

# Macroeconomic Tail Risks and Asset Prices

David Schreindorfer  
ASU

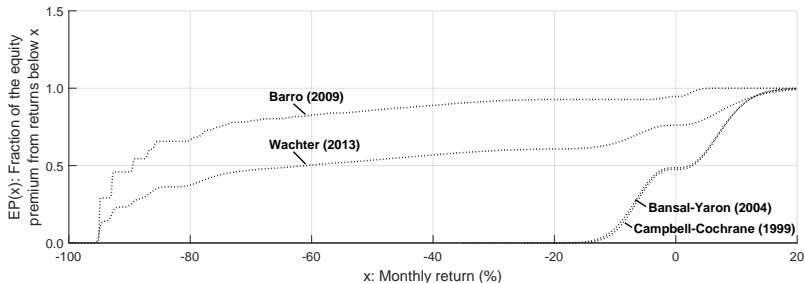
AFA 2020 (San Diego)

# Motivation: Beason and Schreindorfer (2019)

- The unconditional equity premium equals

$$\mathbb{E}[R_{t+1} - R_t^f] = \int_{-1}^{\infty} R[f(R) - f^*(R)]dR$$

- $f$  : unconditional return distribution
  - $f^*$  : average risk-neutral distribution
- Competing asset pricing mechanisms:

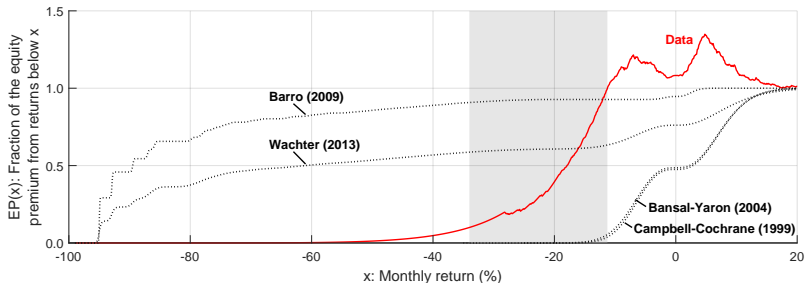


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# This paper

- **I propose a parsimonious consumption-based asset pricing model...**
  - risk premia reflect exposure to (moderately-sized) macroeconomic tail risks
  - analytical asset pricing solutions
  - parsimonious (8 parameters)
- **...that resolves the equity premium and  $R_f$  puzzles...**
  - low risk aversion + small, IID consumption shocks
- **...and is quantitatively consistent with options data**
  - Option prices: IV smirk (Rubinstein 1994), VIX-SVIX (Martin 2017)
  - Option returns: (Broadie et al. 2009)

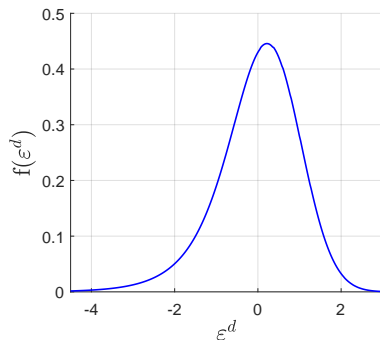
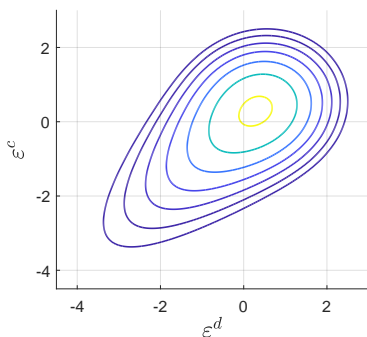
## Model – fundamentals

- Consumption and dividends follow random walks

$$\Delta c_{t+1} = g + \sigma \varepsilon_{t+1}^c$$

$$\Delta d_{t+1} = g + \varphi \sigma \varepsilon_{t+1}^d$$

- ...with correlated  $\sim (0, 1)$  innovations from a **mixture distribution**

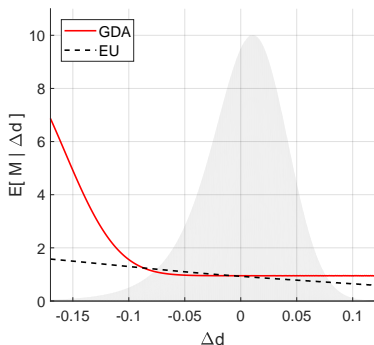
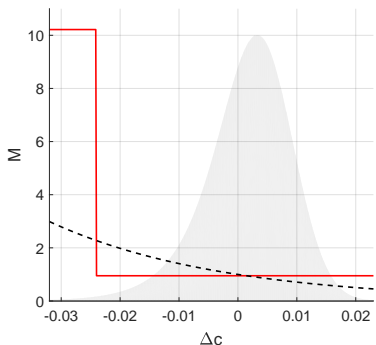


- Tail events in  $\Delta c$  and  $\Delta d$  are likely to coincide

## Model – utility function

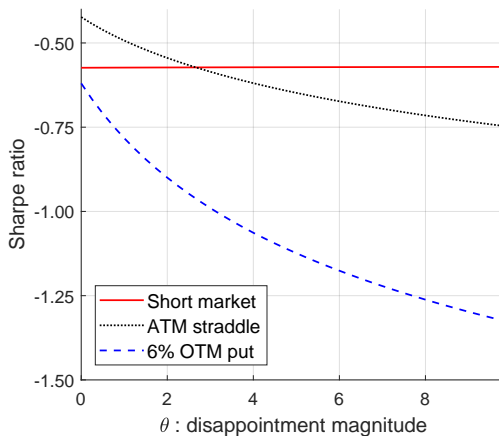
- Recursive utility (CES time aggregator, **GDA** risk aggregator)
- With IID  $\Delta c$ , the GDA pricing kernel is

$$M_{t+1} = \tilde{\beta} e^{(\alpha-1)\Delta c_{t+1}} (1 + \theta \mathbf{1}\{\Delta c_{t+1} \leq \ln(\delta) + \tilde{x}\})$$



- Tail events in  $\Delta c$  and  $\Delta d$  carry a large price of risk

# Mechanism



- Tail sensitive assets earn large Sharpe ratios
- The equity premium predominantly compensated investors for exposure to macroeconomic tail risks

# Many moments

## Annual growth rates

	$\mathbb{E}[\Delta c]$	$\sigma[\Delta c]$	$ac1[\Delta c]$	$corr[\Delta c, \Delta d]$
Data	1.82	2.11	0.50	0.53
Model	1.82	2.11	0.23	0.53
	$\mathbb{E}[\Delta d]$	$\sigma[\Delta d]$	$ac1[\Delta d]$	
Data	1.71	11.02	0.19	
Model	1.82	11.02	0.23	

## Annual asset prices

	$\mathbb{E}[R^f]$	$\mathbb{E}[R - R^f]$	$\sigma[R - R^f]$	$\mathbb{E}[pd]$
Data	0.46	8.02	19.75	3.42
Model	1.04	8.02	14.59	2.83

## Higher moments of monthly market returns

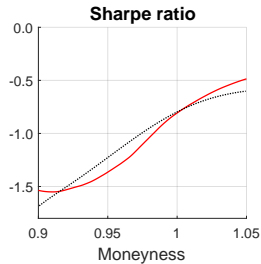
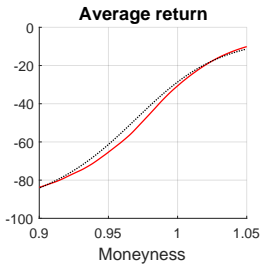
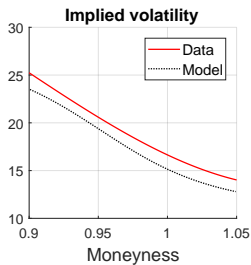
	$skew[R^{ex}]$	$kurt[R^{ex}]$	$\overline{skew}_t^*[R^{ex}]$	$\overline{kurt}_t^*[R^{ex}]$
Data	-0.88	5.09	-1.22	6.21
Model	-0.72	4.19	-1.16	5.31

## Monthly straddle returns

	$\mathbb{E}[R^s]$	$\sigma[R^s]$	$skew[R^s]$	$kurt[R^s]$	$SR[R^s]$
Data	-187.30	217.86	1.30	6.12	-0.86
Model	-176.24	237.31	1.37	5.60	-0.75

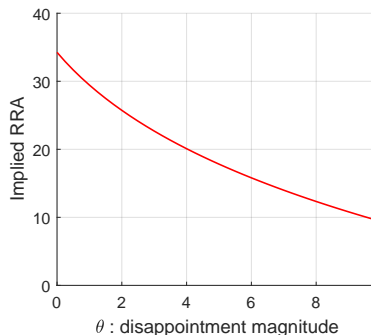
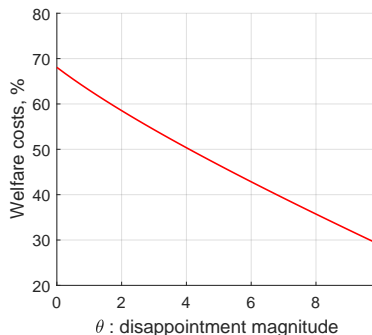


# Equity index options



Martin (2017): VIX-SVIX

# Implied risk aversion



- Left: For each  $\theta$ , pick  $\alpha$  (utility curvature) to match the equity premium
- Right: Then set  $\theta = 0$  and find  $\alpha$  that implies identical welfare costs
- **The benchmark calibration ( $\theta = 9.76$ ) implies the same RRA as a calibration with EU risk preferences and  $1 - \alpha = 9.72$**

# Beason and Schreindorfer (2019)

Table I: Characteristics of important return states:  $R \in [-34\%, -11.3\%]$

	EP fraction	$\int f(R)dR$	$\frac{\int f^*(R)dR}{\int f(R)dR}$
Data, 1990-2018	0.900	0.011	3.236
Campbell-Cochrane (1999)	0.038***	0.004**	1.391*
Bansal-Yaron (2004)	0.055**	0.006	1.311*
Barro (2009)	0.020***	0.001***	1.676
Wachter (2013)	0.046***	0.007	1.245
Bekaert-Engstrom (2017)	0.369**	0.028**	1.511**
Drechsler-Yaron (2011)	0.357**	0.017	1.738
Backus-Chernov-Martin (2011)	0.304*	0.021*	1.378**
Schreindorfer (2019)	0.720	0.008	4.685

- Sources of risk premia in leading asset pricing theories differ substantially (and significantly) from those in the data
- Extensions that increase the **quantity of tail risk** to capture option prices do only slightly better
- A model with a large **price of tail risk** captures sources of the equity premium in addition to its level

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# Take-aways

- GDA preferences can rationalize the equity premium based on
  - ...low risk aversion and
  - ...consumption shocks of the magnitude typically observed during recessions
- The model
  - ...is tightly parameterized
  - ...allows for analytical asset pricing solutions
  - ...is quantitatively consistent with index options data
- Risk premia predominantly reflect exposure to macroeconomic tail risks

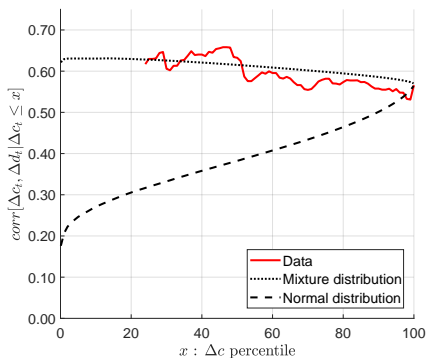


# Mixture distribution

$$\varepsilon_t^c = \sqrt{1 - \omega^2} \eta_t^c + \omega(\eta_t^e - 1)$$

$$\varepsilon_t^d = \sqrt{1 - \omega^2} \eta_t^d + \omega(\eta_t^e - 1)$$

- $\eta^c, \eta^d \sim N(0, 1)$ ,  $\eta^e \sim \exp(1)$ , mutually independent
- nests the Gaussian random walk model:  $\omega = 0$





# Generalized Disappointment Aversion

GDA risk preferences are given by

$$u(\mu_t) = E_t \left[ u(U_{t+1}) \right] - \theta E_t \left[ \left( u(\delta\mu_t) - u(U_{t+1}) \right) \mathbb{D}_{t+1} \right]$$

- Disappointment indicator:  $\mathbb{D}_{t+1} \equiv \mathbf{1}\{U_{t+1} \leq \delta\mu_t\}$
- Period utility function:  $u(U) = \frac{U^\alpha}{\alpha}$
- Nests expected Utility ( $\theta = 0$ ):  $\mu_t = \mathbb{E}[U_{t+1}^\alpha]^\frac{1}{\alpha}$
- Axiomatic foundation, but relaxes independence axiom and allows for asymmetric risk attitudes

## Martin (2017)

$$VIX_t^2 = 2R_t^f \int_0^\infty \frac{O_t(K)}{K^2} dK \quad SVIX_t^2 = \frac{2}{R_t^f} \int_0^\infty \frac{O_t(K)}{S_t^2} dK$$

- VIX puts more weight on OTM put options
- VIX – SVIX captures the magnitude of higher risk-neutral moments
- *“none of [six leading consumption-based models] come close to matching [...] the mean level of VIX minus SVIX observed in the data”*

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	Data, 1990-2017	Campbell-Cochrane '99	Bansal-Yaron '04	Bansal-Kiku-Yaron '12	Bollerslev-Tauchen-Zhou '09	Drechsler-Yaron '11	Wachter '13
VIX-SVIX	0.57	-0.03	-0.01	-0.01	-0.03	0.09	6.44

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