# An Investment-and-Marriage Model with Differential Fecundity 

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## Three Sets of Stylized Facts

## 1. College and Earnings Gender Gaps

- Reversed college gender gap
- Persistent earnings gender gap




## 2. Average Midlife Income by Age at Marriage

- Hump-shaped relationship for men
- Positive relationship for women


3. Average Spousal Income by Age at Marriage for Women

- Hump-shaped relationship
- Changing relationship: early versus late brides



## Previous Explanations

1. More women than men go to college and fewer women than men earn a high income.

- One gender difference in the model can generate these two opposite gender gaps; no paper has done that
- Some empirical studies: Iyigun and Walsh (2007); Chiappori et al. (2009); Ge (2011); Lafortune (2013); Bruze (2015); Greenwood et al. (2016); Chiappori et al. (2017)

2. Relationship between age at marriage and personal midlife income has been persistently hump-shaped for men and positive for women.

- Becker (1974); Keeley (1979): negative for men and negative for women due to marriage frictions;Bergstrom and Bagnoli (1993): positive for men and no relationship for women due to informational frictions

3. Relationship between age at marriage and spousal income for women has been persistently hump-shaped, with a changing marital outcome for early brides versus late brides.

- Low (2017): non-assortative matching

Model

## Model Overview

- Infinite number of periods.
- A unit mass of men and a unit mass of women become adults each period.
- Individuals are born with heterogeneous abilities of succeeding from investments.
- Investments: they make investment and marriage decisions over three periods.
- Differential fecundity: women stay fertile for a shorter period of time than men.
- Marriage market: division of marriage surplus is determined by supply and demand.


## Investments

## Investments

$\theta \bullet$

## Investments

$\theta \bullet$
ages 16-22

## Investments


ages 16-22
ages 23-29
ages 30-39

## Investments


ages 16-22
ages 23-29
ages 30-39

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## Investments



## Differential Fecundity

$$
\begin{gathered}
y+v-c \\
\text { income }+ \text { marital payoff (income, fertility) }- \text { investment costs }
\end{gathered}
$$

- Men who marry in any of the three periods have the same fertility level.
$\rightarrow$ Women who marry in the third period may have a lower fertility level than those who marry in the first two periods.
- Husband's income and wife's income and fertility determine marriage surplus: $s\left(y_{m}, y_{w}, \phi_{w}\right) \equiv s\left(\tau_{m}, \tau_{w}\right)$.
- Surplus is increasing in each argument, supermodular in incomes, and supermodular in income and fertility.


## Marriage Market

- Division of the marriage surplus is endogenously determined: $v_{m \tau_{m}}+v_{w \tau_{w}}=s\left(\tau_{m}, \tau_{w}\right)$ for any married couple $\tau_{m}$ and $\tau_{w}$.
- Marriages are stable: $v_{m \tau_{m}}+v_{w \tau_{w}} \geq s\left(\tau_{m}, \tau_{w}\right)$ for any pair.


## Equilibrium

Investment strategies $\left(\sigma_{m}^{*}, \sigma_{w}^{*}\right)$ and marriage payoffs $\left(v_{m}^{*}, v_{w}^{*}\right)$ form an equilibrium if

- $\sigma_{m}^{*}(\theta), \sigma_{w}^{*}(\theta)$ maximizes each ability- $\theta$ individual's expected payoff.
- $\left(v_{m}^{*}, v_{w}^{*}\right)$ are the stable marriage payoffs in the marriage market $\left(G_{m}^{*}, G_{w}^{*}\right)$ induced by $\left(\sigma_{m}^{*}, \sigma_{w}^{*}\right)$.


## Equilibrium Existence and Uniqueness

## Theorem

There exists an equilibrium. Equilibrium investment strategies $\left(\sigma_{m}^{*}, \sigma_{w}^{*}\right)$ are uniquely determined. Equilibrium marriage payoffs $\left(v_{m}^{*}, v_{w}^{*}\right)$ are uniquely determined up to a constant.

## Proof Steps

0 . Marriage payoffs are determined by payoff difference $\pi_{m} \equiv v_{m H}-v_{m L}$. The mapping in consideration is $\pi_{m} \xrightarrow{f_{\sigma}} \sigma \xrightarrow{f_{G}} G \stackrel{f_{\pi}}{\rightrightarrows} \pi_{m}$.

1. Construct supply function $S\left(\pi_{m}\right)=f_{G}\left(f_{\sigma}\left(\pi_{m}\right)\right)$.
2. Construct demand correspondence $D\left(\pi_{m}\right)=\left\{G: \pi_{m} \in f_{\pi}(G)\right\}$.
3. Show that supply is increasing and demand is decreasing.

# Explanations 

## Men's Midlife Income by Age at Marriage



$$
\theta_{m}=\frac{c_{m}}{\left(y_{m H}-y_{m L}\right)+\left(v_{m H}-v_{m L}\right)} \equiv \frac{c_{m}}{\Delta z_{m}+\pi_{m}}
$$



## Women's Midlife Income by Age at Marriage



$$
\theta_{w 1}=\frac{c_{w}}{\Delta y_{w}+\pi_{w}}<\theta_{w 2}=\frac{c_{w}+v_{w L}-v_{w l}}{\Delta y_{w}+v_{w h}-v_{w l}}
$$



## College and Earnings Gender Gaps

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Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.

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- Only some college-educated women make a career investment.


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- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.


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Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.


- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.
- High-income women are more scarce than high-income men in MM.


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- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.
- High-income women are more scarce than high-income men in MM.
- College generates higher MM returns for women than for men.


## Women's Spousal Income by Age at Marriage



## Fertility-Income Tradeoff



## Early versus Late Brides



## Mandates to Cover/Offer Infertility Treatments in Insurances

Between 1985 and 1995, thirteen states passed mandates to cover/offer infertility treatments in insurances

- Mandate to cover: Maryland (1985), Arkansas, Hawaii, and Massachusetts (1987), Rhode Island (1989), Illinois (1991), Montana (1987), New York (1990), Ohio (1991), West Virginia (1995)
- Mandate to offer: Texas (1987), California (1989), Connecticut (1989)


## Women's Spousal Income by Age at Marriage



## Spousal Total Income Percentile Rank



| men | match | women |
| :---: | :---: | :---: |
| $H$ | $H H$ | $H$ |
|  | $H L$ | $L \uparrow$ |
|  | $H h$ | $\downarrow \downarrow$ |
|  | $L h$ |  |
|  | $L l$ | $l$ |

Fertility more important

| men | match | women |
| :---: | :---: | :---: |
| $H$ | $H H$ | $H$ |
|  | $H h$ | $h \uparrow$ |
|  | $H L$ | $L \downarrow$ |
|  | $L L$ |  |
|  | $L l$ | $l$ |

Income more important

## Supporting Evidence and Calibration

## Evolution of the Marriage Premium



## Age-Income Profiles for Men and Women



## More Career Investments for Low Incomes

Relation between career investment and logincome, men

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| ols79 | logit79 | probit79 | ols97 | $(5)$ <br> $\operatorname{logit97}$ | $(6)$ <br> probit97 |  |
|  |  |  |  |  |  |  |
| logincome | $-0.0969^{* * *}$ | $-0.447^{* * *}$ | $-0.261^{* * *}$ | $-0.0947^{* * *}$ | $-0.406^{* * *}$ | $-0.250^{* * *}$ |
|  | $(0.0142)$ | $(0.0647)$ | $(0.0370)$ | $(0.0161)$ | $(0.0723)$ | $(0.0439)$ |
| age | -0.000539 | 0.00561 | -0.000519 | $-0.0244^{* * *}$ | $-0.108^{* * *}$ | $-0.0664^{* * *}$ |
|  | $(0.00741)$ | $(0.0308)$ | $(0.0188)$ | $(0.00719)$ | $(0.0324)$ | $(0.0199)$ |
| $N$ | 1659 | 1659 | 1659 | 1638 | 1638 | 1638 |

Marginal effects; Standard errors in parentheses
${ }^{*} p<0.05,{ }^{* *} p<0.01,{ }^{* * *} p<0.001$

## Calibration

- Ability distributions are $\operatorname{Beta}\left(\alpha_{m}, \beta_{m}\right)$ and $\operatorname{Beta}\left(\alpha_{w}, \beta_{w}\right)$.
- Low income is average income of the non-college-educated.
- High income is average income of the college-educated.
- Total investment cost is two years of low incomes; annual cost is total cost divided by 40 .
- Surplus in monetary terms is $k$ times estimated surplus in utils.
- Add marriage frictions (possibility of not marrying upon entering MM).
- 19 targeted moments.
- Percentages of early, middle, late grooms/brides (6).
- Average personal income of early, middle, late grooms (3).
- Average personal income of early, middle, late brides (3).
- Average spousal income of early brides (3).
- College enrollment rates of men and women (2).


## Fit of the Model

| moments | 30 s target | 30 s model | difference | 60 s target | 60s model | difference |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $G_{\mathrm{m} 1}$ | 0.48476 | 0.484451 | $-0.0637 \%$ | 0.30756 | 0.307372 | $-0.0613 \%$ |
| $G_{\mathrm{m} 2}$ | 0.411344 | 0.412559 | $0.295 \%$ | 0.451633 | 0.452309 | $0.15 \%$ |
| $G_{\mathrm{m} 3}$ | 0.103896 | 0.102989 | $-0.872 \%$ | 0.240807 | 0.24032 | $-0.202 \%$ |
| $G_{\mathrm{w} 1}$ | 0.740591 | 0.740591 | $0.000051 \%$ | 0.4494 | 0.449534 | $0.0299 \%$ |
| $G_{\mathrm{w} 2}$ | 0.206928 | 0.206847 | $-0.0393 \%$ | 0.381204 | 0.380081 | $-0.295 \%$ |
| $G_{\mathrm{w} 3}$ | 0.0524809 | 0.0525618 | $0.154 \%$ | 0.169396 | 0.170385 | $0.584 \%$ |
| $G_{m, \mathrm{col}}$ | 0.218733 | 0.220363 | $0.745 \%$ | 0.379722 | 0.380819 | $0.289 \%$ |
| $G_{w, \mathrm{col}}$ | 0.119257 | 0.119255 | $-0.00131 \%$ | 0.390058 | 0.389479 | $-0.148 \%$ |
| $y_{\mathrm{m} 1}$ | 40209.7 | 39603.7 | $-1.51 \%$ | 44571.6 | 44730.5 | $0.357 \%$ |
| $y_{\mathrm{m} 2}$ | 43820.8 | 43915.8 | $0.217 \%$ | 56434.2 | 56524.6 | $0.16 \%$ |
| $y_{\mathrm{m} 3}$ | 37442. | 38350.9 | $2.43 \%$ | 48376.5 | 48589.3 | $0.44 \%$ |
| $y_{\mathrm{w} 1}$ | 12049. | 11696.3 | $-2.93 \%$ | 20091. | 20510. | $2.09 \%$ |
| $y_{\mathrm{w} 2}$ | 12457.2 | 12739.2 | $2.26 \%$ | 24627.8 | 25169.9 | $2.2 \%$ |
| $y_{\mathrm{w} 3}$ | 12886.1 | 12421. | $-3.61 \%$ | 26080.1 | 24207.1 | $-7.18 \%$ |
| $x_{\mathrm{w} 1}$ | 41269.2 | 41155.8 | $-0.275 \%$ | 46138.3 | 47051.6 | $1.98 \%$ |
| $x_{\mathrm{w} 2}$ | 45269.5 | 42290.6 | $-6.58 \%$ | 58701.2 | 55594.8 | $-5.29 \%$ |
| $x_{\mathrm{w} 3}$ | 35537.5 | 38066.9 | $7.12 \%$ | 48666.8 | 50699.8 | $4.18 \%$ |
| average |  |  | $1.71 \%$ |  |  | $1.51 \%$ |

## Quantifying Labor-Market Shocks on Marriage Timing

- Estimated ability distributions (labor-market opportunities).

- Labor-market shocks (due to the possibility that one does not receive a high-income offer after college) contribute to $42.7 \%$ of college-educated men and $24 \%$ of college-educated women born in the 1960s delaying marriage (the rest are explained by marriage-market frictions).


## Fit of the Model, Mandated States

| moments | 30s target | 30s model | difference | 60s target | 60s model | difference |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $G_{\mathrm{m} 1}$ | 0.451869 | 0.451556 | $-0.0693 \%$ | 0.271852 | 0.271602 | $-0.092 \%$ |
| $G_{\mathrm{m} 2}$ | 0.430358 | 0.431748 | $0.323 \%$ | 0.462758 | 0.463643 | $0.191 \%$ |
| $G_{\mathrm{m} 3}$ | 0.117773 | 0.116697 | $-0.914 \%$ | 0.26539 | 0.264754 | $-0.239 \%$ |
| $G_{\mathrm{w} 1}$ | 0.712169 | 0.714571 | $0.337 \%$ | 0.40867 | 0.415509 | $1.67 \%$ |
| $G_{\mathrm{w} 2}$ | 0.227668 | 0.221022 | $-2.92 \%$ | 0.403811 | 0.390709 | $-3.24 \%$ |
| $G_{\mathrm{w} 3}$ | 0.0601629 | 0.0644064 | $7.05 \%$ | 0.187518 | 0.193783 | $3.34 \%$ |
| $G_{m, \mathrm{col}}$ | 0.240621 | 0.242344 | $0.716 \%$ | 0.392051 | 0.393502 | $0.37 \%$ |
| $G_{w, \text { col }}$ | 0.131002 | 0.12084 | $-7.76 \%$ | 0.400299 | 0.370931 | $-7.34 \%$ |
| $y_{\mathrm{m} 1}$ | 42549.9 | 41471.4 | $-2.53 \%$ | 45833.3 | 46347.3 | $1.12 \%$ |
| $y_{\mathrm{m} 2}$ | 46013.6 | 46116. | $0.223 \%$ | 59531.3 | 59658.5 | $0.214 \%$ |
| $y_{\mathrm{m} 3}$ | 38934.8 | 40058.4 | $2.89 \%$ | 52070.5 | 52371.7 | $0.579 \%$ |
| $y_{\mathrm{w} 1}$ | 12664.9 | 12918.8 | $2.01 \%$ | 20453.6 | 21866.4 | $6.91 \%$ |
| $y_{\mathrm{w} 2}$ | 13050.4 | 15802.5 | $21.1 \%$ | 25514.7 | 28767.5 | $12.7 \%$ |
| $y_{\mathrm{w} 3}$ | 13429.7 | 12946.1 | $-3.6 \%$ | 27373.5 | 25741.2 | $-5.96 \%$ |
| $x_{\mathrm{w} 1}$ | 43941.9 | 42819.1 | $-2.56 \%$ | 48004.4 | 47777.3 | $-0.473 \%$ |
| $x_{\mathrm{w} 2}$ | 47304.5 | 45972.1 | $-2.82 \%$ | 62317.6 | 60849.6 | $-2.36 \%$ |
| $x_{\mathrm{w} 3}$ | 37059.8 | 39648.9 | $6.99 \%$ | 52485. | 54120.2 | $3.12 \%$ |
| average | $->$ | $->$ | $3.81 \%$ | $->$ | $->$ | $2.94 \%$ |

## Fit of the Model, Nonmandated States

| moments | 30s target | 30s model | difference | 60 s target | 60s model | difference |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $G_{\mathrm{m} 1}$ | 0.50978 | 0.509501 | $-0.0549 \%$ | 0.334886 | 0.334418 | $-0.14 \%$ |
| $G_{\mathrm{m} 2}$ | 0.39688 | 0.397736 | $0.216 \%$ | 0.443119 | 0.444872 | $0.396 \%$ |
| $G_{\mathrm{m} 3}$ | 0.0933392 | 0.0927631 | $-0.617 \%$ | 0.221995 | 0.220711 | $-0.578 \%$ |
| $G_{\mathrm{w} 1}$ | 0.762457 | 0.762457 | $0.000022 \%$ | 0.480704 | 0.485707 | $1.04 \%$ |
| $G_{\mathrm{w} 2}$ | 0.190972 | 0.190905 | $-0.0353 \%$ | 0.363829 | 0.354892 | $-2.46 \%$ |
| $G_{\mathrm{w} 3}$ | 0.0465706 | 0.0466378 | $0.144 \%$ | 0.155467 | 0.159401 | $2.53 \%$ |
| $G_{m, \mathrm{col}}$ | 0.202083 | 0.203549 | $0.725 \%$ | 0.370287 | 0.373063 | $0.75 \%$ |
| $G_{w, \text { col }}$ | 0.11022 | 0.110219 | $-0.000626 \%$ | 0.382188 | 0.36033 | $-5.72 \%$ |
| $y_{\mathrm{m} 1}$ | 38631.7 | 38140.6 | $-1.27 \%$ | 43787.7 | 43444.2 | $-0.785 \%$ |
| $y_{\mathrm{m} 2}$ | 42012. | 42087.2 | $0.179 \%$ | 53959.1 | 54176.3 | $0.402 \%$ |
| $y_{\mathrm{m} 3}$ | 36009.2 | 36372.9 | $1.01 \%$ | 44997. | 45506. | $1.13 \%$ |
| $y_{\mathrm{w} 1}$ | 11606.5 | 11253.5 | $-3.04 \%$ | 19854. | 20950. | $5.52 \%$ |
| $y_{\mathrm{w} 2}$ | 11913. | 12196.3 | $2.38 \%$ | 23871.3 | 26551.5 | $11.2 \%$ |
| $y_{\mathrm{w} 3}$ | 12345.8 | 11857.2 | $-3.96 \%$ | 24881.1 | 22856.7 | $-8.14 \%$ |
| $x_{\mathrm{w} 1}$ | 39414. | 39452.6 | $0.0979 \%$ | 44926.9 | 43993.5 | $-2.08 \%$ |
| $x_{\mathrm{w} 2}$ | 43434.5 | 40533.2 | $-6.68 \%$ | 55639.5 | 55561.4 | $-0.14 \%$ |
| $x_{\mathrm{w} 3}$ | 34045.4 | 36624.4 | $7.58 \%$ | 45155.8 | 47599.5 | $5.41 \%$ |
| average | $->$ | $->$ | $1.65 \%$ | $->$ | $->$ | $2.85 \%$ |

## Mandate Counterfactual Analyses

## Infertility Treatment Insurance Mandate

- If mandated states were not mandated:
- The fraction of late brides in the mandated states would decrease from 19.4 percent to 17.0 percent.
- The average spousal income of early brides would increase by 2.92 percent.
- The average spousal income of late brides would decrease by 0.12 percent.
- If nonmandated states were mandated:
- The fraction of late brides in the mandated states would increase from 15.9 percent to 18.2 percent.
- The average spousal income of early brides would decrease by 2.97 percent.
- The average spousal income of late brides would increase by 0.07 percent.


## Gender Equality Counterfactual Analysis 1

Gender Equality in Fecundity

- 4.96 percent of women would delay their marriage age from between 23 and 29 to between 30 and 39
- Middle brides' average spousal income would increase by 5.43 percent
- Late brides' average spousal income would increase by 3.61 percent
- The average personal income of late brides would not increase, because intermediate-ability women delay marriages


## Gender Equality Counterfactual Analysis 2

## Gender Equality in the Labor Market

- Women's college enrollment rate would decrease from 38.9 percent to 38.3 percent
- Fraction of
- early brides (16-22): would increase by 0.35 percent
- middle brides (23-29): would decrease by 2.94 percent
- late brides (30-39): would increase by 5.64 percent
- Average spousal income of
- early brides would decrease by 0.43 percent
- middle brides would increase by 0.68 percent
- late brides would increase by 0.37 percent


## Gender Equality Counterfactual Analysis 3

## Gender Equality in Investment Opportunities

- Women's college enrollment rate would decrease from 38.9 percent to 38.5 percent
- Fraction of
- early brides (16-22): would increase by 0.23 percent
- middle brides (23-29): would increase by 1.73 percent
- late brides (30-39): would decrease by 4.46 percent
- Average spousal income of
- early brides would decrease by 0.43 percent
- middle brides would increase by 0.68 percent
- late brides would increase by 0.37 percent


## Conclusion

- College and earnings gender gaps.
- Relationships between age at marriage and personal income for men and women.
- Relationship between age at marriage and spousal income for women.
- Differential fecundity, coupled with the equilibrium marriage market, leads to many observed economic and social gender differences.

THANK YOU!

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