.30; and when we reach 20 factors, it is 7.05. At the same time, factor momentum's ability to span UMD improves. The typical one-factor factor momentum strategy leaves UMD with an alpha that is statistically significant with a t-value of 2.96. However, when the number of factors increases to 10, this average t-value has decreased to 1.24; and with all 20 factors, this t-value is 0.07. These estimates suggest that factor momentum's ability to span UMD is not specific to the set of factors used; as the number of factors increases, the autocorrelations found within most sets of factors aggregate to explain individual stock momentum. Figure 3 supports our thesis that individual stock momentum is an aggregation of the autocorrelations found in factor returns; the more factors we identify, the better we capture UMD's return.

4.5 An analysis of momentum crashes

Individual stock momentum sometimes crashes.[13] If the profits of momentum strategies emanate from factor autocorrelations, then momentum crashes should stem from *changes* in these autocorrelations. This represents an additional testable prediction of the proposition that factor momentum drives individual stock momentum. We test for this connection by creating a proxy of the first term in equation (9) for the average factor. We can rewrite this term as a function of factor

[13]See, for example, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016).

autocorrelations:[14]

$$\sum_{f=1}^{F}\left[\operatorname{cov}(r_{-t}^{f}, r_{t}^{f})\,\sigma_{\beta_f}^{2}\right] \approx \frac{1}{12}\sum_{f=1}^{F}\left[\rho_{\text{auto}}^{f}\,\sigma_{\rho_{s,f}\times\sigma_s}^{2}\right], \tag{10}$$

where ρ_{auto}^{f} is the average autocorrelation between factor f's return in month t and its average return from month $t-12$ to $t-1$ and $\sigma_{\rho_{s,f}\times\sigma_s}^{2}$ is the cross-sectional variance of stock s's correlation with factor f ($\rho_{s,f}$) multiplied its volatility (σ_s). If this cross-sectional-dispersion term does not vary significantly across factors, momentum profits directly relate to the summation of factor autocorrelations.

We create an aggregate factor autocorrelation index to proxy for the term in equation (10). We first define factor f's autocorrelation in month t as

$$\rho_{\text{auto},t} = \frac{r_{-t}\,r_t - \mu_{-t}\,\mu_t}{\sigma_{r_t}\,\sigma_{r_{-t}}} \approx \frac{r_{-t}\,r_t - \mu^2}{\sigma^2/\sqrt{12}}, \tag{11}$$

where μ and σ are the factor's mean and standard deviation over the sample period. The aggregate factor autocorrelation index in month t is the cross-sectional average of these autocorrelations. A positive autocorrelation index in month t indicates that the average factor in month t moved in the same direction as its return during the past year.

In Figure 4 we divide the sample into two regimes based on the sign of the autocorrelation index and then draw UMD's return distribution conditional on the regime. In the positive-autocorrelation regime, in which the average factor continues to move in the same direction as it did over the prior

[14] Equation (10) can be derived as follows. Because $r_{-t} = \frac{1}{12}\big(r_{t-12} + r_{t-11} + \cdots + r_{t-1}\big)$, factor variance can be removed from the expression:

$$\begin{aligned}
\sum_{f=1}^{F}\left[\operatorname{cov}(r_{-t}^{f}, r_{t}^{f})\,\sigma_{\beta_f}^{2}\right] &= \sum_{f=1}^{F}\Big[\Big(\frac{1}{12}\sum_{k=1}^{12}\rho_{r_{t-k},r_t}^{f}\,\sigma_f^2\Big)\,\frac{1}{N}\sum_{s=1}^{N}\Big(\frac{\rho_{s,f}\sigma_s}{\sigma_f} - \frac{\overline{\rho_{s,f}\sigma_s}}{\sigma_f}\Big)^2\Big] \\
&= \sum_{f=1}^{F}\Big[\Big(\frac{1}{12}\sum_{k=1}^{12}\rho_{r_{t-k},r_t}^{f}\Big)\,\frac{1}{N}\sum_{s=1}^{N}\big(\rho_{s,f}\sigma_s - \overline{\rho_{s,f}\sigma_s}\big)^2\Big].
\end{aligned}$$

We get the presentation in equation (10) by denoting the summation of the autocorrelations from lags 1 to 12 by ρ_{auto}^{f}.

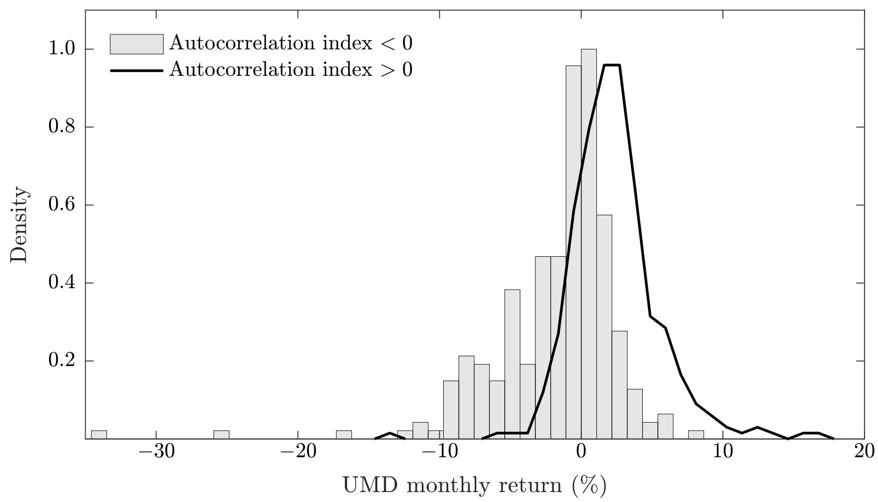

Figure 4: **Distribution of UMD returns conditional on factor autocorrelation.** A factor's autocorrelation in month t is computed as $\frac{r_{-t}\, r_t - \mu^2}{\sigma^2/\sqrt{12}}$, where μ and σ are the factor's mean and standard deviation and the formation period $-t$ runs from month $t-12$ to $t-1$. The aggregate factor autocorrelation index in month t is the cross-sectional average of these autocorrelations. This figure shows the distributions of UMD's monthly returns from July 1964 through December 2015 conditional on the aggregate autocorrelation index being negative (bars; $N = 264$) or positive (solid line; $N = 354$).

year, UMD's returns are typically positive. In these month's UMD's average return is 2.4% and its volatility is 3.3%. In the negative-autocorrelation regime, in which the average factor "turns" against its own past, UMD earns an average return of −1.6% with a standard deviation of 4.4%.[15]

The connection between the sign of the autocorrelation index and UMD is not limited to the first two moments. Figure 4 shows that almost all of UMD's left tail—its crashes—concentrates in months when the autocorrelation index is negative. The 5th percentile of UMD's return in the positive regime is −1.9%; in the negative regime it is −8.8%.

These estimates support the proposition that factor momentum emerges as momentum in the cross section of stock returns. An investor who trades individual stock momentum need to look at the individual positions to know whether the strategy turned a profit or a loss in month t. The amount of continuation in factor returns, or the lack therefore, is already a good indicator of UMD's

[15]In Appendix A.4 we report the mean, standard deviation, skewness, kurtosis, and the percentiles of UMD's return distribution conditional on the factor autocorrelation regime.

performance.

In Appendix A.5, we show that the factor autocorrelation index significantly correlates with momentum crashes and "booms." A one unit increase in this index lowers the probability of a momentum crash by 15% (z-value $= -6.78$). Factor momentum therefore explains all of UMD's returns unconditionally—and the changes in the factor autocorrelations explain when momentum is likely to crash. When factor momentum ceases, the resulting "reversal" in factor returns feeds into stock returns and crashes stock momentum.

4.6 Momentum is not a distinct risk factor

4.6.1 Conditional correlations between factors and the momentum "factor"

If the autocorrelation in factor returns contributes to the profits of these strategies, then these strategies should be more profitable when the "realized" autocorrelation in factor returns is positive.

We first examine the connection between factor momentum and individual stock momentum by measuring factors' correlations with Carhart's (1997) UMD factor. In Table 7 we report three correlations estimates for each factor: unconditional correlation, correlation conditional on the factor's return over the prior one-year period being positive, and correlation conditional on this return being negative.

Table 7 shows that the unconditional correlations between the factors and UMD are low; 11 out of the 20 correlations with the individual factors are positive, and the correlation between UMD and the portfolio of all 20 factors is 0.05. The correlations conditional on past returns, however, are remarkably different. Except for the short-term reversals factor, all factors correlate more with UMD when their past returns are positive. For 17 of these 19 factors, the difference is statistically significant at the 5% level. On the first row, we assign all factors into two groups each month based on their past returns. The estimates show that the basket of factors with positive past returns has

Table 7: Unconditional and conditional correlations with the equity momentum factor (UMD)

This table reports correlations between UMD and factor returns: ρ is UMD's unconditional correlation with the raw factor; ρ^+ is the correlation with the factor conditional on the factor's return over the prior year being positive; and ρ^- is the correlation conditional on the prior-year return being negative. The first row shows the estimates for a diversified factor which is defined as the average of the 20 factors. The conditional correlations on this row are computed by averaging factors with positive or negative returns over the prior year. The last two columns report statistics for the test that the conditional correlations are equal, H_0: $\rho^+ = \rho^-$. This test uses Fisher's (1915) z-transformation, $1 \Big/ \sqrt{\frac{1}{N^+-3} + \frac{1}{N^--3}} \left(\tanh^{-1}(\hat{\rho}^+) - \tanh^{-1}(\hat{\rho}^-)\right) \sim N(0,1)$, where $\tanh^{-1}(x) = \frac{1}{2}\frac{\ln(1+x)}{\ln(1-x)}$ and N^+ and N^- are the number of observations used to estimate ρ^+ and ρ^-.

	Unconditional correlation	Conditional correlations		Test: H_0: $\hat{\rho}^+ = \hat{\rho}^-$	
Anomaly	$\hat{\rho}$	$\hat{\rho}^+$	$\hat{\rho}^-$	z-value	p-value
Diversified	0.05	0.44	−0.50	17.87	0.00
			U.S. factors		
Size	−0.02	0.16	−0.39	6.99	0.00
Value	−0.17	0.23	−0.57	10.55	0.00
Profitability	0.09	0.44	−0.39	10.50	0.00
Investment	−0.01	0.19	−0.35	6.53	0.00
Accruals	0.07	0.24	−0.18	5.04	0.00
Betting against beta	0.17	0.36	−0.13	5.18	0.00
Cash-flow to price	−0.06	0.18	−0.40	7.35	0.00
Earnings to price	−0.14	0.16	−0.55	9.21	0.00
Liquidity	−0.02	0.07	−0.16	2.63	0.01
Long-term reversals	−0.06	0.11	−0.41	6.46	0.00
Net share issues	0.13	0.36	−0.41	9.79	0.00
Quality minus junk	0.24	0.47	−0.41	11.12	0.00
Residual variance	0.20	0.67	−0.56	17.87	0.00
Short-term reversals	−0.29	−0.36	−0.23	−1.50	0.13
			Global factors		
Size	0.08	0.17	−0.02	1.63	0.10
Value	−0.16	0.17	−0.49	5.50	0.00
Profitability	0.26	0.33	−0.17	3.40	0.00
Investment	0.07	0.41	−0.47	7.79	0.00
Betting against beta	0.24	0.27	0.15	0.87	0.38
Quality minus junk	0.41	0.51	−0.14	5.17	0.00

a correlation of 0.44 with UMD; the basket of factors with negative returns has a correlation of −0.50.

Because the unconditional correlations between momentum and the other factors are close to zero, most factor models, such as the five-factor model, explain none of momentum profits. This result, however, does not imply that momentum is "unrelated" to the other factors. Table 7 shows that the unconditional correlations are close to zero only because these correlations are significantly time-varying. Momentum, in fact, appears to relate to *all* factors!

4.6.2 Diversification benefits?

An implication of the connection between factor momentum and individual stock momentum is that the diversification benefits of momentum are more elusive than what the unconditional correlations might suggest. Consider, for example, the interaction between value and momentum. Table 7 shows that UMD's unconditional correlation with the U.S. HML is -0.17 and that with the global HML is -0.16. These negative correlations are consistent with the findings of Asness et al. (2013). This same table, however, shows that these correlations vary greatly depending on HML's performance over the prior year. In the U.S., for example, the correlation is 0.22 following a year in which HML earned a positive return and -0.57 following a negative year. These switching signs matter because HML's return also depends on its prior returns—this is the essence of factor momentum! The U.S. HML earns an average monthly return of 15 basis points after a negative year and a return of 43 basis points after a positive year (see Table 2).

Because of this interaction between individual stock momentum, value, and the autocorrelation in HML's returns, the diversification benefits of mixing momentum and value are limited. Figure 5 illustrates this issue by considering the 50-50 momentum/value strategy of Asness et al. (2013) and an alternative dynamic version of this strategy that sometimes replaces value with cash.[16] If HML

[16] In this analysis, we use the "all equity" value and momentum factors made available by AQR at https://www.aqr.com/Insights/Datasets. The difference between the AQR and Fama-French value factors is that Asness et al. (2013) divide the book values of equity by the end-of-June rather than end-of-December market values of equity (Asness and Frazzini, 2013).

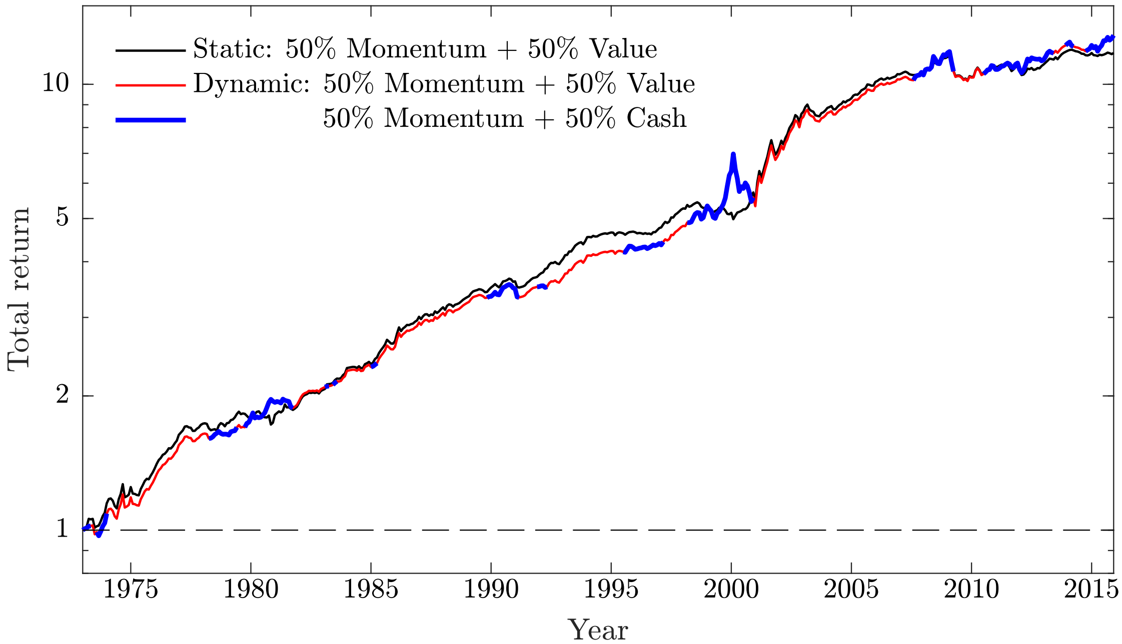

Figure 5: **Returns to diversifying momentum with value versus cash.** This figure shows total cumulative returns to two strategies that invest in momentum, value, and cash. The static strategy always invests 50% in momentum and 50% in value. The dynamic strategy invests 50% in momentum and 50% in value if value's return over the prior one-year period is positive and 50% in momentum and 50% in cash if its return is negative. Because value and momentum are zero-investment portfolios, the return on cash is set to zero. The dynamic strategy is drawn in red when the strategy is momentum-value and in blue when it is momentum-cash.

has earned a positive annual return up to month t, this strategy is the 50-50 momentum/value strategy. However, if HML's return has been negative, this strategy becomes momentum/cash. Here, because both momentum and value are zero-investment portfolios, we set cash's return to zero—this position would finance a purchase of T-bills with T-bills. Figure 5 shows that there is no meaningful difference between the original and dynamic strategies; an investor draws no benefits from the negative correlation.

This result is not specific to value's correlation with momentum. Table 7 shows that almost all factors "diversify" momentum better after they have performed poorly. The same issue thus applies to almost all factors: when a factor correlates negatively with momentum, it typically is also the state in which the factor does not earn a meaningful premium.[17] These state-dependent

[17] The short-term reversals factor is the only exception. Arnott et al. (2019) examine the diversification benefits of

Table 8: Factor returns conditional on sentiment and factor momentum

This table reports average returns on factors conditional on past factor returns and investor sentiment. Investor sentiment is defined in the same way as in Stambaugh et al. (2012). Winner factors are those that earned positive returns over the prior year; losing factors are those that earned negative returns. The winner and loser portfolios are equal-weighted portfolios of factors. The winner-minus-loser is the return difference between the two portfolios.

	Prior factor returns		
Sentiment	Losers	Winners	W − L
Low	−0.22	0.50	0.71
	(−2.10)	(6.11)	(4.79)
High	0.35	0.54	0.18
	(3.42)	(5.83)	(1.32)

correlations are the direct consequence of the nature of momentum—momentum is not something that exists *separate* from the other factors.

5 Investor sentiment and factor momentum

Stambaugh et al. (2012) show that many return anomalies are stronger following high levels of sentiment, and that this effect originates from the anomalies' short legs. They suggest that these findings are consistent with a mispricing interpretation for the anomalies: anomalies may be due to mispricing that is particularly persistent in the presence of short-sale restrictions.

Factor momentum may be a different manifestation of the same mechanism. If a segment of the market—such as growth stocks—becomes overpriced, this overpricing may take time to correct itself because of short-sale restrictions and, more broadly, slow-moving capital (Duffie, 2010). That is, asset prices do not immediately jump back to the fundamental levels but, rather, they converge over time.

Following Stambaugh et al. (2012), we take the residual from a regression of the monthly sentiment index of Baker and Wurgler (2006) against a set of macroeconomic variables as a measure

momentum, (short-term) factor momentum, and short-term reversals of investor sentiment. The intuition is that this residual represents optimism and pessimism that is not justified by the state of the macroeconomy.

In Table 8 we measure the interaction between investor sentiment and factor momentum. We assign month t into a high or low sentiment regime depending on whether month t's sentiment is above or below median; and, similar to the winner and loser portfolios in Table 3, we classify each factor as a winner or loser depending on the sign of its average return over the prior year. The average underperforming factor earns a negative return of -22 basis points (t-value $= -2.10$) in the low-sentiment environment; in the same environment, factors with positive returns over the prior year earn a premium of 50 basis points (t-value $= 6.11$). This 71-basis point difference is significant with a t-value of 4.79. In the high-sentiment environment, however, the difference is just 18 basis points (a t-value of 1.32).[18]

The difference between the two regimes is due to the loser factors. In the high sentiment environment, the average return on prior losers is significantly higher than in the low sentiment environment. This is the Stambaugh et al. (2012) finding: the *average* factor earns a higher return in the high-sentiment environment. The results in Table 8 are consistent with a mispricing interpretation for factor momentum and, by extension, all manifestation of individual stock momentum. A mispricing may take time to build up and, because of impediments to arbitrage, an asset's value does not immediately snap back to its fundamental value when arbitrageurs enter. Factor momentum may stem from asset values *drifting* back towards fundamental values.

[18] In Appendix A.6 we confirm the findings of Stambaugh et al. (2012) by estimating regressions that explain each factor's returns with both investor sentiment and past factor returns. Both sentiment and past factor returns positively predict factor returns, and neither variable subsumes the other.

6 Conclusion

Positive autocorrelation is a pervasive feature of factor returns. Factors with positive returns over the prior year earn significant premiums; those with negative returns earn premiums that are indistinguishable from zero. Factor momentum is a strategy that bets on these autocorrelations in factor returns.

We link factor momentum to individual stock momentum by decomposing stock momentum profits under the assumption that stock returns follow a factor structure. This representation shows that the autocorrelations in factor returns transmit into the cross section of stock returns through the variation in stocks' factor loadings. Consistent with this decomposition, we show that factor momentum explains both the "standard" momentum of Jegadeesh and Titman (1993) and other forms of it: industry-adjusted momentum, industry momentum, intermediate momentum, and Sharpe momentum. By contrast, these other momentum factors do not explain factor momentum. Our results imply that momentum is not a distinct factor; rather, a momentum "factor" is the summation of the autocorrelations found in the other factors. An investor who trades momentum indirectly times factors. The profits and losses of this strategy therefore ultimately depend on whether the autocorrelations in factor returns remain positive.

An additional testable implication of the proposition that individual stock momentum stems from factor momentum relates to momentum crashes. If individual stock momentum is, in the end, about factors, then momentum *crashes* should trace back to the factors as well. Indeed, we show that momentum crashes when the autocorrelations in factor returns abruptly cease. These results can guide future research. A theory of momentum would need to explain why factors are typically positively autocorrelated—and why, sometimes, almost all of these autocorrelations turn negative at the same time.

Our results imply that individual stock momentum is unlikely about firm-specific news—most factors are so well diversified that they likely wash out all remnants of firm-specific information. Our results on the connection between factor momentum and investor sentiment suggest that the autocorrelation in factor returns, and, by extension, individual stock momentum, may stem from mispricing. Factor returns may positively autocorrelate because mispricings slowly mean-revert: prices of assets that have been pushed away from fundamentals must later drift towards these fundamental values as arbitrageurs enter to profit from the mispricings.

References

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Arnott, Rob, Mark Clements, Vitali Kalesnik, and Juhani T. Linnainmaa, 2019, Factor momentum, Working paper.

Asness, Cliff, and Andrea Frazzini, 2013, The devil in HML's details, *Journal of Portfolio Management* 39, 49–68.

Asness, Clifford S., Andrea Frazzini, and Lasse Heje Pedersen, 2017, Quality minus junk, AQR working paper.

Asness, Clifford S., Tobias J. Moskowitz, and Lasse Heje Pedersen, 2013, Value and momentum everywhere, *Journal of Finance* 68, 929–985.

Avramov, Doron, Si Cheng, Amnon Schreiber, and Koby Shemer, 2017, Scaling up market anomalies, *Journal of Investing* 26, 89–105.

Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov, 2013, Anomalies and financial distress, *Journal of Financial Economics* 108, 139–159.

Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *Journal of Finance* 61, 1645–1680.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Baquero, Guillermo, Jenke Ter Horst, and Marno Verbeek, 2005, Survival, look-ahead bias, and persistence in hedge fund performance, *Journal of Financial and Quantitative Analysis* 40, 493–517.

Barillas, Francisco, and Jay Shanken, 2017, Which alpha? *Review of Financial Studies* 30, 1316–

1338.

Barroso, Pedro, and Pedro Santa-Clara, 2015, Momentum has its moments, *Journal of Financial Economics* 116, 111–120.

Basu, Sanjoy, 1983, The relationship between earnings' yield, market value and return for nyse common stocks: Further evidence, *Journal of Financial Economics* 12, 129–156.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Beyhaghi, Mehdi, and Sina Ehsani, 2017, The cross-section of expected returns in the secondary corporate loan market, *Review of Asset Pricing Studies* 7, 243–277.

Bhojraj, Sanjeev, and Bhaskaran Swaminathan, 2006, Macromomentum: Returns predictability in international equity indices, *Journal of Business* 79, 429–451.

Bondt, Werner F. M., and Richard Thaler, 1985, Does the stock market overreact? *Journal of Finance* 40, 793–805.

Boyson, Nicole M., 2008, Hedge fund performance persistence: a new approach, *Financial Analysts Journal* 64, 27–44.

Brown, Stephen J., and William N. Goetzmann, 1995, Performance persistence, *Journal of Finance* 50, 679–698.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Cohen, Randolph B., and Christopher K. Polk, 1998, The impact of industry factors in asset-pricing tests, Kellogg Graduate School of Management working paper.

Conrad, Jennifer, and Gautam Kaul, 1998, An anatomy of trading strategies, *Review of Financial Studies* 11, 489–519.

Cooper, Michael J., Roberto C. Gutierrez, and Allaudeen Hameed, 2004, Market states and momentum, *Journal of Finance* 59, 1345–1365.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under-and overreactions, *Journal of Finance* 53, 1839–1885.

Daniel, Kent, and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Duffie, Darrell, 2010, Presidential address: Asset price dynamics with slow-moving capital, *Journal of Finance* 65, 1237–1267.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2016, Dissecting anomalies with a five-factor model, *Review of Financial Studies* 29, 69–103.

Fisher, Ronald A., 1915, Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population, *Biometrika* 10, 507–521.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2012, Trading costs of asset pricing anomalies, Fama-Miller Working Paper 14-05.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Goyal, Amit, and Narasimhan Jegadeesh, 2017, Cross-sectional and time-series tests of return predictability: What is the difference? *Review of Financial Studies* 31, 1784–1824.

Griffin, John M., Xiuqing Ji, and J. Spencer Martin, 2003, Momentum investing and business cycle risk: Evidence from pole to pole, *Journal of Finance* 58, 2515–2547.

Grinblatt, Mark, Sheridan Titman, and Russ Wermers, 1995, Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior, *American Economic Review* 85, 1088–1105.

Grundy, Bruce D, and J. Spencer Martin Martin, 2001, Understanding the nature of the risks and the source of the rewards to momentum investing, *Review of Financial studies* 14, 29–78.

Hendricks, Darryll, Jayendu Patel, and Richard Zeckhauser, 1993, Hot hands in mutual funds: Short-run persistence of relative performance, 1974–1988, *Journal of Finance* 48, 93–130.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143–2184.

Huang, Dashan, Jiangyuan Li, Liyao Wang, and Guofu Zhou, 2018, Time-series momentum: Is it there? *Journal of Financial Economics,* Forthcoming .

Huberman, Gur, and Shmuel Kandel, 1987, Mean-variance spanning, *Journal of Finance* 42, 873–888.

Jagannathan, Ravi, Alexey Malakhov, and Dmitry Novikov, 2010, Do hot hands exist among hedge fund managers? An empirical evaluation, *Journal of Finance* 65, 217–255.

Jegadeesh, Narasimhan, 1990, Evidence of predictable behavior of security returns, *Journal of Finance* 45, 881–898.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Johnson, Timothy C., 2002, Rational momentum effects, *Journal of Finance* 57, 585–608.

Jostova, Gergana, Stanislava Nikolova, Alexander Philipov, and Christof W. Stahel, 2013, Momentum in corporate bond returns, *Review of Financial Studies* 26, 1649–1693.

Kan, Raymond, and Guofu Zhou, 2012, Tests of mean-variance spanning, *Annals of Economics* *and Finance* 13, 139–187.

Korajczyk, Robert A., and Ronnie Sadka, 2004, Are momentum profits robust to trading costs? *Journal of Finance* 59, 1039–1082.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2018, Interpreting factor models, *Journal of Finance* 73, 1183–1223.

Lee, Jongsub, Andy Naranjo, and Stace Sirmans, 2014, CDS momentum: Slow moving credit ratings and cross-market spillovers, Working paper.

Lesmond, David A., Michael J. Schill, and Chunsheng Zhou, 2004, The illusory nature of momentum profits, *Journal of Financial Economics* 71, 349–380.

Lewellen, Jonathan, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533–564.

Lo, Andrew W., and A. Craig MacKinlay, 1990, When are contrarian profits due to stock market overreaction? *Review of Financial Studies* 3, 175–205.

Loughran, Tim, and Jay R. Ritter, 1995, The new issues puzzle, *Journal of Finance* 50, 23–51.

McLean, R. David, and Jeffrey Pontiff, 2016, Does academic research destroy stock return predictability? *Journal of Finance* 71, 5–32.

Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf, 2012, Carry trades and global foreign exchange volatility, *Journal of Finance* 67, 681–718.

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Miffre, Joëlle, and Georgios Rallis, 2007, Momentum strategies in commodity futures markets, *Journal of Banking and Finance* 31, 1863–1886.

Moskowitz, Tobias J., and Mark Grinblatt, 1999, Do industries explain momentum? *Journal of Finance* 54, 1249–1290.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Novy-Marx, Robert, 2012, Is momentum really momentum? *Journal of Financial Economics* 103, 429–453.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Pástor, Luboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Rachev, Svetlozar, Teo Jašić, Stoyan Stoyanov, and Frank J. Fabozzi, 2007, Momentum strategies based on reward–risk stock selection criteria, *Journal of Banking and Finance* 31, 2325–2346.

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, 1985, Persuasive evidence of market inefficiency, *Journal of Portfolio Management* 11, 9–16.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Sagi, Jacob S., and Mark S. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum, *Journal of Financial Economics* 84, 389–434.

Sloan, Richard, 1996, Do stock prices fully reflect information in accruals and cash flows about future earnings? *The Accounting Review* 71, 289–315.

Stambaugh, Robert F., Jianfeng Yu, and Yu Yuan, 2012, The short of it: Investor sentiment and anomalies, *Journal of Financial Economics* 104, 288–302.

Szakmary, Andrew C., Qian Shen, and Subhash C. Sharma, 2010, Trend-following trading strategies in commodity futures: A re-examination, *Journal of Banking and Finance* 34, 409–426.

Titman, Sheridan, KC John Wei, and Feixue Xie, 2004, Capital investments and stock returns, *Journal of Financial and Quantitative Analysis* 39, 677–700.

Zaremba, Adam, and Jacob Shemer, 2017, Is there momentum in factor premia? Evidence from international equity markets, *Research in International Business and Finance* 1, 10–20.

A Appendix

A.1 Autocorrelations in factor returns

Table 2 in the main text reports estimates from regressions in which the dependent variable is a factor's return in month t and the explanatory variable is an indicator variable that takes the value of one if the factor's return over the prior year is positive and zero other. In Table A1 we measure autocorrelations in factor returns by regressing a factor's return against the factor's realized return over the prior year.

A.2 Cross-sectional and time-series factor momentum strategies with different formation and holding periods

In the main text we form both the cross-sectional and time-series factor momentum strategies using prior one-year returns and rebalance these strategies monthly. Table A2 examines the performance of these strategies with formation and holding periods ranging from one month to two years. When the holding period is longer than a month, we use the overlapping portfolio approach of Jegadeesh and Titman (1993) to correct the standard errors. In month t, the return on a strategy with a k-month formation period is computed as the average return across h portfolios, where h is the length of the holding period. We construct these h strategies every month between months $t-1$ and $t-h$. This approach produces a single time series for each formation period-holding period combination.

Panel A of Table A2 examines the performance of time-series factor momentum strategies. The time-series strategy with the one-month formation and holding periods earns an average return of 35 basis points (t-value = 6.61). These strategies typically remain profitable also with longer formation and holding periods. All time-series momentum strategies are the most profitable when

Table A1: Autocorrelations in factor returns

The table reports estimates from univariate regressions in which the dependent variable is a factor's monthly return and the independent variable is the factor's average return over the prior year. We estimate these regressions using pooled data (first row) and separately for each anomaly (remaining rows). In the pooled regression, we cluster the standard errors by month.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.24	4.34	0.30	3.04
		U.S. factors		
Size	0.19	1.50	0.29	2.28
Value	0.24	2.00	0.23	2.00
Profitability	0.18	2.04	0.28	2.51
Investment	0.23	2.54	0.25	2.11
Momentum	0.71	3.59	−0.01	−0.05
Accruals	0.20	2.48	−0.03	−0.19
Betting against beta	0.41	2.74	0.52	5.52
Cash-flow to price	0.23	1.82	0.16	1.31
Earnings to price	0.26	2.10	0.22	1.89
Liquidity	0.39	2.37	0.11	0.69
Long-term reversals	0.16	1.54	0.38	3.23
Net share issues	0.16	1.58	0.36	3.20
Quality minus junk	0.26	2.46	0.29	2.56
Residual variance	0.10	0.48	0.22	1.80
Short-term reversals	0.49	3.31	0.01	0.04
		Global factors		
Size	0.05	0.38	0.14	0.64
Value	0.21	1.54	0.45	3.67
Profitability	0.29	2.72	0.21	1.15
Investment	0.13	1.15	0.32	2.37
Momentum	0.84	3.30	−0.10	−0.54
Betting against beta	0.56	2.71	0.41	2.75
Quality minus junk	0.36	2.54	0.18	1.07

held for one month; at longer holding periods, this strategy's performance deteriorates because it cannot immediately rebalance away from factors whose average returns turn negative.

Panel B of Table A2 shows that cross-sectional momentum performs the best with one-month formation and holding periods. This strategy's average return is 31 basis points per month with

Table A2: Average returns of time-series and cross-sectional factor momentum strategies

This table reports annualized average returns and t-values for time-series and cross-sectional factor momentum strategies that trade the 20 non-momentum factors listed in Table 1. The time-series factor momentum strategy is long factors with positive returns over a formation period that ranges from one month to two years and short factors with negative returns. The cross-sectional momentum strategy is long factors that earned above-median returns relative to other factors over the same formation period and short factors with below-median returns. We let the rebalancing frequency range from one month to two years. When the holding period is longer than a month, we use the Jegadeesh and Titman (1993) approach correct standard errors for the overlapping holding periods returns.

Panel A: Time-series factor momentum

Holding	Formation period						Formation period					
period	1	3	6	12	18	24	1	3	6	12	18	24
	Average returns						***t*-values**					
1	0.35	0.29	0.33	0.36	0.28	0.28	6.61	5.43	6.44	6.77	5.47	5.60
3	0.06	0.13	0.22	0.28	0.26	0.22	1.25	2.38	4.40	5.26	5.33	4.51
6	0.13	0.08	0.22	0.22	0.21	0.19	2.63	1.70	4.75	4.75	4.40	3.90
12	0.18	0.10	0.12	0.09	0.10	0.09	3.82	2.26	2.79	2.10	2.24	2.07
18	0.08	0.03	0.03	0.06	0.06	0.10	1.78	0.75	0.74	1.38	1.48	2.50
24	0.06	0.04	0.04	0.11	0.12	0.15	1.47	1.10	1.09	2.82	3.20	4.01

Panel B: Cross-sectional factor momentum

Holding	Formation period						Formation period					
period	1	3	6	12	18	24	1	3	6	12	18	24
	Average returns						***t*-values**					
1	0.30	0.24	0.20	0.24	0.17	0.15	5.99	4.91	4.28	4.99	3.80	3.48
3	0.00	0.04	0.07	0.14	0.11	0.09	0.05	0.78	1.57	3.03	2.70	2.14
6	0.07	0.05	0.09	0.09	0.08	0.08	1.50	1.16	1.95	2.04	1.89	1.84
12	0.08	0.07	0.00	−0.04	−0.01	0.01	1.75	1.65	0.00	−0.87	−0.19	0.22
18	0.00	−0.03	−0.06	−0.02	−0.01	0.01	−0.05	−0.70	−1.56	−0.48	−0.36	0.30
24	0.03	0.00	0.00	0.02	0.03	0.02	0.83	−0.06	−0.12	0.59	0.72	0.53

a t-value of 5.98—the largest among all cross-sectional strategies. The profits on this short-term strategy decay quickly: the return on a strategy with one-month formation and three-month holding period is small and insignificant. Some of the strategies with longer formation periods, although less profitable initially, earn statistically significant profits at the three- and six-month holding periods.

A.3 Measuring the effects of the auto- and cross-serial covariances on momentum profits under the Fama-French five-factor model

Section 4.1 shows that the covariance structure of factor returns can induce momentum into the cross section of stocks through autocovariances and cross-serial correlations. Because factor returns are positively autocorrelated, the autocovariance component positively adds to the momentum profits. The effect of the cross-serial correlations, however, depends on the cross-serial covariance in factor returns and the covariance in stocks' factor loadings. This channel adds to the momentum profits only if the cross-serial correlations of returns and the covariances of betas have the same signs. In this Appendix we show that, because of the restrictiveness of this condition, it is unlikely to matter in the data. We use the Fama-French five-factor model as an illustration.

Our approach requires estimating factor betas to measure the cross-sectional variances in factor betas and covariances between betas of different factors. For each stock at time t, we estimate factor betas by estimating rolling five-factor model regressions using one year of daily return data:

$$r_{s,d} = \alpha_s + \beta_{s,d}^{\mathrm{MKTRF}}\mathrm{MKTRF} + \beta_{s,d}^{\mathrm{CMA}}\mathrm{CMA} + \beta_{s,d}^{\mathrm{HML}}\mathrm{HML} + \beta_{s,d}^{\mathrm{RMW}}\mathrm{RMW} + \beta_{s,d}^{\mathrm{SMB}}\mathrm{SMB} + \epsilon_s. \quad \text{(A-1)}$$

We winsorize the beta estimates every month at the 1st and 99th percentiles to mitigate the effect of outliers. The top panel of Table A3 shows the autocovariances (elements on the diagonal of the matrix, boldfaced) and cross-serial covariances in factor returns. Most factors exhibit positive lead-lag relationship with other factors with the exception of the market. High returns on the market between $t-12$ and $t-1$ signal lower returns on the other factors at time t. Similarly, high returns on the size (SMB), profitability (RMW), and investment (CMA) factors are associated with lower future returns on the market.

The lower panel of Table A3 reports the average cross-sectional variances and covariance of

Table A3: Equity momentum profits due to the covariance structure of factor returns under the Fama-French five-factor model.

This table reports estimates of the terms in equation (9) under the assumption that the Fama-French five-factor model governs asset returns. The first panel shows the estimates of the auto- and cross-serial covariances between factor returns, $\left[\text{cov}(r^f_{-t}, r^f_t)\right]$. The second panel shows the covariance matrix of factor betas $\left[\sigma^2_{\beta_f} \text{ and } \text{cov}(\beta^g, \beta^f)\right]$. At the bottom of the table we report the net total effects of the autocovariance-related terms, $\left[\sum_{f=1}^{F} \text{cov}(r^f_{-t}, r^f_t)\sigma^2_{\beta_f}\right]$, and the cross-covariance terms, $\left[\sum_{\substack{f=1 \\ f \neq g}}^{F} \sum_{g=1}^{F} \text{cov}(r^f_{-t}, r^g_t)\text{cov}(\beta^g, \beta^f)\right]$.

Factor auto- and cross-serial covariances					
	CMA_{-t}	HML_{-t}	RMW_{-t}	SMB_{-t}	MKTRF_{-t}
CMA_t	**0.12**	0.08	0.14	0.05	−0.08
HML_t	0.13	**0.23**	0.11	0.08	−0.22
RMW_t	−0.02	−0.03	**0.17**	−0.01	−0.06
SMB_t	0.18	0.36	0.16	**0.29**	−0.29
MKTRF_t	−0.08	0.08	−0.07	−0.10	**0.14**
Covariance matrix of factor betas					
	β_{CMA}	β_{HML}	β_{RMW}	β_{SMB}	β_{MKTRF}
β_{CMA}	**2.52**	−0.86	0.29	0.16	0.11
β_{HML}	−0.86	**2.11**	0.75	0.26	0.34
β_{RMW}	0.29	0.75	**2.66**	0.45	0.14
β_{SMB}	0.16	0.26	0.45	**1.20**	0.47
β_{MKTRF}	0.11	0.34	0.14	0.47	**0.69**
Net auto-covariance, $\left[\sum_{f=1}^{F} \text{cov}(r^f_{-t}, r^f_t)\sigma^2_{\beta_f}\right]$					1.69
Net cross-covariance, $\left[\sum_{\substack{f=1 \\ f \neq g}}^{F} \sum_{g=1}^{F} \text{cov}(r^f_{-t}, r^g_t)\text{cov}(\beta^g, \beta^f)\right]$					−0.13

betas that are estimated each month. With the exception of the investment and value factors, whose betas tend to be negatively correlated, the pairwise covariances are positive. We use these estimates to measure the contributions of the auto-covariances and cross-serial covariances to the profitability of the individual stock momentum strategy. Multiplying each cross-covariance of the top panel by its corresponding covariance in the lower panel, and summing across all factor pairs, gives a small and *negative* estimate of −0.13% for the net effect of the cross-covariance component. In contrast, the contribution of the autocovariance term is significantly higher. Under the five-factor model, it is the autocorrelation in factor returns that turns into momentum in the cross

Table A4: Distribution of momentum profits conditional on factor autocorrelation

This table reports the distributions of UMD returns conditional on the aggregate factor autocorrelation index. This index is the cross-sectional average of factors' autocorrelations (see equation (11)). We report UMD's unconditional distribution as well as the distribution conditional on the autocorrelation index being positive or negative. The sample begins in July 1964 and ends in December 2015.

		Aggregate factor autocorrelation index	
Statistic	Unconditional	< 0	> 0
Mean	0.70	−1.59	2.41
Standard deviation	4.27	4.40	3.26
Skewness	−1.37	−2.58	0.72
Kurtosis	13.59	16.95	7.66
Percentiles			
5th	−6.65	−8.81	−1.92
10th	−4.05	−6.91	−0.69
25th	−0.73	−3.16	0.36
50th	0.78	−0.48	2.07
75th	2.93	0.78	3.76
90th	4.99	2.54	5.93
95th	6.54	3.21	7.95
Number of months	618	264	354

section of stock returns.

A.4 Distribution characteristics of UMD as a function of the aggregate factor autocorrelation index.

Figure 4 in the text shows UMD's return distribution conditional on the sign of the aggregate factor autocorrelation index. Table A4 reports the first four moments and the percentiles of UMD's return distribution.

A.5 Momentum crash and booms probit regressions

Figure 4 suggests that individual stock momentum is likely to crash when the autocorrelations in factor returns turn negative. In this Appendix we measure the strength of this association. In Table A5 we reports estimates of probit models in which the dependent variable (Crash) is an indicator variable that takes the value of one when UMD's return is below the 10th percentile of its distribution and zero otherwise:

$$\Pr\big(\text{Crash}_t = 1 |\ \rho_{\text{auto},t}\big) = F(\alpha + \beta\rho_{\text{auto},t}), \tag{A-2}$$

where $\rho_{\text{auto},t}$ is the factor autocorrelation computed using equation (11), $\Pr(\cdot|\cdot)$ denotes the conditional probability, and $F(\cdot)$ is the cumulative normal distribution. We also report estimates of another set of probit models in the dependent variable (Boom) takes the value of one if UMD's return is above the 90th percentile of its distribution and zero otherwise. Because the probit model is nonlinear, we report the marginal effects implied by the slope estimates. In addition to the 20 non-momentum factors listed in Table 1, we also add the market factor to the list of factors due to its role in generating momentum crashes (Daniel and Moskowitz, 2016).

Table A5 shows that the conditional probability of a momentum crash decreases in the autocorrelation across most factors. For the 11 factors with statistically significant estimates, a one-unit increase in the autocorrelation decreases the probability of a momentum crash between -1.1% (global investment) and -7.1% (liquidity). The first row measures the effect of the aggregate autocorrelation index on the likelihood of a crash. A one-unit increase in the index associates with a 15% lower probability of a crash (z-value $= -6.78$). This estimate exceeds that of all individual factors and it supports the view that equity momentum emerges as the summation of factor autocorrelations.

Just as momentum underperforms when factor returns are negatively autocorrelated, momentum returns are higher when the autocorrelations intensify. The boom estimates are similar to the crash estimates but with the opposite signs. A one-unit increase in the aggregate autocorrelation index increases the probability of a "boom" by 15%. The aggregate autocorrelation index there-

Table A5: Factor autocorrelation and momentum crashes and booms

This table reports estimates for probit regressions that measure the relationship between momentum crashes and booms and factor autocorrelations. Momentum crash is an indicator variable that takes the value of one if UMD's return is below the 10th percentile of its distribution and zero otherwise; momentum boom takes the value of one if UMD's return is above the 90th percentile. Each row, except for the first one, measures the association between momentum crashes and booms and the autocorrelation in one of the factors. The independent variable in these regressions is the factor's autocorrelation in month t computed from equation (11). The independent variable on the first row is the aggregate factor autocorrelation index, which is the cross-sectional average of factors' autocorrelations. This table reports the marginal effects associated with the autocorrelations, that is, the effect of a one-unit increase in the autocorrelation on the likelihood of a crash or a boom in percentage points, the z-values associated with the slope estimates, and McFadden's pseudo R^2s.

	Momentum crash			Momentum boom		
Factor	$\hat{\beta}$	z-value	R^2	$\hat{\beta}$	z-value	R^2
Aggregate autocorrelation index	−15.18	−6.78	38%	15.46	7.23	24%
			U.S. factors			
Size	−3.89	−3.56	3%	3.38	3.91	4%
Value	−1.35	−1.88	1%	0.67	0.98	0%
Profitability	0.30	0.83	0%	0.06	0.20	0%
Investment	−2.33	−4.15	10%	1.91	4.02	13%
Market	−2.02	−1.78	1%	0.76	0.62	0%
Accruals	−2.46	−4.33	8%	−1.11	−2.39	3%
Betting against beta	−0.40	−0.62	0%	5.29	5.85	7%
Cash-flow to price	−2.51	−2.48	1%	1.67	1.88	1%
Earnings to price	−4.92	−4.18	4%	2.51	2.93	2%
Liquidity	−7.86	−5.62	7%	7.06	5.08	6%
Long-term reversals	−2.01	−2.47	2%	3.53	4.63	7%
Net share issues	−4.16	−4.98	11%	2.45	4.40	13%
Quality minus junk	−3.04	−4.75	11%	4.11	5.90	20%
Residual variance	−1.57	−1.55	0%	1.82	1.80	1%
Short-term reversals	−0.62	−0.75	0%	0.95	1.38	0%
			Global factors			
Size	0.84	0.77	0%	1.58	1.94	4%
Value	0.19	0.39	0%	0.67	1.26	1%
Profitability	−0.87	−1.83	32%	0.38	1.39	24%
Investment	−0.33	−0.78	1%	0.31	0.74	1%
Betting against beta	−4.36	−4.02	12%	6.54	4.44	10%
Quality minus junk	−3.38	−3.58	34%	1.53	1.84	56%

fore has remarkably balanced effects on the crashes and booms: a one-standard deviation shock to aggregate autocorrelations has almost identical effects on the *probabilities* of booms and busts.

A.6 Time-series regressions of factor returns on sentiment and lagged returns

We regress each factor's returns on lagged sentiment and past returns:

$$r_t = \alpha + \beta_s S_{t-1} + \beta_m r_{-t} + \epsilon_t, \tag{A-3}$$

where S_{t-1} is the sentiment index at time $t-1$ and r_{-t} is the average factor return from month $t-12$ to $t-1$. We also estimate alternative regressions which replace both the investor sentiment and prior average returns with indicator variables. The investor sentiment indicator variables takes the value of one when the sentiment is above the sample median and zero otherwise; the prior-return indicator variables takes the value of one when the prior return is positive and zero otherwise. This definition of the sentiment indicator variable is the same as that in Stambaugh et al. (2012).

Table A6 confirms the findings of Stambaugh et al. (2012). Anomaly returns are higher following periods of high sentiment, and often significantly so. The only statistically significant exception is the size factor. The first row reports the results from a pooled regression which clusters the standard errors by month. Both sentiment and momentum are statistically significant. The right-hand side of Panel A shows that these conclusions do not change when we replace the explanatory variables with indicator variables. In the pooled regression, high sentiment and positive factor return both signal higher future factor returns. High sentiment increases factor returns by 27 basis points (t-value = 2.75); positive factor return by 49 basis points (t-value = 4.19). Both sentiment and prior factor returns therefore independently predict factor returns.

Table A6: Time-series regressions of factor returns on sentiment and prior factor returns

We report coefficient estimates from predictive time-series regressions in which the dependent variable is the return on a factor and the explanatory variables are the lagged investor sentiment and the factor's prior one-year return: $r_t = \alpha + \beta_s S_{t-1} + \beta_m r_{t-12,t-1} + \epsilon_t$. The second set of columns shows the results from regressions that replace the sentiment and return variables with indicator variables: $r_t = a + b_s \mathbb{1}_{S_{t-1}>\text{median}} + b_m \mathbb{1}_{r_{-t}>0} + \epsilon_t$. The sentiment indicator variable takes the value of one when investor sentiment is above the sample median and zero otherwise. The return indicator variable takes the value of one when the factor's average return over the prior year is positive and zero otherwise. The pooled regression on the first line cluster standard errors by month.

	Regression 1: Continuous variables				Regression 2: Indicator variables			
	Sentiment		Prior returns		Sentiment		Prior returns	
Factor	$\hat{b}_s$	$t(\hat{b}_s)$	$\hat{b}_m$	$t(\hat{b}_m)$	$\hat{b}_s$	$t(\hat{b}_s)$	$\hat{b}_m$	$t(\hat{b}_m)$
Pooled	0.14	2.00	0.25	2.56	0.26	2.75	0.49	4.19
U.S. factors								
Size	−0.32	−2.50	0.22	1.70	−0.33	−1.30	0.63	2.41
Value	0.11	0.90	0.22	1.83	0.49	2.09	0.18	0.73
Profitability	0.29	3.17	0.17	1.41	0.47	2.73	0.31	1.70
Investment	0.12	1.48	0.21	1.77	0.26	1.61	0.22	1.28
Momentum	0.04	0.20	−0.01	−0.07	0.00	0.00	−0.10	−0.23
Accruals	−0.07	−0.82	−0.03	−0.18	−0.03	−0.16	0.14	0.88
Betting against beta	0.13	0.94	0.50	5.03	0.54	2.04	1.12	3.44
Cash-flow to price	−0.08	−0.62	0.16	1.24	0.05	0.22	0.28	1.12
Earnings to price	0.00	−0.03	0.21	1.85	0.34	1.44	0.23	0.92
Liquidity	0.16	1.05	0.08	0.52	0.19	0.63	0.41	1.31
Long-term reversals	0.05	0.53	0.38	3.20	0.02	0.12	0.71	3.41
Net share issues	0.37	3.51	0.20	1.68	0.41	2.13	0.07	0.33
Quality minus junk	0.45	4.26	0.07	0.56	0.50	2.52	0.47	2.25
Residual variance	0.95	4.23	0.01	0.10	1.16	2.73	0.85	2.00
Short-term reversals	−0.13	−0.99	−0.03	−0.17	0.10	0.38	0.21	0.70
Global factors								
Size	−0.31	−1.53	0.04	0.19	−0.39	−1.58	0.20	0.81
Value	0.84	3.67	0.19	1.36	0.68	2.65	0.52	1.89
Profitability	0.13	0.98	0.22	1.21	0.13	0.76	0.34	1.63
Investment	0.60	3.41	0.20	1.50	0.50	2.36	0.53	2.44
Betting against beta	−0.01	−0.03	0.41	2.74	−0.11	−0.33	1.14	2.91
Quality minus junk	0.46	2.32	0.15	0.88	0.28	1.13	0.21	0.77
Momentum	0.54	1.56	−0.11	−0.61	0.03	0.07	0.30	0.54