Title: The Financial Structure Implicit in the Sraffa Pasinetti Framework

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Abstract:

The aim of the present paper is twofold: (1) Show that there is a financial structure implicit in the Sraffa-Pasinetti input-output production model and (2) use this financial structure in order to connect the Sraffa-Pasinetti framework to Keynes' inter-sector monetary analysis of production. The combination of these two perspectives offers a method for the joint study of the financial and productive structures of the economy. These objectives are achieved through the role of the *production commitment*.

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The main goal of the paper is to show that if the monetary value of all financial assets and their corresponding expected cash flows are estimated, these monetary values can be compared with the monetary values and flows implied by a given (existing or expected) structure of production, and that these two monetary structures should match if the structure of production is to be sustained. Otherwise, the economy is either over-leveraged or under-leveraged, and measures must be taken in the monetary and productive sides of the economy in order to re-stablish the compatibility. A financial structure is uncovered using the fundamental notion of the production commitment proposed by Cantillo (2016) and Cantillo (2019). The introduction of the notion of the production commitment is essential for the consideration of the structure of production as a financial structure. Finance is broadly understood here as a credit/debit relation that may or may not use money and as one that is present in capitalist and non-capitalistic economies.

Under the current capitalist system finance drives production. For its part, production has a structure determined by the technical relationships between inputs and outputs. The present paper offers a way to connect these two realms, using the inter-industry model of production of commodities by means of commodities and the inter-sectoral analysis from Keynes' *General theory*. The connection between these two dimensions of the economy and their corresponding theoretical explanations is made through the concept of production commitments. Production commitments are the most basic form of finance². They are a direct derivation of the process of specialization. Production commitments extract finance directly from the process of production.

Using the notion of vertical integration derived from Sraffa's (1963) subsystems, Pasinetti (1973) proposed a way to merge the inter-sectoral analysis of the *General Theory* with the inter-industry analysis of the model of production of commodities by means of commodities. Nevertheless, the way he uses the notion of vertical integration is still incompatible with key aspects of Keynes' *General Theory*. Expectations and finance are excluded from consideration under the Sraffa-Pasinetti framework. One of the main objectives of the present paper is to show that the model of production of commodities by means of commodities is compatible with these two aspects of *The General Theory*.

On the financial side, Keynes' general theory is predicated on investors' decisions to purchase assets with varying degrees of liquidity such as currency and bonds leaving aside the inclusion of other types of financial assets like stocks. Sraffa's inter-industry analysis on the other hand, is compatible with investment alternatives differentiated by the industry that they refer to. With the consideration of the production commitments within Sraffa's input-output framework it is possible to include financial assets like stocks which are industry-specific within the Sraffa-Pasinetti framework. The joint analysis of Keynes' inter-sectoral approach along with Sraffa's inter-industry method is fertile ground for a more complete understanding of finance and production. Both Keynes' *General Theory* and Sraffa's *Production of Commodities* take certain aspects of the economic phenomenon as given. By allowing the factors that are taken as given to change, one theory can be transformed into the other. Keynes took as given factors related to distribution and allocations between industries in order to analyze changes in the general level of employment and the accompanying changes in aggregate consumption, investment, savings and expectations. Sraffa, on the other hand, assumes that these factors are given, and focused on the

² Ibid.

distributional and inter-industry aspects of the economy. With the notion of production commitments, the present paper bridges these two methodologies.

The Sraffa-Pasinetti framework, identified here with the model of production of commodities by means of commodities in conjunction with Sraffa's subsystems and Pasinetti's vertical integration, is extended to include expectations and finance. As noted in Cantillo (2016) and Cantillo (2004) economic theories do not unveil truths. The main role of economic theory is the origination expectations. Thus, the formulation of the structure of production based on the Sraffa-Pasinetti framework constitutes a way to communicate an impression about how the productive system is currently perceived, and how it is expected to evolve. The production process is framed in an analytical period. Multiple industries can only exist if there is specialization, and if there is specialization, there must be production commitments. Production commitments are an expectation and a financial credit/debit relation. Following Cantillo (2019), substantive money emerges as a result of the issuance of paper notes on the outstanding production commit to produce and use them to make purchases. More complex financial instruments can be issued based on those paper notes. Any configuration of monetary and financial assets will have value only if the underlying production commitments are effectively fulfilled (Cantillo, 2019).

The monetary value of financial assets and their cash flows in a monetary economy can be compared with the monetary flows and changes in monetary values of physical commodities implied by the productive structure. The former are production commitments expressed in monetary terms if the economy is not over-leveraged. The credit/debit relationships between different parts of the economy (for instance, industries) implied by the monetary production commitments of the structure of production can be compared with the credit/debit relationships between different entities within the economy, and their respective payment flows. The production commitments are fulfilled as the production process progresses. The multiple ways in which the Leontief inverse matrix can be interpreted imply that, for a given structure of production, there are multiple ways of formulating commitments. It is possible to find an interpretation of the derived subsystems that matches the existing configuration of monetary production commitments and their corresponding monetary flows. The notions of sub-systems and vertical integration are not used here retrospectively to estimate the direct and indirect quantities of labor and capital required to produce the net output. On the contrary, vertical integration is used prospectively, in the form of the production commitments of capital and labor required for the sustenance of an expected or desired structure of production.

The paper is divided in 8 sections plus the appendix: The first and the second sections stablish the analytical framework, the third section explains the multiple interpretations of the Leontief Inverse Matrix. The fourth section does the same for the vertically integrated units of productive capacity and labor. A bridge is stablished between the inter-industry analysis of the Sraffa-Pasinetti framework and the inter-sectoral analysis of the *General Theory*. The fifth section shows how multiple interpretations of the Leontief Inverse Matrix and the concomitant vertically integrated magnitudes can be understood at the light of the production commitments. Section 6 shows how the way cashflows work in the Sraffa-Pasinetti framework. Section 7 changes the elements that are held constant in both theories and develops a connection between Keynes and Sraffa. The appendix offers a short explanation of the model of production of commodities by means of commodities for the unfamiliar reader.

1. Intra-period Vs. Inter-period Analysis, Production Commitments and Expectations³

Production is an interwoven flow of input-output relations at various levels, each of them more or less repetitive in relation with a period of reference. A process of production is formed by sub-processes which are made of smaller sub-processes. A going process of production requires the continuous replenishment of inputs at all levels. This periodic character of production makes necessary the analysis of the conditions under which the economy reproduces itself. In the Sraffa-Pasinetti framework those conditions take the form of replacement prices⁴. This analysis must be complemented with the uncovering of its implicit expectational character. As a result, an alternative concept of finance is proposed based on the notion of production commitments. This framework uses the inter-industry and inter-sectoral dimensions to establish the compatibility between structures of production and their respective financial structures. In short, the notion of production commitments among the producers involved. No producer would specialize in the production of a commodity if they did not rely in other producers' commitment to produce other necessary goods and services. In this way, each producer in the process of specialization owes their specialized product to other specialized producers. This is the most fundamental form of credit because it springs directly from the productive system.

2. The Self-Replacement Mechanism in the Sraffa-Pasinetti Framework with no Growth

In the Sraffa-Pasinetti framework described in Pasinetti (1973), the period of production (the year t) has two moments: The beginning of the *year* and the end of the *year*. Production occurs between those two moments and is explained by the relationships between them. No time elapses between the end of a period and the beginning of the next one. $X_{(t)}$ and $Y_{(t)}$ are respectively the total and net production generated at the end of the period. Net production $(Y_{(t)})$ is obtained by subtracting from $X_{(t)}$, the replacements necessary to restore the initial stocks of commodities for the next period's production. Stocks of labor $L_{(t)}$ and capital (produced means of production) $S_{(t)}$ must be available at the beginning of each period so that production of labor. The remaining part goes to 'new investment'. In this basic model 'new investment' does not increase the stock of capital. Capital is made available in the form of input commodities. 'Old investment' restores capital to its initial level by compensating for depreciation. A stationary economy is assumed in which there is no change in technology or population.

In Pasinetti's basic model, each industry j produces one type of commodity. Each industry is defined by the inputs required for the production of its particular commodity and the corresponding production coefficients. The *j*th column of the technical coefficients a_{ij}^{Θ} represents the quantities of each direct input commodity *i* to be spent during the year by industry *j* in the production of one unit of commodity *j*. Each industry must have available a quantity a_j of labor per unit of output at the beginning of the year in order to deliver x_j units at the end of the period. Industry *j* must begin the period with a stock of capital commodities equal to a fraction a_{ij} of the final output x_j to be produced. The fixed portion of the stock of capital in industry *j* depreciates throughout the year at a constant rate δ_j . The amount of capital

³ This paper uses extensive parts of the Ph.D. dissertation of my authorship quoted here as Cantillo (2016).

⁴ The reader unfamiliar with the Sraffa-Pasinetti framework is directed to the appendix for a brief explanation.

depreciated, along with the circulating capital spent in production, comprises the quantity of input commodities spent in production. This quantity is denoted by a_{ij}^{θ} . This is the amount of input commodities that has to be replaced in each industry in order to reproduce the economy for the next period.

3. Alternative Interpretations of the Leontief Inverse Matrix

If equation (A.1) in the appendix is rearranged so that the level of total output is expressed as a function of the net output, the following expression is obtained:

$$X_{(t)} = (I - A^{\theta})^{-1} Y_{(t)}$$
(1)

The net output $Y_{(t)}$ is multiplied by a matrix of coefficients known as the Leontief inverse matrix. A variety of possible interpretations of the matrix have been proposed (Pasinetti,1973, p. 5). The following argument proposes two kinds of interpretations: Intra-periodic and inter-periodic. The self-replacement assumption projects the intra-periodic inter-industry relations onto the inter-periodic realm.

The effective self-replacement of the structure of production requires the fulfillment of the implicit production commitments. Production commitments are made at the beginning of the production period. The producers involved may implicitly or explicitly agree to generate a structure of production in accordance with the model presented above. In capitalist economies this agreement seems to be tacit, implicit or even hidden. Thus, commitments correspond to the structure of production specified by the model of production of commodities by means of commodities specified above. Each industry commits to deliver a given amount of total output X_t at the end of the production period and as a result, the economy generates a given amount of net output Y_t . The unveiling of those production commitments uncovers the expectational character of the Leontief inverse matrix. Multiple interpretations of the Leontief inverse matrix illustrate the various ways in which production commitments can be made given a particular structure of production.

The first interpretation of the Leontief Inverse Matrix is the one proposed by Pasinetti in (1973 and 1977)⁵. According to this interpretation, the Leontief inverse matrix represents the quantity of commodities required for the production of each unit of net output Y_t . Each component $\alpha_{ij}^{\Theta^*}$ of the Leontief inverse matrix represents the direct and indirect quantities of commodity *i* required, *in the economic system as a whole,* for the net production of one unit of commodity *j*. Commodity *i* is required directly as input in industry *j* and indirectly as input in other industries that provide inputs to industry *j*. The economy must produce a quantity of commodity *i* large enough to replace the direct and indirect expenditure of commodity *i*, and to produce the required net output. This is an intra-periodic interpretation because it refers to a single period of production.

Under this interpretation, the summation of the components in row i of the Leontief inverse matrix represents the total quantity of commodity i required directly and indirectly for the production of one unit of net product in all industries. In other words, it is the total quantity of commodity i that needs to be spent in the whole economy in order to produce one unit of net output in all industries. Each column

⁵ See equations A.4-A.6 in the appendix.

j represents the quantities of heterogeneous commodities that the economy needs to spend directly and indirectly in the economic system as a whole for the net-production of one unit of commodity *j*.

Pasinetti (1977) provides a second interpretation in which the Leontief matrix is calculated with a Taylor expansion. According to this procedure, under a set of assumptions⁶ Pasinetti (Pasinetti 1977, p.66) states that the Leontief inverse matrix can be calculated with the following formula:

$$(I - A^{\theta})^{-1} = I + A^{\theta} + A^{\theta^2} + A^{\theta^3} + \dots$$
(2)

By replacing (2) in (1) the following expression is obtained:

$$X_{t} = (I + A^{\Theta} + A^{\Theta^{2}} + A^{\Theta^{3}} \dots)Y_{t} = IY_{t} + A^{\Theta} Y_{t} + A^{\Theta^{2}}Y_{t} + A^{\Theta^{3}}Y_{t} \dots$$
(3)

Equation (3) shows that, in addition to producing each unit in the net output (that is, when Y_t is assumed to be a column vector of ones), the economy must also produce the direct production requirements whose production coefficients are given by matrix A^{θ} . The production of the direct requirements is also ruled by the technical coefficients described in matrix A^{θ} . In order to produce A^{θ} units of production requirements, it is necessary to spend A^{θ} inputs indirectly. This is the equivalent of A^{θ^2} units of product required indirectly for the production of one unit of each commodity in the form of net output. The remaining terms in the sequence follow the same chain of reasoning. Equation (3) is interpreted by Pasinetti as showing the stages or rounds of production directly and indirectly involved in the production of a required amount of net output. Each year the economy advances one stage until it produces the required unit vector of net output. This is an inter-period interpretation because the input-output relations occur between periods.

A third interpretation based on the aforementioned Taylor expansion can be proposed in which the series represents, within a single year, the different stages required for the production of the commodities in the net output Y_t . Before becoming part of the net output, each product is a mere *orientation* of productive resources; a stage in its productive chain. Products work their way from their early stages of production, to the final stage when they become part of the net output. Production occurs gradually and in stages as described by equation (3). Each period comprises all stages of production of all and each one of the final commodities. Due to the assumption of self-replacement, the maturation of inputs into outputs that each commodity experiences from period to period (second interpretation) is equivalent to the third interpretation in which all stages of production are represented simultaneously within a single period. This is an intra-periodic interpretation. Unlike in the second interpretation in which the final production described by equation (1) is the end result of multiple stages, in this third interpretation, the full structure is already in place, and what the Taylor expansion of the Leontief inverse matrix describes is is the maturation steps that inputs follow to become outputs. Thus within the same period, there is simultaneously, inputs in which are in multiple stages in the production process.

A fourth interpretation consists in understanding equation (3) as a description of the production of the required unit vector of net production in gradual increments of production. Each stage represents a percentage of completion of the production goal. This alternative focuses on the production of each stage

⁶ The mathematical conditions are not relevant for the present analysis.

as a proportion of the final output instead of the generation of inputs that will be used for production in a subsequent stage. This is an intra-periodic interpretation.

A fifth interpretation can be proposed in which the Taylor expansion shows the continuous stream of input-output inter-industry flows within each production year. Each stage represents the time rate at which inputs are continuously being supplied. The rates are determined by the terms of equation (3) which denote the inter-industry flows of commodities per unit of time. The coefficients in equation (3) are defined in terms of quantities per year. This standard measurement of time can be replaced by a fraction based on a different time-scale simply by dividing the matrix of coefficients of the Leontief inverse matrix by the equivalent units of the new standard. Producer industries can thus be understood as continuously receiving a stream of inputs and providing a stream of output. From this point of view, industries do not accumulate inventories. Unlike the first alternative, in this interpretation industries do not wait until the end of the year in order to replace the spent inputs.

The interpretations above show that the same equations can be used to explain what occurs inside a period of production and what occurs between periods. Once the intra-periodic relationships of production are established, the inter-periodic relations are obtained by default or vice versa. If the production coefficients are the same from period to period, and provided that the self-replacement assumption holds, the input-output relations within a single period are the same as the input-output relations from period to period. This allows multiple intra and inter-periodic interpretations.

Different interpretations have different implications in regards to the actual workings of the system. For instance, from the point of view of the fifth interpretation, all inputs are used in the same year in which they are produced. From the point of view of the first interpretation, the outputs of one period will be the inputs of the next one. In the latter case, all inputs advance one stage per year passing from the producer industry to the user industry. The importance of this extrapolation of the intra-periodic input-output relations into the inter-periodic relations of production will be more evident below, where the vertically integrated quantities of labor and capital will be explained.

4. Division of Labor among Industries and the Nature of Production Commitments in the Context of a Self-replacing Economy

By definition, specialization implies that a particular producer will dedicate its resources to the production of a particular output. By doing so, it excludes herself from the production of other commodities. Producers depend on each other for the provision of inputs. Repeated specialization requires the commitment of productive resources, counting on other producers' commitments. Pasinetti's basic model describes an economy with division of labor in which each industry is in charge of the production of one commodity. The effective division of labor amongst industries requires the formulation of reliable production commitments by all industries. In this type of economy, industries are dependent on each other for the provision of inputs and the absorption of their production. A self-replacing and viable economy of this type requires the fulfillment of the commitments of production made for the supply of inputs and the absorption of output. Those commitments are necessary for the division of labor and the survival of the productive entities amongst which labor is divided. Production commitments are expectational in the sense that they are promises for future fulfillment. In this section, a notion of commitment of production in connection with the Sraffa-Pasinetti framework is introduced. In accordance with the first interpretation of the Leontief inverse matrix presented in the previous section, each industry *j* has to make the commitment of producing the direct and indirect quantity of commodity *j* required in the production of a given amount net output. This means that industry *j* must make a commitment to produce $\alpha_j^{\Theta} = \sum_{i=1}^n \alpha_{ij}^{\Theta}$ units of product for delivery at the end of the period for each unit of the corresponding net output.

The second interpretation of the Leontief inverse matrix implies that all industries make the commitment to offer their product in progressive quantities each year as described by the Taylor expansion. In order to do so, they have to use the inputs produced in each previous year. If inputs are available from the previous stage, production can take place in the subsequent stage as described by the Taylor expansion. All industries start their production far back in time with a negligible quantity of inputs in the proportions indicated by the coefficients of production. The chain of production ends with the desired unit of output produced in the final stage.

The third interpretation implies a commitment of production by each industry j to advance one stage per year in the production of commodity j in the form described by the Taylor expansion. In the fourth interpretation on the other hand, all industries make the commitment to accomplish a percentage of the production goal in successive stages. The fifth interpretation demands the commitment of continuous replenishment of inputs by all industries. Produced inputs are used as they are produced in a continuous flow. There is no accumulation of inventories.

The aforementioned five ways of interpreting the Leontief inverse matrix at the light of the production commitments are not the only possible ones. Once a particular interpretation is identified as the one that better reflects the characteristics of a given productive system, the sub-commitments of production that occur inside each industry must conform with the more aggregated structure. This is the case of commitments made by firms within each industry, and commitments made by divisions within each firm. In the case of a monetary economy, the input-output coefficients can be re-expressed in terms of the monetary prices current or expected. If this is the case, the production commitments become monetary production commitments which can be interpreted in the aforementioned way.

5. Vertical Integration and Production Commitments

Equations (A.4)-(A.6) describe the concept of vertical integration. For each one of the commodities in the net output, it is possible to identify the self-replacing structure that produces commodity i, while restoring the initial conditions of production for the next period. This is also Sraffa's notion of sub-system. Pasinetti (1973) notes that the Leontief inverse matrix provides equations (A.5) and (A.6) with a special meaning. In particular, the following two expressions are singled out in Pasinetti's analysis:

$$a_{[n]}(I - A^{\Theta^2})^{-1} = V$$
 (i)
$$A(I - A^{\Theta^2})^{-1} = H$$
 i = 1,2, ... m. (ii)

Where V is a $1 \times m$ vector formed by components v_i , and H is an $m \times m$ matrix formed by column vectors denoted as h_i . Definition (*i*) is interpreted by Pasinetti as the direct and indirect quantity of labor required for the production of commodity *i* in the net output. Since the Leontief inverse matrix represents the direct and indirect quantities of commodities required for the production of commodity *i* in the final

output, and $a_{[n]}$ is the row vector of quantities of labor required per unit of total output, the multiplication of these two terms is equal to the direct and indirect quantities of labor required for the production of vector $Y_{i(t)}$. Similarly, since A arranges the stocks of capital required per unit of output, the multiplication of the Leontief inverse matrix by A is equal to the direct and indirect quantities of capital required for the production of $Y_{i(t)}$. Each component v_i of the row vector V represents the vertically integrated quantities of labor used to produce $Y_{i(t)}$. Each column vector h_i represents the set of heterogeneous commodities required in the form of capital, in the economy as a whole, for the production of each commodity in the net output. Each one of these sets is a vertically integrated unit of productive capacity. Finally, since consumption and investment goods are both part of the net output, it is possible to identify the subsystems underlying both types of commodities. Through the use of the notion of vertical integration, it is possible to analytically divide the commodities produced in two sectors: consumption and investment. Consumption takes the form of wages. New investment in this stationary model takes the form of luxury or non-basic commodities.

The interpretation of the notion of vertical integration proposed here differs from Pasinetti (1973, 1985, and 1988) and Bortis' (2002, 2003 and 2012). Instead of using it to try to calculate the quantities of capital and labor required for the production of commodities, here is used as a method for measuring the quantities of labor and capital that must be committed for a given structure of production. Commitments of production can be reformulated in terms of vertically integrated quantities of labor and capital. Instead of expressing the commitments required by all industries, production promises can be redefined in terms of vertically integrated quantities of the scalar vector V represents the quantity of labor that must be committed directly and indirectly in order to produce one unit of the *j*th commodities that must be committed in the form of capital for the production of one unit of commodity *j*. The different ways in which labor and capital may be committed can be described by using the interpretations of the Leontief inverse matrix presented above.

For instance, under the first interpretation of the Leontief inverse matrix, the commitments to offer labor and capital in all industries are made at the beginning of the year in order to make those resources available at the end of the year or, equivalently, at the beginning of the next one. The quantities of capital and labor are specified by H and V. Wages restore the labor capacity of workers who have no other option but to offer their labor. Wages provide the conditions under which labor is to be executed. In essence, the producers of wage goods make the commitment to produce the amount of consumption commodities required for the payment of wages at the end of the period. The Leontief inverse matrix specifies the quantity of wage goods that would provide the direct and indirect quantity of labor required by all sub-systems in the next period. From this point of view it can be said that the quantity of labor available for the next period is indirectly promised by the producers of wage goods through their respective production commitments. Likewise, the producers of capital commodities promise to make available the amount new and old investment necessary for the replacement of the economy and the generation of the corresponding net output. This is a way to describe the division of labor by the sectors that produce consumption and investment goods. This is an inter-sectoral representation of production which re-classifies production in the form of Consumption or wage goods to be committed, and Investment goods.

6. Dynamics within the Sraffa-Pasinetti Framework and the Production Commitments

According to Pasinetti's period of production, the supply of commodities is decided at the beginning of the period (Pasinetti, 1981, pp. 29-49). Demand, on the other hand, is linked to the payment of wages and profits at the end of the period (ibid). From this analysis Pasinetti extracts a full employment condition which is, according to the terms in which his analysis is conducted, macroeconomic in nature. Simply put, this condition states that

... to achieve full employment, the only requirement that is imposed is that the *sum* of all types of demand be such as to imply a total over-all expenditure equal to total potential national income.(Pasinetti, 1981, p. 46)

Put differently, Pasinetti (ibid) asserts that

Each sector i must be endowed with that stock of productive capacity which is necessary to produce the amount of commodity i which is demanded. (Pasinetti, 1981, p. 47)

The fulfillment of this condition is sufficient for the estimation of a viable system, that is, one that produces more than what it takes in the form of commodity inputs. It also implies that the system has solutions with economic meaning. Pasinetti derives these conclusions from the original production schema (Equations *A1-A3*). The full employment condition captures an essential aspect of *The General Theory*. It states that there is a fundamental breach between individual decisions and macroeconomic outcomes. This breach between the way commitments are made, and individual and aggregated outcomes is concomitant with the one that exists between the beginning of the period and the end of the production period. The level of employment is the result of all the inter-industry and inter-sector interdependencies in the economy acting together.

Under a capitalist market economy, production commitments take the form of financial/monetary arrangements. A decision to commit production is a determination made mostly by firms. The outcome of those decisions depends on the aggregate behavior of the economy. The latter is the result of decisions to commit production. Those decisions, to a considerable extent, need to be made simultaneously. No individual industry or firm has the capacity to modify the aggregate level of employment at will for it is the result of the joint interaction of the inter-industry and inter-sector relations. In other words, it is the result of the economy's decision to save, consume, invest in conjunction with the decision about what to produce.

In accordance with Pasinetti's full employment condition, if the amount and composition of the wage bill is equal to the quantity and composition of the production of consumption goods, the vertically integrated quantity of labor expressed in terms of wage goods must equate the direct and indirect quantity of consumption goods required for the production of a given commodity. From this perspective, Pasinetti's inter-industry condition of full employment is equivalent to his inter-sector condition of full employment according to which workers do not save; the wage is completely allotted to consumption. The rest of the

surplus must be allocated to new investment. Production commitments must be formulated in accordance with the required vertically integrated quantities of capital and labor.

The fulfillment of the production commitments made at the beginning of the period is a necessary condition for the self-replacement of the economy. However, nothing ensures that commitments are going to be fulfilled. The generalization of the Sraffa-Pasinetti framework does not lie on extending its analysis from the case of fixed production coefficients to the case in which coefficients may vary. Sraffa warns the reader that this ought not to be the case. Instead, the generalization proposed here lies on the idea that self-replacement may not occur. This is the case in which production commitments are not effectively fulfilled. Self-replacement is a particular case that can be used to formulate expectations and production commitments. The notion of vertical integration which transcends the period of production can be used as a structured expectation based on the quantities of capital and labor directly and indirectly required for the production of the net output, provided that the assumption of self-replacement holds. Vertical integration is based on what is observed during a period of production which is then extrapolated to other periods through the self-replacement assumption. Hence, in practice, the assumption of selfreplacement and the subsequent fulfillment of the production commitments are mere figments of imagination at the beginning of the period. This does not render the notion of vertical integration useless, but it deprives it from its long run character and puts it in the expectational realm. The current observation of the structure of production and its perceived inertia grants vertical integration the status of a structured expectation.

In a monetary economy production commitments are made with monetary assets. If the vertically integrated quantities of labor and capital are expressed in terms of the same numeraire, it is possible to establish the functionality or dysfunctionality of a given structure of production commitments with respect to a current or expected productive structure. In a monetary economy the commitments of production are formulated in terms of monetary contracts or finance allocated to the payment of wages or to capital formation. Each one of those commitments have a monetary value attached to them. Along with production commitments there are financial commitments whose content is based on monetary assets and not on production. Financial commitments however must be backed with new production or with the distribution of a given amount of production. All financial promises expire and all holders of financial promises have a purpose for them. Financial assets cannot be repaid without new production or redistribution of existing production. If the amount of financial commitments implies a larger quantity of monetary production commitments in relation to the amount required by a given (expected) structure of production, the economy is overleveraged with respect to that particular structure. In this way, the identification of the production commitments within the Sraffa-Pasinetti framework shows that there may be a disruption between the beginning of the period and the end of the period of production and between individual decisions and aggregate outcomes. Yet, the assumption of self-replacement can be used as part of the formation of a grounded expectation. In fact this is what Keynes did in The General Theory when he asserted in chapter 12 that

It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty. For this reason the facts of the existing situation enter, in a sense disproportionately, in to the formation of our long term expectations; our usual practice being to take the existing situation and project it into the future, modified only

to the extent that we have more or less definite reasons for expecting a change. (Keynes, 1936, p.148)

The analysis of monetary assets is more complete when the structure of production is taken into consideration. Let the financial aspect of production commitments expressed in terms of money be defined as the relation between the monetary value of inputs and the monetary value of the output that is to be produced with those inputs. This includes the monetary value of the inter-industry transactions, the hiring of workers and their inter-sector equivalents expressed by the monetary value of the vertically integrated quantities of labor and capital. The financial aspect of production commitments expressed in terms of money can be legitimately compared with the monetary value of X dollars in production commitments represented by the cash outflows required for the purchase of input commodities and hired labor, and the inflows of cash corresponding to the sale of output with a monetary value of say Z dollars, can be compared with a financial asset whose current price and cash outflows amount to X dollars and whose future price and cash inflows are thought to be Y dollars. In this way the financial structure of any financial asset and of the economy as a whole can be compared with the financial aspect of production commitments terms and the financial aspect of production commitments measured with money. Both the productive structure expressed in monetary terms and the financial structure can be measured with national accounting techniques.

The financial aspect of the structure of production can be expressed with the vertically integrated quantities of labor and capital making it closer to Keynes. The notion of vertical integration connects the direct and indirect quantities of labor and capital required for the production of commodities. This division of the economy between labor and capital, aggregate consumption and aggregate investment and the concomitant cash/bonds relationships. It also allows for the analysis of the distributional aspect of the economy; that is, the distribution of the surplus in wages and profits.

7. Interpretation of the Quantities of Labor and Capital

Previous sections have made explicit the production commitments in the Sraffa-Pasinetti framework, and have shown a connection between those production commitments and the financial structure of the economy. This section develops a closer relation between the Sraffa-Pasinetti framework and Keynes' general theory. Pasinetti (1973) proposed that the connection between Sraffa's inter-industry analysis and Keynes' inter-sector analysis can be based on Sraffa's concept of sub-systems and Pasinetti's vertical integration. Bortis (1997, 2003) tried to develop a closer connection between Pasinetti and Keynes through the Marxian reproduction schemes. The latter was not achieved without cost. Bortis had to exclude from his analysis the role of expectations.

Bortis (1997, p. 221) contended that, in order to find the linkages between Keynes' and Sraffa's analysis, it was necessary to eliminate the crucial role of expectations and uncertainty. Bortis argued that institutions replace the role of uncertainty in the analysis of long-run production trends. Bortis (2002, p.84-5) argues that the connection between Keynes and Sraffa must use Pasinetti's vertical integration, but in order to do so, it is necessary to dispose of Keynes' marginal efficiency of capital, and replace it instead with the long-run normal rate of profit which is more coherent with chapter 17 of *The General Theory*. Bortis (2003, p.419) complements this argument by stating that the analytical focus should

abstract short term behavioral fluctuations, and focus in a notion of long term equilibrium that is institutional in character, evolves at a comparatively slower pace, and is based in expectations based on present circumstances and are less liable to sudden changes. Bortis envisions the focus on the long run trend of normal levels as an analytical device driven by expectations that are less liable to change. However, Bortis (2003, p. 422) again disposes of Keynes' notion of the marginal efficiency of capital asserting that:

... is associated with uncertainty and expectations. Indeed, investment decisions are now decisively based on comparisons between the *objectively given* realized and normal profit rates, which enables us to evacuate largely the subjective and psychological elements of Keynes' analysis that Sraffa disliked so intensely, and provides a very strong link between Sraffa and Keynes. Bortis (2003, p. 422)

Thus, even though Bortis acknowledges the need for putting together the inter-industry and inter-sectoral sides of the economy through Pasinetti's vertical integration under a Keynesian framework, he is eliminating key aspects of the latter. This view is re-enforced latter in Bortis (2012, pp. 146-147) where the author concludes that uncertainty, which drives the subjective aspects of *The General Theory*, and Sraffa's prices of re-production are incompatible. The idea is then, to replace the latter with a mark-up pricing principle which in turn brings about the Marxian scheme of reproduction in its basic form : $M - C \dots P \dots C' - M'$ (Bortis, 2012, p.167). Bortis' idea is to use long period analysis as the part of the economy that changes slowly, and use prices, wages and profits as the distributional aspect decided by society. Nonetheless, as Earnest Mandel notices in the introduction to *Capital Volume 2*, this is a misuse of Marxian reproduction schemas, for it was in the evolution of the structure of production that Marx was interested the most. In addition, the focus of this dissertation is not to assume that there are long-run coefficients in Sraffa's framework. Sraffa's analytical device was used by him in order to analyze distributional issues, but it can be extended to the analysis of changes in the aggregate level of output and employment whose dynamics are produced by the changing character of short and long-run expectations. Thus, M - C - M' is not tied here, to a mark-up theory but to the notion of the production commitment.

The alternative proposed here aims at making a more comprehensive integration between the Sraffa-Pasinetti framework and Keynes *General Theory* inclusive of expectations and uncertainty. The quantities of committed labor and capital play a crucial role in the present argument. In this section, an additional step is taken by explaining what is meant by wages and the quantities of labor and capital in both approaches.

Sraffa considered that wages are paid *post factum*:

We shall also hereafter assume that the wage is paid *post factum* as a share of the annual product, thus abandoning the classical economists' idea of a wage 'advanced' from capital. (Sraffa, 1963, p.10)

The notion of production commitments applied to wages makes a bridge between the classics and Sraffa at this respect. At first sight, the payment of wages seems to be a compensation for the laborers' efforts. However, from the point of view of the productive system as a whole, wages constitute the promise to supply the labor required for the use of capital in a subsequent period. The promise to supply labor is a

promise to the suppliers of capital commodities. In turn, the commitment to produce capital commodities is not made to the workers, but to the suppliers of wage goods who will sell their output to the hired workers. The self-replacement assumption plays a key role in this interpretation. In terms of the actual workings of the economy, the wage and its converse, the supply of labor and consumption, justify the use of capital commodities. The promise to produce wage commodities for the payment of wages is a promise to supply labor. The employment of that labor requires the fulfillment of the promise to produce the corresponding capital commodities.

In the model of production of commodities by means of commodities, the employment unit is the set of goods (the wage) required in order to make available the labor force at the beginning of the period of production. An abstract unit of labor, the wage unit, is constructed to measure the standard quantity of labor. In spite of its apparent denial of the institutional factors that differentiate labor, the wage unit allows for a social definition of an employed man. The measurement of the quantity of labor in employment units takes into account the survival needs of the human body and the social definition of employment. From this point of view, full time workers may be, in fact, only partially employed if they are underpaid. A worker is socially underpaid if the existing wage does not allow them to fully express their humanity. The latter, like the wage unit, is not restricted to the physical boundaries of the individual's human body.

This institutional character of the wage unit is also compatible with the coefficients of production in the Sraffian framework, which are considered a long run relationship between the quantity of labor and output. Institutional factors and production coefficients are both expected to change relatively slowly. Sraffa and Keynes define the wage unit following the same method but with a different angle. Whereas the former takes an inter-industry point of view, the latter takes an inter-sector point of view. Sraffa asserts that:

The quantity of labour employed in each industry has now to be represented explicitly, taking the place of the corresponding quantities of subsistence. We suppose labour to be uniform in quality or, what amounts to the same thing, we assume any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage.p10 (Sraffa, 1963, p.10)

Later he asserts that,

We call *w* the wage per unit of labour [the wage unit], which like prices will be expressed in terms of the chosen standard. (Sraffa, 1963, p.11)

For his part Keynes asserts that:

For, in so far as different grades and kinds of labour and salaried assistance enjoy a more or less fixed relative remuneration, the quantity of employment can be sufficiently defined for our purpose by taking an hour's employment of ordinary labor as our unit and weighting an hour's employment of special labor in proportion to its remuneration; i.e. an hour of special labour remunerated at double ordinary rates will count as two units. We shall call the unit in which the quantity of employment is measured the labour unit; and the money-wage of a labour unit we

shall call the wage unit. Thus, if E is the wages (and salaries) bill, W the wage-unit, and N the quantity of employment, $E = N \cdot W$. (Keynes, 1936, p. 41)

Whereas Sraffa is standardizing over "differences in quality" Keynes is standardizing over different degrees of "special labor". The former focused on an inter-industry approach. The latter was carrying his analysis from an inter-sector perspective. Sraffa, from an inter-industry perspective, focused on the fluctuations of the wage unit in relation to the appropriation of a given surplus. Keynes, from an intersector perspective, took the wage unit as given, so that he could analyze the fluctuations in the aggregate level of employment. Whereas Sraffa took as given the standard in which the wage unit is measured, Keynes with the use of Sraffa's 'own rates of interest explained how the economy selects that standard. Like the wage unit, the monetary unit is the standard in which other financial instruments are measured. Whereas Sraffa concentrates on issues regarding prices and distribution, Keynes focused on the impact of economic decisions on the level of employment. Sraffa (1963, p.33) shows how profits can be taken as given so that their effect on prices can be analyzed. Profits in turn are "susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest" (Ibid, 1963). This paragraph also shows the eclectic and survey-like methodology of Sraffa's inquiry. He takes the profits rate, as opposed to the wage rate, as the independent variable. Profits can be determined before prices, and are in general driven by factors outside the system like the money interest rate. The latter was the task undertaken by Keynes in the General Theory. The fact that profits can be set before prices in Sraffa, opens the door for the role of expectations. Sraffa like Keynes also uses the Marshallian one-thing-at-atime method.

In regards to the measurement of capital, Pasinetti uses the units of vertically integrated productive capacity. This measurement takes into account the quantities of circulating and fixed capital. In self-replacing state, this quantity of heterogeneous commodities is equivalent to the direct and indirect quantity of commodities required for production. For his part, Keynes (Keynes, 1936, p.43) is skeptical of using the quantity of capital as part of his theoretical analysis. Since he is analyzing the "the behavior of the economic system as a whole", he uses two units: money and labor.

But when we are aggregating the activities of all firms, we cannot speak accurately except in terms of quantities of employment applied to a given equipment. (Keynes, 1936, p. 40)

Thus, 'fresh capital' and consumption are measured in hours of labor paid for, given an existing amount and composition of capital (Ibid, p. 44). Here there seems to be a bigger difference between the Sraffa-Pasinetti framework and Keynes. Nonetheless, this is another example of Keynes' inter-sector approach versus Sraffa's inter-industry approach. Keynes takes as given the existing capital structure in order to analyze the fluctuations in the aggregate level of employment. Such fluctuations are not explained under the assumption of self-replacement. In Sraffa, from an inter-industry perspective, self-replacement is driven by the price system. In Keynes' inter-sector analysis, self-replacement is not relevant. In the interindustry analysis proposed by Sraffa built upon self-replacement, expectations do not play a crucial role. Both types of analysis are not opposite. They are complementary. It is proposed here that Sraffa's selfreplacing mechanism does not rule out expectations. It takes them as given meaning that their change is not being analyzed. Thus, Sraffa's prices are not long run prices. They are a system of prices given by expectations based on a particular structure of production. In Keynes, the inter-industry relations of production are not ruled out as irrelevant. They are taken as given meaning that their change is not being analyzed. Keynes focuses on the fluctuations of the aggregate level of employment due to changes in expectations.

8. The Sectors Consumption and Investment, commitments of production and vertical integration in Keynes' General Theory

"All production is for the purpose of ultimately satisfying a consumer." (Keynes, 1936, p. 46)

The amount of employment, as determined by the amount of private investment, depends on the factors that explain investment expenditure. These are the money interest rate in conjunction with the marginal efficiency of investment. Once the prospective stream of proceeds of an alternative project of investment is established, firms compare it with other investments. The marginal efficiency of an investment project cannot fall below the money interest rate if it is to be undertaken. Otherwise, the investor would not part with liquidity in order to engage in the investment project. Transaction and precautionary motives for holding money depend on the level of income. This epitomizes the given character of inter-industry relations. Uncertainty about the inter-industry relations is assumed to be zero so that the uncertainty about the level of employment can be analyzed. The speculative motive in conjunction with animal spirits explain the current investment expenditure and the level of employment derived from it. Investors who part with liquidity in order to invest it, accept the promise of a future stream of income derived from their sales to other firms. The existence of a well-structured financial market allow investors to make monetary promises which need not to be backed with future production. Likewise, employment based on current investment need not to be backed by the future use of produced capital equipment. The advantage of producing unproductive goods and services as a result of government policies in order to increase the current level of employment is that such policy does not imply a reduction in the future prices of capital commodities. In this way, it allows the private sector to increase investment in the future which in turn prompts an increase in current demand.

Keynes is concentrating on the expectational and monetary aspect of the productive system. This method unveils the important interaction between money and expectations from the inter-sector point of view. The form in which income is saved is important in order to establish whether such type of decision implies the creation of employment. Being two sides of the same transaction, savings and investment are always equal. Whoever saves part of their monetary income, is purchasing an asset. Assets have value for as long as they can be repaid, and that can only occur with the generation of the corresponding income. In turn, income can only be produced with employment.

Keynes' inter-sector focus has implicit in it an implied notion of production commitments. The interindustry dimension of the commitments of production is given by the supply side of Keynes analysis. The latter is taken as given through an existing level of capital equipment to which various levels of employment are associated. The uncertainty about the demand of specific kinds of commodities is not tackled by Keynes' analysis. Keynes concentrates on the quantities to be demanded of those commodities. Thus, in terms of the quantity of employment, what matters is the quantities of consumption and investment commodities to be produced. The inter-sector dimension of the production commitments are implied by the fact that current employment is the promise of future consumption and that current employment depends on the expectations of future employment. This is analogous to the way financial markets are organized in terms of current and future interest rates. Current interest rates depend on the expectations about future interest rates. The configuration between the two mechanisms does not necessarily procure full employment. Monetary assets promise to be repaid, investment productive assets promise to be used and employment promises to be paid. All those promises depend on each other for their fulfillment. Nothing guaranties that they will be kept. In order to have investment it is necessary to have people willing to hold assets. In order to invest, it is necessary to part with liquidity. Workers must accept the promise of a monetary payment. Those promises must bear a relationship with each other in such a way that the balance is kept so as to sustain a given level of employment. Nothing ensures that the economy acting on its own accounts will keep that balance.

Keynes (1936, p.164) acknowledges that, in addition to conducting monetary and fiscal policy, the government must calculate the marginal efficiencies of the various industries. This can be done with the analysis of the structure of production presented above. In conjunction with Keynes' analysis, this theoretical framework based on the notion of *the production commitment* connects Keynes' monetary inter-sector analysis with Pasinetti's production analysis.

The possibility of hoarding money for speculation opens up the possibility to withholding the commitment part of the structure of production, without committing any production at all, and thus jeopardizing the re-placement of the structure of production. These two sides of the productive process: The commitment part and the technical relations of production, is what must be reconciled in a financial accounting system. What the input-output analysis in conjunction with the production commitments does, is to generalize this whole argument, to the case in which the economy is further partitioned, in order to consider, not only the savings and consumption sectors, but also the multiplicity of industries within them. Thus, what Keynes did in terms of his criticism of the loanable funds theory in relation to production of consumption and investment commodities, is generalized here to the case of multiple industries within those two sectors and the interrelation among them. In the same way reducing consumption does not imply increasing investment, decreasing expenditure on one industry does not imply increasing expenditure in the others. Keynes talked about this possible inconsistency in terms of bonds/money decisions. The generalized structure includes other types of assets like stocks. In this way, the economy is thought to have not only term-specific savings (Bonds of various maturities), but assets that represent both termspecific and industry-specific savings. Remember that the value of a capital asset depends on what it promises to produce and sell, which in turn depends on its role within the division of labor within the structure of production. It is proposed here, that a contribution is made by providing language by which decision-makers can keep a harmonic evolution between the structure of production commitments, expectations of monetary proceeds, production and employment.

APPENDIX 1

SRAFFA-PASINETTI'S BASIC MODEL OF VERTICAL INTEGRATION

Pasinetti (1973) proposes that the productive structure of the economy can be modeled with the following system of equations:

$$(I - A^{\Theta})X_{(t)} = Y_{(t)}$$
 (A.1)

$$a_{[n]}X_{(t)} = L_{(t)} (A.2)$$

$$AX_{(t)} = S_{(t)} \tag{A.3}$$

Where *I* is the identity matrix, $X_{(t)}$ is an $m \times 1$ vector whose components are the total quantities produced of each commodity *i* (i = 1, 2, ..., m). In this economy, commodities are produced by means of commodities. A^{Θ} is an $m \times m$ matrix whose components $[a_{ij}^{\Theta}]$ represent the quantities of commodities per unit of total output ($X_{(t)}$) spent in the form of circulating capital (completely spent within the year of production *t*) and fixed capital (partially spent during the production period) in the production of commodity *i* by the industry *j*. A constant fraction δ_j of fixed capital drops out of the production process every year. $Y_{(t)}$ is an $m \times 1$ column vector that represents the net output of the economy after taking into account the replacements of the circulating and fixed capital spent in production. This economy is assumed to be viable, meaning that it produces a net output.

The components of the $1 \times m$ row vector $a_{[n]}$ represent the quantities of labor measured in *men-years* required for the production of one unit of the respective commodities in the total output $X_{(t)}$. Hence, the scalar $L_{(t)}$ is the total amount of labor required for the production of $X_{(t)}$. A is an $m \times m$ matrix whose components $[a_{ij}]$ represent the total stock of capital required at the beginning of the year for the production of one unit of each commodity in $X_{(t)}$. Each column in matrix A represents the commodities directly required for the production of one unit of one unit of capital stocks required at the beginning of year t for the production of $X_{(t)}$.

Within this framework, Pasinetti isolates each commodity in the net output in order to identify the *self-replacing part* that produces each of them. This is done by proposing the following set of equations derived from equations 1.1-1.3:

$$X_{(t)}^{(i)} = (I - A^{\theta})^{-1} Y_{i(t)}$$
(A.4)

$$L_{(t)}^{(l)} = a_{[n]}(I - A^{\Theta})^{-1}Y_{i(t)}$$
(A.5)

$$S_{(t)}^{(i)} = A(I - A^{\theta})^{-1} Y_{i(t)}$$
(A.6)

Where $Y_{i(t)}$ is an $m \times 1$ column vector whose components are all zeroes except the i^{th} one. Thus, $X_{(t)}^{(i)}$, $L_{(t)}^{(i)}$ and $S_{(t)}^{(i)}$ are the quantities of total output, labor and capital stocks required for the production of commodity i in the net output. These quantities include the replacements necessary to restore the initial conditions of production. Together, equations A.4-A.6 represent a sub-system for commodity i.

The term $(I - A^{\theta})^{-1}$ is known as the Leontief matrix. Each one of its elements represents the quantities of all commodities required in the whole economic system for the production of commodity *i* as a final product $Y_{i(t)}$ (meaning that it can be used as a consumption or investment commodity). Pasinetti proposes that $a_{[n]}(I - A^{\theta})^{-1}$ and $A(I - A^{\theta})^{-1}$ can be interpreted as the direct and indirect quantities of labor (in the case of the former) and capital (in the case of the latter) required for the production of $Y_{i(t)}$.

Pasinetti renames the two aforementioned terms in the following way:

$$a_{[n]}(I - A^{\theta})^{-1} \equiv V$$
(A.7)
$$A(I - A^{\theta})^{-1} \equiv H$$
(A.8)

Each one of the components of vector V, $[v_i]$, represents the direct and indirect quantities of labor required in the whole economic system in order to produce commodity i as a final output while restoring the initial production capabilities. Likewise, each one of the column vectors that form matrix H, $[h_i]$, represents the heterogeneous collection of commodities required directly and indirectly in the form of capital stocks, for the production of commodity i as a final product. Together, v_i and h_i are the vertically integrated sectors of commodity i.

Pricing system in the basic model:

$$p = a_{[n]}w + pA^{\Theta} + pA\pi \tag{A.9}$$

Where p is an $m \times 1$ column vector of prices, w is a scalar that represents the wage rate and π is the uniform rate of profits.

Meaningful solutions, full employment and viability conditions:

According to Pasinetti (1977, p. 62-63) the condition of viability is necessary for the Sraffa-Pasinetti framework to have meaningful solutions (non-zero solution for prices). Viability implies the generation of a positive net output. Pasinetti (1981, p. 33-35) and Pasinetti (1981, p. 46-48) is more explicit in asserting that viability implies full employment, as total expenditure (C+I) must equal potential national income. Pasinetti (1977) and (1981) coincide in defining the condition as the capacity of the system to generate a surplus. Due to the assumption of self-replacement, such capacity must be matched the corresponding expenditure in consumption and investment goods. In Pasinetti (1981, p. 34) Structural change Pasinetti is interpreting the demand coefficients as proportions of the total level of employment.

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